N 70 41457

NASA TECHNICAL MEMORANDUM

NASA TM X-52881

NASA TM X-52881

LAMINATION RESIDUAL STRESSES IN CROSS-PLIED FIBER COMPOSITES

by Christos C. Chamis
Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at the 26th Annual Conference of the Society of Plastic Industries, Inc.
Washington, D. C., February 9-12, 1971

E-5848

LAMINATION RESIDUAL STRESSES IN CROSS-PLIED FIBER COMPOSITES

by Christos C. Chamis*

National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio

ABSTRACT

Residual stresses arising from the lamination fabrication process are investigated using linear laminate theory. An equation to predict the residual stresses is given. The pertinent variables which influence residual stresses are identified. Several composite systems with various ply lay-up configurations are examined. Results are presented to illustrate the dependence of the residual stresses on the pertinent variables. The residual stresses are very sensitive to constituent material properties, ply stacking sequence and orientation, fiber content, and cure temperature. It is found that ply transverse and in-plane shear residual stresses can reach magnitudes comparable to corresponding ply strengths and cause transply cracks in the composites. Residual stresses can also cause interply delamination. Ways to prevent transply cracks and interply delamination are reccommended. The effects of the residual stresses on the design of a composite compressor blade are illustrated.

INTRODUCTION

Multilayered fiber composites are readily adaptable to simultaneous component and material designs. The design approach allows the designer to take advantage of the high stiffness and strength-to-density ratios of fiber composites. The result is an efficient utilization of fiber composite material in structural components. The composite is usually designed by specifying the fiber direction in each ply, the number of plies, and the fiber volume ratio to meet the given loading requirements.

However, having different fiber directions in different plies can cause transply cracks such as have been observed in graphite fiber composites. These cracks are caused by the large thermal residual stresses present in the composite. The residual stresses result from the fabrication process used to make fiber composites. The residual stresses arising from the fabrication process are examined quantitatively herein using linear laminate theory.

*Aerospace Technologist, NASA Lewis Research Center, Cleveland, Ohio.

Multilayered fiber composites consist of several unidirectional layers (plies) oriented at specified angles relative to some set of axes. The geometry of such a composite is depicted in figure 1. The plies consist of parallel fibers embedded in a matrix. The geometry of a ply is depicted in figure 2. Fiber composites such as glass or graphite will have many filaments through the ply thickness.

In general composites achieve their structural integrity by being cured at elevated temperature under pressure. This process induces residual stresses in the composite when the composite is at a temperature different from its cure temperature. The difference between the cure temperature and the use temperature will be referred to as temperature difference herein. The resulting residual stresses can be either micro or macro residual stresses.

The micro residual stresses are influenced by the constituents' local geometry within the ply. They arise from the temperature difference and the difference between the thermal coefficients of expansion of the constituents. The matrix phase change shrinkage also contributed to the micro residual stress. Adams et al. (ref. 1) and Haener et al. (ref. 2), for example, treat this type of residual stress analytically. Marloff and Daniel (ref. 3) and Koufopoulos and Theocaris (ref. 4) examined micro residual stress experimentally. The ply depicted in figure 2 will have micro residual stresses.

Macro residual stresses (ref. 5) are integrated averages through the ply thickness. These stresses have an effect on the ply similar to an equivalent thermal or mechanical load applied to the composite. They arise from the difference between the ply longitudinal and transverse thermal coefficients of expansion. Macro residual stresses are present in those composites which have plies oriented at different angles and whose temperature differs from the cure temperature. These stresses will be present in the composite depicted in figure 1.

The significance of the lamination residual stress was realized when transply cracks were observed in $0^\circ/90^\circ$ graphite composites. Doner and Novak (ref. 5) presented several photomicrographs showing transply cracks. Decrescente and Novak (ref. 6) and Winters (ref. 7) described similar phenomena.

Some analytical treatment of the lamination residual stress has also been reported. Doner and Novak (ref. 5) considered the residual stresses in $0^\circ/90^\circ$ composites. Edighoffer et al. (ref. 8) examined the residual stresses in a boron/epoxy composite with laminations $\pm\theta/90$ where the $\pm\theta$ plies are considered as an equivalent 0° ply. This author (ref. 9) used linear laminate theory to predict residual stresses in graphite composites without restriction to ply orientation.

The lamination theory used in reference 9 was developed in reference 10. It was implemented in references 11 to 13 and in a computer code (ref. 14).

The computer code of reference 14 provides a method for micromechanics, lamination and stress analyses of multilayered fiber composites. The inputs to this code are constituent material properties, fiber and void contents, correlation coefficients, composite geometry, temperature difference, (difference between composite temperature and cure temperature) and composite stress resultants or displacements.

The computer code of reference 14 permits the variation of several basic parameters in fiber composites. It is utilized herein to investigate the residual stresses in laminated fiber composites. The following systems are examined: Boron/Epoxy, S-Glass/Epoxy, Thernel-50/Epoxy, and Modmor-I/Polyimide and Thornel-50/Polyimide. The significant parameters in these systems are ply orientation, fiber and void contents, matrix modulus and thermal coefficients of expansion, and the temperature difference. To the author's knowledge, the effects of some of these parameters on the residual stresses are investigated herein for the first time. Also investigated here for the first time are: (1) possible ply delamination due to residual stress, and (2) residual stresses in composites which are unsymmetric with respect to bending.

The ply orientations examined are shown in table I together with the notation adopted herein for convenience. Theoretical thermal and elastic properties for the composite systems considered are listed in table II. These properties are for 55 percent fiber volume content. The corresponding theoretical strength properties are given in table III. The results in tables II and III are those predicted by the computer code, reference 14.

The significant parameters mentioned previously are varied in the computations. The results discussed later will illustrate the dependence of the ply residual stresses on these parameters. The plots are grouped by composite system. The composite systems Boron/Epoxy, S-Glass/Epoxy and Thornel-50/Epoxy are used in some basic ply orientations. Results for various ply orientations in composites unsymmetric with respect to bending are also presented for these systems. The composite system Modmor-I/Polyimide and Thornel-50/Polyimide are used in simulated structural components. Some results are presented for Modmor-I system to demonstrate possible ply delamination due to residual stresses.

The results presented herein are for two temperature differences: 300° F (589.9 K) for the first four systems and 600° F (589.9 K) for the Modmor-I/Polyimide system. Residual stresses for any other temperature differences are obtained from the plots by direct proportion. This is because a linear theory is used and the residual stress is linearly dependent on the temperature difference.

GOVERNING EQUATIONS

The laminate theory equation for predicting the ply residual stresses is given by

$$\left\{\sigma_{\ell i}\right\} = \left[E_{\ell i}\right]^{-1} \left\langle \left[R_{\ell i}\right] \left\{\epsilon_{cox}\right\} - z_{\ell i}\left[R_{\ell i}\right] \left\{\kappa_{cx}\right\} - \Delta T_{\ell i} \left\{\alpha_{\ell i}\right\} \right\rangle \quad (1)$$

the reference plane strains $\left\{\epsilon_{cox}\right\}$ and the curvature changes $\left\{\kappa_{cx}\right\}$ for a free composite (free of external loads and boundary constraint) are computed from

$$\begin{Bmatrix} \left\{\epsilon_{cox}\right\} \\ \left\{\kappa_{cx}\right\} \end{Bmatrix} = \begin{bmatrix} \left[A_{cx}\right] & \left[C_{cx}\right] \\ \left[C_{cx}\right] & \left[D_{cx}\right] \end{bmatrix}^{-1} \begin{Bmatrix} \left\{N_{c\Delta Tx}\right\} \\ \left\{M_{c\Delta Tx}\right\} \end{Bmatrix} \quad (2)$$

Equations (1) and (2) show that the ply residual stress is a function of the following factors: (1) the composite structural stiffnesses, $[A_{cx}]$, $[C_{cx}]$ and $[D_{cx}]$, (2) the ply spatial location in the composite $z_{\ell i}$ and $[R_{\ell i}]$, (3) the ply stress-strain relation $[E_{\ell i}]$, (4) the ply thermal coefficients of expansion $\left\{\alpha_{\ell i}\right\}$ and (5) the temperature difference between the ply and the reference value $\Delta T_{\ell i}$. This difference equals the ply temperature minus the cure temperature in computing residual and thermal stresses. The afore discussion leads to the identification of the independent variables which influence the ply residual stress. These are: Constituent materials' elastic properties, (2) Constituent materials' thermal properties, (3) Fiber volume ratio (4) Void content, (5) Ply distance from reference plane, (6) Ply orientation relative to composite structural axes and (7) Difference between ply temperature and cure temperature.

