STANFORD RESEARCH INSTITUTE
Menlo Park, California 94025 · U.S.A

16 June 1970

Quarterly Technical Report 2
Covering the Period 4 March to 4 June 1970
Stanford Research Institute Project 8327

AVALANCHE-DIODE OSCILLATOR CIRCUIT WITH TUNING AT MULTIPLE FREQUENCIES

by

Don Parker

Contract NAS 12-2231

Prepared for

National Aeronautics and Space Administration
Electronics Research Center
575 Technology Square

Cambridge, Massachusetts 02139

Attn: TEM/Microwave Technology Branch

Copy No. 2

PURPOSE OF CONTRACT

The purpose of this contract is to determine theoretically the operating characteristics of an avalanche-diode oscillator when several currents, harmonically related, are flowing in the diode, and to design, construct, and test an avalanche diode circuit that will permit independent tuning of three harmonically related frequencies.

CONTENTS

ILLUSTRATIONS

TABLES

I INTRODUCTION

The effort on this contract has so far been directed almost entirely to increasing the theoretical understanding of the high-efficiency modes of oscillation in an avalanche diode. During the second quarter the models of the TRAPATT mode in a PIN diode developed during the first quarter were further developed and used to make calculations of diode operating characteristics.[1]

Reported in Section II are the results of sample calculations of an avalanche diode oscillator operating in the normal TRAPATT mode, computed by the analytical solution derived in Quarterly Technical Report 1.[1] These results indicate that, although the square-wave drive current simplifies the analytical solution, in most instances this current does not correspond to the diode feeding a passive circuit. Power is required not only at dc but at several higher harmonics.

Two other significant advances have been made. First, the computer program SMPAL has been modified for more accurate calculation of the diode voltage while the plasma is being extracted from the ends of the depletion layer. This computer program now constitutes a simple but relatively accurate model of the terminal characteristics of a PIN diode operating in the TRAPATT mode. It can be used with either a passive circuit model in a time domain analysis or arbitrary drive current waveforms, at a cost two to three orders of magnitude less than that of more complete computer programs. The modifications are outlined in Section III, and sample calculations for some non-square-wave current waveforms are presented. Second, an approximate analytical solution has been derived for another

---

[1] References are listed at the end of this report.

class of high-efficiency waveforms, which differ significantly from the normal **TRAPATT** mode. This class of waveforms is discussed in Section IV, and the analytical solution is derived. The solution is derived by considering seven different portions of the voltage waveform during one cycle. An expression for the terminal voltage is derived for each portion of the cycle, assuming the external current to be a square wave.

II CALCULATION OF TRAPATT OSCILLATIONS USING THE ANALYTICAL SOLUTION

An analytical solution for the normal TRAPATT mode in a PIN diode was derived in Quarterly Technical Report 1.[1] We have applied this solution to several different values of drive current and diode saturation current for a 10-micron-wide depletion layer. The ionization coefficient α and the carrier velocities were assumed to be equal, and values consistent with electrons in silicon were used. The normalized low-field velocity, v, was assumed to be ten times the electric field. The expression of Scharfetter and Gummel for the electron ionization coefficient was assumed to correspond to the experimental data for the ionization coefficient.[2]

Figure 1 shows the normalized fundamental impedance of a PIN diode operating in a TRAPATT mode for three levels of a square-wave drive current. Four different levels of the diode saturation current were considered. These levels of the saturation current correspond to the thermal generation of carriers in the diode; the lower the level of the saturation current, the smaller the thermal generation. It is evident from Figure 1 that the higher the level of the drive current, the smaller the variation in the fundamental impedance over several orders of magnitude of the saturation current.

One must interpret the results of Figure 1 very carefully, for two reasons. First, the fundamental frequency for each data point is different; in fact, for a given drive current the frequency varies over an octave, with the range of saturation currents shown. Second, the impedance of the diode at higher harmonics changes appreciably. For a square-wave drive current, the real part of the diode impedance at higher harmonics may be

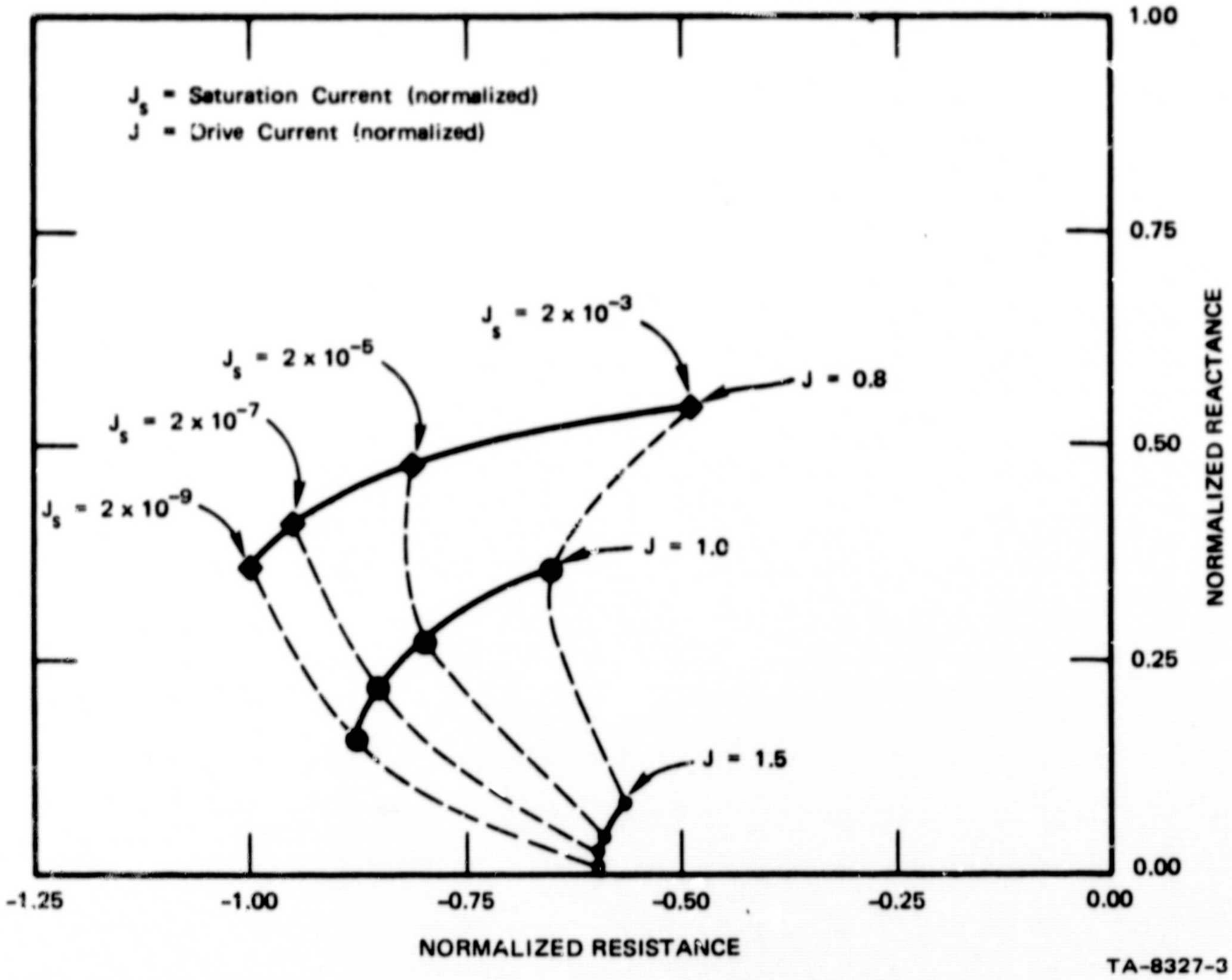


FIGURE 1 NORMALIZED FUNDAMENTAL IMPEDANCE OF PIN DIODE OPERATING IN A TRAPATT MODE WITH A SQUARE-WAVE CURRENT DRIVE

positive, indicating that power must be furnished to the diode at that frequency. Tables I, II, and III show the normalized period, T, power, P_i, efficiencies, η_i, and impedances, Z_i, for the first few harmonics of the diode, corresponding to the results shown in Figure 1. The results shown in these tables indicate that at the higher harmonics the diode furnishes power for small saturation currents and high drive levels, but that ac power must be furnished to it for large saturation currents and low drive currents. The choice of a square-wave drive current was arbitrary and made to simplify the analysis in the analytical solution. In practice, the diode would always be placed in a passive circuit, and the diode impedances would change accordingly. The results in Tables I, II,

Table I

NORMALIZED PERIOD, T, POWERS, P_i, EFFICIENCIES, η_i, AND IMPEDANCES, Z_i, FOR A PIN AVALANCHE DIODE OPERATING IN THE TRAPATT MODE, WITH A SQUARE-WAVE CURRENT DRIVE, AS COMPUTED BY THE ANALYTICAL SOLUTION[1]

