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THE MAGNETIC FIELD OF THE SUN AND THE GENERATION OF SUNSPOTS

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SUMMARY

It is possible to construct migratory dynamo waves in a prismatic volume of a conducting fluid, as it exists in the solar convective zone. Such waves can explain the principal features of the observed solar magnetic activity, particularly the sunspots.

LIST OF SYMBOLS

Symbol	Definition
$\vec{A}$	vector potential
$\vec{B}$	magnetic induction
$\vec{D}$	electric induction
e_{ϵ}	unit vector in ϵ-direction
$\vec{E}$	electric field strength
f	measure for the violence of cyclones
$\vec{F}$	Lorentz force
$\vec{H}$	magnetic field intensity
I	line integral
$\vec{J}$	conduction current density
k	wave number
K	constant
L	separation of periphery from a line C
p	hydrostatic pressure
q	electric charge density
Q	rate of addition of heat per unit mass
S	cross section
t	time

Symbol	Definition
T	temperature
U	internal energy per unit mass
$\vec{v}$	velocity of fluid
$\vec{X}$	external force of nonelectromagnetic origin
Γ	periphery of S
$(\delta\,\epsilon\,\zeta)$	Cartesian coordinate system
ϵ	dielectric constant
η	coefficient of viscosity
κ	heat conductivity
λ	magnetic diffusivity
μ	magnetic permeability
ρ	density of fluid
σ	electric conductivity
ω	angular frequency
Ω	expression for angular frequency

I. INTRODUCTION

Certain motions of electrically conducting fluids are capable of maintaining various magnetic field configurations, such as the magnetic fields of the Earth and of the Sun. These mechanisms of fluid-magnetic field interaction are called hydromagnetic dynamos. This paper demonstrates by means of appropriate wave equations that there could exist a plasma fluid-velocity $\vec{v}$ having a magnetic field $\vec{B}$ which would be maintained by the mutual interaction of magnetic field and plasma. The wave equations constitute a hydromagnetic dynamo model. The model is shown to relate to observed sunspot formation and activity, and accompanying solar magnetic phenomena.

II. THE EQUATIONS OF MAGNETOFLUID DYNAMICS

The behavior of the solar plasma is governed by the equations of fluid dynamics and Maxwell's electrodynamic equations. They are defined as follows:

A. Fluid Dynamic Equations

(1) equation of continuity

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho\vec{v}) = 0$$

(2) equation of momentum

$$\rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right] = \rho\vec{X} + \vec{F} - \vec{\nabla}p + \frac{4}{3}\eta\nabla(\mathrm{div}\,\vec{v}) + \eta\nabla^2\vec{v} \quad (1)$$

(3) equation of energy

$$\rho\left[\frac{\partial U}{\partial t} + (\vec{v}\cdot\nabla)U\right] = -p\,\mathrm{div}\,\vec{v} + \kappa\nabla^2 T + \frac{\vec{J}^2}{\sigma} + \rho Q + \eta[\nabla^2\vec{v}^2 - 2\,\mathrm{div}(\vec{v}\times\mathrm{curl}\,\vec{v}) + (\mathrm{curl}\,\vec{v})^2 - 2\vec{v}(\nabla\mathrm{div}\,\vec{v}) - \frac{2}{3}(\mathrm{div}\,\vec{v})^2]$$

B. Maxwell's Electrodynamic Equations

$$\mathrm{curl}\,\vec{E} = -\partial\vec{B}/\partial t \quad (2)$$

$$\mathrm{curl}\,\vec{H} = \vec{J} + \partial D/\partial t \quad (3)$$

$$\mathrm{div}\,\vec{B} = 0 \quad (4)$$

$$\mathrm{div}\,\vec{D} = q, \quad (5)$$

for which constitutive equations are

$$\vec{B} = \mu\vec{H} \quad (6)$$

$$\vec{D} = \epsilon\vec{E} \quad (7)$$

$$\vec{J} = \sigma(\vec{E} + \vec{v}\times\vec{B}). \quad (8)$$

C. Assumptions

The following simplifications are made so that a practical model can be obtained.