RESIDUAL STRESSES IN BORON, S-GLASS, AND THORNEL-50/EPOXY COMPOSITES

The residual stresses in these composites are computed from a 300° F (167° K) temperature difference, room temperature constituent material properties and the linear theory available in reference 14. This theory is briefly described in the INTRODUCTION. Five reasons justify this approach: (1) the residual stresses are computed at room temperature, (2) polymerization shrinkage (matrix phase change), though not accounted for directly in the theory, is compensated for by the higher temperature, (3) the epoxy modulus decreases with increasing temperature but the epoxy coefficient of expansion increases, (4) the residual stress results presented herein can be scaled down proportionally to any desired temperature difference, and (5) composites designed to withstand the residual stress at room temperature predicted by this approach will be conservatively designed.

Boron/Epoxy Composites

The residual transverse stresses in the $\pm\theta$-plies of the composite $8[2(0), 2(\pm\theta), 2(\mp\theta), 2(0)]$ are plotted against θ and three fiber volume ratios (FVR) in figure 3.

Ply residual transverse stresses for the 0°, $\pm 0^\circ$ and 90°-plies of the composites $8[0, 2(\pm\theta), 2(90), 2(\mp\theta), 0]$ are plotted against θ in figure 4. The FVR is 0.55 for these composites. The results in this figure at $\theta = 0^\circ$ and 90° illustrate the effect on residual stress of the numbers of plies in each orientation.

The residual transverse stress in the outer ply of the composites $8[4(+\theta), 4(-\theta)]$, $8(\pm\theta)$, and $8[4(\pm\theta), 4(\mp\theta)]$ is plotted against θ in figure 5. The FVR is 0.55 for these composites. Note the composites $8[4(+\theta), 4(-\theta)]$ are nonsymmetric with respect to bending and have noninterspersed ply stacking sequence. The composites $8(\pm\theta)$ are nonsymmetric with respect to bending but have interspersed ply stacking sequence. The composites $8[4(\pm\theta), 4(\mp\theta)]$ are symmetric and have interspersed ply sequence.

The transverse strength for the unidirectional composite (ply) at 0.55 FVR is 8.6 ksi (59.3 MN/m^2) from table III. This value is exceeded only by the residual stresses in the outer plies of the composite $8[4(+\theta), 4(-\theta)]$ (fig. 5). However, the margin of safety is about 0.3 in many of the other composites.

Several interesting points are illustrated by the results in figures 3 to 5. (1) The ply residual transverse stress in boron/epoxy composites is relatively high when compared with the ply transverse strength for some composite geometries. (2) The ply residual stress is quite sensitive to the ply angle, but not as sensitive to FVR. (3) The ply transverse residual stresses in the composites 8[0, 2(±θ), 2(90), 2(∓θ), 0] are practically invariant with θ. (4) Nonsymmetric, noninterspersed-ply composites experience high ply residual stresses when compared with other composite geometries. (5) Nonsymmetric, interspersed-ply and symmetric, interspersed-ply composites experience ply residual stresses of about the same magnitude.

S-Glass/Epoxy Composites

The residual stress results for these composites are presented in figures 6 to 8. The composite geometries and other variables investigated are the same as those for the Boron/Epoxy and are indicated in the figures.

The transverse strength for the S-Glass/Epoxy unidirectional composite (ply) at 0.55 FVR is 4.4 ksi (30.4 MN/m^2) from table III. This value is very close to the ply residual stress of several composites. It is exceeded in the outer plies of some composites 8[4(+θ), 4(-θ)]. This implies that S-Glass/Epoxy composites of the geometries and FVR examined in figures 6 to 8 could experience transply cracks. This will not be the case if the transverse strength of the plies is much greater than 4.4 ksi (see for example table I of ref. 5).

Several interesting points are illustrated by the results in figures 6 to 8. (1) Calculated ply residual transverse stress could exceed the ply transverse strength. The extent that this strength is exceeded by the calculated residual stress depends on composite geometry. (2) The ply residual stress is quite sensitive to ply orientation angle in certain composite geometries. (3) The ply residual stress is relatively insensitive to FVR in composites with less than 0.55 FVR. (4) The ply residual stress is sensitive to the number of plies in each orientation. (5) Nonsymmetric, noninterspersed-ply composites experience large ply residual stresses when compared with other composite geometries. (6) Nonsymmetric, interspersed-ply and symmetric, interspersed-ply composites experience ply residual stresses of about the same magnitude.

Thornel-50/Epoxy Composites

The residual stress results for these composites are presented in figures 9 to 11. The composite geometries and other variables investigated are the same as those for the Boron/Epoxy and are indicated in the figures.

The transverse strength for the Thornel-50/Epoxy unidirectional composite (ply) at 0.55 FVR is 3.7 ksi (25.5 MN/m^2) from table III. This value is exceeded, or nearly so, by several composites. This implies that plies in Thornel-50/Epoxy composites of the geometries and FVR examined in figures 9 to 11 will experience transply cracks. This has been observed experimentally (refs. 5 to 7). However, this will not be the case if the transverse strength of the plies is about 8 ksi (55.2 MN/m^2).

Several interesting points are illustrated by the results in figures 9 to 11. (1) The calculated ply residual transverse stress exceeds the ply transverse strength. The extent that this strength is exceeded by the calculated residual stress depends on composite geometry. (2) The ply residual stress is quite sensitive to ply orientation angle in certain composite geometries. (3) The ply residual stress is sensitive to FVR. (4) The ply residual stress is insensitive to the number of plies in each orientation. It is relatively insensitive to θ in the composites $8[0, 2(\pm\theta), 2(90), 2(\mp\theta), 0]$. (5) Nonsymmetric, noninterspersed-ply composites experience about 30 percent higher ply residual stresses when compared with other composite geometries. (6) Nonsymmetric, interspersed-ply and symmetric, interspersed-ply composites experience ply residual stresses of about the same magnitude.

SIMULATED COMPOSITE COMPONENTS

The residual stresses in the plies of two simulated composite components were studied. The composite geometries are $24[4(\pm 30), 2(\pm 15), 12(0), 2(\mp 15), 4(\mp 30)]$ and $24[4(\pm 45), 2(\pm 22.5), 12(0), 2(\mp 22.5), 4(\mp 45)]$. The composite system is Modmor I/Polyimide. The cure temperature for this system is about 600° F (589.9 K).

The dependence of the ply residual stresses on several factors is examined. These factors are: (1) fiber volume ratio (FVR), void content, (2) variation of the FVR from ply to ply within the same composite, (3) different groups of plies with different FVR, (4) variations in the matrix thermal coefficient of expansion, and (5) variations in the matrix modulus. Also the effect of the residual stresses on the relative rotation of adjacent plies is investigated.

The computations are based on 600° F (589.9K) temperature difference and room temperature constituent materials properties. The justification for this approach is the same as that given for the other fiber/epoxy composite systems.

The ply residual transverse stress is plotted against FVR in figure 12 for both components. Results for the ply residual longitudinal stress are shown in figure 13. Note the residual longitudinal stress in the 0°

plies of both components is tensile. Results for ply residual shear stress are plotted against FVR in figure 14. The results in these figures show that the ply residual stresses are sensitive to both composite geometry and FVR.

The void content effect on the ply transverse residual stress is illustrated in figure 15. This effect is negligible for all practical purposes. One important point to keep in mind is that the ply transverse strength decreases with increasing void content (ref. 12). Therefore, the decrease in ply transverse strength could produce transply cracks at smaller temperature differences. Proper consideration of the void effects is necessary to avoid transply cracking (see eq. (3) in this section).

The effects of variable ply FVR on the ply transverse residual stresses are shown in figure 16 for one component only. These effects are negligible as can be seen in the figure. Having different groups of plies at different FVR has negligible effects on the ply residual transverse stress. This is illustrated in figure 17. Negligible effects were also found when the ply temperature was varied similar to the fiber volume ratio. These results are not presented here.

