AREAS OF CONTACT AND PRESSURE DISTRIBUTION IN BOLTED JOINTS

H.H. Gould

B.B. Mikic

Final Technical Report

Prepared for

George C. Marshall Space Flight Center
Marshall Space Flight Center
Alabama 35812

under

Contract NAS 8-24867

June 1970

DSR Project 71821-68

Engineering Projects Laboratory
Department of Mechanical Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

ABSTRACT

When two plates are bolted (or riveted) together these will be in contact in the immediate vicinity of the bolt heads and separated beyond it. The pressure distribution and size of the contact zone is of considerable interest in the study of heat transfer across bolted joints.

The pressure distributions in the contact zones and the radii at which flat and smooth axisymmetric, linear elastic plates will separate were computed for several thicknesses as a function of the configuration of the bolt load by the finite element method. The radii of separation were also measured by two experimental methods. One method employed autoradiographic techniques. The other method measured the polished area around the bolt hole of the plates caused by sliding under load in the contact zone. The sliding was produced by rotating one plate of a mated pair relative to the other plate with the bolt force acting.

The computational and experimental results are in agreement and these yield smaller zones of contact than indicated by the literature. It is shown that the discrepancy is due to an assumption made in the previous analyses.

In addition to the above results this report contains the finite element and heat transfer computer programs used in this study. Instructions for the use of these programs are also included.

ACKNOWLEDGMENTS

This report was supported by the NASA Marshall Space Flight Center under contract NAS 8-24867 and sponsored by the Division of Sponsored Research at M.I.T.

TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

NOMENCLATURE

A, B, C	radii
D	thickness
E	modulus of elasticity
G	shear modulus
h_c, h_f	heat transfer coefficients
H	hardness
k, k_1, k_2	thermal conductivities
P, p	pressure
r	coordinate
R_o	radius of separation
u,w	displacement in r and z directions
x	coordinate
X_c	length of contact
y	coordinate
y'	slope
z	coordinate
δ	deflection
ε	dilation
ε_r, ε_t, ε_{rz}	strains
σ, σ_1, σ_2	standard deviations
σ_r, σ_t, σ_z	stresses

λ , μ	Lame's constants
ν	Poisson's ratio
τ	shear stress
θ	angle

Subscripts

r	radial direction
t	tangential direction
z	z-direction

Chapter I

INTRODUCTION

When two plates are bolted (or riveted) together, these will be in contact in the immediate vicinity of the bolt heads and separated beyond it. The pressure distribution in the contact area and the separation of the plates is of considerable interest in the study of heat transfer across joints. Cooper, Mikic and Yovanovich [1] show that with assumed Gaussian distribution of surface heights, the microscopic contact conductance is related to the interface pressure, surface characteristics and the hardness of the softer material in

$$h_c = 1.45 \frac{\tan\theta}{\sigma} k\left(\frac{P}{H}\right)^{0.985} \tag{1.1}$$

where

$$k \equiv \frac{2k_1 k_2}{k_1+k_2} \tag{1.2}$$

and k_1 and k_2 represent the thermal conductivities of two bodies in contact; σ is the combined standard deviation for the two surfaces which can be expressed as

$$\sigma = (\sigma_1^2 + \sigma_2^2)^{1/2} \tag{1.3}$$

where σ_1 and σ_2 are the individual standard deviation of height for the respective surfaces; $\tan\theta$ is the mean of the absolute value of slope for the combined profile and it is related, for normal distribution of slope, to the individual mean of absolute values of slopes as

$$\tan\theta = (\tan\theta_1^2 + \tan\theta_2^2)^{1/2} \qquad (1.4)$$

where

$$\tan\theta_i = \lim_{L\to\infty} \frac{1}{L} \int_o^L |y_i'| \, dx; \quad i = 1,2 \qquad (1.5)$$

and y' is the slope of the respective surface profiles; P represents the local interface pressure; and H is the hardness of the softer material.

Relation (1.1), as written above, is applicable for contact in a vacuum. One can modify the expression by simply adding to it

$$h_f \equiv \frac{\text{conductivity of interstitial fluid}}{\text{average distance between the surfaces}} \qquad (1.6)$$

in order to account approximately for the presence of the interstitial fluid.

All parameters in relation (1.1), except for the pressure, are functions of the material and geometry and can be easily obtained. The determination of the pressure distribution and the extent of the contact area between two plates present both mathematical and experimental difficulties. From the mathematical point of view, the difficulty stems from the fact that the theory of elasticity will yield a three dimensional (axisymmetric) problem with mixed boundary conditions. Experimentally, the discrimination between contact and gaps of the order of millionths of an inch is required.

Roetscher [2] proposed in 1927, a rule of thumb that the pressure distribution of two bolted plates, Fig.1, is limited to the two frustums of the cones with a half cone angle of 45 degrees as shown in Fig. 2 and that at any level within the cone the pressure is constant. Also, for symmetric plates, according to Roetscher, separation will occur at the circle which is defined by the contact plane and the 45 degree truncated cone emanating from the outer radius of the bolt head.

Since 1961 Fernlund [3], Greenwood [4] and Lardner [5] among others reported solutions based upon the theory of elasticity. Although their solutions also yield separation radii at approximately 45 degrees as in Roetscher's rule, their solutions yield a much more reasonable pressure distribution as compared to Roetscher's constant pressure at each level of the frustrum. These investigators have made use of the Hankel transform method demonstrated by Sneddon [6] in his solution for the elastic stresses produced in a thick plate of infinite radius by the application of pressure to its free surfaces. The basic assumption in their approach is that two bolted plates can be represented by a single plate of the same thickness as the combined thickness of the two plates under the same external loading. It then follows that the z-stress distribution at the parting plane can be approximated by the z-stress distribution in the same plane of the single plate. It also follows that separation will occur at the smallest radius in that plane for which the z-stress is tensile. In the case of two plates of equal thickness the σ_z stress at the midplane of the equivalent single plate is the stress of interest.

Fernlund [3], for example, used the method of superposition in the sequence shown in Figs. 3(a) to 3(c) to obtain annular loading. Then by superposition of shear and radial stresses at radius A, Figs. 3(d) and 3 (e), opposite in sign of those due to the annular loading at the free surfaces, Fernlund obtained the solution for a single plate with a hole under annular loading (Fig. 3(f)).

Experimental work in this area included Bradley's [7] measurements of the stress field by three dimensional photoelasticity techniques, and the use of introducing pressurized oil at various radii in the contact zone and measuring the pressure at which oil leaks out from the joint [3,8]. Both of these experimental methods have uncertainties as indicated by the authors.

Because of the cumbersomness of the Hankel transform solution and experimental difficulties, the body of work in this area has been very limited and definite verification of analytical results by experiment is not cited in the literature.

The research described in the succeeding chapters was undertaken with the following primary objectives:

a) To provide a method of solution for the case of two bolted plates without the simplifying assumption of the single plate substitution.

b) To devise a test to validate the two plate analysis.

c) To test the validity of the single plate substitution.

A finite element computer program has been assembled for the analytical solution of two-plate problems. Experiments have been performed to verify the analytical results. Since in heat transfer calculations the extent of the radius of contact is of primary importance, and since by restricting the experimental effort to the verification of only this parameter, (rather than the verification of the entire pressure distribution,) many experimental uncertainties should be eliminated, the experiments were designed only for the determination of the contact area.

Agreement between analysis and experiment was obtained and the results show that the single plate substitution is not justified and the 45 degree rule is not valid for the flat and smooth surfaces studied.

Chapter II

ANALYSIS

A. Problem Statement

The objective of the anlaysis was to solve the linear elasticity problem of two plates in contact defined mathematically by the following equations for each plate:

The equations of equilibrium

$$\frac{\partial}{\partial r}(r\sigma_r) - \sigma_t + r\frac{\partial \tau}{\partial z} = 0$$

$$\frac{\partial}{\partial r}(r\ \tau) + \frac{\partial}{\partial z}(r\ \sigma_z) = 0 \tag{2.1}$$

where $\tau_{rz} = \tau_{zr} = \tau$ and $\tau_{rt} = \tau_{tr} = \tau_{zt} = \tau_{tz} = 0.$

The stress - strain relations, using standard notation for stress and strain,

$$\sigma_r = \lambda\varepsilon + 2\ \mu\varepsilon_r$$

$$\sigma_t = \lambda\varepsilon + 2\ \mu\varepsilon_t$$

$$\sigma_z = \lambda\varepsilon + 2\ \mu\varepsilon_z \tag{2.2}$$

$$\tau = 2\ \mu\varepsilon_{rz}$$

where λ and μ are Lame's constants and

$$\lambda = \frac{2\ G\nu}{1 - 2\nu}$$
$$\mu = G \tag{2.3}$$

if G is the modulus of elasticity in shear and ν is Poisson's ratio; and ε the volume expansion is defined by

$$\varepsilon = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \tag{2.4}$$

where u is the displacement in the radial direction and w is the displacement in the axial direction.

The strain - displacement relations

$$\varepsilon_r = \frac{\partial u}{\partial r}$$
$$\varepsilon_t = \frac{u}{r}$$
$$\varepsilon_z = \frac{\partial w}{\partial z} \tag{2.5}$$
$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)$$

The above equations can be combined to yield the equilibrium equations in terms of displacements

$$\nabla^2 u - \frac{u}{r^2} + \frac{1}{1 - 2\nu}\ \frac{\partial \varepsilon}{\partial r} = 0$$
$$\nabla^2 w + \frac{1}{1 - 2\nu}\ \frac{\partial \varepsilon}{\partial z} = 0 \tag{2.6}$$

The applicable boundary conditions are (see Fig. 11)

$$
\begin{aligned}
\sigma_r^{(1)}(A,z) &= \sigma_r^{(2)}(A,z) = 0 \\
\tau^{(1)}(A,z) &= \tau^{(2)}(A,z) = 0 \\
\sigma_r^{(1)}(C,z) &= \sigma_r^{(2)}(C,z) = 0 \\
\tau^{(1)}(C,z) &= \tau^{(2)}(C,z) = 0 \\
\tau^{(1)}(r,D_1) &= \tau^{(2)}(r,-D_2) = 0, \\
\tau^{(1)}(r,0) &= \tau^{(2)}(r,0), \qquad A \le r \le R_o \\
\sigma_z^{(1)}(r,D_1) &= \sigma_z(r,-D_2) = 0, \qquad B \le r \le C \\
\tau^{(1)}(r,0) &= \tau^{(2)}(r,0) = 0, \qquad R_o \le r \le C \\
\sigma_z^{(1)}(r,0) &= \sigma_z^{(2)}(r,0), \qquad A \le r \le R_o \\
\sigma_z^{(1)}(r,0) &= \sigma_z^{(2)}(r,0) = 0, \qquad R_o \le r \le R \\
\sigma_z^{(1)}(r,D_1) &= \sigma_z^{(2)}(r,-D_2) = P(r), \qquad A \le r \le B \\
w^{(1)}(r,0) &= w^{(2)}(r,0), \qquad A \le r \le R_o \\
2\pi \int_A^B Pr\ dr &= 2\pi \int_A^{R_o} pr\ dr
\end{aligned}
\tag{2.7}
$$

Inspection of the above equations shows that the above constitutes a mixed boundary value problem and the most appropriate technique for solution is the finite element method.

B. Method of Analysis

A finite element computer program was assembled for the analytical solution of bolted plates. Descriptions of the finite element method are given in references [9,10], but for completeness, an outline of the mathematical formulation for this case is presented in Appendix A. A listing of the cumputer program and instructions for its use may be found in Appendix B. Appendix C contains user's instructions and a listing of the finite element program modified to include thermal strains.

As in the previous work axial symmetry and isotropic linear elastic material behavior were assumed. However, the computer programs accommodate plates with different material properties in a bolted pair.

The basic concept of the finite element method is that a body may be considered to be an assemblage of individual elements. The body then consists of a finite number of such elements interconnected at a finite number of nodal points or nodal circles. The finite character of the structural connectivity makes it possible to obtain a solution by means of simultaneous algebraic equations. When the problem, as is the case here, is expressed in a cylindrical coordinate system and in the presence of axial symmetry in geometry and load, tangential displacements do not exist, and the three-dimensional annular ring finite element is then reduced to the characteristics of a two-dimensional finite element.

The analysis consists of (a) structural idealization, (b) evaluation of the element properties, and (c) structural analysis of the assemblage of the elements. Items (b) and (c) are covered in the appendices and in the references quoted. The structural idealization and the criteria for acceptable solutions will be described in this chapter.

Fig. 4(a) shows two circular plates in contact under arbitrary axisymmetric loading. The plates are subdivided into a number of annular ring elements which are defined by the corner nodal circles (or node points when represented in a plane) as shown in Figs. 4(b) and 4(c). Unlike the cases described in Chapter I, which have been solved by the Hankel transform method, all plates solved by the finite element method have finite radii. The cross sections of each annular ring element is either a general quadrilateral or triangle. To improve accuracy smaller elements are used in zones where rapid variations in stress are anticipated than in zones of constant stress; thus the different size elements shown in Fig. 4(b). (However, the total number of elements allowable are subject to computer capacity.)

Figure 4(b) shows the two plates in contact for the radial distance X_c and separated beyond it. It is to be noted that the nodal points on the parting line and within the length of contact X_c are common to elements in both plates. The other elements adjacent to the parting line on each plate are separated from their corresponding elements in the mating plate and these elements have no common nodal points. Physically, it is equivalent to the welding together of the two plates in the contact zone. Mathematically, we are imposing the condition that in the contact zone the displacements in the z and r directions be identical for both plates. In the case of bolted plates of equal thickness, i.e. in the presence of symmetry about the parting plane, these conditions apply exactly. Furthermore, because of this symmetry, one needs to analyze only one plate, as shown in Fig. 5(b), with the imposed boundary conditions on the contact zone of zero displacement in the z-direction and freedom to displace in the r-direction. It can also be observed that the solution of two plates with symmetry about the parting plane is equivalent to the solution of one of these plates under the same loading conditions, but resting on a frictionless infinitely rigid plane. Also, under the above conditions the shear stress in the contact zone is identically zero.

In the case of bolted plates of unequal thickness the model includes both plates as shown in Fig. 5(c). This model is an approximation because, in general, two plates of unequal thickness do not have the same displacement in the r-direction on the contact surface. The solution yields, therefore, a shearing stress distribution in the contact zone. The solution, however, should be exactly compatible with the physical model if the frictional forces in the joint prevent sliding.

The critical aspect of the approach used herein is the determination of the largest nodal circle on the parting plane which is common to an element on each plate. This nodal circle defines the contact zone and the radius, R_o, at which separation occurs.

The output of the finite element computer program includes the displacement of each node in the r and z directions and the average

σ_z, σ_r, σ_t and τ_{rz} stresses for each element.

The computation is iterative and the objective is to achieve the lowest possible compressive σ_z stress in the outermost elements bordering the contact zone. Unacceptable solutions are shown in Fig. 6(a) and 6(b). If R_o for a given external load distribution is too small, then the solution will show that the two plates intersect (Fig. 6(a)). On the other hand, if R_o is assumed too large, the solution will show that the outer portion of the contact zone sustains a tensile σ_z stress (Fig. 6(b)). Neither of these two situations is physically feasible. In general, the procedure employed was to commence the iterations with a value for R_o which would yield a tensile σ_z stress in the outer elements adjacent to the contact zone and then move R_o inward. The iteration ended as soon as no tensile σ_z stress was present at the contact zone. For example, for the case shown in Fig. 5(b), if the σ_z stress for the element in the last row and to the left of the last roller is tensile, then the following iteration will proceed without the last roller. Thus, the resolution is one nodal interval. Finer resolution can be obtained by reducing the interval between nodal circles by introducing more elements or shifting the grid locally. The same criteria apply to the model shown in Fig. 5(c).

In the finite element analysis of the Fernlund (3) model, i.e. single plate with external loads at the faces $z = \pm D$ no iteration is required and the rollers shown in Fig. 5(c) would extend to the outer radius of the plate. (Although Fernlund's computations are based on infinite plates, computations show that there is no distinction between infinite plates and plates of radius greater than five times of the outer radius, B, of the load. See Fig. 5(a).

Convergence was tested by subdividing elements further, with nodal points in the coarser grid remaining nodal points in the finer grid. Changing the mesh from 180 elements to 360 elements have shown no improvement in accuracy. Meshes from 180 to 300 elements were used in this analysis. Typical spacings between nodal points were 0.015 inch radially and 0.03 inch in the z-direction.

Chapter III

EXPERIMENTAL METHOD

The objective of the experiment was to determine the extent of contact between two plates when bolted together. Sixteen type 304 stainless steel plates, 4 inches in diameter, were machined to nominal thicknesses of 1/16, 1/8, 3/16 and 1/4 inch, 4 plates for each thickness. After rough machining these plates were stress relieved at 1875°F and ground flat to 0.0002 inch. One side of each plate was then lapped flat to better than one fringe of sodium light (11 micro-inches) in the case of the 1/8, 3/16 and 1/4 inch plates, and to better than two fringes in the case of the 1/16 inch plates. Disregarding scratches, the finish of the lapped surfaces was 5 micro-inches rms. Each plate had a central hole, 0.257 inch in diameter, for a 1/4 - 20 bolt, and two notches and two holes on the periphery (see Fig. 7). Two techniques were employed in determining the area of contact when two of these plates were bolted together. The first technique entailed the following procedure (see Fig. 7):

(a) The plates were cleaned with alcohol and lens tissue.

(b) One plate was placed on the base of the fixture shown in Fig.7, lapped surface up and the two holes on the periphery of the plates engaged with two pins on the fixture. Spacers between the fixture base and plate prevented the pins from extending beyond the top surface of the plate.

(c) A second plate was placed on top of the first plate, lapped surfaces mating. The notches on the two plates were lined up with each other and with notches in the base of the fixture. Thus, rotation of the plates was prevented.

(d) A standard 1/4 - 20 hex-nut with its annular bearing surface (0.42 inch O.D.) lapped flat was engaged on a high strength 1/4 - 20 bolt. The nut was located about two threads away from the head of the bolt and served in lieu of the bolt head. The lapped surface of the nut faced away from the bolt head and since the nut was not sent home against the bolt head, the looseness of fit between nut and bolt offered a degree of self alignment.

(e) The bolt and nut assembly described in (d) above was then inserted through the 1/4 inch central holes of the two plates and a second 1/4 - 20 lapped nut was engaged on the bolt. Thus the two plates were captured by the two 1/4 -20 nuts with the lapped surfaces of the nuts bearing against the plates.

(f) With the torque wrench shown on the right in Fig. 7, the nuts were torqued down to 70 pound-inches of torque to yield a 1100 pound force in the bolt [11].

(g) The position of the keys was changed to engage with only the lower plate and the fixture and a special spanner wrench, as shown in Fig. 7, was engaged with the top plate. The spanner wrench was restrained to move in the horizontal plane and it was set into motion by the screw pressing against the wrench handle.

(h) With the aid of the spanner wrench the upper plate was rotated relative to the lower plate several times approximately $\pm$ 5 degrees.

Thus, the above procedure allowed for the rubbing of one plate relative to its mate while under a bolt force of approximately 1100 lbs. The remaining steps were the disassembly and the measurement of the extent of the contact zone which was defined by the shine due to the rubbing in the contact zone. It is to be noted that the boundaries of the contact zone as measured by the naked eye and by searching for marks of "polished" or "damaged" surface under a 10.5 power magnification are essentially the same.

The above test was performed on 5 pairs of specimen. These were

1. One 0.07 in. plate mated to a 0.65 in. plate
2. One 0.126 in. plate mated to a 0.126 in. plate
3. One 0.191 in. plate mated to a 0.192 in. plate
4. One 0.253 in. plate mated to a 0.256 in. plate
5. One 0.124 in. plate mated to a 0.257 in. plate

The identical tests were repeated for

1. One 0.124 in. plate mated to a 0.126 in. plate; and
2. One 0.191 in. plate mated to a 0.192 in. plate,

but in lieu of the 1/4 - 20 nuts in direct contact with the plates special washers, 1.000 in. O.D., 0.257 in. I.D. and 0.620 in. high, were interposed between the bolt head and nut.

The diameters of the contact zones were measured with a machinist ruler with 100 divisions to the inch and with a Jones and Lamston Vertac 14 Optical Comparator.

The second technique used the same parts and fixture, but it involved autoradiographic measurements.

Four plates, 1/4, 3/16, 1/8 and 1/16 inch thick were sent to Tracerlab, Inc., Waltham, Mass., for electrolytic plating with radioactive silver Ag 110^{M} (half life of 8 months). Each plate was masked except for an area on the lapped face one inch in radius. The plates then received a plating of copper about 5 microinches thick and then approximately a 5 microinch plating of silver containing the radioactive isotope. The resultant activity on each plate was about 2 millicuries.

These plates were then mated to plates of equal thickness (not plated) and assembled in a shielded hood as indicated in steps (a) to (h) above except that in the case of the pair of 1/4 inch plates care was taken not to rotate the plates during and after assembly and in the remaining cases the rotation specified in step (h) was done only once in one direction.

The plates were then disassembled and the radioactive contamination on the plates which were in contact with the radioactive plates measured. The transferred activity was:

1/4 in. plate	approximately	0.05	microcuries
3/16 in. plate	approximately	3.	microcuries
1/8 in. plate	approximately	0.1	microcuries
1/16 in. plate	approximately	0.4	microcuries

It was also observed in handling that the adhesion of the silver on the 3/16 in. plate was poor.

Kodak type R single coated industrial x-ray film was then placed on the contaminated plates under darkroom conditions. The sensitive side of the film was pressed against the radioactive sides of the plates with a uniform load of about five pounds and left for exposure for three days. After three days, the film was removed and developed. The results are shown in Fig. 10.

Chapter IV

RESULTS

A. Pressure Distribution and Radii of Separation from Single Plate and Two Plate Finite Element Models.

Using the finite element procedure described in Chapter II, the midplane stress distribution of single circular plates of thickness 2D, outer radii of 1.54 in., inner radii of 0.1 in., Poisson ratio of 0.3, and loaded by a constant pressure between radii A and B, Fig. 3(f), was computed. Computations were performed for D values of 0.1, 0.1333 and 0.2 in. For each value of D the radius B, which defines the region of the symmetric external load, assumed the values of 0.31, 0.22, 0.16 and 0.13 in. The σ_z stress distribution at the midplane, from the inner radius to the radius at which the above stress is no longer compressive, is shown in Figs. 12, 13 and 14 as a function of radius.

The identical cases were then recomputed, using again the finite element method, in accordance with the two plate model shown in Figs. 4(b) and 5(b). These results are given in Figs. 15, 16 and 17.

Inspection of the above figures show that the two plate model yields a somewhat different stress distribution in the contact zone than the stress distribution approximated from the single plate model, and more significantly, from the heat transfer point of view, the two plate model yields a lower value for the radius of separation, R_o, which results in a reduction in area for heat transfer. Table 1 gives a comparison of the values for R_o obtained from the two models.

It may be observed that the single plate result of Fernlund (Ref. 3, pp. 56, 124) is in fair agreement with the finite element results obtained for the single plate model.

B. Radii of Separation from Experiment and Their Predicted Values from the Two Plate Finite Element Computation.

As described in Chapter III, stainless steel circular plate specimen (Fig. 7) were bolted together, rotated relative to each other with the bolt force acting, and after disassembly the contact area of the joint was determined by measuring the footprints (the shiny, polished areas) on each plate due to the plates rubbing against each other. Photographs of these footprints are shown in Fig. 8. Fig. 9 also shows a typical footprint of the annular bearing surface of the 1/4 - 20 nut against a plate. All plates tested were of 304 stainless steel, 4 inch O.D., .257 I.D., and the nominal thicknesses of the plates were 1/16, 1/8, 3/16 and 1/4 inch. In addition to the plates fastened with standard nuts which gave a loading circle of radius B (Fig. 5) of 0.211 inch, plates fastened by the special nuts described in Chpater III for which B was 0.5 inch were also tested.

Figure 10 shows the results of the autoradiographic tests described in Chapter III. For all plate pairs tested, i.e. 1/16, 1/8, 3/16 and 1/4 inch nominal, the value of B was 0.211 inch.

The pressure distributions and radii of separation for all the above test cases were computed independently by the two plate model finite element analysis. Table 2 gives the test and analytical results for all test cases. The test results are an average of all measurements (minimum of six readings). A description of the analyses follows.

Figure 18 shows the results of a two plate and a single plate model analysis for the 0.253 inch bolted test specimen. For Figure 19 the external pressure distribution between radii A and B is triangular. (The total force, however, is equal to the force exerted in the case of uniform pressure.) In one case, the peak external pressure is at A, Fig. 20(a), and in the other case at B, Fig. 20(b). Results of another computation which assumed a uniform displacement of 50 microinches under each nut is shown in Fig. 21. It is interesting to note that the point of separation obtained by using the two plate model for all variations of loading given above occurs in the range of r/A values of 2.73 to 2.93 while the two plate model yields separation at a value for r/A of 3.5. The computed deflections under the nuts are given in Fig. 22.

The finite element analysis results for the 0.191 in. plate pair specimen are given in Fig. 23. Figures 24 and 25 show the computed pressure distribution and deflection patterns in the joint, respectively, for the 1/8 in. plate pair. In order to investigate the possible influence misalignments of the spanner wrench, i.e. vertical forces or restraints exerted at edge of plate, may have on the results of the experiment, the extreme case of fixing the outer edges of the plate as shown in Fig. 20(c) was considered. As Fig. 24 shows, within the resolution of the finite element grid size, the effect is negligible. This model, Fig. 20(c), and result also indicate that the influence of additional fasteners 2 inches away would not have an influence on the contact zone for the geometry considered. (However, if the distance between bolts is considerably reduced, then the contact area should increase.) The computed results for the 1/16 inch plate pair is given in Fig. 26.

Figure 27 gives the finite element analysis results for the asymmetric case of a 1/8 in. plate bolted to a 1/4 in. plate. The model shown in Fig. 5(c) was used and as discussed in Chapter II, this model is strictly valid only if the friction in the joint prevents sliding between the plates. Nevertheless, the percent discrepancy between the computed value and tested value (see Table 2) falls within the range of the symmetric cases analyzed and tested.

In summary, the results obtained from the two plate finite element model and from experiment are in good agreement (Fig. 28).

Chapter V

APPLICATION

An application of the above results for the evaluation of the thermal contact conductance, h_c, and the determination of the heat transferred in a specific, but typical, lap joint section is illustrated in this chapter.

An aluminum lap joint in a vacuum environment, the relevant section and boundary conditions as shown in Fig. 29, was analyzed by means of a nodal analysis. The plate thickness was 0.1 in. and the hole diameter, 2A, was 0.2 in. The bearing surface of the bolt, 2B, was 0.26 in. in diameter. Because of the high conductivity and small thickness of the plates, no z dependence (see Fig. 29) was assumed for the temperature in the main body of the plate. However, heat flow in the z-direction in the nodes above and below the contact zone is considered. Qualitatively, the heat flow in the joint proceeds in the x-y plane from the left end (Fig. 29) toward the 0.2 in. diameter hole. In the vicinity of the hole, a macroscopic constriction for heat flow is encountered because the flow is being channeled toward the small contact zone. The flow of heat then encounters the microscopic constrictions at the contacting asperities (which determine h_c) in the contact zone; spreads out in the x-y directions in the second plate; and continues to the right edge of the lap joint.

The material properties assumed were (refer to equation 1.1):

$H = 150{,}000$ psi

$k = 100$ Btu/hr-°F-ft $\quad (k_1 = k_2 = 100)$

$\sigma = 5.9 \times 10^{-6}$ ft. $\quad (\sigma_1 = \sigma_2 = 50 \times 10^{-6}$ in.)

$\tan\theta = 0.1$

Assuming further, a uniform load of 46,500 psi on the loading surface (#10 screw; 1000 lb. bolt force) and referring to Fig. 15, curve $\frac{B}{A} = 1.3$, the following interface stresses, σ_z, contact heat transfer coefficient, h_c, and conductance, (area)$\cdot$(h_c), were obtained as a function of inner and outer radii. (These radii define increments of area, the sum of which define one quarter of the contact zone.):

r_{outer} inch	r_{inner} inch	σ_z psi	h_c Btu/hr-°F-ft^2	Area x h_c Btu/hr-°F-ft^2
.13	.1	27,900	446,000	16.6
.16	.13	14,000	223,000	10.6
.175	.16	3,950	63,100	1.7

The conductance between nodal points were then computed and with the aid of the steady state heat transfer program listed in Appendix D, the nodal temperatures for the conditions given in Fig. 29 were computed. The heat transferred from the edge maintained at 20°F to the edge at 0°F (Fig. 29) for this case was 2.88 Btu/hour. The same computation was repeated for the case of a bearing surface between the plate and the bolt (2B) of 0.44 in. in diameter, but the bolt force was left unchanged. The heat transferred from the 20°F edge to the 0°F edge in this case was 3.15 Btu/hour. In the absence of the joint the heat transfer along an equivalent 7 inch length of solid aluminum would have been 3.58 Btu/hour. This data shows that the thermal resistance of the contact zone (not entire 7 inch lap joint) was decreased from 1.52 to 0.92 °F-hr/Btu by the increase of the effective bolt head diameter from .26 to .44 in. It should be observed that the change in thermal resistance of the joint is primarily due to the increase in contact area and the resulting decrease in macroscopic constriction resistance at the hole. Also, the heat flux in this example is mainly controlled by the 7 inch length and 0.1 inch thickness rather than the joint resistance. This emphasizes the importance of a balanced thermal design.

For large heat fluxes where thermal strains may have an influence on the radii of separation, the finite element program given in Appendix C may be used. Also, in a non-vacuum environment the effect of the interstitial fluid is added in two ways. Firstly, equation (1.6) is applied to account for the presence of interstitial fluid in the contact zone, and secondly, conduction across the gaps between the plates and convection from the plates is considered. (Radiation heat transfer, if applicable, should also be included.)

Chapter VI

CONCLUSIONS

The finite element technique used in this work for the analysis of the pressure distribution and deformation of smooth and flat bolted plates under conditions of axial symmetry predicts contact areas in joints considerably lower than reported previously in the literature. These results were verified experimentally. The discrepancy between the previously reported results and the results reported here is due to the simplifying assumption made by earlier researchers that a joint can be modeled as a single plate.

The computer programs listed in the appendices will also accommodate joints made up of plates of dissimilar materials and the presence of thermal gradients.

Of the eleven tests performed, only one (case 3, autoradiographic) yielded inconsistent results. (This data point could probably be ignored because of the poor adhesion of the plating material which manifested itself by the high radioactive contamination count during test.)

The finite element analysis performed for the test specimen show that the gap between the 1/4 inch bolted steel specimen is 98.6 microinches at the outer radius of the plate of 2 inches, and 1/32 of an inch away from the radius of separation (0.35 in.), the gap is only 3 microinches for the test load. This data indicates the difficulties previous workers have encountered in their experiments. (This also explains the oval shape of several of the footprints.) Furthermore, this data shows that the effects of surface roughness and the lack of flatness could have a significant effect on the size of contour area.

An application of the above work to a heat transfer problem is illustrated in Chapter V.

REFERENCES

1. Cooper, M.G., Mikic, B.B., and Yovanovich, M.M., "Thermal Contact Conductance," International Journal of Heat and Mass Transfer, Vol. 12, No. 3 (March 1969), 279-300.

2. Roetscher, F., Die Maschinenelemente, Erster Band. Berlin: Julius Springer, 1927.

3. Fernlund, I., "A Method to Calculate the Pressure Between Bolted or Riveted Plates," Transactions of Chalmers, University of Technology. Gothenburg, Sweden, No. 245, 1961.

