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BROUWER-LYDDANE ORBIT GENERATOR ROUTINE

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INTRODUCTION

This document contains a complete description of the Brouwer-Lyddane Orbit generator routine (BROWR0) which was written as a part of the Definitive Orbit Determination System (DODS). The routine accepts as input a set of Brouwer Mean elements at a specified epoch and outputs the position and velocity vectors at requested times along with intermediate data if requested. The formulation for the routine was derived from the following sources:

1. Brouwer, Dirk. "Solution of the Problem of Artificial Satellite Theory Without Drag." The Astronomical Journal, Vol. 64, (Oct. 1959), 378-397.
2. Lyddane, R. H. "Small Eccentricities or Inclinations in the Brouwer Theory of Artificial Satellite." The Astronomical Journal, Vol. 68, (June 1963), 555-558.
3. Siry, J. W. "A Mathematical Formulation of the Brouwer-Lyddane Orbit Theory."

In addition to the basic Brouwer Theory, the BROWR0 routine calls subroutines for computing Delta L Drag and complementary perturbations in order to allow for the inclusion of these effects on the satellite's orbit. This document contains a description of these subroutines.

The Brouwer-Lyddane Orbit generator routine was checked out first in a stand-alone environment and then within the DODS system. DODS is a Fortran language system of programs that execute under MVT on the S/360 75 and 95 computers. The size of the system made it necessary to structure it in segments that could overlay each other in the computer as functional needs changed during execution. Thus, instead of one complete program or set of programs residing in core storage during the entire running time, only those portions needed are in core. Because interfacing with DODS was a major consideration in the programming, some techniques are utilized that would not be necessary in a stand-alone environment. For example, in BROWR0 many variables which could be assumed to contain valid data once they had been computed in a stand-alone environment had to be recomputed when running under DODS since there was a possibility of their being destroyed by the DODS overlay between successive calls to the subroutine. In the complementary perturbation subroutine indicators that determine the logical flow of the program and the time-element array had to be saved before each exit and restored at each entry for the same reason. Since the Fortran "COMMON" statements could not be used in DODS all subroutine arguments had to be passed through the argument list.

A detailed description of the Brouwer-Lyddane Orbit generator routine and the subroutines called by BROWR0 follows. The subroutines referenced but not described in this document are part of the DODS system and their description can be found in the appropriate DODS documentation.

DEFINITIVE ORBIT DETERMINATION SYSTEM

BROWR0 – Brouwer-Lyddane Orbit Generation Subroutine

I. LANGUAGE:

Fortran IV Level G and Level H

II. PURPOSE:

The objective of this subroutine is to compute the position vector (x, y, z) and the velocity vector ($\dot{x}$, $\dot{y}$, $\dot{z}$) of an artificial satellite at any time T, when given a set of elements at a time T_0 called epoch.

- a_0 – Semi-major axis
- e_0 – Eccentricity
- i_0 – Inclination
- ℓ_0 – Mean anomaly
- g_0 – Argument of perigee
- h_0 – Longitude of Ascending node

III. INTERFACE INFORMATION

A. The Calling Module is ORBGN0

B. The called modules are:

(1) DRAG – Computes $\Delta\ell$ drag
(2) REDUCE – Reduces angle between 0 & 2π
(3) KEPLRI – solves Kepler's equation
(4) DATAN0 – Computes the Double Precision Arctan (y/x)
(5) PERTF0 – Reads complementary Perturbations tape for Brouwer
(6) MPIDO0 – Output intermediate data is requested.
(7) MPERR0 – Error handling routine.

C. Calling Sequence

SUBROUTINE BROWR0 (EPHEM, K, ELEP, SATID, DPT, ZONALS, CF, PLN, IDOBE, IDVICE, IERR, SAVE, INDA).

Table I
Calling Sequence Arguments

Argument Name	Analytic Symbol	I/O	Description	Units	Format	Dimension
EPHEM		I/O	Input: The times (t_k-t_0) for which position and velocity vectors are to be computed. Output: Position and velocity vectors corresponding to input times; Brouwer double primed elements for Input times.	DUT	LF	(31, K)
K		I	Number of Input Times	None	LI	–
ELEP		I	Brouwer mean elements at epoch		LF	(7)
SATID		I	Satellite I.D. number	None	LI	–
DPT		I	Drag Parameters Table	DUT, Rad/DUT^2, Rad/DUT^3	LF	(60)
ZONALS		I	Zonal Harmonics Coefficients		SF	(4)
CF		I	Constant Table		LF	(75)
PLN		I	Pertape Logical Number	None	LI	–
IDOBE		I	Indicators for IDO of Brouwer Elements	None	LI	(8)
IDVICE		I	Logical Output Unit Designation	None	LI	(3)
IERR		0	Error Return Indication Normal Return = 0	–	–	–
SAVE		0	Save Area	–	LF	(20)
INDA		I	Indicator for First Pass of an Iteration	None	SI	(1)

DEFINITION OF ARRAYS: EPHEM (I, J)

J \ I	1	2	3	4	5	6	7	8 31
1	T_1	blank →						
2	T_2							
3	T_3						INPUT ARRAY	
.								
.								
.								
K	T_k							

OUTPUT ARRAY

J \ I	1	2	3	4	5	6	7	8	9	10	11	12	13	14 31
1	T_1	X_1	Y_1	Z_1	$\dot{X}_1$	$\dot{Y}_1$	$\dot{Z}_1$	a_1''	e_1''	i_1''	g_1''	h_1''	l_1''	→
2	T_2	X_2	Y_2	Z_2	$\dot{X}_2$	$\dot{Y}_2$	$\dot{Z}_2$	a_2''	e_2''	i_2''	g_2''	h_2''	l_2''	
3	T_3	X_3	Y_3	Z_3	$\dot{X}_3$	$\dot{Y}_3$	$\dot{Z}_3$	a_3''	e_3''	i_3''	g_3''	h_3''	l_3''	
.														
.														
.														
K	T_k	X_k	Y_k	Z_k	$\dot{X}_k$	$\dot{Y}_k$	$\dot{Z}_k$	a_k''	e_k''	i_k''	g_k''	h_k''	l_k''	

ELEP(1) = T_0	Epoch Time in DUT	
ELEP(2) = a_0	In DUL	
ELEP(3) = e_0		
ELEP(4) = i_0	In Radians	
ELEP(5) = g_0	In Radians	
ELEP(6) = h_0	In Radians	
ELEP(7) = ℓ_0	In Radians	

DRAG PARAMETERS ARRAY

DPT(1) – DPT (20) } $t_0, t_1, \ldots, t_{19}$ DUT
DPT(21) – DPT (40) } $N_{2,0}, N_{2,1}, \ldots, N_{2,19}$ Rad/DUT^2
DPT (41) – DPT (60) } $N_{3,0}, N_{3,1}, \ldots, N_{3,19}$ Rad/DUT^3

ZONALS(1) = $C_{2,0}$ 2nd zonal harmonic from Harmonic Coeff. file
ZONALS(2) = $C_{3,0}$ 3rd zonal harmonic from Harmonic Coeff. file
ZONALS(3) = $C_{4,0}$ 4th zonal harmonic from Harmonic Coeff. file
ZONALS(4) = $C_{5,0}$ 5th zonal harmonic from Harmonic Coeff. file

PLN – Perturbation Tape Logical Number

$>$ 0 Read Pert Tape From Unit Number PLN
$<$ 0 Do not read Pert Tape
= 0 Error

IDOBE(N) = Intermediate data output (IDO) indicator
IDOBE(N) = 0, No IDO
IDOBE(N) = i, Output data at every ith iteration

N = 1	Brouwer elements with secular, long period and short period terms. a, e, i, g, h, l
N = 2	Brouwer elements with secular terms only. a'', e'', i'', g'', h'', l''
N = 3	$(\dot{L}' + \dot{G}' + \dot{H}')$
N = 4	$\dot{L}, \dot{G}, \dot{H}$
N = 5	Contributions from secular, long, and short period terms δe, δi, $(\sin i''/2)\,\delta h$, $e''\delta\ell$
N = 6	Complementary Perturbations at Request time $\Delta a, \Delta e, \Delta i, \Delta g, \Delta h, \Delta\ell$

N = 7 True Anomaly (f); Eccentric Anomaly (E) at Request time
N = 8 K2, K3, K4, K5

NOTE: (1) All IDO will have time as an output.
(2) IDO's will be Long Format
(3) [i] will be constant for all outputs, i.e., [i] will not vary if more than one IDO requested.

IDVICE(1) – Sysout logical file number
$>$ 0 use sysout
$<$ 0 do not use
= 0 error
IDVICE(2) – Dedicated printer logical file number. (Treat value same as above)
IDVICE(3) – Tape logical file number (treat value same as above).

SAVE(1) – SAVE (20) Save area

Table 2
Logical Record Format Working Constants Pool

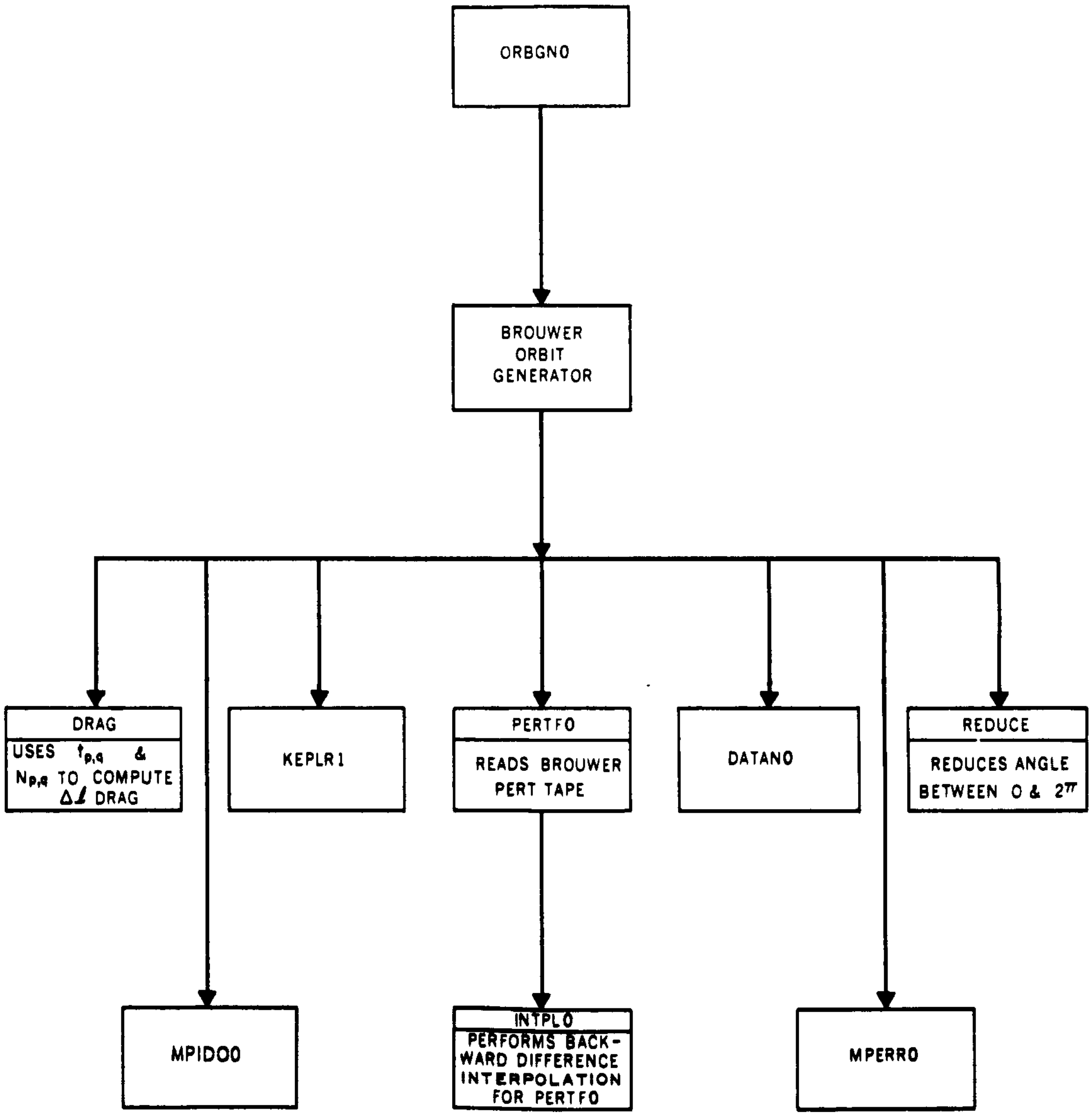
This is the definition of Array CF

Entry Description	Internal			Input		Report	
	Bytes	Format	Units	Format	Units	Format	Units
Conversion Section – 200 Bytes							
CF(1) Meter/Int. Foot	8	LF	None	D-Code	None	D-Code	None
CF(2) Meter/Nau. Mile	8	LF	None	D-Code	None	D-Code	None
CF(3) Kilometer/DUL	8	LF	None	D-Code	None	D-Code	None
CF(4) Kilometer/A.U.	8	LF	None	D-Code	None	D-Code	None
CF(5) Int. Foot/Nau. Mile	8	LF	None	D-Code	None	D-Code	None
CF(6) Nau. Mile/DUL	8	LF	None	D-Code	None	D-Code	None
CF(7) S. Mile/DUL	8	LF	None	D-Code	None	D-Code	None
CF(8) (Km/Sec)/(DUL/DUT)	8	LF	None	D-Code	None	D-Code	None
CF(9) Seconds/DUT	8	LF	None	D-Code	None	D-Code	None
CF(10)-CF(25) Blank							
Mathematical Constants and Tolerance – 200 Bytes							
CF(26) PI(π)	8	LF	None	D-Code	None	D-Code	None
CF(27) J_2 Normalization factor	8	LF	None	D-Code	None	D-Code	None
CF(28) Normal Eqn. Tol.	8	LF	None	D-Code	None	D-Code	None
CF(29)-CF(50) Blank							
Astrodynamic Constants							
CF(51) Mean Radius of Moon	8	LF	DUL	D-Code	Km	D-Code	Km
CF(52) Earth Equatorial Radius (Mean), R_e	8	LF	DUL	D-Code	Km	D-Code	Km

