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KUNITOMO SAKURAI

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GYROSYNCHROTRON RADIATION AND ITS TRANSFER IN A MAGNETOACTIVE PLASMA

by

Kunitomo Sakurai*
Radio Astronomy Branch
Laboratory for Extraterrestrial Physics
NASA-Goddard Space Flight Center
Greenbelt, Maryland 20771

ABSTRACT

Gyrosynchrotron radiation fields from midly relativistic electrons in a magnetoactive plasma are asymptotically calculated using the Green tensor and the Fourier transformation. These fields consist of the two components which correspond to the ordinary and extraordinary modes. Taking into account these fields, the emissivities and the absorption coefficients from an arbitrary distribution of electrons are calculated in order to discuss the intensity, spectrum and polarization of gyrosynchrotron radiation. In general, the transfer of electromagnetic energy takes place along a direction different from that of the wave normal since the radiation fields have a non-vanishing component along the direction of the wave normal. A consideration is given on the problem of radiative transfer in relation to the Stokes parameters.

*NAS-NRC Associate with NASA

INTRODUCTION

The problem of gyrosynchrotron radiation radiation from midly relativistic electrons in a magnetoactive plasma occurs in astrophysics, radio astronomy and magnetospheric physics. The radiation from these electrons moving along helical trajectories has some unusual properties due to the dispersion and anisotropy of the magnetoactive plasma as considered by many authors (e.g., Bunkin, 1957; Eidman, 1958; McKenzie, 1964; Liemohn, 1965; Mansfield, 1967; Pakhomov et al., 1962, 1963; Sakurai and Ogawa, 1969; Fung, 1969a). They have studied the gyrosynchrotron radiation fields from an electron moving in a helical trajectory in a magnetoactive plasma from various points of view. Eidman (1958) and Liemohn (1965), for example, have solved the problem in a cold and collisionless magnetoactive plasma by applying the well known Hamiltonian method of quantum electrodynamics. By introducing the Green tensor and a Fourier transformation, Bunkin (1957) and McKenzie (1964) have found different asymptotic solutions of the radiation fields in a cold and collisionless magnetoactive plasma. Similar method have been further developed by Mansfield (1967) using Fourier transformations. In the case of a collisionless thermal magneto-active plasma, Pakhomov, Aleksin and Stepanov (1962, 1963) have tried to calculate an asymptotic formula for the radiation fields.

Recently, Ramaty and Lingenfelter (1967) and Ramaty (1968) have studied the influence of the ambient plasma on synchrotron and gyrosynchrotron radiation from energetic electrons in order to interpret the suppression of type IV radio bursts at low frequencies. In this case, they have assumed the ambient plasma to be isotropic and homogeneous since it appears that $\omega >> \omega_p$ and ω_H and $\omega_p >> \omega_H$, where ω, ω_p and ω_H are the angular radiation, plasma and gyro-frequencies, respectively. At present, this suppression of synchrotron radiation from relativistic electrons has been applied to interpret the observational spectra of solar and stellar radio emission (e.g., Ramaty, 1968; McCray, 1967; Fung, 1969a; Fung and Yip, 1966; Scheuer, 1965).

The physical condition which are actually encountered near large sunspot groups, however, seems to be quite different from the simple case where $\omega >> \omega_p$ and ω_H. In fact, the type IV radio bursts are usually emitted by midly relativistic electrons which are moving within a plasma medium where the strength of the sunspot magnetic fields is very high, say 1000 gauss (e.g., Kiepenheuer, 1953; Ellison, 1963; Bray and Doughead, 1965). We must, therefore, consider the effect of magnetic fields in dealing with the radiation characteristics of type IV radio bursts (Sakurai, 1964, 1965, 1970), and so we must solve the gyrosynchrotron radiation fields from midly relativistic electrons moving in helical trajectories in a magnetoactive plasma which take account of the influence of angular gyro-frequency.

From this point of view, we deal with the problem of gyrosynchrotron radiation from midly relativistic electrons in a magnetoactive plasma by means of the method similar to that of Bunkin (1957) and McKenzie (1964). In our method, an asymptotic property of radiation fields is investigated by using a Green tensor and a Fourier transformation. In this paper, we assume that the medium is homogeneous, cold and collisionless and immersed in a static external magnetic field. The absorption and polarization of gyrosynchrotron radiation in this medium are considered in relation to the problem of radiative transfer.

FUNDAMENTAL EQUATIONS AND THE DERIVATION OF RADIATION FIELDS

a) Fundamental Equations

Maxwell's electromagnetic equations in a magnetoactive plasma are given, by using the complex dielectric tensor [K] (Stix, 1962), as follows:

$$\mathrm{curl}\ \vec{E} = i \frac{\omega}{c} \vec{B} \qquad (2\text{-}1)$$

$$\mathrm{curl}\ \vec{B} = - i \frac{\omega}{c} [K]\cdot\vec{E} + \frac{4\pi}{c} \vec{j} \qquad (2\text{-}2)$$

$$\mathrm{div}\ \vec{B} = 0 \qquad (2\text{-}3)$$

$$\text{div } [K]\cdot\vec{E} = 4\pi\rho \qquad (2\text{-}4)$$

$$\vec{D} = [K]\cdot\vec{E},\ \vec{B} = \vec{H}, \qquad (2\text{-}5)$$

where $\vec{E}$, $\vec{H}$, $\vec{D}$ and $\vec{B}$ are the electric and magnetic fields and the electric and magnetic inductions, respectively, all of which are assumed to change with exp $(-i\omega t)\cdot\vec{j}$, ρ and c are the electric current and charge density and the speed of light, respectively.

A magnetoactive plasma is electromagnetically dispersive and anisotropic and may be characterized by a complex dielectric tensor [K]. This tensor is a function of the angular propagation frequency and of the angle between the wave normal and the static external magnetic field in case of axial symmetry (Stix, 1962). It is necessary for the following analysis to assume that this tensor is Hermitian

$$[K] = [K]^{*}, \qquad (2\text{-}6)$$

where the asterisk denotes the complex conjugate and transpose. Furthermore, in our treatment, it is assumed that the magnetic susceptibility of the medium vanishes so that $\vec{B} = \vec{H}$.

By the use of the electromagnetic potential, $\vec{A}$ and ϕ, the electromagnetic fields are expressed as

$$\vec{E} = i\,\frac{\omega}{c}\,\vec{A} - \text{grad }\phi, \qquad (2\text{-}7)$$

$$\vec{H} = \text{curl }\vec{A}. \qquad (2\text{-}8)$$

Thus, Eqs. (2-1) to (2-6) reduce to

$$\text{curl curl } \vec{A} + \frac{1}{c} [K] \cdot \text{grad } \phi - \frac{\omega^2}{c^2} [K] \cdot \vec{A} = \frac{4\pi}{c} \vec{j} \qquad (2\text{-}9)$$

$$- \text{grad } ([K] \cdot \text{grad } \phi) - \text{div } \frac{1}{c} [K] \cdot \vec{A} = 4\pi\rho \qquad (2\text{-}10)$$

Since the potentials are not uniquely determined by Eqs. (2-7) and (2-8), a Coulomb gauge condition may be imposed on them (e.g., Jackson, 1962; Sakurai and Ogawa, 1969):

$$\text{div } [K] \cdot \vec{A} = 0. \qquad (2\text{-}11)$$

In this gauge, ϕ is just the static potential of the source charge and so only denotes the corresponding static field $\vec{D}_{st} = [K] \cdot \text{grad } \phi$. Consequently, the radiation fields $\vec{E}$ and $\vec{H}$ are derivable from the vector potential $\vec{A}$ alone. Thus the vector potential $\vec{A}$ satisfies the following equation:

$$\nabla^2 \vec{A} + \frac{\omega^2}{c^2} [K] \cdot \vec{A} = - \frac{4\pi}{c} \vec{j}, \qquad (2\text{-}12)$$

which will be solved in the following discussion.