1. The solar plasma is considered as an electrically conducting single-component fluid.

2. The magnetic permeability, μ , of photosphere and chromosphere is assumed to be that of free space, μ_0 , and is, therefore, a constant ($\mu = \mu_0$ = constant).

3. Viscous forces are neglected (i.e., the last two terms of Equation (1) are set equal to zero).

4. The displacement current, $\partial D/\partial t$ (in Equation 3), and $\sigma \vec{E}$ (in Equation 8) are neglected, as well as other electric field action on free charges, as in Equation (9).

D. Deriviation of the Equation of Motion

Starting with simplified equations, an equation of motion is next derived for the convective and magnetic forces which dominate action of the hydromagnetic dynamo model.

The force per unit volume exerted by an electromagnetic field on a current, neglecting the action of the electric field on the free charges (i.e., neglecting the term $q\vec{E}$), is

$$\vec{F} = \vec{J} \times \vec{B}. \tag{9}$$

In order that the equation of motion for the magnetic field may be obtained, $\vec{E}$ is eliminated between Equations (2) and (8). Then Equation (3) is introduced and the identity

$$\text{curl curl } \vec{B} = \nabla \text{div } \vec{B} - \nabla^2 \vec{B}$$

is applied. When Equations (3) and (5) are used, the equation of motion is obtained in the form:

$$\frac{\partial \vec{B}}{\partial t} = \mathrm{curl}\,(\vec{v} \times \vec{B}) + \frac{1}{\sigma\mu}\ \nabla^2 \vec{B}\,. \tag{10}$$

The equation of motion of an electrically conducting fluid is obtained through substitution of Equation (9) in the simplified equation of motion (1):

$$\rho\,\frac{d\vec{v}}{dt} = \rho\left[\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\,\vec{v}\right] = -\nabla p + \rho\vec{X} + \vec{J} \times \vec{B}\,. \tag{11}$$

III. DYNAMO EFFECTS

Fluid velocity $\vec{v}$ and magnetic field intensity $\vec{H}$ are determined by a solution of Equations (4), (6), (10), and (11). The question to be answered is whether the conduction current $\vec{J}$, produced from field $\vec{B}$ according to Equation (8), can generate the field $\vec{B}$ according to Equation (2) by which $\vec{J}$ originated. When such action does occur it is called a dynamo process. By making the following substitutions of terms in Equation (10)

$$\lambda = \frac{1}{\sigma\mu}$$

$$\mathrm{curl}\,\vec{A} = \vec{B}\,.$$

Equation (10) may be expressed in the form:

$$\frac{\partial \vec{A}}{\partial t} = \vec{v} \times \mathrm{curl}\,\vec{A} + \lambda\nabla^2 \vec{A}\,. \tag{12}$$

According to a theorem by Cowling, a magnetic field symmetric about an axis cannot be maintained by a fluid motion symmetric about the same axis. To show this, let C be a line on which $\vec{B} = 0$ and about which the lines of induction form closed loops which encircle C in the same direction everywhere. Let S be a meridian section of C whose periphery Γ is a closed line of induction surrounding C, and let L denote the maximum separation of Γ from C. When Stokes' theorem and Equations (3) and (8) are applied to Γ, the following result is obtained:

$$\oint \vec{B} \cdot d\vec{\Gamma} = \int \operatorname{curl} \vec{B} \cdot d\vec{S} = \mu \int \vec{J} \, d\vec{S} = \frac{1}{\lambda} \int (\vec{v} \times \vec{B}) d\vec{S},$$

therefore,

$$\oint \vec{B} \cdot d\vec{\Gamma} \leq \frac{v_{max}}{\lambda} \int B \, dS. \tag{13}$$

