SPERRY RAND

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STRAPDOWN CALIBRATION AND ALIGNMENT STUDY

VOLUME 3

LABORATORY PROCEDURES MANUAL

Prepared for

GUIDANCE LABORATORY
ELECTRONICS RESEARCH CENTER
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ABSTRACT

This is Volume 3 of three volumes which report the results of a strapdown calibration and alignment study performed by the Univac Federal Systems Division for the Guidance Laboratory of NASA/ERC.

This study develops techniques to accomplish laboratory calibration and alignment of a strapdown inertial sensing unit (ISU) being configured by NASA/ERC. Calibration is accomplished by measuring specific input environments and using the relationship of known kinematic input to sensor outputs, to determine the constants of the sensor models. The environments used consist of inputs from the earth angular rate, the normal reaction force of gravity, and the angular rotation imposed by a test fixture in some cases. Techniques are also developed to accomplish alignment by three methods. First, Mirror Alignment employs autocollimators to measure the earth orientation of the normals to two mirrors mounted on the ISU. Second, Level Alignment uses an autocollimator to measure the azimuth of the normal to one ISU mirror and accelerometer measurements to determine the orientation of local vertical with respect to the body axes. Third, Gyrocompass Alignment determines earth alignment of the ISU by gyro and accelerometer measurement of the earth rate and gravity normal force vectors.

The three volumes of this study are composed as follows:

- Volume 1 – Development Document. This volume contains the detailed development of the calibration and alignment techniques. The development is presented as a rigorous systems engineering task and a step by step development of specific solutions is presented.
- Volume 2 – Procedural and Parametric Trade-off Analyses Document. This volume contains the detailed trade-off studies supporting the developments given in Volume 1.
- Volume 3 – Laboratory Procedures Manual. In Volume 3 the implementation of the selected procedures is presented. The laboratory procedures are presented by use of both detailed step-by-step check sheets and schematic representations of the laboratory depicting the entire process at each major step in the procedure. The equations to be programmed in the implementation of the procedures are contained in this volume.

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GLOSSARY

As an aid to understanding the symbolism, we present the following rules of notation:

- Wherever possible symbols will be used which suggest the name of the parameter involved.
- Lower case subscripts are used almost exclusively for indexing over several items, of the same kind. Examples are the indexes used to identify the three gyros, the three accelerometers, the two pulse trains of each accelerometer, the two clock scale factors, etc.
- Lowercase superscripts are used to index over different positions.
- Uppercase superscripts and subscripts will be used to distinguish between parameters of the same kind. For example, T is used to identify a transformation matrix. Lettered superscripts such as BE in T^{BE} identify the particular transformation.
- An underline will identify a vector.
- Unit vectors are used to identify lines in space such as instrument axes and the axes of all frames of reference.
- Components of any vector along with any axis is indicated by a dot product of that vector with the unit vector along the axis of interest.
- The Greek sigma (Σ) will be used for summations. Where the limits of summation are clear from the context, they will not be indicated with the symbol.
- The Greek Δ is always used to indicate a difference.
- $S\,\phi$ and $C\,\phi$ are sometimes used to identify the sine and cosine of the angle ϕ.
- A triple line symbol ($\equiv$) will be used for definitions.
- A superior "$\sim$" denotes a prior estimate of the quantity.
- A superior "$\wedge$" denotes an estimate of the quantity from the estimation routine.

$\underline{a}$	Applied acceleration vector.
$(\underline{A}_i \cdot \underline{B}_j)$	Elements of $(Q^A)^{-1}$.
$\underline{A}_i$	Unit vector directed along the input axis of the ith accelerometer $i = 1, 2, 3$.
$\underline{b}$	A vector determined by the Alignment Parameter Evaluation Procedure and input to the Estimation Routine.
$\underline{B}_i$	Unit vector directed along the ith Body Axis $i = 1, 2, 3$.
B_I, B_O, B_S	Gyro unbalance coefficients.
$C_{II}, C_{SS}, C_{IS}, C_{IO}, C_{OS}$	Gyro Compliance Coefficients.
Counters	The six frequency counters used as data collection devices during calibration.
D_0	Accelerometer bias.
D_1	Accelerometer scale factor.
D_2	Accelerometer second order coefficient.
D_3	Accelerometer third order coefficient.
$\underline{E}$	Unit vector directed East $(\underline{E}_2)$.
$\underline{E}_i$	Unit vector directed along the ith Earth Axis.
Eq	Quantization error.
f_1 , f_2	Frequencies of accelerometer strings 1 and 2, in zero crossings per second.
$\underline{F}_i$	A triad of orthogonal unit vectors attached to the base of the table.
$\underline{G}_i$	Unit vector directed along the ith input axis of the gyro.
$(\underline{G}_i \cdot \underline{B}_j)$	Elements of $(Q^G)^{-1}$.
$\underline{g}$	The vector directed up that represents the normal force to counteract gravity in a static orientation. Corresponding to popular convention, this is referred to as the "gravity vector".
I/O	Input/Output.

$\underline{I}_i$	Triad of orthogonal unit vectors attached to the inner axis of test table.
IEU	Interface Electronics Unit – system interface device for the laboratory computer.
ISU	Inertial Sensing Unit.
J	Gyro angular rate coefficient.
K	Number of samples of accelerometer and gyro data taken in Alignment.
m	Position index used in calibration (superscript).
M	Matrix generated by Alignment Parameter Evaluation and used by Alignment Estimation Routine.
$\underline{M}_i$	Unit normal to ith mirror.
$\underline{N}$	Unit vector directed North ($\underline{E}_3$).
N_1, N_2	Count of output pulses from strings 1 and 2 of accelerometer.
n_A	Instrument noise in accelerometer.
n_G	Instrument noise in gyro.
Σn^{ϕ}	Count of output pulses from strings 1 and 2 of accelerometer.
Σn_1^T	Count of timing pulses from master oscillator to frequency counters.
Σn_2^T	Count of timing pulses from master oscillator to IEU.
$\underline{O}$	Unit vector directed along the output axis of gyro.
$\underline{O}_i$	Triad of orthogonal unit vectors attached to the outer axis of the table.
$\underline{P}$	Unit vector in the direction of the projection of $\underline{M}_i$ in the plane formed by $\underline{E}$ and $\underline{N}$.
P_k^A	Defined on Chart 4-12 of the Development Document.
P_k^G	Defined on Chart 4-4 of the Development Document.
Q^A	The transformation from accelerometer input axes to body axes.
Q^G	The transformation from gyro input axes to body axes.

Q_{II}, Q_{IS}	Gyro dynamic coupling coefficients.
$\underline{r}$	Position vector.
R	Gyro bias.
$\underline{R}_i$	Triad of orthogonal unit vectors attached to rotary axis of table.
Resolver	Angular resolvers on each axis of the test table.
$\underline{S}_i$	Unit vector directed along the ith gyro spin axis.
$\underline{S}^{\phi}$	Scale factor associated with pulsed output from test table rotary axis.
S_1^T	Scale factor associated with timing pulses accumulated by the frequency counters.
S_2^T	Scale factor associated with timing pulses to the IEU.
t	Time.
T	In alignment, the determined alignment matrix to transform from body to earth axes. T is equivalent to T^{BE}.
T^{BI}	Transform from ISU body axes to inner axis frame.
T^{BRm}	Transform from ISU Body Frame Axes to Rotary Axis Frame in the mth orientation.
$\underline{T}_i$	Triad of orthogonal unit vectors attached to the trunnion axis of the test table.
$\underline{U}$	Unit vector directed up (E_1).
$\underline{V}$	Velocity vector.
$\underline{W}$	Unit vector directed along $\underline{\omega}^E$.
X-Y	Dual input on frequency counter that will difference two pulse trains for comparison with a third input (Z).
Z	Input on frequency counter for pulse train.
α_i	The azimuth angle of the normal to the ith mirror.
$(\Sigma\gamma)_{ij}$	Pulsed output from the jth string of the ith accelerometer.
$(\Sigma\delta)_i$	Pulsed output of the ith gyro.
$\Delta\Phi$	Gyro scale factor.

ϵ_C	The clock quantization error.
ϵ_T	In instantaneous alignment estimation techniques, this symbol represents the length of time after completion of the last measurement to the time at which the prediction is made.
θ_i	The zenith angle of the normal to the ith mirror.
λ	Local colatitude.
σ_g	The estimated rms error in the magnitude of $\underline{g}$.
σ_θ	The estimated rms error in the direction of "up".
ϕ_1^m	Angular displacement about the trunnion axis of the test table for calibration position m.
ϕ_2^m	Angular displacement about the rotary axis of the test table for calibration position m.
ϕ_3^m	Angular displacement about the outer axis of the test table for calibration position m.
ϕ_4^m	Angular displacement about the inner axis of the test table for calibration position m.
$\phi_n(t)$	Covariance function of accelerometer noise used in Alignment Parameter Evaluation.
$\phi_\alpha(t)$	Covariance function of translational acceleration noise used in Alignment Parameter Evaluation.
$\phi(t)$	Covariance function of rotational noise used in Alignment Parameter Evaluation.
$\underline{\omega}$	Angular velocity vector.
$\underline{\omega}^T$	Angular velocity of the test table rotary axis.
$\underline{\omega}^E$	Earth rotation vector.
$\underline{\omega}^E \cos \lambda$	Component of earth rotation vector along the vertical.
$\underline{\omega}^E \sin \lambda$	Component of earth rotation vector along north.

INTRODUCTION

INTRODUCTION

This document, in conjunction with two other volumes, describes the achievements of a six-month study conducted for the:

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by the.

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The purpose of the study is to develop techniques and outline procedures for the laboratory calibration and alignment of a strapdown inertial sensor unit. The Development Document, Volume 1, presents a detailed analysis of the calibration and alignment problem and develops a specific solution. The nucleus of the study output is the contents of Volume 1. The Procedural and Parametric Trade-off Analyses, Volume 2, is a set of addendums which serve to justify decisions made and conclusions reached in the development of specific calibration and alignment techniques. This document, Volume 3, describes specific procedures for an operational implementation of the solutions obtained in Volume 1. Volume 3 is an extension of the results of Volume 1 into an operational laboratory situation, and is intended to be a handbook for use in the laboratory.

The complete study was performed for NASA/ERC as part of their evaluation of advanced guidance and navigation concepts associated with the use of a strapdown inertial sensing unit. Concurrently with defining the ISU, NASA/ERC is developing a laboratory facility and the requisite procedures to analyze and demonstrate the ISU. Calibration and alignment are essential elements of these procedures.

As noted above, this document is intended to be a handbook for use in the laboratory. The calibration and alignment procedures are defined in this document in as much detail as is possible at the present state of laboratory design. After the ERC laboratory design has been finalized, some of the detailed procedures listings contained herein should be updated to reflect elements particular to the detailed design of the laboratory.

The document is divided into four parts. Part I describes the activities that must be performed prior to calibration and alignment. Part II describes the calibration procedures. Part III contains procedures to accomplish alignment of the ISU. Part IV contains descriptive material on the ISU and laboratory configurations along with duplicates of the forms to be used in the procedures.

The procedures described in Part I, Preliminary Activities, are the laboratory turn-on procedures, the system survey procedures, and the alignment parameter evaluation procedures. Section 1, Turn-on Procedures, is reserved for procedures to turn on the laboratory equipment at the beginning of each day. The System Survey Procedures, Section 2, determine the orientation of the ISU on the test table, and the test table settings required to orient the ISU in calibration. Section 3 describes procedures to generate parameter data required in certain alignment techniques.

Part II contains detailed procedures to accomplish calibration. Sections 1 through 4 contain procedure listings for obtaining measurement data and performing the computations. Section 1 describes the calibration of gyro angular velocity sensitive terms. Section 2 describes the calibration of gyro acceleration sensitive terms. Section 3 covers the complete calibration of accelerometers; while Section 4 contains a listing of the calibration computations in sufficient detail to be used as the computer program definition. In Section 5, Fundamental Modes, the procedures are presented schematically through the use of flow diagrams. Section 6 contains material describing the predetermination of information which is supplied to work sheets prior to calibration.

In Part III, three methods of accomplishing alignment are presented. The Mirror alignment presented in Section 1 uses autocollimators to locate the normals of the ISU mirrors from which the alignment matrix is determined. Section 2 describes the alignment which uses accelerometer measurements and mirror azimuth in the computation of the alignment matrix. Gyrocompass alignment, Section 3, uses accelerometer and gyro measurements of the gravity and earth rotational rate vectors, respectively to determine the alignment matrix.

Part IV, Diagrams and Forms, contains charts and illustrations from the Development Document that may be useful for the laboratory personnel in obtaining an understanding of procedures or explaining the procedures to laboratory visitors.

——//——

The following discussion will help the reader of the manual to understand its structure, and the laboratory user to take maximum advantage of its organization. The procedures presented in this manual were developed so that they could be accomplished in an orderly and straightforward manner. The procedures described in Parts I, II and III can be considered as a set of experiments – some rather short – and others quite complex. There are four types of material presented in the discussion of each experiment:

1. Introductory Remarks
2. Detailed Procedures List
3. Computations
4. Fundamental Modes.

The introduction relates the particular experiment to the overall problem.

The detailed procedures list describes steps to be accomplished in the experiment under discussion. These steps are listed on a form called the Laboratory Procedures Sheet. There is one such form for each experiment, and they are intended to be used as a check sheet during the procedures. Additional forms were designed to allow recording of both intermediate data and results during the experiments. (Blank copies of the various sheets are available in Part IV for duplication.)

The computations are presented in sufficient detail to allow the computations sections to be used in specifying the computer program.

The fundamental modes discussion utilizes laboratory flow diagrams (see Part IV, Section 1) to pictorially represent the steps performed in the procedures. These sections should be referred to by all readers who are not familiar with the procedures, before he attempts to conduct any experiment. The laboratory flow diagrams are intended to keep the user of the manual oriented with respect to the "big picture" of what is going on at each step.

PART I
PRELIMINARY ACTIVITIES

LABORATORY PROCEDURES MANUAL
PART I
PRELIMINARY ACTIVITIES

PART I
PRELIMINARY ACTIVITIES

This part contains instructions for performing the activities which are required before calibration or alignment can be performed. In Section 1, the turn-on procedures are delineated. Section 2 contains the system survey procedures, which develop information that is a prerequisite to calibration. Section 3 lists procedures to generate parameters used in alignment.

PART I
SECTION 1
TURN ON

The laboratory turn-on procedures include prepower checks, safety checks, and turn-on verification procedures. The specifics of these procedures are contained in Chart I-1.

CHART I-1

LABORATORY PROCEDURES		
LABORATORY TURN-ON PROCEDURES		
INSTRUCTIONS	✓	**COMMENTS**
		Note: This sheet will be completed by laboratory personnel after the laboratory design has been finalized.

PART I
SECTION 2
SYSTEM SURVEY

After the ISU has been bolted to the test table, a system survey is performed for the purposes of determining

- The orientation of the ISU with respect to the inner axis of the test table
- The test table resolver settings to be used during calibration to achieve the desired ISU orientations (with respect to the earth and the test table rotary axis)
- The value of the T^{BR} matrix (which transforms from the ISU body axes frame to a frame fixed to the test table rotary axis) for all calibration positions.

The operator should also verify that current estimates of the magnitude of gravity, earth rate, and latitude are available. The procedures, computations, and laboratory flow diagrams for all of these tasks are contained in I-2.1, I-2.2, and I-2.3, respectively.

I-2.1 System Survey Procedures

The survey procedures to be accomplished are listed in Chart I-2. The Survey Results Sheet (Chart I-3) is used during the survey procedure to record the results which will later be used in the calibration procedures. The matrix T^{BI} (the transform from the ISU body frame to a frame fixed to the inner axis of test table) is not explicitly required in subsequent procedures, but is included on the form for information. A master copy of Chart I-3 is provided in Part IV, which can be duplicated for use in System Survey experiments.

The reader may refer to Section I-2.3 for the laboratory flow diagrams which schematically illustrate the procedures outlined in this section.

CHART I-2

LABORATORY PROCEDURES

SYSTEM SURVEY

INSTRUCTIONS	✓	COMMENTS
1) Verify turn-on procedures completed.		1) See Part I, Section 1 for turn-on procedures.
2) Align autocollimators with mirrors.		2) Accomplished by trial and error.
3) Obtain from survey current estimates of g , ω^E , and λ and record on Survey Results Sheet.		
4) Record test table resolver settings $(\phi_1^o, \phi_2^o, \phi_3^o, \phi_4^o)$ on Survey Results Sheet.		
5) Record azimuth and zenith angles $(\alpha_1, \alpha_2, \theta_1, \theta_2)$ from autocollimators on Survey Results Sheet.		
6) Enter computer with System Survey Program.		
7) Enter data from step 4 and 5 into computer.		
8) Initiate computation of T^{BI}, ϕ_3^m, ϕ_4^m, T_{12}^{BRm}, T_{22}^{BRm}, T_{32}^{BRm}.		8) m=1, 3, 5, 13, 14, 15.
9) Record data on Survey Results Sheet and identify T^{BRm} output.		6 to 9) Computations may be accomplished manually.

CHART I-3

SURVEY RESULTS SHEET

Identification:

Date:

Operator:

Current Laboratory Survey Information

$|g| =$ ____________

$\omega^E \sin\lambda =$ ____________

$\omega^E \cos\lambda =$ ____________

AUTOCOLLIMATOR READINGS

Position 0 Readings

$\phi_1^0 =$ ____________

$\phi_2^0 =$ ____________

$\phi_3^0 =$ ____________

$\phi_4^0 =$ ____________

Calculated Results

$$T^{BI} = \begin{pmatrix} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{pmatrix}$$

Calibration Positions

m	ϕ_3^m	ϕ_4^m
1		
3		
5		
13		
14		
15		

m	T_{12}^{BRm}	T_{22}^{BRm}	T_{32}^{BRm}
1			
3			
5			
13			
14			
15			

I-2.2 System Survey Computations

The computations to be performed during System Survey are shown on Charts I-4, I-5, and I-6. These charts define the program for these computations.

Chart I-4 lists the computations which derive the fixed transform from the ISU body frame to the test table inner axis frame. Inputs to this computation are autocollimator readouts, which describe the orientation of the mirror normals relative to the laboratory; and test table resolver settings, which describe the orientation of the test table relative to the laboratory. (See Part IV for a description of the geometry.)

Chart I-5 contains the computations for determining the resolver settings to be used during calibration. The settings for the outer and inner axis resolvers (ϕ_3, ϕ_4), for Positions 1, 3, 5, 13, 14, and 15, are the only angles which must be evaluated.

Chart I-6 lists the computations which derive the transformations (T^{BRm}) from the ISU body axes to the rotary axis frame for all calibration positions. The second column of the matrices are required in the computation of gyro bias and compliance. The other columns are computed for information only.

Outputs of the survey computations are T^{BI}, the test table resolver settings, and the elements of T^{BRm}. These will be recorded on the Survey Results Sheet. The T^{BRm} elements should additionally be output in a form (such as paper tape) suitable for re-entry into a computer.

T^{BI} Determination

Given: a set of resolver readings ϕ_1^0, ϕ_2^0, ϕ_3^0, and ϕ_4^0 from position zero and the transformation from body to earth axes via autocollimator readings ($T^{BS0}\ T^{SE0}$)

Find: the matrix T^{BI} which transforms from the body axes to the axes fixed to the inner gimbal of the test table.

1. From the laboratory geometry definitions described in Section 3 we have:

$$T^{BI} = \left(T^{BS0}T^{SE0}\right)\left(T^{IO0}T^{OR0}T^{RT0}T^{TF0}T^{FE}\right)^{-1}$$

where $T^{BS0}T^{SE0}$ is given by the autocollimators and

$$T^{IO0} = \begin{bmatrix} 0 & 1 & 0 \\ \cos\phi_4^0 & 0 & -\sin\phi_4^0 \\ -\sin\phi_4^0 & 0 & -\cos\phi_4^0 \end{bmatrix}$$

$$T^{OR0} = \begin{bmatrix} 0 & 1 & 0 \\ \cos\phi_3^0 & 0 & -\sin\phi_3^0 \\ -\sin\phi_3^0 & 0 & -\cos\phi_3^0 \end{bmatrix}$$

$$T^{RT0} = \begin{bmatrix} 0 & 1 & 0 \\ \cos\phi_2^0 & 0 & -\sin\phi_2^0 \\ -\sin\phi_2^0 & 0 & -\cos\phi_2^0 \end{bmatrix}$$

$$T^{TF0} = \begin{bmatrix} 0 & 1 & 0 \\ \cos\phi_1^0 & 0 & -\sin\phi_1^0 \\ -\sin\phi_1^0 & 0 & -\cos\phi_1^0 \end{bmatrix}$$

$$T^{FE} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

CHART I-5

TEST TABLE RESOLVER SETTINGS

	Position 1	Position 2	Position 3	Position 4	Position 5	Position 6
ϕ_1	90°	90°	90°	90°	90°	90°
ϕ_2	Rotating	Rotating	Rotating	Rotating	Rotating	Rotating
ϕ_3	From $\cos\phi_3^1 = T_{11}^{BI}$ $\sin\phi_3^1 \sin\phi_4^1 = T_{12}^{BI}$ $\sin\phi_3^1 \cos\phi_4^1 = T_{13}^{BI}$	$\phi_3^1 + 180°$	From $\cos\phi_3^3 = T_{21}^{BI}$ $\sin\phi_3^3 \sin\phi_4^3 = T_{22}^{BI}$ $\sin\phi_3^3 \cos\phi_4^3 = T_{23}^{BI}$	$\phi_3^3 + 180°$	From $\cos\phi_3^5 = T_{31}^{BI}$ $\sin\phi_3^5 \sin\phi_4^5 = T_{32}^{BI}$ $\sin\phi_3^5 \cos\phi_4^5 = T_{33}^{BI}$	$\phi_3^5 + 180°$
ϕ_4		ϕ_4^1		ϕ_4^3		ϕ_4^5

	Position 7	Position 8	Position 9	Position 10	Position 11	Position 12
ϕ_1	0°	0°	0°	0°	0°	0°
ϕ_2	90°	90°	90°	90°	90°	90°
ϕ_3	ϕ_3^1	ϕ_3^2	ϕ_3^3	ϕ_3^4	ϕ_3^5	ϕ_3^6
ϕ_4	ϕ_4^1	ϕ_4^2	ϕ_4^3	ϕ_4^4	ϕ_4^5	ϕ_4^6

	Position 13	Position 14	Position 15
ϕ_1	0°	0°	0°
ϕ_2	90°	90°	90°
ϕ_3	From $\cos\phi_3^{13} = \sqrt{1/2}\left[T_{11}^{BI} + T_{21}^{BI}\right]$ $\sin\phi_3^{13} \sin\phi_4^{13} = \sqrt{1/2}\left[T_{12}^{BI} + T_{22}^{BI}\right]$ $\sin\phi_3^{13} \cos\phi_4^{13} = \sqrt{1/2}\left[T_{13}^{BI} + T_{23}^{BI}\right]$	From $\cos\phi_3^{14} = \sqrt{1/2}\left[T_{11}^{BI} + T_{31}^{BI}\right]$ $\sin\phi_3^{14} \sin\phi_4^{14} = \sqrt{1/2}\left[T_{12}^{BI} + T_{32}^{BI}\right]$ $\sin\phi_3^{14} \cos\phi_4^{14} = \sqrt{1/2}\left[T_{13}^{BI} + T_{33}^{BI}\right]$	From $\cos\phi_3^{15} = \sqrt{1/2}\left[T_{21}^{BI} + T_{31}^{BI}\right]$ $\sin\phi_3^{15} \sin\phi_4^{15} = \sqrt{1/2}\left[T_{22}^{BI} + T_{32}^{BI}\right]$ $\sin\phi_3^{15} \cos\phi_4^{15} = \sqrt{1/2}\left[T_{23}^{BI} + T_{33}^{BI}\right]$
ϕ_4			

T^{BRm} Determination

Given: ϕ_3^m, ϕ_4^m, and T^{BI}

Find: T^{BRm}

1. We know that

$$T^{BRm} = T^{BI} T^{IOm} T^{ORm}$$

where T^{BI} is given from a prior survey and

$$T^{IOm} T^{ORm} = \begin{bmatrix} \cos\phi_3^m & 0 & -\sin\phi_3^m \\ \sin\phi_3^m \sin\phi_4^m & \cos\phi_4^m & \cos\phi_3^m \sin\phi_4^m \\ \sin\phi_3^m \cos\phi_4^m & -\sin\phi_4^m & \cos\phi_3^m \cos\phi_4^m \end{bmatrix}$$

2. The first column of T^{BRm} is the calibration control parameter. The values of this column have already been included in the calibration equations.

3. The second column of T^{BRm} which is the only column required for inclusion into the calibration equations, is:

$$T_{12}^{BRm} = T_{12}^{BI} \cos\phi_4^m - T_{13}^{BI} \sin\phi_4^m$$

$$T_{22}^{BRm} = T_{22}^{BI} \cos\phi_4^m - T_{23}^{BI} \sin\phi_4^m$$

$$T_{32}^{BRm} = T_{32}^{BI} \cos\phi_4^m - T_{33}^{BI} \sin\phi_4^m$$

These equations need only be solved for m = 1, 3, 5, 13, 14, and 15.

4. The third column of T^{BRm}, which is not required in the calibration equations, is given for information·

$$T_{13}^{BRm} = -T_{11}^{BI} \sin\phi_3^m + T_{12}^{BI} \cos\phi_3^m \sin\phi_4^m + T_{13}^{BI} \cos\phi_3^m \cos\phi_4^m$$

$$T_{23}^{BRm} = -T_{21}^{BI} \sin\phi_3^m + T_{22}^{BI} \cos\phi_3^m \sin\phi_4^m + T_{23}^{BI} \cos\phi_3^m \cos\phi_4^m$$

$$T_{33}^{BRm} = -T_{31}^{BI} \sin\phi_3^m + T_{32}^{BI} \cos\phi_3^m \sin\phi_4^m + T_{33}^{BI} \cos\phi_3^m \cos\phi_4^m$$

I-2.3 Fundamental Modes

The steps required to perform the system survey procedures are illustrated in Laboratory Flow Diagrams 1 through 3. It is assumed that the laboratory turn-on procedures have been performed before these steps are begun.

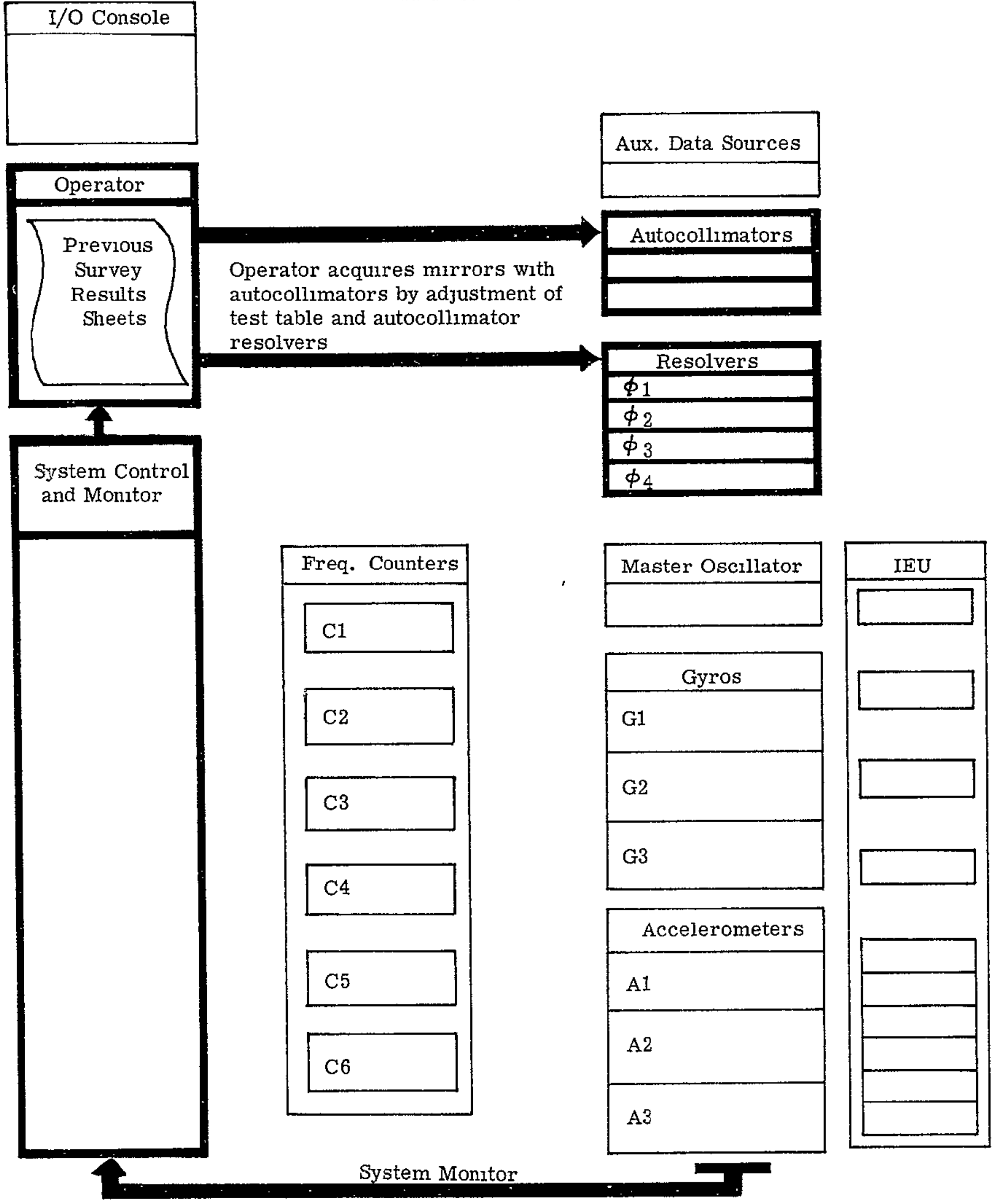
The operator first locates the mirrors on the ISU by use of the autocollimators (as shown on Flow Diagram 1). This is accomplished by a trial and error adjustment of the test table's resolvers and the autocollimators. (The specific procedure depends upon the as yet undefined autocollimator design.) After these system survey procedures have once been performed, previous settings may be used as a starting point for subsequent trial and error procedures.

Next, the operator records (see Flow Diagram 2) the test table resolvers and autocollimator readings on the Survey Results Sheet (Chart I-1). The computer is turned on, and the system survey program entered into memory. This program may be written for any general purpose computer. The resolver and autocollimator readings are entered into the computer; and the computations contained in Section I-2.2 are initiated. The recording of current survey information is also shown in Flow Diagram 2, including the magnitude (g) of the gravitational vector and the components of earth rotational rate along north, $(\omega^E \sin \lambda)$, and vertical, $(\omega^E \cos \lambda)$. This step will be only infrequently required.

The computations are represented in Flow Diagram 3 as three subroutines. The autocollimator and resolver information is used to determine the transformation (T^{BI}) from the ISU body axes to the inner axis frame of the test table. This matrix is then used to determine the test table resolver settings which are required for orientation control during calibration. The resolver settings, additionally, are required for the determination of the T^{BRm} matrices, which are used in the calibration computations.