Table 2—Continued

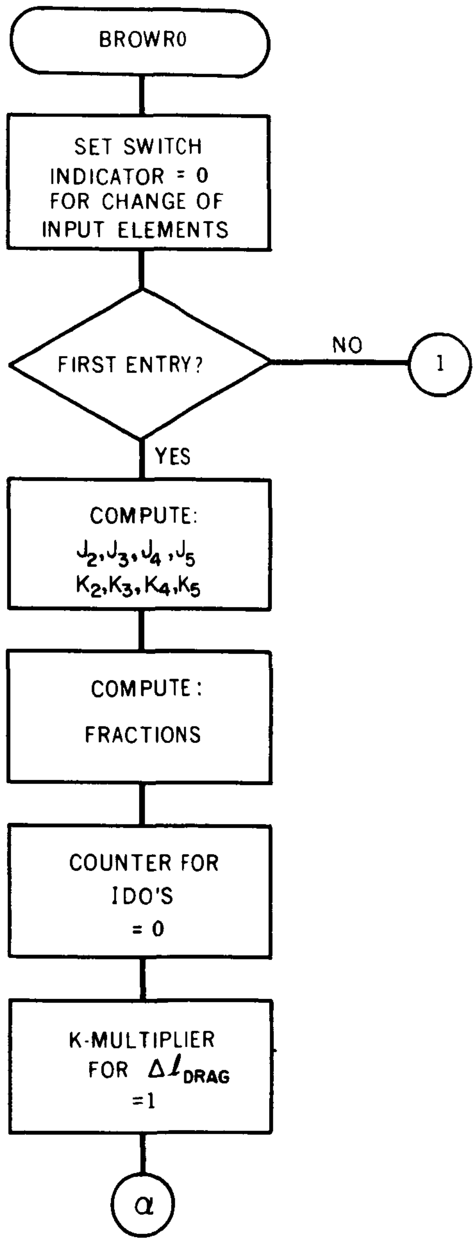
Entry Description	Internal			Input		Report	
	Bytes	Format	Units	Format	Units	Format	Units
CF(53) Mean Rotational Rate of Earth, ω_e	8	LF	Rad/DUT	D-Code	Rad/Sec	D-Code	Rad/Sec
CF(54) Flat. Coef. of Reference Ellipsoid	8	LF	None	D-Code	None	D-Code	None
CF(55) Mass-Ratio-Earth to Moon	8	LF	None	D-Code	None	D-Code	None
CF(56) Mass-Ratio-Sun to Earth and Moon	8	LF	None	D-Code	None	D-Code	None
CF(57) Mass Ratio-Sun to Earth	8	LF	None	D-Code	None	D-Code	None
CF(58) 1 Astronomical Unit A.U.	8	LF	DUL	D-Code	Km	D-Code	Km
CF(59) Polar Radius-Earth	8	LF	DUL	D-Code	Km	D-Code	Km
CF(60) Eccentricity (e)-Earth	8	LF	None	D-Code	None	D-Code	None
CF(61) Normal Gravity-Earth	8	LF	DUL/DUT^2	D-Code	Km/Sec^2	D-Code	Km/Sec^2
CF(62) Speed of Light, C	8	LF	DUL/DUT	D-Code	Km/Sec	D-Code	Km/Sec
CF(63) Mean Motion of Sun, Tau	8	LF	Rad/DUT	D-Code	Deg/Day	D-Code	Deg/Day
CF(64) Obliquity of Ecliptic, ϵ	8	LF	Rad	D-Code	Degrees	D-Code	Degrees
CF(65) μ = GM (Grav. Const. times Mass of Earth)	8	LF	DUL^3/DUT^2	D-Code	Km^3/Sec^3	D-Code	Km^3/Sec^3
CF(66) Julian Date for Space Epoch	8	LF	Days	D-Code	Days	D-Code	Days
CF(67) KSUBC Critical Inclination tolerance	8	LF	Rad	D-Code			
CF(68 - CF(73) Blank							