In Equation (13) v_{max} denotes the maximum value of v anywhere in the solar volume, and the incremental area dS, lying between the lines of induction L and L + dL, cannot in the limit exceed $d\Gamma \, dL$ in area. If I(L) denotes the line integral of $\vec{B}$ over Γ, it follows from Equation (13) that

$$I(L) \leq \frac{v_{max}}{\lambda} \int_0^L I(l) \, dl .$$

The line integral I(L) is a continuous function of the maximum separation L, so that I tends to zero as L tends to zero. Let L_0 denote the point of the interval (0, L) where I attains its maximum value; when Equation (13) is applied to L_0, the result is:

$$I(L_0) \leq \frac{v_{max}}{\lambda} L_0 \, I(L_0).$$

If $I(L_0) \neq 0$, the following relationship is obtained:

$$\frac{\lambda}{v_{max}} \leq L_0 .$$

But L is arbitrary, and if it is assigned a value less than λ / v_{max}, a contradiction has been established. Therefore, the induction field $\vec{B}$ cannot be maintained in the neighborhood of C.

For a complete solution of the dynamo problem it would be necessary to solve the fluid dynamic and electromagnetic equations simultaneously. Because the difficulties of a complete solution are too great, the more restricted problem is usually investigated to establish whether a motion exists which can maintain a magnetic field.

A magnetic field which is everywhere perpendicular to meridional planes is called "toroidal," and a magnetic field whose field lines are everywhere in meridional planes is called "poloidal."

A rectangular Cartesian coordinate system (δ, ϵ, ζ) will be applied to the fluid. The vector potential $\vec{A}$ is considered to extend in the ϵ-direction. Writing A for the ϵ-component of $\vec{A}$, we can express Equation (12) in the form:

$$\frac{\partial A}{\partial t} = f B + \lambda \nabla^2 A . \qquad (14)$$

It will be assumed that a moving fluid rotates about the ζ-axis. Dynamically, such a fluid motion is related to the cyclones and anticyclones observed in the Earth's atmosphere. Such a cyclonic motion results when the Coriolis force component adds to the primary driving force of radial convection. Similarly, deflection along the length of a vertically rising and diverging column of solar plasma would tend to produce a cyclonic configuration. The factor f in Equation (14) is a measure of the violence of such cyclonic motions.

If fluid motion is present:

$$\vec{v} = e_\epsilon v;$$

then, representing the nonuniform rotation by Equation (10), the toroidal field $\vec{B}$ is generated as a function of the magnetic field potential $\vec{A}$:

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times [\vec{v} \times (\nabla \times \vec{A})] + \lambda \nabla^2 \vec{B}.$$

Since $\vec{A}$ and $\vec{v}$ are independent of ϵ and have only ϵ-components, $\frac{\partial \vec{B}}{\partial t}$ is in the ϵ-direction, and the last equation reduces to:

$$\frac{\partial B}{\partial t} = \left(\frac{\partial v}{\partial \zeta} \frac{\partial A}{\partial \delta} - \frac{\partial v}{\partial \delta} \frac{\partial A}{\partial \zeta} \right) + \lambda \nabla^2 B. \tag{15}$$

In order to solve the dynamo Equations (14) and (15), we assume uniform shearing. If v varies linearly with ζ:

$$\frac{\partial v}{\partial \zeta} = K = \text{constant}, \quad \text{and} \quad \frac{\partial v}{\partial \delta} = 0.$$

Then Equation (15) reduces to the form:

$$\frac{\partial B}{\partial t} = K \frac{\partial A}{\partial \delta} + \lambda \nabla^2 B. \tag{16}$$

If the cyclones are distributed uniformly throughout the space, K is a constant, and the dynamo equations reduce to two simultaneous linear equations with constant coefficients.

The following trial solutions are made regarding the solutions of Equations (14) and (16):

$$A = A_0 \, e^{i(\omega t + k\delta)}$$
$$B = B_0 \, e^{i(\omega t + k\delta)} \, . \tag{17}$$