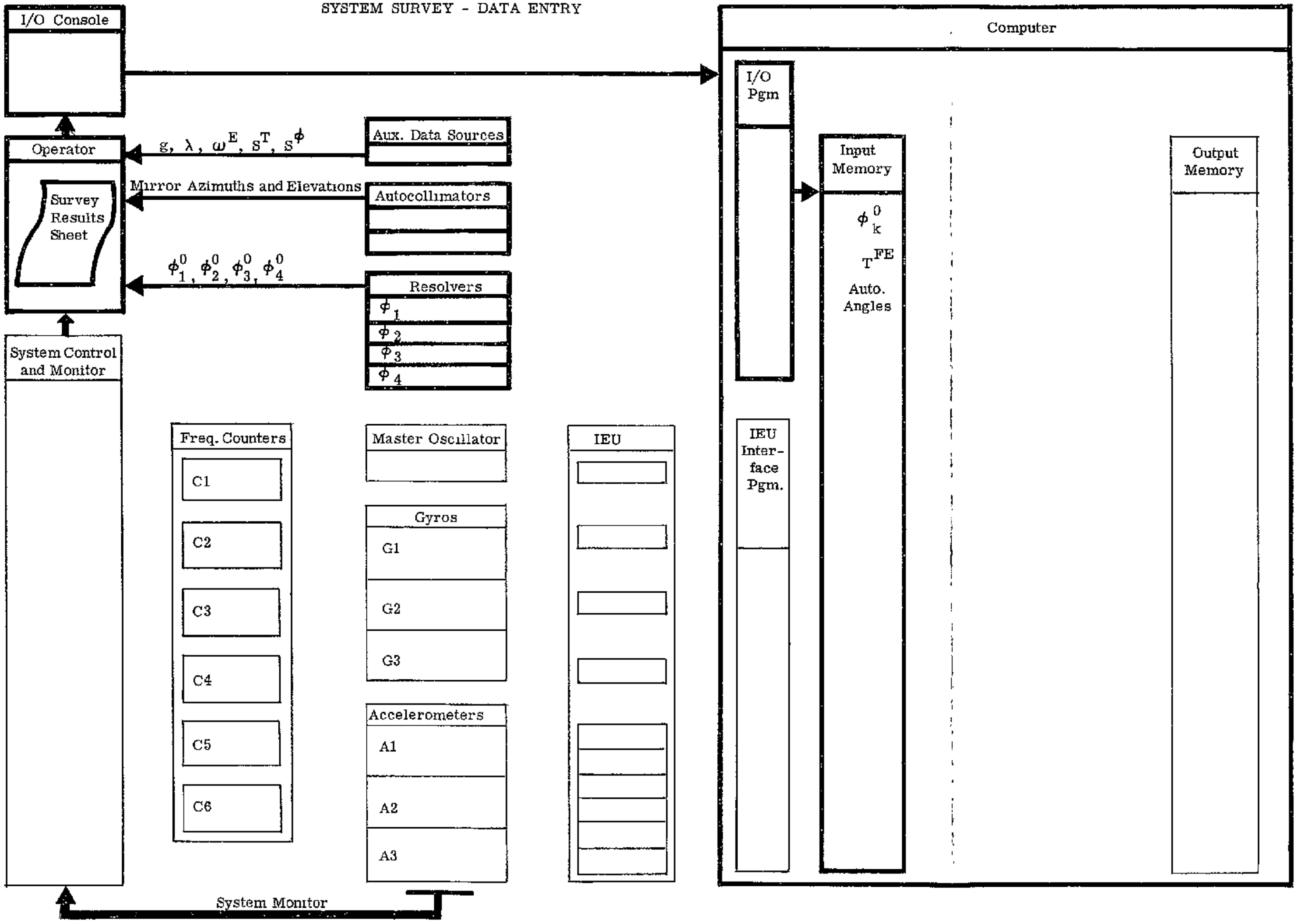
The computational outputs include the calibration resolver settings, T^{BRm}, and T^{BI}. The T^{BI} matrix is not required for calibration, but could prove useful for the purpose of checking the computations. The output data is recorded on the Survey Results Sheet (Chart I-2). The T^{BRm} matrices should also be output on paper or magnetic tapes for the purpose of re-entry to the computer before the computation of calibration constants.

LABORATORY FLOW DIAGRAM #1
SYSTEM SURVEY - ACQUISITION OF MIRRORS WITH AUTOCOLLIMATORS

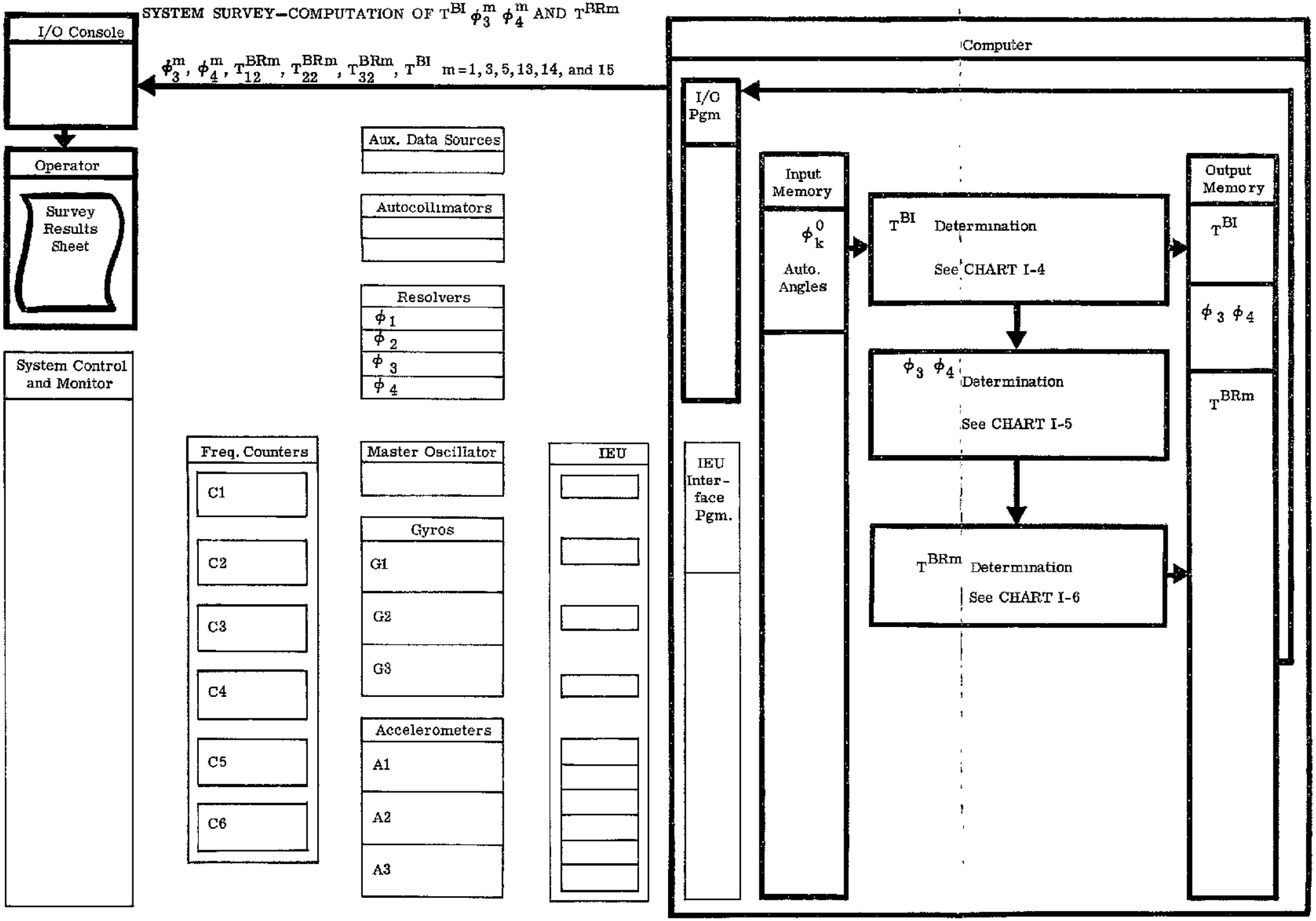


Operator acquires mirrors with autocollimators by adjustment of test table and autocollimator resolvers

LABORATORY FLOW DIAGRAM #2
SYSTEM SURVEY - DATA ENTRY



LABORATORY FLOW DIAGRAM #3
SYSTEM SURVEY—COMPUTATION OF T^{BI} ϕ_3^m ϕ_4^m AND T^{BRm}



PART I

SECTION 3

ALIGNMENT PARAMETER EVALUATION

As a part of both Level and Gyrocompass Alignment, an estimation is made of the body axis components of the gravity vector. (Gyrocompass additionally processes an earth rate measurement.) One of the two gravity estimation techniques outlined in this document requires parameters that are not evaluated as part of the real-time program. Because of the complexity of the computations and their repeated use, these parameters are evaluated at some time before alignment. The procedures to generate these parameters are contained in this section. The procedure listings are in the first subsection, and the computations in the second.

This procedure need be performed only when one of the input parameters shown on the Alignment Parameter Evaluation Sheet (Chart I-8) are changed.

I-3.1 Alignment Parameter Evaluation Procedures

The procedures to obtain parameters to be used by the Alignment Estimation Routine are listed in Chart I-7.

Before these procedures can be accomplished, the parameters shown on Chart I-8 must be determined. These parameters include a nominal value of the alignment matrix. This value can be obtained in several ways including.

1. Determination from the autocollimators using Chart III-3,
2. Determination from the test table resolvers and T^{BI} using the mathematics shown on Chart I-4,
3. By a quick Gyrocompass (or Level Alignment using simple average).

CHART I-7

LABORATORY PROCEDURES

ALIGNMENT PARAMETER EVALUATION

INSTRUCTIONS	✓	COMMENTS
1) Verify input parameters are specified on Alignment Parameter Evaluation Sheet.		
2) Load parameter evaluation program into computer.		
3) Enter input parameters from Alignment Parameter Evaluation Sheet.		
4) Initiate computations.		
5) After output, mark output reel and record reel identification on Alignment Parameter Evaluation Sheet.		

CHART I-8

ALIGNMENT PARAMETER EVALUATION

Operator: ____________________ Output Reel No ________

Date: ________________

INPUT PARAMETERS

K ____________ Δt ____________ Sec

$|g|$ ____________ Ft./Sec^2 σ_g ____________ Ft./Sec^2

σ_θ ____________ Rad.

T^{EB}(EST.) = ________ ________ ________

________ ________ ________

________ ________ ________

NOISE COVARIANCE FUNCTIONS

Time (Seconds)	Accelerometer Noise ϕ_n	Environmental Acceleration ϕ_α	Environmental Rotation ϕ_θ

I-3.2 Alignment Parameter Evaluation Computations

The computations to be performed in order to determine the alignment parameters are shown in Charts I-9 and I-10. These computations require, as inputs, the parameters shown on the Alignment Parameter Evaluation Sheet, and an additional parameter, ϵ_T. The constant ϵ_T equals the interval of time between the completion of the last measurement and the time at which the value of the alignment matrix is to be predicted. ϵ_T will be equal to the minimum time required to complete the alignment computations.

The covariance (see Section 5 of the Development Document) of the instrument and environment noise are input by use of tabular data. They are numerically integrated and used in covariance matrices as functions of the number of samples and intersample time. The nature of this data may dictate that it is more convenient to determine analytic approximations of the covariance functions and to integrate the functions in closed form. Or it may be desirable to input the matrices for each intersample time and maximum K to be selected.

The outputs of the computations are a matrix M and a vector $\underline{b}$. These are output in paper tape format for entry into the computer at the time of alignment.

ESTIMATION MATRIX COMPUTATIONS – LEVEL

Inputs: intersample time, Δt (sec)

number of samples, K

estimate of gravity, $\tilde{g}$ (ft/sec^2)

rms error in gravity estimate $\sigma_{\tilde{g}}$ (ft/sec^2)

estimate of T^{EB}, $\tilde{T}_1$

rms angular error in prior estimate of vertical, σ_θ (radians)

noise covariance functions (tabular)

- accelerometer noise $\phi_n(t)$ (ft^2/sec^4)
- translational acceleration noise $\phi_\alpha(t)$ (ft^2/sec^4)
- rotational noise $\phi_\theta(t)$ (radian^2)

prediction time ϵ_t (sec)

Outputs: alignment parameters M and $\underline{b}$

The intermediate quantities Σ_α, Σ_n, $\Sigma_{\phi\phi}$, c_0, c_j and $\Sigma_{\tilde{g}}$ are computed from the inputs.

- Σ_α is K x K matrix with components

$$(\Sigma_\alpha)_{ij} = \int_0^{\Delta t} [\Delta t - u] \phi_\alpha (u + (j-i)\Delta t) du$$

$$+ \int_{-\Delta t}^{0} [\Delta t + u] \phi_\alpha (u + (j-i)\Delta t) du$$

- Σ_n is K x K matrix with components

$$(\Sigma_n)_{ij} = \int_0^{\Delta t} [\Delta t - u] \phi_n (u + (j-i)\Delta t) du$$

$$+ \int_{-\Delta t}^{0} [\Delta t + u] \phi_n (u + (j-i)\Delta t) du$$

CONTINUATION OF CHART I-9

- $\Sigma_{\phi\phi}$ is a K x K matrix with components

$$(\Sigma_{\phi\phi})_{ij} = \int_0^{\Delta t} [\Delta t - u]\, \phi_\theta (u + (j-i)\,\Delta t)\, du$$

$$+ \int_{-\Delta t}^{0} [\Delta t + u]\, \phi_\theta (u + (j-i)\,\Delta t)\, du$$

- $c_0 = \phi_0 (0)$

- $c_j = \int_0^{\Delta t} \phi_\theta (u + (j-1)\Delta t - K\Delta t - \epsilon_t)\, du \qquad j = 1,\ 2,\ \cdots,\ K$

  where the integrals are evaluated by a convenient integration technique such as trapezoidal rule or Simpson's rule.

- $\Sigma_{\tilde g}$ is a 3 x 3 matrix

$$\Sigma_{\tilde g} = \tilde T_1 \begin{bmatrix} \sigma_g^2 & & 0 \\ & \tilde g^2 \sigma_\theta^2 & \\ 0 & & \tilde g^2 \sigma_\theta^2 \end{bmatrix} \tilde T_1^T$$

From these intermediate quantities, Σ_N, Σ_Z, V, A, B, $\underline{a}_1$, and $\underline{a}_2$ are computed.

- Σ_N is a 3K x 3K matrix

$$\Sigma_N = \begin{bmatrix} \Sigma_n + \Sigma_\alpha & & 0 \\ & \Sigma_n + \Sigma_\alpha & \\ 0 & & \Sigma_n + \Sigma_\alpha \end{bmatrix}$$

CONTINUATION OF CHART I-9

- Σ_Z is a $(5K + 5) \times (5K + 5)$ matrix

$$\Sigma_Z = \begin{array}{|c|c|c|c|c|c|c}
\Sigma_{\tilde{g}} & & & & 0 & & 3 \\ \hline
 & \Sigma_N & & & & & 3K \\ \hline
 & & c_0 & c_1 \cdots c_K & & & 1 \\ \hline
 & & \begin{matrix} c_1 \\ \vdots \\ c_K \end{matrix} & \Sigma_{\phi\phi} & & & K \\ \hline
 & & & & c_0 & c_1 \cdots c_K & 1 \\ \hline
0 & & & & \begin{matrix} c_1 \\ \vdots \\ c_K \end{matrix} & \Sigma_{\phi\phi} & K \\ \hline
3 & 3K & 1 & K & 1 & K &
\end{array}$$

Numbers at edges of matrices denote dimension of submatrices.

- Matrices A (3 x 3) and B (3 x 3K) are submatrices

$$\begin{bmatrix} A & B \\ B^T & D \end{bmatrix} = (V \Sigma_Z V^T)^{-1}$$

where matrix V is the (3K + 3) x (5K + 5) matrix given on the following chart.

- $$\underline{a}_1 = \begin{bmatrix} \tilde{g}(\tilde{T}_1)_{11} \\ \tilde{g}(\tilde{T}_1)_{21} \\ \tilde{g}(\tilde{T}_1)_{31} \end{bmatrix} \qquad \underline{a}_2 = \begin{bmatrix} \left.\begin{matrix} \Delta t\, \tilde{g}(\tilde{T}_1)_{11} \\ \vdots \\ \Delta t\, \tilde{g}(\tilde{T}_1)_{11} \end{matrix}\right\} K \\ \left.\begin{matrix} \Delta t\, \tilde{g}(\tilde{T}_1)_{21} \\ \vdots \\ \Delta t\, \tilde{g}(\tilde{T}_1)_{21} \end{matrix}\right\} K \\ \left.\begin{matrix} \Delta t\, \tilde{g}(\tilde{T}_1)_{31} \\ \vdots \\ \Delta t\, \tilde{g}(\tilde{T}_1)_{31} \end{matrix}\right\} K \end{bmatrix}$$

CONTINUATION OF CHART I-9

Then, the outputs are given by:

- $M = -A^{-1}B$

  Note that A^{-1} is the covariance matrix of the estimate.

- $\underline{b} = \underline{a}_1 + A^{-1}B\,\underline{a}_2$

V MATRIX

$$
V = \begin{array}{cc}
\left[\begin{array}{ccc|ccc|c|ccc|c|ccc}
1 & & & & & & \tilde{g}(\tilde{T}_1)_{13} & & & & -\tilde{g}(\tilde{T}_1)_{12} & & & \\
& 1 & & & 0 & & \tilde{g}(\tilde{T}_1)_{23} & & 0 & & -\tilde{g}(\tilde{T}_1)_{22} & & 0 & \\
& & 1 & & & & \tilde{g}(\tilde{T}_1)_{33} & & & & -\tilde{g}(\tilde{T}_1)_{32} & & & \\
\hline
\Delta t & & & 1 & & & & \tilde{g}(\tilde{T}_1)_{13} & & & & -\tilde{g}(\tilde{T}_1)_{12} & & \\
\vdots & & & & 1 & & 0 & & \ddots & & 0 & & \ddots & \\
\Delta t & & & & & \ddots & & & & \tilde{g}(\tilde{T}_1)_{13} & & & & -\tilde{g}(\tilde{T}_1)_{12} \\
\hline
& \Delta t & & & \ddots & & & \tilde{g}(\tilde{T}_1)_{23} & & & & -\tilde{g}(\tilde{T}_1)_{22} & & \\
& \vdots & & & & \ddots & 0 & & \ddots & & 0 & & \ddots & \\
& \Delta t & & & & \ddots & & & & \tilde{g}(T_1)_{23} & & & & -\tilde{g}(\tilde{T}_1)_{22} \\
\hline
& & \Delta t & & & \ddots & & \tilde{g}(\tilde{T}_1)_{33} & & & & -\tilde{g}(\tilde{T}_1)_{32} & & \\
& & \vdots & & & \ddots & 0 & & \ddots & & 0 & & \ddots & \\
& & \Delta t & & & 1 & & & & \tilde{g}(\tilde{T}_1)_{33} & & & & -\tilde{g}(\tilde{T}_1)_{32}
\end{array}\right]
&
\begin{array}{c} 3 \\ \\ \\ K \\ \\ \\ K \\ \\ \\ K \\ \\ \end{array}
\\
\begin{array}{cccccc} 3 & \underset{\text{(diagonal)}}{3K} & 1 & \underset{\text{(diagonal)}}{K} & 1 & \underset{\text{(diagonal)}}{K} \end{array} &
\end{array}
$$

All missing entries are zero.

$(\tilde{T}_1)_{ij}$ denotes the (i, j) component of $\tilde{T}_1$.

Numbers at the edge of the V matrix denote the dimension of the submatrices.

CHART I-10

PART II
CALIBRATION

LABORATORY PROCEDURES MANUAL
PART II
CALIBRATION

PART II
CALIBRATION

This part of the document contains the instructions to perform a calibration of the ISU. Calibration is the process of determining the instrument model coefficients and the elements of the matrices to transform from the instrument frames to the body frame. It is accomplished by: 1) taking measurements with the instruments contained in the ISU in a known input environment; and 2) using the relationship between the known kinematic inputs and digital outputs of the instruments to determine the instrument model coefficients and the elements of the matrices. The kinematic inputs consist of the earth's "gravity" vector, $\underline{g}$; the earth's rotational vector, $\underline{\omega}^E$; noise; and, in some cases, the angular velocity of the test table, $\underline{\omega}^T$. By judicious choices of ISU orientations relative to these vectors, it is possible to isolate terms in the models. Computation of the calibration constants can then be greatly simplified, and higher accuracy may be achieved.

After the ISU has been mounted on the test table, the system survey procedures contained in Part I, Section 2 are performed to determine the orientation of the ISU relative to the table, and to determine the test table resolver settings for all calibration positions. The calibration procedures contained within this part of the document cannot be performed until after those activities have been accomplished.

The calibration presentation is divided into six sections. The procedures which dictate the collection of data for determining the constants related to angular velocity sensitive terms are contained in Section 1.

Sections 2 and 3 contain the procedures for determining the constants related to acceleration sensitive terms in the gyro and the accelerometer, respectively. The calibration computations are contained in Section 4; and in Section 5 there are schematic descriptions of all calibration procedures. Section 6 is included as an aid to the analyst in developing the information required on various calibration work sheets.

Information required to perform the various calibration procedures, and the forms used to supply this information, include

1. Resolver settings (Test Table Resolver Settings Sheet)
2. Selected elements of the matrix transforming from the body frame to the test table rotary axis frame at certain positions (T^{BRm}), for use in calibration computations (computer entry format)

3. Measurement counter connections (Counter Connections Sheet), and
4. Measurement counter settings (Counter Settings Sheet).

The test table resolver settings are determined in the System Survey Procedure (see Section I-2). The operator will obtain the information required in the Resolver Settings Sheet (Chart II-1) from the Survey Results Sheet described in Section I-2. The accuracies to which the resolver settings are required are discussed in Section II-6.

The required T^{BRm} settings should be available from the current system survey procedure in a format suitable for computer input (probably paper tape). If not, they may be obtained from the Survey Results Sheet.

The counter connections to be used in calibration are shown in the Counter Connections Sheet (Chart II-2). The selections of counter connections are prompted by the requirement to minimize possible error and to minimize the need for reconnection. If, at a later date, it is necessary to change these connections, the Counter Settings and Outputs Sheets must also be changed.

The counter settings are specified by the Counter Settings Sheet, Chart II-3. The values to be assigned to these settings are functions of the calibration accuracy requirements which are a function of measurement time. The determination of these settings is discussed in Section II-6.

Additional work sheets include the Counter Outputs Sheet (Chart II-4), which is used to record intermediate data; and the Calibration Results Sheet (Chart II-5), which is used to record the calibration results. Since all of the above mentioned sheets are common to the entire calibration procedure, they are consolidated here rather than being scattered throughout this part of the document. Master copies of each of the sheets are included in Part IV. which will be duplicated for use in conducting experiments.

CHART II-1

TEST TABLE RESOLVER SETTINGS

	Position 1	Position 2	Position 3	Position 4	Position 5	Position 6
ϕ_1	90°	90°	90°	90°	90°	90°
ϕ_2	Rotating	Rotating	Rotating	Rotating	Rotating	Rotating
ϕ_3		$\phi_3^1 + 180^\circ$		$\phi_3^3 + 180^\circ$		$\phi_3^5 + 180^\circ$
ϕ_4		ϕ_4^1		ϕ_4^3		ϕ_4^5

	Position 7	Position 8	Position 9	Position 10	Position 11	Position 12
ϕ_1	0° *	0° *	0° *	0° *	0° *	0° *
ϕ_2	90°	90°	90°	90°	90°	90°
ϕ_3	ϕ_3^1 *	ϕ_3^2 *	ϕ_3^3 *	ϕ_3^4 *	ϕ_3^5 *	ϕ_3^6 *
ϕ_4	ϕ_4^1	ϕ_4^1	ϕ_4^3	ϕ_4^3	ϕ_4^5	ϕ_4^5

	Position 13	Position 14	Position 15
ϕ_1	0° *	0° *	0° *
ϕ_2	90°	90°	90°
ϕ_3	*	*	*
ϕ_4			

* Requires bubble level correction.

CHART II-2

COUNTER CONNECTIONS

Experiment: Date:

Positions		Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6
GYRO 1-6	X-Y Connection	Master Oscillator	Gyro 1	Gyro 2	Gyro 3	Not Required	Not Required
	Z Connection	Table Rotation	Table Rotation	Table Rotation	Table Rotation		
	Select Mode	Measure Z	Measure Z	Measure Z	Measure Z		
GYRO 7-15	X-Y Connection	Not Required	Gyro 1	Gyro 2	Gyro 3		
	Z Connection		Master Oscillator	Master Oscillator	Master Oscillator		
	Select Mode		Measure Z	Measure Z	Measure Z		
ACCELEROMETER 7-15	X-Y Connection	Accelerometer 1 Line 1	Accelerometer 1 Line 2	Accelerometer 2 Line 1	Accelerometer 2 Line 2	Accelerometer 3 Line 1	Accelerometer 3 Line 2
	Z Connection	Master Oscillator	Master Oscillator	Master Oscillator	Master Oscillator	Master Oscillator	Master Oscillator
	Select Mode	Measure X-Y	Measure X-Y	Measure X-Y	Measure X-Y	Measure X-Y	Measure X-Y

CHART II-3

COUNTER SETTINGS

Experiment:

Date:

Positions		Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6
GYRO		$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$		
	1					Not Required	Not Required
	2						
	3						
	4						
	5						
	6						
GYRO		Not Required	$(\Sigma n_1^T)_1^{Gm}$	$(\Sigma n_1^T)_2^{Gm}$	$(\Sigma n_1^T)_3^{Gm}$		
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						
ACCELEROMETER		$(\Sigma\gamma)_{11}^m$	$(\Sigma\gamma)_{12}^m$	$(\Sigma\gamma)_{21}^m$	$(\Sigma\gamma)_{22}^m$	$(\Sigma\gamma)_{31}^m$	$(\Sigma\gamma)_{32}^m$
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						

COUNTER OUTPUTS

Experiment:

Date:

Positions		Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6
GYRO		$(\Sigma n_1^T)^{Gm}$	$(\Sigma\delta)_1^m$	$(\Sigma\delta)_2^m$	$(\Sigma\delta)_3^m$		
	1					Not Required	Not Required
	2						
	3						
	4						
	5						
	6						
GYRO		Not Required	$(\Sigma\delta)_1^m$	$(\Sigma\delta)_2^m$	$(\Sigma\delta)_3^m$		
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						
ACCELEROMETER		$(\Sigma n_1^T)_{11}^{Am}$	$(\Sigma n_1^T)_{12}^{Am}$	$(\Sigma n_1^T)_{21}^{Am}$	$(\Sigma n_1^T)_{22}^{Am}$	$(\Sigma n_1^T)_{31}^{Am}$	$(\Sigma n_1^T)_{32}^{Am}$
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						

CHART II-5

CALIBRATION RESULTS SHEET

Experiment:
Principal Operator:
Date:

PROCEDURE DESCRIPTION

Survey Results Sheet Used
Resolver Settings Sheet Used
Counter Settings Sheet Used
Counter Outputs Sheet Used
Special Procedures

RESULTS

Parameter	Units	Instrument 1	Instrument 2	Instrument 3
Gyros				
$\Delta\ \Phi$	Deg/Pulse			
$(\underline{G}_i \cdot \underline{B}_1)$				
$(\underline{G}_i \cdot \underline{B}_2)$				
$(\underline{G}_i \cdot \underline{B}_3)$				
R	Deg/Hr.			
B_I	Deg/Hr. -g			
B_O	Deg/Hr. -g			
B_S	Deg/Hr. -g			
C_{II}	Deg/Hr. -g^2			
C_{SS}	Deg/Hr. -g^2			
C_{IS}	Deg/Hr. -g^2			
C_{OS}	Deg/Hr. -g^2			
C_{IO}	Deg/Hr. -g^2			
Q_{II}	Hr. /Deg.			
Q_{IS}	Hr. /Deg.			
J	Hours			
Accelerometers				
D_1	(Pulses/Sec)/g			
$(\underline{A}_i \cdot \underline{B}_1)$				
$(\underline{A}_i \cdot \underline{B}_2)$				
$(\underline{A}_i \cdot \underline{B}_3)$				
D_0	g			
D_2	g/g^2			
D_3	g/g^3			

PART II

SECTION 1

CALIBRATION OF GYRO ANGULAR VELOCITY SENSITIVE TERMS

This section describes the procedures to obtain measurement data used in the determination of the constants associated with the angular velocity sensitive terms of the gyros. This set of procedures will probably, but not necessarily, be accomplished before the procedures described in subsequent sections. The measurements described in this section are taken with the rotation from the test table as the principal kinematic input.

Subsection II-1.1 contains the procedures to obtain data to determine the scale factors of the three gyros and the inverse of the Q^G matrix which transforms from the gyro frame to the ISU body frame. Measurements are made in six positions, denoted as Positions 1 through 6.

In II-1.2, the procedures to obtain data for determination of scale factor nonlinearity are described. These procedures will be performed as a separate experiment rather than as a part of a complete calibration.

Subsection II-1.3 contains the procedures for the evaluation of the J terms. These procedures will also be performed as a special experiment, rather than as a part of a complete calibration.

The reader may refer to Section II-5.1, Fundamental Modes, to further his understanding of the procedures described in this section.

II-1.1 Scale Factor and $(Q^G)^{-1}$

The procedures to obtain measurement data used to compute the gyro scale factor and the inverse of the matrix (Q^G) are listed in Chart II-6. These procedures utilize Positions 1 to 6 as shown on Figure II-1.

CHART II-6

LABORATORY PROCEDURES

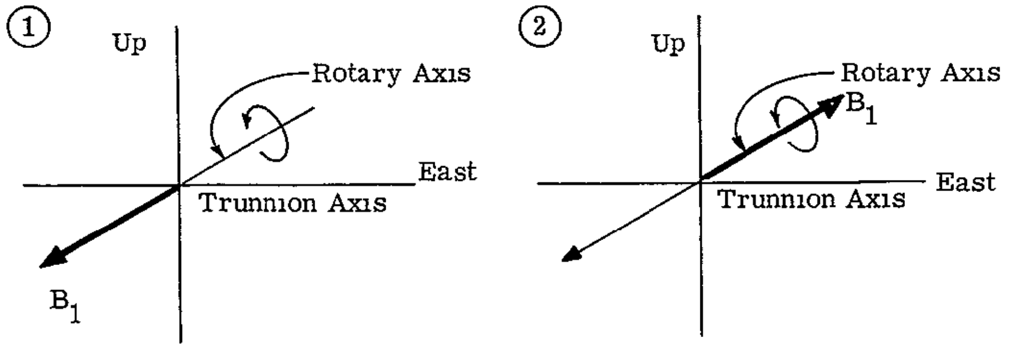
CALIBRATION OF GYRO SCALE FACTOR AND $(Q^G)^{-1}$ – POSITIONS 1 TO 6

INSTRUCTIONS	✓	COMMENTS
1) Verify that the data required for the measurements are available on the Resolver Settings and Counter Settings Sheets.		1) See Section II-6.
2) Verify that the turn-on procedures have been performed and the equipment required is operating.		2) The turn-on procedures are given in Section I-1.
3) Connect gyros, master oscillator, and rotary axis output to frequency counters.		3) See Counter Connections Sheet for connection assignments.
4) Set counters to "measure Z" mode.		
5) Set ϕ_1 to 90^o.		
6) Set $(\Sigma n^{\phi})^{m=1}$ on frequency counters.		6) See Counter Settings Sheet. Note: All four counters will be set to the same value.
7) Set ϕ_3 and ϕ_4 to $\phi_3^{m=1}$ and ϕ_4^{m-1}.		7) Obtain settings from Resolver Settings Sheet.
8) Start table rotating and adjust speed.		8) Speed should be adjusted to just less than gyro saturation level.
9) Start data collection.		9) After table has passed resolver output angle, switch all counters to operate before the output angle is again reached so that time measured on Counter 1 is the measurement time for all gyros.
10) After counters output, switch all counters to standby and stop table.		
11) Record counter outputs on Counter Outputs Sheet.		
12) Rotate ϕ_3 by 180^o.		12) This rotation moves table to Position 2.
13) Start table rotating. Adjust speed to that used for m=1.		13) If ϕ_3 can be rotated accurately with the table in motion, it may not be necessary to stop the table.
14) Perform steps 9 to 11 for m=2.		

