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THE THERMAL STRUCTURE OF THE SUN

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THE THERMAL STRUCTURE OF THE SUN

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ABSTRACT

Under the assumption of radiative equilibrium in the core, of convective equilibrium in the outer parts of the envelope, and of a pronounced inhomogeneity in composition, models of the interior of the Sun are derived. As sources of energy, the proton-proton reaction and the carbon-nitrogen cycle are considered. The derivation of sun models is carried through in three steps. First, a homogeneous model is determined for the initial state of the Sun, when the hydrogen reaction is just beginning. Second, on the basis of this initial model, the transmutation rates and the present composition are computed for every point in the Sun. Finally, using this inhomogeneous composition, a model for the Sun in its present state is constructed.

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LIST OF SYMBOLS

Symbol	Definition
T	temperature
P	total pressure
ρ	density of matter
M(r)	mass of a sphere of radius r
L(r)	radiant power (energy crossing a sphere of radius r per second)
κ	mass absorption coefficient (area per unit mass)
ϵ	power generated per unit mass
Q	energy liberated by thermonuclear reaction
γ	ratio of specific heats
a	radiation density constant
c	velocity of light
H	mass of unit atomic weight
μ	mean molecular weight of matter
k	Boltzmann constant
G	gravitational constant
n	polytropic index

I. INTRODUCTION

Mass M, radius R, and luminosity L of the Sun are considered as known. The distributions of density ρ, pressure P, and temperature T in the interior are to be derived.

The assumption is made that the chemical composition of the solar material may be specified by the abundance of hydrogen and helium, X and Y, respectively, and by the abundance of the heavy elements, Z, which is related to X and Y as shown in the equation:

$$Z = 1 - X - Y.$$

The sources of energy generation in the Sun are thermonuclear reactions.

In the temperature range between 10^6 and 10^9 °K, the main source of stellar energy is provided by the proton-proton (pp) chain, the carbon-nitrogen (CN) cycle, and the helium reactions (3α).

Below 10^7 °K, the pp chain is dominant; above 2×10^7 °K, the CN cycle dominates; in the intermediate range, both reactions compete with each other. The 3α reactions come into operation near 10^8 °K, and therefore do not apply to the Sun. The pp chain occurs in the following stages:

$$\text{(a)} \quad {}_1H^1 + {}_1H^1 = {}_1H^2 + \beta^+ + \nu$$

$$\text{(b)} \quad {}_1H^2 + {}_1H^1 = {}_2He^3 + \gamma$$

$$\text{I (c-1)} \quad {}_2He^3 + {}_2He^3 \rightarrow {}_2He^4 + 2\,{}_1H^1$$

$$\text{II (c-2)} \quad {}_2He^3 + {}_2He^4 = {}_4Be^7 + \gamma$$

$$\text{(d-2)} \quad {}_4Be^7 \rightarrow {}_3Li^7 + \beta^+ + \nu$$

$$\text{(e-2)} \quad {}_3Li^7 + {}_1H^1 = 2\,{}_2He^4$$

III (c-3) $_2He^3 + _2He^4 = _4Be^7 + \gamma$

(d-3) $_4Be^7 + _1H^1 = _5B^8 + \gamma$

(e-3) $_5B^8 \rightarrow _4Be^{8*} + \beta^+ + \nu$

(f-3) $_4Be^{8*} \rightarrow 2\ _2He^4$.

The total amount of energy Q available per pp chain is:

$Q_{pp} = 26.207$ MeV

1 MeV = 1.6021×10^{-6} erg.

transmuted

The energy released per gram of hydrogen destroyed or per gram of helium formed equals:

$$Q^*_{pp} = \frac{Q_{pp}}{4H} = 6.3 \times 10^{18} \text{ erg/gram.}$$

The CN cycle occurs in the following steps:

(a) $_6C^{12} + _1H^1 = _7N^{13} + \gamma$

(b) $_7N^{13} \rightarrow _6C^{13} + \beta^+ + \nu$

(c) $_6C^{13} + _1H^1 = _7N^{14} + \gamma$

(d) $_7N^{14} + _1H^1 = _8O^{15} + \gamma$

(e) $_8O^{15} \rightarrow _7N^{15} + \beta^+ + \nu$

(f) $_7N^{15} + _1H^1 = _6C^{12} + _2He^4$.