The dependence of the ply residual transverse stress on the matrix thermal coefficient of expansion is illustrated in figure 18. Two important points are observed here. (1) The ply residual stress varies linearly with matrix thermal coefficient of expansion. (2) The residual stresses can be reduced by decreasing the magnitude of the thermal coefficient of the matrix. The effects of varying the fiber thermal coefficients of expansion will produce effects opposite to those shown in figure 18 for the matrix coefficient. This implies that increasing the fiber thermal coefficient of expansion is another effective way to reduce the residual stress.

The dependence of the ply residual transverse stress on the matrix modulus is illustrated in figure 19. The results in this figure show that the residual stress increases nonlinearly with increasing matrix modulus. This increase is quite rapid at modulus values of 0.1 to 0.7×10^6 psi (0.69 to 4.83 GN/m^2) and not as rapid at modulus values greater than 0.7×10^6 psi (4.83 GN/m^2).

The results in figure 19 should be examined jointly with the ply transverse strength before any meaningful conclusions are made. The ply transverse strength is given by the expression (ref. 12).

$$S_{\ell 22T} = \beta_{22T} \frac{\epsilon_{mpT}}{\beta_V \phi_{\mu 22}} E_{\ell 22} \qquad (3)$$

The notation in equation (3) is as follows: $S_{\ell 22T}$ is the ply transverse tensile strength; β_{22T} is the correlation factor; ϵ_{mpT} is the allowable matrix tensile strain to denote failure; β_v is the void strain magnification coefficient; $\phi_{\mu 22}$ is the strain magnification factor; and $E_{\ell 22}$ is the ply transverse modulus. The variables ϵ_{mpT}, $\phi_{\mu 22}$ and $E_{\ell 22}$ are not independent of the matrix modulus. For example $\phi_{\mu 22}$ decreases but $E_{\ell 22}$ increases with increasing matrix modulus (ref. 10), and ϵ_{mpT} increases with increasing matrix modulus in general.

The following point can be made in view of the preceding discussion. The ply transverse residual stresses increase with increasing matrix modulus. The ply transverse strength also increases with increasing matrix modulus. Transply cracking may occur if the ply residual stress increases at a higher rate than the ply transverse strength. Limited results from this investigation showed that the ply strength increases at a higher rate than the residual stress.

The effect of the residual stress on the adjacent ply relative rotation is illustrated in figure 20. Relative rotation is plotted against fiber volume ratio (FVR) for selected adjacent plies of the two components. The estimated allowable is also plotted in the figure. The prediction of the estimated allowable is discussed in references 10 and 11. The results in figure 20 show that residual stresses could cause interply delamination. The delamination depends on the composite geometry, the ply angles of the two adjacent plies and the fiber content. It is interesting to note that the component with the $\pm 30^\circ$ and $\pm 15^\circ$ plies has a higher probability of interply delamination than the component with the $\pm 45^\circ$ and $\pm 22.5^\circ$ plies. The component with the $\pm 30^\circ$ and $\pm 15^\circ$ plies is superior in all other respects with respect to residual stress. To the author's knowledge the significance of the results in figure 20 has not been recognized by other investigators. The results in figures 12 to 20 show that components designed by neglecting the residual stresses will be unconservative in some loading conditions.

The effects of introducing transitional plies or decreasing the number of off-axis plies on the residual stress are illustrated in figure 21. The transverse residual stresses in the $\pm 45^\circ$-plies are plotted against FVR. The composite with the transitional plies is denoted by the dot-dash curve. The composite in which the transitional plies have been incorporated with the face (surface) plies is denoted by the solid-line curve. The composite in which the transitional plies have been incorporated with the core plies is identified by the dashed-line curve.

The results in figure 21 show that both reduction of the number of off-axis plies and introduction of transitional plies reduce the residual stress by small amounts. The number of off-axis plies is controlled by strength and stiffness requirements and changes might not be permitted in specific designs.

RESIDUAL AND THERMAL STRESS EFFECTS IN DESIGNING ACTUAL COMPONENTS

The residual and thermal stress effects in actual designs are illustrated by examining the possible application of fiber composites in high-tip-speed (2200 ft/sec) compressor blades. Only partial results of the blade stress analysis are sufficient to illustrate the point.

The composite geometry and loads are given in table IV. The blade consists of surface, transition and core plies which are different at different stations.

The stress analysis results for the ply longitudinal stress are plotted as a function of blade length and four different load conditions in figure 22. These load conditions are: mechanical (M); combined mechanical and thermal (M+T); combined mechanical, thermal and residual (M+T+R); and residual (R). As can be seen from the curves in figure 22, the ply longitudinal stress can be overestimated by as much as 30% when the incorrect load condition is used.

Analogous results for the ply transverse stress are shown in figure 23. These results show that the ply transverse stress can be underestimated by a factor of three depending on the load condition considered.

The results of figures 22 and 23 illustrate the importance of the residual and thermal stresses in the design of actual components from fiber composites. The effects of time on lamination residual stresses is not known at this time.

WAYS TO MINIMIZE AND/OR ELIMINATE LAMINATION RESIDUAL STRESSES

Several alternatives are possible for minimizing or eliminating the lamination residual stresses. These alternatives may be grouped into two categories. The first category is general while the second is for specific designs. They are summarized briefly.

In general, residual stresses at room temperature can be reduced or eliminated by:

1. Use of room or low-temperature cure matrices.

2. Decreasing the difference between fiber and matrix thermal coefficients of expansion.

3. Increasing the allowable elongation of the matrix at failure with a simultaneous increase in the matrix modulus.

In specific designs, the residual stress can be reduced by:

1. Proper selection of composite system.
2. Suitable ply stacking sequence and orientation.
3. Proper selection of the number of plies in each direction.
4. Introduction of transitional plies.
5. Decreasing the number of off-axis plies if stiffness and/or strength requirements permit it.
6. Increasing the fiber volume ratio.
7. Any possible combination of the above.

EXPERIMENTAL WORK NEEDED

Both theory and experiment show that the ply residual transverse stresses due to lamination are of magnitudes comparable to those of the ply transverse strengths. However, four important questions need to be answered before the effect of residual stresses on the composite structural response can be realistically assessed. These are: (1) decay of residual stress with time, (2) effect of transply cracks on composite stiffness, (3) effect of transply cracks on composite strengths, (4) effect of transply cracks on composite degradation by the environment.

These questions can be answered best by simple but strategic experimental investigation. Flexural tests in fatigue and creep should provide qualitative answers to these questions.

CONCLUSIONS

The results of this theoretical investigation, which is based on linear laminate theory and on a semiempirical micromechanics theory, lead to the following conclusions.

1. The ply transverse residual stress is generally tensile. Its magnitude can be comparable to that of the ply's transverse strength. This depends on the composite system, ply orientation and fiber volume ratio. It is critical for Graphite-Fiber/Epoxy composites.

2. The ply in-plane residual shear stress can be comparable to the ply's shear strength.

3. The ply residual transverse and/or shear stress can be comparable to the corresponding ply strengths in simulated components. This can cause transply cracking. It can also reduce the ply capacity to carry transverse tensile and in-plane shear loads.

4. The ply residual stresses are very sensitive to ply orientation, number of plies in each direction and to fiber volume ratio.

5. The ply residual stresses are relatively insensitive to void content, and to moderate fiber content or cure temperature variations from ply to ply.

6. The ply residual stresses generally increase with increasing ply orientation angle.

7. The ply residual stresses increase nonlinearly with increasing matrix modulus. They increase linearly with increasing matrix coefficients of thermal expansion.

8. It is possible to orient plies in simulated components so that some plies have tensile longitudinal residual stress. This stress is usually compressive. Its magnitude is usually about 15 to 20 percent of the ply's corresponding compressive strength.

9. Transply cracking due to residual stress, in general, can be prevented by increasing the ply transverse and shear strength and decreasing the cure temperature. It can be prevented by decreasing the thermal coefficients of the matrix and/or by increasing those of the fibers sufficiently. It can also be prevented by combinations of these.

10. In specific designs, transply cracking can be prevented by suitable ply orientation, variation of the number of plies in each direction, introduction of transitional plies, reduction of the number of off-axis plies, increasing the fiber content or by combinations of these.

