TECHNICAL REPORT

MANEUVERING THE DUAL MODE MANNED/AUTOMATED LUNAR ROVING VEHICLE

By W.F. Wood and G.M. Lewandowski

CAL No. VS-2860-D

GEORGE C. MARSHALL SPACE FLIGHT CENTER
HUNTSVILLE, ALABAMA

PERFORMANCE PERIOD
JUNE 1969 - MARCH 1970

FINAL TECHNICAL REPORT
CONTRACT NO. NAS8-25110
DCN 1-X-40-91084(1)
MARCH 23, 1970

CORNELL AERONAUTICAL LABORATORY, INC.
OF CORNELL UNIVERSITY, BUFFALO, N. Y. 14221

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ABSTRACT

Cornell Aeronautical Laboratory, under contract to George C. Marshall Space Flight Center of NASA, has produced digital maps of hazards to movement for the dual mode Lunar Roving Vehicle. The area mapped from vertical orbiter photography and astronaut's geologic maps is located in Sinus Medii. Hazards to movement located within grid squares 50 meters on a side have been transformed into digital data for computer access. Maps produced via computer have been prepared showing these hazards.

FOREWARD

The work reported herein was conducted under Contract NAS8-25110, for NASA, Huntsville. It was started on 24 June 1969. A companion effort to use the data generated here was performed by WNRE Inc., under Contract NAS8-2517. The effort by WNRE is reported separately.

The contract specifies that the informal monthly status reports are to be written in such a manner as to form parts of the final report. This was done, and in preparing the final report, pages and paragraphs from the monthly reports are included substantially without change.

ACKNOWLEDGMENTS

Appreciation is accorded to the sponsor, NASA, Huntsville, for providing lunar maps, vehicle mobility requirements and vehicle characteristics pertinent to this study. NASA, Langley Research Center, provided the lunar photography used in the conduct of this research.

The following report was authored by Dr. Walter F. Wood and Gregory M. Lewandowski. The authors also wish to acknowledge the contributions of the following people who have participated in this study.

A. Gundel	-	Card punching and correction.
R. P. Leonard	-	Digitization of surface descriptors.
H. T. McAdams	-	Mr. McAdams has had a major role in developing the basic methodology of computer map intersection. This methodology was developed in previous work. He has also contributed the attached Appendix, "Statistics of Crater Occurrence on the Moon".
P. A. Reese	-	Computer programming, automated data handling, and production of computer maps.

I. INTRODUCTION

The purpose of the research reported here is to provide lunar surface data which needs to be considered in the design of the dual mode manned/automated Lunar Rover. Lunar rovers will be designed to meet the same hazards of radiation and temperature as other equipment expected to operate on the moon. The rovers,though, will have to contend with variation in surface geometry and soil properties encountered in traversing the lunar surface. It is admitted that a rover cannot be expected to go everywhere on the moon. It is also agreed that unless provisions are made for overcoming all surface hazards which cannot be avoided, a rover could be delivered to the moon only to fail without accomplishing its mission.

The total weight of vehicle and scientific package is the overriding consideration. Overemphasis on vehicle performance could lead to a design which leaves too little room for the scientific package. Over-emphasis on the scientific package could force design of a vehicle with very limited mobility.

Furnishing the vehicle designers with a large amount of lunar surface data will not assure optimum vehicle design. However, optimum vehicle design could not hope to take place without it.

A. Study Objectives

The overall objective of this program is to prepare maps of specific areas of the lunar surface from sponsor-furnished photographs. These maps are to identify specific areas where a vehicle may be stopped or its progress seriously hampered. Tentative sets of vehicle mobility requirements and vehicle characteristics were furnished by the sponsor, but information yet to become available may make it advisable to build a dual mode LRV considerably different from that now contemplated. Therefore,the mapping has been done without any specific vehicle in mind, and is parametric to the extent that one could deduce from the maps, the expected performance characteristics of any vehicle which may be seriously contemplated.

Specifically, the objective of CAL's part of this study is to prepare, from the sponsor furnished photographs, maps which will indicate lunar surface parameters which might effect the operation of a dual mode rover. These parameters fall into a series of categories some of which appear below.

- Lunar terrain elevations
- Surface type, such as mare or mountains and relative amounts of cratering per region.
- Details on the lunar landscape, such as the number, size and types of craters; the number of blocks present; fault scarps; slopes and soil strengths, and crater diameters.

B. Contractor Interaction

The Introduction relates that the purpose of this research is to match moon surface parameters and vehicle performance parameters so that a near optimum Dual Mode Roving Vehicle can be designed. Contracts with identical work statements were issued to CAL and WNRE, Inc. There is, however, a division of labor which is agreed to by the sponsor and the two contractors (Figure 1).

The primary responsibility of CAL is to map the surface parameters and the primary responsibility of WNRE is the evaluation of vehicle performance on areas represented by CAL's maps. On the flow diagram just cited, CAL's work is shown on the left side and WNRE's on the right. The flow pattern indicates only in a partial way the considerable interaction which has taken place.

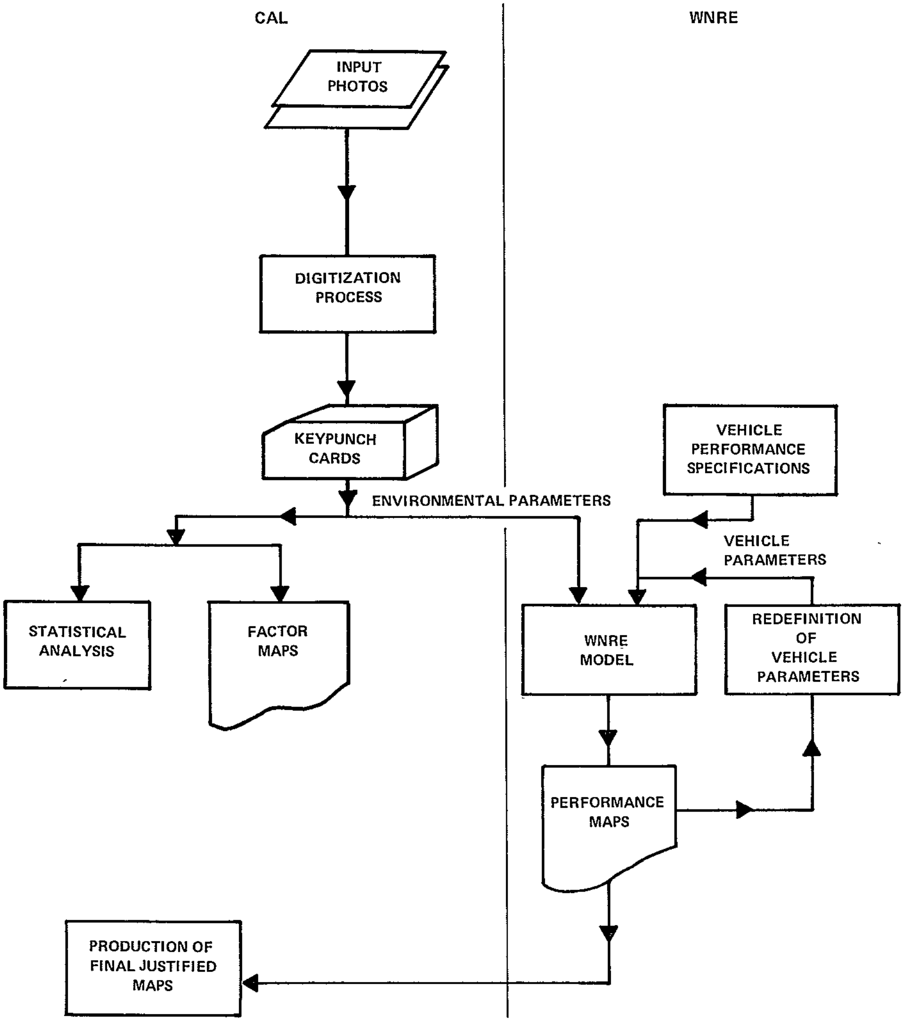


Figure 1 FLOW CHART OF CAL-WNRE DIVISION OF LABOR

II. DISCUSSION

The conduct of this study falls generally into two parts : the gathering, organization and analysis of specific lunar surface data, and the preparation of digital lunar maps of the study area.

As pointed out in the study objectives, tentative vehicle mobility requirements (Table 1) and vehicle characteristics (Table 2) were furnished by the sponsor. However, information available now and likely to become available before deployment of a dual mode LRV (Table 3) might suggest that it is premature now to begin conducting research to support the design of the dual mode rover.

Because of the lack of detailed information on quantitative properties of the lunar surface, such as soil strength, slope and concentrations of blocks, it is necessary to arrive at such estimates by deduction. Ranger, Surveyor and Orbiter projects have furnished photography which has been used by geologists to establish a stratigraphy of the moon surface. Geologists and photogrammetrists have also derived a classification of craters based on relative age. It is fairly certain that primary craters when first formed have depth-to-diameter and depth-to-rim-height ratios which are fairly constant. Craters age with time and eventually disappear. Very sharp rim crests, bright ray material and a scarcity of small primary craters in the immediate vicinity indicate a young crater. With time, colors darken, rims become subdued and the number of smaller craters in the immediate surroundings become as numerous as the general surroundings. We can be certain that, as craters age, slopes tend to flatten out and soils become more firm. Blocks in rim deposits are the last evidence of cratering to disappear. Assigning soil and slope classes to very new or very old craters is not too difficult, but craters fitting neither of these classifications present a problem.

Investigators at WNRE have other problems, aside from having to work with inexact surface data. Even if CAL's data were precise, surmises covering vehicle-soil interaction would still have to be based in large part, on the meager information supplied by Surveyor, Orbiter and Apollo landings 11 and 12.

TABLE 1

VEHICLE MOBILITY REQUIREMENTS

Slope Climbing Ability	35° slope 25 ft. in length 25° slope 1500 ft. in length
Obstacle Negotiation	3 ft. step -- both wheels 3 ft. bottomless crevasse -- both wheels
Break Angle	35° lateral and longitudinal
Turn Radius	15 ft.

TABLE 2

VEHICLE CHARACTERISTICS

Wheels		
	Number	4 and 6
	Diameter	25" → 40"
	Tread	60" → 120"
	Base	50" → 100"
		(base – tread for 4 wheel configuration)
	Width	4" → 10"
	Steering	Articulated or Ackerman
Weight (earth)		750 → 2500 lbs
Ground Clearance		15" → 40"
Center of Gravity		
	Height	5" → 20" above bottom of chassis
	Lateral and Longitudinal	Center of vehicle

TABLE 3

INFORMATION AVAILABLE FOR LUNAR MOBILITY MAPPING

(Source)	Available Now Primary Information	Processed Information	Information to Become Available Before Operation of Dual Mode LRV
SURVEYOR	Very large scale photography (TV) of small areas.	Geologic interpretations including geologic maps. Contour maps.	Project finished. More processed information of the type already available.to come.
	Data from physical and chemical analysis of Lunar materials conducted on site but remotely controlled from earth.	Interpretations, especially particle size, cohesiveness and *in situ* strength of the soil.	Data analysis not complete.
ORBITER	Small to medium scale photography. Most medium scale photography is near the equator and is of Apollo landing sites.	Geologic interpretations and contour maps.	Project finished. Much more processed information of the type already available will be coming.
APOLLO	Photography which is TV noise-free taken in orbit,landing and landed.	Same as for SURVEYOR and ORBITOR	Project continuing.
	Man-conducted on-site tests of surface properties. Samples of Lunar material returned to earth.	Additions to, refinements and/or correction of previously derived information.	Project continuing
	Operation of manned Rover.	After-the-fact analysis of operation of a real vehicle over a real surface.	Conclusions can be drawn about the suitability of specific design features, and what might have happened with different designs.

Hopefully, there will soon be some results from experiments to be performed with simulated moon soils and prototype rover wheels. However, enough is known to develop the research methodology and demonstrate it, as is done here. In this case, many of the methods may be borrowed and adapted from other sources, such as References 1 and 2.

A. Location of the Study Area

The specific area of the moon to be treated is that contained on the "Map of Selected Geologic Features in Part of Apollo Landing Site 3 (Lunar Orbiter Site II Q-8)", Scale 1:5000, by N.J. Trask, 1969 (Map Ref. 1). The base from which this sheet was made is uncontrolled, therefore only the approximate location in lunar coordinates can be given. These coordinates are:

1°16' to 1°28' West
and
0°18' to 0°26' North

Although this area is comparatively level and does not contain all possible types of lunar features, the system of digital mapping will be applicable to any part of the moon.

Maps at other scales (1:25,000 and 1:100,000) furnished by the sponsor are of the Orbiter 2, P-8 site (Maps Ref. 2 and 3). A map at a scale of 1:1,000 containing the Surveyor VI landing site was also provided. (Map Ref. 4).

B. Data Collection

The data collection plan was designed so that it may be used on any part of the lunar surface. However, it is inevitable that parts of the plan will be more useful on some types of surfaces than on others.