$J = 1.5 \quad E_A = 1.0 \quad R = 4.5$

Normalized Parameter	$J_s = 2 \times 10^{-9}$	$J_s = 2 \times 10^{-7}$	$J_s = 2 \times 10^{-5}$	$J_s = 2 \times 10^{-3}$
Period	37.7	30.4	23.1	15.6
P_0	0.453	0.459	0.469	0.49
P_1	-0.270	-0.269	-0.267	-0.259
P_3	-0.024	-0.023	-0.020	-0.013
P_5	-0.006	-0.005	-0.002	+0.003
P_7	-0.002	-0.000	+0.002	+0.00[illegible]
η_1	-59.6%	-58.6%	-56.9%	-53.0%
η_3	- 5.4%	- 5.0%	- 4.2%	- 2.6%
η_5	- 1.4%	- 1.1%	- 0.5%	+ 0.6%
η_7	- 0.4%	- 0.1%	+ 0.3%	+ 1.0%
Z_0	0.806	0.817	0.834	0.870
Z_1	-0.593 + j 0.007	-0.591 + j 0.022	-0.585 + j 0.043	-0.567 + j 0.082
Z_3	-0.480 + j 0.201	-0.451 + j 0.233	-0.394 + j 0.274	-0.253 + j 0.322
Z_5	-0.349 + j 0.326	-0.272 + j 0.356	-0.132 + j 0.376	+0.165 + j 0.315
Z_7	-0.173 + j 0.399	-0.042 + j 0.4	+0.176 + j 0.344	+0.533 + j 0.063

Table II

NORMALIZED PERIOD, T, POWERS, P_i, EFFICIENCIES, η_i, AND IMPEDANCES, Z_i, FOR A PIN AVALANCHE DIODE OPERATING IN THE TRAPATT MODE, WITH A SQUARE-WAVE CURRENT DRIVE, AS COMPUTED BY THE ANALYTICAL SOLUTION[1]

$J = 1.0, \quad E_A = 1.0, \quad R = 2.4$

Normalized Parameter	$J_s = 2 \times 10^{-9}$	$J_s = 2 \times 10^{-7}$	$J_s = 2 \times 10^{-5}$	$J_s = 2 \times 10^{-3}$
Period	23.9	19.6	15.0	10.6
P_0	0.323	0.333	0.349	0.384
P_1	-0.177	-0.172	-0.162	-0.134
P_3	-0.008	-0.004	+0.003	+0.014
P_5	+0.003	+0.005	+0.007	+0.006
P_7	+0.004	+0.004	+0.003	-0.002
η_1	-55.0%	-51.7%	-46.2%	-34.8%
η_3	- 2.5%	- 1.3%	+ 0.8%	+ 3.7%
η_5	+ 0.8%	+ 1.5%	+ 2.1%	+ 1.5%
η_7	+ 1.1%	+ 1.2%	+ 0.9%	- 0.5%
Z_0	1.291	1.331	1.398	1.53
Z_1	-0.875 + j 0.182	-0.849 + j 0.218	-0.797 + j 0.271	-0.659 + j 0.353
Z_3	-0.365 + j 0.537	-0.185 + j 0.54	+0.118 + j 0.471	+0.636 + j 0.109
Z_5	+0.327 + j 0.454	+0.611 + j 0.242	+0.910 - j 0.219	+0.710 - j 1.052
Z_7	+0.879 - j 0.060	+1.0 - j 0.525	+0.755 - j 0.193	-0.481 - j 0.973

Table III

NORMALIZED PERIOD, T, POWERS, P_i, EFFICIENCIES, η_i, AND IMPEDANCES, Z_i, FOR A PIN AVALANCHE DIODE OPERATING IN THE TRAPATT MODE, WITH A SQUARE-WAVE CURRENT DRIVE, AS COMPUTED BY THE ANALYTICAL SOLUTION[1]

$J = 0.8, \quad E_A = 1.0, \quad R = 2.0$

Normalized Parameter	$J_s = 2 \times 10^{-9}$	$J_s = 2 \times 10^{-7}$	$J_s = 2 \times 10^{-5}$	$J_s = 2 \times 10^{-3}$
Period	20.0	16.5	13.0	9.3
P_0	0.276	0.289	0.308	0.348
P_1	-0.132	-0.123	-0.106	-0.063
P_3	+0.001	+0.006	+0.013	+0.015
P_5	+0.006	+0.006	+0.004	-0.003
P_7	+0.003	+0.001	-0.002	-0.000
η_1	-47.7%	-42.6%	-34.3%	-18.1%
η_3	+ 0.5%	+ 2.2%	+ 4.1%	+ 4.3%
η_5	+ 2.1%	+ 2.2%	+ 1.3%	- 1.0%
η_7	+ 1.0%	+ 0.3%	- 0.7%	- 0.01%
Z_0	1.727	1.803	1.927	2.173
Z_1	-1.017 + j 0.357	-0.948 + j 0.408	-0.816 + j 0.475	-0.487 + j 0.544
Z_3	+0.104 + j 0.621	+0.436 + j 0.460	+0.871 + j 0.059	+1.033 - j 0.914
Z_5	+1.127 - j 0.237	+1.219 - j 0.819	+0.799 - j 1.510	-0.647 - j 0.990
Z_7	+1.006 - j 1.47	+0.322 - j 1.79	-0.820 - j 1.158	-0.175 + j 0.456

and III can be used, however, to find the general operating characteristics of the diode.

The results shown in Figure 1 and Tables I, II, and III are based upon a particular choice of values for some of the parameters used in the analytical solution. These parameter values were chosen by comparing the results of the analytical solution with results obtained by a more exact computer simulation of the diode over the first few transit times.[1] The analytical solution assumes an exponential function for the ionization coefficient, α, which is valid only over a limited range of electric fields. Figure 2 shows the exponential function used for this ionization coefficient compared with the more exact form.[2] The parameters α_o and λ for the exponential approximation were chosen by equating the two functions at E = 2.0 and E = 1.5. The value E = 2.0 was used because the maximum value of the E-field is of this order. The second point E = 1.5 was chosen because the two curves are then approximately equal over the range of E-fields where most of the avalanche occurs.

Another parameter used in the analytical solution is R, the ratio of the maximum value of the ionization coefficient, α_m, to that value of the ionization coefficient, α_c, below which we assume that avalanche stops and the ionization coefficient drops to zero. Quarterly Technical Report 1 notes that R is of the order of 2.0.[1] However, by comparing the results of a computer simulation (UNSAT) with those of the analytical solution, over the initial charge-up and generation phases of a TRAPATT cycle, we find that the value of R differs for different drive currents. Figure 3(a), (b), and (c) compares the amount of total charge generation as a function of the saturation current as calculated by the computer simulation (UNSAT) and by the analytical solution. Note that for J = 0.8, R = 2.0; whereas for J = 1.0 and 1.5, R = 2.4 and 4.5, respectively. Incidentally, the diode fundamental period of oscillation for a square-wave drive current is equal to twice the generation divided by the current magnitude.