The result of the cycle is the conversion of four protons into a helium nucleus; the carbon nucleus with which the cycle starts is recovered unchanged. The total amount of energy obtained from the cycle is:

$Q_{CN} = 25.026$ MeV.

The energy released per gram of hydrogen destroyed or per gram of helium formed equals:

$$Q^*_{CN} = \frac{Q_{CN}}{4H} = 6.0 \times 10^{18} \text{ erg/gram}.$$

The physical picture of a spherically symmetric star in equilibrium is expressed mathematically by four simultaneous, nonlinear, ordinary differential equations of the first order, which represent the radial gradients of P, M(r), T, and L(r). Since there are more than four unknowns, additional relations between the unknowns are determined from the physical properties of the material. These relations are called the "constitutive equations," which involve usually the chemical composition. The four fundamental equations of stellar structure are:

(1) the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2},$$

which balances the gravitational force by the pressure gradient;

(2) the equation of conservation of mass:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho,$$

which expresses the mass of a spherical shell of radius r and thickness dr;

(3) the luminosity equation:

$$\frac{dL(r)}{dr} = 4\pi r^2 \rho \epsilon,$$

which expresses the radiant energy of a spherical shell of radius r and thickness dr;

(4) the temperature gradient equations:

$$\frac{dT}{dr} = -\frac{3\kappa\rho}{4ac}\,\frac{1}{T^3}\,\frac{L(r)}{4\pi r^2} \quad \text{for radiative transport,}$$

$$\frac{dT}{dr} = \frac{\gamma - 1}{\gamma}\,\frac{T}{P}\,\frac{dP}{dr} \quad \text{for convective transport,}$$

which express the law of conservation of energy.

The constitutive equations comprise the equation of state, the equation of opacity, and the equation of energy generation.

Since everywhere in the interior of the Sun the radiative pressure is so small in comparison with the gas pressure that it can be neglected, the equation of state for a perfect gas applies to a sufficient degree of approximation:

$$P = \frac{k\rho}{\mu H} T$$

$$\mu = 1/[2X + \tfrac{3}{4} Y + \tfrac{1}{2} Z].$$

It is not necessary to consider the degeneracy of matter.

II. DERIVATION OF SUN MODELS

In the beginning, the pp chain will be considered as the source of energy generation, and the assumption will be made that the chemical composition is uniform throughout the Sun. This is true only when it just begins to generate energy by thermonuclear reactions. As the Sun ages, thermonuclear reactions produce chemical inhomogeneity between the core and the envelope. At first, a homogeneous composite model will be considered. An inhomogeneous model will be discussed later.

A. Homogeneous Composite Model

Solar models incorporating the proton-proton reaction and computed under the assumption of radiative equilibrium give a computed luminosity much higher than the observed luminosity. The reason for this discrepancy is that a hydrogen convection zone extends into the interior.

The central temperature is smaller in a star with an outer convection zone than in a star with no such convection zone. In a convective region the temperature gradient is lower than the radiative temperature gradient. An outer convection zone will reduce the central temperature and luminosity.

At the surface, the atmosphere of the Sun is in radiative equilibrium. At some depth, the radiative temperature gradient becomes unstable against convection.

The radiative core equations are:

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L(r)}{4\pi r^2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2\rho \tag{1}$$

$$\frac{dL(r)}{dr} = 4\pi r^2\rho\epsilon(r)$$

$$P = \frac{k}{\mu H}\rho T$$

$$\kappa = 6.52\times 10^{24}\left(Z+\frac{X+Y}{59.3}\right)(1+X)^{0.75}\rho^{0.75}T^{-3.5}$$

$$\epsilon = 2.8\times 10^{-33}X^2\rho T^{4.5}.$$

The convective envelope equations are:

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

$$\frac{dM(r)}{dr} = 4\pi r^2\rho$$

$$\frac{dL(r)}{dr} = 4\pi r^2\rho\epsilon(r) \tag{2}$$

$$P = \frac{k}{\mu H}\rho T$$

$$P = KT^{\frac{\gamma}{\gamma-1}} = KT^{2.5}.$$

In order to reduce the equations to dimensionless form, the Schwarzschild variables p, t, q, f, and x will be introduced:

$$P = p\frac{GM^2}{4\pi R^4} \qquad M(r) = qM$$

$$T = t\frac{\mu H}{k}\frac{GM}{R} \qquad L(r) = fL \tag{3}$$

$$r = xR.$$

This transformation reduces the equations (2) to the form:

$$\frac{dp}{dx} = -\frac{pq}{tx^2}$$

$$\frac{dq}{dx} = \frac{px^2}{t} \tag{4}$$

$$p = Et^{2.5}$$

$$E = 4\pi K \left(\frac{H}{k}\right)^{2.5} G^{1.5} M^{0.5} R^{1.5} \mu^{2.5} ,$$

and the equations (1) to the form:

$$\frac{dp}{dx} = -\frac{pq}{tx^2} \qquad \frac{dt}{dx} = -C\frac{p^{1.75}f}{x^2\, t^{8.25}}$$

$$\frac{dq}{dx} = \frac{px^2}{t} \qquad \frac{df}{dx} = D\, p^2 x^2 t^{2.5} \tag{5}$$

$$C = \frac{3\times 6.52\times 10^{24}\left(Z + \frac{X+Y}{59.3}\right)(1+X)^{0.75}}{(4\pi)^{2.75}\, 4ac} \left(\frac{k}{HG}\right)^{7.5} \frac{L R^{1.25}}{M^{5.75}\, \mu^{7.5}}$$

$$D = \frac{2.8\times 10^{-33} X^2}{4\pi} \left(\frac{GH}{k}\right)^{4.5} \frac{M^{6.5}}{L R^{7.5}} \mu^{4.5} .$$

Since the structure of the outermost parts of the Sun is uncertain, the model of the envelope is varied. A series of models is computed corresponding to a series of values of E. Finally, the value of E is fixed by the condition that the overall model of the Sun fits the observational data.

To describe the structure of the outer convective zone, the equations (4) are integrated with the boundary conditions:

at $x = 1$: $q = 1$, $p = 0$.

Eight homogeneous models were constructed, covering the range $E = 1.68$ to $E = 44.83$. Since the models refer to the homogeneous initial state while the observed luminosity and radius refer to the present state of the Sun, no reliable results will be obtained in this way. These models are applied to the observed mass, luminosity, the radius of the Sun by selecting a value for the hydrogen content X, and then determining the values of the helium content Y, and the convection parameter E, with the help of the mass-luminosity relation and the energy-output relation.

The main result obtained is the fact that a value for Z of 0.024, in agreement with the spectroscopic data, is reached when E has the small value of 0.86, that is, for a convective envelope extending only over 10.9 percent of the radius. The completely radiative models yield much smaller values for the abundance of the heavier elements than the one which is observed spectroscopically.

The Sun is in radiative equilibrium at its center. It does not have a convective core. Therefore the equations of radiative equilibrium may be integrated from the center outward to the point at which they become convectively unstable according to the criterion:

$$(n+1)_{rad} = \left(\frac{d \log P}{d \log T}\right)_{rad} \geq \left(\frac{d \log P}{d \log T}\right)_{adiab} = (n+1)_{adiab}.$$

From this point outward the structure is convective.

The boundary conditions at the center are:

at $x = 0$: $q = 0$, $f = 0$.