For purposes of data recording and digital mapping, the surface was subdivided by a square grid approximately 50 meters on a side. The reason for not making the grid exactly 50 meters is that near the equator of the moon, the vicinity which will contain all large scale mapping for some time to come, a minute of latitude or longitude has a length of 506 meters and every tenth of a minute, 50.6 meters. Although this first study, which is a demonstration of the method, uses a nominal 50-meter grid, it is anticipated that, in the future, mapping with a larger or smaller grid will be done, and it will be more convenient to superimpose the mapping done with different grid sizes if some of the same intersections of latitude and longitude always serve as grid corners. Digital mapping on the moon differs from that on earth where use can be made of the Universal Transverse Mercator or other rectangular grid.

In actual practice the recording grid to be used is exactly 20 centimeters on a side and always placed on a 1:5000 base in such a way that all odd numbered minutes of latitude and longitude are correctly located at a grid corner. This placement of the grid allows 400 small grid squares to be contained within each square kilometer. The grid is transparent and information on the base is written in a numerical code directly on the grid. After a grid is filled, it is cut into east and west halves and pasted on a specially prepared coding form from which data cards are punched. (Figure 2).

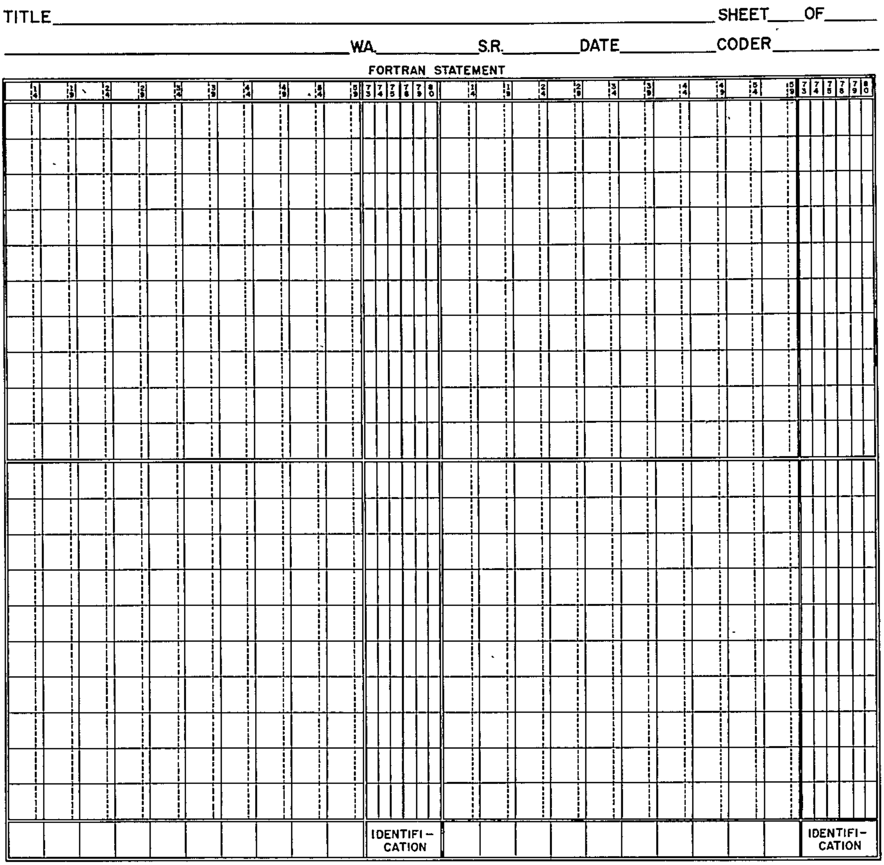
Each 20 cm.x 20 cm. grid furnishes the data for punching 40 cards. Each individual grid square is allowed five spaces on a card. One or two of these spaces are not used and are available for other data when the necessity arises.

Information collected for this study was recorded on computer punch cards. Five data sets were developed as shown in Table 4.

TABLE 4

DATA SETS USED TO DESCRIBE THE LUNAR SURFACE

1. Gross Surface Geometry
2. Regional Statistics of Surface Type
3. Surface Detail
4. Number of Craters 5-50 Meter Diameter
5. Large Crater Configuration


TITLE ______ SHEET ___ OF ___

______ W.A. ______ S.R. ______ DATE ______ CODER ______

FORTRAN STATEMENT

IDENTIFI-CATION

IDENTIFI-CATION

Figure 2   DATA CODING FORM

The following describes, in detail, the legend or plan used in each of the five data sets. It is reiterated that the plan was designed to work anywhere on the moon and the plan for each data set was laid out before any data were gathered.

1. Gross Surface Geometry

This is a recording of elevation information represented by contour lines on Lunar Aeronautical Charts, ORB II 8a and 8c(25), February 1968. With the 20 x 20 grid positioned as described above, data is recorded so that the value appearing in each square is the elevation of the northwest corner of that square. Five digits are allowed for each elevation.

Standard mapping practice calls for the use of contour intervals such that elevations read from the map will be within one-half the contour interval 90% of the time. Anyone who has gained a little experience with digitizing a contour map in this manner soon learns to record elevations along major contours first, wherever they may be. By contrast, an inexperienced operator is likely to start by recording the elevations by rows or columns. Squares lying half-way between two contours are given a value which is the average of the two. All elevations are considered to be the same as a contour or a contour value plus 1/2 the contour interval.

2. Regional Statistics of Surface Types

With the grid positioned on the base as described previously, the following information is recorded in each individual grid square. The corresponding position, starting from the left, within the five spaces allocated to individual grid squares is also presented.

Positions 1 and 2	These positions are left blank.
Position 3.	If the area is a mare, a "0" is recorded. (Other digits are to be assigned other terrain types as they are encountered.)
Position 4.	A "0" is recorded for the most highly cratered surface, a "1" for the next most cratered, "2" for the next, and so on. An "8" is recorded for a lineated mare and a "9" for a mare ridge. (Note: Logarithms of numbers of craters larger than a specified diameter have an inverse linear relationship to logarithms of diameters.)
Position 5.	Length, in kilometers, is recorded for all linear features on which vertical displacement may or may not have occurred . This is done by area subtype unit as designated in position 4. A "1" is recorded for 1 km. per km^2 and less, "2" for 2 and less on up to "8" for 8 and less. A "9" is recorded for all cases greater than 8 km. per km^2.

This data set (2) contains limited useful information. Legends on the smaller scale geological maps claimed that there were four different mare surface regions. Regionalization was based on crater counts and length of lineaments. A count of craters by regions within the area covered by the 1:5000 base map revealed no significant differences in crater density. The reason for taking note of gross regional differences, is to facilitate the identification of analog surfaces. Once it is recognized that two specific areas of the moon surface are analogs, conclusions regarding vehicle performance on one surface are equally applicable to the other surface.

Two of the four regions contained significantly more linear features per unit area than the other two. This may possibly indicate somewhat softer soil, but whether or not the soil is really different will have to be based on updated information. What is important, at this stage, is that these two regions are differentiated in terms of surface configuration; weight commensurate with knowledge available at some time in the future, can be given to these differences.

3. Surface Details

The cards within this data set (3) contain detailed information on the surface characteristics within every 50m x 50m grid square. Not enough precise information is known to code the data in quantitative terms. Rather, the supposed worst hindrance to vehicular movement in each grid square is noted in genetic terms. This allows for future updating as more information becomes available.

The following surface information is recorded in the five positions allotted if it is contained within the specific grid square.

Positions 1 and 2	These positions are left blank.
Position 3.	"0" is recorded if the 50m x 50m grid square has encountered undifferentiated old crater material.
	"1" is recorded if all or part of a young crater with eight or more blocks greater than two meters in diameter is present.
	"2" is recorded if all or part of a young crater surrounded by a halo is present.
	"3" is recorded if all or part of a young crater with a sharp rim crest is present.
	"4" is recorded if all or part of an elongated crater is present.
	"5" is recorded if all or part of a chain crater is present.
	"6" is recorded if all or part of crater cluster is present.
	"7" is recorded if dimple craters are present.
	"8" is recorded if blocks, away from an indicated crater rim, are present.

Position 4

This position is used for indication of elongate features with or without vertical displacement. Starting with the most severe hazard and moving toward the least severe, the elongated features are listed as follows:

"1" is recorded for an old incomplete crater rim

"2" is recorded for the outer limits of new crater material

"3" is recorded for an inferred geologic contact

"4" is recorded for a probable fault

"5" is recorded for a shallow grove, subdued scarp, or a line of highly subdued craters

"6" is recorded for a scarp, which may be a flow front of lava or debris

"7" is recorded for a shallow trough in the rim of a crater

"8" is recorded for abundant resolvable semi-continuous block fields within the rim of a large subdued crater.

Position 5

This position indicates the most likely compass orientation of the "up" side and "down" side, if any, of the linear feature noted in Position 4.

If displacement is possible, imagine an observer standing on the higher land looking off over the lower land. The direction he is looking is recorded as follows:

"1" North
"2" Northeast
"3" East
"4" Southeast
"5" South

"6" Southwest

"7" West

"8" Northwest

Position 3 deals specifically with craters and blocks. If there was nothing unusual to be seen in a square, it was given the designation of undifferentiated old crater material. About 80% of the area fell into this category. The recording of a 1, 2 or 3 indicated a primary crater exhibiting youthful characteristics of blocks on the rim, halos and sharp rims. A primary crater is one believed to have been caused by the impact of a body landing on the moon. Secondary craters indicated by a 3, 4 or 5 are believed to have been caused by rocks thrown out during the formation of a primary crater elsewhere on the moon. The impacting body may be buried in the bottom of the crater, may be lying on one side, or may have rolled some distance from the crater. Such craters may be elongated, and all secondary craters have a tendency to occur in chains or clusters. Dimple craters indicated by a 7 are distinguished by having no rims, a fact which has led to the theory that they are collapse features caused by the sinking of surface material into a void. Blocks away from a crater rim are indicated by an 8. Their most likely place of origin is nearby craters.

It is quite certain that the new craters will, with varying degree, offer hazards of soft soils, steep slopes and blocks. To a lesser degree, the secondary craters will offer some of the same hazards, but are also likely to have more subresolution blocks.

Position 4 of this data set (3) indicates elongate features which may prove to offer hindrances to vehicle movement. It is recognized that on a mare surface such features will be more or less softened because of the few meters of loose surface material. The linear features noted in the study area are not expected to present faces with slopes steeper than vehicle design criteria, but loose soil creeping down slope could present a surface which is not firm. After a study of the photographs and assimilation of technical reading a tentative array is presented as eight different kinds of elongate features in Position 4 of this data set. This array is set up according to overall difficulty of traverse.

4. Small Crater Statistics

This data set (4) deals with numbers of craters per cell with diameters up to 50 meters. Craters as small as 10 meters and thought to present an unusual hazard have already been treated in data set 3, position 3. Heavy concentrations of small subdued craters may provide a rough ride and require slower speeds than an area with fewer craters.

Position 1	This position is left blank.
Positions 2 and 3	These positions contain the number of craters with diameters between 5 and 50 meters.
Positions 4 and 5	These positions contain the number of craters with diameters between 20 and 50 meters.

NOTE: Craters overlapping grid lines are assigned to the cell in which the largest part lies.

Data set 4 deals with numbers of craters 5 to 50 meters in diameter as can be seen above. Although craters down to 10 meters in diameter which had anything unusual about them have been treated separately on data set 3, they have also been counted here. In considering what occurred in the coding of data set 3 it should be remembered that around 80% turned out to be undifferentiated old crater material. Data set 4 points up differences which actually do exist within this largest class of surface. There was wide variation in numbers of craters among the cells, and there was a tendency for squares with few craters to bunch together. The same is true for squares with several craters. There are many times more craters with diameters 5 to 20 meters than there are with diameters from 20 to 50 meters. In fact, only about 12% of the cells contain craters in the latter size range while essentially all cells have craters in the 5 to 50 meter range. This will be illustrated with maps and tables later, but it seems that a rover will seldom be required to cross 20 to 50 meter craters. On the other hand, craters in the 5 to 20 meter range will present a continuous challenge to either being crossed or circumvented.

An Appendix has been prepared to show how the matter of crater size and number may be handled on a probabilistic basis. It is considered as a tentative way of treating the range of variability among the 50 meter squares which have a "0" in position 3 of data set 3 indicating "undifferentiated of crater material".

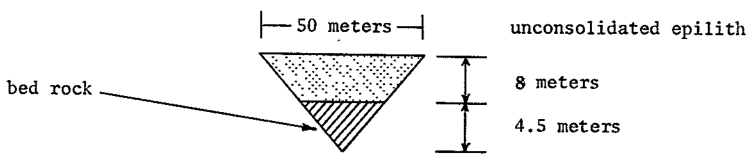
5. Large Crater Configuration:

This data set concerns itself with craters having diameters of more than 50 meters. At some diameter reasonably close to 50 meters on this mare surface, blocks, whether or not discernable, could very well have been thrown out and might lie close to the rim. The reasoning is as follows.

a) The deepest estimate of unconsolidated epilith in this area is 8 meters

b) A depth to diameter ratio of 1:4 is possible in freshly formed craters

The above strongly suggests that cratering could have gone on in the consolidated rock zone of craters with a diameter of 50 meters.



