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TRAJECTORY APPLICATION METHOD (TAM)

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ABSTRACT

A simulation technique (TAM) for the post-flight evaluation of the propulsion system performance has been developed which incorporates the time history trajectory parameters from the post-flight observed trajectory as input. This technique represer a significant reduction in time required to perform stage propulsion system evaluatior The development and some advantages and disadvantages of this technique are given. The propulsion system evaluation was performed on the S-IB stage of AS-201, AS-203, ar AS-202 utilizing both the proposed technique and a conventional simulation technique; the results and comparison of both methods are presented. Additional detailed specifi tions of the TAM program are given in the Appendices.

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TABLE OF CONTENTS

DEFINITION OF SYMBOLS

Symbol	Definition
*AA	Average $*A_m$ between two time intervals printed out at the latter time point.
A_E	Nozzle exit area (Engines 1 through n).
ALISPO	Average instantaneous sea level specific impulse.
*ALT	Altitude (from OMPT).
A_m	Total calculated platform (inertial) acceleration.
$*A_m$	Total platform (inertial) acceleration from Observed Mass Point Trajectory (OMPT).
*AMACH	Average Mach number between t and t_{-1} as observed.
AREA	Cross sectional area of vehicle.
A_T	Throat area (Engines 1 through n).
AVDFLOX	Average propellant mass loss rate.
AVDMM	Average total weight loss rate of the vehicle.
AVFJ10	Average total sea level longitudinal thrust.
BDRAG	Base drag (input as table or equation).
CD	Drag coefficient (table look-up vs. Mach).
CCDD	Average CD between two time points.
CF	Sea level thrust coefficient.
CFV	Vacuum thrust coefficient (pulled from data tape).
CXCA	Required CD as based on acceleration difference (DA).
CXCVE	Required CD as based on earth-fixed velocity difference (DDVE).
CXCVI	Required CD as based on integrated acceleration differences (DDIAT).
DA	Difference in total acceleration (calculated minus measured).
DCXA	Required drag coefficient change at any time as based on acceleration difference (DA).
DCXVE	Required CD change based on DDVE.
DCXVI	Required CD change based on DDIAT.

DEFINITION OF SYMBOLS (CONT'D)

Symbol	Definition
DD	Average vehicle aerodynamic longitudinal drag between t and t_{-1}
DDVE	Difference of DVE between two time points.
DDIAT	Difference between DIAT from one time point to another divided by time difference.
DFA	Required thrust change at any time as based on acceleration difference (DA).
DFLOX	Propellant mass loss rate (from data tape).
DFVE	Required thrust change based on DDVE.
DFVI	Required thrust change based on DDIAT.
DIAT	Integrated difference between A_m and $^*A_m$.
DMA	Required mass change at any time as based on acceleration difference (DA).
DMASS ($\dot{w}$)	Total vehicle mass loss rate (from data tape).
DMVE	Required mass change based on DDVE.
DMVI	Required mass change based on DDIAT.
DRAG	Total vehicle aerodynamic longitudinal drag.
DVE	Difference between calculated and measured earth-fixed velocity
FAI	Total longitudinal drag force.
FB	Vertical buoyancy force.
$FE_{(1-n)}$	Local individual engine turbine exhaust thrust (from data tape)
$FEI_{(1-n)}$	Sea level turbine exhaust for engines (1-n).
$F\text{-}ENG_{(1-n)}$	Local engine thrust.
FF	Average local thrust (FJI) between t and t_{-1}.
FJI	Total local longitudinal thrust.
FJIO	Total sea level longitudinal thrust.
FLTTT	Flight time as measured from first motion.
FMI(F)	Total local longitudinal effective force.
$FO_{(1-n)}$	Individual engine sea level thrust.

DEFINITION OF SYMBOLS (CONT'D)

Symbol	Definition
GCTTT (T)	Time from guidance reference release.
GRR	Guidance reference release time.
IAT	Integral of A_m.
*IAT	Integral of $*A_m$.
K	Vehicle firing direction East of North.
KVAL	Thrust multiplier (local thrust correction constant).
LISPO	Instantaneous sea level specific impulse.
*MACH	Mach number as pulled from OMPT.
MASS (M)	Instantaneous mass.
M_i	Initial vehicle mass.
MM	Average mass between t and t_{-1}.
OMPT	Observed Mass Point Trajectory or Measured Trajectory.
PAF	Partial derivative of thrust with respect to acceleration difference.
PAM	Partial derivative of mass with respect to acceleration difference.
PAW	Partial derivative of flow rate with respect to acceleration difference.
$P_{C_{(1-n)}}$	Individual engine chamber pressure (from data tape).
PO (P_o)	Sea level pressure.
PO-*P	Local pressure difference.
*PRESS (*P)	Ambient pressure from OMPT.
PVF	Partial derivative of thrust with respect to velocity differe
PVM	Partial derivative of mass with respect to velocity differenc
PVW	Partial derivative of flow rate with respect to velocity difference.
*Q	Dynamic pressure from OMPT.
*QQ	Average dynamic pressure between time points.

DEFINITION OF SYMBOLS (CONT'D)

Symbol	Definition
RADD	Radial distance from pad.
*RADD	Radial distance from pad from OMPT.
*RHO	Local atmospheric density from OMPT.
R_o	Radial distance from geocentric center of the earth to launch pad.
$\vec{r}_o$	Initial components of the vehicle position vector referenced to the geocentric center of the earth.
RTIME (t)	Range time.
TT	Midpoint time between two data points.
VE (V_e)	Calculated earth-fixed velocity.
VE (V_e)	Measured earth-fixed velocity.
VOLUME	Total vehicle volume.
ϕ_o	Geodetic latitude of the launch site.
ψ_o	Geocentric latitude of launch site.
ω	Angular rotational velocity of earth.
$\ddot{X}_m$, $\ddot{Y}_m$, $\ddot{Z}_m$	Calculated platform (inertial) acceleration components.
*$\ddot{X}_m$, *$\ddot{Y}_m$, *$\ddot{Z}_m$	Measured platform (inertial) acceleration components.

DEFINITION OF SYMBOLS (CONT'D)

MATRIX IDENTIFICATION

$$[K] = \begin{vmatrix} \sin K & 0 & \cos K \\ 0 & 1 & 0 \\ -\cos K & 0 & \sin K \end{vmatrix}$$

$$[\phi_o] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -\cos\phi_o & -\sin\phi_o \\ 0 & \sin\phi_o & -\cos\phi_o \end{vmatrix}$$

$$[\omega] = \begin{vmatrix} \cos\omega T & -\sin\omega T & 0 \\ \sin\omega T & \cos\omega T & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$[\dot{\omega}] = \begin{vmatrix} -\omega\sin\omega T & -\omega\cos\omega T & 0 \\ \omega\cos\omega T & -\omega\sin\omega T & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$[\dot{\omega}_o] = \begin{vmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

$$\vec{r}_o = \begin{vmatrix} -R_o\cos K \sin B_o \\ R_o \cos B_o \\ R_o \sin K \sin B_o \end{vmatrix} = \begin{vmatrix} X_o \\ Y_o \\ Z_o \end{vmatrix}$$

where:

$B_o = (\phi_o - \psi_o)$

1.0 INTRODUCTION

The post-flight propulsion system analysis on each Saturn stage is usually performed by two methods. The first method of determining the stage propulsion system flight performance is a reconstruction of the telemetered flight data including calculated propellant residuals. This flight reconstruction method is a mathematical model of the stage propulsion system utilizing a table of influence coefficients to determine engine performance. The second method utilizes a trajectory simulation to generate adjustments that are enforced on the results from the flight reconstruction method so that the resulting calculated trajectory will match the observed trajectory. This second method, trajectory simulation, will be the point of discussion in this report.

The post-flight propulsion system performance simulation on each Saturn stage has been accomplished using either three-degree or six-degree-of-freedom trajectory computer program. These, combined with a weighted least squares program, provide the linear adjustment of the post-flight propulsion system parameters (thrust, mass loss rate and/or initial mass) and the predicted aerodynamic drag coefficient required for the trajectory parameters of the computed trajectory to match those derived from the tracking data observed during flight. (See Reference 1).

All of the tracking data observed during flight are converted from the various tracker measurements with the origin at the tracking site to the trajectory parameters in a coordinate system with the origin at the launch site. All of these tracking data are used in conjunction with the guidance system outputs to obtain a best estimate of the trajectory. The meteorological data observed at the launch time are combined with the best estimate of trajectory to yield what is called the Observed Mass Point Trajectory (OMPT) or measured trajectory. In all the propulsion system simulations performed on Saturn stages flown, an attempt has been made to compute a trajectory which matches the measured trajectory. There are several difficult problems (which will be discussed later) associated with this procedure; however, these problems could possibly be circumvented to a large degree if some of the trajectory parameters representing the altitude-time history of the measured trajectory are used as input to the simulation program.

A simulation program called the Trajectory Application Method (TAM) was developed to investigate the advantages and disadvantages of this approach.