11. Neglecting residual stresses in design can lead to unconservative designs.

12. Experimental work is needed for a realistic assessment of residual stresses on composite structural response.

SYMBOLS

A_{cx} array of composite axial stiffnesses referred to composite structural axes

C_{cx} array of composite coupling stiffnesses referred to composite structural axes

D_{cx}	array of composite bending (flexural) stiffnesses referred to composite structural axes
$E_{\ell i}$	array of strain-stress relations (elastic constants for the i^{th} ply)
$E_{\ell 11}$	ply longitudinal modulus
$E_{\ell 22}$	ply transverse modulus
$G_{\ell 12}$	ply shear modulus
k_f, k_v	fiber and void volume ratios, respectively
$M_{c\Delta Tx}$	vector of unbalanced thermal moments referred to composite structural axes
$N_{c\Delta Tx}$	vector of unbalanced thermal forces referred to composite structural axes
$R_{\ell i}$	array of transformation coefficients for the i^{th} ply
$S_{\ell 11T}$	ply longitudinal tensile strength
$S_{\ell 11C}$	ply longitudinal compressive strength
$S_{\ell 22T}$	ply transverse strength
$S_{\ell 22C}$	ply transverse compressive strength
$S_{\ell 12S}$	ply shear strength
$\Delta T_{\ell i}$	temperature difference for the i^{th} ply
x,y,z	structural axes
$z_{\ell i}$	distance from reference plane to centroid of the i^{th} ply
1,2,3	material axes
$\alpha_{\ell i}$	vector of thermal coefficients of expansion of the i^{th} ply
$\alpha_{\ell 11}$	ply longitudinal thermal coefficient of expansion
$\alpha_{\ell 22}$	ply transverse thermal coefficient of expansion
β_{22T}	correlation coefficient for ply transverse strength
β_V	void strain magnification coefficient

ϵ_{cox} vector of composite strains referred to composite structural axes at the reference plane

$\epsilon_{\ell i}$ vector of strains for the i^{th} ply referred to ply's material axes

ϵ_{mpT} matrix allowable strain in tension

θ ply angle measured from the composite structural axes to the ply material axes

κ_{cx} composite local curvatures referred to composite structural axes

$\nu_{\ell 12}$ ply major Poisson's ratio

$\sigma_{\ell i}$ vector of ply stresses referred to ply's material axes

$\phi_{\mu 22}$ strain magnification factor for ply transverse strength

TABLE I. - COMPOSITE PLY ORIENTATION (GEOMETRY) AND

NOTATION IDENTIFICATION

Composite geometry									Notation
Ply	1	2	3	4	5	6	7	8	
Ply angle	0°	0°	$+\theta$	$-\theta$	$-\theta$	$+\theta$	0°	0°	$8[2(0),2(\pm\theta),2(\mp\theta),2(0)]$
Ply angle	0°	$+\theta$	$-\theta$	90°	90°	$-\theta$	$+\theta$	0°	$8[0,2,(\pm\theta),2(90),2(\mp\theta),\theta]$
Ply angle	$+\theta$	$-\theta$	$+\theta$	$-\theta$	$-\theta$	$+\theta$	$-\theta$	$+\theta$	$8[4(\pm\theta),4(\mp\theta)]$
Ply angle	$+\theta$	$-\theta$	$+\theta$	$-\theta$	$+\theta$	$-\theta$	$+\theta$	$-\theta$	$8(\pm\theta)$
Ply angle	$+\theta$	$+\theta$	$+\theta$	$+\theta$	$-\theta$	$-\theta$	$-\theta$	$-\theta$	$8[4(+\theta),4(-\theta)]$

TABLE II. - THEORETICAL UNIAXIAL PLY ELASTIC AND THERMAL PROPERTIES OF VARIOUS COMPOSITES AT ROOM TEMPERATURE. NORMALIZED AT 55% FIBER VOLUME CONTENT AND ZERO VOIDS. (Refs. 12 and 13)

Composite System	Elastic and Thermal Properties at Room Temperature					
	$E_{\ell 11}$ 10^6 psi	$E_{\ell 22}$ 10^6 psi	$G_{\ell 12}$ 10^6 psi	$\nu_{\ell 12}$	$\alpha_{\ell 11}$ 10^{-6} in./in/°F	$\alpha_{\ell 22}$ 10^{-6} in./in/°F
Boron/Epoxy	33.2	2.1	1.0	0.24	3.0	14.2
S-Glass/Epoxy	7.0	1.7	0.9	.25	3.7	14.1
Thornel-50/Epoxy	27.8	1.0	.7	.24	-0.1	20.6
Modmor-I/Polyimide	33.2	0.9	.6	.24	- .3	20.5

TABLE III. - THEORETICAL UNIAXIAL STRENGTHS OF VARIOUS COMPOSITES AT ROOM TEMPERATURE. NORMALIZED AT 55% FIBER VOLUME CONTENT AND ZERO VOIDS (Refs. 12 and 13)

Composite System	Uniaxial Strengths (ksi) at Room Temperature				
	$S_{\ell 11T}$	$S_{\ell 11C}$	$S_{\ell 22T}$	$S_{\ell 22C}$	$S_{\ell 12S}$
Boron/Epoxy	214	188	8.6	30.1	13.0
S-Glass/Epoxy	204	143	4.4	28.3	8.3
Thornell-30/Epoxy	106	56.6	3.7	17.6	2.6
Modmor-I/Polyimide	138	136	6.1	19.7	7.8

TABLE IV. - HIGH-TIP-SPEED COMPOSITE COMPRESSOR BLADE

GEOMETRY AND LOADS

[Thornel 50/Polyimide at 50% FVR. density ≈ 0.05 #/in.^{3} tip speed = 2200 Ft/sec ply stacking sequence at max. thickness]

% Blade length	7.9	26.6	50.0	73.1	95.5
Blade temperature (^{o}F)	215	245	300	360	440
Number of plies	17	16	14	11	8
Surface	4(±30)	4(±30)	4(±30)	4(±30)	4(±30)
Transitional	4(±15)	4(±15)	4(±15)	4(±15)	4(±15)
Core	9(0)	8(0)	6(0)	3(0)	0(0)

REFERENCES

1. Adams, Donald F.; Doner, Douglas R.; and Thomas, Rodney L.: Mechanical Behavior of Fiber-Reinforced Composite Materials. Rep. U-4093, Aeronutronic (AFML-TR-67-96, DDC No. AD-654056), May 1967.

2. Haener, Juan; Ashbaugh, Noel; Chia, Chuen-Yuan; and Feng, Ming-Yuan: Investigation of Micromechanical Behavior of Fiber Reinforced Plastics. Whittaker Corp. (USAAVLABS-TR-67-66, DDC No. AD-667901), Feb. 1968.

3. Marloff, R. H.; and Daniel, I. M.: Three-Dimensional Photoelastic Analysis of a Fiber-Reinforced Composite Model. Exp. Mech., vol. 9, Apr. 1969, pp. 156-162.

4. Koufopoulos, T.; and Theocaris, P. S.: Shrinkage Stresses In Two-Phase Materials. J. Composite Mat., vol. 3, Apr. 1969, pp. 308-320.

5. Doner, D. R.; and Novak, R. C.: Structural Behavior of Laminated Graphite Filament Composites. Proceedings of the 24th Annual Technical Conference, Society of the Plastics Industry, 1969, pp. 2-D-1 to 2-D-8.

6. Decrescente, M. A.; and Novak, R.: Fabrication Stresses In Graphite-Resin Composites. Presented at the 15th Annual International Gas Turbine Conference and Products Show, Palais Du Centenaire, Brussels, Belgium, May 24-28, 1970.

7. Winters, W. E.: Thermal Anisotropic Behavior In Graphite Epoxy Composites. Presented at the 15th Annual International Gas Turbine Conference and Products Show, Palais Du Centenaire, Brussels, Belgium, May 24-28, 1970.

8. Edighoffer, H.; Ravnhall, R.; and Juneau, P. W., Jr.: Analysis and Structural Verification of Boron Filament Wound Composites. Proceedings of the 25th Annual Conference, Society of the Plastics Industry, 1970, Sec. 12-D.

9. Chamis, Christos C.: Design and Analysis of Fiber Composite Structural Components. Aerospace Structural Materials. NASA SP227, 1970, pp. 217-228.

10. Chamis, Christos C.: Design Oriented Analysis and Synthesis of Multilayered-Filamentary Structural Panels. Ph.D. Thesis, Case Western Reserve Univ., 1967.

11. Chamis, Christos C.: Important Factors In Fiber Composite Design. Proceedings of the 24th Annual Conference Society of the Plastics Industry, 1969, pp. 18-E-1 to 18-E-13.

