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THREE DIMENSIONAL MIXING OF JETS

By

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LIST OF SYMBOLS

A - Area

AG - Iteration multiplier

D - Turbulent diffusion coefficient

f - Final; or defined in Equation (35)

H - Total enthalpy

h_i - Static enthalpy of ith specie

i - x-direction subscript; or ith specie

j - y-direction subscript; or refers to jet

jsm - y-direction subscript one less than jstop

jsp - y-direction subscript on greater than jstop

jstop - y-direction subscript of y=constant boundary

k - z-direction subscript

ksm - z=direction subscript one less than kstop

ksp - z=direction subscript one greater than kstop

kstop - z=direction subscript of z=constant boundary

Le - Lewis number

M_i - Molecular weight of ith specie

p - Pressure

Pr - Prandtl number

Q - Any flow variable

R - Universal gas constant

r - Radius

s - Coordinate along streamline (Equation (19))

LIST OF SYMBOLS (Continued)

T	- Temperature
u	- x component of velocity
V	- Total velocity (Equation (7) or (12))
v	- y-component of velocity
w	- z-component of velocity
x	- Computational direction
y	- Normal coordinate (horizontal)
z	- Normal coordinate (vertical)
α_i	- Specie mass fraction of ith specie
$\tilde{\alpha}_i$	- Elemental mass fraction of ith specie
β	- Sweepback angle
Δ	- Incremental difference
θ	- Variable area streamline inclination
κ	- Turbulent conductivity
μ	- Turbulent viscosity
ν	- Turbulent kinematic viscosity
ρ	- Density
ϕ	- Equivalence ratio
∇	- Gradient operator

I. INTRODUCTION

The analysis and design of combustion chambers for scramjet engines requires a sound understanding of combustion controlled by diffusion. In an effort to simplify the design of present combustion chambers, fuel is usually injected into the combustor at a series of discrete points, either at the walls or in the center of the flow, by means of streamline injectors. The analysis of the diffusion and subsequent combustion of the fuel injected with this system can be divided into two separate parts:

(a) The analysis of the flow downstream of a single injector outside of the region of interference between adjacent injectors.

(b) The analysis of the interference region between adjacent injectors.

The first of these problems has received considerable attention both for the case of normal injection as well as tangential injection. The analytical model, especially for the case of tangential injection, is now well understood and considerable progress has been made recently for the determination of downstream effects for normal injection. Numerical programs are now available based on mixing theory. In addition, the considerable amount of experimental data presently available for such flows has encouraged current efforts to refine the model of the mixing process.

The second portion of the problem is the analysis of the diffusion process in the region where adjacent injectors interact. These regions of the flow are of extreme importance for actual combustion chambers of scramjet engines. Because of the discrete number of injectors used in an engine, the fuel introduced by each injector is related to a given air streamtube: this streamtube being bounded by the stream surfaces of adjacent injection regions. At the point where adjacent mixing regions begin to interact, present methods of analysis become invalid. These regions of the flow are of extreme importance because it is here that the fuel-air mixture must reach an average uniform distribution for combustion to become completed. Unfortunately, no method is presently available for the analysis of these regions and as a result the burner length required to achieve a given level of uniformity in fuel distribution still cannot be calculated. In addition, no analysis has been performed to determine the effect of the resulting non-uniform flow entering the nozzle on losses due to the resulting diffusion process. Both effects require additional attention.

The problem of interaction between adjacent mixing regions, in other words, three dimensional mixing, can be illustrated with the aid of Figures (1) and (2). Figure (1) shows jets located so that all jet exits lie in the same plane, while Figure (2) portrays jets located in a plane with a certain "sweepback." The dashed lines commencing at the jet exits represent that region of the flow where the fuel concentration (both burnt and unburnt) has reached a measurable level, for example 0.5%. Two cross sectional views, one where the jet mixing regions just begin to touch and the other where a certain amount of interaction has taken place are shown with each figure. An approximate estimate of the value of the equivalence ratio, ϕ, for the downstream cross section is also presented. For the case with the jets positioned with sweepback (Figure (2)) the onset of the interaction takes place along a line HH with the same sweepback. This results in non-symmetric interaction as is portrayed in the cross sections and fuel concentration distributions.

Present methods of analysis of the mixing region are only valid between the point of injection and station C, the axial location where the mixing lines of adjacent injectors cross. At this station, the actual fuel air distribution is still far

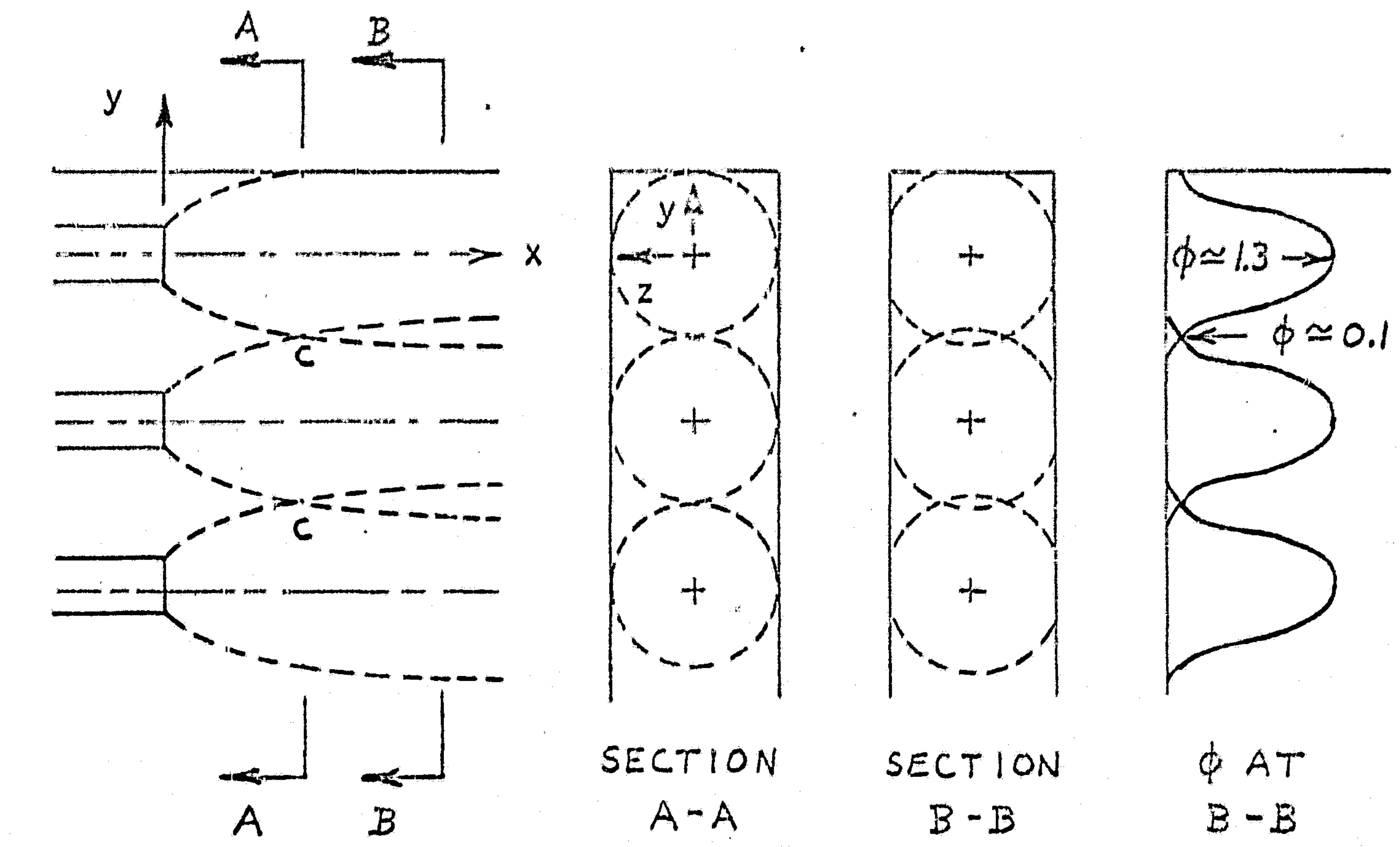


FIGURE 1. JET INTERACTION WITH NO SWEEPBACK

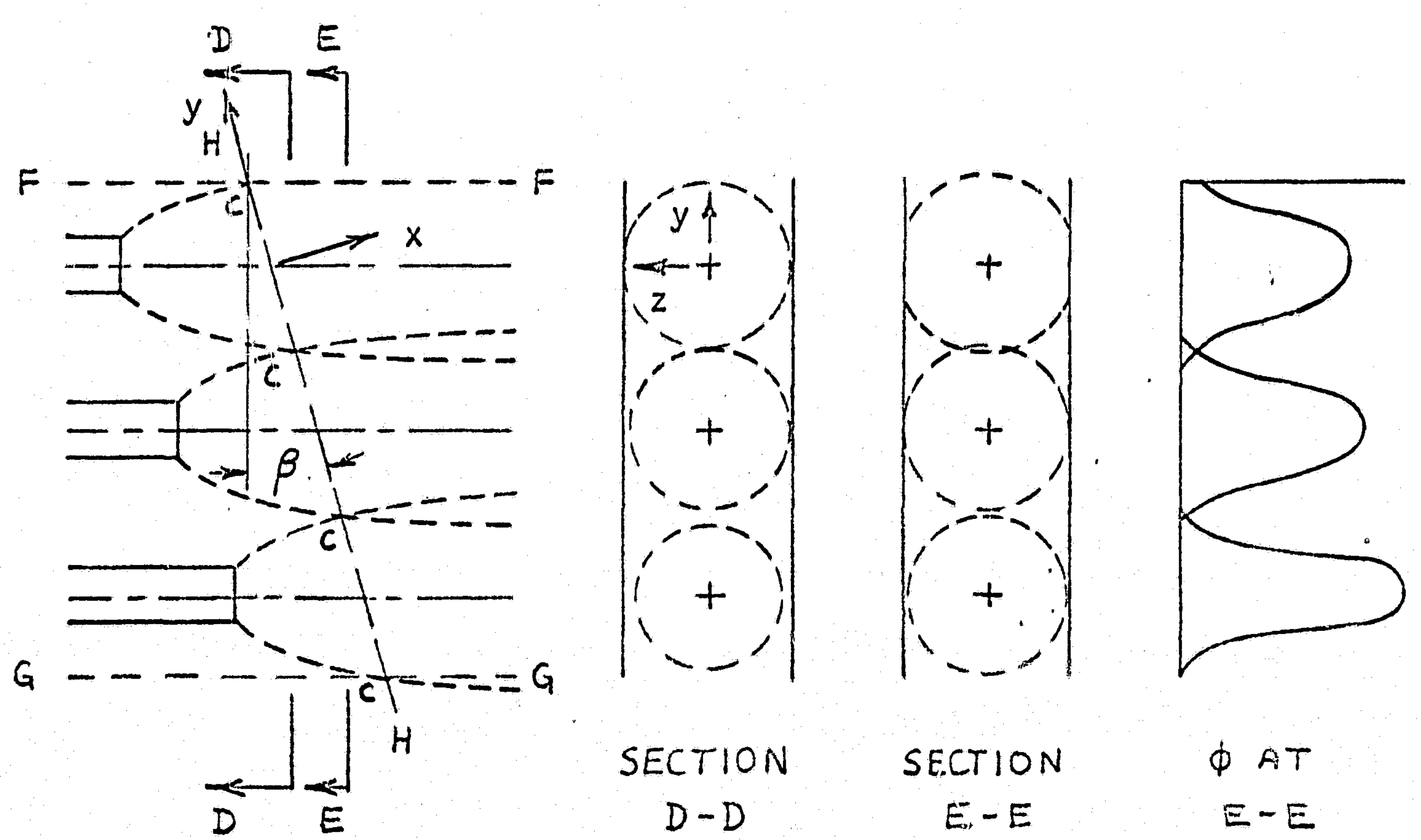


FIGURE 2. JET INTERACTION WITH SWEEPBACK

from uniform. As shown in the figure, the concentration of fuel at point C corresponds to an equivalence ratio of the order of 0.05 while the concentration of fuel along the axis of the mixing region can have an equivalence ratio from 1.3 to 1.5. When the average equivalence ratio for the engine is close to 1, the fuel at this point is far from completely burned and a substantial amount of the combustion process must take place downstream of station C in the region of interaction. It is the diffusion and combustion processes occurring downstream of C which will define the length of the burner.