If we make use of a Green Tensor $[G(\vec{r}, \vec{r}_e)]$ defined in an infinite space with no boundary, that satisfies the equation

$$\text{curl curl } [G] - \frac{\omega^2}{c^2} [K] \cdot [G] = \delta (\vec{r} - \vec{r}_e), \qquad (2\text{-}13)$$

the formal solution for the vector potential $\vec{A}(\vec{r},t)$ is expressed mathematically as

$$\vec{A} = \frac{4\pi}{c} \int_{\vec{r}',t'} [G(\vec{r},\vec{r}')]\, \vec{j}(\vec{r},t')\, d\vec{r}'\, dt, \qquad (2\text{-}14)$$

where $\vec{r}$ and $\vec{r}'$ are the position vectors from the null point of the coordinate system to the point of observation and to the location of the radiating electrons, respectively. t and t' are the times with respect to the point of observation and the location of the radiating electrons, respectively.

Taking into account that the Green tensor which satisfies Eq. (2-13) is a function of the difference in the coordinate $\vec{r} - \vec{r}'$ alone, the Fourier transformation of [G] is given by

$$[G] = \frac{1}{(2\pi)^3} \int [\tilde{G}(\vec{k},\omega)] \exp\{i\, \vec{k}\cdot(\vec{r} - \vec{r}')\}\, dk', \quad (2\text{-}15)$$

where $[\tilde{G}(\vec{k},\omega)]$ is the Fourier component of [G].
The Dirac δ-function is also Fourier-transformed as

$$\delta(\vec{r} - \vec{r}') = \frac{1}{(2\pi)^3} \int \exp\{i\, \vec{k}\cdot(\vec{r} - \vec{r}')\} d\vec{k} \qquad (2\text{-}16)$$

By the use of Eqs. (2-15) and (2-16), Eq. (2-13) is reduced to

$$\vec{k}\,(\vec{k}\cdot[G]) - k^2\,[\tilde{G}] + \frac{\omega^2}{c^2}\,[K]\cdot[\tilde{G}] = -1 \qquad (2\text{-}13)$$

When a static external magnetic field $\vec{H}_o$ lies along the Z-axis in the Cartesian coordinate system, the complex dielectric tensor is given by (Stix, 1962)

$$[K] = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \qquad (2\text{-}17)$$

where

$$S = \frac{1}{2}(R + L), \qquad D = \frac{1}{2}(R - L)$$

$$R = 1 - \frac{X}{1-Y}, \; L = 1 - \frac{X}{1+Y}, \qquad P = 1-X$$

and

$$X = \omega p^2/\omega^2, \qquad Y = \omega_H/\omega.$$

By substituting Eqs. (2-17) into (2-13)′, we obtain

$$[\tilde{G}(\vec{k},\omega)] = -[Q_{ij}]/Q_o, \qquad (2\text{-}18)$$

where

$$Q_o = \frac{\omega^2}{c^2} P (k_{\parallel}^{\;2} - k_{\parallel,1}^{2})(k_{\parallel}^{\;2} - k_{\parallel,2}^{2}) \qquad (2\text{-}19a)$$

$$\left.\begin{matrix} k_{\parallel,1}^{\;2} \\ k_{\parallel,2}^{\;2} \end{matrix}\right\} = \frac{1}{2P}\Big\{2SP\frac{\omega^2}{c^2} - (S + P)\,k_\perp^2 \pm\Big[(S-P)^2\,k_\perp^4 - 4PD^2\,k_\perp^2\,\frac{\omega^2}{c^2} + 4P^2D^2\,\frac{\omega^4}{c^4}\Big]^{\frac{1}{2}}\Big\} \qquad (2\text{-}19b)$$

and the elements of $[Q_{ij}]$ are

$$Q_{xx} = -k_y^2 k_z^2 + (k_x^2 + k_y^2 - S \frac{\omega^2}{c^2}) (k_\perp^2 - P \frac{\omega^2}{c^2})$$

$$Q_{xy} = k_x k_y k_z^2 + (k_x k_y - i D \frac{\omega^2}{c^2}) (k_\perp^2 - P \frac{\omega^2}{c^2})$$

$$Q_{xz} = k_y k_z (k_x k_y - i D \frac{\omega^2}{c^2}) + k_x k_z (k_x^2 + k_y^2 - S \frac{\omega^2}{c^2})$$

$$Q_{yx} = k_x k_y k_z^2 + (k_x k_y + i D \frac{\omega^2}{c^2}) (k_\perp^2 - P \frac{\omega^2}{c^2})$$

$$Q_{yy} = (k_y^2 + k_z^2 - S \frac{\omega^2}{c^2}) (k_\perp^2 - P \frac{\omega^2}{c^2}) - k_x^2 k_z^2$$

$$Q_{yz} = k_x k_z (k_x k_y + i D \frac{\omega^2}{c^2}) + k_y k_z (k_y^2 + k_z^2 - S \frac{\omega^2}{c^2})$$

$$Q_{zx} = k_y k_z (k_x k_y + i D \frac{\omega^2}{c^2}) + k_x k_z (k_x^2 + k_z^2 - S \frac{\omega^2}{c^2})$$

$$Q_{zy} = k_x k_z (k_x k_y - i D \frac{\omega^2}{c^2}) + k_y k_z (k_y^2 + k_z^2 - S \frac{\omega^2}{c^2})$$

$$Q_{zz} = (k_y^2 + k_z^2 - S \frac{\omega^2}{c^2}) (k_x^2 + k_z^2 - S \frac{\omega^2}{c^2}) -$$

$$(k_x k_y - i D \frac{\omega^2}{c^2}) (k_x k_y + i D \frac{\omega^2}{c^2}), k^2 = k_x^2 + k_y^2 + k_z^2$$

and $k_\perp^2 = k_x^2 + k_y^2$, $k_\parallel = k_z$.

Using Eqs. (2-15), (2-18) and (2-19), Eq. (2-14) is calculated as follows:

$$\vec{A} (\vec{r},t) = - \frac{1}{4\pi^3 c} \int \frac{c^2 [Q_{ij}] \vec{j}' (\vec{r}',t')}{P \omega^2 (k_\parallel^2 - k_{\parallel,1}^2) (k_\parallel^2 - k_{\parallel,2}^2)}$$
$$\exp [i \{\vec{k}.\vec{r}-\vec{r}') - \omega (t-t')\}]$$
$$\times d\vec{k} \, d\omega \, d\vec{r}' \, dt' \qquad (2\text{-}20)$$

This gives the vector potential due to the current density $\vec{j}\,(\vec{r}',t')$. In order to calculate the gyrosynchrotron radiation from a midly relativistic electron, we must define the current density by taking into account the helical orbit of this electron.

b) Derivation of Gyrosynchrotron Radiation Fields

The Cartesian coordinate system as shown in Fig. 1 is used here, where the static external magnetic field $\vec{H}_o$ is along the Z-axis. The position vector $\vec{r}$ is assumed to be in the (y-z) plane and therefore $\vec{r} = (0,\ r \sin \alpha,\ r \cos \alpha)$ where α is the angle between $\vec{H}_o$ and the position vector. The wave vector $\vec{k}$ is decomposed as $\vec{k} = (k_\perp \cos \phi,\ k_\perp \sin \phi,\ k_z)$, where ϕ is the angle between $\vec{H}_o$ and $\vec{k}$.

The orbit of a radiating electron in a helical trajectory is given as shown in Fig. 2, where $\vec{r}_e\,(t')$ denotes the position vector of the electron. Since the radiating electron produces the electric current, the current density $\vec{j}\,(\vec{r}',t')$ in Eq. (2-20) is given by

$$\vec{j}\,(\vec{r}',t') = e\,\vec{V}\,\delta\,(\vec{r}' - r_e'\,(t')), \qquad (2\text{-}21)$$

where e and $\vec{V}$ are the electronic charge and the velocity of the electron. The velocity $\vec{V}$ and the position vector $\vec{r}_e\,(t')$ of the electron with pitch angle Ψ are expressed by

$$\vec{V} = - \vec{a}_x V_\perp \sin \frac{\omega_H}{\gamma} t' + \vec{a}_y V_\perp \cos \frac{\omega_H}{\gamma} t' + \vec{a}_z V_{\parallel} \qquad (2\text{-}22a)$$

and

$$\vec{r}_e(t') = \vec{a}_x \frac{\gamma V_\perp}{\omega_H} \cos \frac{\omega_H}{\gamma} t' + \vec{a}_y \frac{\gamma V_\perp}{\omega_H} \sin \frac{\omega_H}{\gamma} t' + \vec{a}_z V_{\parallel} t', \qquad (2\text{-}22b)$$

where $V_\perp = V \sin \psi$, $V_{\parallel} = V \cos \psi$, $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ and $\beta = \frac{V}{c}$. $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$ are the unit vectors as shown in Fig. 2.