4. Greenwood, J.A., "The Elastic Stresses Produced in the Mid-Plane of a Slab by Pressure Applied Symmetrically at its Surface," Proc. Camb. Phil. Soc. Cambridge, England, Vol. 60, 1964, 159-169.

5. Lardner, T.J., "Stresses in a Thick Place with Axially Symmetric Loading," J. of Applied Mechanics, Trans. ASME, Series E, Vol. 32, June 1965 (458-459).

6. Sneddon, I.N., "The Elastic Stresses Produced in a Thick Plate by the Application of Pressure to its Free Surfaces," Proc. Camb. Phil. Soc. Cambridge, England, Vol. 42, 1946 (260-271).

7. Bradley, T.L., "Stress Analysis for Thermal Contact Resistance Across Bolted Joints," M.S. Thesis, Massachusetts Institute of Technology, Mechanical Engineering Department, Cambridge, Mass. August 1968.

8. Louisiana State University, Division of Engineering Research, The Thermal Conductance of Bolted Joints. NASA Grant No. 19-001-035, C.A. Whitehusrt, Dir. Baton Rouge: LSU Div. of Eng. Res., May 1968.

9. Zienkiewicz, O.C., The Finite Element Method in Structural and Continuum Mechanics. London: McGraw-Hill Publishing Co., Ltd., 1967.

10. Przemieniecki, J.S., Theory of Matrix Structural Analysis. New York: McGraw-Hill Book Co., 1968.

11. Cobb, B.J., "Preloading of Bolts," Product Engineering, August 19, 1963 (62-66).

12. Wilson, E.L., "Structural Analysis of Axisymmetric Solids," AIAA Journal, Vol. 3, No. 12, (Dec. 1965), 2269-2274.

13. Jones, R.M. and Crose, J.G., SAAS II Finite Element Stress Analysis of Axisymmetric Solids. United States Air Force Report No. SAMSO-TR-68-455 (Sept. 1968).

14. Christian, J.T. and others, FEAST-1 and FEAST-3 Programs. Cambridge: Computer Program Library, M.I.T. Department of Civil Engineering, Soil Mechanics Division, 1969.

15. Clough, R.W., "The Finite Element Method in Structural Mechanics," Ch. 7, Stress Analysis, Zienkiewicz, O.C. and Holister, G.S., editors. London: John Wiley and Sons, Ltd., 1965.

APPENDIX A

FINITE ELEMENT ANALYSIS OF AXISYMMETRIC SOLIDS

The finite element method and the equations which govern the stresses and displacements in axisymmetric solids is given in the literature [9,10,12,13,15] and the procedure will be briefly summarized in this appendix.

The procedure for the standard stiffness analysis method is as follows [15]:

(a) The internal displacements, v, are expressed as

$$\{v(r,z)\} = [M(r,z)]\,\{\alpha\} \tag{A.1}$$

where M is a displacement function and α are the generalized coordinates representing the amplitudes of the displacement functions.

(b) The nodal displacements v_i are expressed in terms of the generalized coordinates

$$\{v_i\} = [A]\,\{\alpha\} \tag{A.2}$$

where A is obtained by substituting the coordinates of the nodal points into M.

(c) The generalized coordinates are expressed in terms of the nodal displacements

$$\{\alpha\} = [A]^{-1}\,\{v_i\} \tag{A.3}$$

(d) The element strains, ε, are evaluated

$$\{\varepsilon\} = [B(r,z)]\ \{\alpha\} \tag{A.4}$$

where B is obtained from the appropriate differentiation of M.

(e) The element stresses are expressed in terms of the stress-strain relation D

$$\{\sigma(r,z)\} = [D]\ \{\varepsilon\} = [D]\ [B]\ \{\alpha\} \tag{A.5}$$

(f) Assuming a virtual strain $\bar{\varepsilon}$ and a generalized virtual coordinate displacement $\bar{\alpha}$ the internal virtual work, W_i, in the differential volumn, dV, is given by

$$dW_i = \{\bar{\varepsilon}\}^T\ \{\sigma\}\ dV = \{\bar{\alpha}\}^T\ [B]^T\ [D]\ [B]\ \{\alpha\}\ dV \tag{A.6}$$

and the total internal virtual work is

$$W_i = \{\bar{\alpha}\}^T\ [\int_{Vol} [B]^T\ [D]\ [B]\ \ dV\ ]\ \ \alpha \tag{A.7}$$

(g) The external work, W_e. associated with the generalized displacement $\bar{\alpha}$ is

$$W_e = \{\alpha\}^T\ \{\beta\} \tag{A.8}$$

where β are generalized forces corresponding with the displacements α.

(h) After equating W_i and W_e and setting the $\bar{\alpha}$ displacement to unity

$$\{\beta\} = [\int_{Vol} [B]^T [D] [B]]\, \alpha = [\bar{k}] \{\alpha\} \qquad (A.9)$$

where $$[\bar{k}] = \int_{Vol} [B]^T [D] [B] \; dV \qquad (A.10)$$

and which transforms to the nodal point surfaces

$$k = [A^{-1}] [\bar{k}] [A^{-1}] \qquad (A.11)$$

(i) The stiffness matrix for the complete system is then

$$[K] = \sum_{m=1}^{n} [k]_m \qquad (A.12)$$

where n equals the number of elements and the equilibrium relationship becomes

$$\{Q\} = [K] \{v_i\} \qquad (A.13)$$

where

$$\{Q\} = \sum_{m=1}^{n} \{R\}_m \qquad (A.14)$$

$$\{R\} = \int_{Area} [A^{-1}]^T [M]^T \{P\}_m \; dA \qquad (A.15)$$

and P are the surface forces.

The above procedure applies with minor modification to problems with thermal and body force loading.

The expression

$$\{Q\} = [K] \{v_i\} \tag{A.16}$$

represents the realtionship between all nodal point forces and all nodal point displacements. Mixed boundary conditions are considered by rewriting this equation in the partitioned form

$$\left\{ \begin{array}{c} Q_a \\ \hline Q_b \end{array} \right\} = \left[\begin{array}{c|c} K_{aa} & K_{ab} \\ \hline K_{ba} & K_{bb} \end{array} \right] \left\{ \begin{array}{c} u_a \\ \hline u_b \end{array} \right\} \tag{A.17}$$

where $v_i = u$.

The first part of the partitioned equation can be written as

$$\{Q_a\} = [K_{aa}] \{u_a\} + [K_{ab}] \{u_b\} \tag{A.18}$$

and then expressed in the reduced form

$$\{Q^*\} = [K_{aa}] \{u_a\} \tag{A.19}$$

where

$$\{Q^*\} = \{Q_a\} - [K_{ab}] \{u_b\} \tag{A.20}$$

The matrix equation (A.19) is solved for the nodal point displacements by standard techniques. Once the displacement are known the strains are evaluated from the strain displacement relationship and the stresses in turn are evaluated from the stress strain relations.

Both triangular and quadrilateral elements are used. The displacements in the r-z plane in the element are assumed to be of the form

$$
\begin{aligned}
v_r &= \alpha_1 + \alpha_2 r + \alpha_3 z \\
v_z &= \alpha_4 + \alpha_5 r + \alpha_6 z
\end{aligned}
\tag{A.21}
$$

This linear displacement field assures continuity between elements since lines which are initially straight remain straight in their displaced position. Six equilibrium equations are developed for each triangular element.

A quadrilateral element is composed of four triangular elements and ten equilibrium equations correspond to each element.

APPENDIX B

FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF ISOTROPIC ELASTIC AXISYMMETRIC PLATES (ref. 13,14)

Input Instructions:

Card Sequence	Item	Format	Columns
1	Title	18A4	1-72
2	Total number of nodal points	I5	1-5
	Total number of elements	I5	6-10
	Total number of materials	I5	11-15
	Normalizing stress (NORM)	I5	16-20
	Number of pressure cards	I5	21-25

(If NORM = 0, put in value of E in material card;
if NORM = 1, put in value $E/\sigma_{vertical}$;
if NORM = -1, put in value $E/\sigma_{octahedral}$;
NOTE: Use NORM = 0 for this application.)

Card Sequence	Item	Format	Columns
3	(Material property cards - one set of (a) and (b) for each material)		
	(a) 1st card		
	Material No.	I5	1-5
	Initial σ_z stress	F10.0	6-15
	Initial σ_r stress	F10.0	16-25
	(b) Second Card		
	E	F10.0	1-10
	ν	F10.0	11-20

Card Sequence	Item	Format	Column
4	Nodal point information (One for each node)	2I5,4F10.0	
	Node number		1-5
	CODE		6-10
	r-coordinate		11-20
	z-coordinate		21-30
	XR		31-40
	XZ		41-50

If the number in column 10 is

		Condition
0	XR is the specified R-load and XZ is the specified Z-load	free
1	XR is the specified R-displacement and XZ is the specified Z-load	
2	XR is the specified R-load and XZ is the specified Z-displacement.	
3	XR is the specified R-displacement and XZ is the specified Z-displacement.	fixed

Remarks

The following restrictions are placed on the size of problems which can be handled by the program.

Item	Maximum Number
Nodal Points	450
Elements	450
Materials	25
Boundary Pressure Cards	200

All loads are considered to be total forces acting on a one radian segment. Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the defined nodal points. The boundary code (column 10), XR and XZ are set equal to zero.

If the number in columns 6-10 of the nodal point cards is other than 0, 1, 2 or 3, it is interpreted as the magnitude of an angle in degrees. The terms in columns 31-50 of the nodal point card are then interpreted as follows:

XR is the specified load in the s-direction
XZ is the specified displacement in the n-direction

The angle must always be input as a negative angle and may range from -.001 to -180 degrees. Hence, +1.0 degree is the same as -179.0 degrees. The displacements of these nodal points which are printed by the program are

u_r = the displacement in the s-direction
u_z = the displacement in the n-direction

Element cards must be in element number sequence. If element cards are omitted, the program automatically generates the omitted information by incrementing by one the preceding I, J, K and L. The material identification code for the generated cards is set equal to the value given on the last card. The last element card must always be supplied.

Triangular elements are also permissible; they are identified by repeating the last nodal point number (i.e. I, J, K, K).

One card for each boundary element which is subjected to a normal pressure is required. The boundary element must be on the left as one progresses from I to J. Surface tensile force is input as a negative pressure.

Printed output includes:

1. Reprint of input data.
2. Nodal point displacement
3. Stresses at the center of each element.

Nodal point numbers must be entered counterclockwise around the element when coding element data.

The maximum difference between the nodal point numbers on an element must be less than 25. However, on a nodal diagram elements and nodes need not be numbered sequentially.

Listing:

```
C     ********************************************************************
C      FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF ISOTROPIC ELASTIC
C      AXYSYMMETRIC PLATES  REF  FEAST 1,3 SAAS 2
C     ********************************************************************
C
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
     1 DEPTH(25),E(10,25),SIG(7),R(450),Z(450),UR(450),
     2 UZ(450),STOTAL(450,4),KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 FE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
      COMMON/PRESS/ IBC(200),JBC(200),PR(200),NUMPC
      DATA STRS /'****'/
C     ******************************************************************
C     READ AND PRINT CONTROL INFORMATION
C     ******************************************************************
   50 READ (5,1000,END=950) HED
      WRITE (6,2000) HED
C
      READ(5,1001) NUMNP,NUMEL,NUMMAT,NORM,NUMPC
      WRITE (6,2006) NUMNP,NUMEL
      IF (NORM) 65,65,66
   66 WRITE (6,2041)
C     ********************************************************************
C     READ AND PRINT MATERIAL PROPERTIES
C     ********************************************************************
   65 CONTINUE
C
      DO 80 M=1,NUMMAT
      READ (5,1002) MTYPE,          SIGIZ(MTYPE),SIGIR(MTYPE)
      WRITE (6,2007) MTYPE,SIGIZ(MTYPE),SIGIR(MTYPE)
      READ (5,1003) (E(J,MTYPE),J=1,2)
```

```
      WRITE (6,2051) (E(J,MTYPE),J=1,2)                                 FENT0037
   80 CONTINUE                                                          FENT0038
C     ************************************************************      FENT0039
C     READ AND PRINT NODAL POINT DATA                                   FENT0040
C     ************************************************************      FENT0041
  100 WRITE (6,2013)                                                    FENT0042
      L=0                                                               FENT0043
  105 READ (5,1006) N,ICODE(N),R(N),Z(N),UR(N),UZ(N)                    FENT0044
  106 NL=L+1                                                            FENT0045
      IF (L.EQ.0) GO TO 110                                             FENT0046
      ZX=N-L                                                            FENT0047
      DR=(R(N)-R(L))/ZX                                                 FENT0048
      DZ=(Z(N)-Z(L))/ZX                                                 FENT0049
  110 L=L+1                                                             FENT0050
      IF (N-L) 113,112,111                                              FENT0051
  111 ICODE(L)=0                                                        FENT0052
      R(L)=R(L-1)+DR                                                    FENT0053
      Z(L)=Z(L-1)+DZ                                                    FENT0054
      UR(L)=0.0                                                         FENT0055
      UZ(L)=0.0                                                         FENT0056
      GO TO 110                                                         FENT0057
  112 WRITE (6,2014) (K,ICODE(K),R(K),Z(K),UR(K),UZ(K),K=NL,N)          FENT0058
      IF (NUMNP-N) 113,120,105                                          FENT0059
  113 WRITE (6,2015) N                                                  FENT0060
      GO TO 900                                                         FENT0061
C     ************************************************************      FENT0062
C     READ AND PRINT ELEMENT PROPERTIES                                 FENT0063
C     ************************************************************      FENT0064
  120 WRITE (6,2016)                                                    FENT0065
      N=0                                                               FENT0066
  130 READ (5,1007) M,(IX(M,I),I=1,5)                                   FENT0067
  140 N=N+1                                                             FENT0068
      IF (M-N) 170,170,150                                              FENT0069
  150 IX(N,1)=IX(N-1,1)+1                                               FENT0070
      IX(N,2)=IX(N-1,2)+1                                               FENT0071
      IX(N,3)=IX(N-1,3)+1                                               FENT0072
```

```
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
  170 WRITE (6,2017) N,(IX(N,I),I=1,5)
      IF (M-N) 180,180,140
  180 IF (NUMEL-N) 300,300,130
C     *****************************************************************
C     READ AND PRINT  THE PRESSURE CARDS
C     *****************************************************************
  300 IF(NUMPC) 290,210,290
  290 WRITE(6,9000)
      DO 200 L=1,NUMPC
      READ(5,9001) IBC(L),JBC(L),PR(L)
  200 WRITE(6,9002) IBC(L),JBC(L),PR(L)
  210 CONTINUE
C     ***************************************************************
C     DETERMINE BAND WIDTH
C     ***************************************************************
      J=0
      DO 340 N=1,NUMEL
      DO 340 I=1,4
      DO 325 L=1,4
      KK=IX(N,I)-IX(N,L)
      IF (KK.LT.0) KK=-KK
      IF  (KK.GT.J)  J=KK
  325 CONTINUE
  340 CONTINUE
      MBAND=2*J+2
C     ***************************************************************
C     SOLVE FOR DISPLACEMENTS AND STRESSES
C     ***************************************************************
      KSW=0
      CALL STIFF
      IF (KSW.NE.0) GO TO 900
C
      CALL BANSCL
      WRITE(6,2052)
```

```
      WRITE (6,2025) (N,B     (2*N-1),B     (2*N),N=1,NUMNP)
C
  450 CALL STRESS(SPLOT)
C     ****************************************************************
C     PROCESS ALL DECKS EVEN IF ERROR
C     ****************************************************************
      GO TO 910
  900 WRITE (6,4000)
  910 WRITE (6,4001) HED
C
  920 READ (5,1000) CHK
      IF (CHK.NE.STRS) GO TO 920
      GO TO 50
  950 CONTINUE
      WRITE (6,4002)
      CALL EXIT
C     ******************************************************************
C     ******************************************************************
 1000 FORMAT (18A4)
 1001 FORMAT (12I5)
 1002 FORMAT ( I5,2F10.0)
 1003 FORMAT(2F10.0)
 1004 FORMAT (2F10.0)
 1005 FORMAT (3F10.0)
 1006 FORMAT (2I5,4F10.0)
 1007 FORMAT (6I5)
C     ****************************************************************
 2000 FORMAT (1H1,20A4)
 2006 FORMAT (28HONUMBER OF NODAL POINTS----- I3/
     1 28H NUMBER OF ELEMENTS--------- I3)
 2007 FORMAT (20HOMATERIAL NUMBER---- I3/
     1 25H INITIAL VERTICAL STRESS= F10.3 ,5X,
     2 26HINITIAL HORIZONTAL STRESS= F10.3)
 2013 FORMAT (12H1NODAL POINT ,4X, 4HTYPE ,4X, 10HR-ORDINATE ,4X,
     1 10HZ-ORDINATE ,10X, 6HR-LOAD ,10X, 6HZ-LOAD )
 2014 FORMAT (I12,I8,2F14.3,2E16.5)
```

```
 2015 FORMAT (26HONODAL POINT CARD ERROR N= I5)
 2016 FORMAT (49H1ELEMENT NO.       I     J     K     L    MATERIAL  )
 2017 FORMAT (1I13,4I6,1I12)
 2025 FORMAT (12HONODAL POINT ,6X, 14HR-DISPLACEMENT ,6X, 14HZ-DISPLACEM
     1ENT / (I12,1P2D20.7))
 2041 FORMAT (76HOMODULUS AND YIELD STRESS NORMALIZED WITH RESPECT TO IN
     1ITIAL VERTICAL STRESS )
 2051 FORMAT(1H0,10X,'E',8X,'NU',/,3X,F11.1,F10.4/)
 2052 FORMAT(1H1)
C     ******************************************************************
 3003 FORMAT (16I5)
C     ******************************************************************
 4000 FORMAT (//// ' ABNORMAL TERMINATION')
 4001 FORMAT (//// ' END OF PROBLEM ' 20A4)
 4002 FORMAT (////' END OF JOB')
C     ******************************************************************
 9000 FORMAT(29HOPRESSURE BOUNDARY CONDITIONS/ 24H     I     J    PRESSU
     1RE )
 9001 FORMAT(2I5,F10.0)
 9002 FORMAT(2I6,F12.3)
      END
      SUBROUTINE STIFF
C
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON     STTCP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
     1 DEPTH(25),E(10,25),SIG(7),R(450),Z(450),UR(450),
     2 UZ(450),STOTAL(450,4),KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 EE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
      COMMON/PRESS/ IBC(200),JBC(200),PR(200),NUMPC
      DIMENSION CODE(450)
C     ******************************************************************
```

```
C     INITIALIZATION                                                    FENT0181
C     ******************************************************************FENT0182
      NB=27                                                             FENT0183
      ND=2*NB                                                           FENT0184
      ND2=2*NUMNP                                                       FENT0185
      DO 50 N=1,ND2                                                     FENT0186
      B(N)=0.0                                                          FENT0187
      DO 50 M=1,ND                                                      FENT0188
   50 A(N,M)=0.0                                                        FENT0189
C     ******************************************************************FENT0190
C     FORM STIFFNESS MATRIX                                             FENT0191
C     ******************************************************************FENT0192
      DO 210 N=1,NUMEL                                                  FENT0193
C                                                                       FENT0194
C                                                                       FENT0195
   90 CALL QUAD(N,VOL)                                                  FENT0196
      IF (VOL) 142,142,144                                              FENT0197
  142 WRITE (6,2003) N                                                  FENT0198
      KSW=1                                                             FENT0199
      GO TO 210                                                         FENT0200
C                                                                       FENT0201
  144 IF (IX(N,3)-IX(N,4)) 145,165,145                                  FENT0202
  145 DO 150 II=1,9                                                     FENT0203
      CC=S(II,10)/S(10,10)                                              FENT0204
      DO 150 JJ=1,9                                                     FENT0205
  150 S(II,JJ)=S(II,JJ)-CC*S(10,JJ)                                     FENT0206
C                                                                       FENT0207
      DO 160 II=1,8                                                     FENT0208
      CC=S(II,9)/S(9,9)                                                 FENT0209
      DO 160 JJ=1,8                                                     FENT0210
  160 S(II,JJ)=S(II,JJ)-CC*S(9,JJ)                                      FENT0211
C                                                                       FENT0212
C     ADD ELEMENT STIFFNESS TO TOTAL STIFFNESS                          FENT0213
C                                                                       FENT0214
  165 DO 166 I=1,4                                                      FENT0215
  166 LM(I)=2*IX(N,I)-2                                                 FENT0216
```

```
C                                                                       FENT0217
      DO 200 I=1,4                                                      FENT0218
      DO 200 K=1,2                                                      FENT0219
      II=LM(I)+K                                                        FENT0220
      KK=2*I-2+K                                                        FENT0221
      DO 200 J=1,4                                                      FENT0222
      DO 200 L=1,2                                                      FENT0223
      JJ=LM(J)+L-II+1                                                   FENT0224
      LL=2*J-2+L                                                        FENT0225
      IF (JJ) 200,200,175                                               FENT0226
  175 IF (ND-JJ) 180,195,195                                            FENT0227
  180 WRITE (6,2004) N                                                  FENT0228
      KSW=1                                                             FENT0229
      GO TO 210                                                         FENT0230
  195 A(II,JJ)=A(II,JJ)+S(KK,LL)                                        FENT0231
  200 CONTINUE                                                          FENT0232
  210 CONTINUE                                                          FENT0233
      IF(KSW.EQ.1) GO TO 500                                            FENT0234
C                                                                       FENT0235
C     ADD CONCENTRATED FORCES                                           FENT0236
C                                                                       FENT0237
      DO 250 N=1,NUMNP                                                  FENT0238
      K=2*N                                                             FENT0239
      B(K)=B(K)+UZ(N)                                                   FENT0240
      B(K-1)=B(K-1)+UR(N)                                               FENT0241
  250 CONTINUE                                                          FENT0242
C                                                                       FENT0243
C                                                                       FENT0244
C     PRESSURE BOUNDARY CONDITIONS                                      FENT0245
C                                                                       FENT0246
      IF(NUMPC) 260,310,260                                             FENT0247
  260 DO 300 L=1,NUMPC                                                  FENT0248
      I=IBC(L)                                                          FENT0249
      J=JBC(L)                                                          FENT0250
      CODE(I)=ICODE(I)                                                  FENT0251
      CODE(J)=ICODE(J)                                                  FENT0252
```

```
      PP=PR(L)/6.                                                       FENT0253
      DZ=(Z(I)-Z(J))*PP                                                 FENT0254
      DR=(R(J)-R(I))*PP                                                 FENT0255
      RX=2.0*R(I)+R(J)                                                  FENT0256
      ZX=R(I)+2.0*R(J)                                                  FENT0257
  264 II=2*I                                                            FENT0258
      JJ=2*J                                                            FENT0259
  270 SINA=0.0                                                          FENT0260
      COSA=1.0                                                          FENT0261
      IF(CODE(I)) 271,272,272                                           FENT0262
  271 SINA=DSIN(CODE(I))                                                FENT0263
      COSA=DCOS(CODE(I))                                                FENT0264
  272 B(II-1)=B(II-1)+RX*(COSA*DZ+SINA*DR)                              FENT0265
      B(II)=B(II)-RX*(SINA*DZ-COSA*DR)                                  FENT0266
  290 SINA=0.0                                                          FENT0267
      COSA=1.0                                                          FENT0268
      IF(CODE(J)) 291,292,292                                           FENT0269
  291 SINA=DSIN(CODE(I))                                                FENT0270
      COSA=DCOS(CODE(I))                                                FENT0271
  292 B(JJ-1)=B(JJ-1)+ZX*(COSA*DZ+SINA*DR)                              FENT0272
      B(JJ)=B(JJ)-ZX*(SINA*DZ-COSA*DR)                                  FENT0273
  300 CONTINUE                                                          FENT0274
  310 CONTINUE                                                          FENT0275
C     DISPLACEMENT B.C.                                                 FENT0276
C                                                                       FENT0277
      DO 400 M=1,NUMNP                                                  FENT0278
      U=UR(M)                                                           FENT0279
      N=2*M-1                                                           FENT0280
      KX=ICODE(M)+1                                                     FENT0281
      GO TO (400,370,390,380),KX                                        FENT0282
  370 CALL MODIFY(N,U,ND2)                                              FENT0283
      GO TO 400                                                         FENT0284
  380 CALL MODIFY(N,U,ND2)                                              FENT0285
  390 U=UZ(M)                                                           FENT0286
      N=N+1                                                             FENT0287
      CALL MODIFY(N,U,ND2)                                              FENT0288
```

```
  400 CONTINUE
C
  500 RETURN
C     ******************************************************************
 2003 FORMAT (26HONEGATIVE AREA ELEMENT NO. I4)
 2004 FORMAT (29HOBAND WIDTH EXCEEDS ALLOWABLE I4)
C     ******************************************************************
      END
      SUBROUTINE QUAD(N,VOL)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON      STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
     1 DEPTH(25),E(10,25),SIG(7),R(450),Z(450),UR(450),
     2 UZ(450),STOTAL(450,4),KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 FE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
C     ******************************************************************
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
C
      I1=1
      I2=2
      I3=3
      I4=4
      I5=5
C     ********************************************************************
C     DETERMINE ELASTIC CONSTANTS AND STRESS-STRAIN RELATIONSHIP
C     ********************************************************************
C
      CALL MPROP(N)
```

```
C                                                                       FENT0325
C     ***************************************************************  FENT0326
C     FORM QUADRILATERAL STIFFNESS MATRIX                               FENT0327
C     ***************************************************************  FENT0328
  210 RRR(5)=(R(I)+R(J)+R(K)+R(L))/4.0                                  FENT0329
      ZZZ(5)=(Z(I)+Z(J)+Z(K)+Z(L))/4.0                                  FENT0330
      DO 94 M=1,4                                                       FENT0331
      MM=IX(N,M)                                                        FENT0332
      IF(R(MM).EQ.0..AND.ICODE(MM).EQ.0) ICODE(MM)=1                    FENT0333
   93 RRR(M)=R(MM)                                                      FENT0334
   94 ZZZ(M)=Z(MM)                                                      FENT0335
C                                                                       FENT0336
      DO 100 II=1,10                                                    FENT0337
      DO 95 JJ=1,6                                                      FENT0338
   95 HH(JJ,II)=0.0                                                     FENT0339
      DO 100 JJ=1,10                                                    FENT0340
  100 S(II,JJ)=0.0                                                      FENT0341
      IF (K-L) 125,120,125                                              FENT0342
  120 CALL TRISTF(I1,I2,I3)                                             FENT0343
      RRR(5)=(RRR(1)+RRR(2)+RRR(3))/3.0                                 FENT0344
      ZZZ(5)=(ZZZ(1)+ZZZ(2)+ZZZ(3))/3.0                                 FENT0345
      VOL=XI(1)                                                         FENT0346
      GO TO 160                                                         FENT0347
  125 VOL=0.0                                                           FENT0348
      CALL TRISTF(I4,I1,I5)                                             FENT0349
      IF(XI(1).EQ.0.) WRITE(6,2000) N                                   FENT0350
      VOL=VOL+XI(1)                                                     FENT0351
      CALL TRISTF(I1,I2,I5)                                             FENT0352
      IF(XI(1).EQ.0) WRITE(6,2000) N                                    FENT0353
      VOL=VOL+XI(1)                                                     FENT0354
      CALL TRISTF(I3,I4,I5)                                             FENT0355
      IF(XI(1).EQ.0) WRITE(6,2000) N                                    FENT0356
      VOL=VOL+XI(1)                                                     FENT0357
      CALL TRISTF(I2,I3,I5)                                             FENT0358
      IF(XI(1).EQ.0) WRITE(6,2000) N                                    FENT0359
      VOL=VOL+XI(1)                                                     FENT0360
```

```
C
      DO 140 II=1,6
      DO 140 JJ=1,10
  140 HH(II,JJ)=HH(II,JJ)/4.0
C
C
  160 RETURN
C     ****
 2000 FORMAT (' ZERO AREA    ELEMENT',I5)
      END
      SUBROUTINE TRISTF(II,JJ,KK)
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
     1 DEPTH(25),E(10,25),SIG(7),R(450),Z(450),UR(450),
     2 UZ(450),STOTAL(450,4),KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 EE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
C     *********************************************************************
C     INITIALIZATION
C
      LM(1)=II
      LM(2)=JJ
      LM(3)=KK
C
      RR(1)=RRR(II)
      RR(2)=RRR(JJ)
      RR(3)=RRR(KK)
      RR(4)=RRR(II)
      ZZ(1)=ZZZ(II)
      ZZ(2)=ZZZ(JJ)
      ZZ(3)=ZZZ(KK)
      ZZ(4)=ZZZ(II)
```

```
C
   85 DO 100 I=1,6
      DO 90 J=1,10
      F(I,J)=0.0
   90 H(I,J)=0.0
      DO 100 J=1,6
  100 D(I,J)=0.0
C
C     FORM INTEGRAL (G)T*(C)*(G)
      CALL INTER(XI,RR,ZZ)
C
      D(2,6)=XI(1)*(C(1,2)+C(2,3))
      D(3,5)=XI(1)*C(4,4)
      D(5,5)=D(3,5)
      D(6,6)=XI(1)*C(2,2)
      D(1,1)=XI(3)*C(3,3)
      D(1,2)=XI(2)*(C(1,3)+C(3,3))
      D(1,3)=XI(5)*C(3,3)
      D(1,6)=XI(2)*C(2,3)
      D(2,2)=XI(1)*(C(1,1)+2.0*C(1,3)+C(3,3))
      D(2,3)=XI(4)*(C(1,3)+C(3,3))
      D(3,3)=XI(6)*C(3,3)+XI(1)*C(4,4)
      D(3,6)=XI(4)*C(2,3)
      DO 110 I=1,6
      DO 110 J=1,6
  110 D(J,I)=D(I,J)
C
C
C     FORM COEEFFICIENT-DISPLACEMENT MATRIX
C
      COMM=RR(2)*(ZZ(3)-ZZ(1))+RR(1)*(ZZ(2)-ZZ(3))+RR(3)*(ZZ(1)-ZZ(2))
      DD(1,1)=(RR(2)*ZZ(3)-RR(3)*ZZ(2))/COMM
      DD(1,2)=(RR(3)*ZZ(1)-RR(1)*ZZ(3))/COMM
      DD(1,3)=(RR(1)*ZZ(2)-RR(2)*ZZ(1))/COMM
      DD(2,1)=(ZZ(2)-ZZ(3))/COMM
      DD(2,2)=(ZZ(3)-ZZ(1))/COMM
```

```
      DD(2,3)=(ZZ(1)-ZZ(2))/COMM                                        FENT0433
      DD(3,1)=(RR(3)-RR(2))/COMM                                        FENT0434
      DD(3,2)=(RR(1)-RR(3))/COMM                                        FENT0435
      DD(3,3)=(RR(2)-RR(1))/COMM                                        FENT0436
C                                                                       FENT0437
      DO 120 I=1,3                                                      FENT0438
      J=2*LM(I)-1                                                       FENT0439
      H(1,J)=DD(1,I)                                                    FENT0440
      H(2,J)=DD(2,I)                                                    FENT0441
      H(3,J)=DD(3,I)                                                    FENT0442
      H(4,J+1)=DD(1,I)                                                  FENT0443
      H(5,J+1)=DD(2,I)                                                  FENT0444
  120 H(6,J+1)=DD(3,I)                                                  FENT0445
C                                                                       FENT0446
C     FORM STIFFNESS MATRIX (H)T*(D)*(H)                                FENT0447
C                                                                       FENT0448
      DO 130 J=1,10                                                     FENT0449
      DO 130 K=1,6                                                      FENT0450
      IF (H(K,J)) 128,130,128                                           FENT0451
  128 DO 129 I=1,6                                                      FENT0452
  129 F(I,J)=F(I,J)+D(I,K)*H(K,J)                                       FENT0453
  130 CONTINUE                                                          FENT0454
C                                                                       FENT0455
      DO 140 I=1,10                                                     FENT0456
      DO 140 K=1,6                                                      FENT0457
      IF (H(K,I)) 138,140,138                                           FENT0458
  138 DO 139 J=1,10                                                     FENT0459
  139 S(I,J)=S(I,J)+H(K,I)*F(K,J)                                       FENT0460
  140 CONTINUE                                                          FENT0461
C                                                                       FENT0462
C     FORM STRAIN TRANSFORMATION MATRIX                                 FENT0463
C                                                                       FENT0464
      DO 410 I=1,6                                                      FENT0465
      DO 410 J=1,10                                                     FENT0466
  410 HH(I,J)=HH(I,J)+H(I,J)                                            FENT0467
C                                                                       FENT0468
```

```
C
  500 RETURN
      END
      SUBROUTINE MPROP(N)
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
     1 DEPTH(25),E(10,25),SIG(7),R(450),Z(450),UR(450),
     2 UZ(450),STOTAL(450,4),KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 EE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
C     ******************************************************************
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
      MTYPE=IX(N,5)
C
      DO 5 II=1,4
      DO 5 JJ=1,4
    5 C(II,JJ)=0.0
C     ******************************************************************
C     DETERMINE ELASTIC CONSTANTS
C     ******************************************************************
C
   40 DO 55 KK=1,2
   55 EE(KK)=E(KK,MTYPE)
   60 IF (NORM) 65,75,65
   65 EE(1)=EE(1)*SIGIZ(MTYPE)
C     ******************************************************************
C     FORM STRESS STRAIN RELATIONSHIP
C     ******************************************************************
   75 COFF=EE(1)/(1.-EE(2)-2.*EE(2)*EE(2))
```

```
      C(1,1)=COEF*(1.-EE(2))                                            FENT0505
      C(1,2)=COEF*EE(2)                                                 FENT0506
      C(1,3)=EE(2)*COEF                                                 FENT0507
      C(2,1)=C(1,2)                                                     FENT0508
      C(2,2)=C(1,1)                                                     FENT0509
      C(2,3)=C(1,2)                                                     FENT0510
      C(3,1)=C(1,3)                                                     FENT0511
      C(3,2)=C(1,2)                                                     FENT0512
      C(3,3)=C(1,1)                                                     FENT0513
      C(4,4)=COEF*(0.5-EE(2))                                           FENT0514
      RETURN                                                            FENT0515
      END                                                               FENT0516
      SUBROUTINE MODIFY(N,U,ND2)                                        FENT0517
C                                                                       FENT0518
      IMPLICIT REAL*8 (A-H,O-Z)                                         FENT0519
      IMPLICIT INTEGER*2(I-N)                                           FENT0520
      COMMON /BANARG/ B(900),A(900,54),MBAND                            FENT0521
      DO 250 M=2,MBAND                                                  FENT0522
      K=N-M+1                                                           FENT0523
      IF (K) 235,235,230                                                FENT0524
  230 B(K)=B(K)-A(K,M)*U                                                FENT0525
      A(K,M)=0.0                                                        FENT0526
  235 K=N+M-1                                                           FENT0527
      IF (ND2-K) 250,240,240                                            FENT0528
  240 B(K)=B(K)-A(N,M)*U                                                FENT0529
      A(N,M)=0.0                                                        FENT0530
  250 CONTINUE                                                          FENT0531
      A(N,1)=1.0                                                        FENT0532
      B(N)=U                                                            FENT0533
      RETURN                                                            FENT0534
      END                                                               FENT0535
      SUBROUTINE BANSOL                                                 FENT0536
C                                                                       FENT0537
      IMPLICIT REAL*8 (A-H,O-Z)                                         FENT0538
      IMPLICIT INTEGER*2(I-N)                                           FENT0539
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25), FENT0540
```

```
    1 DEPTH(25),E(10,25),SIG(7),R(450),7(450),UR(450),
    2 UZ(450),STOTAL(450,4),KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /BANARG/ B(900),A(900,54),MBAND
      ND2=2*NUMNP
C
      DO 280 N=1,ND2
      DO 260 L=2,MBAND
      C=A(N,L)/A(N,1)
      I=N+L-1
C
      IF (ND2.LT.I) GO TO 260
C
      J=0
      DO 250 K=L,MBAND
      J=J+1
  250 A(I,J)=A(I,J)-C*A(N,K)
      B(I)=B(I)-C*B(N)
  260 A(N,L)=C
  280 B(N)=B(N)/A(N,1)
C
C     BACKSUBSTITUTION
C
      N=ND2
  300 N=N-1
C
      IF (N.LE.0) GO TO 500
      DO 400 K=2,MBAND
      L=N+K-1
      IF (ND2.LT.L) GO TO 400
      B(N)=B(N)-A(N,K)*B(L)
  400 CONTINUE
C
      GO TO 300
C
  500 RETURN
```

```
      END                                                               FENT0577
      SUBROUTINE STRESS(SPLOT)                                          FENT0578
      IMPLICIT REAL*8 (A-H,O-Z)                                         FENT0579
      IMPLICIT INTEGER*2(I-N)                                           FENT0580
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25), FENT0581
     1 DEPTH(25),E(10,25),SIG(7),R(450),Z(450),UR(450),                 FENT0582
     2 UZ(450),STOTAL(450,4),KSW                                        FENT0583
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)   FENT0584
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),          FENT0585
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),  FENT0586
     2 EE(10),IX(450,5)                                                 FENT0587
      COMMON /BANARG/ B(900),A(900,54),MBAND                            FENT0588
C     ***************************************************************** FENT0589
C     COMPUTE ELEMENT STRESSES AND STRAINS                              FENT0590
C     ****************************************************************** FENT0591
      DO 300 N=1,NUMEL                                                  FENT0592
      CALL QUAD(N,VOL)                                                  FENT0593
C                                                                       FENT0594
C     FIND ELEMENT COORDINATES                                          FENT0595
C                                                                       FENT0596
      I1=IX(N,1)                                                        FENT0597
      J1=IX(N,2)                                                        FENT0598
      K1=IX(N,3)                                                        FENT0599
      L1=IX(N,4)                                                        FENT0600
C                                                                       FENT0601
      IF (K1-L1.EQ.0) GO TO 50                                          FENT0602
      RRR(5)=(R(I1)+R(J1)+R(K1)+R(L1))/4.0                              FENT0603
      ZZZ(5)=(Z(I1)+Z(J1)+Z(K1)+Z(L1))/4.0                              FENT0604
      GO TO 100                                                         FENT0605
   50 RRR(5)=(R(I1)+R(J1)+R(K1))/3.0                                    FENT0606
      ZZZ(5)=(Z(I1)+Z(J1)+Z(K1))/3.0                                    FENT0607
C                                                                       FENT0608
C     COMPUTE STRAINS                                                   FENT0609
C                                                                       FENT0610
  100 DO 120 I=1,4                                                      FENT0611
      II=2*I                                                            FENT0612
```

```
      JJ=2*IX(N,I)                                                      FENT0613
      P(II-1)=B(JJ-1)                                                   FENT0614
  120 P(II)=B(JJ)                                                       FENT0615
C                                                                       FENT0616
      P(9)=0.0                                                          FENT0617
      P(10)=0.0                                                         FENT0618
  130 DO 150 I=1,2                                                      FENT0619
      RR(I)=P(I+8)                                                      FENT0620
      DO 150 K=1,8                                                      FENT0621
  150 RR(I)=RR(I)-S(I+8,K)*P(K)                                         FENT0622
C                                                                       FENT0623
      COMM=S(9,9)*S(10,10)-S(9,10)*S(10,9)                              FENT0624
      IF (COMM) 155,160,155                                             FENT0625
  155 P(9)=(S(10,10)*RR(1)-S(9,10)*RR(2))/COMM                          FENT0626
      P(10)=(-S(10,9)*RR(1)+S(9,9)*RR(2))/COMM                          FENT0627
C                                                                       FENT0628
  160 DO 170 I=1,6                                                      FENT0629
      TP(I)=0.0                                                         FENT0630
      DO 170 K=1,10                                                     FENT0631
  170 TP(I)=TP(I)+HH(I,K)*P(K)                                          FENT0632
C                                                                       FENT0633
      RR(1)=TP(2)                                                       FENT0634
      RR(2)=TP(6)                                                       FENT0635
      RR(3)=(TP(1)+TP(2)*RRR(5)+TP(3)*ZZZ(5))/RRR(5)                    FENT0636
      RR(4)=TP(3)+TP(5)                                                 FENT0637
C                                                                       FENT0638
C     COMPUTE STRESSES                                                  FENT0639
C                                                                       FENT0640
      DO 180 I=1,4                                                      FENT0641
      SIG(I)=0.0                                                        FENT0642
      DO 180 K=1,4                                                      FENT0643
  180 SIG(I)=SIG(I)+C(I,K)*RR(K)                                        FENT0644
C                                                                       FENT0645
C     COMPUTE PRINCIPLE STRESSES                                        FENT0646
C                                                                       FENT0647
      CC=(SIG(1)+SIG(2))/2.0                                            FENT0648
```

```
      BB=(SIG(1)-SIG(2))/2.0                                            FENT0649
      CR=DSQRT(BB**2+SIG(3)**2)                                         FENT0650
      SIG(5)=CC+CR                                                      FENT0651
      SIG(6)=CC-CR                                                      FENT0652
C     ******************************************************************FENT0653
C     CALCULATE ROTATION OF PRINCIPLE PLANES                            FENT0654
C     ******************************************************************FENT0655
  500 IF(DABS(SIG(4)).LT.1.0E-09) SIG(4)=0.0                            FENT0656
      IF(DABS(BB).GT.1.0E-09) GO TO 510                                 FENT0657
      BB=0.0                                                            FENT0658
  510 IF ((SIG(4).NE.0.).OR.(BB.NE.0.)) GO TO 520                       FENT0659
      ANG=0.0                                                           FENT0660
      GO TO 530                                                         FENT0661
  520 ANG=DATAN2(SIG(4),BB)/2.0                                         FENT0662
  530 SIG(8)=57.396*ANG                                                 FENT0663
      SIG(7)=(SIG(5)-SIG(6))/2.0                                        FENT0664
C     ******************************************************************FENT0665
C     OUTPUT STRESSES                                                   FENT0666
C     ******************************************************************FENT0667
      IF(N.NE.1) GO TO 615                                              FENT0668
      WRITE (6,2000)                                                    FENT0669
  615 WRITE (6,2001) N,RRR(5),ZZZ(5),(SIG(I),I=1,4)                     FENT0670
  300 CONTINUE                                                          FENT0671
 2000 FORMAT (8H1ELEMENT,8X,'R',8X,'Z',6X,'SIG(R)',6X,'SIG(Z)',5X,'SIG(TFENT0672
     1)',4X,'TAU(RZ)')                                                  FENT0673
 2001 FORMAT (I8,2F9.3,   1P7D12.3,        0P1F10.2)                    FENT0674
      RETURN                                                            FENT0675
      END                                                               FENT0676
      SUBROUTINE INTER(XI,RR,ZZ)                                        FENT0677
      IMPLICIT REAL*8 (A-H,O-Z)                                         FENT0678
      IMPLICIT INTEGER*2(I-N)                                           FENT0679
      DIMENSION RR(1),ZZ(1),XI(1)                                       FENT0680
      DIMENSION XM(7),R(7),Z(7),XX(9)                                   FENT0681
C                                                                       FENT0682
      XX(1)=.1259391805448                                              FENT0683
      XX(2)=XX(1)                                                       FENT0684
```

```
      XX(3)=XX(1)
      XX(4)=.1323941527884
      XX(5)=XX(4)
      XX(6)=XX(4)
      XX(7)=.225
      XX(8)=.696140478028
      XX(9)=.410426192314
      R(7)=(RR(1)+RR(2)+RR(3))/3.
      Z(7)=(ZZ(1)+ZZ(2)+ZZ(3))/3.
C
      DO 100 I=1,3
      J=I+3
      R(I)=XX(8)*RR(I)+(1.-XX(8))*R(7)
      R(J)=XX(9)*RR(I)+(1.-XX(9))*R(7)
      Z(I)=XX(8)*ZZ(I)+(1.-XX(8))*Z(7)
  100 Z(J)=XX(9)*ZZ(I)+(1.-XX(9))*Z(7)
C
      DO 200 I=1,7
  200 XM(I)=XX(I)*R(I)
C
      DO 300 I=1,6
  300 XI(I)=0.
C
      AREA=.5*(RR(1)*(ZZ(2)-ZZ(3))+RR(2)*(ZZ(3)-ZZ(1))+RR(3)*(ZZ(1)-ZZ(2
     1)))
C
      DO 400 I=1,7
      XI(1)=XI(1)+XM(I)
      XI(2)=XI(2)+XM(I)/R(I)
      XI(3)=XI(3)+XM(I)/(R(I)**2)
      XI(4)=XI(4)+XM(I)*Z(I)/R(I)
      XI(5)=XI(5)+XM(I)*Z(I)/(R(I)**2)
  400 XI(6)=XI(6)+XM(I)*(Z(I)**2)/(R(I)**2)
C
      DO 500 I=1,6
  500 XI(I)=XI(I)*AREA
```

```
C                                                              FENT0721
      RETURN                                                   FENT0722
      END                                                      FENT0723
```

