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LUMPED ELEMENT MODELING OF
TWO-DIMENSIONAL DISTRIBUTED RC NETWORKS

Prepared under Grant NGL-03-002-136 for the
Instrumentation Division of the Ames Research Center
National Aeronautics and Space Administration

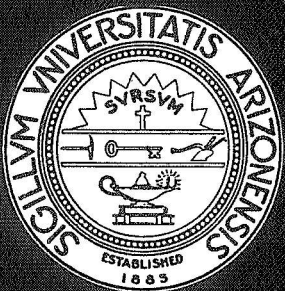
by

D. E. Robinson

and

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Abstract: This report describes the use of lumped network elements to model a uniform two-dimensional distributed RC network. Such modeling makes it possible to select arbitrary terminal topologies on the resistive surfaces of two- and three-layered distributed networks. An implementation of the theory on the digital computer is presented, and the results obtained from analyzing several terminal topologies are given. One of these topologies illustrates the feasibility of using a single distributed RC network as a null device.

January 1970

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Lumped-Element Models for Two-Dimensional Distributed RC Networks

I. Introduction

This is one of a series of reports concerning the use of digital computational techniques in the analysis and synthesis of DLA (distributed-lumped-active) networks. This class of networks consists of three distinct types of elements, namely distributed elements (modeled by partial differential equations), lumped elements (modeled by algebraic equations and ordinary differential equations), and active elements (modeled by algebraic equations). Such a characterization is especially applicable to the broad class of circuits referred to as linear integrated circuits, since the required fabrication techniques for such circuits readily produce elements which may be referred to as "distributed", as well as producing elements which may be characterized as "lumped" and/or "active". The DLA class of networks is capable of realizing network functions with a wide range of properties. In addition, such realizations usually have fewer components and superior characteristics than realizations using only lumped elements, or realizations using lumped elements and active elements. The analysis problem for this class of networks, however, is considerably more complex than the analysis problem for more restricted classes of networks. Thus, one is led to implement the analysis problem by the use of suitable models for specific types of distributed networks. A model which has been shown to be of considerable value in representing a one-dimensional distributed RC network is the lumped-element model. Its properties have been discussed in a previous report.¹ It is based on the use of a cascade of a finite number of "L" network sections each composed of a series resistor and a shunt capacitor. Such a model has been shown to give analysis results that are more accurate and that require less computation time than models generated by using considerably more sophisticated mathematical techniques.² In this report an extension is made of the lumped-element modeling technique so that it may be applied to a two-dimensional RC network. Such a network may be considered either as a two-layer structure consisting of resistive, dielectric, and conducting layers as shown in Fig. 1 or as a three-layer structure containing two resistive layers as shown in Fig. 2. The modeling techniques that are developed in this report are designed so as to permit the implementation of arbitrary terminal configurations on the surface of the resistive layers of such distributed RC networks.

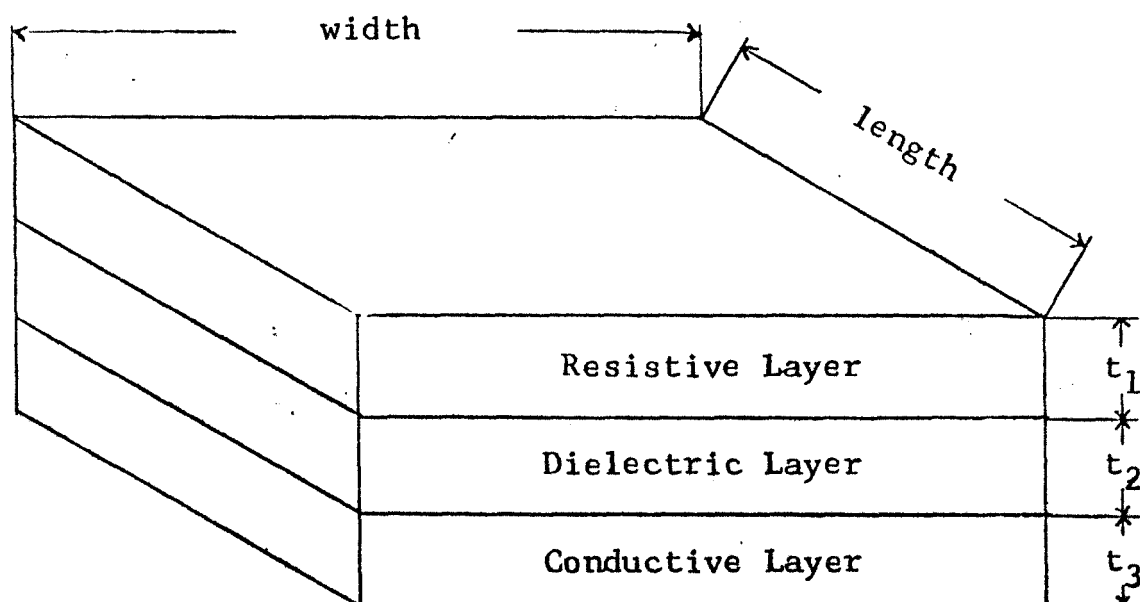


Fig. 1. Two-layer distributed RC network

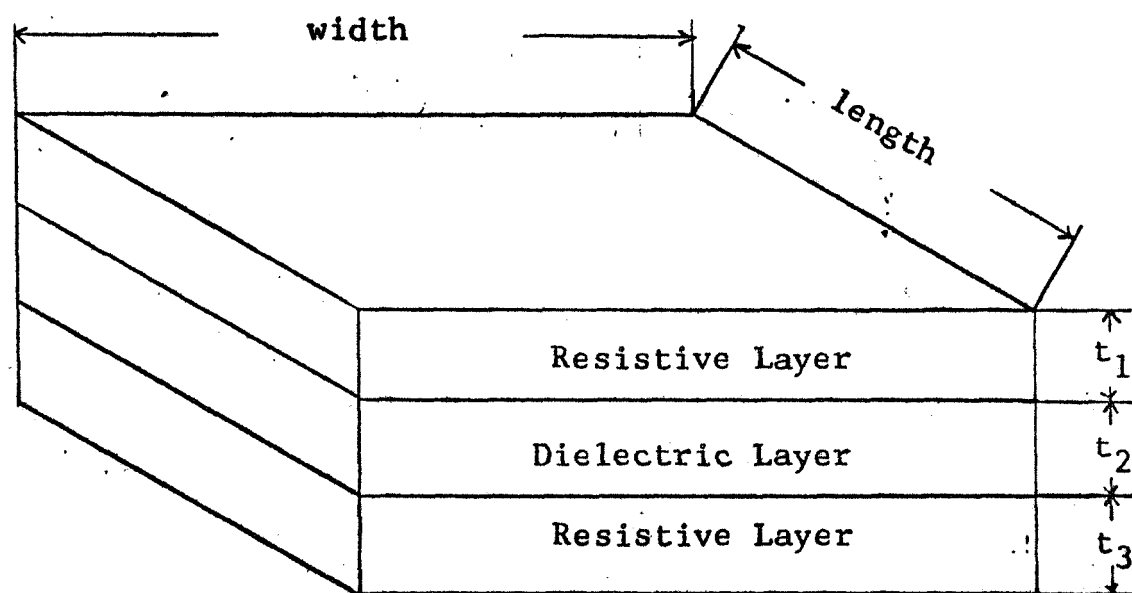


Fig. 2. Three-layer distributed RC network

Thus, the techniques presented here permit the user to perform analysis studies of a wide range of new and novel distributed RC networks configurations. Some examples of these are included as a part of this report.

II. Admittance Matrices for Lumped-Element Models

Most studies of distributed RC networks assume simple terminal configurations in which the terminals are connected to conducting strips which run completely across each end of the resistive layer. There are two advantages to such an assumption. First, when determining the two-port parameters of a uniformly distributed RC network with such a terminal configuration, the network may be described by a second-order, linear, partial differential equation, and the solution for such an equation follows readily.³ Second, the nonuniform distributed network may be modeled by a cascade of a finite number of lumped "L" sections, and the ensuing analysis consists primarily of matrix multiplication. Thus both uniform and non-uniform one-dimensional distributed RC networks are readily treated. Unfortunately, the simple terminal configuration assumption of these methods discounts any possibility of the alteration of a distributed RC network's electrical properties by varying its terminal configuration.

