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ON THE SHARPNESS OF CRACKS COMPARED WITH WELLS' COD *

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Wells' (1,2,3) concept of crack opening displacement (originally dislocation) COD has been widely employed in research on fracture, especially in Britain (4). For sufficiently small gross deformations of a body containing a crack, the COD is simply a multiple (of order unity) of the ratio of crack extension force $\mathcal{G}$ to yield strength. It is in fact a feature of the <u>equivalent</u> <u>elastic</u> <u>crack</u>, fig. 1, which is longer than the actual crack to which it is (approximately) equivalent in respect of near-tip stress and displacement fields. Specifically, it is the distance of separation of the faces of the equivalent elastic crack at the position corresponding to the tip of the real crack. However, the COD is assumed to represent a well defined distance of separation of the faces of the real crack, at or very near the tip, which is therefore regarded as very blunt. As a working hypothesis the COD is regarded as a quantity which characterizes the potentiality for crack extension to occur, related to the quantities $\mathcal{G}$ and K that serve the same purpose in linear elastic fracture mechanics, but less restricted in scope.

Many specialists in fracture mechanics have strong reservations about Wells' COD concept. The experimental and theoretical evidence to support the model seems to us to be unconvincing, and we are studying the deformation of cracks in finite element models of specimens in some detail. Our main purpose is to relate displacements measured at convenient gage locations on practical specimens to displacements elsewhere, particularly near the crack tip, in the expectation that some function of the near-tip displacements will be useful to characterize the potentiality for crack extension to occur. To date we have obtained

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results for a standard ASTM threepoint bend specimen (5) and for a matching centrally cracked tension specimen, both in plane strain, by the finite element method (6). The procedure admits real material characteristics, and we have used those of a maraged steel with a yield strength of 186 ksi and tensile strength of 200 ksi, evidently low work hardening. The element map for half the bend specimen is identical to that for a quadrant of the tension specimen, fig. 2. The triangular elements vary in size, the smallest internodal distance (nearest the crack tip) being 1/320 of the crack length which, in turn, is one half of the map width. The specimens are loaded through prescribed boundary displacements which are matched so that the stress intensity factors for the two specimens are the same at a given step in the early stages of loading. We are thus able to compare directly the deformation patterns resulting from imposed tension and bending.

In the present context the most important indication from our results is that the crack tip is much sharper than Wells' COD concept seems to imply. That is, the distance from the crack tip at which the COD occurs is much greater than the magnitude of the COD. Furthermore, the COD region is not a very distinctive feature of the crack profile; consequently there is no obvious way to determine the COD from inspection of the crack profile (or, indeed, by any other means). These points are illustrated in fig. 3 by the selected set of crack tip halfprofiles in true proportion for the tension specimen. The lowest curve is for a stage of loading where δ/b(yield strength) is about 0.002, somewhat beyond the ASTM E-399-70T criterion for valid plane strain toughness tests (5). The imposed displacement d/2b (see fig. 2) is shown for each curve in fig. 3 for comparison. The dashed line is the locus of an estimated lower bound on the position of Wells' COD. An objective method of estimation was used which involved comparison of the shape of each curve with the shape of the curve for linear elastic behaviour (which is independent of load when displacements are normalized with respect to the displacement at the center of the crack). This choice of method is somewhat arbitrary, but the COD values so obtained are of the order $\mathcal{G}$/n(yield strength), where n is an illdefined constraint factor which may be taken as 2 for plane strain, and $\mathcal{G}$ is calculated as proportional to the square of the imposed displacement, not from the load in terms of force. Since the positions shown by the locus are closer to the crack tip than the corresponding positions of the elastoplastic enclave boundary, the locus is considered to represent a lower bound on the COD position for the Wells' model.

The crack tip shapes obtained for bending (not shown here) are quite similar to those for tension, but the COD values are smaller by a factor between 2 and 4 at corresponding stages of loading, tending to converge most at the lowest loads for which satisfactory estimates of COD could be made. The tension results shown here are only an illustrative selection from a much larger body of data which will be published in due course.