2.0 SIMULATION PROBLEMS

Two basic problems are associated with the usual simulation programs that do not use the observed trajectory parameters representing altitude-time history from the measured trajectory or OMPT.

ALTITUDE EFFECTS - The meteorological data (atmospheric density, pressure, temperature, and wind data) are independently observed functions of altitude. These data are combined with trajectory parameters from the best estimate of trajectory to compute several altitude-dependent parameters, such as dynamic pressure, Mach number, and the thrust gain from increased altitude. The relationship between altitude and the meteorological data is observed independently and is assumed correct. However, the altitude from conventional simulation programs will initially be incorrect since the initial values of the propulsion system parameters have not been adjusted. Thus, meteorological data, which are referenced to the correct altitude, are introduced into the trajectory computation at either an earlier or later time (depending on the propulsion parameter adjustments required) than needed to satisfy the measured trajectory parameters. This scheme will eventually converge to the correct relationship as the adjustments to the propulsion system parameters and the aerodynamic drag coefficient converge to the appropriate solution.

ATTITUDE EFFECTS - The simulation programs often compute the attitude of the vehicle in the same fashion used in the precalculated or operational trajectory programs, except that the flight sequence and attitude program are considered fixed. The vehicle attitude as computed in this way is a function of the propulsion and trajectory parameters. Thus, if the propulsion parameters are incorrect, the resultant altitude-time history and attitudes will be incorrect. These, too, will eventually converge to the correct relationships as the propulsion parameters and aerodynamic drag coefficient adjustments converge to the appropriate solution. Also, the telemetered attitudes are sometimes used as inputs to the simulation programs. The problems associated with this approach are the time and bias shifts which may be inherent in telemetered data.

The effect of these two problems cannot be separated. After several iterations these problems can be resolved, but a large number of both man-hours and computer hours are required. Since this type of simulation requires a substantial amount of computer time for a single run, the turn-around time on the computer is longer than would be necessary with a more simplified simulation.

3.0 TAM DEVELOPMENT

The Trajectory Application Method was developed to circumvent some of the problems associated with the usual propulsion system simulations. In addition, a capability to estimate the instantaneous adjustments required was built into the program. The advantages and disadvantages of the TAM approach and some flight results are presented in subsequent paragraphs.

3.1 ALTITUDE AND ATTITUDE EFFECTS

The altitude and attitude effect problems are handled in the following way:

ALTITUDE EFFECTS - The following parameters, representing the altitude-time history, are input directly as a function of time: ambient pressure, density, Mach number, dynamic pressure and altitude. These parameters are used wherever they are required in the various computations insuring an altitude-time history compatible with the measured trajectory. Observed ambient temperature and wind effects are included in the calculation of dynamic pressure and Mach number. This eliminates the convergence problems associated with an incorrect altitude-time history, thereby placing the vehicle in its proper environment for drag considerations although adjustments to propulsion parameters may yet be required.

ATTITUDE EFFECTS - The measured platform (inertial) acceleration components from the OMPT or measured trajectory are input as a function of time. The unit vector of the acceleration is established.

$$*A_m = \left(*\ddot{X}_m^{\,2} + *\ddot{Y}_m^{\,2} + *\ddot{Z}_m^{\,2} \right)^{1/2}$$

$$\overrightarrow{*A_m'} = \frac{*\ddot{X}_m}{*A_m}\, i + \frac{*\ddot{Y}_m}{*A_m}\, j + \frac{*\ddot{Z}_m}{*A_m}\, k$$

The calculated platform acceleration components using the components of $\overrightarrow{*A_m'}$ are computed from the total acceleration $\left(\frac{FMI}{MASS}\right)$ proportional to those observed in the measured trajectory. This method replaces the transformation from the body-fixed to platform coordinate system.

$$\ddot{X}_m = \left(\frac{*\ddot{X}_m}{*A_m}\right) \left(\frac{FMI}{MASS}\right)$$

$$\ddot{Y}_m = \left(\frac{*\ddot{Y}_m}{*A_m}\right) \left(\frac{FMI}{MASS}\right)$$

$$\ddot{Z}_m = \left(\frac{*\ddot{Z}_m}{*A_m}\right) \left(\frac{FMI}{MASS}\right)$$

This scheme eliminates the necessity for either computing the attitude or directly inputting the telemetered attitude information and also insures compatibility with the measured trajectory parameter components.

3.2 GRAVITY CONSIDERATIONS

The trajectory parameters, after a correct solution for propulsion system parameters and aerodynamic drag coefficient adjustments have been obtained, should be identical to those in the measured trajectory or OMPT. Therefore, the gravity contributions to the trajectory parameters needed to convert from inertial space-fixed coordinates are identical. The components of acceleration, velocity, and position due to gravity are input directly from the measured trajectory as a function of time, thus forcing the gravity contributions to the trajectory parameters in TAM to be equivalent to those in the measured trajectory. This approach reduces the system of second order differential equations to be solved in the usual types of simulations to simple linear equations.

3.3 ESTIMATED INSTANTANEOUS ADJUSTMENTS

Redundant instantaneous adjustments to the propulsion system parameters and aerodynamic coefficients are made by comparing the (1) inertial acceleration,

(2) inertial velocity, and (3) earth-fixed velocity computed in TAM with those input from the measured trajectory at each time point. There is a requirement for partial derivatives of the trajectory parameters with respect to the parameters for which required adjustments are sought. These partial derivatives can be obtained by conventional perturbation methods. However, a highly simplified approach to determining these partial derivatives can be developed, when the appropriate simplifying assumption is made. The assumption to be made is that the deviations produced in inertial and earth-fixed acceleration resulting from a deviation in either the propulsion parameters or aerodynamic drag coefficient are approximately equivalent. The partial derivatives shown below can be used for both inertial and earth-fixed accelerations.

$$\left(\frac{\partial A_m}{F}\right)_t = -\left(\frac{1}{MASS}\right)_t$$

$$\left(\frac{\partial A_m}{M_i}\right)_t = \left(\frac{A_m}{MASS}\right)_t$$

$$\left(\frac{\partial A_m}{C_x}\right)_t = \left(\frac{*Q \cdot AREA}{MASS}\right)_t$$

These partial derivatives which are used for both inertial and earth-fixed accelerations have proven quite adequate in several test cases. Also, these partial derivatives may be integrated with respect to time to yield partial derivatives that may be used with velocity and position differences. Since position and velocity data are generally quite smooth, it may be desirable to use these in lieu of (or in addition to) acceleration data.

The instantaneous adjustments to the propulsion system parameters and the aerodynamic drag coefficient are determined by dividing the difference between the trajectory parameters computed in TAM and input from the measured trajectory or OMPT by these partial derivatives at each time point. It must be assumed that the entire difference in the trajectory parameters is a result of any one of the adjustments.

Engineering judgement combined with a priori knowledge of the accuracy of the parameters being adjusted can be used to give an estimate of how much each of the parameters to be adjusted contribute to the difference between the computed and measured trajectory parameters. The estimated instantaneous adjustments can be used to determine if any significant trends or discontinuities could exist in the parameters to be adjusted. (If any discontinuities exist in the trajectory parameter input from OMPT, these would also be reflected in the estimated instantaneous adjustments.)

Usually initial corrections are applied to the propulsion parameters before any attempt is made to establish the adjustment to the aerodynamic drag coefficient. Normally, only a constant shift to the propulsion parameters are considered even though estimated instantaneous adjustments are available. The instantaneous adjustments for the aerodynamic drag coefficient are considered applicable.

3.4 OVERALL ADJUSTMENTS

The difference between the trajectory parameters computed in TAM and input from OMPT can also be used with a conventional weighted least squares program to solve for an overall constant shift or bias in the propulsion system parameters. The partial derivatives required can either be those established using perturbation techniques or those partial derivatives determined in the simplified approach discussed under Paragraph 3.3. Usually, the a priori knowledge of the accuracy of the parameters to be adjusted is included in the least squares solution. Any conventional least squares computer program can be used with TAM to obtain propulsion adjustments such as the one shown below.

$$P = \left[W_O^{-1} + \sum \left(C^T W_{IP}^{-1} C \right) \right]^{-1} \left[\sum \left(C^T W_{IP}^{-1} R \right) \right]$$

where:

P = $(n \times 1)$ matrix of the propulsion adjustments.

W_O = $(n \times n)$ diagonal matrix consisting of the squares of the accuracies associated with measured propulsion parameters.

C = $(m \times n)$ matrix of partial derivatives of the trajectory data with respect to the parameters P.

W_{IP} = $(m \times m)$ diagonal matrix consisting of the squares of the accuracies associated with trajectory data.

R = $(m \times 1)$ matrix of the difference between the calculated and observed trajectory usually referred to as the residual matrix.

The diagonal elements of the covariance matrix $\left[W_O^{-1} + \sum \left(C^T W_{IP}^{-1} C \right) \right]^{-1}$ are the statistical variances of the parameter adjustments, and the off-diagonal elements are the covariances of the parameter adjustments. The square roots of the diagonal elements are the standard deviations of the adjustments and the off-diagonal elements are an indication of the correlation between the adjustments.