In this section we shall introduce the concept of a lumped-element model for a two-dimensional distributed RC network. Such a model has the advantage that it is readily amenable to arbitrary terminal configurations.

To see how the lumped-element concept may be extended to a two-layer distributed network, consider the six node structure shown in Fig. 3. The admittance matrix $\underline{Y}$ for this network is

$$\underline{Y} = \begin{bmatrix} Y_{11} & -Y_{12} & 0 & -Y_{14} & 0 & 0 \\ -Y_{12} & Y_{22} & -Y_{23} & 0 & -Y_{25} & 0 \\ 0 & -Y_{23} & Y_{33} & 0 & 0 & -Y_{36} \\ -Y_{14} & 0 & 0 & Y_{44} & -Y_{45} & 0 \\ 0 & -Y_{25} & 0 & -Y_{45} & Y_{55} & -Y_{56} \\ 0 & 0 & -Y_{36} & 0 & -Y_{56} & Y_{66} \end{bmatrix} \quad (1)$$

In general, the admittance matrix $\underline{Y}$ for such a network has the following properties

1. The number of columns and the number of rows of the admittance

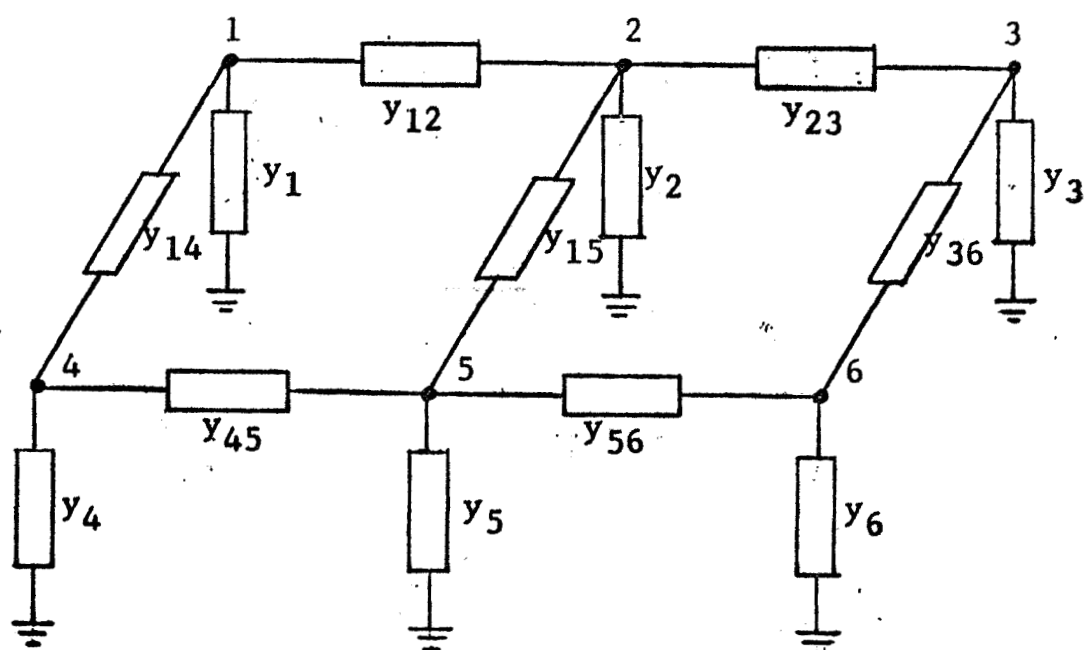


Fig. 3 Lumped-element 6 node model for two-layer distributed network

matrix are equal to the product of the nodal array length and width.

2. The resulting matrix is a 5-banded square matrix.
3. Band one, or the "above" band, is removed from the main diagonal by the nodal array length, ℓ , and it is located below the main diagonal of the matrix. Each individual element, $Y_{i-\ell,i}$, is equal to the negative of the node-to-node admittance between the node i and the node above it, node $i-\ell$.
4. Band two, or the "left" band, is removed from the main diagonal by 1, and it is located below the main diagonal of the matrix. Each individual element, $Y_{i-1,i}$, is equal to the negative of the node-to-node admittance connected between the node i and the node to the left of it, node $i-1$.
5. Band three is the main diagonal, and each element, Y_{ii} , is equal to the sum of the node-to-ground admittance, and the node-to-node admittance connected between node i and the adjacent nodes to the left, right, above, and below it.
6. Band four, or the "right" band, is similar to band two, except that it represents the negative of the node-to-node admittance connected between node i and the node to the right of it, node $i+1$.
7. Band five, or the "below" band, is similar to band 1, except that it represents the negative of the node-to-node admittance connected between node i and node $i+\ell$.
8. If ℓ = length of the nodal array and w = width of the nodal array, then bands one and five each contain $\ell(w-1)$ elements, bands two and four each contain $w(\ell-1)$ elements, and band three contains ℓw elements.
9. Since the network is reciprocal, the matrix is symmetric.

The numbered items given above may be considered as a basis for developing an algorithm which will generate the admittance matrix for a lumped network modeling a two-layer distributed RC structure. The degree of approximation obtained is, of course, improved as the number of nodes used in the lumped structure is increased. The rules are based on the convention that the nodes are numbered in increasing order from "left to right" and from "top to bottom", assuming that the network is drawn as shown in Fig. 3.

The results given above are readily extended to determine the admittance matrix for a lumped network model for a three-layer device. A 12 node example of such a model is shown in Fig. 4. The admittance matrix for this network is

$$\begin{aligned}
 \underline{\underline{Y}} = & \begin{bmatrix}
 Y_{11} & -y_{12} & 0 & -y_{14} & 0 & 0 & -y_{17} & 0 & 0 & 0 & 0 & 0 \\
 -y_{12} & Y_{22} & -y_{23} & 0 & -y_{25} & 0 & 0 & -y_{28} & 0 & 0 & 0 & 0 \\
 0 & -y_{23} & Y_{33} & 0 & 0 & -y_{36} & 0 & 0 & -y_{39} & 0 & 0 & 0 \\
 -y_{14} & 0 & 0 & Y_{44} & -y_{45} & 0 & 0 & 0 & 0 & -y_{40} & 0 & 0 \\
 0 & -y_{25} & 0 & -y_{45} & Y_{55} & -y_{56} & 0 & 0 & 0 & 0 & -y_{51} & 0 \\
 0 & 0 & -y_{36} & 0 & -y_{56} & -y_{66} & 0 & 0 & 0 & 0 & 0 & -y_{62} \\
 \hline
 -y_{17} & 0 & 0 & 0 & 0 & 0 & Y_{77} & -y_{78} & 0 & -y_{70} & 0 & 0 \\
 0 & -y_{28} & 0 & 0 & 0 & 0 & -y_{78} & Y_{88} & -y_{89} & 0 & -y_{81} & 0 \\
 0 & 0 & -y_{39} & 0 & 0 & 0 & 0 & -y_{89} & Y_{99} & 0 & 0 & -y_{92} \\
 0 & 0 & 0 & -y_{40} & 0 & 0 & -y_{70} & 0 & 0 & Y_{00} & -y_{01} & 0 \\
 0 & 0 & 0 & 0 & -y_{51} & 0 & 0 & -y_{81} & 0 & -y_{01} & Y_{11} & -y_{12} \\
 0 & 0 & 0 & 0 & 0 & -y_{62} & 0 & 0 & -y_{92} & 0 & -y_{12} & Y_{22}
 \end{bmatrix} \\
 & = \begin{bmatrix}
 \underline{\underline{M}}_{11} & \underline{\underline{M}}_{12} \\
 \underline{\underline{M}}_{21} & \underline{\underline{M}}_{22}
 \end{bmatrix} \quad (2)
 \end{aligned}$$

The numbering scheme used for the model of the two-layer device has been modified to accommodate the model for the three-layer device as follows: the nodes are numbered from left-to-right, top-to-bottom, and first layer-to-second-layer. Notice that the $\underline{\underline{M}}_{11}$ submatrix in (2) is similar to the admittance matrix given in (1); the only difference is that there is no node-to-ground admittance for the network of Fig. 4.