12. Chamis, Christos C.: Failure Criteria For Filamentary Composites. NASA TN D-5367, 1969.

13. Chamis, Christos C.: Characterization and Design Mechanics For Fiber-Reinforced Metals. NASA TN D-5784, 1970.

14. Chamis, Christos C.: Multilayered Filamentary Composite Analysis Computer Code - User's Manual. Proposed NASA Technical Note.

E-5848

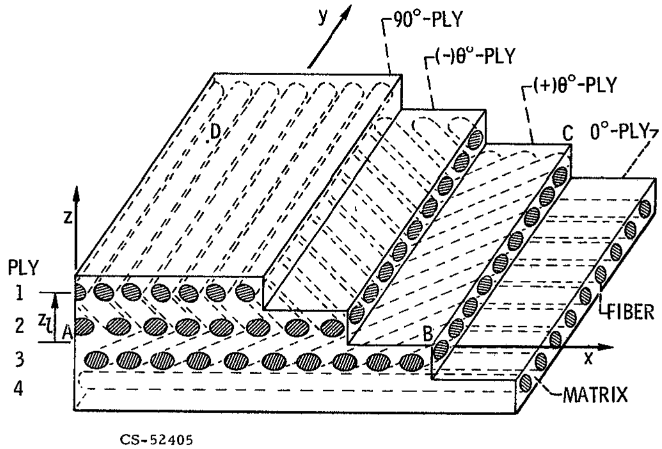

Figure 1. - Typical fiber composite geometry.

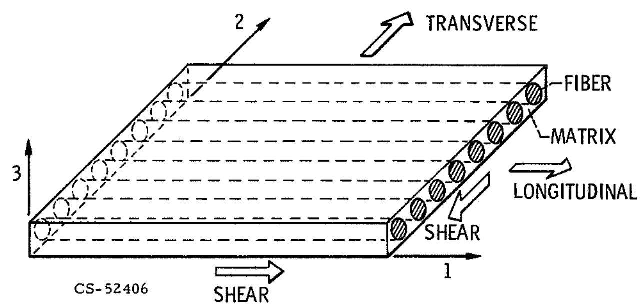

Figure 2. - Single ply.

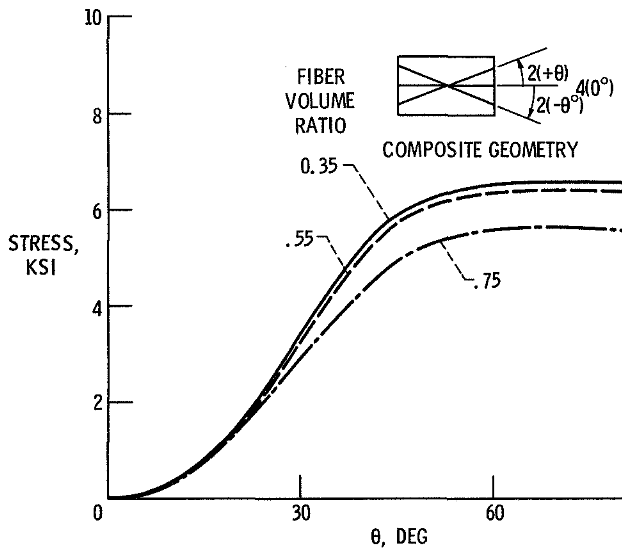

Figure 3. - ±θ-Ply transverse residual stress for boron/epoxy composite. Ply stacking sequence 8[2(0°), 2(±θ°), 2(∓θ), 2(0)]. Temperature difference = -300° F.

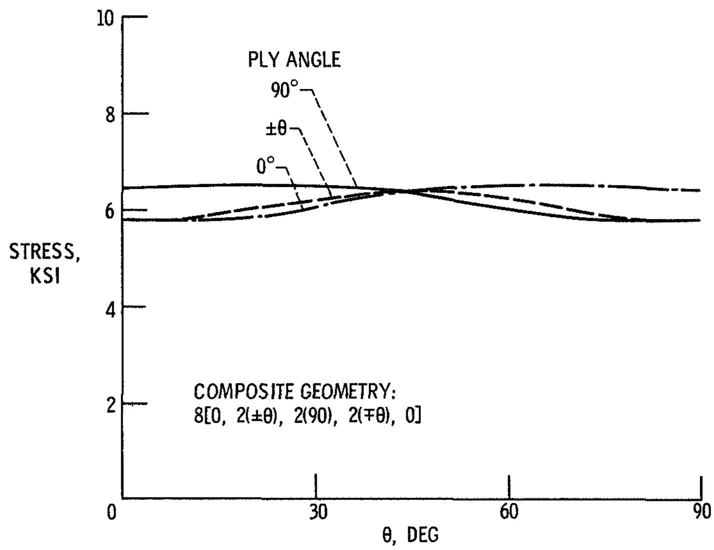

Figure 4. - Ply residual transverse stress for boron/epoxy composites. Fiber volume ratio = 0.55. Temperature difference = -300° F.

E-5848

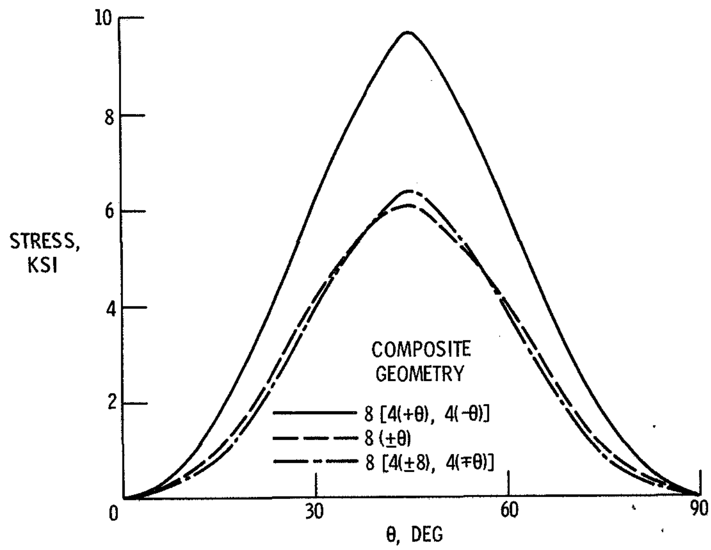

Figure 5. - Outer-ply residual transverse stress for boron/epoxy composites. Fiber volume ratio 0.55. Temperature difference = = -300° F.

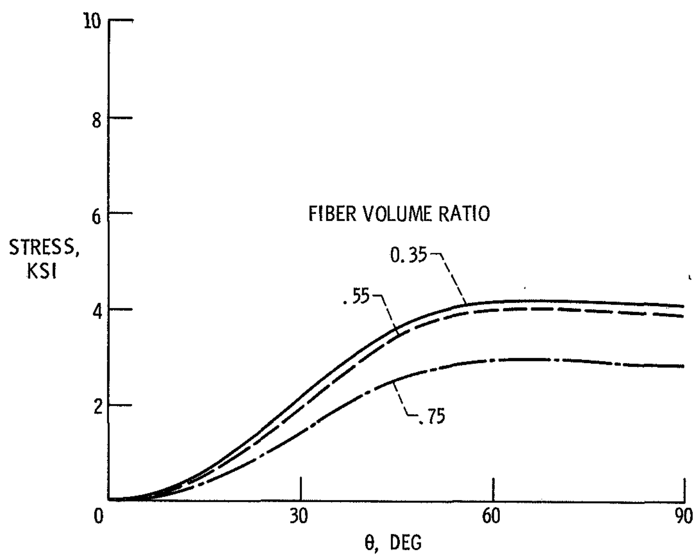

Figure 6. - ±θ-Ply residual transverse stress for S-glass/epoxy composite. Ply stacking sequence 8[2(0), 2(±θ), 2(∓θ), 2(0)] Temperature difference = -300° F.