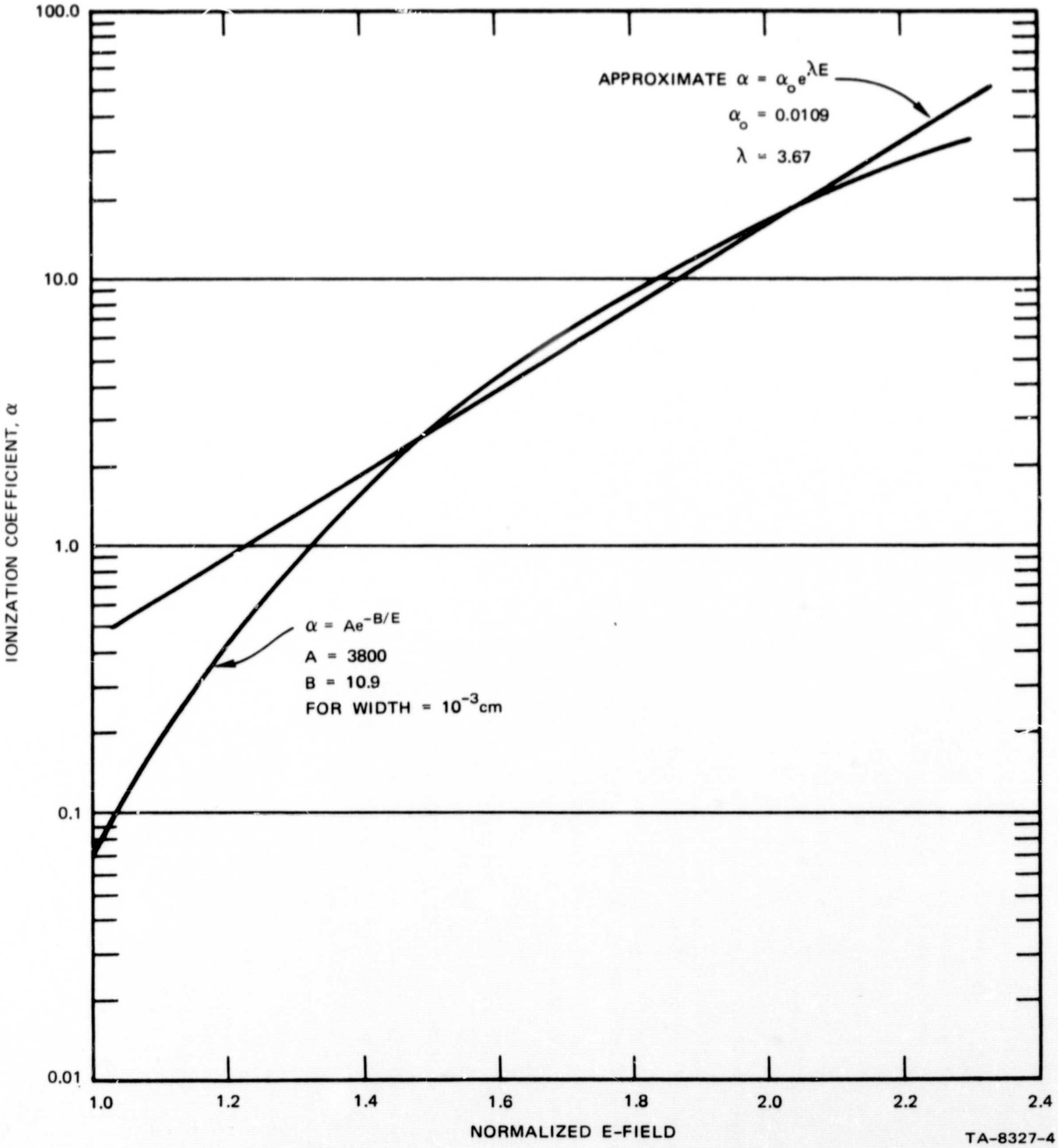


FIGURE 2 COMPARISON OF EXPONENTIAL APPROXIMATION OF THE IONIZATION COEFFICIENT FOR ELECTRONS IN SILICON WITH A MORE EXACT FORM[2]

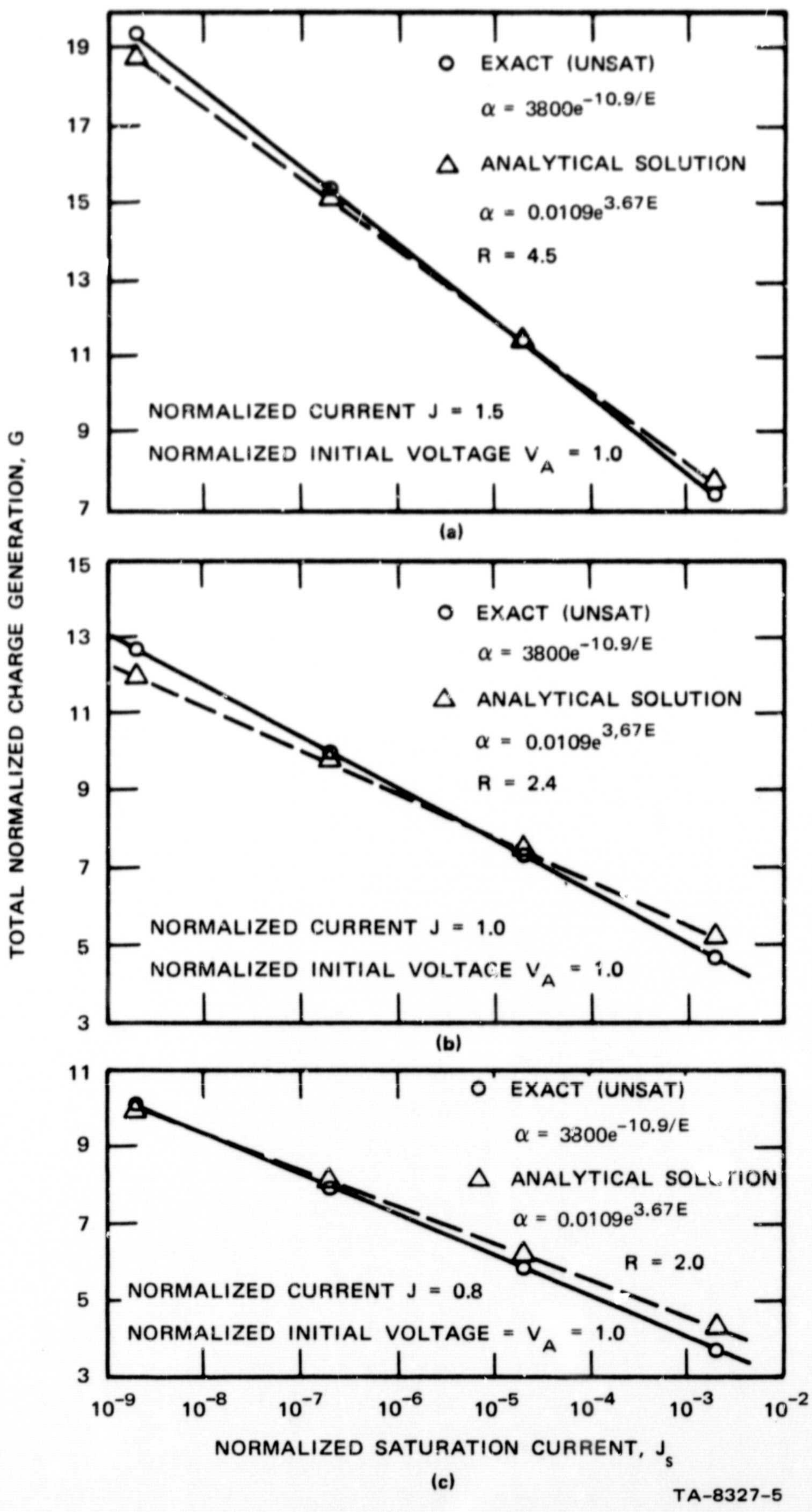


FIGURE 3 THE TOTAL NORMALIZED CHARGE GENERATION DURING ONE CYCLE OF THE TRAPATT MODE IN A PIN DIODE AS COMPUTED BY AN EXACT COMPUTER SIMULATION (UNSAT) AND BY THE ANALYTICAL SOLUTION USING AN EXPONENTIAL IONIZATION COEFFICIENT[1]

We have also made a comparison of the diode voltage as a function of time as computed by the approximate analytical solution and by the exact computer simulation (UNSAT) over the initial charge-up and charge generation phases of the TRAPATT cycle. These results are shown in Figure 4(a) and (b) for two different drive levels. The results from the approximate analytical solution are quite good. A comparison was not made over a longer portion of the cycle because of the excessive running times that would be required for the exact computer simulation.

