ASRL TR 153-3

A COMPARISON BETWEEN EXPERIMENTAL DATA AND A LIFTING SURFACE THEORY CALCULATION OF VORTEX INDUCED LOADS

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FOREWORD

This report was prepared by the Aeroelastic and Structures Research Laboratory, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, Massachusetts. The work was sponsored by NASA-Langley Space Research Center under Grant Number NGR-22-009-303.

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ABSTRACT

A comparison is made between experimental data and theoretical calculations using a lifting surface theory solution for the loads induced by a free vortex on a single-bladed rotor operating at high advance ratio. Good correlation between experiment and theory is obtained when the vortex is not close to the rotor hub.

CONTENTS

NOMENCLATURE

b	blade semichord
c	blade chord
c_l	section coefficient
h	vortex/blade vertical separation
L	section lift
r	rotor radial coordinate
r_c	vortex core radius
R	blade radius
y_G	lateral location of vortex
w	downwash
Γ_∞	free vortex strength
μ	rotor advance ratio: forward speed divided by tip speed
ρ	air density
ψ	rotor azimuthal coordinate
Ω	rotor rotational speed

INTRODUCTION

A lifting surface theory solution has been developed for a model problem for vortex induced airloads (Ref. 1). The model consists of an infinite aspect ratio wing in a subsonic compressible free stream, and a straight, infinite vortex at an arbitrary angle with the wing (Fig. 1). The vortex is a distance h below the plane of the wing and is convected past the blade by the free stream. Using the exact numerical results from linear lifting surface theory, an approximate, closed form solution for the vortex induced loads was obtained. The solution in this form is suitable for application to the calculation of rotary wing airloads; procedures for this application were developed in Ref. 1. In order to evaluate the lifting surface solution, it was compared with lifting line theory in the calculation of the loads in a simplified vortex/blade configuration (Fig. 2). This configuration is a single-bladed rotor encountering a free vortex from an upstream wing. This comparison showed that the lifting surface theory results differed substantially from the lifting line results; it was concluded that the lifting surface theory solution should be used for any vortices closer than about five chord lengths to the blade.

Experimental data has become available for the rotor/vortex configuration shown in Fig. 2 (Ref. 2). This report compares that experimental data with the loads calculated using the lifting surface theory solution.

In what follows all quantities are nondimensional, based on the fluid density, the rotor radius, and the rotor rotational speed (ρ, R, Ω).

DESCRIPTION OF THE EXPERIMENT AND THE THEORETICAL MODEL

The experiment of Ref. 2 consisted of a single-bladed rotor at high advance ratio encountering a free vortex from a fixed upstream airfoil (Fig. 2). The rotor had no flapping or lagging hinges and operated at zero collective pitch. The blade radius was twelve inches and the rotor hub was covered with a spinner about 2.5 inches in diameter.

For the theoretical model the blade was considered to have no flapping or torsional freedom of motion, so the tip-path plane remained undistorted. The vortex was considered an undistorted straight line parallel to the rotor tip-path plane, at a distance y_G from the rotor hub (in the notation of Ref. 2, $y_G \equiv Z$), and at a distance h below the tip-path plane. The location of the vortex generator tip (the upstream end of the vortex -- see Fig. 2) was given by

$$\vec{r}_G = x_G \vec{i'} + y_G \vec{j'} + h \vec{k'}$$

The parameters describing the blade, the vortex, and the calculation procedures are given below.

a) rotor rotational speed: Ω = 1500 RPM and 2000 RPM

b) advance ratio:

$$\mu = \begin{cases} .700 & (\Omega = 1500) \\ .526 & (\Omega = 2000) \end{cases}$$

c) advancing tip Mach number:

$$M_{1.0,90} = \begin{cases} .241 & (\Omega = 1500) \\ .288 & (\Omega = 2000) \end{cases}$$

d) free vortex strength: $\Gamma_\infty = -15.25\ \text{ft}^2/\text{sec}$

thus

$$\Gamma_\infty/\Omega R^2 = \begin{cases} -.0972 & (\Omega = 1500) \\ -.0730 & (\Omega = 2000) \end{cases}$$

e) blade semichord to radius ratio: b = .08333

f) vortex generator position:

$$\vec{r}_G = -1.5\ \vec{i'} + y_G \vec{j'} + h\ \vec{k'}$$

$$y_G = -0.75 \text{ to } 0.0$$

$$h/c = -1.5 \text{ to } 1.0$$

g) collective pitch: $\theta^\circ = 0$

tip-path plane inclination angle: $i = 0$

h) azimuth increment in the calculation procedure: $\Delta\psi$ = 10 degrees
number of trailers in the vortex net representing the wake: N = 4
extent of blade wake: $\Delta\phi$ = 220 degrees

TREATMENT OF EFFECT OF VORTEX CORE

In order to calculate the loads for small values of the vortex/blade vertical separation, h, it is necessary to consider the viscous core of the vortex. Inside a core radius r_c, the induced tangential velocity of the vortex decreases due to viscosity, rather than continuing to increase as in the potential flow model. The downwash (the component of the induced velocity normal to the wing surface) at a blade a distance h above the vortex usually has peaks located a distance h either side of the vortex line, and the presence of the core has little or no effect on the variation of the downwash along the span. However, if $\sqrt{2}\,h < r_c$, then the downwash peaks are located at the edge of the viscous core, and the magnitude of the peak downwash is less than that of the potential flow model. Without the effect of the core the downwash induced by the vortex at the blade is given by:

$$\omega = \frac{\Gamma_\infty}{2\pi} \frac{-d}{d^2 + h^2} \tag{1}$$

where d is the perpendicular distance from the projection of the vortex on the plane of the blade. It is convenient to use this same analytic expression for the downwash also when core effects are included. This is accomplished by replacing the vortex by an equivalent vortex, of strength Γ_{eq} and at a distance h_{eq}; requiring that the positions and magnitudes of the downwash peaks be the same gives:

$$\Gamma_{eq} = 2\Gamma_\infty \left(1 - h^2/r_c^2\right) \tag{2}$$

$$h_{eq} = \sqrt{r_c^2 - h^2} \tag{3}$$

which are used if $\sqrt{2}\, h < r_c$. The derivation of these expressions is given in Ref. 1.