The matrix $\left[\sum (C^T W_{IP}^{-1} R) \right]$ represents the sum of the weighted squares of the residuals that are to be minimized subject to the constraints imposed by W_O.

3.5 ADVANTAGES AND DISADVANTAGES

There are several advantages and at least one disadvantage of the TAM approach over conventional simulation techniques.

ADVANTAGES - The advantages, other than those for which the scheme was originally devised, are discussed as follows:

Altitude Effects

a). The altitude-dependent functions are given versus time thereby eliminating tape interpolation.

b). Fewer equations are required, thus eliminating unnecessary computations.

Attitude Effects

a). Control equations are eliminated.

b). Moment and angular motion equations are eliminated.

c). Computation or input of attitude angles is eliminated.

Gravity Considerations

a). The system of second order differential equations, usually required in most simulations, is reduced to simple linear equations.

b). Complex integration schemes are not required.

Partial derivatives

The simplified approach for computing the partial derivatives eliminates the necessity of consecutive computer runs usually required for the conventional perturbation schemes.

All of these effects aid in separating the propulsion parameter and aerodynamic drag coefficient adjustments and also reduce the number of iterations required to obtain a valid solution. The TAM simulation technique is far less complex and more economical with respect to machine time than either the six-degree-of-freedom or three-degree-of-freedom simulation programs. Both the man-hours and machine-hours required for an evaluation are significantly reduced through the use of this program.

DISADVANTAGES - The TAM program was devised for use in post-flight evaluation of propulsion system performance with a high degree of dependence on input data obtained from the measured trajectory. This is the principal limitation and disadvantage of this approach. Since the altitude-time history and vehicle attitudes are used as inputs, TAM cannot be used to show the effects of propulsion parameter and aerodynamic drag perturbations on trajectory parameters.

3.6 FLIGHT RESULTS

The flight results using this simplified technique are compared with the flight results using a conventional three-degree-of-freedom (3D) simulation program for three S-IB stage Saturn IB flights in the table below. This table shows the excellent result obtainable with the TAM simulation technique. This approach is as efficient and reliable for use with the latter or upper stages as with those stages for which drag effects are of more concern.

		AVERAGE SEA LEVEL LONG. THRUST (LB)	AVERAGE TOTAL PROPELLANT FLOW-RATE (LB/SEC)	AVERAGE SEA LEVEL LONG. ISP (SEC)
AS-201	3D	1,613,560	6153.98	262.20
	TAM	1,612,754	6151.89	262.16
	% DEV.	-.05%	-.034%	-.015%
AS-203	3D	1,660,471	6285.18	264.19
	TAM	1,659,928	6283.40	264.18
	% DEV.	-.03%	-.03%	-.005%
AS-202	3D	1,631,558	6234.70	261.69
	TAM	1,631,374	6234.86	261.65
	% DEV.	-.01%	-.003%	-.001%

$$\% \text{ DEV.} = \frac{\text{TAM}-\text{3D}}{\text{3D}} \times 100$$

3.7 PROGRAM DESCRIPTION

The complete set of TAM equations is given in Appendix A. The integrations called for in these equations can be accurately accomplished using either Simpson's Rule or the Trapezoidal Rule since no complex equations of motion are present. The coordinate systems utilized in the equations are defined in Appendix B. The symbols and matrices used in Appendix A are defined on pages iv through viii. The input parameters required to satisfy the equations are given in Appendix C.

Output formats are generally considered arbitrary; however, in order to illustrate the Trajectory Application Method, a sample print format is given in Appendix D. This sample print is extracted from a typical S-IB stage calculated trajectory. This table shows the residuals between the measured and calculated trajectory parameters along with the instantaneous corrections to the propulsion parameters. Any one of these corrections will explain the difference between the two trajectories.

4.0 CONCLUSIONS

The TAM simulation technique, described under Paragraph 3.0, yields results which are well within the accuracy tolerances of the more conventional simulation schemes. The use of this program for the trajectory simulation represents a significant reduction in both man-hours and machine-hours required for an evaluation of the propulsion system performance. The TAM simulation technique is not a tool for studying the effects of propulsion system parameters and aerodynamic drag coefficient perturbations upon the trajectory parameters representing the altitude-time history or vehicle-attitude, but represents a most efficient means of obtaining the post-flight evaluation of the propulsion system performance.

APPENDIX A

EQUATIONS:

#1 $$CF_{(1-n)} = CFV_{(1-n)} - \left(\frac{A_{E_{1-n}} P_o}{A_{T_{1-n}} P_{C_{1-n}}} \right)$$

#2 $$FO_{1-n} = (CF)_{1-n} (A_T)_{1-n} (P_C)_{1-n}$$

#3 $$F\text{-}ENG_{1-n} = FO_{1-n} + A_{E_{1-n}} (P_o - {}^*PRESS)$$

#4 $$^{**}FJI = [\cos 6° \, \Sigma \, (F\text{-}ENG_{1-4} + FE_{1-4}) + \cos 3° \, \Sigma \, (F\text{-}ENG_{5-8}) + \Sigma \, FE_{5-8}] \, KVAL$$

#5 $$FMI = FJI + FAI + FB$$

#6 $$FB = ({}^*RHO) \, (VOLUME)$$

#7 $$FAI = - (BDRAG) - (DRAG)$$

#8 $$DRAG = (CD) \, ({}^*Q) \, (AREA)$$

#9 $$^{**}FJIO = \cos 6° \, \Sigma \, [FO_{1-4} + FEI_{1-4}] + \cos 3° \, \Sigma \, [FO_{5-8}] + \Sigma \, FEI_{5-8}$$

#10 $$LISPO = \left(\frac{FJIO}{DFLOX} \right)$$

#11 $$ALISPO = \left(\frac{AVFJIO}{AVDFLOX} \right)$$

#12 $$AVFJIO = \left(\frac{1}{FLTTT} \right) \int_{t_i}^{t_n} (FJIO) \, dt$$

* Input data from tape

** The equations illustrate the S-IB stage, Saturn IB, where the 4 inboard engines are canted 3° and the 4 outboard engines are canted 6°. However, on stages where engines are not canted these considerations can be dropped.

#13 $AVDFLOX = \left(\frac{1}{FLTTT}\right)\int_{t_i}^{t_n} (DFLOX)\, dt$

#14 $MASS = M_i + \int_{t_i}^{t_n} (DMASS)\, dt$

#15 $A_m = \left(\frac{FMI}{MASS}\right)$

#16 $\ddot{X}_m = \left(\frac{*\ddot{X}_m}{*A_m}\right) A_m,\qquad \ddot{Y}_m = \left(\frac{*\ddot{Y}_m}{*A_m}\right) A_m,\qquad \ddot{Z}_m = \left(\frac{*\ddot{Z}_m}{*A_m}\right) A_m$

#17 $\vec{\dot{X}}_m = \int_{t_i}^{t_n} (\vec{\ddot{X}}_m)\, dt,\qquad \vec{X}_m = \iint_{t_i}^{t_n} (\vec{\ddot{X}}_m)\, dt$

#18 $IAT = \int_{t_i}^{t_n} (A_m)\, dt \qquad *IAT = \int_{t_i}^{t_n} (*A_m)\, dt$

#19 $DIAT = (IAT - *IAT)$

#20 $\vec{\ddot{X}}_s = \vec{\ddot{X}}_m + *\vec{\ddot{X}}_G$

$\vec{\dot{X}}_s = \vec{\dot{X}}_{so} + \vec{\dot{X}}_m + *\vec{\dot{X}}_G$

$\vec{X}_{sc} = \vec{r}_o + \vec{\dot{X}}_{so}\, T + \vec{X}_m + *\vec{X}_G$

$\vec{X}_s = \vec{X}_{sc} - \vec{r}_o$

#21 $\vec{\dot{X}}_e = \{[K]^T[\phi_o]^T[\omega]^T[\phi_o][K]\, \vec{\dot{X}}_s\} + \{[K]^T[\phi_o]^T[\dot{\omega}]^T[\phi_o][K]\, \vec{X}_{sc}\}$

$\vec{X}_e = \{[K]^T[\phi_o]^T[\omega]^T[\phi_o][K]\, \vec{X}_{sc}\} - \vec{r}_o$

#22 $DA = \left| A_m - {}^*A_m \right|$

#23 $V_e = \left(\dot{X}_e^{\,2} + \dot{Y}_e^{\,2} + \dot{Z}_e^{\,2} \right)^{1/2}$

#24 $DVE = \left| V_e - {}^*V_e \right|$

#25 ${}^*A_m = \left({}^*\ddot{X}_m^{\,2} + {}^*\ddot{Y}_m^{\,2} + {}^*\ddot{Z}_m^{\,2} \right)^{1/2}$