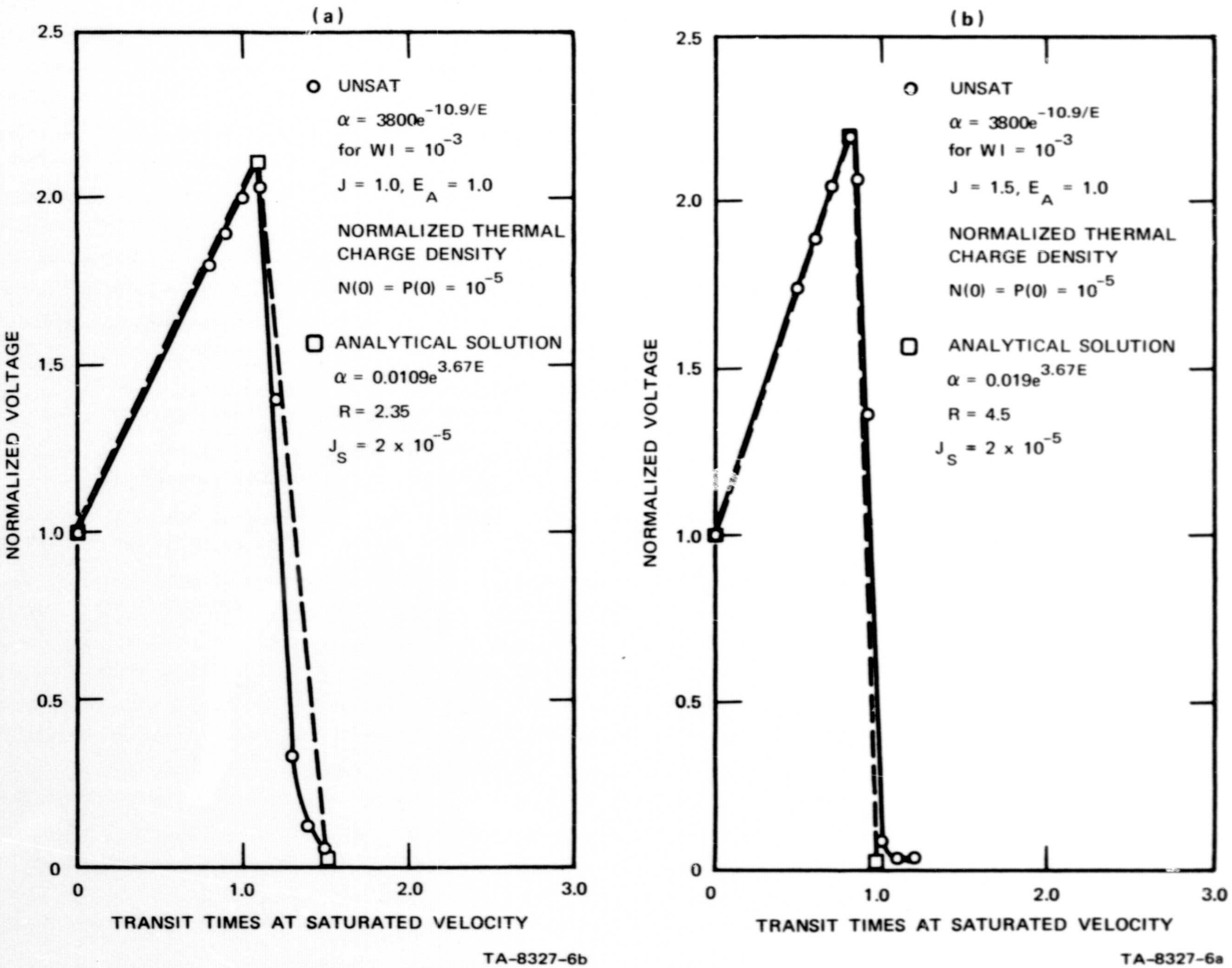
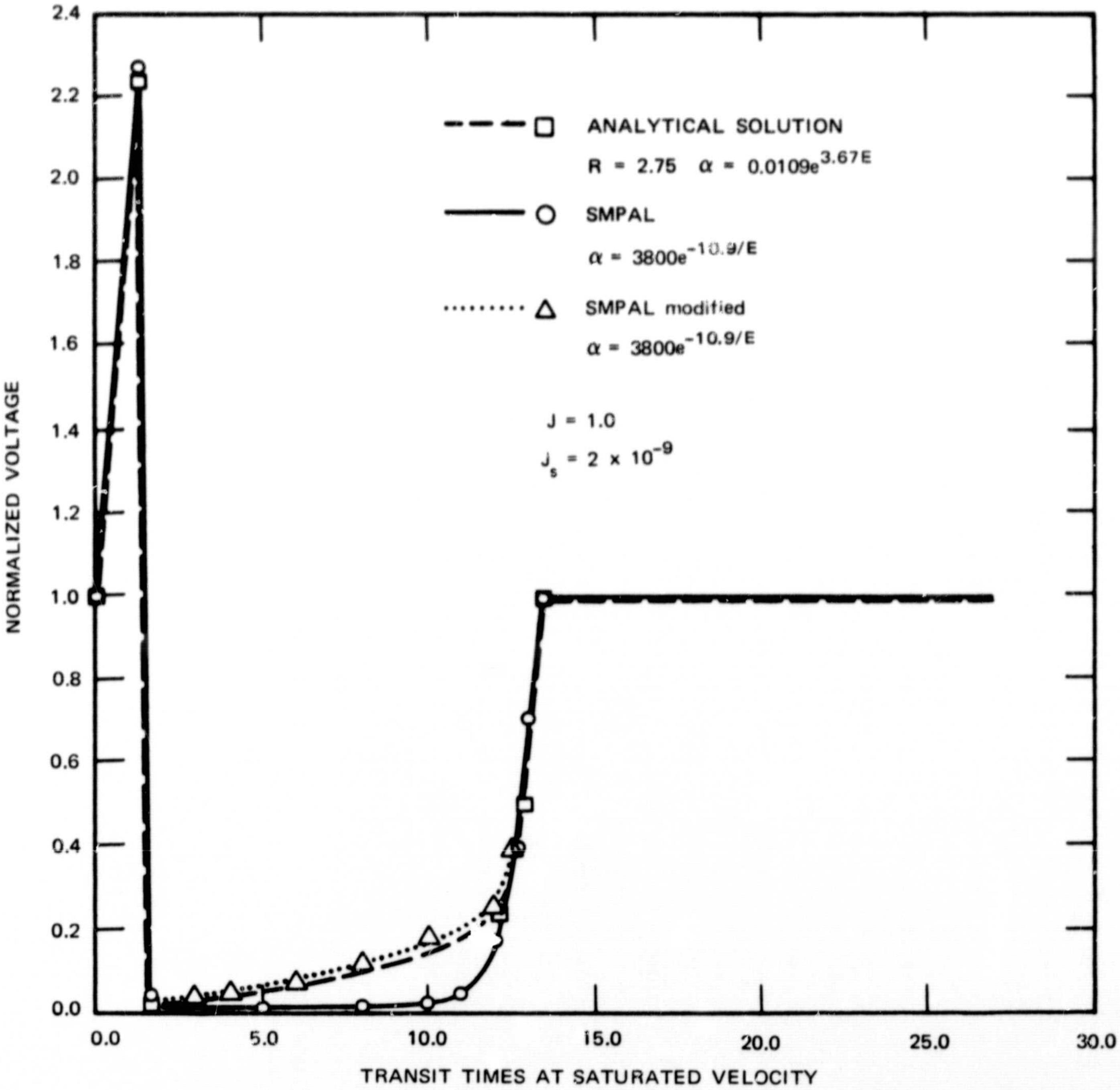


FIGURE 4 COMPARISON OF THE NORMALIZED VOLTAGE AS A FUNCTION OF TIME DURING THE INITIAL CHARGE-UP AND GENERATION PHASES OF THE TRAPATT MODE IN A PIN DIODE, AS COMPUTED BY AN EXACT COMPUTER SIMULATION (UNSAT) AND BY THE ANALYTICAL SOLUTION USING AN EXPONENTIAL IONIZATION COEFFICIENT[1]

III MODIFICATION OF PROGRAM SMPAL

We have reported previously[1] that program SMPAL exhibits most of the essential features of the TRAPATT mode yet has a computer running time two to three orders of magnitude less than that of the exact simulation program (UNSAT). One of the limitations of SMPAL, however, is that during the plasma extraction phase of the TRAPATT cycle the voltage is too low, causing the efficiencies predicted by this program to be too high. We have modified this program so that the voltage during the plasma extraction phase of the cycle is more nearly correct. For example, Figure 5 shows a comparison of the diode normalized voltage as a function of time as computed by the unmodified and modified versions of SMPAL and by the analytical solution for a square-wave current. Table IV gives the corresponding powers, efficiencies, and impedances. The results from the analytical solution and those from the modified version of SMPAL are in close agreement. This agreement is significant for two reasons. First, the analytical solution is limited to square-wave current drives, whereas SMPAL can handle arbitrary current waveforms. Second, since SMPAL in essence simultaneously integrates two coupled nonlinear ordinary differential equations, it can be used in conjunction with passive circuit elements in a time domain analysis of the TRAPATT mode.

An example of the type of current waveform that can be handled with reasonable accuracy by the modified version of SMPAL is the first two nonzero harmonics of the square-wave current. The normalized voltage and current waveforms as a function of time were computed by SMPAL and the modified version of SMPAL; see Figure 6. The corresponding values of normalized power, impedance, and efficiency at the two harmonics are given in Table V. Note that in the modified SMPAL computation the efficiency dropped from an optimistic value of 49 percent to 40 percent



TA-8327-11

FIGURE 5 COMPARISON OF THE NORMALIZED VOLTAGE OVER ONE CYCLE OF THE TRAPATT MODE OF A PIN DIODE AS COMPUTED BY THE ANALYTICAL SOLUTION, PROGRAM SMPAL, AND BY A MODIFIED VERSION OF SMPAL FOR A SQUARE-WAVE DRIVE CURRENT

Table IV

COMPARISON OF NORMALIZED OPERATING PARAMETERS FOR THE TRAPATT MODE IN A PIN DIODE AS COMPUTED FOR A SQUARE-WAVE DRIVE CURRENT BY THE ANALYTICAL SOLUTION, BY PROGRAM SMPAL, AND BY A MODIFIED VERSION OF SMPAL--NORMALIZED DRIVE CURRENT, J = 1.0 AND NORMALIZED SATURATION CURRENT $J_s = 2 \times 10^{-9}$

Normalized Parameter	Analytic Solution (R = 2.75)	SMPAL	SMPAL Modified
Period	26.93	26.75	26.75
P_0	0.316	0.305	0.32
P_1	-0.180	-0.186	-0.171
P_3	-0.011	-0.0108	-0.00967
P_5	+0.001	+0.0015	+0.0013
η_1	-57.1%	-60.7%	-53.5%
η_3	- 3.4%	- 3.5%	- 3.0%
η_5	+ 0.3%	+ 0.48%	+ 0.4%
Z_0	1.263	1.244	1.305
Z_1	-0.889 + j 0.150	-0.916 + j 0.145	-0.84 + j 0.11
Z_3	-0.479 + j 0.504	-0.479 + j 0.395	-0.43 + j 0.474
Z_5	+0.109 + j 0.534	+0.18 + j 0.438	+0.15 + j 0.518