The defining properties of the admittance matrix for the lumped element model for the three-layer device are:

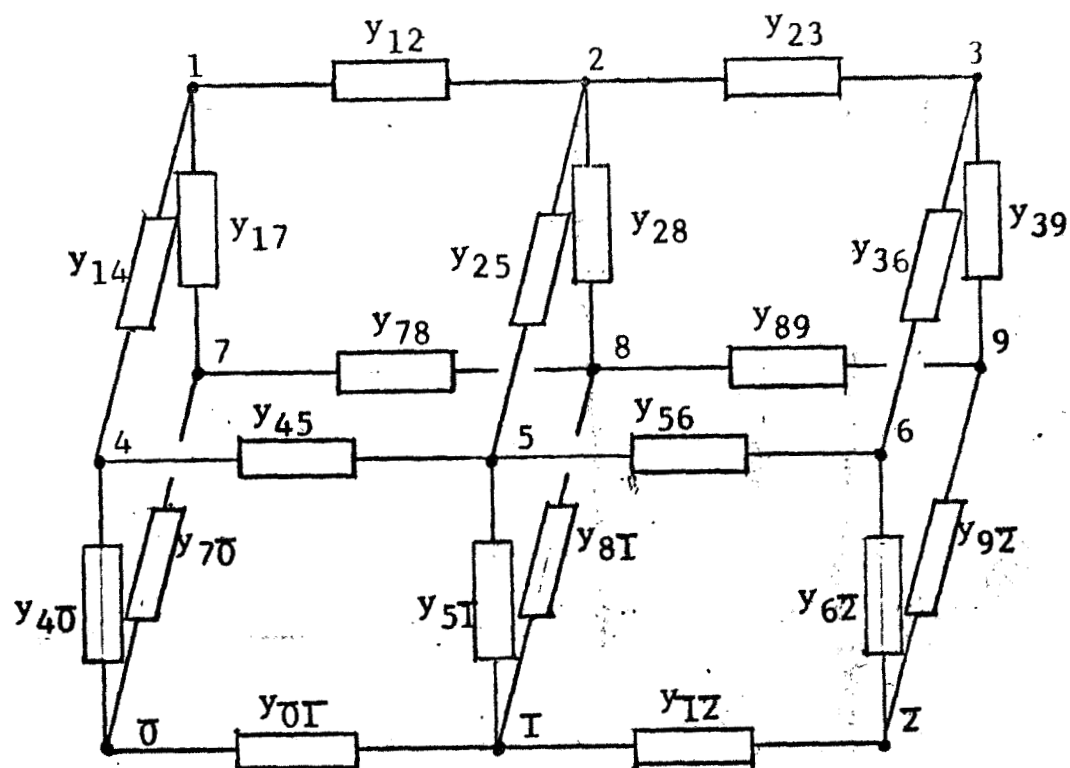


Fig. 4 Lumped-element 12 node model for three-layer distributed network

1. The upper left-hand sub matrix, M_{11} , is the admittance matrix for the upper plane elements.
2. The lower right-hand matrix, M_{22} , is the admittance matrix for the lower plane elements.
3. The upper right-hand matrix, M_{12} , and the lower left-hand matrix, M_{21} , are diagonal matrices which represent the plane-to-plane admittances. The two matrices are equal because the network is reciprocal.

The numbered items given above illustrate that the admittance matrix for a lumped-element model for a three-layer device may be directly obtained from the admittance matrix for a lumped-element model for a two-layer device by a relatively simple procedure. Thus, these items, together with those given for the two-layer lumped-element model, form the basis of an algorithm which will generate the admittance matrix for a lumped network modeling a three-layer distributed RC structure.

III. Digital Computer Program for Analyzing Lumped Element Models

The properties of the admittance matrices described in the previous section are readily used to define algorithms which may be implemented on a digital computer to form the basis for a general program capable of performing sinusoidal steady state analysis of lumped-element structures modeling two-dimensional two- and three-layer distributed RC networks. Such a program has been prepared at the University of Arizona. It is called TDSDN (for Two-Dimensional Study of Distributed Networks). The program is subdivided into four major computational subroutines named JCG4, JCN4, and JCY4, and a pair of input and output subroutines JCR4 and JCP4. These subroutines are described more fully in the following paragraphs.

The subroutine JCG4, generates the complex admittance matrix for a lumped-element network in which resistors are connected between adjacent nodes and capacitors are connected from the nodes to ground. It does this by making use of the banding properties described in Sec. II. In the algorithm each node is assigned a positional number, which allows the JCG4 subprogram to operate in a fashion that might be described as a binary iterative routine. Essentially, this binary iterative routine starts with a particular number, performs a desired operation, subtracts an appropriate power of two from the original

number, and returns to the starting instruction until zero is obtained, at which time the iterative routine is terminated.

The JCT4 subprogram generates the admittance matrix for a lumped-element network with two planes of resistors connected between adjacent nodes, and with capacitors connected between the planes. It does this by using the JCG4 subroutine discussed above. The JCT4 subroutine finds the admittance matrix of the lower plane of resistors by calling the JCG4 subroutine. The results are inserted into the M_{22} submatrix given in (2). The admittance matrix of the upper plane of resistors is found by again calling the JCG4 subprogram, and inserting the result into the M_{11} submatrix. Diagonal matrices are then generated and stored in the M_{12} and M_{21} upper right-hand and lower left-hand submatrices of (2).

Since the TDSDN program must analyze lumped-element networks having an arbitrary terminal configuration, some method must be devised to alter the original admittance matrix to reflect the effects of the specified terminal configuration. The JCN4 subroutine performs this operation. Specifically, it adds the appropriate rows and columns of the admittance matrix to take account of terminal connections which short the related nodes. Two specific details of the subprogram are of interest. The first is the terminal connection vector. This vector is an n-tuple, whose size corresponds to the size of the original matrix. The value of each element of the n-tuple is equal to a node number of the original matrix, and each element is in numerical sequence. Whenever a higher-numbered node is connected to a lower-numbered node the element of the node connection vector that corresponds to the original higher-numbered node is set to zero, and the values of the next elements of the node connection vector (the elements that correspond to the higher-numbered nodes) are reduced by one. As an example of this, consider a 3 by 3 model for a two-layer device. The original node connection vector would be

$$(1, 2, 3, 4, 5, 6, 7, 8, 9) \quad (3)$$

Should node 5 be connected to node 2 and node 8 be connected to node 4, the resultant node connection vector would be

$$(1, 2, 3, 4, 0, 5, 6, 0, 7) \quad (4)$$

The node connection vector provides a means by which the JCN4 subprogram

can determine which terminals have been connected and the position of the unconnected ones. This vector is also used by the JCY4 subroutine described in later paragraphs.

A second detail concerning the JCN4 subroutine is a limitation imposed by the algorithm in that only higher-numbered nodes may be connected to lower-numbered nodes. In addition, once a node has been used in a role as a higher-numbered node, it may not subsequently be used as a lower-numbered node. This detail does not limit the versatility of the TSDSN program, but it does impose a restriction which must be observed by the user.