Two specific problems are investigated. One problem deals with jets or sets of jets having no sweepback while the other considers jets or a set of jets with sweepback. These problems are also distinct from one another due to the fact that the first problem contains only planes of symmetry, while the second has not only planes of symmetry but also planes of equivalence, that is planes which repeat themselves periodically.

II. THE GOVERNING EQUATIONS

The governing equations for a steady three dimensional flow with flame sheet equilibrium chemistry were derived. With the introduction of the assumptions Le = Pr = 1, these equations are:

Continuity: $$\nabla\cdot(\rho u) = 0 \qquad (1)$$

Momentum: $$V\cdot\nabla V = \frac{1}{\rho} \quad -\nabla p + \nabla.(\rho\nu\nabla V) \qquad (2)$$

Energy: $$V\cdot\nabla H = \frac{1}{\rho} \quad \nabla.(\rho\kappa\nabla V) \qquad (3)$$

Species: $$V\cdot\nabla\tilde{\alpha}_i = \frac{1}{\rho} \quad \nabla.(\rho D\nabla\tilde{\alpha}_i) \qquad (4)$$

State: $$p = \rho RT \sum_i \frac{\alpha_i}{M_i} \qquad (5)$$

Enthalpy: $$H = \sum_i \alpha_i \; h_i(T) + \frac{1}{2} V.V \qquad (6)$$

Total Velocity: $$V\cdot V = u^2 + v^2 + w^2 \qquad (7)$$

As these equations stand, there are nine equations (Equation (2) is actually three equations) and nine unknowns (if $\tilde{\alpha}_i$ is known so is α_i, as shown later), and as such the system is closed and should be solvable. However, the solution of the complete set is a monumental task, and various simplifications can be applied.

By restricting the present analysis to regions where the streamline curvature is not very large, normal pressure gradients are found to be quite small. An order of magnitude analysis of the type commonly used for mixing problems indicates that the two normal momentum equations may be eliminated and Equations (2), (3), (4), (6) and (7) may be reduced to the following form:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \frac{1}{\rho}\left[- \frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\rho\nu \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\rho\nu \frac{\partial u}{\partial z}\right)\right] \tag{8}$$

$$u \frac{\partial H}{\partial x} + v \frac{\partial H}{\partial y} + w \frac{\partial H}{\partial z} = \frac{1}{\rho}\left[\frac{\partial}{\partial y}\left(\rho\kappa \frac{\partial H}{\partial y}\right) + \frac{\partial}{\partial z}\left(\rho\kappa \frac{\partial H}{\partial z}\right)\right] \tag{9}$$

$$u \frac{\partial \tilde{\alpha}_i}{\partial x} + v \frac{\partial \tilde{\alpha}_i}{\partial y} + w \frac{\partial \tilde{\alpha}_i}{\partial z} = \frac{1}{\rho}\left[\frac{\partial}{\partial y}\left(\rho D \frac{\partial \tilde{\alpha}_i}{\partial y}\right) + \frac{\partial}{\partial z}\left(\rho D \frac{\partial \tilde{\alpha}_i}{\partial z}\right)\right] \tag{10}$$

$$H = \sum_i \alpha_i \, h_i(T) + \frac{1}{2} V^2 \tag{11}$$

$$V^2 = u^2 + v^2 + w^2 \tag{12}$$

Let us consider the case of no sweepback and constant area. In view of the fact that the initial profiles are in a region where a certain amount of decay of sharp gradients has taken place, and also because the streamlines have reached a region of small inclination relative to the x direction, the simplification that normal velocity components are small ($v, w << u$) and can be neglected, i.e.,

$$v, w \approx 0 \tag{13}$$

is introduced. This implies that the flow process is governed by diffusion; and we again have a closed system of seven equations and seven unknowns. Since two of the variables appearing in Equation (1), the differential continuity equation, have been eliminated, this equation is replaced by the overall continuity equation:

$$\int_{A(x)} \rho u \, dy \, dz = \text{constant} \tag{14}$$

For the sweepback case, instead of solving the problem along the direction of jet axes, the solution will be carried out normal to the planes of constant sweepback. This is done since the pressure is assumed constant along lines of constant sweepback. For zero sweepback this obviously reduces to the unswept problem. In either case, the solution plane is defined by the y and z coordinates and x is the normal direction in which the solution progresses. For the sweptback case the z component of velocity is still negligible as given in Equation (13) and we can say

$$w \approx 0 \tag{15}$$

However, the y component of velocity is determined so that streamlines of the jet remain parallel to the jet axes. The result is that

$$v = u \tan \beta \tag{16}$$

where β is the sweepback angle. For zero sweepback ($\beta = 0$) this reduces to the previous case of $v = 0$ (Figure (2) shows β).

The case of non-constant area, with β arbitrary, is such that the area enlarges only in one direction, namely the z direction; thus we must introduce a z component of velocity. This velocity component is assumed to be such that the streamlines are inclined at angles proportional to their distance from the axis and having their maximum inclination at the diverging or converging wall. This leads to a velocity component

$$w = u \tan \theta(z) \tag{17}$$

where

$$\theta(z) = \theta(z_{MAX}) \frac{z}{z_{MAX}} \tag{18}$$

Obviously this reduces to $w = 0$ when $\theta(z_{MAX}) = 0$; which is the assumption for constant area (Figure (5) shows θ).

In solving a specific problem the sweepback effect will be included by the term $v \frac{\partial}{\partial y}$ in each of the equations. However, for area variation we must look along a line s where

$$\Delta s = (\Delta x^2 + \Delta z^2)^{\frac{1}{2}} \tag{19}$$

if we wish to keep the number of grid points fixed. If Q represents any quantity to be solved for,

$$Q_2 = Q_1 + \frac{\partial Q}{\partial s} \Delta s \tag{20}$$

where

$$\frac{\partial}{\partial s} = \frac{\partial x}{\partial s} \frac{\partial}{\partial x} + \frac{\partial z}{\partial s} \frac{\partial}{\partial z} \tag{21}$$

$$\frac{\partial x}{\partial s} = \cos \theta \tag{22}$$

$$\frac{\partial z}{\partial s} = \sin \theta \tag{23}$$

and

$$\Delta s = \frac{\Delta x}{\cos \theta}$$

By combining these results we find

$$Q_2 = Q_1 + \left[\frac{\partial Q}{\partial x} + \tan \quad \frac{\partial Q}{z} \right] \quad \Delta x \tag{24}$$

but

$$\tan \theta(z) = \frac{w}{u} \tag{25}$$

and

$$\frac{\partial Q}{\partial x} = \frac{\nabla(\rho\mu \ \nabla Q)}{\rho u} - \frac{v}{u}\frac{\partial Q}{\partial y} - \frac{w}{u}\frac{\partial Q}{\partial z} \tag{26}$$

Therefore,

$$Q_2 = Q_1 + \left[\frac{\nabla(\rho\mu \ \nabla Q)}{\rho u} - \frac{v}{u}\frac{\partial Q}{\partial y}\right] \Delta x \tag{27}$$

Hence, let us write simply

$$\frac{\partial Q}{\partial x} = \frac{\nabla(\rho\mu \ \nabla Q)}{\rho u} - \frac{v}{u}\frac{\partial Q}{\partial y} \tag{28}$$

which is the general expression valid for a constant number of grid points with sweepback and variable area.

The concept Le = Pr = 1 which is an excellent approximation for turbulent flow yield the equations

$$\mu = \rho\nu = \rho\kappa = \rho D \tag{29}$$

where μ is a turbulent viscosity model assumed to be a function of x only (it is pointed out that these equations are also valid for laminar flow, but the above simplification is especially good for turbulent flow). The final equations for the swept problem with variable area are:

$$u \frac{\partial u}{\partial x} = \frac{1}{\rho} \left[- \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \right] - v \frac{\partial u}{\partial y} \tag{30}$$

$$u \frac{\partial H}{\partial x} = \frac{1}{\rho} \left[\mu \left(\frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} \right) \right] - v \frac{\partial H}{\partial y} \tag{31}$$

$$u \frac{\partial \tilde{\alpha}_i}{\partial x} = \frac{1}{\rho} \left[\mu \left(\frac{\partial^2 \tilde{\alpha}_i}{\partial y^2} + \frac{\partial^2 \tilde{\alpha}_i}{\partial z^2} \right) \right] - v \frac{\partial \tilde{\alpha}_i}{\partial y} \tag{32}$$

and Equations (5), (11), (12) and (14).

In order to expedite the chemical analysis, a simple chemical reaction using a flame sheet equilibrium chemistry model is assumed for this program. The reaction assumed is

$$2H_2 + O_2 = 2H_2O \tag{33}$$

This simplification is acceptable in the problem because the main goal of the analysis is to determine the diffusion process and this process is affected only slightly by the presence of small amounts of H, O, and OH radicals. Furthermore, the presence of these radicals would result in a slight decrease of the temperature and molecular weight of the mixture and these two influences have the opposite effect on density which thus remains relatively unaffected. Consequently, streamlines essentially retain their locations and so normal derivatives are also virtually unaffected.

