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DISCUSSION OF A MODEL OF THE APPARENT TEMPERATURE OF NATURAL SURFACES IN THE MICROWAVE RANGE

by

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ABSTRACT

Many important characteristics of a natural surface affect the emission of electromagnetic energy from this surface. In this paper the emission characteristics are related to the differential scattering coefficients of the surface. Basic electromagnetic properties are defined and commonly used simplifying assumptions are stated. Peake's derivation of a general form of the Kirchhoff Radiation Law which predicts the angular and polarization dependence of the emitted energy is discussed. This model relates the emission and absorption characteristics to two measurable quantities, the backscatter and the apparent radiometric temperature of the surface. Comparison of theoretical and experimental values for both specular and diffuse surfaces show a good approximation is possible.

DISCUSSION OF A MODEL OF THE APPARENT
TEMPERATURE OF NATURAL SURFACES IN THE MICROWAVE RANGE

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Technical Report RSC-10

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INTRODUCTION

All objects above absolute zero temperature emit electromagnetic energy because of thermal agitation of charged particles. This emitted energy is of concern because many important characteristics of the material affect the emission process. The primary method of sampling the emitted energy is through the sampling of the apparent surface temperature. This temperature is defined as the temperature at which a perfect black body would radiate the same amount of energy.

In the microwave region very few natural materials have emissivities that are close to one. The majority of the natural materials have emissivities over the range of 0.6 to 0.9. Since the emissivity of the materials is not unity and, as will be shown, the emissivity and the absorptivity must be equal to satisfy

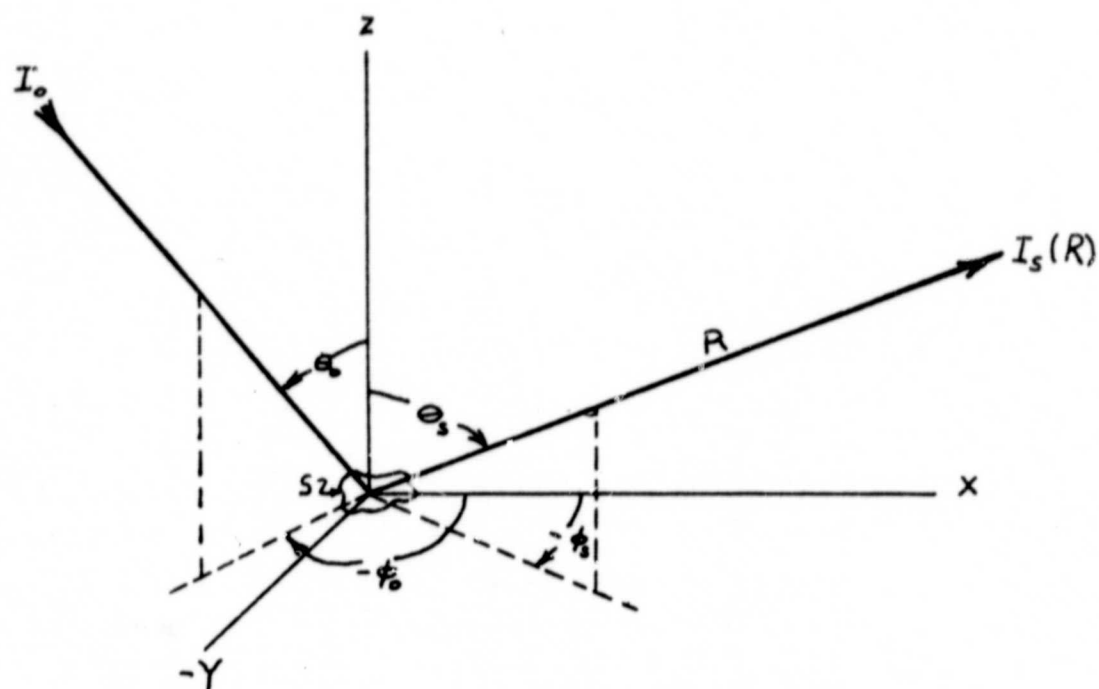
thermodynamic constraints, the energy not absorbed must be reflected. Because of this reflected energy, the scattering properties of the material must be considered when the apparent temperature is determined. In the microwave region it may be difficult, if not impossible, to separate the several contributions to the apparent temperature by measurement of thermal radiation alone. Because of this difficulty it is necessary to establish the interdependence of the emissivity, scattering properties, and apparent temperature. This relation is expressed in a form of Kirchhoff's Radiation Law, which allows for the angular and polarization dependence of the apparent temperature for a general surface.

ASSUMPTIONS AND DEFINITIONS

Since the earth's surface is considered in the derivation, several assumptions can be made which cause only small errors up to grazing angles. Because of the large radius of curvature of the earth, the surface is considered to be a plane which is infinitely wide. A small region of this plane may have any com-

plexity of surface. Finally this surface is considered to be of infinite depth, eliminating any radiation transmitted through the plane.