APPENDIX C

FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF ISOTROPIC ELASTIC AXISYMMETRIC PLATES - THERMAL STRAINS INCLUDED (Ref. 13, 14)

Program Capabilities:

The following restrictions are placed on the size of problems which can be handled by the program.

Item	Maximum Number
Nodal Points	450
Elements	450
Materials	25
Boundary Pressure Cards	200

Printed output includes:

1. Reprint of Input Data
2. Nodal Point Displacements
3. Stresses at the center of each element.

Input Data Format:

A. Identification card - (18A4)

Columns 1 to 72 of this card contain information to be printed with results.

B. Control card - (5I5,F10.0)

Columns	1 - 5	Number of nodal points
	6 - 10	Number of elements
	11 - 15	Number of different materials

16 - 20	Normalizing stress (see NORM, Appendix B)
21 - 25	Number of boundary pressure cards
26 - 35	Reference temperature (stress free temperature)

C. Material Property information

The following group of cards must be supplied for each different material:

First Card - (2I5, 2F10.0)

Columns	1 - 5	Materials identification - any number from 1 to 12.
	6 - 10	Number of different temperatures for which properties are given = 8 maximum.
	11 - 20	Initial Z stress.
	21 - 30	Initial R stress.

Following Cards - (4F10.0) One card for each temperature

Columns	1 - 10	Temperature
	11 - 20	Modulus of elasticity - E
	21 - 30	Poisson's ratio - ν
	31 - 40	Coefficient of thermal expansion

D. Nodal Point Cards - (2I5, 5F10.0)

One card for each nodal point with the following information:

Columns	1 - 5	Nodal point number
	10	Number which indicates if displacements or forces are to be specified.
	11 - 20	R - ordinate
	21 - 30	Z - ordinate
	31 - 40	XR
	41 - 50	XZ
	51 - 60	Temperature

If the number in column 10 is

		Condition
0	XR is the specified R-load and XZ is the specified Z - load	free
1	XR is the specified R-displacement and XZ is the specified Z-load.	
2.	XR is the specified R-load and XZ is the specified Z-displacement.	
3	XR is the specified R-displacement and XZ is the specified Z- displacement.	fixed

All loads are considered to be total forces acting on a one radian segment. Nodal point cards must be in numerical sequence. If cards are omitted, the omitted nodal points are generated at equal intervals along a straight line between the defined nodal points. The necessary temperatures are determined by linear interpolation. The boundary code (column 10), XR and XZ are set equal to zero.

Skew Boundaries:

If the number in columns 5-10 of the nodal point cards is other than 0, 1, 2 or 3, it is interpreted as the magnitude of an angle in degrees. The terms in columns 31-50 of the nodal point card are then interpreted as follows:

XR is the specified load in the s-direction
XZ is the specified displacement in the n-direction

The angle must always be input as a negative angle and may range from -.001 to -180 degrees. Hence, +1.0 degree is the same as -179.0 degrees. The displacements of these nodal points which are printed by the program are

u_r = the displacement in the s-direction

u_z = the displacement in the n-direction

E. Element Cards - (6I5)

One card for each element

Columns	1 - 5	Element number	1. Order nodal points counter-clockwise around element.
	6 - 10	Nodal Point I	
	11 - 15	Nodal Point J	
	16 - 20	Nodal Point K	2. Maximum difference between nodal point I.D. must be less than 25.
	21 - 25	Nodal Point L	
	26 - 30	Material Identification	

Element cards must be in element number sequence. If element cards are omitted, the program automatically generates the omitted information by incrementing by one the preceding I, J, K and L. The material identification code for the generated cards is set equal to the value given on the last card. The last element card must always be supplied.

Triangular elements are also permissible; they are identified by repeating the last nodal point number (i.e., I, J, K, K).

F. Pressure Cards - (2I5, 1F10.0)

One card for each boundary element which is subjected to a normal pressure.

Columns	1 - 5	Nodal Point I
	6 - 10	Nodal Point J
	11 - 20	Normal Pressure

The boundary element must be on the left as one progresses from I to J. Surface tensile force is input as a negative pressure.