By substituting Eqs. (2-21) into (2-20) and using Eq. (2-22), Eq. (2-20) can be integrated as follows (Ogawa and Sakurai, 1969):

$$\vec{A}(\vec{r},t) = - \frac{e}{2\pi^2 c} \int d\vec{k} d\omega \frac{c^2 [Q_{ij}] \exp\{i(k_\perp r_\perp \sin \phi + R_{\parallel} r_{\parallel} - \omega t)\}}{P\omega^2 (k_{\parallel}^2 - k_{\parallel,1}^2)(k_{\parallel}^2 - k_{\parallel,2}^2)}$$

$$\times \sum_{n=-\infty}^{\infty} [R_n] \, \delta \, (\omega - n\omega_{H/\gamma} - k_{\parallel} V_{\parallel}) \qquad (2\text{-}23)$$

where

$$[R_n] = -i \frac{V_\perp}{2}\{J_{n-1}(z)\exp\{i(n-1)(\phi - \tfrac{\pi}{2})\} - J_{n+1}(z)\exp\{i(n+1)(\phi - \tfrac{\pi}{2})\}$$

$$\frac{V_\perp}{2}\{J_{n-1}(z)\exp\{i(n-1)(\phi - \tfrac{\pi}{2})\} + J_{n+1}(z)\exp\{i(n+1)(\phi - \tfrac{\pi}{2})\}$$

$$V_{\parallel} J_n (z) \exp i n(\phi - \tfrac{\pi}{2})$$

and $Z = \gamma \frac{k_\perp V_\perp}{\omega_H}$. Here $J_n(z)$ is the Bessel function of the first kind of n-th order. The right hand side of Eq. (2-23) must be integrated with respect to $\vec{k}$. In integrating this equation, we can decompose $d\vec{k}$ into $k_{\perp} dk_{\perp} dk_{\parallel} d\phi$. The integration over ϕ gives the radiation fields in the (y-z) plane. In order to obtain the radiation fields, all the terms of the order higher than $r^{-\frac{1}{2}}$ are excluded in the integrand. The manipulative method for integrating over $k_{\parallel}$ is given in the literature (Ogawa and Sakurai, 1969).

By integrating Eq. (2-23) with respect to $k_{\perp}$, we obtain an asympototic expression of the radiation fields by applying the laborious method of steepest descents which will now be described. As has been defined in Fig. 1, $k_{\perp}$ and $k_{\parallel}$ are given by

$$\begin{aligned} k_{\perp} &= k_j\ (\theta) \sin\theta \\ k_{\parallel} &= k_j\ (\theta) \cos\theta, \end{aligned} \tag{2-24}$$

where $j = 1,2$ and θ is the angle between $\vec{H}_o$ and $\vec{k}$. According to the theory of the propagation of electromagnetic waves in a magnetoactive plasma, $k_j(\theta)$ is given as (e.g., Ginzburg, 1964).

$$k_j^2\ (\theta) = \frac{\omega^2}{c^2}\ \frac{\epsilon_2 \pm \sqrt{\epsilon_2^2 - 4\epsilon_1\epsilon_3}}{2\epsilon_1} \tag{2-25}$$

where

$$\epsilon_1 = P \cos^2 \theta + S \sin^2 \theta$$

$$\epsilon_2 = PS (1 + \cos^2 \theta) + (S^2 - D^2) \sin^2 \theta$$

and

$$\epsilon_3 = P (S^2 - D^2)$$

Taking into account these relations between k_j $(j = 1, 2)$ and θ, we can transform the integration over $k_\perp$ into one over θ. Thus, the integration over θ leads to the following form:

$$F(r) = \int_{c_\theta} \psi(\theta) \exp\{r\Phi(\theta)\} d\theta \qquad (2\text{-}26)$$

where c_θ denotes the contour of the integration in the θ-plane and $\Phi(\theta)$ is expressed as $ik_j(\theta) \cos(\theta - \alpha)$. When the distance from the point of observation to that of the electron is much longer than the wave length of radiation fields, i.e., $|k_j(\theta) r| >> 1$, the integration of Eq. (2-26) is approximately carried out by the method of steepest descents. The result is

$$F(r) \simeq \sqrt{-\frac{2\pi}{r\Phi''(\theta_s)}}\ \psi(\theta_s) \exp\{r\Phi(\theta_s)\} \qquad (2\text{-}27)$$

where $\Phi''(\partial_s) = \partial^2 \Phi / \partial^2 \theta |_{\theta = \theta_s}$ and θ_s is the angle at the saddle point which can be determined by the equation

$$\frac{\partial \Phi}{\partial \theta} = i \frac{\partial k_j(\theta)}{\partial \theta} \cos(\theta - \alpha) - ik_j(\theta) \sin(\theta - \alpha) = 0,$$

from which we obtain the well-known equation,

$$\frac{1}{k_j} \frac{\partial k_j}{\partial \theta} = \tan(\theta - \alpha)$$

or

$$\frac{1}{\mu_j} \frac{\partial \mu_j}{\partial \theta} = \tan \chi, \quad \chi = \theta - \alpha$$

which gives the relation between the wave normal direction θ and the ray direction α (e.g., Stix, 1962; Bekefi, 1966), where μj ($j = 1,2$) defines the phase refractive index in the propagation of electromagnetic waves ($\mu j = ck_j/\omega$).

Taking Eq. (2-27) into consideration, the asymptotic solution of Eq. (2-23) integrated with respect to $\vec{k}$ is given by

$$\vec{A}(\vec{r},t) = i \frac{e}{r} \sum_j \sum_n (-1)^{j+1} \int d\omega \exp\{i(\frac{\mu_j \omega}{c} r \cos \psi - \omega t)\} I_j(\theta)$$

$$\times \begin{pmatrix} -i U_{nj} \\ V_{nj} \\ W_{nj} \end{pmatrix} \delta(\omega - n \frac{\omega_H}{\gamma} - \mu_j \omega \beta_{\parallel} \cos \theta) \Big|_{\theta = \theta_s} \qquad (2\text{-}28)$$

where

$$I_j(\theta) = \frac{\cos \alpha(\mu_j \sin \theta)^{\frac{1}{2}}}{\cos \chi \cos \theta} [\sin \alpha\{(\mu_j'' - \mu_j) \cos \chi - 2\mu_j' \sin \chi\}$$

$$\times \{(S - P)^2 \mu_j \sin^4 \theta - 4PD^2 (\mu_j \sin^2 \theta - P)\}]^{-\frac{1}{2}},$$

$$U_{nj} = \beta_\perp \{\mu_j^2 (P \cos^2 \theta + S \sin^2 \theta) - SP\} J_n' (z)$$

$$+ D \{\frac{n\beta_\perp}{z} (P - \mu_j^2 \sin^2 \theta) - \beta_{\parallel} \mu_j^2 \sin \theta \cos \theta\} J_n (z),$$

$$V_{nj} = \beta_\perp D(P - \mu_j^2 \sin^2 \theta) J_n' (z)$$

$$+ (\mu_j^2 - S) \{\frac{n\beta_\perp}{z} (P - \mu_j^2 \sin^2 \theta)$$

$$- \beta_{\parallel} \mu_j^2 \sin \theta \cos \theta\} J_n (z)$$

and

$$W_{nj} = - \beta_\perp D \mu_j^2 \sin \theta \cos \theta J_n' (z)$$

$$- [\frac{n\beta_\perp}{z} \mu_j^2 (\mu_j^2 - S) \sin \theta \cos \theta$$

$$+ \beta_{\parallel} \{(\mu_j^2 - S)(\mu_j^2 \cos^2 \theta - S) - D^2\}] J_n (z).$$

