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TRACKING ACCURACY STUDIES OF GEOS-C ORBITS FOR ALTIMETRY USING RADAR AND OPTICAL DATA

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ABSTRACT

Tracking accuracy studies of orbits of the type envisioned for GEOS-C indicate that gravity model and station position uncertainties preclude determination of spacecraft altitude within 10-20 meters. However, the variation of height error is low (1 meter in 20-30 minutes) with intensive tracking so surveys of relative changes in sea level over short arcs appear feasible.

The low inclination of 20° for GEOS-C appears to incur little or no penalty in orbit determination accuracy, particularly if SAO Baker-Nunn or Unified S-Band tracking is used. In spite of limited coverage by STADAN Optical stations, such data will be an important supplement to previous GEOS analyses.

TRACKING ACCURACY STUDIES OF GEOS-C ORBITS FOR ALTIMETRY USING RADAR AND OPTICAL DATA

I. Introduction

The altimeter experiment of the GEOS-C spacecraft is one of the important facets of the mission. This is a fundamentally new way of gathering geodetic data. Because of this newness, very accurate GEOS-C orbits must be obtained for calibration and evaluation of the altimeter. While this does not entirely preclude the use of altimeter data to determine the orbit, much more satisfactory and error-free analyses will be obtained if other tracking systems are used for definitive orbit determination. The altimeter data can then be regarded as measurements from a known (to some precision) spacecraft position to the surface.

The current orbit planned for GEOS-C (Apogee height = 1481.6 kilometers, Perigee height = 1111.2 kilometers) is similar to the GEOS-II orbit with the exception of inclination. For GEOS-C an inclination of 20° is planned.

A low inclination like 20° is favorable for GEOS-C for several reasons. First, it is well known that present determinations of the zonal harmonics of the geopotential are deficient because of a lack of satellite data of geodetic quality with inclination below 30°. Second, the effect of the geopotential is generally reduced at low inclinations, particularly in regard to resonance. Since a resonance with a beat period of at least 2 days is always present, a low inclination that minimizes (indeed eliminates in this case) resonance as a problem is an important factor.

II. Tracking Systems

In this study we have considered tracking with STADAN/MOTS, Unified S-Band (USB), C-Band, and SAO Baker-Nunn systems. Figures 1 through 3 show the positions of these instruments on world maps. In the case of the C-Band radars, we have studied only a partial system (Fig. 2) A small number of C-Band radars is known to be able to determine very accurate orbits as shown by the GEOS-II C-Band tracking project.

The world maps show that the USB and SAO Baker-Nunn systems give excellent coverage of the GEOS-C orbit whether it is inclined at 20° or at a more conventional value, e.g., 30°. However, at an inclination of 20°, the STADAN/MOTS optical system is degraded and certain important C-Band stations, i.e.,