The above argument is only valid when there is a definite boundary between the viscous and potential regions of the vortex. The distribution of vorticity actually found in a vortex decreases the magnitude of the downwash peaks as h decreases to zero. The potential flow formula (Eq. 1) for the downwash gives the peak downwash:

$$w_{max} = \frac{\Gamma_{eq}}{4\pi h_{eq}} \tag{4}$$

at $d = h_{eq}$. For $h = 0$, Eq. 3 gives $h_{eq} = r_c$ so then $\Gamma_{eq} = 4\pi r_c w_{max}$. Thus the quantities w_{max} and r_c give the equivalent vortex strength that should be used when $h = 0$ to match both the position and magnitude of the peaks of the measured downwash distribution. These quantities are available from experimental measurements of the tangential velocity of vortices (which is exactly the downwash at the blade when $h = 0$); the results are given in the table below. Note that Eq. 2 gives $\Gamma_{eq} = 2\Gamma_\infty$ when $h = 0$.

source	w_{max} (ft/sec)	Γ_∞ (ft^2/sec)	r_c (ft)	$\Gamma_{eq} = 4\pi r_c w_{max}$ (ft^2/sec)	$\Gamma_{eq}/\Gamma_\infty$
ref. 2	52	15.25	.0312	20.4	1.34
ref. 2	33	9.3	.0312	12.9	1.39
ref. 3	31.75	8.25	.025	10.0	1.21
ref. 4	51.5	19.4	.041	26.4	1.36

Thus the following expression should actually be used for the equivalent vortex strength:

$$\Gamma_{eq} = \left[1.35 + 0.65(\sqrt{2}\,h/r_c)\right]\Gamma_\infty(1 - h^2/r_c^2) \quad (5)$$

This is the expression that was used for the theoretical calculation to be discussed below. The data of Ref. 2 also give a core radius of r_c = 0.031 (r_c/b = 0.375). The use of the core correction gives good agreement with the experimental results even down to h/b = 0. It is of course likely that the vortex is actually pushed aside by the blade, rather than exactly cut by it when h = 0, but the accuracy of the experimental data is not sufficient to allow this feature to be separated from the results.

COMPARISON OF THE EXPERIMENTAL AND THEORETICAL RESULTS

Figures 3 to 10 compare the experimental data and the lifting surface theory prediction of the peak-to-peak lift coefficient for the vortex/blade intersection on the rear half of the disk. These figures show the variation of the loads with h/c at a rotor radial station of r = .95. The section lift coefficient, c_l, is based on the local velocity relative to the blade:

$$c_\ell = \frac{L}{\frac{1}{2}\rho U_T^2 c} = \frac{L}{(r + \mu\sin\psi)^2 b}$$

The figures are for y_G = 0.0, -.25, -.50, and -.75, and Ω = 1500 and 2000 RPM (μ = .700 and .526). Figures 6 and 10 include for comparison the results of a lifting line calculation.

Good agreement is shown between the experimental data and the lifting surface theory results when the vortex is away from the rotor hub, particularly for y_G = -.75 and -.50. When the vortex is nominally passing through the rotor hub, $y_G = 0.0$, poor agreement is shown between the experiment and the theory, the theory overpredicting the actual loads. It is believed that this discrepancy is due to interference between the vortex and the hub with its large spinner. The vortex cannot pass through hub, so its position is not actually known for this case.

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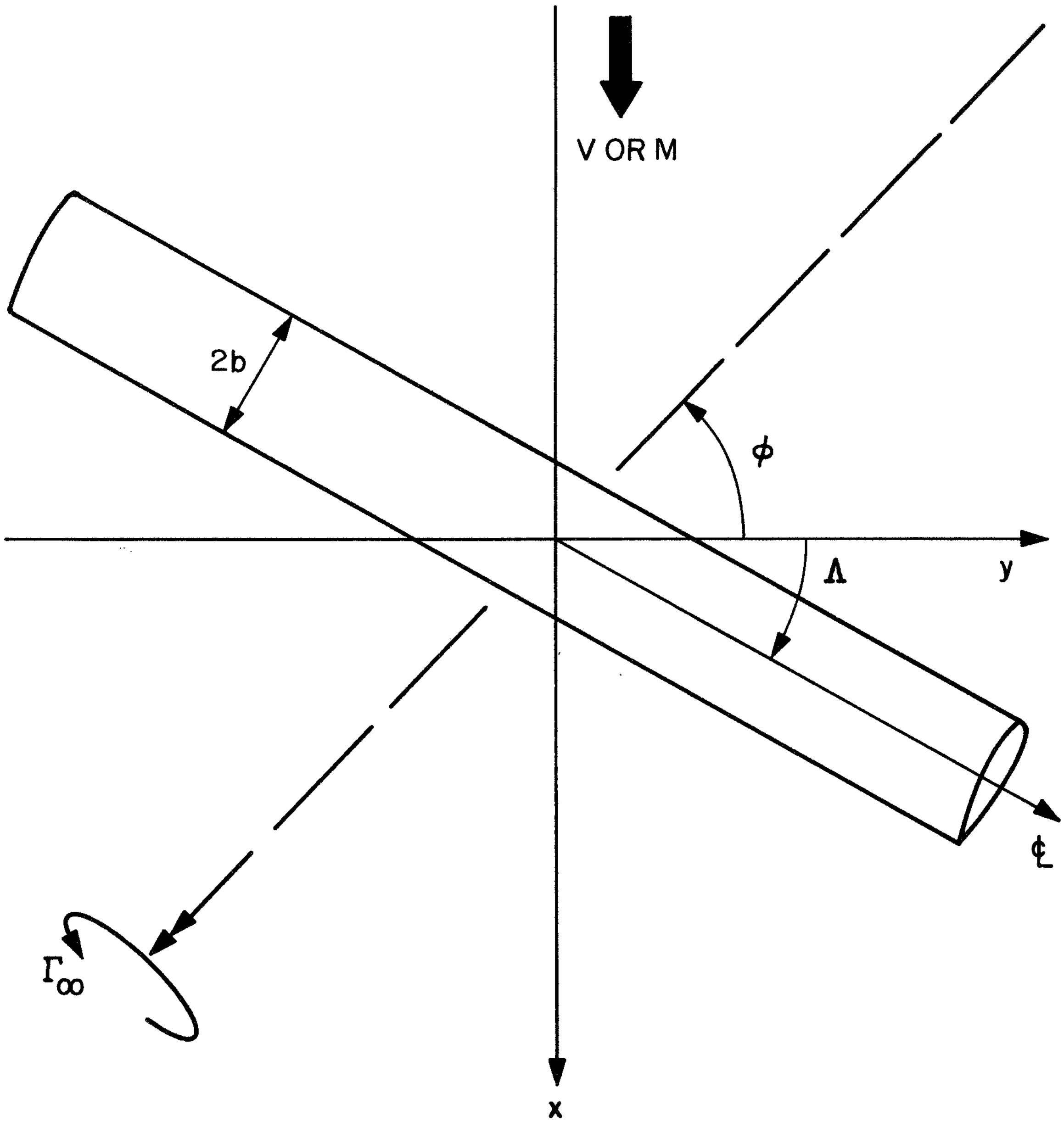