Figure 1 - Geometry for Scattering Relations



If radiation of intensity I_0 (watts per meter) falls at an angle of incidence θ_0 and azimuth of ϕ_0 on a given element of this surface area, S , in the plane of the earth and if the intensity of the scattered radiation in the direction θ_s, ϕ_s at a distance R from S is I_s , then the differential scattering coefficient $\gamma(\theta_0, \phi_0; \theta_s, \phi_s)$ is defined by:

$$\gamma(\theta_0, \phi_0; \theta_s, \phi_s) = (4\pi R^2 I_s) / (I_0 S \cos \theta_0)$$

This general surface equation can be divided into two parts: a "specular", or coherent part, and a "diffuse", or incoherent part. For a perfectly flat surface (specular)

$$\gamma(\theta_0, \phi_0; \theta_s, \phi_s) = 4\pi |R_i| (C_{sc} \theta_s) \delta(\theta_s - \theta_0) \delta(\phi_s - \phi_0)$$

where θ_0, ϕ_0 represents the specular direction of reflection and $|R_i|$ represents the Fresnel reflection coefficient. For the diffuse part, I_s is proportional to S and inversely proportional to R^2 at large distances. Therefore, γ is independent of R and S . However, S must be much larger than any significant structural feature of the surface if γ is to be independent of the particular area illuminated.

For any electromagnetic radiation the polarization of both the incident and scattered radiation must be known. For the two orthogonal polarization states the customary vertical (v) and horizontal (h) states will be used. Thus the scattering coefficient will be written in the form $\gamma_{ij}(o,s)$; the first subscript indicates the incident polarization state and the second subscript indicates the polarization state of the scattered radiation. The letter "o" is an abbreviation for the angles θ_0, ϕ_0 and the letter "s" is an abbreviation for the angles θ_s, ϕ_s .

Two parameters common in radiometry can be defined in terms of the γ 's. The albedo, A , is defined as the fraction of incident electromagnetic radiation that is reflected by a surface from the direction θ_0, ϕ_0 at a specific polarization and frequency. The albedo is

$$A(\theta_0, \phi_0) = \int \frac{I_r R^2 d\Omega_r}{S I_0 \cos \theta_0}$$

where the integration is over the upper hemisphere. Taking into account the polarization properties of the surface, the albedo for incident radiation of a specific polarization is

$$A_i(\theta_0, \phi_0) = 1/4\pi \int [\gamma_{ii}(\theta, \phi) + \gamma_{ij}(\theta, \phi)] d\Omega,$$

$$i = h \text{ or } v$$

$$j = v \text{ or } h$$

The absorption coefficient is defined as the fraction of power of a given polarization and frequency incident on an area from the direction θ_0, ϕ_0 that is absorbed by the surface. Since the surface under consider-

ation is assumed to be of infinite thickness, there is no energy transmitted through the body and the absorption coefficient is unity minus the albedo.

$$a_l(\theta, \phi) = 1 - A_l(\theta, \phi)$$

$$l = h \text{ or } v$$

RELATION OF EMISSIVITY TO SCATTERING COEFFICIENTS

One of the basic theorems in electromagnetic field theory is the reciprocity theorem. A simple statement of this theorem is that the field measured due to a source at one end of a path is equal to the field measured at the opposite end of the same path due to the same source, if the positions of the source and the detector are reversed. As a result of the reciprocity theorem, the scattering coefficients satisfy the four relations,

$$\cos \theta_o \gamma_{ij}(o, s) = \cos \theta_s \gamma_{ji}(s, o)$$

where i, j are either vertical or horizontal.

A complete description of the interaction of electromagnetic waves with nonuniform surfaces requires

an understanding of the emissivity of the surface. Since it is easier to measure the scattering coefficients than it is to separate the different contributions to the apparent temperature, an understanding of the relation of the scattering coefficient to the emissivity is necessary to understand the relation of a natural surface to its apparent temperature. In order to calculate the emissivity from the scattering coefficients the Kirchhoff Radiation Law in the most general form must be used.

A basic definition used in the derivation of Kirchhoff's Radiation Law is that of the emission coefficients $e_h(\theta_0, \phi_0)$, where

$$e_h(\theta_0, \phi_0) = \frac{\text{Power emitted with horizontal polarization by a unit area of surface into an element of solid angle } d\Omega_0 \text{ in the direction } \theta_0, \phi_0.}{\text{Power emitted with horizontal polarization by a unit area of black body at the same temperature into the same element of solid angle in the same direction.}}$$

and the thermal radiation is understood to occupy a narrow band of frequencies Δf .

