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SOLAR FLOW PARAMETERS

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George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

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ABSTRACT

The solar plasma is assumed to be a fully ionized mixture of two species. Nine important nondimensional parameters are obtained from the fundamental equations of plasma flow. Some thermodynamic properties of the solar plasma are presented in tables which are based on a solar model computed by R. L. Sears.

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LIST OF SYMBOLS

Symbol	
a	Velocity of sound
a_r	Radiation density constant
α	Friction coefficient
$\vec{B}$	Magnetic induction
C_V	Specific heat at constant pressure
C	Specific heat at constant volume
c	Velocity of light
$\vec{D}$	Electric induction
D_d	Coefficient of diffusion
D_r	Diffusion coefficient of radiation
d	Molecular diameter
δ	Thickness of the viscous boundary layer
$\vec{E}$	Electric field strength
e	Electron charge
e_m	Total energy of the plasma per unit mass
e_1	Total energy of the electron gas per unit volume

LIST OF SYMBOLS (Continued)

Symbol	
ϵ	Dielectric constant
$\epsilon(AB)$	Energy generation of process (AB), energy m^{-1}
ϵ_1	Total energy rate of the electron gas per unit volume
$\vec{F}_e$	Electromagnetic force
$\vec{F}_g$	Gravitational force
F_r	Froude number
ϕ	Characteristic velocity
G	Gravitation constant
γ	Ratio of specific heats
$\vec{H}$	Magnetic field strength
$\vec{J}$	Electric current density
$\vec{K}$	Flux of matter
k	Coefficient of heat conductivity
κ	Mass absorption coefficient
L	Luminosity of sphere of radius R
L_r	Luminosity of sphere of radius r
ℓ	Length
ℓ_0	Characteristic length

LIST OF SYMBOLS (Continued)

Symbol	
ℓ_D	Debye length
ℓ_f	Mean free path
ℓ_l	Larmor radius
ℓ_{le}	Larmor radius for the electron
ℓ_{lp}	Larmor radius for the proton
M	Mass within sphere of radius R
M_r	Mass within sphere of radius r
m	Mean mass of a particle in the plasma
m_e	Mass of the electron
m_p	Mass of the proton
μ	Apparent molecular weight of the plasma
μ_e	Magnetic permeability
μ_v	Coefficient of viscosity
n	Number of density of the plasma
n_1	Number density of the electron gas
ν_h	Magnetic viscosity
ω_c	Cyclotron angular frequency
ω_{ce}	Cyclotron frequency of the electron

LIST OF SYMBOLS (Continued)

Symbol	
ω_{cp}	Cyclotron frequency of the proton
ω_e	Electrical decay angular frequency
ω_f	Friction angular frequency
ω_p	Plasma angular frequency
P_r	Prandtl number
p	Pressure of the solar plasma
p_0	Average pressure of the sun
p_r	Radiation pressure
p_1	Pressure of the electron gas
p_t	Total pressure, $p + p_r$
Π_i	Nondimensional parameters
$\vec{Q}$	Heat flux in the plasma
$\vec{Q}_1$	Energy flux due to heat in the electron gas
q	Concentration
R	Radius of sphere
R_a	Boltzmann's constant
R_e	Reynolds number
R_c	Relativity parameter

LIST OF SYMBOLS (Continued)

Symbol	
R_E	Electrical field parameter
R_h	Magnetic pressure number
R_σ	Magnetic Reynolds number
R_p	Gas constant of the plasma, $\frac{R_a}{m}$
r	Radius of sphere, $0 \leq r \leq R$
ρ	Density of the solar plasma
ρ_0	Characteristic density
ρ_e	Electric charge density
S	Collision cross section
S_c	Schmidt number
σ	Electrical conductivity
T	Temperature of the solar plasma
T_0	Average temperature of the sun
T_1	Temperature of the electron gas
t	Time
t_0	Characteristic time
τ	Stress tensor of the plasma
τ_1	Stress tensor of the electron gas

LIST OF SYMBOLS (Concluded)

Symbol	
$\vec{u}$	Flow velocity of the ions
$\vec{u}_1$	Flow velocity of the electrons
$< u >$	Average thermal velocity
u_s	Flow velocity at the boundary layer
V_h	Alfvèn velocity
X	Fractional abundance by mass of hydrogen
Y	Fractional abundance by mass of helium
Z	Fractional abundance by mass of heavy elements

SOLAR FLOW PARAMETERS

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SUMMARY

The solar plasma is assumed to be a fully ionized mixture of two species. Nine important nondimensional parameters are obtained from the fundamental equations of plasma flow. Some thermodynamic properties of the solar plasma are presented in tables which are based on a solar model computed by R. L. Sears.

