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INVOLUTE STRIP DEVELOPMENT METHOD FOR FABRICATION OF ROTATIONALLY SYMMETRICAL DOUBLE CURVED SURFACES OF REVOLUTION

by Rene E. Chambellan and Eugene J. Pleban

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16. Abstract

A double curved surface of revolution can be approximated by a series of coaxial, single curved surfaces. These surfaces can then be subdivided, in the plane of development, by families of involute curves which define flat curved ribbons. These ribbons can be joined to form the surface of revolution. Applied to heat-exchanger design, the surface of revolution also can be covered by a set of constant-diameter, equispaced cooling tubes. The tubes can be either internal or external to the shell.

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INVOLUTE STRIP DEVELOPMENT METHOD FOR FABRICATION OF ROTATIONALLY SYMMETRICAL DOUBLE CURVED SURFACES OF REVOLUTION

by Rene E. Chambellan and Eugene J. Pleban

Lewis Research Center

SUMMARY

Many methods are available for the approximate development of double curved surfaces of revolution onto a plane. The method presented in this report differs from others in that a set of constant-width curved strips (ribbons) or tubes can be used to reproduce any surface of revolution. This feature is of particular interest in the design of heat exchangers, where a uniform distribution of heat-transfer tubes may be required over a surface of revolution. Two important examples of this type are tube-type rocket nozzles and fuel-cooled nozzle plugs for jet engines.

In application to a tube-type heat exchanger, the tubes are preformed into a curve matching the elemental curved strip from the shell development. The tube-to-elemental-strip joint is easily made and inspected. After joining, the tube-strip assembly can be formed over a mandrel and joined at the strip edges to form the shell. The tubes can be either internal or external to the shell.

The surface development is performed in two steps. First, the surface of revolution is divided into a series of coaxial intersecting right-circular cones, cylinders, and annulii, as appropriate. These single curved surfaces are then developed onto a single plane where each of these surface elements must have a particular orientation with respect to its contiguous element. Second, these surface elements are then further subdivided into curved strips of constant width. Winding these strips over a mandrel, representing the original surface of revolution, produces the surface of revolution. The development can be accomplished either graphically or by means of a digital computer program; both methods are presented in this report.

The development procedures presented in this report are limited to rotationally symmetrical surfaces of revolution. However, extension of the principle can be made to include more complicated surfaces. It is not possible to completely cover, with constant-width strips with no overlapping, closed surfaces of revolution such as spheres and streamlined bodies. The curved strips have a finite width and therefore require a finite pole circle whose circumference is equal to the strip width times the number of strips. An exception to this is the development of a toroid, since it is possible in this case to completely cover the surface.

INTRODUCTION

This report presents a method for developing a double curved surface of revolution into flat ribbons. The development method is presented both as a graphical procedure and as a digital computer program. The development is accomplished by developing a series of cones, cylinders, and plane annulii, which are used to closely approximate the original surface, and by then subdividing the developed surfaces into curved ribbons.

Many methods are available for producing double curved thin shells of revolution (refs. 1 and 2). Generally, the material for the formation of a double curved shell of revolution is initially in the form of flat sheet stock, conical or cylindrical shells, small preformed shell segments, or thick shells As appropriate, these material forms can be spun, stretch-formed, bent, machined, and welded to produce the desired shape.

In the course of developing a construction method for a fuel-cooled jet engine nozzle plug, where the cooling tubes are to be located and uniformly spaced on the inside surface of the plug shell, it became apparent that direct access to the tube-shell joint would be necessary to ensure an adequate connection. A method, described in this report, has been evolved whereby a rotationally symmetric shell can be developed approximately into identical planar ribbon strip-elements. In the case of a nozzle plug, these strips are joined with the cooling tubes, and then these subassemblies are formed to produce the plug shell. The identical procedure can be used to produce subassemblies for the construction of tube-type rocket nozzles.

The development method presented herein is general and not restricted to heat-exchanger-type structures. Surfaces where the axis of symmetry pierces the surface of revolution require a pole circle of finite size. The holes in the surface of revolution represented by the pole circles can be closed, in practice, by suitably formed shallow double curved surfaces. A toroidal surface does not have this limitation.

SYMBOLS

A	area
$a, b, \ldots$	geometrical construction points
f	operator, dependent on stated conditions
h	projected length of element of surface of revolution on axis of symmetry
N	number of ribbons
n	ordinal number
r	radius of surface of revolution about axis

S	curvilinear length of composite involute
u, v, w, x, y	orthogonal coordinate axes
γ	angle between radius vector and axis normal to evolute where involute starts
Δ	increment
η	angle between tangent to an involute and axis normal to evolute where involute starts
θ	angle about u-axis measured in plane parallel to v-w plane
ν	number of elemental composite involute strips
ρ	slant height of conical element or radius vector in plane of development
φ	one-half vertex angle of right-circular cone
ψ	increment in the angle θ
ω	width of elemental composite involute strip

Subscripts:

i	index number, 1 or 2
j	index number, 1 or 2, identifying extremities of element of surface of revolution
m	integer, identifying intermediate radii of cone n
n	integer identifying particular element of subdivided surface of revolution
s	identifies locations between $j = 1$ and $j = 2$, $1 \leqq s \leqq 2$
0	indicates involute generating circle

SUBDIVISION OF SURFACES OF REVOLUTION INTO INVOLUTE STRIPS

Subdivision of Plane Surfaces

The development of right-circular truncated cones and cylinders onto a plane results in geometric figures which are segments of annulii or rectangles, respectively. Plane annulii and segments of annulii can be subdivided by families of involute curves whose evolutes (circles herein) are concentric and equal to or smaller than the inner circle of the annulus considered. If the spacing of the involutes in the family is made uniform, a plane annulus can be subdivided into a number of identical curved strips whose sides are

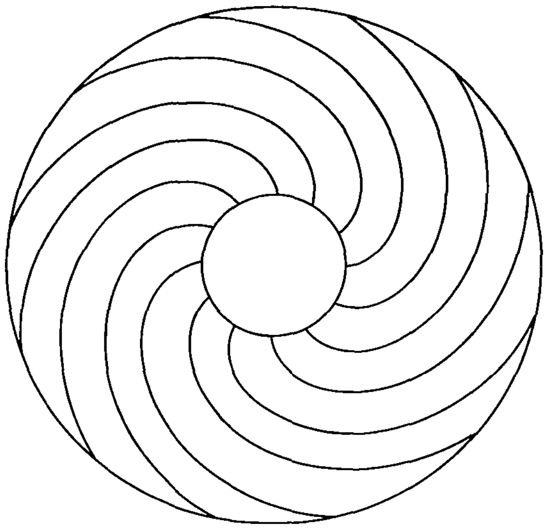


Figure 1. - Plane circular annulus subdivided by involute strips.

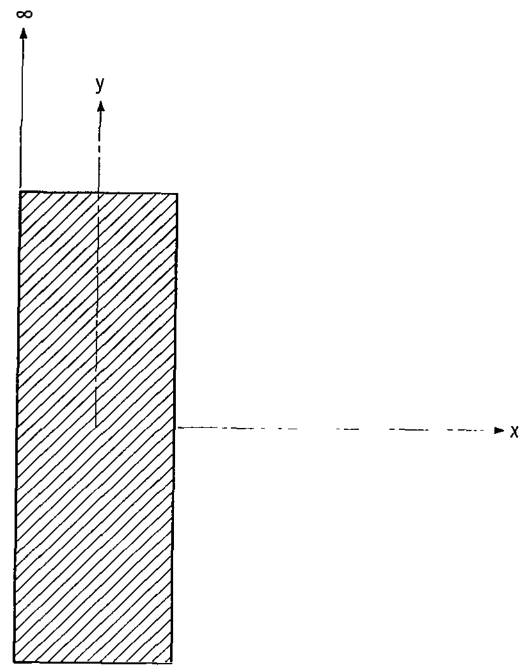


Figure 2. - Rectangle subdivided by involute strips, with center of evolute circle at infinity.

parallel (see ref. 1), as shown in figure 1. Recognizing that the cylinder is the limiting case of a cone (with zero apex angle), then the development will be a rectangle and the family of involute curves will degenerate into straight parallel lines, as shown in figure 2. These straight lines are developed helices and are the limiting case for the involute, since the developed helix can be generated by an involute generating circle located at infinity.

Subdivision of Double Curved Surfaces of Revolution

Double curved surfaces of revolution cannot be exactly developed onto a plane. These surfaces can be approximated by a series of colinear right-circular cones, right-circular cylinders, and plane annulii, as appropriate. Since these latter surfaces are single curved, they can be developed. The larger the number of surface-approximating elements, the greater the accuracy of the development.

A typical example of the development of a double curved surface of revolution into curved ribbons is shown in figure 3. The top of the figure shows a 15^{o}-15^{o} isometric view of the surface, and the lower portion of the figure shows the developed curved ribbon.

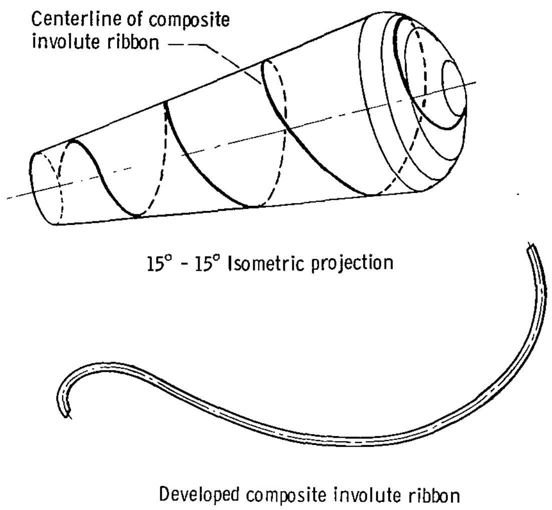


Figure 3. - Open-ended streamlined surface of revolution (jet engine nozzle plug).

Development of Typical Surfaces of Revolution

In the development of surfaces of revolution such as partial spheres, annulii, rocket thrust chambers, and open-ended streamlined bodies, the smallest pole circle that can be covered by the strips, with no overlapping, will have a circumference equal to the product of the strip widths and the number of strips. Examples of these representative bodies of revolution are shown in figures 3, 4, 5, and 6 which are an open-ended streamlined surface, an open-ended sphere, a toroid, and a rocket thrust chamber, respectively. These figures show an outline of the body, the trace of one involute ribbon strip centerline and the developed involute strip. These figures were drawn from data generated by a computer program described in appendix B based on equations derived in appendix A.

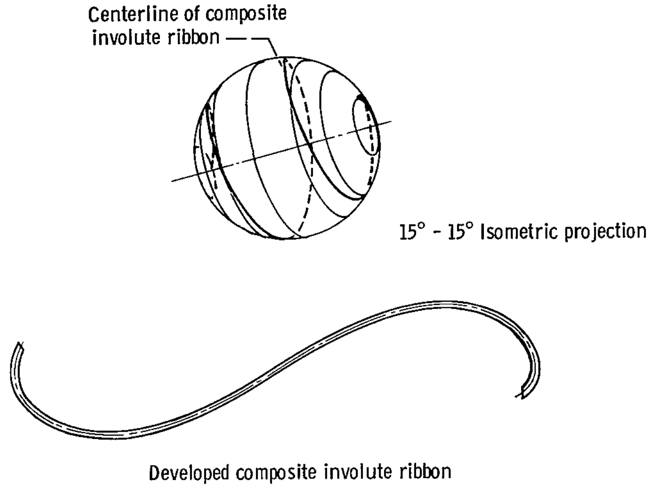


Figure 4. - Open-ended spherical surface.