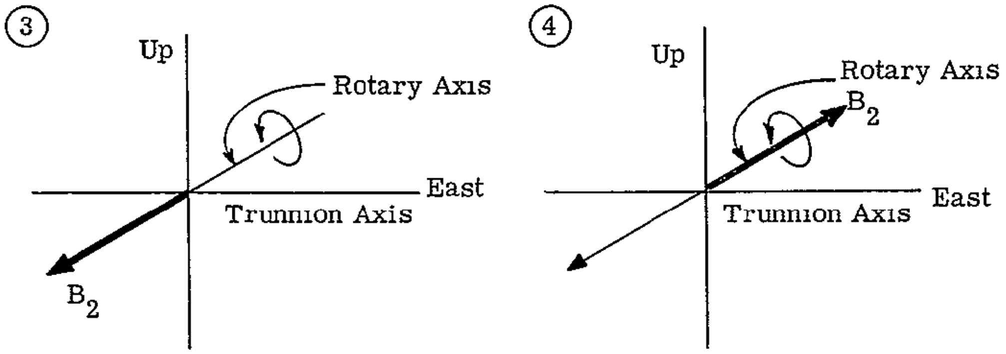
LABORATORY PROCEDURES

CALIBRATION OF GYRO SCALE FACTOR AND $(Q^G)^{-1}$ – POSITIONS 1 TO 6 (Continued)

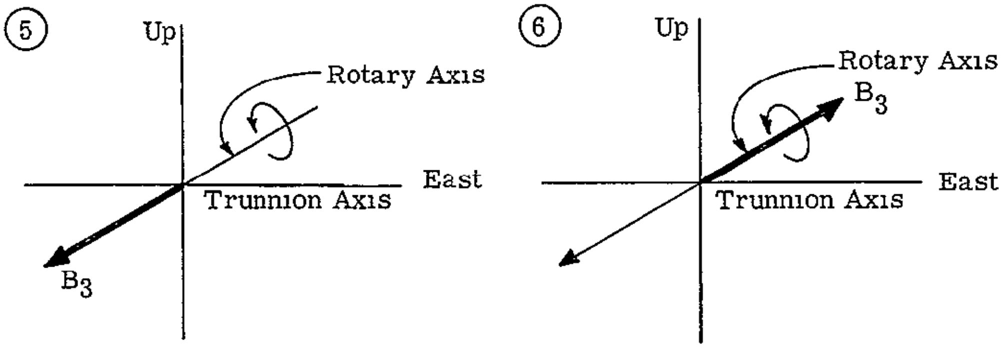
INSTRUCTIONS	✓	COMMENTS
15) Perform steps 6 to 14 for m=3, 4, and m=5, 6.		



Positions 1 and 2. B_1 aligned with rotary axis in plus and minus sense



Positions 3 and 4. B_2 aligned with rotary axis in plus and minus sense



Positions 5 and 6. B_3 aligned with rotary axis in plus and minus sense

In these positions, ϕ_1 is set at 90° so that the rotary axis is horizontal. The table is rotating

Figure II-1. Test Table Orientations For Positions 1 to 6

II-1.2 Nonlinearity Experiments

This section describes the procedures to perform the experiments to determine the nonlinearity of the gyro scale factors. The derivation of this experiment is described in Section 4.2.4 of the Development Document.

Basically, the experiment consists of measuring gyro outputs for several speeds of the test table, plotting the frequency of the gyro output as a function of the angular velocity, and determining the functional relationship between the angular velocity input and the gyro output from the plot.

The measurements from each gyro are taken when its input axis is nominally directed along the table rotary axis. Positions 1, 3 and 5 (see the preceding subsection) will be used in these experiments. The test table is brought to a constant angular velocity and measurements are made for N whole turns, where N is selected to minimize the quantization and noise errors. Frequency counters are used to measure the time and the count of gyro output pulses for the N turns. The process is repeated at several different angular velocities for each gyro.

Next, the total angular velocity and the frequency of gyro output is computed and plotted for each measurement. A separate plot is made for each gyro. From these plots, the relationship between frequency of output and angular velocity can be determined.

The Counter Settings Sheet and the Counter Outputs Sheet are modified as shown in Charts II-8 and II-9 for use during this experiment. A listing of laboratory procedures for the nonlinearity experiment is contained on Chart II-7. Master copies of Charts II-8, and II-9 are available in Part IV.

CHART II-7

LABORATORY PROCEDURES

NONLINEARITY EXPERIMENTS

INSTRUCTIONS	✓	COMMENTS
1) Verify that the data required for the measurements are available on the Resolver Settings and Counter Settings Sheets.		1) See Section II-6.
2) Verify that the turn on procedures have been performed and the equipment required is operating.		2) The turn-on procedures are given in Section I-1.
3) Connect gyros, master oscillator and rotary axis output to frequency counters.		3) See Counter Connections Sheet (Positions 1 to 6) for connection assignments.
4) Set counters to "measure Z" mode.		4) See Resolver Settings Sheet. Position 1 is used for gyro 1 experiment.
5) Set ϕ_3 and ϕ_4 to ϕ_3^m and ϕ_4^m. Set ϕ_1 to 90°.		5) m=1 for gyro 1, m=3 for gyro 2, and m=5 for gyro 3. Settings obtained from Resolver Settings Sheet.
6) Set (Σn^{ϕ}) on counters for this gyro and this speed.		6) Use Counters 1 and 2 for gyro 1, Counters 1 and 3 for gyro 2, and Counter 1 and 4 for gyro 3. Set from Counter Settings Sheet.
7) Start table and adjust speed for measurement 1.		7) Obtain speed setting from Counter Settings Sheet.
8) Switch counters to operate.		8) Make sure that a rotary axis output pulse does not occur between the time the two counters are switched.
9) After counter output, switch counters to standby.		
10) Record counter outputs on Counter Outputs Sheet.		
11) Repeat steps 6 to 10 for all speeds for this gyro.		
12) Stop table.		
13) Repeat steps 5 to 12 for gyros 2 and 3.		
14) Compute $\omega = S_1^T \left(\frac{\text{counter setting}}{\text{counter 1 output}}\right) + \omega^E \sin\lambda$.		14) $\omega^E \sin\lambda$ from Survey Results Sheet. S_1^T is scale factor for master oscillator output. Data may be recorded on Counter Outputs Sheet.

CHART II-7 (Cont.)

LABORATORY PROCEDURES

NONLINEARITY EXPERIMENTS (Continued)

INSTRUCTIONS	✓	COMMENTS
15) Compute $\dot{P} = S_1^T \left(\frac{\text{counter i output}}{\text{counter 1 output}}\right)$ i = 2, 3, or 4 for gyros 1, 2, or 3, respectively		15) Record on Counter Outputs Sheet.
16) Plot $\dot{P}$ against ω on graph paper for each gyro.		
17) Analyze plots to find f where $\dot{P} = f(\omega)$.		

CHART II-8

COUNTER SETTINGS

Experiment: Gyro Scale Factor Nonlinearity

Date.

Positions	Counter 1	Counter 2	Counter 3	Counter 4		Angular Velocity
					Not Required	
					↓	

CHART II-9

COUNTER OUTPUTS

Experiment Gyro Scale Factor Nonlinearity Date:

Positions	Counter 1	Counter 2	Counter 3	Counter 4	ω	P

II-1.3 J Term Experiments

The positions to be used during the experiments which determine the value of the J term are the same as Positions 7 and 11 with the modification that the test table will be rotating. Gyro output data are collected over a period during which the angular velocity of the table has changed. This experiment is the only calibration experiment that requires evaluation of the integrals:

- $\int \sin \varphi_2 \, dt$
- $\int \cos \varphi_2 \, dt$

which vary as a function of time. The manner in which these integrals are determined will depend upon the manner in which the test table angular acceleration is commanded during the experiment. This cannot be specified until the test table is evaluated to discern its ability to control angular accelerations. For this reason, the detailed procedure listing for the J term has not been completed. After sufficient data has been obtained on the operation of the test table, an approach to the evaluation of these integrals can be selected.

CHART II-10

LABORATORY PROCEDURES

J TERM EXPERIMENT

INSTRUCTIONS	✓	COMMENTS
		Note: This sheet will be completed by laboratory personnel after the test table angular acceleration control has been specified.

PART II

SECTION 2

CALIBRATION OF GYRO ACCELERATION SENSITIVE TERMS

This section describes the procedures for obtaining data to determine the gyro acceleration sensitive coefficients. The positions used in these experiments are identical with those used in accelerometer calibration (see Section II-3). Section II-2.1 lists the procedures for the determination of the bias, unbalance, and square compliance terms. Section II-2.2 contains procedures for the determination of the product compliance terms.

II-2.1 Bias, Unbalance, and Square Compliance Terms

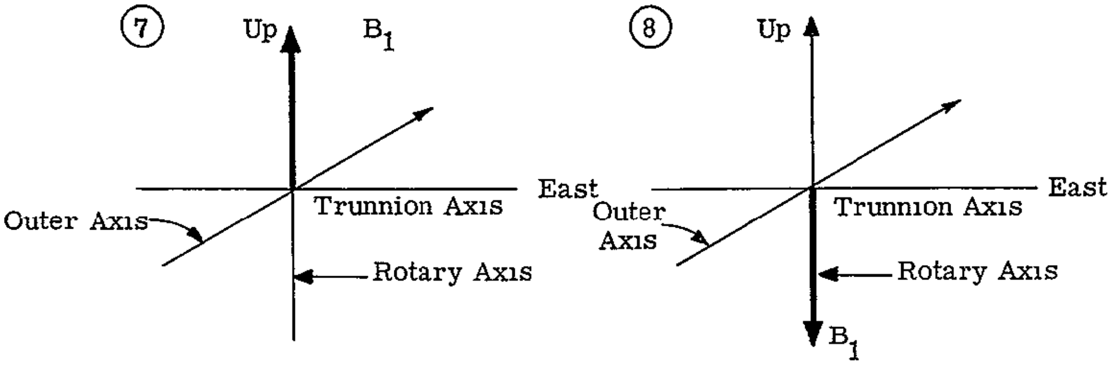
The procedures to obtain measurement data to be used to compute gyro bias (R), unbalance (B_I, B_O, B_S) and square compliance (C_{II}, C_{SS}) terms are listed in Chart II-11. These procedures utilize test table Positions 7 through 12 shown on Figure II-2.

CHART II-11

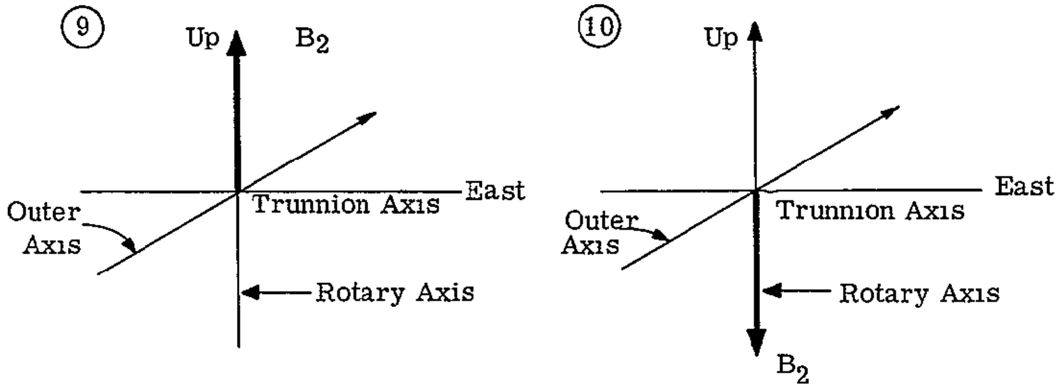
LABORATORY PROCEDURES

CALIBRATION OF GYRO BIAS, UNBALANCE, AND SQUARE COMPLIANCE TERMS – POSITIONS 7 TO 12

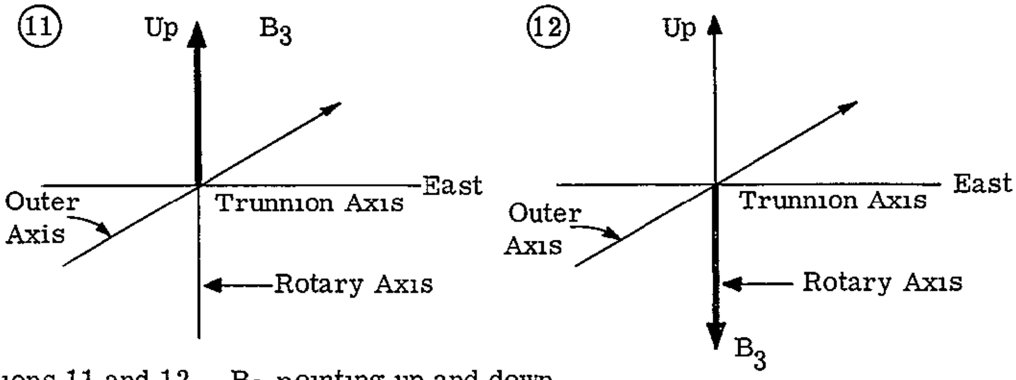
INSTRUCTIONS	✓	COMMENTS
1) Verify that the data required for the measurements are available on the Resolver Settings and Counter Settings Sheets.		1) See Section II-6.
2) Verify that the turn-on procedures have been performed and the equipment required in operating.		2) The turn-on procedures are given in Section I-1.
3) Connect gyros and master oscillator to frequency counters.		3) See Counter Connections Sheet (Positions 7 to 15, gyro) for connection assignments.
4) Set counters to "measure Z" mode.		
5) Set ϕ_1 to 0^o plus level correction and ϕ_2 to 90^o.		5) Level corrections are obtained from test table bubble levels.
6) Set ϕ_3 to $\phi_3^{m=7}$ plus level correction and ϕ_4 to $\phi_4^{m=7}$.		6) Settings obtained from Resolver Settings Sheet.
7) Set $(\Sigma n_1^T)_i^{Gm=7}$		7) These settings are on the Counter Settings Sheet.
8) Start measurement by switching counters to operate. After counter output, switch counters to standby.		
9) Record data $(\Sigma\delta)_i^{m=7}$ on Counter Outputs Sheet.		9) This number may be positive or negative.
10) Rotate ϕ_3 by 180^o.		10) This rotation moves table to Position 8.
11) Repeat 7 to 9 for m=8.		
12) Repeat 6 to 11 for m=9, 10; and m=11, 12.		



Positions 7 and 8. B_1 pointing up and down.



Positions 9 and 10. B_2 pointing up and down.



Positions 11 and 12. B_3 pointing up and down

In these positions, ϕ_1 is set to 0°, and ϕ_2 is set to 90°

Figure II-2. Test Table Orientations For Positions 6 to 12

II-2.2 Product Compliance Terms

The procedures to obtain measurement data to be used to compute gyro product compliance terms (C_{IO}, C_{IS}, C_{OS}) are listed in Chart II-12. These procedures use test table Positions 13, 14 and 15 shown on Figure II-3.

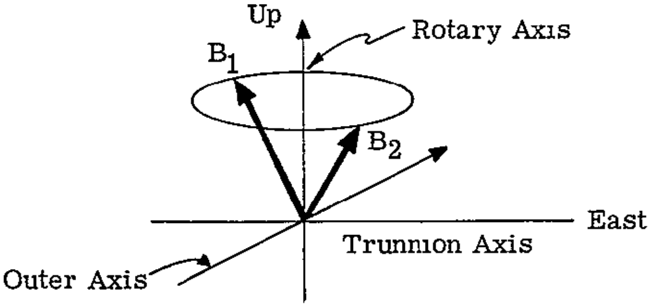
CHART II-12

LABORATORY PROCEDURES

CALIBRATION OF GYRO PRODUCT COMPLIANCE TERMS – POSITIONS 13 TO 15

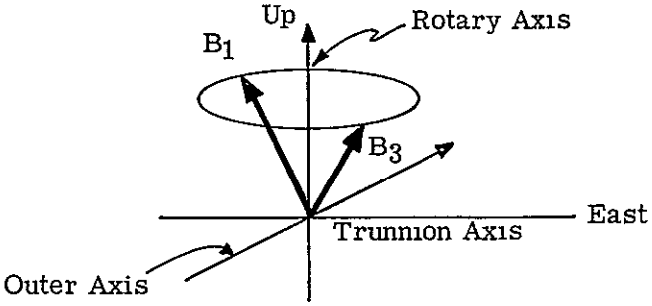
INSTRUCTIONS	✓	COMMENTS
1) Verify that the data required for the measurements are available on the Resolver Settings and Counter Settings Sheets.		1) See Section II-6.
2) Verify that the turn-on procedures have been performed and the equipment required is operating.		2) The turn-on procedures are given in Section I-1.
3) Connect gyros and master oscillator to frequency counters.		3) See Counter Connections Sheet (Positions 7 to 15, gyro) for connection assignments.
4) Set counters to "measure Z" mode.		
5) Set ϕ_1 to 0^o plus level correction and ϕ_2 to 90^o.		5) Level corrections are obtained from test table bubble levels.
6) Set ϕ_3 to $\phi_3^{m=13}$ plus level correction ϕ_4 to $\phi_4^{m=13}$.		6) Settings obtained from Resolver Settings Sheet.
7) Set $(\Sigma n_1^T)_i^{Gm=13}$ on frequency counters.		7) These settings are on the Counter Settings Sheet.
8) Start measurements by switching counters to operate. After counter output, switch counters to standby.		
9) Record data $(\Sigma\delta)_i^{m=13}$ on Counter Outputs Sheet.		9) These numbers may be positive or negative.
10) Repeat steps 6 to 9 for m=14; and m=15.		

⑬



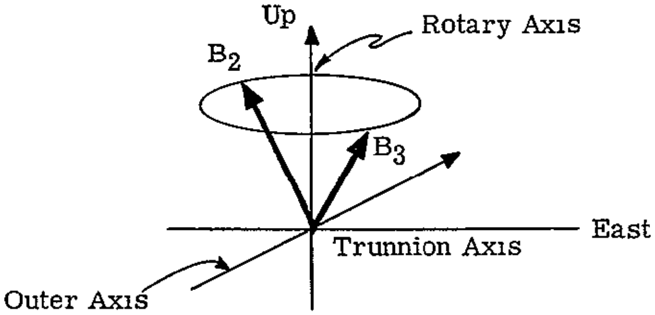
Position 13. B_1 and B_2 45° off the vertical

⑭



Position 14. B_1 and B_3 45° off the vertical

⑮



Position 15. B_2 and B_3 45° off the vertical

In these positions, ϕ_1 is set at 0°, and ϕ_2 at 90°.

Figure II-3. Test Table Orientations For Positions 13, 14, and 15

PART II
SECTION 3
CALIBRATION OF THE ACCELEROMETERS

Accelerometer calibration measurements are made at the same positions as used to determine the gyro acceleration sensitive terms (see Section II-2). Section II-3.1 describes the procedures for obtaining data for the determination of all accelerometer constants other than the third order term. Procedures to obtain data to determine the third order term are contained in II-3.2. The flow diagram descriptions of all procedures are contained in II-5.3.

II-3.1 Scale Factor, Bias, Second Order Terms and $(Q^A)^{-1}$

The procedures to obtain measurement data to be used to compute each accelerometer's scale factor (D_1), bias (D_0), second order term (D_2), and the transform from accelerometer to body axis frames are listed in Chart II-13. These procedures use test table Positions 7 through 12 which are illustrated in Figure II-2 in Section II-2.1.

CHART II-13

LABORATORY PROCEDURES

CALIBRATION OF ACCELEROMETER BIAS, SCALE FACTORS, SECOND ORDER TERMS $(QA)^{-1}$ – POSITIONS 7 TO 12

INSTRUCTIONS	✓	COMMENTS
1) Verify that the data required for the measurements are available on the Resolver Settings and Counter Settings Sheets.		1) See Section II-6.
2) Verify that the turn-on procedures have been performed and the equipment required is operating.		2) The turn-on procedures are given in Section I-1.
3) Connect accelerometers and master oscillator to frequency counters.		3) See Counter Connections Sheet (Positions 7 to 15, accelerometer) for connection assignments.
4) Set frequency counters to "measure X-Y" mode.		
5) Set ϕ_1 to 0^{o} plus level correction and ϕ_2 to 90^{o}.		5) Level corrections are obtained from the test table bubble levels.
6) Set ϕ_3 to $\phi_3^{m=7}$ plus level correction and ϕ_4 to $\phi_4^{m=7}$.		6) Settings obtained from Resolver Settings Sheet.
7) Set $(\Sigma\gamma)_{k\ell}^{m=7}$ on frequency counters.		7) These settings are on the Counter Settings Sheet.
8) Start measurement by switching counter to operate. After counter output, switch counters to standby.		
9) Record data $(\Sigma n_1^T)_{k\ell}^{Am=7}$ on Counter Outputs sheet.		
10) Rotate ϕ_3 by 180^{o}.		10) This rotation moves table to Position 8.
11) Repeat steps 7 to 9 for m=8.		
12) Repeat steps 6 to 11 for m=9, 10; and m=11, 12.		

II-3.2 Third Order Terms

The procedures to obtain measurement data to be used to compute the third order term (D_3) of each accelerometer are listed in Chart II-14. These procedures use test table Positions 13 and 14, which are shown on Figure II-3 in Section II-2.2.

CHART II-14

LABORATORY PROCEDURES

CALIBRATION OF ACCELEROMETER THIRD ORDER TERMS-POSITIONS 13 AND 14

INSTRUCTIONS	✓	COMMENTS
1) Verify that the data required for the measurements are available on the Resolver Setting and Counter Settings Sheets.		1) See Section II-6.
2) Verify that the turn-on procedures have been performed and the equipment required is operating.		2) The turn-on procedures are given in Section I-1.
3) Connect accelerometers and master oscillator to frequency counters.		3) See Counter Connections Sheet (positions 7 to 15 accelerometer) for connection assignment.
4) Set frequency counters to "measure X-Y" mode.		
5) Set ϕ_1 to 0^o plus level correction and ϕ_2 90^o.		5) Level corrections are obtained from the test table bubble levels.
6) Set ϕ_3 to $\phi^{m=13}$ plus level correction and ϕ_4 to $\phi_4^{m=13}$.		6) Settings obtained from Resolver Settings Sheet.
7) Set $(\Sigma\gamma)_{k\ell}^{m=13}$ on frequency counters.		7) These settings are on the Counter Settings Sheet.
8) Start measurement by switching counters to operate. After counter output, switch counters to standby.		
9) Record data $(\Sigma n_1^T)_{k\ell}^{Am=13}$ on Counter Outputs Sheet.		
10) Repeat steps 5 to 8 for m=14.		

PART II
SECTION 4
CALIBRATION COMPUTATIONS

Contained herein are the procedures to accomplish the calibration computations, and the equations to be programmed to determine the calibration constants. The first subsection contains the procedures listing to accomplish the computations. The equations for the gyro and the accelerometer calibration constants are presented in II-4.2 and II-4.3, respectively. These equations are presented in sufficient detail to define the program. The computations may be programmed to run on any general purpose computer. The program should be constructed so that it is general enough to allow computations for a partial calibration. Schematic illustrations of the procedures to accomplish the calibration computations are contained in Section II-5.4.

II-4.1 Calibration Computation Procedures

The procedures to perform calibration computations are listed in Chart II-15. Chart II-16 contains a list of positions at which measurements must be made in order that a given constant can be evaluated. Many constants are determined from measurements at only one or two positions. Many others require a knowledge of other calibration constants. In all cases, the complete set of required positions are shown on the chart, including those positions required in the determination of prerequisite constants, which are shown in parentheses. The scale factor ($\Delta\Phi$ or D_1) is also required for each other term.

CHART II-15

LABORATORY PROCEDURES

CALIBRATION COMPUTATIONS

INSTRUCTIONS	✓	COMMENTS
1) Prepare data for entry.		
2) Enter calibration program into computer.		
3) Enter system constants from Survey Results Sheet.		3) Includes g, $\omega^E \cos\lambda$, $\omega^E \sin\lambda$ and scale factors not permanently in program.
4) Enter T_{12}^{BRm}, T_{22}^{BRm}, T_{32}^{BRm} for m = 1, 3, 5, 13, 14, 15 from system survey procedure.		
5) Enter counter settings from Counter Settings Sheet.		5) It will be desirable to perform partial calibration so that in some cases partial data lists will be entered.
6) Enter counter outputs from Counter Outputs Sheet.		
7) If it is desired to calibrate the accelerometer without performing positions 13 and 14, enter last values of $(D_1 D_3)$ for all accelerometers.		
8) Computer computes and outputs constants.		
9) Record all data that is valid on Calibration Results Sheet.		9) For partial calibration, refer to Chart II-16 to determine which constants are valid.

CHART II-16

CALIBRATION RESULTS SHEET

Experiment:
Principal Operator:
Date:

PROCEDURE DESCRIPTION

Survey Results Sheet Used ____________________
Resolver Settings Sheet Used ____________________
Counter Settings Sheet Used ____________________
Counter Outputs Sheet Used ____________________
Special Procedures ____________________
____
____

RESULTS

Parameter	Units	Instrument 1	Instrument 2	Instrument 3
Gyros				
$\Delta \Phi$	Deg/Pulse	1-6	1-6	1-6
$(\underline{G}_i \cdot \underline{B}_1)$		1, 2	1, 2	1, 2
$(\underline{G}_i \cdot \underline{B}_2)$		3, 4	3, 4	3, 4
$(\underline{G}_i \cdot \underline{B}_3)$		5, 6	5, 6	5, 6
R	Deg/Hr.	11, 12 (1 to 4)	7, 8(3to6)	7, 8, (3 to 6)
B_I	Deg/Hr. -g	7, 8 (1, 2)	9, 10(3, 4)	11, 12 (5, 6)
B_O	Deg/Hr. -g	11, 12 (5, 6)	7, 8(1, 2)	7, 8 (1, 2)
B_S	Deg/Hr. -g	9, 10 (3, 4)	11, 12(5, 6)	9, 10 (3, 4)
C_{II}	Deg/Hr. $-g^2$	7, 8(1to 6, 11, 12)	9, 10(1to8)	11, 12 (1 to 8)
C_{SS}	Deg/Hr. $-g^2$	9, 10(1to6, 11, 12)	11, 12(1to8)	9, 10 (1 to 8)
C_{IS}	Deg/Hr. $-g^2$	13 (1 to 12)	15 (1 to 12)	15 (1 to 12)
C_{OS}	Deg/Hr. $-g^2$	15(1to6, 9to12)	14 (1to8, 11, 12)	13 (1 to 10)
C_{IO}	Deg/Hr. $-g^2$	14(1to8, 11, 12	13 (1 to 10)	14 (1to8, 11, 12)
Q_{II}	Hr. /Deg.	Special Experiments		
Q_{IS}	Hr. /Deg.	Special Experiments		
J	Hours	Special Experiments		
Accelerometers				
D_1	(Pulses/Sec)/g	7,8,13(9,10,Do)	9, 10, 13, (7,8,Do)	11,12,14(7,8,Do)
$(\underline{A}_i \cdot \underline{B}_1)$		7,8	7, 8	7, 8
$(\underline{A}_i \cdot \underline{B}_2)$		9, 10	9, 10	9, 10
$(\underline{A}_i \cdot \underline{B}_3)$		11, 12	11, 12	11, 12
D_0	g	9, 10 or 11, 12	7, 8 or 11, 12	7, 8 or 9, 10
D_2	g/g^2	7, 8 (Do)	9, 10 (Do)	11, 12 (Do)
D_3	g/g^3	Same as D_1	Same as D_1	Same as D_1

II-4.2 Gyro Computations

The gyro computations are shown on the following pages. The reader should note that all the equations except those for product compliance (C_{IS} or C_{IO}) contain terms of the form

$$\frac{(\Sigma\,\delta)_i^m}{(\Sigma n_1^T)_i^{Gm}} \pm \frac{(\Sigma\,\delta)_i^{m+1}}{(\Sigma n_1^T)_i^{G(m+1)}}$$

where m = 1, 3, 5, 7, 9, or 11. This suggests that it may be convenient to perform these computations as a special routine.

