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ON THE CAPTURE OF SYNCHRONOUS ROTATION FOR THE MOON

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ABSTRACT

The dynamic behavior of the librational motion for the Moon near the synchronous spin rate can not be covered with a single solution of an averaged equation of motion as a simple pendulum. The pitfalls of the averaging procedure in the theory of capture probability are pointed out. It is shown that the capture process of the synchronous rotation of the Moon is not a probabilistic affair and that the value of the theory of capture has been trapped into the pitfalls of the averaging method.

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ON THE CAPTURE OF SYNCHRONOUS ROTATION FOR THE MOON

INTRODUCTION

Goldreich and Peale (Goldreich and Peale, 1966) have discussed the capture probabilities of resonance rotation in the solar system by analogy with the motion of a pendulum. They have argued that the rotation of the Moon can not be captured near the synchronous spin rate with an assumed constant tidal torque and that the probability of capture is about seven chances in ten for several assumed models of tidal torques. It is well known, however, that the librational response of the Moon near the synchronous spin rate cannot be covered with a single solution of an averaged equation as a simple pendulum. Therefore the logic of their arguments may contain serious flaws. Does the combined effect of the initial conditions of the rotational state and the models of tidal torques really make the capture process of the synchronous rotation of the Moon a probabilistic affair? The present investigation is devoted to answering this question.

In the following, we shall point out the pitfalls in the averaged equation of motion. In order to check the invalidity of the theory of capture probability for the Moon, the complete equations of motion are integrated by a computer using double precision arithmetic. The computer results were shown to disagree totally with the conclusions derived from the averaged equation of motion.

EQUATIONS OF MOTION

The equation of rotation of the Moon is governed by (Goldreich and Peale, 1966, Equation 34.)

$$\frac{d^2\psi}{df^2} - 2\left(\frac{r}{a}\right)\frac{e \sin f}{1-e^2}\frac{d\psi}{df} + \frac{3}{2}\cdot\frac{B-A}{C}\cdot\left(\frac{r}{a}\right)\frac{\sin 2\psi}{1-e^2} = 2\left(\frac{r}{a}\right)\frac{e \sin f}{1-e^2} - q \tag{1}$$

where a is semimajor axis, A, B and C are principal moments of inertia of the Moon, e orbital eccentricity, f the true anomaly, q lunar tidal torque and ψ the angle between the long axis of the Moon and the instantaneous radius r.

For an elliptic orbit

$$r = \frac{a\left(1 - e^2\right)}{1 + e \cos f} \tag{2}$$

Substituting Equation (2) into Equation (1), the result is

$$\frac{d^2\psi}{df^2} - 2\frac{e \sin f}{1 + e\cos f}\left(\frac{d\psi}{df} + 1\right) + \frac{3}{2} \cdot \frac{B - A}{C} \cdot \frac{\sin 2\psi}{1 + e\cos f} = -q \tag{3}$$

The angle of rotation for the Moon has the form

$$\theta = f + \psi \tag{4}$$

in which ψ is the solution of Equation (3).

Goldreich and Peale (Goldreich and Peale, 1966) were concerned with the spin rates having values near pn, where p is a half-integer and n is the orbital mean motion. They have introducted a new angle γ such that

$$\gamma = \theta - pM \tag{5}$$

where $M = nt$ is the mean anomaly.

As it stands, Equation (3) is insoluble analytically for $(B - A)/C \gtrless 0$. Therefore, in order to investigate the dynamic behavior of the angle γ near the synchronous spin rate with assumed tidal torques, it is necessary to integrate Equation (3) numerically.

NUMERICAL SOLUTIONS

To investigate the the dynamic response of the angle γ , we have developed two computer programs for integrating Equation (3): (1) a Fortran program using double precision arithmetic (2) a Mimic program with a graphical terminal. The problem—oriented Mimic language is designed to provide required information and pertinent data to check the results derived from the algorithmic Fortran language. In these computer programs, the true anomaly f has been transformed to the mean anomaly M according to Keppler's equation

$$M = E - e \sin E \tag{6}$$

where E is the eccentric anomaly which is related to f by

$$\tan \frac{E}{2} = \left(\frac{1 - e}{1 + e}\right)^{1/2} \tan \frac{f}{2} \tag{7}$$

For the constants or parameters, the following values are used: $p = 1$, $e = 0.0549$, $B - A / C = 0.000208$.

We have chosen different initial spin rates and constant tidal torques to illustrate the oscillatory response of γ . The computation is to last about 500 revolutions. A portion of the results from the Fortran program is shown in Table 1, Table 2 and Table 3.

From Tables 1, 2 and 3, we observe that the period and amplitude of the oscillatory response of γ depend on the assumed initial spin rates. This means that the dynamic behavior of γ near the synchronous spin rate is not governed by a single solution of an averaged equation as a pendulum. In Table 1, γ does not increase indefinitely when the Moon rotates with an assumed initial spin rate $d\theta/dt|_{t=0} = 1.022845633n$ which is faster than the synchronous one. With an assumed initial spin rate $d\theta/dt|_{t=0} = 0.978131070\,n$ which corresponds to a rotation rate less than the synchronous one, γ (in Table 3) oscillates at perigee about the Earth direction even with an assumed constant tidal torque $q = 10^{-7}$.

In the first sight, these results are quite offensive to one's sensibilities. For a special case $(B - A)/C = 0$, Equations (3) and (5) can be solved analytically. We have tested the accuracy of the computer programs and have found that the computer results are perfectly correct in comparison with these analytical solutions. Therefore, we are forced to face the facts.