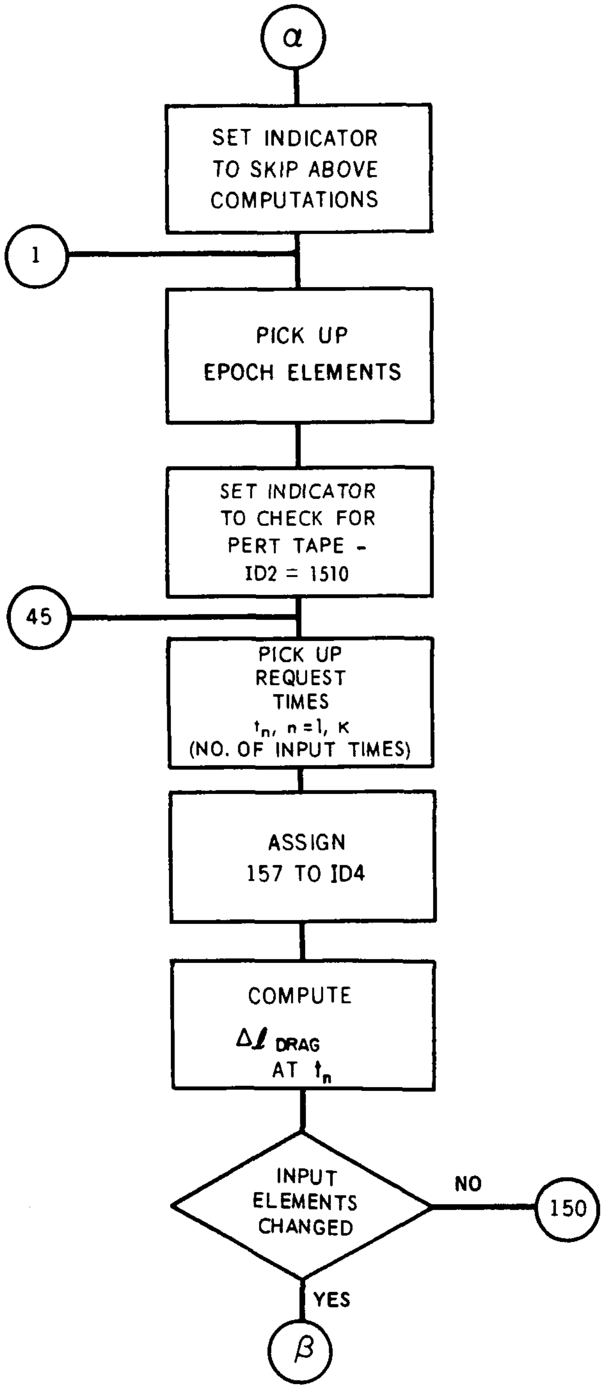
INTERFACE BLOCK DIAGRAM

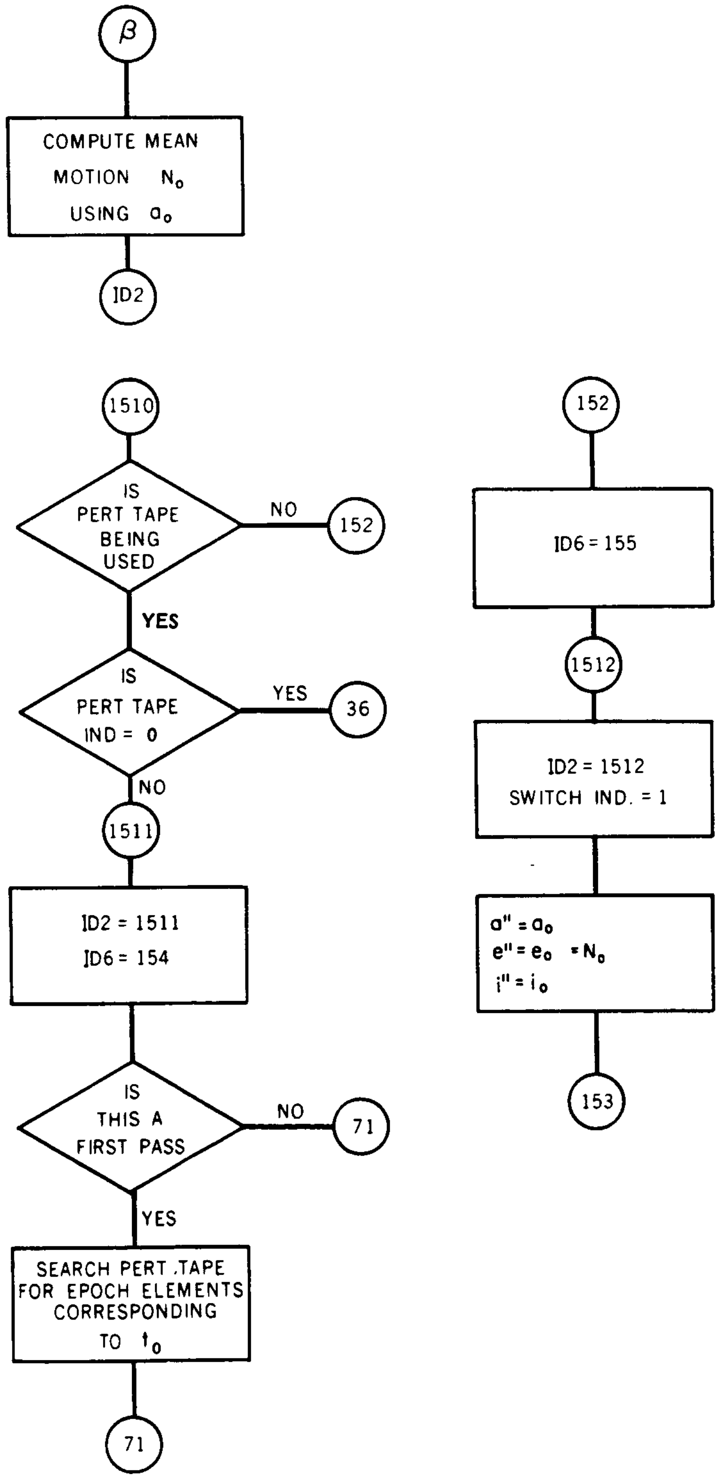


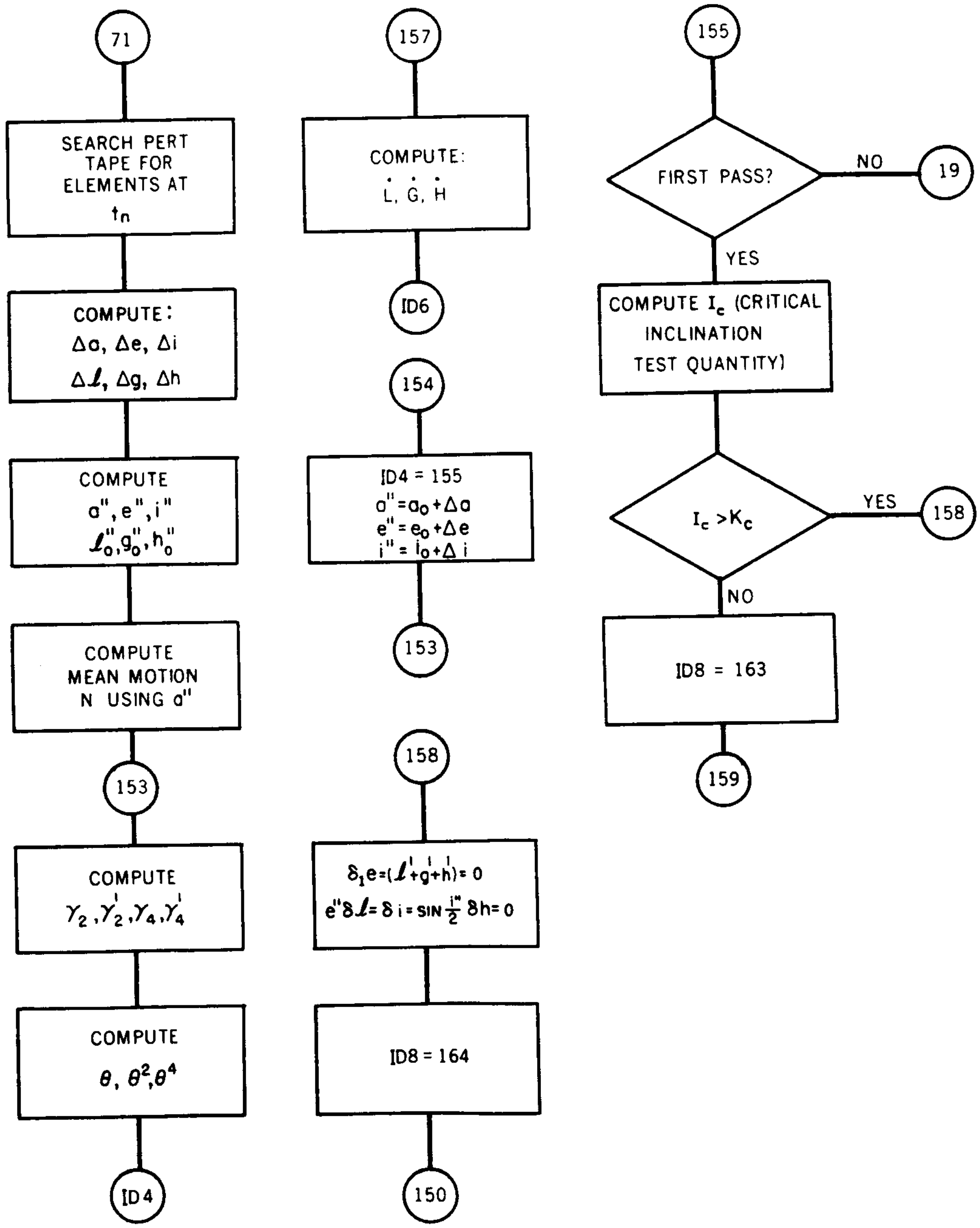
FUNCTIONAL ANALYSIS

BROUWER– LYDDANE FLOW CHART

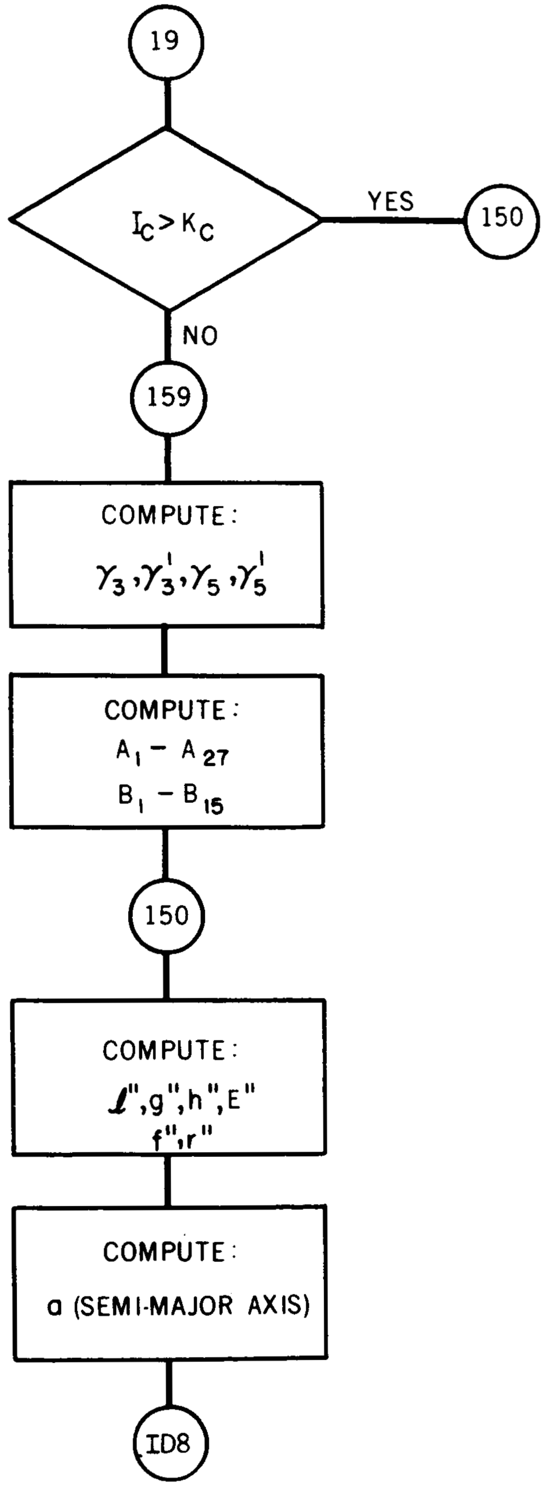


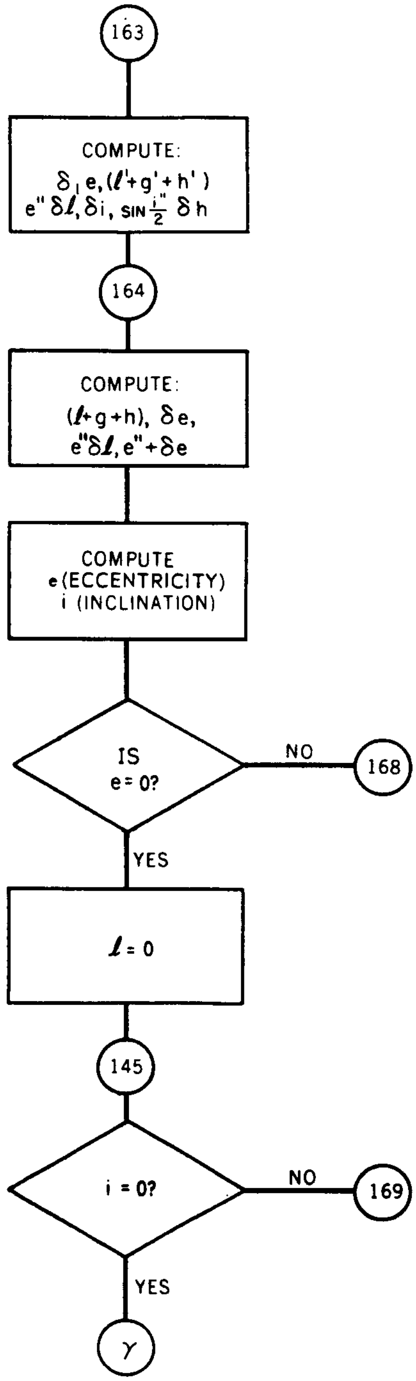

α
SET INDICATOR TO SKIP ABOVE COMPUTATIONS
1
PICK UP EPOCH ELEMENTS
SET INDICATOR TO CHECK FOR PERT TAPE - ID2 = 1510
45
PICK UP REQUEST TIMES t_n, n = 1, K (NO. OF INPUT TIMES)
ASSIGN 157 TO ID4
COMPUTE $\Delta \ell_{DRAG}$ AT t_n
INPUT ELEMENTS CHANGED
NO
150
YES
β

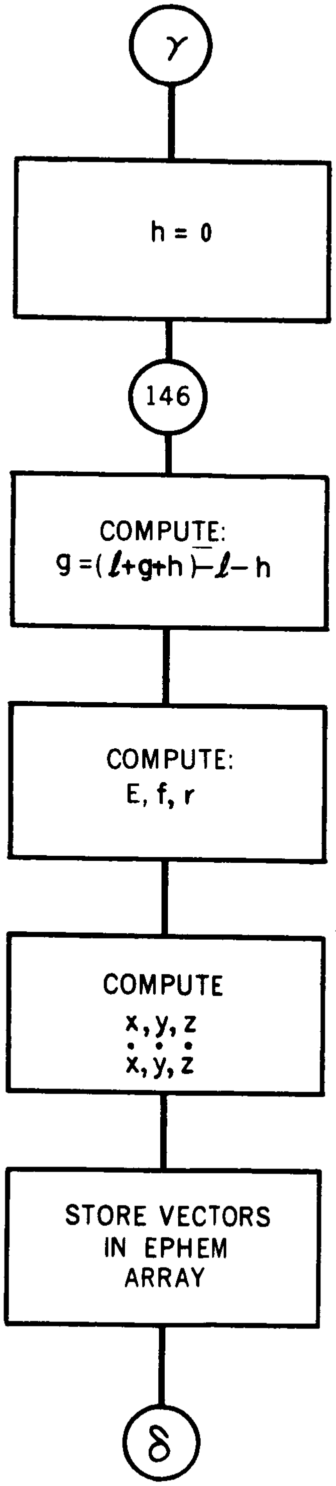

β
COMPUTE MEAN MOTION N_0 USING a_0
ID2
1510
IS PERT TAPE BEING USED
NO
152
YES
IS PERT TAPE IND = 0
YES
36
NO
1511
ID2 = 1511
ID6 = 154
IS THIS A FIRST PASS
NO
71
YES
SEARCH PERT TAPE FOR EPOCH ELEMENTS CORRESPONDING TO t_0
71
152
ID6 = 155
1512
ID2 = 1512
SWITCH IND. = 1
$a'' = a_0$
$e'' = e_0$ $*N_0$
$i'' = i_0$
153


71
SEARCH PERT TAPE FOR ELEMENTS AT t_n
COMPUTE: Δa, Δe, Δi, $\Delta \ell$, Δg, Δh
COMPUTE a'', e'', i'', ℓ_0'', g_0'', h_0''
COMPUTE MEAN MOTION N USING a''
153
COMPUTE γ_2, γ_2', γ_4, γ_4'
COMPUTE θ, θ^2, θ^4
ID4
157
COMPUTE: $\dot{L}$, $\dot{G}$, $\dot{H}$
ID6
154
ID4 = 155
$a'' = a_0 + \Delta a$
$e'' = e_0 + \Delta e$
$i'' = i_0 + \Delta i$
153
158
$\delta_1 e = (\ell' + g' + h') = 0$
$e'' \delta \ell = \delta i = \sin \frac{i''}{2} \delta h = 0$
ID8 = 164
150
155
FIRST PASS?
NO
19
YES
COMPUTE I_c (CRITICAL INCLINATION TEST QUANTITY)
$I_c > K_c$
YES
158
NO
ID8 = 163
159

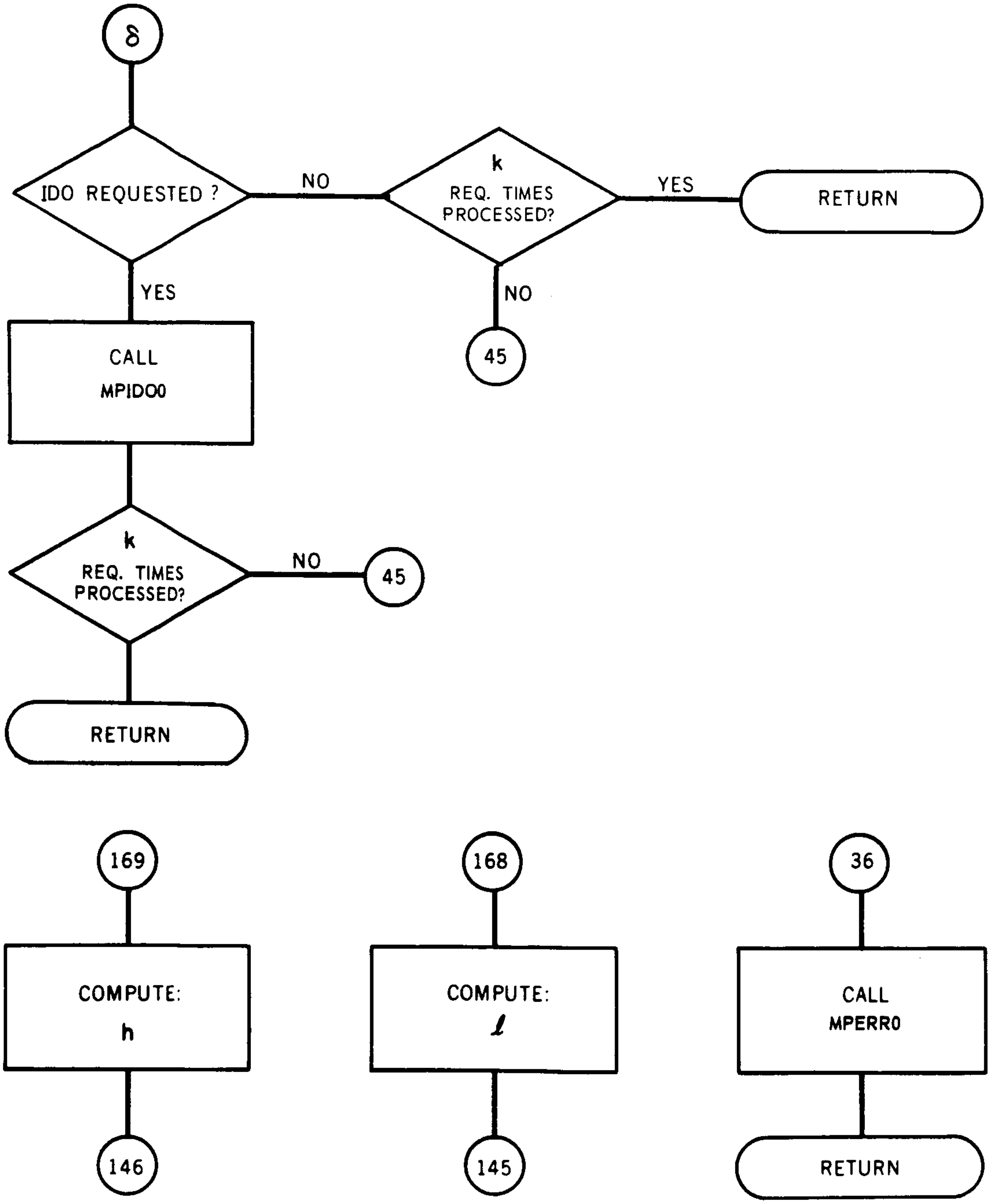
PAGE: 13-28
DATE: 20 June, 1969
SYMBOL: BROWR0
CONTRIBUTOR: E. A. Galbreath
GSFC