FIGURE I MODEL PROBLEM FOR VORTEX INDUCED AIRLOADS

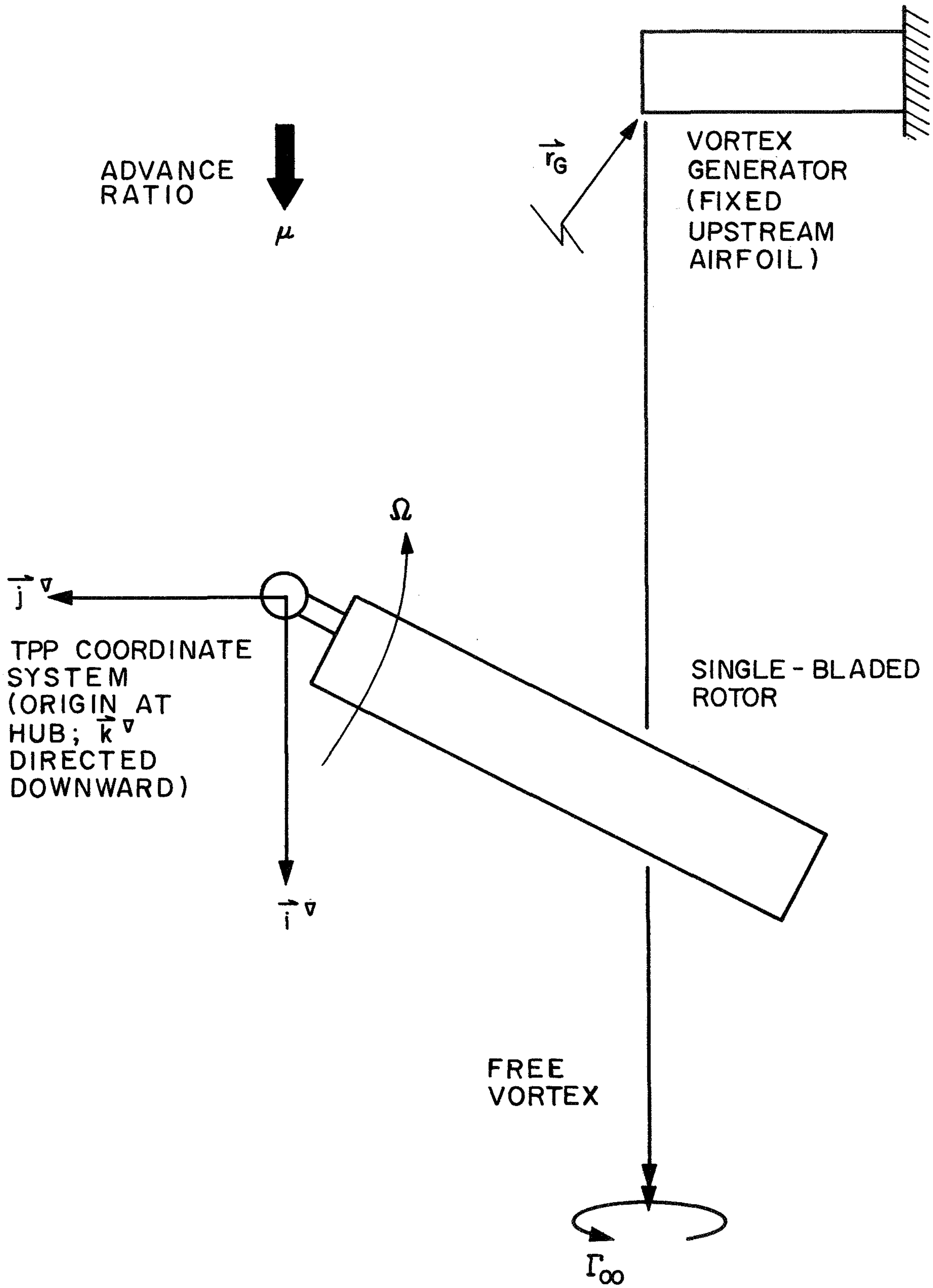

FIGURE 2 SIMPLIFIED VORTEX / BLADE CONFIGURATION

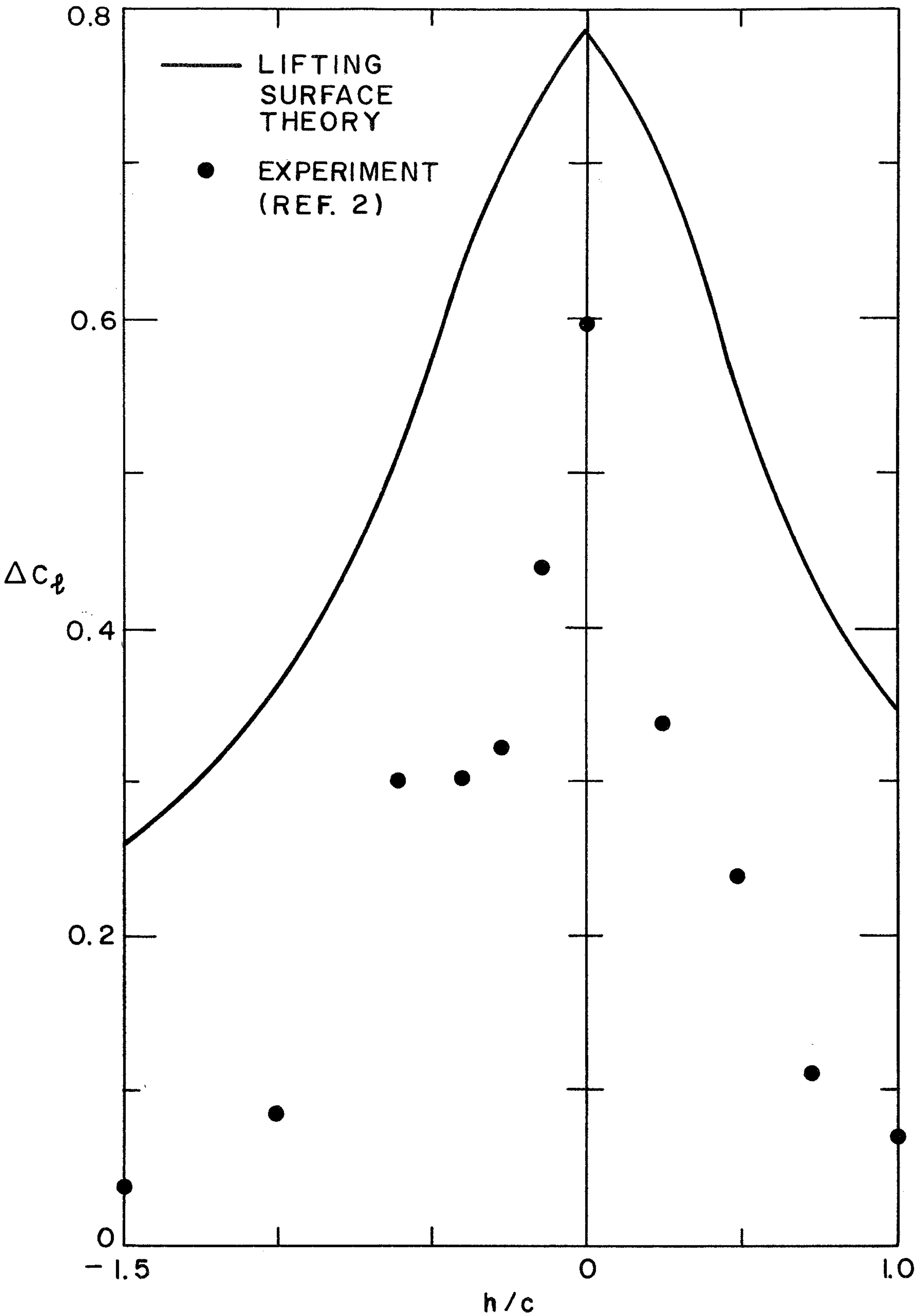

FIGURE 3 COMPARISON OF EXPERIMENTAL AND THEORETICAL PEAK-TO-PEAK SECTION LIFT COEFFICIENT: $y_G = 0$, $\mu = .700$, $r = .95$

