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Interim Report

TOWARD A UNIFIED SET OF LOW-THRUST MISSION ANALYSIS TECHNIQUES

Jerry L. Horsewood

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Analytical Mechanics Associates, Inc.
9430 Lanham Severn Road
Seabrook, Maryland 20801

Toward a Unified Set of
Low-Thrust Mission Analysis Techniques*

Jerry L. Horsewood**
Analytical Mechanics Associates, Inc.
Seabrook, Maryland

Abstract

This paper presents one view of the current status of low-thrust mission analysis techniques and computer programs. Major problem areas facing the mission analyst are discussed and possible approaches to the solutions of some of these problems are suggested. Also, extensions to currently available computer programs and techniques that will be required for the analysis of non-standard missions representing potentially useful applications of electric propulsion are indicated. Where applicable, the discussions of the major problem areas include a brief description of current research activities within Analytical Mechanics Associates, Inc. that are pertinent to that problem area.

Introduction

In May 1968, a compilation of low-thrust trajectory analysis computer programs in use throughout the country was prepared and distributed by Mascy[1]. This document is important in that it gives a brief description of virtually our entire low-thrust mission analysis capability as of about one year ago. Several months prior to the publication of this compilation, Flandro and Barber[2] outlined the need for a "graded set" of low-thrust mission analysis programs, the set being comprised of four discrete levels of sophistication and complexity. At the lowest level are programs designed for first-cut performance estimate and mission opportunity definition studies employing rapid and economical approximate analytical techniques. The Level 2 programs provide full optimization capability and more precision in the calculations at the expense of running time by replacing the analytical techniques with numerical integration. Programs at this level are used for preliminary mission definition studies. Advanced mission analyses and targeting are accomplished with the Level 3 programs which include N-body perturbations and a limited optimization capability. Level 4 programs are precise path simulators with little or no optimization capability and are used for final targeting and mission support.

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**Senior Analyst and Manager, Mission Analysis

Upon reviewing the individual program descriptions contained in the compilation, one is left with the disturbing impression that our present mission analysis techniques are somewhat shallow when viewed in the light of the requirements as seen by Flandro and Barber. In an attempt to assess the spectrum of available techniques, I separated the programs listed in the compilation into the four suggested categories on the basis of the information given in the descriptions. Of the fifty-five programs included in the compilation, seven fall within the Level 1 category, thirty-two are of Level 2, eight are of Level 3, and six are of Level 4. Two programs listed are not low-thrust programs and were not categorized. The number of programs categorized as Levels 3 and 4 is somewhat misleading. To qualify for Level 3, I required only that the program possess optimization capabilities and at least a three-body formulation. Any program with at least a three-body formulation without optimization was placed in the Level 4 category. A more thorough evaluation of the precision of these programs would probably result in the reassignment of some to Level 2. Furthermore, comments accompanying the descriptions indicate that many of the programs classed as Level 3 and 4 are actually in the development stage and are not capable of production work. Others acknowledge that their programs are not well documented; consequently, it would be very difficult to make them available for general use. Of the eight programs categorized as Level 3, only one N-body program, the Lewis N-body code, was described without reservations as to its availability, documentation, or operational status. Some results obtained using this program and the performance of the program in generating these results are discussed in subsequent paragraphs.

In the following paragraphs several of what I believe to be the major problem areas in the development of a complete and unified set of low-thrust mission analysis techniques are discussed. It will be noted that most of the subjects discussed fit well within the framework of the four levels of sophistication and complexity suggested by Flandro and Barber. The major exception is the subject of error analysis which was not covered by them. For each subject covered, the nature of the problem is discussed, recent work representative of the current state-of-the-art is cited, and, in certain cases, current research that appears to hold particular promise of yielding a solution to the problem is indicated. It is to be emphasized that the references cited with respect to any specific problem area do not constitute a complete bibliography on the subject, but rather are representative of the present status of the subject. Also, it should be noted that the principal applications in mind when writing this paper were primary propulsion for solar system missions.

Major Problem Areas

In subsequent paragraphs five distinct problem areas are discussed. The first four problem areas may, to an extent, be correlated with the four grades of program capabilities discussed above. The fifth problem area, mission error analysis, has not been discussed previously in the literature, at least in the context of the terminology employed here.

Analytical Techniques.

One of the more frequent laments of the low-thrust mission analyst is the lack of a rapid, easy-to-use, approximate analytical technique for performing first-cut mission analyses, feasibility studies, gross vehicle designs, etc. The computer time requirements of numerical integration techniques are simply too great to permit investigation of all the cases one would like. Consequently, the analyst is forced to commence his study at a higher level of detail than he would prefer and the subsequent phases of the study suffer from the lack of quantitative approximate data that would be available had the proper analytical tools been available.