Listing:

```
C       ******************************************************************
C        FINITE ELEMENT PROGRAM FOR THE ANALYSIS OF ISOTROPIC ELASTIC
C        AXYSYMMETRIC PLATES  REF  FEAST 1,3 SAAS 2
C       ******************************************************************
C
        IMPLICIT REAL*8 (A-H,O-Z)
        IMPLICIT INTEGER*2(I-N)
        COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
       1DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
       2 UZ(450),STOTAL(450,4),
       3 T(450),TEMP,Q,KSW
        COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
        COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
       1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
       2 EE(10),IX(450,5)
        COMMON /BANARG/ B(900),A(900,54),MBAND
        COMMON/PRESS/ IBC(200),JBC(200),PR(200),NUMPC
        DATA STRS /'****'/
C       ****************************************************************
C       READ AND PRINT CONTROL INFORMATION
C       ****************************************************************
   50 READ (5,1000,END=950) HED
        WRITE (6,2000) HED
C
        READ(5,1001) NUMNP,NUMEL,NUMMAT,NORM,NUMPC,Q
        WRITE(6,2006) NUMNP,NUMEL,NUMMAT,NUMPC,Q
        IF (NORM) 65,65,66
   66 WRITE (6,2041)
C       ******************************************************************
C       READ AND PRINT MATERIAL PROPERTIES
C       ******************************************************************
   65 CONTINUE
C
        DO 80 M=1,NUMMAT
        READ(5,1012) MTYPE,NUMTC,SIGIZ(MTYPE),SIGIR(MTYPE)
        WRITE(6,2011)MTYPE,NUMTC,SIGIZ(MTYPE),SIGIR(MTYPE)
```

```
      READ(5,1011) ((E(I,J,MTYPE),J=1,4),I=1,NUMTC)                     FEWT0037
      WRITE(6,2010) ((E(I,J,MTYPE),J=1,4),I=1,NUMTC)                    FEWT0038
      DO 81 I=NUMTC,8                                                   FEWT0039
      DO 81 J=1,4                                                       FEWT0040
   81 E(I,J,MTYPE)=E(NUMTC,J,MTYPE)                                     FEWT0041
   80 CONTINUE                                                          FEWT0042
C     ******************************************************************FEWT0043
C     READ AND PRINT NODAL POINT DATA                                   FEWT0044
C     ******************************************************************FEWT0045
  100 WRITE (6,2013)                                                    FEWT0046
      L=0                                                               FEWT0047
  105 READ(5,1006) N,ICODE(N),R(N),Z(N),UR(N),UZ(N),T(N)                FEWT0048
  106 NL=L+1                                                            FEWT0049
      IF (L.EQ.0) GO TO 110                                             FEWT0050
      ZX=N-L                                                            FEWT0051
      DR=(R(N)-R(L))/ZX                                                 FEWT0052
      DZ=(Z(N)-Z(L))/ZX                                                 FEWT0053
      DT=(T(N)-T(L))/ZX                                                 FEWT0054
  110 L=L+1                                                             FEWT0055
      IF (N-L) 113,112,111                                              FEWT0056
  111 ICODE(L)=0                                                        FEWT0057
      R(L)=R(L-1)+DR                                                    FEWT0058
      Z(L)=Z(L-1)+DZ                                                    FEWT0059
      T(L)=T(L-1)+DT                                                    FEWT0060
      UR(L)=0.0                                                         FEWT0061
      UZ(L)=0.0                                                         FEWT0062
      GO TO 110                                                         FEWT0063
  112 WRITE(6,2014) (K,ICODE(K),R(K),Z(K),UR(K),UZ(K),T(K),K=NL,N)      FEWT0064
      IF (NUMNP-N) 113,120,105                                          FEWT0065
  113 WRITE (6,2015) N                                                  FEWT0066
      GO TO 900                                                         FEWT0067
C     ******************************************************************FEWT0068
C     READ AND PRINT ELEMENT PROPERTIES                                 FEWT0069
C     ******************************************************************FEWT0070
  120 WRITE (6,2016)                                                    FEWT0071
      N=0                                                               FEWT0072
```

```
  130 READ (5,1007) M,(IX(M,I),I=1,5)
  140 N=N+1
      IF (M-N) 170,170,150
  150 IX(N,1)=IX(N-1,1)+1
      IX(N,2)=IX(N-1,2)+1
      IX(N,3)=IX(N-1,3)+1
      IX(N,4)=IX(N-1,4)+1
      IX(N,5)=IX(N-1,5)
  170 WRITE (6,2017) N,(IX(N,I),I=1,5)
      IF (M-N) 180,180,140
  180 IF (NUMEL-N) 300,300,130
C     *****************************************************************
C     READ AND PRINT  THE PRESSURE CARDS
C     *****************************************************************
  300 IF(NUMPC) 290,210,290
  290 WRITE(6,9000)
      DO 200 L=1,NUMPC
      READ(5,9001) IBC(L),JBC(L),PR(L)
  200 WRITE(6,9002) IBC(L),JBC(L),PR(L)
  210 CONTINUE
C     ****************************************************************
C     DETERMINE BAND WIDTH
C     ****************************************************************
      J=0
      DO 340 N=1,NUMEL
      DO 340 I=1,4
      DO 325 L=1,4
      KK=IX(N,I)-IX(N,L)
      IF (KK.LT.0) KK=-KK
      IF  (KK.GT.J)  J=KK
  325 CONTINUE
  340 CONTINUE
      MBAND=2*J+2
C     ****************************************************************
C     SOLVE FOR DISPLACEMENTS AND STRESSES
C     ****************************************************************
```

```
      KSW=0
      CALL STIFF
      IF (KSW.NE.0) GO TO 900
C
      CALL BANSOL
      WRITE(6,2052)
      WRITE (6,2025) (N,B     (2*N-1),B     (2*N),N=1,NUMNP)
C
  450 CALL STRESS(SPLOT)
C     *****************************************************************
C     PROCESS ALL DECKS EVEN IF ERROR
C     *****************************************************************
      GO TO 910
  900 WRITE (6,4000)
  910 WRITE (6,4001) HED
C
  920 READ (5,1000) CHK
      IF (CHK.NE.STRS) GO TO 920
      GO TO 50
  950 CONTINUE
      WRITE (6,4002)
      CALL EXIT
C     *******************************************************************
C     *******************************************************************
 1000 FORMAT (18A4)
 1001 FORMAT(5I5,F10.0)
 1002 FORMAT ( I5,2F10.0)
 1003 FORMAT(2F10.0)
 1004 FORMAT (2F10.0)
 1005 FORMAT (3F10.0)
 1006 FORMAT(2I5,5F10.0)
 1007 FORMAT (6I5)
 1011 FORMAT(4F10.0)
 1012 FORMAT(2I5,2F10.0)
C     *****************************************************************
 2000 FORMAT (1H1,20A4)
```

```
 2006 FORMAT (//,
     1 30H0 NUMBER OF NODAL POINTS------ I3 /
     2 30H0 NUMBER OF ELEMENTS---------- I3 /
     3 30H0 NUMBER OF DIFF. MATERIALS--- I3 /
     4 30H0 NUMBER OF PRESSURE CARDS---- I3 /
     5 30H0 REFERENCE TEMPERATURE------- E12.4)
 2010 FORMAT (15H0  TEMPERATURE 15X 5HE    15X 6HNU     15X 6HALPHA   9X
     1/4F20.8)
 2011 FORMAT (17H0MATERIAL NUMBER= I3, 30H, NUMBER OF TEMPERATURE CARDS=
     1 I3,25H INITIAL VERTICAL STRESS= F10.3,5X,
     2 27H INITIAL HORIZONTAL STRESS= F10.3)
 2013 FORMAT (12H1NODAL POINT ,4X, 4HTYPE ,4X, 10HR-ORDINATE ,4X,
     1 10HZ-ORDINATE ,10X,6HR-LOAD ,10X, 6HZ-LOAD,10X,4HTEMP )
 2014 FORMAT(I12,I8,2F14.3,2E16.5,F14.3)
 2015 FORMAT (26H0NODAL POINT CARD ERROR N= I5)
 2016 FORMAT (49H1ELEMENT NO.      I     J     K     L    MATERIAL  )
 2017 FORMAT (1I13,4I6,1I12)
 2025 FORMAT (12H0NODAL POINT ,6X, 14HR-DISPLACEMENT ,6X, 14HZ-DISPLACEM
     1ENT / (I12,1P2D20.7))
 2041 FORMAT (76H0MODULUS AND YIELD STRESS NORMALIZED WITH RESPECT TO IN
     1ITIAL VERTICAL STRESS )
 2051 FORMAT(1H0,10X,'E',8X,'NU',/,3X,F11.1,F10.4/)
 2052 FORMAT(1H1)
C     ******************************************************************
 3003 FORMAT (16I5)
C     ******************************************************************
 4000 FORMAT (//// ' ABNORMAL TERMINATION')
 4001 FORMAT (//// ' END OF PROBLEM ' 20A4)
 4002 FORMAT (////' END OF JOB')
C     ******************************************************************
 9000 FORMAT(29H0PRESSURE BOUNDARY CONDITIONS/ 24H      I     J    PRESSU
     1RE )
 9001 FORMAT(2I5,F10.0)
 9002 FORMAT(2I6,F12.3)
      END
      SUBROUTINE STIFF
```

```
C
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNCT(25),
     1DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
     2 UZ(450),STOTAL(450,4),
     3 T(450),TEMP,Q,KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 EE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
      COMMON/PRESS/ IBC(200),JBC(200),PR(200),NUMPC
      DIMENSION CODE(450)
C     ******************************************************************
C     INITIALIZATION
C     ******************************************************************
      NB=27
      ND=2*NB
      ND2=2*NUMNP
      DO 50 N=1,ND2
      B(N)=0.0
      DO 50 M=1,ND
   50 A(N,M)=0.0
C     ******************************************************************
C     FORM STIFFNESS MATRIX
C     ******************************************************************
      DO 210 N=1,NUMEL
C
C
   90 CALL QUAD(N,VOL)
      IF (VOL) 142,142,144
  142 WRITE (6,2003) N
      KSW=1
      GO TO 210
C
```

```
  144 IF (IX(N,3)-IX(N,4)) 145,165,145                                FEWT0217
  145 DO 150 II=1,9                                                   FEWT0218
      CC=S(II,10)/S(10,10)                                            FEWT0219
      P(II)=P(II)-CC*P(10)                                            FEWT0220
      DO 150 JJ=1,9                                                   FEWT0221
  150 S(II,JJ)=S(II,JJ)-CC*S(10,JJ)                                   FEWT0222
C                                                                     FEWT0223
      DO 160 II=1,8                                                   FEWT0224
      CC=S(II,9)/S(9,9)                                               FEWT0225
      P(II)=P(II)-CC*P(9)                                             FEWT0226
      DO 160 JJ=1,8                                                   FEWT0227
  160 S(II,JJ)=S(II,JJ)-CC*S(9,JJ)                                    FEWT0228
C                                                                     FEWT0229
C     ADD ELEMENT STIFFNESS TO TOTAL STIFFNESS                        FEWT0230
C                                                                     FEWT0231
  165 DO 166 I=1,4                                                    FEWT0232
  166 LM(I)=2*IX(N,I)-2                                               FEWT0233
C                                                                     FEWT0234
      DO 200 I=1,4                                                    FEWT0235
      DO 200 K=1,2                                                    FEWT0236
      II=LM(I)+K                                                      FEWT0237
      KK=2*I-2+K                                                      FEWT0238
      B(II)=B(II)+P(KK)                                               FEWT0239
      DO 200 J=1,4                                                    FEWT0240
      DO 200 L=1,2                                                    FEWT0241
      JJ=LM(J)+L-II+1                                                 FEWT0242
      LL=2*J-2+L                                                      FEWT0243
      IF (JJ) 200,200,175                                             FEWT0244
  175 IF (ND-JJ) 180,195,195                                          FEWT0245
  180 WRITE (6,2004) N                                                FEWT0246
      KSW=1                                                           FEWT0247
      GO TO 210                                                       FEWT0248
  195 A(II,JJ)=A(II,JJ)+S(KK,LL)                                      FEWT0249
  200 CONTINUE                                                        FEWT0250
  210 CONTINUE                                                        FEWT0251
      IF(KSW.EQ.1) GO TO 500                                          FEWT0252
```

```
C                                                                       FEWT0253
C     ADD CONCENTRATED FORCES                                           FEWT0254
C                                                                       FEWT0255
      DO 250 N=1,NUMNP                                                  FEWT0256
      K=2*N                                                             FEWT0257
      B(K)=B(K)+UZ(N)                                                   FEWT0258
      B(K-1)=B(K-1)+UR(N)                                               FEWT0259
  250 CONTINUE                                                          FEWT0260
C                                                                       FEWT0261
C                                                                       FEWT0262
C     PRESSURE BCUNDARY CONDITICNS                                      FEWT0263
C                                                                       FEWT0264
      IF(NUMPC) 260,310,260                                             FEWT0265
  260 DO 300 L=1,NUMPC                                                  FEWT0266
      I=IBC(L)                                                          FEWT0267
      J=JBC(L)                                                          FEWT0268
      CODE(I)=ICODE(I)                                                  FEWT0269
      CODE(J)=ICODE(J)                                                  FEWT0270
      PP=PR(L)/6.                                                       FEWT0271
      DZ=(Z(I)-Z(J))*PP                                                 FEWT0272
      DR=(R(J)-R(I))*PP                                                 FEWT0273
      RX=2.0*R(I)+R(J)                                                  FEWT0274
      7X=R(I)+2.0*R(J)                                                  FEWT0275
  264 II=2*I                                                            FEWT0276
      JJ=2*J                                                            FEWT0277
  270 SINA=0.0                                                          FEWT0278
      COSA=1.0                                                          FEWT0279
      IF(CODE(I)) 271,272,272                                           FEWT0280
  271 SINA=DSIN(CODE(I))                                                FEWT0281
      COSA=DCCS(CODE(I))                                                FEWT0282
  272 B(II-1)=B(II-1)+RX*(COSA*DZ+SINA*DR)                              FEWT0283
      B(II)=B(II)-RX*(SINA*DZ-COSA*DR)                                  FEWT0284
  290 SINA=C.0                                                          FEWT0285
      COSA=1.0                                                          FEWT0286
      IF(CODE(J)) 291,292,292                                           FEWT0287
  291 SINA=DSIN(CODE(I))                                                FEWT0288
```

```
      COSA=DCOS(CODE(I))
  292 B(JJ-1)=B(JJ-1)+ZX*(COSA*DZ+SINA*DR)
      B(JJ)=B(JJ)-ZX*(SINA*DZ-COSA*DR)
  300 CONTINUE
  310 CONTINUE
C     DISPLACEMENT B.C.
C
      DO 400 M=1,NUMNP
      U=UR(M)
      N=2*M-1
      KX=ICODE(M)+1
      GO TO (400,370,390,380),KX
  370 CALL MODIFY(N,U,ND2)
      GO TO 400
  380 CALL MODIFY(N,U,ND2)
  390 U=UZ(M)
      N=N+1
      CALL MODIFY(N,U,ND2)
  400 CONTINUE
C
  500 RETURN
C     *************************************************************
 2003 FORMAT (26HONEGATIVE AREA ELEMENT NO. I4)
 2004 FORMAT (29HOBAND WIDTH EXCEEDS ALLOWABLE I4)
C     *************************************************************
      END
      SUBROUTINE QUAC(N,VOL)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
     1DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
     2 UZ(450),STOTAL(450,4),
     3 T(450),TEMP,Q,KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
```

```
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 EE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
C     ******************************************************************
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
C
      I1=1
      I2=2
      I3=3
      I4=4
      I5=5
C       THERMAL STRESSES
      TEMP=(T(I)+T(J)+T(K)+T(L))/4.0
      DO 103 M=2,8
      IF(E(M,1,MTYPE)-TEMP) 103,104,104
  103 CONTINUE
  104 RATIO=0.0
      DEN=E(M,1,MTYPE)-E(M-1,1,MTYPE)
      IF(DEN) 70,71,70
   70 RATIO=(TEMP-E(M-1,1,MTYPE))/DEN
   71 DO 105 KK=1,3
  105 EE(KK)=E(M-1,KK+1,MTYPE)+RATIO*(E(M,KK+1,MTYPE)-E(M-1,KK+1,MTYPE))
      TEMP=TEMP-Q
C     ******************************************************************
C     DETERMINE ELASTIC CONSTANTS AND STRESS-STRAIN RELATIONSHIP
C     ******************************************************************
C
      CALL MPROP(N)
C
   88 DO 110 M=1,3
  110 TT(M)=(C(M,1)+C(M,2)+C(M,3))*EE(3)*TEMP
C     ******************************************************************
C     FORM QUADRILATERAL STIFFNESS MATRIX
```

```
C     ****************************************************************** FEWT0361
  210 RRR(5)=(R(I)+R(J)+R(K)+R(L))/4.0                                   FEWT0362
      ZZZ(5)=(Z(I)+Z(J)+Z(K)+Z(L))/4.0                                   FEWT0363
      DO 94 M=1,4                                                        FEWT0364
      MM=IX(N,M)                                                         FEWT0365
      IF(R(MM).EQ.0..AND.ICODE(MM).EQ.0) ICODE(MM)=1                     FEWT0366
   93 RRR(M)=R(MM)                                                       FEWT0367
   94 ZZZ(M)=Z(MM)                                                       FEWT0368
C                                                                        FEWT0369
      DO 100 II=1,10                                                     FEWT0370
      P(II)=0.0                                                          FEWT0371
      DO 95 JJ=1,6                                                       FEWT0372
   95 HH(JJ,II)=0.0                                                      FEWT0373
      DO 100 JJ=1,10                                                     FEWT0374
  100 S(II,JJ)=0.0                                                       FEWT0375
      IF (K-L) 125,120,125                                               FEWT0376
  120 CALL TRISTF(I1,I2,I3)                                              FEWT0377
      RRR(5)=(RRR(1)+RRR(2)+RRR(3))/3.0                                  FEWT0378
      ZZZ(5)=(ZZZ(1)+ZZZ(2)+ZZZ(3))/3.0                                  FEWT0379
      VOL=XI(1)                                                          FEWT0380
      GO TO 160                                                          FEWT0381
  125 VOL=0.0                                                            FEWT0382
      CALL TRISTF(I4,I1,I5)                                              FEWT0383
      IF(XI(1).EQ.0.) WRITE(6,2000) N                                    FEWT0384
      VOL=VOL+XI(1)                                                      FEWT0385
      CALL TRISTF(I1,I2,I5)                                              FEWT0386
      IF(XI(1).EQ.0) WRITE(6,2000) N                                     FEWT0387
      VOL=VOL+XI(1)                                                      FEWT0388
      CALL TRISTF(I3,I4,I5)                                              FEWT0389
      IF(XI(1).EQ.0) WRITE(6,2000) N                                     FEWT0390
      VOL=VOL+XI(1)                                                      FEWT0391
      CALL TRISTF(I2,I3,I5)                                              FEWT0392
      IF(XI(1).EQ.0) WRITE(6,2000) N                                     FEWT0393
      VOL=VOL+XI(1)                                                      FEWT0394
C                                                                        FEWT0395
      DO 140 II=1,6                                                      FEWT0396
```

```
      DO 140 JJ=1,10
  140 HH(II,JJ)=HH(II,JJ)/4.0
C
C
  160 RETURN
C     ****
 2000 FORMAT (' ZERO AREA    ELEMENT',I5)
      END
      SUBROUTINE TRISTF(II,JJ,KK)
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
     1DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
     2 UZ(450),STOTAL(450,4),
     3 T(450),TEMP,Q,KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 EE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
C     ****************************************************************
C     INITIALIZATION
C
      LM(1)=II
      LM(2)=JJ
      LM(3)=KK
C
      RR(1)=RRR(II)
      RR(2)=RRR(JJ)
      RR(3)=RRR(KK)
      RR(4)=RRR(II)
      ZZ(1)=ZZZ(II)
      ZZ(2)=ZZZ(JJ)
      ZZ(3)=ZZZ(KK)
      ZZ(4)=ZZZ(II)
C
```

```
   85 DO 100 I=1,6
      DO 90 J=1,10
      F(I,J)=0.0
   90 H(I,J)=0.0
      DO 100 J=1,6
  100 D(I,J)=0.0
C
C     FORM INTEGRAL (G)T*(C)*(G)
      CALL INTER(XI,RR,ZZ)
C
      D(2,6)=XI(1)*(C(1,2)+C(2,3))
      D(3,5)=XI(1)*C(4,4)
      D(5,5)=D(3,5)
      D(6,6)=XI(1)*C(2,2)
      D(1,1)=XI(3)*C(3,3)
      D(1,2)=XI(2)*(C(1,3)+C(3,3))
      D(1,3)=XI(5)*C(3,3)
      D(1,6)=XI(2)*C(2,3)
      D(2,2)=XI(1)*(C(1,1)+2.0*C(1,3)+C(3,3))
      D(2,3)=XI(4)*(C(1,3)+C(3,3))
      D(3,3)=XI(6)*C(3,3)+XI(1)*C(4,4)
      D(3,6)=XI(4)*C(2,3)
      DO 110 I=1,6
      DO 110 J=1,6
  110 D(J,I)=D(I,J)
C
C
C     FORM COEEFFICIENT-DISPLACEMENT MATRIX
C
      COMM=RR(2)*(ZZ(3)-ZZ(1))+RR(1)*(ZZ(2)-ZZ(3))+RR(3)*(ZZ(1)-ZZ(2))
      DD(1,1)=(RR(2)*ZZ(3)-RR(3)*ZZ(2))/COMM
      DD(1,2)=(RR(3)*ZZ(1)-RR(1)*ZZ(3))/COMM
      DD(1,3)=(RR(1)*ZZ(2)-RR(2)*7Z(1))/COMM
      DD(2,1)=(ZZ(2)-ZZ(3))/COMM
      DD(2,2)=(ZZ(3)-ZZ(1))/COMM
      DD(2,3)=(ZZ(1)-ZZ(2))/COMM
```

```
      DD(3,1)=(RR(3)-RR(2))/COMM                                        FEWT0469
      DD(3,2)=(RR(1)-RR(3))/COMM                                        FEWT047C
      DD(3,3)=(RR(2)-RR(1))/COMM                                        FEWT0471
C                                                                       FEWT0472
      DO 120 I=1,3                                                      FEWT0473
      J=2*LM(I)-1                                                       FEWT0474
      H(1,J)=DD(1,I)                                                    FEWT0475
      H(2,J)=DD(2,I)                                                    FEWT0476
      H(3,J)=DD(3,I)                                                    FEWT0477
      H(4,J+1)=DD(1,I)                                                  FEWT0478
      H(5,J+1)=DD(2,I)                                                  FEWT0479
  120 H(6,J+1)=DD(3,I)                                                  FEWT0480
C                                                                       FEWT0481
C     FORM STIFFNESS MATRIX (H)T*(D)*(H)                                FEWT0482
C                                                                       FEWT0483
      DO 130 J=1,10                                                     FEWT0484
      DO 130 K=1,6                                                      FEWT0485
      IF (H(K,J)) 128,130,128                                           FEWT0486
  128 DO 129 I=1,6                                                      FEWT0487
  129 F(I,J)=F(I,J)+D(I,K)*H(K,J)                                       FEWT0488
  130 CONTINUE                                                          FEWT0489
C                                                                       FEWT0490
      DO 140 I=1,10                                                     FEWT0491
      DO 140 K=1,6                                                      FEWT0492
      IF (H(K,I)) 138,140,138                                           FEWT0493
  138 DO 139 J=1,10                                                     FEWT0494
  139 S(I,J)=S(I,J)+H(K,I)*F(K,J)                                       FEWT0495
  140 CONTINUE                                                          FEWT0496
      TP(1)=XI(2)*TT(3)                                                 FEWT0497
      TP(2)=XI(1)*(TT(1)+TT(3))                                         FEWT0498
      TP(3)=XI(4)*TT(3)                                                 FEWT0499
      TP(4)=0.0                                                         FEWT0500
      TP(5)=0.0                                                         FEWT0501
      TP(6)=XI(1)*TT(2)                                                 FEWT0502
      DO 160 I=1,10                                                     FEWT0503
      DO 160 K=1,6                                                      FEWT0504
```

```
  160 P(I)=P(I)+H(K,I)*TP(K)                                               FEWT0505
C                                                                          FEWT0506
C     FORM STRAIN TRANSFORMATION MATRIX                                    FEWT0507
C                                                                          FEWT0508
      DO 410 I=1,6                                                         FEWT0509
      DO 410 J=1,10                                                        FEWT0510
  410 HH(I,J)=HH(I,J)+H(I,J)                                               FEWT0511
C                                                                          FEWT0512
C                                                                          FEWT0513
  500 RETURN                                                               FEWT0514
      END                                                                  FEWT0515
      SUBROUTINE MPROP(N)                                                  FEWT0516
      IMPLICIT REAL*8 (A-H,O-Z)                                            FEWT0517
      IMPLICIT INTEGER*2(I-N)                                              FEWT0518
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),    FEWT0519
     1DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),              FEWT0520
     2 UZ(450),STOTAL(450,4),                                              FEWT0521
     3 T(450),TEMP,Q,KSW                                                   FEWT0522
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)      FEWT0523
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),             FEWT0524
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),     FEWT0525
     2 EE(10),IX(450,5)                                                    FEWT0526
      COMMON /BANARG/ B(900),A(900,54),MBAND                               FEWT0527
C     ****************************************************************    FEWT0528
      I=IX(N,1)                                                            FEWT0529
      J=IX(N,2)                                                            FEWT0530
      K=IX(N,3)                                                            FEWT0531
      L=IX(N,4)                                                            FEWT0532
      MTYPE=IX(N,5)                                                        FEWT0533
C                                                                          FEWT0534
      DO 5 II=1,4                                                          FEWT0535
      DO 5 JJ=1,4                                                          FEWT0536
    5 C(II,JJ)=0.0                                                         FEWT0537
C     ****************************************************************    FEWT0538
C     DETERMINE ELASTIC CONSTANTS                                          FEWT0539
C     ****************************************************************    FEWT0540
```

```
C
   60 IF (NORM) 65,75,65
   65 FE(1)=EE(1)*SIGIZ(MTYPE)
C     ******************************************************************
C     FORM STRESS STRAIN RELATIONSHIP
C     ******************************************************************
   75 COEF=EE(1)/(1.-EE(2)-2.*EE(2)*EE(2))
      C(1,1)=COEF*(1.-EE(2))
      C(1,2)=COEF*EE(2)
      C(1,3)=EE(2)*COEF
      C(2,1)=C(1,2)
      C(2,2)=C(1,1)
      C(2,3)=C(1,2)
      C(3,1)=C(1,3)
      C(3,2)=C(1,2)
      C(3,3)=C(1,1)
      C(4,4)=COEF*(0.5-EE(2))
      RETURN
      END
      SUBROUTINE MODIFY(N,U,ND2)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON /BANARG/ B(900),A(900,54),MBAND
      DO 250 M=2,MBAND
      K=N-M+1
      IF (K) 235,235,230
  230 B(K)=B(K)-A(K,M)*U
      A(K,M)=0.0
  235 K=N+M-1
      IF (ND2-K) 250,240,240
  240 B(K)=B(K)-A(N,M)*U
      A(N,M)=0.0
  250 CONTINUE
      A(N,1)=1.0
      B(N)=U
```

```
      RETURN                                                            FEWT0577
      END                                                               FEWT0578
      SUBROUTINE BANSOL                                                 FEWT0579
C                                                                       FEWT0580
      IMPLICIT REAL*8 (A-H,O-Z)                                         FEWT0581
      IMPLICIT INTEGER*2(I-N)                                           FEWT0582
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25), FEWT0583
     1DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),           FEWT0584
     2 UZ(450),STOTAL(450,4),                                           FEWT0585
     3 T(450),TEMP,Q,KSW                                                FEWT0586
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)   FEWT0587
      COMMON /BANARG/ B(900),A(900,54),MBAND                            FEWT0588
      ND2=2*NUMNP                                                       FEWT0589
C                                                                       FEWT0590
      DO 280 N=1,ND2                                                    FEWT0591
      DO 260 L=2,MBAND                                                  FEWT0592
      C=A(N,L)/A(N,1)                                                   FEWT0593
      I=N+L-1                                                           FEWT0594
C                                                                       FEWT0595
      IF (ND2.LT.I) GO TO 260                                           FEWT0596
C                                                                       FEWT0597
      J=0                                                               FEWT0598
      DO 250 K=L,MBAND                                                  FEWT0599
      J=J+1                                                             FEWT0600
  250 A(I,J)=A(I,J)-C*A(N,K)                                            FEWT0601
      B(I)=B(I)-C*B(N)                                                  FEWT0602
  260 A(N,L)=C                                                          FEWT0603
  280 B(N)=B(N)/A(N,1)                                                  FEWT0604
C                                                                       FEWT0605
C     BACKSUBSTITUTION                                                  FEWT0606
C                                                                       FEWT0607
      N=ND2                                                             FEWT0608
  300 N=N-1                                                             FEWT0609
C                                                                       FEWT0610
      IF (N.LE.0) GO TO 500                                             FEWT0611
      DO 400 K=2,MBAND                                                  FEWT0612
```

```
      L=N+K-1
      IF (ND2.LT.L) GO TO 400
      B(N)=B(N)-A(N,K)*B(L)
  400 CONTINUE
C
      GO TO 300
C
  500 RETURN
      END
      SUBROUTINE STRESS(SPLOT)
      IMPLICIT REAL*8 (A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      COMMON     STTOP,HED(18),SIGIR(25),SIGIZ(25),GAMMA(25),ZKNOT(25),
     1DEPTH(25),E(8,4,25),SIG(7),R(450),Z(450),UR(450),TT(3),
     2 UZ(450),STOTAL(450,4),
     3 T(450),TEMP,Q,KSW
      COMMON /INTEGR/ NUMNP,NUMEL,NUMMAT,NDEPTH,NORM,MTYPE,ICODE(450)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),LM(4),DD(3,3),
     1 HH(6,10),RR(4),ZZ(4),C(4,4),H(6,10),D(6,6),F(6,10),TP(6),XI(6),
     2 EE(10),IX(450,5)
      COMMON /BANARG/ B(900),A(900,54),MBAND
C     *****************************************************************
C     COMPUTE ELEMENT STRESSES AND STRAINS
C     *******************************************************************
      DO 300 N=1,NUMEL
      CALL QUAD(N,VOL)
C
C     FIND ELEMENT COORDINATES
C
      I1=IX(N,1)
      J1=IX(N,2)
      K1=IX(N,3)
      L1=IX(N,4)
C
      IF (K1-L1.EQ.0) GO TO 50
      RRR(5)=(R(I1)+R(J1)+R(K1)+R(L1))/4.0
```

```
      ZZZ(5)=(Z(I1)+Z(J1)+Z(K1)+Z(L1))/4.0
      GO TO 100
   50 RRR(5)=(R(I1)+R(J1)+R(K1))/3.0
      ZZZ(5)=(Z(I1)+Z(J1)+Z(K1))/3.0
C
C     COMPUTE STRAINS
C
  100 DO 120 I=1,4
      II=2*I
      JJ=2*IX(N,I)
      P(II-1)=B(JJ-1)
  120 P(II)=B(JJ)
C
      P(9)=0.0
      P(10)=0.0
  130 DO 150 I=1,2
      RR(I)=P(I+8)
      DO 150 K=1,8
  150 RR(I)=RR(I)-S(I+8,K)*P(K)
C
      COMM=S(9,9)*S(10,10)-S(9,10)*S(10,9)
      IF (COMM) 155,160,155
  155 P(9)=(S(10,10)*RR(1)-S(9,10)*RR(2))/COMM
      P(10)=(-S(10,9)*RR(1)+S(9,9)*RR(2))/COMM
C
  160 DO 170 I=1,6
      TP(I)=0.0
      DO 170 K=1,10
  170 TP(I)=TP(I)+HH(I,K)*P(K)
C
      RR(1)=TP(2)
      RR(2)=TP(6)
      RR(3)=(TP(1)+TP(2)*RRR(5)+TP(3)*ZZZ(5))/RRR(5)
      RR(4)=TP(3)+TP(5)
C
C     COMPUTE STRESSES
```

```
C                                                                       FEWT0685
      DO 180 I=1,4                                                      FEWT0686
      SIG(I)=0.0                                                        FEWT0687
      DO 180 K=1,4                                                      FEWT0688
  180 SIG(I)=SIG(I)+C(I,K)*RR(K)                                        FEWT0689
C                                                                       FEWT0690
C     COMPUTE PRINCIPLE STRESSES                                        FEWT0691
C                                                                       FEWT0692
      CC=(SIG(1)+SIG(2))/2.0                                            FEWT0693
      BB=(SIG(1)-SIG(2))/2.0                                            FEWT0694
      CR=DSQRT(BB**2+SIG(3)**2)                                         FEWT0695
      SIG(5)=CC+CR                                                      FEWT0696
      SIG(6)=CC-CR                                                      FEWT0697
C     ******************************************************************FEWT0698
C     CALCULATE ROTATION OF PRINCIPLE PLANES                            FEWT0699
C     ******************************************************************FEWT0700
  500 IF(DABS(SIG(4)).LT.1.0E-09) SIG(4)=0.0                            FEWT0701
      IF(DABS(BB).GT.1.0E-09) GO TO 510                                 FEWT0702
      BB=0.0                                                            FEWT0703
  510 IF ((SIG(4).NE.0.).OR.(BB.NE.0.)) GO TO 520                       FEWT0704
      ANG=0.0                                                           FEWT0705
      GO TO 530                                                         FEWT0706
  520 ANG=DATAN2(SIG(4),BB)/2.0                                         FEWT0707
  530 SIG(8)=57.396*ANG                                                 FEWT0708
      SIG(7)=(SIG(5)-SIG(6))/2.0                                        FEWT0709
C     ******************************************************************FEWT0710
C     OUTPUT STRESSES                                                   FEWT0711
C     ******************************************************************FEWT0712
      IF(N.NE.1) GO TO 615                                              FEWT0713
      WRITE (6,2000)                                                    FEWT0714
  615 WRITE (6,2001) N,RRR(5),ZZZ(5),(SIG(I),I=1,4)                     FEWT0715
  300 CONTINUE                                                          FEWT0716
 2000 FORMAT (8H1ELEMENT,8X,'R',8X,'Z',6X,'SIG(R)',6X,'SIG(Z)',5X,'SIG(TFEWT0717
     1)',4X,'TAU(RZ)')                                                  FEWT0718
 2001 FORMAT (I8,2F9.3,   1P7D12.3,         0P1F10.2)                   FEWT0719
      RETURN                                                            FEWT0720
```

```
      END
      SUBROUTINE INTER(XI,RR,ZZ)
      IMPLICIT REAL*8 (A-H,C-Z)
      IMPLICIT INTEGER*2(I-N)
      DIMENSION RR(1),ZZ(1),XI(1)
      DIMENSION XM(7),R(7),Z(7),XX(9)
C
      XX(1)=.1259391805448
      XX(2)=XX(1)
      XX(3)=XX(1)
      XX(4)=.1323941527884
      XX(5)=XX(4)
      XX(6)=XX(4)
      XX(7)=.225
      XX(8)=.696140478028
      XX(9)=.410426192314
      R(7)=(RR(1)+RR(2)+RR(3))/3.
      Z(7)=(ZZ(1)+ZZ(2)+ZZ(3))/3.
C
      DO 100 I=1,3
      J=I+3
      R(I)=XX(8)*RR(I)+(1.-XX(8))*R(7)
      R(J)=XX(9)*RR(I)+(1.-XX(9))*R(7)
      Z(I)=XX(8)*ZZ(I)+(1.-XX(8))*Z(7)
  100 Z(J)=XX(9)*ZZ(I)+(1.-XX(9))*Z(7)
C
      DO 200 I=1,7
  200 XM(I)=XX(I)*R(I)
C
      DO 300 I=1,6
  300 XI(I)=0.
C
      AREA=.5*(RR(1)*(ZZ(2)-ZZ(3))+RR(2)*(ZZ(3)-ZZ(1))+RR(3)*(ZZ(1)-ZZ(2
     1)))
C
      DO 400 I=1,7
```

```
      XI(1)=XI(1)+XM(I)                                                 FEWT0757
      XI(2)=XI(2)+XM(I)/R(I)                                            FEWT0758
      XI(3)=XI(3)+XM(I)/(R(I)**2)                                       FEWT0759
      XI(4)=XI(4)+XM(I)*Z(I)/R(I)                                       FEWT0760
      XI(5)=XI(5)+XM(I)*Z(I)/(R(I)**2)                                  FEWT0761
  400 XI(6)=XI(6)+XM(I)*(Z(I)**2)/(R(I)**2)                             FEWT0762
C                                                                       FEWT0763
      DO 500 I=1,6                                                      FEWT0764
  500 XI(I)=XI(I)*AREA                                                  FEWT0765
C                                                                       FEWT0766
      RETURN                                                            FEWT0767
      END                                                               FEWT0768
```

APPENDIX D

STEADY STATE HEAT TRANSFER PROGRAM FOR BOLTED JOINT

Program Capacity: 50 nodal points

Output Data:

(a) Input data

(b) Inverse of matrix

(c) Nodal temperature

(d) Given and calculated augmenting vector and residual error

Input Data Sequence:

A. Case identification (12A4) followed by two blank cards

B. Card (I1) with a 1

C. Card (I7) with dimension of matrix

D. Card (I1) with a 1

E. Cards (I1, 3(2I3, E15.8)) with node indices started in the first I3 field followed by conductance between these nodes. Only input from lower node number to higher node number required (since the conductance from node i to j equals the conductance from j to i.) Each card has three groups of z node numbers followed by a conductance value except the last card. Last card could have 1, 2 or 3 groups and has a 1 in column 1.

F. Cards (I1, 3(I6, E15.8) with number of node followed by conductance from the node to ground node which is at specified temperature. Each card has 3 groups of node number followed by conductance. The I1 field is skipped except for the last card for ground conductances which can have 1, 2 or 3 fields and the first column has a 1. A node can be connected to only one ground node.

G. Same as F above, but code temperature specified for ground node instead of the conductance value.

H. Same as F above, but code internal power dissipation for the particular node instead of the conductance value.

Listing:

```
C      STEADY STATE HEAT TRANSFER PROGRAM    BOLTED JOINT
          DIMENSION IDENT(12),A(050,050),AA(050,050),B( 50),BI( 50 ),
     1BC( 50),RES( 50),ACON( 50),TACON( 50),Q( 50)
  101 WRITE(6,23)
   41 READ(5,51) K,IDENT
   51 FORMAT(I1,12A4)
      WRITE(6,111) IDENT
  111 FORMAT(12A6)
      IF(K .NE. 1) GO TO 41
      READ(5,55) N,K
   55 FORMAT(I7/I1)
      M = N+1
      DO 3 I = 1,N
      DO 3 J = 1,N
      AA(I,J) = 0.0
      ACON(I)=0.
      Q(I) = 0.
      TACON(I) = 0.
    3 CONTINUE
C     READ IN COEFF. MATRIX ELEMENTS
   42 READ(5,52) K,(I,J,AA(I,J),JM=1,3)
   52 FORMAT(I1,3(2I3,E15.8))
      IF(K .NE. 1) GO TO 42
   43 READ(5,53) K,(I,ACON(I),JM=1,3)
      IF(K .NE. 1) GO TO 43
   44 READ(5,53) K,(I,TACON(I),JM=1,3)
      IF(K .NE. 1) GO TO 44
   45 READ(5,53) K,(I,Q(I),JM=1,3)
      IF(K .NE. 1) GO TO 45
   53 FORMAT(I1,3(I6,E15.8))
      DO 500 I=1,N
  500 B(I) = -(Q(I) + ACON(I) * TACON(I))
      DO 1000 I=1,N
      DO 1000 J=I,N
 1000 AA(J,I) = AA(I,J)
      DO 3000 I=1,N
```

```
      IN=I                                                              SSHT0037
      AA(I,I) =0.                                                       SSHT0038
      DO 2001 J=1,N                                                     SSHT0039
      JN=J                                                              SSHT0040
      IF (JN .EQ. IN) GO TO 2001                                        SSHT0041
 2000 AA(I,I) = AA(I,I) + AA(I,J)                                       SSHT0042
 2001 CONTINUE                                                          SSHT0043
 3000 AA(I,I) =(AA(I,I)+ACON(I)) * (-1.)                                SSHT0044
      WRITE(6,26)                                                       SSHT0045
   26 FORMAT(1H1,27X,1HI,12X,1HQ,15X,10HGRD. COND.,10X,10HGRD. TEMP.//) SSHT0046
      WRITE(6,25)(I,Q(I),ACON(I),TACON(I),I=1,N)                        SSHT0047
   25 FORMAT(1H ,26X,I3,7X,F10.5,10X,F10.5,10X,F10.5,10X)               SSHT0048
      MM = 1                                                            SSHT0049
      DO 4 I=1,N                                                        SSHT0050
      DO 4 J = 1,N                                                      SSHT0051
        A(I,J) = AA(I,J)                                                SSHT0052
    4 CONTINUE                                                          SSHT0053
      WRITE(6,5)                                                        SSHT0054
    5 FORMAT( 1H1,39X17HA = COEFF. MATRIX //                            SSHT0055
     1            40X21HB = AUGMENTING VECTOR  //                       SSHT0056
     2            40X19HT = SOLUTION VECTOR    //                       SSHT0057
     3            40X16HAI= INVERSE OF A     //                         SSHT0058
     4            40X33HBC = AUGMENTING VECTOR CALCULATED   //          SSHT0059
     5            40X21H( A ) * ( T ) = ( B )   /// )                   SSHT0060
      DO 7 I = 1,N                                                      SSHT0061
    7 WRITE(6,6)(       I,J,A(I,J), J = 1,N )                           SSHT0062
    6 FORMAT(1H / (4(  5H   A(I3,1H,I3,2H)=F10.5,5X)))                  SSHT0063
      DO 8 I = 1,N                                                      SSHT0064
      BI(I) = B(I)                                                      SSHT0065
    8 CONTINUE                                                          SSHT0066
      CALL MAT(N,M,A,B )                                                SSHT0067
      WRITE(6,22)                                                       SSHT0068
   22 FORMAT( 1H1 )                                                     SSHT0069
C     WRITE INVERSE MATRIX                                              SSHT0070
      DO 9 I = 1,N                                                      SSHT0071
    9 WRITE(6,10)(       I,J,A(I,J),J = 1,N )                           SSHT0072
```

```
   10 FORMAT(1H / (4(   5H   AI(I3,1H,I3,2H)=E15.8)))                        SSHT0073
      WRITE(6,23)                                                            SSHT0074
   23 FORMAT(1H1)                                                            SSHT0075
C     WRITE SOLUTION VECTOR                                                  SSHT0076
   12 WRITE(6,11)(        J,   B(J )   ,J = 1,N   )                          SSHT0077
   11 FORMAT(1H / 4(5H    T(I3,2H)=F10.5,9X))                                SSHT0078
      DO 13 I = 1,N                                                          SSHT0079
       BC(I)  =0.0                                                           SSHT0080
       DO 13 J = 1,N                                                         SSHT0081
       BC(I)  = BC(I) + (AA(I,J) * B(J))                                     SSHT0082
   13  CONTINUE                                                              SSHT0083
      DO 15 J = 1,N                                                          SSHT0084
      RES(J) = ABS(BI(J)) - ABS( BC(J))                                      SSHT0085
   15 CONTINUE                                                               SSHT0086
      WRITE(6,16)                                                            SSHT0087
   16 FORMAT(1H1,30X76H AUGMENTING VECTOR       CALCU.AUGMENTING VECTOR      SSHT0088
     1             RESIDUAL ERROR   // )                                     SSHT0089
   17 WRITE (6,18) (J,I,BI(J),J,I,BC(J),RES(J),J=1,N)                        SSHT0090
   18 FORMAT(25X4H  B(I3,1H,    I3,2H)=E15.8,2X4H BC(I3,1H,I3,2H)=E15.8,     SSHT0091
     1             6XE15.8 /)                                                SSHT0092
      GO TO 101                                                              SSHT0093
      END                                                                    SSHT0094
      SUBROUTINE MAT    (N,M,A,B)                                            SSHT0095
C     M          = N + 1                                                     SSHT0096
C     N          = SIZE OF MATRIX TO BE INVERTED                             SSHT0097
C     TO SOLVE  AX = B, WHERE INPUT A = A, INPUT B = B                       SSHT0098
C                             OUTPUT B= X, OUTPUTA = A INVERSE               SSHT0099
      DIMENSION A(50,50),B(50)                                               SSHT0100
      N1         = N - 1                                                     SSHT0101
      TEMP 15 = A(1,1)                                                       SSHT0102
      A(M,N)   = 1.0 / TEMP 15                                               SSHT0103
      B(M)     = A(1,2) / TEMP 15                                            SSHT0104
      DO  1  I=2,N1                                                          SSHT0105
      A(M,I-1)= A(1,I+1) / TEMP 15                                           SSHT0106
    1 CONTINUE                                                               SSHT0107
      A(M,N1) = B(1) / TEMP 15                                               SSHT0108
```

```
      DO 10  I=1,N1                                       SSHT0109
      TEMP 6   = A(I+1,1)                                 SSHT0110
      B(I)    = A(I+1,2) - TEMP 6 * B(M)                  SSHT0111
      DO 5   J=2,N1                                       SSHTC112
      A(I,J-1)= A(I+1,J+1) - TEMP 6 * A(M,J-1)            SSHT0113
    5 CONTINUE                                            SSHT0114
      A(I,N1) = B(I+1) - TEMP 6 * A(M,N1)                 SSHT0115
      A(I,N)  = -TEMP6 / TEMP 15                          SSHT0116
   10 CONTINUE                                            SSHT0117
      B(N)      = B(M)                                    SSHT0118
      DO 15 I=1,N                                         SSHT0119
      A(N,I)  = A(M,I)                                    SSHT0120
   15 CONTINUE                                            SSHT0121
C     REPEATS N - 1 TIMES                                 SSHT0122
      DO 100 K=1,N1                                       SSHT0123
      TEMP 15 = B(1)                                      SSHTC124
      A(M,N)  = 1.0 / TEMP 15                             SSHT0125
      B(M)      = A(1,1) / TEMP 15                        SSHT0126
      DO 51 I=2,N                                         SSHT0127
      A(M,I-1)= A(1,I) / TEMP 15                          SSHT0128
   51 CONTINUE                                            SSHT0129
      DO 60 I =1,N1                                       SSHT0130
      TEMP 6   = B(I+1)                                   SSHT0131
      B(I)      = A(I+1,1) - TEMP 6 * B(M)                SSHT0132
      DO 55  J=2,N                                        SSHT0133
      A(I,J-1)= A(I+1,J) - TEMP 6* A(M,J-1)               SSHTC134
   55 CONTINUE                                            SSHT0135
      A(I,N)  = -TEMP 6  / TEMP 15                        SSHT0136
   60 CONTINUE                                            SSHT0137
      B(N)      = B(M)                                    SSHT0138
      DO 65 I=1,N                                         SSHT0139
      A(N,I)  = A(M,I)                                    SSHTC140
   65 CONTINUE                                            SSHTC141
  100 CONTINUE                                            SSHT0142
      RETURN                                              SSHT0143
      END                                                 SSHT0144
```

TABLE 1

Separation Radius Comparison - Single and Two Plate Models

(see Figs. 12 - 17)

$\frac{A}{B}$	$\frac{B}{A}$	R_o/A Single Plate Model	R_o/A Two Plate Model	Percent Discrepancy Between Models
1	3.1	4.2	3.7	13.5
	2.2	3.3	2.7	22.2
	1.6	2.7	2.1	28.6
	1.3	2.4	1.7	41.7
.75	3.1	4.5	3.8	18.5
	2.2	3.6	2.8	28.9
	1.6	3.0	2.2	36.4
	1.3	2.7	2.0	35.0
.5	3.1	5.1	4.1	24.4
	2.2	4.2	3.2	31.3
	1.6	3.6	2.8	28.6
	1.3	3.3	2.5	32.0

TABLE 2

Test and Analytical Results for Radii of Separation of Bolted Plates (see Fig. 5)

Case	D in.	2B in.	Separation Diameters, 2 R_o – in.					% Discrepancy Between Computed Values and Tested Values	
			"Rubbing Test"		Autoradiographic Test		Computed		
			Range	Average	Range	Average		Rub. Test	Autorad. Test
1	.065	.422	.42-.48	.45	.41-.46	.44	.488	7.8	9.8
2	.124	.422	.50-.53	.51	.4 - .6	.55	.554	7.9	.7
3	.191	.422	.58-.64	.62	.76-.81	.78*	.620	0	25.8
4	.253	.422	.70-.76	.72	.68-.73	.7**	.700	2.9	0
5.	Unmatched Pair .124/ .257	.422	.54-.58	.56	—	—	.588	4.8	—
6.	.124	1.0	1.06-1.10	1.09	—	—	1.104	1.3	—
7.	.191	1.0	1.11-1.17	1.16	—	—	1.210	4.1	—

*Original x-ray film shows hole in plate and 0.6 inch diameter zone more distinctly than remainder of area sensitized by the radioactive contamination. Loose radiographic contamination observed during test.