#26 $DDVE = \left[\frac{(DVE_t - DVE_{t-1})}{(t - t_{-1})} \right]$

#27 $DDIAT = \left[\frac{(DIAT_t - DIAT_{t-1})}{(t - t_{-1})} \right]$

#28 $TT = \frac{1}{2} \; (t + t_{-1})$

#29 $FF = \frac{1}{2} \; (FJI_t + FJI_{t-1})$

#30 $DD = \frac{1}{2} \; (DRAG_t + DRAG_{t-1})$

#31 ${}^*AA = \frac{1}{2} \; ({}^*A_{m_t} + {}^*A_{m_{t-1}})$

#32 ${}^*QQ = \frac{1}{2} \; ({}^*Q_t + {}^*Q_{t-1})$

#33 $MM = \frac{1}{2} \; (MASS_t + MASS_{t-1})$

#34 $CCDD = \frac{1}{2} \; (CD_t + CD_{t-1})$

#35 $DFA = -(DA)\ (MASS)$

#36 $DFVE = -(DDVE)\ (MM)$

#37 $DFVI = -(DDIAT)\ (MM)$

#38 $$DMA = \left(\frac{FMI}{*A_m}\right) - MASS$$

#39 $$DMVE = \left(\frac{FF - DD}{*AA - DDVE}\right) - MM$$

#40 $$DMVI = \left(\frac{FF - DD}{*AA - DDIAT}\right) - MM$$

#41 $$*AMACH = \frac{1}{2}\ (*MACH_t + *MACH_{t-1})$$

#42 $$DCXA = \left[\frac{(DA)\ (MASS)}{*Q\ (AREA)}\right]$$

#43 $$DCXVE = \left[\frac{(DDVE)\ (MM)}{*QQ\ (AREA)}\right]$$

#44 $$DCXVI = \left[\frac{(DDIAT)\ (MM)}{*QQ\ (AREA)}\right]$$

#45 $CXCA = CD + DCXA$

#46 $CXCVE = CCDD + DCXVE$

#47 $CXCVI = CCDD + DCXVI$

#48 PVM = $\frac{\partial M}{\partial DVE} = \left(\frac{MM}{*AA}\right)$

#49 PAM = $\frac{\partial M}{\partial A_m} = \left(\frac{MASS}{A_m}\right)$

#50 PVF = $\frac{\partial F}{\partial DVE} = -\ MM$

#51 PAF = $\frac{\partial F}{\partial A_m} = -\ MASS$

#52 PVW = $\frac{\partial \dot{w}}{\partial DVE} = \left[\frac{(MM)}{(*AA)\ (TT)}\right]$

#53 PAW = $\frac{\partial \dot{w}}{\partial A_m} = \left(\frac{MASS}{A_m t}\right)$

#54 RADD = $\left(X_e^{\ 2} + Y_e^{\ 2} + Z_e^{\ 2}\right)^{1/2}$

#55 $\vec{\dot{X}}_{so} = [K]^T[\phi_o]^T[\dot{\omega}_o][\phi_o][K]\ \vec{r}_o$

APPENDIX B

PLUMBLINE COORDINATE SYSTEM DEFINITIONS

1. Earth-Fixed Coordinate System. The earth-fixed coordinate system is defined as a right-handed Cartesian system with the projection of the center of gravity of the complete vehicle at or prior to First Motion (FLTTT = 0) on the reference ellipsoid as the origin.

The X-Z plane is tangent to the reference ellipsoid at the origin of the coordinate system. The positive X-axis is oriented in the flight azimuth direction; the positive Y-axis is above and normal to the X-Z plane; the positive Z-axis is in a right-handed relation to the X-Y axes. The origin of this earth-fixed system rotates with an angular velocity equal to that of the earth.

Launch pad coordinates are defined with respect to the reference ellipsoid chosen to represent the earth and its gravitational field. The elevation of the launch site above mean sea level and the position of the center of gravity of the complete vehicle are treated as an elevation above the reference ellipsoid.

$$\vec{X}_e = \begin{vmatrix} X_E \\ Y_E \\ Z_E \end{vmatrix} = \text{Earth-Fixed Cartesian displacement components} = \begin{vmatrix} \text{XXXE} \\ \text{YYYE} \\ \text{ZZZE} \end{vmatrix}$$

$$\vec{\dot{X}}_e = \begin{vmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \end{vmatrix} = \text{Earth-Fixed Cartesian velocity components} = \begin{vmatrix} \text{DXXE} \\ \text{DYYE} \\ \text{DZZE} \end{vmatrix}$$

$$\vec{\ddot{X}}_e = \begin{vmatrix} \ddot{X}_E \\ \ddot{Y}_E \\ \ddot{Z}_E \end{vmatrix} = \text{Earth-Fixed Cartesian acceleration components} = \begin{vmatrix} \text{DDXE} \\ \text{DDYE} \\ \text{DDZE} \end{vmatrix}$$

2. Space-Fixed Coordinate System. The orientation of the space-fixed coordinate system is identical to the earth-fixed system at and prior to guidance reference release (GCTTT = 0). The origin is a point fixed in space, and the coordinate system remains fixed in space as oriented at guidance reference release (GRR).

$$\vec{X}_s = \begin{vmatrix} X_S \\ Y_S \\ Z_S \end{vmatrix} = \text{Space-Fixed Cartesian displacement components} = \begin{vmatrix} XXXS \\ YYYS \\ ZZZS \end{vmatrix}$$

$$\vec{\dot{X}}_s = \begin{vmatrix} \dot{X}_S \\ \dot{Y}_S \\ \dot{Z}_S \end{vmatrix} = \text{Space-Fixed Cartesian velocity components} = \begin{vmatrix} DXXS \\ DYYS \\ DZZS \end{vmatrix}$$

$$\vec{\ddot{X}}_s = \begin{vmatrix} \ddot{X}_S \\ \ddot{Y}_S \\ \ddot{Z}_S \end{vmatrix} = \text{Space-Fixed Cartesian acceleration components} = \begin{vmatrix} DDXS \\ DDYS \\ DDZS \end{vmatrix}$$

3. Inertial Platform Coordinate System. The inertial platform is a gyro stabilized reference element oriented at guidance reference release (GRR) time identical to the earth-fixed and space-fixed coordinate systems. The coordinate system remains fixed in inertial space as oriented at GRR. Coordinates in the inertial system do not include the effects of gravity and the initial rotational velocity of the earth.

$$\vec{X}_m = \begin{vmatrix} X_M \\ Y_M \\ Z_M \end{vmatrix} = \text{Platform displacement components} = \begin{vmatrix} XXXM \\ YYYM \\ ZZZM \end{vmatrix}$$

$$\vec{\dot{X}}_m = \begin{vmatrix} \dot{X}_M \\ \dot{Y}_M \\ \dot{Z}_M \end{vmatrix} = \text{Platform velocity components} = \begin{vmatrix} DXXM \\ DYYM \\ DZZM \end{vmatrix}$$

$$\vec{\ddot{X}}_m = \begin{vmatrix} \ddot{X}_M \\ \ddot{Y}_M \\ \ddot{Z}_M \end{vmatrix} = \text{Platform acceleration components} = \begin{vmatrix} DDXM \\ DDYM \\ DDZM \end{vmatrix}$$

4. Gravitational Components

$$\vec{X}_g = \begin{vmatrix} X_G \\ Y_G \\ Z_G \end{vmatrix} = \text{Gravitational displacement components (Initiated at GRR Time)} = \begin{vmatrix} XXXG \\ YYYG \\ ZZZG \end{vmatrix}$$

4. Gravitational Components (Cont'd)

$$\vec{\dot{X}}_g = \begin{vmatrix} \dot{X}_G \\ \dot{Y}_G \\ \dot{Z}_G \end{vmatrix} = \text{Gravitational velocity components (Initiated at GRR Time)} = \begin{vmatrix} DXXG \\ DYYG \\ DZZG \end{vmatrix}$$

$$\vec{\ddot{X}}_g = \begin{vmatrix} \ddot{X}_G \\ \ddot{Y}_G \\ \ddot{Z}_G \end{vmatrix} = \text{Gravitational acceleration components} = \begin{vmatrix} DDXG \\ DDYG \\ DDZG \end{vmatrix}$$

APOLLO COORDINATE SYSTEM (SEE REFERENCE 2).