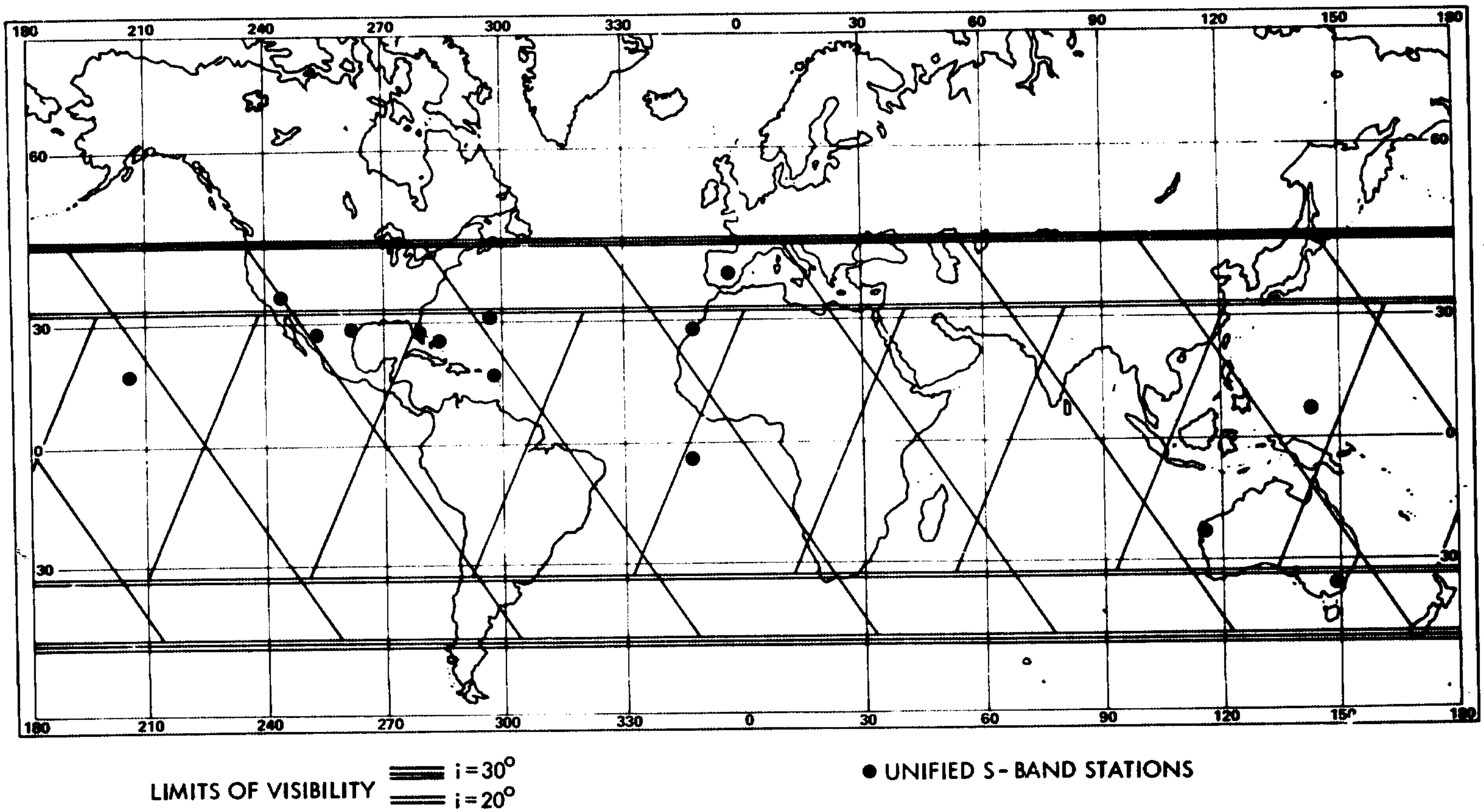

Figure 1. Station Visibility Map - Unified S-Band Radar

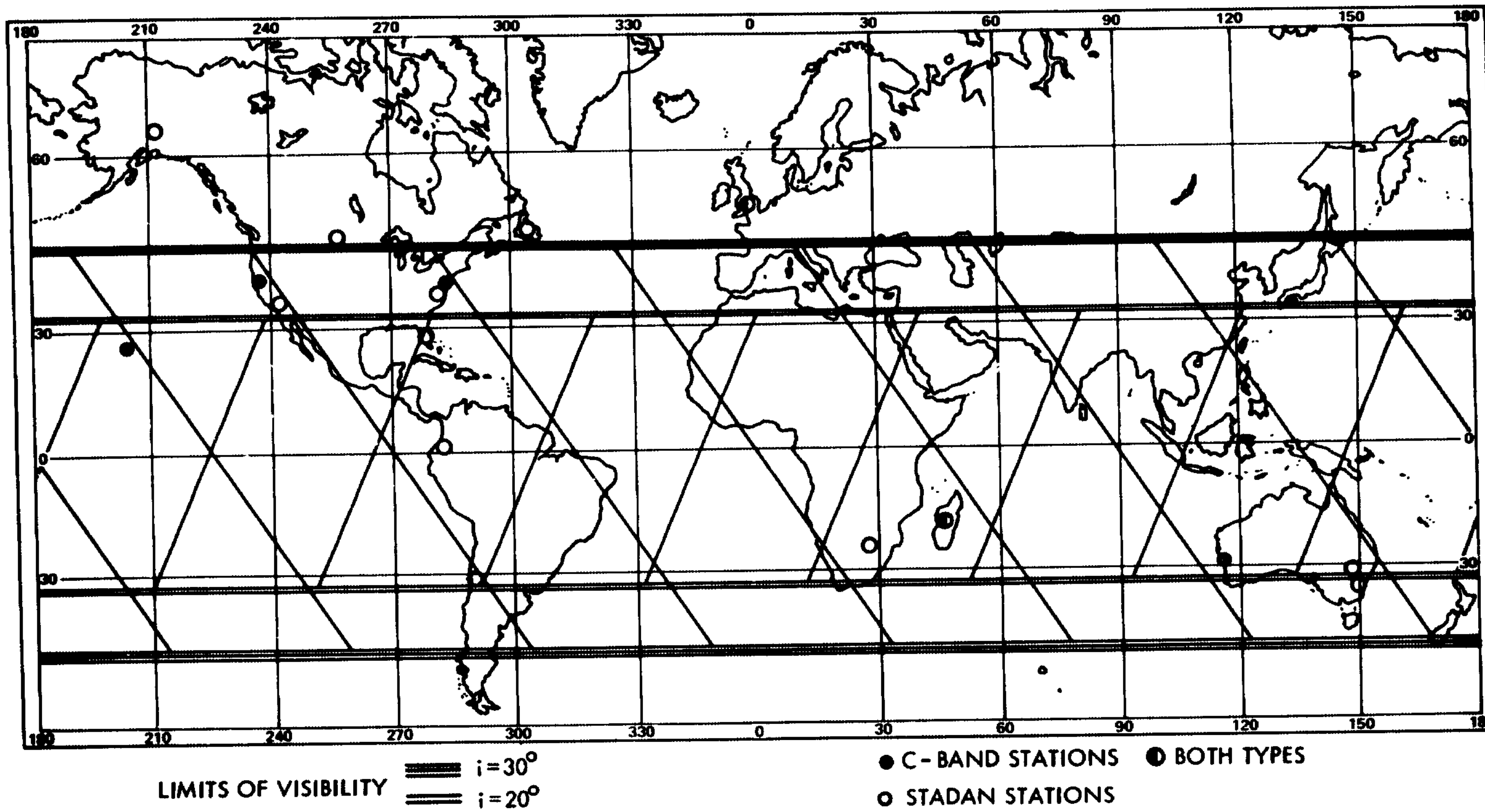

Figure 2. Station Visibility Map - C-Band Radar, STADAN Optical

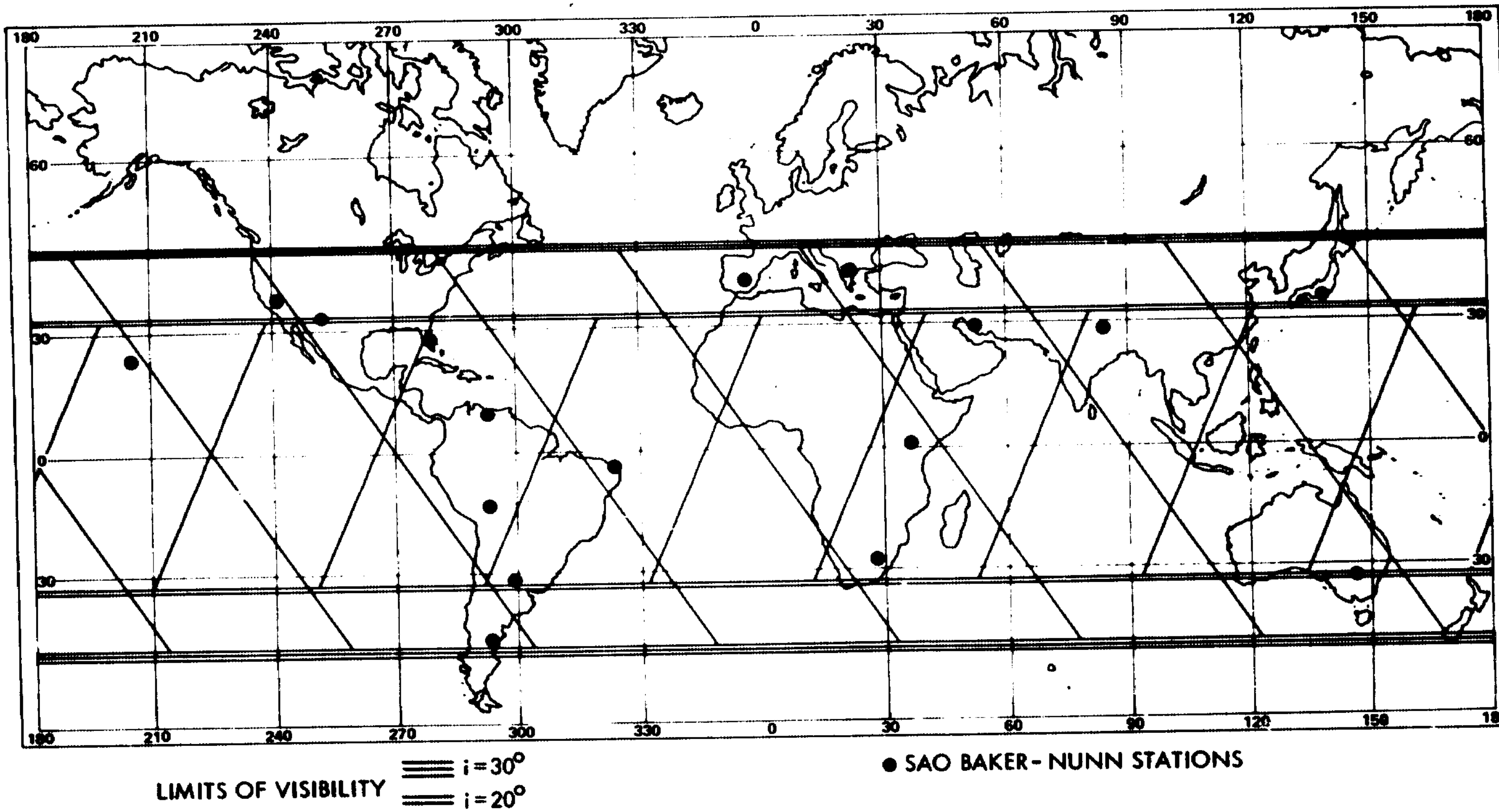

Figure 3. Station Visibility Map - SAO Baker-Nunn Optical

Wallops Station, can no longer track. However, a number of these sites will be able to track and will supplement previous GEOS analyses. A large amount of STADAN optical data are available for GEOS-I and II. These data plus SAO Baker-Nunn tracking data on the GEOS satellites has been used with success to determine the locations of the STADAN optical stations. For the system as a whole, the center of mass position errors are 20 meters or less (Reference 1). A third well-observed geodetic satellite will enable us to reduce this figure for some stations when the data is combined with data from GEOS-I and II.

III. Tracking Accuracy Studies

To evaluate the performance of the altimeter and use the data for geodesy requires that the geocentric distance of the satellite be known accurately. Thus we analyzed the effect of data noise, data bias, gravity model uncertainty, and station position error on determination of the geocentric distance of a GEOS-C type satellite. The method used is explained in the Appendix and also in Reference 2. Table 1 shows the error model assumed for each of the STADAN optical, USB, C-Band, and SAO Baker-Nunn tracking systems. The optical data rates shown assume that the satellite has flashing lights.