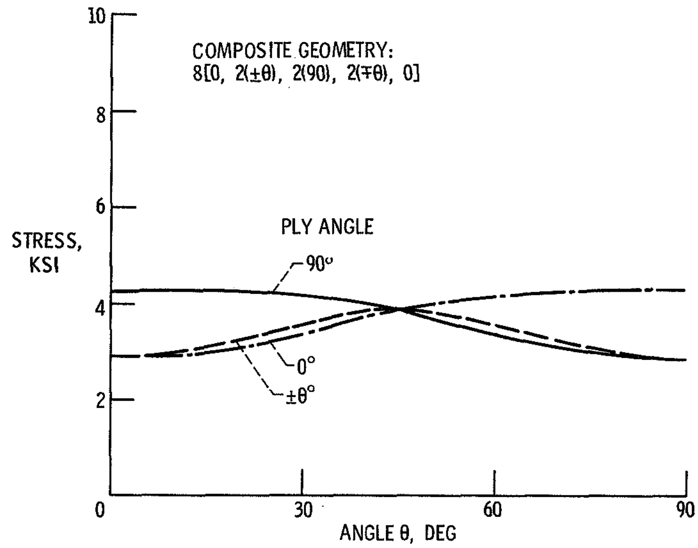

Figure 7. - Ply residual transverse stress for S-glass/epoxy composites. Fiber volume content = 0.55. Temperature difference, -300° F.

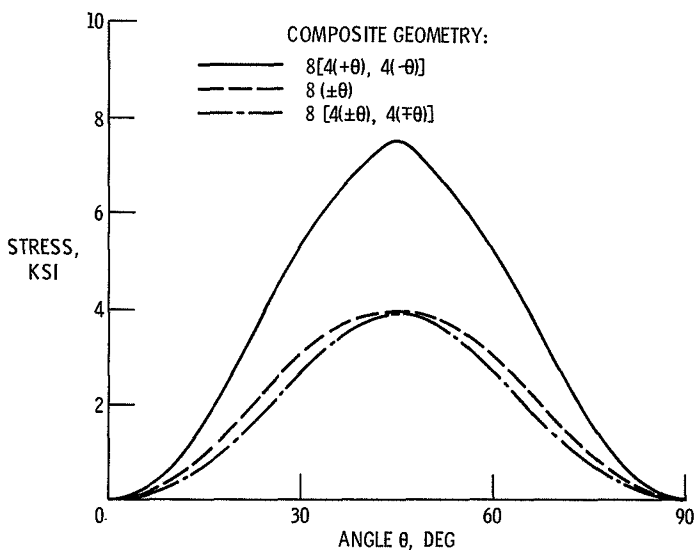

Figure 8. - Outer-ply residual transverse stress for S-glass/epoxy composites. Fiber volume content = 0.55. Temperature difference = -300° F.

E-5848

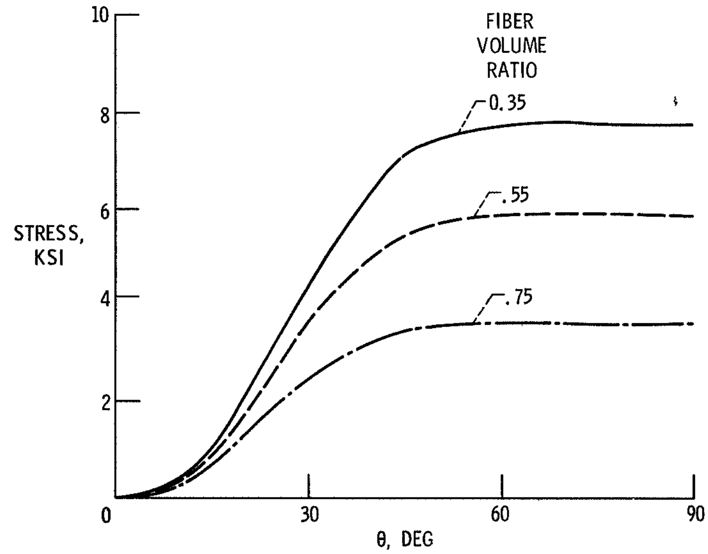

Figure 9. - ±θ-Ply residual transverse stress for graphite Thornel-50/epoxy composite. Ply stacking sequence 8 [2(0), 2±θ), 2(∓θ), 2(0)]. Temperature difference = -300° F.

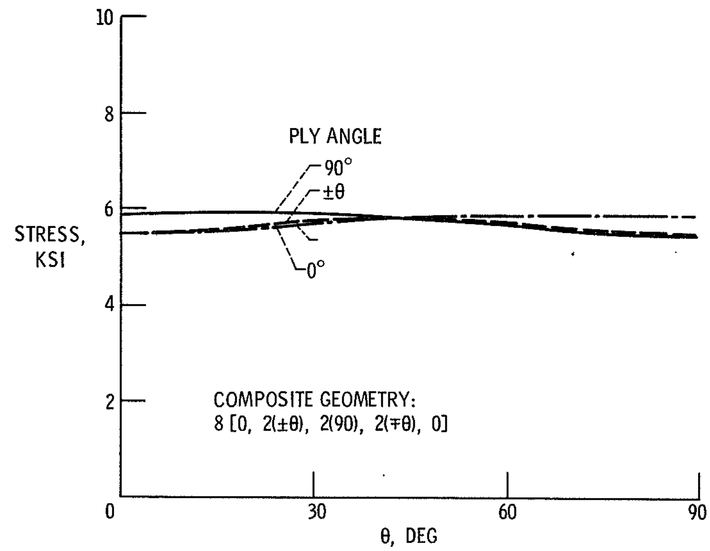

Figure 10. - Ply residual transverse stress for Thornel-50/epoxy composites. Fiber volume ratio = 0.55. Temperature difference = -300° F.

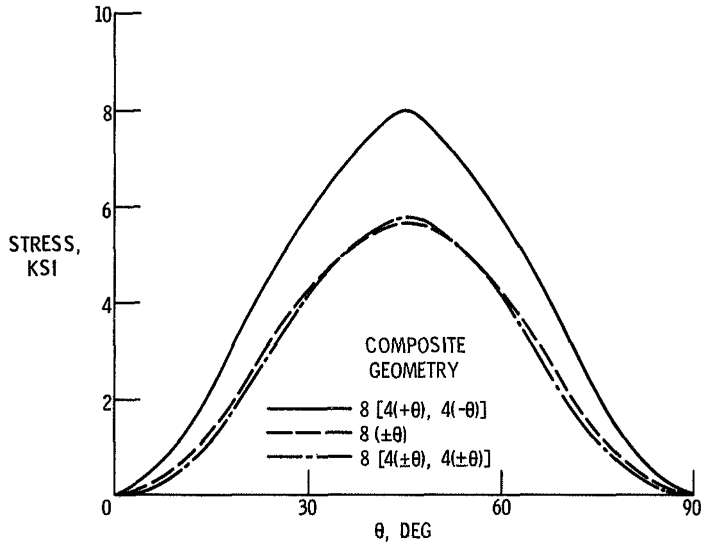

Figure 11. - Outer-ply residual transverse stress for Thornel-50/epoxy composites. Fiber volume ratio 0.55. Temperature difference = -300° F.

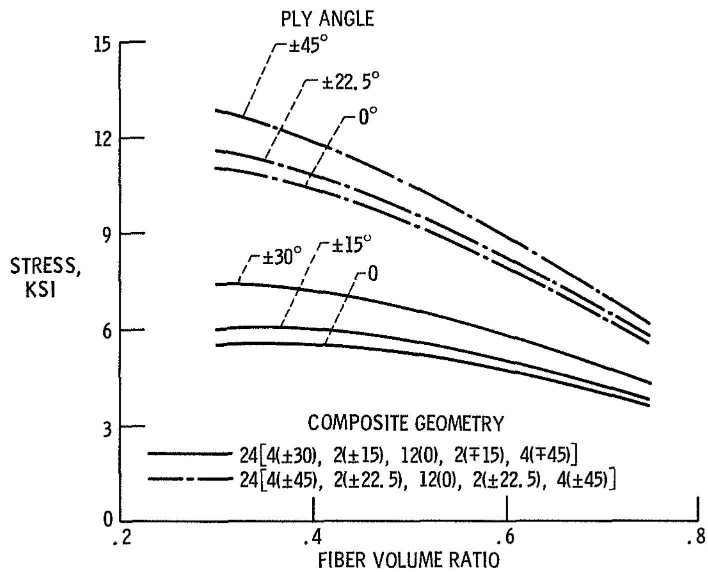

Figure 12. - Ply residual transverse stress for Modmor-I/polyimide composites. Temperature difference = -600° F.

