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DESIGN SENSITIVITIES OF MULTISTAGE LAUNCH VEHICLES

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DEFINITION AND SYMBOLS

SYMBOL	DEFINITION
G	Growth factor $= \dfrac{\text{overall launch weight}}{\text{payload weight}}$
f	Stage mass fraction $= \dfrac{\text{propellant weight}}{\text{propellant \& structure weight}}$
$\sigma = 1-f$	Stage structure fraction
r	Mass ratio of stage $= \dfrac{\text{ignition weight}}{\text{cut-off weight}}$
g	Sea level acceleration of gravity
Isp	Average stage specific impulse
Δv	Stage ideal velocity increment
Δv_{DRAG}	Stage drag loss
λ	Stage payload ratio $= \dfrac{\text{payload weight}}{\text{ignition weight}}$
ζ	Stage propellant ratio $= \dfrac{\text{propellant weight}}{\text{ignition weight}}$
ϵ	Stage structure ratio $= \dfrac{\text{structure weight}}{\text{ignition weight}}$
p	General stage parameter
i	General stage nomenclature
k	Particular stage under consideration

DESIGN SENSITIVITIES OF MULTISTAGE LAUNCH VEHICLES

SUMMARY

The performance sensitivities of a launch vehicle concept in combination with design parameter magnitudes and dispersions permit the discussion of general feasibility within a given "state of the art," and of the need for R& D efforts in order to reduce development risk.

This paper presents the derivation and application of a closed-form expression for design feasibilities of multistage launch vehicles.

INTRODUCTION

Launch vehicle design sensitivities are a measure of confidence in the ability to fulfill systems performance specifications at a given "state of the art."

Establishment of concept feasibility involves limiting the impact of potential parameter variations on systems characteristics.

This does not necessarily call for a preference of concepts having low sensitivities (some of the most promising concepts exhibit high sensitivities), but it means that through research and development efforts design parameters must be refined to such a degree that the remaining uncertainties combined with the inherent systems sensitivities lead to a tolerable dispersion of systems characteristics. Only then is a concept ready for serious consideration, i.e., it is within the "state of the art."

This report describes the derivation and application of a closed-form expression for design sensitivities applicable to any stage of a multistage launch vehicle, to replace the previous practice of generating slopes by re-running vehicle performance several times.

The procedure is simple and fast and allows the discussion of several conceptual approaches in a common frame of reference.

Frequently Used Relations Between Parameters

$$1 = \lambda + \zeta + \epsilon$$

$$\lambda = 1 - \frac{\zeta}{f} = \frac{1 - r(1-f)}{rf} = \frac{1 - r\sigma}{r(1-\sigma)}$$

$$\zeta = 1 - \frac{1}{r} = 1 - e^{-\frac{\Delta v}{g\,Isp}} \approx \frac{\Delta v}{g\,Isp} \quad \text{(for small } \Delta v\text{)}$$

$$r = \frac{1}{1-\zeta} = e^{\frac{\Delta v}{g\,Isp}}$$

$$\Delta v = Isp\; g \ln r = Isp\; g \ln \frac{1}{1-\zeta}$$

$$\epsilon = \zeta\left(\frac{1}{f} - 1\right) = \frac{\zeta\sigma}{1-\sigma}$$

$$f = \frac{\zeta}{1-\lambda} = \frac{\zeta}{\epsilon + \zeta} = 1 - \sigma$$

ANALYSIS

For the purpose of this analysis the convention shall be made that an individual stage consists of payload, main stage propellant and structure.* By this set-up, all other propellants, as reserves, residuals, and flyback propellants are accounted for as structural weights.

*Krause, H. G. L., General Theory of Multi-Stage Rockets and Performance Theory of an N-Stage Satellite Carrier with a specific turning program, presented to ARS Space Flight Report to the Nation, Oct. 8-14, 1961, New York, N.Y.