After substitution of Equations (17) into Equations (14) and (16), the following results are obtained:

$$A_0 = (i\omega + \lambda k^2) - B_0 f = 0$$

and

$$-A_0 ikK + B_0 (i\omega + \lambda k^2) = 0.$$

Setting the determinant of the coefficients equal to zero yields the result:

$$(i\omega + \lambda k^2)^2 - ikKf = 0,$$

which can be expressed in the form:

$$i\omega + \lambda k^2 = \pm (ikKf)^{\frac{1}{2}} = \Omega (1 \pm i);$$

therefore,

$$\Omega = \left(\frac{kKf}{2}\right)^{\frac{1}{2}},$$

$$i\omega = (\Omega - \lambda k^2) \pm i\Omega,$$

and

$$A_0 = -B_0 \Omega \left(\frac{1 \pm i}{kK}\right).$$

We now use the last two equations with the trial solutions (Equations 17) of the dynamo equations (Equations 14 and 15) to obtain equations describing the migratory dynamo waves:

$$B = B_0 e^{(\Omega - \lambda k^2)t} e^{i(k\delta \pm \Omega t)}$$

$$A = -B_0 \frac{\Omega}{kK} (1 \pm i) e^{(\Omega - \lambda k^2)t} e^{i(k\delta \pm \Omega t)}.$$

The solar dynamo, if it exists, would be located in the convective zone in a shell sufficiently thin ($\approx 10^5$ km) that its curvature may be neglected. Application of the dynamo equations to such a flattened space yields just the migratory dynamo.

Dynamo waves migrate from the polar to the equatorial regions in the convective zone just below the surface of the Sun. Because the liquid-dense photosphere has a relatively high sonic and magnetic conductivity in comparison with the adjacent gaseous chromosphere, the dynamo waves are constrained from diffusing into the chromosphere. Thus only in regions of most intense cyclonic motions or of nonuniform rotation will significant wave strands be carried beyond the normal radial limit of the photosphere. It is in such regions that sunspot activity would be observed.

The Sun's magnetic field is composed of traveling magnetic waves. The traveling wave is assumed to consist of two components, toroidal and poloidal, as indicated diagramatically in Figure 1. The sense of the latitudinal toroidal bands alternates from one band to the next. The poloidal field components are in meridional planes rotated 90 degrees from the planes of the toroidal components.

In each hemisphere, there are two or three toroidal bands at any one time. About 22 years are required for migration of a band from pole to equator, so that new bands approach the equator at 11-year intervals. The poloidal component of the traveling wave predominates from pole to middle latitudes, the toroidal component dominates below the middle latitudes, and both components vanish at the equator.

An occasional strand of plasma, as a result of particularly intense local wave amplification, moves to or above the surface of the photosphere, but the main field configuration remains essentially unaffected. Sunspot activity peaks at 11-year intervals, as new toroidal bands in each hemisphere move into the region in the lower latitudes of most intense field amplification. Strands thrown out below the middle latitudes are predominantly a result of action of local toroidal field intensification, while strands occurring nearer the pole result from local poloidal field buildup.

Below the middle latitudes the spots occur in pairs. These paired spots correspond to the exit and entrance locations of inverted U-shaped distortions of the toroidal flux tubes which have been pushed above the normal radial limit of the photosphere.

The east-west orientation of the sunspots in the lower latitudes results from the east-west direction of the dominant toroidal component of the flux tube. The migration of a spot toward the equator from its region of formation is a result of the migration of the associated toroidal field. The reversal of polarity of the paired spots corresponds to the alternation of field each half cycle (that is, with each successive toroidal band).

IV. RESULTS AND CONCLUSIONS

The explanation of the Sun's magnetic field given in this paper is derived from studies by E. N. Parker, and may be summarized in the following way.

The Sun's magnetic field is generated by, and moves in unison with, the electrically conducting solar plasma. Hence, fields of major intensity are confined to the convective zone.

Field and moving plasma continuously interact to modify their interdependent configurations. The field is azimuthal at low altitudes, and progressively changes toward a meridional plane near the poles.

The field is maintained as a result of nonuniform axial rotation of successive layers of the Sun's fluid mass and vortex effects in the convecting plasma. The nonuniform rotation amplifies the azimuthal component of the force field. Vortex effects in the convecting plasma produce field components in poloidal planes. The net effect is to twist the azimuthal field loop by a process of reenforcing the loop on one side and reducing field density on the other. Simultaneously, the dynamo effects intensify the field. Similarly, the magnetic force-field differential causes surface loops to move toward the equator and subsurface loops to move toward the pole while dynamo effects act to intensify the fields.