REMARKS ON THE AVERAGING METHODS

The existence of resonances in Equation (3) for p = half-integer and q = 0 has been proved mathematically by Chernous'ko (Chernous'ko, 1963). By introducing Equation (5) into Equation (3), the result is (Chernous'ko, 1963)

$$\frac{d^2 \gamma}{dt^2} + \frac{3(B - A)}{2C} n^2 \Phi_{2p} \left(e\right) \sin 2\gamma = 0 \tag{8}$$

for q = 0 For synchronous rotation, p = 1, the value of Φ_2 is $1 - 5/2\, e^2 + \ldots$. The values of Φ_{2p} for other resonance spin rates were also given by Chernous'ko (Chernous'ko, 1963). In the procedure of derivation of Equation (8), Chernous'ko has made two important assumptions i.e.

(1) 2 p is an integer
(2) The coefficents in Equation (8) have been averaged over an orbital period by holding γ fixed.

These assumptions are pitfalls of the averaging methods. One should be afraid of being trapped.

If the Moon rotates with a spin rate which is faster or slower than the synchronous one, then $p \neq 1$, and the value of γ will increase or decrease substantially during each orbit (see Table 1 and Table 3). Therefore, the assumptions which were made in the derivation of Equation (8) prohibit us from using Equation (8) to describe the motion of γ when the Moon rotates with a spin rate in the regimes other than the synchronous one. This is a fundamental mathematic principle in physics. One should not violate this principle to make calculations. In the discussion of the capture process of the synchronous rotation for the Moon, Goldreich and Peale (Goldreich and Peale, 1966) have based all their arguments on the averaged equation of motion. Clearly, their calculations have been trapped into the pitfalls of the averaging methods.

SUMMARY

The period and amplitude of the librational response for the Moon near the synchronous spin rate depend on the initial rotational states. The dynamic behavior of such librational motion cannot be covered with a single solution of an averaged equation of motion as a simple pendulum. The apparent circulational motion of the Moon can be converted to a librational motion near the synchronous spin rate even if the lunar tidal torque is assumed to be a constant. In this article we have shown that the capture process of the synchronous rotation for the Moon is not a probabilistic affair and that the value of the theory of capture is resonantly trapped into the pitfalls of the averaging method.

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Table 1

Values of $\gamma = \theta - M$ for initial spin rate $d\theta/dt|_{t=0} = 1.002845633n$ with assumed constant tidal torques $q = 0$, $q = 10^{-10}$ and $q = 10^{-7}$.

f (2π)	γ (rad.)		
	$q = 0$	$q = 10^{-10}$	$q = 10^{-7}$
0	0.000000000	0.000000000	0.000000000
1	+0.143448113	+0.143448111	+0.143446131
5	+0.655242261	+0.655242213	+0.655194250
10	+1.045582604	+1.045582409	+1.045387604
15	+1.170503613	+1.170503105	+1.169995599
20	+1.068564387	+1.068563248	+1.067425685
25	+0.706839283	+0.706837096	+0.704653653
30	+0.070668207	+0.070665213	+0.067678356
35	-0.600779925	-0.600782335	-0.603183785
40	-1.020034597	-1.020035912	-1.021345873
45	-1.168386103	-1.168386705	-1.168986806
50	-1.089064019	-1.089064259	-1.089306380
55	-0.755543773	-0.755543842	-0.755618690
60	-0.140925331	-0.140925333	-0.140937015
65	+0.543588966	+0.543588935	+0.543549528

Table 2

Values of $\gamma = \theta - M$ for initial spin rate $d\theta/dt|_{t=0} = 1.000488352n$ with assumed constant tidal torques $q = 0$, $q = 10^{-10}$ and $q = 10^{-7}$.

$\frac{f}{(2\pi)}$	γ (rad.)		
	$q = 0$	$q = 10^{-10}$	$q = 10^{-7}$
0	0.000000000	0.000000000	0.000000000
1	+0.003510063	+0.003510061	+0.003508081
5	+0.015881031	+0.015880984	+0.015833863
10	+0.022541400	+0.022541238	+0.022380097
15	+0.016115400	+0.016115124	+0.015839214
20	+0.000332699	+0.000332374	+0.000007606
25	-0.015643189	-0.015643469	-0.015922756
30	-0.022536490	-0.022536656	-0.022702590
35	-0.016346273	-0.016346323	-0.016396873
40	-0.000665327	-0.000665327	-0.000665402
45	+0.015401953	+0.015401909	+0.015358109

Table 3

Values of $\gamma = \theta - M$ for initial spin rate $d\theta/dt|_{t=0} = 0.978131070n$ with assumed constant tidal torques $q = 0$, $q = 10^{-10}$ and $q = 10^{-7}$.

f (2π)	γ (rad.)		
	$q = 0$	$q = 10^{-10}$	$q = 10^{-7}$
0	0.000000000	0.000000000	0.000000000
1	-0.136422505	-0.136422507	-0.136424487
5	-0.622609661	-0.622609709	-0.622657594
10	-0.983528090	-0.983528282	-0.983720257
15	-1.062248255	-1.062248741	-1.062734356
20	-0.874857727	-0.874858752	-0.875883530
25	-0.395907798	-0.395909500	-0.397611029
30	+0.270217744	+0.270215949	+0.268420487
35	+0.803529362	+0.803528200	+0.802365322
40	+1.046846398	+1.046845827	+1.046274874
45	+1.020279823	+1.020279587	+1.020045223
50	+0.718036706	+0.718036638	+0.717970993
55	+0.135104413	+0.135104411	+0.135106328
60	-0.515868948	-0.515868980	-0.515896834