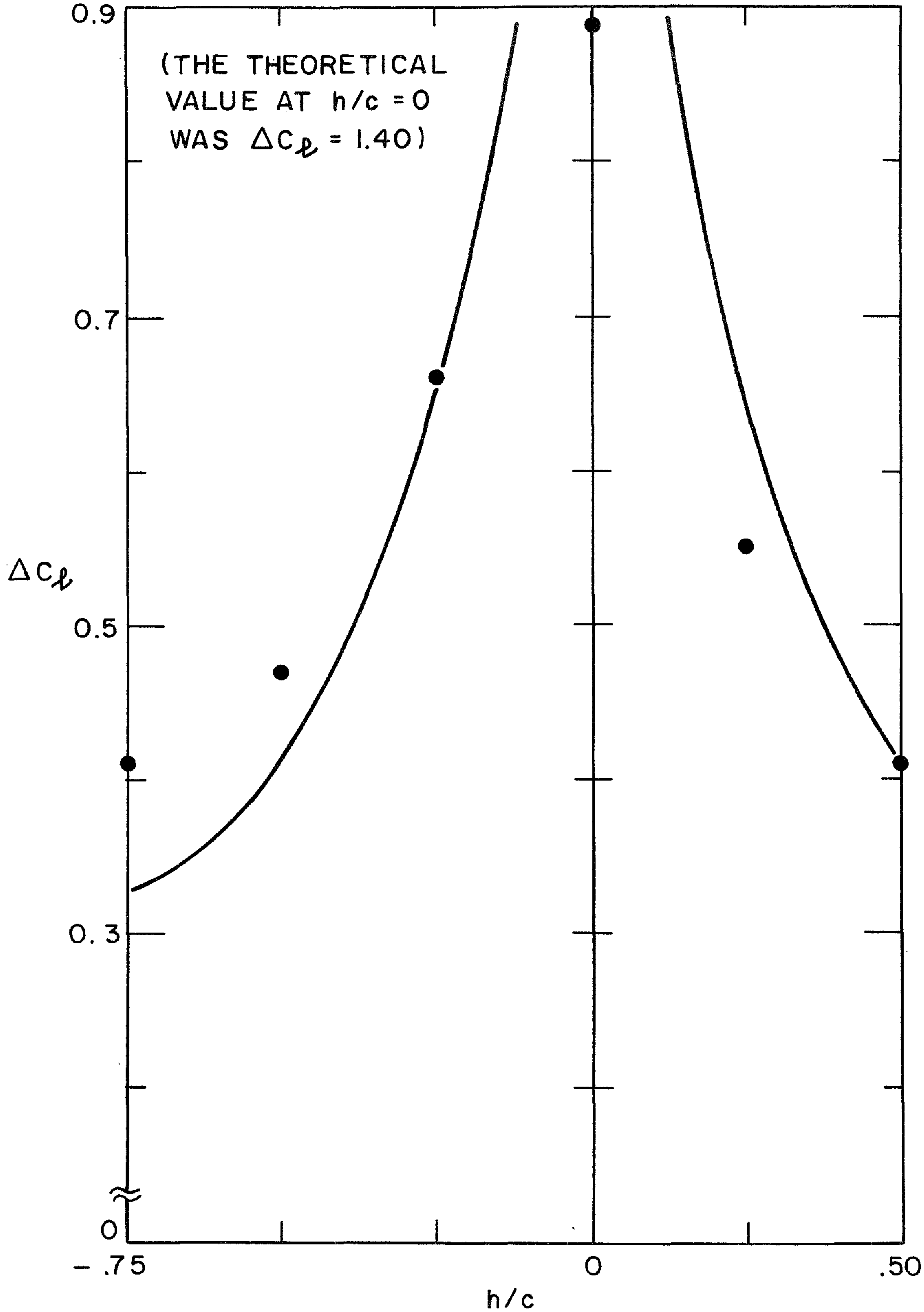

FIGURE 4 $y_G = -.25$, $\mu = .700$, $r = .95$

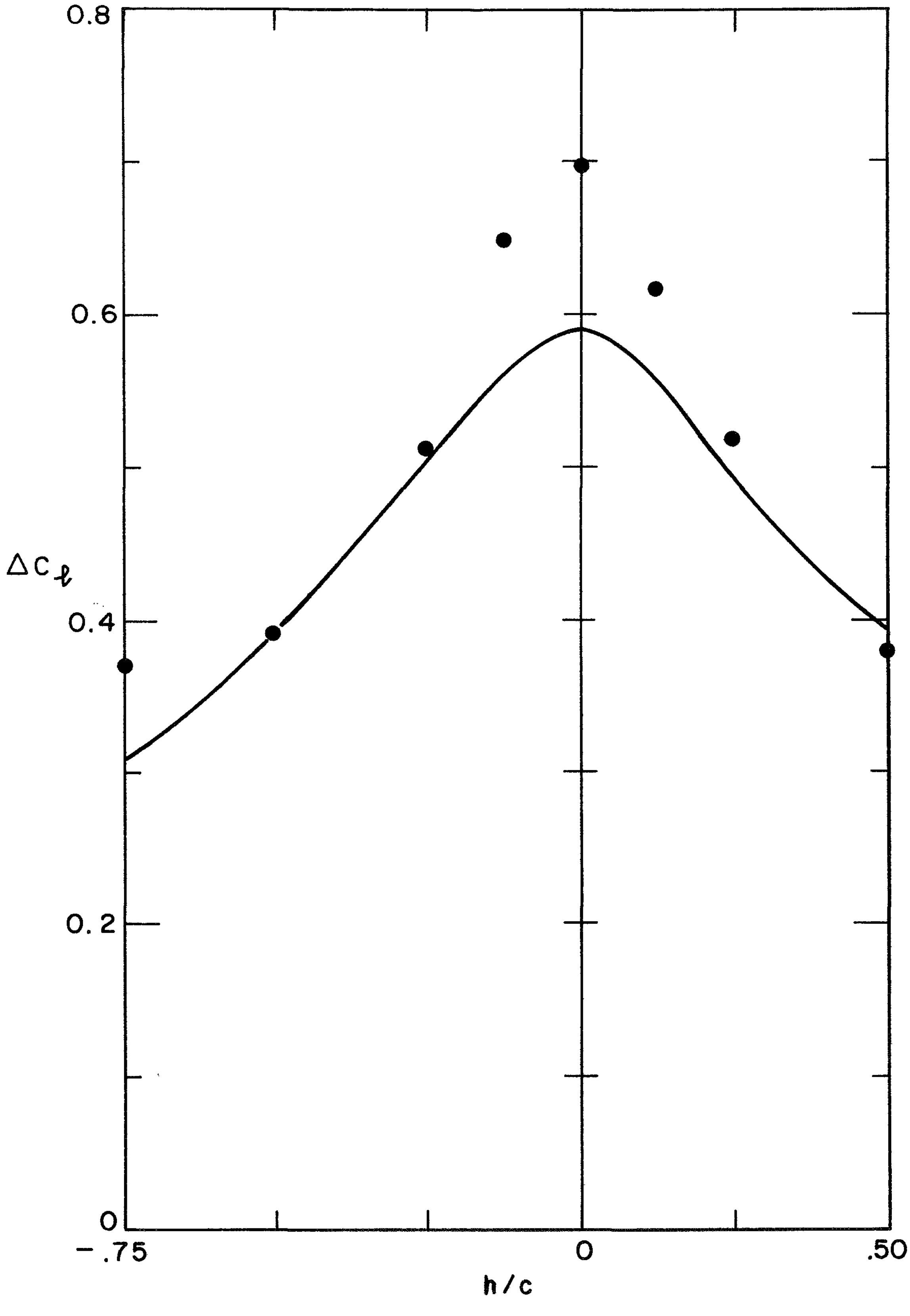