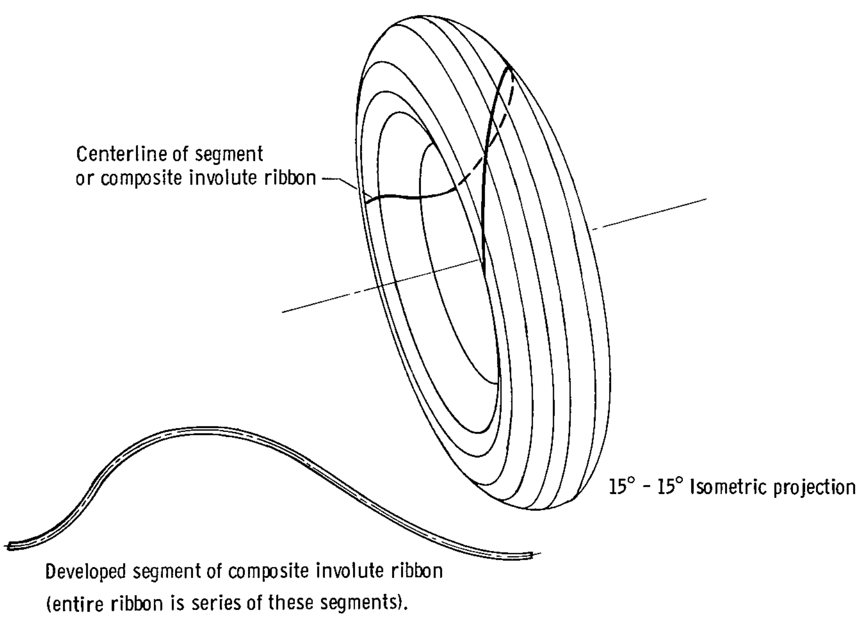


Figure 5. - Toroid.

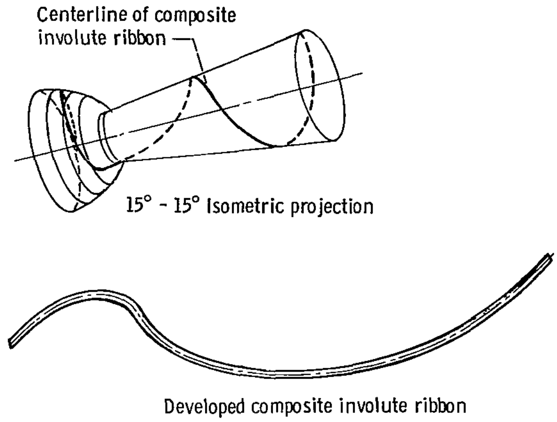


Figure 6. - Rocket thrust chamber assembly.

Accuracy of Method

The accuracy of the surface of revolution constructed from the developed involute ribbon strips is dependent on the strip width and the number of single curved surface-approximating elements such as cones, cylinders, and plane annulii. The greater the number of strips and elemental surfaces of revolution, the more accurate the development will be. Thus, in practice, the original double curved surface of revolution can be approximated as closely as desired.

The precision of the development procedure is greatly enhanced when a numerical computation method is used in place of a graphical method. Graphical methods are useful for checking results and for preliminary design; however, a final design should be done by the numerical computation method. The precision of the graphical method where very small cone apex angles are encountered suffers because very large radii of curvature are required in the plane development and the involute generating circle is far removed, in many cases far off the drawing board. In either method of development, errors in the development process are cumulative; however, in the numerical method, these errors can be reduced by increasing the precision of the numbers used. In the graphical method, the only way to improve precision is to enlarge the scale of the drawing, which quickly becomes impractical.

Method of Development

The single curved surface of revolution approximating elements can be established so as to obtain an equivalent surface where either the external or the internal envelope is preserved. In either case, the axis of symmetry is subdivided into lengths which are not necessarily equal. In the former case (external envelope), the radii to the actual surface are obtained at the selected points along the axis. In the latter case (internal envelope), the radii to the intersections of contiguous approximating elements, which are tangent to the actual surface, are obtained. The relative magnitudes and positions of consecutive radii establish whether the approximating element is a cone, a cylinder, or an annulus. If a mathematical formula is available, describing the shape of the surface, the former approximation is less cumbersome, assuming that the determination of radii is a computational instead of a graphical procedure. Neither approach is superior for graphical measurements.

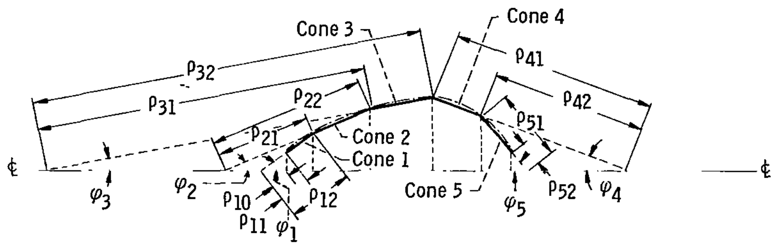
The development of surfaces of revolution can be more lucidly explained as a graphical procedure. Expressing the procedure in terms of mathematical symbolism is then more easily accomplished by the use of geometric relations.

An open-ended streamlined surface of revolution has been selected as an example and developed by a graphical method. Figure 7(a) shows a longitudinal cross section of the surface of revolution. The actual surface is indicated by a dot-dash line. Truncated cones, shown as solid lines in the cross section, are used to approximate the original surface. These cones are the surface-approximating elements and are shown as segments of plane annulii in the plane of development (fig. 7(b)). The relative positions of the developed cones are dependent on the conditions which require that the segments of the composite involute curve be continuous and tangent at the juncture of adjacent developed cones. The developed cones must also be tangent at the junction point.

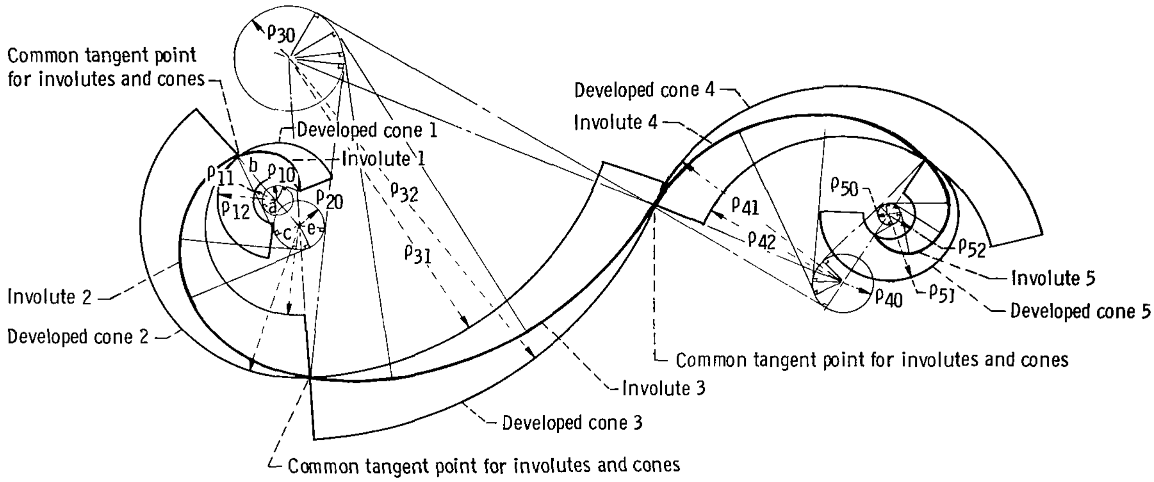
The reason for using involute curves for the subdivision of the developed surface-approximating elements is that a family of involutes are all parallel (ref. 3). Thus, a plane surface can be subdivided into any arbitrary number of similar contiguous areas. The use of many involutes in the family produces subdivisions which are ribbon-like in configuration.

Since the surface-approximating elements (cones in our example) are single curved, their development is exact and, therefore, any subdivision of the development will suffer no distortion when the cones are transformed to surfaces of revolution. This conclusion leads to the concept that the conical surface can also be constructed of strips defined by a family of involutes in the development plane.

A complication arises when the involute strips are to be made continuous (in the development plane) in going from one surface-approximating element (cones) to the next. Since the developed contiguous surface elements can be tangent at a single point only on each boundary, the edges of the involute strip in proceeding from one element to the next would suffer a discontinuity. However, a smooth and continuous, single composite-involute curve can be defined which passes through each juncture point of the developed



(a) Half cross section of open-ended streamlined surface of revolution showing surface-approximating cones where $\rho_{11} \geqq \rho_{10} \leqq \rho_{52}$.



(b) Graphical development of double curved surface of revolution shown in figure 7(a) (see fig. 10 for method of generating involute).

Figure 7. - Graphical development of open-ended streamlined surface of revolution. (Circles designated by ρ_{n0} (n = 1, 2, 3, . . .) are involute generating circles.)

surface-approximating elements. This composite curve is then used as the centerline of an involute ribbon strip and to locate all the points of tangency of the developed surface-approximating elements.

In this example, the development is started with cone 1, which is shown as a segment of a circular annulus in the plane of development. The radius of the involute generating circle is determined from an assumed number and width of strips, consistent with the requirement that the product of number of strips times strip width cannot exceed the smallest circumference of the surface of revolution. In appendix A it is shown that, in general, $\rho_{n0} = \dfrac{\nu\omega}{2\pi \sin \varphi_n}$ (eq. (A9)); therefore, the radius of the first involute general circle is

$$\rho_{10} = \frac{r_{10}}{\sin \varphi_1}$$

where $r_{10} = \nu\omega/2\pi$ (from eq. (A7)) and is limited by the requirement that $r_{51} \geqq r_{10}$ and $r_{11} \geqq r_{10}$.

The center of each involute generating circle is coincident with the arc center of its corresponding developed cone. The first involute curve is drawn, in the plane of development, passing through the inner cone arc on to the exterior arc of the cone where it is terminated. This termination point is where the developed cone 2 is then made tangent to cone 1. This means that the center of the arc for developed cone 2 is colinear with a line drawn through the point of cone tangency and the center of cone 1. In figure 7(b) this line is indicated as $\overline{be}$.

At the arc center for cone 2, a new involute generating circle is drawn whose radius is ρ_{20}. Graphically, this radius is obtained by extending the involute generating line $\overline{ba}$ to c. Point c is where a line perpendicular to $\overline{ba}$, extended, passes through the arc center of developed cone 2. The line segment $\overline{ec}$ is then the radius ρ_{20}, obtained from equation (A9).

The point b on the line $\overline{bc}$ traces the involute of circle e on the developed cone 2, as the extended tangent line $\overline{bc}$ rolls around circle e. This involute segment is generated until it intersects the outer arc of developed cone 2. This process is repeated for successive cones. The arc of developed cone 4 has its center on the other side of the composite involute curve for cones 1, 2, and 3; which is due to the fact that the slope of cone 4, in figure 7(a), is opposite to that for cones 1, 2, and 3. In general, this alternation of the developed cone centers will occur whenever the composite involute curve crosses its tangent. Using this composite involute curve as a centerline, a ribbon of width ω can now be drawn which is a developed surface element of the double curved surface of revolution.

Two deviations from the original surface of revolution occur in the surface as constructed from the developed involute ribbon strips. These deviations are that the involute strips form single curved surfaces and introduce a slight kink at the juncture of each surface-approximating element in the surface of revolution. These errors can be reduced to produce any degree of accuracy desired by increasing the number of surface-approximating elements and involute ribbon strips.

DISCUSSION

Applications to Heat-Exchanger Design

There are many methods available for the approximate graphical development of double curved surfaces of revolution onto a plane (refs. 1 and 2). The method presented in this report differs from others in that a number of identical constant-width strips can be used to reproduce many surfaces of revolution. This feature should be of particular interest in the construction of heat exchangers where a uniform distribution of heat-transfer tubes is required over a double curved surface of revolution. Two important structures of this type are tube-type rocket nozzles and the fuel-cooled nozzle plug for a jet engine.

Regeneratively cooled tube-type rocket nozzles, where the involute ribbon strip method of development is used, can be constructed of constant-diameter tubing from the front end of the combustion chamber, through the throat, to the end of the expansion section. The number and size of tubes will be dictated by the requirements of critical heat-transfer regions in the nozzle assembly. The tube assembly, as in conventional designs, must be supported by a pressure-tight structure in order to contain the chamber pressure.