The overall lift-off-to-payload weight ratio (growth factor G) of an n-stage launch vehicle is defined as:

$$G = \prod_{i=1}^{n} \frac{1}{\lambda_i} \tag{1}$$

The vehicle growth-factor sensitivities to changes of a general design parameter (p): (in the vicinity of the design point and for any one stage), are found by partial differentiation:

$$\left(\frac{\partial G}{\partial p}\right)_i = \left(\frac{\partial G}{\partial \lambda}\right)_i \left(\frac{\partial \lambda}{\partial p}\right)_i \tag{2}$$

After carrying out the differentiation and normalizing,

$$\frac{\frac{\Delta G}{G}\,\%}{\frac{\Delta p_i}{p_i}\,\%} = - \frac{p_i}{\lambda_i} \left(\frac{\partial \lambda}{\partial p}\right)_i \tag{3}$$

The stage payload sensitivities to stage parameter variations are found by partial differentiation. The sensitivity to structural weight is derived from:

$$\lambda = \frac{1 - r\sigma}{r\,(1 - \sigma)} \tag{4}$$

at constant Δv; Isp, and r:

$$\frac{\partial \lambda}{\partial \sigma} = - \frac{r - 1}{r\,(1-\sigma)^2} = - \frac{r - 1}{rf^2} \tag{5}$$

This expression is introduced into equation (3) to yield the vehicle growth factor sensitivity to changes of stage structure fraction:

$$\frac{\frac{\Delta G}{G}\,\%}{\frac{\Delta \sigma_i}{\sigma_i}\,\%} = \left[\frac{\sigma\,(r-1)}{(1-\sigma)(1-r\sigma)}\right]_i = \left[\frac{(1-f)\,(r-1)}{f\,[1-r\,(1-f)]}\right]_i \tag{6}$$

This closed-form expression is plotted in Figure 1 and due to normalization is applicable to any one stage of a multistage vehicle configuration. The stage payload/ sensitivity to changes in stage specific impulse is determined by differentiation of:

$$\lambda = \frac{1-r\,(1-f)}{rf} = \frac{1}{rf} - \frac{1-f}{f} \tag{7}$$

$$= \frac{1}{fe^{\Delta v/\,g\,Isp}} - \frac{1-f}{f}$$

at constant Δ v, and f:

$$\frac{\partial\lambda}{\partial Isp} = \frac{\ln r}{rf\;Isp} \tag{8}$$

This expression is introduced into equation (3) to yield the vehicle growth factor sensitivity to changes of stage specific impulse:

$$\frac{\frac{\Delta G}{G}\%}{\frac{\Delta\,Isp_i}{Isp_i}\%} = -\left[\frac{\ln r}{1-r(1-f)}\right]_i \tag{9}$$

It can be shown that the growth factor sensitivities to changes in specific impulse and ideal velocity increment are equal but have opposite signs:

$$\frac{\frac{\Delta G}{G}\%}{\frac{\Delta\,(\Delta v)_i}{\Delta v_i}\%} = \left[\frac{\ln r}{1-r(1-f)}\right]_i \tag{10}$$

This expression in turn can be utilized to discuss growth factor sensitivities to changes in aerodynamic drag coefficient:

$$\frac{\frac{\Delta G}{G}\%}{\frac{\Delta C_{D_i}}{C_{D_i}}\%} \cdot \frac{\Delta v_i}{\Delta v_{DRAGi}} = \frac{\frac{\Delta G}{G}\%}{\frac{\Delta\,(\Delta v)_i}{\Delta v_i}\%} \tag{11}$$