163
COMPUTE:
$\delta_1 e, (\ell' + g' + h')$
$e'' \delta \ell, \delta i, \sin \frac{i''}{2} \delta h$
164
COMPUTE:
$(\ell + g + h), \delta e,$
$e'' \delta \ell, e'' + \delta e$
COMPUTE
e (ECCENTRICITY)
i (INCLINATION)
IS
e = 0?
NO
168
YES
$\ell = 0$
145
i = 0?
NO
169
YES
γ


γ
h = 0
146
COMPUTE:
g =(ℓ+g+h)−ℓ− h
COMPUTE:
E, f, r
COMPUTE
x, y, z
ẋ, ẏ, ż
STORE VECTORS
IN EPHEM
ARRAY
δ

CONTRIBUTOR: E. A. Galbreath
GSFC



PAGE: 17-28
DATE: 20 June, 1969
SYMBOL: BROWR0
CONTRIBUTOR: E. A. Galbreath
GSFC

FORMULATION

I. ZONAL HARMONICS DETERMINATION

$$J_2 = -\bar{C}_{2,0}\sqrt{5}; \quad K_2 = 1/2\, J_2 R_e^2$$

$$J_3 = -\bar{C}_{3,0}\sqrt{7}; \quad K_3 = -J_3 R_e^3$$

$$J_4 = -\bar{C}_{4,0}\sqrt{9}; \quad K_4 = -3/8\, J_4 R_e^4$$

$$J_5 = -\bar{C}_{5,0}\sqrt{11}; \quad K_5 = -J_5 R_e^5$$

where $\bar{C}_{i,0}$ is the normalized form of Harmonic Coefficients taken from the DODS Harmonic Coefficients Array.

R_e = Radius of earth taken from Constants File

II. CALL DRAG TO COMPUTE $\Delta\ell_{DRAG}$ AT TIME T.

$$\Delta\ell_{DRAG} = \sum_{q=0}^{m} \sum_{p=2}^{3} N_{p,q} (t - t_q)^p$$

where $m = 0, 1, 2, \cdots 19$

III. COMPUTE MEAN MOTION

1. Without pert tape $\quad N_0 = \sqrt{\dfrac{\mu}{a_0^3}}$

2. With pert tape $\quad N = \sqrt{\dfrac{\mu}{(a_0'')^3}}$ where $a_0'' = a_0 + \Delta a/2$

IV. COMPUTE:

$$\eta = \sqrt{1 - e''^2} \qquad \gamma_4 = K_4/a''^4$$

$$\gamma_2 = K_2/a''^2 \qquad \gamma_4' = \gamma_4/\eta^8$$

$$\gamma_2' = \gamma_2/\eta^4 \qquad \gamma_5 = K_5/a''^5$$

$$\gamma_3 = K_3/a''^3$$

$$\gamma_3' = \gamma_3/\eta^6 \qquad \gamma_5'' = \gamma_5/\eta^{10}$$

$$\theta = \text{Cos } i''$$

$$\dot{\ell} = \eta N \left\{ \gamma_2' \left[\frac{3}{2}(3\theta^2 - \delta) + \gamma_2' \left(\frac{3}{32}\right) [\theta^2 (-96\eta + 30 - 90\eta^2) \right.\right.$$

$$\left. + (16\eta + 25\eta^2 - 15) + \theta^4 (144\eta + 25\eta^2 + 105)] \right]$$

$$\left. + e''^2 \gamma_2' \left(\frac{15}{16}\right) (3 + 35\theta^4 - 30\theta^2) \right\}$$

$$\dot{g} = N \left\{ \frac{5}{16} \gamma_4' \quad [[\theta^2 (126\eta^2 - 27) + \theta^4 (385 - 189\eta^2)] \right.$$

$$- 9\eta^2 + 21] \quad + \gamma_2' \left[\frac{3}{32} \gamma_2' [\theta^4 (45\eta^2 + 360\eta + 385) \right.$$

$$+ \theta^2 (90 - 192\eta - 126\eta^2) + (24\eta + 25\eta^2 - 35)]$$

$$\left.\left. + \frac{3}{2}(5\theta^2 - 1) \right] \right\}$$

$$\dot{h} = N \left\{ \gamma_4' \left(\frac{5}{4}\right) \theta (3 - 7\theta^2)(5 - 3\eta^2) + \gamma_2' \left[\gamma_2' \left(\frac{3}{8}\right) \right.\right.$$

$$\times [\theta (12\eta + 9\eta^2 - 5) - \theta^3 (5\eta^2 + 36\eta + 35)]$$

$$\left.\left. - 3\theta \right] \right\}$$

V. Critical inclination test quantity (I_c) is compared to K_c, where $K_c = .01$

$$I_c = 25\,\theta^5\,\gamma_2\,e''^2/(1 - 5\,\theta^2)^2.$$

If $I_c > K_c$ then

$$\delta_1 e = (\ell' + g' + h') = e''\,\delta\ell = \delta_1 i = 0$$

$$\left(\mathrm{Sin}\,\frac{i''}{2}\right)\delta h = 0$$

If $I_c \leq K_c$ then compute

$$\delta_1 e,\ \ell' + g' + h',\ e''\,\delta\ell,$$

$$\delta_1 i,\ \left(\mathrm{Sin}\,\frac{i''}{2}\right)\delta h$$

VI. COMPUTE A1-A10

$$A_1 = \left(\frac{1}{8}\gamma_2'\,\eta^2\right)\ \{1 - 11\,\theta^2 - [(40\,\theta^4)/(1 - 5\,\theta^2)]\}$$

$$A_2 = \left(\frac{5}{12}\right)\left(\frac{\gamma_4'}{\gamma_2'}\right)\eta^2\ \{1 - [8\,\theta^4/(1 - 5\,\theta^2)] - 3\,\theta^2\}$$

$$A_3 = \left(\frac{\gamma_5'}{\gamma_2'}\right)(3\,e''^2 + 4)$$

$$A_5 = \left\{\frac{\gamma_5'}{\gamma_2'}\,(3\,e'' + 4)\right\}\ \left\{1 - \frac{24\,\theta^4}{(1 - 5\,\theta^2)} - 9\,\theta^2\right\}$$

$$A_4 = \frac{\gamma_5'}{\gamma_2'}\left\{1 - \frac{(24\,\theta^4)}{(1 - 5\,\theta^2)} - 9\,\theta^2\right\}$$

$$A_{10} = \text{Sin}\,(i'')\,\eta^2$$

$$A_6 = (\gamma_3'/\gamma_2')\,(1/4)$$

$$A_7 = A_6 \times A_{10}$$

$$A_8 = \left(\frac{\gamma_5'}{\gamma_2'}\right) e''^2 \left\{1 - \frac{16\,\theta^4}{(1 - 5\,\theta^2)} - 5\,\theta^2\right\}$$

COMPUTE B13-B15

$$B_{13} = e''\,(A_1 - A_2)$$

$$B_{14} = A_7 + (5/64)\,(A_5)\,A_{10}$$

$$B_{15} = A_8\,(A_{10})\,(35/384)$$

COMPUTE A11-A27

$$A_{11} = 2\,e''^2$$

$$A_{12} = 3\,e''^2 + 2$$

$$A_{13} = \theta^2\,(A_{12})$$

$$A_{14} = (5\,e''^2 + 2)\,[\theta^4/(1 - 5\,\theta^2)]$$

$$A_{17} = \theta^4/(1 - 5\,\theta^2)^2$$

$$A_{15} = \theta^6\,e''^2/(1 - 5\,\theta^2)^2$$

$$A_{16} = \theta^2/(1 - 5\,\theta^2)$$

$$A_{18} = e''\,\text{Sin}\,(i'')$$

$$A_{19} = A_{18}/(1 + \eta)$$

$$A_{20} = (1 + \theta) \text{ Sin } i''$$

$$A_{21} = e'' \theta$$

$$A_{22} = e''^2 \theta$$

$$A_{23} = (A_{21}) \text{ Tan } \left(\frac{i''}{2}\right)$$

$$A_{24} = e'' \eta^2 \text{ Sin } i''$$

$$A_{25} = A_{12} + 2$$

$$A_{26} = 16 (A_{16}) + 40 (A_{17}) + 3$$

$$A_{27} = A_{22} (1/8) [11 + 200 (A_{17}) + 80 (A_{16})]$$

COMPUTE B1-B12

$$B_1 = \eta (A_1 - A_2) - \Big\{[A_{11} - 400 (A_{15}) - 40 (A_{14}) - 11 (A_{13})] \left(\frac{1}{16}\right) [11 + 200 (A_{17}) + 80 (A_{16})] (A_{22}) \left(\frac{1}{8}\right)\Big\} \times \gamma_2' + \Big\{[- 80 (A_{15}) - 8 (A_{14}) - 3 (A_{13}) + A_{11}] \left(\frac{5}{24}\right) + \frac{5}{12} (A_{26}) (A_{22})\Big\} \left(\frac{\gamma_4'}{\gamma_2'}\right)$$

$$B_2 = (A_6) (A_{19}) (2 + \eta - e''^2) + \left(\frac{5}{64}\right) (A_5) (A_{19}) \eta^2 - \left(\frac{15}{32}\right) A_4 (A_{18}) \eta^3 + \left[\left(\frac{5}{64}\right) A_5 + A_6\right] (A_{21}) \text{ Tan } \left(\frac{i''}{2}\right) + (9 e''^2 + 26) \left(\frac{5}{64}\right) A_4 (A_{18}) + \frac{15}{32} (A_3) A_{21} (A_{26}) \times \text{ Sin } i'' (1 - \theta)$$

$$B_3 = \left\{ [80\ (A_{17}) + 5 + 32\ (A_{16})]\ (A_{22})\ \text{Sin i}''\ (\theta - 1) \right.$$

$$\left. \times \left(\frac{35}{576}\right) \frac{\gamma_5'}{\gamma_2'}\, e'' \right\} - \left\{ \left[(A_{22})\ \text{Tan}\left(\frac{i''}{2}\right) + [2\, e''^2 + 3\ (1 - \eta^3)] \right. \right.$$

$$\left. \left. \text{Sin i}'' \right] \left(\frac{35}{1152}\right) \left(\frac{A_8}{e''}\right) \right\}$$

$$B_4 = \eta\, e''\ (A_1 - A_2)$$

$$B_5 = \left[(9\, e''^2 + 4)\ (A_{10})\ A_4 \left(\frac{5}{64}\right) + A_7 \right] \eta$$

$$B_6 = \left(\frac{35}{384}\right) A_8\ (\eta^3)\ \text{Sin}\ (i'')$$

$$B_7 = [(\eta^2\, A_{18})/(1 - 5\,\theta^2)] \left[\frac{1}{8}\gamma_2'\ (1 - 15\,\theta^2) + (1 - 7\,\theta^2) \times \left(\frac{\gamma_4'}{\gamma_2'}\right) \left(-\frac{5}{12}\right) \right]$$

$$B_8 = \left(\frac{5}{64}\right) \left\{ (A_3)\ \eta^2 \left[1 - 9\,\theta^2 - \left[\frac{24\,\theta^4}{(1 - 5\,\theta^2)} \right] \right] \right\}$$

$$B_9 = A_8 \left(\frac{35}{384}\right) \eta^2$$

$$B_{10} = \text{Sin i}'' \left[(A_{22})\ (A_{26}) \left(\frac{\gamma_4'}{\gamma_2'}\right) \left(\frac{5}{12}\right) - A_{27}\ (\gamma_2') \right]$$

$$B_{11} = A_{21} \left[A_5 \left(\frac{5}{64}\right) + A_6 + A_3\ (A_{26}) \left(\frac{15}{32}\right) \text{Sin}^2\ i'' \right]$$

$$B_{12} = -\left\{ [80 (A_{17}) + 32 (A_{16}) + 5] \left[(A_{22}) e'' \operatorname{Sin}^2 i'' \times \left(\frac{35}{576}\right) \left(\frac{\gamma_5'}{\gamma_2'}\right) \right] \right\} + \left[A_8 (A_{21}) \left(\frac{35}{1152}\right) \right]$$

VII. COMPUTE DOUBLE PRIMED ELEMENTS

$$\ell'' = \dot{\ell} (t - t_0) + N_0 (t - t_0) + \ell_0 + \Delta \ell_{DRAG}$$

$$g'' = \dot{g} (t - t_0) + g_0$$

$$h'' = \dot{h} (t - t_0) + h_0$$

$$f'' = \operatorname{Arctan} \left[\frac{\operatorname{Sin} f''}{\operatorname{Cos} f''} \right]$$

where

$$\operatorname{Sin} f'' = \eta \operatorname{Sin} E''$$

$$\operatorname{Cos} f'' = \operatorname{Cos} E'' - e$$

$$r'' = a'' (1 - e'' \operatorname{Cos} E'')$$

E'' is double primed eccentric anomaly computed from Kepler's equation.