Cooling a jet engine nozzle plug with liquid fuel can be accomplished by attaching cooling tubes to the shell which forms the plug. Since the plug is in a high-velocity, high-temperature gas stream, it is desirable to protect the coolant tubes, which generally will be thin-walled and small in diameter. A direct approach is to install the tubes on the inside surface of the plug. One method for accomplishing this installation is to fasten the tubes to the developed involute ribbon strips for the plug shell before assembly into the shell form. In this way the fastening is simplified and inspection of the tube-strip joint is facilitated. If the tube-strip assembly is too difficult to shape over a mandrel, the tube and the strip can be preformed individually in a fixture to match a space curve identical to the composite involute on the surface of revolution. Assembly of the shell with the tube-strip elements would require a certain amount of springing over the mandrel for the plug. The final operation required to complete the plug would then be welding of the faying surfaces of adjoining strips.

Construction Problems

Partially closed spherical shells and streamlined body shells or closed toroidal shells have a common assembly problem. In each case, some sort of internal removable support or mandrel is required for the retention of the involute strip during assembly and joining. The mandrel must be able to withstand welding or brazing operations. If a low-melting-point material is used for the mandrel structure, provisions must be made to insulate the material at those locations where welding or brazing is done. For a furnace-braze-type of joint, the mandrel could be made from materials such as plaster of paris which could be dissolved out of the assembly after joining. Since in all these structures, with the exception of the annulus, two holes must be left in the shell (where the composite involutes start and end), it is conceivable, depending on hole size, that collapsible mandrels might be devised. In this event, the mandrels could be reusable.

Where shell thicknesses are large, the involute strips would have to be preformed to the final shape of the space curve, sprung into place over a temporary support, and tack welded to the adjacent strip. Before the shell is completely formed, the temporary support may have to be removed, unless sufficient access was provided at the end holes.

SUMMARY OF RESULTS

The results of this investigation of the involute strip development method for fabrication of rotationally symmetrical double curved surfaces of revolution can be summarized as follows:

1. Four types of rotationally symmetrical surfaces of revolution have been developed by the involute ribbon method and presented in this report. These surfaces are (1) a partially closed streamlined body, (2) a rocket thrust chamber assembly, (3) a partially closed sphere, and (4) an annulus. The involute ribbons are the plane developed elements of the parent double curved surfaces of revolution.

2. The generation of the specific composite involute ribbon for a double curved surface of revolution is dependent on the approximation of the surface shape by means of a series of contiguous cones, cylinders, and plane annulii, as appropriate. The greater the number of these approximating surface elements, the better the accuracy of the development. Since all these approximating elements are single curved surfaces, they can be developed exactly onto a plane surface. In a plane, one set of curves which are parallel is a family of involutes. A developed helix is the limiting case for an involute. In a practical application, an involute ribbon would be used in place of the strip just defined. The composite involute curve obtained in going from one surface-approximating element to the next would then be the centerline of the ribbon. The development method can be accomplished graphically or by means of numerical computations. In the computational approach, the graphical method is reduced to the use of mathematical formulas where the composite involute curve is described in a suitable coordinate system.

3. The computational approach produces better results than the graphical development method. In either method, accuracy can be improved by increasing the number of surface-approximating elements (cones and cylinders).

4. The graphical involute development method can be very complicated because a multitude of required construction lines obfuscate the drawing. In addition, cones with small apex angles require the use of very long radii of curvature in the plane of development.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 16, 1970,
720-03.

APPENDIX A

DEVELOPMENT OF THEORY

DERIVATION OF EQUATIONS

The detailed derivation of the equations for developing a rotationally symmetrical surface of revolution is presented in the following text. The equations for defining the centerline of a composite involute strip will be developed for a general surface of revolution which is composed of the basic surface elements of cone and cylinder. The plane annulus is a special case of the cone with a vertex angle of π radians. In figure 8 are shown the basic elements for approximating any surface of revolution. By using many

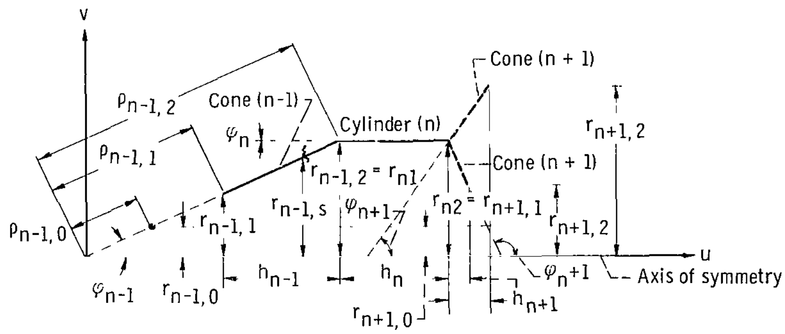


Figure 8. - Basic elements for approximating any surface of revolution.

small approximating elements, the actual surface of revolution can be approached as closely as desired. On the right side of figure 8, the cone (n + 1) is shown going either way so as to accommodate either an increasing or a decreasing slope. The left-hand cone, cone (n - 1), could have been shown this way too; however, the figure would then be too complicated.

EQUATIONS FOR SINGLE CONICAL ELEMENT

Before proceeding with the derivation of the equations for the general case, the method for a single conical element will be exposed. Figure 9 shows a developed approximating surface in the form of a truncated right-circular cone. This cone is labeled

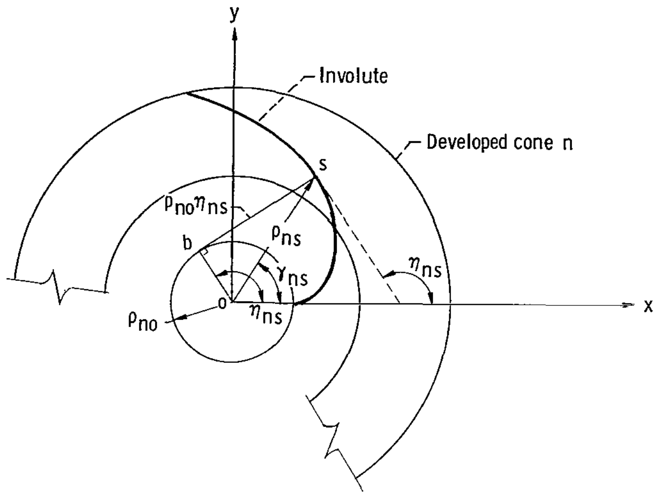


Figure 9. - Developed conical surface and involute.

n and is developed in a plane x-y. The x-y plane, representing rectangular Cartesian axes, is tangent to the cone surface, with the x-axis coincident with any arbitrary generator of the cone surface.

Coordinates of any point on the involute can be read directly from figure 9 as follows

$$\left.\begin{aligned} x_{ns} &= \rho_{n0}(\cos \eta_{ns} + \eta_{ns} \sin \eta_{ns}) \\ y_{ns} &= \rho_{n0}(\sin \eta_{ns} - \eta_{ns} \cos \eta_{ns}) \end{aligned}\right\} \tag{A1}$$

or as

$$\left.\begin{aligned} \rho_{ns} &= \rho_{n0} \sqrt{1 + \eta_{ns}^2} \\ \gamma_{ns} &= \eta_{ns} - \arctan \eta_{ns} \end{aligned}\right\} \tag{A2}$$

For a particular width of involute strip, the surface area of the surface of revolution can be obtained once the strip length is known. The length of the involute from the inner boundary of the developed cone and a point designated by S is obtained from figure 9. A differential element of length is $dS = \sqrt{\overline{dx}^2 + \overline{dy}^2}$ and from equations (A1):

$$dS = \rho_{n0}\eta_{ns}\, d\eta_{ns}$$

whence

$$S = \int_{\eta_{n1}}^{\eta_{ns}} \rho_{n0}\eta_{ns}\, d\eta_{ns} = \frac{\rho_{n0}}{2}\left(\eta_{ns}^2 - \eta_{n1}^2\right) \tag{A3}$$

The area of the surface of revolution in figure 9 is very closely

$$A = \nu\omega S$$

$$A = \frac{\nu\omega}{2}\rho_{n0}\left(\eta_{ns}^2 - \eta_{n1}^2\right) \tag{A4}$$

A general surface of revolution, as approximated by the elements shown in figure 8, has been developed in figure 10, where for the sake of clarity the cone (n + 1) is assumed to decrease in diameter in going from left to right. In some cases, a plane annulus may be required as an approximating element for the surface, but this produces no complications since this is a special case for a cone. The dimensions r and Δu are, in general, readily available from a drawing or description of the surface of revolution. And

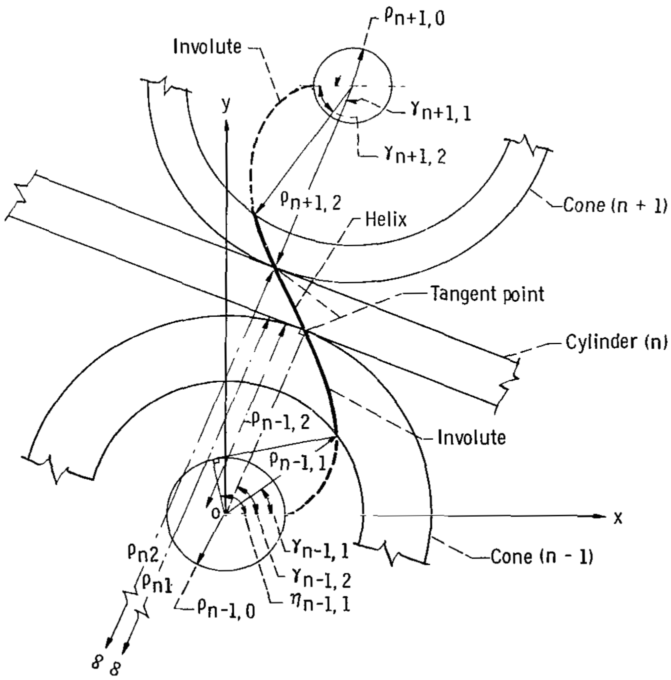


Figure 10. - Development of surface shown in figure 8.

the quantities ρ and φ are readily derived from r and Δu as follows:

$$\varphi_n = \arcsin\left[\frac{\sum_{j=1}^{2} (-1)^j r_{nj}}{\sqrt{\overline{\Delta u}^2 + \left(\sum_{j=1}^{i} (-1)^j r_{nj}\right)^2}}\right]$$

$$= \arcsin\left[\frac{r_{ns} - r_{n1}}{\sqrt{(u_{ns} - u_{n1})^2 + (r_{ns} - r_{n1})^2}}\right] \tag{A5}$$

and

$$\rho_{ns} = \frac{r_{ns}}{\sin \varphi_n} \tag{A6}$$

In this work the surfaces are rotationally symmetrical; therefore, the involutes will be generated from circles (the evolutes). The radii of the involute generating circles are determined from an arbitrary selection of strip width, the number of strips, and the requirement that a smooth transition occur in going from one surface-approximating element to its contiguous element. To preclude overlapping of the strip elements on the surface of revolution, it is necessary that the product $\nu\omega$ not exceed the circumference of the smallest pole circle for the surface of revolution; that is, $r_{n0} \leqq r_{ns}$.

The radius of the circle in the surface of revolution (see fig. 8) for determining the size of the involute generating circle for any cone n is

$$r_{n0} = \frac{\nu\omega}{2\pi} \tag{A7}$$

Thus, r_{n0} is a constant dependent only on the selection of ν, ω and the requirement that adjacent involutes have a common tangent, which means that

$$\left.\begin{aligned} &\eta_{n1} = \eta_{n-1,2} \\ \text{and} \\ &\eta_{n2} = \eta_{n+1,1} \end{aligned}\right\} \tag{A8}$$

where negative subscripts are excluded. The radius of the involute generating circle for the n^{th} surface element is

$$\left.\begin{aligned} \rho_{n0} &= \frac{r_{n0}}{\sin \varphi_n} \\ \rho_{n0} &= \frac{\nu\omega}{2\pi \sin \varphi_n} \end{aligned}\right\} \tag{A9}$$

which is inversely proportional to $\sin \varphi_n$ for any particular selection of ν and ω. When φ_n is zero, ρ_{n0} becomes infinite. For this case, $\rho_{n0} = \rho_{ns} = \infty$, the surface element is a cylinder and the involute becomes a straight line in the plane of development. On the cylinder this line becomes a helix. For $\varphi_n = \pi/2$ radians, the surface element is a plane annulus. Therefore, in general, a surface of revolution can be developed into a series of involute and helical strips.