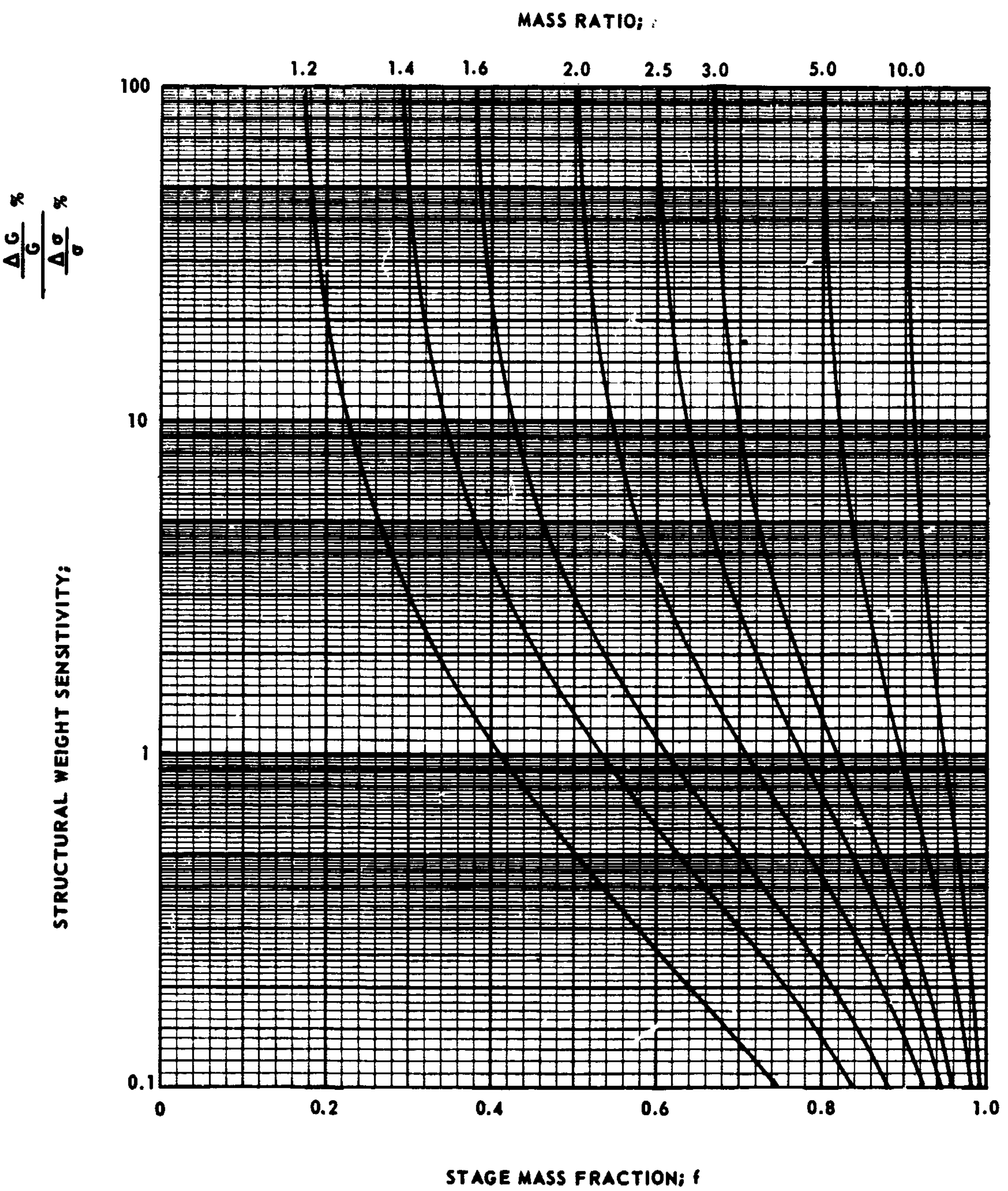

FIG. 1 GROWTH FACTOR SENSITIVITY TO CHANGES OF STRUCTURE FRACTION.

Equations (9), (10), and (11) can be read from Figure 2, and are also applicable to any one stage of a multistage vehicle configuration.

DISCUSSION

Figures 3 and 4 show design sensitivities of various concepts and their propulsive stages indicative of present state of the art, for low orbital injection due east out of AMR and for vehicles in the 10, 000 pounds of useful payload class. For comparison, the sensitivities of a Boeing 707-320B airplane (53, 000 pounds payload) are entered at its maximum design condition. The "Existing Expendable Systems" have LOX/RP first stages and LOX/ H_2 upper stages. The "Lifting Rocket First Stages" are assumed to use LOX/ RP with an average specific impulse of 286 sec. The "Lifting Rocket Second Stages" use advanced LOX/ H_2 propulsion with an assumed specific impulse of 455 sec. The advanced airbreathing concepts are assumed to use liquid hydrogen as propellant. Obviously, the design trends plotted in these graphs represent typical systems only. Particular concepts may differ considerably depending on design considerations like trajectory profile, staging conditions, and staging arrangement (tandem, parallel, nested), which determine loads and temperatures and thereby structural weights and further depending on operational considerations like expected number of flights per vehicle, abort and ferry provisions, etc., which have an influence on the required degree of overdesign.