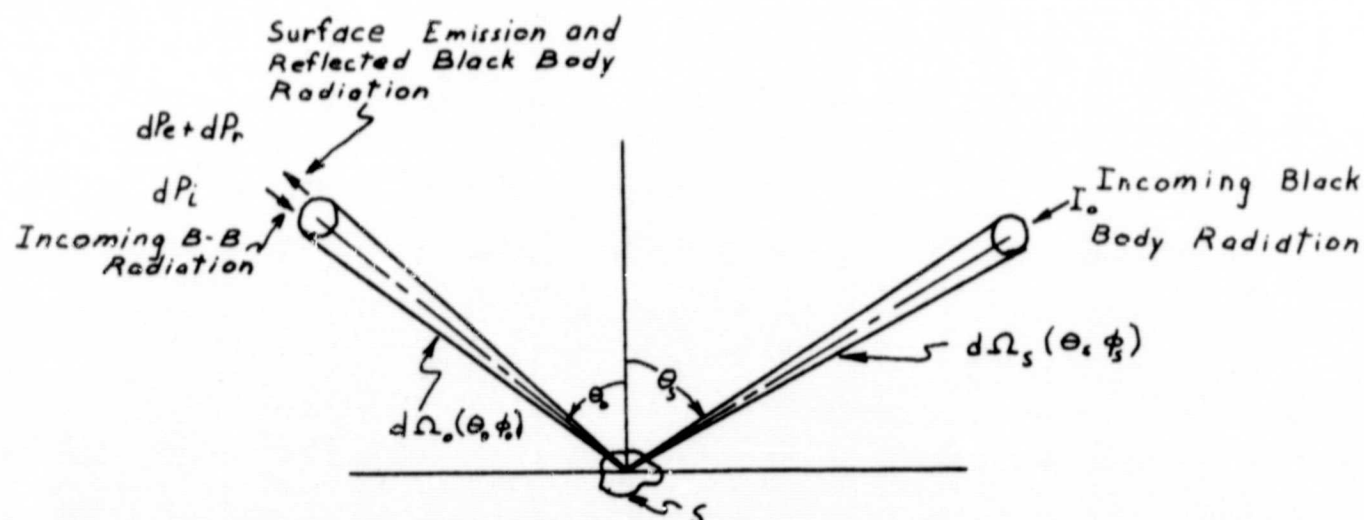


Figure 2 - Geometry for Kirchhoff's Law Derivation

Consider a surface in thermal equilibrium with the black body radiation in the half space above it. Under this condition it is assumed that just as much energy of a given polarization leaves the surface in a given direction as falls on it from the same direction. That is, the power increment dP_i incident with horizontal polarization on an elemental surface S with a solid angle range $d\Omega_0$ in direction θ_0, ϕ_0 is

$$dP_i = i_0 d\Omega_0 S \cos \theta_0$$

where i_0 is the power density per unit solid angle of black body radiation in a specific polarization state. The power emitted by the same surface from the definition of emission coefficient is

$$dP_e = e_h(\theta_0, \phi_0) i_0 d\Omega_0 S \cos \theta_0$$

In calculating the power reflected by the surface, the differential scattering coefficient $\gamma(\theta_s, \phi_s; \theta_0, \phi_0)$ must be calculated for the solid angle $d\Omega_0$ into which the power is reflected. Therefore $\gamma_{ij}(d\Omega_0)$ becomes

$$\gamma_{ij}(d\Omega_o) = \gamma_{ij} \frac{d\Omega_o}{4\pi}$$

or the differential scattering per unit solid angle. The power reflected by the surface into $d\Omega_o$, as a function of the incident solid angle $d\Omega_s$, is the incident power times the differential scattering coefficient.

$$dP_r^2 = dP_i \gamma_{ij}(d\Omega_s)$$

$$dP_r^2 = (i_o d\Omega_s S \cos \theta_s) \left\{ [\gamma_{vh}(s, 0) + \gamma_{hh}(s, 0)] \frac{d\Omega_o}{4\pi} \right\}$$

Therefore the power dP_r reflected into $d\Omega_o$ with horizontal polarization is

$$dP_r = \frac{i_o d\Omega_o S}{4\pi} \int [\gamma_{hh}(s, 0) + \gamma_{vh}(s, 0)] \cos \theta_s d\Omega_s$$

Using the ideal black body, where the incident power is balanced by the sum of the emitted and reflected power,

$$dP_i = dP_e + dP_r$$

gives

$$\cos \theta_0 \, i_0 \, d\Omega_0 S = e_h(\theta_0, \phi_0) i_0 \, d\Omega_0 S \cos \theta_0 + \frac{i_0 d\Omega_0 S}{4\pi} [\gamma_{hh}(s, 0) + \gamma_{vh}(s, 0)] \cos \theta_s \, d\Omega_s$$

By algebraic manipulation, this equation gives

$$1 = e_h(\theta_0, \phi_0) + \frac{1}{4\pi} \int [\gamma_{hh}(s, 0) + \gamma_{vh}(s, 0)] \frac{\cos \theta_s}{\cos \theta_0} \, d\Omega_s$$

Comparing the part of the equation concerned with the reflection, i.e.