The parameters C and D can be removed from the equations (5') by the following transformations:

$$p = p_0 \; p^*$$
$$q = q_0 \; q^*$$
$$f = f_0 \; f^*$$
$$t = t_0 \; t^*$$
$$x = x_0 \; x^*.$$

In terms of these variables, the equations (5) assume the form:

$$\frac{dp^*}{dx^*} = -\frac{p^* q^*}{t^* x^{*2}} \qquad \frac{q_0}{t_0\, x_0} = 1$$

$$\frac{dq^*}{dx^*} = \frac{p^* x^{*2}}{t^*} \qquad \frac{p_0\, x_0^3}{t_0\, q_0} = 1$$

$$\frac{dt^*}{dx^*} = -\frac{p^{*1.75} f^*}{t^{*8.25}\, x^{*2}} \qquad C\,\frac{p_0^{1.75}\, f_0}{t_0^{9.25}\, x_0} = 1 \tag{6}$$

$$\frac{df^*}{dx^*} = p^{*2}\, x^{*2}\, t^{*2.5} \qquad D\,\frac{p_0^2\, t_0^{2.5}\, x_0^3}{f_0} = 1.$$

If the value of one of the five unknowns p_0, q_0, f_0, t_0, x_0 is selected arbitrarily, the remaining four can be determined by the conditions in the equations (6) in terms of the chosen parameter and of C and D. The central value of t is chosen:

$$t_0 = t_c.$$

In addition to the variables x, p, t, q, and f, the following functions are computed:

$$U = \frac{d \log M(r)}{d \log r} = \frac{4\pi r^3 \rho}{M(r)}$$

$$V = -\frac{d \log P}{d \log r} = \frac{\rho}{P}\,\frac{GM(r)}{r}$$

$$W = \frac{d \log L(r)}{d \log r} = 4\pi r^3 \rho\,\frac{\epsilon(r)}{L(r)} \tag{7}$$

$$n + 1 = \frac{d \log P}{d \log T} = \frac{16\pi ac}{3}\,\frac{GM(r)\,T^4}{P(r)\,\kappa(r)\,L(r)} \quad \text{for radiative equilibrium,}$$

$$n + 1 = \frac{\gamma}{\gamma + 1} = 2.5 \quad \text{for convective equilibrium.}$$

The actual numerical integration of the equations (6) is performed by employing logarithmic variables:

$$\log p^* = \lambda \qquad \log f^* = \phi$$

$$\log q^* = \psi \qquad \log x^* = y. \tag{8}$$

$$\log t^* = \tau$$

In terms of these variables, the equations (6) are reduced to the form:

$$\begin{aligned}
\log(-d\lambda/dy) &= \psi - \tau - y \\
\log(d\psi/dy) &= \lambda - \psi - \tau + 3y \\
\log(-d\tau/dy) &= 1.75\lambda - 9.25\tau + \phi - y \\
\log(d\phi/dy) &= 2\lambda + 2.5\tau - \phi + 3y.
\end{aligned} \tag{9}$$

The combination of the equations (7), (8), and (9) yields:

$$\begin{aligned}
U &= d\psi/dy \\
V &= -d\lambda/dy \\
W &= d\phi/dy \\
n + 1 &= [d\lambda/dy] / [d\tau/dy].
\end{aligned} \tag{10}$$

The equations (9) are integrated under the boundary conditions at the center:

$$\text{at } x^* = 0: \quad q^* = 0, \; f^* = 0, \; t^* = 1, \; p^* = p_c^*$$

for a number of values of p_c^*, which is treated as a parameter. The values of U, V, W, and n + 1 are tabulated together with those of q^*, f^*, t^*, and p^* against the values of x^*, starting from $x^* = 0$. The outward integrations are stopped at $x^* = x_i^*$, where n + 1 attains the value 2.5. In this way the U - V curves are obtained along which n + 1 attains the value 2.5 for the radiative solutions. This procedure yields the value of the inner solution at the interface U_{ic}, V_{ic}, p_{ic}^*, q_{ic}^*, t_{ic}^*, f_{ic}^*, x_{ic}^* at the point where the convective envelope has to be fitted.