The flame sheet chemistry model allows us to determine the species composition by evaluating only one elemental specie equation, say for $\tilde{\alpha}_{O_2}$. It is then possible to find $\tilde{\alpha}_{N_2}$ by making use of the fact that the ratio of the number of molecules of oxygen to the number of molecules of nitrogen remains fixed, i.e.

$$\tilde{\alpha}_{N_2} = f\,\tilde{\alpha}_{O_2} \tag{34}$$

where

$$f = \left\{\frac{\tilde{\alpha}_{N_2}}{\tilde{\alpha}_{O_2}}\right\}_{air} \tag{35}$$

The remaining elemental specie, $\tilde{\alpha}_{H_2}$, is found from the definition

$$\sum_i \tilde{\alpha}_i = 1 \tag{36}$$

Then, depending on the relative values of $\tilde{\alpha}_{H_2}$ and $\tilde{\alpha}_{O_2}$ we evaluate the individual species as follows:

if $\frac{\tilde{\alpha}_{H_2}}{M_{H_2}} < \frac{2\tilde{\alpha}_{O_2}}{M_{O_2}}$, excess oxygen, then

$$\alpha_{H_2} = 0$$

$$\alpha_{H_2O} = \tilde{\alpha}_{H_2} \frac{M_{H_2O}}{M_{H_2}} \tag{37a}$$

and $$\alpha_{O_2} = \tilde{\alpha}_{O_2} - \alpha_{H_2O} \frac{M_{O_2}}{M_{H_2O}}$$

if $\frac{\tilde{\alpha}_{H_2}}{M_{H_2}} > \frac{2\tilde{\alpha}_{O_2}}{M_{O_2}}$, excess hydrogen, then

$$\alpha_{O_2} = 0$$

$$\alpha_{H_2O} = 2\tilde{\alpha}_{O_2} \frac{M_{H_2O}}{M_{O_2}} \tag{37b}$$

and $$\alpha_{H_2} = \tilde{\alpha}_{H_2} - \alpha_{H_2O} \frac{M_{H_2}}{M_{H_2O}};$$

or if $\frac{\tilde{\alpha}_{H_2}}{M_{H_2}} = \frac{2\tilde{\alpha}_{O_2}}{M_{O_2}}$, stoichiometric, then

$$\alpha_{O_2} = 0$$

$$\alpha_{H_2} = 0 \tag{37c}$$

and $$\alpha_{H_2O} = \tilde{\alpha}_{O_2} \frac{M_{H_2O}}{M_{O_2}}$$

and for each of the above

$$\alpha_{N_2} = \tilde{\alpha}_{N_2}. \tag{38}$$

The α_i to be used in Equations (5) and (11) are thus determined.

If frozen flow with water present is to be studied, then one additional equation of the form of Equation (32) must be introduced for α_{O_2} the mass fraction of oxygen, and the individual species are then determined as before except there are no tests required and

$$\alpha_{H_2O} = (\tilde{\alpha}_{O_2} - \alpha_{O_2}) \frac{2M_{H_2O}}{M_{O_2}} \tag{39}$$

$$\alpha_{H_2} = \tilde{\alpha}_{H_2} - (\tilde{\alpha}_{O_2} - \alpha_{O_2}) \frac{2M_{H_2}}{M_{O_2}} \tag{40}$$

III. SOLUTION DOMAIN OF THE PROBLEM

The problem will be analyzed with respect to two considerations regarding the solution domain:

(1) The solution is bounded only by symmetry planes.

(2) The solution is bounded by symmetry planes and planes of equivalence.

In case (1), complete symmetry is obtained so long as only walls and planes of symmetry can be made to bound the problem. A wall is equivalent to a plane of symmetry in this analysis since the boundary layer along a wall is neglected. The boundary layer is neglected because the primary concern in this analysis is the study of the diffusion process associated with the interaction of adjacent jet mixing regions. The neglect of the boundary layer results in inviscid and adiabatic walls which implies zero normal derivatives at the wall. This, of course, is the property of a plane of symmetry.

A symmetry plane can be inserted along any plane such that the flow on one side of it is a mirror image of the flow on the other side. Examples of various jet arrangements with complete symmetry showing the locations of suitable symmetry planes and indicating corresponding y-z planes used in the calculations are presented in Figure (3).

Case 2, which contains not only symmetry planes but also planes of equivalence, is obtained when jet or sets of jet exits are located along a sweptback line. Two specific examples showing even as well as uneven jet spacing are shown in Figure (4). The first arrangement shows even spacing of jets, while the other shows repeated uneven spacing. Planes AA and BB, and AA'A and BB'B are symmetry planes for these two respective configurations, while planes AB and also planes A'B' are equivalence planes.

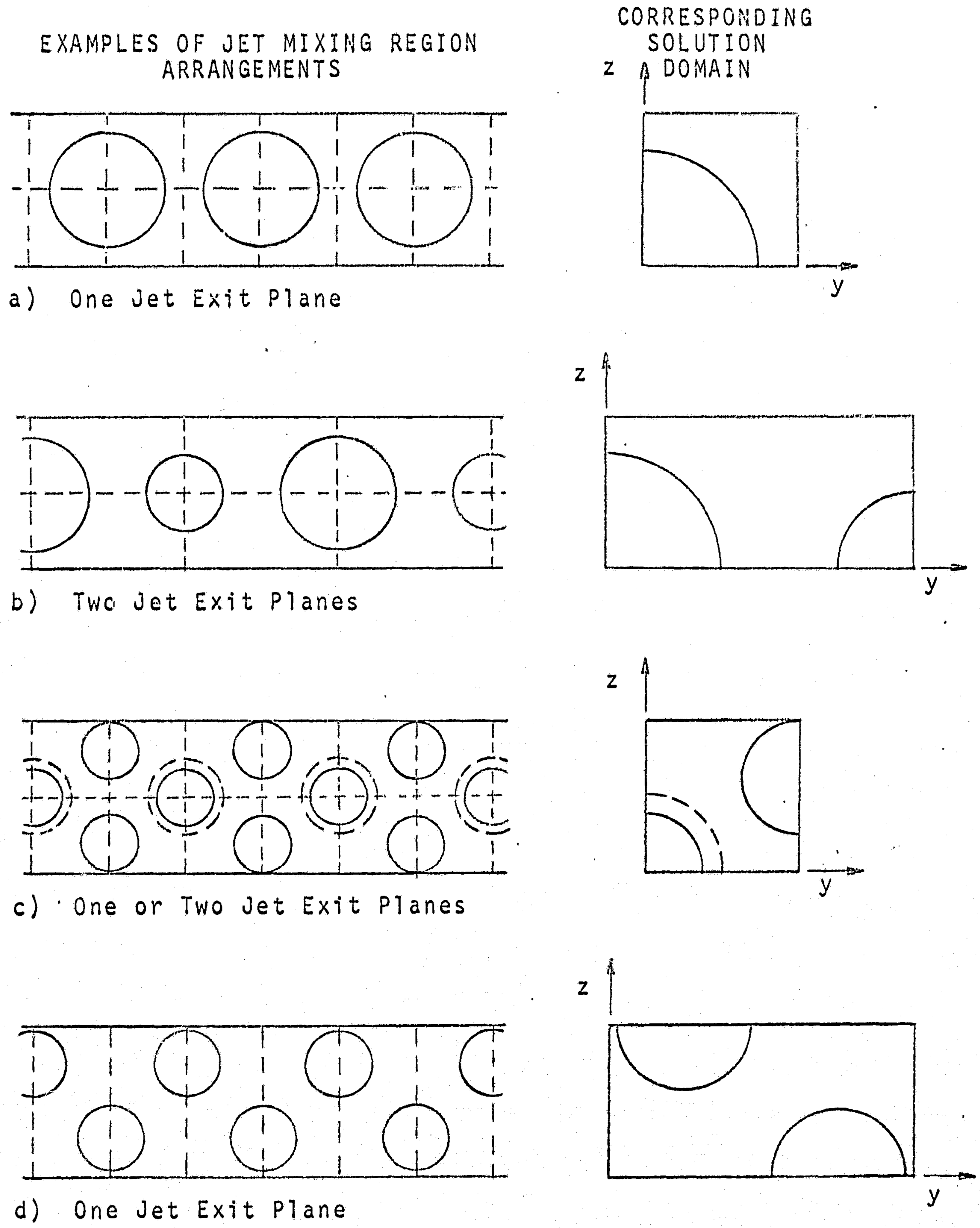
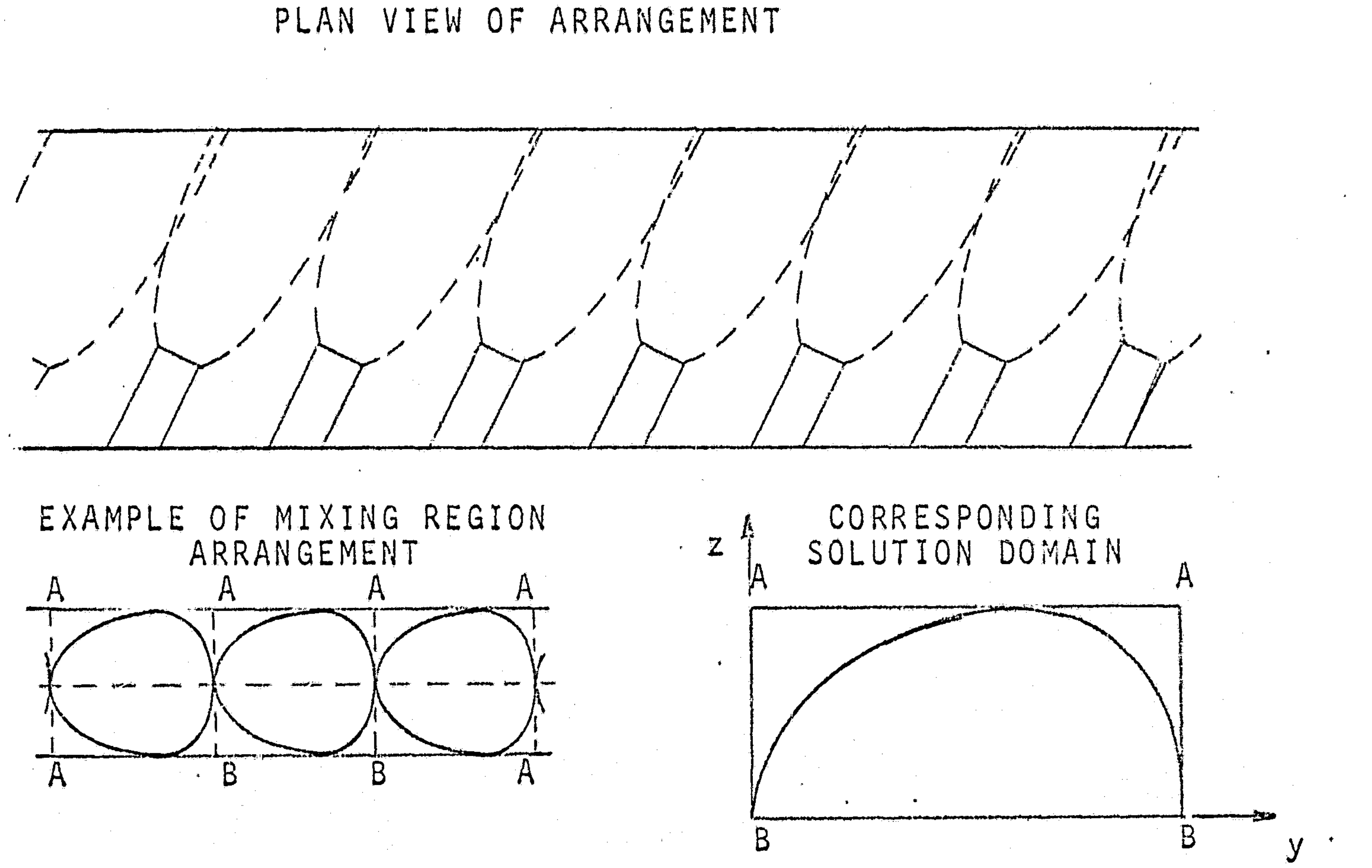
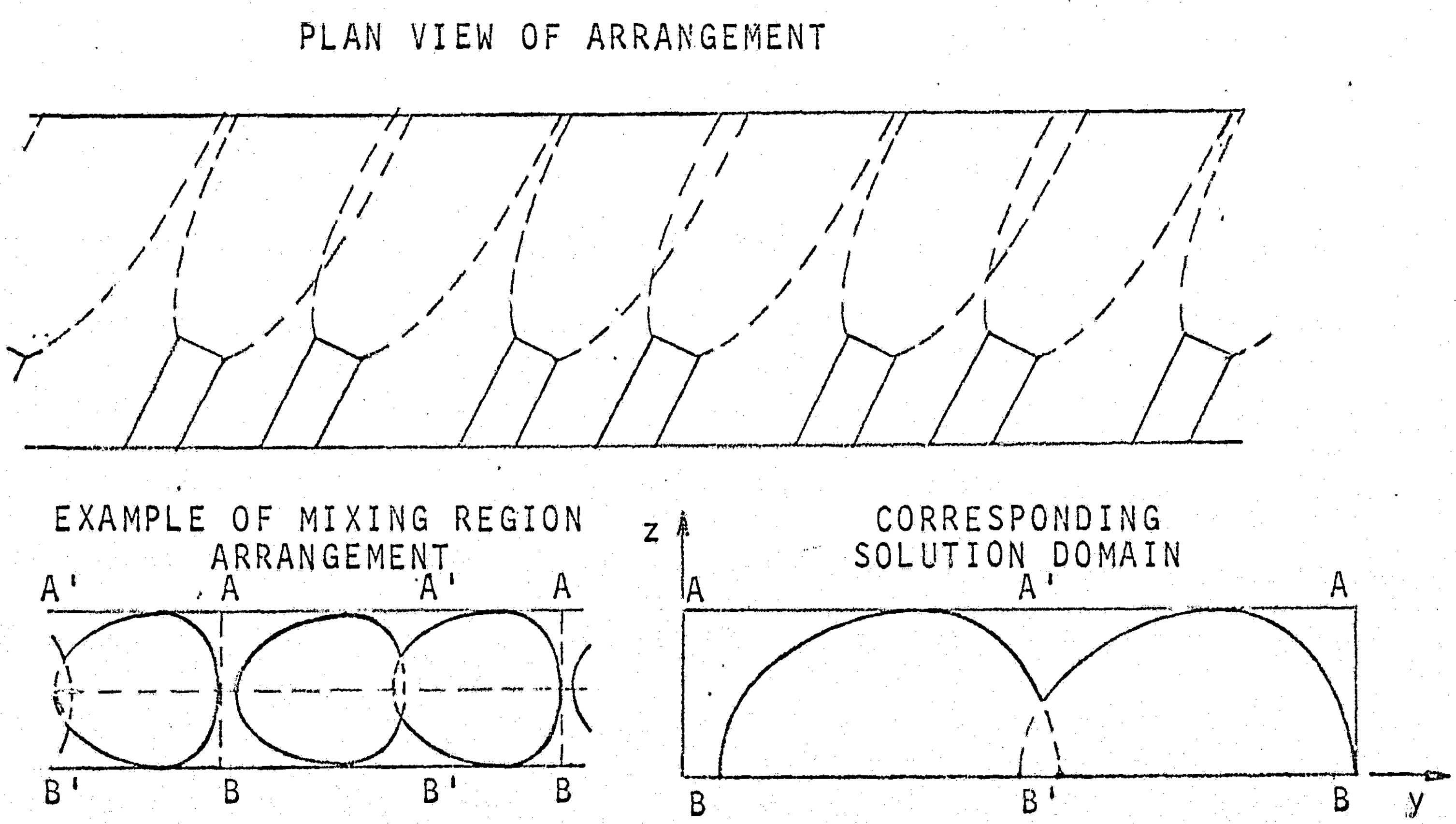