$$\frac{1}{4\pi} \int [\gamma_{hh}(s, 0) + \gamma_{vh}(s, 0)] \frac{\cos \theta_s}{\cos \theta_0} \, d\Omega_s,$$

with the form of the albedo derived earlier, a relation between the emissivity and the absorption coefficient may be derived. In order to compare the two values, the definitions of incident and reflected angles are defined

in terms of the angles shown in Fig. 1. This changes the reflected term in the relation of emissivity to the absorption coefficient to

$$\frac{1}{4\pi} \int [\gamma_{hh}(0,s) + \gamma_{vh}(0,s)] \frac{\cos \theta_s}{\cos \theta_o} d\Omega_o$$

By applying the reciprocity relation,

$$\gamma_{ji}(s,o) = \gamma_{ij}(o,s) \frac{\cos \theta_o}{\cos \theta_s}$$

to the reflected term in the relation above, the form is altered to

$$\frac{1}{4\pi} \int [\gamma_{hh}(s,o) + \gamma_{hv}(s,o)] d\Omega_o$$

For an isotropic, statistically distributed surface, the reciprocity law states

$$\gamma_{hh}(o,s) d\Omega_s = \gamma_{hh}(s,o) d\Omega_o$$

and

$$\gamma_{hv}(o,s) d\Omega_s = \gamma_{hv}(s,o) d\Omega_o$$

Using this form of the reciprocity relations, the reflected term becomes

$$4\pi \int [\gamma_{hh}(0,s) + \gamma_{vh}(0,s)] \frac{\cos \theta_0}{\cos \theta_s} d\Omega_s = 4\pi \int [\gamma_{hh}(0,s) + \gamma_{hv}(0,s)] d\Omega_s$$

Since the albedo, $A_i(\theta_0, \phi_0)$ is defined by

$$A_h(\theta_0, \phi_0) = 4\pi \int [\gamma_{hh}(0,s) + \gamma_{hv}(0,s)] d\Omega_s$$

the relation of the emissivity to the absorption coefficient becomes

$$i = e_h(\theta_0, \phi_0) + A_h(\theta_0, \phi_0)$$

Since

$$a_h(\theta_0, \phi_0) = 1 - A_h(\theta_0, \phi_0)$$

it is shown that

$$e_h(\theta_0, \phi_0) = a(\theta_0, \phi_0)$$

or the emissivity is equal to the absorption coefficient, and a similar relation for vertical polarization

$$e_v(\theta, \phi) = a_v(\theta, \phi)$$

can be derived using the same method.

Using these two relations and the equations defining the albedo and the absorption coefficient, the emission coefficients can be found from the scattering coefficients, γ , of the surface.

$$e_h = 1 - \frac{1}{4\pi} \int [\gamma_{hh}(\theta, \phi) + \gamma_{hv}(\theta, \phi)] d\Omega,$$

$$e_h = 1 - A_h(\theta, \phi)$$

APPARENT TEMPERATURE OF A NATURAL SURFACE

Definition of Apparent Temperature - Since natural terrain is not an ideal black body, the total radiation emanating from the surface of this terrain will not be the same as the radiation emitted by a black body. This difference gives rise to an apparent temperature which is different from the actual temperature of the target. At sufficiently

high temperatures and long wavelengths the radiation can be said to have temperature T because of the unique relationship between T and the radiation given by the Rayleigh - Jeans approximation,

$$i_o(T, \lambda) = \frac{k T \Delta f}{\lambda^2} \text{ watts-meters}^{-2}\text{-steradian}$$

Using this unique relationship between temperature and radiation, the apparent temperature of an arbitrary radiation, with frequencies in the small range Δf , is defined as the temperature of the black body radiation which has the same power density per unit solid angle. By similar logic, the apparent temperature of a surface is simply equal to the apparent temperature of the radiation emanating from the surface in the prescribed direction with the prescribed polarization.