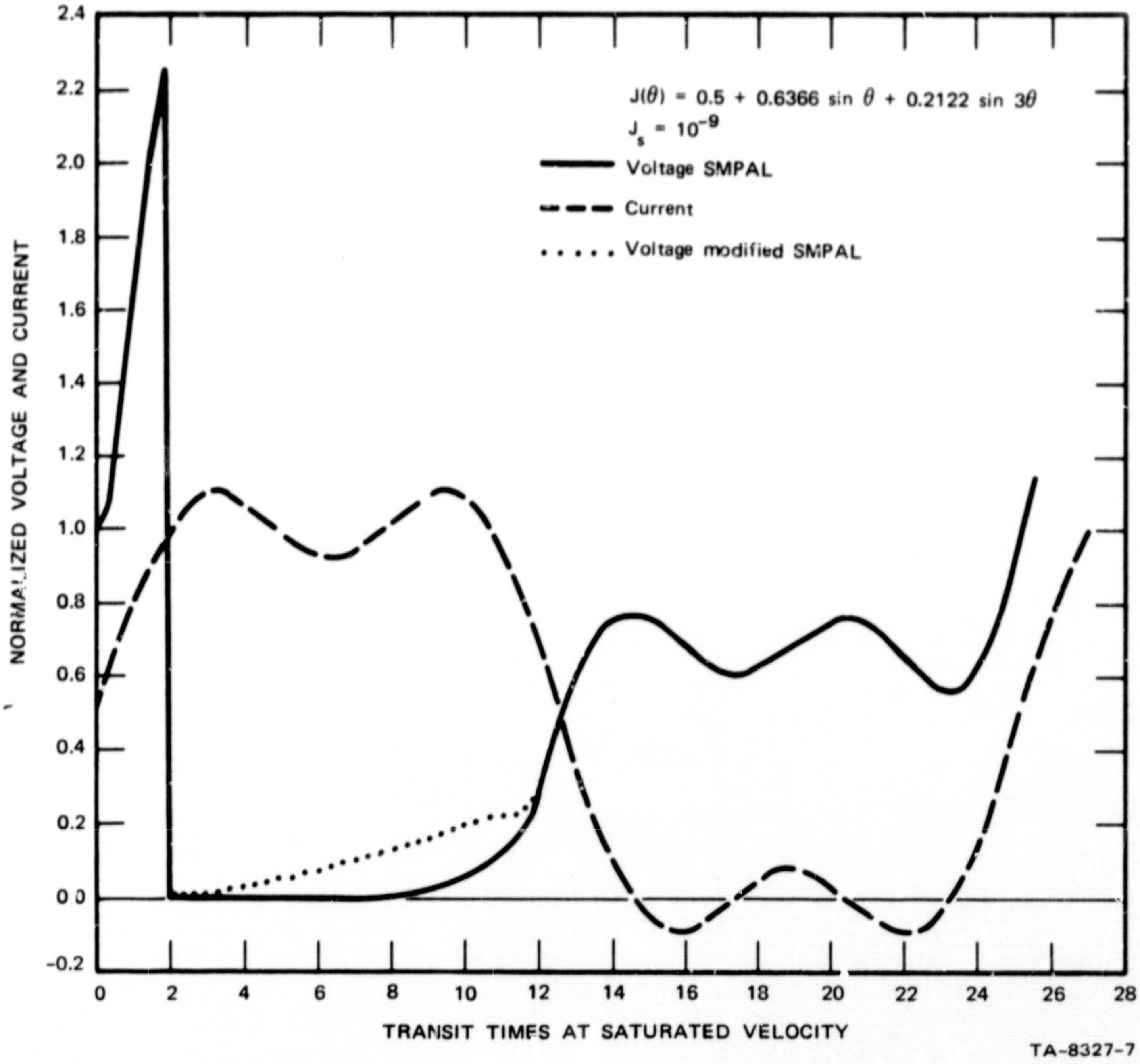


FIGURE 6 NORMALIZED VOLTAGE AND CURRENT WAVEFORMS FOR THE TRAPATT MODE IN A PIN DIODE WITH A NORMALIZED DRIVE CURRENT $J(\theta) = 0.5 + 0.6366 \sin\theta + 0.2122 \sin 3\theta$ AS COMPUTED BY SMPAL AND BY THE MODIFIED VERSION OF SMPAL

Table V

COMPARISON OF OPERATING CHARACTERISTICS FOR TRAPATT OSCILLATIONS IN A PIN DIODE FOR THE NORMALIZED DRIVE CURRENT, $J = 0.5 + 0.6366 \sin \theta + 0.2122 \sin 3\theta$, AS COMPUTED BY SMPAL AND BY THE MODIFIED VERSION OF SMPAL

$V_A = 1.0$

Normalized Parameter	Program: SMPAL	SMPAL Modified
Period	25.15	25.15
P_0	0.238	0.253
P_1	-0.117	-0.1025
P_3	+0.0015	+0.003
η_1	-49.0%	-40%
η_3	+ 0.63%	+ 1.9%
Z_0	0.955	1.015
Z_1	$-0.576 + j\,0.271$	$-0.506 + j\,0.227$
Z_3	$+0.067 + j\,0.57$	$+0.133 + j\,0.646$

and that whereas SMPAL indicated that the diode impedance at the third harmonic was reactive, modified SMPAL showed that it has a positive real part and that power must be fed to the diode at this frequency. Actually, the power required is small, and an adjustment of the parameters should make the third-harmonic impedance reactive or make it have a negative real part.

Below we outline the modifications that were made in program SMPAL to correct for the nonuniform E-field during the field depression and plasma extraction phases of the TRAPATT cycle. In program SMPAL[1] we assumed that for a PIN diode the E-field was essentially uniform across the depletion layer throughout the TRAPATT cycle. To account for the drifting of carriers out of the depletion region, we assumed a relaxation in the total charge. These assumptions lead to two nonlinear coupled differential equations for the total charge in the junction and in the terminal voltage of the junction. Equations (26) and (28) from Quarterly Technical Report 1 are repeated here for reference:

$$\frac{dQ}{dt} = 2v\ (\alpha - 1)Q \tag{1}$$

$$\frac{dE}{dt} = vQ + J \quad . \tag{2}$$

Program SMPAL was written to integrate these equations for different drive currents. The model is inadequate only for that portion of the cycle when the trapped plasma is being extracted; this inadequacy arises because the model assumes that the plasma is removed uniformly across the depletion layer. Actually, the charge is removed from the ends of the plasma, and the electric field is larger in those portions of the depletion region where the plasma has been extracted. The net effect is that, during the plasma-extraction phase, the simple model predicts terminal voltages that are too low.

Program SMPAL was modified by incorporating a new expression for the terminal voltage during the plasma-extraction phase of a TRAPATT cycle. At the end of the generation phase, a plasma has been created that is essentially uniform throughout the entire depletion width. The plasma then begins to flow out the ends of the depletion layer, leaving a residual charge of electrons on one side and a residual charge of holes on the other. Let x be the distance from the right-hand edge of the depletion layer to the edge of the trapped plasma. Since we have assumed equal velocities for both holes and electrons, x is also the distance from the left-hand edge to the plasma. Let E_p be the value of the E-field in the plasma at any instant of time; then, during the field depression phase of the TRAPATT cycle, x and the terminal voltage are given by the following two equations:

$$\frac{dx}{dt} = v \tag{3}$$

$$V = n_r x^2 + E_p \quad , \tag{4}$$

where n_r is the magnitude of the residual charge.[1] In developing the analytical solution for the TRAPATT mode we showed that to a first approximation the residual charge is equal to J/v_s if the time variations are not too rapid. This has also been verified by examination of the results of more accurate computer simulations using program UNSAT.[1] Thus Eq. (4) becomes

$$V(t) = \frac{J(t)}{v_s} x^2 + E_p(t) \quad . \tag{5}$$

When the initial plasma extraction and field depression phase of the cycle is complete, E_p reaches a minimum value determined by the plasma velocity and continues at this low value until the plasma has been completely removed. During the plasma extraction phase, the edges of the plasma move at a plasma velocity, v_p, equal to the external current divided by the plasma charge density, Q_p. Thus, during the plasma extraction phase, Eqs. (3) and (5) become

$$\frac{dx}{dt} = v_p = \frac{J}{Q_p} \tag{6}$$

$$V(t) = \frac{J(t)}{v_s} x^2 + E_p \quad . \tag{7}$$

Equations (3) through (7) are valid when $Q > J/v_s$. When the charge density is so small that the particle current (at saturated velocity) is less than the external current, the E-field is nearly uniform across the depletion layer, and the normalized terminal voltage is just equal to the magnitude of the E-field.

Program SMPAL was modified by incorporating Eqs. (3) through (7) into the calculations for the field depression and plasma extraction phases of the cycle. The results of this program were compared with those of the analytical solution for a square-wave current, as shown in Figure 5.

Several other current waveforms were investigated by using the modified SMPAL. Figures 7 and 8 are plots of the voltage and current waveforms for up to the fifth and seventh harmonic terms of a square wave, respectively. The magnitude of the saturation current and that of the drive current were chosen to be the same as those in the first column of Table I, where the real parts of the impedances were negative for the odd harmonics. The operating power, efficiency, and impedances corresponding to Figures 7 and 8 are given in Table VI.