The JCY4 subroutine reduces the admittance matrix which has been generated and constrained by the subroutines described above, into a form that is useful for describing the response of the network. The JCY4 subprogram uses a matrix reduction algorithm which is expressed by the matrix relationship

$$\underline{y}^{(n-1)} = \underline{M}_{11} - \underline{M}_{12} \underline{M}_{22}^{-1} \underline{M}_{21} \quad (5)$$

Equation (5) is a matrix equation, and the individual matrices are defined as

$$\underline{y}^{(n)} = \left[\begin{array}{c|c} \begin{matrix} \overbrace{\underline{M}_{11}}^{n-1} & \underline{M}_{12} \\ \hline \underline{M}_{21} & \underline{M}_{22} \end{matrix} & \begin{matrix} 1 \\ \hline 1 \end{matrix} \end{array} \right]^{n-1} = \left[\begin{array}{ccc|c} y_{11} & \cdots & & y_{1n} \\ \vdots & \ddots & & \vdots \\ \hline y_{n1} & \cdots & & y_{nn} \end{array} \right] \quad (6)$$

where $\underline{y}^{(n)}$ is an admittance matrix of order n , and $\underline{y}^{(n-1)}$ is the reduced admittance matrix obtained from $\underline{y}^{(n)}$ by the matrix reduction algorithm. As indicated in (6) the $\underline{M}_{12}$ submatrix is a $(n-1) \times 1$ or column matrix, and the $\underline{M}_{22}$ matrix is a 1×1 matrix, i.e., the element y_{nn} . Successive applications of this algorithm may be used to reduce the original admittance matrix to a size corresponding with the number of actual terminals specified for the network. Thus the JCY4 subprogram will reduce an $n \times n$ matrix to an $m \times m$ matrix, where n ranges from 4 to 64 and m ranges from 2 to 10. Essentially, the JCY4 subroutine reorders the rows and columns of the admittance matrix by utilizing the node connection vector described above before applying (5). The algorithm requires that a restriction be observed on the choice of the output terminals. This is that any group of output terminals, whose node numbers are less than the total number of output terminals, must be in increasing numerical order. For example, if it was desired to reduce a 16×16 matrix to a 4×4 matrix and nodes 1 and 3 were to be included in the output terminal group, then the terminals must be ordered 1 and 3 rather than 3 and 1. This restriction should not impose any

serious limitation upon a user, other than the proper specification of input data.

The two remaining subroutines JCR4 and JCP4 are concerned respectively with the input and output of data associated with the TDSDN program. Options are provided for the user to specify the size of the grid being used in the lumped-element model, whether a two- or three-layer device is to be analyzed, the desired terminal configurations to be imposed on the grid of nodes, and whether a single-frequency analysis or a multiple-frequency analysis is desired. In the latter case, an on-line plotting capability, included as part of the TDSDN program, may be used.

IV. Analysis of Specific Two- and Three-layer Distributed RC Networks

In this section we present some of the results obtained from applying the TDSDN program discussed in the preceding section to the analysis of several two- and three-layer distributed RC network configurations. Each network was analyzed at 60 different sinusoidal frequencies chosen so as to include a representative spectrum of the most interesting portion of the open-circuit voltage transfer function for the network. A discussion of nine of the networks which produced the most interesting characteristics together with the magnitude curves for these networks is given in the following paragraphs. The networks are shown in Figs. 5-12. A 3×3 grid structure of 9 nodes was used to model each of the distributed network configurations. Thus, an admittance matrix of order nine was solved at each frequency for the single two-layer network described, and an admittance matrix of order 18 was solved at each frequency for each of the eight three-layer networks which are discussed. The total computation time for the analysis of the nine networks was approximately one minute on a CDC 6400 computer.

The first type of network to be considered is the lowpass type. The response of four lowpass networks is shown in the curves of Fig. 13. Curve A shows the response of a two-layer network (Fig. 5). In the model used for this network, the open circuit voltage transfer ratio should have two poles, as is verified by the -40 db/decade slope of the high-frequency section of Curve A. The second lowpass network considered is the Castro-Happ type 2 shown in Fig. 6.⁴ As shown by Curve B in Fig. 13, it has a +20 db/decade slope at high frequencies. The third lowpass network is shown in Fig. 7. It has the