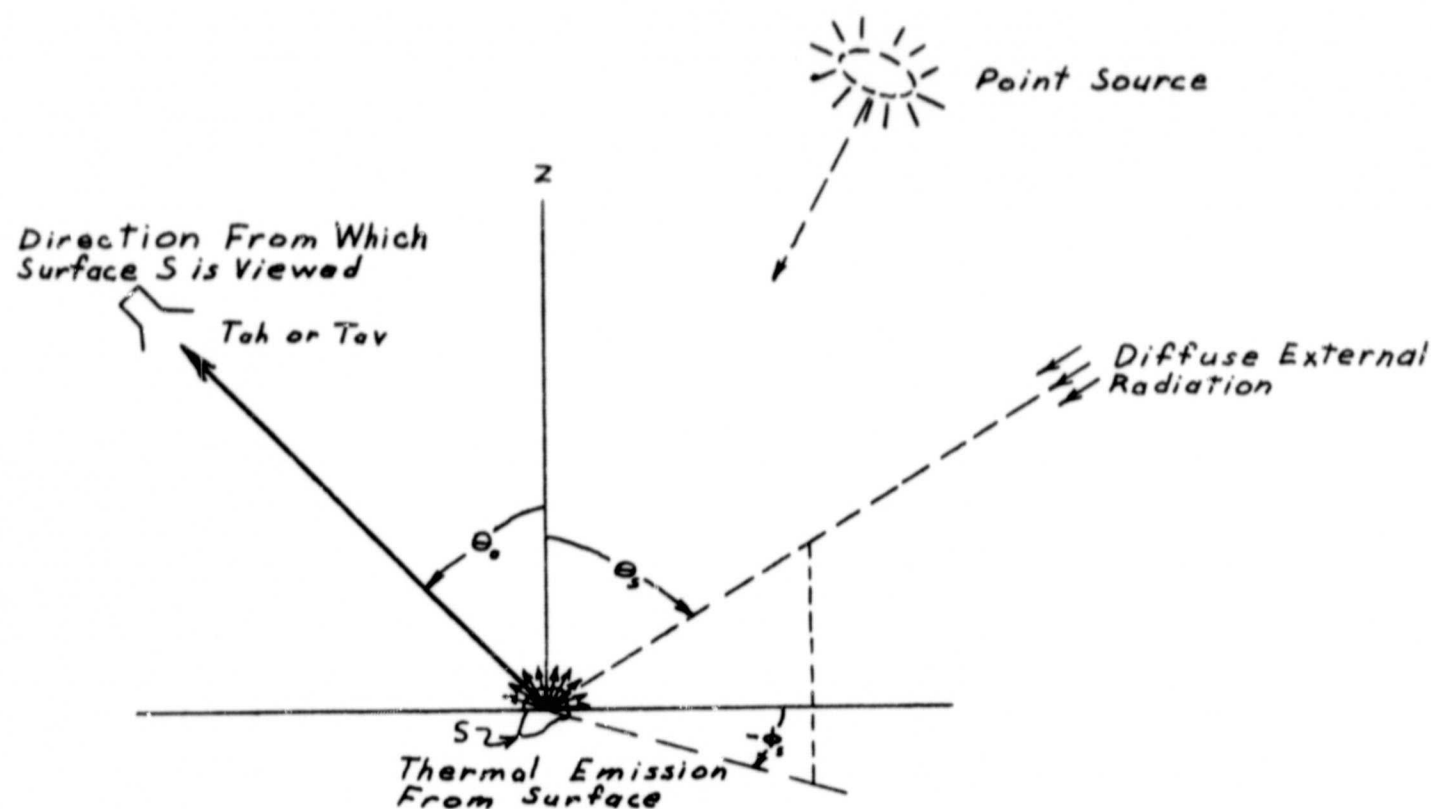


Figure 3
Contributions to Apparent Temperature of a Surface

Contributions to Apparent Temperature - The total apparent surface temperature for either vertical or horizontal polarization is the sum of four major sources of radiation.

The first source is the thermal radiation of the surface. It is assumed that the emission of this surface is independent of the conditions external to the surface and that it has a well defined temperature. The second source of radiation is the reflection of radiation caused by quasi-point sources such as the

sun. Calculations of the magnitude of the apparent temperature caused by such reflections in the microwave region show that it is small enough to be ignored unless extreme accuracy is required.

The third source of radiation is the reflection of diffuse radiation by the surface. This diffuse radiation is caused primarily by radiation of the earth's atmosphere at microwave wavelengths shorter than 10 cm and by extraterrestrial radio noise at wavelengths longer than 100 cm.

The final part of the total apparent temperature is direct atmospheric radiation. This radiation comes from the atmosphere along the path between the antenna and the surface. Therefore the total apparent temperature of a surface can be described as

$$T_{ah} = T_g(e_h)[t(h)] + T_{air}[1-t(h_c)][t(h)]A_h + T_{air}[1-t(h)]$$

where

T_g - actual, or thermometer, temperature of the ground

T_{air} - actual, or thermometer, temperature of the atmosphere

$t(h)$ - transmission coefficient for the layer of atmosphere between the antenna and the ground

$t(h_c)$ - transmission coefficient for the layer of atmosphere between the ground and the location of cosmic noise sources.

Models of Apparent Temperature - Since surfaces are generally classified as mostly rough or mostly smooth, further derivation will be divided into these two categories.

Smooth Surfaces -

Using the previously derived relations the apparent temperature equations become

$$T_{ah} = T_g [1 - A_h] [t(h)] + T_{air} [1 - t(h_c)] [t(h)] A_h + T_{air} [1 - t(h)]$$

$$T_{av} = T_g [1 - A_v] [t(h)] + T_{air} [1 - t(h_c)] [t(h)] A_v + T_{air} [1 - t(h)]$$

At this point the Fresnel reflection coefficients for a smooth surface may be used since they adequately describe the reflection characteristics of a smooth surface. The only consideration necessary in their use is remembering that the Fresnel coefficients describe reflection of fields, while the albedo is a description of the reflected power. Therefore, in substituting the Fresnel coefficients for the albedo, the square of the Fresnel coefficients must be used. This gives the apparent temperatures as

$$T_{oh} = T_g [1 - |R_h|^2] [t(h)] + T_{air} [1 - t(h_c)] [t(h)] |R_h|^2 + T_{air} [1 - t(h)]$$

$$T_{ov} = T_g [1 - |R_v|^2] [t(h)] + T_{air} [1 - t(h_c)] [t(h)] |R_v|^2 + T_{air} [1 - t(h)]$$