In figure 10 the composite involute curve is shown starting at the x-axis. The radius of the first involute generating circle is $\rho_{n-1,0}$ and is obtained from equation (A9), namely,

$$\rho_{n-1,0} = \frac{\nu\omega}{2\pi \sin \varphi_{n-1}}$$

The first segment of the composite involute, starting at the x-axis is extended through cone (n - 1). It is more convenient to use polar coordinates in defining the involute because the radius vector to the involute also corresponds to a slant height along the cone. The definition of the angular argument γ is readily obtained by reference to figure 9 and equation (A2). Equation (A8) implies that

$$\gamma_{n1} = \gamma_{n-1,2}$$

$$\gamma_{n2} = \gamma_{n+1,1}$$

where negative subscripts are excluded.

It will be shown that equation (A8), establishing the common tangency of contiguous elements of the composite involute curve, is a consequence of r_{n0} being constant. By rearranging equation (A2) and exchanging j for s,

$$\eta_{nj} = \sqrt{\left(\frac{\rho_{nj}}{\rho_{n0}}\right)^2 - 1} \tag{A10}$$

and, therefore,

$$\eta_{n+(-1)^j, j-(-1)^j} = \sqrt{\left(\frac{\rho_{n+(-1)^j, j-(-1)^j}}{\rho_{n+(-1)^j, 0}}\right)^2 - 1} \tag{A11}$$

From figure 9, noting that j is exchanged for s,

$$\frac{\rho_{nj}}{\rho_{n0}} = \frac{r_{nj}}{r_{n0}} \tag{A12}$$

and, similarly,

$$\frac{\rho_{n+(-1)^j, j-(-1)^j}}{\rho_{n+(-1)^j, 0}} = \frac{r_{n+(-1)^j, j-(-1)^j}}{r_{n+(-1)^j, 0}} \tag{A13}$$

Also from figure 9 it can be seen that

$$r_{nj} = r_{n+(-1)^j, j-(-1)^j} \tag{A14}$$

and from equation (A7)

$$r_{n+(-1)^j, 0} = r_{n0} \tag{A15}$$

Substituting equations (A14) and (A15) into equation (A13) yields

$$\frac{\rho_{n+(-1)^j, j-(-1)^j}}{\rho_{n+(-1)^j, 0}} = \frac{r_{nj}}{r_{n0}} \tag{A16}$$

Equation (A16) substituted into equation (A12) shows that

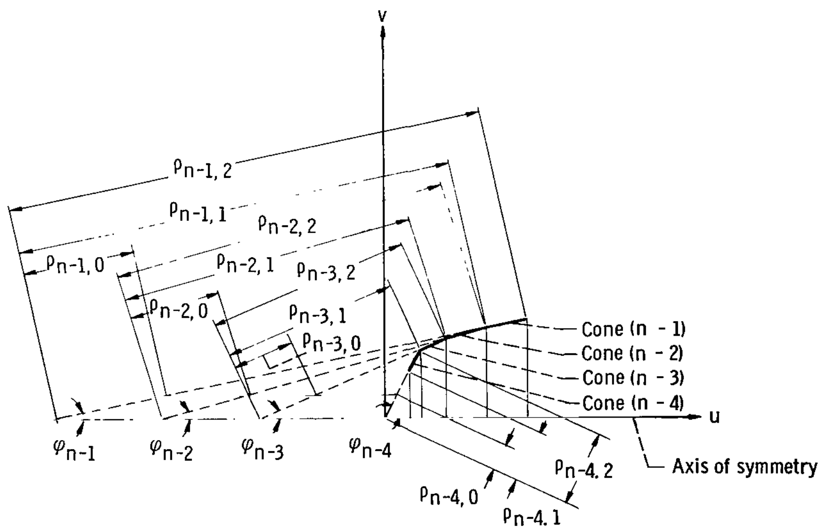
$$\frac{\rho_{n+(-1)^j, j-(-1)^j}}{\rho_{n+(-1)^j, 0}} = \frac{\rho_{nj}}{\rho_{n0}}$$

Thus, equations (A10) and (A11) are identical; and therefore,

$$\left.\begin{aligned}\eta_{nj} &= \eta_{n+(-1)^j,\, j-(-1)^j} \\ &\text{or} \\ \eta_{n1} &= \eta_{n-1,2} \\ \eta_{n2} &= \eta_{n+1,1}\end{aligned}\right\} \tag{A8}$$

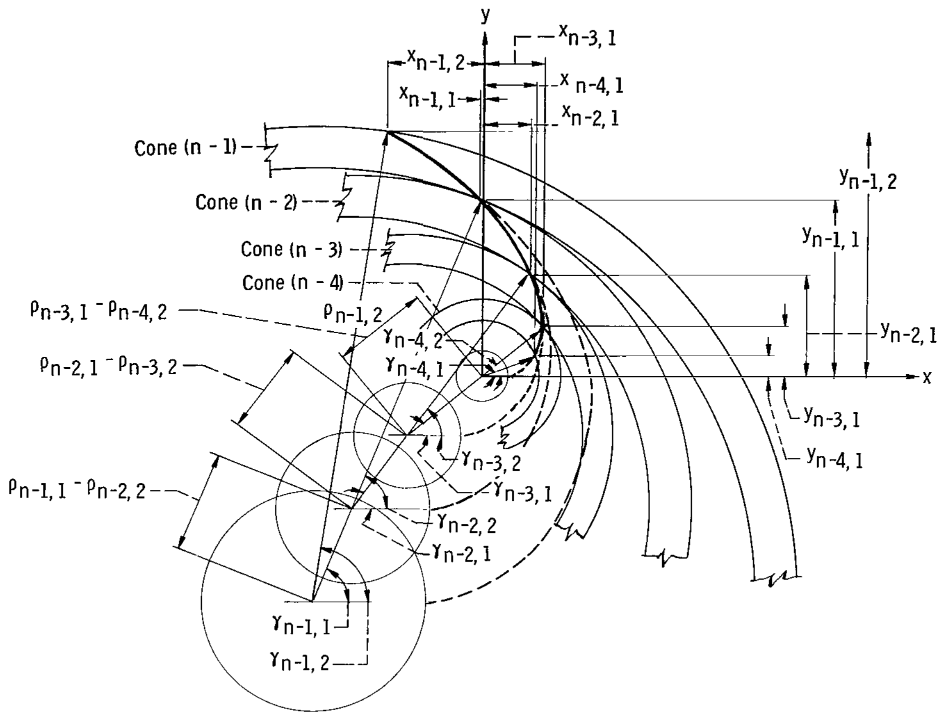
EQUATIONS FOR SERIES OF CONICAL ELEMENTS OF MONOTONICALLY INCREASING SIZE

The development, in the x-y plane, of a series of contiguous cones to the left of the cylinder in figure 8 is shown in figures 11(a) and (b). To be consistent with figure 8, the cones are labeled (n - 3), (n - 2), and (n - 1). Cone (n - 4) is the initial cone and is used here to establish a reference. Generally, the development can be started anywhere; however, it is convenient to start at an end cone (such as at the left-hand end).



(a) Cross section of part of surface of revolution for $0 \leq \varphi_n \leq \pi/2$.

Figure 11. - Development of series of contiguous cones to left of cylinder shown in figure 8.



(b) Composite involute generated on developed contiguous cones.

Figure 11. - Concluded.

From figure 11(b), it can be seen that successive values of the composite involute coordinates x and y can be written in terms of the immediately preceding values. Establishing the relations provides recursion formulas for determining points on the involute curves for each cone. From figure 11(b), it is apparent that

$$\left.\begin{aligned} x_{n-2,1} &= x_{n-3,2}; \; y_{n-2,1} = y_{n-3,2} \\ x_{n-2,2} &= x_{n-1,1}; \; y_{n-2,2} = y_{n-1,1} \end{aligned}\right\} \tag{A17}$$

Recalling that

$$\gamma_{n1} = \gamma_{n-1,2}$$

$$\gamma_{n2} = \gamma_{n+1,1}$$

and algebraically adding x and then y increments, the following equations are obtained:

$$\left.\begin{aligned} x_{n2} &= x_{n-1,2} - (\rho_{n1} \cos \gamma_{n1} - \rho_{n2} \cos \gamma_{n2}) \\ y_{n2} &= y_{n-1,2} - (\rho_{n1} \sin \gamma_{n1} - \rho_{n2} \sin \gamma_{n2}) \end{aligned}\right\} \tag{A18}$$

EQUATIONS WHERE CONES ONLY ARE USED FOR APPROXIMATING A STREAMLINED SHAPE

Before proceeding to the case where a cylinder is one of the surface-approximating elements, the case where the surface of revolution has a tear-drop shape and is approximated by cones only will be considered. In the latter case, the composite involute curve will close (radii of curvature will decrease) when the approximating cones have a vertex half angle $\varphi_n > \pi/2$ because the radius vectors defining the involutes will cross over to the other side, as can be seen in figure 10 for cone $(n + 1)$.

In equation (A18), the algebraic sign of the second term is dependent on whether r_{n2} is greater or less than r_{n1}. Multiplying the second term of equations (A18) by

$$\frac{r_{n1} - r_{n2}}{|r_{n1} - r_{n2}|}$$

where the ratio 0/0 is defined to be zero, yields the equations for x_{n2} and y_{n2}, which for a tear-drop shape approximated by a series of colinear and contiguous cones become

$$\left.\begin{aligned} x_{n2} &= x_{n1} + \left(\frac{r_{n1} - r_{n2}}{|r_{n1} - r_{n2}|}\right)(\rho_{n1} \cos \gamma_{n1} - \rho_{n2} \cos \gamma_{n2}) \\ y_{n2} &= y_{n1} + \left(\frac{r_{n1} - r_{n2}}{|r_{n1} - r_{n2}|}\right)(\rho_{n1} \sin \gamma_{n1} - \rho_{n2} \sin \gamma_{n2}) \end{aligned}\right\} \tag{A19}$$

EQUATIONS WHERE CONES AND CYLINDERS ARE USED

FOR ANY SURFACE OF REVOLUTION

The last step in the development of the equations for x_{n2} and y_{n2} is the addition of a term to account for a cylindrical element in the surface of revolution. The increments in x and y can be read directly from the diagram of figure 12. The developed

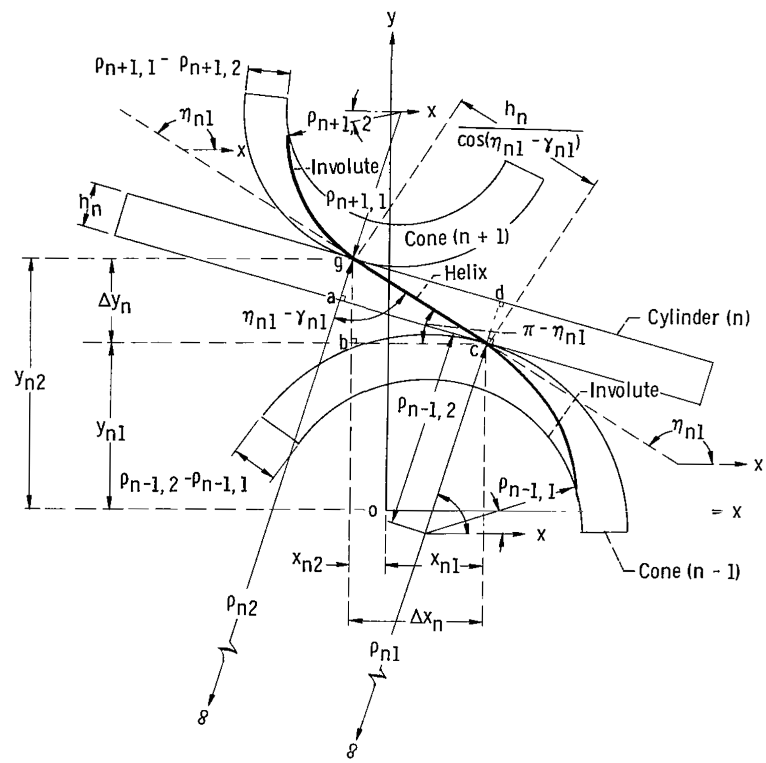


Figure 12. - Development of cone-cylinder-cone surface of revolution showing composite involute curve.