Comparison of Tables I and VI shows that the fundamental period of oscillation for a current waveform using only the first five or seven harmonics is considerably less than that for the corresponding square-wave current waveform. This is a consequence of the fact that the

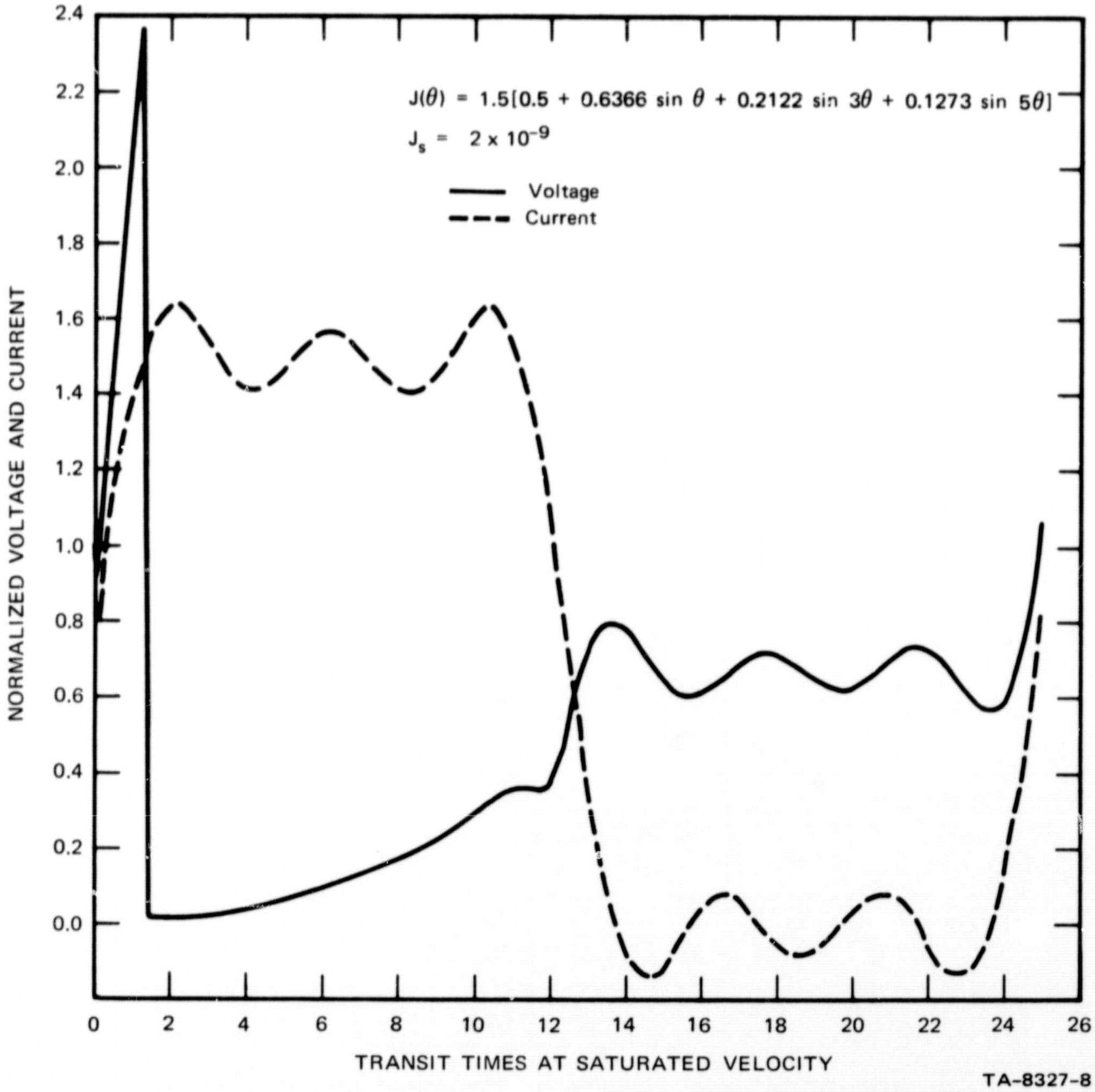


FIGURE 7 NORMALIZED VOLTAGE AND CURRENT WAVEFORMS FOR THE TRAPATT MODE IN A PIN DIODE WITH A DRIVE CURRENT USING THE FIRST, THIRD, AND FIFTH HARMONIES OF A SQUARE WAVE AS COMPUTED BY THE MODIFIED VERSION OF SMPAL

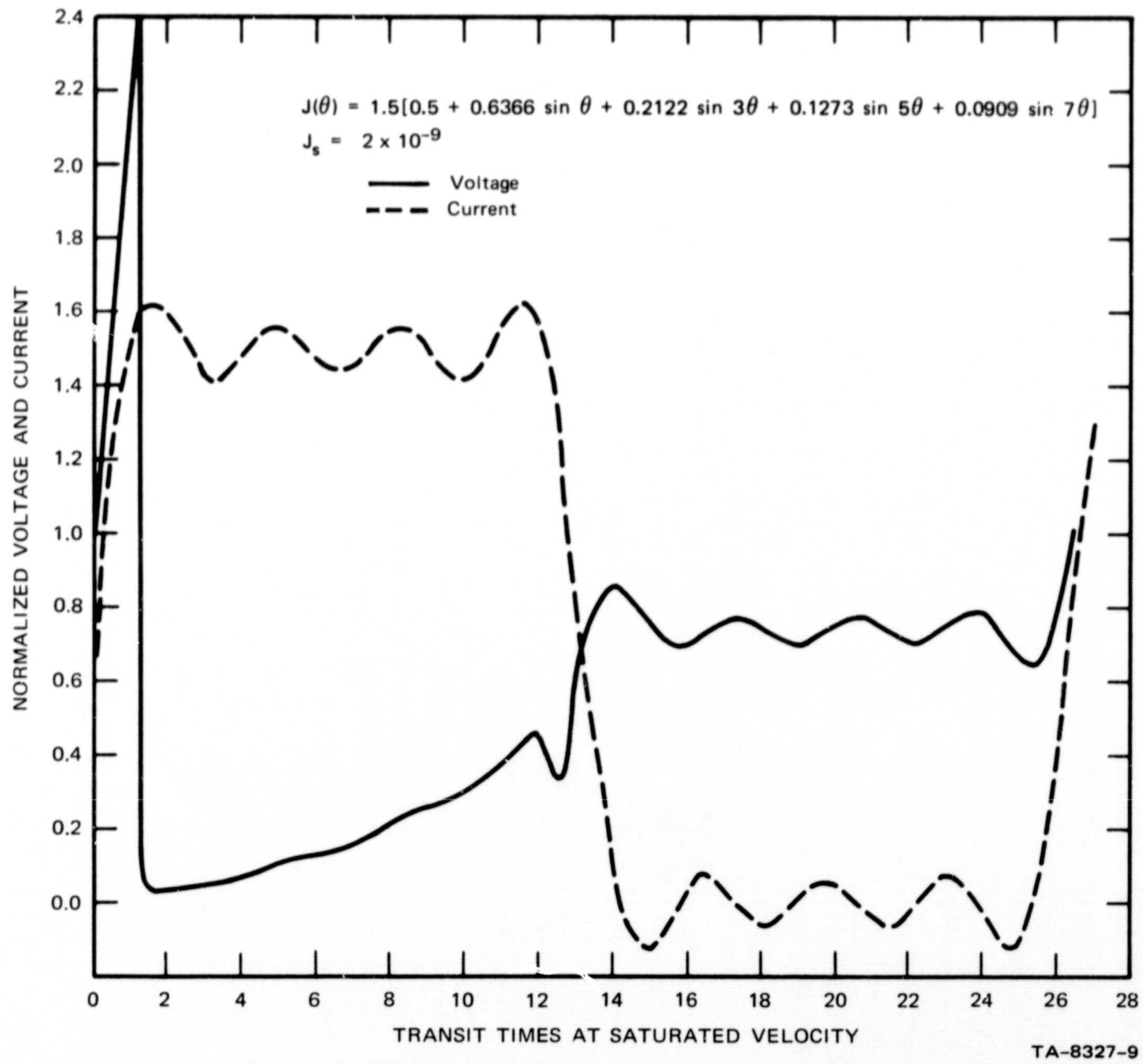


FIGURE 8 NORMALIZED VOLTAGE AND CURRENT WAVEFORMS FOR THE TRAPATT MODE IN A PIN DIODE WITH A DRIVE CURRENT USING THE FIRST, THIRD, FIFTH, AND SEVENTH HARMONICS OF A SQUARE WAVE AS COMPUTED BY THE MODIFIED VERSION OF SMPAL

Table VI

OPERATING CHARACTERISTICS FOR TRAPATT OSCILLATIONS IN A PIN DIODE FOR A NORMALIZED DRIVE CURRENT USING UP TO THE FIFTH AND UP TO THE SEVENTH HARMONICS OF A SQUARE WAVE, AS COMPUTED BY THE MODIFIED SMPAL--$V_A = 1.0$

Normalized Parameter	Five Harmonics	Seven Harmonics
Period	24.90	26.4
P_0	0.383	0.407
P_1	-0.146	-0.158
P_3	-0.00015	-0.0057
P_5	+0.007	+0.0037
P_7		+0.0051
η_1	-38.1%	-38.8%
η_3	0.0%	-1.4%
η_5	+1.83%	+0.91%
η_7		1.25%
Z_0	0.68	0.729
Z_1	-0.319 + j 0.083	-0.347 + j 0.038
Z_3	-0.0029 + j 0.403	-0.111 + j 0.347
Z_5	+0.400 + j 0.434	+0.196 + j 0.438
Z_7		+0.498 + j 0.322

current shown in Figures 7 and 8 does not charge up the diode as rapidly as does a square-wave current. Therefore, the diode is not driven as far into avalanche, and less charge is generated. It is also interesting that for the assumed operating conditions power must be fed into the diode at the fifth and seventh harmonics for the non-square-wave currents.