FIGURE 5 $y_G = -.50$, $\mu = .700$, $r = .95$

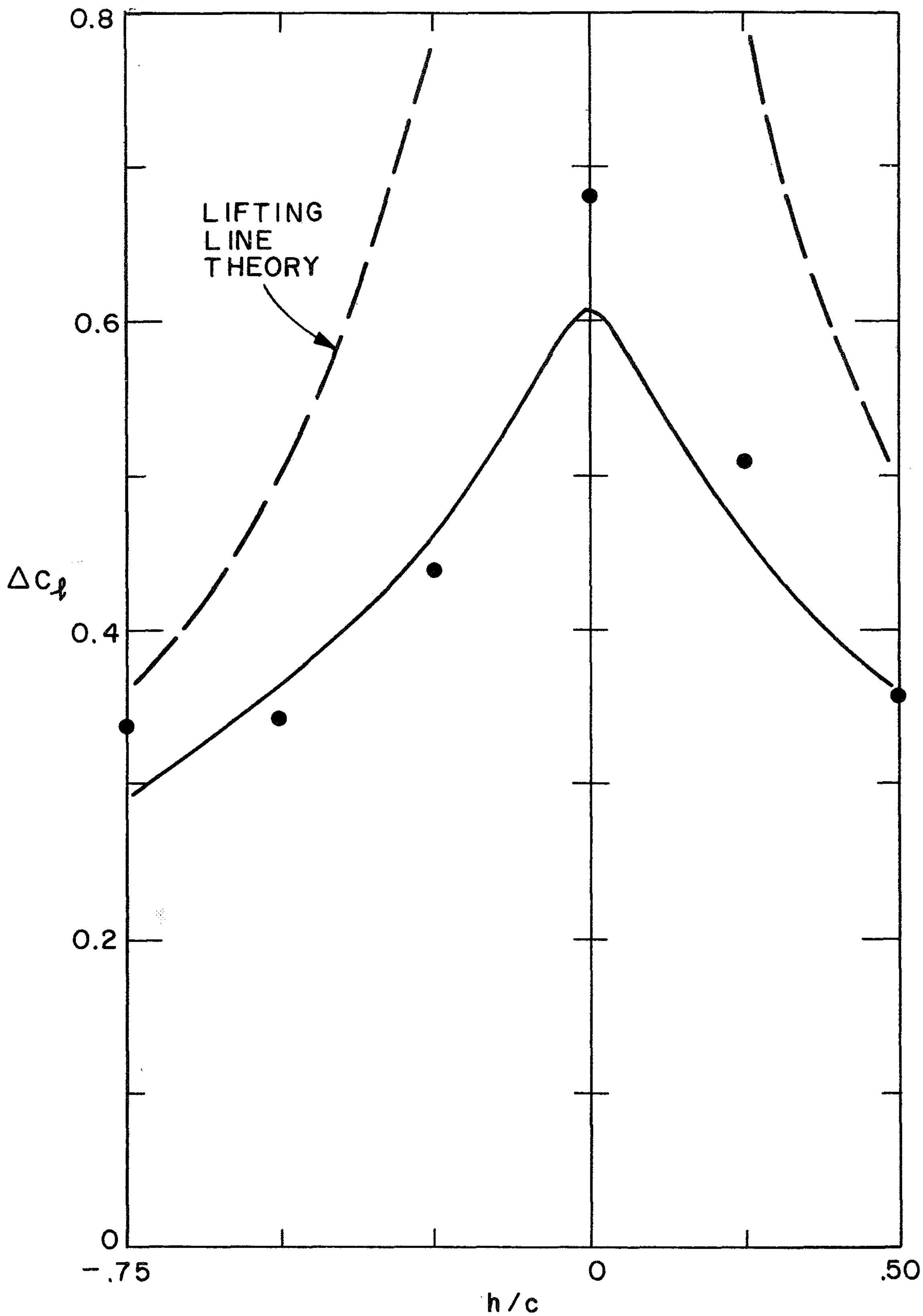

FIGURE 6 $y_G = -.75$, $\mu = .700$, $r = .95$

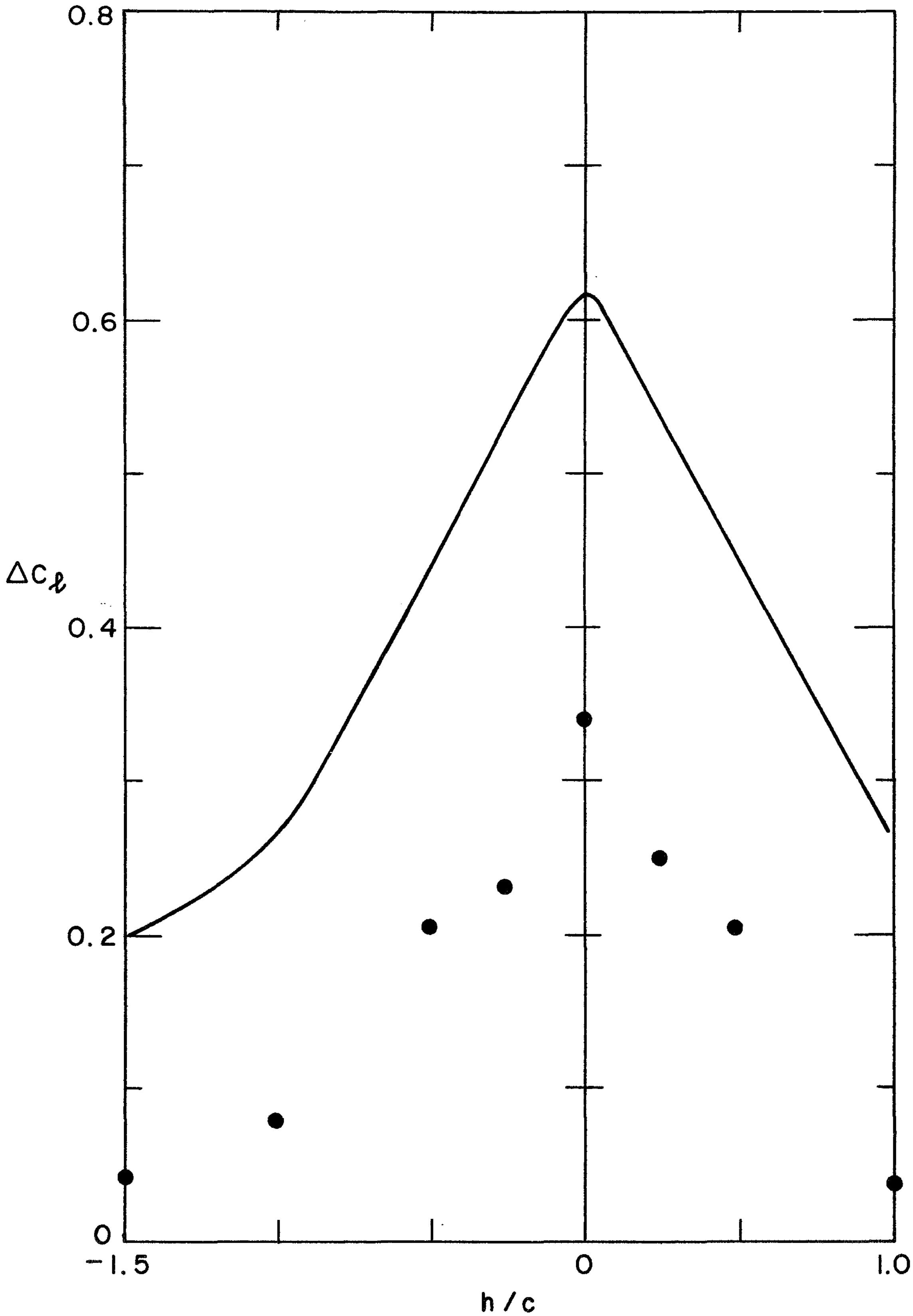