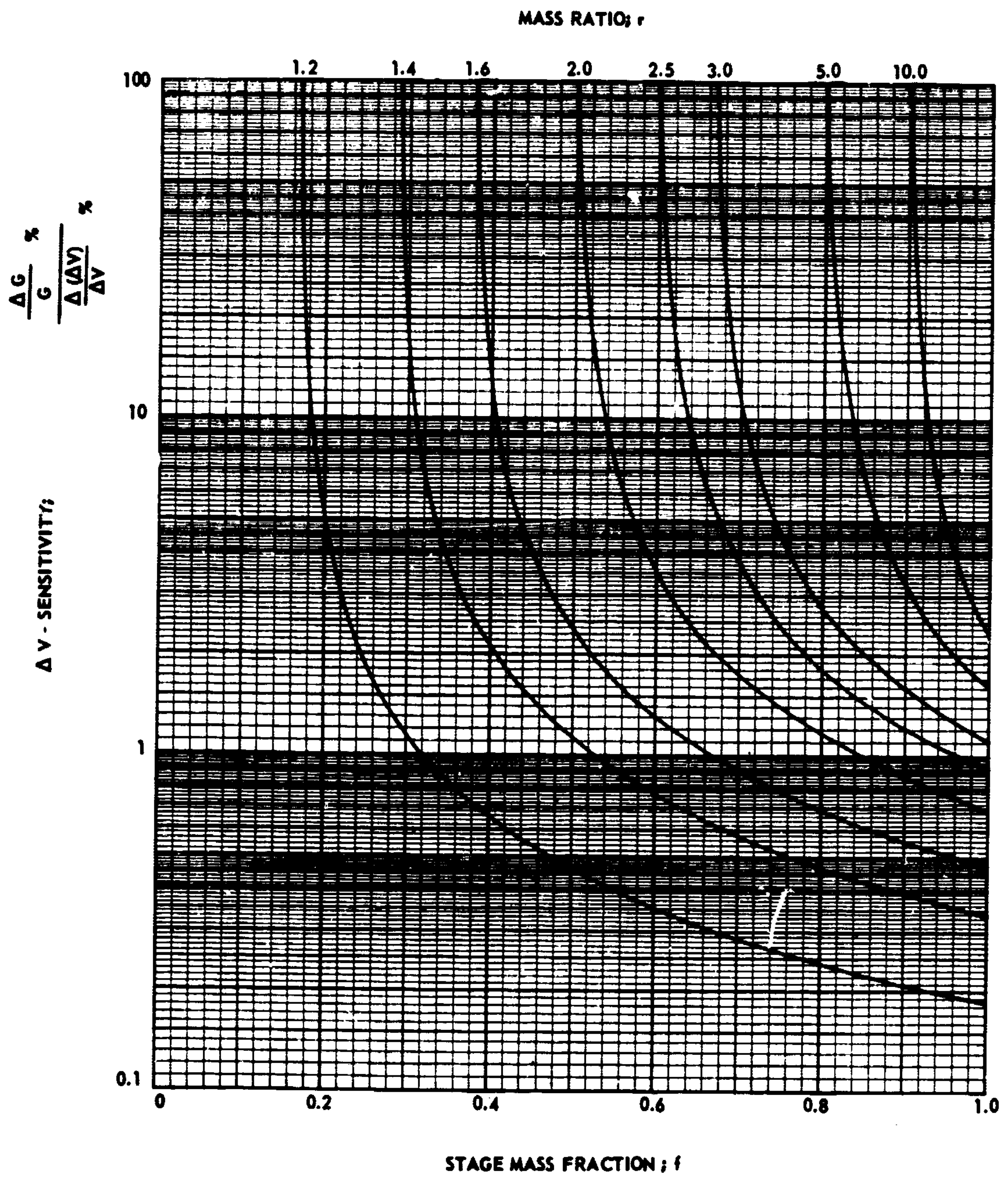

FIG. 2 GROWTH FACTOR SENSITIVITY TO CHANGES OF STAGE VELOCITY INCREMENT