**Assembled and disassembled radioactive and non-radioactive plates without rotating plates relative to each other.

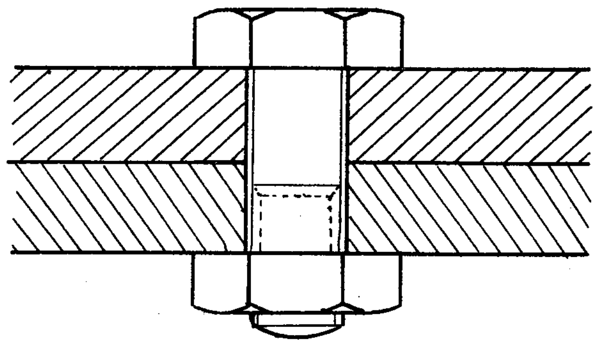


FIG. 1. BOLTED JOINT

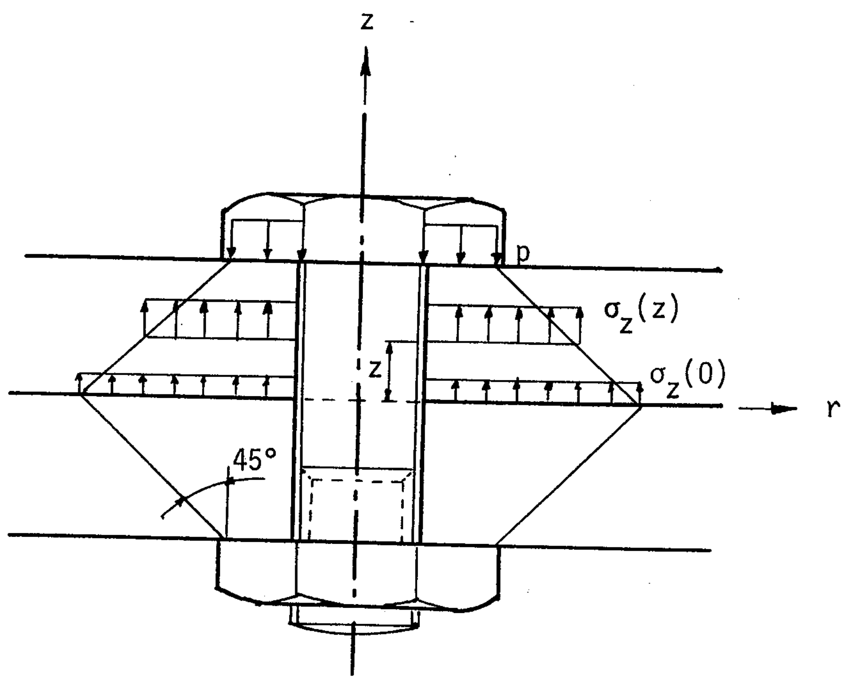


FIG. 2. ROETSCHER's RULE OF THUMB FOR PRESSURE DISTRIBUTION IN A BOLTED JOINT

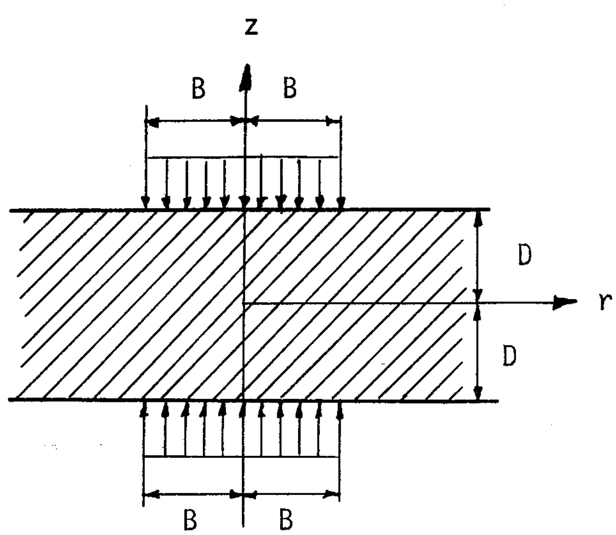


FIG. 3(a)

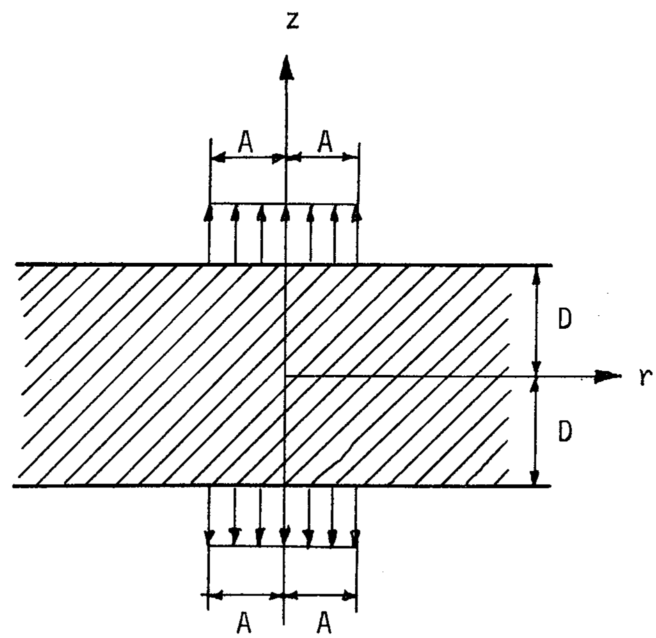


FIG. 3(b)

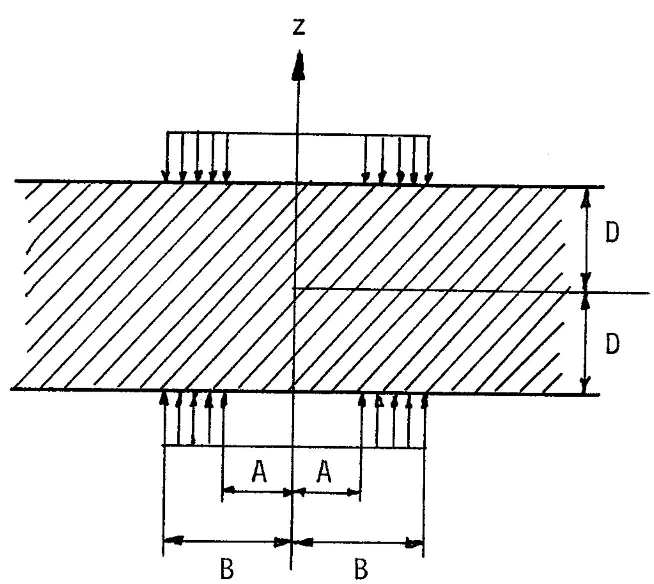


FIG. 3(c)

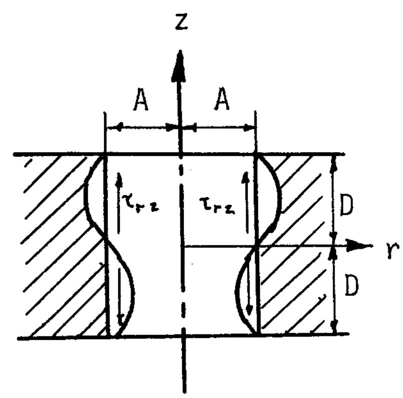


FIG. 3(d)

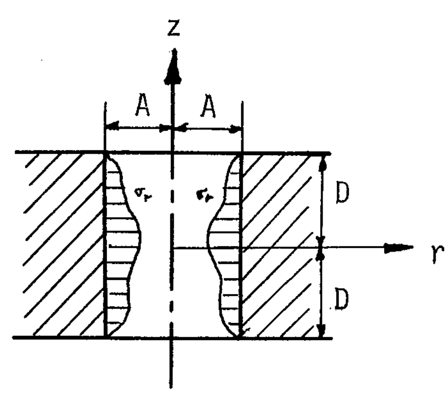


FIG. 3(e)

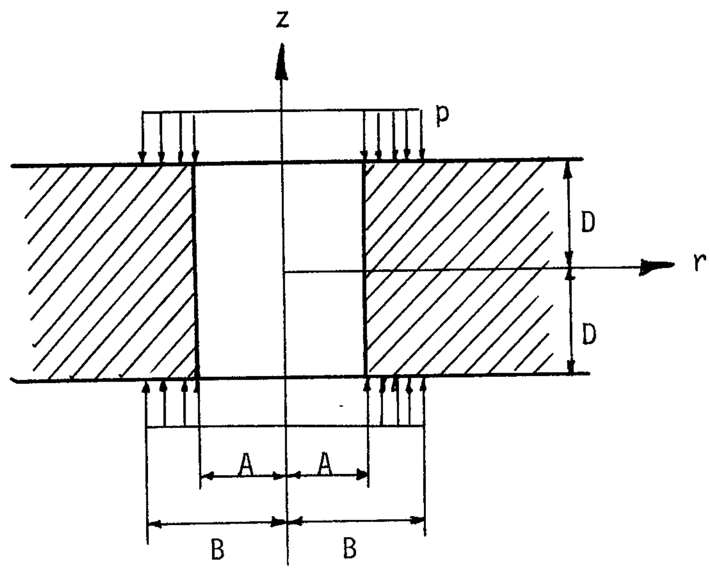
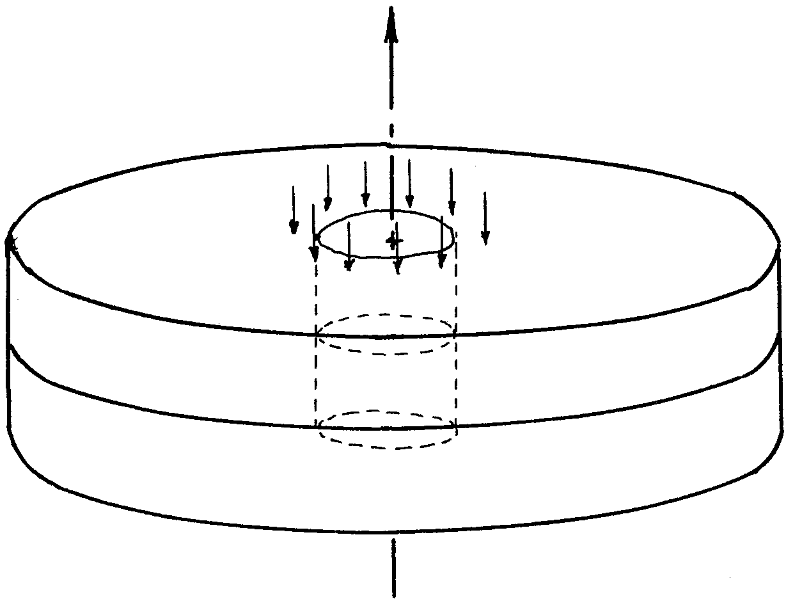
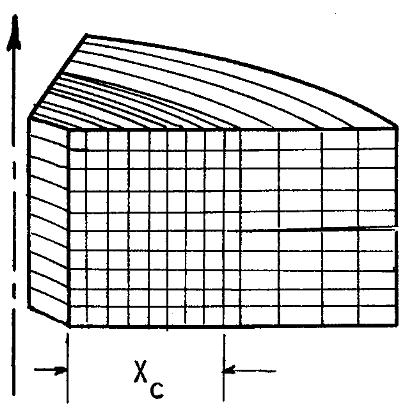


FIG. 3(f)

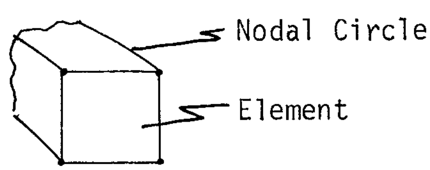
FIG. 3. FERNLUND'S SEQUENCE OF SUPERPOSITION



(a) Actual Plates

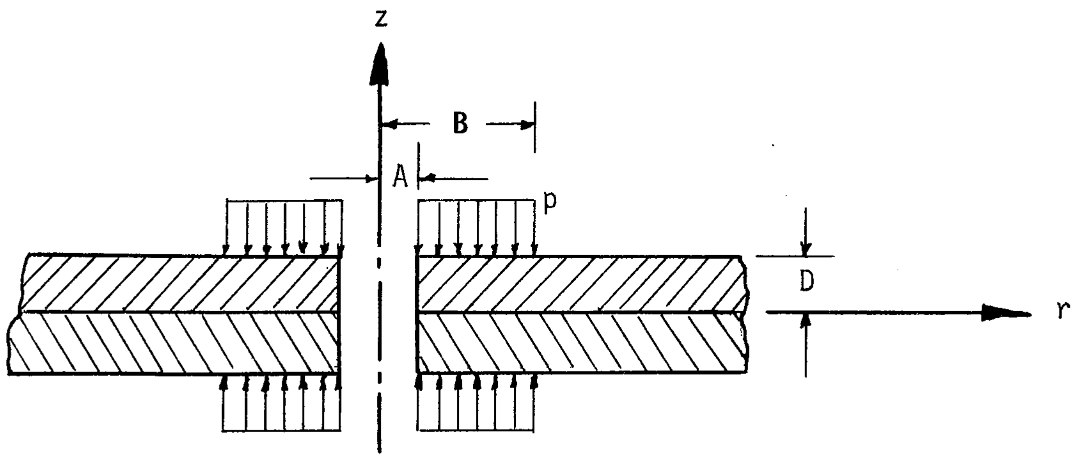


(b) Finite Element Idealization

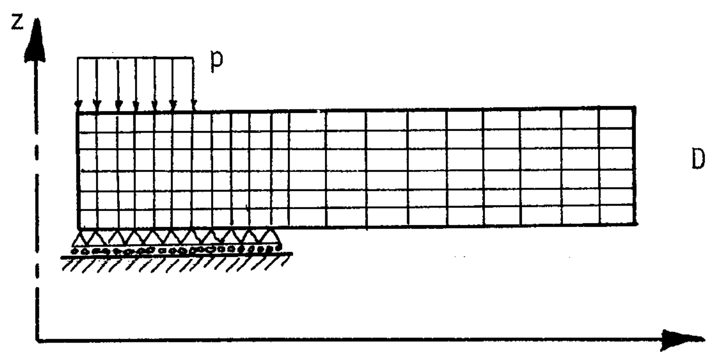


(c) Single Annular Ring Element

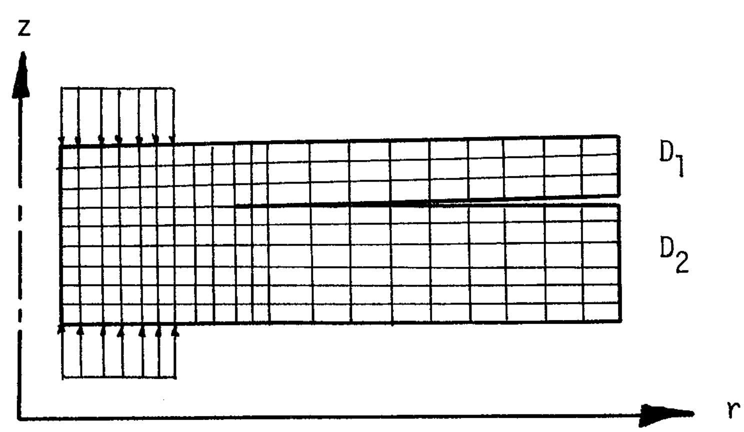
FIG. 4. FINITE ELEMENT IDEALIZATION OF TWO PLATES IN CONTACT



(a) Plates of Equal Thickness Under Load

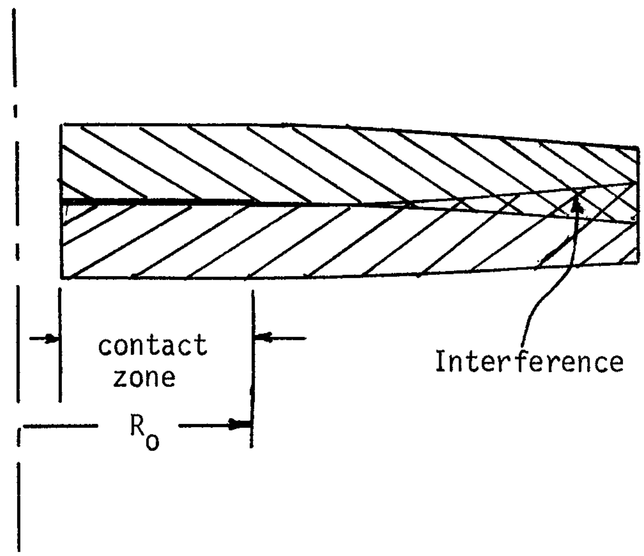


(b) Finite Element Model for Plates of Equal Thickness

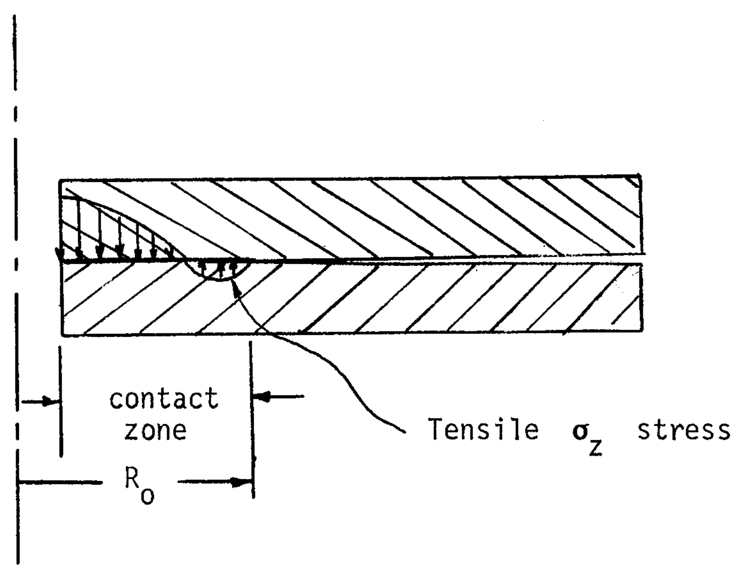


(c) Finite Element Model for Plates of Unequal Thickness

FIG. 5. FINITE ELEMENT MODELS



(a) Plates Intersect, R_o too small



(b) Contact Zone Sustains Tension, R_o too large

FIG. 6. EXAMPLES OF UNACCEPTABLE SOLUTIONS

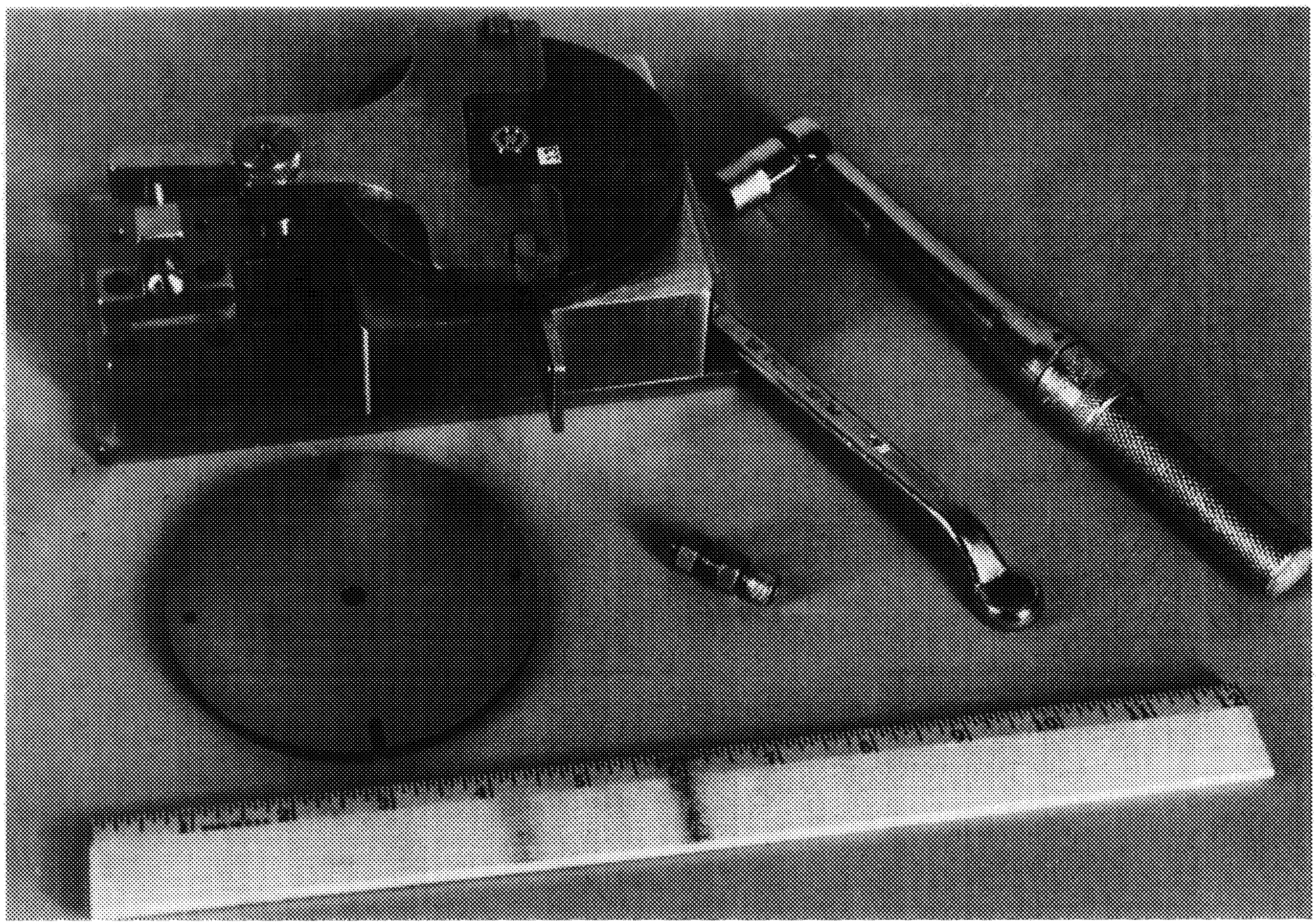


FIG. 7. PLATE SPECIMEN, BOLT AND NUTS, FIXTURE AND TOOLS.

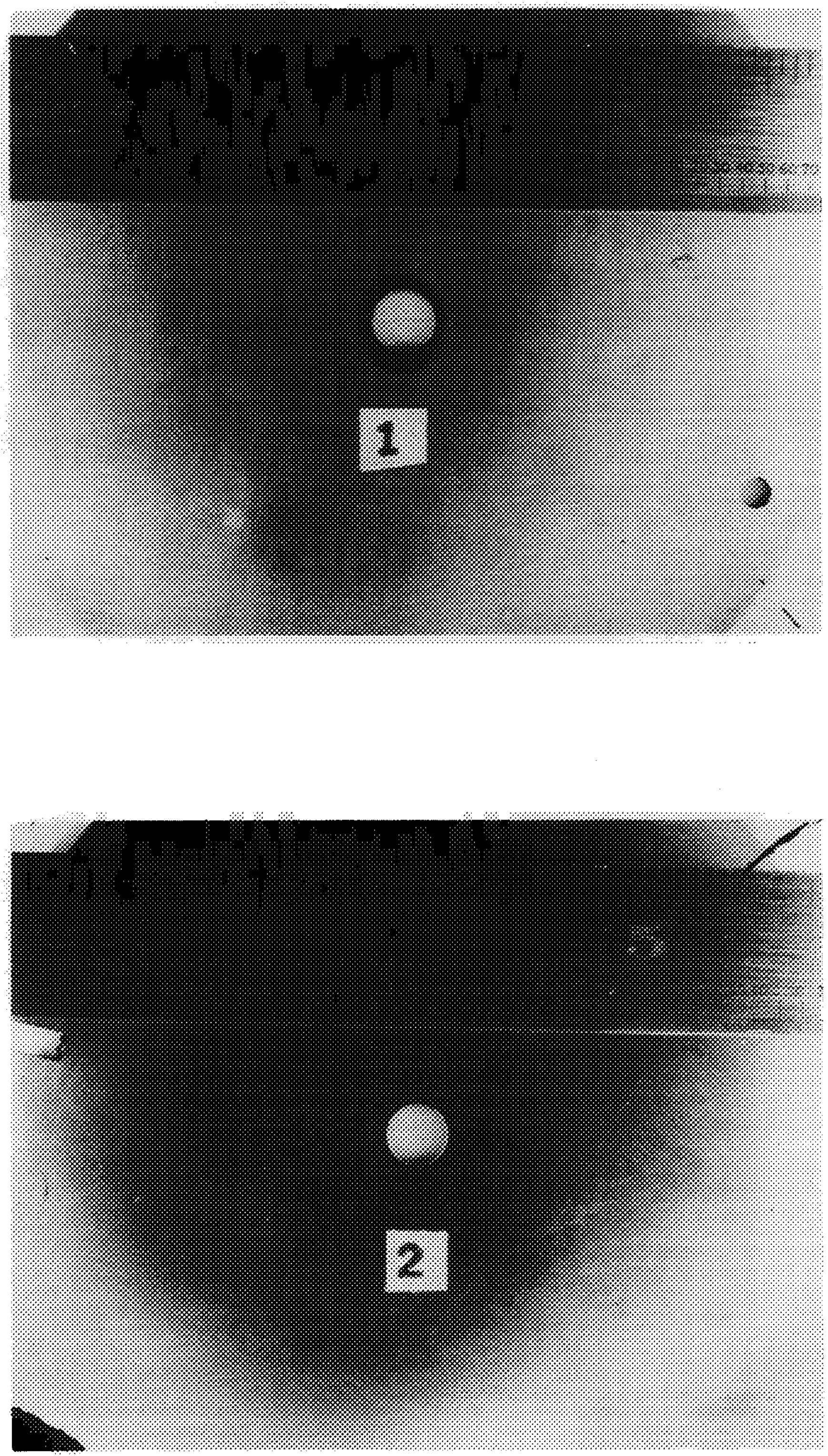


FIG. 8(a). FOOTPRINTS ON MATED PAIR OF 1/16 INCH PLATES.