There are several reasons other than speed and economy in generating preliminary performance data that the development of approximate analytical techniques is so important. One is that the simple straightforward approach tends to enhance the understanding of the overall problem providing, of course, that the simplicity is not achieved by obscuring the correlation between the model and the problem (e.g., curve-fitting). Also, an analytical technique, if sufficiently accurate, can usefully be employed as a first-guess generator for a more sophisticated program. Finally, analytical techniques may, because of their simplicity, speed, and economy, have other applications more closely associated with the hardware such as real time control, on-board calculations, or even forming the basis of the guidance policy.

The number of analytical approximate techniques that have received notable attention is extremely limited. One that has been the subject of much discussion, both pro and con, is the characteristic length method proposed by Zola[3] and more recently promoted by Mickelsen[4]. This method is based on the premise that a parameter known as the characteristic length, defined as the distance a vehicle would travel in field-free space, is invariant with respect to the propulsion system for a given mission. The characteristic length is easily calculated for impulsive thrust missions and may then be used for determining the mass requirements for any other thrust program. The primary objections to the method are that the theoretical justifications for the basic premise are somewhat tenuous, that the method is not sufficiently accurate, and that it offers none of the additional features discussed in the preceding paragraph.

There are two potentially useful analytical approximate techniques presently under study. One of these is being studied at JPL and involves the use of an asymptotic series obtained from perturbation techniques with the low-thrust considered as the perturbation. The approach combines the perturbation solution with an optimization technique to yield, supposedly, a very good approximation to the optimal solution[5]. The second technique is under study at Princeton University and also employs perturbation expansion methods. By expanding a finite thrust trajectory about the optimal impulsive trajectory, a series solution representing the approximate optimal low-thrust solution is obtained. This approach is discussed in detail in the preceding paper of this section.

Extensions to Two-Body Optimization Programs.

As shown previously, the developmental effort of low-thrust trajectory analysis programs has been concentrated on the two-body, numerical integration programs with full optimization capability. Irrespective of this concentration of effort, there still remains to be done much challenging development work to extend the capabilities of these Level 2 programs. For example, present techniques are generally incapable of handling missions other than the standard flyby and rendezvous interplanetary missions and the solar probe missions. Non-standard mission profiles have been the subject of much interest in high-thrust mission studies. Of particular interest is the planetary swingby mode because it permits significant reductions in flight time and/or propulsion requirements compared to the standard mode for many missions. There is no reason to believe that the same benefits would not be realizable for many low-thrust missions.

The development of a low-thrust swingby trajectory optimization program is not at all a straightforward procedure. The primary difficulties in developing a workable approach will arise from the sensitivities of the post-encounter trajectory to the swingby geometry. Consequently, the two point boundary value problem will be quite severe. Dickerson and Smith[(6)] presented a low-thrust swingby trajectory past Venus to the orbit of Jupiter. However, they circumvented the sensitivity problem by treating the encounter as a discontinuity in velocity whenever Venus' orbit was crossed, without regard as to whether Venus was actually there. This approach is not entirely acceptable since the actual relative orientations of all the planets involved may never match those required by the solution. A more desirable approach would be to account for the ephemeris of the swingby planet, but reduce the sensitivity of the problem by starting the solution at the swingby planet, and proceeding in both directions to the launch and target planets. This approach removes much of the sensitivity of the problem by increasing the dimensionality of the boundary value problem. This approach is currently under study at Analytical Mechanics Associates, Inc. (AMA, Inc.) .

A side benefit from the development of the swingby program is that, with only a few minor changes, the program would be capable of treating round-trip missions. Most of the round-trip mission analyses performed in the past were accomplished by combining optimum one-way trajectories. But for realistic propulsion system models, the initial conditions for the return trajectory are dependent upon the arrival conditions of the outbound trajectory. Consequently, to properly optimize the entire mission the two segments should be considered together.

Other missions which will require further development of our present techniques include those for which the basic assumption of two-body motion is not strictly valid. Examples would include trips to and around libration and Trojan points. Such missions are important from a communications standpoint and appear to be well suited for electric propulsion applications. It is entirely possible that the most useful techniques for analyzing this type of mission will not come from the extension of our two-body techniques, but rather from either the Level 1 analytical techniques or the Level 3 or 4 N-body techniques. The general problem of motion in the transition region between two attracting bodies is also currently under study at AMA, Inc.