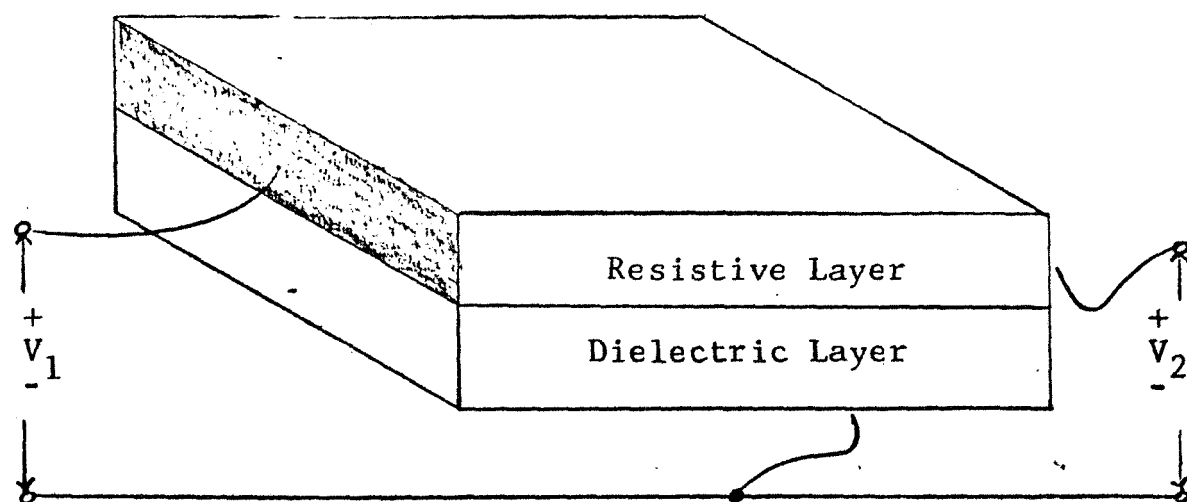


Fig. 5. Two-layer RC network

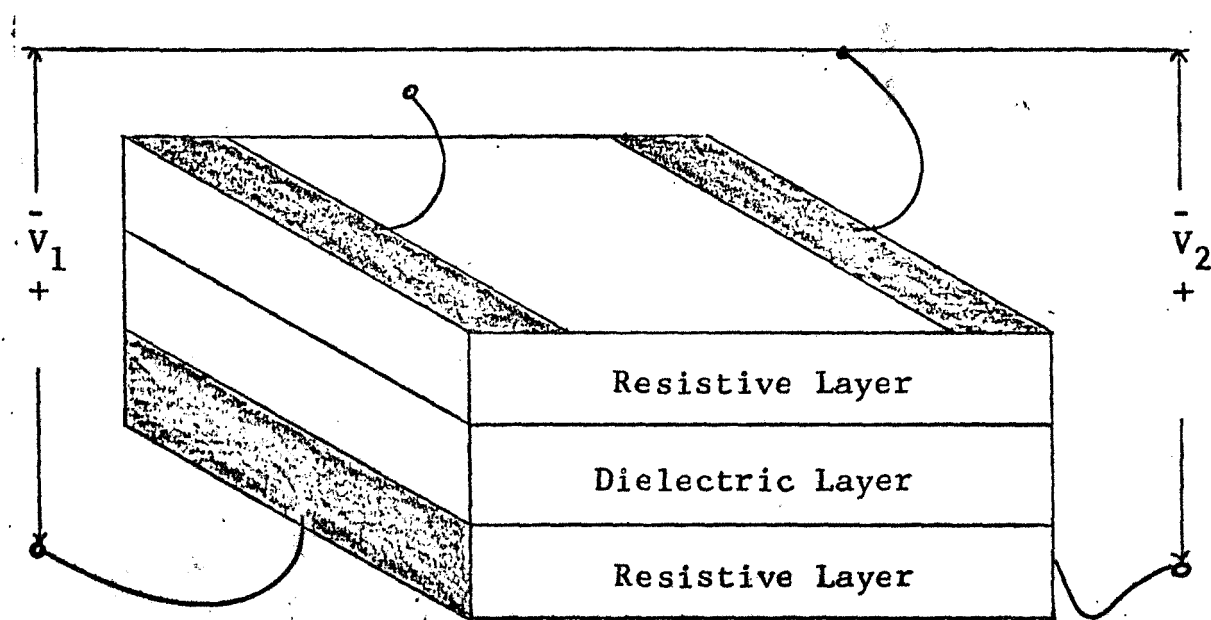


Fig. 6. Castro-Happ type 2 network

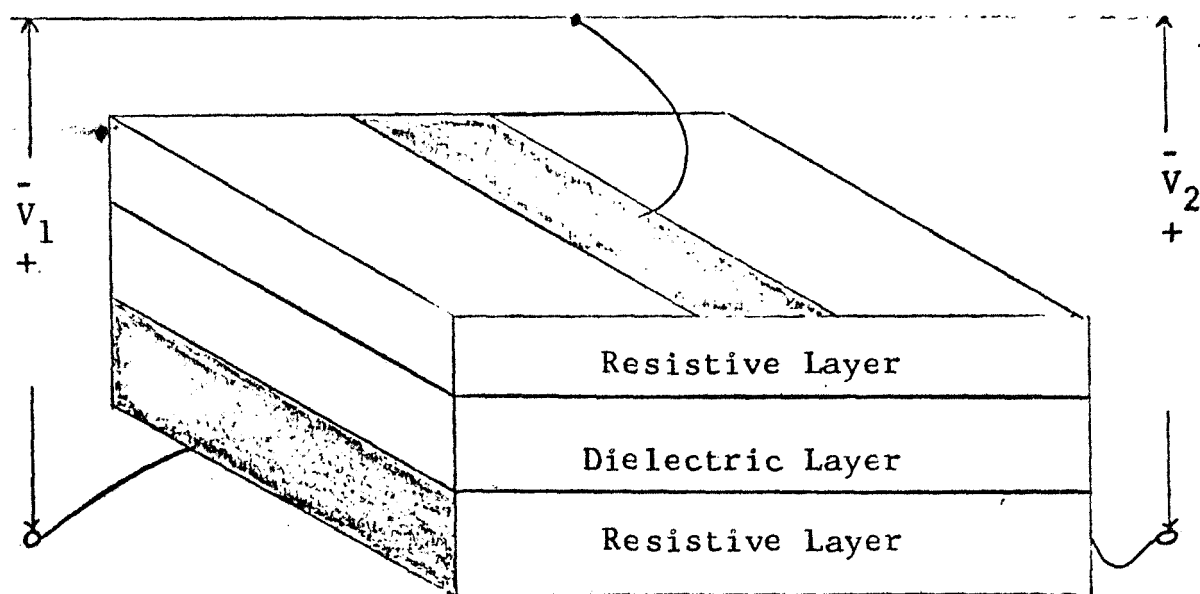


Fig. 7. Three terminal network with grounded terminal bisecting input-output terminals

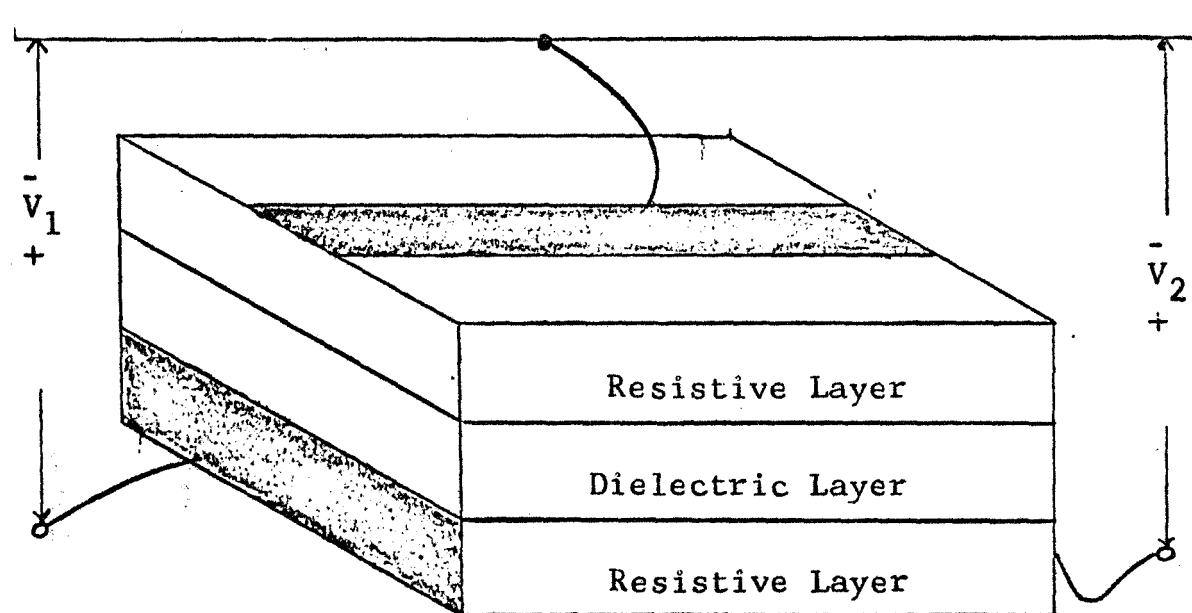


Fig. 8. Three terminal network with grounded terminal perpendicular to input terminal