helix is tangent to the connecting involutes at each end, establishing its slope in the x-y planes as the tangent of the angle η_{n1}. From the figure then

$$\Delta x_n = \frac{h_n}{\sin \angle acg} \cos \angle gcb$$

$$\Delta y_n = \frac{h_n}{\sin \angle acg} \sin \angle gcb$$

The magnitudes of angle qcb and angle acq can be read directly from figure 12 as

$$\angle qcb = \pi - \eta_{n1}$$

$$\angle acq = \angle qcb - \angle acb$$

$$= \frac{\pi}{2} - \eta_{n1} + \gamma_{n1}$$

In the determination of the x and y increments for the cylindrical element,

$$\eta_{n1} = \eta_{n2}$$

and

$$\gamma_{n1} = \gamma_{n2}$$

whence,

$$\Delta x_n = \frac{-h_n \cos \eta_{n1}}{\cos(\eta_{n1} - \gamma_{n1})}$$

$$\Delta y_n = \frac{h_n \sin \eta_{n1}}{\cos(\eta_{n1} - \gamma_{n1})}$$

The denominators of these equations can be modified by the use of equation (A2); then

$$\left.\begin{aligned} \Delta x_n &= \frac{-h_n \cos \eta_{n1}}{\cos \arctan \eta_{n1}} \\ \Delta y_n &= \frac{h_n \sin \eta_{n1}}{\cos \arctan \eta_{n1}} \end{aligned}\right\} \tag{A20}$$

The addition of the x and y increments from equation (A20) to equation (A19) will give the values of x and y when a cylinder also is used in the approximation of a surface of revolution. When this addition is made, the generality of the equation can be preserved by multiplying the term contributed by the cylinder by an operator f_n which has the following values:

$$f_n = 0 \quad \text{when } r_{n1} \neq r_{n2}$$

$$f_n = 1 \quad \text{when } r_{n1} = r_{n2}$$

Making these substitutions results in a pair of general recursive equations for x and y for developing double curved surfaces of revolution, namely,

$$\left.\begin{aligned} x_{n2} &= x_{n1} + \left(\frac{r_{n1} - r_{n2}}{\left|r_{n1} - r_{n2}\right|}\right)(\rho_{n1} \cos \gamma_{n1} - \rho_{n2} \cos \gamma_{n2}) + f_n \frac{h_n \cos \eta_{n1}}{\cos \arctan \eta_{n1}} \\ y_{n2} &= y_{n1} + \left(\frac{r_{n1} - r_{n2}}{\left|r_{n1} - r_{n2}\right|}\right)(\rho_{n1} \sin \gamma_{n1} - \rho_{n2} \sin \gamma_{n2}) + f_n \frac{h_n \sin \eta_{n1}}{\cos \arctan \eta_{n1}} \end{aligned}\right\} \tag{A21}$$

For the determination of points on the composite involute curve at locations, on the cylinder, designated by the subscript ns, the equations are modified to read as follows:

$$\left.\begin{aligned} x_{ns} &= x_{n1} + \left(\frac{r_{n1} - r_{n2}}{\left|r_{n1} - r_{n2}\right|}\right)(\rho_{n1} \cos \gamma_{n1} - \rho_{n2} \cos \gamma_{n2}) + f_n \frac{\Delta u_{ns} \cos \eta_{n1}}{\cos \arctan \eta_{n1}} \\ y_{ns} &= y_{n1} + \left(\frac{r_{n1} - r_{n2}}{\left|r_{n1} - r_{n2}\right|}\right)(\rho_{n1} \sin \gamma_{n1} - \rho_{n2} \sin \gamma_{n2}) + f_n \frac{\Delta u_{ns} \sin \eta_{n1}}{\cos \arctan \eta_{n1}} \end{aligned}\right\} \tag{A22}$$

LENGTH OF COMPOSITE INVOLUTE CURVE

A formula for computing the length of the composite involute curve to any location denoted by the subscript ns is developed here. Equation (A3) can be modified by the substitution suggested by equation (A2), where

$$\eta_{nj} = \sqrt{\left(\frac{\rho_{nj}}{\rho_{n0}}\right)^2 - 1}$$

then

$$S = \frac{1}{2}\left(\frac{\rho_{n2}^2 - \rho_{n1}^2}{\rho_{n0}}\right) \tag{A23}$$

For a set of contiguous involutes plus a segment of the n^{th} involute

$$S = \frac{1}{2} \sum_{i=1}^{n-1} \frac{\rho_{i2}^2 - \rho_{i1}^2}{\rho_{i0}} + \frac{1}{2}\left(\frac{\rho_{ns}^2 - \rho_{n1}^2}{\rho_{n0}}\right) \tag{A24}$$

Complications arise where a helix, resulting from the use of a cylinder as an approximating element, is interspersed in the composite curve. Referring to figure 12 it is seen that the length of the helical element is

$$\frac{h_n}{\cos \arctan \eta_{n1}}$$

Since it may be desired to obtain the length of the helix to any point designated by the subscript s on the helix, the segment of the cylinder is more conveniently defined by substituting $U_{ns} - U_{n1}$ for h. Thus the length of helix for the n^{th} cylinder is

$$S_n = \frac{u_{ns} - u_{n1}}{\cos \arctan \eta_{n1}}$$

In order to combine the lengths of the helical elements with the lengths of the involute elements, use of the operators applied in equations (A22) can be made here, namely,

$$\frac{r_{n1} - r_{n2}}{|r_{n1} - r_{n2}|} \quad \text{and} \quad f_n = \begin{vmatrix} 0 \\ 1 \end{vmatrix} \quad \begin{matrix} \text{when } r_{n1} \neq r_{n2} \\ \text{when } r_{n1} = r_{n2} \end{matrix}$$

For application to equation (A24), the first operator can be written as

$$\frac{r_{i1} - r_{i2}}{|r_{i1} - r_{i2}|}$$

and

$$\frac{r_{ns} - r_{n2}}{|r_{ns} - r_{n2}|}$$

where $0/0$ is defined as equal to zero. The second operator can be rewritten as f_i where

$$f_i = \begin{vmatrix} 0 & \text{when } r_{i1} \neq r_{i2} \\ 1 & \text{when } r_{i1} = r_{i2} \end{vmatrix}$$

These operators have the purpose, as in equations (A21) and (A22) of admitting only those terms which represent actual length increments.

Combining the lengths of all the involute elements with the lengths of all the helical elements yields the following formula for the length of the composite curve to any desired location:

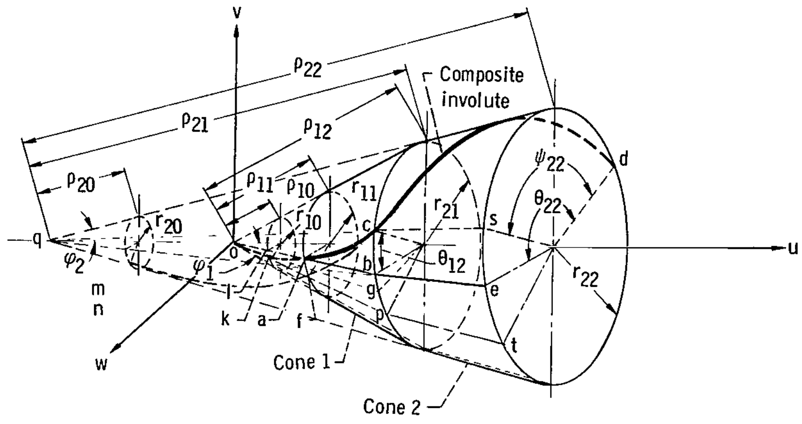
$$S = \frac{1}{2}\left[\sum_{i=1}^{n-1}\left(\frac{r_{i1} - r_{i2}}{|r_{i1} - r_{i2}|}\right)\left(\frac{\rho_{i2}^2 - \rho_{i1}^2}{\rho_{i0}}\right) + \left(\frac{r_{ns} - r_{n1}}{|r_{ns} - r_{n1}|}\right)\left(\frac{\rho_{n2}^2 - \rho_{n1}^2}{\rho_{n0}}\right) + \sum_{i=1}^{n+1} f_i\left(\frac{u_{is} - u_{i1}}{\cos \arctan \eta_{i1}}\right)\right] \tag{A25}$$

noting that $\eta_{n+1,1} = \eta_{n2}$.

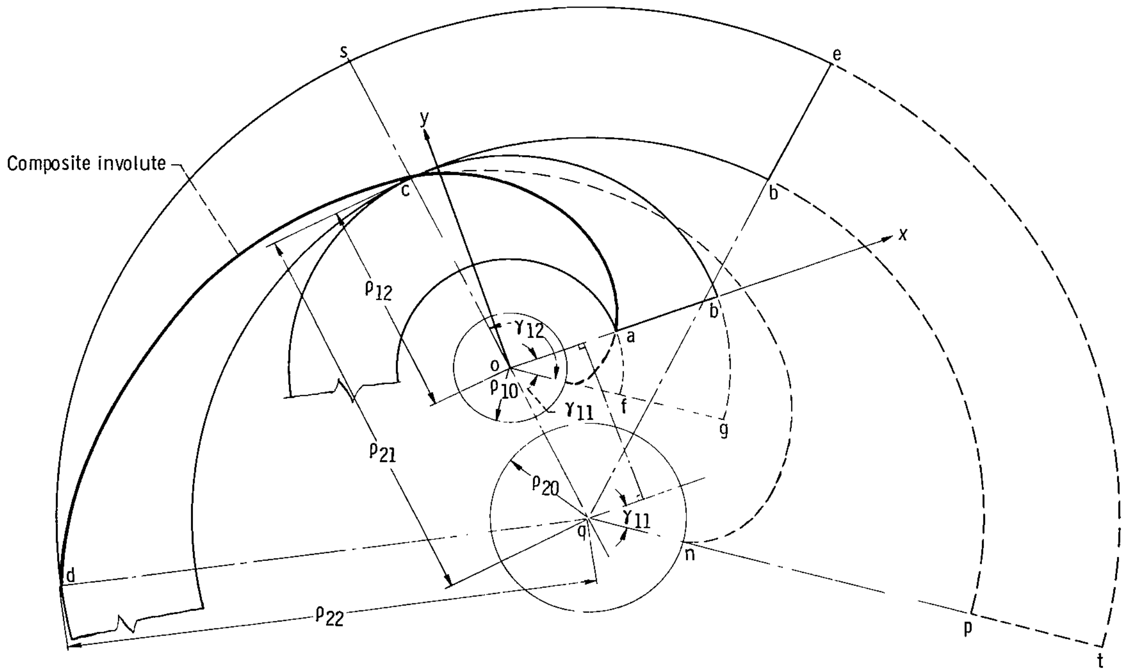
TRACE OF COMPOSITE INVOLUTE CURVE ON SURFACE OF REVOLUTION

Formulas defining the trace of the composite involute curve on the surface of revolution are developed here. The axes are labeled u, v, and w in cyclic order for a right-hand orthogonal system. The u-axis is made coincident with the axis of symmetry of the surface of revolution. These axes and the other pertinent dimensions are shown in figure 13. Figure 13(b) shows the first two approximating cone elements developed in the x-y plane, as defined previously, where the x-axis is located in the u-w plane and the x-y origin is located at the beginning of the first conical element. For the sake of clarity, the addition of a cylindrical element will be made later.

The u, v, w coordinates are obtained as functions of ρ, φ, and θ. In order to simplify the derivation, the value of θ_{11} is assumed to be equal to zero. This gives



(a) Two contiguous cones approximating a surface of revolution. Curve kac is involute for cone 1; curve nfcd is involute for cone 2.



(b) Planar development of surface shown in figure 13(a). Corresponding points have the same notation.