Here

$$z = \frac{\mu_j \gamma \omega \beta_\perp \sin \theta}{\omega_H}, \quad \beta_\perp = \beta \sin \psi, \quad \beta_{\parallel} = \beta \cos \psi,$$

$$\mu_j' = \partial\mu_j/\partial\theta, \quad \mu_j'' = \partial^2\mu_j/\partial\theta^2 \text{ and } J_n' (z) = \partial J_n (z)/\partial z.$$

We must finally integrate Eq. (2-28) with respect to ω. Since the augument of the Dirac delta function in Eq. (2-28) is a function of the variable of integration, we therefore obtain the following relation,

$$\int G(\omega)\ \delta\ (f\ (\omega) - \omega_o)\ d\omega = \frac{G\ (\omega)}{\left|\frac{df\ (\omega)}{d\omega}\right|}\ \Bigg|_{f\ (\omega) = \omega_o} \qquad (2\text{-}29)$$

Since $f\ (\omega) = \omega - \omega\ \mu_j\ \beta \cos\psi \cos\theta$ and $\omega_o = n\omega_H/\gamma$ ($n = [-\infty, \infty]$) in our case, it follows that

$$\omega_n = \frac{n\ \omega_H/\gamma}{1 - \mu_j\ \beta \cos\psi \cos\theta} \qquad (2\text{-}30)$$

which gives the well-known Doppler-shifted angular wave frequency.

The vector potential $\vec{A}$ thus obtained is expressed as

$$\vec{A}\ (\vec{r},t) = \sum_{j=1}^{2} \sum_{n=-\infty}^{\infty} A_{nj}\ (\vec{r},t) \qquad (2\text{-}31a)$$

$$\vec{A}_{nj}(\vec{r},t) = i\ \frac{e}{r}\ (-1)^{j+1}\ \Lambda\ \exp\{i(\frac{\mu_j\omega}{c}\ r\cos\psi - \omega t)\} \begin{vmatrix} -iU_{nj} \\ V_{nj} \\ W_{nj} \end{vmatrix}\ \Bigg|_{\substack{\omega = \omega_n \\ \theta = \theta_s}} \qquad (2\text{-}31b)$$

and

$$\Lambda_j = I_j(\theta)/\left| 1 - \beta \cos\theta \cos\psi \frac{\partial(\omega \mu_j)}{\partial\omega} \right| \tag{2-31c}$$

$\vec{A}_{nj}(\vec{r},t)$ denotes the n-th harmonic of the j-th mode of the vector potential. From Eq. (2-25), the phase refractive index $\mu_j(\omega,\theta)$ is deduced to be

$$\mu_j^2 = 1 - \frac{X(1-X)}{1-X - \frac{Y^2 \sin^2\theta}{2} \pm \left\{\frac{Y^4 \sin^4\theta}{4} + (1-X)^2 Y^2 \cos^2\theta\right\}^{\frac{1}{2}}} \tag{2-32}$$

where the upper sign (+), corresponding to the case j = 1, denotes the ordinary mode of electromagnetic wave propagations and the lower sign (-) (j = 2), the extraordinary mode.

As has been considered earlier in this paper, the radiation fields can be determined by using only the vector potential $\vec{A}$. Thus the fields of the n-th harmonic of the j-th mode are easily calculated from the equations

$$\vec{E}_{nj} = + i \frac{\omega}{c} \vec{A}_{nj}, \quad \vec{B}_{nj} = \text{curl } \vec{A}_{nj}. \tag{2-33}$$

c) The Emissivities

By calculating the Poynting vector, the flow of electromagnetic wave energy of the n-th harmonic of the j-th mode is given by

$$\vec{P}_{nj} = \frac{c}{8\pi} (\vec{E}_{nj} \times \vec{H}^{*}_{nj} + \vec{E}^{*}_{nj} \times \vec{H}_{nj})$$

$$= \frac{e^2 \mu_j \omega_j^2}{4\pi c r^2} \cos \chi \, \Lambda_j^2 \, (\xi_{nj} \, \vec{n} + \eta_{nj} \, \vec{a}_x \times \vec{n}) \Big|_{\substack{\omega = \omega_n \\ \theta = \theta_s}} , \qquad (2\text{-}34)$$

$$(j = 1,2)$$

where the asterisk denotes the complex conjugate and

$$\xi_{nj} = U_{nj}^2 + Z_{nj}^2$$

$$\eta_{nj} = (V_{nj}^2 - W_{nj}^2) \sin \alpha_j \cos \alpha_j + V_{nj} W_{nj} (\cos^2 \alpha_j - \sin^2 \alpha_j)$$

$$Z_{nj} = V_{nj} \cos \alpha_j - W_{nj} \sin \alpha_j$$

$\vec{n}$ is the unit vector in the direction of $\vec{r}$ (Fig. 3). The first term of the Poynting vector is along the direction $\vec{n}$ which subtends the angle α_j from the external magnetic field $\vec{H}_o$ as shown in Fig. 3, whereas the second term is in the (y - z) plane and perpendicular to the vector $\vec{n}$. Since the second term does not give the energy flux passing through the point of observation along the vector $\vec{r}$, we do not need to take into account this term in calculating the observational flux.

The angular distribution of the radiation from a single electron at the n-th harmonic of the j-th mode at the point of the electron is thus calculated from Eq. (2-34) as follows:

$Q_{nj}\ (\theta, \beta, \psi)$

$$= \frac{e^2 \mu_j \omega^2}{4\pi c} \cos \chi_j \Lambda_j^2 \xi_{nj} \left(1 - \beta \cos \theta \cos \chi \frac{\partial(\mu_j \omega)}{\partial \omega}\right) \Big|_{\substack{\omega = \omega_{nj} \\ \theta = \theta_s}} \quad (2\text{-}35)$$

$(j = 1, 2)$

In deriving the above result, we have considered the correction discussed by Scheuer (1968) and Ginzburg and Syrovatskii (1969). This result gives the emissivity from a radiating electron at the n-th harmonic of the j-th mode. Then, the frequency and angular distribution of the radiation from a single electron, into the ordinary and extraordinary modes, is given by

$$\eta_j\ (\omega, \theta, \beta, \psi)$$

$$= \frac{e^2 \omega^2}{4\pi c} \cos \chi \sum_{n=1}^{\infty} \Lambda_j^2 \xi_{nj} \left(1 - \beta \cos \theta \cos \psi \frac{\partial(\mu_j \omega)}{\partial \omega}\right) \Big|_{\substack{\omega = \omega_{nj} \\ \theta = \theta_s}} \quad (2\text{-}36)$$

This result is very similar to that obtained by Ramaty (1969), but different from his case because the present result includes the effect due to the longitudinal component of radiation electric fields. This difference arises from the fact that the radiation energy does not propagate along the direction of the wave normal $\vec{k}$ because of the existence of the longitudinal component.

The emission-coefficient, $j_j\ (\omega, \theta)$ are thus given by the following general expression (Bekefi, 1966; Ramaty, 1969):

$$j_j(\omega, \theta) = \int \eta_j(\omega, \theta, \beta, \psi)\ f\ (\vec{p}')\ d^3p' \text{ ergs/sec str } H_z \text{ cm}^3$$

$$(j = 1,2) \qquad (2\text{-}37)$$

where $f\ (\vec{p})\ d^3p$ is the number of electrons per unit volume, with momenta in d^3p around $\vec{P}$.