The following data are encoded.

Position 1	This position is left blank
Position 2	"0" is recorded if no crater with a diameter greater than 50 meters is involved "1" is recorded if the square contains part of a crater rim with no discernable blocks

"2" is recorded if the square contains part of a crater rim and 1-4 blocks inclusive

"3" is recorded if the square contains part of a crater rim and 5-8 blocks inclusive

"4" is recorded if the square contains part of a crater rim and 9-12 blocks inclusive

"5" is recorded if the square contains part of a crater rim and 13-16 blocks inclusive

"6" is recorded if the square contains part of a crater rim and 17-24 blocks inclusive

"7" is recorded if the square contains part of a crater rim and 25-28 blocks inclusive

"8" is recorded if the square contains part of a crater rim and over 29 blocks inclusive

NOTE: Under optimum lighting conditions, the smallest block which can be resolved, on the photographs used, has a height or diameter of between 1 and 2 meters.

By far the most common place for blocks is on crater r
realistic to assume that the blocks are located in a line or narrow zone and estimation of mobility should be made in this light.

Position 3

If the grid square contains part of the inside wall of a crater, it is identified by a mean slope class. The following integers are recorded for the average slopes measured over a 100 meter base length listed below:

"1" less than 2°
"2" 2°-4° inclusive
"3" 5°-6° inclusive
"4" 7°-8° inclusive
"5" 9°-10° inclusive
"6" 11°-12° inclusive
"7" 13°-14° inclusive
"8" over 14°

NOTE: An extensive "look-up" table is provided later in the discussion to permit translating mean slope with a hundred meter base to mean slope at other bases and slope frequencies at selected percentile levels.

Position 4. This position contains information which attempts to describe the surface material of crater bottoms in terms of a soil strength gradation. Background mare material is assumed to be the firmest soil. Soil on the oldest crater bottoms appears to approach the undisturbed mare surface. Assumptions of firmness are based on patterning and structuring. Firmness is rated on a scale 1 to 6, where 1 is the firmest.

In general, the more patterning and structuring there is, the less packed and sorted the soil is. In most severe cases, what passes for turbulence of soil movement on the moon brings a few resolvable blocks to the surface and it may be assumed that more subresolution blocks are present.

Position 5. This position contains information which indicates the rim-to-rim diameter of craters.

"1" indicates craters of 50-100 meters inclusive
"2" indicates craters of 101-150 meters inclusive
"3" indicates craters of 151-200 meters inclusive
"4" indicates craters of 201-250 meters inclusive.

"5" indicates craters of 251-300 meters inclusive
"6" indicates craters of 301-350 meters inclusive
"7" indicates craters of 351-400 meters inclusive
"8" indicates craters of 401 meters or greater

Slope Considerations

Slope is a very important consideration to a vehicle traveling over it and has been given a considerable amount of attention. The reliability of slope measurements on the moon is and will remain a matter of uncertainty for some time. The measurement of slope on earth usually takes advantage of contour maps which have been constructed using a geodetic control. A geodetic net based on celestial observation ,satellite observation and actual measurement consists of a hierarchy of points and lines on the earth's surface. Monuments are established giving latitude and longitude and in some cases also elevation above sea level. A good topographic sheet must contain a few such points. The best topographic maps are at very large scales and exist only for areas containing public works involving considerable earth movement. Mass production of topographic maps for the past few decades has depended on stereo models based on aerial photographs. In spite of everything done to assure elevation accuracy, according to the Army Map Service accuracy specifications, only 90% of the points on a map are guaranteed to have an accuracy to within 1/2 contour interval. Of course, there is no comparable geodetic net on the moon. Lunar aeronautical charts drawn from earth, based on telescopic observations, established a kind of geodetic system. Orbiter was supposed to help refine the geodetics, but fell somewhat short of expectations because the shape of the moon turned out to be different than supposed, and the mass of the moon is unequally distributed. Periods of orbit and exact position of the satellite at any given time are not sufficiently predictable to permit the precise location of lunar landmarks from Orbiter photos.

Measurement of slopes on the moon by stereo photo interpretation has two handicaps. Stereo coverage at high resolution scales is scarce, and there is no assurance that both photos of a stereo pair were taken at the same elevation. If slope measurement is done with a pair of photographs taken at unequal elevations, a bias is introduced.

If photographs are taken at low sun angles, slopes may be estimated by length of shadows cast by the object which obstructs the sun. Since the moon has no atmosphere, the contrast between light and shadow is greater on the moon than on the earth. At low sun angles, the amount of light reflected by a surface slanted toward the sun is greater than that reflected by a surface slanting away from the sun. Photoclinometry makes use of this principle in measuring slopes from photographs. In order for the method to be successful, there must be a uniform albedo for the area on which the slopes are measured. Since precise measurement of albedo can be done only on site, an element of uncertainty exists with regard to slope readings from this source. However, there is no reason to suspect that albedo could vary widely over an area on which photoclinometry is being done without evidence appearing on the photographs. A more serious handicap of photo clinometry is that slopes can be measured only in directions from which or toward which the sun is shining. The output of the photoclinometric process is a two dimensional plot of elevations along a straight base line.

NASA has expended a substantial amount of funds on slope determination of the moon surface and it is not within the scope of this contract to evaluate work previously done. It was felt necessary though to summarize very briefly the state of the art. No slope measurements as such were made under this contract. The photography furnished from Langley Research Center were in the AMS Fall Frame Format which were enhanced to bring out planimetric detail. This process resulted in point to point gray scale changes and made them unfit for any sort of photometrics.

At this point it is helpful to refer to some earth based slope-measuring experience. Geographers make use of the concept of average slope. The term "average slope" has come to have many meanings, but one of the early concepts is based on the number of elevation contours crossed by a traverse line drawn on a map. C.K. Wentworth (Ref. 4) in 1930, derived a method which is still used.

Wentworth's formula is:

$$S_m = \frac{I \cdot M}{3361}$$

where: S_m = average slope in tangent form

I = contour interval in feet

M = average number of contours crossed per mile or random traverse

The constant 3361 is a correction factor used to reconcile units and to compensate for the fact that contour lines do not intersect a traverse at 90°, for the most part. Wentworth assumed that in the random placement of a traverse, all values of the intersecting angle (θ) are equally likely to obtain, and he further assumed that the average of various values of the sine of the angle θ for a traverse of considerable length will approach the theoretical mean value of the random value of sin θ.

> "...The mean value of all possible values of sin θ can be determined by integrating the expression
>
> $$y = \sin \theta,$$
>
> determining the value of the definite integral between the limits 0 and $\pi/2$ radians and dividing by $\pi/2$. This gives the value $2/\pi$, which is equal to .6366."

Therefore, if the average number of contour lines crossed per mile of traverse is determined, the average slope is given by the following formula

$$S_m = \frac{I \cdot N}{5280\ (.6366)} = \frac{I \cdot N}{3361}$$

where: I = contour interval in feet

N = number of contours crossed per mile of traverse.

What the Wentworth method says is that the average slope of short segments along a random traverse on a terrestrial map is only 63.66 percent as steep as the average which might be obtained by sampling slope at a large number of random points on the same map. In the latter case, of course, the base line used to compute the slope tangent through a sample point between two contours would be the shortest one which could be drawn. If such a pair of contours are parallel, the base length for measuring slope would be perpendicular to the contours. If such a pair of contours were not parallel the shortest distance through the sample point would be used even though the base line was not perpendicular to either contour.

The distributions of such slopes on maturely dissected terrains, measured in the way just described, often approaches the Gaussian, although each terrain has its own individual pecularities. Arthur N. Strahler (Ref.5), who is well known for his work in slope analysis, has pointed up some of these departures from normality and ascribed reasons. Bimodality is associated with structural benching in the Catskills. Strong skewness of slope with a high mean is associated with youthful erosion on the badlands of Perth Amboy, New Jersey. Near Emporia, Pennsylvania, the slope histogram is "smoothly symmetrical with almost normal form." Structure and youth are thus pinpointed as causes of departure from normality. From what we know about the moon, similar effects should be much less prevalent, and it is permitted, now at least, to hope for normal distribution of slopes on much of the moon's area.

There is a type of slope measurement applied to terrestrial surfaces which, in spite of the different method of measurement, is analogous to photoclinimetric slope measurements on the moon. In that photoclinimetric slope measurements are gathered in straight lines; so are terrestrial slope measurements made on terrain profiles. The terrain profiles are constructed on pieces of cross-section paper laid on straight lines drawn on contour maps. The widest use of such terrain profiles comes in line-of-sight and aircraft terrain avoidance problems. If contour lines are counted and multiplied by the contour interval, and the result divided by the length of traverse, an average cross-section slope tangent is computed. If this result is multiplied by $\frac{\pi}{2}$ the Wentworth average slope results.

Fortunately, normalized distributions (individual slope measurements divided by the mean) of photoclinometric lunar slopes and normalized distributions of valley side slopes from earth measurements are nearly identical. This fact lends a measure of confidence when using earth derived generalizations in lieu of hard information on lunar slopes. Table 5 and Figure 3, originally compiled by Pike (Ref. 3), are repeated here. Lunar slopes were measured in many different apparently homogeneous regions. The mean for each such sample group was then computed and each individual slope measurement divided by the mean. When cumulative distribution curves were plotted and superimposed, they were found to be quite similar and a composite curve was drawn to represent all such cumulative slope distributions. Such a distribution is, of course, strongly skewed toward the low side. Pike so treated slope populations using methods from previous work in which he was involved (Ref. 6). In this case, slopes from ridge crest to valley bottom were measured on samples of terrain from 24 different physiographic regions within the United States. The cumulative curve of terrestrial slopes are seen to be similar to the curve for lunar slopes. Maximum slopes on the terrestrial curve depart further from the mean on the upper end, partly at least, because of the use of tangents in the terrestrial curve and angles in the lunar curve.

Thus, the disadvantages of measuring slopes without respect to the direction of inclination have been stressed. Since measuring slope without respect to direction is the situation which must be lived with on the moon (at least for the time being), it is necessary to stress what can be inferred from a cumulative distribution of slopes measured by photoclinometry.

Even relatively small samples of 50 or more provide rather stable means. Since this is true it is possible to estimate the absolute maximum slope as well as the steepest few percent of all slopes. Knowledge of the surface on the moon in the area of immediate concern will indicate the nature of the sites where such slopes will be found, i.e., new craters, escarpments, etc.

TABLE 5

COMPARISON OF LUNAR AND TERRESTRIAL SLOPE DISTRIBUTION MODELS*

	Model Percentages of Mean Slope Angle	
	Lunar Slope Angle	Terrestrial Slope Angle
Cumulative percentage of cases (%N)	(in tangents at a constant base length of about 0.6m - 1.0m from photo-clinometry of Lunar Orbiter I, II, III and V imagery)	(tangent of valley side slopes comprising linear segments between major slope reversals; variable base length; data from aerial photos of scale 1:20,000)
	Total N is 171,025	Total N is 2876
(100)	(450)	(531)
98	346	369
95	273	269
90	216	210
80	152	150
70	116	115
60	96	91
50	76	71
40	58	57
30	44	45
20	28	34
10	15	22

Correlation coefficient, r, for above data (omitting 100th percentile) is 0.99774.

*(Table 1 in Pike - Ref. 3)

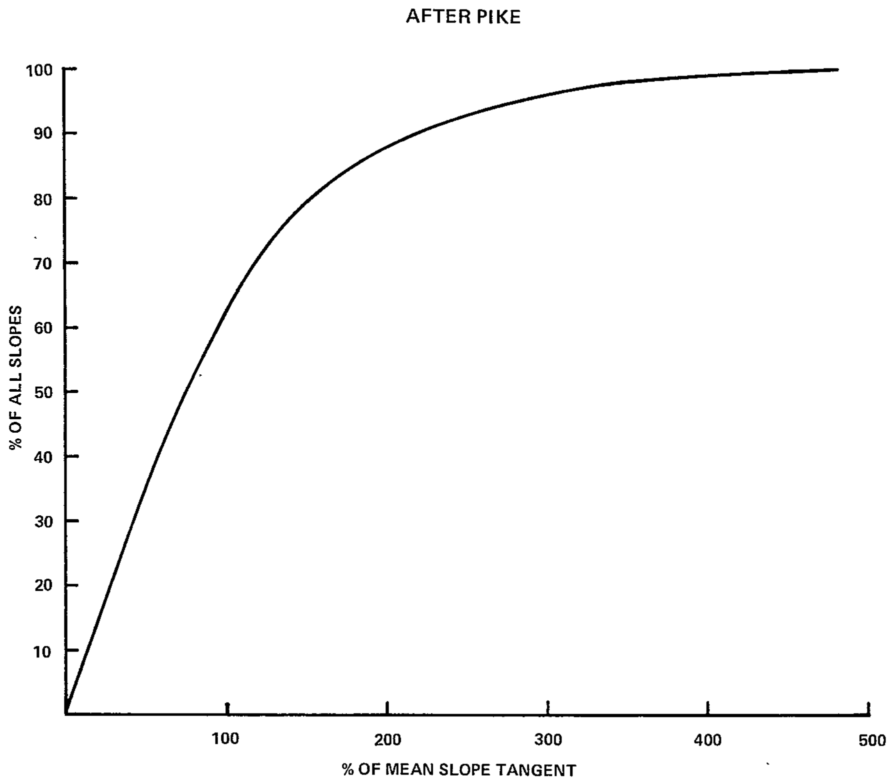


Figure 3 CUMULATIVE DISTRIBUTION OF LUNAR SLOPE VERSUS MEAN SLOPE TANGENT

Crater Frequency

Whatever other processes are active on the moon, such as tectonic folding or faulting and volcanic eruptions, impact cratering is a process which has apparently affected the whole lunar surface. A systematic hierarchy exists among crater sizes and their distribution. Moore (Ref. 3) has presented a visual summary of cumulative distributions of crater numbers in relation to crater diameter (Figure 4). The logarithms of diameters are inversely proportional to numbers of craters per unit area. The primary interest of this study is the cratering on mare surfaces.