The Q^G matrix is defined from the misalignment terms as follows:

$$Q^G = \left| \begin{array}{ccc} 1 & -(\underline{G}_1 \cdot \underline{B}_2) & -(\underline{G}_1 \cdot \underline{B}_3) \\ -(\underline{G}_2 \cdot \underline{B}_1) & 1 & -(\underline{G}_2 \cdot \underline{B}_3) \\ -(\underline{G}_3 \cdot \underline{B}_1) & -(\underline{G}_3 \cdot \underline{B}_2) & 1 \end{array} \right|$$

The Q^A matrix is defined from the misalignment terms as follows:

$$Q^A = \left| \begin{array}{ccc} 1 & -(\underline{A}_1 \cdot \underline{B}_2) & -(\underline{A}_1 \cdot \underline{B}_3) \\ -(\underline{A}_2 \cdot \underline{B}_1) & 1 & -(\underline{A}_2 \cdot \underline{B}_3) \\ -(\underline{A}_3 \cdot \underline{B}_1) & -(\underline{A}_3 \cdot \underline{B}_2) & 1 \end{array} \right|$$

GYRO CALIBRATION EQUATIONS

Scale Factor and Misalignments

$$[\Delta\Phi]_k = \left\{ \left[\frac{\underline{G}_k \cdot \underline{B}_1}{\Delta\Phi}\right]_k^2 + \left[\frac{\underline{G}_k \cdot \underline{B}_2}{\Delta\Phi}\right]_k^2 + \left[\frac{\underline{G}_k \cdot \underline{B}_3}{\Delta\Phi}\right]_k^2 \right\}^{-1/2}$$

Gyro One

$$[(\underline{G}_1 \cdot \underline{B}_1)/\Delta\Phi]_1 = \frac{\left[\dfrac{(\Sigma\delta)_1^1}{(\Sigma n_1^T)^1} - \dfrac{(\Sigma\delta)_1^2}{(\Sigma n_1^T)^2}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^1}{(\Sigma n_1^T)^1} + \dfrac{(\Sigma n^{\phi})^2}{(\Sigma n_1^T)^2}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

$$[\underline{G}_1 \cdot \underline{B}_2)/\Delta\Phi]_1 = \frac{\left[\dfrac{(\Sigma\delta)_1^3}{(\Sigma n_1^T)^3} - \dfrac{(\Sigma\delta)_1^4}{(\Sigma n_1^T)^4}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^3}{(\Sigma n_1^T)^3} + \dfrac{(\Sigma n^{\phi})^4}{(\Sigma n_1^T)^4}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

$$[(\underline{G}_1 \cdot \underline{B}_3)/\Delta\Phi]_1 = \frac{\left[\dfrac{(\Sigma\delta)_1^5}{(\Sigma n_1^T)^5} - \dfrac{(\Sigma\delta)_1^6}{(\Sigma n_1^T)^6}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^5}{(\Sigma n_1^T)^5} + \dfrac{(\Sigma n^{\phi})^6}{(\Sigma n_1^T)^6}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

Gyro Two

$$[(\underline{G}_2 \cdot \underline{B}_1)/\Delta\Phi]_2 = \frac{\left[\dfrac{(\Sigma\delta)_2^1}{(\Sigma n_1^T)^1} - \dfrac{(\Sigma\delta)_2^2}{(\Sigma n_1^T)^2}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^1}{(\Sigma n_1^T)^1} + \dfrac{(\Sigma n^{\phi})^2}{(\Sigma n_1^T)^2}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

GYRO CALIBRATION EQUATIONS (Continued)

Gyro Two (Continued)

$$[(\underline{G}_2 \cdot \underline{B}_2)/\Delta\Phi]_2 = \frac{\left[\dfrac{(\Sigma\delta)_2^3}{(\Sigma n_1^T)^3} - \dfrac{(\Sigma\delta)_2^4}{(\Sigma n_1^T)^4}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^3}{(\Sigma n_1^T)^3} + \dfrac{(\Sigma n^{\phi})^4}{(\Sigma n_1^T)^4}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

$$[(\underline{G}_2 \cdot \underline{B}_3)/\Delta\Phi]_2 = \frac{\left[\dfrac{(\Sigma\delta)_2^5}{(\Sigma n_1^T)^5} - \dfrac{(\Sigma\delta)_2^6}{(\Sigma n_1^T)^6}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^5}{(\Sigma n_1^T)^5} + \dfrac{(\Sigma n^{\phi})^6}{(\Sigma n_1^T)^6}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

Gyro Three

$$[(\underline{G}_3 \cdot \underline{B}_1)/\Delta\Phi]_3 = \frac{\left[\dfrac{(\Sigma\delta)_3^1}{(\Sigma n_1^T)^1} - \dfrac{(\Sigma\delta)_3^2}{(\Sigma n_1^T)^2}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^1}{(\Sigma n_1^T)^1} + \dfrac{(\Sigma n^{\phi})^2}{(\Sigma n_1^T)^2}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

$$[(\underline{G}_3 \cdot \underline{B}_2)/\Delta\Phi]_3 = \frac{\left[\dfrac{(\Sigma\delta)_3^3}{(\Sigma n_1^T)^3} - \dfrac{(\Sigma\delta)_3^4}{(\Sigma n_1^T)^4}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^3}{(\Sigma n_1^T)^3} + \dfrac{(\Sigma n^{\phi})^4}{(\Sigma n_1^T)^4}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

GYRO CALIBRATION EQUATIONS (Continued)

Gyro Three (Continued)

$$[(\underline{G}_3 \cdot \underline{B}_3)/\Delta\Phi]_3 = \frac{\left[\dfrac{(\Sigma\delta)_3^5}{(\Sigma n_1^T)^5} - \dfrac{(\Sigma\delta)_3^6}{(\Sigma n_1^T)^6}\right]}{\left[S^{\phi}\left(\dfrac{(\Sigma n^{\phi})^5}{(\Sigma n_1^T)^5} + \dfrac{(\Sigma n^{\phi})^6}{(\Sigma n_1^T)^6}\right) - 2S_1^T \omega^E \sin\lambda\right]}$$

Bias

$$[R]_k = [R/\Delta\Phi]_k \, [\Delta\Phi]_k$$

Gyro One

$$\begin{aligned}[R/\Delta\Phi]_1 = \frac{1}{2S_1^T} & \left[\frac{(\Sigma\delta)_1^{11}}{(\Sigma n_1^T)_1^{G11}} + \frac{(\Sigma\delta)_1^{12}}{(\Sigma n_1^T)_1^{G12}}\right] \\ & - \omega^E \sin\lambda \left[T_{12}^{BR5} [(\underline{G}_1 \cdot \underline{B}_1)/\Delta\Phi]_1 + T_{22}^{BR5} [(\underline{G}_1 \circ \underline{B}_2)/\Delta\Phi]_1\right]\end{aligned}$$

Gyro Two

$$\begin{aligned}[R/\Delta\Phi]_2 = \frac{1}{2S_1^T} & \left[\frac{(\Sigma\delta)_2^7}{(\Sigma n_1^T)_2^{G7}} + \frac{(\Sigma\delta)_2^8}{(\Sigma n_1^T)_2^{G8}}\right] \\ & - \omega^E \sin\lambda \left[T_{22}^{BR1} [(\underline{G}_2 \cdot \underline{B}_2)/\Delta\Phi]_2 + T_{32}^{BR1} [(\underline{G}_2 \cdot \underline{B}_3)/\Delta\Phi]_2\right]\end{aligned}$$

Gyro Three

$$\begin{aligned}[R/\Delta\Phi]_3 = \frac{1}{2S_1^T} & \left[\frac{(\Sigma\delta)_3^7}{(\Sigma n_1^T)_3^{G7}} + \frac{(\Sigma\delta)_3^8}{(\Sigma n_1^T)_3^{G8}}\right] \\ & - \omega^E \sin\lambda \left[T_{32}^{BR1} [(\underline{G}_3 \cdot \underline{B}_3)/\Delta\Phi]_3 + T_{22}^{BR1} [(\underline{G}_3 \cdot \underline{B}_2)/\Delta\Phi]_3\right]\end{aligned}$$

GYRO CALIBRATION EQUATIONS (Continued)

Unbalance

$$[B]_k = [B/\Delta\Phi]_k \, [\Delta\Phi]_k$$

Gyro One

$$[B_I/\Delta\Phi]_1 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\delta)_1^7}{(\Sigma n_1^T)_1^{G7}} - \frac{(\Sigma\delta)_1^8}{(\Sigma n_1^T)_1^{G8}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_1 \cdot \underline{B}_1)/\Delta\Phi]_1$$

$$[B_O/\Delta\Phi]_1 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\delta)_1^{11}}{(\Sigma n_1^T)_1^{G11}} - \frac{(\Sigma\delta)_1^{12}}{(\Sigma n_1^T)_1^{G12}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_1 \cdot \underline{B}_3)/\Delta\Phi]_1$$

$$[B_S/\Delta\Phi]_1 = \frac{-1}{2gS_1^T}\left[\frac{(\Sigma\delta)_1^9}{(\Sigma n_1^T)_1^{G9}} - \frac{(\Sigma\delta)_1^{10}}{(\Sigma n_1^T)_1^{G10}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_1 \cdot \underline{B}_2)/\Delta\Phi]_1$$

Gyro Two

$$[B_I/\Delta\Phi]_2 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\delta)_2^9}{(\Sigma n_1^T)_2^{G9}} - \frac{(\Sigma\delta)_2^{10}}{(\Sigma n_1^T)_2^{G10}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_2 \cdot \underline{B}_2)/\Delta\Phi]_2$$

$$[B_O/\Delta\Phi]_2 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\delta)_2^7}{(\Sigma n_1^T)_2^{G7}} - \frac{(\Sigma\delta)_2^8}{(\Sigma n_1^T)_2^{G8}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_2 \cdot \underline{B}_1)/\Delta\Phi]_2$$

$$[B_S/\Delta\Phi]_2 = \frac{-1}{2gS_1^T}\left[\frac{(\Sigma\delta)_2^{11}}{(\Sigma n_1^T)_2^{G11}} - \frac{(\Sigma\delta)_2^{12}}{(\Sigma n_1^T)_2^{G12}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_2 \cdot \underline{B}_3)/\Delta\Phi]_2$$

GYRO CALIBRATION EQUATIONS (Continued)

Gyro Three

$$[B_I/\Delta\Phi]_1 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\delta)_3^{11}}{(\Sigma n_1^T)_3^{G11}} - \frac{(\Sigma\delta)_3^{12}}{(\Sigma n_1^T)_3^{G12}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_3 \cdot \underline{B}_3)/\Delta\Phi]_3$$

$$[B_O/\Delta\Phi]_3 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\delta)_3^{7}}{(\Sigma n_1^T)_3^{G7}} - \frac{(\Sigma\delta)_3^{8}}{(\Sigma n_1^T)_3^{G8}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_3 \cdot \underline{B}_1)/\Delta\Phi]_3$$

$$[B_S/\Delta\Phi]_3 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\delta)_3^{9}}{(\Sigma n_1^T)_3^{G9}} - \frac{(\Sigma\delta)_3^{10}}{(\Sigma n_1^T)_3^{G10}}\right] - \frac{\omega^E \cos\lambda}{g}\,[(\underline{G}_3 \cdot \underline{B}_2)/\Delta\Phi]_3$$

Square Compliance

$$[C]_k = [C/\Delta\Phi]_k[\Delta\Phi]_k$$

Gyro One

$$[C_{II}/\Delta\Phi]_1 = \frac{1}{2g^2S_1^T}\left[\frac{(\Sigma\delta)_1^{7}}{(\Sigma n_1^T)_1^{G7}} + \frac{(\Sigma\delta)_1^{8}}{(\Sigma n_1^T)_1^{G8}}\right] - \frac{1}{g^2}[R/\Delta\Phi]_1$$
$$- \frac{\omega^E \sin\lambda}{g^2}\left[T_{22}^{BR1}\,[(\underline{G}_1 \cdot \underline{B}_2)/\Delta\Phi]_1 + T_{32}^{BR1}\,[(\underline{G}_1 \cdot \underline{B}_3)/\Delta\Phi]_1\right]$$

$$[C_{SS}/\Delta\Phi]_1 = \frac{1}{2g^2S_1^T}\left[\frac{(\Sigma\delta)_1^{9}}{(\Sigma n_1^T)_1^{G9}} + \frac{(\Sigma\delta)_1^{10}}{(\Sigma n_1^T)_1^{G10}}\right] - \frac{1}{g^2}[R/\Delta\Phi]_1$$
$$- \frac{\omega^E \sin\lambda}{g^2}\left[T_{12}^{BR3}\,[(\underline{G}_1 \cdot \underline{B}_1)/\Delta\Phi]_1 + T_{32}^{BR3}\,[(\underline{G}_1 \cdot \underline{B}_3)/\Delta\Phi]_1\right]$$

GYRO CALIBRATION EQUATIONS (Continued)

Gyro Two

$$[C_{II}/\Delta\Phi]_2 = \frac{1}{2g^2S_1^T}\left[\frac{(\Sigma\delta)_2^9}{(\Sigma n_1^T)_2^{G9}} + \frac{(\Sigma\delta)_2^{10}}{(\Sigma n_1^T)_2^{G10}}\right] - \frac{1}{g^2}[R/\Delta\Phi]_2$$

$$- \frac{\omega^E \sin\lambda}{g^2}\left[T_{12}^{BR3}\,[(\underline{G}_2\cdot\underline{B}_1)\Delta\Phi]_2 + T_{32}^{BR3}\,[(\underline{G}_2\cdot\underline{B}_3)/\Delta\Phi]_2\right]$$

$$[C_{SS}/\Delta\Phi]_2 = \frac{1}{2g^2S_1^T}\left[\frac{(\Sigma\delta)_2^{11}}{(\Sigma n_1^T)_2^{G11}} + \frac{(\Sigma\delta)_2^{12}}{(\Sigma n_1^T)_2^{G12}}\right] - \frac{1}{g^2}[R/\Delta\Phi]_2$$

$$- \frac{\omega^E \sin\lambda}{g^2}\left[T_{12}^{BR5}\,[(\underline{G}_2\cdot\underline{B}_1)/\Delta\Phi]_2 + T_{22}^{BR5}\,[(\underline{G}_2\circ\underline{B}_2)/\Delta\Phi]_2\right]$$

Gyro Three

$$[C_{II}/\Delta\Phi]_3 = \frac{1}{2g^2S_I^T}\left[\frac{(\Sigma\delta)_3^{11}}{(\Sigma n_1^T)_3^{G11}} + \frac{(\Sigma\delta)_3^{12}}{(\Sigma n_1^T)_3^{G12}}\right] - \frac{1}{g^2}[R/\Delta\Phi]_3$$

$$- \frac{\omega^E \sin\lambda}{g^2}\left[T_{12}^{BR5}\,[(\underline{G}_3\cdot\underline{B}_1)/\Delta\Phi]_3 + T_{22}^{BR5}\,[(\underline{G}_3\circ\underline{B}_2)\Delta\Phi]_3\right]$$

$$[C_{SS}/\Delta\Phi]_3 = \frac{1}{2g^2S_1^T}\left[\frac{(\Sigma\delta)_3^9}{(\Sigma n_1^T)_3^{G9}} + \frac{(\Sigma\delta)_3^{10}}{(\Sigma n_1^T)_3^{G10}}\right] - \frac{1}{g^2}[R/\Delta\Phi]_3$$

$$- \frac{\omega^E \sin\lambda}{g^2}\left[T_{12}^{BR3}\,[(\underline{G}_3\cdot\underline{B}_1)/\Delta\Phi]_3 + T_{32}^{BR3}\,[(\underline{G}_3\cdot\underline{B}_3)/\Delta\Phi]_3\right]$$

Product Compliance

Gyro One

$$[C_{IS}/\Delta\Phi]_1 = -\frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_1^{13}}{(\Sigma n_1^T)_1^{G13}}\right] + \frac{2}{g^2}[R/\Delta\Phi]_1 + \frac{\sqrt{2}}{g}\left[[B_I/\Delta\Phi]_1 - [B_S/\Delta\Phi]_1\right]$$

$$-\frac{2\omega^E}{g^2}\left[[\underline{G}_1\cdot\underline{B}_1/\Delta\Phi]_1\left[\sqrt{1/2}\cos\lambda + T_{12}^{BR13}\sin\lambda\right] + [(\underline{G}_1\cdot\underline{B}_2)/\Delta\Phi]_1\right.$$

$$\left.\left[\sqrt{1/2}\cos\lambda + T_{22}^{BR13}\sin\lambda\right] + [\underline{G}_1\circ\underline{B}_3/\Delta\Phi]_1\left[T_{32}^{BR13}\sin\lambda\right]\right]$$

$$+ [C_{II}/\Delta\Phi]_1 + [C_{SS}/\Delta\Phi]_1$$

$$[C_{IO}/\Delta\Phi]_1 = +\frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_1^{14}}{(\Sigma n_1^T)_1^{G14}}\right] - \frac{2}{g^2}[R/\Delta\Phi]_1 - \frac{\sqrt{2}}{g}\left[[B_I/\Delta\Phi]_1 + [B_O/\Delta\Phi]_1\right]$$

$$-\frac{2\omega^E}{g^2}\left[[\underline{G}_1\cdot\underline{B}_1/\Delta\Phi]_1\left[\sqrt{1/2}\cos\lambda + T_{12}^{BR14}\sin\lambda\right] + [\underline{G}_1\cdot\underline{B}_2/\Delta\Phi]_1\right.$$

$$\left.\left[T_{22}^{BR14}\sin\lambda\right] + [\underline{G}_1\cdot\underline{B}_3/\Delta\Phi]_1\left[\sqrt{1/2}\cos\lambda + T_{32}^{BR14}\sin\lambda\right]\right] - [C_{II}/\Delta\Phi]_1$$

$$[C_{OS}/\Delta\Phi]_1 = -\frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_1^{15}}{(\Sigma n_1^T)_1^{G15}}\right] + \frac{2}{g^2}[R/\Delta\Phi]_1 + \frac{\sqrt{2}}{g}\left[-[B_S/\Delta\Phi]_1 + [B_O/\Delta\Phi]_1\right]$$

$$+\frac{2\omega^E}{g^2}\left[[\underline{G}_1\cdot\underline{B}_1/\Delta\Phi]_1\left(T_{12}^{BR15}\sin\lambda\right) + [\underline{G}_1\cdot\underline{B}_2/\Delta\Phi]_1\right.$$

$$\left.\left(\sqrt{1/2}\cos\lambda + T_{22}^{BR15}\sin\lambda\right) + [\underline{G}_1\cdot\underline{B}_3/\Delta\Phi]_1\left[\sqrt{1/2}\cos\lambda + T_{32}^{BR15}\sin\lambda\right]\right]$$

$$+ [C_{SS}/\Delta\Phi]_1$$

Gyro Two

$$
\begin{aligned}
[C_{IS}/\Delta\Phi]_2 = {} & -\frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_2^{15}}{(\Sigma n_1^T)_2^{G15}}\right] + \frac{2}{g^2}[R/\Delta\Phi]_2 + \frac{\sqrt{2}}{g}\Big[[B_I/\Delta\Phi]_2 - [B_S/\Delta\Phi]_2\Big] \\
& + \frac{2\omega^E}{g^2}\Big[[\underline{G}_2 \cdot \underline{B}_1/\Delta\Phi]_2 \left[T_{12}^{BR15} \sin\lambda\right] + [\underline{G}_2 \cdot \underline{B}_2/\Delta\Phi]_2 \\
& \left[\sqrt{1/2}\cos\lambda + T_{22}^{BR15} \sin\lambda\right] + [\underline{G}_2 \cdot \underline{B}_3/\Delta\Phi]_2 \left[\sqrt{1/2}\cos\lambda + T_{32}^{BR15} \sin\lambda\right]\Big] \\
& + [C_{II}/\Delta\Phi]_2 + [C_{SS}/\Delta\Phi]_2
\end{aligned}
$$

$$
\begin{aligned}
[C_{IO}/\Delta\Phi]_2 = {} & \frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_2^{13}}{(\Sigma n_1^T)_2^{G13}}\right] - \frac{2}{g^2}[R/\Delta\Phi]_2 - \frac{\sqrt{2}}{g}\Big[[B_I/\Delta\Phi]_2 + [B_O/\Delta\Phi]_2\Big] \\
& - \frac{2\omega^E}{g^2}\Big[[\underline{G}_2 \cdot \underline{B}_1/\Delta\Phi]_2 . \left[\sqrt{1/2}\cos\lambda + T_{12}^{BR13} \sin\lambda\right] \\
& + [\underline{G}_2 \cdot \underline{B}_2/\Delta\Phi]_2 \left[\sqrt{1/2}\cos\lambda + T_{22}^{BR13} \sin\lambda\right] + [\underline{G}_2 \cdot \underline{B}_3/\Delta\Phi]_2 \\
& \left[T_{32}^{BR13} \sin\lambda\right]\Big] - [C_{II}/\Delta\Phi]_2
\end{aligned}
$$

$$
\begin{aligned}
[C_{OS}/\Delta\Phi]_2 = {} & -\frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_2^{14}}{(\Sigma n_1^T)_2^{G14}}\right] + \frac{2}{g^2}[R/\Delta\Phi]_2 + \frac{\sqrt{2}}{g}\Big[-[B_S/\Delta\Phi]_2 + [B_O/\Delta\Phi]_2\Big] \\
& + \frac{2\omega^E}{g^2}\Big[[\underline{G}_2 \cdot \underline{B}_1/\Delta\Phi]_2 \left[\sqrt{1/2}\cos\lambda + T_{12}^{BR14} \sin\lambda\right] + [\underline{G}_2 \cdot \underline{B}_2/\Delta\Phi]_2 \\
& \left[T_{22}^{BR14} \sin\lambda\right] + [\underline{G}_2 \cdot \underline{B}_3/\Delta\Phi]_2 \left[\sqrt{1/2}\cos\lambda + T_{32}^{BR14} \sin\lambda\right]\Big] \\
& + [C_{SS}/\Delta\Phi]_2
\end{aligned}
$$

Gyro Three

$$[C_{IS}/\Delta\Phi]_3 = \frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_3^{15}}{(\Sigma n_1^T)_3^{G15}}\right] - \frac{2}{g^2}[R/\Delta\Phi]_3 - \frac{\sqrt{2}}{g}\left[[B_I/\Delta\Phi]_3 + [B_S/\Delta\Phi]_3\right]$$

$$-\frac{2\omega^E}{g^2}\left[[\underline{G}_3 \cdot \underline{B}_1/\Delta\Phi]_3 \left[T_{12}^{BR15}\sin\lambda\right] + [\underline{G}_3 \cdot \underline{B}_2/\Delta\Phi]_3\right.$$

$$\left.\left[\sqrt{1/2}\cos\lambda + T_{22}^{BR15}\sin\lambda\right] + [\underline{G}_3 \cdot \underline{B}_3/\Delta\Phi]_3 \left[\sqrt{1/2}\cos\lambda + T_{32}^{BR15}\sin\lambda\right]\right]$$

$$- [C_{II}/\Delta\Phi]_3 - [C_{SS}/\Delta\Phi]_3$$

$$[C_{IO}/\Delta\Phi]_3 = \frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_3^{14}}{(\Sigma n_1^T)_3^{G14}}\right] - \frac{2}{g^2}[R/\Delta\Phi]_3 - \frac{\sqrt{2}}{g}\left[[B_I/\Delta\Phi]_3 + [B_O/\Delta\Phi]_3\right]$$

$$-\frac{2\omega^E}{g^2}\left[[\underline{G}_3 \cdot \underline{B}_1/\Delta\Phi]_3 \left[\sqrt{1/2}\cos\lambda + T_{12}^{BR14}\sin\lambda\right] + [\underline{G}_3 \cdot \underline{B}_2/\Delta\Phi]_3\right.$$

$$\left.\left[T_{22}^{BR14}\sin\lambda\right] + [\underline{G}_3 \cdot \underline{B}_3/\Delta\Phi]_3 \left[\sqrt{1/2}\cos\lambda + T_{32}^{BR14}\sin\lambda\right]\right] - [C_{II}/\Delta\Phi]_3$$

$$[C_{OS}/\Delta\Phi]_3 = \frac{2}{g^2 S_1^T}\left[\frac{(\Sigma\delta)_3^{13}}{(\Sigma n_1^T)_3^{G13}}\right] - \frac{2}{g^2}[R/\Delta\Phi]_3 - \frac{\sqrt{2}}{g}\left[[B_S/\Delta\Phi]_3 + [B_O/\Delta\Phi]_3\right]$$

$$-\frac{2\omega^E}{g^2}\left[[\underline{G}_3 \cdot \underline{B}_1/\Delta\Phi]_3 \left[\sqrt{1/2}\cos\lambda + T_{12}^{BR13}\sin\lambda\right] + [\underline{G}_3 \cdot \underline{B}_2/\Delta\Phi]_3\right.$$

$$\left.\left[\sqrt{1/2}\cos\lambda + T_{22}^{BR13}\sin\lambda\right] + [\underline{G}_3 \cdot \underline{B}_3/\Delta\Phi]_3 \left[T_{32}^{BR13}\sin\lambda\right]\right] - [C_{SS}/\Delta\Phi]_3$$

GYRO CALIBRATION EQUATIONS (Continued)

J Term Equations

The J term equations will be supplied at the time when the procedures for J term calibration measurements are specified. At that time this page will be replaced by a page containing those equations. A discussion of the procedures to be performed to calibrate the J term is presented in Section 4.2.4 of the Development Document.

II-4.3 Accelerometer Computations

The computations to be performed to determine the accelerometer calibration constants are shown on the following pages. The misalignments are the off diagonal terms of the inverse of the matrix (Q^A) which transforms from the accelerometer input axes frame to the body axes frame.

The form of these equations suggest that it might be convenient to compute the terms

$$\frac{(\Sigma\gamma)^m_{i2}}{(\Sigma n^T_1)^{Am}_{i2}} - \frac{(\Sigma\gamma)^m_{i1}}{(\Sigma n^T_1)^{Am}_{i1}}$$

and the sums and differences of these terms in a special routine that is performed before the specific calibration constant determination.

The accelerometer third order term cannot be separated from the scale factor by a choice of positions. In the following set of equations there are two equations given for each accelerometer that relate the scale factor term $[D_1(\underline{A}_i \cdot \underline{B}_i)]_i$ to the third order term $(D_1D_3)_i$. If a simultaneous solution of the two equations is used to determine the scale factor and the third order term, then the scale factor will be sensitive to errors in the bias and the second order term. These terms appear on the equation listed second in each of the three sets of two equations. This may be avoided by determining the third order term $(D_1D_3)_i$ (by simultaneous solution or other methods) and using this value to solve the first equation for $[D_1(\underline{A}_i \cdot \underline{B}_i)]_i$. This value is subject to the accuracy of other terms only through the extremely small term containing D_3. (D_1) is then given by the square root of the sum of the squares of $[D_1(\underline{A}_i \cdot \underline{B}_j)]$ for $j = 1, 2, 3$.

ACCELEROMETER CALIBRATION EQUATIONS

Scale Factor and Cubic Term

Accelerometer One

$$[D_1(\underline{A}_1 \cdot \underline{B}_1)]_1 + g^2 [D_1 D_3]_1 = \frac{1}{2gS_1^T} \left[\frac{(\Sigma\gamma)_{12}^7}{(\Sigma n_1^T)_{12}^{A7}} - \frac{(\Sigma\gamma)_{11}^7}{(\Sigma n_1^T)_{11}^{A7}} - \frac{(\Sigma\gamma)_{12}^8}{(\Sigma n_1^T)_{12}^{A8}} + \frac{(\Sigma\gamma)_{11}^8}{(\Sigma n_1^T)_{11}^{A8}} \right]$$

$$[D_1(\underline{A}_1 \cdot \underline{B}_1)]_1 + \frac{g^2}{2} [D_1 D_3]_1 = \frac{\sqrt{2}}{gS_1^T} \left[\frac{(\Sigma\gamma)_{12}^{13}}{(\Sigma n_1^T)_{12}^{A13}} - \frac{(\Sigma\gamma)_{11}^{13}}{(\Sigma n_1^T)_{11}^{A13}} \right] - [D_1(\underline{A}_1 \cdot \underline{B}_2)]_1 - \frac{\sqrt{2}}{g} [D_1 D_0]_1$$

$$- \frac{\sqrt{2}\,g}{2} [D_1 D_2]_1$$

Accelerometer Two

$$[D_1(\underline{A}_2 \cdot \underline{B}_2)]_2 + g^2 [D_1 D_3]_2 = \frac{1}{2gS_1^T} \left[\frac{(\Sigma\gamma)_{22}^9}{(\Sigma n_1^T)_{22}^{A9}} - \frac{(\Sigma\gamma)_{21}^9}{(\Sigma n_1^T)_{21}^{A9}} - \frac{(\Sigma\gamma)_{22}^{10}}{(\Sigma n_1^T)_{22}^{A10}} + \frac{(\Sigma\gamma)_{21}^{10}}{(\Sigma n_1^T)_{21}^{A10}} \right]$$

$$[D_1(\underline{A}_2 \cdot \underline{B}_2)]_2 + \frac{g^2}{2} [D_1 D_3]_2 = \frac{\sqrt{2}}{gS_1^T} \left[\frac{(\Sigma\gamma)_{22}^{13}}{(\Sigma n_1^T)_{22}^{A13}} - \frac{(\Sigma\gamma)_{21}^{13}}{(\Sigma n_1^T)_{21}^{A13}} \right] - [D_1(\underline{A}_2 \cdot \underline{B}_1)]_2$$

$$- \frac{\sqrt{2}}{g} [D_1 D_0]_2 - \frac{\sqrt{2}\,g}{2} [D_1 D_2]_2$$

Accelerometer Three

$$[D_1(\underline{A}_3 \cdot \underline{B}_3)]_3 + g^2 [D_1 D_3]_3 = \frac{1}{2gS_1^T} \left[\frac{(\Sigma\gamma)_{32}^{11}}{(\Sigma n_1^T)_{32}^{A11}} - \frac{(\Sigma\gamma)_{31}^{11}}{(\Sigma n_1^T)_{31}^{A11}} - \frac{(\Sigma\gamma)_{32}^{12}}{(\Sigma n_1^T)_{32}^{A12}} + \frac{(\Sigma\gamma)_{31}^{12}}{(\Sigma n_1^T)_{31}^{A12}} \right]$$

$$[D_1(\underline{A}_3 \cdot \underline{B}_3)]_3 + \frac{g^2}{2} [D_1 D_3]_3 = \frac{\sqrt{2}}{gS_1^T} \left[\frac{(\Sigma\gamma)_{32}^{14}}{(\Sigma n_1^T)_{32}^{A14}} - \frac{(\Sigma\gamma)_{31}^{14}}{(\Sigma n_1^T)_{31}^{A14}} \right] - [D_1(\underline{A}_3 \cdot \underline{B}_1)]_3 - \frac{\sqrt{2}}{g} [D_1 D_0]_3$$

$$- \frac{\sqrt{2}\,g}{2} [D_1 D_2]_3$$

Bias and Second Order Term

Accelerometer One

$$[D_1 D_0]_1 = \frac{1}{2S_1^T}\left[\frac{(\Sigma\gamma)_{12}^{9}}{(\Sigma n_1^T)_{12}^{A9}} - \frac{(\Sigma\gamma)_{11}^{9}}{(\Sigma n_1^T)_{11}^{A9}} + \frac{(\Sigma\gamma)_{12}^{10}}{(\Sigma n_1^T)_{12}^{A10}} - \frac{(\Sigma\gamma)_{11}^{10}}{(\Sigma n_1^T)_{11}^{A10}}\right]$$

$$= \frac{1}{2S_1^T}\left[\frac{(\Sigma\gamma)_{12}^{11}}{(\Sigma n_1^T)_{12}^{A11}} - \frac{(\Sigma\gamma)_{11}^{11}}{(\Sigma n_1^T)_{11}^{A11}} + \frac{(\Sigma\gamma)_{12}^{12}}{(\Sigma n_1^T)_{12}^{A12}} - \frac{(\Sigma\gamma)_{11}^{12}}{(\Sigma n_1^T)_{11}^{A12}}\right]$$

$$[D_1 D_2]_1 + \frac{1}{g^2}[D_1 D_0]_1 = \frac{1}{2g^2 S_1^T}\left[\frac{(\Sigma\gamma)_{12}^{7}}{(\Sigma n_1^T)_{12}^{A7}} - \frac{(\Sigma\gamma)_{11}^{7}}{(\Sigma n_1^T)_{11}^{A7}} + \frac{(\Sigma\gamma)_{12}^{8}}{(\Sigma n_1^T)_{12}^{A8}} - \frac{(\Sigma\gamma)_{11}^{8}}{(\Sigma n_1^T)_{11}^{A8}}\right]$$

Accelerometer Two

$$[D_1 D_0]_2 = \frac{1}{2S_1^T}\left[\frac{(\Sigma\gamma)_{22}^{7}}{(\Sigma n_1^T)_{22}^{A7}} - \frac{(\Sigma\gamma)_{21}^{7}}{(\Sigma n_1^T)_{21}^{A7}} + \frac{(\Sigma\gamma)_{22}^{8}}{(\Sigma n_1^T)_{22}^{A8}} - \frac{(\Sigma\gamma)_{21}^{8}}{(\Sigma n_1^T)_{21}^{A8}}\right]$$

$$= \frac{1}{2S_1^T}\left[\frac{(\Sigma\gamma)_{22}^{11}}{(\Sigma n_1^T)_{22}^{A11}} - \frac{(\Sigma\gamma)_{21}^{11}}{(\Sigma n_1^T)_{21}^{A11}} + \frac{(\Sigma\gamma)_{22}^{12}}{(\Sigma n_1^T)_{22}^{A12}} - \frac{(\Sigma\gamma)_{21}^{12}}{(\Sigma n_1^T)_{21}^{A12}}\right]$$

$$[D_1 D_2]_2 + \frac{1}{g^2}[D_1 D_0]_2 = \frac{1}{2g^2 S_1^T}\left[\frac{(\Sigma\gamma)_{22}^{9}}{(\Sigma n_1^T)_{22}^{A9}} - \frac{(\Sigma\gamma)_{21}^{9}}{(\Sigma n_1^T)_{21}^{A9}} + \frac{(\Sigma\gamma)_{22}^{10}}{(\Sigma n_1^T)_{22}^{A10}} - \frac{(\Sigma\gamma)_{21}^{10}}{(\Sigma n_1^T)_{21}^{A10}}\right]$$

ACCELEROMETER CALIBRATION EQUATIONS (Continued)