Then the equations (4) are integrated under the boundary conditions:

$$\text{at } x = 1: \quad t = 0, \; q = 1,$$

for a number of values of the parameter E. The values of p, q, t, U, and V are tabulated against x. In general, no convective envelope U - V curve passes through the end point U_{ic}, V_{ic} of the radiative core U - V curve. Then an interpolation is performed in the U - V plane between the convective outer solutions to find the value of E for the solution which passes through the point U_{ic}, V_{ic}. This procedure determines the envelope solution uniquely. For this value of E, the values of the outer solution at the interface p_{is}, t_{is}, q_{is}, and x_{is} are found from the envelope solution at the point $U_{is} = U_{ic}$, $V_{is} = V_{ic}$. Therefore:

$$x_{is} = x_0\, x_{ic}^{*};\ p_{is} = p_0\, p_{ic}^{*};\ t_{is} = t_0\, t_{ic}^{*};\ q_{is} = q_0\, q_{ic}^{*} \ .$$

With the values known for x_{is}, x_{ic}^{*}, p_{is}, p_{ic}^{*}, t_{is}, t_{ic}^{*}, q_{is}, and q_{ic}^{*}, the values x_0, p_0, t_0, and q_0 are determined. The equation for df/dx in equations (5) is then integrated for the envelope, using the solution for the value of E which fits it at the end point U_{ic}, V_{ic} of the inner solution for the assumed value of p_c^{*}. Since at

$$x = 1,\ f = 1, \text{ and at } x = x_{is},\ f = f_{is}:$$

$$1 - f_{is} = D \int_{x_{is}}^{1} p^2 t^{2.5} x^2 dx$$

$$f_{is} = 1 - D \int_{x_{is}}^{1} p^2 t^{2.5} x^2\, dx.$$

This equation determines f_{is} at the interface for a value of D, which is determined by the following procedure. The continuity of f at the interface yields:

$$f_{is} = f_{ic} = f_0\, f_{ic}^{*}$$

$$f_0\, f_{ic}^{*} = 1 - D \int_{x_{is}}^{1} p^2 t^{2.5} x^2\, dx\ .$$

By solving this equation and the condition equation for D in equations (6), with the knowledge of values of x_0, p_0, t_0, f_{ic}^{*}, the values of

f_0 and D are obtained:

$$f_0 = 1/[f_{ic} + \frac{1}{p_0^2 t_0^{2.5} x_0^3} \int_{x_{is}}^{1} p^2 t^{2.5} x^2 dx]$$

$$D = f_0 / [p_0^2 t_0^{2.5} x_0^3].$$

The equation for C in equations (5) yields the value of C. Terms p_0 and p_c^* yield p_c, and $t_s = t_c$ gives the central value of t.

The method of fitting the solutions indicates that a definite initial value of p_c^* determines C, D, and E uniquely. On the other hand, the fitting of the radiative core at the point U_{is}, V_{is} beneath a convective envelope with a prescribed value of E determines C, D, and p_c^* uniquely. The term E is, therefore, the parameter which determines the model uniquely.

B. Inhomogeneous Composite Model

The initial homogeneous model of the Sun consists of a radiative core and a convective envelope whose depth depends on the parameter E. Energy is generated in the radiative core where the material remains unmixed. The rate of energy generation is proportional to the density and to some power of temperature. It decreases with increasing distance from the center, so that the hydrogen abundance varies at different rates at different points on a radius inside the core.

Since at the center the transmutation rate from hydrogen to helium is about ten times higher than the average rate, a pronounced inhomogeneity in composition exists at present in the Sun. Therefore, an inhomogeneous model has to be constructed.

The core equations are the same as the equations (1). For a chemically homogeneous envelope, the equations are the same as equations (2) except that μ has to be replaced by μ_e. For a given value of E, the envelope equations can be integrated under the boundary conditions:

at $x = 1$: $q = 1$, $t = 0$,

to obtain a unique solution.

The energy and opacity laws assume the form:

$$
\begin{aligned}
\epsilon &= \epsilon_{0e}\,\mu_e\,\frac{H}{k}\; i\;P\,T^{3.5} \\
\kappa &= \kappa_{0e}\,\mu_e^{0.75}\left(\frac{H}{k}\right)^{0.75} j^{0.75} P^{0.75}\,T^{-4.25} \\
\epsilon_{0e} &= 2.8\times 10^{-33}\,X^2 \\
\kappa_{0e} &= 6.52\times 10^{24}\left(Z+\frac{1-Z^2}{59.3}\right)(1+X_e)^{0.75} \\
i &= \ell\left(\frac{X}{X_e}\right)^2 \\
j &= \ell\,\frac{1+X}{1+X_e} \\
\ell &= \frac{\mu}{\mu_e}\,.
\end{aligned}
\tag{11}
$$