ABSORPTION

The absorption coefficient, α_j for an arbitrary velocity and pitch-angle distribution can be obtained from the following general expression (Bekefi, 1966):

$$\alpha_j(\omega, \theta, \beta, \psi) = - \frac{8\pi^3\ c^2}{\eta_{rj}^2\ \hbar\ \omega^3} \int \eta_j(\omega, \theta, \beta, \psi)[f(\vec{p}') - f(p)]\ d^3p' \qquad (3\text{-}1)$$

where $\hbar$ and η_{rj} are, respectively, Planck's constant divided by 2π and the "ray" refractive index given by Bekefi (1966). In the above equation, the sign is defined such that a photon is emitted when the momentum of an electron changes from $\vec{p}'$ to $\vec{p}$. Since the photon energy $\hbar\omega$ is assumed to be much smaller than the electron energy $E\ (= \sqrt{m_o^2\ c^4 + p^2\ c^2})$, it follows that

$$f\ (\vec{p}') - f\ (p) = \frac{E\ \hbar\ \omega}{pc^2}\ \frac{\partial f}{\partial p} + \frac{\hbar\ \omega}{p^2\ c^2\ \sin\psi\ \cos\psi}$$

$$\times\ \{E - 2\ n_j\ pc\ \cos\theta\ \cos\psi\}\ \frac{\partial f}{\partial \psi} \qquad (3\text{-}2)$$

The coefficient of the second term on the right hand side of the above equation gives the change in the pitch-angle of the radiating electron resulting from the emission of a photon of energy $\hbar\omega$ into the direction θ.

In case of an isotropic distribution ($\partial f/\partial\psi = 0$), a necessary condition for negative absorption is $\partial f/\partial p > 0$ as can be seen from Eqs. (3-1) and (3-2). For an anisotropic distribution, however, the negative absorption can occur even if $\partial f/\partial p < 0$. A necessary condition for this is given by

$$\frac{1}{\sin\psi\cos\psi}\{E - 2n_j\, pc\cos\theta\cos\psi\}\frac{\partial f}{\partial\psi} > 0. \tag{3-3}$$

Since this equation contains the phase refractive index, n_j, the condition for negative absorption is to some extent different between the ordinary and extraordinary modes. For ultra-relativisitic electrons, the emission is strongly concentrated into the direction of the instantaneous velocity and so the above condition can be approximated as

$$\frac{1 - 2n_j\cos^2\psi}{\sin\psi\cos\psi}\frac{\partial f}{\partial\psi} > 0$$

since $E \simeq pc$ and $\theta \simeq \psi$. In the case of $\psi < \pi/2$ and $\partial f/\partial\psi > 0$, for example, the pitch-angle of radiating electrons is limited within the range given by $0 < \cos\psi < \sqrt{1/(2n_j)}$, in order to amplify synchrotron radiation. We must, however, remark that maser action from a moderately anisotropic distribution of radiating electrons will be confined mainly to the first few harmonics of gyrosynchrotron radiation (Heyvaerts, 1968).

The momentum and pitch-angle distribution of radiatirg electrons is, therefore, very important in calculating the absorption coefficient and hence the source function in relation to the equation of radiative transfer of gyrosynchrotron radiation from midly relativistic electrons in a magnetoactive plasma.

POLARIZATION AND RADIATIVE TRANSFER

The shape of polarization ellipse, R_{nj}, for the n-th harmonic of the j-th mode is given by

$$R_{nj} = \frac{(\vec{k} \times \vec{a}_x) \cdot \vec{E}_{nj}}{\vec{a}_x \cdot \vec{E}_{nj} \cdot |\vec{k}|} = \frac{(\vec{k} \times \vec{a}_x) \cdot \vec{A}_{nj}}{\vec{a}_x \cdot \vec{A}_{nj} \cdot |\vec{k}|}$$

$$= i\,\frac{V_{nj}\cos\theta - W_{nj}\sin\theta}{U_{nj}} \quad (j = 1,2) \tag{4-1a}$$

$$= i\,\frac{Z_{nj}}{U_{nj}} \quad (j = 1,2) \tag{4-1b}$$

Taking into account that U_{nj} and Z_{nj} are both real, the polarization of both ordinary and extraordinary waves is elliptic with the axes of $\vec{a}_x$ and $\vec{k} \times \vec{a}_x/|\vec{k}|$ (Fig. 4).

In a magnetoactive plasma, the electric fields $\vec{E}_{nj}$ which are given by Eq. (2-33) generally have a component along the direction $\vec{k}$. In order to consider the radiative transfer in such a medium, we cannot, therefore, use the Stokes parameters defined for and applicable to the transverse polarized electromagnetic waves. It is convenient to consider the electric induction $\vec{D}$ instead of using the electric field $\vec{E}$ in this case because the former does not have a component along the direction $\vec{k}$ (e.g., Zheleznyakov, 1968; Fung, 1969b). After some algebra, the electric induction $\vec{D}$ is calculated as follows:

$$\vec{D}_{nj} = (D_{nj\,x}, D_{nj\,\theta}, D_{nk\,k})$$
$$= \left(\frac{\omega}{c}\mu_j\right)^2 (E_{nj\,x}, -E_{nj\,\theta}, 0)(j=1,2) \qquad (4\text{-}2)$$

Here $D_{nj\,\theta}$ and $E_{nj\,\theta}$ are the components along the unit vector $\vec{a}_\theta$ as shown in Fig. 4. The definition of the polarization, Eq. (4-1), is thus rewritten as

$$R_{nj} = \frac{E_{nj\,\theta}}{E_{nj\,x}} = -\frac{D_{nj\,\theta}}{D_{nj\,x}}$$
$$= i\,\frac{Z_{nj}}{U_{nj}} \qquad (j = 1,2) \qquad (4\text{-}3)$$

Since $\vec{D}_{nj}$ does not have the component $D_{nj\,k}$, we can define the Stokes parameters such as I, Q, U and V in an analogous way as the vacuum case, in which $E_{nj\,k} = 0$ since $V_{nj}\sin\theta + W_{nj}\cos\theta = 0$.

By adding both components of the ordinary and extraordinary modes, we can define the polarization tensor $I_{\alpha\beta}$ $(\alpha, \beta = x, \theta)$ of the radiation in terms of the components of $\vec{D}$, which are given by

$$D_x = \sum_j D_{nj}\ x,$$
$$D_\theta = \sum_j D_{nj}\ \theta$$

The tensor $I_{\alpha\beta}$ is, therefore, defined as

$$I_{\alpha\beta} = D_\alpha\ D_\beta^*,\ (\alpha,\ \beta = x,\ \theta) \tag{4-4}$$

where the asterisk indicates the complex conjugate of the components. The Stokes parameters are then expressed in terms of $I_{\alpha\beta}$ as follows:

$$\begin{aligned} I &= I_{xx} + I_{\theta\theta} \\ Q &= I_{xx} - I_{\theta\theta} \\ U &= I_{\theta x} + I_{x\theta} \\ V &= i\ (I_{\theta x} - I_{x\theta}) \end{aligned} \tag{4-5}$$

From Eq. (4-3), we can define the polarization coefficients in terms of the components of the electric induction of the radiation fields as

$$\frac{D_{\theta j}}{D_{xj}} = - i \frac{V_{nj}}{U_{nj}} = i \alpha_{\theta j} \cdot (j = 1,2) \tag{4-6}$$

Taking into account the "apparent" radiation intensities of the ordinary and extraordinary modes, I_1 and I_2, calculated from the electric induction and the phase difference δ between both modes, D_x and D_θ can be written as

$$\binom{D_x}{D_\theta} = \left(\frac{2\pi}{c}\right)^{\frac{1}{2}} [I_1^{\frac{1}{2}} \vec{a}_1 + e^{-i\delta} I_2^{\frac{1}{2}} \vec{a}_2], \tag{4-7}$$

where the unit vectors are given by

$$\vec{a}_j = (1 + \alpha_{\theta j})^{-\frac{1}{2}} \binom{1}{i \alpha_{\theta j}}. \quad (j = 1,2)$$

Consequently, we can express the Stokes parameters, Eq. (4-5), in terms of I_j and $\alpha_{\theta j}$ $(j = 1,2)$. In these expressions, we do not need to consider the component along the direction $\vec{k}$ of the electric fields. Therefore, we can develop the problem of radiative transfer in a similar way as has been considered by Zheleznyakov, (1968) and Fung (1969b).

The degree of polarization, p, the ellipticity, ϵ, and the angle χ between the major axis and the x-direction can be written as (Born and Wolf, 1964; Bekefi, 1966):

$$p = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

$$\epsilon = \tan\beta ; \ \sin 2\beta = \frac{V}{\sqrt{Q^2 + U^2 + V^2}} \tag{4-8}$$

$$\tan 2\chi = \frac{U}{Q}$$

Taking into account Eq. (4-5), the degree of polarization p is expressed as

$$p = \frac{|I_1 - I_2|}{I} \tag{4-9}$$

This means that p is always less than the unity when both modes can escape from the source region.