Accelerometer Three

$$[D_1D_0]_3 = \frac{1}{2S_1^T}\left[\frac{(\Sigma\gamma)_{32}^{7}}{(\Sigma n_1^T)_{32}^{A7}} - \frac{(\Sigma\gamma)_{31}^{7}}{(\Sigma n_1^T)_{31}^{A7}} + \frac{(\Sigma\gamma)_{32}^{8}}{(\Sigma n_1^T)_{32}^{A8}} - \frac{(\Sigma\gamma)_{31}^{8}}{(\Sigma n_1^T)_{31}^{A8}}\right]$$

$$= \frac{1}{2S_1^T}\left[\frac{(\Sigma\gamma)_{32}^{9}}{(\Sigma n_1^T)_{32}^{A9}} - \frac{(\Sigma\gamma)_{31}^{9}}{(\Sigma n_1^T)_{31}^{A9}} + \frac{(\Sigma\gamma)_{32}^{10}}{(\Sigma n_1^T)_{32}^{A10}} - \frac{(\Sigma\gamma)_{31}^{10}}{(\Sigma n_1^T)_{31}^{A10}}\right]$$

$$[D_1D_2]_3 + \frac{1}{g^2}[D_1D_0]_3 = \frac{1}{2g^2S_1^T}\left[\frac{(\Sigma\gamma)_{32}^{11}}{(\Sigma n_1^T)_{32}^{A11}} - \frac{(\Sigma\gamma)_{31}^{11}}{(\Sigma n_1^T)_{31}^{A11}} + \frac{(\Sigma\gamma)_{32}^{12}}{(\Sigma n_1^T)_{32}^{A12}} - \frac{(\Sigma\gamma)_{31}^{12}}{(\Sigma n_1^T)_{31}^{A12}}\right]$$

Misalignments

Accelerometer One

$$[D_1(\underline{A}_1\cdot\underline{B}_2)]_1 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\gamma)_{12}^{9}}{(\Sigma n_1^T)_{12}^{A9}} - \frac{(\Sigma\gamma)_{11}^{9}}{(\Sigma n_1^T)_{11}^{A9}} - \frac{(\Sigma\gamma)_{12}^{10}}{(\Sigma n_1^T)_{12}^{A10}} + \frac{(\Sigma\gamma)_{11}^{10}}{(\Sigma n_1^T)_{11}^{A10}}\right]$$

$$[D_1(\underline{A}_1\cdot\underline{B}_3)]_1 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\gamma)_{12}^{11}}{(\Sigma n_1^T)_{12}^{A11}} - \frac{(\Sigma\gamma)_{11}^{11}}{(\Sigma n_1^T)_{11}^{A11}} - \frac{(\Sigma\gamma)_{12}^{12}}{(\Sigma n_1^T)_{12}^{A12}} + \frac{(\Sigma\gamma)_{11}^{12}}{(\Sigma n_1^T)_{11}^{A12}}\right]$$

Accelerometer Two

$$[D_1(\underline{A}_2\cdot\underline{B}_1)]_2 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\gamma)_{22}^{7}}{(\Sigma n_1^T)_{22}^{A7}} - \frac{(\Sigma\gamma)_{21}^{7}}{(\Sigma n_1^T)_{21}^{A7}} - \frac{(\Sigma\gamma)_{22}^{8}}{(\Sigma n_1^T)_{22}^{A8}} + \frac{(\Sigma\gamma)_{21}^{8}}{(\Sigma n_1^T)_{21}^{A8}}\right]$$

$$[D_1(\underline{A}_2\cdot\underline{B}_3)]_2 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\gamma)_{22}^{11}}{(\Sigma n_1^T)_{22}^{A11}} - \frac{(\Sigma\gamma)_{21}^{11}}{(\Sigma n_1^T)_{21}^{A11}} - \frac{(\Sigma\gamma)_{22}^{12}}{(\Sigma n_1^T)_{22}^{A12}} + \frac{(\Sigma\gamma)_{21}^{12}}{(\Sigma n_1^T)_{21}^{A12}}\right]$$

ACCELEROMETER CALIBRATION EQUATIONS (Continued)

Accelerometer Three

$$[D_1(\underline{A}_3 \cdot \underline{B}_1)]_3 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\gamma)_{32}^{7}}{(\Sigma n_1^T)_{32}^{A7}} - \frac{(\Sigma\gamma)_{31}^{7}}{(\Sigma n_1^T)_{31}^{A7}} - \frac{(\Sigma\gamma)_{32}^{8}}{(\Sigma n_1^T)_{32}^{A8}} + \frac{(\Sigma\gamma)_{31}^{8}}{(\Sigma n_1^T)_{31}^{A8}}\right]$$

$$[D_1(\underline{A}_3 \circ \underline{B}_2)]_3 = \frac{1}{2gS_1^T}\left[\frac{(\Sigma\gamma)_{32}^{9}}{(\Sigma n_1^T)_{32}^{A9}} - \frac{(\Sigma\gamma)_{31}^{9}}{(\Sigma n_1^T)_{31}^{A9}} - \frac{(\Sigma\gamma)_{32}^{10}}{(\Sigma n_1^T)_{32}^{A10}} + \frac{(\Sigma\gamma)_{31}^{10}}{(\Sigma n_1^T)_{31}^{A10}}\right]$$

PART II
SECTION 5
FUNDAMENTAL MODES

In the previous sections, the procedures to be-performed to accomplish calibration were delineated by use of Laboratory Procedures Sheets. Those listings of procedures present the detailed steps to be performed. They do not, however, present the total picture of what is being accomplished in the laboratory, for they do not present an integrated view of the entire system. This section is included to illustrate that integrated view.

The partitioning of the presentation corresponds to the preceding Sections 1 through 4 as follows:

- Subsection II-5.1 Calibration of Gyro Angular Velocity Sensitive Terms
- Subsection II-5.2 Calibration of Gyro Acceleration Sensitive Terms
- Subsection II-5.3 Calibration of the Accelerometers
- Subsection II-5.4 Calibration Computations

The presentation utilizes laboratory flow diagrams to schematically illustrate the procedures. (A master copy of the diagram and definitions of its elements are contained in Section 1.2 of Part IV.) In a mode of operation where the computer is not being used, that portion of the diagrams has been omitted. In the calibration computation discussions, the laboratory computer is schematically represented, although the computations might be accomplished on any general-purpose computer. The system monitor functions shown in diagrams for measurement or setup modes represent monitoring devices which are not specifically defined at the time of this writing. They are, therefore, not discussed in detail.

As shown in the calibration diagrams, all calibration measurements are made using the frequency counters, rather than the computer interface. Frequency counters are required for all accelerometer measurements to allow sampling of the leading edge of output pulses, for the purpose of reducing quantization error. During gyro angular velocity term determination, the test table rotation data must also be collected by the frequency counters to reduce quantization error. To completely free the laboratory computer during calibration, NASA/ERC has decided to use frequency counters for all other gyro measurements, which otherwise could be accomplished by the computer interface.

Six frequency counters are available for data collection. This is an insufficient number to allow for a simultaneous measurement of gyro and accelerometer data in those positions which are common to both calibrations. Therefore, measurements to determine gyro acceleration sensitive terms and accelerometer constants are performed in series; although they use the same test table positions (7 to 15), and could be performed simultaneously if enough data collection devices were available. This would result in a considerable savings in calibration time at no loss of accuracy.

II-5.1 Calibration of Gyro Angular Velocity Sensitive Terms

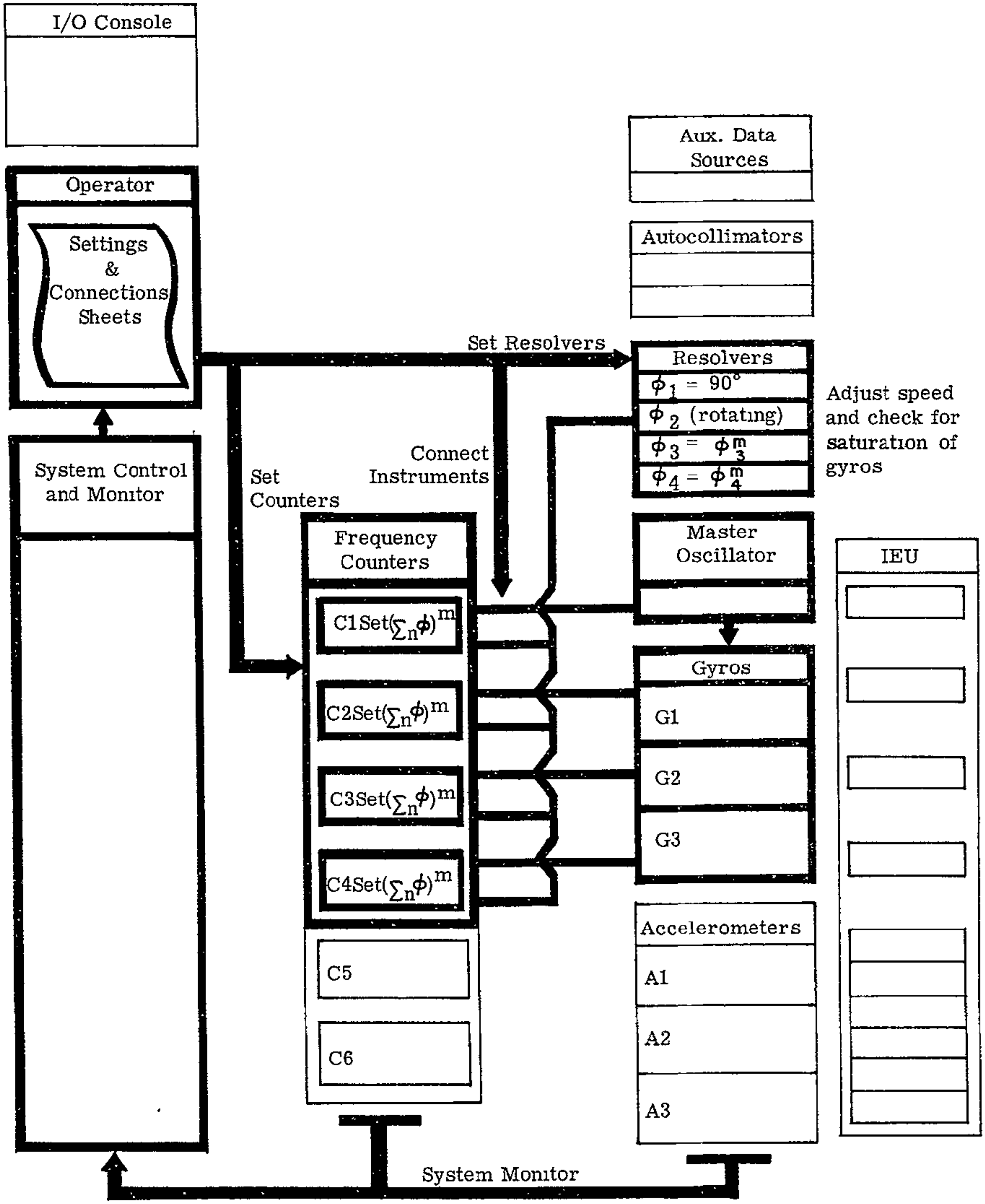
In order to determine the gyro angular velocity sensitive terms, measurements are made utilizing the test table rotation as the principal kinematic input. There are three experiments which utilize the rotational capability of the table. The first experiment obtains measurement data in six positions (denoted as Positions 1 to 6) required for the determination of the gyro scale factors and $(Q^G)^{-1}$. The second experiment obtains data from each gyro at several choices of test table angular velocity for the purpose of determining the scale factor nonlinearity. The third experiment evaluates the J term by a use of the accelerating capability of table. The procedures to be accomplished for either of the first two experiments in any position or for any angular velocity are illustrated in Laboratory Flow Diagrams 4 and 5. Flow Diagram 4 illustrates the setup steps required before measurement, and Flow Diagram 5 illustrates the data flow during the measurement process.

After the operator has verified that the system is properly operating, and he has the information he requires on the Resolver Settings and Counter Settings Sheets, he begins the experiment by connecting instruments to the frequency counters, setting the frequency counters, and setting the test table resolvers as shown in Flow Diagram 4. The gyros, master oscillator, and rotary axis resolver are connected to the frequency counters as shown. The output from the rotary axis resolver is connected to the Z input of Counters 1 to 4. The master oscillator is connected to the X input of Counter 1, and the bipolar outputs of gyros 1 to 3 are connected to the X-Y inputs of Counters 2 to 4.

Each counter is set to the mode which measures (in terms of counts on the X-Y input) the duration of N counts of the Z input. Counters 2 to 4 will be used to measure the gyro output pulses for a number (N) of complete revolutions of the test table; Counter 1 will be used to measure the time period over which the measurements are made (the time required for the N revolutions). In order to make the time counted on Counter 1 valid for the measurements on Counters 2 to 4, all of these counters should be set to the same count as Counter 1. The count is obtained from the Counter Settings Sheet.

Next, the test table is set to the correct position. The settings for outer and inner axis resolvers (ϕ_3 and ϕ_4) for each position are obtained from the Resolver Settings Sheet. The trunnion axis resolver, ϕ_1, is set to 90^o. The table is then commanded to rotate and adjusted to the desired speed. For measurements at Positions 1 to 6 this speed should be just less than the speed causing saturation of the gyro output, and equal for each pair of measurement positions (1 and 2, 3 and 4, and 5 and 6). The speed of each measurement will be specified on the Counter Settings Sheet for the scale factor nonlinearity experiments.

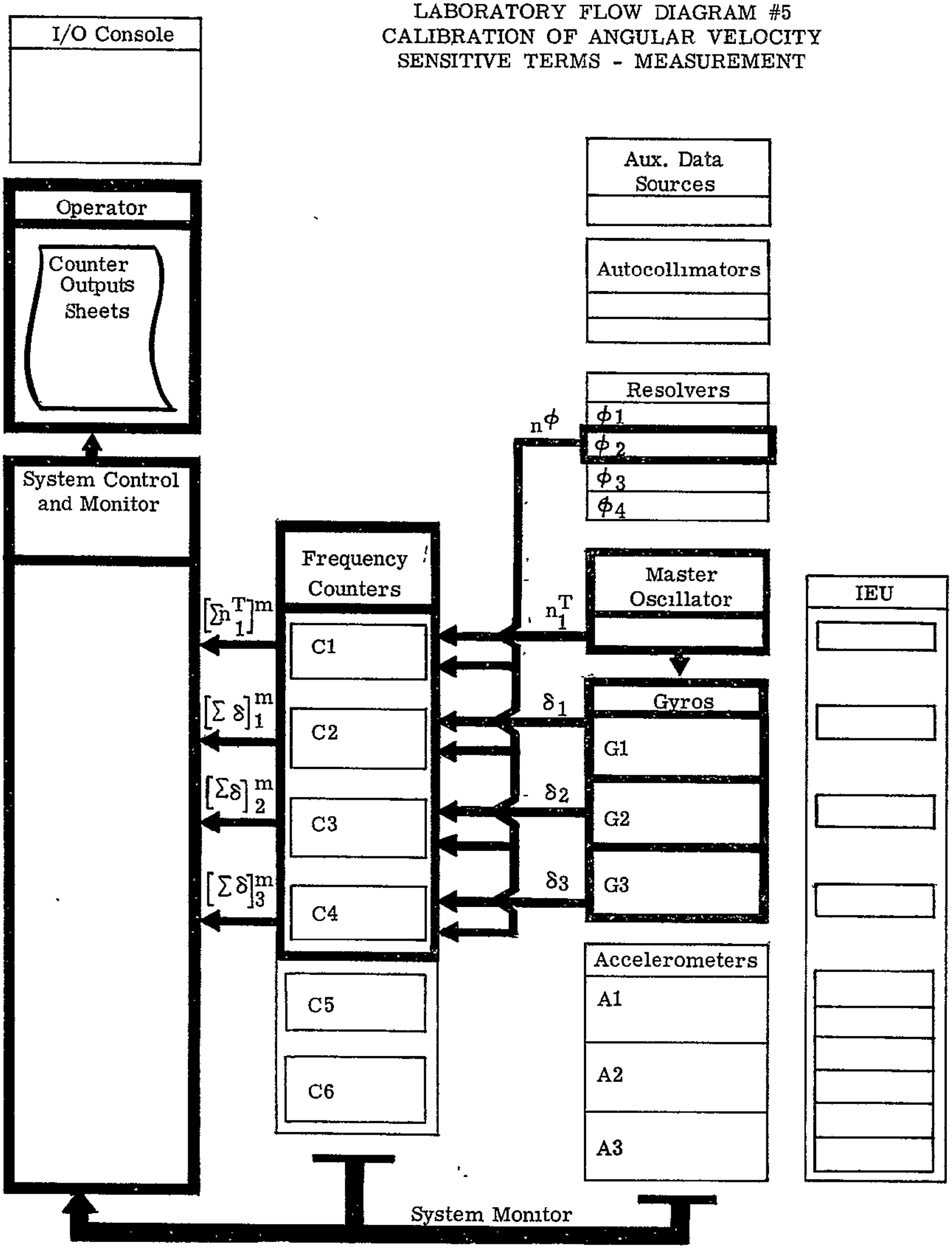
LABORATORY FLOW DIAGRAM #4
CALIBRATION OF ANGULAR VELOCITY SENSITIVE TERMS - SETUP



The measurements are then initiated by switching the counters from standby to operate. The operator should switch all counters at some time between consecutive output pulses from the rotary axis resolver. This is required so that all measurements are made over the same time period. The frequency counters collect information as shown in Flow Diagram 5. After the first output pulse from the rotary axis resolver, each input pulse train is summed in the frequency counters until N rotary axis pulses have been received. When this occurs, the frequency counters will print the counts of their X-Y inputs, clear their counters, and continue counting. The operator then terminates the procedure by switching the counters to standby and he then records the output of each counter on the Counter Outputs Sheet. The output from Counter 1 is a count of time pulses which represent an elapsed time. Counters 2 to 4 outputs are pulse train counts from gyros 1 to 3 for the N test table revolutions.

The operator repeats the above steps until data has been obtained from all required positions (or angular velocities in the case of nonlinearity experiments).

LABORATORY FLOW DIAGRAM #5
CALIBRATION OF ANGULAR VELOCITY SENSITIVE TERMS - MEASUREMENT



II-5.2 Calibration of Gyro Acceleration Sensitive Terms

In order to determine the acceleration sensitive terms of the gyros, gyro measurements are made using earth environment inputs only. The procedures to accomplish these measurements are listed in Section 2. The steps to obtain measurement data at each position are shown in Laboratory Flow Diagrams 6 and 7.

After the operator has ascertained that the data required for the experiment is available on the Resolver Settings and Counter Settings Sheets, and that the equipment is ready for operation, he connects the instruments to the frequency counters, and sets the frequency counters and test table resolvers.

The operator connects the gyros and the master oscillator to the frequency counters as shown on Flow Diagram 6. The master oscillator line is connected to the Z input of Counters 2 to 4. Gyros 1 to 3 are connected to the X-Y inputs of Counters 2 to 4, respectively, as they were in the calibration of angular velocity terms for the gyros. each counter is set to count gyro pulses during the time period required to receive N time pulses. N is then set on each counter from the Counter Settings Sheet. The time of measurement equals N times the period of the input pulses from the master oscillator.

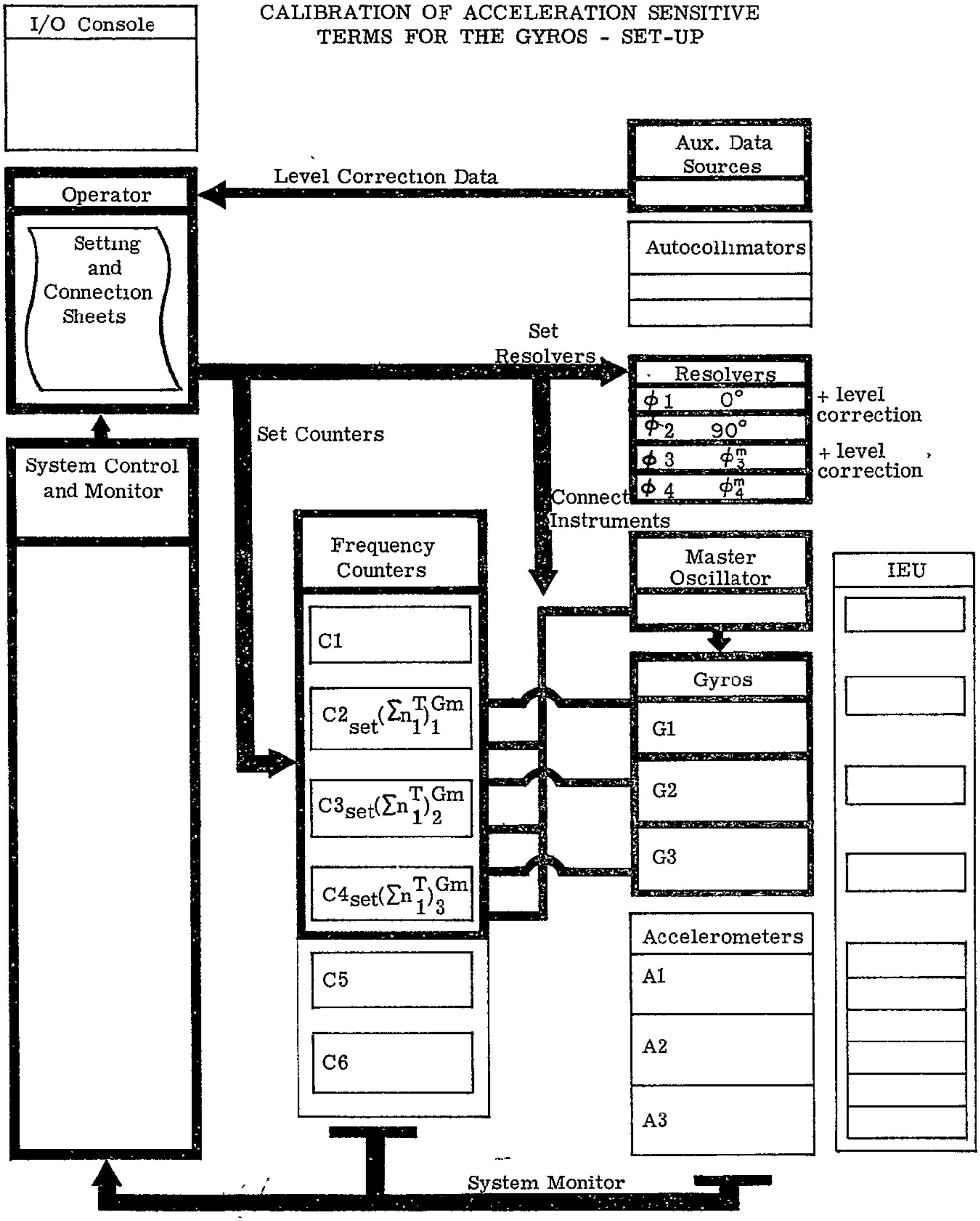
The counters are being used to count gyro data over a fixed time period. The alternative of counting the time for a fixed number of gyro counts would have a smaller quantization error; but in many cases the fixed number to be used would be difficult to predetermine. The predetermination would require a good prior knowledge of the constant to be calibrated.

The test table trunnion axis resolver (ϕ_1) is set to 0^o and corrected by the bubble level correction. The rotary axis resolver (ϕ_2) is set to 90^o, and the inner and outer axis resolvers (ϕ_3 and ϕ_4) are set from the Resolvers Settings Sheet. ϕ_3 is corrected for level from the bubble level data shown as auxiliary data sources in Flow Diagram 6.

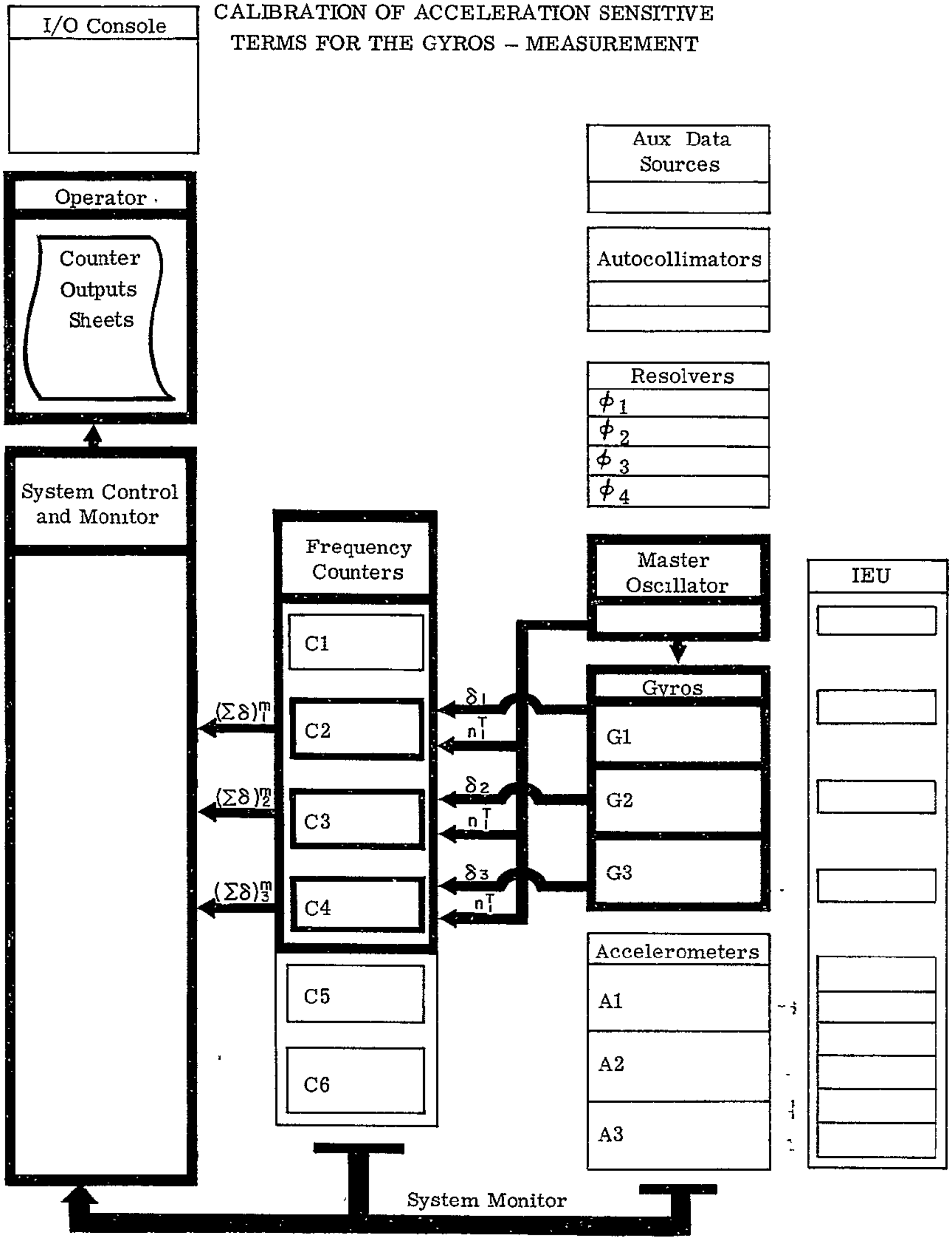
The measurement procedure is started by switching the counters to operate and data is collected as shown in Flow Diagram 7. Each counter counts gyro pulses until receipt of N time pulses. Then the counter prints out, clears its counters, and continues counting. The operator terminates the measurement process by switching the counters to standby. The data is then recorded on the Counter Outputs Sheet.

The steps above are repeated for all positions required. After all required measurement data has been obtained, the calibration computations (Section II-5.4) will be performed.

LABORATORY FLOW DIAGRAM #6
CALIBRATION OF ACCELERATION SENSITIVE TERMS FOR THE GYROS - SET-UP



LABORATORY FLOW DIAGRAM #7
CALIBRATION OF ACCELERATION SENSITIVE TERMS FOR THE GYROS – MEASUREMENT



II-5.3 Calibration of the Accelerometers

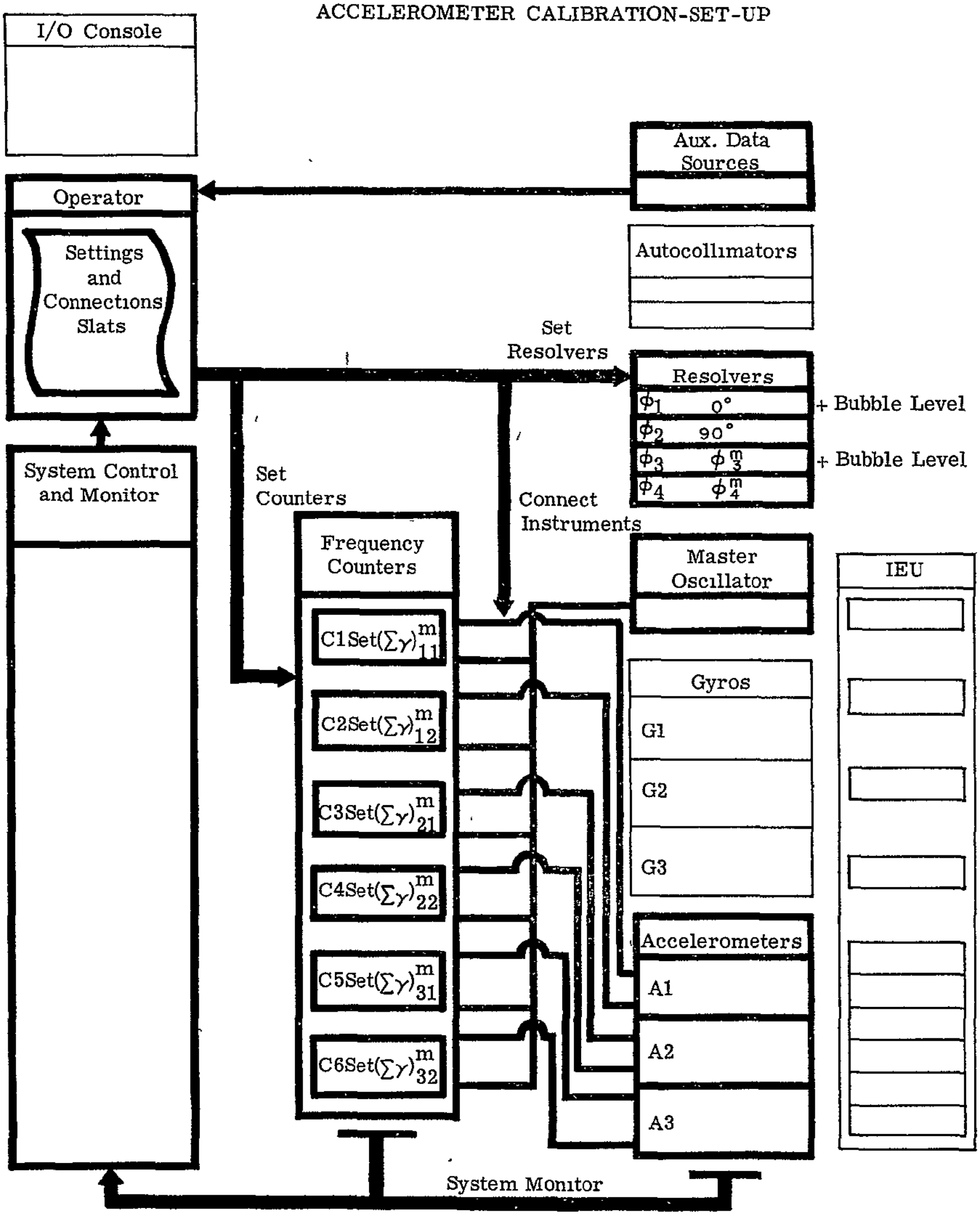
The measurements made for accelerometer calibration are taken from Positions 7 to 14. (Recall that these positions are used in the calibration of the acceleration sensitive terms in the gyro.) The procedures listings for accelerometer calibration measurements are continued in Section II-3. The steps involved in obtaining measurements are shown in Laboratory Flow Diagrams 8 and 9. The following discussions describe the contents of these diagrams.