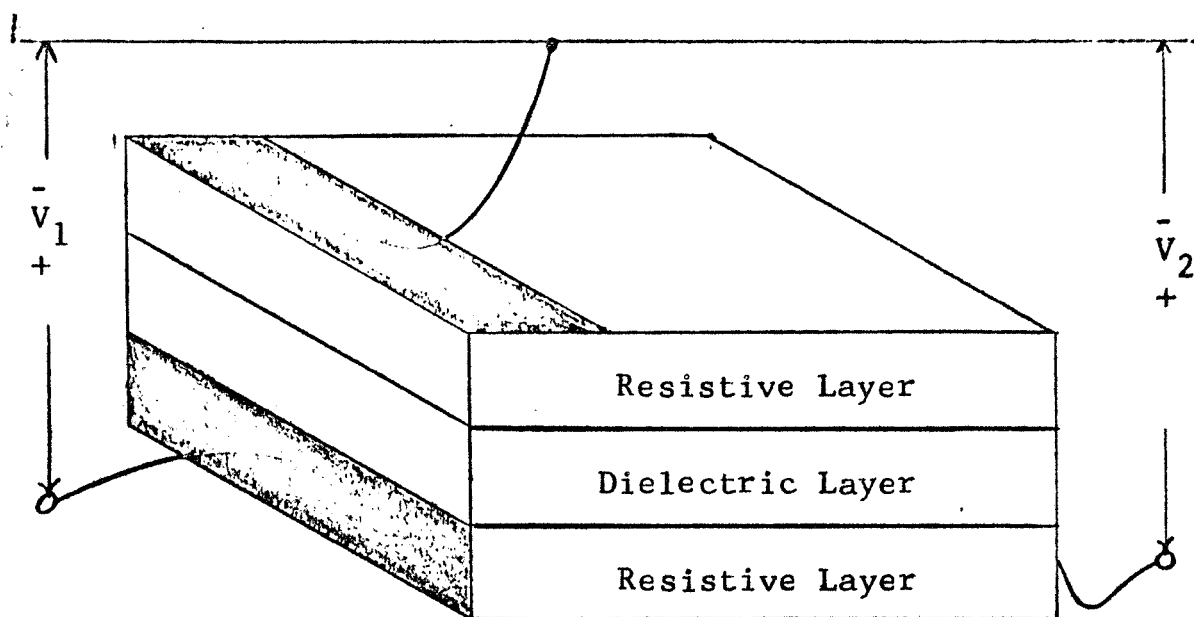


Fig. 9. Castro-Happ type 1 network

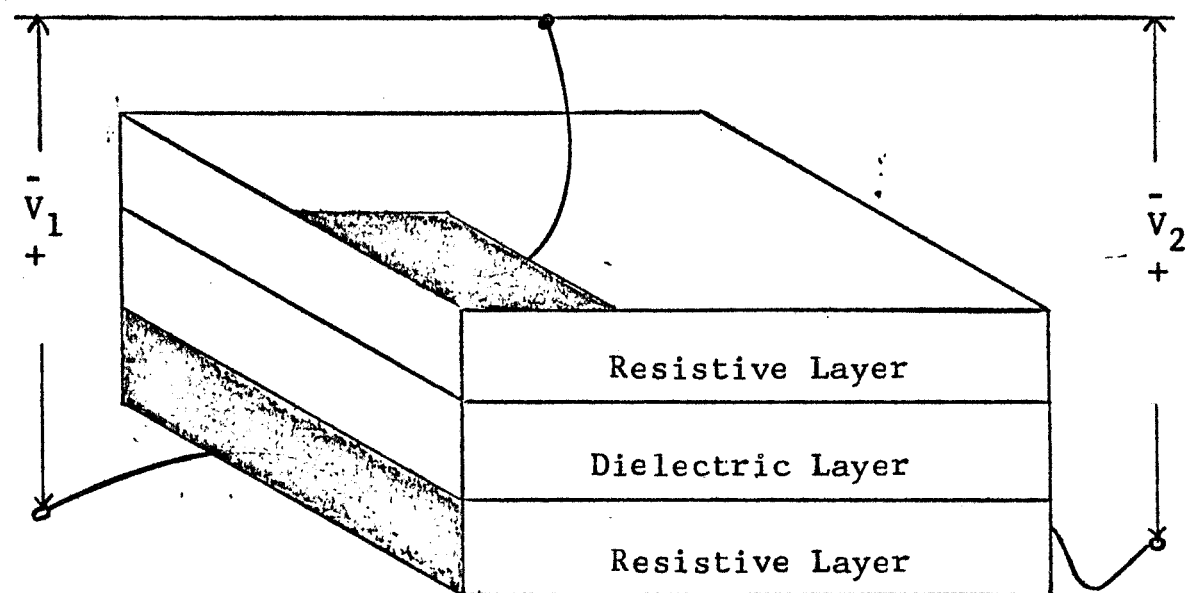


Fig. 10. Three terminal network with shortened grounded terminal towards the input terminal

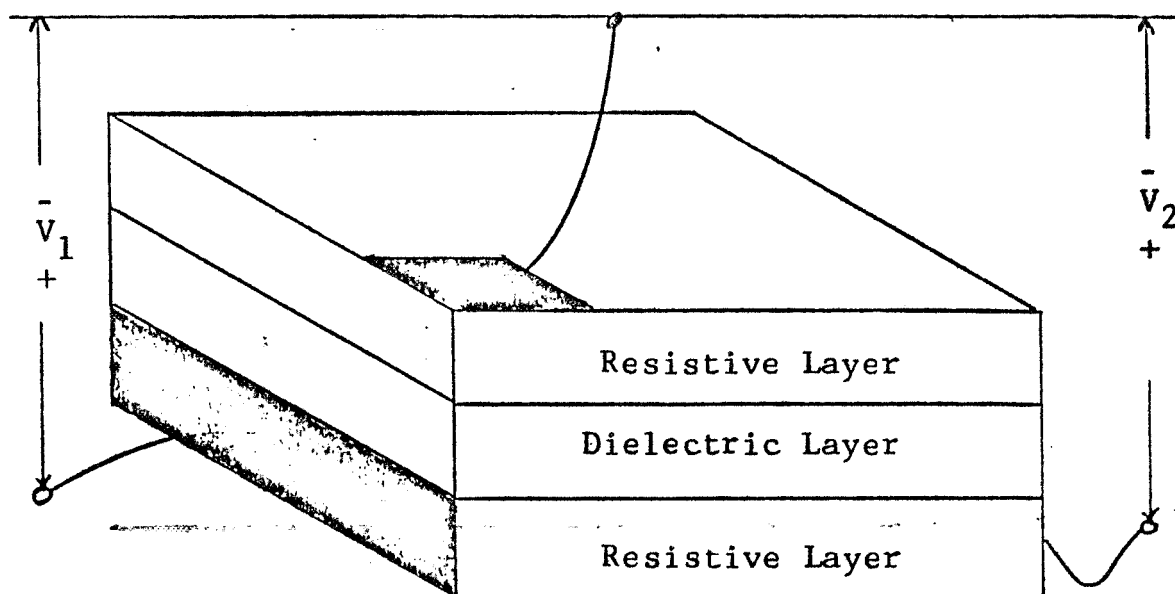


Fig. 11. Three terminal network with a point grounded terminal at corner of input end

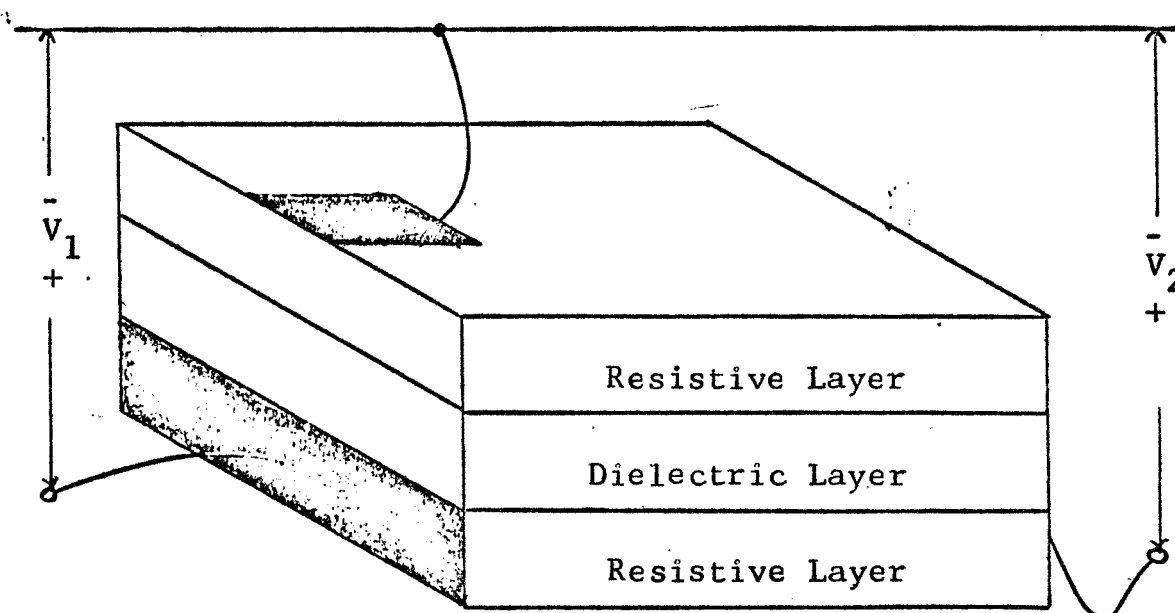


Fig. 12. Three terminal network with a point grounded terminal in center of input end