FIGURE 3. EXAMPLES OF ARRANGEMENTS WITH SYMMETRY BOUNDARY PLANES



a) Infinite Set of Jets with Repeated Equal Spacing



b) Infinite Sets of Jets with Repeated Unequal Spacing

FIGURE 4. EXAMPLES OF ARRANGEMENTS WITH PLANES OF EQUIVALENCE BOUNDARIES

IV. THE NUMERICAL GRID

The three dimensional growth and interaction of a jet can be represented as shown in Figure (5). The growth proceeds from the original tangency with one wall or plane of symmetry. The ultimate limit in the x direction would be determined by the uniformity of the final profiles. In the present schematic representation this point has not yet been achieved. A typical y-z plane for the numerical approach is pictured in Figure (6) where various types of grid points as well as their location and identification is specified.

Boundary grid points are those lying on the boundaries determined by $y = z = 0$, $y = y_{max}$ and $z = z_{max}$. Interior grid points are those inside the boundary but not on the boundary, and internal grid points are those interior grid points, one mesh point inside the boundary. Finally, exterior grid points lie one mesh point outside the boundary, exactly opposite the corresponding interior grid point whose value they will assume for symmetry planes as discussed in the boundary condition section.

The grid point identification subscript for exterior grid points are $j = k = 1$, $j = jsp$ and $k = ksp$, for boundary grid points are $j = k = 2$, $j = jstop$ and $k = kstop$ and for interior grid points are $j = k = 3$, $j = jsm$ and $k = ksm$. Note that