FIGURE 7 $y_G = 0.0$, $\mu = .526$, $r = .95$

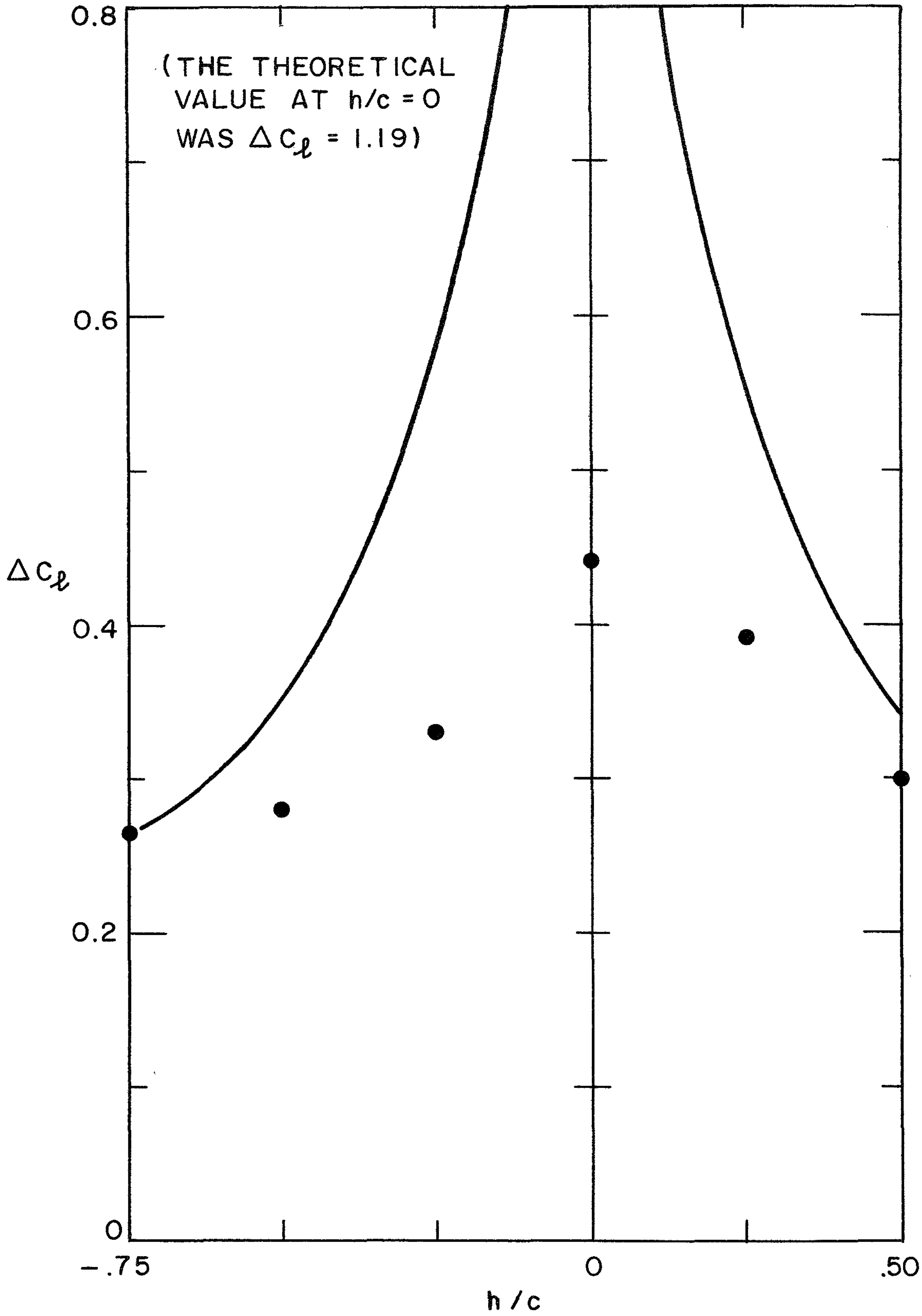

FIGURE 8 $y_G = -.25$, $\mu = .526$, $r = .95$