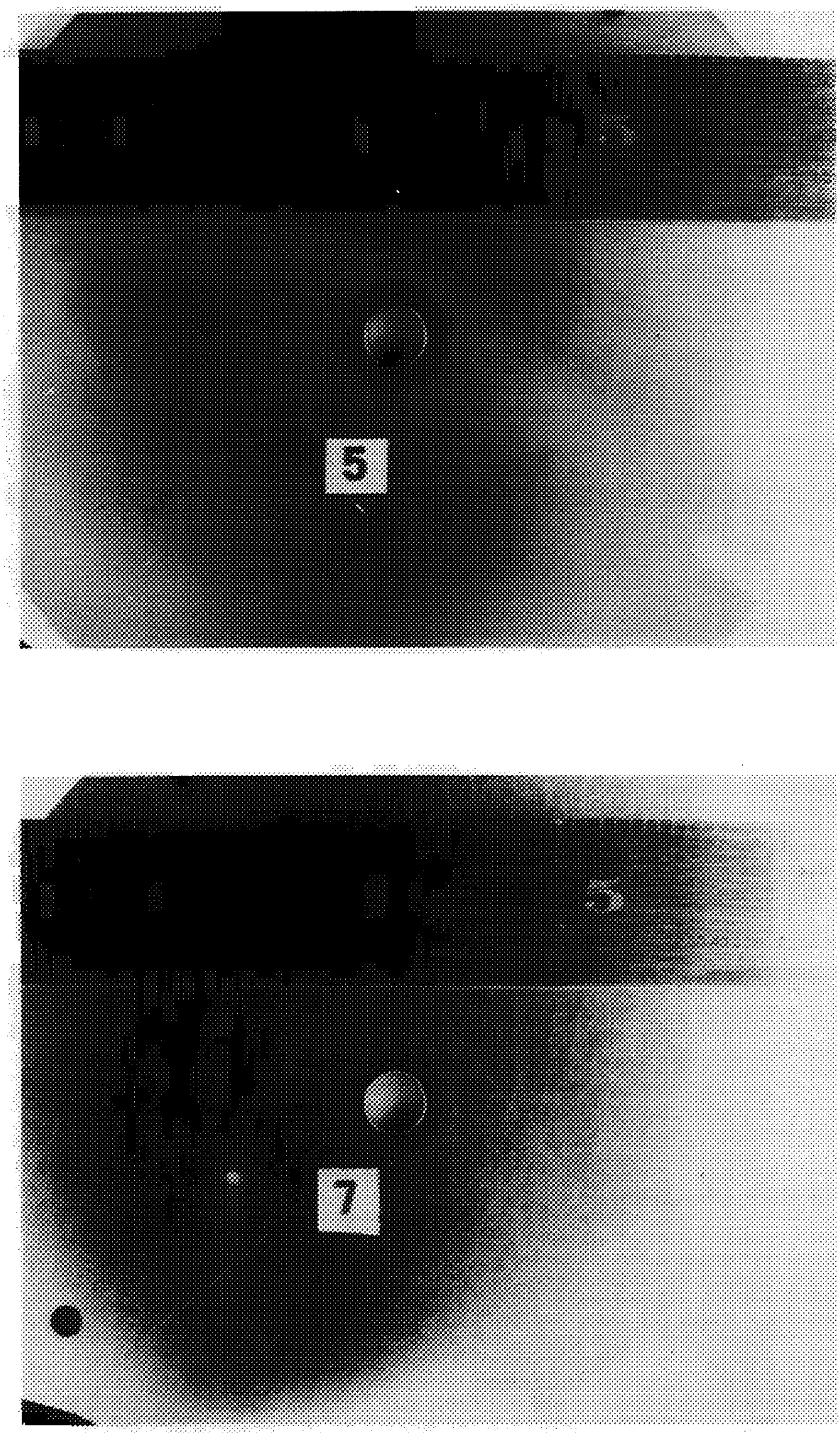


FIG. 8(b). FOOTPRINTS ON MATED PAIR OF 1/8 INCH PLATES.

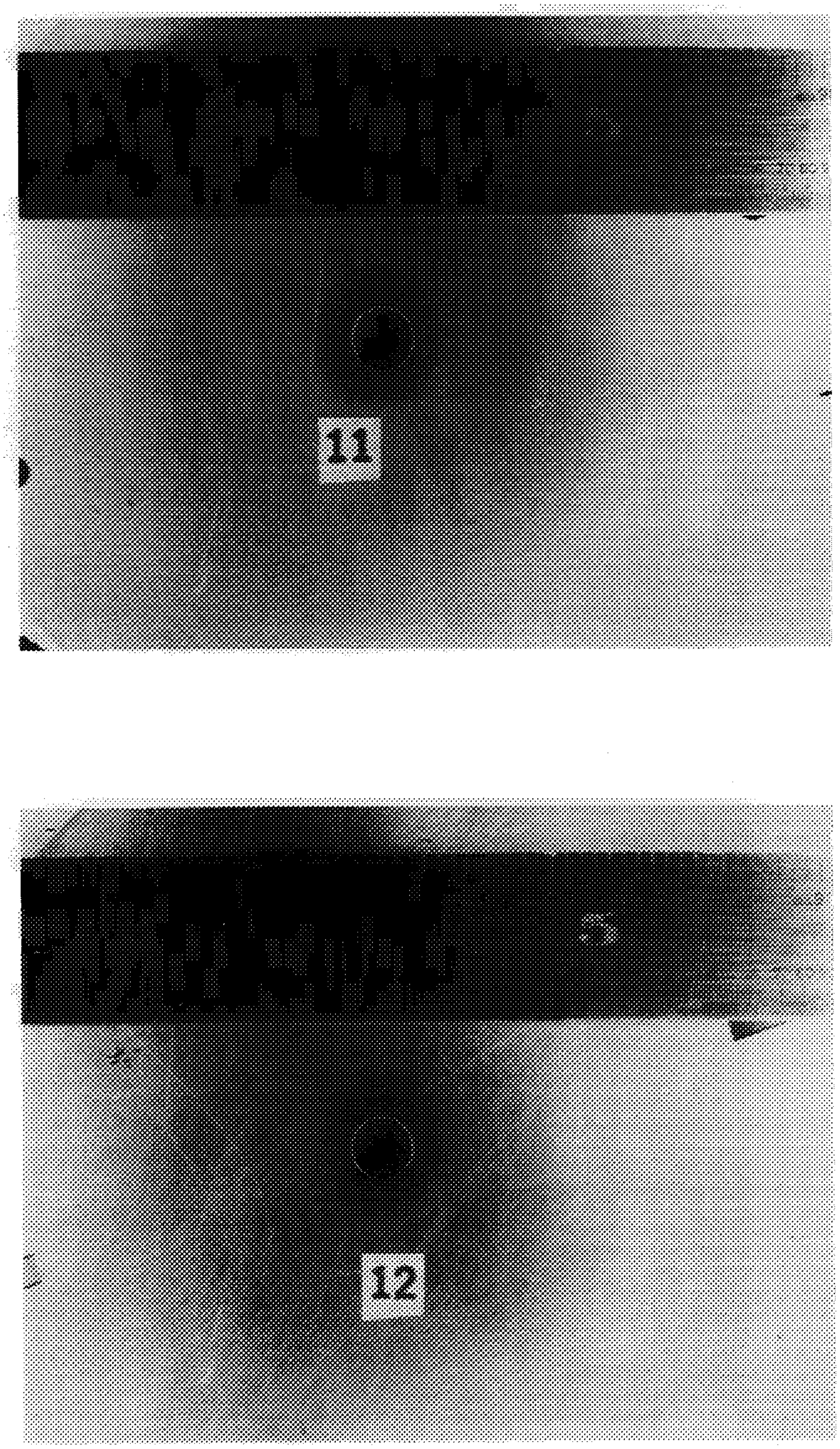


FIG. 8(c). FOOTPRINTS ON MATED PAIR OF 3/16 INCH PLATES.

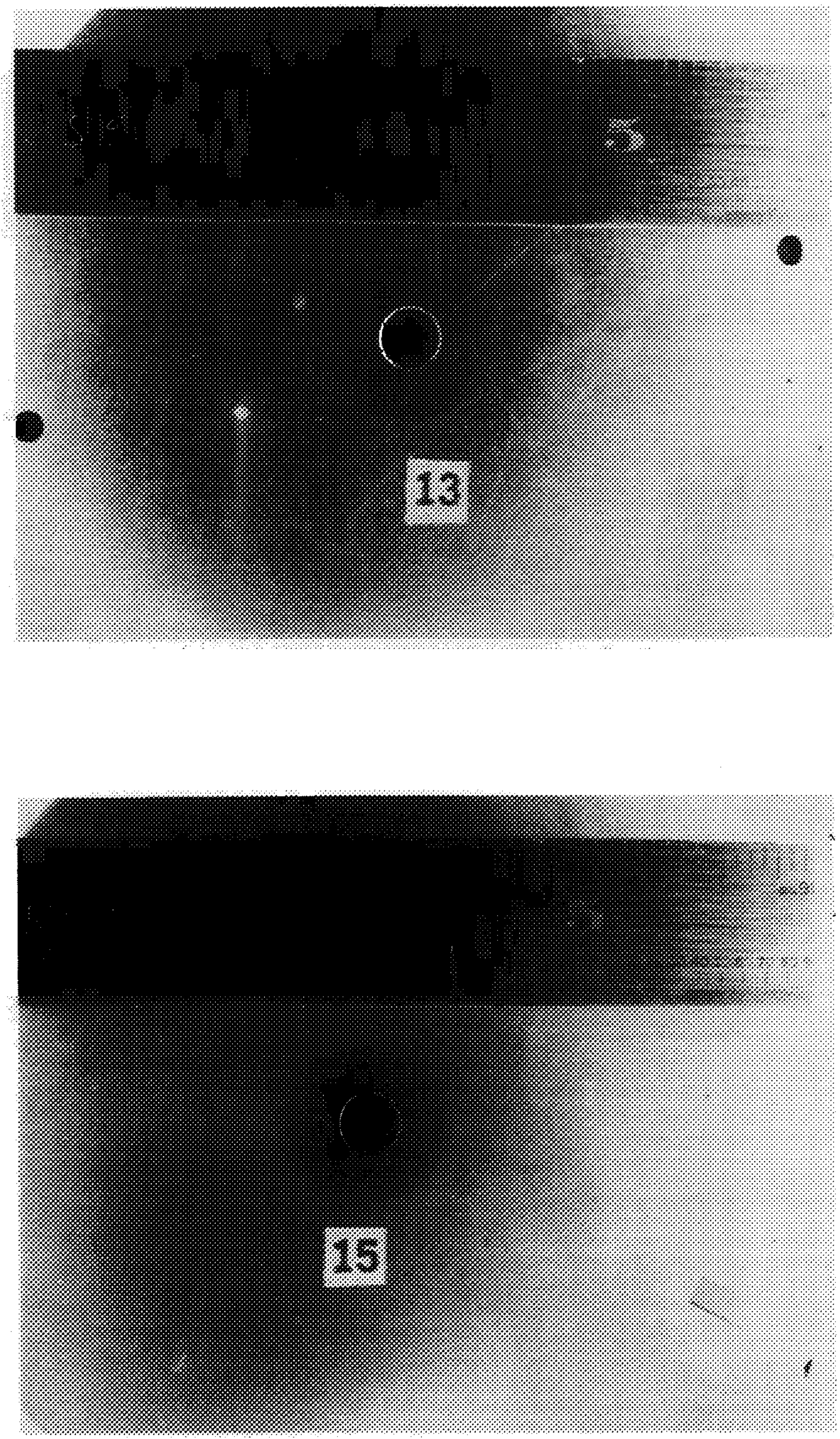


FIG. 8(d). FOOTPRINTS ON MATED PAIR OF 1/4 INCH PLATES.

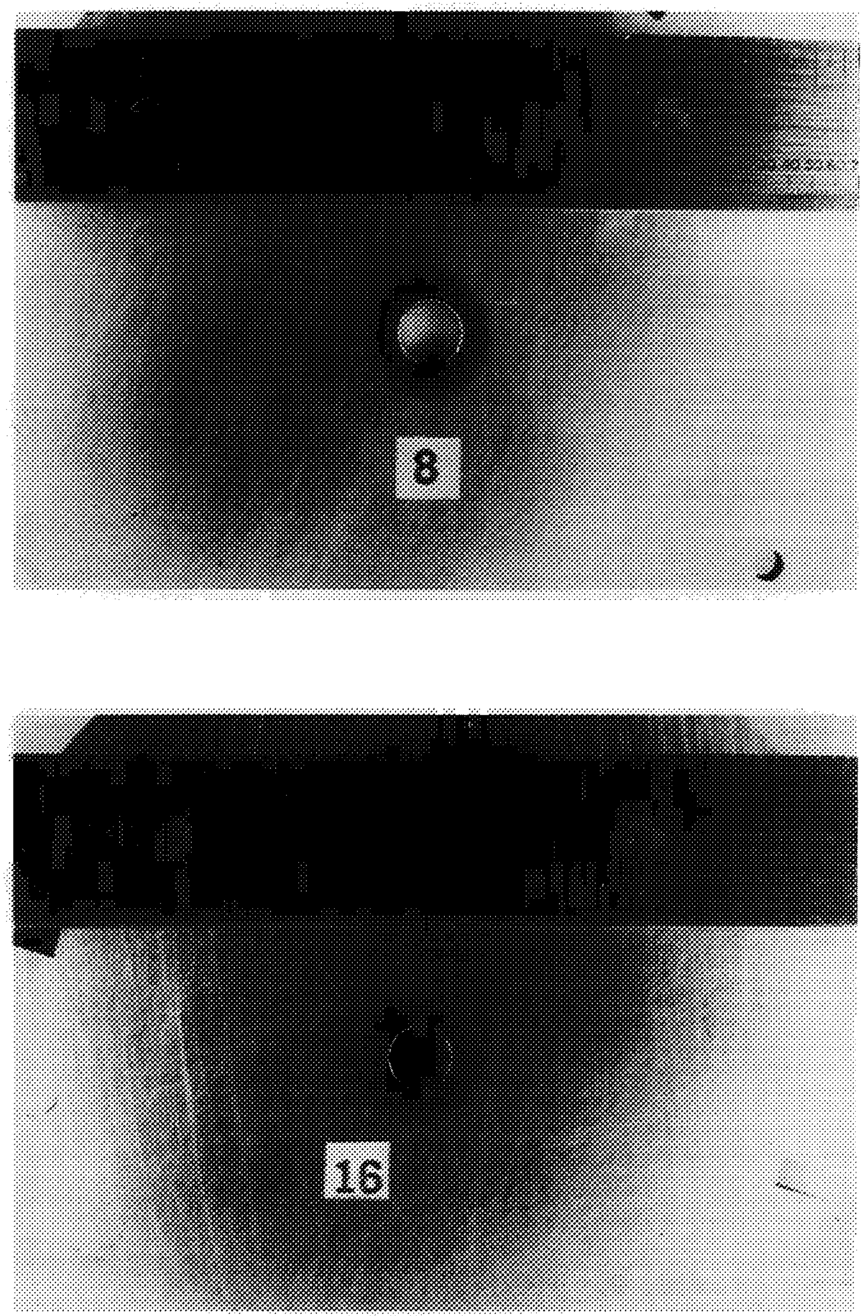


FIG. 8(e). FOOTPRINTS ON MATED PAIR OF 1/8 AND 1/4 INCH PLATES.

FIG. 8. FOOTPRINTS ON THE MATING SURFACES OF 1/16 - 1/16, 1/8 - 1/8, 3/16 - 3/16, 1/4 - 1/4, and 1/8 - 1/4 PAIRS. (A = .128, B = .21)

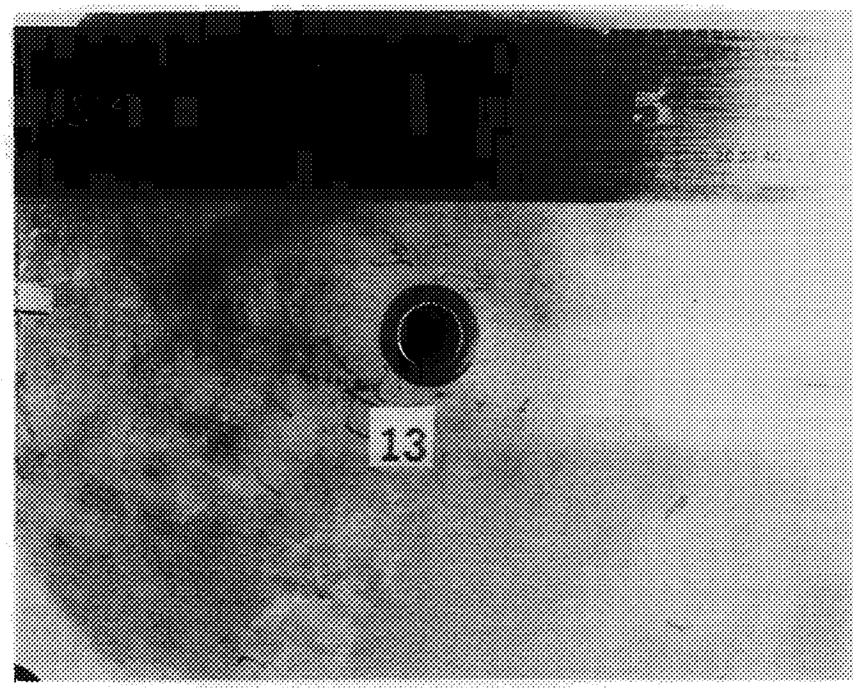


FIG. 9. FOOTPRINT OF NUT ON PLATE.

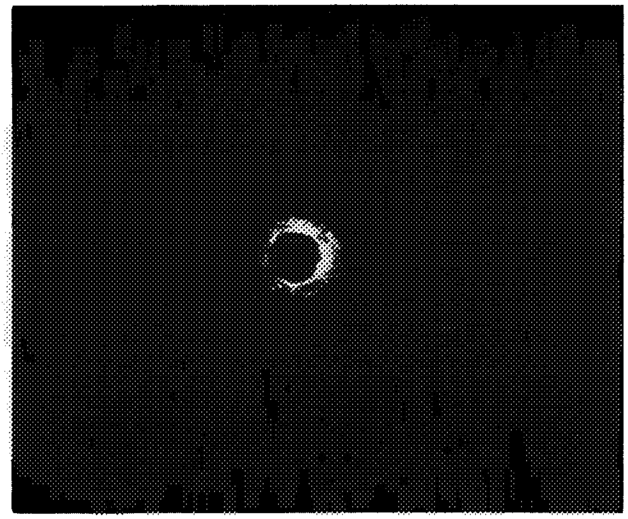


FIG. 10 (a). 1/16 INCH PAIR

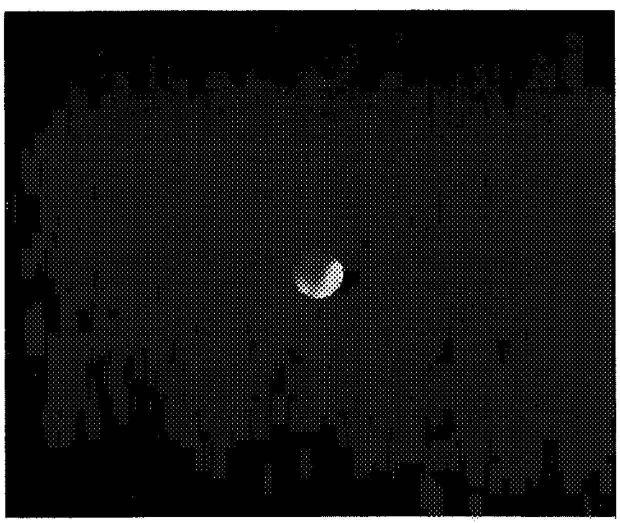


FIG. 10(b). 1/8 INCH PAIR

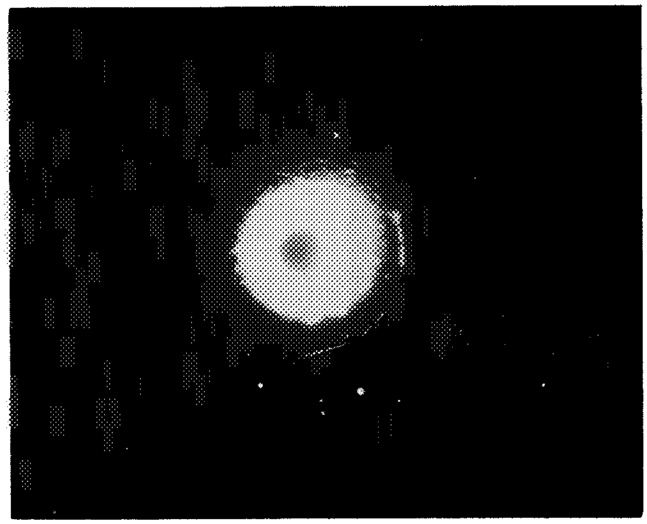


FIG. 10(c). 3/16 INCH PAIR

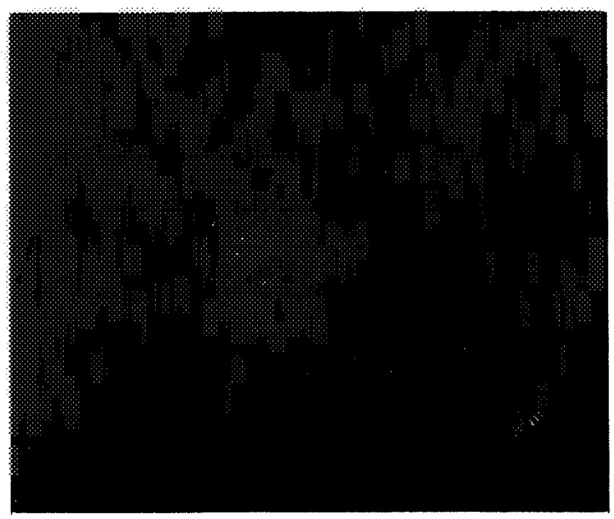


FIG. 10(d). 1/4 INCH PAIR

FIG. 10. X-RAY PHOTOGRAPHS OF CONTAMINATION TRANSFERRED FROM RADIOACTIVE PLATE TO MATED PLATE. 1/16, 1/4, 3/16, 1/4 INCH PAIRS. (A = .128 in., B = .21 in.)

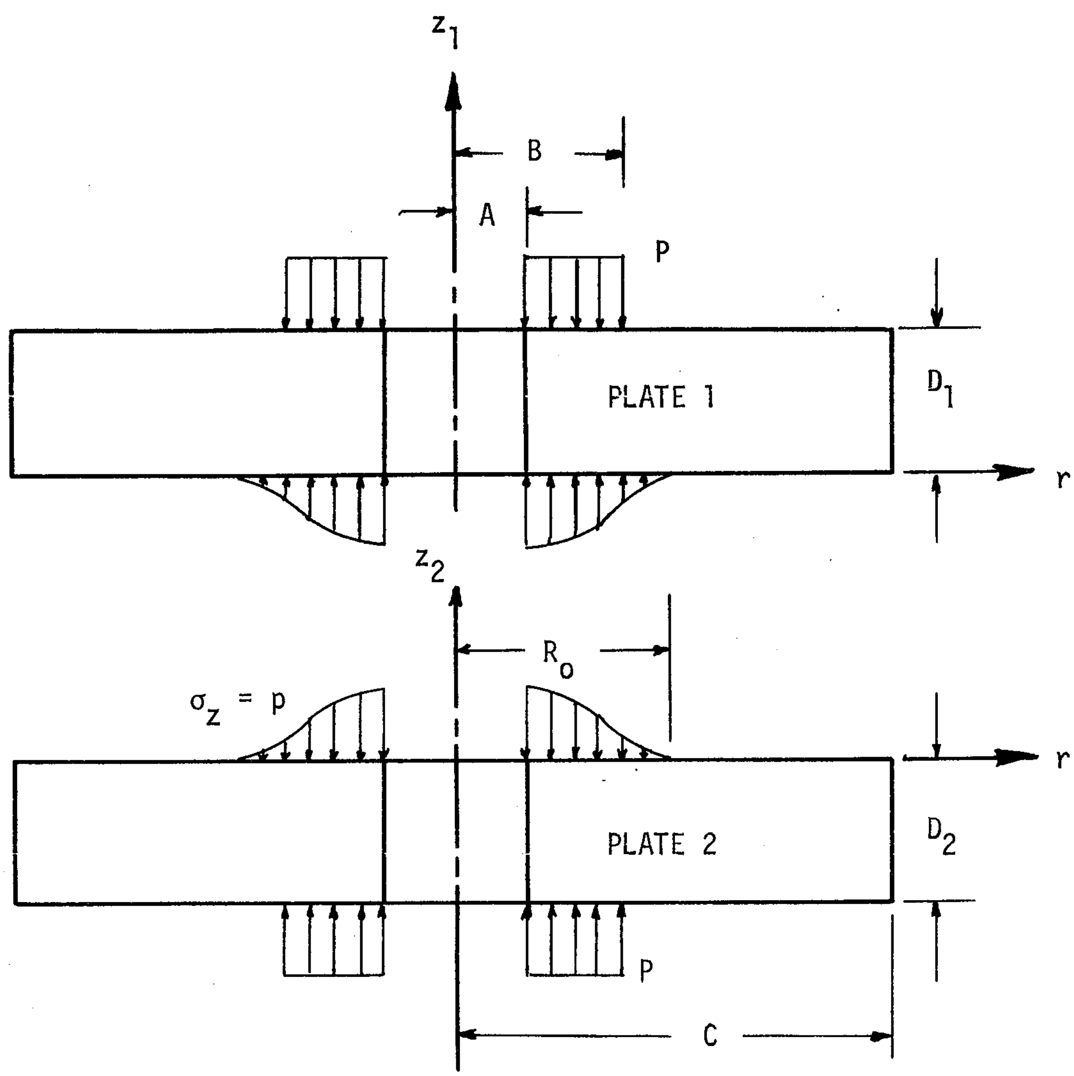


FIG. 11. FREE BODY DIAGRAM FOR TWO PLATES IN CONTACT.

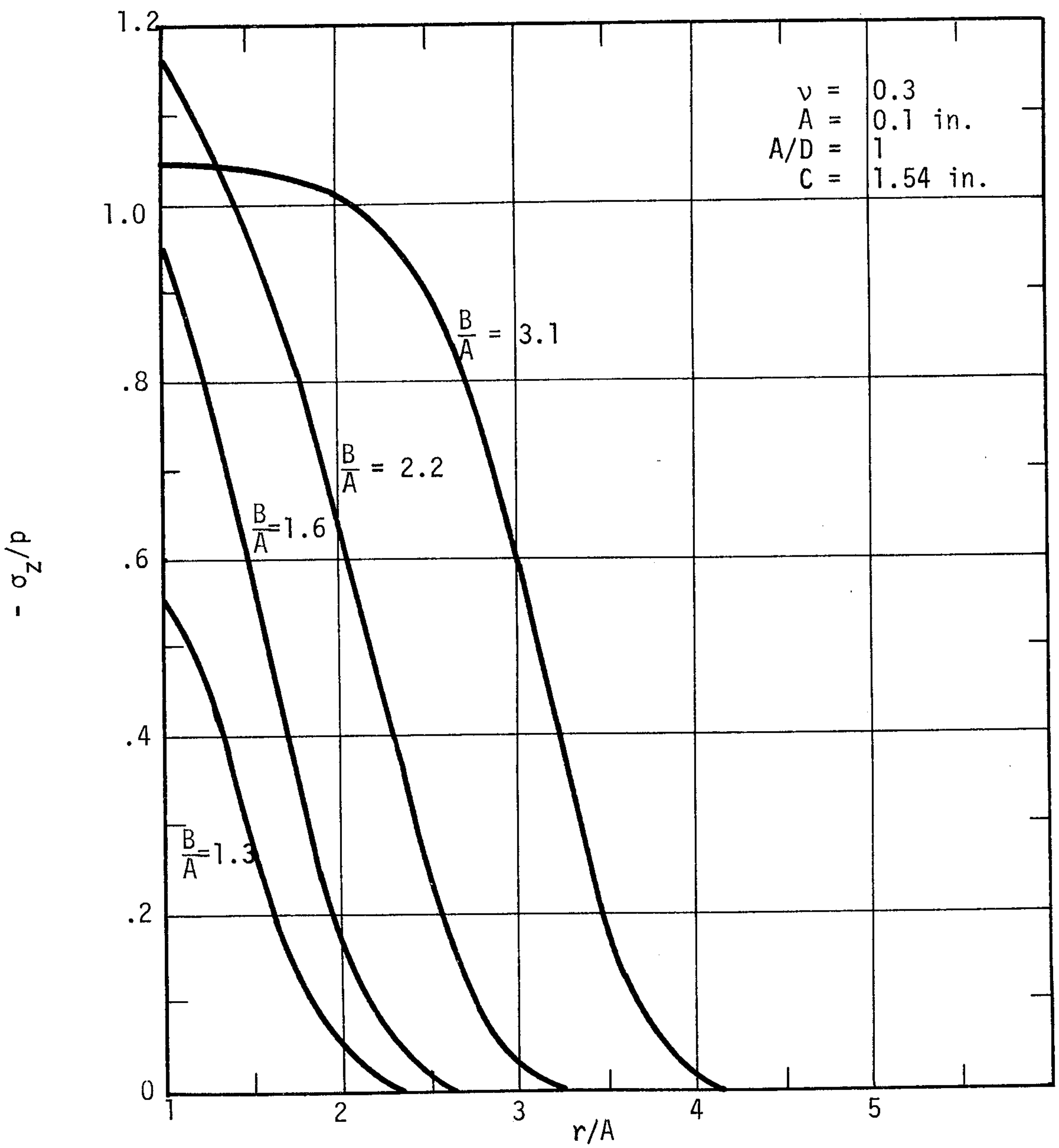


FIG. 12. SINGLE PLATE ANALYSIS-MIDPLANE σ_z STRESS DISTRIBUTION (D = 0.1 in.)

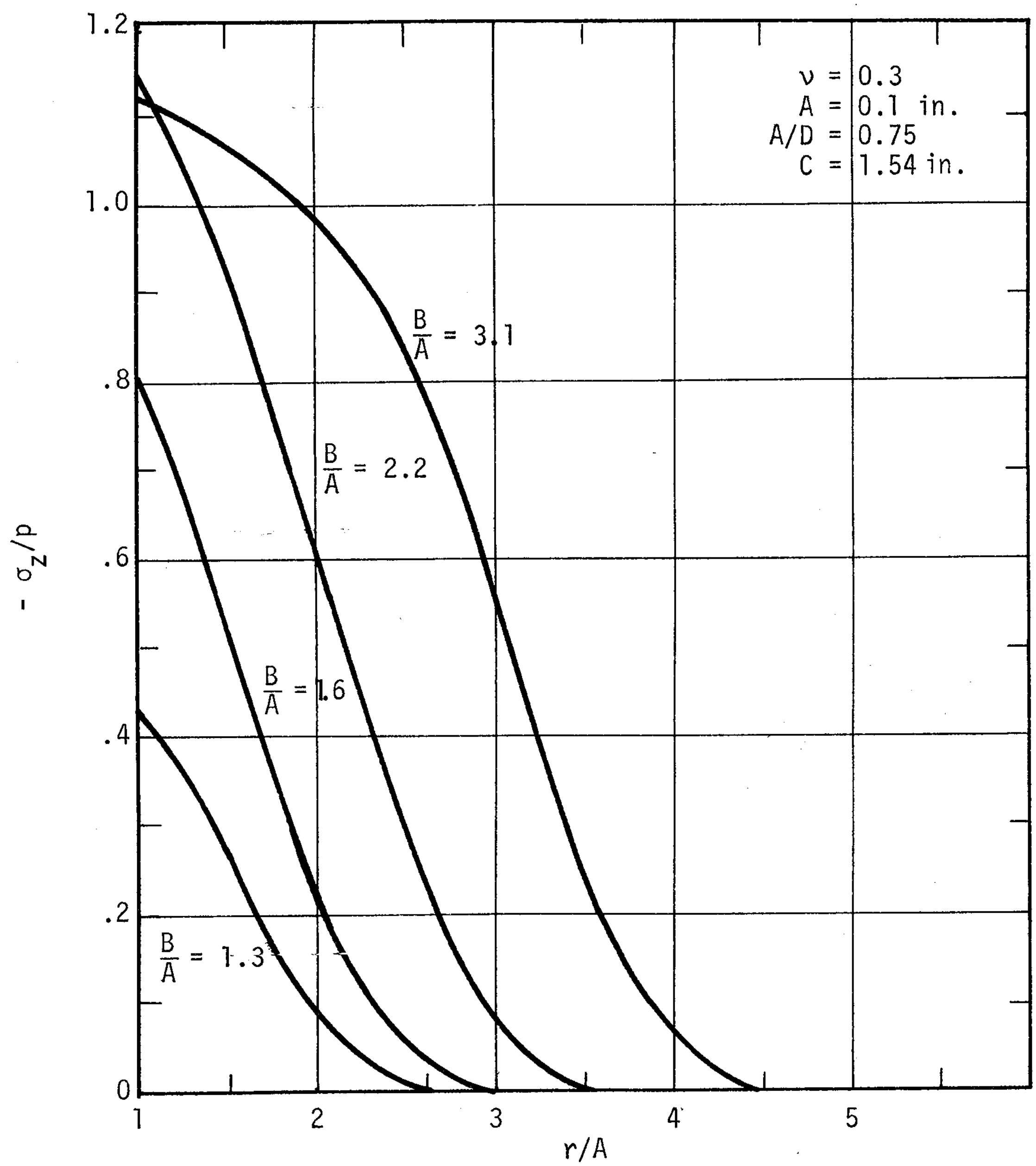


FIG. 13. SINGLE PLATE ANALYSIS-MIDPLANE σ_z STRESS DISTRIBUTION (D = 0.133 in.)