If Pert tape is being used,

$$a'' = a_0 + \Delta_p a \qquad \ell_0 = \ell_0 + \Delta_p \ell$$

$$e'' = e_0 + \Delta_p e \qquad g_0 = g_0 + \Delta_p g$$

$$i'' = i_0 + \Delta_p i \quad , \quad h_0 = h_0 + \Delta_p h$$

If Pert tape is not used,

$$a'' = a_0$$

$$e'' = e_0$$

$$i'' = i_0$$

COMPUTE A (SEMI-MAJOR AXIS):

$$a = a'' \left\{ 1 + \gamma_2 \left[\left(3\theta^2 - 1\right) \left(\frac{e''^2}{\eta^6}\right) \left(\eta + \left(\frac{1}{1+\eta}\right)\right) + \left(\frac{3\theta^2 - 1}{\eta^6}\right) \left(e'' \operatorname{Cos} f''\right) \right.\right.$$

$$\left.\left. \times \left(3 + 3e'' \operatorname{Cos} f'' + e''^2 \operatorname{Cos}^2 f''\right) + 3\left(1 - \theta^2\right) \left(\frac{a''}{r''}\right)^3 \operatorname{Cos}\left(2g'' + 2f''\right) \right] \right\}$$

$$\delta_1 e = \text{B14} \operatorname{Sin} g'' + \text{B13} \operatorname{Cos} 2g'' - \text{B15} \operatorname{Sin} 3g''$$

COMPUTE $\ell' + g' + h'$:

$$\ell' + g' + h' = \ell'' + g'' + h'' + \text{B3} \operatorname{Cos} 3g'' + \text{B1} \operatorname{Sin} 2g'' + \text{B2} \operatorname{Cos} g''$$

$$e'' \delta\ell = \text{B4} \operatorname{Sin} 2g'' - \text{B5} \operatorname{Cos} g'' + \text{B6} \operatorname{Cos} 3g''$$

$$- \frac{1}{4} \eta^3 \gamma_2' \left\{ 2\left(3\theta^2 - 1\right) \left[\left(\frac{a''}{r''}\right)^2 \eta^2 + \frac{a''}{r''} + 1\right] \operatorname{Sin} f'' + 3\left(1 - \theta^2\right) \left[\left[-\left(\frac{a''}{r''}\right)^2 \eta^2 \right.\right.\right.$$

$$\left.\left.\left. - \frac{a''}{r''} + 1\right] \operatorname{Sin}(2g'' + f'') + \left[\left(\frac{a''}{r''}\right)^2 \eta^2 + \frac{a''}{r''} + \frac{1}{3}\right] \operatorname{Sin}(3f'' + 2g'')\right]\right\}$$

$$\delta I = \frac{1}{2} \gamma_2' \theta \operatorname{Sin} i'' \left\{ e'' \operatorname{Cos}(3f'' + 2g'') + 3\left[e'' \operatorname{Cos}(2g'' + f'') + \operatorname{Cos}(2g'' + 2f'')\right]\right\}$$

$$- \frac{\text{A21}}{\eta^2} \left(\text{B8} \operatorname{Sin} g'' + \text{B7} \operatorname{Cos} 2g'' - \text{B9} \operatorname{Sin} 3g''\right)$$

$$\operatorname{Sin}\left(\frac{1}{2} I''\right) \delta h = \left(\frac{1}{\operatorname{Cos} \frac{i''}{2}}\right) \left\{ \frac{1}{2} \left[\text{B12} \operatorname{Cos} 3g'' + \text{B11} \operatorname{Cos} g'' + \text{B10} \operatorname{Sin} 2g'' - \left[\frac{1}{2} \gamma_2' \theta \operatorname{Sin} i'' \right.\right.\right.$$

$$\left.\left.\left. \times \left[6(e'' \operatorname{Sin} f'' - \ell'' + f'') - \left[3\left(\operatorname{Sin}(2g'' + 2f'') + e'' \operatorname{Sin}(2g'' + f'')\right) + e'' \operatorname{Sin}(3f'' + 2g'')\right]\right]\right]\right]\right\}$$

COMPUTE $\ell + g + h$

$$\ell + g + h = \ell' + g' + h' + \Bigg\{ \left(\frac{1}{\eta+1}\right) \frac{1}{4} e'' \gamma_2' \eta^2 \Bigg[3\left(1-\theta^2\right) \Bigg[\operatorname{Sin}(3f'' + 2g'')$$

$$\times \left\{ \frac{1}{3} + \left(\frac{a''}{r''}\right)^2 \eta^2 + \frac{a''}{r''} \right\} + \operatorname{Sin}(2g'' + f'') \left(1 - \left(\eta^2 \left(\frac{a''}{r''}\right)^2 + \frac{a''}{r''} \right) \right) \Bigg]$$

$$+ 2 \operatorname{Sin} f'' \left(3\theta^2 - 1\right) \left(\eta^2 \left(\frac{a''}{r''}\right)^2 + \left(\frac{a''}{r''}\right) + 1 \right) \Bigg] \Bigg\} + \gamma_2' \left(\frac{3}{2}\right) \left[\left(-2\theta - 1 - 5\theta^2\right) \right.$$

$$\left. \times \left(e'' \operatorname{Sin} f'' + f'' - \ell''\right) \right] + \left(3 + 2\theta - 5\theta^2\right) \Bigg\{ \gamma_2' \left(\frac{1}{4}\right) \Big[e'' \operatorname{Sin}(3f'' + 2g'')$$

$$+ 3 \left[\operatorname{Sin}(2g'' + 2f'') + e'' \operatorname{Sin}(2g'' + f'') \right] \Big] \Bigg\}$$

$$\delta e = \delta_1 e + \Bigg\{ \frac{1}{2} \eta^2 \Bigg[\Bigg[3\left(\frac{1}{\eta^6}\right) \gamma_2 \left(1-\theta^2\right) \operatorname{Cos}(2g'' + 2f'')$$

$$\times \left\{ 3e'' \operatorname{Cos}^2 f'' + 3 \operatorname{Cos} f'' + e''^2 \operatorname{Cos}^3 f'' + e'' \right\} \Bigg]$$

$$- \left\{ \gamma_2' \left(1-\theta^2\right) \left[3 \operatorname{Cos}(2g'' + f'') + \operatorname{Cos}(3f'' + 2g'') \right] \right\}$$

$$+ \left(3\theta^2 - 1\right) \gamma_2 \left(\frac{1}{\eta^6}\right) \left\{ e'' \eta + \left(\frac{e''}{1+\eta}\right) + 3e'' \operatorname{Cos}^2 f'' + 3 \operatorname{Cos} f'' + e''^2 \operatorname{Cos}^3 f \right\} \Bigg] \Bigg\}$$

COMPUTE e (ECCENTRICITY)

$$e = \sqrt{(e'' \, \delta\ell)^2 + (e'' + \delta e)}$$

COMPUTE i (INCLINATION)

$$i = \text{Arc Sin}\left(\sqrt{\left(\text{Sin}\,\frac{i''}{2}\;\delta h\right)^2 + \left(\delta I\;\text{Cos}\,\frac{i''}{2}\left(\frac{1}{2}\right) + \text{Sin}\,\frac{i''}{2}\right)^2}\right)$$

If $e = 0$, set $\ell = 0$

If $e \neq 0$, compute ℓ (mean anomaly)

$$\ell = \text{Arctan}\left(\frac{e''\;\delta\ell\,\text{Cos}\,\ell'' + (e'' + \delta e)\,\text{Sin}\,\ell''}{(e'' + \delta e)\,\text{Cos}\,\ell'' - e''\;\delta\ell\,\text{Sin}\,\ell''}\right)$$

If $i = 0$ set $h = 0$

If $i \neq 0$ compute h (longitude of the ascending node)

$$h = \text{Arctan}\left(\frac{\text{Sin}\,\frac{i''}{2}\;\delta h\,(\text{Cos}\,h'') + \text{Sin}\,h''\left(\frac{1}{2}\;\delta I\,\text{Cos}\,\frac{i''}{2} + \text{Sin}\,\frac{i''}{2}\right)}{\text{Cosh}\,h''\left(\frac{1}{2}\;\delta I\;\text{Cos}\,\frac{i''}{2} + \text{Sin}\,\frac{i''}{2}\right) - \text{Sin}\,h''\left(\text{Sin}\,\frac{i''}{2}\;\delta h\right)}\right)$$

COMPUTE g (ARGUMENT OF PERIGEE)

$$g = (\ell + g + h) - \ell - h$$

COMPUTE E (ECCENTRIC ANOMALY) USING KEPLER'S EQUATION.

COMPUTE f (TRUE ANOMALY)

$$f = \text{Arctan}\left(\frac{\text{Sin}\,E\sqrt{(1 - e^2)}}{\text{Cos}\,E - e}\right)$$

COMPUTE r (RADIUS VECTOR)

$$\frac{r}{a} = 1 - e\,\text{Cos}\,E$$

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DATE: 20 June, 1969
SYMBOL: BROWR0
CONTRIBUTOR: E. A. Galbreath
GSFC

COMPUTE POSITION VECTOR (x, y, z)

$$x = r\ [\mathrm{Cos}\ h\ \mathrm{Cos}\ (g + f) - \mathrm{Sin}\ h\ \mathrm{Cos}\ i\ \mathrm{Sin}\ (g + f)]$$

$$y = r\ [\mathrm{Cos}\ h\ \mathrm{Cos}\ i\ \mathrm{Sin}\ (g + f) + \mathrm{Sin}\ h\ \mathrm{Cos}\ (g + f)]$$

$$z = r\ [\mathrm{Sin}\ i\ \mathrm{Sin}\ (g + f)]$$

COMPUTE VELOCITY VECTOR ($\dot{x}$, $\dot{y}$, $\dot{z}$)

$$\dot{x} = \mathrm{Sin}\ E\ (e \sqrt{a\mu}/r)\ [\mathrm{Cos}\ h\ \mathrm{Cos}\ (g + f) - \mathrm{Sin}\ h\ \mathrm{Cos}\ i$$

$$\times\ \mathrm{Sin}\ (g + f)] - \sqrt{1 - e^2} \sqrt{a\mu}/r\ [\mathrm{Sin}\ h\ \mathrm{Cos}\ i\ \mathrm{Cos}\ (g + f)$$

$$+ \mathrm{Cos}\ h\ \mathrm{Sin}\ (g + f)]$$

$$\dot{y} = e\ \mathrm{Sin}\ E\ (\sqrt{a\mu}/r)\ [\mathrm{Cos}\ h\ \mathrm{Cos}\ i\ \mathrm{Sin}\ (g + f)$$

$$+ \mathrm{Sin}\ h\ \mathrm{Cos}\ (g + f)] - \sqrt{1 - e^2}\ (\sqrt{a\mu}/r)\ [\mathrm{Sin}\ h\ \mathrm{Sin}\ (g + f)$$

$$- \mathrm{Cos}\ h\ \mathrm{Cos}\ i\ \mathrm{Cos}\ (g + f)]$$

$$\dot{z} = e\ \mathrm{Sin}\ E\ (\sqrt{a\mu}/r)\ \mathrm{Sin}\ i\ \mathrm{Sin}\ (g + f)$$

$$+ [\sqrt{1 - e^2}\ (\sqrt{a\mu}/r)\ \mathrm{Sin}\ i\ \mathrm{Cos}\ (g + f)]$$

RESTRICTIONS AND LIMITATIONS

I. Mathematical Restrictions: None

II. Data Restrictions: None

III. Hardware Restrictions: None

IV. Programming Language Restrictions: None

CONTRIBUTOR: E. A. Galbreath
GSFC

DEFINITIVE ORBIT DETERMINATION SYSTEM

PERTF0 – Complementary Perturbations Tape Read Routine

I. LANGUAGE:

Fortran IV, Level G and Level H

II. PURPOSE:

This subroutine reads the complementary perturbations tape for the Brouwer-Lyddane Orbit generator.