E-5848

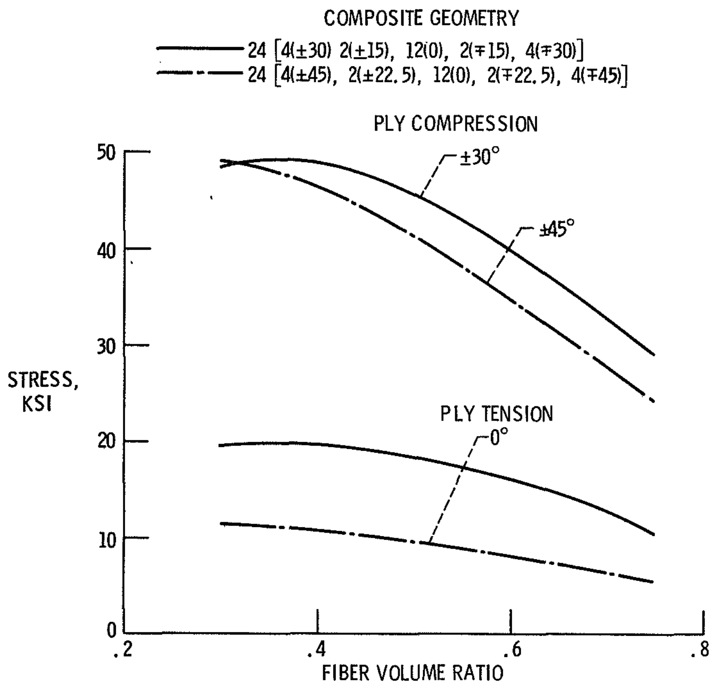

Figure 13. - Ply residual longitudinal stress for graphite Modmor-I/polyimide composites. Temperature difference = -600° F.

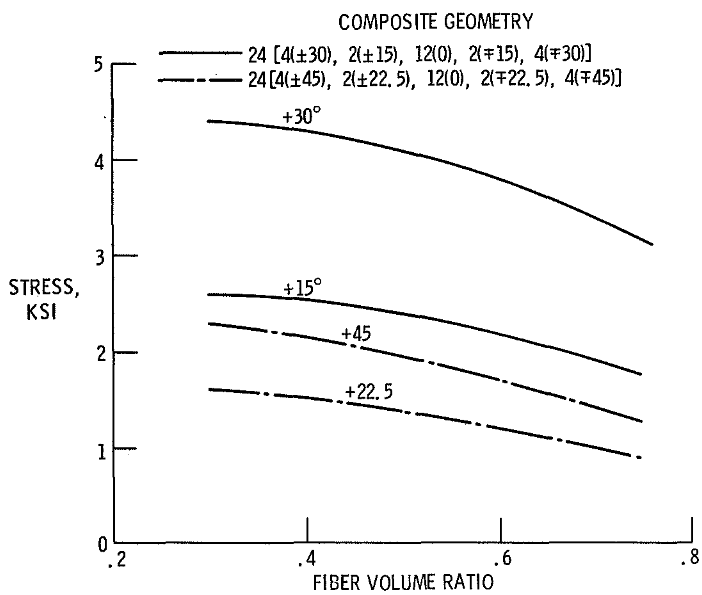

Figure 14. - Ply residual shear stress for graphite Modmor-I/polyimide composites. Temperature difference = -600° F.

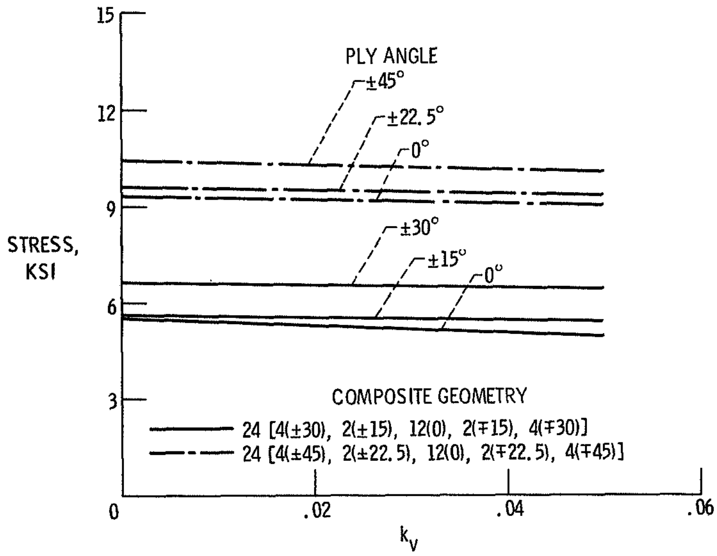

Figure 15. - Effects of voids on ply transverse residual stress. Modmor-I/polyimide composites. Fiber volume ratio = 0.50. Temperature difference = -600° F.

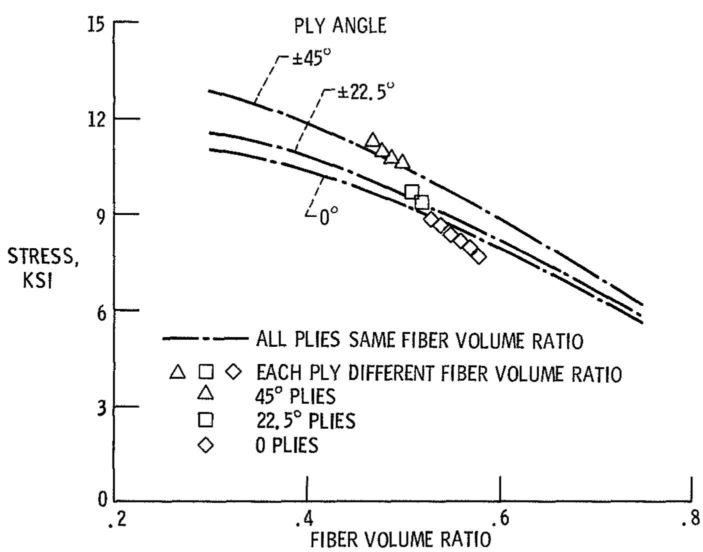

Figure 16. - Effects of variable fiber volume ratio on ply residual stress in Modmor-I/polyimide composite: 24[4(±45), 2(22.5), 12(0), 2(∓22.5), 4(∓45)]. Void volume ratio = 0. Temperature difference = -600° F.

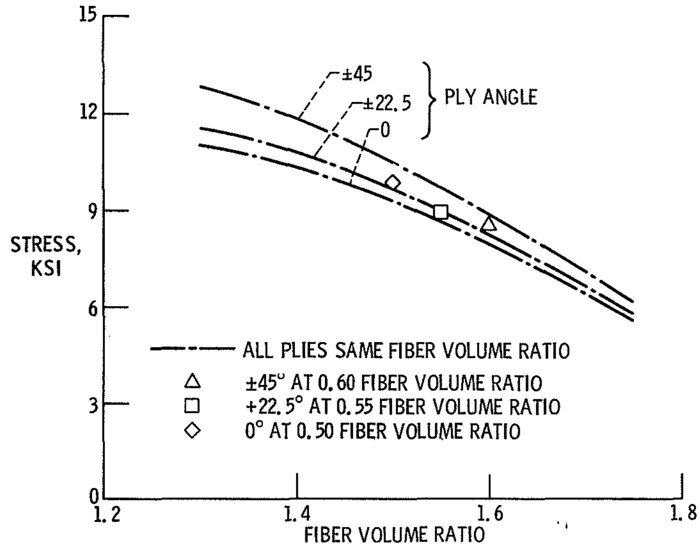

Figure 17. - Effect of having different groups of plies A_7 different fiber content on the ply transverse residual stress. Modmor I/ Polyimide composite: 24[4(±45), 2(±22.5), 12(0), 2(∓22.5), 4(∓45)]. Zero void content. Temperature difference = -600° F.