Another contribution from Moore (Figure 5) is introduced here concerning the stages a crater goes through from formation to disappearance. Moore gives depth to diameter ratios for fresh, young, mature and old age craters. Fresh craters have depth to diameter ratios within the narrow range of 1/4.4 to 1/4. If a pessimistic view (from the standpoint of lunar rover operation) is taken, the slope from the rim of the crater down a radius to the bottom is 26.6 degrees, i.e. depth/radius = 1/2 = .5 which is the slope tangent of the interior wall (the tangent of 26.6 = .50076). Even though 26.6° is the overall slope from rim to bottom, the actual challenge to a vehicle would be considerably worse. In actuality, chaos prevails. Blocks are strewn about, and the slope goes down toward the bottom in a series of discontinuous steps.

In coding the slopes associated with large craters, note that average slope is specified. This is average slope in the Wentworth sense. The meaning in terms of the fresh crater cited above is that the crater would be considered to have an average slope of approximately 31.8%,i.e., $(\frac{.5}{1.57}) = .318$. Expressed as an angle, this amounts to about 17.6° and an 8 would be punched in position 3 of data set 5.

Increase in Slope With Decrease in Base Length

As the base length over which slope is measured decreases, slopes become steeper. The primary cause is that large numbers of craters, especially small ones exist. Using data furnished by Pike (Ref. 3), Figure 6

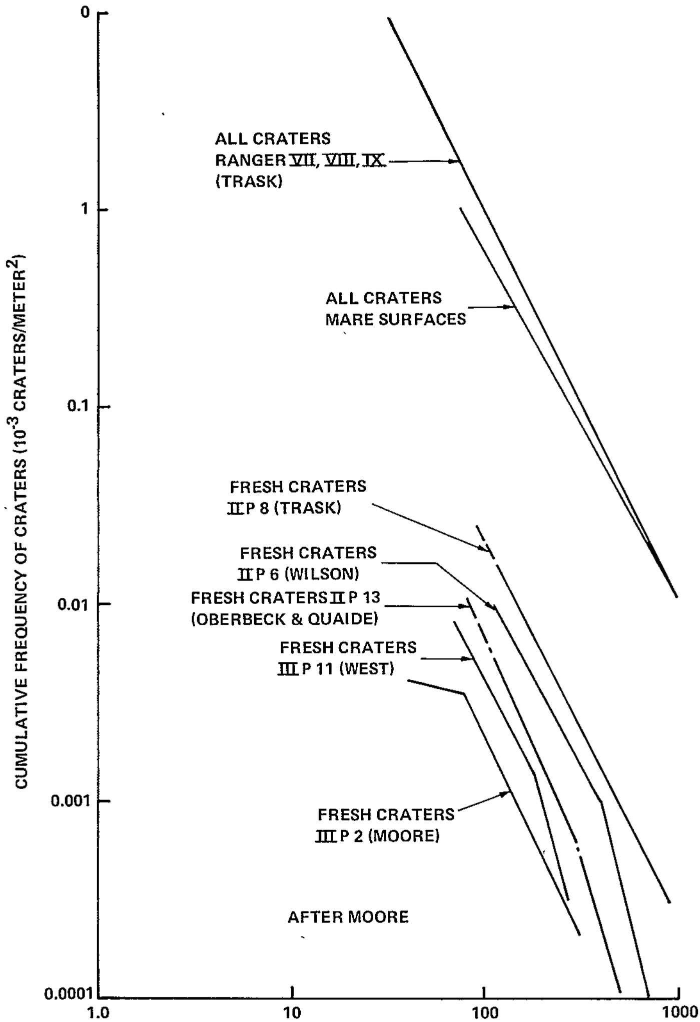


Figure 4 CUMULATIVE FREQUENCY DISTRIBUTION OF CRATERS VERSUS CRATER DIAMETER

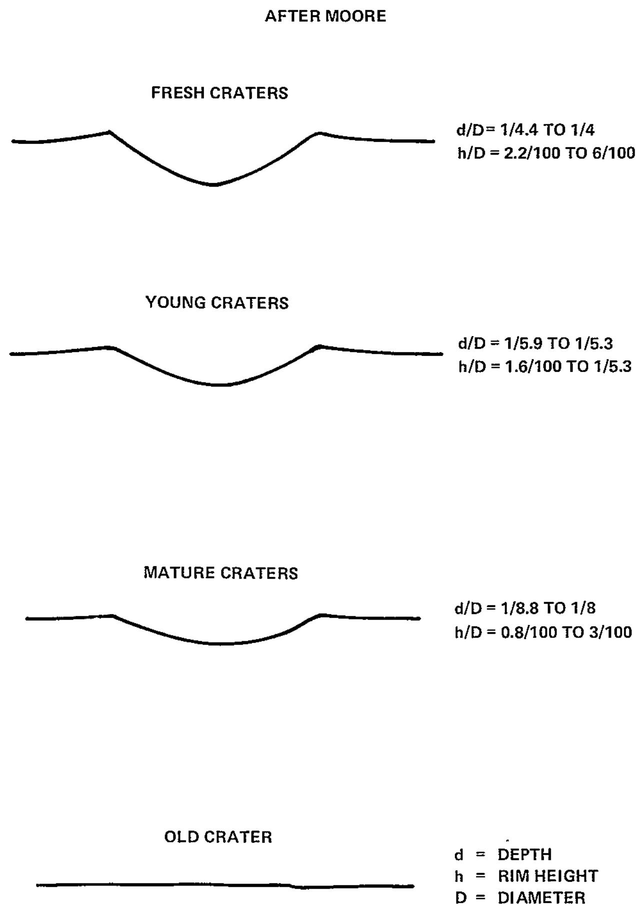
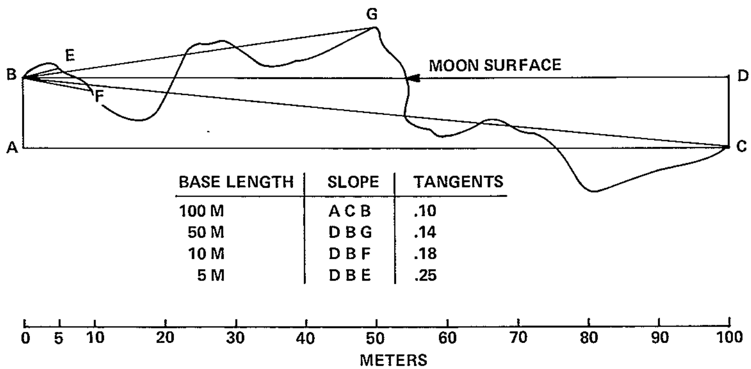


Figure 5 CRATER STAGES - FROM FORMATION TO DISAPPEARANCE



BASE LENGTH	SLOPE	TANGENTS
100 M	A C B	.10
50 M	D B G	.14
10 M	D B F	.18
5 M	D B E	.25

Figure 6 MEAN SLOPE DECREASES WITH BASE LENGTH INCREASE

shows why this is true. B and C mark the end points of a hundred meter base length on a hypothetical piece of moon surface which has an elevation difference of 10 meters, giving a tangent of .10. As the base length shortens to 5 meters, the average slope along the cross-section increases to .25.

Table 6, also based on principles set forth by Pike, gives a more complete statement of this principle. Since the great preponderance of Pike's slope data was concerned with areas of low slope, which would produce almost identical plots of slope distributions regardless of whether angles were stated as angles or tangents, it was felt permissible to make a slight change and assume that Pike's data applied to tangents as well as angle. Table 5 of this report says that the 98th percentile slope along a two-dimensional photoclinometrically derived cross section is 3.46 times the average slope for any given base length. For example, consider the case of such an average slope of 15° along a 100 meter base length. According to Table 6 the average slope along a one-meter base length, which corresponds to the 15° average slope on a 100 meter base, is 39.7°. If the 98th percentile slope is calculated, (i.e.) 3.46 times the average slope (39.7°), the result of 137.36° is clearly impossible. By contrast, if the tangent of 39.7°(.831) is multiplied by 3.46, a tangent of 2.875, equivalent to 70.8 degrees, results. 70.8 degrees is possible and substantially consistent with the facts as far as information on lunar slopes is known.

The Assignment of Slope and Soil Strength Categories

The 1:25,000 map (Map Ref.2) used in this study makes a classification of craters from the youngest to the oldest. Even the youngest of these craters are thought to be too old to be considered really fresh. Table 7 lists descriptive phrases which, when compared with Moore's (Ref. 3) description of craters, permits the making of slope and soil strength estimates of a sort. However, estimates of slope may well vary by as much as one class. Soil strength categories were based on the following. Class 1 is considered to be firm soil such as was found at the Apollo 11 site. The softer soils in the vicinity of the Apollo 12 landings are considered to be Class 2 or 3. Class 6 is used for dimple craters thought

TABLE NO. 6

CHANGES IN AVERAGE SLOPE* WITH CHANGES IN LENGTH OF BASE LINE

Average slope by tangents (first line), and degrees (second line) by base length

Base Length m																		
100	.017	.035	.052	.070	.087	.105	.123	.140	.158	.176	.194	.213	.231	.249	.268	.287	.306	.325
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
50	.024	.049	.073	.098	.122	.147	.172	.197	.222	.247	.272	.298	.323	.349	.375	.401	.428	.455
	1.4	2.8	4.2	5.6	7.0	8.4	9.8	11.1	12.5	13.9	15.2	16.6	17.9	19.2	20.6	21.9	23.2	24.5
10	.031	.063	.094	.126	.157	.189	.221	.253	.285	.317	.350	.383	.416	.449	.482	.516	.550	.585
	1.8	3.6	5.4	7.2	8.9	10.7	12.5	14.2	15.9	17.6	19.3	21.0	22.6	24.2	25.7	27.3	28.8	30.3
7.5	.033	.066	.100	.133	.166	.200	.233	.267	.301	.335	.369	.404	.439	.474	.509	.545	.581	.617
	1.9	3.8	5.7	7.6	9.4	11.3	13.1	14.9	16.8	18.5	20.3	22.0	23.7	25.4	27.0	28.6	30.2	31.7
5	.037	.073	.110	.147	.184	.221	.258	.295	.333	.370	.408	.446	.485	.524	.562	.602	.642	.682
	2.1	4.2	6.3	8.4	10.4	12.5	14.5	16.4	18.4	20.3	22.2	24.0	25.9	27.7	29.3	31.0	32.7	34.3
2.5	.044	.087	.131	.175	.219	.263	.307	.351	.396	.441	.486	.531	.577	.623	.670	.717	.764	.812
	2.5	5.0	7.5	9.9	12.4	14.7	17.1	19.4	21.6	23.8	25.9	28.0	30.0	31.9	33.8	35.6	37.4	39.1
1	.054	.108	.162	.217	.271	.326	.381	.436	.491	.547	.603	.659	.716	.773	.831	.889	.948	1.007
	3.1	6.2	9.2	12.2	15.2	18.1	20.9	23.6	26.2	28.7	31.1	33.4	35.6	37.7	39.7	41.6	43.5	45.2

* A base line of given length is laid down randomly and the difference in elevation of the end points is computed. This number is divided by the length of the base line to obtain a tangent.