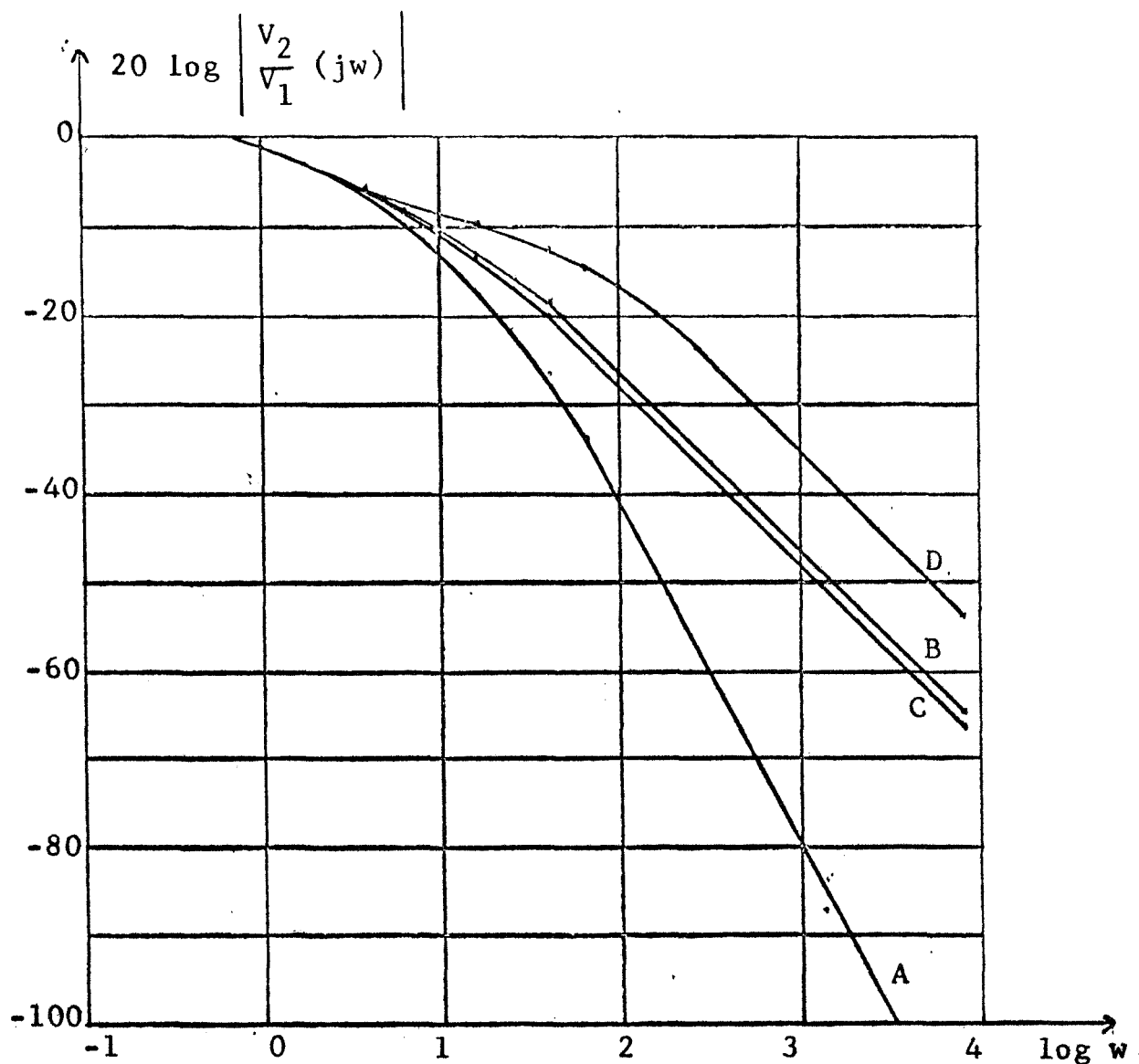


Fig. 13. Low pass characteristic networks

- A: Two-layered two-section RC Model (See Fig. 18)
- B: Two-section Castro-Happ type 2 network, $\sigma = 0.795$ (See Fig. 19)
- C: Two-section three terminal network with grounded terminal bisecting input-output terminals, $\sigma = 0.795$ (See Fig. 20)
- D: Two-section three terminal network with grounded terminal network with grounded terminal perpendicular to input terminal, $\sigma = 0.795$ (See Fig. 21)

characteristic given as Curve C in Fig. 13. The last network of the lowpass type to be considered is a network in which the terminal on the bottom layer is perpendicular to the Castro-Happ type 2 as shown in Fig. 8. This network exhibits a shift of one of the zeros and one of the poles of the Castro-Happ type 2 network, as Curve D of Fig. 13 shows.

The results of two of the Castro-Happ type 1 networks shown in Fig. 9 are shown in Fig. 14. These networks may be considered as lag-lead networks rather than lowpass ones. The lag-lead characteristic is caused by an identical number of finite poles and zeros in the network functions, and, as shown by Curve B in Fig. 14, the effect of increasing the ratio of resistance of the two resistive layers is to increase the level of response of the high frequency end of the curve.

Three types of networks were found which displayed a null type characteristic. The networks are shown in Figs. 10, 11, and 12. The characteristics are shown in Fig. 15. The analysis of other test problems has shown that an increase in σ , the ratio of the resistances of the two layers, flattens the null of each network. It should be noted that the best of the three null-producing networks is the one shown in Fig. 12, in which a short grounded terminal is centered between the sides of the array at its input end. An investigation of the use of additional sections in the lumped-element model of this network is planned.

V. Comments on Results Obtained

The discussion given in the preceding section has indicated the feasibility of using the lumped-element modeling technique as a tool for the analysis of two-dimensional distributed RC networks. Since the results were intended to be mainly indicative of the capabilities of this approach only a very gross model utilizing a 3×3 grid of nodes was used in the tests verifying the utility of the approach. Obviously, for a better approximation to the actual distributed network, a larger grid size should be used. In this section we consider some of the problems associated with using such larger grids.

To begin our discussion we note that for an $n \times n$ grid, n^2 nodes are required to model the two-layer distributed network and $2n^2$ nodes are required to model the three-layer device. The admittance matrices for these two cases require n^4 and $4n^4$ storage elements respectively. For example, a 50×50 grid would require 6.25×10^6 storage locations for the admittance matrix of the

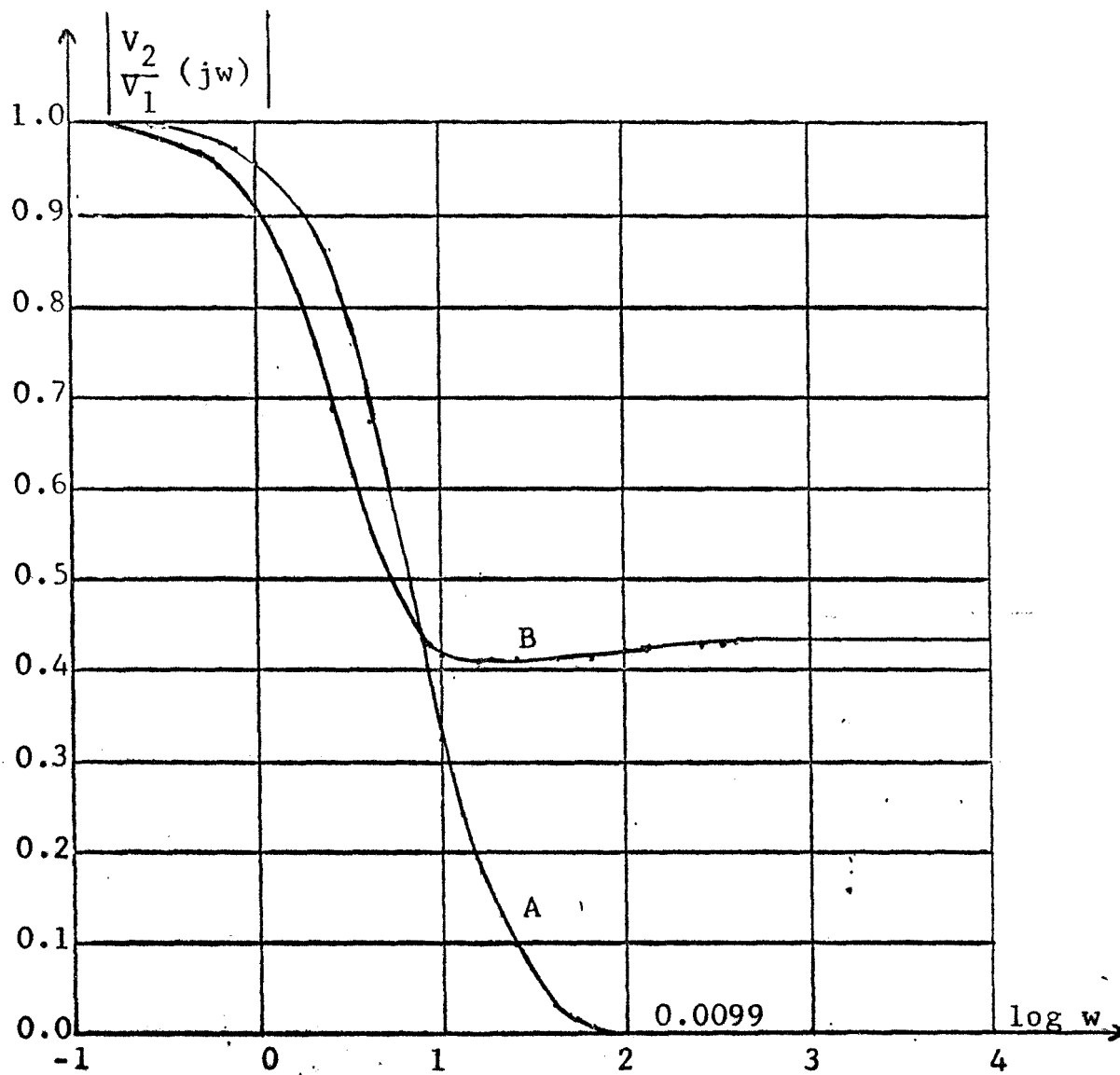


Fig. 14. Castro-Happ type 1 networks (See Fig. 22)