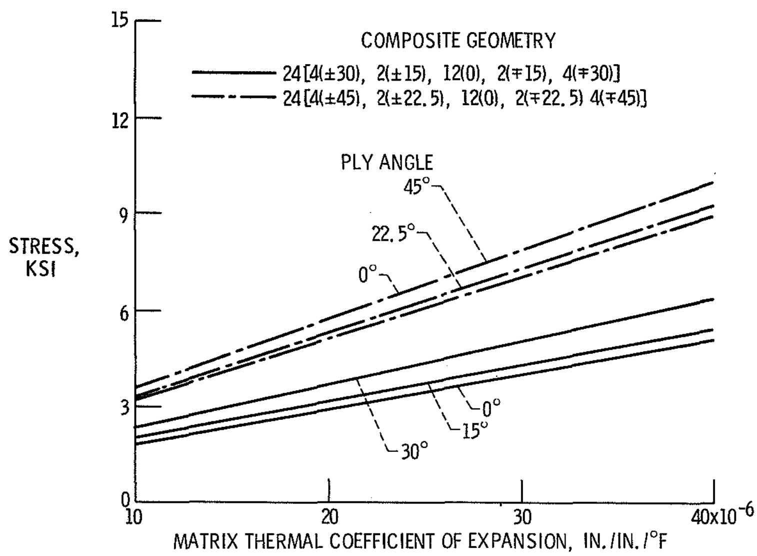

Figure 18. - Effects of the matrix thermal coefficient of expansion on the ply transverse residual stress. Modmor-I/polyimide composites. Zero void content. 0.50 Fiber volume ratio. Temperature difference = -600° F. Matrix modulus = $0.5x10^6$ psi.

E-5848

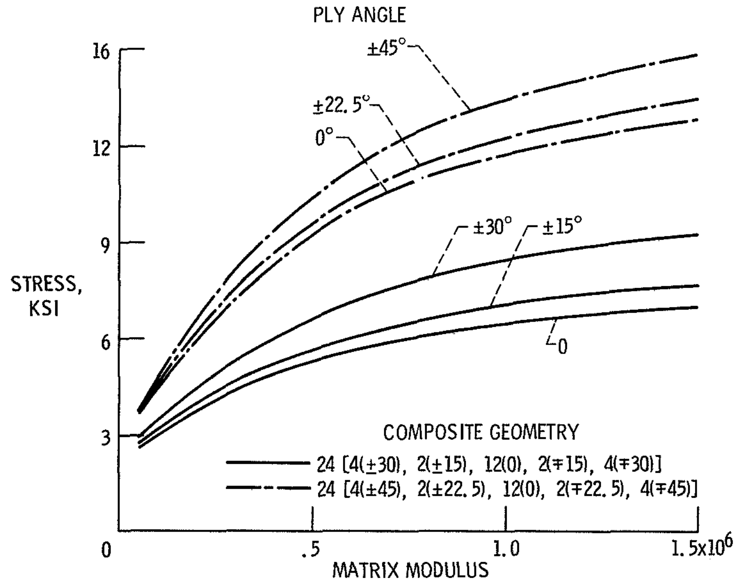

Figure 19. - Effects of matrix modulus on the ply transverse residual stress. Modmor-I/polyimide composites. Zero void content. 0.50 Fiber volume ratio. Temperature difference = -600° F.

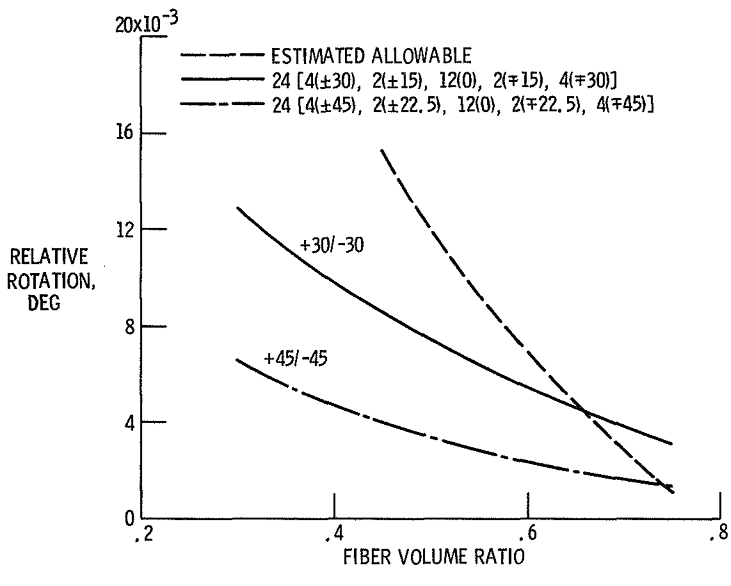

Figure 20. - Adjacent ply relative rotation due to residual stress for graphite Modmor-I/polyimide composites. Temperature difference = -600° F.

E-5848

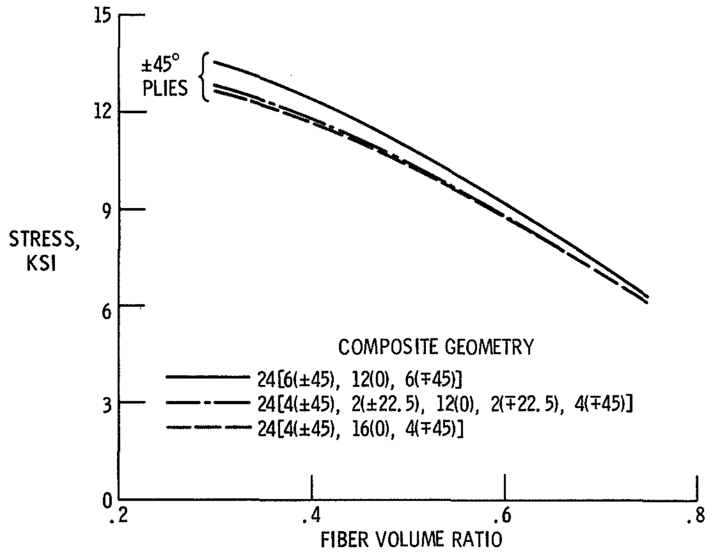

Figure 21. - Transitional plies and decreasing number of off-axis plies effects on the ±45°-ply residual transverse stress. Modmor-I/polyimide. Temperature difference = -600° F.

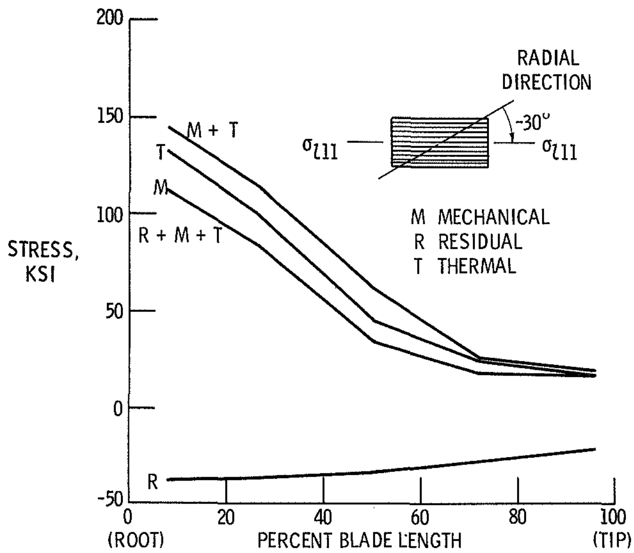

Figure 22. - 30° Surface ply longitudinal stress of a high-tip-speed composite compressor blade. (See table IV for composite geometry and loads.)

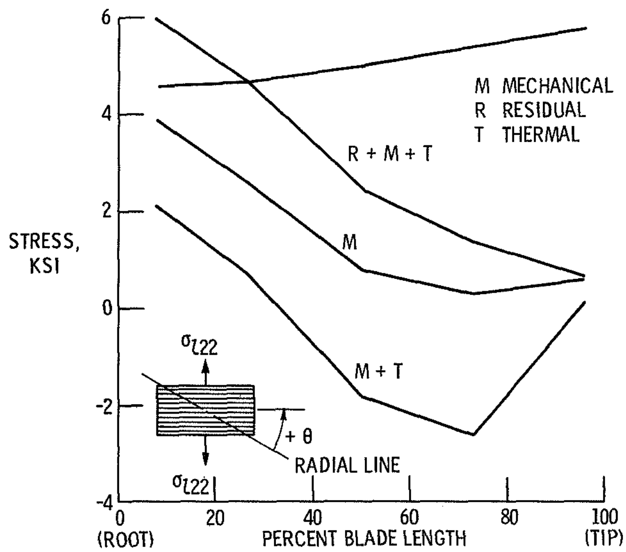

Figure 23. - +30° Surface ply transverse stress of a high-tip-speed composite compressor blade. (See table IV for composite geometry and loads.)