If the atmospheric density is assumed to decay exponentially as the height above the earth's surface increases, then the transmission coefficients can be expressed as

$$t(h) = \text{EXP}(-0.2303 \alpha h_e \text{ Sec } \theta_o)$$

$$t(h_c) = \text{EXP}(-0.2303 \alpha H \text{ Sec } \theta_o)$$

In these equations, α is the ground-level attenuation of the atmosphere in db/km, and h_e is the effective height of the atmosphere, defined as

$$h_e = H [1 - \text{EXP}(-h/H)]$$

where H is the scale height of the atmosphere, taken to be 8 km. The values for α can be found in various published works.

Rough Surfaces -

In discussing the radiating characteristics of a rough surface the total apparent temperature may again be expressed as

$$T_{ah} = T_g [1 - A_h] [t(h)] + T_{air} [1 - t(h_c)] [t(h)] A_h + T_{air} [1 - t(h)]$$

with a similar equation for T_{av} . By substituting the definitive value for the albedo and for $t(h_c)$, the equation becomes

$$T_{ah} = T_g [t(h)] \left\{ 1 - \frac{1}{4\pi} \int [\gamma_{hh}(0,s) + \gamma_{hv}(0,s)] d\Omega_s \right\} + T_{air} [1 - t(h)]$$

$$+ T_{air} \left\{ [t(h)] \frac{1}{4\pi} \int [1 - \exp(-0.2303 \alpha H \sec \theta_s)] [\gamma_{hh}(0,s) + \gamma_{hv}(0,s)] d\Omega_s \right\}$$

For most rough surfaces, the scattering coefficient tends to be approximately independent of polarization, therefore the apparent surface temperature is assumed to be the same for both states of polarization.

In order to evaluate the integrals the complete biastic scattering pattern, namely, $\gamma(o,s) = \gamma_{hh}(o,s) + \gamma_{hi}(o,s)$ must be expressed as a fairly simple function of a measurable quantity. A simple heuristic choice for $\gamma(o,s)$ consistent with measurements and the reciprocity principle is

$$\gamma(o,s) = \gamma_o (\cos \theta_o + \cos \theta_s) / (2 \cos \theta_o)$$

where γ_o is a constant which can be estimated from back-scatter data of the surface.

Using this value in the equation for apparent temperature, a manageable integration is acquired.

$$\begin{aligned} T_a = & T_g [t(h)] \left\{ 1 - \frac{1}{4\pi} \int \left[\frac{\gamma_o (\cos \theta_o + \cos \theta_s)}{2 \cos \theta_o} \right] d\Omega_s \right\} \\ & + T_{air} [t(h)] \left\{ \frac{1}{4\pi} \int [1 - \exp(-0.2303 \alpha H \sec \theta_s)] \left[\frac{\gamma_o (\cos \theta_o + \cos \theta_s)}{2 \cos \theta_o} \right] d\Omega_s \right\} \\ & + T_{air} [1 - t(h)] \end{aligned}$$

Evaluating this integral T_a becomes

$$T_e(\theta_e) = T_g[t(h)] [1 - \gamma_o/4 - \gamma_o/(8 \cos \theta_e)] + T_{air}[t(h)] [\gamma_o F_1(\chi)/4 + (\gamma_o F_2(\chi)/8 \cos \theta_e)] \\ + T_{air}[1-t(h)]$$

in which

$$F_1(\chi) = \int_0^{\pi/2} (1 - \chi^{\sec \theta_s}) \sin \theta_s \, d\theta_s$$

$$F_1(\chi) = 1 - \chi + (\ln \chi) Ei(\ln \chi)$$

$$F_2(\chi) = 2 \int_0^{\pi/2} (1 - \chi^{\sec \theta_s}) \cos \theta_s \sin \theta_s \, d\theta_s$$

$$F_2(\chi) = 1 - \chi - \chi \ln \chi + (\ln \chi)^2 Ei(\ln \chi)$$

$$\chi = \exp(-0.2303 \alpha H)$$

$$Ei(\ln \chi) = \int_0^{\chi} \frac{dt}{\ln t}$$

COMPARISON OF CALCULATED TO EXPERIMENTAL RESULTS

Smooth Surface -

Using the model derived for a smooth surface,

a comparison can be made between calculated and measured values of the apparent temperature of a smooth surface. The apparent temperature was calculated assuming a dielectric constant of 4.0 and an attenuation constant of .25, and plotted in Figure 4 along with Peake's predictions and radiometric measurements of asphalt at the 4.3 mm wavelength made by Straiton, Tolbert, and Britt. There appears to be a good correlation between the measurements made by Straiton et al. and the predictions of the model. However, the atmospheric attenuation at this wavelength is approximately 1.0 db/km. Using this value of attenuation the fourth curve in Figure 4 is obtained. Preliminary investigation indicated that perhaps the effects of atmospheric attenuation is exaggerated and that another model for the atmospheric absorption should be used at this frequency.