A: $\sigma = 0.01$
 B: $\sigma = 0.795$

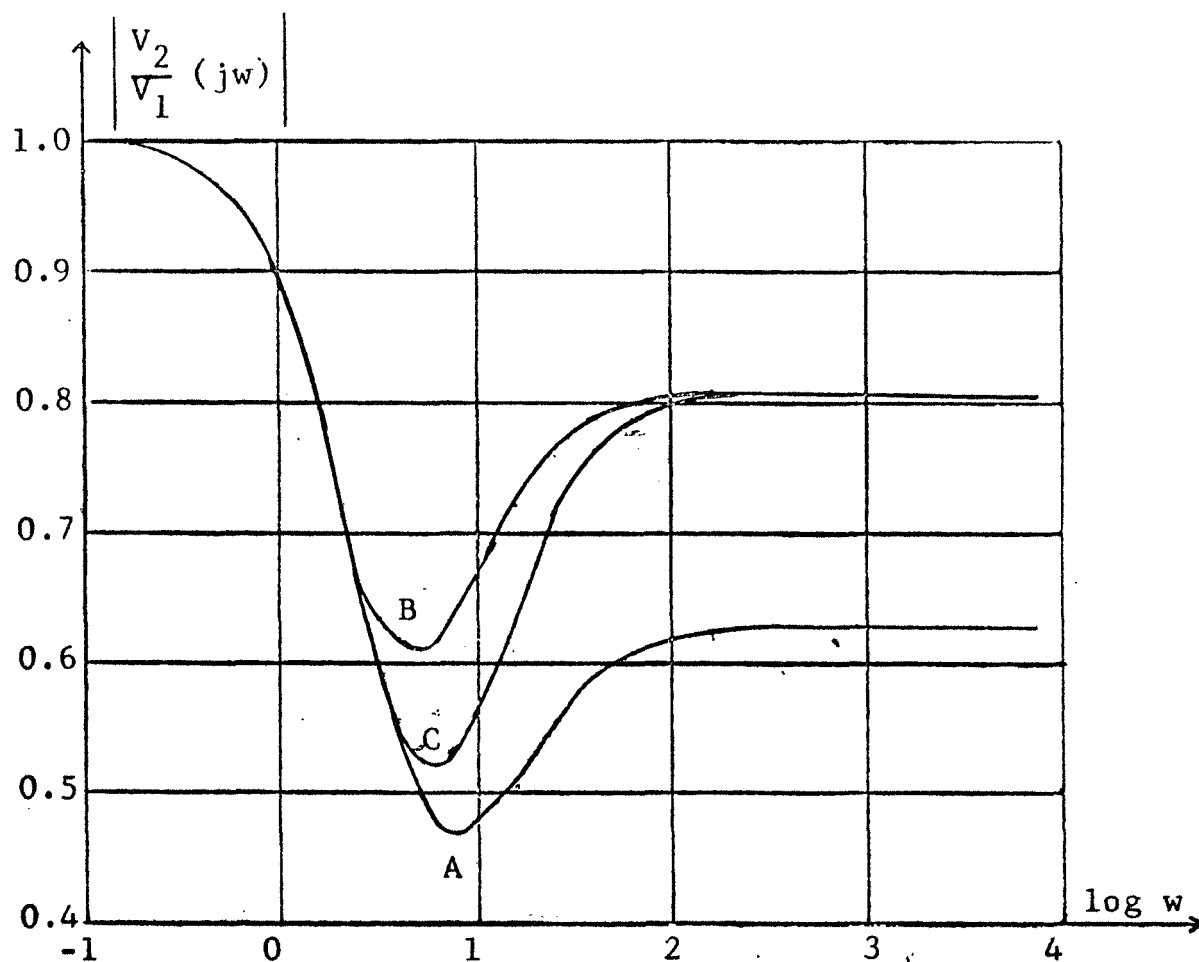


Fig. 15. Null type networks

- A: Two-section three terminal network with shortened grounded terminal towards the input terminal, $\sigma = 0.795$ (See Fig. 23)
- B: Two-section three terminal network with a point grounded terminal at corner of input end, $\sigma = 0.795$ (See Fig. 24)
- C: Two-section three terminal network with a point grounded terminal in center of input end, $\sigma = 0.795$ (See Fig. 25)

associated network, a prohibitively large amount. One interesting property of these admittance matrices, however, is the number of zero terms in each matrix, i.e., the sparseness of the matrix. The number of non-zero terms for a lumped-element model for a two-layer distributed RC network can be shown to be:

$$\begin{aligned}\text{Number of Non-zero Terms} &= 2lw + 2w(l+1) + 2l(w-1) \\ &= 5lw - 2(l+w)\end{aligned}\tag{7}$$

where "l" is the nodal array length, and "w" is the nodal array width. Table 1 tabulates the percentage of necessary computer space needed to store the non-zero elements of the admittance matrices for various sizes of nodal arrays. This table shows that the percentage decreases drastically as the size of the nodal array increases. A similar analysis for the sparsity of the admittance matrix for the lumped-element model of a three-layer distributed RC network shows that

$$\begin{aligned}\text{Number of Non-zero Terms} &= 2lw + 2[5lw - 2(l+w)] \\ &= 12lw - 4(l+w)\end{aligned}\tag{8}$$

Table 2 gives the data for some representative cases for this type of network. Obviously, considering these results, the problem of storage can be attacked by utilizing algorithms which store only the non-zero elements of the matrix array. The resulting program complexity provides a certain amount of trade-off between the use of large amounts of core storage on the one hand, and the requirement for large amounts of computing time on the other.

The second factor to be considered when using larger size nodal arrays in the modeling process is the extra computational effort required to solve such arrays. Assuming that the same algorithms are used, a preliminary study of this problem indicates that the analysis of a distributed network using a 4 x 4 nodal array requires approximately three times as much computing time as the analysis of a 3 x 3 array, and that this increase rises drastically for even larger arrays. Obviously, this places another constraint on the use of larger nodal arrays in the modeling process.

VI. Conclusion

This report has demonstrated the feasibility of applying a lumped-element modeling concept to the analysis of two-layer and three-layer two-dimensional

Table 1. Table of the sparsity of the admittance matrix for a lumped-element model of a two-layer distributed RC network

Length of nodal array	Width of nodal array	Number of matrix elements	Number of non-zero elements	% non-zero elements
2	2	16	12	75
3	2	36	20	56
4	2	64	28	44
5	2	100	36	36
3	3	81	35	43
4	3	144	44	31
5	3	225	59	26
4	4	256	64	25
5	5	625	105	17
10	10	10K	460	4.6
20	20	160K	1.92K	1.2
50	50	6.25M	12.3k	0.2

Table 2. Table of the sparsity of the admittance matrix for the lumped-element model for a three-layer distributed PC network

Length of nodal array	Width of nodal array	Number of matrix elements	Number of non-zero elements	% non-zero elements
2	2	64	32	50
3	2	144	52	36
4	2	256	72	28
5	2	400	92	23
3	3	264	84	32
4	3	576	116	20
5	3	900	148	16
4	4	1024	160	16
5	5	2.5K	260	10
10	10	40K	1160	2.9
20	20	640K	4640	0.7
50	50	25M	29.6K	0.1

distributed RC networks. One of the major advantages of such an approach is that it permits the use of a completely unrestricted choice of terminal configurations, thus permitting the development of network characteristics which are not otherwise attainable with the more usual one-dimensional treatment of such networks. Although this study was restricted to uniform distributed networks, the techniques developed are readily adaptable to handle non-uniform networks, by making a suitable modification of the portion of the program used to generate the admittance matrix. Finally, the analysis results generated by the techniques described in this report are of a form which may be readily incorporated into the general program for the analysis of networks with distributed, lumped, and active components described in a previous report.⁵ Thus they provide a means whereby the analysis capabilities of that program may be extended to include two-dimensional distributed RC networks with arbitrary terminal configurations. Such an extension is planned as a future research effort. In addition, considerable further application of the TDSDN program is planned to analyze specific two- and three-layer distributed RC network configurations.

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