The operator first verifies that the turn-on procedures are completed. He then verifies that the information required on the Resolvers Settings Sheet and the Counter Setting Sheet are available. After these steps, he performs the measurement setups as shown on Flow Diagram 8. The counters are set to measure the time required to receive a preset number of counts from each string of the accelerometer. The master oscillator output is connected to the Z input of Counters 1 to 6. The output from string 1 of accelerometer 1 is connected to the X input of Counter 1 and the output from string 2 is connected to the X input of Counter 2. Similarly, outputs from strings 1 and 2 of accelerometer 2 are connected to Counters 3 and 4, respectively; and outputs from strings 1 and 2 of accelerometer 3 are connected to Counters 5 and 6. The test table resolvers are set as described in the previous subsection. ϕ_1 is set to 0^o plus the level correction as given by the test table bubble level. The angle ϕ_2 is set to 90^o. Angles ϕ_3 and ϕ_4 are set from the Resolver Settings Sheet for each position and ϕ_3 is corrected for level.

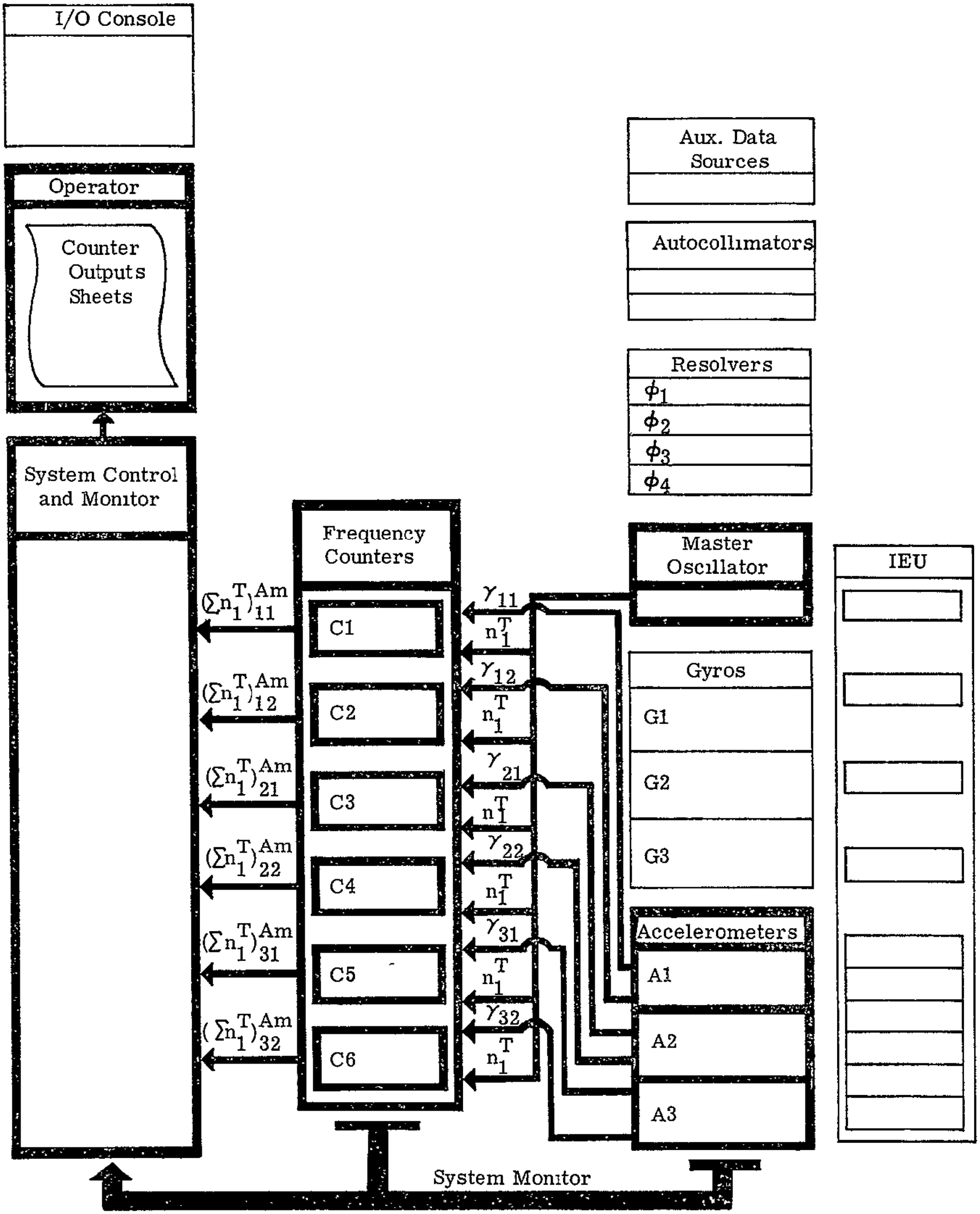
Measurements are then initiated by switching the counters from standby to operate. The data is collected as shown in Flow Diagram 9. Each counter outputs the number of time pulses (elapsed time) required to accumulate the preset number of accelerometer counts. This data is recorded on the Counter Outputs Sheet.

The above process must be repeated for each position. After all measurements have been completed, the operator will initiate the calibration computations by performing the steps described in the following section.

LABORATORY FLOW DIAGRAM #8
ACCELEROMETER CALIBRATION-SET-UP



LABORATORY FLOW DIAGRAM #9
ACCELEROMETER CALIBRATION - MEASUREMENT



II-5.4 Calibration Computations

After all measurement data has been obtained, the operator will initiate the computations of the calibration constants. The procedure listings to accomplish this task are contained in Section 4.

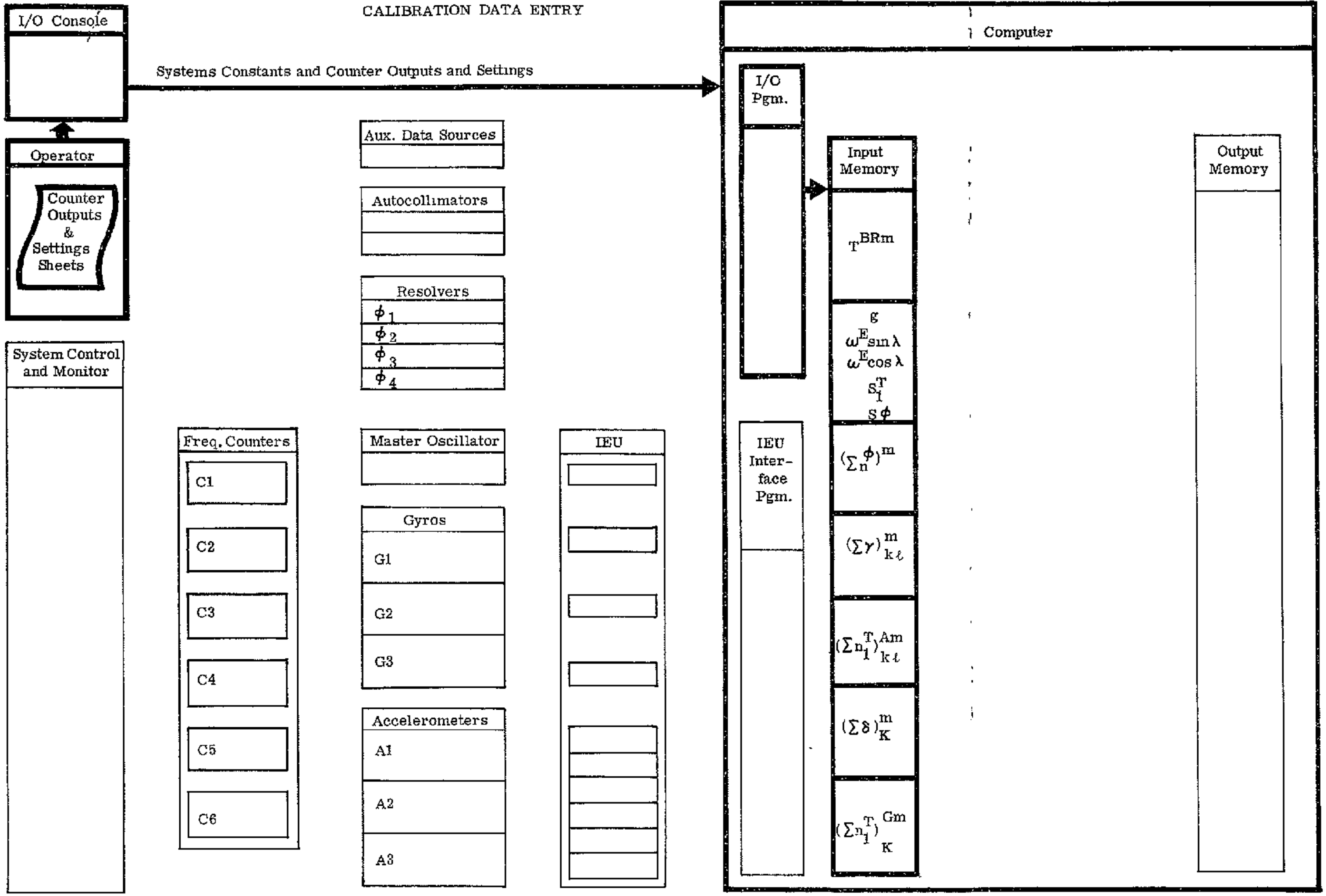
The operator initiates these modes of operation by verifying that he has all required data including:

1. g, $\omega^E \sin\lambda$, $\omega^E \cos\lambda$ from the Survey Results Sheet
2. The value of any resolver or clock scale factors not permanently in the program
3. Elements of the body to rotary axis frame for selected positions T_{12}^{BRm}, T_{22}^{BRm}, and T_{32}^{BRm}, $m = 1, 3, 5, 13, 14, 15$ from the current System Survey Procedure
4. Counter settings from the Counter Settings Sheet
5. Counter outputs from the Counter Outputs Sheet.

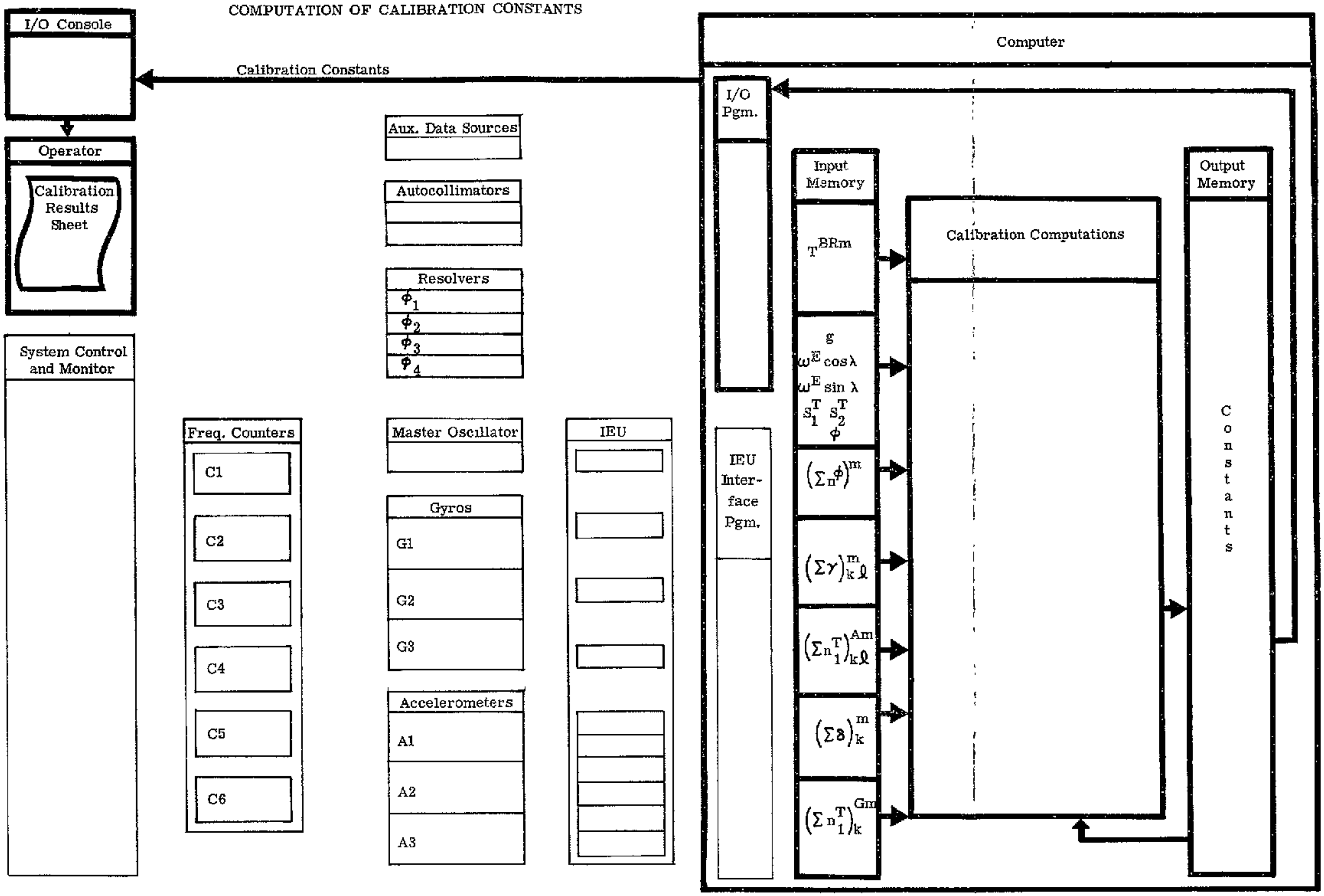
The calibration computations may be programmed for any general-purpose computer. To initiate the computations, the operator prepares the data by putting it in the proper entry format, loads the computer with the calibration computations program, loads the data, and initiates the run (see Flow Diagram 10).

The computation subroutines are indicated in Flow Diagram 11. The specific computations are listed in Section II-4. The outputs of the program are the calibration constants, which are recorded on the Calibration Results Sheet.

LABORATORY FLOW DIAGRAM #10
CALIBRATION DATA ENTRY



LABORATORY FLOW DIAGRAM #11
COMPUTATION OF CALIBRATION CONSTANTS



PART II
SECTION 6
RESOLVER AND COUNTER SETTINGS

This section is concerned with two important activities required prior to calibration data collection. The first subsection discusses the accuracies required in setting the test table resolver angles. The second subsection describes the selection of frequency counter settings for each calibration position.

II-6.1 Table Resolver Settings

This subsection is concerned with the precision requirements on the test table resolver settings. The least allowable precision is dictated by the required precision of calibration; more specifically, the precision of the Q matrices. The most desirable precision is the most precise setting which is attainable by the test table resolvers. The actual requirements will probably lie somewhere between these two limits; assuming, of course, that the maximum attainable precision is greater than the precision dictated by the calibration requirements. In the following paragraphs we will discuss how the actual precision requirements will be arrived at.

In deciding upon the actual requirements there are a number of points to consider.

- From a precision standpoint the resolvers should be set as accurately as possible. It is certainly advisable to make the calibration as insensitive as possible to table setting imprecisions.
- From a logistic standpoint it would be advisable for the setting precisions to be the same for all calibration positions. This requirement would tend to minimize human error.
- From a time standpoint there may be a limit to the amount of time available for settings. In other words, the maximum attainable precision may take an unfortunate amount of time.
- Setting precision requirements (as given by the specifications of calibration accuracies) vary between an order of one degree to an order of an arc second. The least precision is required for gyro scale factor calibration, and the most is required for the determination of the Q^A matrix.
- The gyro calibration, in total, requires a lesser precision in resolver settings than the accelerometer calibration.
- All calibrations, and consequently, the table settings will vary in their precision requirements from week to week depending upon the experiment.

These considerations result in a multitude of precision requirement alternatives. It appears, however, that the following options should be the only ones considered:

If:

- The maximum possible precision is attainable in a reasonable time.

Then:

- All setting precisions should be the same, and equal to the maximum precision.

If not:

- The accelerometer calibration will nevertheless require the highest precision settings, in order to achieve the calibration precision specifications. (This assumes, of course, that the resolvers can be set to an order of one second of arc.)

With the "if not" condition, the gyro calibration positions require the following setting options.

If:

- There exists a setting precision which is higher than the Q^G matrix precision requirements, and which is attainable in a <u>reasonable</u> time.

Then:

- All gyro calibration positions should utilize that precision.

If not:

- The gyro calibration, with the exception of the scale factor calibration, will nevertheless require table setting precisions which are higher than the Q^G matrix requirements. The scale factor positions can be set to a precision of the order of one degree.

The above comments result in either one, two, or three separate precision requirements. It must be noted that the position used to determine the scale factor for any given gyro is used for a Q^G matrix element for the other two. Therefore, the requirement for a small precision in the settings for gyro scale factor determination assumes that the scale factors are the only constants of interest.

On Chart II-17, which contains the magnitudes of the resolver settings, there are references to bubble level corrections for Positions 7 through 15. These corrections must be consistent with the table resolver setting requirements. In other words, all uses of Positions 7 through 12 for accelerometer calibration should always implement bubble level corrections. Additionally, assuming that the maximum amplitude of the table motion relative to the earth is below the gyro calibration precision requirement, these corrections need not be implemented during gyro calibration. It might be noted, however, that it would be advisable to get in the habit of making such corrections for all positions, so as to minimize the chance that he will overlook the corrections when needed. See Section 4.4.4 of the Development Document for further discussion of bubble level corrections.

CHART II-17

TEST TABLE RESOLVER SETTINGS

	Position 1	Position 2	Position 3	Position 4	Position 5	Position 6
ϕ_1	90°	90°	90°	90°	90°	90°
ϕ_2	Rotating	Rotating	Rotating	Rotating	Rotating	Rotating
ϕ_3		$\phi_3^1 + 180°$		$\phi_3^3 + 180°$		$\phi_3^5 + 180°$
ϕ_4		ϕ_4^1		ϕ_4^3		ϕ_4^5

	Position 7	Position 8	Position 9	Position 10	Position 11	Position 12
ϕ_1	0° *	0° *	0° *	0° *	0° *	0° *
ϕ_2	90°	90°	90°	90°	90°	90°
ϕ_3	ϕ_3^1 *	ϕ_3^2 *	ϕ_3^3 *	ϕ_3^4 *	ϕ_3^5 *	ϕ_3^6 *
ϕ_4	ϕ_4^1	ϕ_4^1	ϕ_4^3	ϕ_4^3	ϕ_4^5	ϕ_4^5

	Position 13	Position 14	Position 15
ϕ_1	0° *	0° *	0° *
ϕ_2	90°	90°	90°
ϕ_3	*	*	*
ϕ_4			

* Requires bubble level correction.

II-6.2 Counter Settings

All calibrations in this document utilize the frequency counters for inertial instrument, clock, and resolver data collection. Each of these counters have the ability to count the number of pulses from any device occurring in the interval of time in which a predetermined number of pulses are outputted from any other device. The control of each calibration experiment is given by those predetermined numbers set in the counters. The predetermined numbers must be recorded in the Counter Settings Sheet before the calibration procedures are initiated. In this subsection we describe how these numbers are found.

The basis of any counter setting is a required accuracy of calibration. In Section 2.2 of the trade-off document it was shown that calibration accuracy is a function of data collection time. The counter settings can, therefore, always be dictated by a duration of time. The mechanics of determining a required setting is as follows: 1) decide upon a required accuracy of calibration, 2) use the accuracy versus time charts to determine the time of data collection, 3) equate the time to the predetermined number of counts, and 4) record the predetermined number of the Counter Settings Sheet. In the following paragraphs we will show how these steps are accomplished for each calibration. As a reference for discussion, the Counter Settings Sheet and the accuracy versus time charts are included as Charts II-18 through II-24.

Positions 1 through 6 are used exclusively for the determination of gyro angular velocity coefficients. The time versus accuracy chart for this gyro calibration is Chart II-18. We see that relatively high precisions can be attained in very short times. This is due primarily to the high speed attainable by the table. The control setting is a number of whole turns in Positions 1 through 6. Each whole turn equates to approximately 24 seconds at the speeds used in these positions. We see from Chart II-18 that only one turn is required for the required precision of 10^{-4}. Since the noise curve in Chart II-18 represents a standard deviation and not a worst case, and since time is no problem in these positions, it is probably advisable to take a number of whole turns of data.

Gyro Positions 7 through 15 are used in determining the gyro acceleration sensitive terms. Chart II-19 yields the time versus accuracy information for those positions. A reasonable data collection time of ten minutes yields a precision on the order of 0.02 deg/hr for R, Bg, and Cg^2. (We say "on the order of" because the noise curves represent standard deviations.) The counter settings in Chart II-20 are in terms of a number of clock counts. The time is converted to counts by simply dividing the time of data collection by the scale factor of the clock. The number of counts in each counter for each position will nearly always be the same.

$$\beta_2 = \frac{\Delta(1/_{\Delta}\Phi)}{(1/_{\Delta}\Phi)} = \Delta(\underline{G}_i \bullet \underline{B}_j) = \Delta Q\omega$$

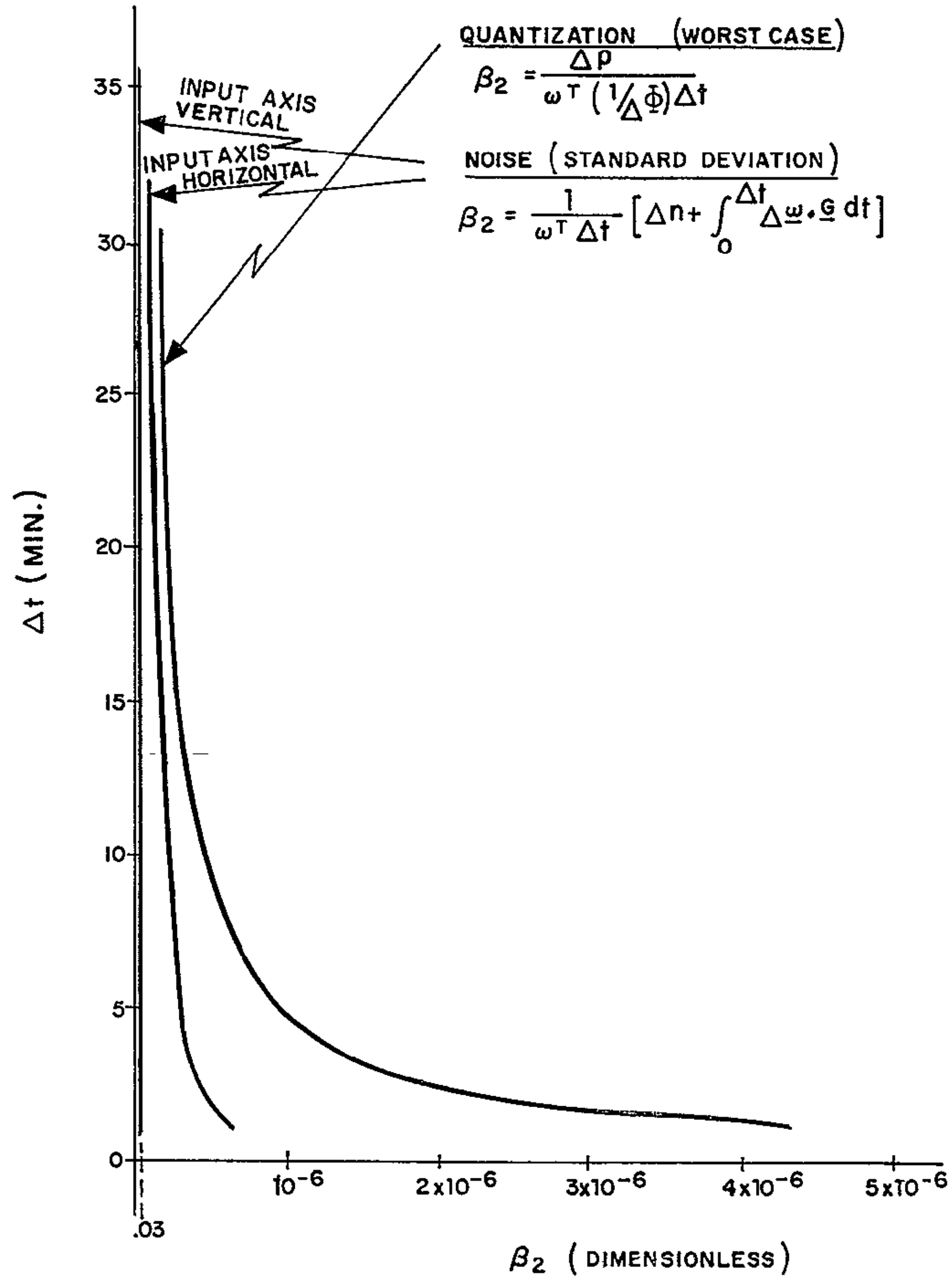


Chart II-18. Gyro Scale Factor Error vs Time

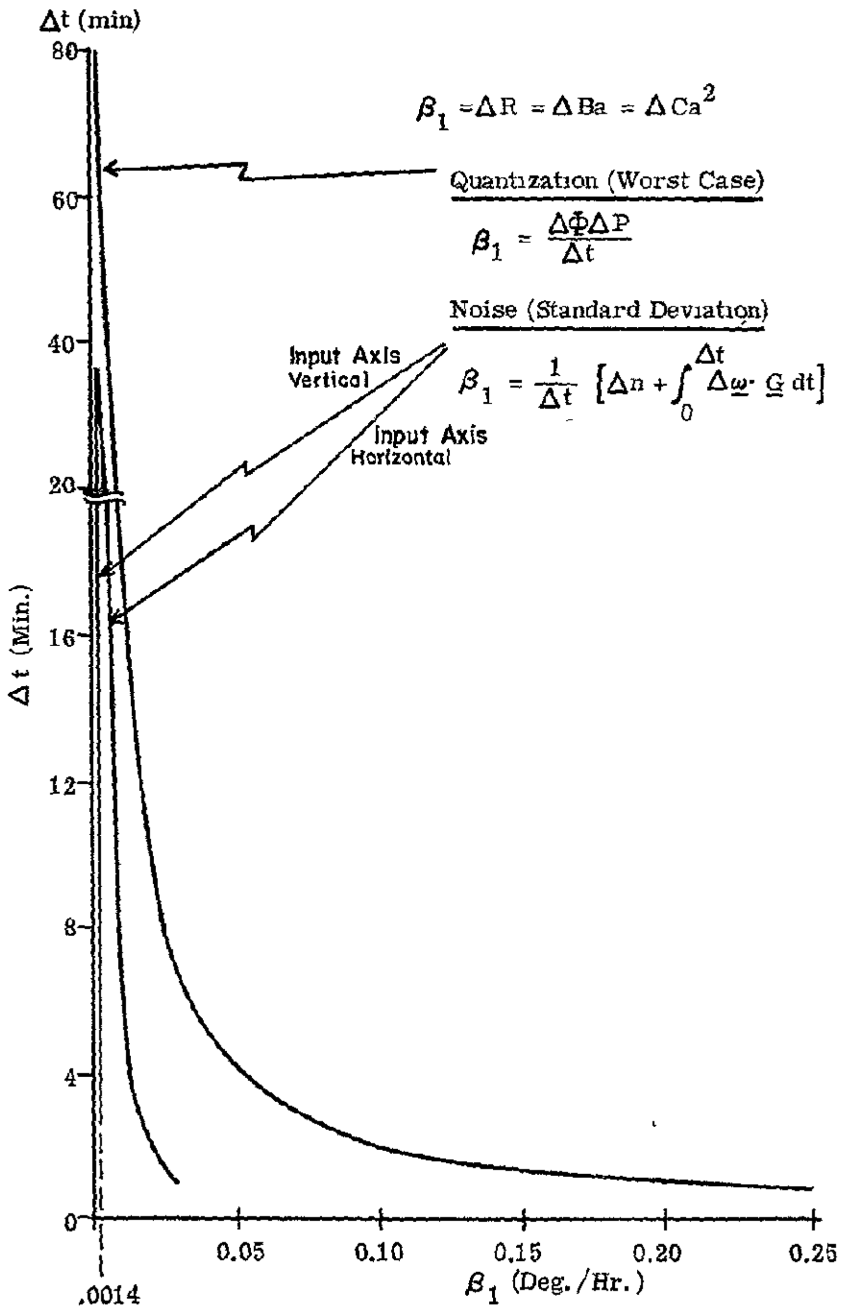


Chart II-19. Gyro Bias Error vs Time

COUNTER SETTINGS

CHART II-20

Experiment:

Date:

Positions		Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6
GYRO		$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$		
	1					Not Required	Not Required
	2						
	3						
	4						
	5						
	6						
GYRO		Not Required	$(\Sigma n_1^T)_1^{Gm}$	$(\Sigma n_1^T)_2^{Gm}$	$(\Sigma n_1^T)_3^{Gm}$		
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						
ACCELEROMETER		$(\Sigma\gamma)_{11}^m$	$(\Sigma\gamma)_{12}^m$	$(\Sigma\gamma)_{21}^m$	$(\Sigma\gamma)_{22}^m$	$(\Sigma\gamma)_{31}^m$	$(\Sigma\gamma)_{32}^m$
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						

The determination of the accelerometer settings are a little more difficult to explain. This is primarily due to two facts.

1. The noise errors predominate over the quantization errors for horizontal positions.
2. The times must be converted to accelerometer output counts.

The second fact is dictated by the necessity for reading the leading edges of the accelerometer pulses. Reading the leading edges results in a quantization error an order of magnitude less than if a clock was used as the control.

Positions 7 through 12 have accelerometers in either a horizontal or vertical orientation. The vertical positions are used exclusively for scale factor determination; and the horizontal positions are used for bias, Q^A matrix, and second-order term determination. Accelerometer one is in a vertical orientation in Positions 7 and 8, accelerometer two in Positions 9 and 10, and accelerometer three in Positions 11 and 12. Referring to` Chart II-21 we see that ten minutes of data collection will yield a precision of less than two parts in a million for vertical instruments. To convert that time to pulse counts we simply multiply the time by the basic frequency of the accelerometer (in units of pulses/unit of time). An example would be:

$$tf = 10 \text{ minutes} \times \frac{20,000 \text{ counts}}{\text{sec}} = 1.2 \times 10^6 \text{ counts}$$

(Note that the basic frequency, and not the accelerometer scale factor is the conversion factor.)

For the horizontal instruments, Charts II-21 and II-22 show precisions of approximately 100 parts in a million where bubble corrections are not applied. That accuracy is (apparently) not a function of time. Fortunately, the low frequency noise can be reduced by use of bubble levels. In Charts II-23 and II-24, we see a reduction in the error by two different assumptions as to how the bubble level corrections reduce the noise. (These reductions are explained in Section 2.2 of the trade-off document.) The time to be used for data collection depends upon a subjective decision as to the accuracy of these assumptions. In any event we see that calibration precisions of six parts in a million are probably available in less than 30 minutes of data collection. Whichever time is used, the conversion to pulse counts is the same as indicated in the preceding paragraph.