Other current waveforms were tried where, for the assumed operating conditions, a trapped plasma occurred, but the diode exhibited no negative resistance. For example, only a fundamental component of current and a dc component were assumed and then these two components plus a second harmonic were used. The explanation for the lack of negative resistance here seems to be that too much of the charge required from the external current to neutralize the avalanche charge occurred during the second half of a period. Thus the voltage remained low after avalanche until the very last portion of the cycle. Consequently, the efficiency was very low, and in some instances power had to be furnished to the diode at all frequencies. The higher harmonics are very important in shaping the current waveform for high efficiency. The current waveform should be nearly zero over the entire last half of the cycle to keep the voltage high.

IV ANALYTICAL SOLUTION FOR A SECOND HIGH-EFFICIENCY MODE

A. Description of the Second High-Efficiency Mode

It was suggested in Quarterly Technical Report 1 that other modes of oscillation would provide efficient dc-to-RF energy conversion.[1] Figure 9 shows a set of voltage and current waveforms that are representative of one of these modes of oscillation. In this section an approximate analytical solution for the voltage waveform for this mode is derived for a square-wave current. First, the essential characteristics of this mode of operation are briefly described.

This mode is distinguished from the TRAPATT mode[3,4] by the fact that initially the diode is biased into avalanche but the external current is assumed to be zero, whereas in the normal TRAPATT mode the diode is initially biased below avalanche and the external current is charging up the diode. (See Figure 1 of Ref. 1.) At Point A in Figure 9, the voltage is above breakdown, but there is very little particle current in the depletion layer. The charge density builds up owing to the avalanche process and drifts out, thereby depressing the internal E-field and the terminal voltage. Since the initial internal charge is so small, considerable time is required to generate sufficient charge to depress the E-field. At Point B enough charge has built up that the ionization process acts very rapidly, and a large amount of charge is generated during the short time between Points B and C. At Point C the field has been depressed sufficiently to stop further ionization of charge carriers. We assume that at this point the external current begins to flow. The exact instant of time is somewhat arbitrary and depends upon the external circuit. For the present discussion it is sufficient to assume that the external current comes on when the E-field drops to the critical value E_c below which the avalanche process stops.

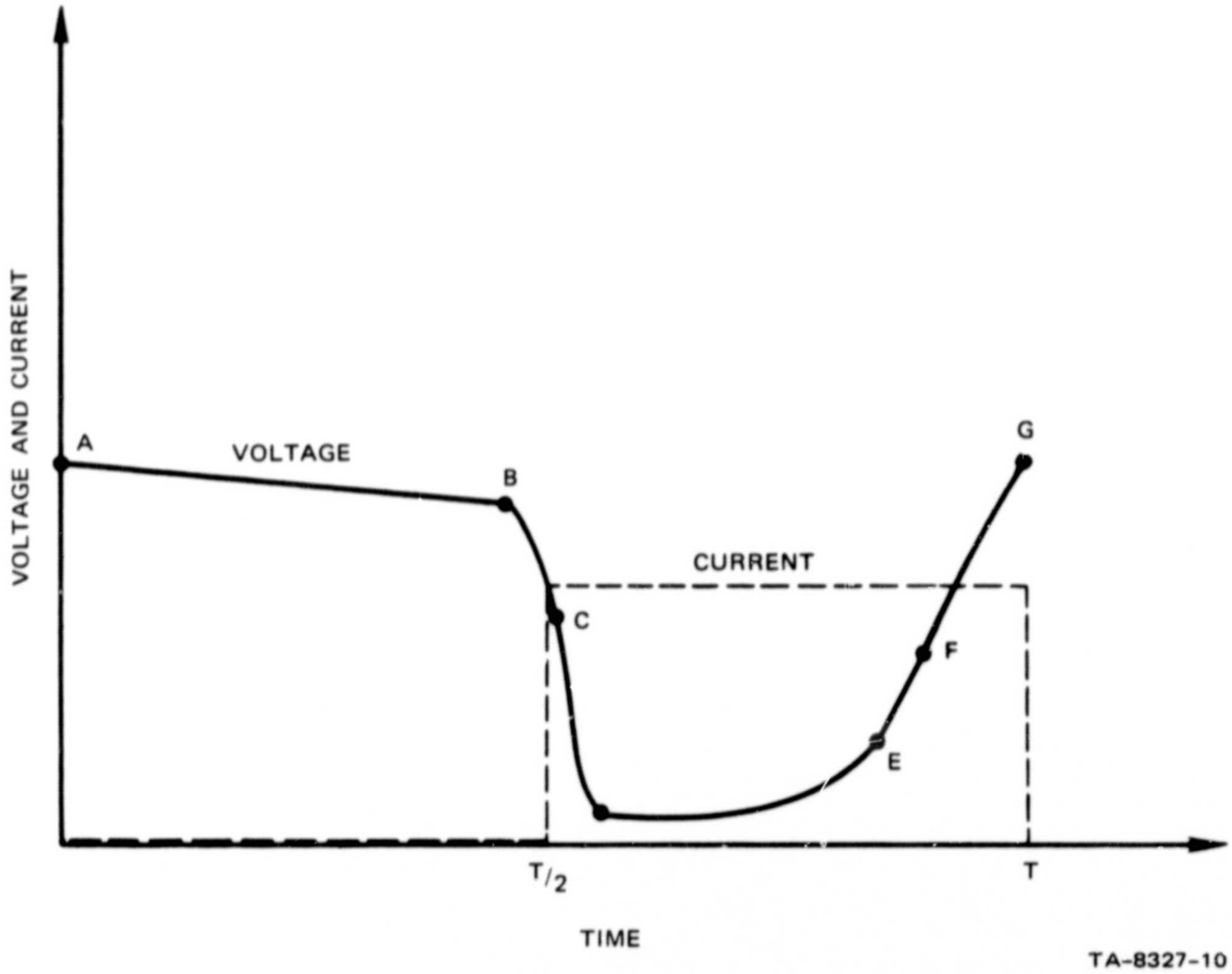


FIGURE 9 VOLTAGE WAVEFORM OF A HIGH-EFFICIENCY MODE OF AN AVALANCHE DIODE FOR A SQUARE-WAVE DRIVE CURRENT

As more charge drifts out the ends of the depletion region, the field in the center is further depressed. The voltage drops to point D, and the plasma is "trapped" in the middle of the depletion region. This portion of the cycle and the portions over to Point F are the same as the corresponding portions in the ordinary TRAPATT mode if the plasma density is sufficiently large.[1] Thus from Point D to Point E the plasma is extracted by the external current, and several transit times will be required for large charge densities.

At Point E the plasma has been extracted, leaving a residual charge of electrons in one half of the depletion layer and a residual charge of holes in the other half. As the residual charge is removed, the voltage increases more or less linearly from Point E to Point F. At Point F all the charge that has been generated internally has been removed from the depletion layer. This charge must be greater than or equal to that supplied by the external current or the voltage will exceed that at Point A. For analysis it is convenient to assume that Point F lies below the critical field required for avalanche. Then all the charges except those due to thermal generation will have been swept out before the diode is charged back up to Point G. The assumption that only a background of thermally generated charges exists at Point F enables one to calculate the charges generated in going from Point F to Point G. The charge in the depletion layer at Point G must be the same as that at Point A to have a periodic solution.

As in the case of our analysis of the ordinary TRAPATT mode, we assume that we have a PIN diode with equal ionization coefficients, α, and equal velocities, v, for both electrons and holes. We showed in Ref. 1 that, when the charge density is small or when the depletion region is filled with a plasma, the E-field is essentially uniform. These conditions are satisfied over certain portions of a cycle of the high-efficiency mode discussed here. Under the assumption of an essentially uniform E-field, Eqs. (1) and (2) are valid, and we use them for appropriate phases of the cycle of oscillation.