LEGEND

EXPERIMENTAL (HOR.)	x
EXPERIMENTAL (VERT.)	o
PEAKE	*
CALCULATED ($\alpha = .25$)	—
CALCULATED ($\alpha = 1.0$)	---

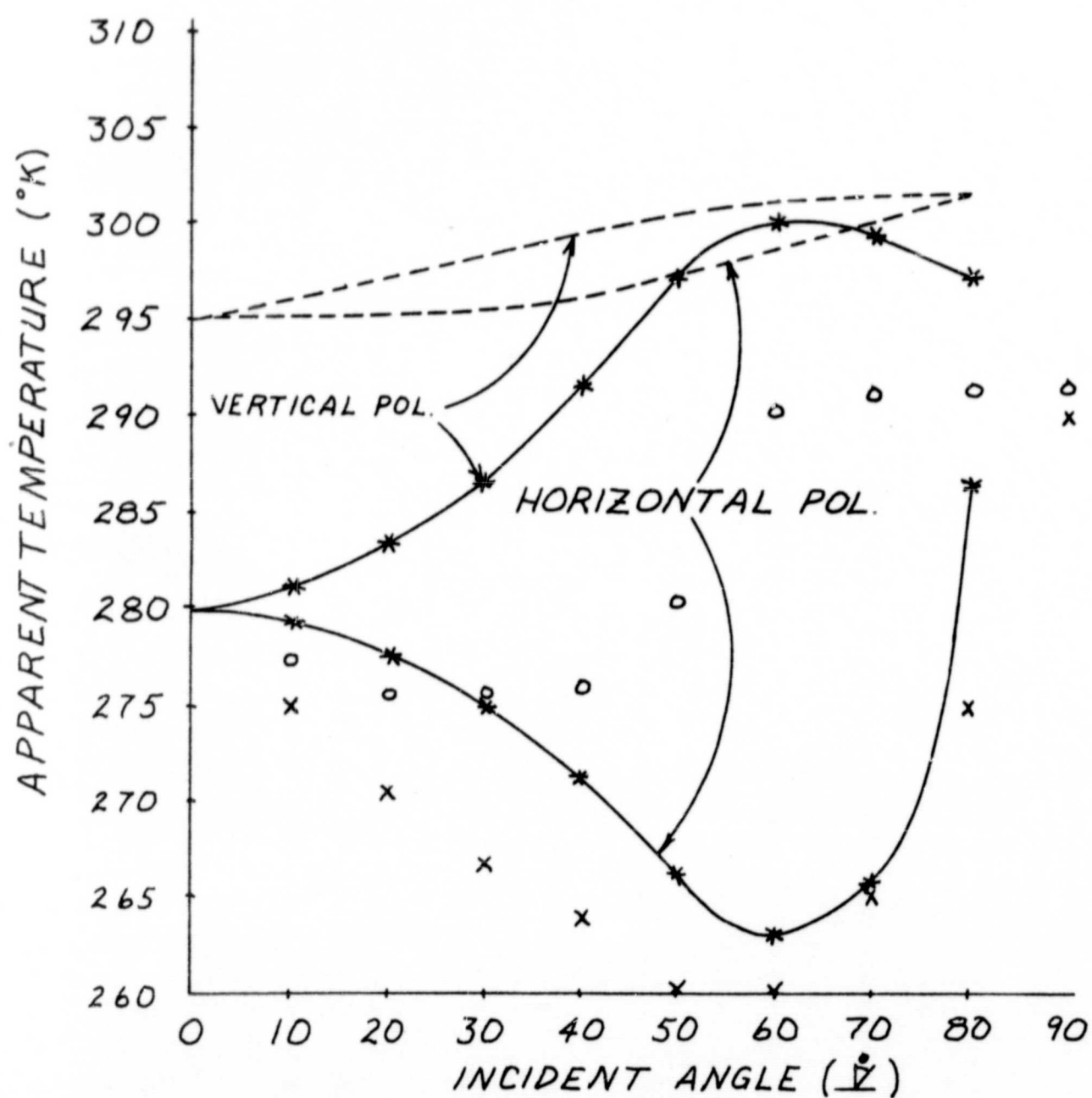


Figure 4

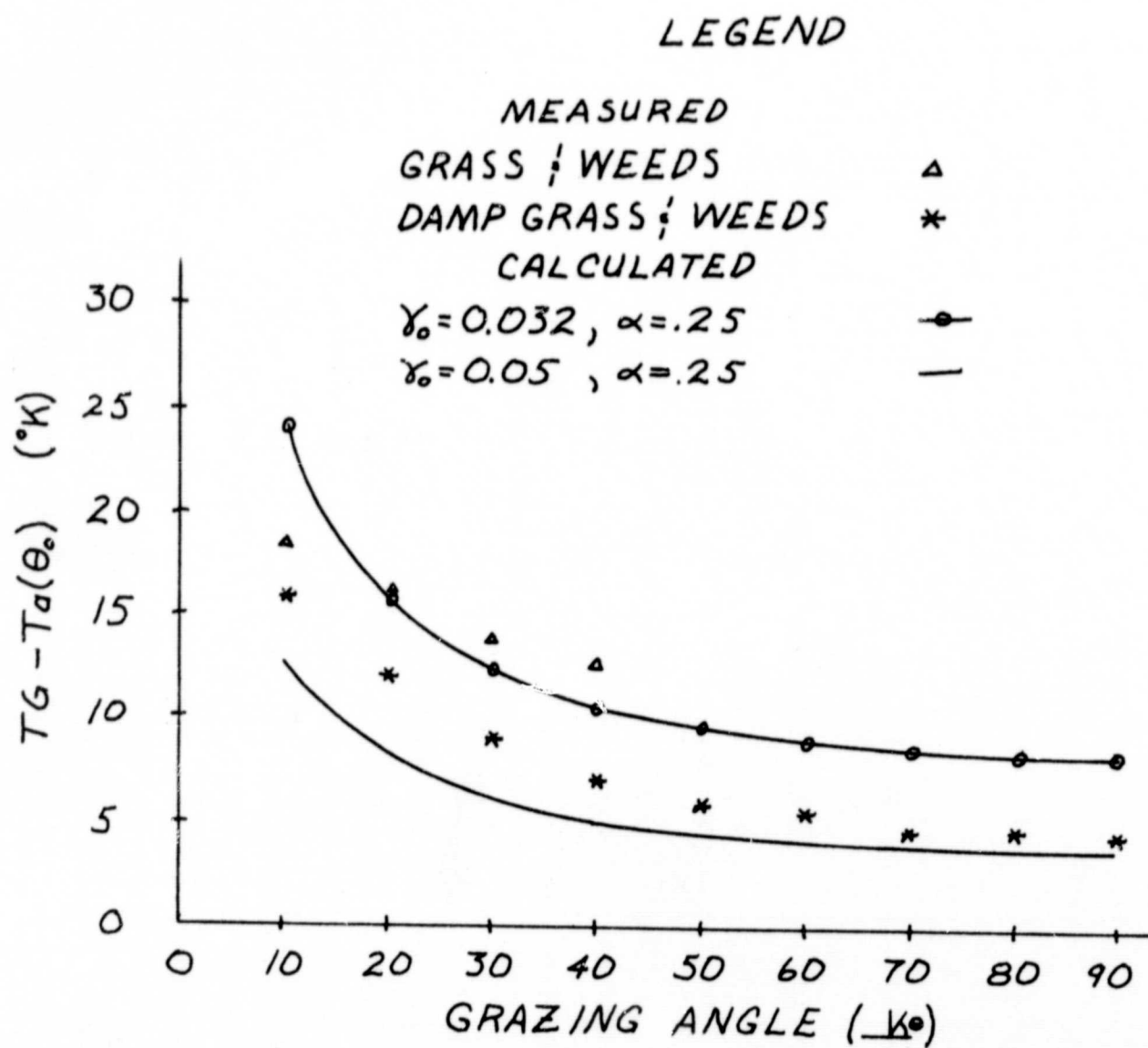
Comparison of Calculated to Experimental Results: Smooth Surface

Rough Surfaces -

In order for a surface to be considered rough, the backscatter must be relatively independent of incident angle and both the direct return and the cross polarized return should be similar in amplitude and angular dependence. Radar return measurements by Taylor and Campbell indicate that a surface composed of grass and weeds satisfy these conditions at the 8 - 6 mm wavelength. Using the model derived for a rough surface the apparent temperature was calculated using the value of .25 db/cm atmospheric attenuation at the 4.3 mm wavelength. A comparison of measurements of the apparent temperature of wet and dry grass at the 4.3 mm wavelength to the results of the rough surface model are shown in Figure 5. The effect of atmospheric attenuation was again exaggerated.

Figure 5

Comparison of Calculated to Experimental Results: Rough Surface



CONCLUSIONS

Thus using the reciprocity theorem, a form of the Kirchhoff Radiation Law which predicts the angular and polarization dependence of the emitted energy of a natural surface was derived. This model relates two measurable quantities, backscatter and apparent temperature, to the important emission and absorption characteristics. By using these experimentally obtained quantities, a theoretical estimation of the properties of a material may be made using the model derived. Comparisons of theoretical and experimental values for both specular and diffuse surfaces show a good approximation is possible. The exaggeration of atmospheric effects is probably due to the high attenuation at the frequency used for measurements. The atmospheric model used was derived for frequencies which have small values of atmospheric attenuation. A study of radiometric measurements at wavelengths having small values of attenuation will determine the accuracy of the atmospheric approximation used.

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