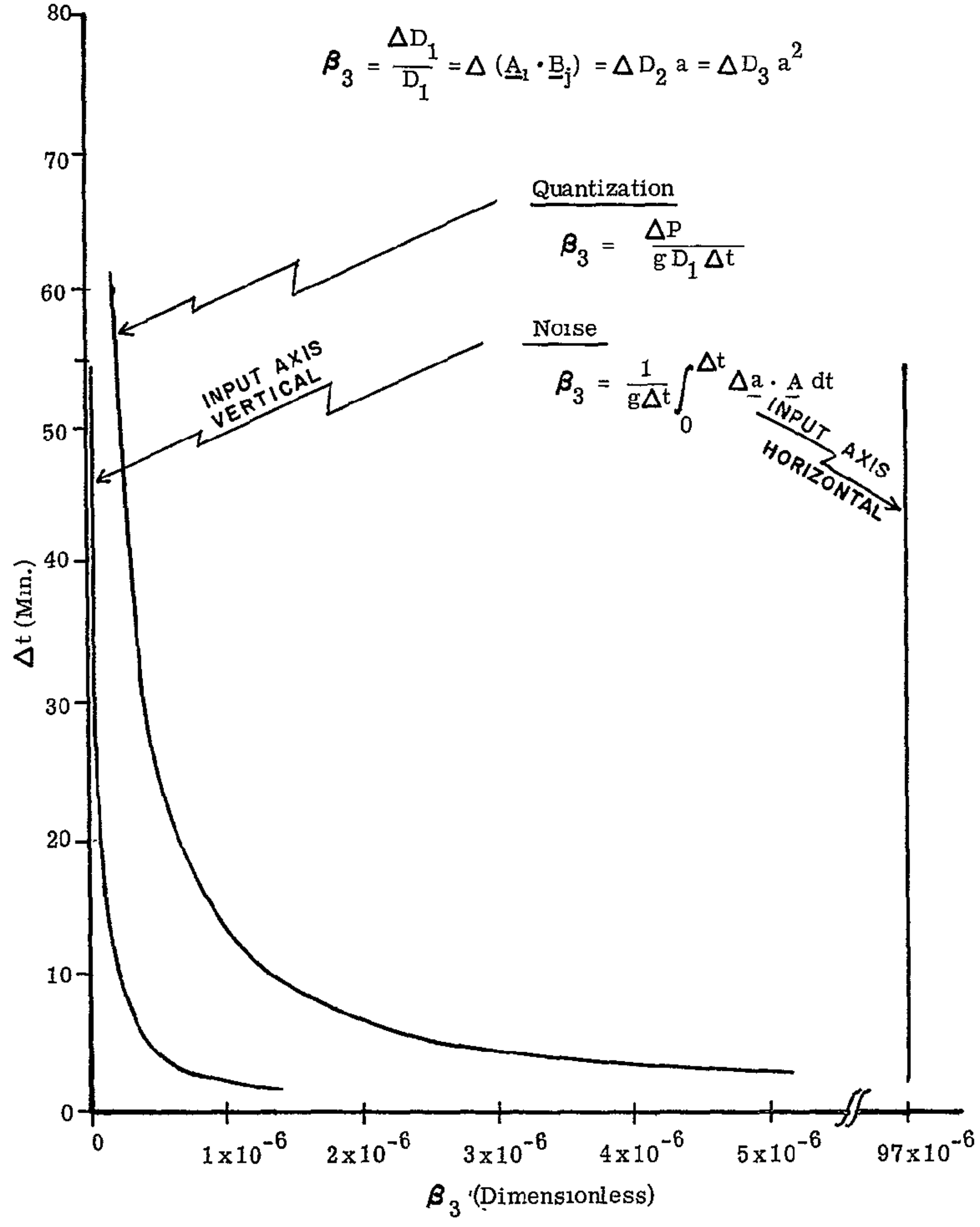


Chart II-21. Accelerometer Scale Factor Error vs Time

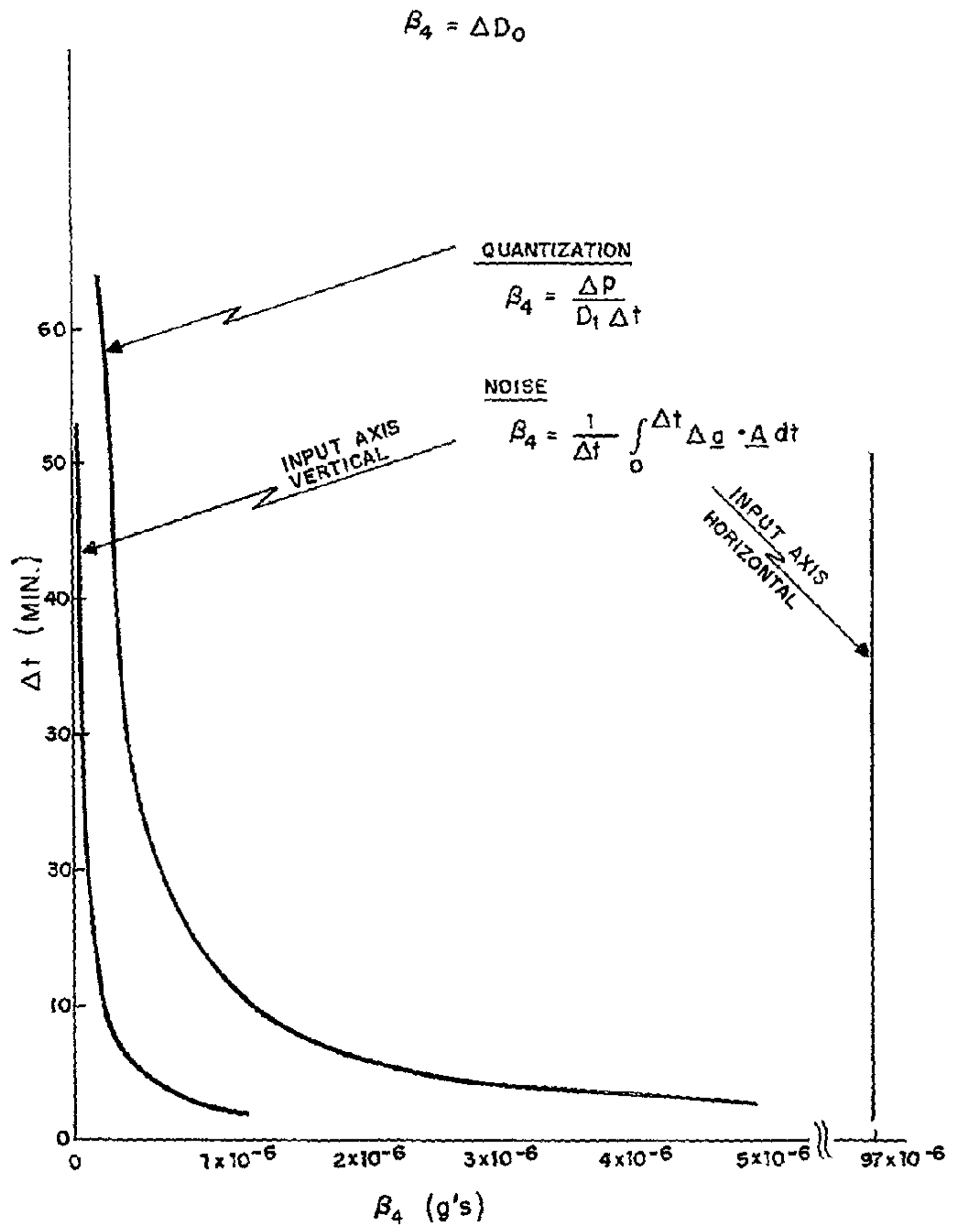
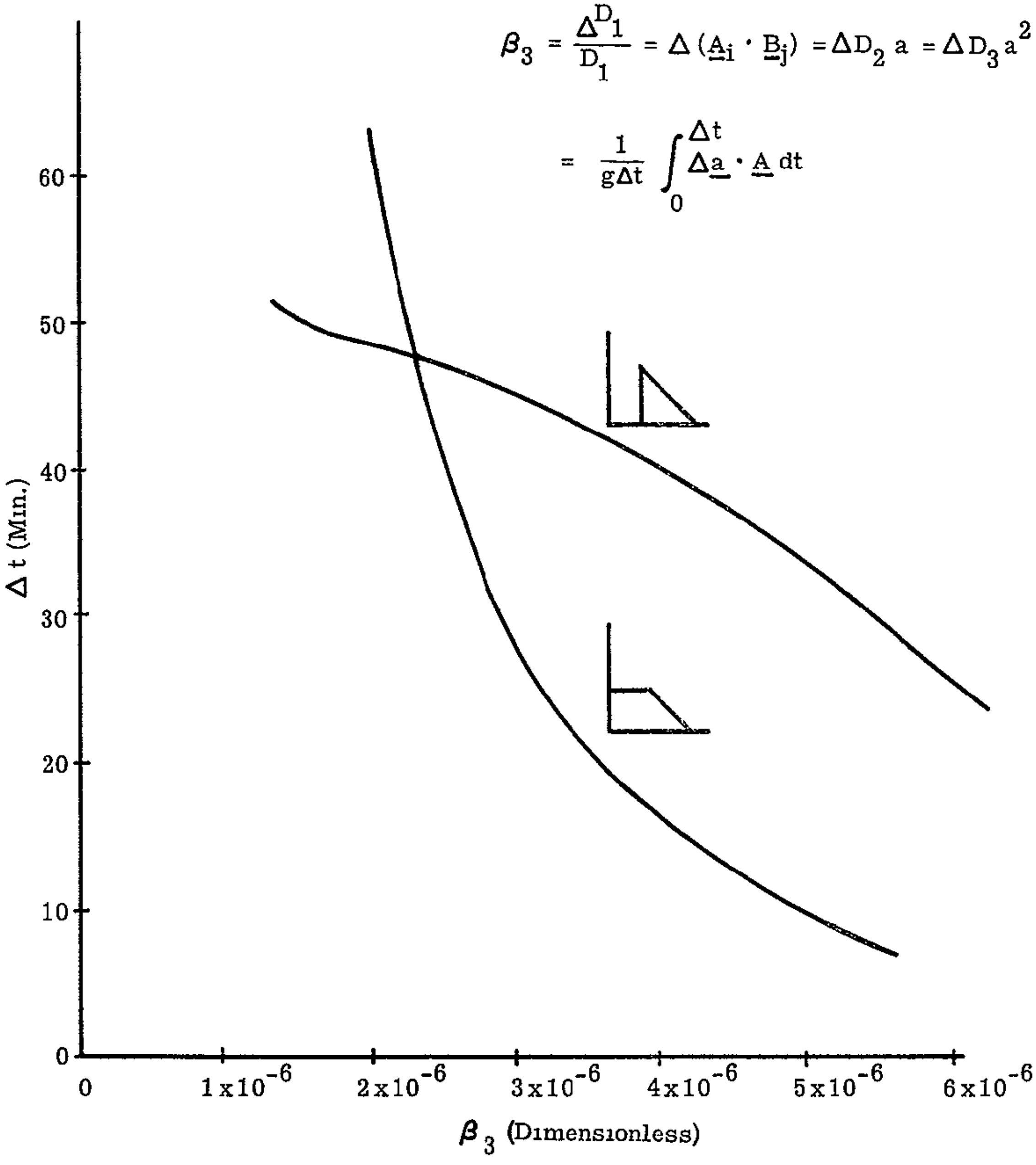


Chart II-22. Accelerometer Bias Error vs Time

- With Bubble Level Compensation
- Input Axis Horizontal



$$\beta_3 = \frac{\Delta^{D_1}}{D_1} = \Delta\,(\underline{A}_i \cdot \underline{B}_j) = \Delta D_2\, a = \Delta D_3\, a^2$$

$$= \frac{1}{g\Delta t}\int_0^{\Delta t} \Delta\underline{a} \cdot \underline{A}\, dt$$

Chart II-23. Accelerometer Scale Factor Error vs Time

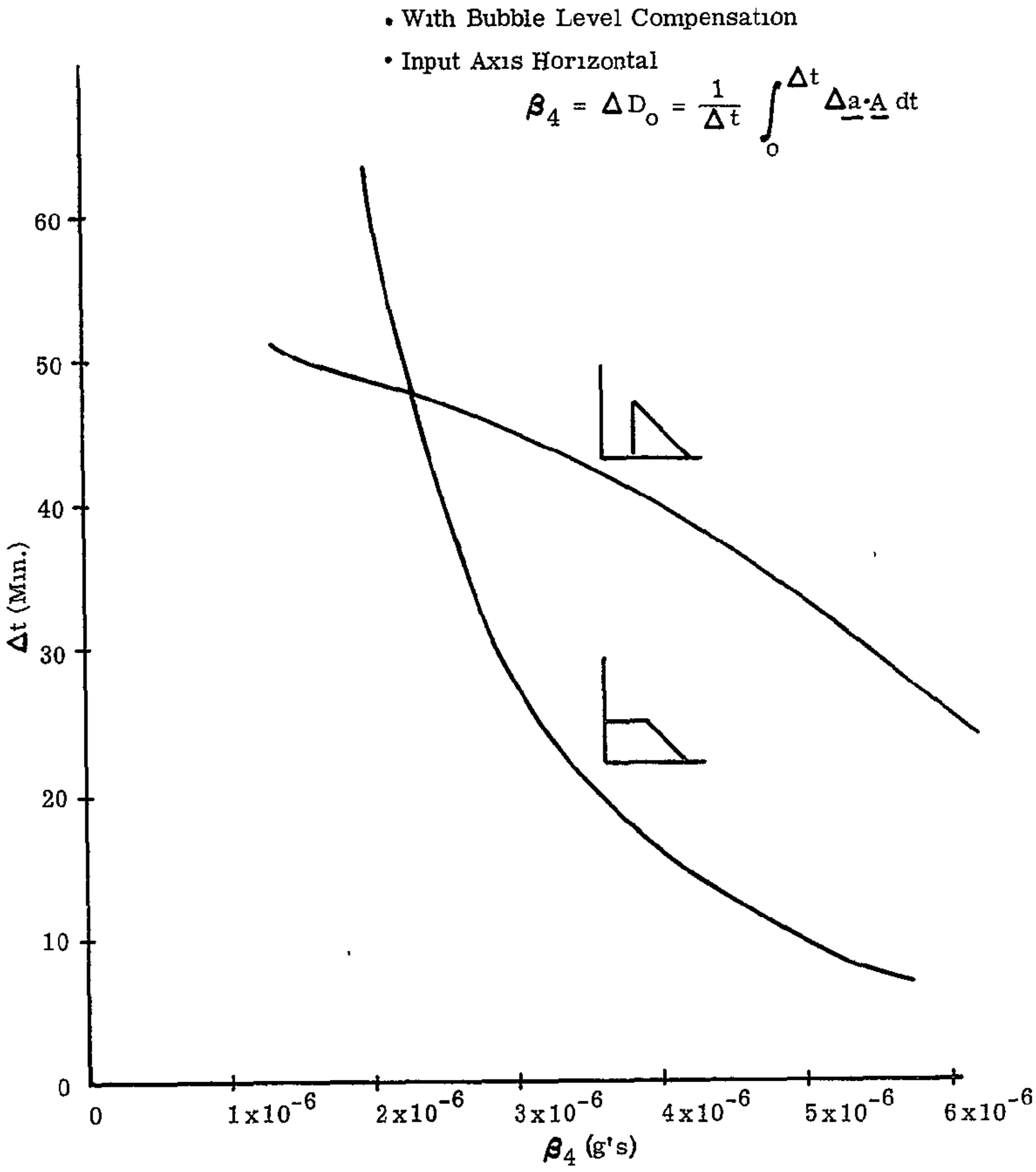


Chart II-24. Accelerometer Bias Error vs Time

In Positions 13 and 14 the environment noise would be somewhere between the limits shown in Chart II-21 for horizontal and vertical positions. (Two of the accelerometers are at 45° orientations in each of these three positions.) It is suggested, however, that a worst case is assumed, and the horizontal orientation noise graphs be utilized.

In summary, the accelerometers for all positions can be assumed to be in either a horizontal or vertical position. In the vertical positions, ten minutes of data can be assumed to yield precisions of two parts in a million. In the horizontal positions, 30 minutes of data will probably yield six parts in a million. To convert those times to counter settings, simply multiply the time by the basic frequency of the instrument (in pulses per unit of time).

LABORATORY PROCEDURES MANUAL
PART III
ALIGNMENT

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The function of alignment is to determine the matrix (T) which transforms instrument outputs from the ISU body frame to the earth fixed frame. These frames of reference are defined in Section IV-1. Alignment is accomplished by one of three techniques.

- Mirror Alignment
- Level Alignment
- Gyrocompass Alignment.

Mirror Alignment uses autocollimators to measure angles of two ISU mirror normals with respect to the earth axes. This procedure is described in Section 1.

Section 2 describes Level Alignment. Accelerometer data determines the body frame components of the gravity vector which, together with an azimuth angle of an ISU mirror, determines the alignment.

Gyrocompass Alignment, described in Section 3, uses accelerometer and gyro data to find the body frame components of the gravity and earth rate vectors, which are then used to determine the alignment.

The alignment program must be able to interface with the navigation program for the purpose of initializing the altitude matrix. This requires that the computer Interface Electronics Unit be used for alignment data sampling, rather than the frequency counters used in calibration.

Each of the alignment procedures utilizes the Alignment Results Sheet, Chart III-1. On this sheet the inputs for the type of alignment to be performed are recorded as are the table resolver settings, table bubble level corrections, and the T matrix.

CHART III-1

ALIGNMENT RESULTS SHEET

Experiment:

Principal Operator:

Date:

Procedure (Check one): Accel. Level ☐ Gyrocompass ☐ Mirror Align. ☐

GYROCOMPASS INPUT

Calibration Constants Identification ____________________

Filtering Parameter Identification ____________________

Intersample Time, Δt ____________________

Number of Measurements, K ____________________

ACCELEROMETER LEVEL INPUT

Calibration Constants Identification ____________________

Filtering Parameter Identification ____________________

Intersample Time, Δt ____________________

Number of Measurements, K ____________________

Azimuth Angle, $\hat{\alpha}_1$ ____________________

MIRROR ALIGNMENT INPUT

Autocollimator 1	Autocollimator 2
Azimuth, $\hat{\alpha}_1$ ____________	Azimuth, α_2 ____________
Elevation, θ_1 ____________	Elevation, θ_2 ____________

TABLE POSITION

ϕ_1 ____________	ϕ_2 ____________
ϕ_3 ____________	ϕ_4 ____________

Bubble Level 1 ____________

Bubble Level 2 ____________

RESULTS

$$T = \begin{pmatrix} \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \\ \_\_\_\_\_\_ & \_\_\_\_\_\_ & \_\_\_\_\_\_ \end{pmatrix}$$

PART III
SECTION 1
MIRROR ALIGNMENT

Mirror Alignment is accomplished by obtaining the azimuth and zenith angles of the normals of two mirrors on the ISU and utilizing this information to compute the alignment matrix, T. The procedures are contained in Section III-1.1 and the computations in III-1.2. Section III-1.3, Fundamental Modes, is included as an aid to understanding the Mirror Alignment procedures.

III-1.1 Mirror Alignment Procedures

The procedures to be performed for Mirror Alignment are given on the Laboratory Procedures Sheet for Mirror Alignment, Chart III-2.

LABORATORY PROCEDURES		
MIRROR ALIGNMENT		
INSTRUCTIONS	✓	**COMMENTS**
1) Verify turn-on procedures are completed.		1) See Part I, Section 1 for turn-on procedures.
2) Align autocollimators to mirrors.		
3) Turn on computer and load mirror alignment program.		
4) Enter azimuths (α_1 and α_2) and zeniths (θ_1 and θ_2) from autocollimators on Alignment Results Sheet.		
5) Enter α_1, α_2, θ_1, and θ_2 into computer.		
6) Computer computes and outputs alignment matrix (T).		
7) Record T, test table resolver settings, and bubble levels on Alignment Results Sheet.		

III-1.2 Mirror Alignment Computations

The computations for Mirror Alignment are shown on Chart III-3. These computations use, as inputs, azimuth and zenith angles of the normals to two ISU mirrors. The output is the matrix, T.

MIRROR ALIGNMENT MATRIX

Inputs $\theta_1, \alpha_1, \theta_2$ and α_2

From these quantities the alignment matrix is given by:

$$\begin{bmatrix} \\ T \\ \\ \end{bmatrix} = \begin{bmatrix} (\underline{U}\cdot\underline{M}_1) & \dfrac{(\underline{M}_1 \times \underline{U})\cdot(\underline{M}_1 \times \underline{M}_2)}{|\underline{M}_1 \times \underline{M}_2|} & \dfrac{(\underline{E} \times \underline{N})\cdot(\underline{M}_1 \times \underline{M}_2)}{|\underline{M}_1 \times \underline{M}_2|} \\ (\underline{E}\cdot\underline{M}_1) & \dfrac{(\underline{M}_1 \times \underline{E})\cdot(\underline{M}_1 \times \underline{M}_2)}{|\underline{M}_1 \times \underline{M}_2|} & \dfrac{(\underline{N} \times \underline{U})\cdot(\underline{M}_1 \times \underline{M}_2)}{|\underline{M}_1 \times \underline{M}_2|} \\ (\underline{N}\cdot\underline{M}_1) & \dfrac{(\underline{M}_1 \times \underline{N})\cdot(\underline{M}_1 \times \underline{M}_2)}{|\underline{M}_1 \times \underline{M}_2|} & \dfrac{(\underline{U} \times \underline{E})\cdot(\underline{M}_1 \times \underline{M}_2)}{|\underline{M}_1 \times \underline{M}_2|} \end{bmatrix}$$

where

$$|\underline{M}_1 \times \underline{M}_2| = [1 - (\underline{M}_1 \cdot \underline{M}_2)^2]^{1/2}$$

$$(\underline{M}_1 \cdot \underline{M}_2) = (\underline{M}_1 \cdot \underline{U})(\underline{M}_2 \cdot \underline{U}) + (\underline{M}_1 \cdot \underline{E})(\underline{M}_2 \cdot \underline{E}) + (\underline{M}_1 \cdot \underline{N})(\underline{M}_2 \cdot \underline{N})$$

$$\begin{bmatrix} (\underline{U}\cdot\underline{M}_1) \\ (\underline{E}\cdot\underline{M}_1) \\ (\underline{N}\cdot\underline{M}_1) \end{bmatrix} = \begin{bmatrix} \cos\theta_1 \\ \cos\alpha_1 \sin\theta_1 \\ \sin\alpha_1 \sin\theta_1 \end{bmatrix}, \quad \begin{bmatrix} (\underline{U}\cdot\underline{M}_2) \\ (\underline{E}\cdot\underline{M}_2) \\ (\underline{N}\cdot\underline{M}_2) \end{bmatrix} = \begin{bmatrix} \cos\theta_2 \\ \cos\alpha_2 \sin\theta_2 \\ \sin\alpha_2 \sin\theta_2 \end{bmatrix}$$

An optional technique might utilize the value of $|\underline{M}_1 \times \underline{M}_2|$ from a previous alignment thus eliminating the aforementioned dot product and square root operations.

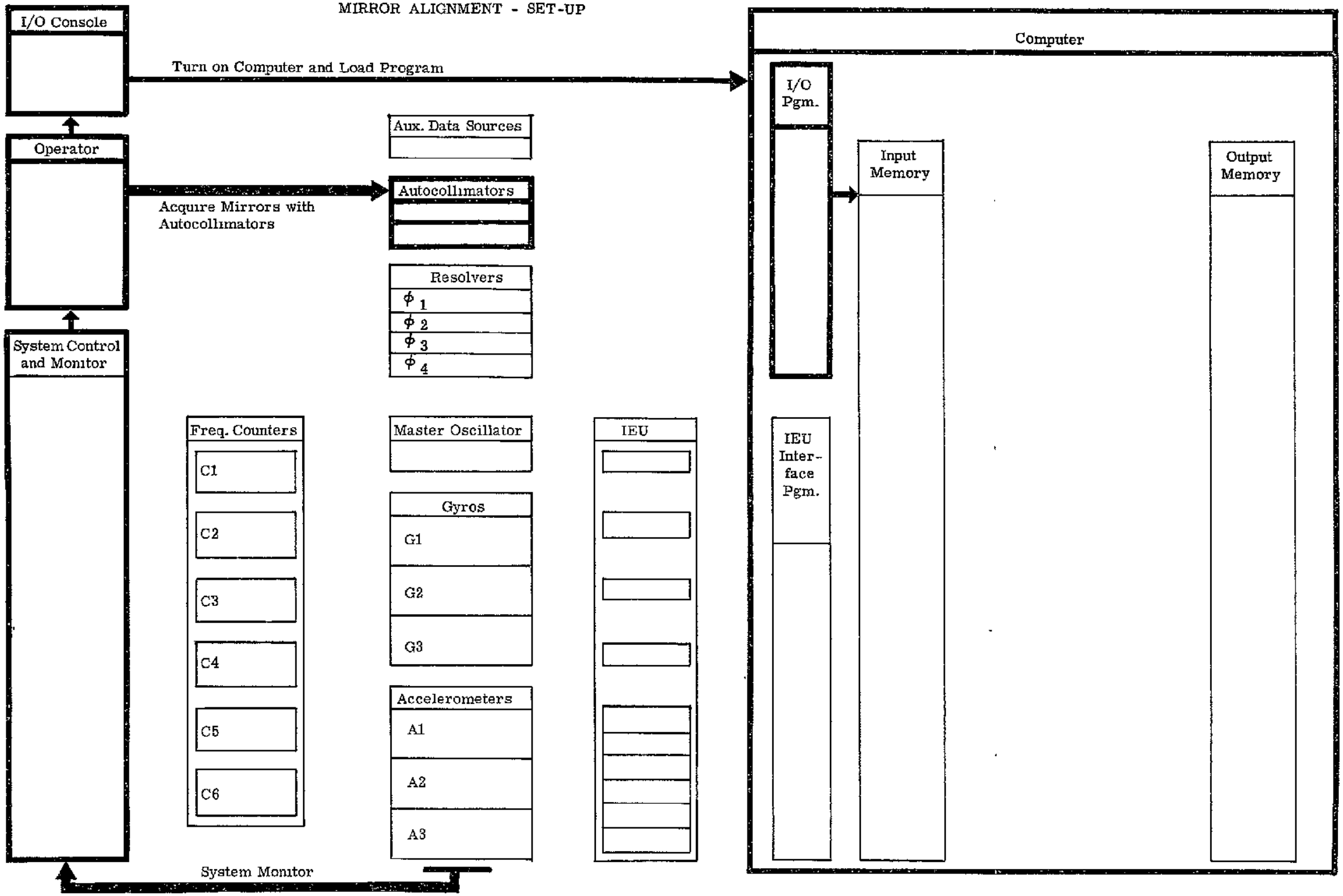
III-1.3 Mirror Alignment Fundamental Modes

The steps to be performed during Mirror Alignment are shown in Laboratory Flow Diagrams 12 to 14.

The set-up procedure includes acquiring the mirror with the autocollimators by a trial and error procedure and loading the program into the laboratory computer as shown in Flow Diagram 12. Next, the azimuth and zenith angles of the normals of Mirror One and Mirror Two are read out, recorded on the Alignment Results Sheet, and input to the computer (see Flow Diagram 13).

The computations are performed as shown in Flow Diagram 14 and the T matrix is output. T may also be supplied to the navigation loop program. The T matrix, the bubble levels, and the resolver settings are then recorded on the Alignment Results Sheet (Chart III-1). Bubble levels and resolvers settings have been included on this form for convenience in using the results of the Alignment. They are not required by the Alignment procedure.