We have considered two orbits, one inclined at 20° and the other at 30°. The results are quite general for orbits in the range of 10° - 40°. The major difference, as seen in Figures 1 - 3, is one of coverage by the observing systems.

Table 1
Tracking Systems and Model Errors Assumed for Error Analyses

Type	Frequency	Noise	Bias	Station Position Error
SAO Baker-Nunn	10/min*	2 arc seconds	none	20 meters
STADAN/MOTS	10/min*	2 arc seconds	none	20 meters
C-Band	60/min	5 meters	2 meters	20 meters
Unified S-Band-range	10/min	10 meters	40 meters	20 meters
USB range-rate	10/min	1 cm/sec	none	20 meters
Gravity model error: SAO M1-SAO COSPAR (1969) to (8,8) + 20% error in resonant coefficients				
GM error: $1:10^6$				

*Flash data, one sequence per pass.

With the exception of the USB ranges, the tracking systems studied are known to have low noise, and small biases. The most important sources of error are thus likely to be the gravity model and station position uncertainty.

For the effects of gravity model error we considered resonant and non-resonant terms separately. The estimate of the error of the non-resonant part of the geopotential was taken to be the difference between the SAO M1 (Reference 3) and SAO COSPAR (Reference 4) models to (8,8). The results obtained in Reference 1 indicate that this estimate of error is probably pessimistic. The error in the 13th order resonant coefficients was taken to be 20% of their value. This is probably pessimistic, but in any case it has been found that resonance is no problem at inclination near 20°. The effect of resonance drops off dramatically at inclinations below 45° and can be further minimized by choosing a small beat period. Also, the effect of resonance radially is small for a circular orbit. To see this, we can write for geocentric distance, r,

$$r = a(1 - e \cos E) = a(1 - e \cos M) + o(e^2), \tag{1}$$

Where a is semi-major axis, e is eccentricity, E is eccentric anomaly and M is mean anomaly. To find the effect of perturbations of the elements, take

$$\Delta r = \Delta a(1 - e \cos M) - a\Delta e \cos M + ae \sin M \Delta M. \tag{2}$$

For a near circular orbit ($e \sim 0$)

$$\Delta r \approx \Delta a - a\Delta e \cos M \lesssim \Delta a - a\Delta e. \tag{3}$$

In contrast to ΔM, neither of Δa or Δe contain the square of the beat period as a factor (Reference 5). Thus for circular orbits the effect of resonance on r is something like the square root of the along-track effect. For a two-day beat period at 20° or 30°, the total effect of resonance along-track is 50 m or less. Thus modeling resonance adequately for GEOS-C type orbits will be simple.