As has been discussed earlier in this paper, the electromagnetic energy emitted from electrons is usually transported in the direction making the angle α_j with the static external magnetic field which is not generally coincident with the angle θ_j as shown in Fig. 3. The Stokes parameters defined in this paper are very useful for dealing with the problem of radiative transfer, especially the polarization treatment, but are not so much helpful in the study of the transport of the electromagnetic energy in a magnetoactive plasma because of the reason just mentioned.

The transport of the electromagnetic energy in a magnetoactive plasma has recently been considered by Bekefi (1966) and Enomé (1969). According to them, the equation of the transfer of electromagnetic energy is given by

$$\frac{\partial}{\partial s}\left(\frac{I_j^*}{n_{rj}^2}\right) + \frac{1}{\omega_{gj}}\frac{\partial}{\partial t}\left(\frac{I_j^*}{n_{rj}^2}\right) = \frac{j_j}{n_{rj}^2} - \alpha_j \frac{I_j^*}{n_{rj}^2}, \quad (j = 1,2) \qquad (4\text{-}10)$$

where the scalar quantity I_j^* is known as the specific intensity of radiation and usually not equal to I_j as defined earlier. Here $d\vec{S}$ is the element of the length along the ray direction $\vec{S}$ (Fig. 3). ω_{gj} is the group velocity.

In dealing with the transfer of radiation, let us assume the wave normal vectors, $\vec{k}_1$ and $\vec{k}_2$ to be initially in the same direction, i.e., $\vec{k}_1 || \vec{k}_2$. Even in this case, the ray direction of the ordinary mode α_1, in general, does not coincide with that of the extraordinary mode, α_2, i.e., $\alpha_1 \neq \alpha_2$. This result can be proved by calculating the equation

$$\tan(\theta - \alpha_j) = \frac{1}{\mu_j}\frac{\partial \mu_j}{\partial \theta} \quad (j = 1,2)$$

and $\theta = \theta_1 = \theta_2$. Accordingly, the two directions of the intensities I_1^* and I_2^* which are obtained by solving Eq. (4-10), are generally not coincident with each other.

The case in which I_j^* and n_{rj} are independent of time, Eq. (4-10) is reduced to

$$\frac{\partial}{\partial s}\left(\frac{I_j^*}{n_{rj}^2}\right) = \frac{I_j^*}{n_{rj}^2} - \alpha_j \frac{I_j^*}{n_{rj}^2}, \quad (j = 1,2) \tag{4-11}$$

or, by dividing both sides with α_j,

$$\frac{\partial}{\partial \tau_j}\left(\frac{I_j^*}{n_{rj}^2}\right) = \frac{I_j^*}{n_{rj}^2} - S_j, \quad (j = 1,2) \tag{4-12}$$

where $d\tau_j = - \alpha_j \; dS$ and

$$S_j = \frac{I}{n_{rj}^2} \; \frac{I_j^*}{\alpha_j} \quad (j = 1,2) \tag{4-13}$$

which is known as the source function and plays an important role in the analysis of the radiation from the medium (e.g., Chandrasekhar, 1960). The above equation (4-12) shows that the effective path length is generally different between the two modes.

When the phase difference between the ordinary and extraordinary modes is completely random, i.e., when the Faraday rotation is very large, the two modes of waves propagate independently. In this case, we can separately solve the equation of radiative transfer, Eqs. (4-11) or (4 -12) for each mode. For a homogeneous source region, we can integrate Eq. (4-11) as given by

$$I_j^* = \frac{j_j}{\alpha_j} [1 - \exp(-\alpha_j L_j)], \quad (j = 1,2) \qquad (4\text{-}14)$$

where L_j is the depth of the source. The intensity of the escaping radiation can thus be given by

$$I_j^* = \frac{n_{rj}^2 \omega_j^2 \int \eta_j f(\vec{p}') d^3p'}{8\pi^3 \int \eta_j \left(-\frac{E}{p}\frac{\partial f}{\partial p} + \frac{1}{p^2 \sin\psi \cos\psi}\{2n_j pc \cos\theta \cos\psi - E\}\frac{\partial f}{\partial \psi}\right) d^3p'}$$

$$\times \left[1 - L_j \frac{8\pi^3}{n_{rj}^2 \omega_j^2} \int \eta_j \left(-\frac{E}{p}\frac{\partial f}{\partial p} + \frac{1}{p^2 \sin\psi \cos\psi}\right.\right.$$

$$\left.\left. \times \{2 n_j pc \cos\theta \cos\psi - E\} \frac{\partial f}{\partial \psi}\right) d^3p'\right] \quad (j = 1,2) \qquad (4\text{-}15)$$

In arriving at this result, we have used Eqs. (2-37), (3-1) and (3-2).

If the inequality

$$-\frac{E}{p}\frac{\partial f}{\partial p} + \frac{1}{p^2 \sin\psi \cos\psi} \{2 n_j pc \cos\theta \cos\psi - E\} \frac{\partial f}{\partial \psi} < 0 \qquad (4\text{-}16)$$

is satisfied in some frequency range, the radiation can be amplified and then is observed to be enhanced in the same frequency range.

The source function S_j is given, by using Eqs. (2-37), (3-1) and (3-2), as follows:

$$S_j = \frac{\omega_j^2}{8\pi^3} \frac{\int \eta_j f(\vec{p}') d^3p'}{\int \eta_j (-\frac{E}{p}\frac{\partial f}{\partial p} + \frac{1}{p^2 \sin\psi \cos\psi}\{2n_j pc \cos\theta \cos\psi - E\}\frac{\partial f}{\partial \psi}) d^3p'}$$

$$(j = 1,2) \tag{4-17}$$

The source function S_j is sometimes conveniently written in the form

$$S_j = \frac{1}{n_{rj}^2} \frac{j_j}{\alpha_j} = \frac{\omega_j^2}{8\pi^3 c^2} k\, T_j, \quad (j = 1,2) \tag{4-18}$$

where k is Boltzmann's constant and T_j is a quantity with dimensions of temperature. This is usually referred to as the radiation temperature of the medium and is a function of frequency, the direction of the ray in the medium and the elementary emission processes. The temperature is also usually different between the two modes, and is very important in estimating the radiation processes associated with solar and galactic radio emissions. This temperature occasionally becomes negative when maser action works in the medium where the radiation and propagation take place.

If we assume that $\omega >> \omega_p$, ω_H and $\omega_p >> \omega_H$, the refractive index n_j is reduced approximately to $(1 - \omega_p^2/\omega^2)^{\frac{1}{2}}$. If follows that $n_1 = n_2$, $n_{r1} = n_{r2}$ and $x_j = \theta_j - \alpha_j = 0$. Taking into the above assumption, the results obtained here are reduced to the same forms as currently used (e.g., Ramaty, 1969). It is clear that the former are much more complicated than the latter. However, the equations of radiative transfer and their solution are similar to each other except for some essential differences on the solution of radiation fields.

SUMMARY

We have asymptotically calculated the gyrosynchrotron radiation fields from electrons in a magnetoactive plasma by using the Green tensor and the Fourier transformation. The medium has been assumed to be cold and collisionless and immersed in a static external magnetic field. The radiation fields obtained in this paper would be useful in studying the interpretation of solar radio type IV radio bursts and stellar and galactic radio spectra.

Taking these radiation fields into consideration, we have developed the detailed calculation of the emissivity and absorption coefficients for a given direction of observation from an ensemble of electrons with arbitrary momentum and pitch angle distribution.

In order to study the problem of radiative transfer, we have used the electric induction $\vec{D}$ in place of the electric field $\vec{E}$. The former is very convenient in formulating the Stokes parameters since it does not have a component along the

wave normal $\vec{k}$. The electromagnetic energy is, however, usually transported in a direction different from the direction $\vec{k}$. In order to calculate the transport of this energy, we must then consider the electric and magnetic fields, $\vec{E}$ and $\vec{H}$. When the longitudinal component of the electric fields cannot be neglected, the theory of radiative transfer, therefore, becomes much complicated in comparison with the theory currently used.