Figure 13. - Development of composite involute for general surface of revolution approximated by cones.

the trace for one composite curve which starts with the coordinates

$$u_{11} = \rho_{11} \cos \varphi_1$$

$$v_{11} = 0$$

$$w_{11} = \rho_{11} \sin \varphi_1$$

The traces of other composite involutes which would be the centerlines of other strips can be obtained by setting $\theta_{n1} = 2\pi n/\nu$ radians, the initial angle about the u-axis for the n^{th} strip.

In figure 13, corresponding points on the surface of revolution and on the plane of development are labeled identically. Equivalence of arc lengths in the u, v, w coordinate system and the x-y plane will be used to calculate the values of θ and its increments ψ. From figure 13(a)

$$\widehat{bc} = \rho_{12}\theta_{12} \sin \varphi_1$$

and from figure 13(b)

$$\widehat{bc} = \rho_{12}(\gamma_{12} - \gamma_{11})$$

whence

$$\theta_{12} = \frac{\gamma_{12} - \gamma_{11}}{\sin \varphi_1}$$

For continuity of values of θ

$$\theta_{n+1,1} = \theta_{n2}$$

Similarly, for the arc ds from figure 13(a)

$$\widehat{ds} = \rho_{22}\psi_{22} \sin \varphi_2$$

and from figure 13(b)

$$\widehat{ds} = \rho_{22}(\gamma_{22} - \gamma_{21})$$

whence

$$\psi_{22} = \frac{\gamma_{22} - \gamma_{21}}{\sin \varphi_2}$$

Note that since $\theta_{11} = 0$, then $\psi_{12} = \psi_{21} = \theta_{12}$. Again from figure 13(a), it can be seen that

$$\theta_{22} = \theta_{12} + \frac{\gamma_{22} - \gamma_{21}}{\sin \varphi_2}$$

$$= \frac{\gamma_{12} - \gamma_{11}}{\sin \varphi_1} + \frac{\gamma_{22} - \gamma_{21}}{\sin \varphi_2}$$

and further that

$$\theta_{32} = \theta_{22} + \psi_{32}$$

$$= \frac{\gamma_{12} - \gamma_{11}}{\sin \varphi_1} + \frac{\gamma_{22} - \gamma_{21}}{\sin \varphi_2} + \frac{\gamma_{32} - \gamma_{31}}{\sin \varphi_3}$$

and that, generally,

$$\theta_{n2} = \theta_{n-1,2} + \psi_{n2}$$

or written in terms of γ and φ

$$\theta_{nj} = \sum_{i=1}^{n-2+j} \frac{\gamma_{i2} - \gamma_{i1}}{\sin \varphi_i}$$

and

$$\theta_{ns} = \sum_{i=1}^{n-1} \frac{\gamma_{i2} - \gamma_{i1}}{\sin \varphi_i} + \frac{\gamma_{ns} - \gamma_{n1}}{\sin \varphi_n}$$

for intermediate points.

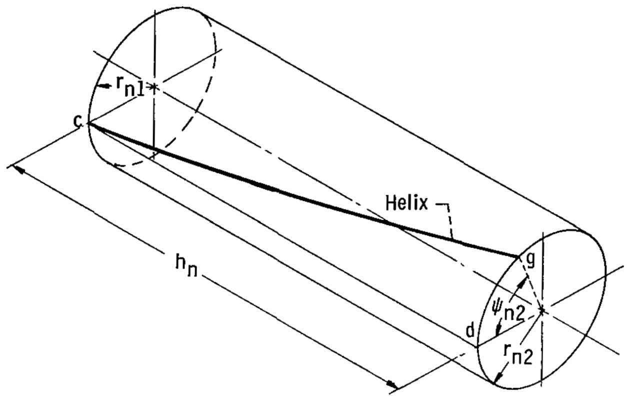


Figure 14. - Surface of revolution in form of right-circular cylinder.

A formula for θ_{n2} can now be written in more general form where, in addition to the cones used in the approximation, cylinders can also be used. In figure 14 a surface-approximating element in the form of a right-circular cylinder is shown. The same cylinder in the plane of development is shown in figure 12 as element n. From figure 14

$$\widehat{gd} = r_{n2}\psi_{n2}$$

and from figure 12

$$\overline{gd} = \frac{h_n}{\cos(\eta_{n1} - \gamma_{n1})} \cos\left(\frac{\pi}{2} - \eta_{n1} + \gamma_{n1}\right)$$

whence

$$\overline{gd} = h_n \tan(\eta_{n1} - \gamma_{n1})$$

From equation (A2) it is seen that

$$\eta_{n1} - \gamma_{n1} = \arctan \eta_{n1}$$

therefore

$$\overline{gd} = h_n \tan \arctan \eta_{n1}$$

$$= h_n \eta_{n1}$$

The arc $\widehat{gd}$ is equal to the straight-line segment $\overline{gd}$; then

$$\psi_{n2} r_{n2} = h_n \eta_{n1}$$

whence

$$\psi_{n2} = \frac{h_n}{r_{n2}} \eta_{n1}$$

This is the increment in the argument θ in going from one end of the cylinder, along a helix, to the other.

An equation for θ_{n2} which includes both conical and cylindrical elements can now be written. The operators used in equations (A21) can be used to advantage here, resulting in

$$\theta_{n2} = \sum_{i=1}^{n} \left(\frac{r_{i1} - r_{i2}}{|r_{i1} - r_{i2}|}\right)\left(\frac{\gamma_{i2} - \gamma_{i1}}{\sin \varphi_i}\right) + f_i \frac{h_i}{r_{i2}} \eta_{i1} \tag{A26}$$

Referring to figure 13(a), equations for the u, v, w coordinates of the composite involute can now be written as the following set of recursive relations.

$$\left.\begin{aligned} u_{n2} &= u_{n1} + \left(\frac{r_{n1} - r_{n2}}{|r_{n1} - r_{n2}|}\right)(\rho_{n2} - \rho_{n1}) \cos \varphi_n + f_n h_n \\ v_{n2} &= r_{n2} \sin \theta_{n2} \\ w_{n2} &= r_{n2} \cos \theta_{n2} \end{aligned}\right\} \tag{A27}$$

APPENDIX B

COMPUTER PROGRAM

A geometry calculation code, STRIP, was programed to compute the planar coordinates needed to define the basic involute ribbon of width ω. N number of these ribbons are used to fabricate a particular rotationally symmetric surface by a method described in this report. Specific equations used by the program are shown in the order of their use in the program. Specifications of input data are given. Outputs from a sample problem are included to demonstrate the program usage. The problem was programed entirely in FORTRAN IV language for a 7044/7094 DCS computer.

PROBLEM DESCRIPTION

To solve for the plane coordinates that define a ribbon, the body must be divided into a number of cones along the axis of symmetry. Each cone is numbered consecutively from left to right and is associated with a left cone radius and the cone height (see fig. 15). The cylinder is considered to be a special case of the cone. When a large cone

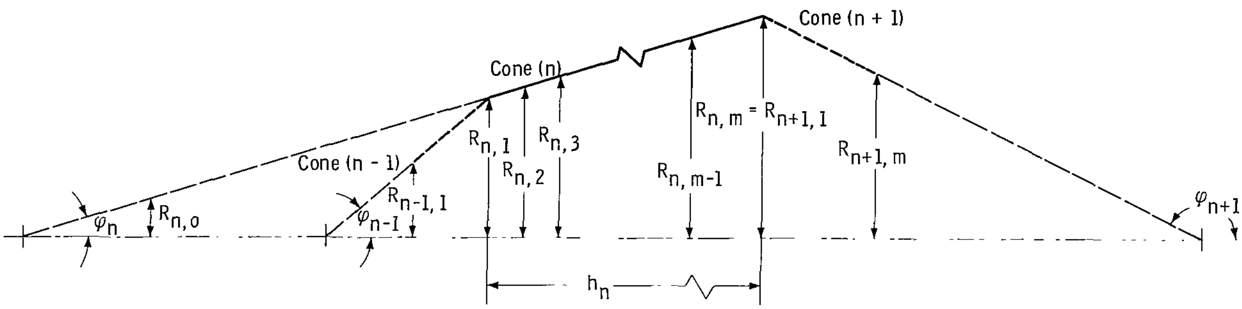


Figure 15. - Basic element for surface of revolution showing cone (n).

height is specified, up to eight intermediate cones can be specified to help smooth the ribbon profile.

Equations

The calculations in this program concern themselves with defining the ribbon in terms of ρ, γ, x, and y. The equations used are developed in appendix A and rearranged in this appendix to facilitate programing. Separate equations for the cylinder case were developed.

Cone equations. - The cone equations are as follows:

(1) Cone constants:

$$\sin \varphi_n = \frac{\left| r_{n,1} - r_{n+1,1} \right|}{\sqrt{h_n^2 + (r_{n,1} - r_{n+1,1})^2}} \tag{B1}$$

$$\rho_{n0} = \frac{\nu\omega}{2\pi \sin \varphi_n} \tag{B2}$$

$$\rho_{n,m} = \frac{r_{n,1}}{\sin \varphi_n} \tag{B3}$$

(2) Polar data:

$$\eta_{n,m} = \sqrt{\left(\frac{\rho_{n,m}}{\rho_{n0}}\right)^2 - 1} \tag{B4}$$

$$\gamma_{n,m} = \eta_{n,m} - \arctan \eta_{n,m} \tag{B5}$$

(3) Cartesian data:

$$x_{n,m} = x_{n-1,m} + \rho_{n,1} \cos \gamma_{n,1} - \rho_{n,m} \cos \gamma_{n,m} \tag{B6}$$

$$y_{n,m} = y_{n-1,m} + \rho_{n,1} \sin \gamma_{n,1} - \rho_{n,m} \sin \gamma_{n,m} \tag{B7}$$

where m designates the intermediate radii for cone n. No intermediate radii are allowed for the cylinder.

Cylinder equations. - The cylinder equations are as follows:

(1) Polar data, for the cylinder in the initial position (n = 1):

$$\eta_{1,1} = \arcsin\left(\frac{\nu\omega}{2\pi r_{1,1}}\right) \tag{B8}$$

$$\gamma_{1,1} = \eta_{1,1} - \arctan \eta_{1,1} \tag{B9}$$

(2) Polar data, for the cylinder in any position except initial position:

$$\eta_{n,1} = \eta_{n-1,m} = \eta_{n,2} \tag{B10}$$

$$\gamma_{n,1} = \gamma_{n-1,m} = \gamma_{n,2} \tag{B11}$$

(3) Cartesian data, for the cylinder in the initial position:

$$\left.\begin{aligned} x_{1,1} &= 0 \\ x_{1,2} &= \rho_{1,2} \cos \gamma_{1,2} \end{aligned}\right\} \tag{B12}$$

$$\left.\begin{aligned} y_{1,1} &= 0 \\ y_{1,2} &= \rho_{1,2} \sin \gamma_{1,2} \end{aligned}\right\} \tag{B13}$$

(4) Cartesian data, for the cylinder in any position:

$$\rho_{n,2} = \left| \frac{h_n}{\sin\left(\frac{\pi}{2} + \gamma_{n,1} - \eta_{n,1}\right)} \right| \tag{B14}$$

$$\gamma_{n,2} = \pi - \eta_{n,1} \tag{B15}$$

$$x_{n,2} = x_{n,1} - \rho_{n,2} \cos \gamma_{n,2} \tag{B16}$$

Input Data

The input into STRIP consists of three card formats. The information required to enter into these cards is described here. Included in the description is the number of input cards required. The actual formats of the cards follow the description.

Description. - The following information is required to enter into the STRIP card formats:

(1) Identification: The first card of each case contains a descriptive heading which will appear in the output as a heading in various places.

(2) Ribbon data: The second card specifies

(a) The total number of radii starting from left to right associated with each cone or cylinder, including the final closing radius

(b) The total number of ribbons

(c) The width of ribbon (If the given width is too large to be consistent with the smallest radius, the program will compute and use a corrected ribbon width.)

(3) Cone data: The third and succeeding cards describe the model as it is simulated by contiguous cones. For each cone or cylinder, the following data are specified.

(a) The cone index number

(b) The number of intermediate equal-height cones to be added (A maximum number of eight cones may be added. For the cylinder (a special case of the cone) case, a 1 is entered. For the final most right-hand radius, a 0 (zero) is entered.)