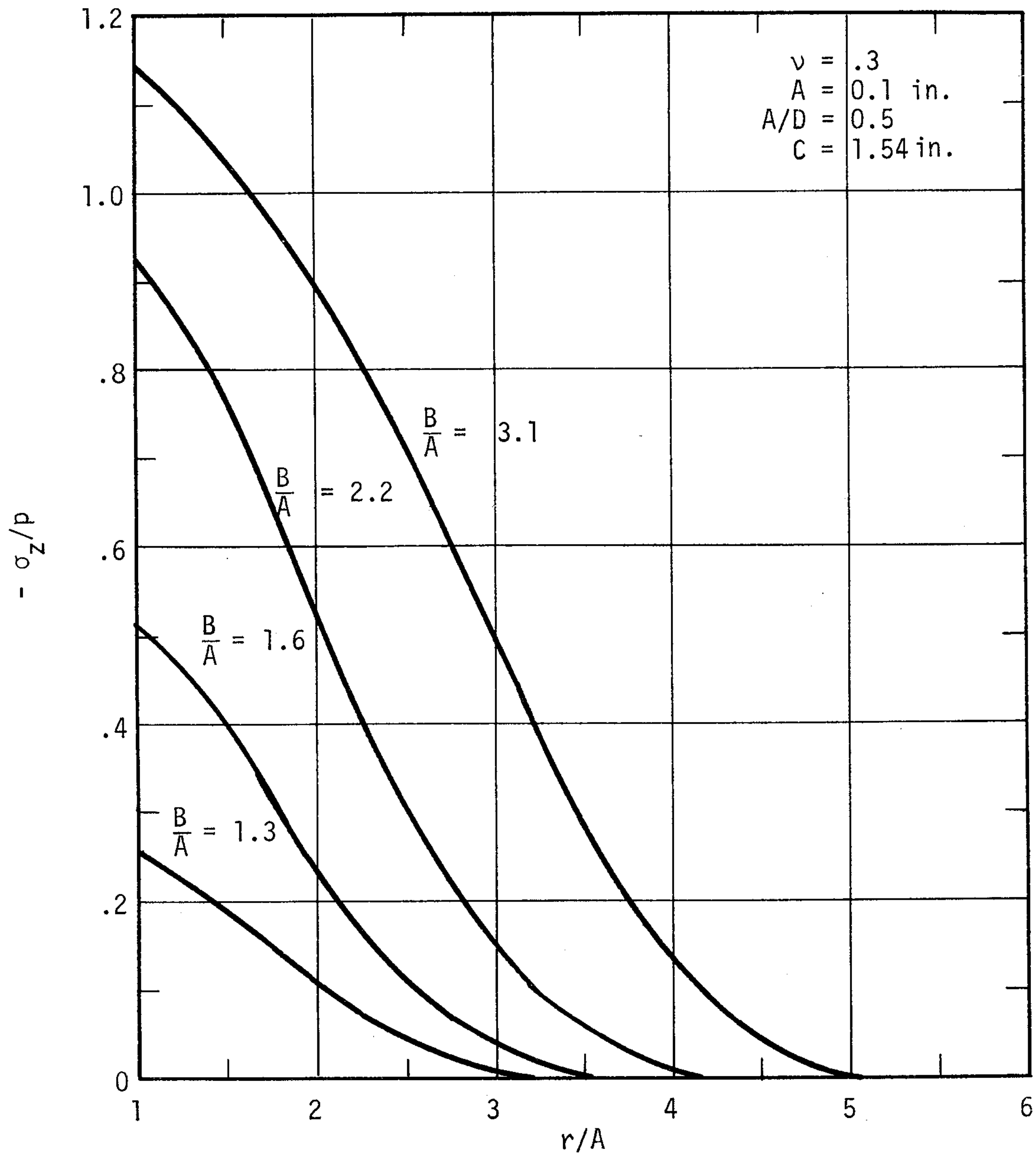


FIG. 14. SINGLE PLATE ANALYSIS-MIDPLANE σ_z STRESS DISTRIBUTION (D = 0.2 in.)

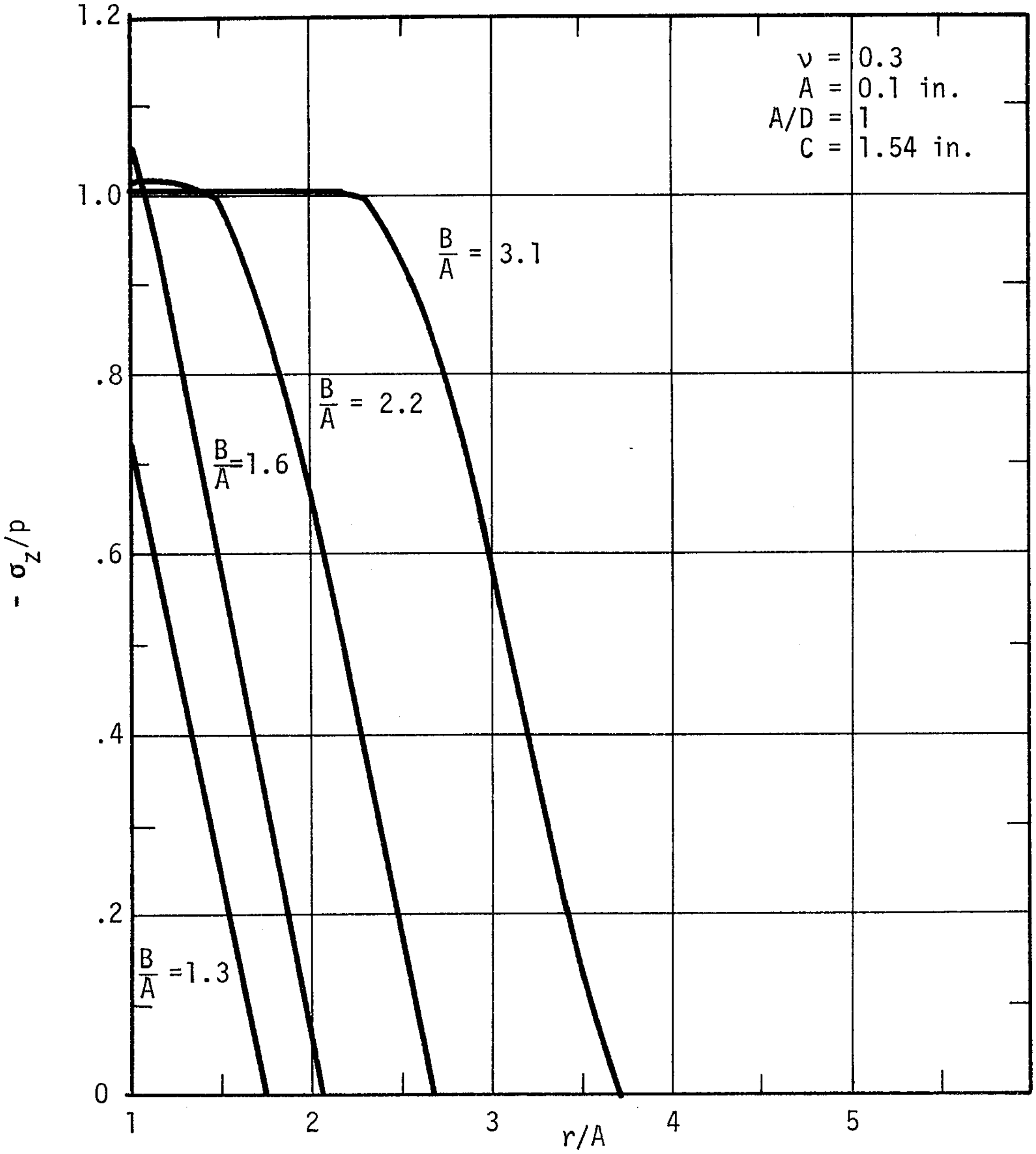


FIG. 15. INTERFACE PRESSURE DISTRIBUTION IN A BOLTED JOINT

(D = 0.1 in.)

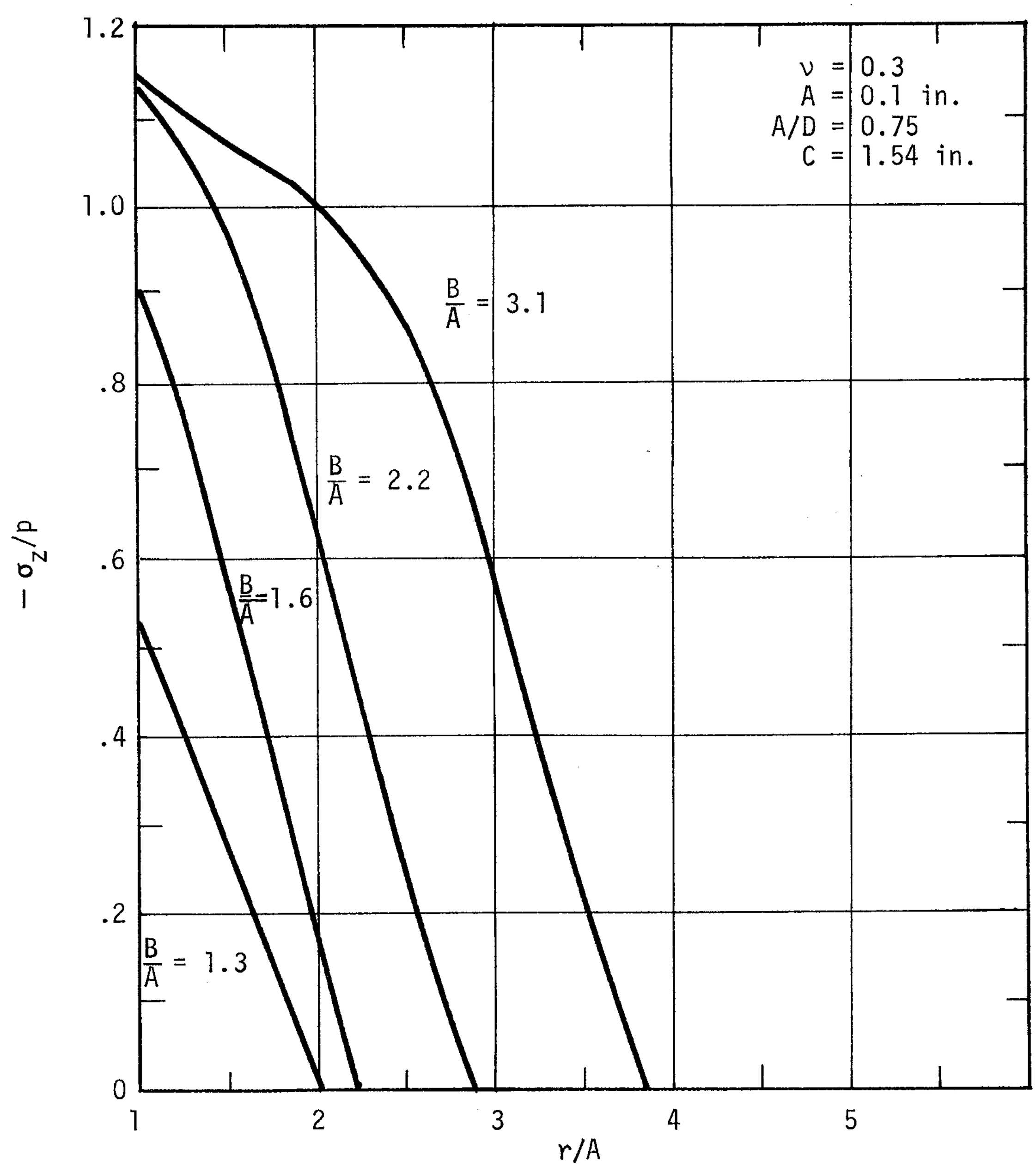


FIG. 16. INTERFACE PRESSURE DISTRIBUTION IN A BOLTED JOINT
(D = .133 in.)

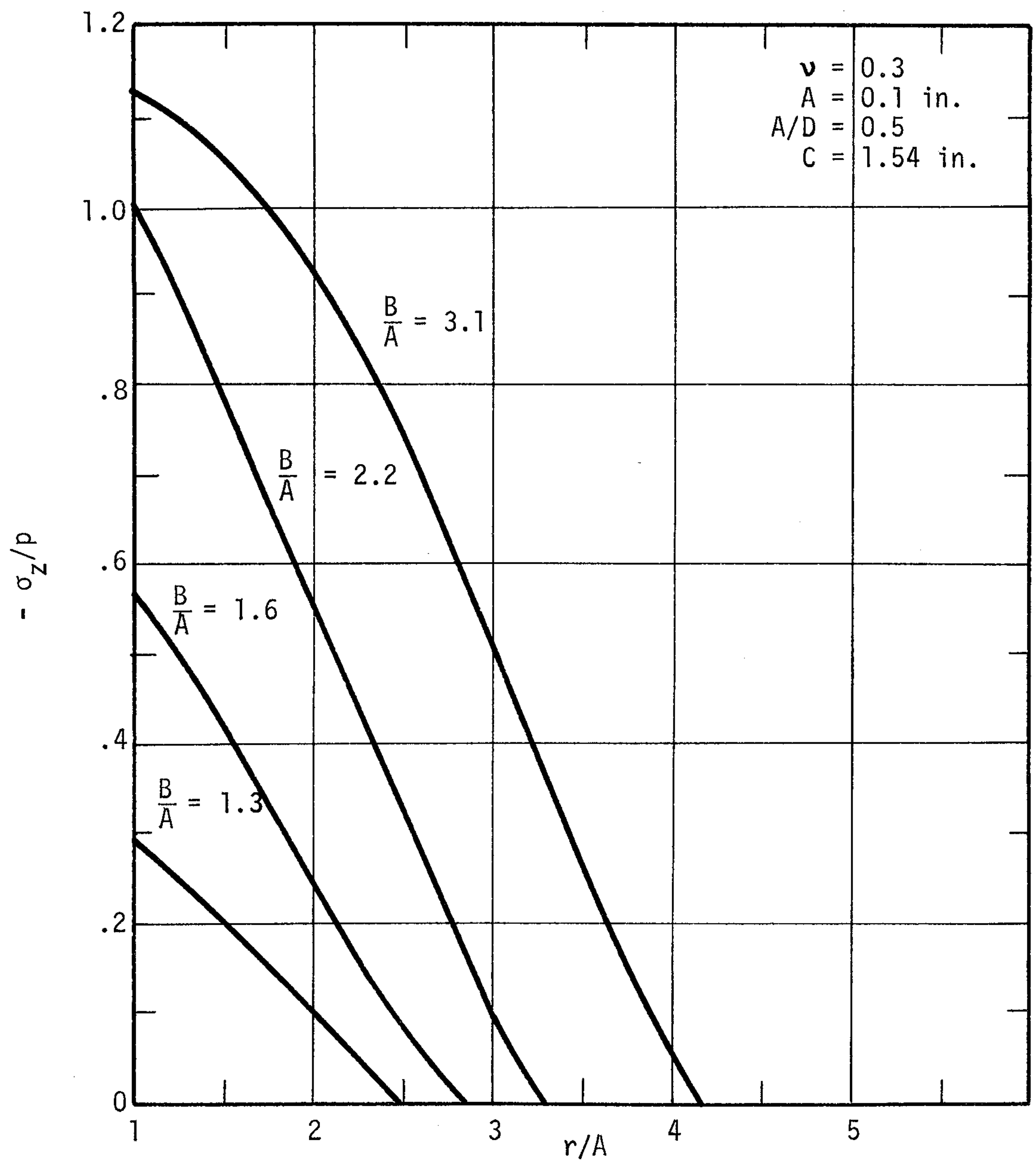


FIG. 17. INTERFACE PRESSURE DISTRIBUTION IN A BOLTED JOINT

(D = 0.2 in.)

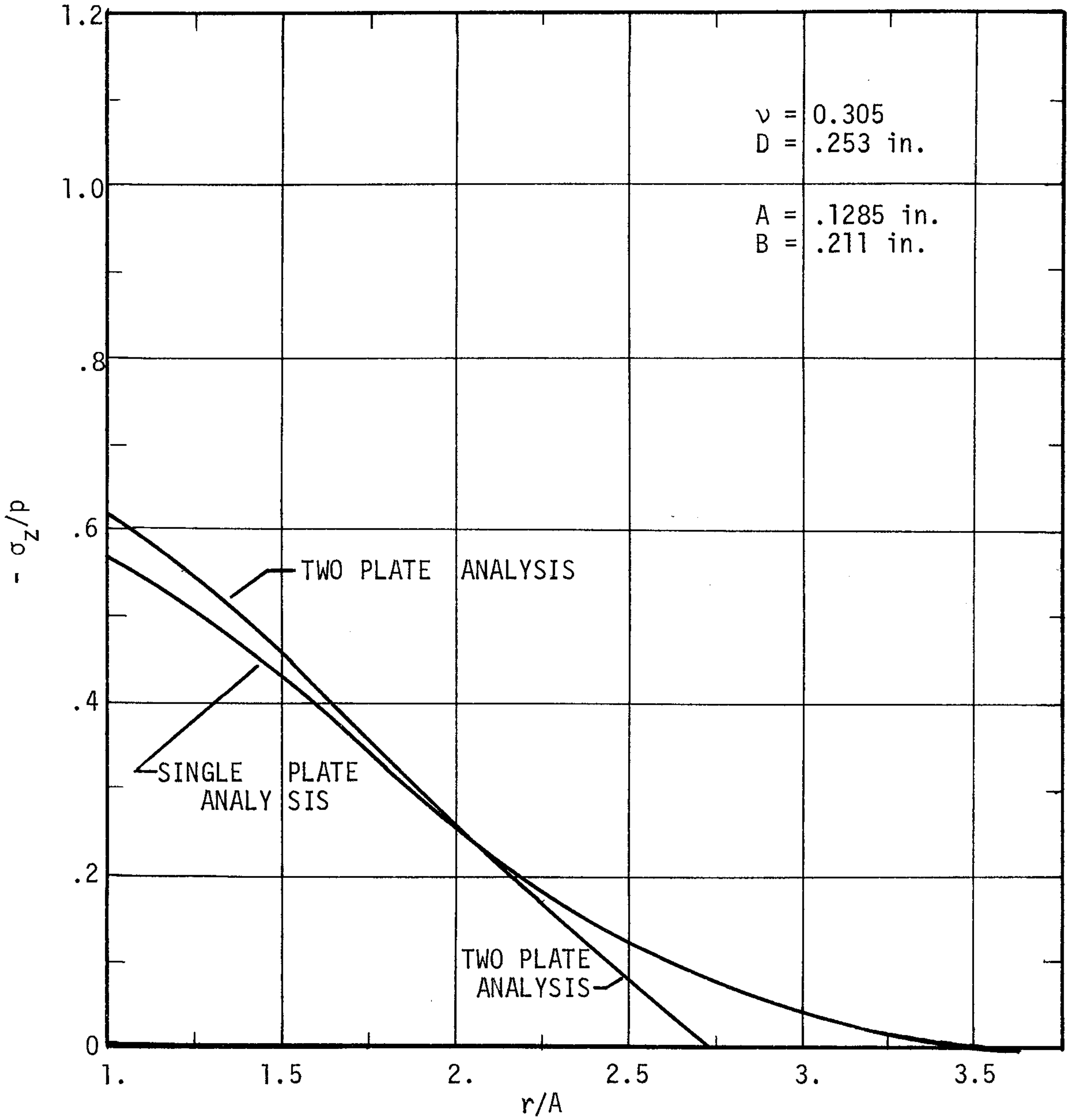


FIG. 18. FINITE ELEMENT ANALYSIS RESULTS FOR 1/4 INCH PLATE PAIR.

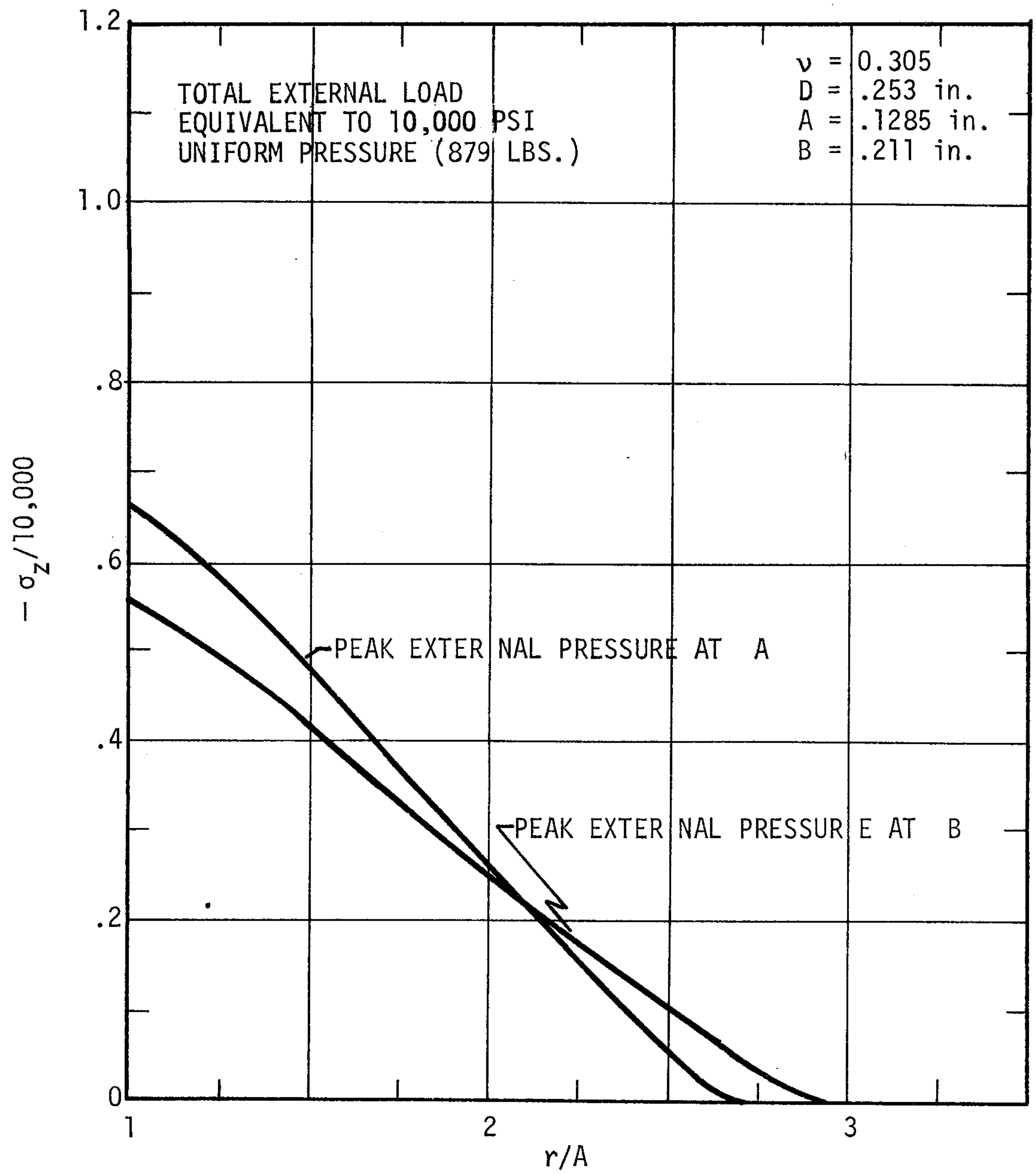


FIG. 19. PRESSURE IN JOINT, TRIANGULAR LOADING

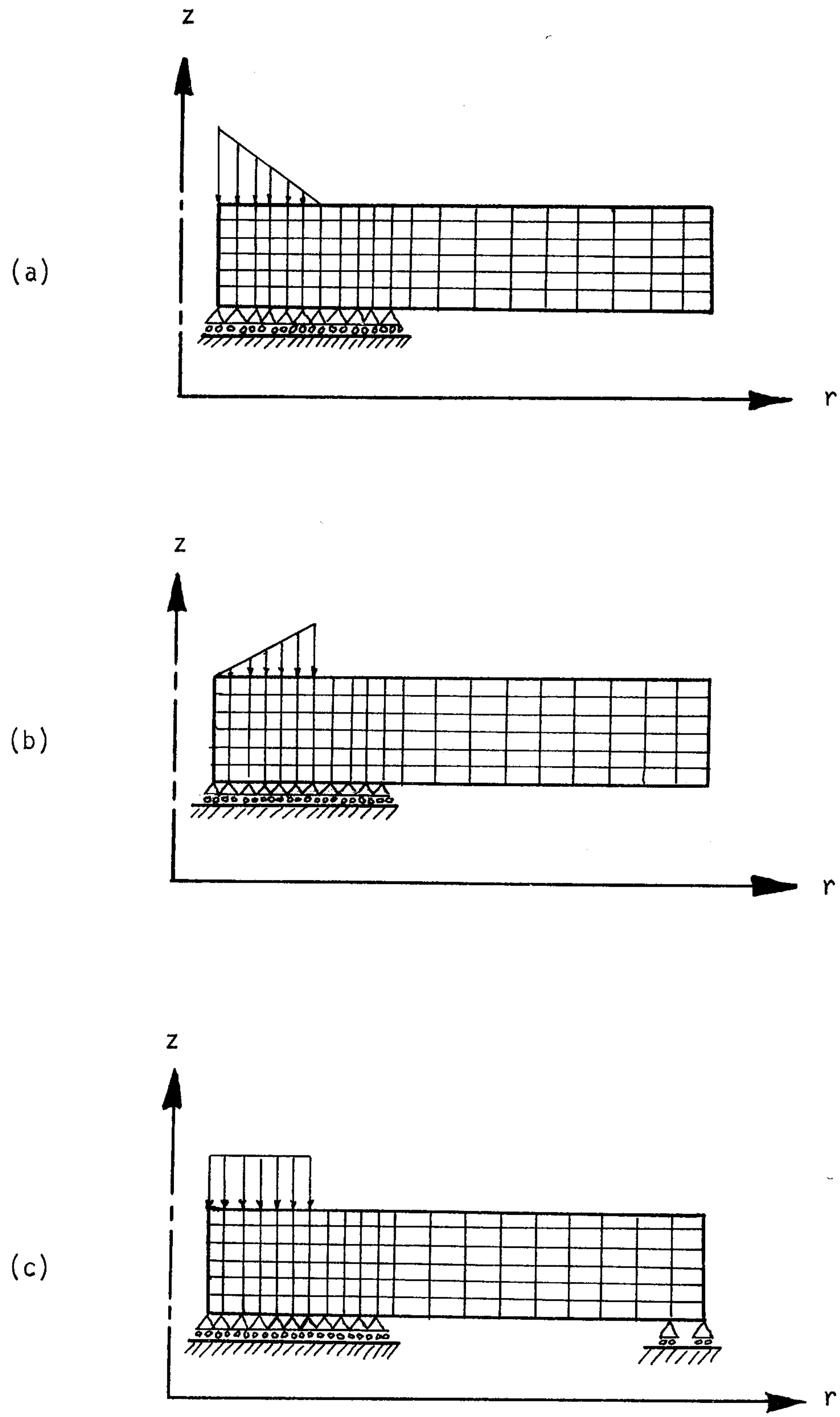


FIG. 20. VARIATIONS OF LOADING AND BOUNDARY CONDITIONS.

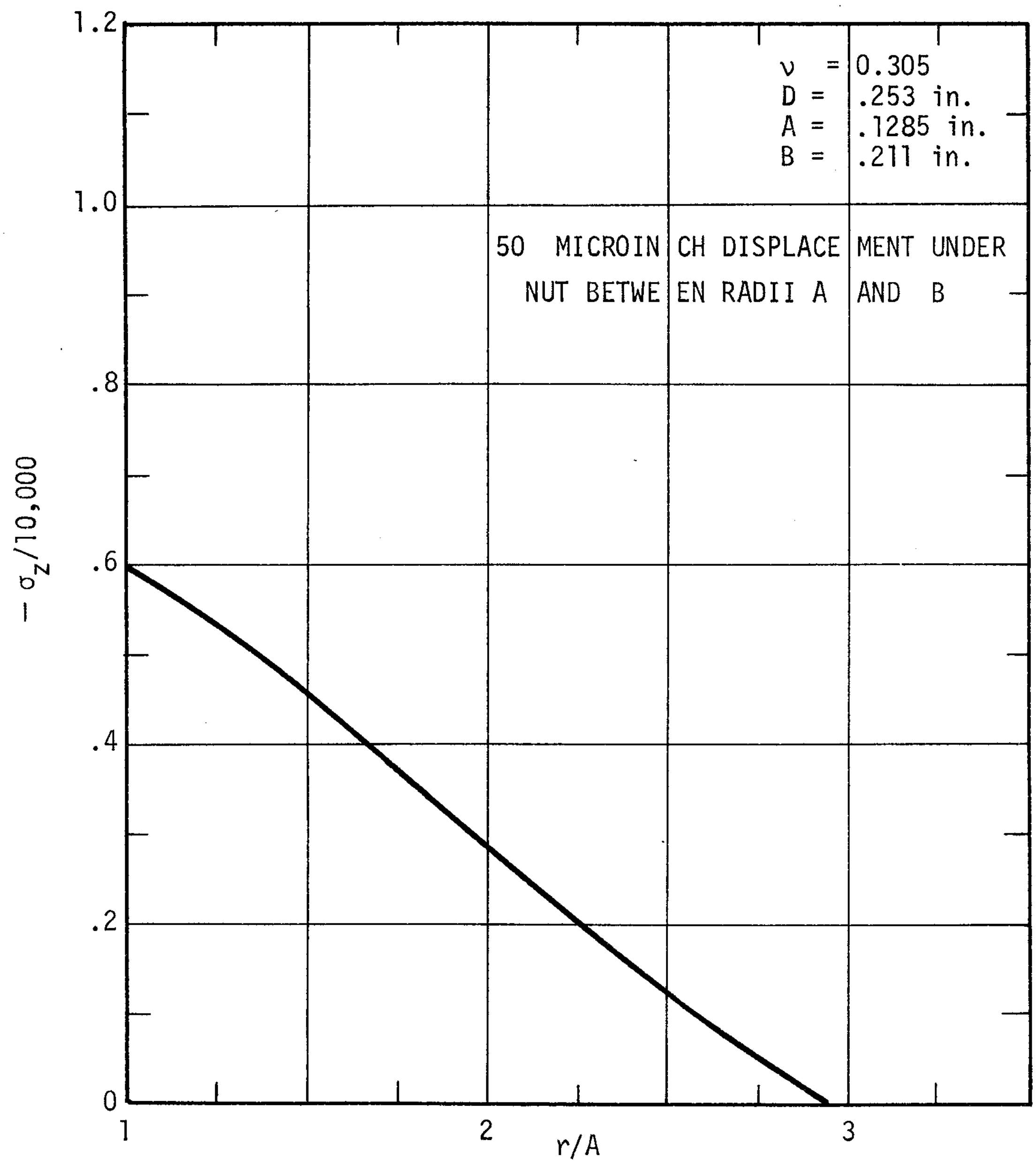


FIG. 21. PRESSURE IN JOINT, UNIFORM DISPLACEMENT UNDER NUT.