In this paper, we have not shown any results of numerical calculation, but this will be studied in a forthcoming paper and applied to problems on solar and galactic radio emissions.

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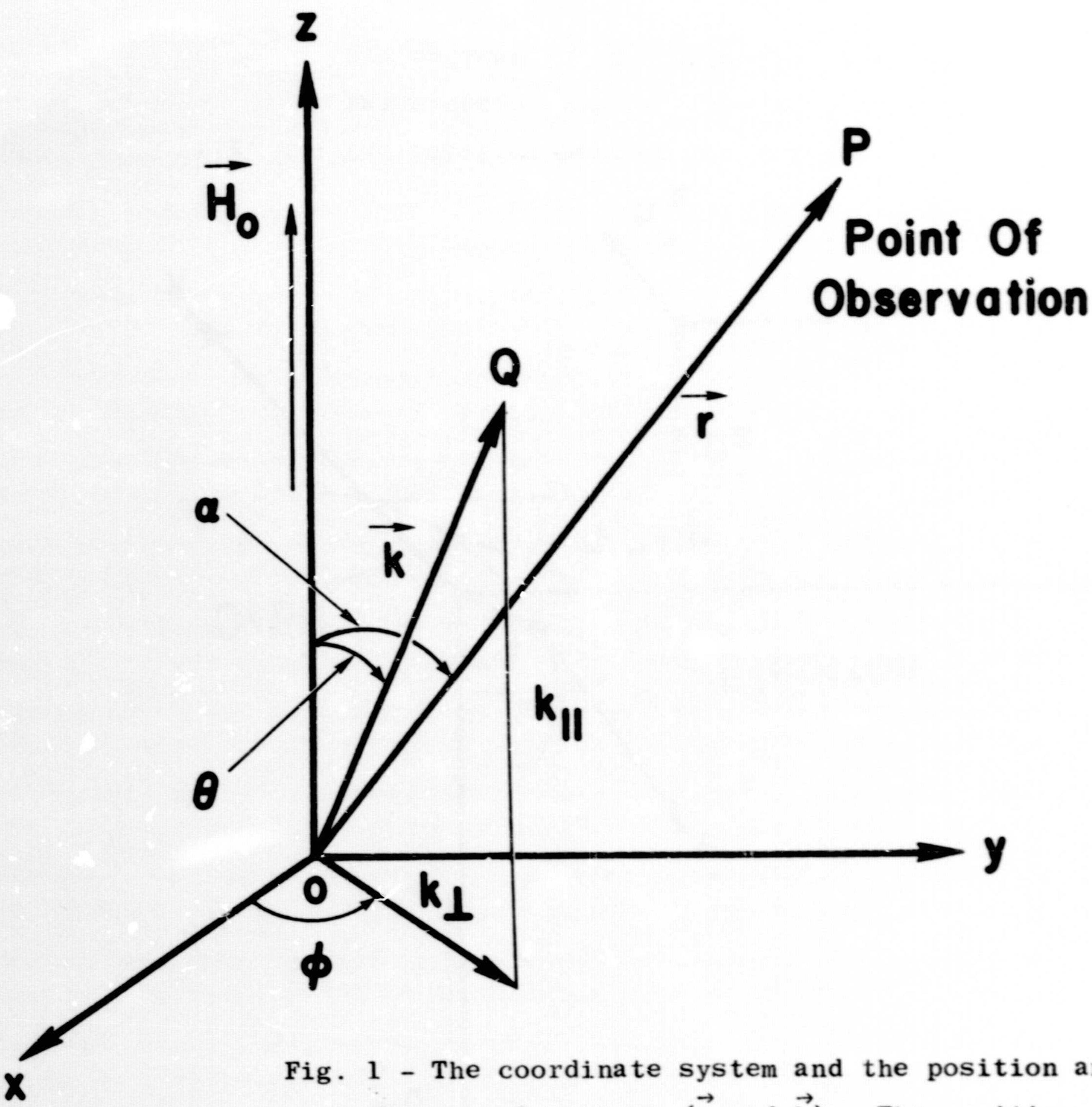

Fig. 1 - The coordinate system and the position and wave normal vectors ($\vec{r}$ and $\vec{k}$). The position vector is in the (y - z) plane.

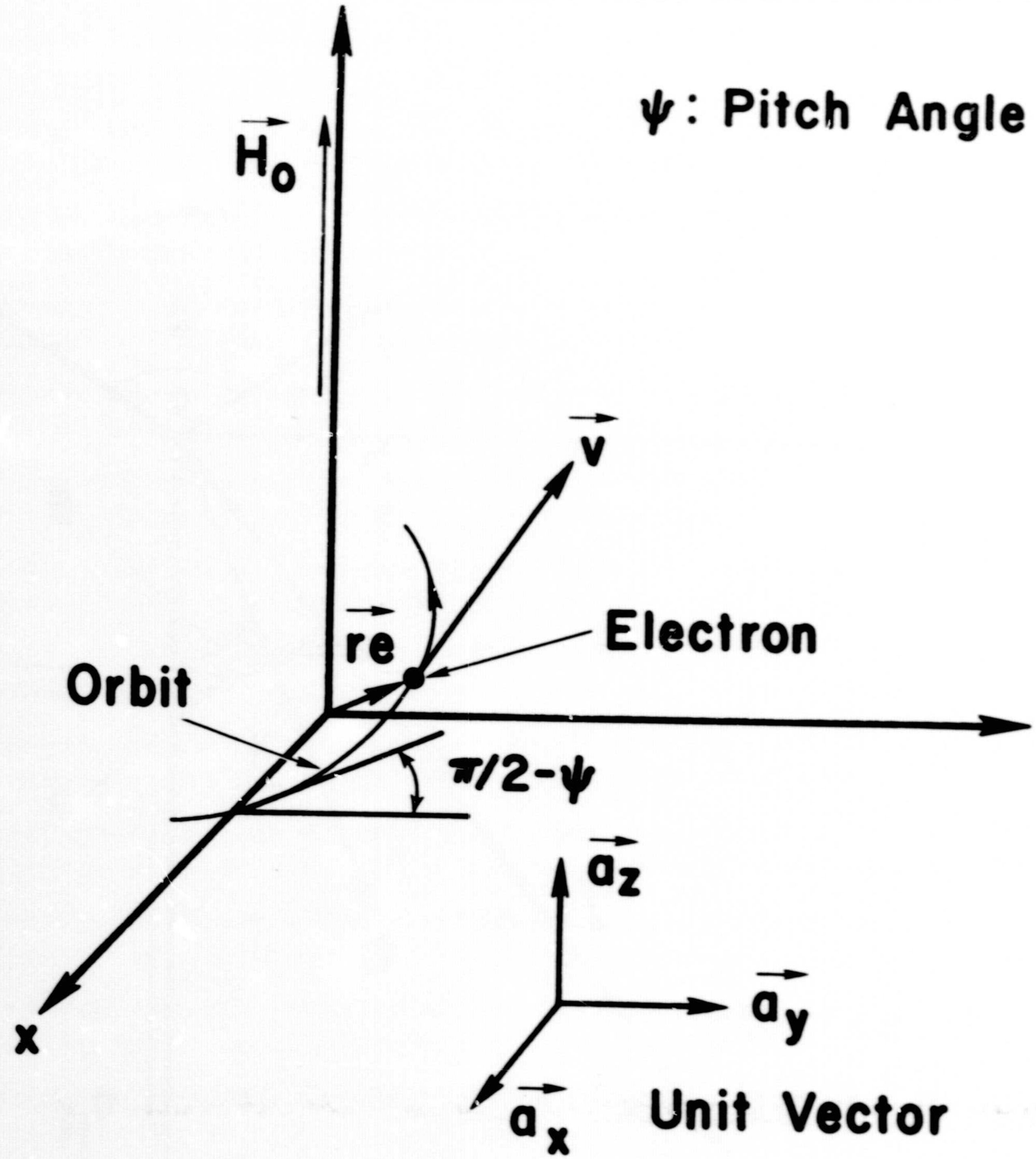

Fig. 2 - The relation between the point of observation and the electron motion. The unit vectors ($\vec{a}_x$, $\vec{a}_y$, $\vec{a}_z$) are defined in the Cartesian coordinate system.