Because of model errors, there exists a definite limitation on the length of data arc that can be used. From our studies we found 1 day solutions to be generally superior to either 12 hour or 5 day solutions. Twelve hour solutions may suffer from lack of data, particularly for tracking systems with poor geographic distribution. In contrast, longer arcs have a more difficult time accommodating model errors.

Figure 4 shows the total radial (and therefore altitude) error that can be expected from model errors and noise for a typical 1-day STADAN optical solution. In this and all cases, gravity model errors other than resonance are by far dominant, although station error effects can occasionally reach 5 m. Note that at $i = 20°$, the altitude error occasionally reaches nearly 30 m. The situation is improved for $i = 30°$ because of better coverage by the tracking system. Ten meters is a typical figure with occasional excursions to 15 or even 20 m.

Note in Fig. 4 (and all succeeding figures) that the periodicity of the errors is relatively large, exceeding one hour. Since the satellite is moving 3 degrees/min., small regions such as the Puerto Rican trench will yield good relative altitude measures, particularly if there is intensive tracking of the satellite while it is in the region of interest.

Figure 5 shows the radial error to be expected with USB tracking at 20° and 30° inclination. Note that for this case, the error is worse at 30° than at 20°. This is almost certainly due to the geometry of this solution which actually yielded less tracking at $i = 30°$ than $i = 20°$. We can conclude that at 20° or 30° inclination, height error with USB tracking will be about 10m on the average with a range from 5 meters to about 15 to 20 meters.

Figure 6 shows the performance of a partial C-Band system (Fig. 2) consisting of Hawaii, Western Test Range, Wallops Station, Winkfield, Madagascar, and Carnavon. This very sparse system can have periods of many hours in a day where there is no tracking as in Figure 6, but the quality of the solution is still very high because of the accuracy of the data. These five C-Band stations do nearly as well as the 14 USB stations. Again, an average error of 10 meters with excursions to perhaps twice that figure will be seen. Even better results may be obtainable with more C-Band instruments.

An error analysis was performed combining the STADAN optical and C-Band systems, but the results were not substantially improved. The C-Band data are so numerous and accurate, that optical data has low weight in a combined solution.

Figure 7 shows the total error in radius of a one day solution for the SAO Baker-Nunn system. Again, errors of 10 m are typical with occasional excursions to 20 m.

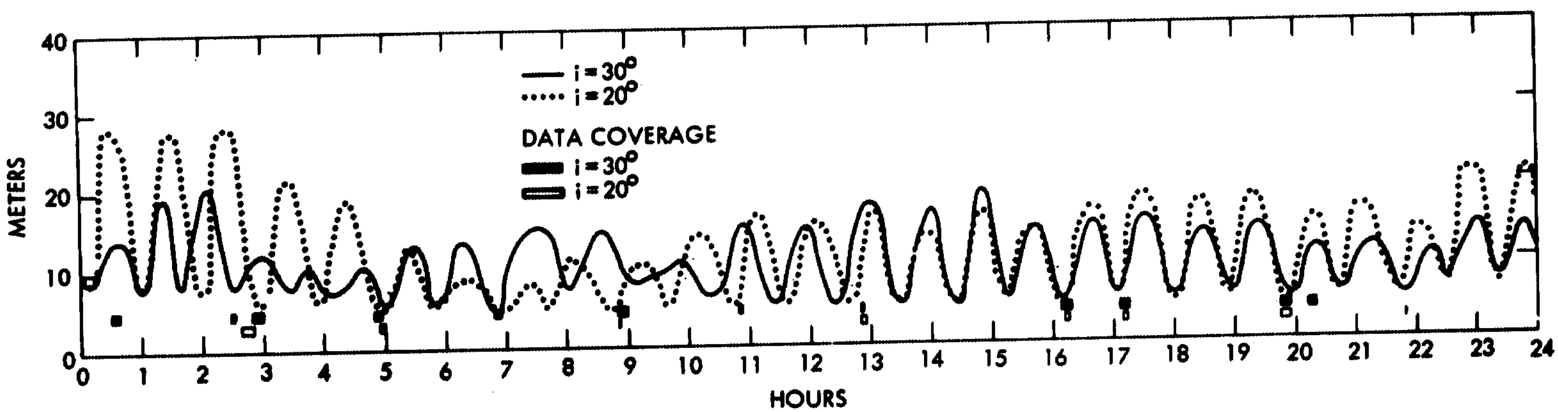

Figure 4. Total Radial Error for a One Day Orbital Solution – STADAN Optical Tracking System