B. Initial Avalanche and Generation Phase

During that portion of the cycle from Point A to Point B (Figure 9) the internal charge is small, and there is no external current. The operation of the diode over this portion of the cycle is described by Eqs. (1) and (2). Between Points B and C a large plasma is generated by the avalanche process. As the charges in this plasma drift, they depress the E-field. However, the avalanche process is essentially uniform across the depletion layer, and the E-field remains relatively uniform. We assume as a first approximation that E remains uniform between Points A and C. At Point B, Q has increased from a small value to a magnitude of about

0.1; during this time E is nearly constant and equal to E_m. Similarly, α is equal to α_m, a maximum value corresponding to E_m. Between Point B and Point C, Q increases rapidly. The E-field begins to decrease, forcing α to decrease. Since the mathematics becomes very complicated for α not a constant, we let $\alpha = \alpha_m$ until E reaches a critical value, E_c, at Point C. For values of E less than E_c, we assume $\alpha = 0$. This assumption is similar to the one used in analytical solutions for the normal TRAPATT mode.[1,2,3] The validity of these assumptions and the value of E_c can be established from a computer program like LAGAV or UNSAT.[1]

Using the above assumptions we can write the equations for the charge density and the electric field during the initial avalanche and generation phases of the high-efficiency mode:

$$\frac{dE}{dt} = -v_s\, Q \tag{9}$$

$$Q = Q_A\, e^{2v_s(\alpha_m - 1)t} \quad , \tag{10}$$

where Q_A is the initial charge density at Point A, v_s is the saturated velocity, and α_m is the ionization coefficient corresponding to the maximum field at Point A. Substituting Eq. (10) into Eq. (9) and integrating we obtain

$$E(t) - E_A = \frac{J_A}{2v_s(\alpha_m - 1)} \left[e^{2v_s(\alpha_m - 1)t} - 1 \right] \qquad 0 < t < t_g \ , \tag{11}$$

where $J_A = v_s\, Q_A$ is the small particle current at the beginning of the cycle, t_g is the time when E drops to the critical field E_c, and the generation phase is complete. We solve for t_g from the above equation by setting $E = E_c$:

$$t_g = \frac{1}{2v_s(\alpha_m - 1)} \ln \left\{ 1 + \frac{2v_s(\alpha_m - 1)}{J_A} (E_A - E_c) \right\} \quad . \tag{12}$$

The time for the generation phase is a function of both E_A and J_A. Since J_A can vary over several orders of magnitude, it can have a dominant effect on the generation time. The voltage over this portion of the cycle is just equal to E(t), since we have normalized and have assumed a uniform electric field. By substituting the value of t_g back into Eq. (10) we can calculate the magnitude of the plasma charge, Q_p, in the depletion region at the end of the generation phase:

$$Q_p = Q_A + 2(\alpha_m - 1)(E_A - E_c) \quad . \tag{13}$$

The plasma charge is a function of only the ionization coefficient and the difference between the magnitude of the E-field at Point A and that at Point C. Thus far no assumptions have been made about the functional dependence of α on E. For the last portion of the cycle from Point F to Point G we assume an exponential function, to simplify the analysis.

By analogy with Eqs. (34) and (39) of Ref. 1, the total charge generated in one cycle, G, is equal to the following:

$$G = \alpha_m (E_A - E_c) + \frac{1}{2} Q_A \quad . \tag{14}$$

The integral of the external current over one period must equal G, the total charge generated internally, for the oscillations to be periodic. If we assume that J is constant over one half of the cycle and zero over the other half and that no additional avalanche occurs, then the fundamental period is $T = 2G/J$. A limit is set on the maximum value of J to prevent a second avalanche within the same period.

We can rewrite Eq. (13) for the total charge at Point C as a function of the generation:

$$Q_p = 2G - 2(E_A - E_c) \quad . \tag{15}$$

As in the normal TRAPATT mode, the plasma charge density is equal to the total charge generated less those charges that have drifted out of the depletion layer. It is the drifting charges that actually depress the E-field and stop further ionization.

C. Field Depression and Charge Removal

The operation of the device over those portions of a cycle from Point C to Point F in this high-efficiency mode is the same as that in the normal TRAPATT mode if $Q_p \gg J$.[1] The discussions and corresponding equations are the same if Eq. (41) in Ref. 1 is replaced by Eq. (15) above. The reader is referred to Ref. 1 for these results. If $Q_p \lesssim J$, then new expressions for the voltage during the field-depression and charge-removal phases of a cycle must be derived. These new expressions will be reported at a later date.

D. Final Charge-up Phase

If the drive current has not exceeded the value that would cause second avalanche, all the charge will be removed from the depletion region at Point F (Figure 9). The E-field will again be uniform across the sample. From Eq. (61) of Ref. 1 the voltage at Point F is V_F:

$$V_F = E_p + \frac{J}{2(v_s + v_p)} \quad . \tag{16}$$

Since the field has been assumed to be below avalanche, the diode charges up like a linear capacitor until the field reaches the point where significant avalanche occurs, i.e., where the normalized ionization coefficient approaches unity. Physically, in the device this would be at the same field magnitude defined earlier as E_c, but with the approximations used to calculate the total generation the point where $\alpha = 1.0$ is not necessarily equal to E_c at Point C (Figure 9). We assume that the diode charges up from Point F like a linear capacitor until $\alpha = 1$. During this time the charge density will remain at a small value, determined by the diode back-biased saturation current, J_s:

$$Q_s = \frac{J_s}{v_s} \ll 1 \quad . \tag{17}$$

The external current will continue to drive the field higher into avalanche until the voltage reaches the value at Point G. Since the charge density is small throughout this portion of the cycle, the first term on the right-hand side of Eq. (2) is negligible, and the time required for the final charge-up phase is given by the following:

$$t_c = (E_A - E_F)/J \quad , \tag{18}$$

where $E_A = E_G$ owing to periodicity.

To calculate the charge Q_A we must assume a particular functional dependence on E for α. As before, we choose an exponential function,

$$\alpha = \alpha_o \, e^{\lambda E} \quad . \tag{19}$$

The constants α_o and λ are determined by fitting the exponential function at two points to the experimental data. Since we have assumed that the E-field increases linearly with the external current, we can solve for the charge from Eq. (1) during the final charge-up phase of the cycle:

$$Q(t) = Q_s \exp\left\{ \frac{2v_s\alpha_1}{\lambda J} \left[e^{\lambda J t} - 1 \right] - 2v_s t \right\} , \tag{20}$$

where α_1 is the value of the ionization coefficient when avalanche starts and hence is equal to one; Q_s is the charge due to thermal generation and corresponds to the diode saturation current. To find Q_A we must substitute for t, the time required for the field to increase from E_1 to E_A where E_1 is the value corresponding to $\alpha_1 = 1$. Thus $t = (E_A - E_1)/J$ and

$$Q_A = Q_s \exp\left\{ \frac{2v_s\alpha_1}{\lambda J} \left[e^{\lambda (E_A - E_1)} \right] - \frac{2v_s}{J} (E_A - E_1) \right\} . \tag{21}$$

For a self-consistent solution the above value of charge should be used in Eq. (11) for the electric field during the generation phase of the cycle. The only quantity that is somewhat arbitrary is E_c in Eqs. (12), (13), and (14). As mentioned earlier, we can determine E_c by comparing our results for the generation as computed by Eq. (14) with those obtained by using a computer program like LAGAV or UNSAT.

V CONCLUSIONS

The results during this second quarter showed that the assumption of a square-wave current in the analytical solutions for the high-efficiency modes is very instructive and provides further understanding of the importance of higher harmonics in these modes. For high efficiency in the normal TRAPATT mode, the higher harmonics are necessary to minimize the amount of charge (integral of the external current) in the last half of the cycle. If the charge is too large, the voltage never rises after avalanche until the very end of the cycle, and the efficiency is low. The highest efficiencies were obtained for the largest drive currents and the smallest saturation currents. From the several different operating conditions considered, it was evident that in many instances the square-wave drive current does not correspond to the diode being embedded in a passive circuit.

It was also demonstrated that other current waveforms for the TRAPATT mode can be investigated relatively economically by using a modified version of SMPAL. A square-wave current was approximated by using the first few terms of its Fourier series. It was found that both the period of oscillation and the efficiency decreased from those for a corresponding square wave. For high efficiency the higher harmonics are necessary, to allow the current waveform to rise very rapidly and drive the diode far into avalanche before appreciable charge has been generated.

The second high-efficiency mode considered in this report appears to be very attractive. Higher harmonic currents may not be as important in this mode as in the normal TRAPATT mode. Also, since in this mode the generation is relatively small, this second mode should be more efficient at higher frequencies than is the normal TRAPATT. This mode will be studied more extensively during the next quarter.

REFERENCES

1. D. Parker, "Avalanche-Diode Oscillator Circuit with Tuning at Multiple Frequencies," Quarterly Technical Report 1, Contract NAS 12-2231, SRI Project 8327, Stanford Research Institute, Menlo Park, California (24 March 1970).

2. D. L. Scharfetter and H. K. Gummel, "Large Signal Analysis of a Silicon Read Diode Oscillator," IEEE Trans. on Electron Devices, Vol. ED-16, No. 1, pp. 64-77 (January 1969).

3. A. S. Clorfeine et al., "A Theory for the High-Efficiency Mode of Oscillation in Avalanche Diodes," RCA Review, Vol. 30, No. 3, pp. 397-421 (September 1969).

4. B. D. DeLoach, Jr., and D. L. Scharfetter, "Device Physics of TRAPATT Oscillators," IEEE Trans. on Electron Devices, Vol. ED-17, No. 1, pp. 9-21 (January 1970).