III. INTERFACE INFORMATION

A. Calling Module is BROWR0

B. Called Modules are

(1) INTPL0 — performs backward difference interpolation for Complementary Perturbations Tape Read Routine

(2) MPERR0 – Error handling routine from DODS

(3) TCONV0 – Time Conversion Routine from DODS

C. Calling sequence

Subroutine PERTF0 (PLN, SATID, TIME, KMULT, B, IERR, EPHEM)

Table I
Calling Sequence Arguments

Argument Name	Analytic Symbol	I/O	Description	Units	Format	Dimension
PLN		I	Perturbation Tape Logical Input Unit	None	LF	
SATID		I	Satellite Identification Number	None	LI	
TIME	*t = t + t0	I	Request time	DUT	LF	
KMULT		I/O	K-multiplier for $\Delta\ell$ drag computation	None	LI	
B		O	Array of elements from perturbation tape for time t	DUL Rad	LF	(6)
EPHEM		I/O	Array to store variables that may be destroyed by DODS overlay	DUL DUT Rad	LF	(31, 50)

*t0 - epoch time

DEFINITION OF ARRAYS:

B ARRAY

$$\left.\begin{array}{l} B(1) = a_p \text{ in DUL} \\ B(2) = e_p \\ B(3) = i_p \quad \text{(Rad)} \\ B(4) = \ell_p \quad \text{(Rad)} \\ B(5) = g_p \quad \text{(Rad)} \\ B(6) = h_p \quad \text{(Rad)} \end{array}\right\} \text{Elements from perturbations tape.}$$

PLN – Perturbations tape unit number

PLN > 0 Read Pert tape on this unit

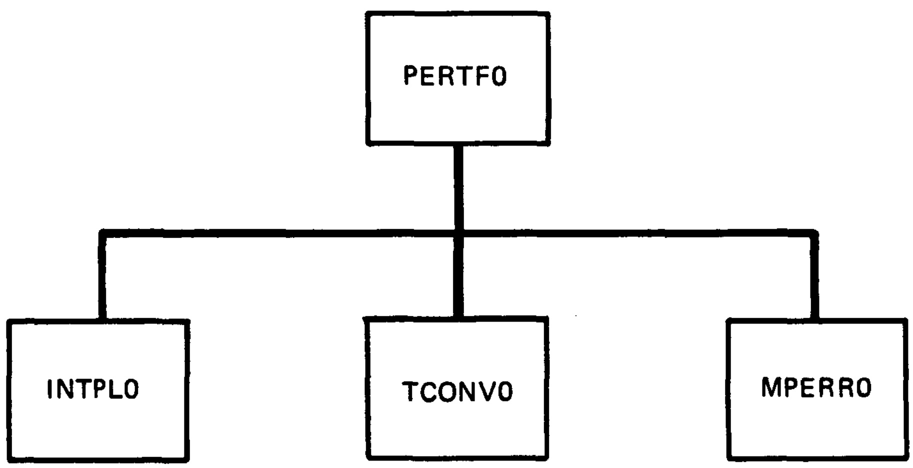
PLN < 0 Do not read Pert tape

PLN = 0 Error

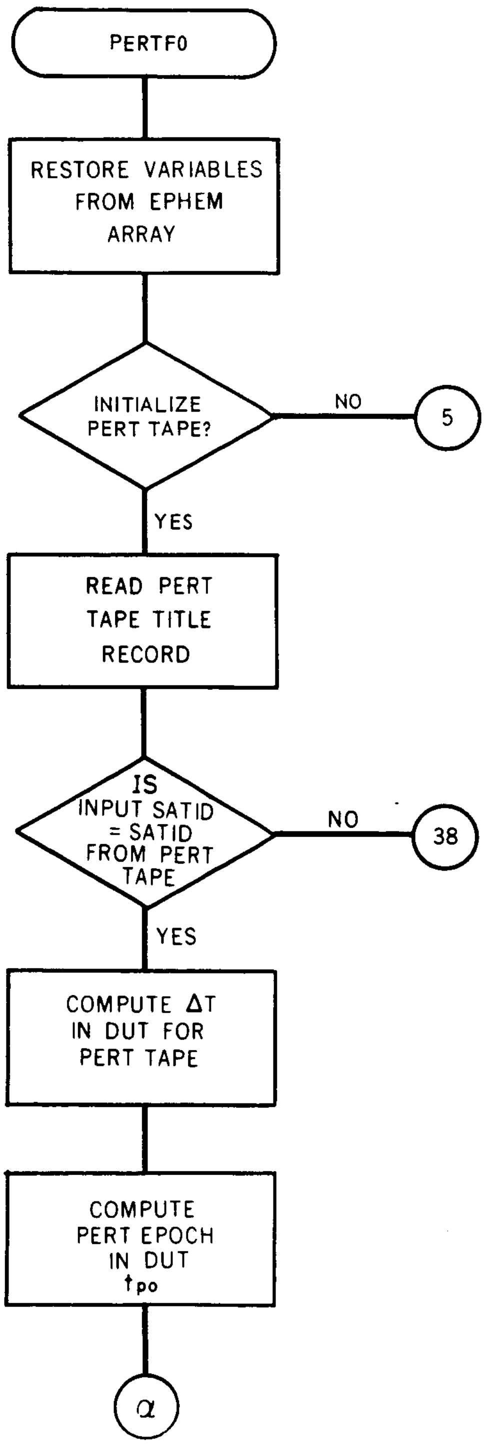
EPHEM ARRAY

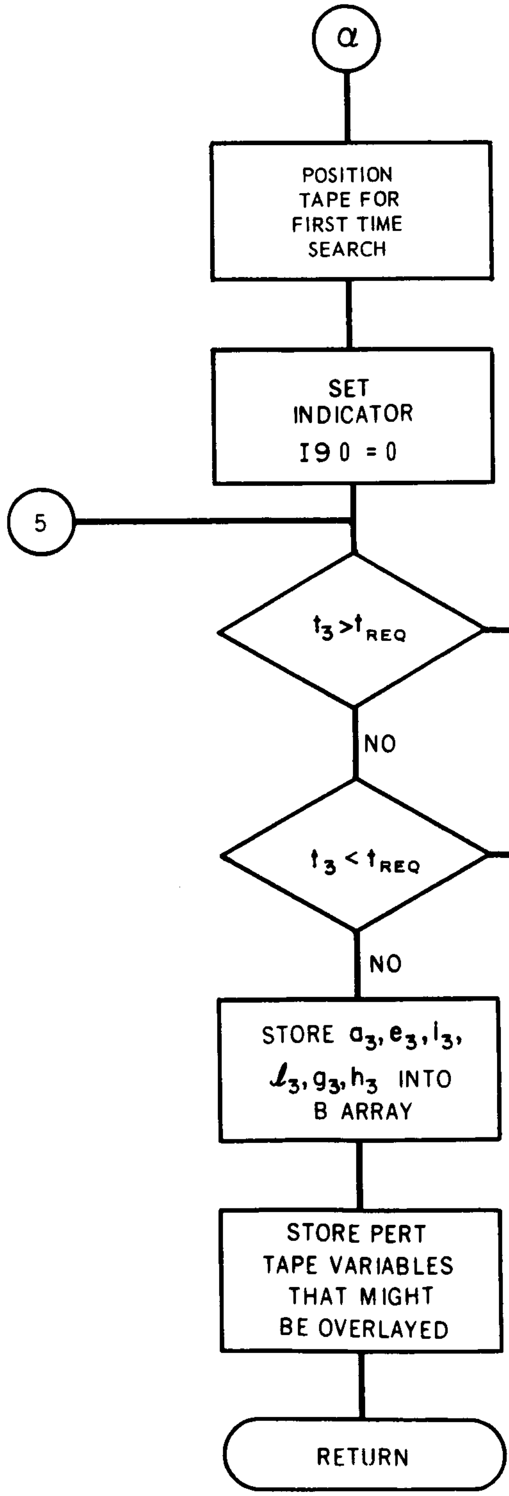
Array to store variables that might be destroyed by DODS overlay.