The previous coordinate systems as described are similar to Project Apollo Coordinate System Standards No. 10, 13, and 12. A simple matrix rotation and, in case of the space-fixed system, a shift of origin will convert the data to Apollo Standard Systems.

$$\vec{X}_e(\text{Apollo}) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \vec{X}_e$$

$$\vec{X}_{sc}(\text{Apollo}) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} (\vec{X}_s + \vec{r}_o)$$

$$\vec{X}_m\ (\text{Apollo}) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \vec{X}_m$$

APPENDIX C

INPUTS:

A. The following parameters are preloaded as constants.

$A_{E_{(I-n)}}$	K	ϕ_o	R_o
AREA	M_i	PO	VOLUME
$A_{T_{(I-n)}}$	ω	ψ_o	$FEI_{(I-n)}$

B. Parameters input from the Observed Mass Point Trajectory (OMPT).

*ALT	*RHO	$*\ddot{X}_g$	$*\dot{X}_g$	$*X_g$
$*A_m$	$*\ddot{X}_m$	$*\ddot{Y}_g$	$*\dot{Y}_g$	$*Y_g$
*MACH	$*\ddot{Y}_m$	$*\ddot{Z}_g$	$*\dot{Z}_g$	$*Z_g$
*PRESS	$*\ddot{Z}_m$	*RADD		
*Q	$*V_e$			

C. Parameters input from the data tape.

$CFV_{(I-n)}$	DMASS	$P_{C_{(I-n)}}$
DFLOX	$FE_{(I-n)}$	

D. Initial trajectory parameters.

$\vec{X}_m$ $\dot{\vec{X}}_m$ $\vec{r}_o$ $\dot{\vec{X}}_{so}$

APPENDIX D

RTIME	DVE	DIAT	DA	DMVE	DMVI	DMA	DFVE	DFVI	DFA	RTIME
.11	.602	.000	.012825	.000	.000	66.562	-.00	-.00	-776.41	.11
1.00	.125	.018	.027691	-2763.151	105.863	142.665	32399.89	-1241.32	-1681.60	1.00
2.00	.153	.046	.027383	142.386	141.335	140.096	-1689.55	-1677.08	-1673.44	2.00
3.00	.180	.076	.027149	135.447	147.745	137.461	-1625.63	-1773.24	-1657.65	3.00
4.00	.207	.103	.027592	132.941	135.824	135.007	-1609.30	-1644.20	-1640.56	4.00
5.00	.234	.132	.027747	131.634	141.070	132.567	-1603.29	-1718.21	-1618.38	5.00
6.00	.261	.160	.027087	130.323	131.193	130.130	-1594.37	-1605.02	-1595.40	6.00
7.00	.288	.186	.026825	128.676	128.363	127.693	-1581.06	-1577.22	-1572.45	7.00
8.00	.315	.213	.026906	126.880	126.551	126.905	-1565.92	-1561.86	-1569.72	8.00
9.00	.342	.239	.027671	128.132	123.730	129.104	-1589.70	-1535.08	-1606.60	9.00
10.00	.371	.268	.028919	131.373	130.753	133.438	-1639.96	-1632.21	-1670.95	10.00
11.00	.400	.297	.030255	135.764	136.358	138.119	-1705.05	-1712.49	-1739.68	11.00
12.00	.431	.328	.032089	141.265	141.096	144.936	-1784.47	-1782.34	-1836.14	12.00
13.00	.465	.361	.034598	149.099	147.869	154.123	-1894.86	-1879.22	-1964.88	13.00
14.00	.501	.397	.037614	159.511	159.644	166.103	-2040.08	-2041.79	-2131.16	14.00
15.00	.540	.437	.041070	171.899	172.992	179.451	-2212.29	-2226.36	-2316.53	15.00
16.00	.583	.480	.045342	186.783	187.330	196.054	-2418.52	-2425.60	-2546.25	16.00
17.00	.630	.528	.050513	205.042	205.856	215.916	-2670.92	-2681.52	-2820.93	17.00
18.00	.684	.581	.056434	226.320	226.865	238.648	-2965.70	-2972.84	-3136.56	18.00
19.00	.743	.640	.061173	247.945	247.076	255.613	-3268.50	-3257.03	-3379.61	19.00
20.00	.804	.701	.061599	253.590	254.176	253.637	-3363.47	-3371.25	-3374.71	20.00
21.00	.865	.762	.060754	249.969	251.152	248.152	-3336.15	-3351.94	-3322.06	21.00
22.00	.924	.822	.058995	242.237	242.750	238.226	-3253.03	-3259.92	-3209.17	22.00
23.00	.981	.879	.055111	228.220	228.903	219.989	-3084.10	-3093.34	-2982.27	23.00
24.00	1.034	.933	.053017	209.743	214.232	209.216	-2852.24	-2913.28	-2853.92	24.00
25.00	1.087	.986	.053113	206.751	207.907	207.225	-2828.89	-2844.71	-2844.01	25.00
26.00	1.139	1.039	.052654	203.557	204.863	203.086	-2802.32	-2820.30	-2804.47	26.00
27.00	1.190	1.091	.050928	196.827	198.534	194.169	-2726.52	-2750.18	-2698.07	27.00
28.00	1.239	1.140	.048310	185.893	187.968	182.045	-2591.25	-2620.18	-2545.64	28.00
29.00	1.285	1.187	.044673	171.710	174.196	166.361	-2408.82	-2443.70	-2341.26	29.00
30.00	1.327	1.229	.039675	153.469	156.587	146.721	-2166.93	-2210.95	-2078.44	30.00
31.00	1.352	1.265	.032346	128.301	132.344	117.581	-1823.58	-1881.04	-1676.79	31.00
32.00	1.389	1.295	.026222	97.358	106.090	94.175	-1392.99	-1517.93	-1351.90	32.00
33.00	1.413	1.321	.025593	86.344	92.916	90.843	-1243.32	-1337.95	-1312.14	33.00
34.00	1.436	1.346	.024792	81.903	88.922	86.933	-1186.70	-1288.41	-1263.51	34.00
35.00	1.457	1.370	.023719	73.621	84.389	82.218	-1073.41	-1230.41	-1202.54	35.00
36.00	1.476	1.392	.020613	65.772	76.432	70.769	-965.08	-1121.49	-1041.71	36.00
37.00	1.492	1.411	.016936	51.663	64.620	57.478	-762.92	-954.26	-851.5	37.00
38.00	1.503	1.426	.012892	39.524	50.060	43.092	-587.45	-744.05	-642.53	38.00
39.00	1.511	1.438	.010099	23.864	39.743	33.345	-357.01	-594.57	-500.43	39.00
40.00	1.516	1.448	.009630	18.591	32.526	32.067	-279.93	-489.74	-484.37	40.00
41.00	1.519	1.457	.007652	10.421	28.300	24.663	-157.90	-428.81	-374.87	41.00
42.00	1.522	1.463	.005047	7.796	20.188	16.071	-118.88	-307.82	-245.82	42.00
43.00	1.520	1.467	.002185	-6.290	11.517	6.874	96.51	-176.72	-105.81	43.00
44.00	1.517	1.467	-.000938	-10.153	2.099	-3.069	156.80	-32.41	47.55	44.00
45.00	1.507	1.464	-.004411	-28.758	-8.503	-13.531	447.03	132.17	211.02	45.00
46.00	1.495	1.457	-.011539	-36.284	-23.701	-34.956	567.75	370.85	548.78	46.00
47.00	1.473	1.441	-.019540	-67.947	-46.737	-58.598	1070.26	736.18	926.07	47.00
48.00	1.444	1.417	-.029019	-85.829	-72.046	-85.711	1360.91	1142.38	1363.54	48.00
49.00	1.404	1.383	-.039476	-116.814	-100.598	-115.109	1864.62	1605.78	1843.56	49.00

APPENDIX D (CONT'D)