Another area which needs additional development effort (i.e., program development, not theoretical) is the inclusion of constraints in the program formulation. There will exist missions which preclude system and trajectory optimization, at least in the classical sense, because of prespecified constraints imposed by, say, hardware considerations or scientific objectives. Examples of these would be missions with thrust direction fixed in space to simplify communications. A few programs are now available which will handle such missions, but this is the exception rather than the rule. Another constraint which is expected to noticeably affect both the optimum vehicle design and the trajectory of a solar electric spacecraft will arise due to limitations in varying the solar array orientation relative to the spacecraft. Current programs do not account for variations in power generated as the angle of incidence of the Sun - spacecraft line to the panels varies. If the arrays are fixed relative to the rest of the spacecraft, then variations in thrust direction will cause variations in power level. This effect should be accounted for in the equations of motion, but this is not necessarily straightforward because the formulation of the problem is directly dependent on the assumed design of the spacecraft. Consequently, care must be taken to formulate the equations so as to provide adequate flexibility for treating a variety of designs.

The preceding problems are, of course, only representative of those which we, as mission analysts, are generally not equipped to handle. As the exploration of our solar system continues, we are certain to be faced with other missions which will require special purpose techniques. Constraints also will become more numerous as the mission analyses become more refined and hardware designs become more firmly fixed.

Verification of Preliminary Data.

The verification of results obtained from two-body trajectory optimization programs is a difficult problem. In the context of this discussion, the term "verification" is meant to imply proof of the validity of results and conclusions obtained from a simplified approach by comparing with results obtained using the most accurate techniques possible. Of great concern to many mission planners is the question of whether real world perturbations that are generally neglected in mission studies will invalidate the results of those studies. Or, more specifically, had the real world perturbations been included in the study, would the results and conclusions have been different?

An acceptable test of the validity of approximate trajectory and performance data could be achieved through the use of an N-body trajectory optimization program with features comparable to those of most two-body programs now available. To this end a small number of such programs have been developed, but without exception these programs have been found to be extremely difficult to operate successfully. Using a very gentle hand and a considerable amount of insight, Strack[7] successfully obtained some three-body optimum low-thrust trajectories to Mars.

The difficulty encountered in generating optimum N-body trajectories is due to extreme sensitivities and very large rates of change of most parameters in the vicinity of the planets. Some insight into this problem can be gained quickly through a simple example.

The second order differential equation for the primer vector, P, for a thrusting vehicle in the combined gravitational field of (N-1) bodies is

$$\ddot{P} = \sum_{i=1}^{N-1} - \frac{\mu_i}{r_i^3} \left[P - \frac{3}{r_i^2} (P \cdot R_i) R_i \right]$$

where μ_i is the gravitational constant of the i^{th} attracting body, R_i is the position of the spacecraft relative to the i^{th} body, and $r_i = |R_i|$. The value of the quantity within the square brackets will be of the same order of magnitude for all planets. Therefore, the relative magnitudes of the contributions to the second derivative of the primer from the several attracting bodies are proportional to the ratios, μ_i/r_i^3, $i = 1, 2, ---, N-1$. Now consider a spacecraft in the vicinity of the Earth where, for the purposes of this discussion, the contributions to $\ddot{P}$ from all bodies except the Earth and Sun are negligible. It is easily shown that the ratios μ_i/r_i^3 for the Sun and Earth are equal when the spacecraft is somewhat over 2×10^6 kilometers from the Earth. Comparing the value of the ratios for the Earth at this large distance to its value near the Earth, say at about 500-1000 km above the Earth's surface, one finds that the ratio is larger near the Earth by a factor of 3×10^7. Consequently, one may expect the second derivatives of the primer to vary possibly several orders of magnitude in an N-body solution as the vehicle moves from the close proximity of a planet into heliocentric space. Since most problems of interest will begin in the vicinity of Earth and terminate in the vicinity of another planet, the extreme sensitivity of the two-point boundary value problem becomes painfully clear.

Because of the severity of the sensitivity problem, it is advisable to take precautionary steps in the design and development of an N-body program to minimize the effects of the sensitivities. Strack[7] was successful in obtaining some optimum N-body trajectories by starting the optimization problem a large distance from the Earth (at least 20 radii). This appears to be the most promising approach. Within a prespecified distance of any planet the thrust could be forced to follow some preprogrammed plan (e.g., tangential thrust) or simply be turned off. At greater distances from the planets, the thrust would be optimally directed.

There are alternatives to the use of an N-body optimization program for verification. One is to improve the approximations of the normal two-body program to more realistically simulate the entire mission by explicitly including the planetocentric phases. This approach has been investigated [7, 8] previously. The missions were formulated as series of two-body segments which were linked together at preselected distances from the planets (spheres of influence). Although this approach does not include the gravitational effects of more than one body at a time, it does include the dominant effect at each point. From this viewpoint it represents a first order improvement over the normal two-body program. In his N-body optimum trajectory studies, Strack showed that the proper selection of the radius of the sphere of influence in the segmented two-body program results in performance estimates identical to those obtained from the N-body program. The path that the vehicle follows, although not precisely the same from the two programs, is very close. Even these small path differences can undoubtedly be reduced by applying the theory of matched asymptotic expansions developed by Breakwell and Rauch[9]. This is an analytic method of including the gravitational effects of both the Sun and planet in the region of transition from one dominant field to the other. In addition to serving as alternative methods of verification, these extensions to the standard two-body program should prove extremely valuable for generating starting conditions for the more sophisticated N-body optimization program.