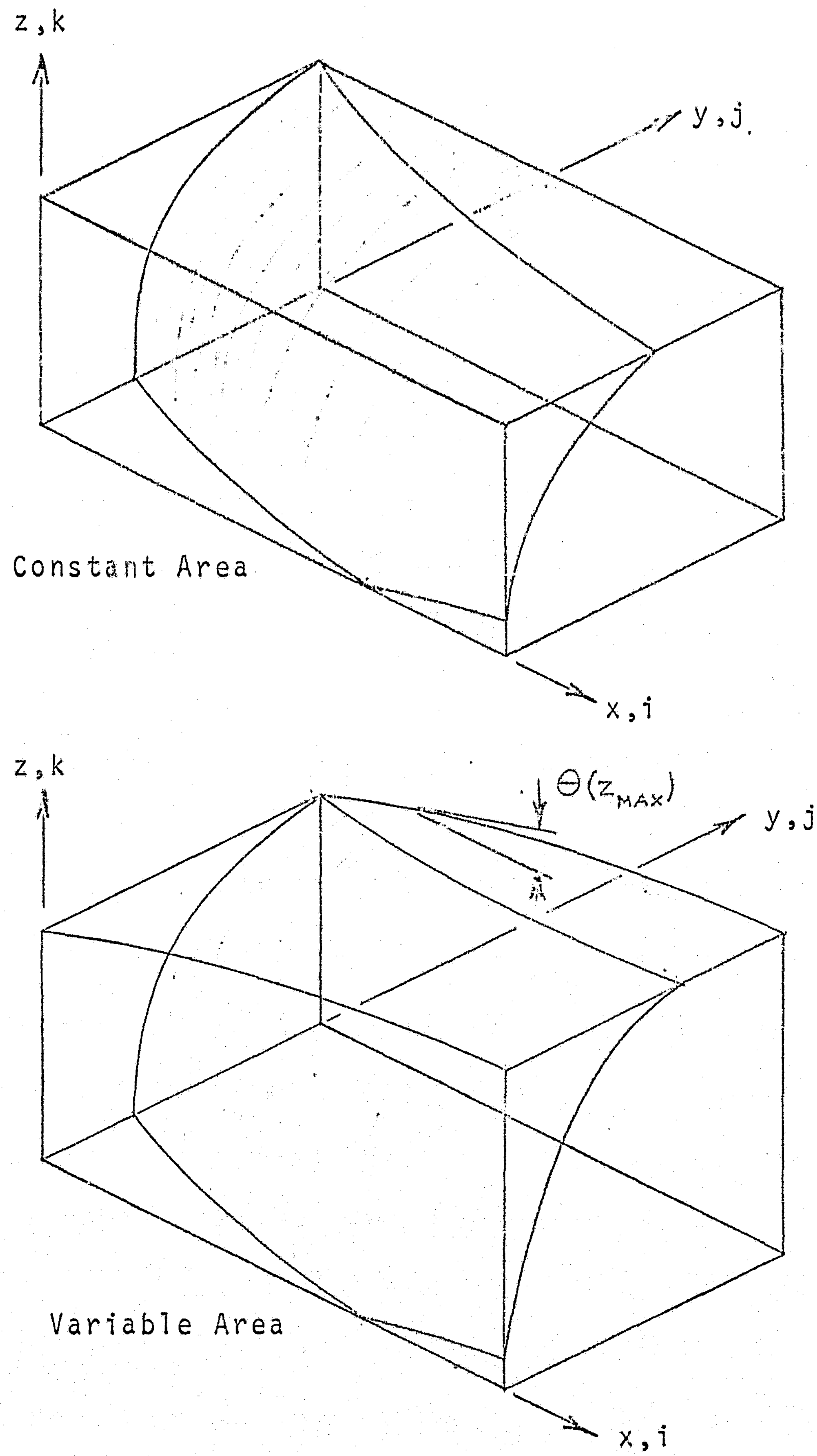


FIGURE 5. SOLUTION DOMAIN FOR THREE DIMENSIONAL INTERACTION OF JET MIXING REGIONS

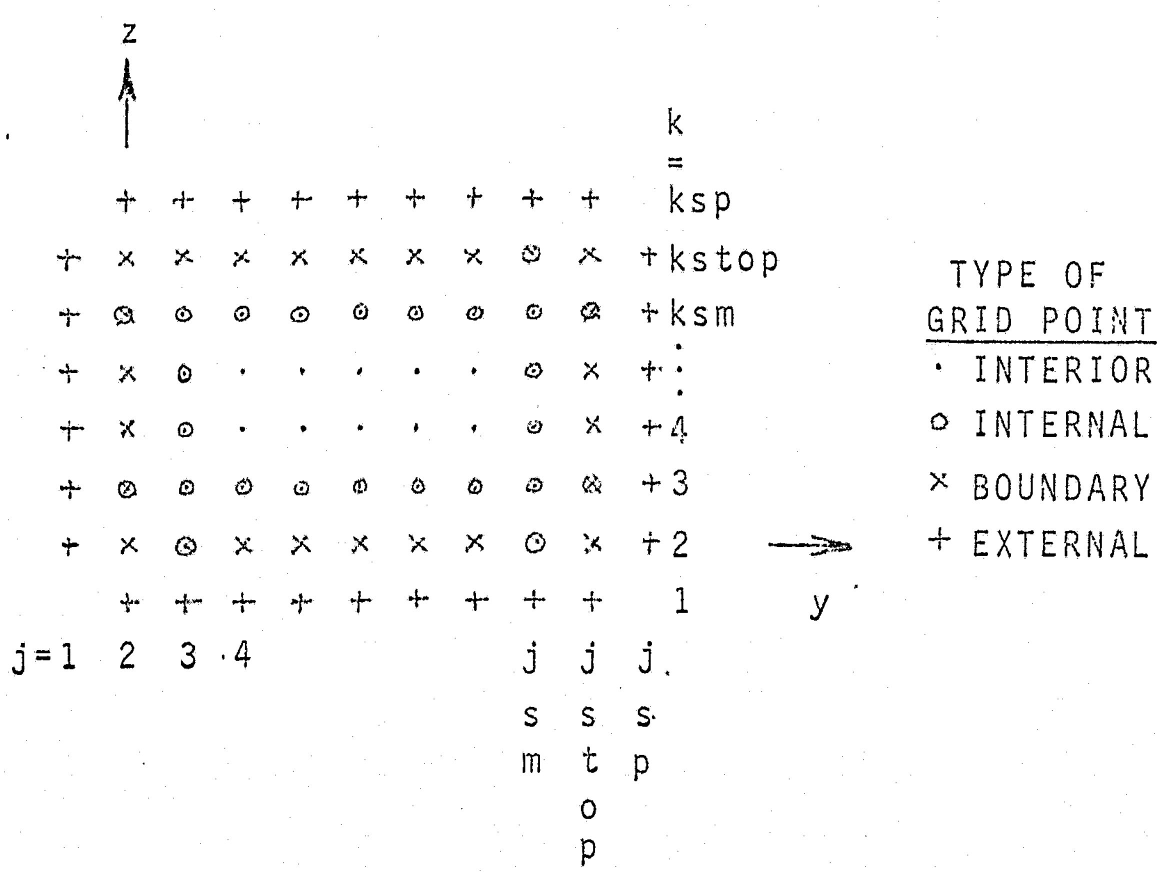


FIGURE 6. THE NUMERICAL GRID

jstop and kstop correspond to y_{max} and z_{max} respectively.

The solution will be carried out in going from one y-z plane at x to another at $x + \Delta x$. An iteration is carried out to predict the values of the variables at $x = x + \Delta x$ based on x derivative values at x. The results are averaged with those at x providing a new set of values for the variables at $x + \frac{\Delta x}{2}$ and consequently also a new set of x derivative values at $x + \frac{\Delta x}{2}$. This is accomplished by means of a Δx multiplier AG which assumes the value of 0.5 for the first iterate and 1.0 for the final calculation. A final set of values of the variables at $x + \Delta x$ can then be found. The dependence of a grid point value at $x + \Delta x$ $(i = 2)$ on those at x $(i = 1)$ or $x + \frac{\Delta x}{2}$ $(i = 2)$ is shown schematically in Figure (7).

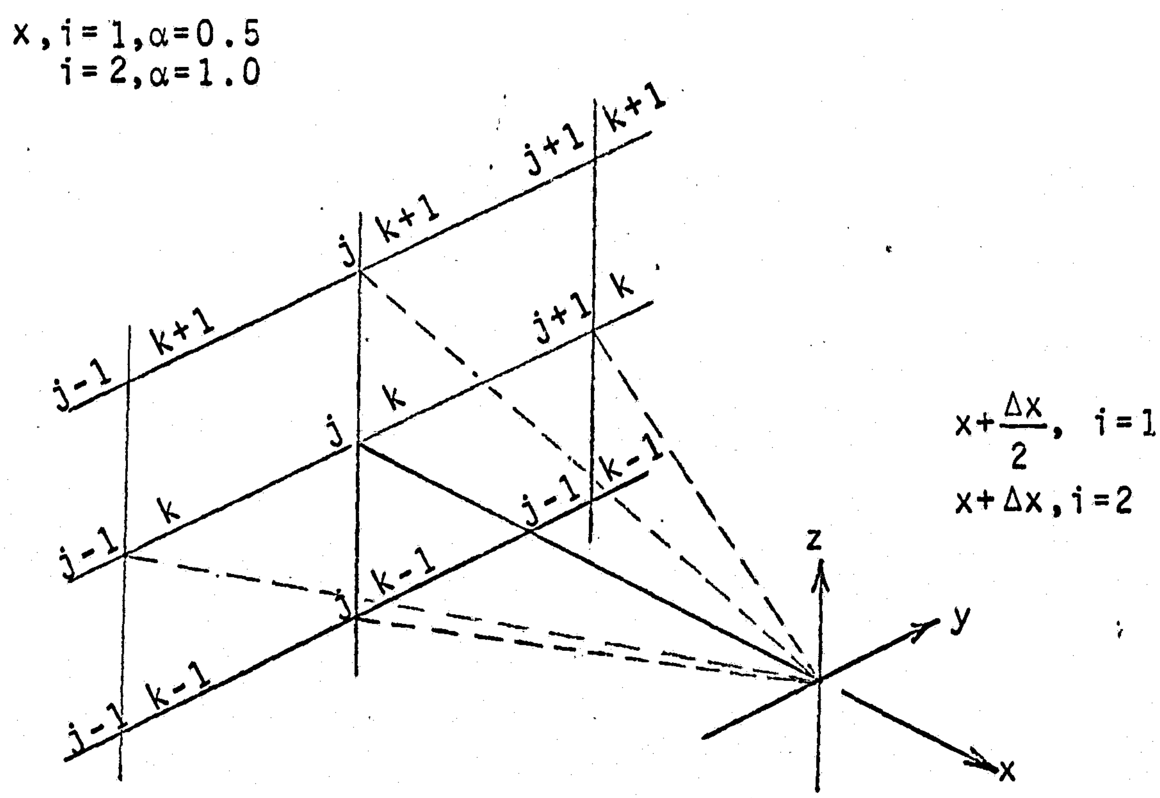


FIGURE 7. THE DEPENDENCE OF A GRID POINT ON GRID POINTS AT THE PREVIOUS STATION

V. DIFFERENCE FORM OF DIFFERENTIAL EQUATIONS

Let the quantity $Q(i)_j^k$ represent any of the variables used in the differential equations. The subscript i refers to the x-direction and has only two values 1 or 2, the subscript j, as a subscript, refers to the y-direction and the subscript k, as a subscript, to the z-direction. The value of 1 for i refers to the known values at station x and the value 2 refers to either the unknown values at x + Δx or the averaged value at $x + \frac{\Delta x}{2}$. A known value is thus written $Q(1)_j^k$. First and second order y- and z- derivatives are thus:

$$\left.\frac{\partial Q(i)}{\partial y}\right|_j^k = \frac{Q(i)_{j+1}^k - Q(i)_{j-1}^k}{2(\Delta y)} \tag{41}$$

$$\left.\frac{\partial Q(i)}{\partial z}\right|_j^k = \frac{Q(i)_j^{k+1} - Q(i)_j^{k-1}}{2(\Delta z)} \tag{42}$$

$$\left.\frac{\partial^2 Q(i)}{\partial z^2}\right|_j^k = \frac{Q(i)_{j+1}^k - 2Q(i)_j^k + Q(i)_{j-1}^k}{(\Delta y)^2} \tag{43}$$

$$\left.\frac{\partial^2 Q(i)}{\partial y^2}\right|_j^k = \frac{Q(i)_j^{k+1} - 2Q(i)_j^k + Q(i)_j^{k-1}}{(\Delta z)^2} \tag{44}$$

a value at $x + \frac{\Delta x}{2}$, or $x + \Delta x$, is found from a value at x according to

$$Q(2)_j^k = Q(1)_j^k + \left.\frac{\partial Q(i)}{\partial(x)}\right|_j^k (\Delta x)\ (AG) \tag{45}$$

where $\left.\frac{\partial Q(i)}{\partial x}\right|_j^k$ is evaluated from the differential Equations (30), (31) and (32) and i in this equation is 1 or 2 depending on whether the x-derivative was found at station x or at $x + \frac{\Delta x}{2}$, and AG is the averaging constant which has the value 0.5 for i = 1 and 1.0 for i = 2. As an example Equation (31) would appear as follows:

$$\left.\frac{\partial H(i)^k}{\partial x}\right|_j = \frac{1}{\rho(i)_j^k \quad u(i)_j^k} \left[\frac{H(i)_{j+1}^k - 2H(i)_j^k + H(i)_{j-1}^k}{(\Delta y)^2} + \frac{H(i)_j^{k+1} - 2H(i)_j^k + H(i)_j^{k-1}}{(\Delta z)^2}\right] \tag{46}$$

and

$$H(2)_j^k = H(1)_j^k + \left.\frac{\partial H(i)}{\partial x}\right|_j^k (\Delta x)\ (AG) \tag{47}$$

VI. SPECIFICATION OF THE BOUNDARY CONDITIONS

As discussed in Section III, walls and symmetry planes are equivalent when the boundary layer on the wall is neglected. At a symmetry plane boundary all variables being used have zero normal derivatives (except that v and w are assumed zero). This manifests itself by a simple reflection of the property at a plane of symmetry. Numerically this is accomplished by providing a set of grid points one mesh point beyond the actual boundary. The values of the variables at these outer or external grid points are equal to the values of the variables one mesh point inside the boundary or interior grid points.