I. INTRODUCTION

The solar plasma is assumed to be a fully ionized mixture of two species. Nondimensional parameters are obtained from the following fundamental equations which describe the plasma flow:

$$p = R_p \rho T$$

$$p_1 = R_a n_1 T_1$$

$$\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{u}) = 0$$

$$\rho \frac{d\vec{u}}{dt} = - \nabla p_t + \nabla \tau + \vec{F}_e + \vec{F}_g$$

$$\rho \frac{de_m}{dt} = - \nabla (\vec{u} p_t) + \nabla (\vec{u} \tau) + \nabla \vec{Q}$$

$$\frac{\partial e_1}{\partial t} + \nabla [e_1 \vec{u}_1 - (\vec{u}_1 \tau_1) - \vec{Q}_1] = \epsilon_1$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \epsilon \vec{E}}{\partial t}$$

$$\nabla \times \vec{E} = - \frac{\partial \mu_e \vec{H}}{\partial t}$$

$$\frac{\partial \rho_e}{\partial t} + \nabla \cdot \vec{J} = 0$$

$$\vec{J} = \sigma (\vec{E} + \mu_e \vec{u} \times \vec{H}) \quad \rho_e \vec{u}$$

The solar model adopted for the computations is shown in Table I. It was published in this form by B. Stroemgren and was computed by R. L. Sears by an evolutionary model sequence for $M = M_{\odot}$, covering the age range between 0 and 4.5×10^9 years. The opacities used are those of Keller and Meyerott, and the energy generation is taken according to Reeves.

It is intended to present the properties of the solar plasma in tables. Because the computed values were obtained by extreme extrapolations, all values given after Table I need corroboration.

II. TYPICAL PHYSICAL QUANTITIES

The following quantities, which characterize solar plasma dynamics, can be introduced into the preceding plasma equations.

1. Length (ℓ_0), which characterizes the dimension of the flow field. All distances may be expressed in terms of ℓ_0. For the sun, the length

$$\ell_0 = 10^7 \text{ m}$$

may be used.

TABLE I. SOLAR MODEL FOR AGE 4.5×10^9 YEARS

$\frac{M_r}{M}$	r	T	ρ	L_r	ϵ(p-p)	ϵ(CN)	κ	X
	m	°K	kg m	watts	watts kg^{-1}	watts kg^{-1}	$m^2 kg^{-1}$	
	10^9	10^6	10^3	10^{26}	10^{-4}	10^{-4}	10^{-1}	
0.0	0.00	15.7	158	0.00	15.9	1.6	1.09	0.36
0.05	0.06	13.8	103	1.30	10.0	0.13	1.32	0.52
0.1	0.08	12.8	83	2.13	6.8	0.023	1.48	0.58
0.2	0.10	11.3	59	3.09	3.3	0.001	1.78	0.65
0.3	0.13	10.1	43	3.55	1.6	0.000	2.09	0.68
0.4	0.15	9.0	31.5	3.77	0.7	0.000	2.42	0.69
0.5	0.17	8.1	22.4	3.86	0.3	0.000	2.79	0.70
0.6	0.20	7.1	15.2	3.90	0.06	0.000	3.2	0.70
0.7	0.23	6.2	9.4	3.90	0.02	0.000	3.8	0.71
0.8	0.26	5.1	5.0	3.90	0.00	0.000	4.5	0.71
0.9	0.32	3.9	1.84	3.90	0.00	0.000	6.0	0.71
0.95	0.38	3.0	0.74	3.90	0.00	0.000	7.4	0.71
0.99	0.48	1.73	0.117	3.90	0.00	0.000	9.6	0.71
0.99955	0.62	0.66	0.0063	3.90	0.00	0.000	—	0.71