(c) The actual value of the radius

(d) The cone height

Data for two cones may be specified on each card. A card need not be full but the first set of columns must be filled. The read operation ends with a blank card or a card with the first data field blank. Cases may be stacked. Each subsequent case starts with an identification card. The input data are independent of the system of length units used.

Card formats. - Examples of card formats are

(1) Card 1: Identification (Format 9A6)

1 54 columns may be used for a descriptive heading

(2) Card 2: Ribbon data (Format 2I5, F10.0)

1 5 Number of radii, NR X	6 10 Number of ribbons, ν X	11 20 Width of ribbon, ω X.XXXX	

(3) Card 3: Cone radius and heat data (Format 2 (2I5, 2F10.0))

5	10	20	30	35	40	50	60
Cone index, n	Number of intermediate radii, m	Cone radius, $r_{n,1}$	Cone height, h_n	n	m	$r_{n,1}$	h_n
X	X	X.XXXX	X.XXXX	X	X	X.XXXX	X.XXXX

(4) Blank card (last card)

In these formats X represents a numerical digit:

(1) X indicates that the field contains an integer and is right justified in the card field.

(2) X.XXXX indicates a fixed decimal point number with the decimal point occurring anywhere in the card field except the first position which is usually reserved for the sign of the number.

Problem limits. - There are size restrictions which must be considered due to core storage size. The maximum number of cones is set at 100. Due to printing limitations, eight intermediate cone radii may be specified to smooth ribbon data for cones with large cone heights.

Cone numbering. - Numbering starts with index number 1 and must be sequential to the maximum number 100. To facilitate changing the model, the cone data cards can be in any order as long as each card has an index number in the first card field. If two of more cone data cards have the same index, the last cone data card read will be used.

Output Data

The printed output consists of the following:

(1) The input

(2) Complete cone data including intermediate radii

(3) Computed polar data, polar radius and angle

(4) Computed x-y coordinate data of the ribbon centerline

(5) Strip length against model length

EXAMPLE

A listing of the program with one complete example of a jet engine nozzle plug (figs. 3 and 16) is presented which includes a listing of input cards and a computer listing of the output data.

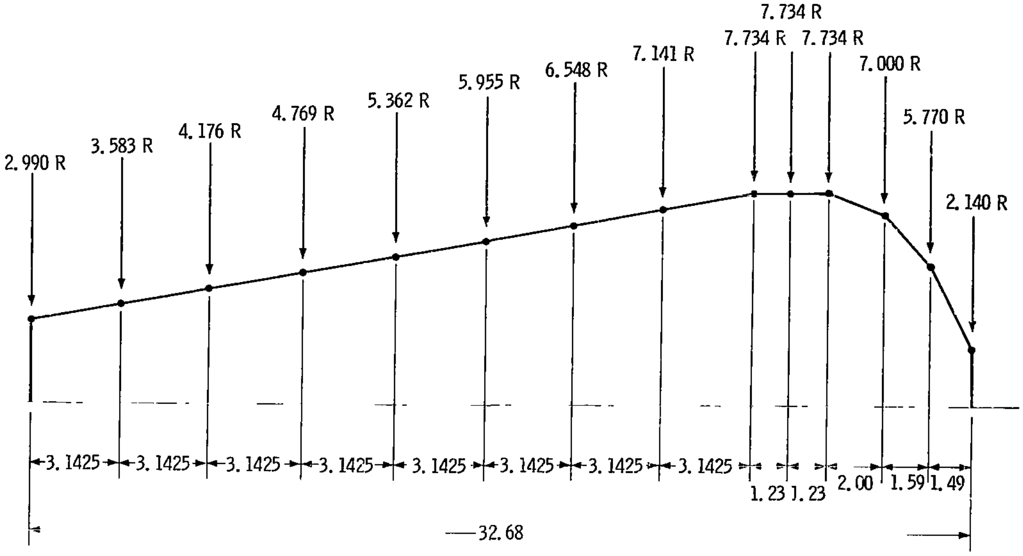


Figure 16. - Input data for sample problem (jet engine nozzle plug).

Computer Program STRIP: Program Listing

```
$IBFTC STRIP
C     ***INPUT CARDS INSTRUCTIONS***
C     CARD 1 - TITLE CARD
C                FORMAT 9A6 (54 ACTIVE COLUMNS)
C     CARD 2 - RIBBON DATA
C                FORMAT 2I5,F10.0
C                NR = NO. OF RADII
C                NU = NO. OF RIBBONS(STRIPS)
C                W  = WIDTH OF RIBBON (CORRECTED BY PROGRAM IF TOO LARGE)
C     CARD 3 TO NR+3 - CONE RADIUS AND HEIGHT DATA
C                FORMAT 2(2I5,2F10.0) (FIRST FIELD BLANK OR BLANK CARD ENDS
C                                      READ 1 OR 2 CONE DATA PER CARD)
C                I  = CONE INDEX NO. FROM 1 TO R
C                J  = NO. OF INTERMEDIATE RADII TO BE ADDED, MAX OF 8
C                     (J=1 FOR CYLINDER, J=0 FOR LAST RADIUS)
C                R  = LEFT RADIUS OF CONE
C                H  = CONE HEIGHT
C               -REPEAT ABOVE FOR SECOND SET ON CARD-
C
C     CASES MAY BE STACKED.   NEXT CASE STARTS WITH TITLE CARD.
C
      COMMON/IDENT /ID(9)
      COMMON/INPUT /R(101,9),H(100),M(100),NU,W,NC
      COMMON/MIDATA/P(100),RO(100,10),ETA(100,10),GAM(100,10)
      COMMON/OUTPUT/X(100,10),Y(100,10)
      REAL MINR
C
```

```
C    INITIALIZATIONS
C
  300 NMAX = 0
      DO 1 I=1,100
      DO 2 J=1,10
      R(I,J) = 0.0
      RO(I,J) = 0.0
      ETA(I,J) = 0.0
      GAM(I,J) = 0.0
      X(I,J) = 0.0
      Y(I,J) = 0.0
    2 CONTINUE
      H(I) = 0.0
      M(I) = 0.0
      P(I) = 0.0
    1 CONTINUE
      TPI = 2.0*3.1415926
C
C    INPUT/OUTPUT ID AND PROBLEM DATA
C
      READ(5,901) ID
      WRITE(6,900)
      WRITE(6,902) ID
      READ(5,903) NC,NU,W
      WRITE(6,905) NC, NU, W
   10 READ(5,904) I,J,A,B,K,L,C,D
      IF(I.EQ.0) GO TO 11
      NMAX = MAX0(NMAX,I,K)
      M(I) = J
      R(I,1) =A
      H(I) = B
      IF(K.EQ.0) GO TO 10
      M(K) = L
      R(K,1) = C
      H(K) = D
      GO TO 10
   11 CONTINUE
C
C    CHECK FOR CONSISTENCY
C
      RU = NU
      MINR = W*RU/TPI
      CHECKR = R(1,1)
      DO 5 I=1,NMAX
      CHECKR = AMIN1(CHECKR,R(I,1))
    5 CONTINUE
      IF(MINR.LT.CHECKR) GO TO 6
      W = CHECKR*TPI/RU-.001
      WRITE(6,915) W
    6 NMAX = NMAX-1
C
C    COMPUTE COMPLETE R MATRIX
C
      DO 12 I=1,NMAX
      L = M(I)
      RI = R(I,1)
      RJ = R(I+1,1)
      RJMI = RJ-RI
```

```
      DEBUG RI,RJ,RJMI,L
      DO 13 J=1,L
      A = J
      B = L
      SLOPE = A/B
      R(I,J+1) = RI+SLOPE*RJMI
   13 CONTINUE
   12 CONTINUE
C
C    OUTPUT COMPUTED PLUS INPUT DATA
C
      WRITE(6,906)
      DO 14 I=1,NMAX
      L = M(I)+1
      WRITE(6,913) I,R(I,1),R(I+1,1),H(I),(R(I,J),J=2,L)
   14 CONTINUE
C
C    COMPUTE POLAR DATA
C
      DO 15 I=1,NMAX
      L = M(I)+1
      RJMI = ABS(R(I+1,1)-R(I,1))
      IF(RJMI.EQ.0.0) GO TO 17
      SPHI = RJMI/SQRT(H(I)**2+RJMI**2)
      RNO = RU*W/(TPI*SPHI)
      DO 16 J=1,L
      RO(I,J) = R(I,J)/SPHI
      ETA(I,J) = SQRT((RO(I,J)/RNO)**2-1.0)
      GAM(I,J) = ETA(I,J)- ATAN (ETA(I,J))
   16 CONTINUE
      GO TO 15
C
C    CYLINDER CASE
C
   17 L = M(I-1)+1
      RO(1,1) = 0.0
      IF(I.EQ.1) GO TO 18
      ETA(I,1) = ETA(I-1,L)
      GAM(I,1) = GAM(I-1,L)
      GO TO 19
   18 ETA(I,1) = ARSIN(RU*W/(TPI*R(I,1)))
      GAM(I,1) = ETA(I,1)-ATAN(ETA(I,1))
   19 RO(I,2) = RO(I,1)
      ETA(I,2) = ETA(I,1)
      GAM(I,2) = GAM(I,1)
   15 CONTINUE
C
C    COMPUTE X-Y COORDINATE DATA
C
      DO 20 I=1,NMAX
      L = M(I)
      TEST = R(I+1,1)-R(I,1)
      IF(TEST) 21,22,23
   21 Q = 1.0
      F = 0.0
      GO TO 24
   22 Q = 0.0
      F = 1.0
      GO TO 24
```

```
   23 Q = -1.0
      F = 0.0
   24 RON1 = RO(I,1)
      RCG = RON1*COS(GAM(I,1))
      RSG = RON1*SIN(GAM(I,1))
      IF(I.NE.1) GO TO 25
      X(I,1) = RCG
      Y(I,1) = RSG
      GO TO 26
   25 K = M(I-1)+1
      X(I,1) = X(I-1,K)
      Y(I,1) = Y(I-1,K)
   26 DO 30 J=1,L
      IF(F.EQ.1.0) GO TO 27
      D = RO(I,J+1)
      A = GAM(I,J+1)
      DX = Q*(RCG-D*COS(A))
      DY = Q*(RSG-D*SIN(A))
      GO TO 28
   27 D = ABS(H(I)/SIN(TPI/4.0+GAM(I,J)-ETA(I,J)))
      A = TPI/2.0-ETA(I,J)
      DX = -D*COS(A)
      IF(I.EQ.1) DX = -DX
      DY = D*SIN(A)
   28 X(I,J+1) = X(I,1)+DX
      Y(I,J+1) = Y(I,1)+DY
   30 CONTINUE
   20 CONTINUE
C
C   OUTPUT COMPUTED POLAR DATA AND X-Y COORDINATE DATA
C
      WRITE(6,907)
      WRITE(6,908)
      DO 100 I=1,NMAX
      L = M(I)
      WRITE(6,914) I,RO(I,1),RO(I,L+1),(RO(I,J),J=2,L)
  100 CONTINUE
      WRITE(6,909)
      WRITE(6,908)
      DO 101 I=1,NMAX
      L = M(I)
      WRITE(6,914) I,GAM(I,1),GAM(I,L+1),(GAM(I,J),J=2,L)
  101 CONTINUE
      WRITE(6,910)
      WRITE(6,908)
      DO 102 I=1,NMAX
      L = M(I)
      WRITE(6,914) I,X(I,1),X(I,L+1),(X(I,J),J=2,L)
  102 CONTINUE
      WRITE(6,912)
      WRITE(6,908)
      DO 103 I=1,NMAX
      L = M(I)
      WRITE(6,914) I,Y(I,1),Y(I,L+1),(Y(I,J),J=2,L)
  103 CONTINUE
      GO TO 300
  900 FORMAT(1H1,  5X,85HSTRIP - A PROGRAM TO APPROXIMATE A BODY OF REVO
     1LUTION BY A SERIES OF CONTIGUOUS CONES/
     2 5X,36HDEVELOPED FROM INVOLUTE CURVE STRIPS)
```

```
901 FORMAT(9A6)
902 FORMAT(1H0, 5X,14A6)
903 FORMAT(2I5 ,F10.0,2I5)
904 FORMAT(2(2I5,2F10.0))
905 FORMAT(1H0, 5X,16HFIXED INPUT DATA/
   1 5X,18HNUMBER OF CONES = ,I5/
   2 5X,39HNUMBER OF STRIPS ON THE CIRCUMFERENCE = ,I5/
   3 5X,29HRIBBON WIDTH OF EACH STRIP = ,F10.5)
906 FORMAT(1H0, 5X,19HVARIABLE INPUT DATA/
   11X,4HCONE,6X,10HCONE RADII, 8X, 4HCONE,30X,18HINTERMEDIATE RADII/
   22X,2HNO,   6X,1H1,9X,1H2, 9X,2HHT, 8X,1H1,9X,1H2,9X,1H3,9X,1H4,9X,
   31H5,9X,1H6,9X,1H7,9X,1H8//)
907 FORMAT(1H0, 5X,19HCOMPUTED POLAR DATA/
   11X,4HCONE,6X,11HPOLAR RADII,7X, 34X,16HINTERMEDIATE RHO)
908 FORMAT(2X,2HNO,6X,1H1,9X,1H2,19X,1H1,9X,1H2,9X,1H3,9X,1H4,9X,1H5,
   19X,1H6,9X,1H7,9X,1H8//)
909 FORMAT(1H0, 5X,19HCOMPUTED POLAR DATA/
   11X,4HCONE,6X,11HPOLAR ANGLE,7X, 34X,18HINTERMEDIATE GAMMA)
910 FORMAT(1H0, 5X,28HCOMPUTED X-Y COORDINATE DATA/
   11X,4HCONE,6X,12HX-COORDINATE,6X,34X,19HINTERMEDIATE X-DATA)
912 FORMAT(1H0, 5X,28HCOMPUTED X-Y COORDINATE DATA/
   11X,4HCONE,6X,12HY-COORDINATE,6X,34X,19HINTERMEDIATE Y-DATA)
913 FORMAT(1H0,I4,1X,11F10.5)
914 FORMAT(1H0,I4,1X,2F10.5,10H   ********,8F10.5)
915 FORMAT(5X,39HCORRECTED RIBBON WIDTH OF EACH STRIP = ,F10.5)
    END
```