RTIME	DVE	DIAT	DA	DMVE	DMVI	DMA	DFVE	DFVI	UFA	RTIME
50.00	1.354	1.338	-.049219	-144.694	-128.231	-141.697	2325.14	2060.60	2284.54	50.00
51.00	1.292	1.284	-.059423	-178.029	-155.253	-168.937	2879.50	2511.12	2741.16	51.00
52.00	1.226	1.220	-.069267	-186.548	-181.441	-194.368	3037.31	2954.15	3175.43	52.00
53.00	1.146	1.146	-.079694	-222.154	-207.239	-220.679	3641.92	3397.42	3630.22	53.00
54.00	1.058	1.060	-.090755	-241.397	-234.198	-248.025	3984.93	3866.08	4108.62	54.00
55.00	.956	.967	-.097678	-277.216	-254.196	-263.547	4607.55	4224.95	4394.99	55.00
56.00	.845	.862	-.111695	-297.548	-280.453	-297.206	4978.21	4692.20	4988.69	56.00
57.00	.721	.743	-.126238	-328.643	-314.492	-331.929	5534.22	5295.92	5607.58	57.00
58.00	.582	.609	-.141571	-362.551	-349.598	-367.400	6144.84	5925.30	6247.22	58.00
59.00	.430	.464	-.146800	-392.907	-374.461	-381.330	6701.31	6386.70	6523.65	59.00
60.00	.275	.315	-.150[illegible]92	-395.803	-380.918	-380.720	6790.61	6535.23	6550.45	60.00
61.00	.115	.159	-.161990	-401.048	-392.553	-405.154	6919.24	6772.68	7009.31	61.00
62.00	-.058	-.008	-.172641	-430.517	-415.213	-426.502	7469.39	7203.86	7420.82	62.00
63.00	-.168	-.112	-.035254	-271.994	-255.305	-86.893	4722.21	4432.45	1505.33	63.00
64.00	-.140	-.090	.079380	68.941	54.544	194.677	-1192.52	-943.50	-3362.47	64.00
65.00	-.055	.001	.101640	209.170	222.329	248.201	-3614.16	-3841.51	-4290.16	65.00
66.00	.058	.123	.140476	274.063	294.843	339.945	-4737.80	-5097.02	-5877.46	66.00
67.00	.220	.291	.194641	390.170	405.294	468.846	-6741.67	-7003.00	-8096.11	67.00
68.00	.363	.432	.087691	342.830	338.246	207.916	-5943.33	-5863.87	-3618.58	68.00
69.00	.383	.455	-.041981	46.229	53.976	-97.888	-808.49	-944.02	1720.27	69.00
70.00	.280	.357	-.154971	-238.175	-227.489	-355.087	4207.67	4018.87	6305.89	70.00
71.00	.089	.169	-.220713	-433.319	-426.917	-496.715	7737.27	7622.96	8917.51	71.00
72.00	-.154	-.073	-.263[illegible]70	-541.345	-540.035	-582.473	9774.61	9750.96	10577.34	72.00
73.00	-.439	-.355	-.299757	-625.256	-616.426	-649.584	11422.97	11261.64	11938.81	73.00
74.00	-.740	-.654	-.296907	-645.535	-640.656	-630.916	11938.15	11847.92	11739.89	74.00
75.00	-1.025	-.934	-.263502	-600.438	-589.631	-549.369	11238.74	11036.46	10343.23	75.00
76.00	-1.275	-1.181	-.232557	-515.898	-511.313	-475.681	9770.40	9683.57	9061.63	76.00
77.00	-1.495	-1.398	-.201659	-444.622	-438.680	-404.641	8520.14	8406.26	7799.65	77.00
78.00	-1.679	-1.579	-.160181	-366.923	-359.448	-315.314	7114.18	6969.24	6149.27	78.00
79.00	-1.826	-1.724	-.129659	-285.911	-283.144	-250.515	5607.04	5552.77	4940.20	79.00
80.00	-1.942	-1.838	-.097801	-221.670	-218.115	-185.406	4396.47	4325.95	3698.22	80.00
81.00	-2.037	-1.932	-.089482	-179.164	-175.862	-166.237	3594.33	3528.09	3354.14	81.00
82.00	-2.120	-2.013	-.073670	-152.016	-149.219	-134.352	3085.59	3028.84	2743.31	82.00
83.00	-2.183	-2.068	-.043767	-114.655	-100.415	-78.271	2355.03	2062.54	1617.19	83.00
84.00	-2.215	-2.101	-.023154	-55.657	-57.488	-40.623	1156.52	1194.57	848.90	84.00
85.00	-2.230	-2.113	-.004430	-26.871	-20.600	-7.624	564.75	432.97	161.15	85.00
86.00	-2.227	-2.111	.006195	5.825	2.757	10.457	-123.84	-58.60	-223.56	86.00
87.00	-2.220	-2.095	.025[illegible]76	11.929	26.640	41.877	-256.40	-572.58	-904.66	87.00
88.00	-2.173	-2.059	.044175	77.301	58.981	71.831	-1679.24	-1281.26	-1568.73	88.00
89.00	-2.115	-2.000	.068086	93.340	95.232	108.563	-2050.23	-2091.78	-2398.31	89.00
90.00	-2.034	-1.917	.094646	127.911	130.405	148.018	-2841.67	-2897.08	-3306.62	90.00
91.00	-1.927	-1.810	.115136	165.412	166.097	176.310	-3717.73	-3733.13	-3986.44	91.00
92.00	-1.807	-1.686	.131662	181.927	187.890	197.841	-4137.27	-4272.87	-4525.10	92.00
93.00	-1.670	-1.550	.140671	203.175	202.539	207.346	-4672.65	-4658.03	-4794.68	93.00
94.00	-1.526	-1.404	.151587	210.526	213.822	218.794	-4895.84	-4972.49	-5116.84	94.00
95.00	-1.368	-1.247	.161791	226.528	224.934	228.551	-5327.77	-5290.28	-5405.67	95.00
96.00	-1.206	-1.085	.160434	226.509	226.359	223.167	-5387.62	-5384.06	-5337.94	96.00
97.00	-1.042	-.923	.162813	226.244	222.813	221.687	-5442.92	-5360.37	-5364.03	97.00
98.00	-.882	-.762	.155056	214.837	217.028	212.361	-5226.94	-5280.25	-5195.01	98.00
99.00	-.724	-.604	.157703	209.290	209.562	206.422	-5148.39	-5155.09	-5105.98	99.00

APPENDIX D (CONT'D)

RTIME	DVE	DIAT	DA	DMVE	DMVI	DMA	DFVE	DFVI	DFA	RTIME
100.00	-.565	-.445	.160981	206.416	206.267	206.593	-5133.88	-5130.19	-5166.37	100.00
101.00	-.404	-.285	.159290	204.936	203.326	200.400	-5153.22	-5112.75	-5066.84	101.00
102.00	-.247	-.127	.157041	195.140	196.860	193.731	-4960.26	-5003.99	-4950.68	102.00
103.00	-.091	.030	.155788	190.096	191.019	188.592	-4881.77	-4905.48	-4866.96	103.00
104.00	.065	.183	.151010	187.027	183.735	179.181	-4853.03	-4767.61	-4674.60	104.00
105.00	.211	.331	.146100	171.397	174.274	169.935	-4495.82	-4571.29	-4481.34	105.00
106.00	.356	.474	.138803	167.103	163.862	158.232	-4430.67	-4344.71	-4218.15	106.00
107.00	.490	.608	.129986	152.048	151.463	145.181	-4075.89	-4060.19	-3913.35	107.00
108.00	.615	.733	.121174	137.495	138.643	132.623	-3726.31	-3757.40	-3613.68	108.00
109.00	.733	.850	.111301	127.841	125.866	119.355	-3502.43	-3448.31	-3287.71	109.00
110.00	.839	.955	.100338	112.534	112.202	105.408	-3116.87	-3107.67	-2935.45	110.00
111.00	.933	1.049	.087404	98.037	97.506	89.971	-2744.76	-2729.88	-2532.29	111.00
112.00	1.012	1.129	.072776	80.419	81.398	73.386	-2275.72	-2303.44	-2087.89	112.00
113.00	1.077	1.194	.058299	65.353	65.033	57.572	-1869.61	-1860.47	-1656.05	113.00
114.00	1.127	1.244	.043428	48.801	48.980	41.023	-1411.60	-1416.78	-1193.23	114.00
115.00	1.164	1.280	.029020	35.055	34.121	27.479	-1025.16	-997.83	-807.93	115.00
116.00	1.186	1.302	.015159	20.603	20.417	14.056	-609.05	-603.55	-417.75	116.00
117.00	1.194	1.310	.001142	7.336	7.224	1.037	-219.21	-215.87	-31.16	117.00
118.00	1.188	1.303	-.012273	-5.441	-5.729	-11.795	164.37	173.07	358.27	118.00
119.00	1.167	1.282	-.027590	-18.151	-18.272	-23.997	554.36	558.05	736.93	119.00
120.00	1.132	1.247	-.041819	-29.987	-30.171	-35.595	925.98	931.65	1105.21	120.00
121.00	1.082	1.197	-.057801	-42.254	-42.266	-48.138	1319.26	1319.66	1511.30	121.00
122.00	1.016	1.131	-.073536	-54.398	-54.449	-59.912	1717.40	1718.99	1901.97	122.00
123.00	.935	1.049	-.089119	-65.818	-65.732	-71.025	2101.11	2098.36	2279.91	123.00
124.00	.837	.952	-.104641	-76.668	-76.728	-81.562	2474.86	2476.80	2647.55	124.00
125.00	.725	.840	-.119431	-86.610	-86.557	-91.034	2827.17	2828.70	2988.12	125.00
126.00	.598	.712	-.134418	-95.982	-96.028	-100.173	3168.35	3169.86	3325.29	126.00
127.00	.455	.570	-.149469	-104.904	-104.960	-108.880	3502.24	3504.12	3655.62	127.00
128.00	.298	.412	-.164424	-113.385	-113.374	-117.048	3828.83	3828.46	3975.18	128.00
129.00	.125	.240	-.179419	-121.293	-121.304	-124.788	4143.30	4143.67	4287.34	129.00
130.00	-.062	.052	-.194559	-128.793	-128.887	-132.179	4450.88	4454.14	4594.53	130.00
131.00	-.264	-.151	-.210685	-135.753	-136.255	-139.792	4746.51	4764.06	4916.30	131.00
132.00	-.483	-.369	-.226052	-143.243	-143.543	-146.454	5067.48	5078.10	5211.55	132.00
133.00	-.718	-.604	-.241586	-150.691	-150.027	-152.790	5394.41	5370.64	5502.07	133.00
134.00	-.968	-.854	-.257345	-156.338	-156.234	-158.836	5663.91	5660.11	5785.99	134.00
135.00	-1.234	-1.120	-.273330	-161.874	-162.218	-164.596	5935.74	5948.37	6072.18	135.00
136.00	-1.517	-1.402	-.289528	-168.167	-167.722	-170.059	6242.21	6225.70	6351.19	136.00
137.00	-1.815	-1.700	-.305834	-172.821	-172.927	-175.187	6494.21	6498.20	6623.54	137.00
138.00	-2.129	-2.015	-.322419	-177.691	-178.131	-180.062	6760.14	6776.88	6892.85	138.00
138.41	-2.326	-2.149	-.340448	-267.565	-181.368	-182.622	10268.81	6960.67	7026.75	138.41
139.00	-2.460	-2.347	-.339418	-124.349	-183.871	-184.769	4602.20	7100.85	7161.73	139.00
140.00	-2.808	-2.696	-.356804	-187.125	-187.609	-189.272	7299.03	7317.90	7429.31	140.00
141.00	-3.140	-2.548	.643665	-175.824	78.324	346.781	6855.09	-3053.72	-13429.01	141.00
142.00	9.927	3.828	-1.331055	8919.303	4352.492	-1280.263	-266743.55	-130167.03	26998.00	142.00