TABLE NO. 6 (Continued)

Base Length m																	
100	.344	.364	.384	.404	.424	.445	.466	.488	.509	.532	.554	.577	.601	.625	.649	.675	.700
	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
50	.482	.510	.537	.566	.594	.623	.653	.683	.713	.744	.776	.808	.841	.875	.909	.944	.980
	25.7	27.0	28.2	29.5	30.7	31.9	33.1	34.3	35.3	36.6	37.8	38.9	40.1	41.2	42.3	43.4	44.4
10	.620	.655	.691	.727	.764	.801	.839	.878	.917	.957	.998	1.039	1.082	1.125	1.169	1.214	1.260
	31.8	33.2	34.6	36.0	37.4	38.7	40.0	41.3	42.5	43.7	44.9	46.1	47.3	48.4	49.5	50.5	51.6
7.5	.654	.692	.729	.768	.806	.846	.886	.927	.968	1.010	1.053	1.097	1.142	1.187	1.234	1.282	1.330
	33.2	34.7	36.1	37.6	38.9	40.2	41.5	42.8	44.1	45.3	46.5	47.6	48.8	49.9	60.0	52.1	53.1
5	.723	.764	.806	.848	.891	.935	.979	1.024	1.070	1.117	1.164	1.212	1.262	1.312	1.364	1.416	1.470
	35.9	37.4	38.9	40.3	41.7	43.1	44.4	45.7	46.9	48.2	49.3	50.5	51.6	52.7	53.8	54.8	55.8
2.5	.861	.910	.960	1.010	1.061	1.113	1.166	1.219	1.274	1.329	1.386	1.443	1.502	1.562	1.624	1.686	1.751
	40.7	42.3	43.8	45.3	46.7	48.1	49.4	50.6	51.9	53.1	54.2	55.3	56.3	57.8	58.4	59.3	60.3
1	1.067	1.128	1.189	1.252	1.316	1.380	1.446	1.512	1.580	1.648	1.718	1.790	1.863	1.937	2.013	2.091	2.171
	46.9	48.4	49.9	50.2	52.8	54.1	55.3	56.5	57.7	58.8	59.8	60.8	61.8	62.7	63.6	64.4	65.3

by some to be collapse features. Other places thought to be collapse features would also be given the same classification.

The notation of rim diameter takes care of all craters in 50 meter increments up to 400 meters in diameter. It can be said, in general, that larger craters create more of a hazard than smaller ones. Larger craters have more and larger blocks and they age more slowly. Also a larger area of surface outside the rim of a larger crater becomes loosened upon impact and presumably the soil would take longer to settle back into its original firmness.

Interpreted Quantitative Information On Areas Not Contained Within Large Craters

Except for the designation of undifferentiated old crater material which includes nearly 80% of the total study area, all other features smaller than 50 meters in diameter are considered to be small size counterparts of the larger craters, and are assigned soil strength and short base length slopes classes wherever a judgment could be made (Table 8). Judgments were made by comparisons between the legends of the 1:5000 and 1:25,000 maps. There is not a 1:1 correspondence between the two legends, but it is felt that wherever numerical designations were made on Table 8 they do designate the same kind of surface conditions as the same number used for larger craters (Table 7). When it could be reasoned that there probably was a relationship between a piece of data in sets A to D, but it could not be expressed quantitatively, positive or negative relationships were expressed as described on the legend for Table 9.

TABLE 7

DESCRIPTIVE TERMS USED TO DESCRIBE CRATERS AND CRATER MATERIALS WITH NUMERICAL INDICATORS OF AVERAGE SLOPE AND SOIL STRENGTH

Map Designation	Descriptive Terms	Slope Class	Soil Class
Cc 6	o Well developed bright rays o Sub-resolution blocks probably abundant o Rim crests very sharp o Interiors sharply structured o Well developed terraces	7	5
6	o Intensely bright rays	7	5
Cc 5	o Weak to well-developed rays o A few blocks are resolveable in the rim o Moderately abundant blocks in large craters o Interiors structured, rough and terraced o Crater rim crests sharp	7	5
5	o Slightly subdued rim crests o Interiors smooth o Few details visible	6	5
Cc 4	o Distinctly higher density of 5-20 m craters o Moderately subdued o Slightly elongate North-South o Mòre sub-resolution blocks than surroundings	5	4
Cc 4	o Rim deposits as bright or slightly brighter than surroundings o Flat or terraced floors slightly subdued o Larger craters have a few resolvable blocks in rims o Rim crests slightly subdued	5	4
4	o Rim deposits only as bright as surroundings o Floors flat or terraced o Rim crests moderately subdued	3	2
Cc 3	o Rim deposits only as bright as surroundings o A few blocks are present in rim-deposits of larger craters o Crater interior subdued o Central mounds occur in some craters o Rim crests distinctly raised, but moderately to strongly subdued.	3	2

TABLE 7 (Cont.)

		Slope Class	Soil Class
Cc 3	o Strongly subdued rims o May have double rim crests o Floors flat or strongly convex upward	3	2
Cc 2	o Strongly subdued rims o A few blocks present in rim deposits of large craters o Sparse blocks and patterned ground occur in wall material of large craters o Interiors range from cup shaped to flat	3	2
2	o Shaped like a shallow bowl o Rim crests raised but rounded	2	2
Cc 1	o Shape of shallow bowl o Patterned ground in wall material o Rim crest raised but slightly rounded or o Strongly subdued rim crests, but relatively deep interiors o No blocks present in rim o Blocks present near tops of walls o Crests raised, but slightly rounded	2	2
1	o Gentle depressions	2	2
Cc d	o Subdued rim crest o Walls steeper at base than near crest (convex upward)	3	6
Ec h	o Northeast trending shallow depressions with crater-like enlargements	2	2
Ec i Ec ci	o Irregularily shaped craters occurring singly and in clusters o Highly subdued form - pan shaped gentle depressions o Slightly raised rim o Patterned ground abundant both in wall and exterior rim deposits o Resolvable blocks present in wall.	2	2
Ecc	o Terrain covered with shallow craters o Craters contiguous o Intercrater areas gently undulating and relatively smooth o Density of small craters same as mare material o Patterned ground on slopes o No resolvable blocks.	2	1

TABLE 7 (Cont)

		Slope Class	Soil Class
Ec	o Range from gentle depressions to the shape of shallow bowls with larger craters pan shaped o Distinctive break in slope at rim crest o Blocks and patterned ground in wall material	2	1

TABLE 8

QUANTITATIVE MEANING OF INFORMATION ON

DATA SETS 1-4

Data Set	Space	Punch	Soil Strength**	Blocks**	Short Base Length Slopes **
1			0	0	0
2	3	0	1	0	+
2	4	1	1	0	+
2	4	8	2	0	+
2	4	9	2	0	+
2	5	>1	-	0	+
3	3	0	1	0	-
3	3	1	5	++	7
3	3	2	5	++	7
3	3	3	4	++	6
3	3	4	2	++	4
3	3	5	2	++	4
3	3	6	2	++	4
3	3	7	6	0	3
3	3	8	0	+	0
3	4	1	-	+	0
3	4	2	-	++	0
3	4	3	-	0	+
3	4	4	-	+	+
3	4	5	0	+	0
3	4	6	+	+	++
3	4	7	-	+	0
3	4	8	-	0	+
3	5	1-7	0	0	+
4	2-3	*	-	+	++
4	4-5	*	-	+	++

* The higher the number, the greater liklihood of correlation

** Meaning of these symbols are found on the next page.

LEGEND

MEANING OF SYMBOLS

0 = No constant relationship. Example: Card 3, space 3, punch 8, blocks away from indicated crater rim; there is no reason to expect that there will be any relationship to soil strength or short base length slopes.

An inverse relationship. Example: Card 3, space 3, punch 0, undifferentiated old crater material. Short base length slopes are likely to be lower than in most areas having the same long base length slope.

+ = Positive relationship. Example: Card 3, space 4, punch 1, old crater rim. Crater rims are one of the most likely place to find blocks.

Double -'s and +'s indicate that the relationship is especially strong.

Integers 1-8 referring to soil strength and short base length slopes have the same meaning as noted for data set 5.

III. SELECTED MAPS OF SURFACE CHARACTERISTICS

This section illustrates the types of maps which can be prepared from the lunar surface data collected during this study. For the most part the maps represent "echo" printouts of the data, and isolate individual factors, or sets of factors, by showing their spatial distribution over the study area.

The maps presented here, with the exception of Figure 11, have been distorted by the printer mechanism, which prints 10 characters per inch horizontally and 6 characters per inch vertically. This gives the impression that the maps have been stretched in the north-south direction and squeezed in the east-west direction. Each individual symbol on the following maps represents data collected in a 50M x 50M grid square. Question marks on the map indicate locations where no data were available. On one of the photographs some of the emulsion had been scraped off the negative and in other cases dense shadows blacked out detail on the insides of craters. Some sun-facing sides of craters showed up on the photographs as all white with no surface detail distinguishable.

A paragraph preceding each map provides an explanation of the map's purpose. A legend, or a page number within this report on which a key to the symbols is found, is noted both in the explanation and on the map.

Figure 7. Elevation Map

This map is a printout of elevations supplied by a 25 meter contour map. All elevations are over 7100 meters above datum. Elevations recorded are confined to those equal to a contour interval or a contour interval plus one half. Symbols and the elevations they represent are: + < 7125, = < 7138, @ < 7150, - < 7163, A < 7175, and blank ≥ 7175. The over-all gross slope on mare surfaces is low with the steepest slope on this map equalling about 4%. This type of information has a place, though, in an over-all system of mobility mapping. (Additional information can be found on page 10.)

LEGEND

		METERS
*	ELEVATION BELOW	7100
/		7113
+		7125
=		7138
@		7150
-		7163
A		7175
BLANK	ELEVATION ≥	7175

(MORE INFORMATION CAN BE FOUND ON PAGE 10)

Figure 7 MAP ELEVATION

Figure 8. Surface Detail

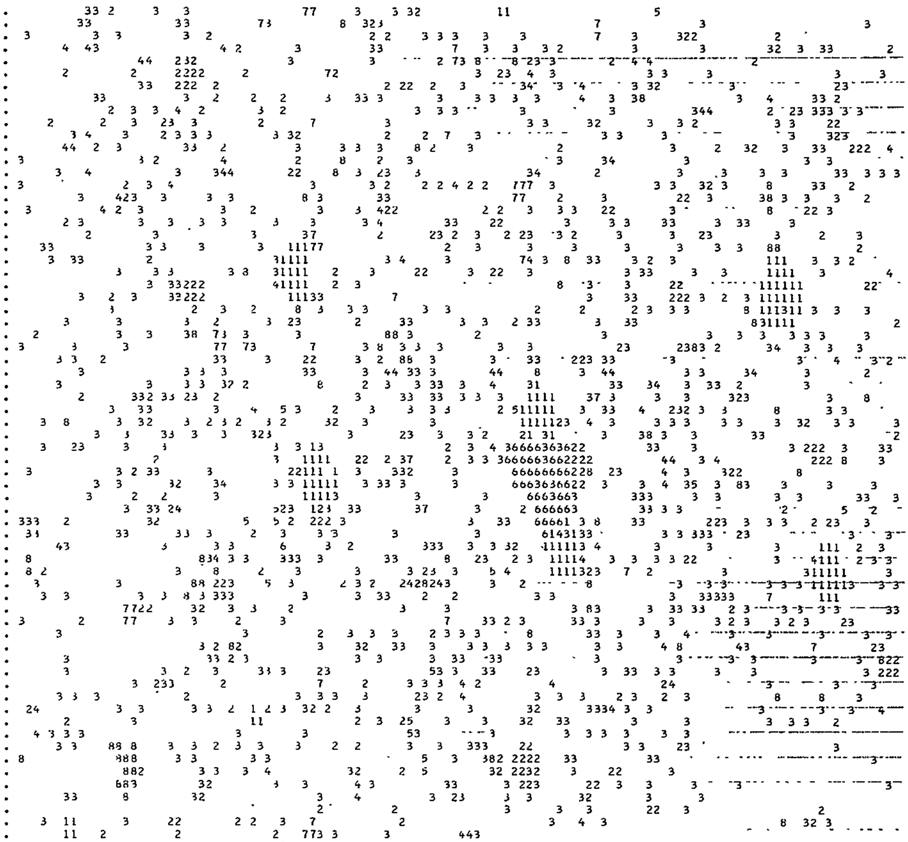
This map is a printout of the data contained in position 3 of data set 3. The blank areas are those given the designation "Undifferentiated Old Crater Material". This type of surface makes up about 80 percent of the area and is the only one which the lunar rover would have to be able to negotiate. If all other surfaces were capable of denying access to a rover (which they are not) travel from north to south or east to west would not have to deviate much from a straight line.

The key to the symbols in Figure 8 is found on page 12.

Figure 9. Elongate Features

This printout is of data in position 4 of data set 3. It includes all features mapped by Trask (1969) at a scale of 1:5000 of the study area. Some features may not be visible or are barely visible on the surface. It is in the nature of aerial photography to reveal some features which cannot be seen on site. Many of these features have their origin in the bedrock below a few meters of loose surface material which serves to mask any sharpness of detail that may have been present at one time. It is thought that these features, with the possible exception of a "4" which designates a probable fault, would not stop a vehicle. The most which any other of these features are thought capable of doing is to cause a detour of a few 100 meters.

The key to the symbols in Figure 9 is found on page 13.