With reference to Figure (6) this implies that for any variable Q the symmetry boundary condition specification becomes Q at 1, jsp or ksp equals Q at 3, jsm or ksm respectively, i.e.

$$Q_1^k = Q_3^k \tag{48}$$

$$Q_{jsp}^k = Q_{jsm}^k \tag{49}$$

$$Q_j^1 = Q_j^3 \tag{50}$$

and

$$Q_j^{ksp} = Q_j^{ksm} \tag{51}$$

For the plane of equivalence boundary conditions, planes AB or A'B' in Figure (4b), the boundary variables given by Equations (48) and (49) are specified instead by

$$Q_{jsp}^k = Q_3^k \tag{52}$$

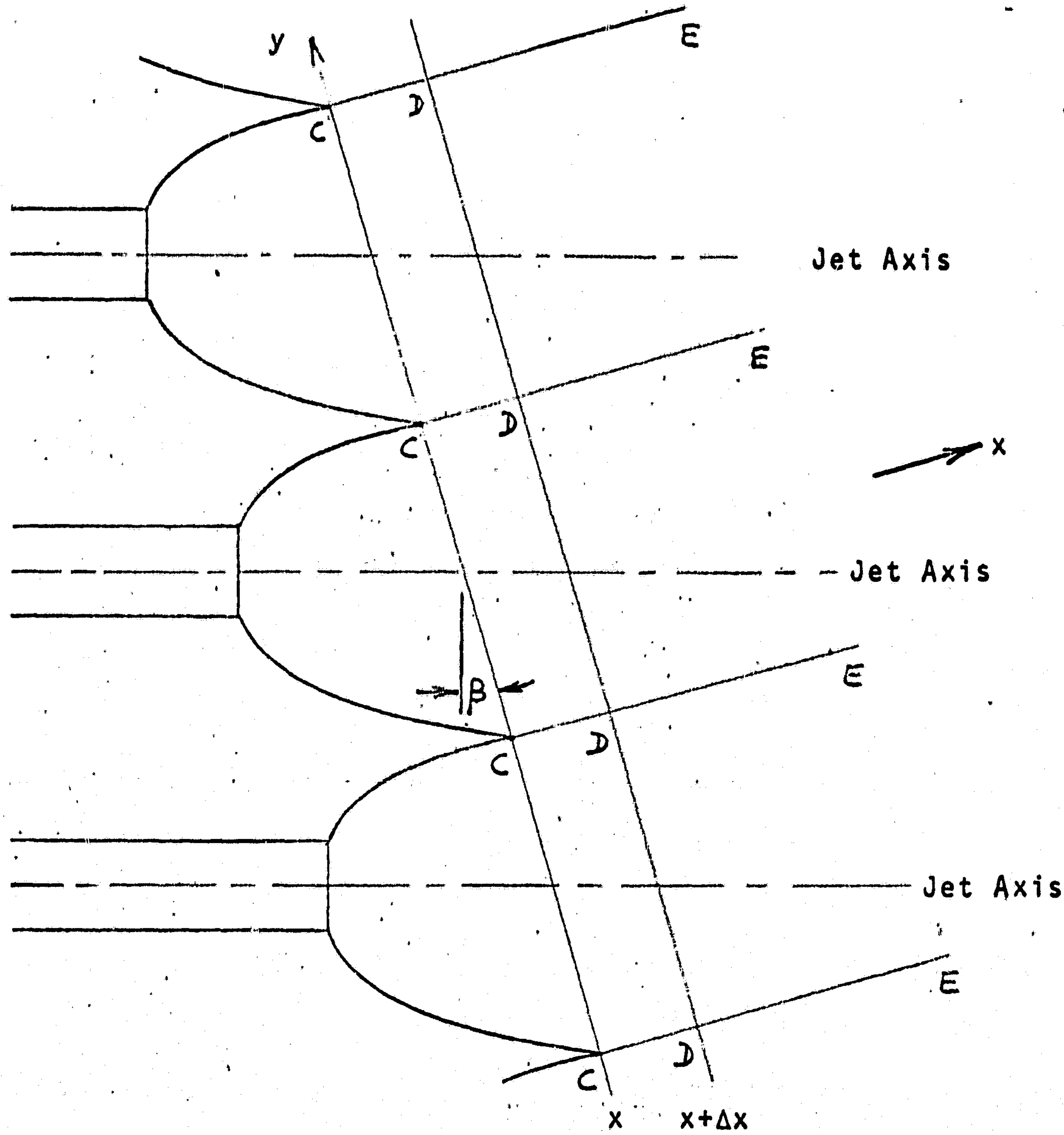
and

$$Q_1^k = Q_{jsm}^k \tag{53}$$

In addition, a consequence of this will also be the fact that

$$Q_2^k = Q_{jstop}^k \ . \tag{54}$$

Figure (8) portrays the solution geometry for the swept case. This plan view identifies CDE as planes of equivalence. It is noted again that area variation can take place only in the z direction.



CDE are Planes of Equivalence

CC and DD are Solution Planes

FIGURE 8. SOLUTION OF THREE DIMENSIONAL MIXING - JET EXITS AND PLANES OF CONSTANT PRESSURE AND DY WITH SWEEPBACK

VII. SOLUTION OF THE GOVERNING EQUATIONS

The final equations can now be solved for several distinct cases:

(1) Prescribed pressure variation.

(2) Prescribed area variation.

Both of these can be divided into two subcategories:

(a) Pressure and Δz constant at constant x, where x is the direction parallel to the jet axes. This case includes jets and sets of staggered jets where complete symmetry can be found.

(b) Pressure and Δz constant in planes of constant sweepback, where x is normal to a swept plane. This case includes jets and sets of jets where symmetry and equivalence planes exist.

These cases are different from one another in only two respects. One of the differences lies in the fact that for case (a) calculations proceed along the axial direction of the jets while in the other case calculations proceed normal to planes of constant sweepback. This leads to a difference in the boundary conditions. The other difference, which has also been pointed out, is that an additional velocity component, namely v in the y direction, must be introduced. This component is calculated simply by Equation (16) and makes use of the assumption that the jet streamlines are parallel to the jet axes.

The two separate cases of prescribed pressure and prescribed area variation in the x direction are handled slightly differently. For the prescribed pressure distribution, the pressure gradient $\frac{\partial p}{\partial x}$ is known and the correct area to conserve the mass flow must be found. This is done by iterating on the area so that Equation (14) holds. For the prescribed area variation the dz is known at each x but $\frac{\partial p}{\partial x}$ is unknown. An iteration is carried out on $\frac{\partial p}{\partial x}$ until Equation (14) is satisfied.

Thus, it can be said that $\frac{\partial p}{\partial x}$ is known. Then since all properties are known at station x, all orders of y- and z- derivatives at x can be calculated. All the derivatives for solving Equations (30), (31) and (32) are evaluated and these equations are then solved for $\frac{\partial u}{\partial x}$ $\frac{\partial H}{\partial x}$ $\frac{\partial \tilde{\alpha}_{O_2}}{\partial x}$ and $\frac{\partial \alpha_{O_2}}{\partial x}$. These x derivatives give rise to new values of u, H, $\tilde{\alpha}_{O_2}$ and α_{O_2} at $x + \Delta x$ (c.f. Equation (45)). Equations (34) through (38) (or Equation (37) replaced by Equations (39) and (40) for frozen flow) provide values for all α_i and this allows the temperature T to be evaluated from Equations (11) and (12) because v and w are determined from u and the density ρ from Equation (5). The first iteration in essence results in a process whereby the values of the variables at $x + \Delta x$ are averaged with those at x. The calculation is then repeated using the same $\frac{\partial p}{\partial x}$ as the first time and x derivatives at $x + \frac{\Delta x}{2}$ which now provide new values of the variables at station $x + \Delta x$. These are taken to be the temporary final values, subject to satisfying conservation of mass flow. As outlined earlier, several cases must be considered with regard to conservation of mass depending on the type of problem being analyzed.

When the mass flow is conserved, the answers found become the final results at $x + \Delta x$. It now remains for the boundary conditions to be applied. This was discussed in Section VI. One additional comment regarding the actual computer calculations is appropriate at this point. To pick a particular parameter, total enthalpy is conserved locally only to within the numerical accuracy of the solution. However, on the average, the overall total enthalpy should also be conserved. Total enthalpy thus may be corrected at each station by adding to each local value of total enthalpy a constant ΔH, i.e.,

$$H' = H + \Delta H \tag{55}$$

where

$$\Delta H = \frac{\overline{H}_s - \overline{H}}{\dot{m}_s} . \tag{56}$$

$\overline{H}$ is the local integrated total enthalpy flux, $\overline{H}_s$ is the reference value of this flux, and $\dot{m}_s$ is the reference mass flow. However, this may lead to distortions in maximum or minimum values of H and/or in the temperature T, ρ etc. It may be wise to accept a small error in the conservation to retain better diffusion of the individual profiles.

The program solution of the problem is discussed in Reference (1), which is a program manual. Detailed flow charts, sample runs and input specifications are presented there. The computer program "JETMIX" (LAR-10792) is available from COSMIC (Computer Software Management and Information Center) at the University of Georgia. Inquiries concerning the program should be addressed to:

COSMIC
Barrow Hall
University of Georgia
Athens, Georgia 30601

VIII. RESULTS OF SAMPLE CASE

Results of computer program calculations for one sample case have been obtained. This case was constant area for jet exit without sweepback with flame sheet equilibrium chemistry. The initial y - z area had an aspect ratio of 1. The solution domain was that pictured in Figure (3) part (a). The initial data was obtained from output of an axisymmetric jet program solution. This constant area solution gave the expected result of pressure rise due to heat addition. Figure (9) shows the variation of pressure with x. The slight wiggle in the curve is a result of the finite tolerance on the mass flow conservation. For this computation the viscosity was assumed to be constant with x, and roughly equal to a product of the jet axis density and velocity and the jet radius modified by a numerical coefficient (i.e., $\mu = K \rho_j u_j r_j$). A value of $\mu = 0.01$ was used (the units of viscosity are Kgm/meter/sec).

The diffusion process is clearly evidenced by the results shown in Figures (10) through (14). Figure (10) shows this result for total enthalpy, H, Figure (11) for the x component of velocity, u, Figure (12) for the elemental mass fraction of oxygen, $\tilde{\alpha}_{O_2}$, Figure (13) for the temperature T, and Figure (14) for the density ρ. The diffusion effect on the points near the axis are more predominant since the periphery carries more