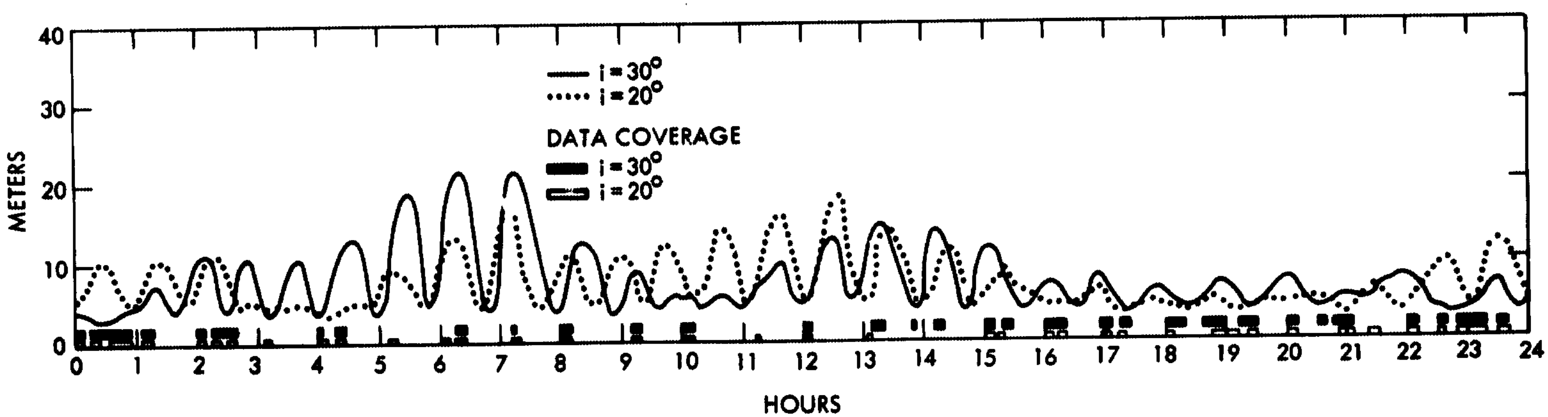

Figure 5. Total Radial Error for a One Day Orbital Solution – Unified S-Band System

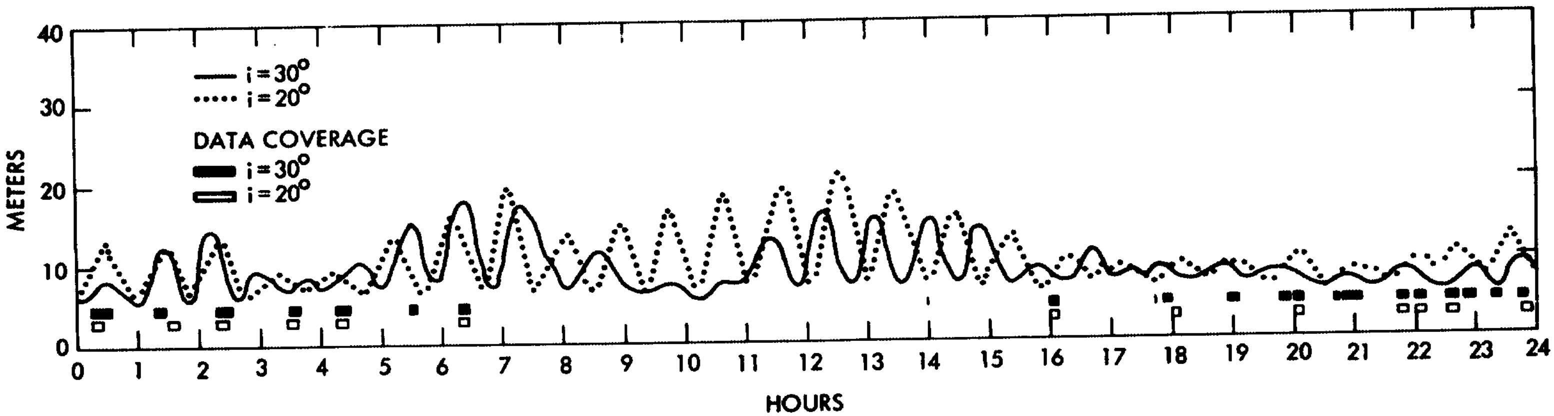

Figure 6. Total Radial Error for a One Day Orbital Solution – C-Band Range System