2. Characteristic velocity (ϕ). Velocities of 6×10^2 m sec^{-1} occur in sunspots, and of 9×10^2 m sec^{-1} in the solar granulation. The velocity of sound may be used as characteristic velocity at 7.1×10^6 °K:

$$\phi = 4 \cdot 10^5 \text{ m sec}^{-1} \quad .$$

3. Time (t_0), which characterizes the interval considered. The travel time for sound waves may be taken as value t_0:

$$t_0 = \frac{\ell_0}{\phi} = 25 \text{ sec} \quad .$$

All phenomena of a time scale much smaller than 10^{-2} m sec^{-1} should be neglected.

4. Number density of the solar plasma n. It is obtained from the equation

$$\rho = m \cdot n$$

Substituting the densities given in Table I, the values in Table II are obtained.

5. Apparent molecular weight of the plasma, μ, which is obtained with sufficient accuracy from the equation

$$\mu = \frac{1}{2X + \frac{3}{4} Y + \frac{1}{2} Z}$$

yielding

$$\mu = 0.614$$

$$m = 1.01912 \cdot 10^{-27} \text{ kg} \quad .$$

6. Temperature of the plasma. The typical temperature T_0 may differ greatly for various flow problems. It may be taken as the average temperature which is with sufficient accuracy:

$$T_0 = 7.1 \times 10^6 \,{}^\circ K$$

7. The typical pressure p_0 may also differ greatly for various flow problems. Using the values of Table I, the pressures obtained by the ideal gas equation are given in Table II. As characteristic pressure may be taken the value for T_0

$$p_0 = 1.46 \times 10^{15} \text{ kg m}^{-1} \text{ sec}^{-2} \;.$$

8. Boltzmann's constant R_a, which characterizes the gas constant of the plasma:

$$R_a = 1.38046 \times 10^{-23} \text{ joule deg}^{-1}$$

$$R_p = \frac{R_a}{m} = 1.35456 \times 10^4 \text{ joule kg}^{-1} \text{ deg}^{-1}$$

9. Coefficient of viscosity μ_v, which characterizes the viscous stress of the plasma. The values of μ_v, ℓ_f, $<u>$, and S, as functions of the solar temperature, are derived from the following expressions of the kinetic theory:

$$\mu_v = \frac{5\pi}{32} \; n \; m \; \ell_p \; < u >$$

$$\ell_f = \frac{1}{\sqrt{2}\, n\, S}$$

$$< u > = \sqrt{\frac{8 R_a T}{\pi\, m}}$$

$$S = 0.8 \left(\frac{e^2}{4\Pi\, \epsilon\, R_a T} \right)^2 \ell n \left(\frac{12\pi\, \epsilon R_a T}{e^2} \sqrt{\frac{\epsilon R_a T}{n_1 e^2}} \right) \;.$$