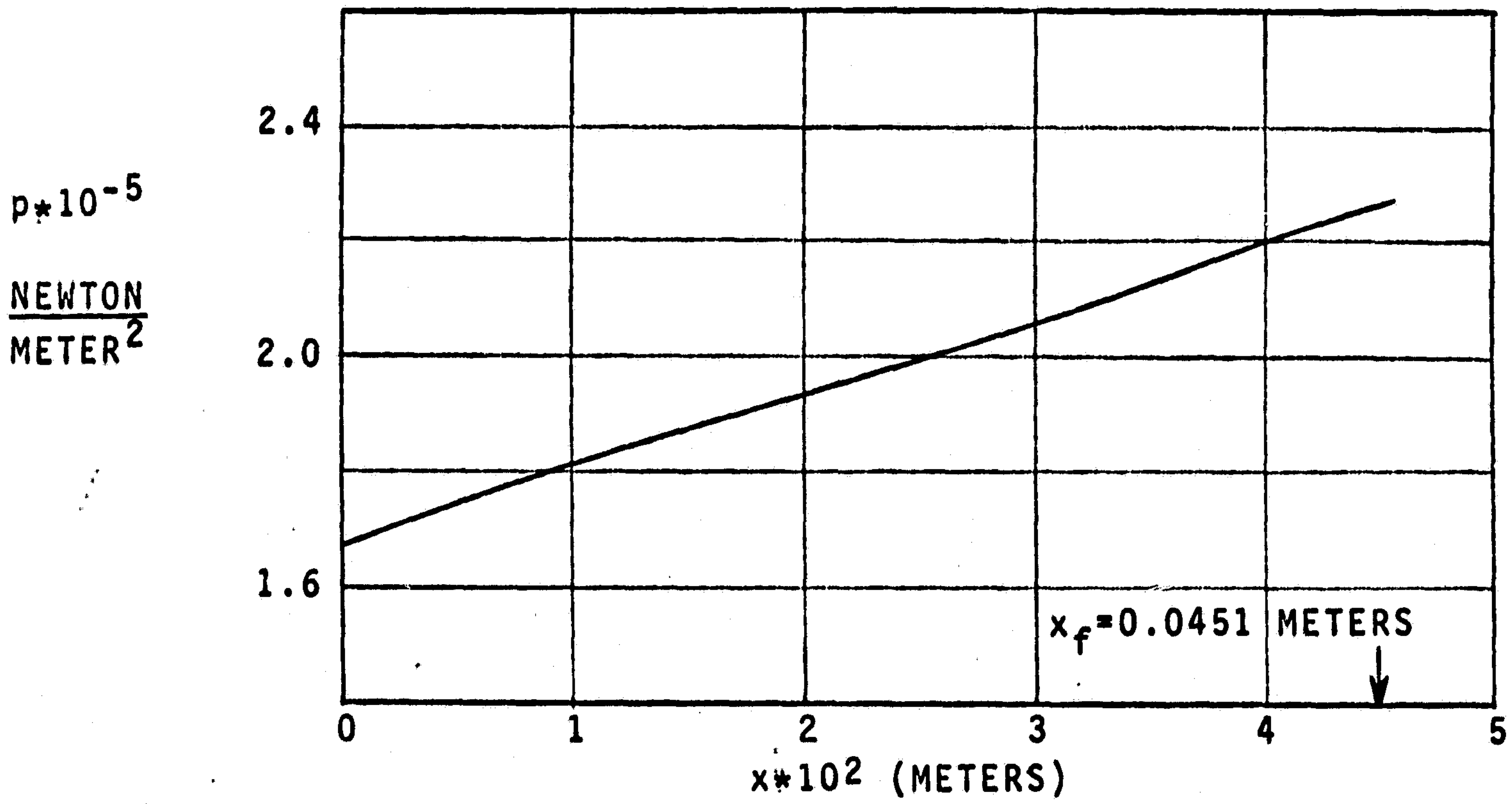


FIGURE 9. PRESSURE VARIATION IN X DIRECTION

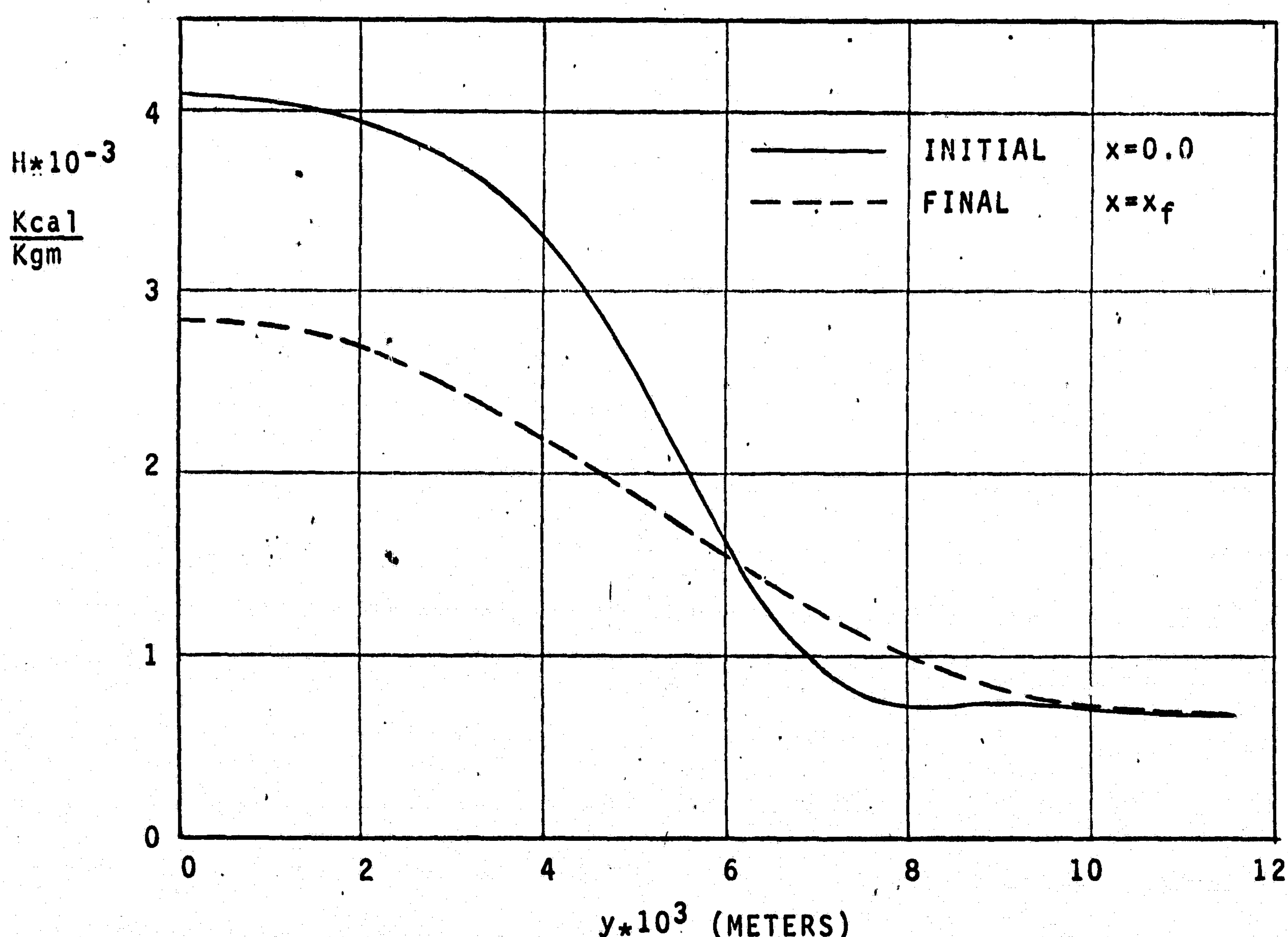


FIGURE 10. TOTAL ENTHALPY DISTRIBUTION

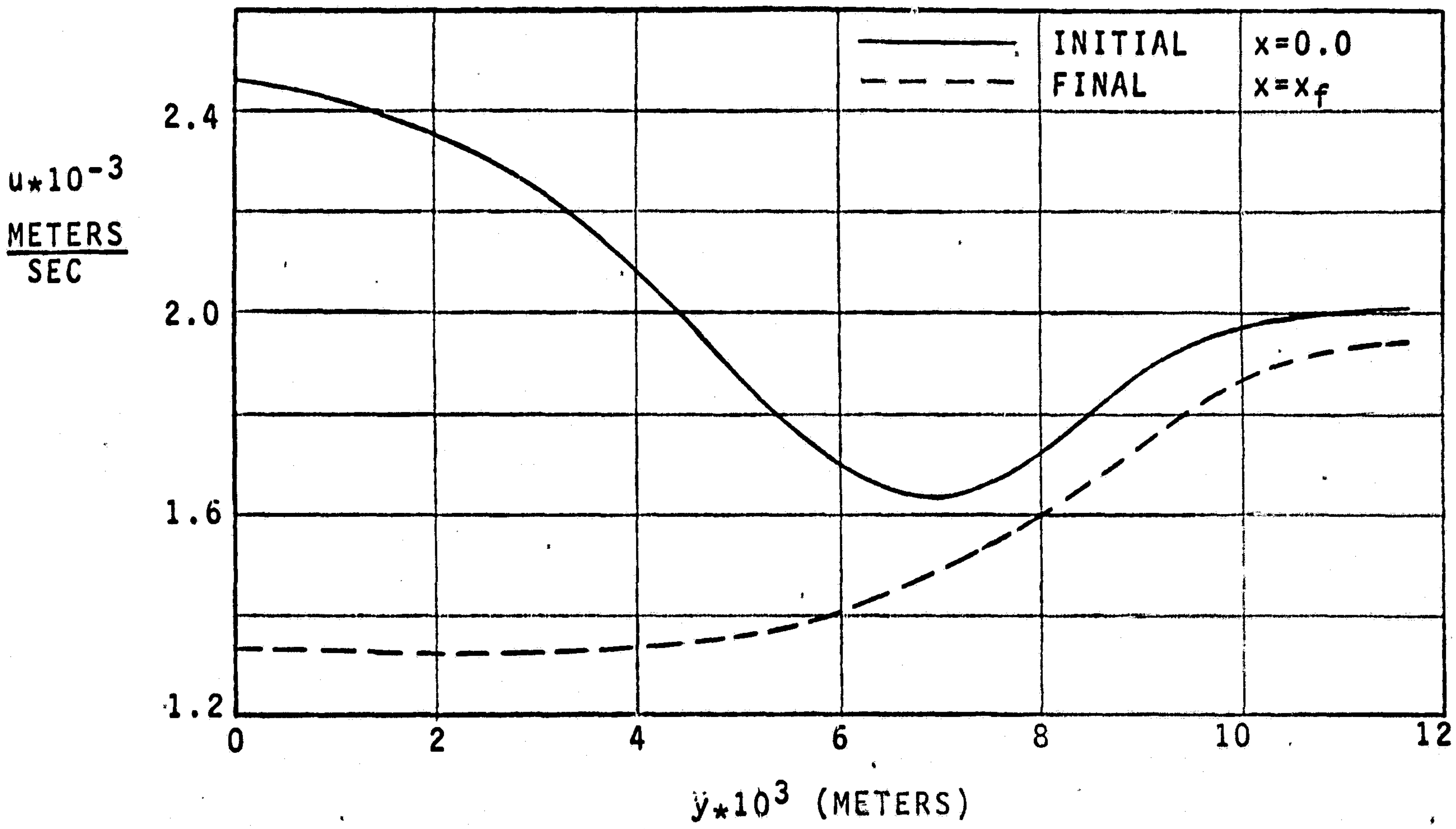


FIGURE 11. X-DIRECTION VELOCITY DISTRIBUTION

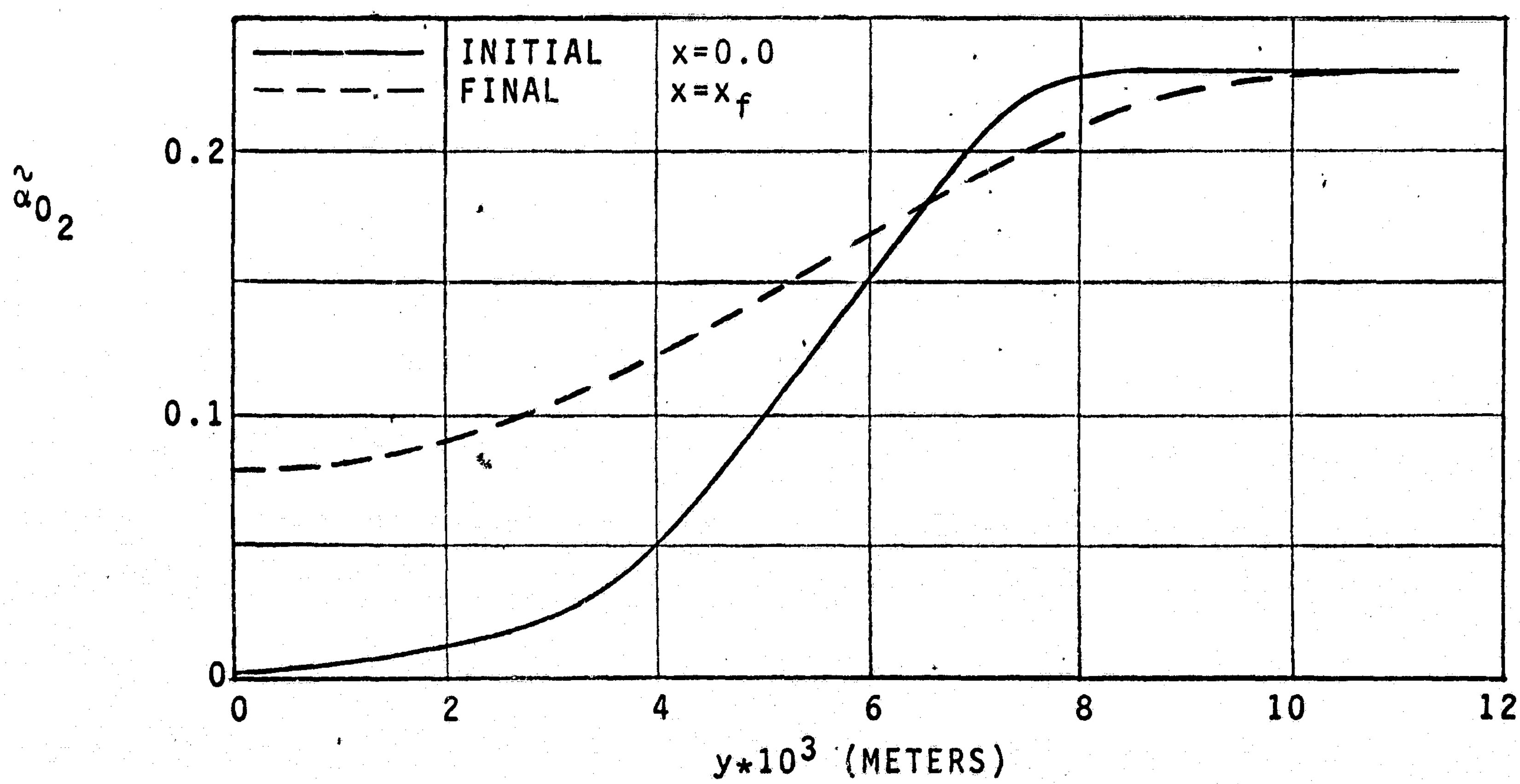


FIGURE 12. ELEMENTAL MASS FRACTION OF OXYGEN DISTRIBUTION

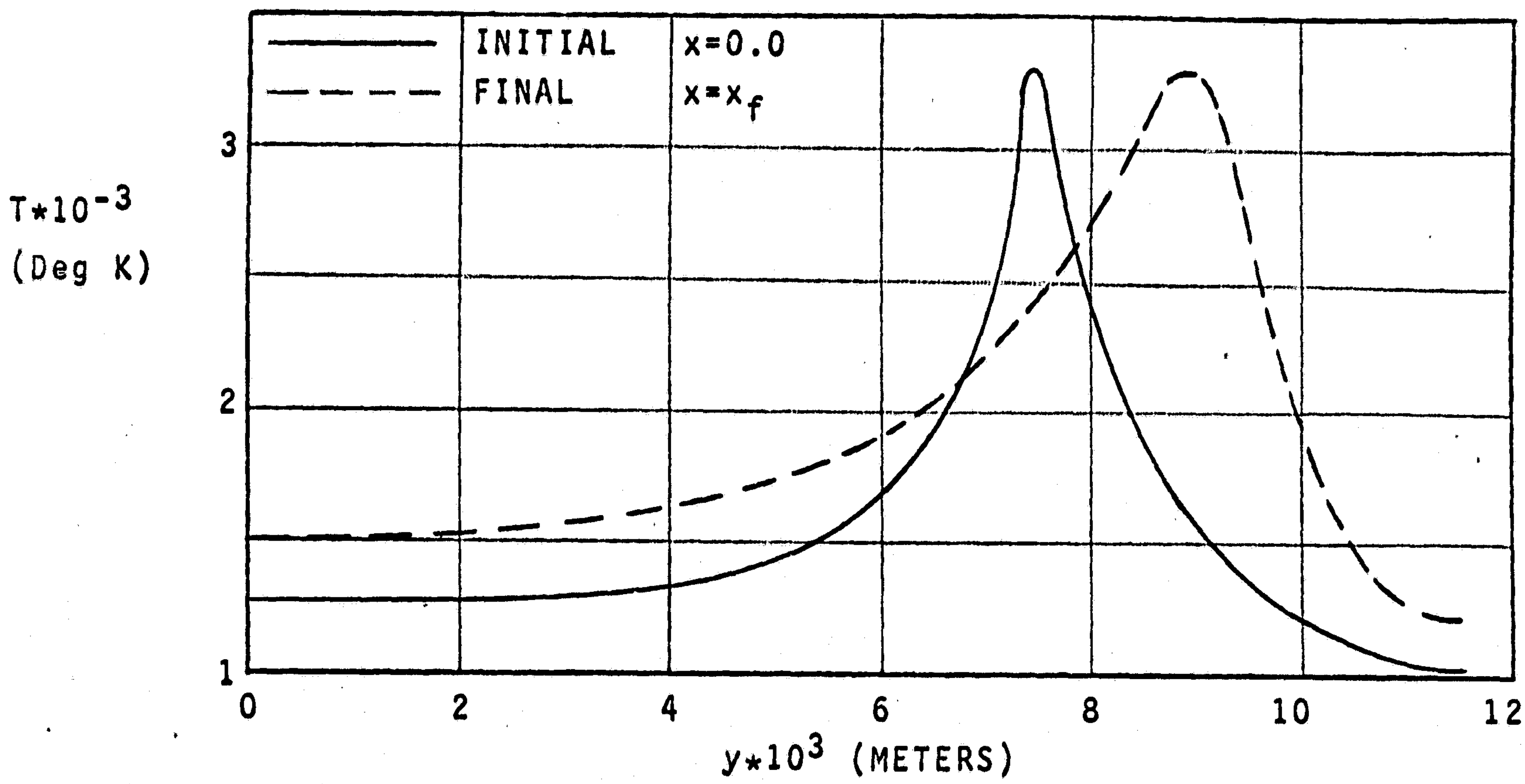


FIGURE 13. TEMPERATURE DISTRIBUTION

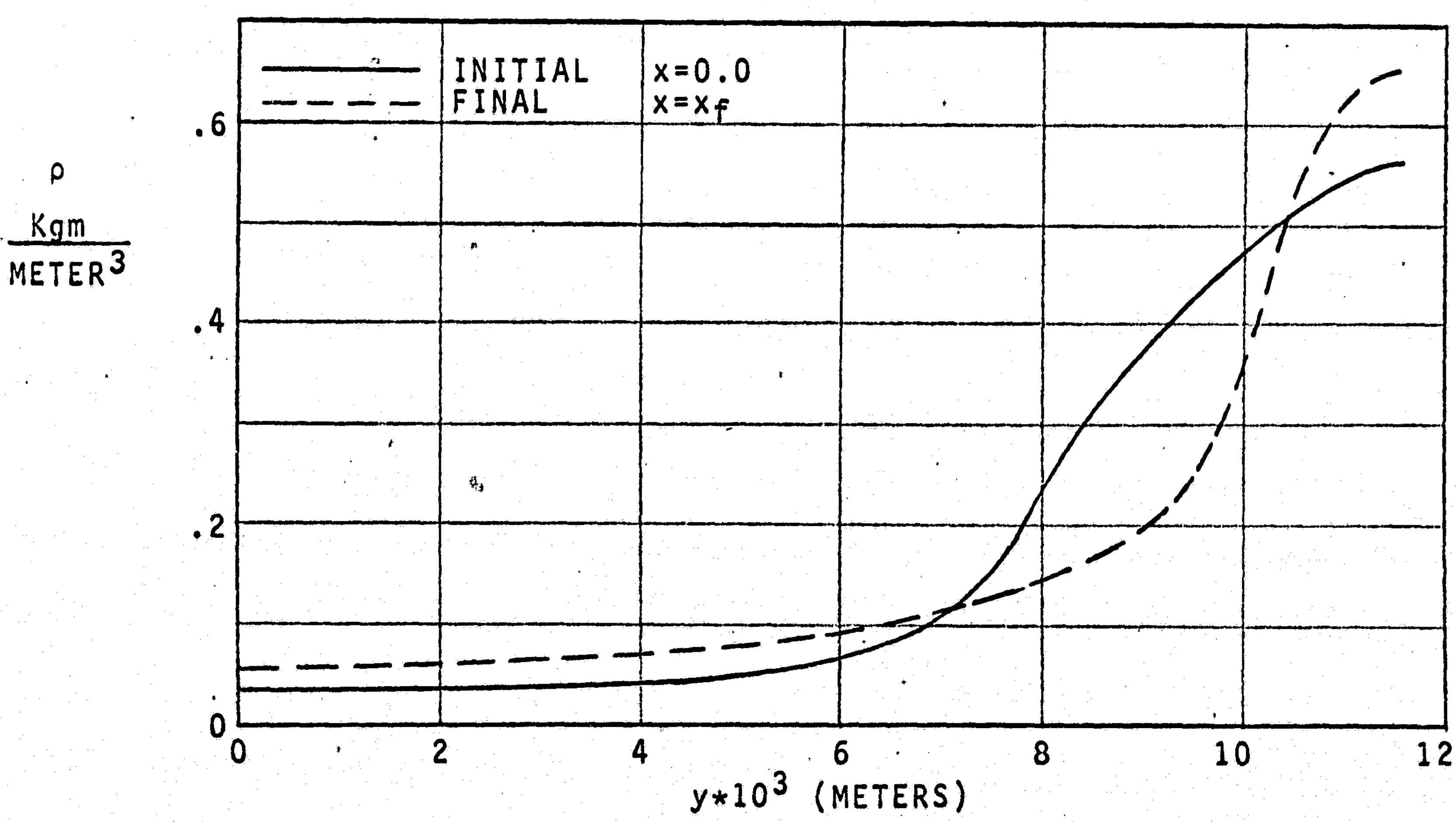


FIGURE 14. DENSITY DISTRIBUTION

"inertia" than the axis. The overall decrease in velocity shown in Figure (11) is a direct consequence of the increase in pressure. Evidence of the diffusion of species is supported by the temperature distributions of Figure (13). The outward motion of the temperature peak reflects the outward motion of the point where the fuel air ratio, ϕ, equals unity. If the calculation were continued, the peak would ultimately reach the boundary at $y = .01156715$. Subsequent to this there would be an increase of the low temperature and a decrease of the high temperature with a tendency for an overall uniform distribution. This trend would also be found in all other variables.

REFERENCES

1. Alzner, E., "Program Manual for Three Dimensional Mixing of Jets," ATL TM 151, July 1970.