The equations (3) are replaced by the transformation:

$$
\begin{aligned}
M(r) &= qM & P &= p\,\frac{GM^2}{4\pi R^4} \\
L(r) &= fL & T &= t\mu_e\,\frac{H}{k}\,\frac{GM}{R} \\
r &= xR & \rho &= \frac{p}{t}\,\frac{\mu}{\mu_e}\,\frac{M}{4\pi R^3}\,.
\end{aligned}
\tag{12}
$$

When equations (12) are substituted in equations (1), the following replacement is obtained for the equation (5):

$$
\begin{aligned}
dp/dx &= -\,\ell\,(pq/tx^2) & dt/dx &= -C\ell\, j^{0.75}\,(p^{1.75}f/t^{8.25}\,x^2) \\
dq/dx &= \ell\,(px^2/t) & df/dx &= D\ell\, i\, p^2 t^{2.5}\,x^2
\end{aligned}
$$

$$
\begin{aligned}
C &= \frac{3\kappa_{0c}}{4ac}\,\frac{1}{(4\pi)^{2.75}}\left(\frac{k}{HG}\right)^{7.5}\frac{LR^{1.25}}{M^{5.75}\,\mu^{7.5}} \\
D &= \frac{\epsilon_{0c}}{4\pi}\left(\frac{GH}{k}\right)^{4.5}\frac{M^{6.5}}{LR^{7.5}}\,\mu_e^{4.6}\,.
\end{aligned}
\tag{13}
$$

In terms of the starred variables, whose values have to be taken from the core solution of the initial model, the equations (13) become:

$$
\begin{aligned}
dp^*/dx^* &= -\ell\,(p^*q^*/t^*x^{*2}) \\
dq^*/dx^* &= \ell\,(p^*x^{*2}\,/t^* \\
dt^*/dx^* &= -\ell\,j^{0.75}(p^{*1.75}f^*/t^{*\,8.25}x^{*2}) \\
df^*/dx^* &= \ell\;ip^{*2}\,t^{*2.5}\,x^{*2}).
\end{aligned}
\qquad (14)
$$

with the boundary conditions:

at $x^* = 0$: $q^* = 0$, $f^* = 0$, $t^* = 1$, $p^* = p^*_c$.

The condition equations are the same as in the equations (6).

In the solution of equations (14), p^*_c is a trial parameter, as in the case of the equations (6) for a homogeneous model. Because of the presence of i, j, and ℓ in equations (14), q_0 has to be considered as an additional parameter. The value of q_0 may be taken from the homogeneous model, and then it may be improved by integrations.

The envelope solution forms a family of curves corresponding to the parameter E. The inner and outer solutions may be fitted in the U - V plane, as in the integration of the homogeneous model.

In order that the effect of the carbon-nitrogen cycle upon the structure of the interior may be taken into account, the energy generation is represented by different laws, the differential equation governing the energy generation is modified, the non-dimensional parameter D is redefined, and a quantity $\delta - 1$ representing the ratio of energy generated by the carbon cycle to that generated by the proton-proton reaction is introduced:

$$\epsilon_{pp} = 9.5 \times 10^{-30} X^2 \rho T^4$$

$$\epsilon_{CN} = 6.3 \times 10^{-143} X X_{CN} T^{20}$$

$$df/dx = i \, \ell \, Dp^2 t^2 x^2 \delta$$

$$\delta - 1 = \left(\frac{\ell}{i}\right)^{\frac{1}{2}} \frac{6.3 \times 10^{-143} Z}{3 \times 9.5 \times 10^{-30} X_e} \left(\frac{HGM}{kR}\right)^{16} \mu_e^{16} \; t^{16}$$

$$D = \left(\frac{HG}{k}\right)^4 \frac{9.55 \times 10^{-30}}{4\pi} \; X_e^2 \; \mu_e^4 \; \frac{M^6}{LR^7} \; .$$

The integration from the center is performed by the same procedure which was explained previously. As in the case of inhomogeneous composition, in addition to $(n + 1)_c$, q_0 becomes a second unknown parameter. The equation for $\delta - 1$ also contains the unknown quantity t_0. By the use of the condition equations in (6), t_0 may be eliminated and $\delta - 1$ expressed in terms of q_0, M, R, X_e, and Z. (Refer to Table I.) The radiative solutions obtained by the integrations are fitted to the convective solutions which were previously obtained.