TABLE II. THERMAL PARAMETERS OF THE SOLAR PLASMA

T	n	p	S	ℓ_f	$\langle u \rangle$	μ_v	D_d	k	D_r	σ	α
°K	m^{-3}	kg m^{-1} sec^{-2}	m^2	m	m sec^{-1}	kg m^{-1} sec^{-1}	m^2 sec^{-1}	kg m sec^{-3} deg	kg m sec^{-3} deg	ohm^{-1} m^{-1}	kg m^{-3} sec^{-1}
10^6	10^{30}	10^{13}	10^{-23}	10^{-9}	10^5		10^{-3}	10^5	10^{10}	10^6	10^{16}
15.7	155.04	6720	0.40622	1.12275	7.35901	64.08259	0.48841	32.55695	6.79411	13.28822	4642.8
13.8	101.07	3850	0.52818	1.32459	6.89936	46.20688	0.54014	23.47195	5.84445	10.90077	2405.2
12.8	81.44	2878	0.61327	1.41578	6.64468	38.32696	0.55565	19.45753	5.16187	9.74870	1746.2
11.3	57.89	1806	0.78403	1.55794	6.24322	28.16805	0.57490	14.31022	4.15413	8.11516	1059.9
10.1	42.19	1176	0.97916	1.71168	5.90242	21.32349	0.59706	10.83144	3.46631	6.87375	664.64
9.0	30.91	763	1.22832	1.86241	5.57174	16.04577	0.61330	8.15055	2.89147	5.80419	422.49
8.1	21.98	492	1.55242	2.07228	5.25308	11.96978	0.66088	6.24555	2.57111	4.93787	251.12
7.1	14.915	292	1.97754	2.39738	4.94878	8.85220	0.70103	4.49556	2.22484	4.05904	140.67
6.2	9.224	158	2.61487	2.93168	4.62450	6.25598	0.80130	3.17783	2.01735	3.28500	66.475
5.1	4.906	69	3.88406	3.71084	4.19425	3.81987	0.91969	1.94607	1.78256	2.43845	25.335
3.9	1.805	19.44	6.78564	5.77321	3.66777	1.91202	1.25124	0.97132	1.62458	1.59616	5.2389
3.0	0.726	6.02	11.62163	8.38072	3.21684	0.97914	1.59304	0.49735	1.49079	1.06261	1.2732
1.73	0.1148	0.548	35.66943	17.26820	2.44283	0.24226	2.49238	0.12303	1.39379	0.45590	0.07420
0.66	0.00618	0.001126	245.8719	46.535[illegible]	1.50883	0.021708	4.14902	0.01103	—	0.10708	0.000915

They are given in Table II.

10. Nonelectric forces, such as the gravitational force. The constant of gravitation equals

$$G = 6.668 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ sec}^{-2} \quad .$$

The surface gravity of the sun is

$$g = 2.7398 \times 10^2 \text{ m sec}^{-2} \quad .$$

11. Specific heat at constant volume C_v, which characterizes the internal energy of the plasma. For a monatomic gas:

$$C_v = \frac{3}{2} \cdot \frac{R_a}{m} \quad .$$

After substitution of the previous values, this equation yields:

$$C_v = 2.03184 \cdot 10^4 \text{ joule kg}^{-1} \text{ deg}^{-1} \quad .$$

12. Coefficient of diffusion D_d. Due to the temperature and pressure differences, the molecules in the solar plasma diffuse from regions of high to regions of low concentration, according to the equation

$$\vec{K} = - D_d \nabla q \ .$$

In the kinetic theory, the value of D_d is given by the expression:

$$D_d = 1.204 \frac{\mu_v}{\rho} \quad .$$

The values are contained in Table II.

13. Coefficient of heat conductivity k, which characterizes the heat conduction of the solar plasma. To a monatomic gas with smooth spherically symmetrical molecules applies the relation:

$$k = \frac{75\pi}{128} n R_a \ell_f < u > = \frac{5}{2} \mu_v C_v \quad .$$

Substitution of the previously given values yields the results contained in Table II.

14. Radiation constant a_r, which characterizes both the radiation pressure and the radiation energy. Its value is

$$a_r = 7.5641 \times 10^{-16} \text{ joule m}^{-3} \text{ kg}^{-4} \quad .$$

15. Diffusion coefficient of radiation D_r, which characterizes the radiation flux. It is given by the relation:

$$D_r = \frac{4}{3} \frac{a_r c T^3}{\kappa \rho} \quad .$$

After substitution of the previously given values, the results contained in Table II are obtained.

16. Magnetic induction B. The general magnetic induction near the sun's pole equals

$$B = 1 \text{ gauss} = 10^{-4} \text{ weber m}^{-2} \quad .$$

17. Electric field strength E. The typical value of the electric field strength is rather arbitrary; for the solar plasma it may be taken as 0.