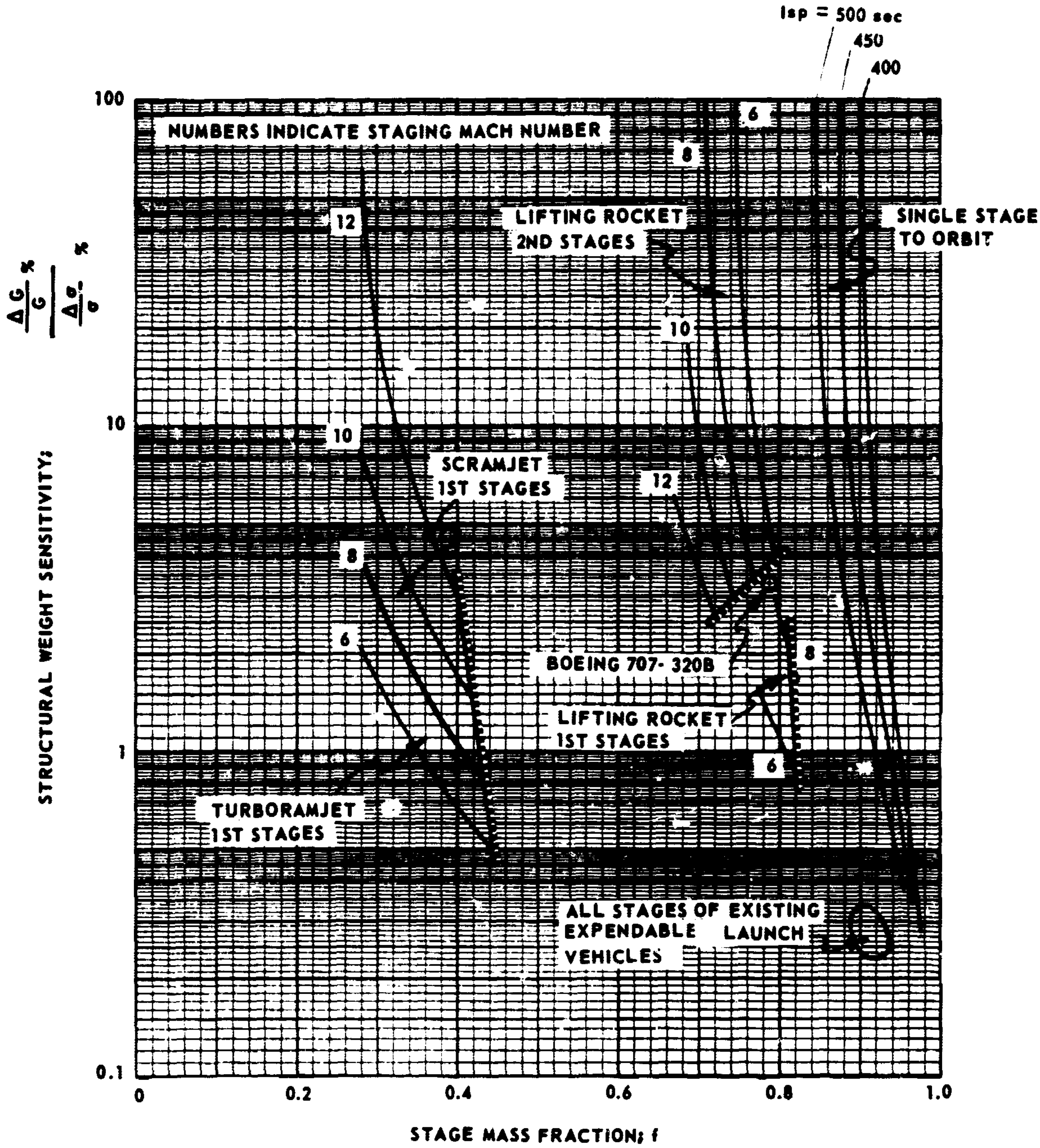

FIG. 3 TRENDS OF GROWTH FACTOR - WEIGHT SENSITIVITIES