A final alternative, and conceptually the most simple, is to use an N-body simulation program. Since this type of program would not have optimization capability, it could not verify that an optimum solution from a two-body program is representative of the N-body optimum solution. It would, however, verify whether a vehicle that was designed for a specific mission using data from a two-body program can actually perform the mission when all perturbing influences are considered. After all, this is truly the important question since the many hardware and software constraints imposed on the flight vehicle will preclude following the optimum thrust programming precisely. In any event, the N-body simulation program represents the only presently available method of verifying results in detail. Because of the serious problems remaining to be solved in the development of N-body optimization programs, the simulation program may prove to be the most effective method of verification for some time to come.

Simulation.

There are presently available throughout the country a small number of precision, N-body, numerical integration trajectory programs with targeting capability that are adequate for precisely simulating the path of a general thrusting vehicle. With no more than minor extensions these programs could probably satisfy all the present low-thrust mission simulation needs. These programs will undoubtedly not be adequate for mission support and mission control requirements however, since, historically, programs for these purposes have been specially designed and closely tied to the specific hardware constraints and design. This special purpose software has been found desirable in the past as it permits the project engineer to generate his necessarily voluminous data in the most efficient manner. Past experience indicates there is little reason to expect a change in approach for low-thrust missions. Consequently, the extension and further development of the precision simulation programs will, for the most part, be undertaken under the authority of the individual project planners and managers.

Conceptually, the precise simulation program is the simplest of all the mission analysis tools, for one just includes all possible effects in the model. No justification for simplifying assumptions is required because no simplifying assumptions are made. In practice, of course, the task is not so simple, but the problems are generally due to the great detail required and the magnitude of the program rather than because of theoretical considerations. As a consequence, precision simulation will not be as great a problem area to the mission analyst as will the other subjects discussed in this paper.

Mission Error Analysis.

A subject that, to this point, has received virtually no attention for low-thrust missions is error analysis. Contributing to this deficiency is, of course, the fact that, for low-thrust applications, studies of the major technologies involved in performing an error analysis, i.e., guidance, control, navigation, and orbit determination, have been very limited. Of the four technologies, control is probably the most advanced because various aspects of this subject have been investigated for auxiliary propulsion applications related to Earth satellites. Guidance, navigation, and orbit determination have each been given only very cursory investigations[10-13] with limited study scopes. These studies have been introspective in nature in that the interrelated effects of the other technologies were neglected. Consequently there has been no attempt to conduct a total mission error analysis nor to develop the necessary techniques to do so.

There will undoubtedly be considerable disagreement among mission analysts on the proper approach to the development of error analysis techniques. AMA, Inc. has recently embarked upon a specific course of action which, it is hoped, will eventually lead to the desired error analysis capability. The initial scope of work for this study calls for the development of an open loop error analysis program which incorporates realistic propulsion system, navigation system, and environmental error models. The error analysis program will be developed to be consistent with a two-body trajectory optimization program. That is, errors will be propagated along a nominal trajectory generated by the two-body program and the accuracy objectives will be to maintain a level consistent with that of the two-body assumption. The two-body trajectory program will be extended to include planetocentric phases by treating the overall problem as a series of two-body segments. The decision to use a two-body technique as a basis for the overall program is based on the premise that, although the trajectory obtained from the two-body program is not precise, the sensitivites to change about that path are good approximations to those about the precise trajectory. Consequently, the requirements for guidance and navigation, based on a two-body approach, should be directly transferrable to the precision trajectory. Subsequent plans are to incorporate guidance and control systems error models which will permit the performance of a closed loop error analysis and the assessment of various combinations of guidance, control, and navigation systems.

Whether the two-body approach to the performance of error analyses is valid remains to be proven. At this point in time, however, there appears to be no alternative in terms of a more precise technique. Until sophisticated N-body optimization programs are developed to a state that one can reasonably expect to consistently converge to solutions, it seems pointless to insist on more accurate error analysis techniques. Furthermore, it seems likely that the success of error analysis techniques would be independent of the manner in which the trajectory were generated; consequently, the methods would be available and checked out when the more precise trajectory program becomes available.

Concluding Remarks

In conclusion, it appears quite evident that the field of low-thrust mission analysis will provide challenging opportunities and problems for some time to come. Several specific problem areas have been defined and, in certain cases, potentially attractive approaches to the solution of the problems are indicated. Also, in those areas in which AMA, Inc. is currently involved in research, the particular approach being pursued is outlined.

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