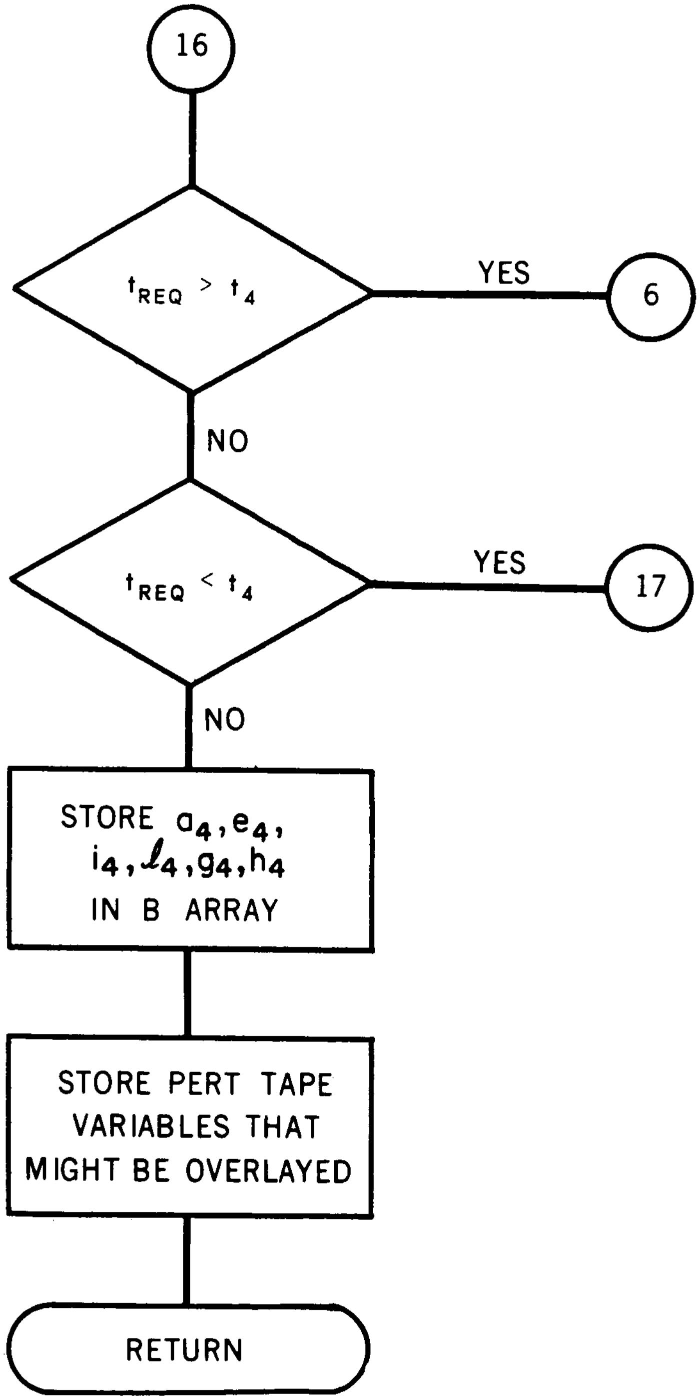
INTERFACE BLOCK DIAGRAM

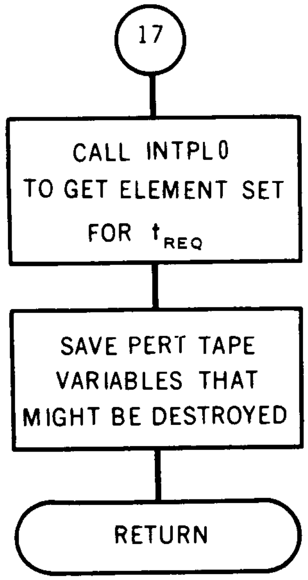


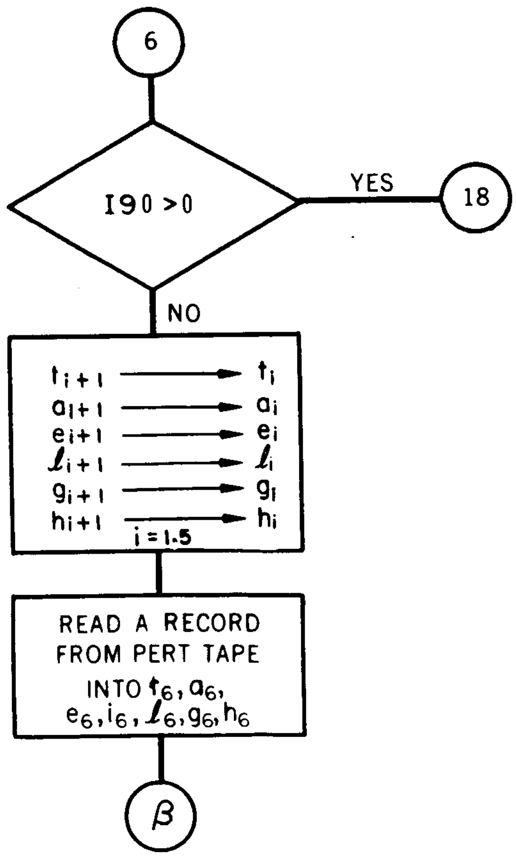
PERTF0 FLOWCHART

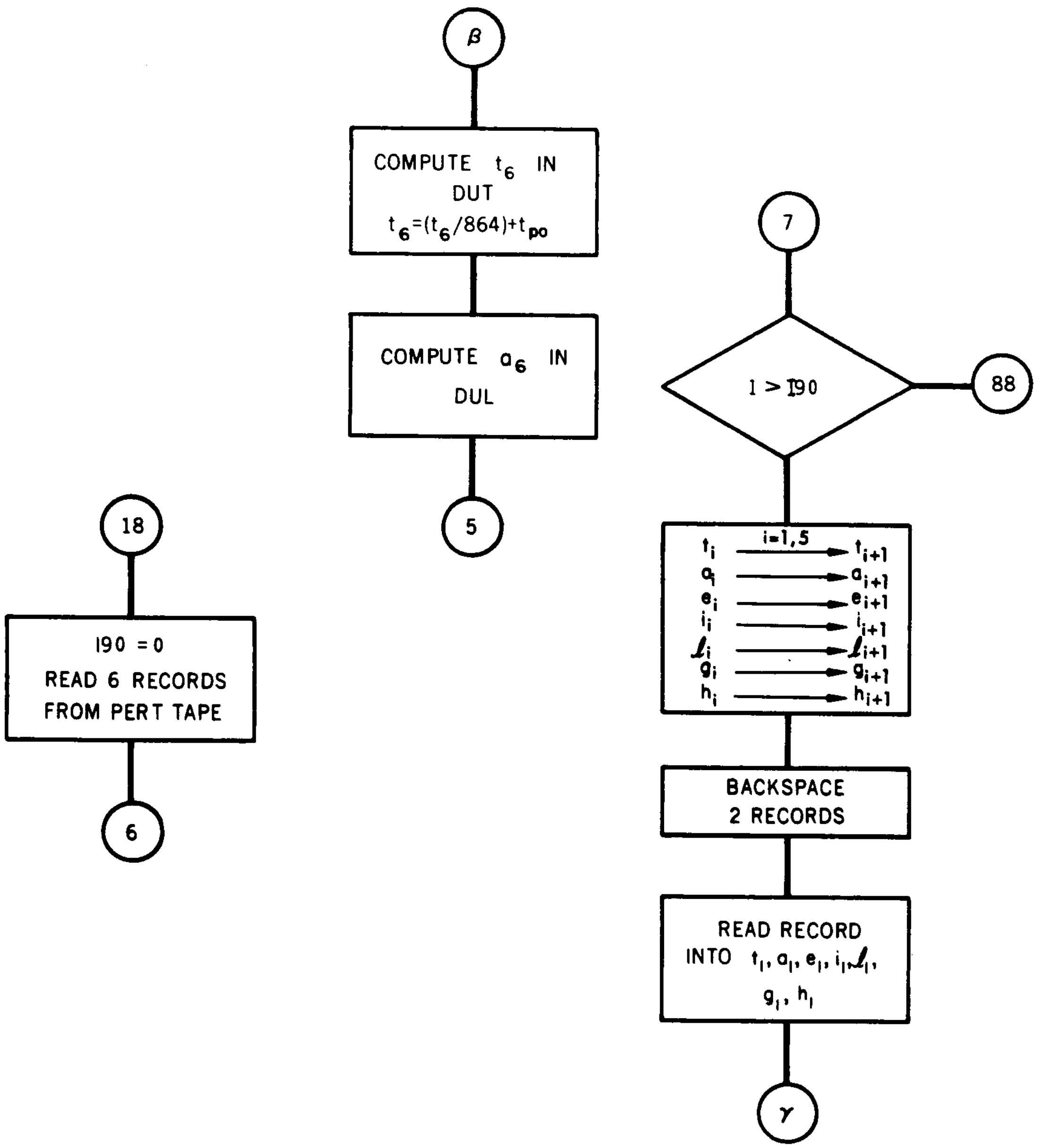


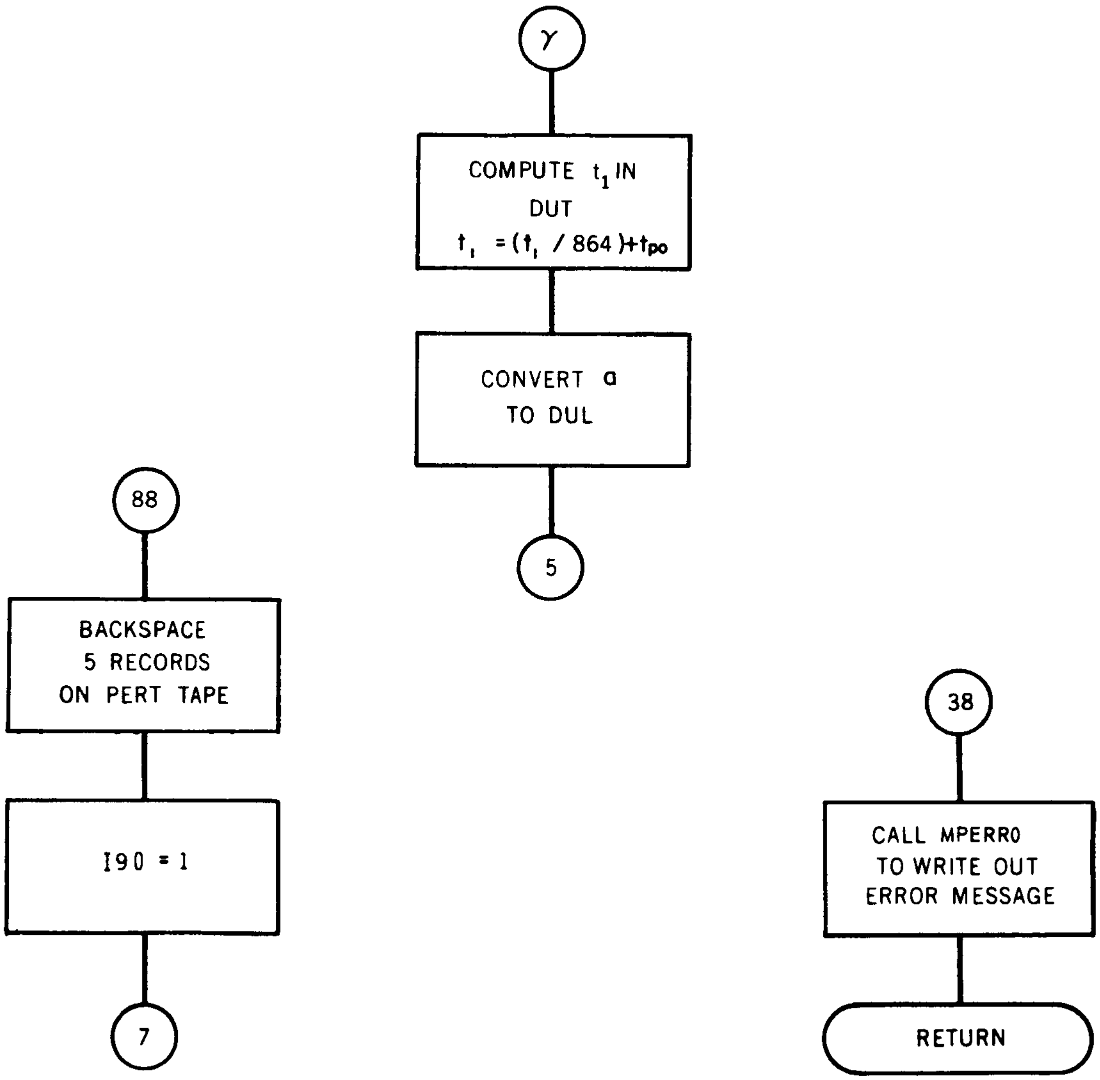

α
POSITION TAPE FOR FIRST TIME SEARCH
SET INDICATOR I90 = 0
5
$t_3 > t_{REQ}$
YES
7
NO
$t_3 < t_{REQ}$
YES
16
NO
STORE $a_3, e_3, i_3, \ell_3, g_3, h_3$ INTO B ARRAY
STORE PERT TAPE VARIABLES THAT MIGHT BE OVERLAYED
RETURN


16
$t_{REQ} > t_4$
YES
6
NO
$t_{REQ} < t_4$
YES
17
NO
STORE $a_4, e_4, i_4, \ell_4, g_4, h_4$ IN B ARRAY
STORE PERT TAPE VARIABLES THAT MIGHT BE OVERLAYED
RETURN


17
CALL INTPL0 TO GET ELEMENT SET FOR t_{REQ}
SAVE PERT TAPE VARIABLES THAT MIGHT BE DESTROYED
RETURN


6
I90 >0
YES
18
NO
$t_{i+1} \rightarrow t_i$
$a_{i+1} \rightarrow a_i$
$e_{i+1} \rightarrow e_i$
$\ell_{i+1} \rightarrow \ell_i$
$g_{i+1} \rightarrow g_i$
$h_{i+1} \rightarrow h_i$
$i = 1\text{-}5$
READ A RECORD FROM PERT TAPE INTO $t_6, a_6, e_6, i_6, \ell_6, g_6, h_6$
β


β
COMPUTE t_6 IN DUT
$t_6=(t_6/864)+t_{po}$
COMPUTE a_6 IN DUL
5
7
$I > I90$
88
i=1,5
t_i → t_{i+1}
a_i → a_{i+1}
e_i → e_{i+1}
i_i → i_{i+1}
ℓ_i → ℓ_{i+1}
g_i → g_{i+1}
h_i → h_{i+1}
BACKSPACE 2 RECORDS
READ RECORD INTO $t_1, a_1, e_1, i_1, \ell_1, g_1, h_1$
γ
18
I90 = 0
READ 6 RECORDS FROM PERT TAPE
6


γ
COMPUTE t_1 IN DUT
$t_1 = (t_1 / 864) + t_{po}$
CONVERT a TO DUL
5
88
BACKSPACE 5 RECORDS ON PERT TAPE
I90 = 1
7
38
CALL MPERRO TO WRITE OUT ERROR MESSAGE
RETURN

PAGE: 11-12
DATE: 12 January, 1970
SYMBOL: PERTF0
CONTRIBUTOR: E. A. Galbreath
GSFC

BROUWER PERTAPE FORMAT

Word No.	Contents
	Record #1 – Header Record
1	Fortran Word Count
2	Time Increment – days
3	Month
4	Day
5	Year
6	Satellite ID number
7	Input Semi-major Axis - E.R.
8	Input Eccentricity
9	Input Inclination - degrees
10	Input right ascension of the ascending node - degrees
11	Input argument of perigee - degrees
12	Input mean anomaly - degrees
13	Input time from midnight - days
14	Input period - minutes
15	No. of records on tape excluding header and trailer
16	Delta mean anomaly option indicator
	(1 - delta drag mean anomaly not computed on tape
	0 - delta drag mean anomaly computed on tape.)
	Record 2 to N
1	Fortran Word count
2	Time in seconds from epoch
3	A (semi-major axis) - ER
4	e (eccentricity)
5	i (Inclination) - $\pi/2 \leqq i \leqq \pi/2$
6	ΔM (mean anomaly change from t_0) $0 \leqq \Delta M \leqq 2\pi$
7	ω (argument of perigee) $0 \leqq \omega \leqq 2\pi$
8	Ω (right ascension of the ascending node) $0 \leqq \Omega \leqq 2\pi$
	Last Record - Trailer Record
1	Fortran Word count
2	$.99999999 \times 10^{30}$
3–8	Dummy Words

RESTRICTIONS AND LIMITATIONS

I. Mathematical Restrictions:

None

II. Data Restrictions:

There must be at least 3 data records on the perturbation tape before the start time for the run and at least 3 data records on tape after the end time for the run.

III. Hardware Restrictions:

None

IV. Programming Language Restrictions:

The perturbation tape may be backspaced during the program, therefore the pert tape must be unblocked. Blocked tapes give undetermined results if backspaced.

DEFINITIVE ORBIT DETERMINATION SYSTEM

INTPL0 – Backward Difference Interpolation Function

I. LANGUAGE:

Fortran IV, Level G and Level H

II. PURPOSE:

This subroutine interpolates for the elements a_p, e_p, i_p, ℓ_p, g_p, h_p when given a request time (t_{REQ}) between two times on the tape.

III. INTERFACE INFORMATION

A. Calling Module is PERTF0

B. Calling Sequence SUBROUTINE INTPL0 (TIME, A, B, TSUB0, DELTA)

Table I
Calling Sequence Arguments

Argument Name	Analytic Symbol	I/O	Description	Units	Format	Dimension
TIME	$t_{REQ}=t+t_0$ EPOCH	I	Request time	DUT	LF	
A		I	Array of times and elements from the perturbation tape	DUT, DUL, Rad	LF	(6,7)
B		O	Array of interpolated elements from perturbation tape for request time.	DUL, Rad	LF	(6)
TSUB0	t_6	I	Sixth time in the time element array A.	DUT	LF	
DELTA	Δt	I	Time increment between times on the perturbation tape	DUT	LF	

DEFINITION OF ARRAYS: A(I, J)

J \ I	1	2	3	4	5	6
1	t_1	t_2	t_3	t_4	t_5	t_6
2	a_1	a_2	a_3	a_4	a_5	a_6
3	e_1	e_2	e_3	e_4	e_5	e_6
4	i_1	i_2	i_3	i_4	i_5	i_6
5	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6
6	g_1	g_2	g_3	g_4	g_5	g_6
7	h_1	h_2	h_3	h_4	h_5	h_6

B ARRAY

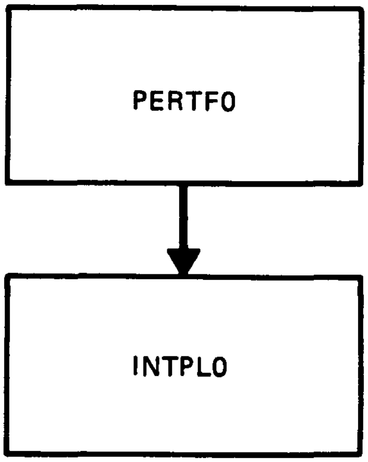
$B(1) = a_p$ in DUL

$B(2) = e_p$

$$\left.\begin{array}{l} B(3) = i_p \\ B(4) = \ell_p \\ B(5) = g_p \\ B(6) = h_p \end{array}\right\} \text{Radians}$$

CONTRIBUTOR: E. A. Galbreath
GSFC

INTERFACE BLOCK DIAGRAM



FUNCTIONAL ANALYSIS

Backward Difference Interpolation

$$\left.\begin{array}{l} t_6 \\ t_5 \\ t_4 \\ t_3 \\ t_2 \\ t_1 \end{array}\right\} \text{times from the perturbation tape}$$

$$\left.\begin{array}{l} f(t_6) \\ f(t_5) \\ f(t_4) \\ f(t_3) \\ f(t_2) \\ f(t_1) \end{array}\right\} \text{elements from the perturbation tape at time } t_i.$$