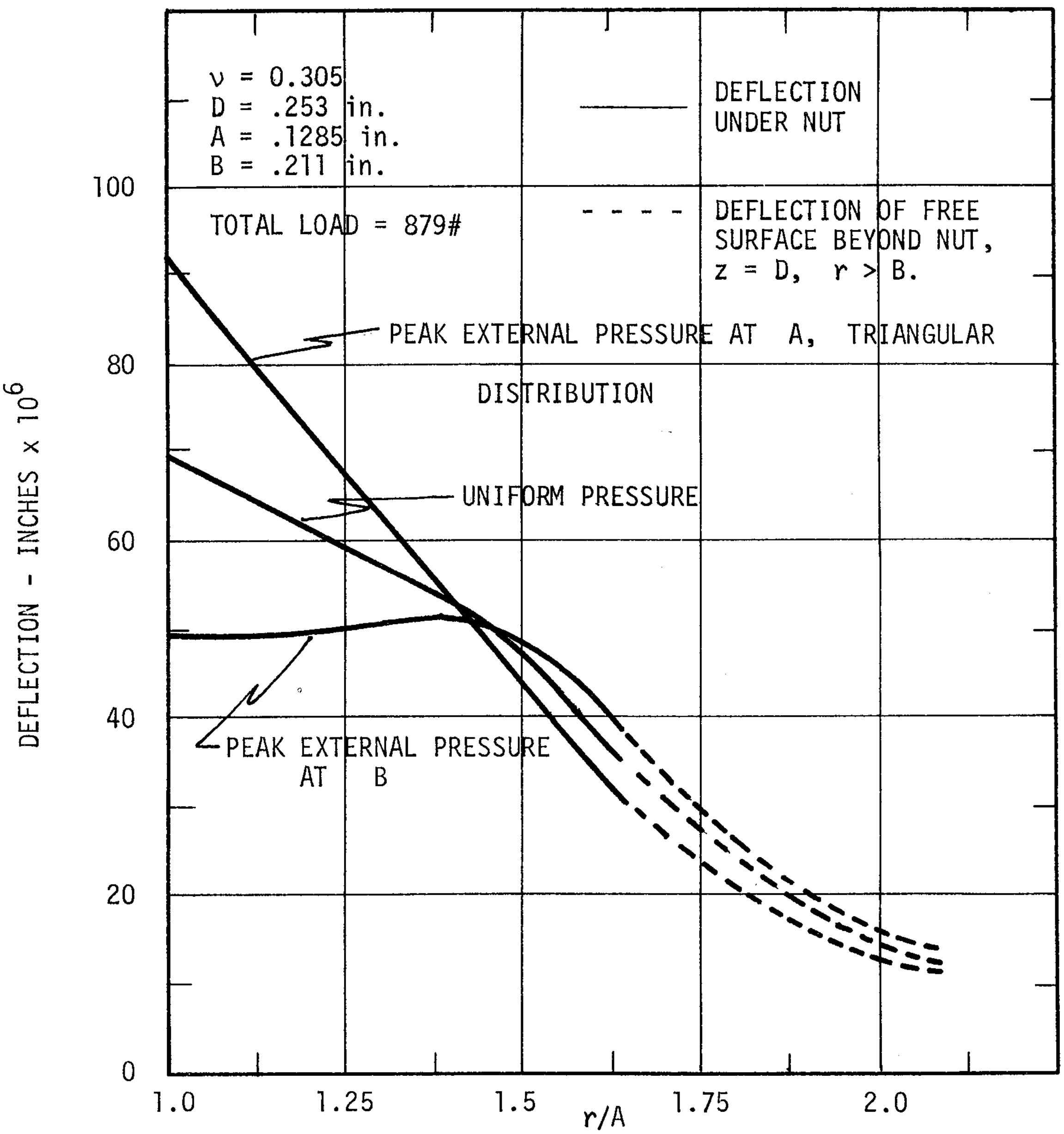


FIG. 22. DEFLECTION OF PLATE UNDER NUT.

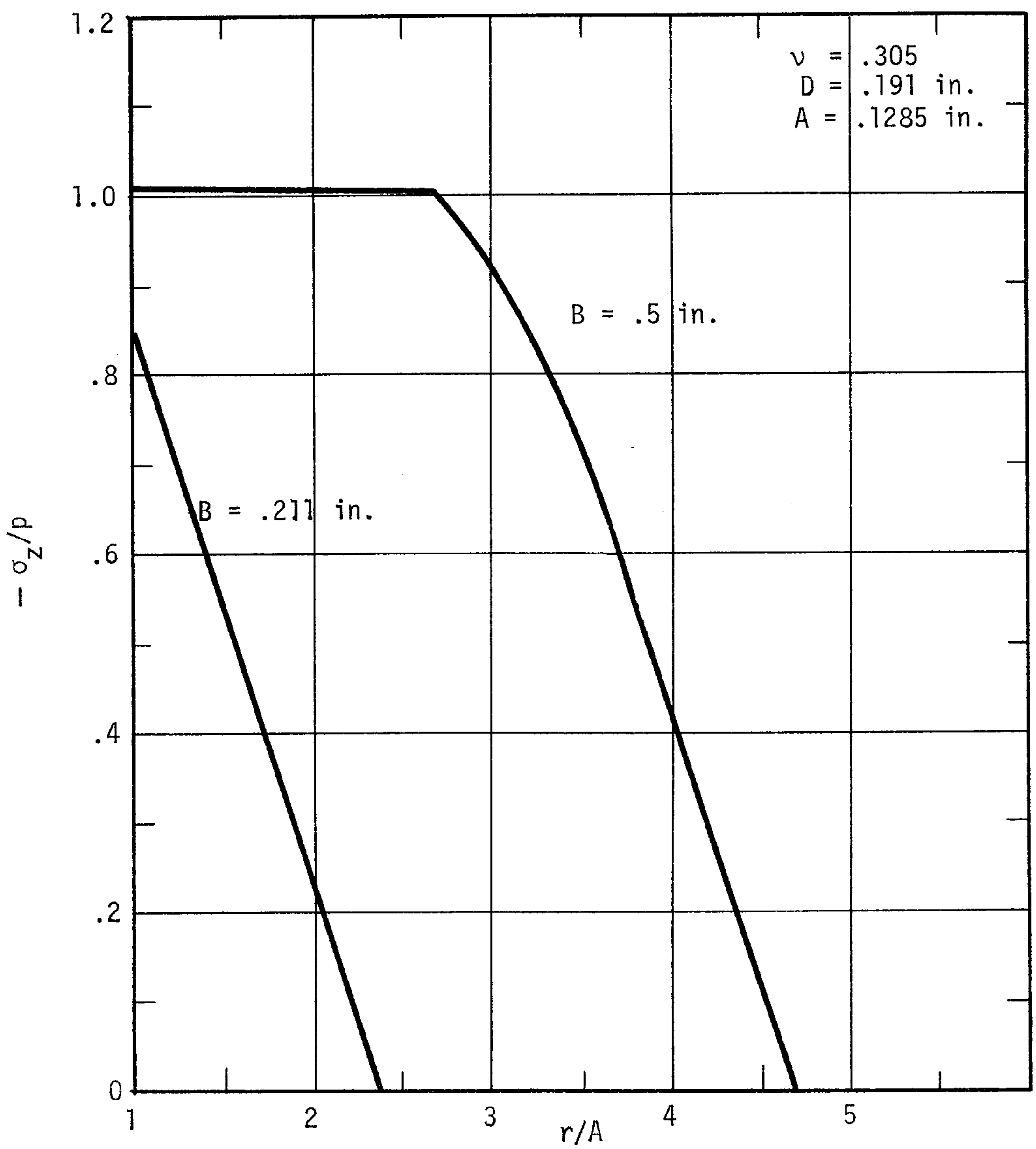


FIG. 23. FINITE ELEMENT ANALYSIS RESULTS FOR 3/16 INCH PLATE PAIR.

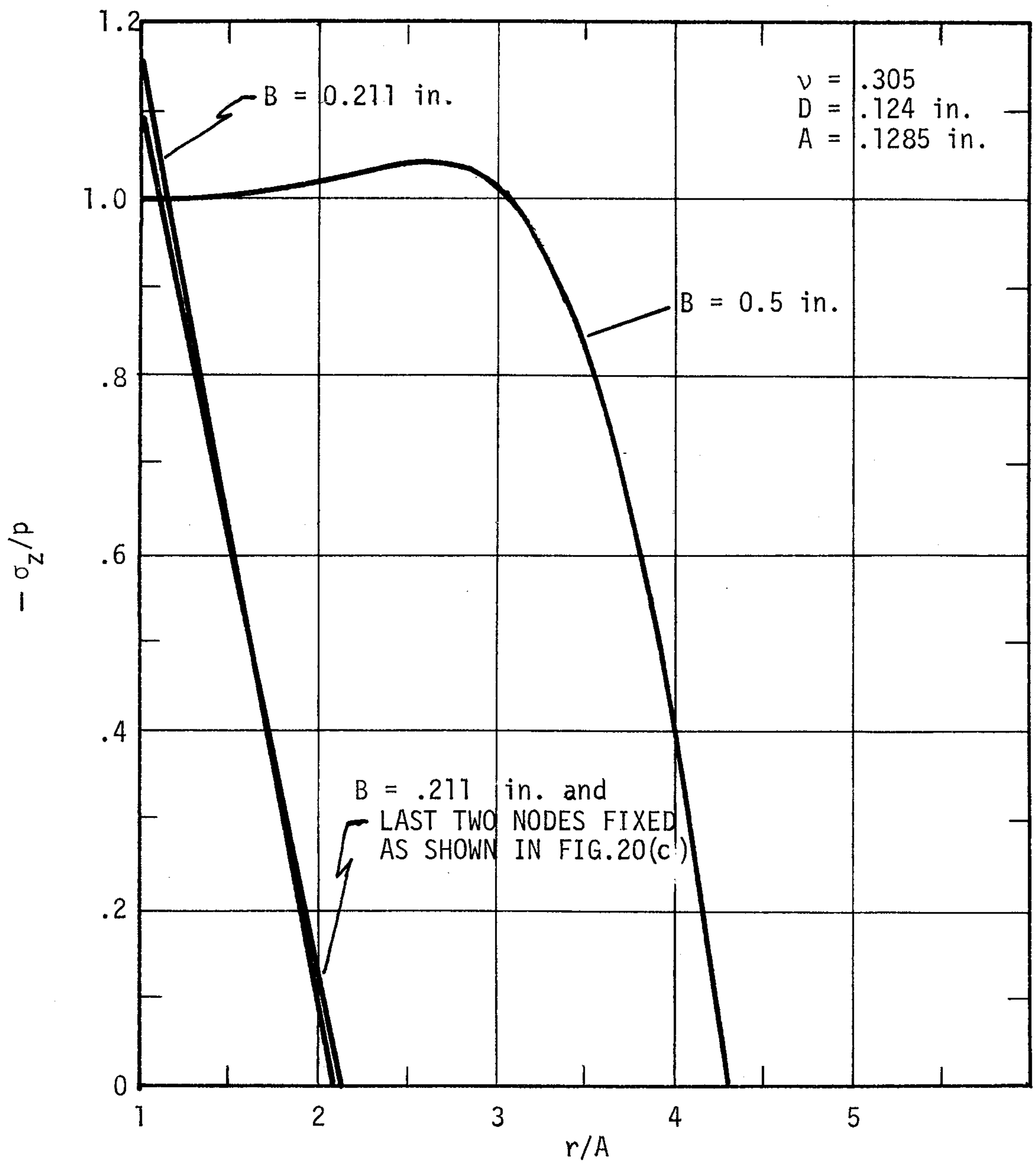


FIG. 24. FINITE ELEMENT ANALYSIS RESULTS FOR 1/8 INCH PLATE PAIR.

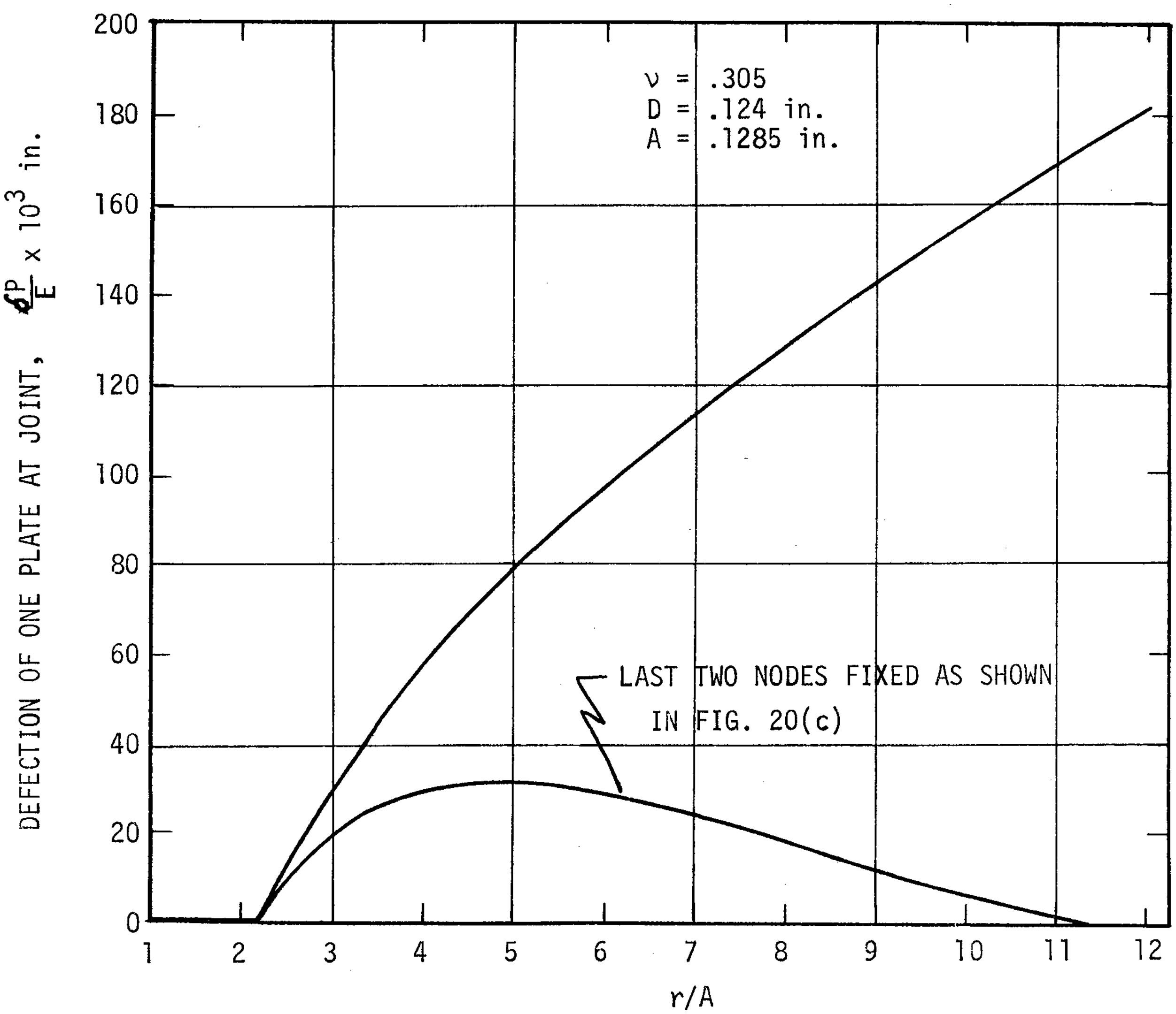


FIG. 25. GAP DEFORMATION FOR FREE AND FIXED EDGES — FINITE ELEMENT ANALYSIS, 1/8 INCH PLATE PAIR.

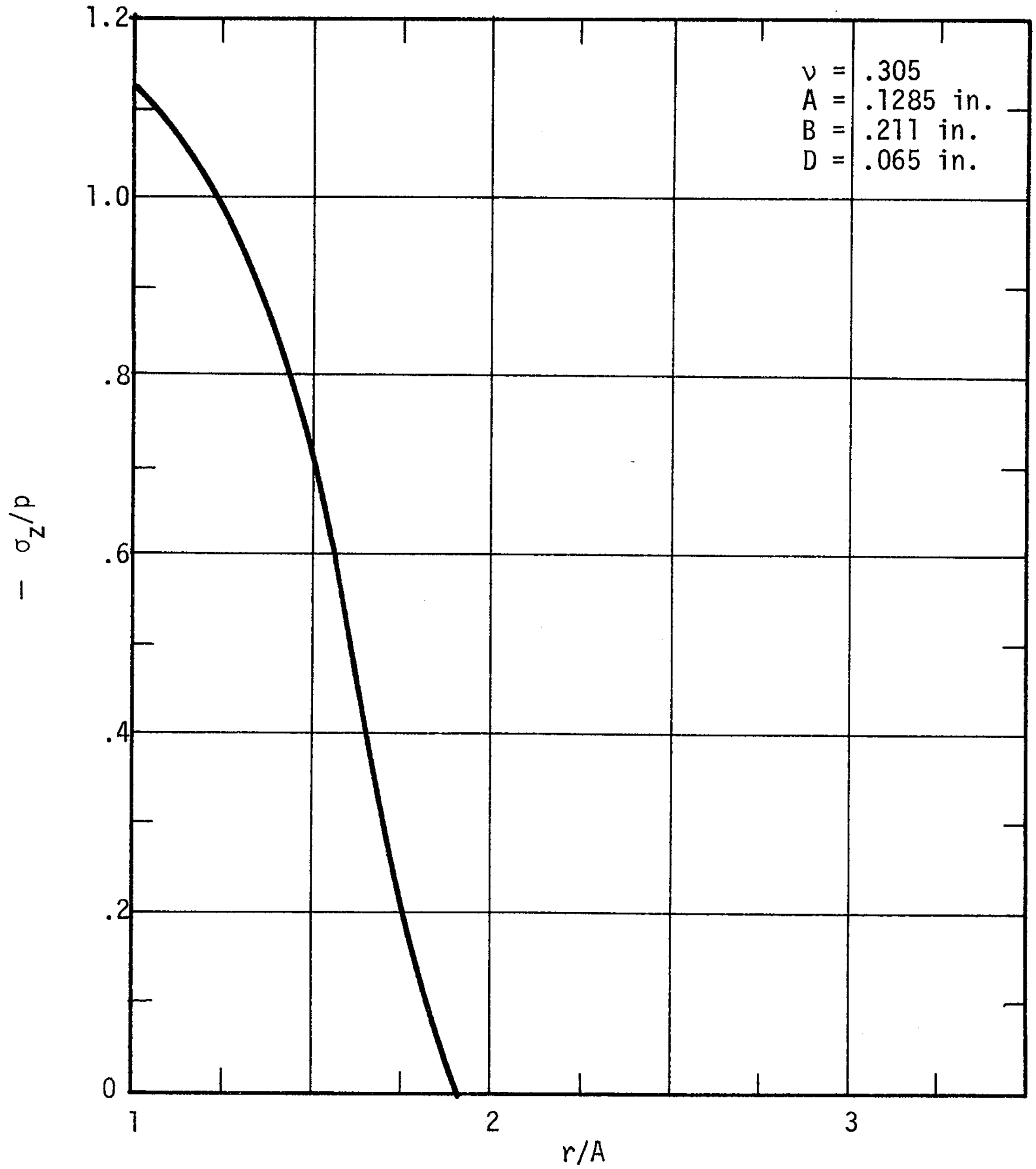


FIG. 26. FINITE ELEMENT ANALYSIS RESULT FOR 1/16 INCH PLATE PAIR.

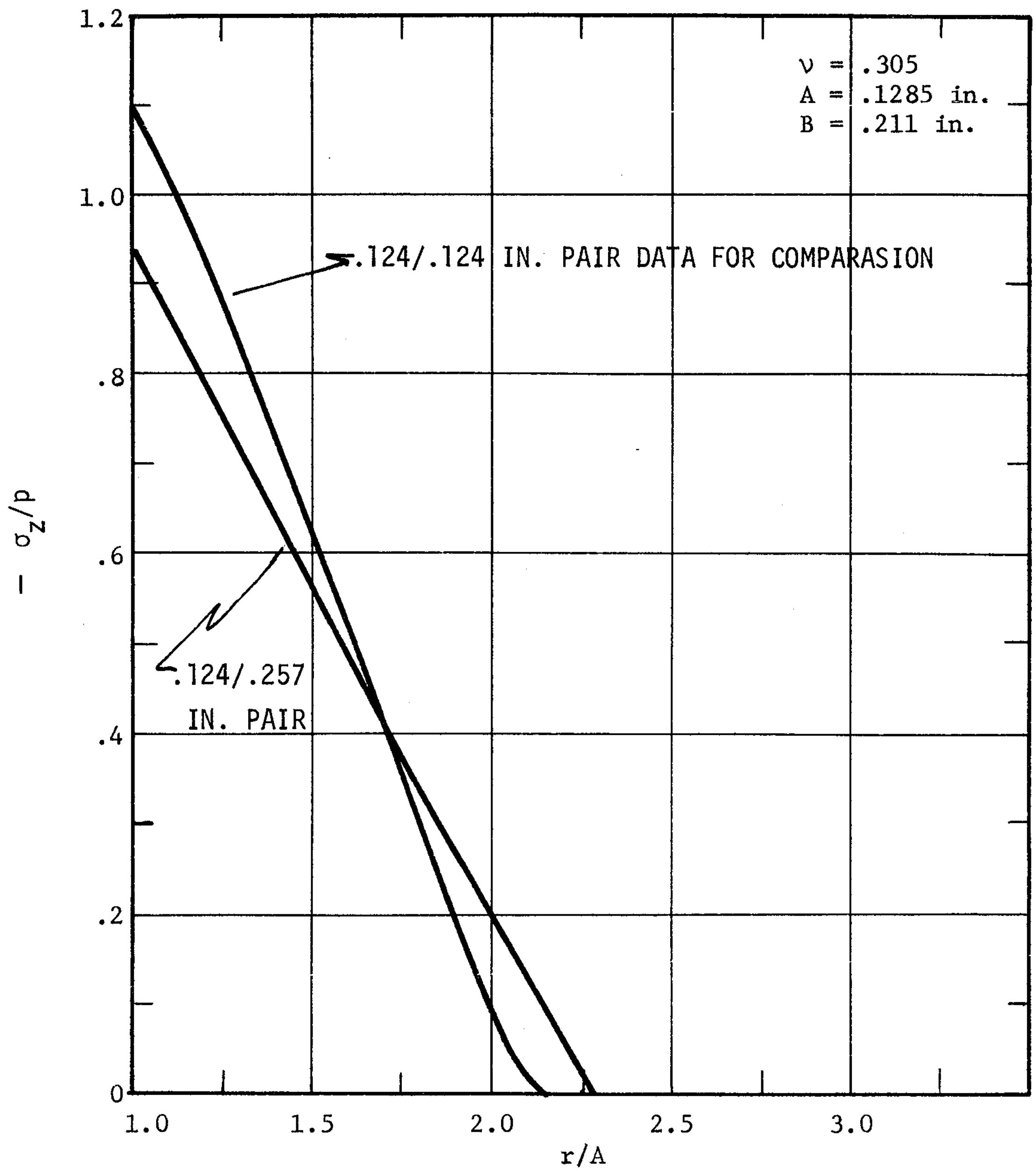


FIG. 27. FINITE ELEMENT ANALYSIS RESULTS FOR 1/8 INCH PLATE MATED WITH 1/4 INCH PLATE.

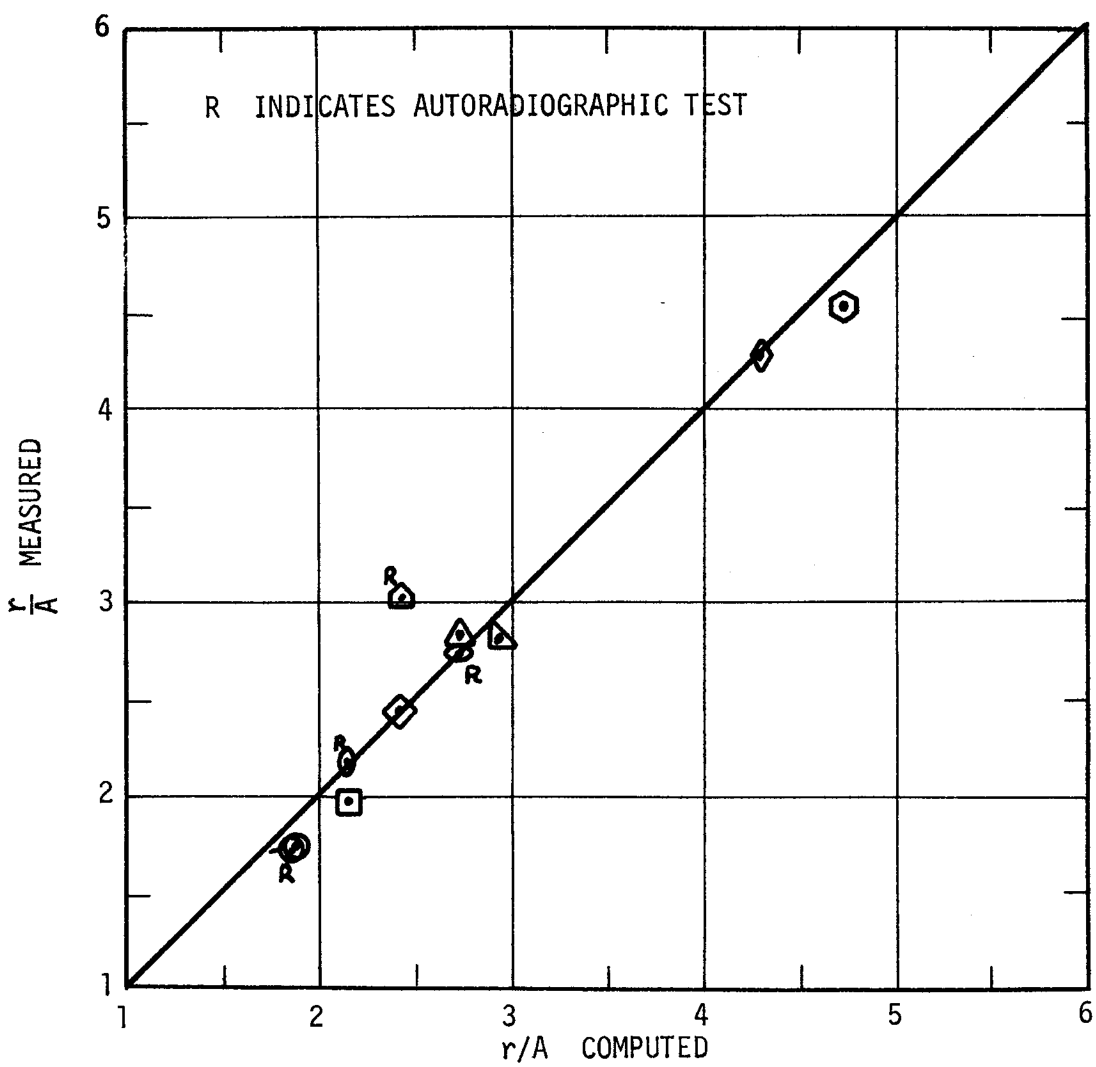


FIG. 28. COMPARISON BETWEEN TESTED AND MEASURED SEPARATION RADII.

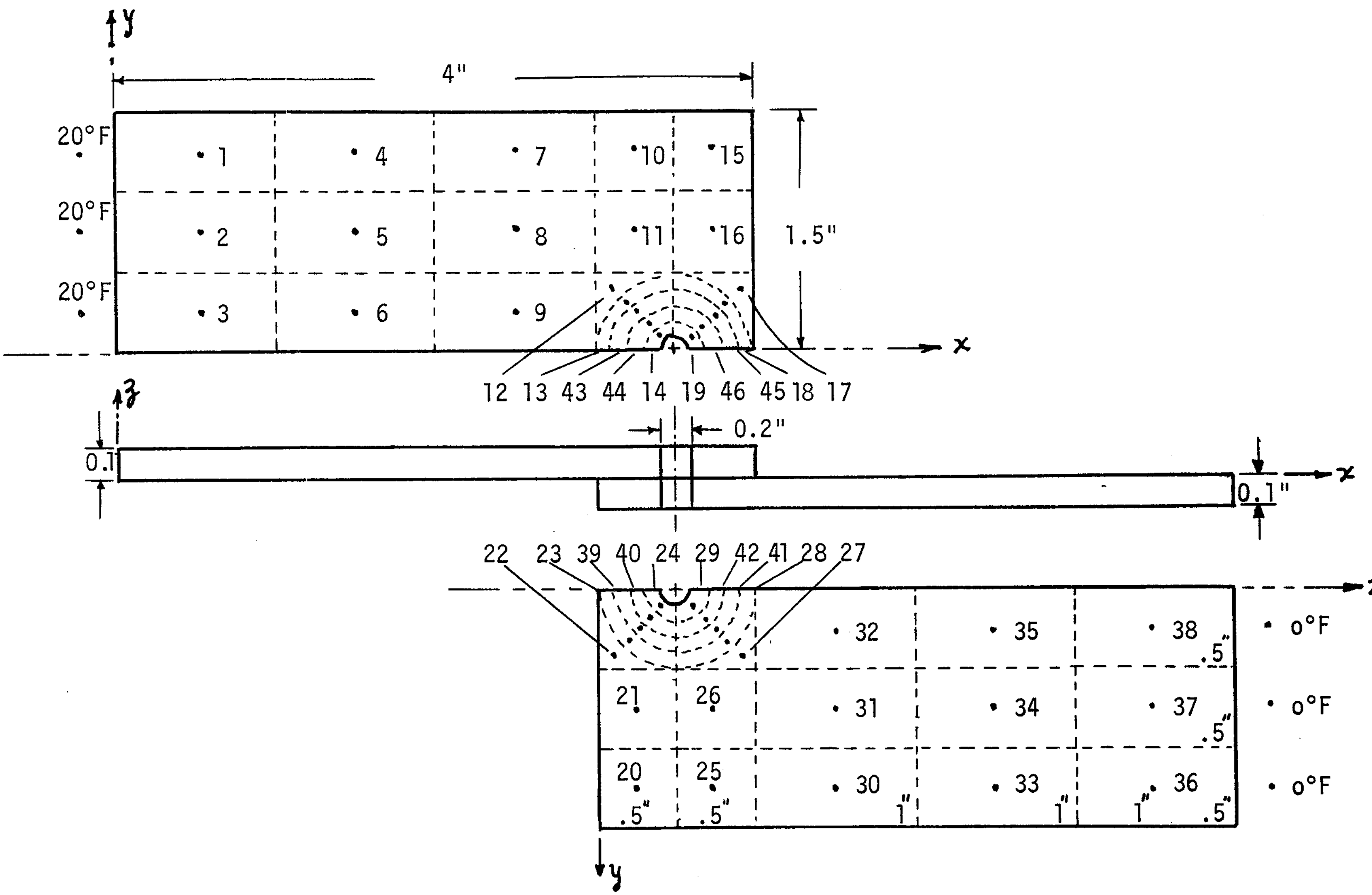


FIG. 29. LOCATION OF NODES — STEADY STATE HEAT TRANSFER ANALYSIS