TABLE I. SOLAR PARAMETERS

$M_\odot = 1.989 \times 10^{30}$ kg	$A_\odot = 4.5 \times 10^9$ years
$R_\odot = 6.9598 \times 10^8$ m	$X = 0.74$
$L_\odot = 3.90 \times 10^{26}$ watt	$Y = 0.24$
$\bar{\rho}_\odot = 1.409$ g/cm^3	$Z = 0.02$
$T_{e\odot} = 5800$ °K	$\mu = 0.599$
$M_{bol\,\odot} = 4.72$	spectral classification dG 2

III. RESULTS AND CONCLUSIONS

The physical properties of the models obtained are summarized in Table II, from which the following conclusions can be drawn. There is a large abundance of elements heavier than hydrogen and helium (Table III). The value $E \approx 2$ lies between 1 and 10. The radiative core contains about 99 percent of the solar mass and extends over 84 percent of its radius. The convective envelope does not penetrate more than 16 to 20 percent of the radius. The central density of the present Sun is 136 grams/cm^3; its central temperature is 14×10^6 °K.

TABLE II. ABUNDANCE OF ELEMENTS IN THE SUN

Atomic No.	Element	Log of Relative Abundance	Atomic No.	Element	Log of Relative Abundance
1	H	12.00	38	Sr	2.80
2	He	11.16	39	Y	3.01
3	Li	0.86	40	Zr	2.16
4	Be	2.35	41	Nb	2.00
6	C	8.56	42	Mo	1.96
7	N	7.98	44	Ru	1.36
8	O	9.00	45	Rh	0.70
9	F	5.4	46	Pd	1.00
10	Ne	8.44	47	Ag	-0.50
11	Na	6.30	48	Cd	1.76
12	Mg	7.28	49	In	0.80
13	Al	6.21	50	Sn	1.2
14	Si	7.60	51	Sb	1.82
15	P	5.44	56	Ba	2.26
16	S	7.17	57	La	1.8
17	Cl	5.4	58	Ce	2.4
18	Ar	6.62	59	Pr	0.6
19	K	4.96	60	Nd	2.0
20	Ca	6.38	62	Sm	1.5
21	Sc	3.20	63	Eu	1.4
22	Ti	4.96	64	Gd	1.1
23	V	4.03	66	Dy	1.6
24	Cr	6.00	68	Er	0.1
25	Mn	5.30	69	Tm	0.5
26	Fe	6.76	70	Yb	1.42
27	Co	4.74	71	Lu	1.0
28	Ni	5.80	72	Hf	0.4
29	Cu	4.99	73	Ta	0.0
30	Zn	4.53	74	W	0.2
31	Ga	2.16	76	Os	0.5
32	Ge	3.31	77	Ir	-0.2
37	Rb	2.28	78	Pt	1.6
			82	Pb	2.68

TABLE III. PHYSICAL PROPERTIES OF THE SUN

X	Y	Z	E	x_i	q_i	T_i (°K × 10^6)	ρ_i	T_c (°K × 10^6)	ρ_c
0.80	0.19	0.01	2.05	0.849	0.998	0.94	0.018	14.1	136
0.74	0.24	0.02	2.14	0.847	0.998	1.00	0.019	14.6	134
0.70	0.27	0.03	2.19	0.846	0.998	1.04	0.020	15.1	132
0.60	0.32	0.08	2.33	0.842	0.998	1.17	0.023	16.4	128

It has been found that most of the energy comes from the region located within 0.25 $R_\odot$ of the center of the Sun. The energy is transferred to the surface of the Sun by radiation and convection. Radiation due to x- and γ-rays is the dominant process out to about 0.7 $R_\odot$, with convection becoming the principal process from 0.7 to 1.0 $R_\odot$.

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