18. The electron charge e. Its value is

$$e = 1.60207 \times 10^{-19} \text{ coulomb} \quad .$$

19. Magnetic permeability μ_e. The characteristic value for the solar plasma may be taken as the value in free space.

$$\mu_e = 4\pi \cdot 10^{-7} \text{ kg m coul}^{-2}$$

20. Dielectric constant ϵ. The characteristic value for the solar plasma may be taken as the one in free space.

$$\epsilon = \frac{1}{36\pi} \cdot 10^{-9} \text{ kg}^{-1} \text{ m}^{-3} \text{ sec}^2 \text{ coul}^2$$

21. Electrical conductivity σ. The conductivity of a completely ionized plasma with a single-charged ions is determined by the relations:

$$\sigma = \frac{0.591\,(\pi\epsilon)^2\,(R_a T)^{3/2}}{m_e^{1/2}\; e^2\; \ell n \left(\frac{h}{b}\right)}$$

$$h = \sqrt{\frac{\epsilon R_a T}{n_1 e^2}}$$

$$b = \frac{e^2}{12\pi \epsilon R_a T} \quad .$$

Substituting the previously given values, the results contained in Table II are obtained.

22. Friction coefficient α, which characterizes the effect of the viscous forces on the electromagnetic force. It is given by the relation

$$\alpha = \frac{n^2 e^2}{\sigma} \quad .$$

Substituting the previously given values, the results contained in Table II are obtained.

These 22 quantities may be used to characterize the solar plasma. The values in Table II, which belong to $T_0 = 7.1 \times 10^6\ ^\circ K$, may be selected as characteristic values.

III. CHARACTERISTIC VELOCITIES, FREQUENCIES, AND LENGTHS

Some velocities, frequencies, and lengths may be derived from the 22 characteristic quantities which will be useful in obtaining nondimensional parameters later.

1. Velocity of light:

$$c = \frac{1}{\sqrt{\epsilon \mu_e}} = 2.99793 \cdot 10^8 \text{ m sec}^{-1} \quad .$$

2. Velocity of sound:

$$a = \sqrt{\gamma R_p T} \quad .$$

Substituting $\gamma = \frac{5}{3}$ and the previously given values, the results contained in Table III are obtained.

TABLE III. CHARACTERISTIC VELOCITIES, FREQUENCIES, AND LENGTHS

T	a	V_h	ω_p	ω_e	ω_f	ℓ_D	ℓ_{Ie}	ℓ_{Ip}
°K	m sec^{-1}	m sec^{-1}	sec^{-1}	sec^{-1}	sec^{-1}	m	m	m
10^6	10^5	10^{-3}	10^{15}	10^{17}	10^{13}	10^{-11}	10^{-2}	
15.7	5.95351	0.22442	21.01457	15.02864	29.3841	2.19448	3.38481	62.18324
13.8	5.58166	0.27796	16.96712	12.32850	23.3510	2.54820	3.17340	58.29934
12.8	5.37562	0.30964	15.23065	11.02553	21.0393	2.73395	3.05626	56.14729
11.3	5.05083	0.36726	12.84103	9.17804	17.9653	3.04678	2.87160	52.75492
10.1	4.77512	0.43019	10.96229	7.77404	15.4578	3.37411	2.71485	49.87518
9.0	4.50759	0.50262	9.38311	6.56439	13.4120	3.72114	2.56275	47.08089
8.1	4.27628	0.59603	7.91244	5.58461	11.2107	4.18632	2.43124	44.66490
7.1	4.00362	0.72355	6.51790	4.59067	9.2546	4.75797	2.27622	41.81702
6.2	3.74127	0.92009	5.12571	3.71525	7.0700	5.65380	2.12706	39.07683
5.1	3.39320	1.26157	3.73817	2.75783	5.0670	7.03113	1.92917	35.44130
3.9	2.96726	2.07969	2.26742	1.80522	2.8480	10.13673	1.68701	30.99244
3.0	2.60246	3.27923	1.43809	1.20179	1.7208	14.01834	1.47960	27.18218
1.73	1.97627	8.24674	0.57184	0.51561	0.63419	26.77049	1.12359	20.64175
0.66	1.22066	35.54052	0.13268	0.12110	0.14528	71.26577	0.69399	12.74955

3. Velocity of the Alfvén wave:

$$V_h = \frac{B}{\sqrt{\mu_e \rho}} .$$

Substituting the previously given values, the results contained in Table III are obtained.

4. The plasma frequency:

$$\omega_p = e\sqrt{\frac{n}{m\epsilon}} .$$

It is the frequency of oscillation caused by the electric field above. Substituting the previously given values, the results contained in Table III are obtained.