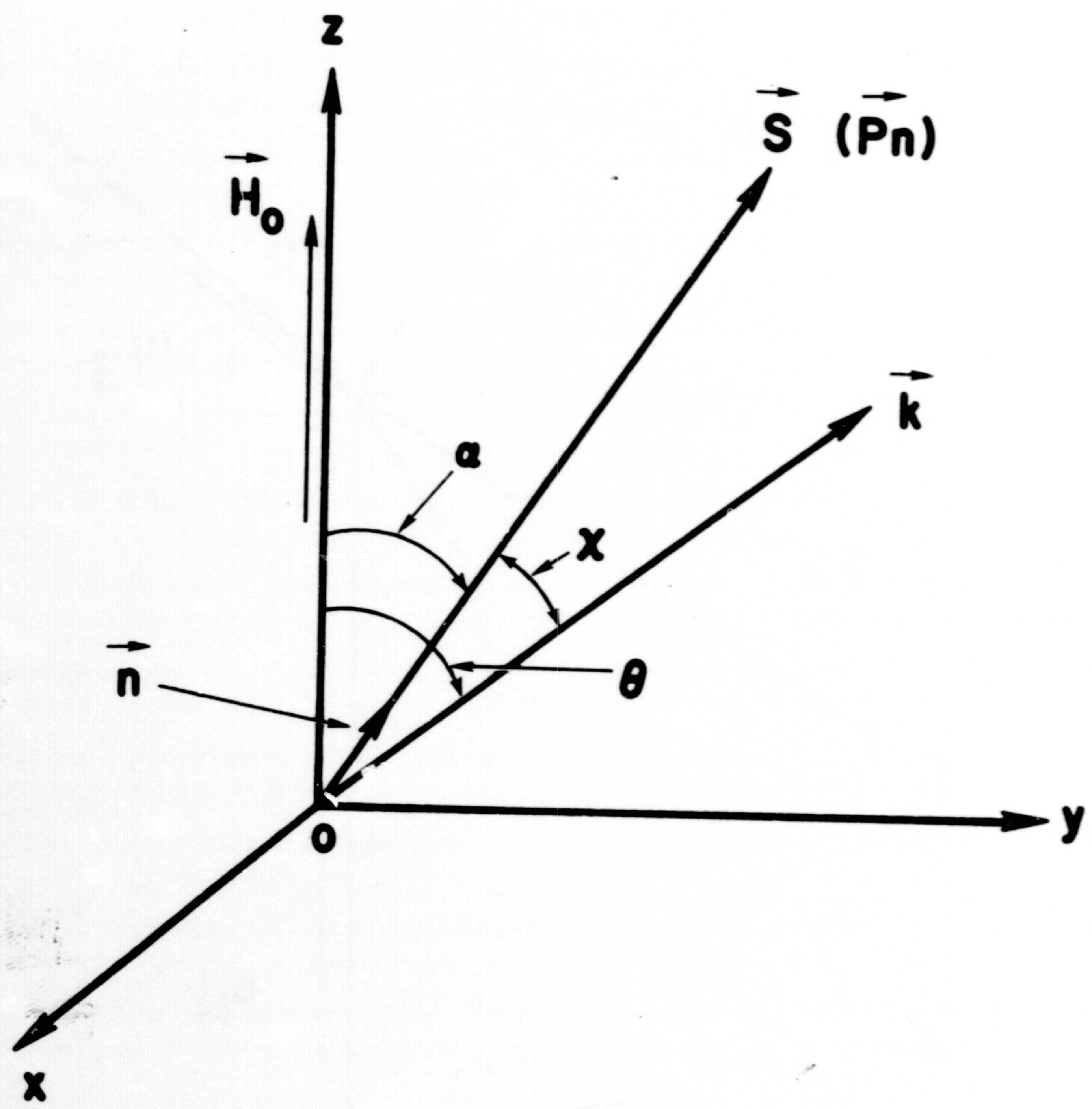

Fig. 3 - The relation between the two directions of the energy transport and the wave normal.

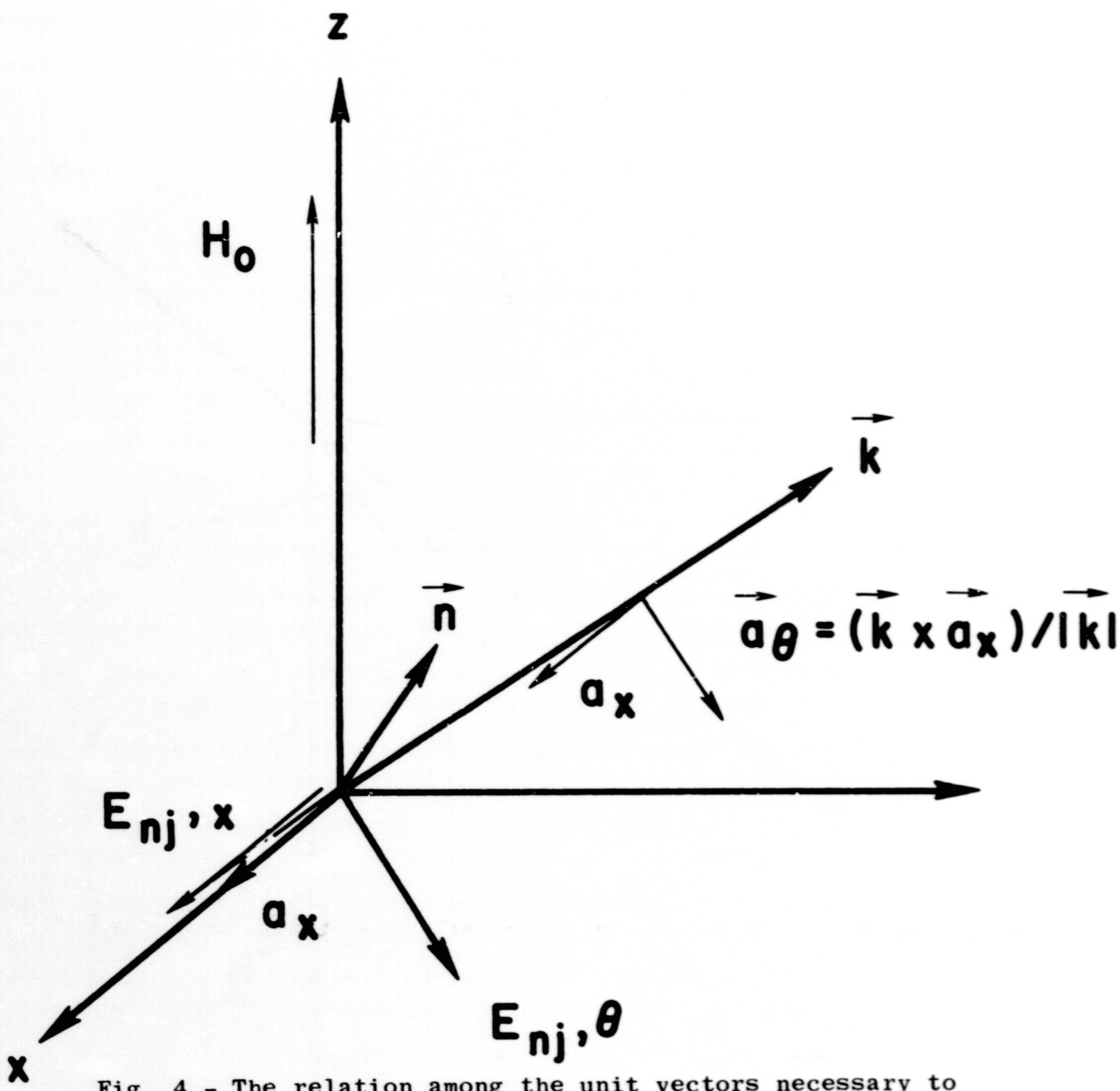

Fig. 4 - The relation among the unit vectors necessary to define the polarization ellipse.