From such results as those shown in fig. 3 it seems to us that the crack tip radius must be much smaller than the Wells' COD. This being so, the tip radius is likely to be a much more significant feature of the deformed crack than the hardly prominent COD. The present results suggest that this radius increases roughly in proportion to the square of the imposed displacement (or any other sufficiently remote displacement), which raises the prospect that the crack tip radius could be used as a measure of the potentiality for crack extension in the same sense that $\mathcal{G}$ and K are such measures in linear fracture mechanics. This is indeed so within linear fracture mechanics since the crack tip radius is then exactly $(4/\pi)\mathcal{G}/E$ for plane stress, or the same multiplied by $(1-\nu^2)$ for plane strain, as first noted by M. L. Williams (7). But the crack tip radius is not subject to the same restrictions as $\mathcal{G}$ and K because it is always well defined, irrespective of the nature of the surrounding stress and displacement fields. Therefore it has the characteristics needed to provide a link between linear fracture mechanics and a more generally applicable method of fracture analysis. The question whether it is the best link, however, can only be resolved empirically.

Wells' COD in linear fracture mechanics can be regarded as the product of the crack tip radius with the ratio of elastic modulus to yield strength (augmented by an appropriate constraint factor), and it is possible that this simple connection might continue to hold to a useful degree of approximation well beyond the useful range of linear fracture mechanics. Nevertheless, there are reasons to prefer the tip radius over the COD even if they were so simply related. Conceptual realism and simplicity are two important considerations. Another is the fact that fatigue crack propagation data for a variety of materials tend to condense into a fairly narrow band when the rate of crack propagation is plotted (on the usual log-log scales) against the ratio of the K-range of the fatigue cycle to the elastic modulus, whereas the same data tend to spread out if the variable chosen is the ratio of the K-range to yield strength.

Finally, we are aware of a number of contributions to the literature of fracture mechanics which may have some connection to our proposition that crack tip radius rather than COD should be considered as a measure of the potentiality for crack extension to occur. We hope to discuss some of these contributions in a later paper.

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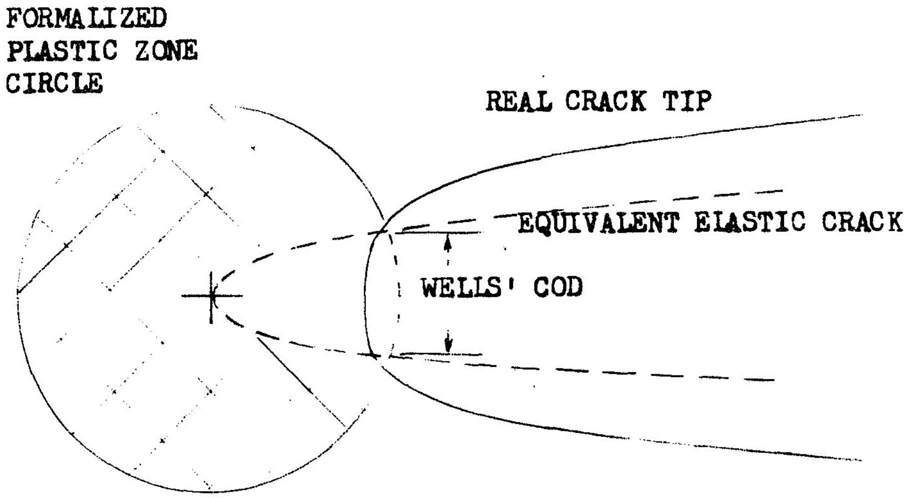

Figure 1. Wells' concept of crack opening displacement, as described in references 1 - 3.