(THE KEY TO THIS MAP IS FOUND ON PAGE 12)

Figure 8 MAPS OF SURFACE DETAILS (CRATER MATERIAL)

(THE KEY TO THIS MAP IS FOUND ON PAGE 13)

Figure 9 MAP OF SURFACE DETAILS (ELONGATE FEATURES)

Figure 10. Soil Strength-Large Craters

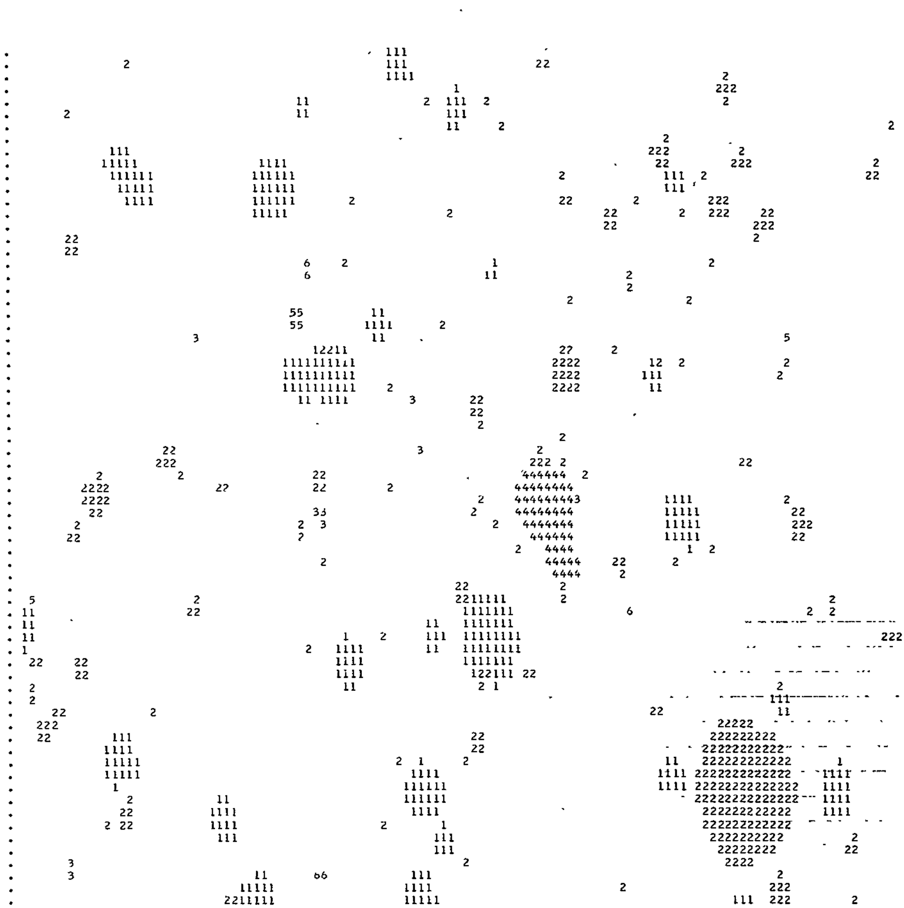
As explained in the body of the text, soil is described in grades from 1, the firmest, to 6, the weakest. There is very little information available that permits the relating of soil strength to tone or pattern on photographs. The only rational explanation for cratering effect on loose soil is lack of density related to recency and severity of disturbance. This, in effect, adjudges size and newness of craters as containing the most reasons for having other than a firm soil. Class 6 is reserved for places where a slightly cohesive soil may overlie a void.

The key to the symbols in Figure 10 is found on page 18.

Figure 11. Large Crater Diameters

This map was made on a plotter and has correct north-south and east-west relationships. The large crater in the south-east corner has a diameter of about 700 meters, and is given the code 8 which represents craters over 400 meters across. Although perfect circles can not result from fitting squares together, the chance distribution of cell boundaries and crater rims makes small craters depart even further from circularity.

The key to the symbols in Figure 11 is found on page 18.



(INFORMATION RELATED TO THIS MAP CAN BE FOUND ON PAGE 18)

Figure 10 MAP OF SOIL STRENGTH AS IT CONCERNS LARGE CRATERS

(THE KEY CAN BE FOUND ON PAGE 18)

Figure 11 MAP OF LARGE CRATER DIAMETERS

Figure 12. More Than 4 Small Craters 50M x 50M Square.

A crater is assigned to the square in which its center lies, and small craters are defined as having diameters of 5 to 50m. Although a count of craters by whole regions on the photographs did not support a hypothesis of 4 regions, there is indeed a difference in their concentration from place to place. On this map considerably more craters appear in the south than elsewhere. If 5 or more craters per cell would be enough to stop or severely hinder an LRV of specific design, there are places where such a vehicle could not go.

Figure 13. Over 20% of Area in Craters

With 20% of the area of a cell in craters, some cells would have a distribution of craters such that it could not be crossed without crossing one of the craters contained. Depending on the design of a vehicle this may or may not be a hindrance.

Since craters are recorded by number and two size classes for each cell, and the need exists to evaluate an array of vehicle designs, a large amount of meaningful mapping can be done concerning craters alone. The value, 20% of the area of a cell in craters, is an arbitrary choice. Other cut-off points could have been used.

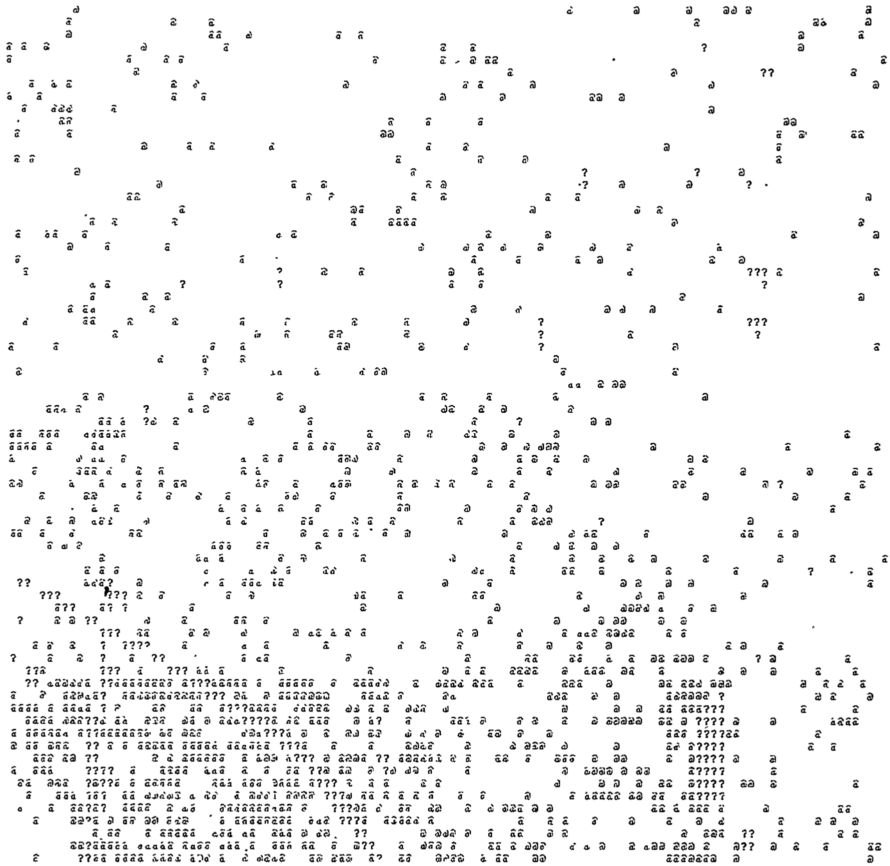
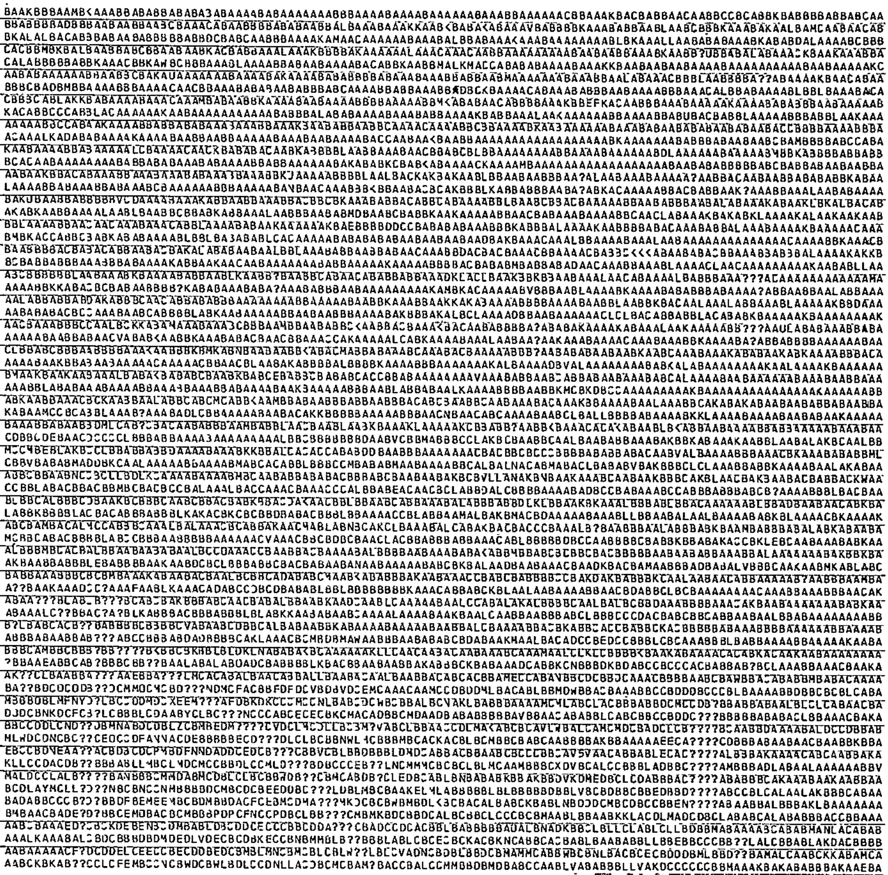


Figure 12 MAP OF 50M X 50M CELLS HAVING MORE THAN 4 SMALL CRATERS

Figure 13 MAP OF 50M X 50M CELLS HAVING 20% OF THEIR AREA CRATERED

Figure 14. Crater Factor Families

This map, though difficult to read, presents all of the information that was gathered on small craters. They key to the symbols presented appears in Table 9. Twenty-four factor families are represented on the table with associated symbols. The number of factor families were derived by association between the number of craters contained within two general classes of crater sizes. For example, while craters may not have been present in the diameter class of 20-50 meters, some craters may have been present in the class of 5-50 meters. The frequencies of the latter class were divided into six intervals; 0-2, 3-4, etc. to 9-10, and greater than 10. Given that either 0, 1, 2, or 3 craters are present in the 20-50 meter diameter class, and one of the six intervals in the 5-50 meter diameter class is present, a total of 24 frequency combinations, or factor families can be derived. The number of times a factor family was encountered, and the percent of total, is listed. When these factor families were generated for the study area, the following map was created. It is noted that squares with few craters predominate. The classes A and B, 0 to 2 and 3 to 4 craters respectively, include 72.5% of all sample cells (41.2 + 31.3 = 72.5).



(THE KEY TO THIS MAP IS FOUND IN TABLE 9)

Figure 14 MAP OF CRATER FACTOR FAMILIES

Table 9

LEGEND FOR CRATER FACTOR FAMILY MAP

FAM	SYM	NUMBER	PERCENT	5-50	20-50	TOTAL
1	A	3353.	41.2	0.- 2.	0.	3353.
2	B	2550.	31.3	3.- 4.	0.	5903.
3	C	793.	9.7	5.- 6.	0.	6696.
4	D	229.	2.8	7.- 8.	0.	6925.
5	E	58.	0.7	9.- 10.	0.	6983.
6	F	15.	0.2	GT 10.	0.	6998.
11	K	328.	4.0	0.- 2.	1.	7326.
12	L	371.	4.6	3.- 4.	1.	7697.
13	M	152.	1.9	5.- 6.	1.	7849.
14	N	46.	0.6	7.- 8.	1.	7895.
15	O	10.	0.1	9.- 10.	1.	7905.
16	P	4.	0.0	GT 10.	1.	7909.
21	U	6.	0.1	0.- 2.	2.	7915.
22	V	31.	0.4	3.- 4.	2.	7946.
23	W	15.	0.2	5.- 6.	2.	7961.
24	X	2.	0.0	7.- 8.	2.	7963.
25	Y	3.	0.0	9.- 10.	2.	7966.
26	Z	1.	0.0	GT 10.	2.	7967.
31	4	0.	0.0	0.- 2.	3.	7967.
32	5	1.	0.0	3.- 4.	3.	7968.
33	6	0.	0.0	5.- 6.	3.	7968.
34	7	0.	0.0	7.- 8.	3.	7968.
35	8	0.	0.0	9.- 10.	3.	7968.
36	9	0.	0.0	GT 10.	3.	7968.

IV. RESULTS OF THE STUDY AND RECOMMENDATIONS FOR FUTURE WORK

1. Results

A. The major result of the work performed under this contract is the methodology devised to produce digital maps of lunar surface parameters considered to be hazardous to lunar vehicle travel. The methodology devised is one whereby numerical values have been assigned to sets of qualitative and quantitative information pertaining to the lunar surface. The plan has the flexibility to allow the numerical values to be refined or updated as more precise knowledge about the surface becomes available.