APPENDIX D (CONT'D)

RTIME	•AMACH	CXCVE	CXCVI	CCDD	•MACH	CXCA	CD	FJ1	FM1	RTIME
.11	.001	.6000	.6000	.600	.002	89.2992	1.200	709716.0	706913.1	.11
1.00	.010	-727.4609	29.1169	1.200	.018	22.1893	1.200	714355.8	712357.3	1.00
2.00	.019	19.5417	19.4063	1.200	.021	17.2732	1.200	719588.4	718560.8	2.00
3.00	.023	14.0722	15.2410	1.200	.025	12.3651	1.200	722112.2	722031.0	3.00
4.00	.027	10.0260	10.2114	1.200	.030	8.7880	1.200	724322.0	724160.0	4.00
5.00	.033	7.3007	7.7330	1.200	.036	6.4307	1.200	724343.1	724069.7	5.00
6.00	.039	5.5178	5.5467	1.200	.042	4.9180	1.200	724116.1	723700.2	6.00
7.00	.045	4.3431	4.3355	1.200	.049	3.9255	1.200	724024.0	723433.2	7.00
8.00	.052	3.5386	3.5325	1.186	.056	3.2533	1.173	723971.5	723192.4	8.00
9.00	.060	2.9957	2.9362	1.146	.063	2.7851	1.119	725091.0	724125.3	9.00
10.00	.067	2.6009	2.5978	1.092	.071	2.4467	1.065	726378.6	725215.4	10.00
11.00	.075	2.3099	2.3134	1.047	.079	2.1961	1.030	727381.8	725986.6	11.00
12.00	.083	2.0922	2.0909	1.012	.087	2.0074	.995	728374.9	726735.0	12.00
13.00	.091	1.9271	1.9192	.977	.096	1.8624	.960	729766.7	727872.5	13.00
14.00	.100	1.7991	1.7998	.942	.104	1.7492	.925	731233.2	729077.7	14.00
15.00	.109	1.6970	1.7020	.912	.114	1.6583	.898	732533.6	730081.3	15.00
16.00	.118	1.6156	1.6178	.885	.123	1.5845	.872	733840.8	731074.3	16.00
17.00	.128	1.5506	1.5533	.859	.133	1.5256	.845	735089.2	732003.9	17.00
18.00	.138	1.4989	1.5015	.832	.143	1.4793	.819	736359.1	732967.0	18.00
19.00	.148	1.4534	1.4512	.809	.153	1.4256	.798	737558.5	733833.8	19.00
20.00	.158	1.3727	1.3741	.788	.163	1.3294	.778	738779.0	734683.8	20.00
21.00	.168	1.2813	1.2837	.768	.174	1.2409	.757	739802.6	735343.4	21.00
22.00	.179	1.1943	1.1953	.747	.184	1.1549	.737	740834.5	735998.2	22.00
23.00	.190	1.1094	1.1105	.729	.195	1.0705	.721	741838.2	736568.9	23.00
24.00	.201	1.0304	1.0372	.714	.206	1.0074	.706	742870.2	737146.9	24.00
25.00	.212	.9823	.98[illegible]9	.698	.217	.9628	.690	743915.1	737722.2	25.00
26.00	.223	.9383	.94[illegible]0	.683	.229	.9192	.675	744997.1	738314.6	26.00
27.00	.235	.8951	.89[illegible]0	.668	.241	.8764	.662	746072.2	738850.5	27.00
28.00	.247	.8523	.8545	.655	.253	.8341	.649	747163.0	739382.6	28.00
29.00	.259	.8102	.8126	.642	.265	.7923	.636	748267.3	739913.6	29.00
30.00	.272	.7682	.7710	.630	.278	.7508	.623	749408.6	740470.5	30.00
31.00	.285	.7261	.7295	.618	.291	.7092	.614	750557.5	740947.7	31.00
32.00	.298	.6855	.6923	.609	.305	.6759	.605	751744.1	741433.7	32.00
33.00	.312	.6633	.6680	.600	.319	.6598	.596	752889.3	741859.2	33.00
34.00	.326	.6471	.6519	.591	.332	.6441	.587	754046.0	742290.3	34.00
35.00	.340	.6293	.6362	.582	.347	.6287	.578	755242.3	742750.5	35.00
36.00	.354	.6145	.6208	.575	.361	.6134	.572	756457.3	743135.5	36.00
37.00	.369	.5986	.6059	.570	.376	.5982	.567	757676.8	743503.7	37.00
38.00	.384	.5850	.5906	.564	.391	.5834	.561	758906.8	743904.3	38.00
39.00	.399	.5704	.5784	.558	.406	.5719	.556	760178.5	744333.0	39.00
40.00	.414	.5617	.5683	.553	.422	.5649	.550	761463.3	744761.0	40.00
41.00	.430	.5532	.5613	.548	.438	.5579	.547	762669.0	744992.5	41.00
42.00	.447	.5488	.5542	.545	.455	.5507	.544	763882.7	745162.5	42.00
43.00	.464	.5399	.5472	.542	.473	.5437	.541	765155.8	745397.5	43.00
44.00	.481	.5356	.5403	.539	.490	.5368	.538	766462.4	745665.2	44.00
45.00	.499	.5258	.5333	.536	.508	.5301	.535	767776.1	745960.0	45.00
46.00	.517	.5223	.5268	.535	.526	.5232	.535	769113.7	746093.3	46.00
47.00	.536	.5124	.5197	.536	.545	.5162	.536	770446.8	746173.8	47.00
48.00	.555	.5080	.5125	.536	.566	.5088	.536	771794.2	746141.7	48.00
49.00	.576	.4999	.5050	.536	.587	.5014	.537	773184.9	746130.7	49.00

APPENDIX D (CONT'D)