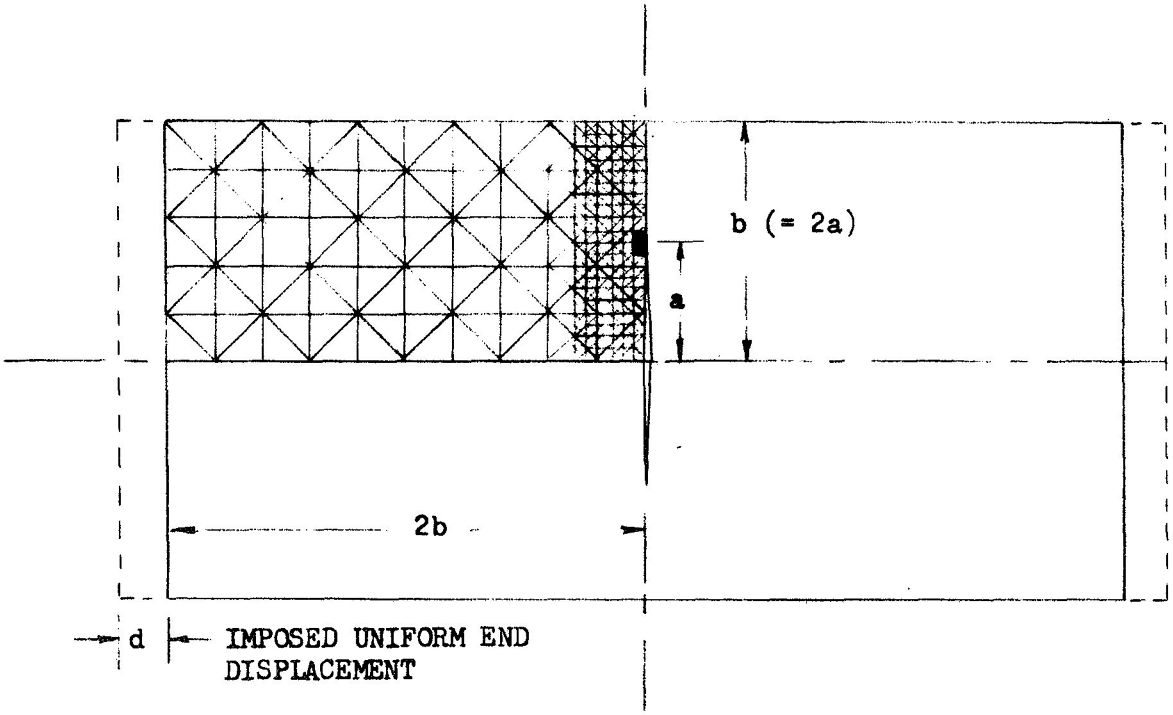

Figure 2. Tension specimen showing finite element map in one quadrant.

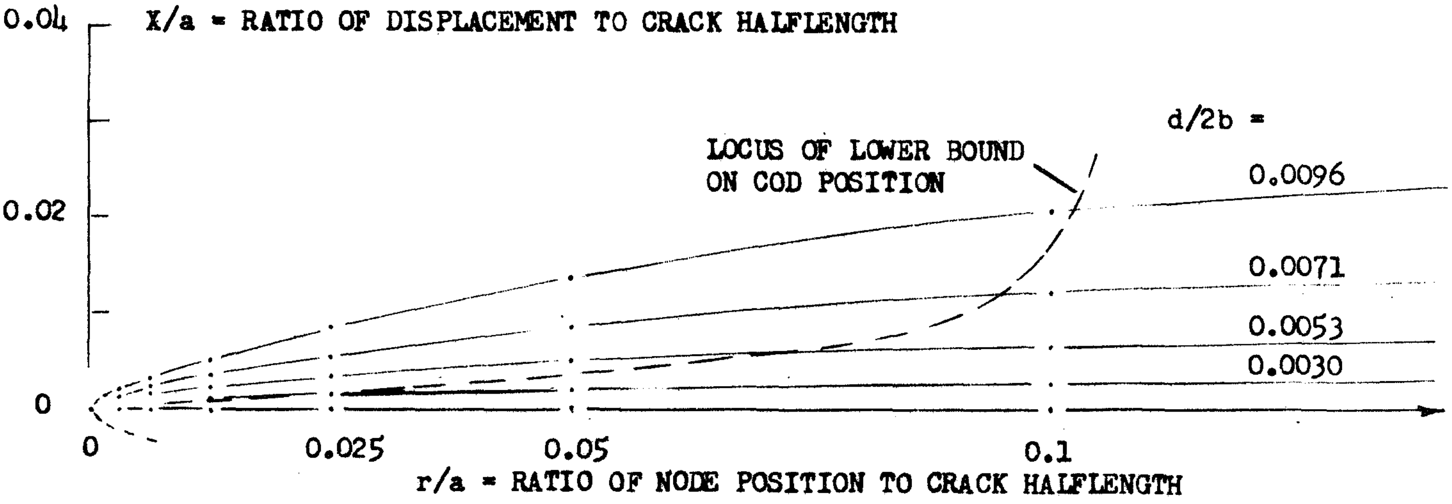

Figure 3. Crack tip halfprofiles <u>in true proportion</u> for tension specimen of fig. 2 in plane strain. Points are at nodal positions.