5. The cyclotron frequency:

$$\omega_c = \frac{eB}{m} .$$

It is equal to the angular frequency with which particles of mass, m, and charge, e, gyrate in a cyclotron. Substituting the previously given values, it becomes for the electron

$$\omega_{ce} = 1.75889 \cdot 10^7 \ \mathrm{sec}^{-1} ,$$

and for the proton

$$\omega_{cp} = 9.57414 \cdot 10^3 \ \mathrm{sec}^{-1} .$$

6. The electrical decay frequency:

$$\omega_e = \frac{J}{D} = \frac{\sigma}{\epsilon} \ .$$

It prescribes the rate of electrical energy converted into joule heat. Substituting the previously given values, the results contained in Table III are obtained.

7. The friction frequency:

$$\omega_f = \frac{\alpha}{mn} \ .$$

It is the frequency of the oscillation if the force of friction is the only external force. Substituting the previously given values, the results contained in Table III are obtained.

8. The Debye length:

$$\ell_D = \sqrt{\frac{R_a T \epsilon}{n e^2}} \ .$$

It is a measure of the distance over which an excess electrical charge may be appreciably different from zero. Substituting the previously given values, the results contained in Table III are obtained. Since the Debye length is very small, the solar plasma tends toward electrical neutrality, and

$$\rho_e = 0$$

is a good approximation.

9. The mean free path:

$$\ell_f = 1.255 \sqrt{\gamma} \ \frac{\mu_v}{a \rho} \ .$$

Substituting $\gamma = \frac{5}{3}$:

$$\ell_f = 1.6202 \frac{\mu_v}{a \rho} .$$

It is a measure of distance travelled between collisions of neutral particles. Since the typical length of the flow field is much larger than the mean free path, the continuum theory can be applied. Substituting the previous by given values, results are obtained which are approximately 1.7 percent lower than those given in Table II.

10. The Larmor radius:

$$\ell_1 = \frac{a}{\omega_c} .$$

It is a measure of the radius of the helical path of a charged particle in a magnetic field. Substituting the previously given values, the results for the electron ℓ_{1e} and the proton ℓ_{1p} are contained in Table III.

IV. NONDIMENSIONAL PARAMETERS OF SOLAR DYNAMICS

The nondimensional parameters which characterize the flow of a fully ionized plasma are obtained by dimensional analysis from the 22 physical quantities given in Chapter II. In the system of units adopted, there are five independent fundamental units: mass, time, length, electrical charge, and temperature. Then by the π -theorem of dimensional analysis, 17 nondimensional parameters may be formed. The following five fundamental units are suggested:

$$\ell_0 = 10^7 \text{ m}$$

$$\rho_0 = 15.2 \times 10^3 \text{ kg m}^{-3}$$

$$t_0 = 25 \text{ sec}$$

$$e = 1.60207 \times 10^{-19} \text{ coulomb}$$

$$T_0 = 7.1 \times 10^6 \, ^\circ\text{K}$$

Nondimensional parameters of all other quantities listed in Chapter II are then derived in terms of these fundamental units. They will be denoted as Π_i .

1. The nondimensional parameter for the velocity:

$$\Pi_1 = \frac{u\,t_0}{\ell_0}$$

2. The nondimensional parameter for the pressure:

$$\Pi_2 = \frac{p}{\frac{\rho u^2}{2}}$$

3. The ratio of specific heats:

$$\Pi_3 = \gamma = \frac{C_p}{C_v} \quad .$$

For a monatomic gas, $\gamma = \frac{5}{3}$; for a diatomic, $\gamma = \frac{7}{5}$.