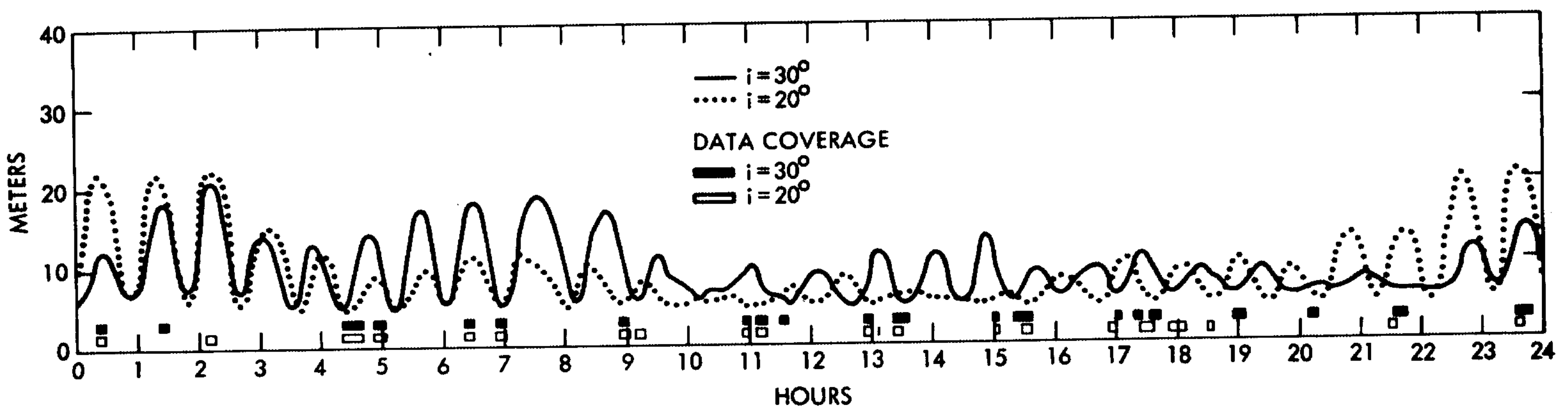

Figure 7. Total Radial Error for a One Day Orbital Solution – SAO Baker-Nunn Optical System

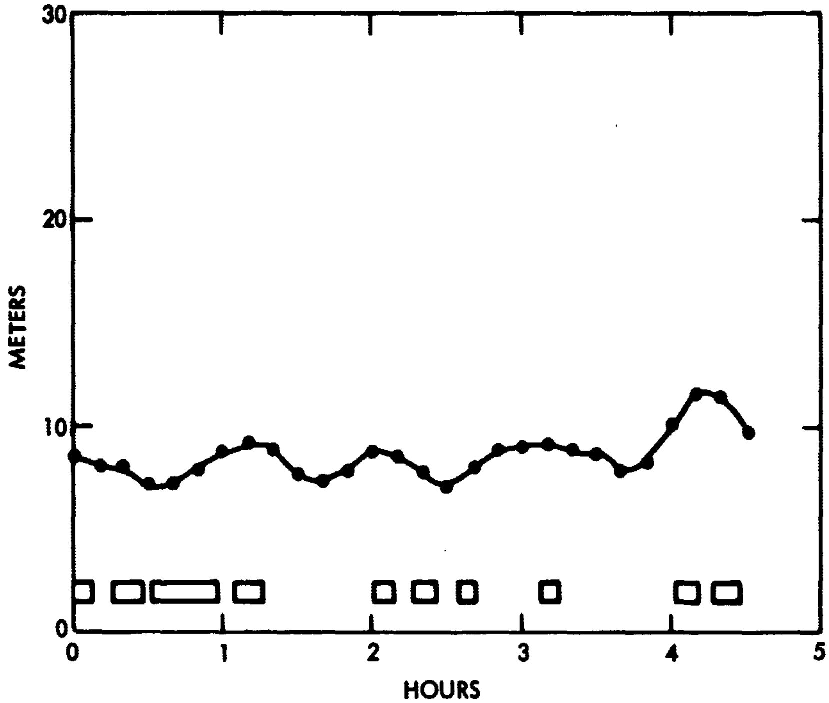

Figure 8. Total Radial Error for a 4-1/2 Hour Orbital Solution - Unified S-Band System - Inclination = 20°, □ = Data Coverage

Figure 8 shows the total radial error for an orbital solution computed with the intensely tracked first 4-1/2 hours of USB data presented in Figure 5. Although the maximum errors in each case are on the order of 10 meters, the rate of change of the radial error for the 4-1/2 hour orbit is much smaller.