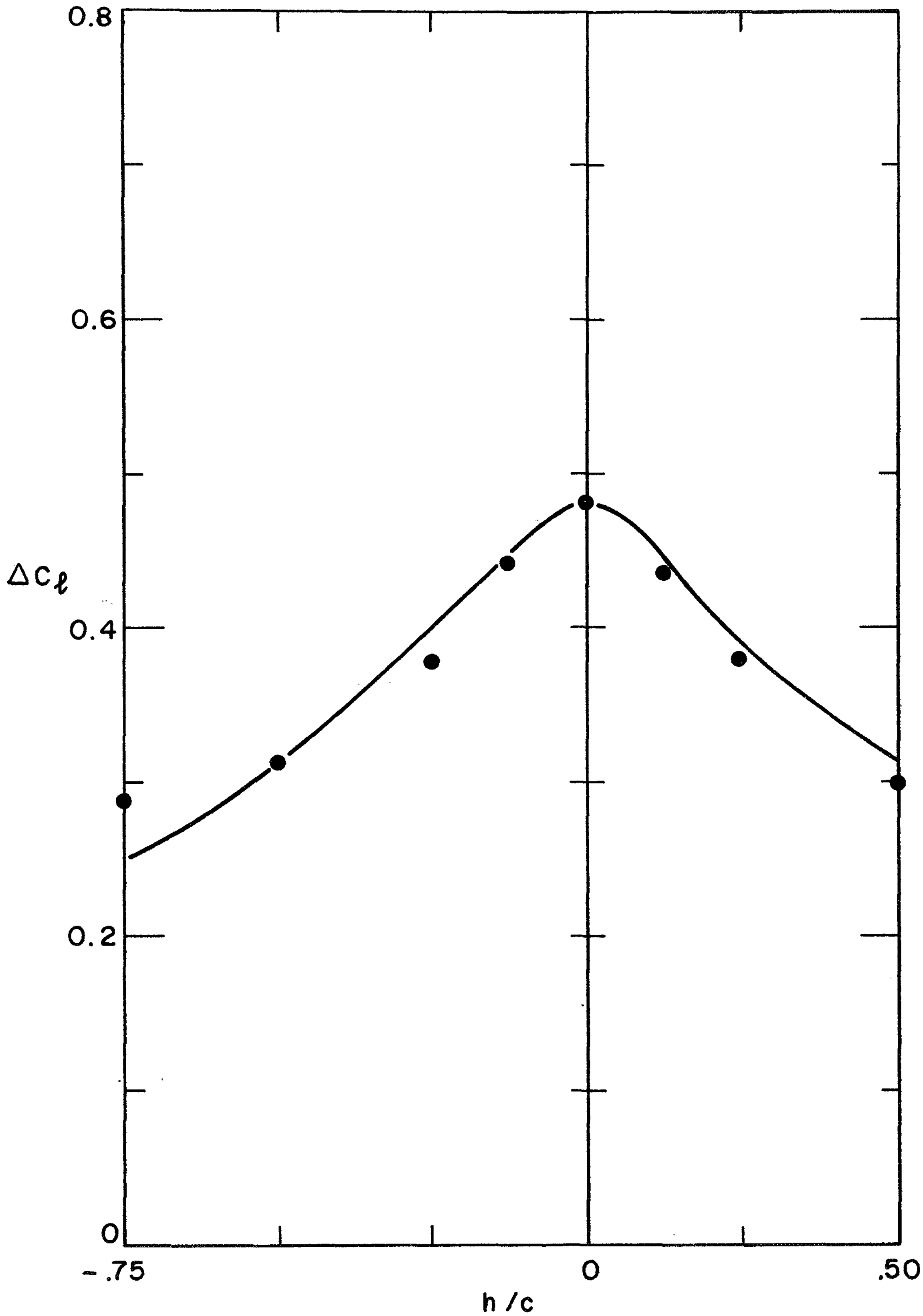

FIGURE 9 y_G = ~~-.75~~ -.5 , μ = .526, r = .95

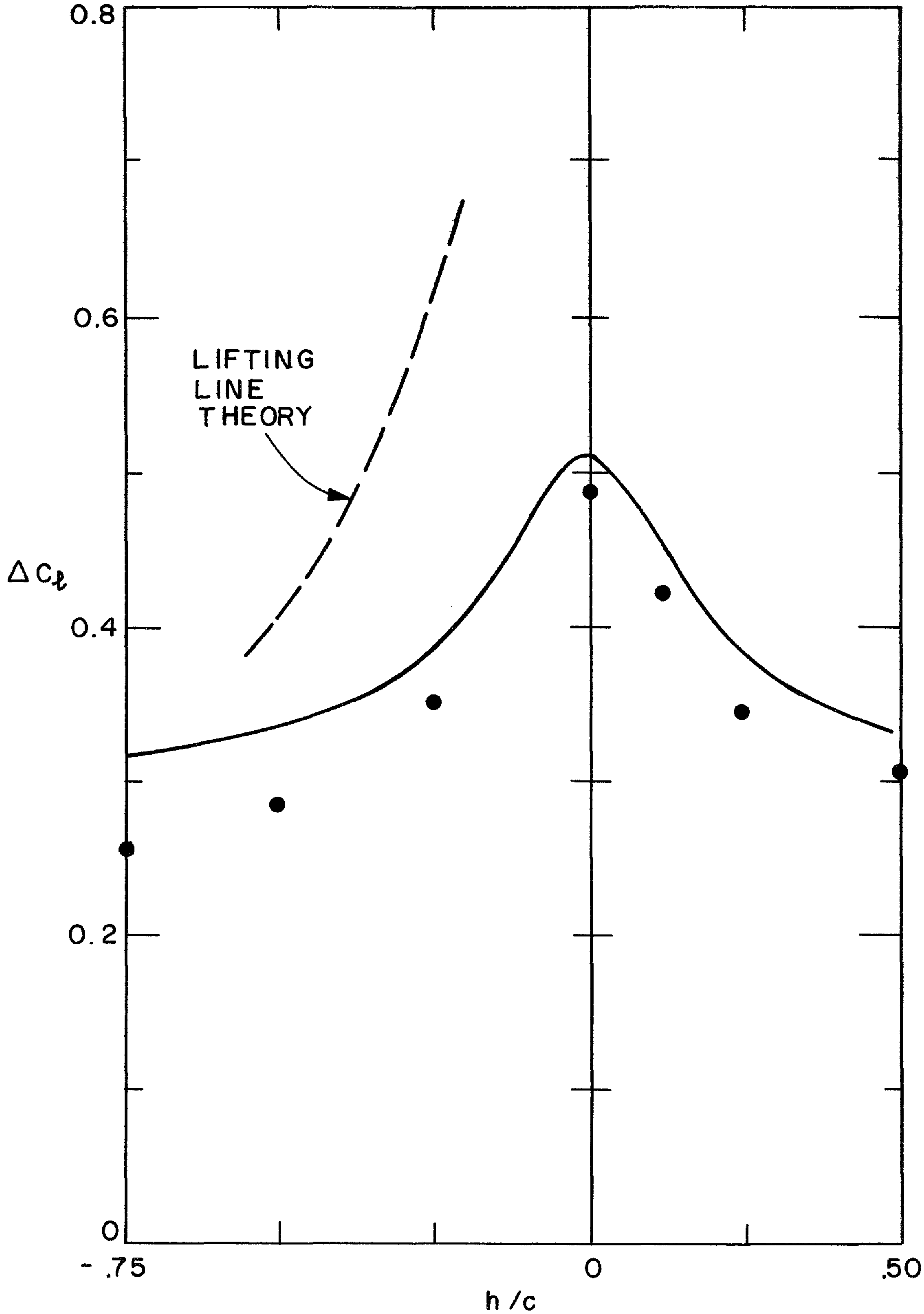

FIGURE 10 $y_G = -.75,\ \mu = .526,\ r = .95$

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13. ABSTRACT

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