STRIP Dictionary

FORTRAN symbol	Definition or equivalent mathematical symbol
ETA	η
GAM	ribbon polar coordinate, γ
H	cone cylinder height, h
ID	title and problem identification data
NR	total number of radii from model data
NU	ν
R	model radii, r
RO	ribbon polar coordinate, ρ
W	ω
X	ribbon Cartesian coordinate, x
Y	ribbon Cartesian coordinate, y

Listing of Input Cards

SAMPLE PROBLEM - PLUG

13 50 .25

1	8	2.99	3.1425	2	8	3.58310	3.1425
3	8	4.17620	3.1425	4	8	4.76931	3.1425
5	8	5.36241	3.1425	6	8	5.95551	3.1425
7	8	6.54861	3.1425	8	8	7.14172	3.1425
9	1	7.73482	1.23	10	1	7.73482	1.23
11	8	7.73482	2.0	12	8	7.0	1.59
13	8	5.77	1.49	14		2.14	

Listing of Output Data

STRIP - A PROGRAM TO APPROXIMATE A BODY OF REVOLUTION BY A SERIES OF CONTIGUOUS CONES
DEVELOPED FROM INVOLUTE CURVE STRIPS

SAMPLE PROBLEM - PLUG

FIXED INPUT DATA
NUMBER OF CONES = 13
NUMBER OF STRIPS ON THE CIRCUMFERENCE = 50
RIBBON WIDTH OF EACH STRIP = 0.25000

VARIABLE INPUT DATA

CONE NO	CONE RADII 1	CONE RADII 2	CONE HT	INTERMEDIATE RADII 1	2	3	4	5	6	7	8
1	2.99000	3.58310	3.14250	3.06414	3.13827	3.21241	3.28655	3.36069	3.43482	3.50895	3.58310
2	3.58310	4.17620	3.14250	3.65724	3.73137	3.80551	3.87965	3.95379	4.02792	4.10206	4.17620
3	4.17620	4.76931	3.14250	4.25034	4.32448	4.39862	4.47275	4.54689	4.62103	4.69517	4.76931
4	4.76931	5.36241	3.14250	4.84345	4.91758	4.99172	5.06586	5.14000	5.21413	5.28827	5.36241
5	5.36241	5.95551	3.14250	5.43655	5.51068	5.58482	5.65896	5.73310	5.80724	5.86137	5.95551
6	5.95551	6.54861	3.14250	6.02965	6.10378	6.17792	6.25206	6.32620	6.40033	6.47447	6.54861
7	6.54861	7.14172	3.14250	6.62275	6.69689	6.77103	6.84516	6.91930	6.99344	7.06758	7.14172
8	7.14172	7.73482	3.14250	7.21586	7.28999	7.36413	7.43827	7.51241	7.58654	7.66068	7.73482
9	7.73482	7.73482	1.23000	7.73482							
10	7.73482	7.73482	1.23000	7.73482							
11	7.73482	7.00000	2.00000	7.64297	7.55111	7.45926	7.36741	7.27556	7.18370	7.09185	7.00000
12	7.00000	5.77000	1.59000	6.84625	6.69250	6.53875	6.38500	6.23125	6.07750	5.92375	5.77000
13	5.77000	2.14000	1.49000	5.31625	4.86250	4.40875	3.95500	3.50125	3.04750	2.59375	2.14000

COMPUTED POLAR DATA

CONE NO	POLAR RADII 1	POLAR RADII 2		INTERMEDIATE RHO 1	2	3	4	5	6	7	8
1	16.12200	19.31998	********	16.52175	16.92150	17.32124	17.72099	18.12074	18.52049	18.92023	
2	19.31998	22.51796	********	19.71973	20.11948	20.51922	20.91897	21.31872	21.71847	22.11821	
3	22.51759	25.71557	********	22.91734	23.31709	23.71684	24.11658	24.51633	24.91608	25.31583	
4	25.71599	28.91397	********	26.11574	26.51549	26.91524	27.31498	27.71473	28.11448	28.51423	
5	28.91397	32.11195	********	29.31372	29.71347	30.11322	30.51296	30.91271	31.31246	31.71221	
6	32.11196	35.30994	********	32.51170	32.91145	33.31120	33.71095	34.11069	34.51044	34.91019	
7	35.30936	38.50734	********	35.70910	36.10885	36.50860	36.90835	37.30810	37.70784	38.10759	

8	38.50797	41.70595	********	38.90771	39.30746	39.70721	40.10696	40.50670	40.90645	41.30620	
9	0.	0.	********	0.							
10	0.	0.	********	0.							
11	22.42824	20.29753	********	22.16190	21.89556	21.62922	21.36288	21.09654	20.83021	20.56387	
12	11.44030	9.43007	********	11.18902	10.93774	10.68646	10.43519	10.18391	9.93263	9.68135	
13	6.23717	2.31326	********	5.74668	5.25619	4.76570	4.27521	3.78473	3.29424	2.80375	

COMPUTED POLAR DATA

CONE NO	POLAR ANGLE 1	2		INTERMEDIATE GAMMA 1	2	3	4	5	6	7	8
1	0.27916	0.51578	********	0.30724	0.33583	0.36487	0.39433	0.42419	0.45439	0.48493	
2	0.51578	0.77142	********	0.54691	0.57831	0.60996	0.64184	0.67394	0.70625	0.73875	
3	0.77142	1.03828	********	0.80427	0.83729	0.87045	0.90376	0.93720	0.97077	1.00447	
4	1.03828	1.31236	********	1.07221	1.10623	1.14036	1.17459	1.20891	1.24331	1.27780	
5	1.31236	1.59140	********	1.34701	1.38173	1.41651	1.45137	1.48628	1.52127	1.55631	
6	1.59140	1.87399	********	1.62655	1.66176	1.69701	1.73232	1.76767	1.80307	1.83851	
7	1.87399	2.15923	********	1.90952	1.94508	1.98069	2.01633	2.05200	2.08771	2.12345	
8	2.15923	2.44647	********	2.19503	2.23087	2.26674	2.30263	2.33855	2.37450	2.41047	
9	2.44647	2.44647	********	2.44647							
10	2.44647	2.44647	********	2.44647							
11	2.44647	2.09087	********	2.40188	2.35732	2.31280	2.26832	2.22389	2.17950	2.13516	
12	2.09087	1.50369	********	2.01685	1.94298	1.86927	1.79574	1.72240	1.64927	1.57635	
13	1.50369	0.01899	********	1.29083	1.08094	0.87499	0.67447	0.48174	0.30089	0.13991	

COMPUTED X-Y COORDINATE DATA

CONE NO	X-COORDINATE 1	2		INTERMEDIATE X-DATA 1	2	3	4	5	6	7	8
1	15.49789	16.80660	********	15.74806	15.97623	16.18099	16.36095	16.51478	16.64116	16.73884	
2	16.80660	16.14354	********	16.84328	16.84775	16.81898	16.75594	16.65772	16.52344	16.35229	
3	16.14354	13.05608	********	15.89651	15.61063	15.28537	14.92031	14.51508	14.06941	13.58311	
4	13.05608	7.38944	********	12.48829	11.87982	11.23081	10.54151	9.81226	9.04347	8.23567	
5	7.38944	-0.66160	********	6.50549	5.58460	4.62764	3.63556	2.60942	1.55036	0.45958	
6	-0.66160	-10.54255	********	-1.81179	-2.98953	-4.19328	-5.42140	-6.67219	-7.94389	-9.23464	
7	-10.54255	-21.37392	********	-11.86562	-13.20184	-14.54909	-15.90523	-17.26806	-18.63532	-20.00472	
8	-21.37392	-32.02901	********	-22.74053	-24.10215	-25.45635	-26.80065	-28.13258	-29.44963	-30.74928	
9	-32.02901	-35.93345	********	-35.93345							
10	-35.93345	-39.83789	********	-39.83789							
11	-39.83789	-46.97554	********	-40.69217	-41.56240	-42.44600	-43.34039	-44.24305	-45.15144	-46.06308	
12	-46.97554	-53.29308	********	-47.83372	-48.68324	-49.51823	-50.33311	-51.12267	-51.88207	-52.60687	
13	-53.29308	-55.18768	********	-54.46276	-55.34787	-55.92966	-56.21394	-56.22882	-56.02107	-55.65119	

COMPUTED X-Y COORDINATE DATA

CONE NO	Y-COORDINATE 1	2		INTERMEDIATE Y-DATA 1	2	3	4	5	6	7	8
1	4.44234	9.52889	********	4.99669	5.57649	6.18071	6.80829	7.45810	8.12897	8.81965	
2	9.52889	15.69856	********	10.25533	10.99757	11.75417	12.52364	13.30444	14.09495	14.89355	
3	15.69856	22.15508	********	16.50825	17.32085	18.13458	18.94760	19.75806	20.56406	21.36371	
4	22.15508	27.95369	********	22.93624	23.70522	24.46007	25.19882	25.91952	26.62020	27.29890	

5	27.95369	32.10503	********	28.58264	29.18385	29.75542	30.29550	30.80228	31.27395	31.70877
6	32.10503	33.69923	********	32.46107	32.77528	33.04610	33.27204	33.45166	33.58358	33.66652
7	33.69923	32.03130	********	33.68056	33.60943	33.48485	33.30590	33.07176	32.78170	32.43505
8	32.03130	26.71159	********	31.56997	31.05071	30.47327	29.83749	29.14333	28.39083	27.58015
9	26.71159	23.95033	********	23.95033						
10	23.95033	21.18908	********	21.18908						
11	21.18908	17.94000	********	20.61498	20.08873	19.61058	19.18061	18.79879	18.46497	18.17885
12	17.94000	18.45884	********	17.77343	17.67878	17.65372	17.69541	17.80060	17.96563	18.18649
13	18.45884	24.63803	********	19.15903	20.04390	21.02410	22.01216	22.92841	23.70564	24.29097

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