$t \Longrightarrow$ request time

$\Delta t \Longrightarrow$ time increment from perturbation tape

$\Delta^k f(t_i) \Rightarrow$ the k^{th} difference of the elements $\Delta^{k-1} f(t_{i+1}) - \Delta^{k-1} f(t_i)$

where $\Delta^{\circ} f(t_i) \equiv f(t_i)$

DIFFERENCE TABLE

t_6	$f(t_6)$					
		$\Delta f(t_5)$				
t_5	$f(t_5)$		$\Delta^2 f(t_4)$			
		$\Delta f(t_4)$		$\Delta^3 f(t_3)$		
t_4	$f(t_4)$		$\Delta^2 f(t_3)$		$\Delta^4 f(t_2)$	
		$\Delta f(t_3)$		$\Delta^3 f(t_2)$		$\Delta^5 f(t_1)$
t_3	$f(t_3)$		$\Delta^2 f(t_2)$		$\Delta^4 f(t_1)$	
		$\Delta f(t_2)$		$\Delta^3 f(t_1)$		
t_2	$f(t_2)$		$\Delta^2 f(t_1)$			
		$\Delta f(t_1)$				
t_1	$f(t_1)$					

$$(\text{element})_p = f(t_6) + \Delta f(t_5)(tt_0) + \Delta^2 f(t_4)\left[\frac{(tt_0)(tt_1)}{2}\right]$$

$$+ \Delta^3 f(t_3)\left[\frac{(tt_0)(tt_1)(tt_2)}{6}\right] + \Delta^4 f(t_2)$$

$$\times \left[\frac{(tt_0)(tt_1)(tt_2)(tt_3)}{24}\right] + \Delta^5 f(t_1)$$

$$\times \left[\frac{(tt_0)(tt_1)(tt_2)(tt_3)(tt_4)}{120}\right]$$

Where

$$tt_0 = t - t_6$$

$$tt_1 = \Delta t + tt_0$$

$$tt_2 = \Delta t + tt_1$$

$$tt_3 = \Delta t + tt_2$$

$$tt_4 = \Delta t + tt_3$$

RESTRICTION AND LIMITATIONS

I. Mathematical Restrictions: None

II. Data Restrictions: None

III. Hardware Restrictions: None

IV. Programming Language Restrictions: None.

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DATE: 29 May, 1969
SYMBOL: DRAG
CONTRIBUTOR: E. A. Galbreath
GSFC

DEFINITIVE ORBIT DETERMINATION SYSTEM

Drag – ΔL Drag Subroutine

I. Language:

Fortran IV, Level G and Level H

II. Purpose:

This subroutine computes $\Delta\ell_{DRAG}$ for time $t = t - t_0$ from input parameters $t_{p,q}$ and $N_{p,q}$.

III. Interface Information

A. Calling Module is BROWR0

B. Called Module is REDUCE which reduces angle between 0 & 2π

C. Calling Sequence

Subroutine Drag (DPT, PI2, DRAGL, T0, T, KMULT)

Table I
Calling Sequence Arguments

Argument Name	Analytic Symbol	I/O	Description	Units	Format	Dimension
DPT		I	Drag Parameters Table	DUT, Rad/DUT^2 Rad/DUT^3	LF	(60)
PI2	2π	I	2π radians	Rad	LF	
DRAGL	$\Delta\ell_{DRAG}$	O	Delta L drag	Rad	LF	
T0	t_0	I	Epoch	DUT	LF	
T	t	I	Request time $t = t - t_0$	DUT	LF	
KMULT		I	K-multiplier		I	

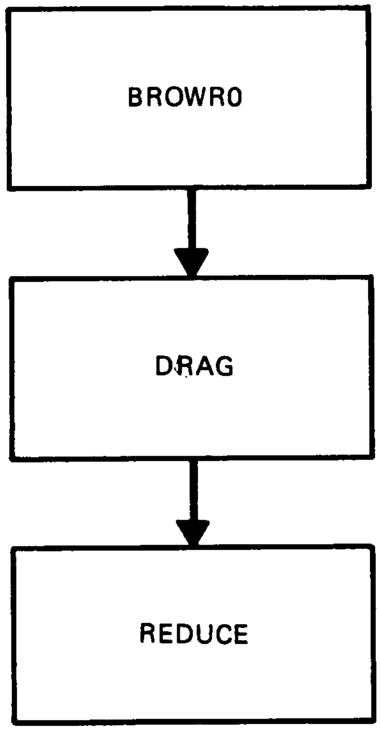
DEFINITION OF ARRAYS

$$\left.\begin{array}{c}\text{DPT}(1)\\ \downarrow\\ \text{DPT}(20)\end{array}\right\} t_0,\ t_1,\ \ldots,\ t_{19}$$

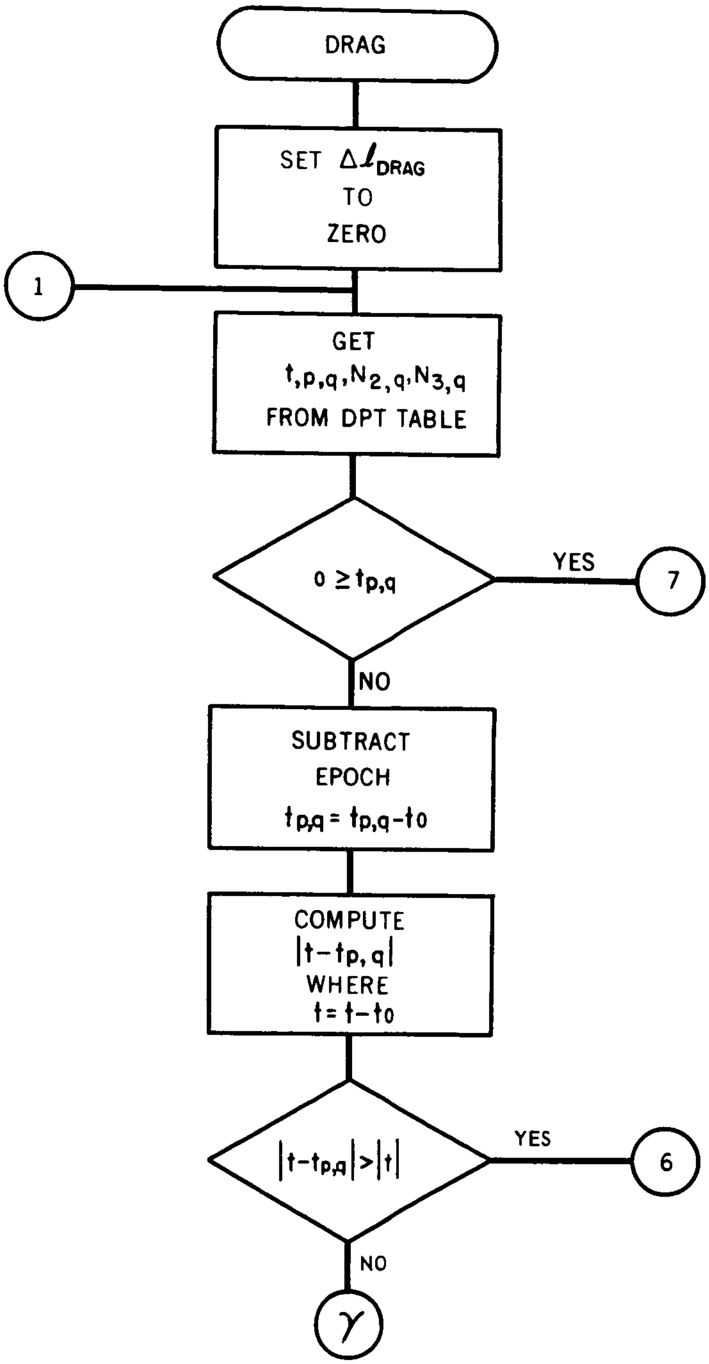
$$\left.\begin{array}{c}\text{DPT}(21)\\ \downarrow\\ \text{DPT}(40)\end{array}\right\} N_{2,0},\ N_{2,1},\ \ldots,\ N_{2,19}$$

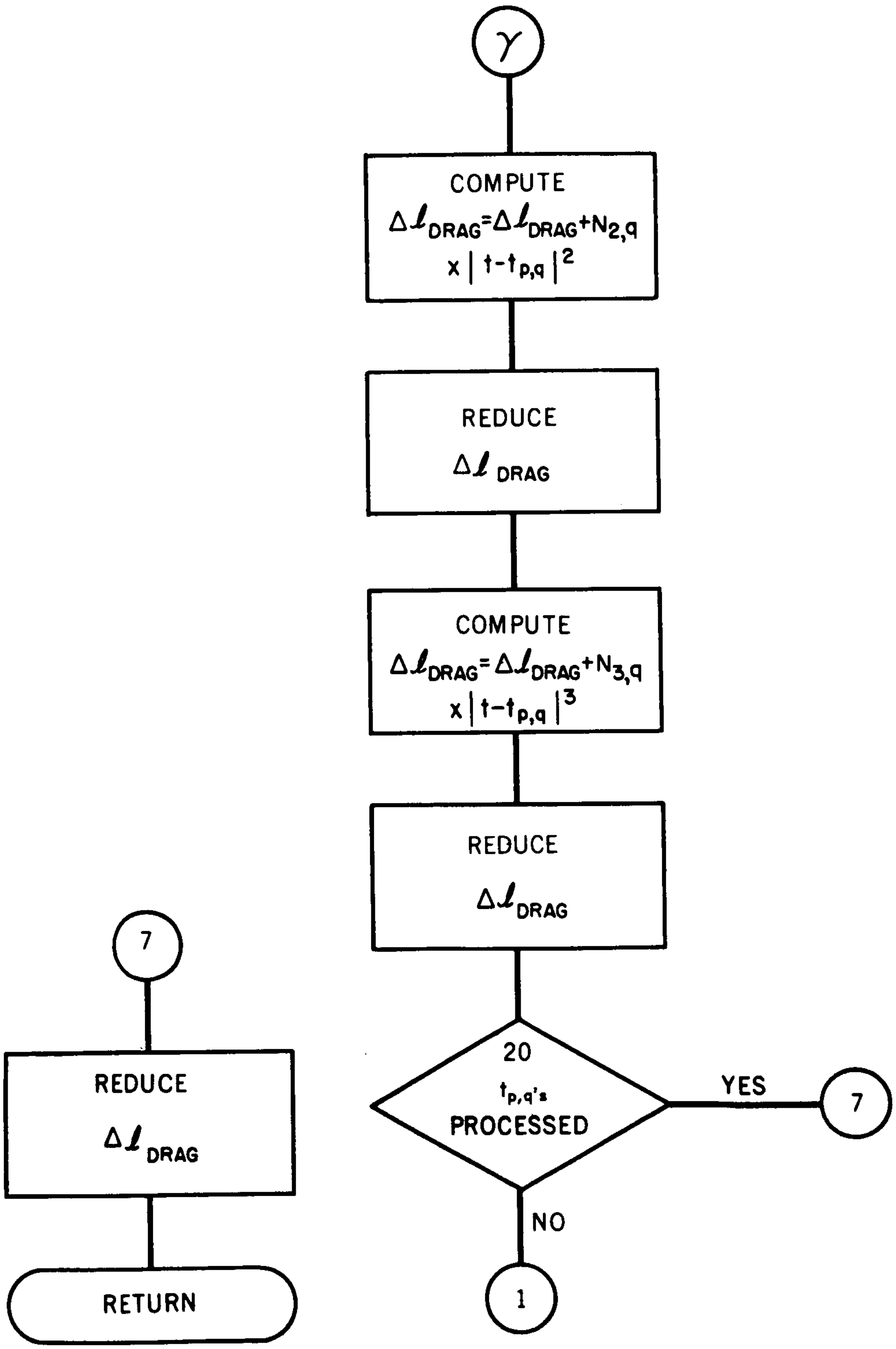
$$\left.\begin{array}{c}\text{DPT}(41)\\ \downarrow\\ \text{DPT}(60)\end{array}\right\} N_{3,0},\ N_{3,1},\ \ldots,\ N_{3,19}$$

INTERFACE BLOCK DIAGRAM



DRAG FLOWCHART




γ
COMPUTE
$\Delta\ell_{DRAG} = \Delta\ell_{DRAG} + N_{2,q}$
$\times \left| t - t_{p,q} \right|^2$
REDUCE
$\Delta\ell_{DRAG}$
COMPUTE
$\Delta\ell_{DRAG} = \Delta\ell_{DRAG} + N_{3,q}$
$\times \left| t - t_{p,q} \right|^3$
REDUCE
$\Delta\ell_{DRAG}$
20
$t_{p,q}$'s
PROCESSED
YES
7
NO
1
7
REDUCE
$\Delta\ell_{DRAG}$
RETURN

FUNCTIONAL ANALYSIS

I. Formula for computing

$$\Delta \ell_{DRAG} = \sum_{q=0}^{m} \sum_{p=2}^{3} N_{p,q} |t - t_q|^p$$

where $m = 0, 1, 2, \cdots 19$

RESTRICTIONS AND LIMITATIONS

I. Mathematical Restrictions: None

II. Data Restrictions: None

III. Hardware Restrictions: None

IV. Programming Language Restrictions: None