B. Tentative surface descriptor values have been set up and have been used to produce lunar surface parameter maps of the mare sample region.

C. The highland regions of the lunar surface have not been studied under this contract, but the methodology developed can be used to treat that surface in the same manner as was done on the mare surface.

D. A cursory examination of the terrain, and also the vehicle characteristics supplied by the sponsor, indicates that the lunar rover should have little difficulty negotiating the surface presented in the test site. Although this study applies to a specific site, it is felt that the data gathered is representative of the mare surface in general.

E. The Appendix of this report is a discussion of crater statistics based on the occurrence of craters on the moon. Results, in this case, are the techniques which, tentatively, can be used to categorize configurations of craters according to the degree of difficulty they present to vehicle movement. These techniques, in turn, can be used to aid in the generation of probabilistic maps of the mare surfaces. Initial considerations suggest that perhaps many configurations of craters might be grouped into categories of relative difficulty and that, indeed, craters at vehicle scale may pose little threat regardless of their configurations.

F. The methodology developed in this study was designed principally to contribute to vehicle design considerations. However, the methods of mapping can be extended to other parameters more suited to operational consideration, such as line-of-sight, traverse energy expended, etc.

2. Recommendations

It is suggested that other mare landing sites, and also selected highland sites be studied, using the methods presented in this report, to gain a knowledge of the mobility hazards which may be encountered.

The maps presented in this report were produced using computer output devices having a limited set of symbols which can be used to display mapped information. It is recommended that an effort be initiated to investigate map design criteria which would enhance and augment the computer output, and also aid in the comprehension of the final maps.

It is recommended that procedures and methods be designed to refine and update digital lunar mobility parameter data as it becomes available. Photometric techniques, which have not been used in the conduct of this study, can be employed to refine the assignment of slope classes, soil strengths and other parameters.

Additional work is needed to refine the probability model dealing with the occurrence of craters. Concern should also be given to the development of models capable of handling large craters, blocks, elongate features, etc. on a probabilistic basis. The vehicle recognizes these features not as craters, but as slopes or discrete obstacles.

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APPENDIX

STATISTICS OF CRATER OCCURRENCE ON THE MOON

By

H. T. McAdams

I. INTRODUCTION

In the process of formulating "factor families" to describe the moon's surface relative to vehicle mobility, one must address the occurrence of craters both from the standpoint of their size and their areal distribution. Average areal densities (number of craters per unit area) and average crater size can be obtained by counting and sizing craters in the region of interest. However, such averages may mean little in assessing vehicle capability in view of the variability of both crater density and size when an area commensurate with the size of the vehicle is considered. It is appropriate, therefore, to examine the sampling distribution of areal density and size for the region when a small sample size is considered. In view of the decision to use a 50-meter grid for digitizing the moon terrain, it may be revealing to see what variation in crater parameters ensues at that sampling scale.

2. SOME ESSENTIAL STATISTICAL CONCEPTS

A simplified approach to the problem under consideration is provided by the binomial distribution, the multinomial distribution, the Poisson distribution, and certain concepts of probability generating functions.

A binomial distribution results when the sample space consists of two mutually exclusive events, such as success and failure, occurrence and non-occurrence, etc. Thus, whether a crater occurs or does not occur in a sample area fits this general requirement.

The probability function for a binomial distribution is

$$f(x) = \binom{n}{x} p^x q^{n-x} \quad (1)$$

where n = number of trials

x = number of successes (occurrences)

p = probability of success on a single trial

q = 1 - p = probability of failure on a single trial

Note that (1) is just the general term in an expansion of the binomial $(q+p)^n$. For,

$$(q+p)^n = q^n + n p q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \cdots + p^n \quad (2)$$

Equation (2) suggests a <u>probability generating function,</u> written

$$P(s) = (q+ps)^n = q^n + n p q^{n-1} s + \binom{n}{2} p^2 q^{n-2} s^2 + \cdots + p^n s^n \quad (3)$$

In this function, the symbol "s" is an "operator", a "dummy variable," or a "bookkeeping device" designed to have the following properties. Whatever the exponent of s in a given term, that term is associated with the corresponding number of successes; the numerical coefficient denotes the probability of realizing that many successes. For example, suppose we toss a coin three times and call heads a "success". Then n = 3 and we have

$$(q+ps)^3 = q^3 + 3q^2ps + 3qp^2s^2 + p^3s^3 \quad (4)$$

Thus $q^3 = (0.5)^3 = 0.125$ = probability of no heads

$3q^2p = 3(0.5)^2\ (0.5) = 0.375$ = probability of one head

$3q\ p^2 = 3(0.5)\ (0.5)^2 = 0.375$ = probability of two heads

$p^3 = (0.5)^3 = 0.125$ probability of three heads.

In addition to its bookkeeping property, the operator s can also be used operationally in computing such quantities as the mean, variance, and other moments of the distribution.

If n is quite large and p is quite small, the binomial distribution can often be approximated reasonably well by the Poisson distribution

$$f(x) = \frac{m^x}{x!} e^{-m} \tag{5}$$

This distribution has probability generating function

$$\mathcal{P}(s) = e^{-m+ms} \tag{6}$$

Writing

$$\mathcal{P}(s) = e^{-m} \cdot e^{ms} \tag{7}$$

and expanding e^{ms} in series, one obtains

$$\begin{aligned} \mathcal{P}(s) &= e^{-m}\left(1 + ms + \frac{m^2s^2}{2!} + \frac{m^3s^3}{3!} + \cdots\right) \\ &= e^{-m} + me^{-m}s + \frac{m^2}{2!}e^{-m}s^2 + \cdots \end{aligned} \tag{8}$$

and it is seen that the coefficients of s^0, s^1, s^2. s^3,... "generate" the probabilities associated with 0, 1, 2, 3, ... successes.

A logical generalization of a binomial distribution is a multinomial distribution. Instead of there being assumed two mutually exclusive events (such as success or failure), there is assumed k mutually exclusive events $E_1, E_2, \ldots, E_k$ Let there be n trials and let

p_1 denote the probability of occurrence of E_1
p_2 " " " E_2
:
p_k " " " E_k

Then the probability function is

$$f(x_1, x_2, \ldots, x_k) = \binom{n}{x_1 \cdots x_k} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$$

$$= \frac{n!}{x_1! x_2! \cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k} \quad (9)$$

where $f(x_1, x_2, \ldots, x_k)$ denotes the probability of obtaining a sequence in which there are x_1 occurrences of the event E_1, x_2 occurrences of the event $E_2, \ldots, x_k$ occurrences of the event E . Note that

$$x_1 + x_2 + \cdots + x_k = n$$

Now it is easy to see that (9) is simply the general term of an expansion of a multinomial, and this fact suggests a form of generating function in which we use operators $s_1, s_2, \ldots, s_k$ to keep track of the number of occurrences of $E_1, E_2, \ldots, E_k$ respectively. For example, a term containing $s_1^2 \; s_2 \; s_3^3$ would denote the probability of obtaining a sequence containing 2 occurrences of E_1, one occurrence of E_2, and 3 occurrences of E_3. (Note: subscript of s identifies the type of event, exponent identifies the number of occurrences.) The generating function then becomes:

$$\Pi(s_1, s_2, \ldots, s_k) = (p_1 s_1 + p_2 s_2 + \cdots + p_k s_k)^n \quad (10)$$

For example, suppose that we consider 3 mutually exclusive events E_1, E_2, E_3, where

E_1 has probability $p_1 = 0.2$ of occurring
E_2 has probability $p_2 = 0.3$ of occurring
E_3 has probability $p_3 = 0.5$ of occurring

and suppose that we let n = 2. The only possible combinations are (E_1, E_1), (E_2, E_2), (E_3, E_3), (E_1, E_2), (E_1, E_3), (E_2, E_3) and these occur according to the multinomial distribution.

Then

$$\mathbb{P}(s_1, s_2, s_3) = (p_1 s_1 + p_2 s_2 + p_3 s_3)^2$$

$$= p_1^2 s_1^2 + p_2^2 s_2^2 + p_3^2 s_3^2 + 2 p_1 p_2 s_1 s_2 + 2 p_1 p_3 s_1 s_3 + 2 p_2 p_3 s_2 s_3 \qquad (11)$$

Thus

$P(E_1, E_1) = p_1^2 = 0.04$

$p(E_2, E_2) = p_2^2 = 0.09$

$p(E_3, E_3) = p_3^2 = 0.25$

$p(E_1, E_2) = 2\ p_1 p_2 = 0.12$

$p(E_1, E_3) - 2\ p_1 p_3 = 0.20$

$p(E_2, E_3) = 2\ p_2 p_3 = 0.30$

A similar analysis is possible for more complicated cases.

Just as a binomial distribution can, under certain circumstances, be approximated by a Poisson distribution, so can a multinomial distribution be approximated by multiple Poisson distribution

$$f(x_1, x_2, \ldots, x_k) = \frac{m_1^{x_1}}{x_1!} e^{-m_1} \cdot \frac{m_2^{x_2}}{x_2!} e^{-m_2} \cdot \ \ldots \ \cdot \frac{m_k^{x_k}}{x_k!} e^{-m_k}$$

$$= e^{-(m_1 + m_2 + \cdots + m_k)} \frac{m_1^{x_1} m_2^{x_2} \cdots m_k^{x_k}}{x_1!\ x_2! \cdots x_k!} \qquad (12)$$

and a probability generating function analogous to (6) can be obtained. Thus:

$$\mathbb{P}(s_1, s_2, \ldots, s_k) = (e^{-m_1} \cdot e^{m_1 s_1})(e^{-m_2} \cdot e^{m_2 s_2}) \cdots (e^{-m_k} e^{m_k s_k})$$

$$= e^{-(m_1 + m_2 + \cdots + m_k) + (m_1 s_1 + m_2 s_2 + \cdots + m_k s_k)} \qquad (13)$$

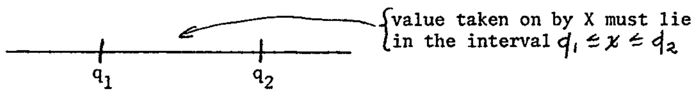
In the sequel, we expect to identify the events E_1, E_2,...E with crater size-class intervals. For example, E_1 might denote that a crater is in the interval 0-1 meter in diameter, E_2 that it is in the interval 1-2 meters, and so on.

For analyzing the number of craters occurring in a sampling area and subsequently the sizes of the craters given that a certain number occur, we will find it convenient to use the notion of conditional probability. Let X and Y denote two random variables. Their joint probability function can be written

$$f_{X,Y}(x,y) = f_X(x/y) \cdot f_Y(y).$$

where $f_X(x/y)$ is the conditional distribution of X given that X = x and $f_Y(y)$ is the unconditional distribution of Y. For example, X might denote the random variable associated with the number of craters occurring in a sample area and Y might denote the assignment of these craters to size classes, given that a certain number of craters occur. The joint distribution, then, specifies the probability associated with a certain number of craters having specific sizes.

One more preliminary remains to be considered. Suppose we have a random variable X which can assume values on the real number continuum between two values q_1 and q_2, as shown below.



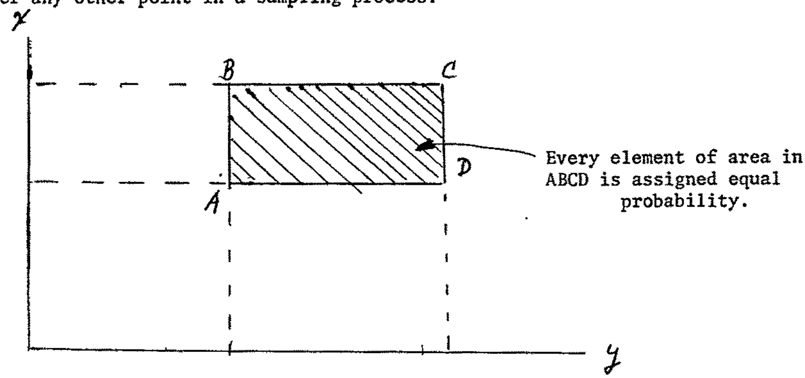
If the value of X has "no preference" for any particular value in the interval, it is said to have a uniform or rectangular probability distribution. The probability density function is

$$f(x) = \begin{cases} \dfrac{1}{q_2 - q_1}, & q_1 \leq x \leq q_2 \\ 0 \text{ otherwise} \end{cases} \qquad (14)$$

The notion of a uniform distribution can be extended to the bivariate (or multivariate) case. For example, consider a point P in the (x,y) plane and let the coordinates of P be assumed to be random variables. If X must take on a value in the interval $q_1 \leq x \leq q_2$ and Y must take on a value in the interval $\omega_1 \leq y \leq \omega_2$, and if X and Y are statistically independent, then

$$f(x,y) = \begin{cases} \dfrac{1}{(q_2-q_1)(\omega_2-\omega_1)}, & q_1 \leq x \leq q_2,\ \omega_1 \leq y \leq \omega_2 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

The equation (15) defines a situation often referred to indiscriminately as "random", in the sense that no point in the rectangle ABCD is preferred over any other point in a sampling process.