IV. Conclusions

We conclude from the foregoing that model errors, particularly gravity model errors and to a lesser extent station position errors, prevent determination of absolute spacecraft altitude to better than 10 m as an average. This error can change as fast as 1 meter/2 min. in areas with little or no tracking. In cases where tracking is intensive, the rate of change of the radial error can drop to 1 meter/30 min. or less. Such errors are tolerable for surveys of the variation of ocean height.

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APPENDIX

ERROR ANALYSIS

The ORAN (Orbital Analysis) computer program used for this study was designed for computing the effects of random and systematic errors on minimum variance orbit determinations. Systematic errors can be in the form of either adjusted or unadjusted parameters, with the effects of the latter broken down into effects of the individual error sources. The program computes the effects of the unadjusted parameters on both the recovered parameters and the orbit, with the orbital effects propagated from epoch to any desired prediction time.

The program is configured for multiple arcs, with some error model parameters such as station positions constrained to be common to all arcs, and other parameters, such as measurement biases, which differ from arc to arc.

Force model errors can arise from uncertainties in geopotential coefficients through degree and order 20. Uncertainties in up to 44 individual coefficients can be carried, and any of these may be either adjusted or their unadjusted effects propagated. Alternately, or in addition, the force model error can be carried as the differences between complete gravity models in which case the restriction to 44 parameters does not apply. The SAO, APL, and NWL models are built into the program and the differences between any two of these three, or any complete model supplied as input, are available as force model errors. Note that the gravity model difference is treated as a single parameter, and 43 geopotential parameters may also be considered as adjustable. Of course, adjusting a geopotential coefficient removes it from the model difference set.

Mathematically, the unmodeled error propagation is based on the following observations. The minimum variance orbit determination uses the basic equation

$$\delta 0 = A \delta a + e \tag{1}$$

to relate discrepancies ($\delta 0$) between measured and calculated observations to discrepancies (δa) between true and a priori estimates of the set of parameters to be recovered. The set δa includes the six orbital elements but may also include other parameters. The matrix A is the set of partial derivatives of the measurements with respect to the adjustable parameters, and e is a vector of measurement "noise." When the least squares criterion is used to solve (1) for the best estimate of a, the result is

$$\delta\hat{a} = (A^T W A)^{-1} A^T W \delta 0, \tag{2}$$

where W is the matrix of measurement weights. For the solution to be minimum variance, the weight matrix must be chosen such that

$$W^{-1} = E(ee^t). \tag{3}$$

That is, W must be the inverse of the variance covariance matrix of measurement noise. In the normal data reduction programs, W is generally so chosen because it actually is measurement random error, in which case W is rather accurately expressed as a diagonal matrix.

For various reasons, the set of parameters adjusted in data reduction programs is only a subset of those parameters having some error. For example, our knowledge of geopotential coefficients is by no means complete. Yet a truncated model is always (of necessity) used, and the error in all coefficients used is ignored in all variance computations. Because the net effect is that e is not random yet contains definite systematic components, we can obtain a more accurate representation of the measurement discrepancy vector by expressing e as

$$e = K\gamma + \epsilon, \tag{4}$$

where γ is a set of errors in parameters previously ignored, K is the matrix of partial derivatives of the measurements with respect to these parameters, and ϵ is the vector of measurement random noise upon which W is still based. Substitution of (4) into (1) gives

$$\delta 0 = A\delta a + K\gamma + \epsilon. \tag{5}$$

If the weight matrix for the measurements is based on ϵ and is the same as that used in the data reduction program, it follows that the solution for $\delta\hat{a}$ actually being obtained is not that given by (2), but actually is a "biased" solution given by

$$\delta \hat{a} = (A^T W A)^{-1} A^T W (\delta 0 - K\gamma). \tag{6}$$

From this relation, we may obtain by differentiation the effects of "unit" values of the set of γ parameters,

$$\frac{\partial \delta \hat{a}}{\partial \gamma} = -(A^T W A)^{-1} A^T W K. \tag{7}$$

It follows that if the matrix K can be obtained, the effects of unit values of the γ parameters are obtained by substituting K for the $\delta 0$ vector used in the data reduction program. A priori estimates of errors in the γ parameters lead to an estimate of the magnitudes of the effects on recovered parameters, and the trajectory, of each γ parameter.

Uncertainties in the γ's are generally uncorrelated. If their correlations are known or can otherwise be accounted for, an estimate of the total or overall accuracy of the orbital solution is readily obtainable. For this study, errors in station locations, GM, and the geopotential were considered.