4. The Mach number:

$$\Pi_4 = M = \frac{u}{a} \quad .$$

5. The Reynolds number:

$$\Pi_5 = R_e = \frac{\rho\, u\, \ell_0}{\mu_v} \quad .$$

6. The Prandtl number:

$$\Pi_6 = P_r = \frac{\mu_v C_p}{k} \quad .$$

7. The Froude number:

$$\Pi_7 = F_r = \frac{u^2}{g \ell_0} .$$

8. The Schmidt number:

$$\Pi_8 = S_c = \frac{\mu_v}{\rho D_d} .$$

9. The relativity parameter:

$$\Pi_9 = R_c = \frac{u^2}{c^2} .$$

10. The electric decay parameter:

$$\Pi_{10} = t_0 \omega_e = t_0 \frac{\sigma}{\epsilon} .$$

11. The plasma frequency parameter:

$$\Pi_{11} = t_0 \omega_p = t_0 e \sqrt{\frac{n}{m \epsilon}} .$$

12. The frictional frequency parameter:

$$\Pi_{12} = t_0 \omega_f = t_0 \frac{\alpha}{m n} .$$

13. The electrical frequency parameter:

$$\Pi_{13} = R_E = \frac{E}{\mu_e u H} .$$

14. The magnetic pressure number:

$$\Pi_{14} = R_h = \frac{V_h^2}{u^2} = \frac{\mu_e H^2}{\rho u^2} .$$

15. The magnetic Reynolds number:

$$\Pi_{15} = R_\sigma = u \ell_0 \sigma \mu_e = \frac{u \ell_0}{\nu_h}$$

$$\nu_h = \frac{1}{\sigma \mu_e} .$$

16. The radiation pressure parameter:

$$\Pi_{16} = \frac{D_r a_r}{k} = \frac{a_r T^4}{3p} .$$

17. The radiation flux parameter:

$$\Pi_{17} = \frac{P_r a_r}{k} T^3 .$$

Solar plasma dynamics is characterized by the following conditions:

$$\Pi_1 \geq 1$$

$$R_E = 0$$

$$R_c \ll 1 .$$

The following nine parameters have considerable influence on the flow problems:

$$\Pi_2, \gamma, M, R_e, P_r, R_h, R_\sigma, \Pi_{16}, \Pi_{17} .$$

The remaining five are unimportant parameters which may be neglected:

$$F_r, \ S_c, \ \Pi_{10}, \ \Pi_{11}, \ \Pi_{12} \quad .$$

Substituting the previously given values and $< u >$ for u, the results for the important nondimensional parameters are given in Table IV.

TABLE IV. NONDIMENSIONAL PARAMETERS FOR THE SOLAR PLASMA

T	Π_2	$\Pi_4 = M$	$\Pi_5 = R_e$	$\Pi_6 = P_r$	$\Pi_{14} = R_h$	$\Pi_{15} = R_\sigma$	Π_{16}	Π_{17}
°K								
10^6			10^{15}	10^{-5}	10^{-18}	10^{12}	10^{-4}	10^9
15.7	1.57020	1.23629	18.14422	6.66665	0.092968	122.9054	4.55927	61.08643
13.8	1.57034	1.23613	15.37975	6.66662	0.16230	94.51396	4.75031	49.49811
12.8	1.57044	1.23618	14.39899	6.66665	0.21712	81.40802	4.70343	42.08294
11.3	1.57022	1.23625	13.07692	6.66663	0.34595	63.67585	4.55262	31.68306
10.1	1.57008	1.23606	11.90246	6.66666	0.53122	50.98328	4.46213	24.94033
9.0	1.57072	1.23608	10.93820	6.66665	0.81376	40.63901	4.30799	19.56215
8.1	1.57230	1.23606	9.62969	6.66666	1.27153	32.79854	4.41205	16.54864
7.1	1.56950	1.23581	8.49755	6.66666	2.13860	25.23698	4.38851	13.39822
6.2	1.57188	1.23609	6.94862	6.66663	3.95842	19.09039	4.71601	11.44414
5.1	1.56936	1.23590	5.49010	6.66660	9.04970	12.85043	4.94422	9.21922
3.9	1.57048	1.23618	3.52957	6.66667	32.1456	7.35740	6.00106	7.50464
3.0	1.57238	1.23605	2.43119	6.66665	103.92	4.29539	6.78507	6.12174
1.73	1.57048	1.23580	1.17980	6.66665	1140.2	1.39919	8.24270	4.43693
0.66	1.57016	1.23608	0.43785	6.66534	55484	0.20303	8.49485	—

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APPROVAL

SOLAR FLOW PARAMETERS

By Klaus Schocken

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC's Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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