3. ELEMENTARY MODEL FOR CRATER DISTRIBUTION

We shall proceed now to use the previously presented concepts in the development of a simple description of the statistics of craters. The craters will be idealized as circles. In this way attention can be confined to two aspects of craters only: (1) the spatial distribution of their locations (as given by the location of their centers), and (2) the magnitudes of their sizes (as given by their diameters).

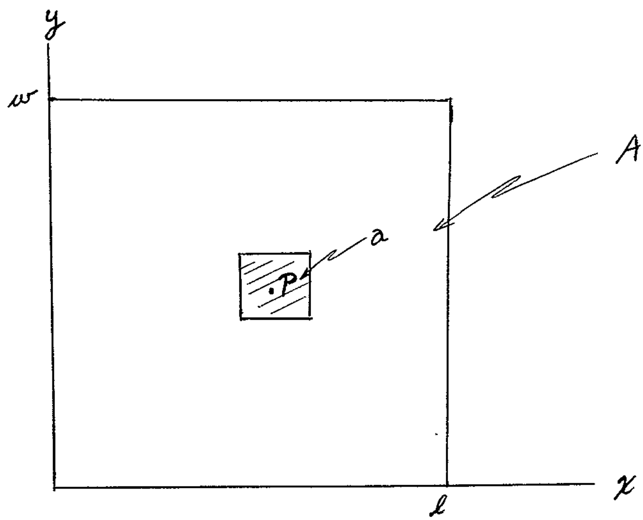


Figure 1

Consider an area $A = \ell w$ and suppose that a point P is assigned randomly to the area in such a way that the x,y coordinates of the point have a bivariate rectangular probability density function

$$f(x, y) = \begin{cases} \frac{1}{\ell w} & 0 \le x \le \ell, \quad 0 \le y \le w \\ 0 & \text{otherwise} \end{cases}$$

Consider a subarea a. The probability that the point falls in the subarea is

$$p = a/A$$

If N points are assigned to A, the number of points falling in a obeys a binomial distribution and has probability generating function

$$P(s) = (q + p\,s)^N$$

where $q = 1 - p = 1 - (a/A) = \frac{A - a}{A}$

and s is an operator.

Now, assume that the point P is the center of a circle of radius r, so that assigning the point P at random is equivalent to assigning a circle of area πr^2 to the area.

The probability that the <u>center</u> of the circle falls in a is exactly

$$p = a/A$$

as before and the number of circles <u>centered</u> in a has probability generating function

$$P(s) = (q + p\,s)^N$$

as before.

What is the probability that the <u>entire</u> circle falls in a? The answer is evident from Figure 2.

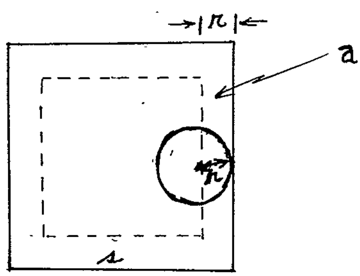


Figure 2

If $\mathfrak{a} = s^2$, a square, then the area of the aperture is reduced to $(s - 2r)^2$, and

$$p = \frac{(s - 2r)^2}{A} , \quad s > 2r .$$

$$= 0 \text{ otherwise}$$

and the same probability generating function applies but with p defined differently.

What is the probability that some part of the circle falls in $\mathfrak{a}$? The answer is evident from Figure 3.

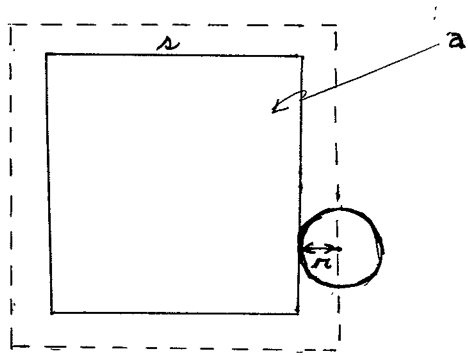


Figure 3.

Here the aperture area is augmented to $(s + 2r)^2$ and $p = \frac{(s+2r)^2}{A}$ with the same comments about the probability generating function.

If one considers only the centering case, this corresponds to the case in which at least half of the circle is captured in the aperture, so long as the circle is small relative to the aperture and if "corner effects" are ignored. At any rate, the centering case provides a logical starting point for introducing the distribution of circle radii.

The probability generating function discussed above gives the distribution of the number of centers falling in a given aperture. However, the radius associated with a given center has a probability distribution. For first approximation, we shall consider the distribution of r to be independent of the distribution of center locations, but in the actual "moon-forming" process there would probably be some dependence because a "big crater" forming subsequently to "little craters" might obliterate the smaller ones.

Let $f(r)$ denote the probability density distribution of r and $F(r)$ its cumulative probability function. How can this be introduced?

Cumulative numerical counts of craters, according to existing reports, assert that the cumulative number of craters is a log-log plot. That is

$$\log m = A + b \log r$$

or $$m(r) = a\, r^{b} \quad \text{where } A = \log a.$$

Now if the area under study has a total of N craters counting all sizes, the c.d.f. would be

$$F(r) = \frac{m(r)}{N} = \frac{1}{N}\, a\, r^{b}$$

and $$f(r) = \left(1/N\right) a\, b\, r^{b-1}$$

Thus the p.d.f. for r is also of the form of a power function of r.

A simple approach, perhaps, is to assume a multinomial distribution in which the craters are divided among k size intervals. Some characteristic length, such as the midrange size for each interval, can be used to identify the various sizes of craters. Thus the problem is idealized as one in which a crater, when it occurs, assumes one of the dimensions $r_1, r_2, \ldots, r_k$. From the size distributions of r (or diameter), one can compute corresponding probabilities $p_1, p_2, \ldots, p_k$.

Let us assume that, in a sampling cell of area a there occurs n crater centers. These n centers can be identified with many sequences of radii drawn from the set $(r_1, r_2, \ldots, r_k)$. If one applies the multinomial distribution, one can postulate a conditional distribution

$$f_1(x_1, x_2, \ldots, x_k / n) = \frac{n!}{x_1!\, x_2! \cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k}$$

with probability generating function

$$\mathcal{P}(s_1, s_2, \ldots, s_k) = (p_1 s_1 + p_2 s_2 + \cdots + p_k s_k)^n$$

This distribution defines how the radii are assigned to classes, given that exactly n centers fell within the sampling interval or grid square. Note, however, that n can be considered to be binomially distributed with unconditional distribution

$$f_2(n) = \binom{N}{n} p^n q^{N-n} = \frac{N!}{n!\,(N-n)!} p^n q^{N-n}$$

where N is the total number of craters in the area A, of which the sampling area a is a subset. Then

$$f(x_1, x_2, \cdots x_k; n) = f_1(x_1, x_2, \cdots, x_k | n) \cdot f_2(n)$$

$$= \frac{n!}{x_1! x_2! \cdots x_k!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k} \cdot \frac{N!}{n!(N-n)!} p^n q^{N-n}$$

$$= \frac{N!}{x_1! x_2! \cdots x_k! (N-n)!} p_1^{x_1} p_2^{x_2} \cdots p_k^{x_k} p^n q^{N-n}$$

This formula gives the probability that exactly n craters will be centered in the sampling aperture and that these n craters will constitute a certain size mix: x_1 of size r_1, x_2 of size $r_2, \ldots,$ x_k of size r_k. A generating function could be provided to generate the various types of groupings or configurations possible together with their probability of occurrence. If it is assumed that some of these configurations would be critical (No-Go) and others non-critical (Go), a rationale could be developed for assessing probability of encountering difficulty within a sampling interval or grid square, given certain parameters associated with the larger area within which the sampling is performed.

Suppose we are considering a region considered to be "homogeneous" with regard to the nature and occurrence of its craters. Suppose this region has an area of the order of 1000 grid squares. Then A = 1000, and p = a/A = 1/1000 = 0.001. Also, if the density is in the vicinity of 25 craters per grid square, the total number of craters centered in the area A would be N = 25 x 1000, or 25,000. Under these conditions, one might argue that the Poisson distribution applies for $f_2(n)$, so that

$$f_2(n) = \frac{m^n}{n!} \cdot e^{-m}$$

where m = Np = (25,000) (0.001) = 25. Then the Poisson approximation gives the following:

n	cumulative probabilities	n	cumulative probabilities
9	.000	26	.629
10	.001	27	.700
11	.001	28	.763
12	.003	29	.818
13	.006	30	.863
14	.012	31	.900
15	.022	32	.929
16	.038	33	.950
17	.060	34	.966
18	.092	35	.978
19	.134	36	.985
20	.185	37	.991
21	.247	38	.994
22	.318	39	.997
23	.394	40	.998
24	.473	41	.999
25	.553	42	.999
		43	1.000

As a typical deduction from the table, it can be seen that the probability that exactly 25 craters would be centered in the sample grid is

$$0.553 - 0.473 = 0.08$$

These 25 craters would then have a (conditional) diameter distribution given by an appropriate multinomial distribution.

At this point, reference is made to Fig. III-15 of JPL Technical Report 32-1262. As an example of how the above theory might be applied, consider the following. Log crater diameter = 0 corresponds approximately to 10^{11} craters per 10^6 km^2 or per 10^{12} m^2. Thus there are 10^{-1} craters per square meter and, since a 50-meter square has an area of 2500 m^2, there would be expected 250 craters of size 1 meter or greater in the grid square. If we choose the ordinate at 10^{10} craters per 10^6 km^2, however, this would correspond to our case of 25 craters per grid square. This point corresponds to log crater diameter $\approx$ 0.5 or craters greater than $10^{0.5}$ or approximately $\sqrt{10} = 3.16$ meters. It might be argued that craters 3 meters or less in diameter, regardless of their numbers, would not pose a vehicle threat. (Actually we might choose the cut-off point differently; we choose it as we have here to make it match the postulated problem). Other points are as follows:

Log diameter(m)	diameter(m)	Cumulative number of Craters per 10^6 km^2	per grid square
0	1	10^{11}	250
0.5	3.2	10^{10}	25
1.0	10	10^9	2.5
1.5	32	10^8	0.25
2.0	100	10^7	0.025

Now we can also argue that--say-- craters larger than 32 m in diameter, we must consider not as craters but as slope obstacles, etc. Then our sample space consists of craters $>$ 3.2 m in diameter, and we could set up size intervals as follows.

Size Interval	No.per grid square	Probability of Occurring
3.2m - 10m.	25- 2.5 = 22.5	$\frac{22.5}{25} = 0.90$
10 m - 32m	2.5-0.25 = 2.25	$\frac{2.25}{25} = 0.09$
$>$32 m.	0.25	$\frac{0.25}{25} = 0.01$

Then $(0.9\, s_1 + 0.09\, s_2 + 0.01\, s_3)^n$

is the generating function which tells the probability of the mix of craters in these three sizes for a given n, or the probabilities of interest could be computed directly from equation (9):

$$f(x_1, x_2, x_3) = \frac{n!}{x_1!\, x_2!\, x_3!}\, p_1^{x_1}\, p_2^{x_2}\, p_3^{x_3}$$

For example, suppose n = 25 and we want to know the probability associated with $x_1 = 20$, $x_2 = 4$, $x_3 = 1$. Then

$$f(20,4,1) = \frac{25!}{20!\,4!\,1!}(0.9)^{20}(0.09)^{4}(0.01)^{1}$$

$$\approx (12075)(0.185)(0.000066)(0.01)$$

$$\approx 0.0015$$

Therefore, of all the times when exactly 25 craters occur, these will be distributed as above (20 in class 1, 4 in class 2, 1 in class 3) with a probability of 0.0015. Since the unconditional probability that n = 25 is 0.08, the joint probability is (0.08)(0.0015) $\approx$ 0.00012.

Quite clearly, any particular configuration will have relatively low probability of occurrence. However, it should be possible to classify configurations into categories of relative difficulty and to compute, by unions of sets, probabilities associated with these categories. Possibly one might arrange these configurations in a monotone sequence of difficulty and perform a statistical assessment accordingly.

The above example is given only for illustration. Certainly the chosen crater cut-off point is not appropriate for the size of vehicle likely to be considered. When the cut-off point is more realistically chosen, it will be necessary to perform more extensive calculations. These, however, could probably be managed by computer without difficulty. In particular, it should not be necessary to compute probabilities for all configurations, but only for those considered a threat to mobility.

4. SUMMARY

An elementary discussion of crater statistics has been presented, a crude probability model developed and an illustrative example shown. Suggestions as to how these developments might be implemented are only tentative. Initial considerations suggest that perhaps many configurations of craters might be grouped into categories of relative difficulty and that, indeed, craters at vehicle scale may pose little threat regardless of their configurations. If further analysis should validate this presumption, then concern can be focused on those large craters, lineal features, etc. which the vehicle recognizes not as a crater but as a slope challenge or discrete obstacle.

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