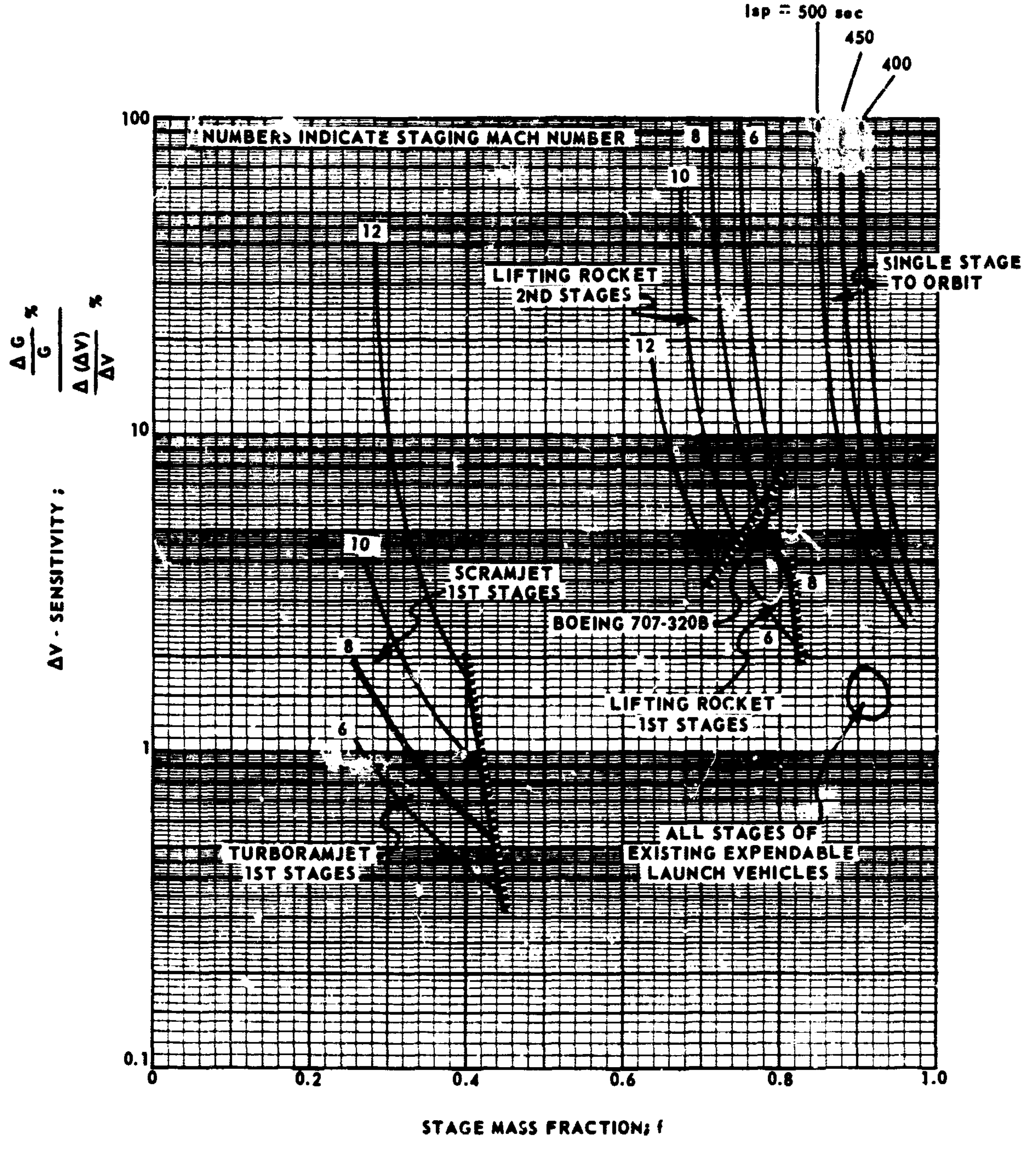

FIG. 4 TRENDS OF GROWTH FACTOR - ΔV SENSITIVITIES