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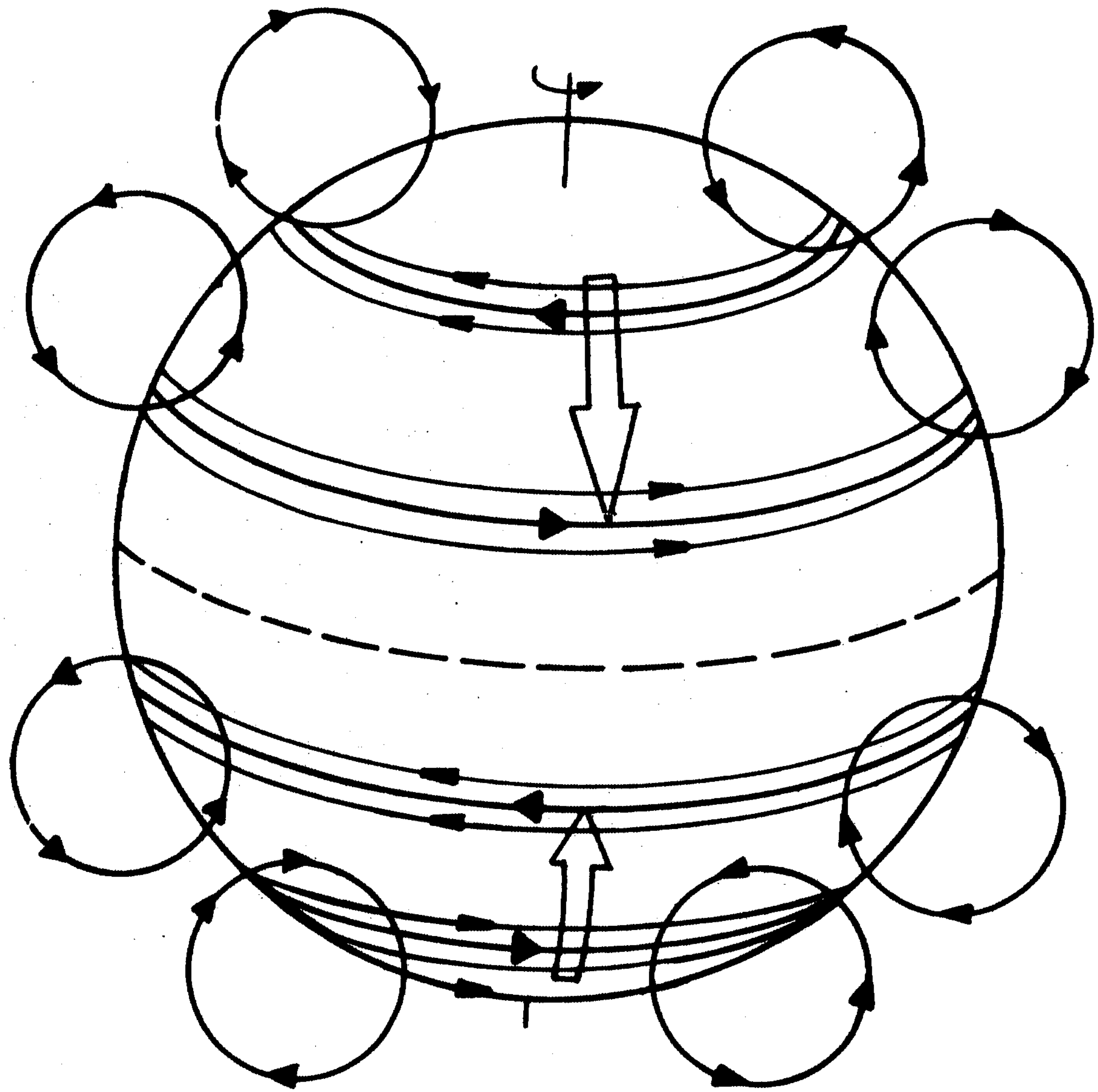

FIGURE 1. Diagram of Solar Toroidal and Poloidal Magnetic Fields in the Outer Half of the Convective Zone (from Parker, Ref. 1)