RTIME	•AMACH	CXCVE	CXCVI	CCDD	•MACH	CXCA	CD	FJ1	FM1	RTIME
50.00	.598	.4935	.4974	.537	.609	.4955	.537	774596.6	746059.8	50.00
51.00	.621	.4873	.4938	.538	.632	.4922	.540	775975.0	745761.3	51.00
52.00	.643	.4894	.4908	.541	.653	.4894	.542	777362.2	745777.8	52.00
53.00	.663	.4840	.4880	.543	.674	.4865	.545	778749.1	745800.4	53.00
54.00	.684	.4832	.4851	.546	.695	.4836	.547	780148.1	745828.3	54.00
55.00	.706	.4784	.4843	.549	.717	.4841	.550	781561.5	745797.5	55.00
56.00	.729	.4807	.4849	.554	.741	.4858	.558	782975.0	745308.0	56.00
57.00	.753	.4833	.4867	.562	.766	.4877	.566	784351.2	744712.1	57.00
58.00	.778	.4856	.4886	.570	.791	.4896	.574	785722.6	744094.1	58.00
59.00	.804	.4889	.4931	.578	.817	.4966	.582	787148.3	743504.5	59.00
60.00	.830	.4986	.5018	.586	.844	.5070	.590	788577.9	742848.9	60.00
61.00	.858	.5107	.5125	.597	.872	.5180	.604	789994.1	741577.6	61.00
62.00	.886	.5206	.5238	.611	.900	.5293	.618	791412.5	740472.9	62.00
63.00	.915	.5741	.5775	.630	.930	.6240	.641	792616.0	738181.3	63.00
64.00	.945	.6669	.6641	.653	.959	.7032	.665	793788.9	735913.8	64.00
65.00	.974	.7243	.7269	.684	.989	.7502	.703	794975.9	732448.0	65.00
66.00	1.003	.7735	.7775	.721	1.017	.8044	.740	796158.4	729263.0	66.00
67.00	1.031	.8338	.8356	.760	1.046	.8684	.781	797345.0	725631.1	67.00
68.00	1.062	.8651	.8643	.801	1.077	.8605	.822	798513.2	721798.5	68.00
69.00	1.094	.8480	.8494	.839	1.110	.8390	.857	799771.3	718419.2	69.00
70.00	1.127	.8248	.8268	.869	1.144	.8150	.880	801031.0	716306.2	70.00
71.00	1.161	.8012	.8024	.880	1.179	.7902	.881	802346.5	716439.9	71.00
72.00	1.196	.7765	.7767	.875	1.214	.7634	.870	803649.7	717899.8	72.00
73.00	1.233	.7464	.7481	.861	1.251	.7329	.852	804924.1	720070.6	73.00
74.00	1.271	.7160	.7169	.834	1.291	.7010	.817	806172.8	724019.8	74.00
75.00	1.308	.6885	.6905	.799	1.326	.6798	.782	807367.4	728689.6	75.00
76.00	1.343	.6701	.6710	.767	1.361	.6617	.752	808542.4	733216.3	76.00
77.00	1.378	.6517	.6528	.736	1.396	.6434	.721	809613.6	737725.2	77.00
78.00	1.413	.6347	.6361	.706	1.430	.6288	.691	810648.7	742523.3	78.00
79.00	1.452	.6191	.6197	.676	1.473	.6107	.660	811615.4	746430.5	79.00
80.00	1.495	.6009	.6016	.645	1.517	.5926	.630	812543.5	750554.7	80.00
81.00	1.540	.5822	.5828	.618	1.564	.5731	.607	813469.0	753797.3	81.00
82.00	1.585	.5641	.5647	.595	1.606	.5559	.584	814376.1	757614.3	82.00
83.00	1.624	.5482	.5512	.572	1.642	.5441	.561	815180.8	761823.0	83.00
84.00	1.665	.5373	.5349	.549	1.689	.5290	.538	815934.1	765289.4	84.00
85.00	1.714	.5205	.5219	.526	1.738	.5133	.515	816659.0	768721.8	85.00
86.00	1.766	.5053	.5046	.504	1.795	.4954	.493	817348.6	771749.4	86.00
87.00	1.828	.4848	.4382	.482	1.861	.4807	.471	818009.1	774443.3	87.00
88.00	1.896	.4781	.4738	.460	1.931	.4659	.449	818630.1	777112.1	88.00
89.00	1.960	.4603	.4607	.438	1.988	.4533	.427	819320.9	780557.8	89.00
90.00	2.018	.4474	.4480	.416	2.049	.4419	.405	819985.1	783866.0	90.00
91.00	2.065	.4390	.4391	.396	2.081	.4347	.388	820480.7	787528.9	91.00
92.00	2.101	.4289	.4305	.379	2.122	.4261	.371	820906.5	790630.5	92.00
93.00	2.157	.4199	.4197	.362	2.193	.4134	.354	821389.2	792963.4	93.00
94.00	2.222	.4070	.4080	.345	2.250	.4023	.337	821840.6	795575.0	94.00
95.00	2.278	.3977	.3972	.328	2.305	.3915	.320	822168.8	798100.8	95.00
96.00	2.333	.3860	.3860	.313	2.362	.3802	.307	822456.0	800176.7	96.00
97.00	2.377	.3773	.3761	.300	2.392	.3719	.294	822662.8	802538.5	97.00
98.00	2.424	.3646	.3654	.287	2.457	.3589	.281	822840.3	804197.1	98.00
99.00	2.484	.3533	.3534	.274	2.512	.3478	.268	823028.9	805976.1	99.00

APPENDIX D (CONT'D)

RTIME	•AMACH	CXCVE	CXCVI	CCDD	•MACH	CXCA	CD	FJ1	FM1	RTIME
100.00	2.537	.3437	.3416	.261	2.562	.3397	.255	823199.9	807730.7	100.00
101.00	2.584	.3362	.3335	.249	2.607	.3316	.244	823352.2	809309.0	101.00
102.00	2.642	.3258	.3265	.238	2.678	.3217	.233	823487.3	810546.6	102.00
103.00	2.742	.3147	.3151	.227	2.806	.3087	.222	823486.8	811090.5	103.00
104.00	2.831	.3055	.3039	.216	2.855	.2992	.211	823461.4	812337.1	104.00
105.00	2.890	.2922	.2936	.205	2.925	.2883	.200	823458.1	813361.9	105.00
106.00	2.953	.2846	.2829	.195	2.981	.2777	.190	823440.7	814343.5	106.00
107.00	2.997	.2727	.27[illegible]4	.185	3.013	.2673	.180	823439.7	815416.0	107.00
108.00	3.049	.2601	.2608	.175	3.086	.2546	.170	823432.0	816204.8	108.00
109.00	3.113	.2495	.2462	.165	3.140	.2418	.160	823353.3	816955.6	109.00
110.00	3.162	.2352	.2350	.155	3.185	.2282	.150	823257.0	817661.1	110.00
111.00	3.225	.2212	.2208	.146	3.264	.2134	.143	823075.4	817980.5	111.00
112.00	3.297	.2046	.2054	.139	3.330	.1974	.136	822876.9	818324.2	112.00
113.00	3.354	.1893	.1890	.132	3.378	.1810	.128	822783.5	818761.9	113.00
114.00	3.396	.1715	.1716	.125	3.415	.1623	.121	822700.8	819205.4	114.00
115.00	3.444	.1540	.1531	.118	3.473	.1436	.114	822450.0	819357.4	115.00
116.00	3.507	.1334	.1332	.110	3.541	.1231	.107	822175.9	819447.3	116.00
117.00	3.574	.1120	.1119	.103	3.607	.1009	.100	821874.3	819483.3	117.00
118.00	3.626	.0889	.0886	.096	3.644	.0763	.092	821563.6	819514.5	118.00
119.00	3.664	.0630	.0628	.089	3.684	.0495	.085	821268.4	819521.4	119.00
120.00	3.703	.0350	.0347	.082	3.722	.0201	.078	820970.3	819492.8	120.00
121.00	3.752	.0045	.0045	.076	3.783	-.0108	.074	820666.8	819358.1	121.00
122.00	3.818	-.0275	-.0276	.072	3.853	-.0438	.070	820359.1	819198.7	122.00
123.00	3.892	-.0615	-.0613	.068	3.932	-.0789	.066	819958.3	818930.3	123.00
124.00	3.968	-.0984	-.0986	.064	4.005	-.1176	.062	819542.9	818639.7	124.00
125.00	4.047	-.1376	-.1377	.060	4.089	-.1574	.057	819058.6	818265.3	125.00
126.00	4.123	-.1805	-.1606	.055	4.156	-.2033	.053	818563.1	817876.9	126.00
127.00	4.183	-.2294	-.2296	.051	4.211	-.2556	.049	818083.9	817497.6	127.00
128.00	4.243	-.2838	-.2838	.047	4.274	-.3117	.045	817604.4	817105.3	128.00
129.00	4.303	-.3432	-.3432	.043	4.331	-.3748	.041	817114.9	816695.1	129.00
130.00	4.377	-.4062	-.4065	.039	4.423	-.4374	.037	816622.5	816267.7	130.00
131.00	4.468	-.4707	-.4726	.035	4.512	-.5081	.034	816039.9	815736.4	131.00
132.00	4.555	-.5456	-.5469	.032	4.598	-.5849	.031	815444.8	815188.5	132.00
133.00	4.645	-.6294	-.6265	.029	4.692	-.6673	.027	814848.0	814633.5	133.00
134.00	4.737	-.7146	-.7141	.026	4.782	-.7602	.024	814249.8	814073.5	134.00
135.00	4.830	-.8100	-.8118	.023	4.877	-.8618	.021	813635.9	813493.5	135.00
136.00	4.981	-.9005	-.8981	.019	5.085	-.9312	.018	813018.9	812901.6	136.00
137.00	5.111	-.9984	-.9990	.016	5.138	-1.0697	.015	812290.8	812203.3	137.00
138.00	5.183	-1.1365	-1.1394	.013	5.229	-1.2079	.011	811548.1	811485.1	138.00
138.41	5.248	-1.8344	-1.2400	.011	5.266	-1.2727	.010	811216.4	811162.5	138.41
139.00	5.293	-.8882	-1.3177	.009	5.320	-1.3622	.008	810724.9	810683.3	139.00
140.00	5.367	-1.4458	-1.4496	.007	5.413	-1.5348	.005	809891.0	809868.1	140.00
141.00	5.470	-1.4680	.6611	.005	5.527	3.0004	.005	809019.2	808998.1	141.00
142.00	5.570	62.0375	30.2759	.005	5.613	-6.5529	.005	400749.1	400730.1	142.00

REFERENCES

1. Jackson, J.W., "Propulsion System Evaluation through Flight Simulation", MTP-AERO-62-69, dated August 31, 1962, MSFC.

2. Office of Manned Spaceflight: Project Apollo Coordinate System Standards, SE-008-001-1, June 1965.