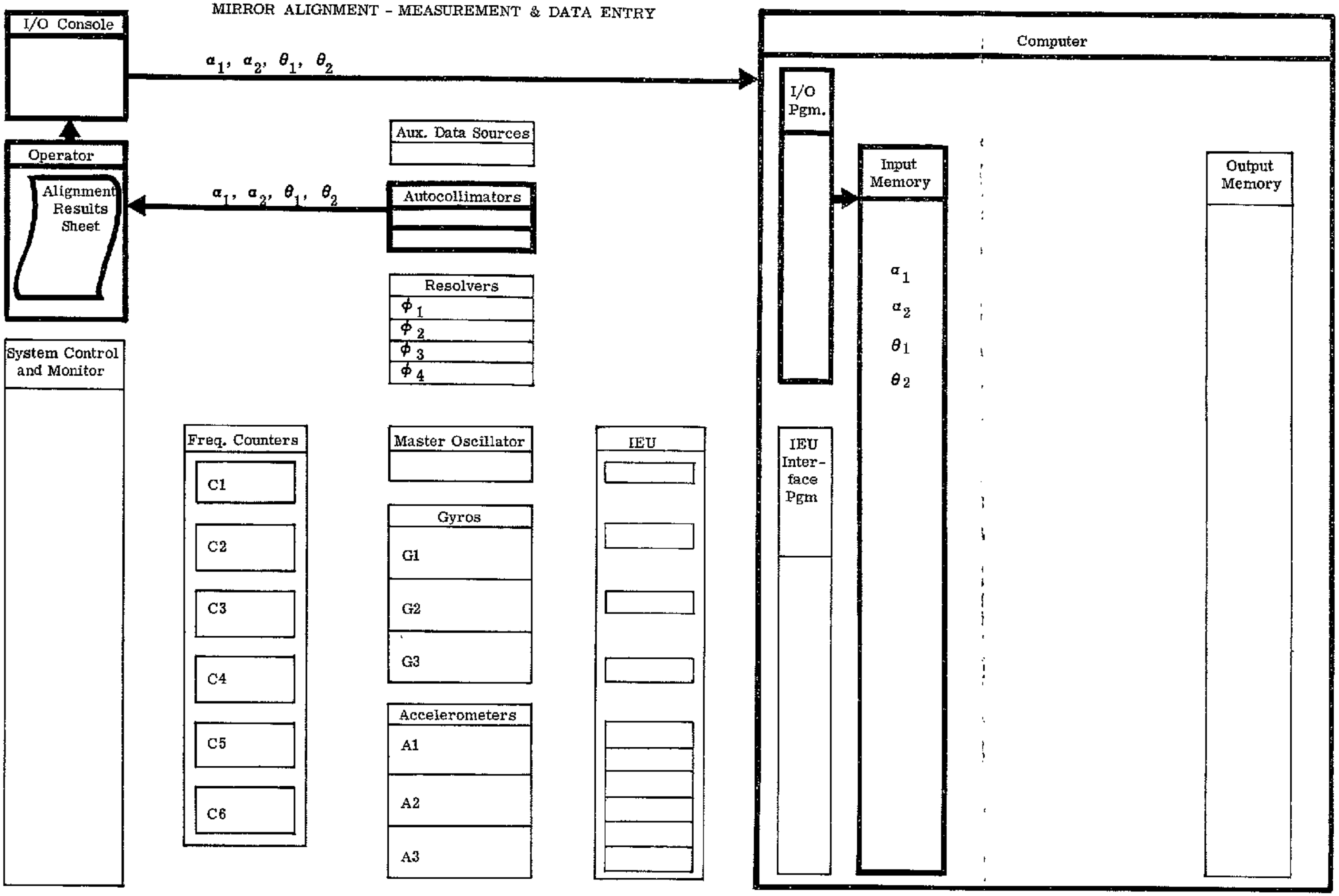
LABORATORY FLOW DIAGRAM #12
MIRROR ALIGNMENT - SET-UP



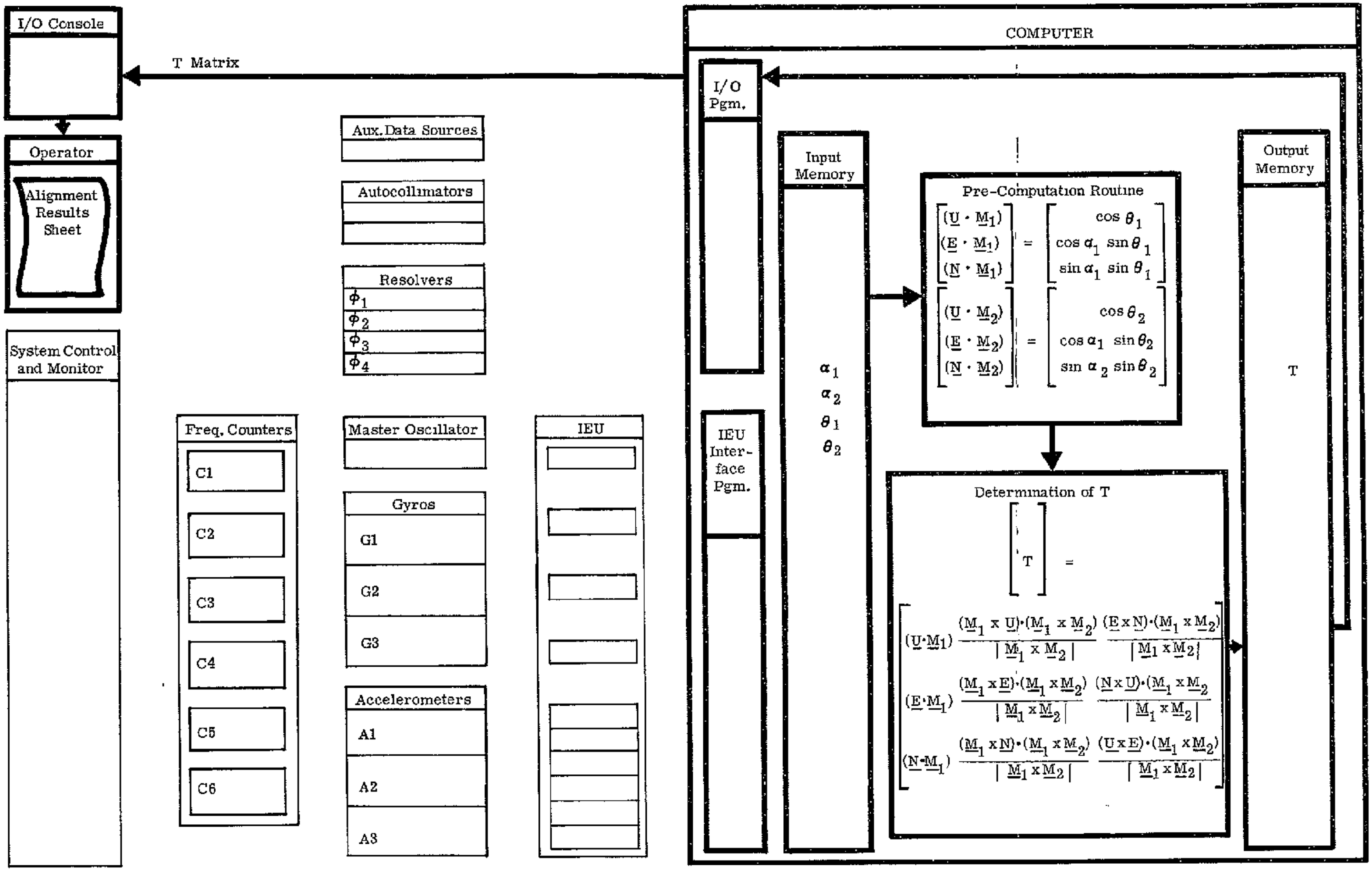
FOLDOUT FRAME

FOLDOUT FRAME 2

LABORATORY FLOW DIAGRAM #13
MIRROR ALIGNMENT - MEASUREMENT & DATA ENTRY



LABORATORY FLOW DIAGRAM #14
MIRROR ALIGNMENT - COMPUTATIONS



PART III
SECTION 2
LEVEL ALIGNMENT

In Level Alignment, data from the accelerometers are used in the estimation of the body axis components of $\underline{g}$. (This determination is sometimes referred to as "leveling".) The level information, along with the azimuth angle of an ISU fixed mirror, is used to compute the alignment matrix. Procedure listings for Level Alignment are contained in Section III-2.1, and computations are listed in Section III-2.2. Section III-2.3 contains schematic diagrams of the procedures being performed.

III-2.1 Level Alignment Procedures

The Laboratory Procedures Sheet (Chart III-4) contains the procedures to be performed for Level Alignment.

CHART III-4

LABORATORY PROCEDURES

LEVEL ALIGNMENT

INSTRUCTIONS	✓	COMMENTS
1) Verify turn-on procedures and Alignment Results Sheet completed.		1) See Part I, Section 1 for turn-on procedures.
2) Verify master oscillator and accelerometers are connected to the IEU.		
3) Set sampling rate on IEU panel.		
4) Acquire ISU mirror one in azimuth with autocollimator.		
5) Load level alignment program.		
6) Load calibration constants from current Calibration Results Sheet.		
7) Enter number of measurements, K, and intersample time, Δt, from Alignment Results Sheet.		
8) Enter parameters from parameter evaluation program for chosen K and Δt.		8) See Part I, Section 3. Not required if average estimation technique is used.
9) Initiate alignment.		
10) Enter azimuth angle, α_1, from autocollimators on Alignment Results Sheet and into computer.		
11) Enter test table bubble levels and resolver readouts on Alignment Results Sheet.		
12) After computer output, enter T matrix on Alignment Results Sheet.		

III-2.2 Level Alignment Computations

The Level Alignment computations are accomplished on the laboratory computer, and utilize the laboratory executive and navigation programs for control, input/output, and sampling of accelerometer and timing data. Inputs to these computations are the calibration constants; the number of consecutive accelerometer data samples to be taken (K); the time duration of each of these samples (Δt); and the azimuth angle of ISU Mirror One (α_1). In addition, the Posterior Mean technique requires the $\underline{M}$ and $\underline{b}$ outputs from the Alignment Parameter Evaluation Routine (see Section I-3).

The computations program is segmented into three routines. The Preprocessing Routine converts accelerometer data into integrals of input acceleration along the ISU axes. The second routine estimates the body components of the gravity vector from these integrals. The final routine computes the alignment matrix from the gravity estimate and α_1. The computations to be performed by the three routines are presented on Charts III-5, III-6, and III-7.

CHART III-5

PREPROCESSING COMPUTATIONS

Inputs $(\Sigma\gamma)_{k2}$, $(\Sigma\gamma)_{k1}$, and $\left(\Sigma n_2^T\right)$ for k = 1, 2, 3

The outputs $\int_t^{t+\Delta t}(\underline{a}\cdot\underline{B}_k)dt$ (k = 1, 2, 3) are given by the following computations:

- $$P_k^A \equiv [(\Sigma\gamma)_{k2} - (\Sigma\gamma)_{k1}]$$
- $$\Delta t \equiv S_2^T\left(\Sigma n_2^T\right)$$
- $$[\overline{(\underline{a}\circ\underline{A}_k)\Delta t}] \equiv P_k^A/(D_1)_k - (D_O)_k\Delta t$$
- $$(\overline{\underline{a}\cdot\underline{A}_k}) \equiv [\overline{(\underline{a}\cdot\underline{A}_k)\Delta t}]/\Delta t$$
- $$\int_t^{t+\Delta t}(\underline{a}\cdot\underline{A}_k)dt = \overline{[(\underline{a}\cdot\underline{A}_k)\Delta t]} - (D_2)_k(\overline{\underline{a}\cdot\underline{A}_k})^2\Delta t - (D_3)_k(\overline{\underline{a}\cdot\underline{A}_k})^3\Delta t$$
- $$\int_t^{t+\Delta t}(\underline{a}\cdot\underline{B}_k)dt = \sum_{\ell} Q_{k\ell}^A \int_t^{t+\Delta t}(\underline{a}\cdot\underline{A}_\ell)dt$$

where

- $$Q^A = \begin{bmatrix} 1 & -(\underline{A}_1\cdot\underline{B}_2) & -(\underline{A}_1\cdot\underline{B}_3) \\ -(\underline{A}_2\cdot\underline{B}_1) & 1 & -(\underline{A}_2\cdot\underline{B}_3) \\ -(\underline{A}_3\cdot\underline{B}_1) & -(\underline{A}_3\cdot\underline{B}_2) & 1 \end{bmatrix}$$

CHART III-6

ESTIMATION ROUTINE COMPUTATIONS – LEVEL

Inputs: Preprocessed accelerometer measurements, $\underline{X}$, estimation matrix, M, and vector, $\underline{b}$

Output: Estimate of acceleration components in body frame, $\widehat{\underline{g} \circ \underline{B}_i}$, $i = 1, 2, 3$ at time t*

The basic estimation computation is

$$\begin{bmatrix} \widehat{\underline{g} \cdot \underline{B}_1}\,(t^*) \\ \widehat{\underline{g} \cdot \underline{B}_2}\,(t^*) \\ \widehat{\underline{g} \cdot \underline{B}_3}\,(t^*) \end{bmatrix} = M\underline{X} + \underline{b}$$

where

$$\underline{X}^T = \left[\int_0^{\Delta t} \underline{a} \cdot \underline{B}_1\, dt,\ \int_{\Delta t}^{2\,\Delta t} \underline{a} \cdot \underline{B}_1\, dt, \cdots \int_{(K-1)\,\Delta t}^{K\Delta t} \underline{a} \cdot \underline{B}_1\, dt, \int_0^{\Delta t} \underline{a} \circ \underline{B}_2\, dt, \cdots, \right.$$

$$\left. \int_{(K-1)\,\Delta t}^{K\Delta t} \underline{a} \cdot \underline{B}_2\, dt, \cdots \int_{(K-1)\,\Delta t}^{K\Delta t} \underline{a} \cdot \underline{B}_3\, dt \right]$$

Δt = Intersample time

K = Number of samples

- Posterior Mean Technique (Instantaneous): Computations of $\underline{b}$ and M from the Estimation Matrix Computation Chart

- Simple Average Technique:

$$\underline{b} = \underline{0}$$

$$M = \begin{bmatrix} (K\Delta t)^{-1} \cdots (K\Delta t)^{-1} & & 0 \\ & (K\Delta t)^{-1} \cdots (K\Delta t)^{-1} & \\ 0 & & (K\Delta t)^{-1} \cdots (K\Delta t)^{-1} \\ \underbrace{\qquad\qquad\qquad}_{K} & \underbrace{\qquad\qquad\qquad}_{K} & \underbrace{\qquad\qquad\qquad}_{K} \end{bmatrix}$$

CHART III-7

LEVEL ALIGNMENT MATRIX

Inputs $(\underline{g}\cdot\underline{B}_1)$, $(\underline{g}\cdot\underline{B}_2)$, $(\underline{g}\cdot\underline{B}_3)$ and α_1

From these quantities the alignment matrix is given by:

$$\begin{bmatrix} \\ T \\ \\ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\alpha_1 & \cos\alpha_1 \\ 0 & -\cos\alpha_1 & \sin\alpha_1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{1}{|\underline{M}_1 x \underline{U}|} \\ \dfrac{1}{|\underline{M}_1 x \underline{U}|} & \dfrac{(\underline{M}_1\cdot\underline{U})}{|\underline{M}_1 x \underline{U}|} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ (\underline{U}\cdot\underline{B}_1) & (\underline{U}\cdot\underline{B}_2) & (\underline{U}\cdot\underline{B}_3) \\ 0 & -(\underline{U}\cdot\underline{B}_3) & (\underline{U}\cdot\underline{B}_2) \end{bmatrix}$$

where

- $(\underline{M}_1\cdot\underline{U}) = (\underline{U}\cdot\underline{B}_1)$
- $|\underline{M}_1 x \underline{U}| = [1 - (\underline{M}_1 \circ \underline{U}^2]^{1/2}$
- $(\underline{U}\cdot\underline{B}_k) = (\underline{g}\cdot\underline{B}_k)/g$
- $g = [(\underline{g}\cdot\underline{B}_1)^2 + (\underline{g}\cdot\underline{B}_2)^2 + (\underline{g}\cdot\underline{B}_3)^2]^{1/2}$

An optional technique might utilize any of the following additional inputs:

- The zenith angle (θ_1) of mirror one might be utilized to find $(\underline{M}_1\cdot\underline{U})$ from

$$(\underline{M}_1\cdot\underline{U}) = \cos\theta_1$$

- The magnitude of gravity (g) might be supplied from a local survey. This piece of information can be utilized to reduce the number of required accelerometers to two.

III-2.3 Level Alignment Fundamental Modes

The steps required to accomplish level alignment are shown in Laboratory Flow Diagrams 15, 16 and 17. The procedures illustrated utilize autocollimator and accelerometer measurements to determine the alignment T matrix. The autocollimator measures the azimuth of the normal to ISU Mirror One while the accelerometers measure components of the gravity vector.

The operator verifies first that system turn-on has been completed, and that values of the parameters required for alignment have been chosen. These include the number of samples to be taken (K), the duration of each of the K samples (Δt), the accelerometer calibration constants, and the estimation routine parameters (see Chart I-8). Next, the set up is performed as shown on Flow Diagram 15. The operator acquires ISU Mirror One in azimuth with an autocollimator. He verifies the master oscillator and accelerometer connections to the IEU, and sets the IEU sampling rate on the IEU panel The alignment program and the navigation loop program (if a navigation experiment is to be done) are loaded into the computer. The current calibration constants, Δt and K are also loaded.

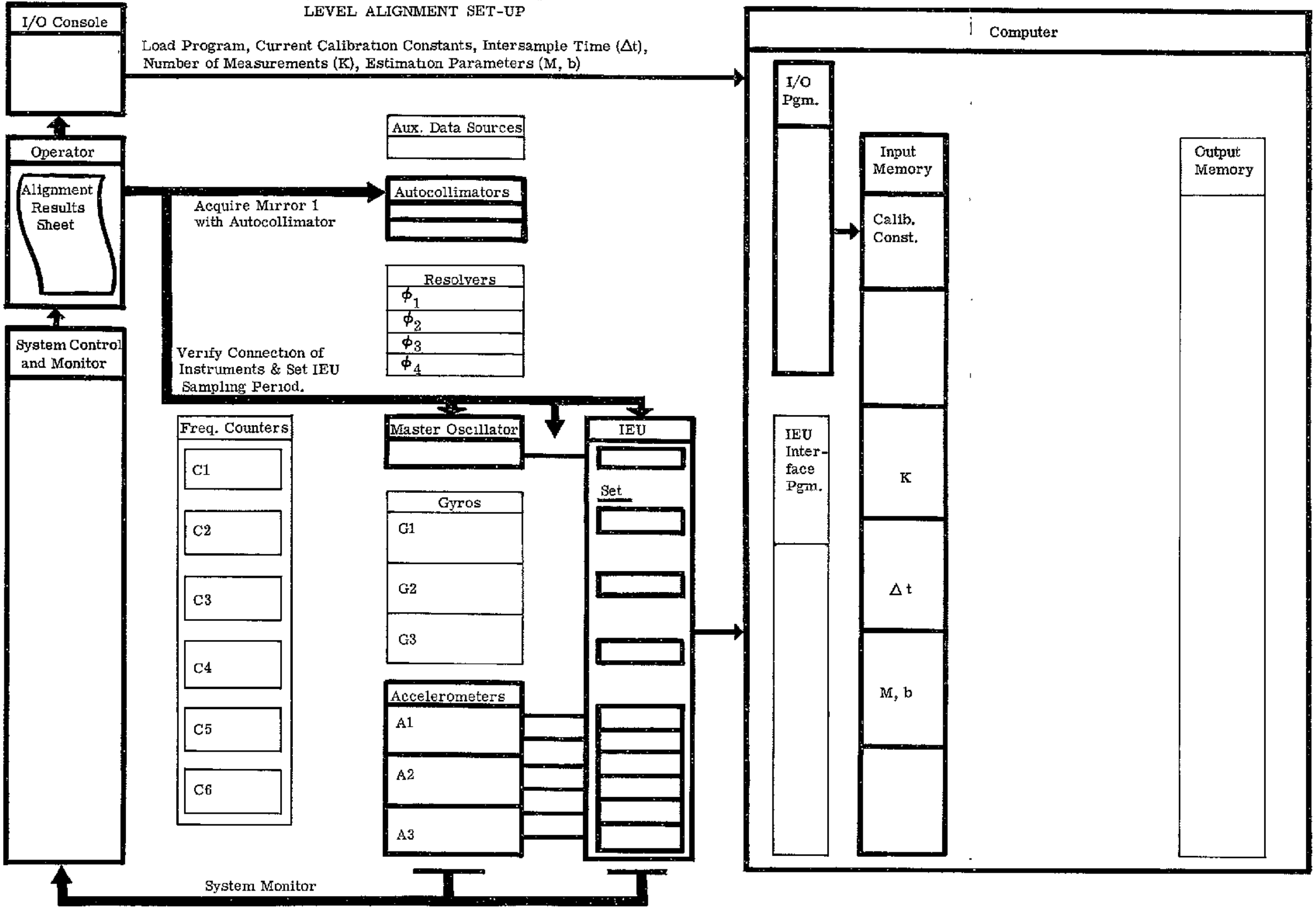
The operator must select one of the two programmed gravity vector ($\underline{g}$) estimation techniques: the posterior mean or simple average.

Selection of the posterior mean requires input to the computer of previously computed elements of the M matrix and $\underline{b}$ vector via magnetic or paper tape. The required tape will be accompanied by an Alignment Parameter Evaluation Sheet (Chart I-8). The operator must verify that the input parameters on this sheet agree with those specified for the subject experiment. If a tape with accompanying sheet cannot be found which does so agree, then the operator should produce one by following the procedures of Section I-3.

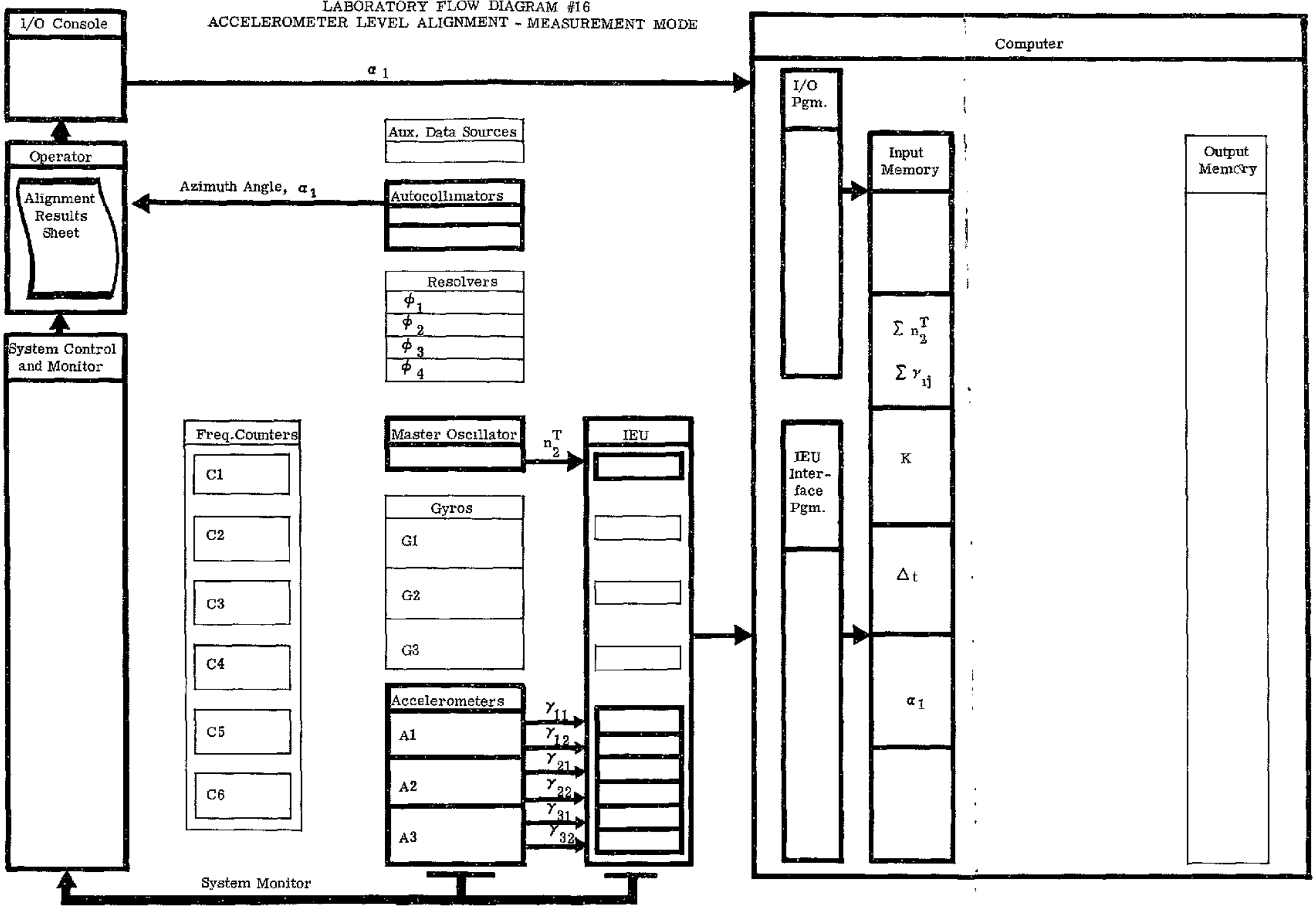
If the simple average technique is selected, the M matrix and $\underline{b}$ vector do not have to be fed into the computer. In this case, the M matrix is only a function (as shown in Chart III-6) of the chosen K and Δt, while the $\underline{b}$ vector is identically zero. The computer can therefore generate M and $\underline{b}$ from K and Δt which was input earlier in the procedure.

Following the initial set up and data entry into the computer, the operator initiates the measurement mode shown in Flow Diagram 16. During this mode, he must record the azimuth angle (α_1) of the normal to Mirror One (as measured by the autocollimator) on the Alignment Results Sheet. This angle must also be entered into the computer.

LABORATORY FLOW DIAGRAM #15
LEVEL ALIGNMENT SET-UP



LABORATORY FLOW DIAGRAM #16
ACCELEROMETER LEVEL ALIGNMENT - MEASUREMENT MODE



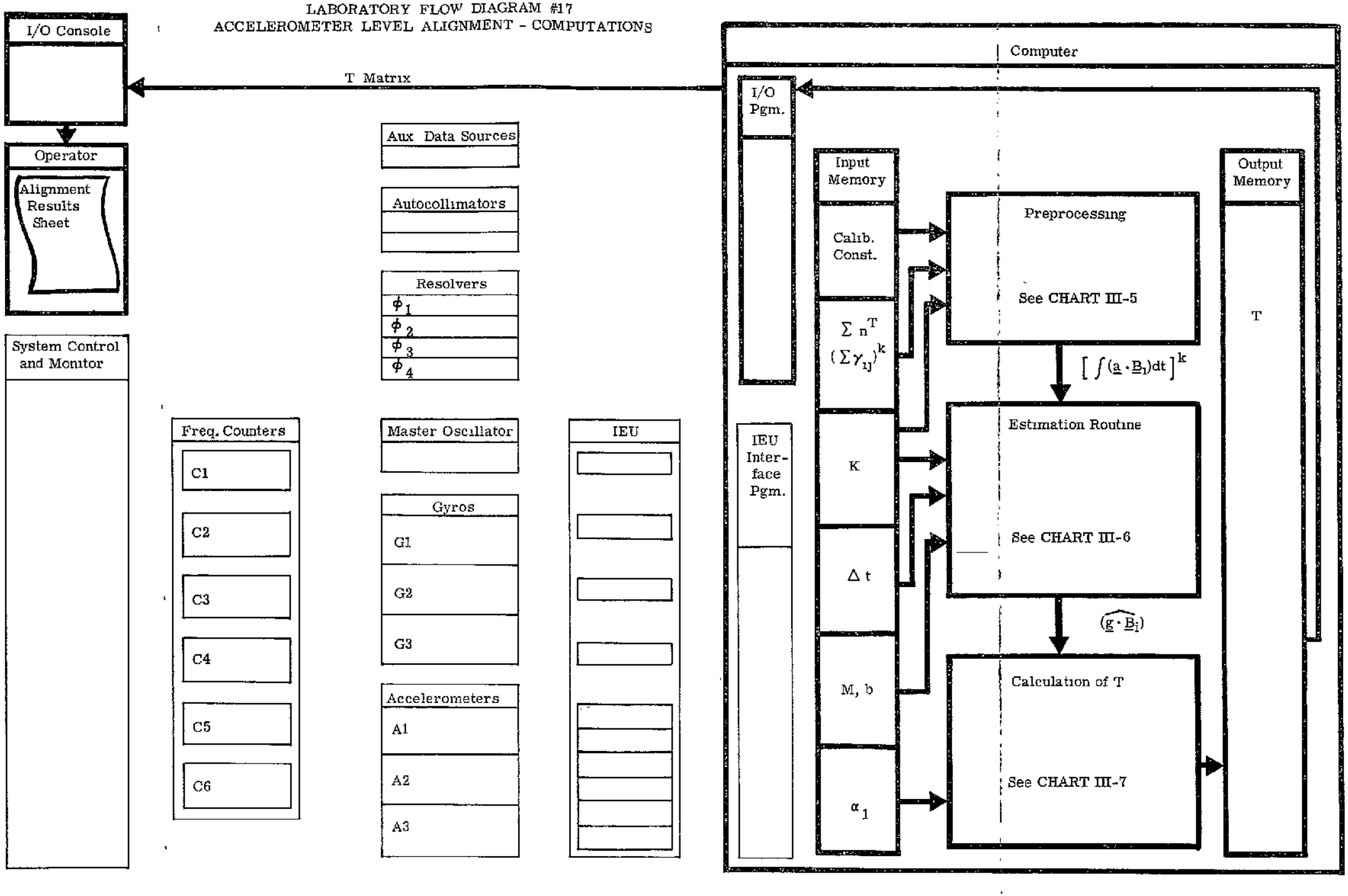
The accelerometer outputs are accumulated in the counters of the IEU until the IEU time counter has counted a number of master oscillator time pulses equal to the selected sampling period. These counts are then automatically input into the computer, where they are summed together until the total elapsed time is equal to Δt. The computer saves this accumulation and begins a new accumulation. This process of accumulating samples over the Δt interval is continued by the computer until K such accumulations have been obtained.

The computations for level alignment are shown in Flow Diagram 17. The preprocessing routine converts the measurement data into integrals of acceleration in body coordinates. This same function is required in a navigation program. Thus, alignment may use the program as developed for navigation. The outputs of this routine are K sets of $\int(\underline{a}\cdot\underline{B}_i)\,dt$, for the K consecutive measurement periods.

The estimation routine receives the outputs of the preprocessing routine and determines an estimate of the $\underline{g}$ vector in the body axes frame $(\widehat{\underline{g}\cdot\underline{B}_i})$ by application of the M matrix and $\underline{b}$ vector.

The final routine computes the T matrix from α_1 and the components of $\underline{g}$. The estimate of this matrix is output by the computer program and is recorded by the operator on the Alignment Results Sheet. This T matrix also initializes the navigation program.

LABORATORY FLOW DIAGRAM #17
ACCELEROMETER LEVEL ALIGNMENT - COMPUTATIONS



PART III
SECTION 3
GYROCOMPASS ALIGNMENT

In Gyrocompass Alignment, the body components of the gravity and earth rotation vectors are estimated from accelerometer and gyro data. These are then used to determine the Alignment matrix. Procedure listings for Gyrocompass Alignment are contained in Section III-3.1 and the computations are given in III-3.2. The Fundamental Modes, Section III-3.3, contain schematic illustrations of the Gyrocompass Alignment Procedures.

III-3.1 Gyrocompass Alignment Procedures

The procedures to be accomplished for Gyrocompass Alignment are shown on Chart III-8.

CHART III-8

LABORATORY PROCEDURES

GYROCOMPASS ALIGNMENT

INSTRUCTIONS	✓	COMMENTS
1) Verify turn-on procedures and Alignment Results Sheet are completed.		1) See Part I, Section 1 for turn-on procedures.
2) Verify master oscillator, gyros and accelerometers are connected to the IEU.		
3) Set sampling rate on IEU panel.		
4) Load gyrocompass program.		
5) Load calibration constants from current Calibrations Result Sheet.		
6) Enter number of samples, K, and inter-sample time, Δt, from Alignment Results Sheet.		
7) Enter parameters from parameter evaluation program for selected K and Δt.		7) See Part I, Section 3. Not required if an average estimation technique is used.
8) Initiate alignment.		
9) Enter test table bubble levels and resolver readouts on Alignment Results Sheet.		
10) After computer output, enter T matrix on Alignment Results Sheet.		

III-3.2 Gyrocompass Alignment Computations

The gyrocompass alignment computations are performed on the laboratory computer. The program will interface with the other laboratory programs which are used for control, input/output, and sampling of instrument data. The alignment computations require the following inputs:

- The Calibration Constants
- The time duration of each sample of instrument data (Δt)
- The number of samples to be used (K)
- Estimation parameters required (only) for the Posterior Mean gravity estimation technique.

The computations are segmented into portions to be performed by three routines. The first routine, Preprocessing, determines integrals of the acceleration and angular velocity applied to the ISU from the accelerometer and gyro output data. The Estimation Routine processes sets of these integrals to obtain estimates of the body axes components of the gravity and earth rate vectors. These estimates are used by the final routine to compute the alignment matrix.

PREPROCESSING COMPUTATIONS

Inputs $(\Sigma\gamma)_{k2}$, $(\Sigma\gamma)_{k1}$, $(\Sigma\delta)_k$, and $\left(\Sigma n_2^T\right)$ for k = 1, 2, 3

The outputs $\int_t^{t+\Delta t}(\underline{\omega}\cdot\underline{B}_k)dt$ and $\int_t^{t+\Delta t}(\underline{a}\cdot\underline{B}_k)dt$ (k = 1, 2, 3) are given by the following computations

- $P_k^A \equiv [(\Sigma\gamma)_{k2} - (\Sigma\gamma)_{k1}]$
- $P_k^G \equiv (\Sigma\delta)_k$
- $\Delta t \equiv S_2^T\left(\Sigma n_2^T\right)$
- $[\overline{(\underline{\omega}\cdot\underline{G}_k)\Delta t}] \equiv P_k^G(\Delta\Phi)_k - (R)_k\Delta t$
- $[\overline{(\underline{a}\cdot\underline{A}_k)\Delta t}] \equiv P_k^A/(D_1)_k - (D_O)_k\Delta t$
- $\overline{(\underline{\omega}\cdot\underline{G}_k)} \equiv [\overline{(\underline{\omega}\cdot\underline{G}_k)\Delta t}]/\Delta t$
- $\overline{(\underline{a}\cdot\underline{A}_k)} \equiv [\overline{(\underline{a}\cdot\underline{A}_k)\Delta t}]/\Delta t$
- $\overline{(\underline{a}\cdot\underline{G}_k)} \equiv \overline{(\underline{a}\cdot\underline{A}_k)}$
- $$\overline{(\underline{a}\cdot\underline{O}_k)} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \overline{(\underline{a}\cdot\underline{A}_1)} \\ \overline{(\underline{a}\cdot\underline{A}_2)} \\ \overline{(\underline{a}\cdot\underline{A}_3)} \end{bmatrix}$$
- $$\overline{(\underline{a}\cdot\underline{S}_k)} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \overline{(\underline{a}\cdot\underline{A}_1)} \\ \overline{(\underline{a}\cdot\underline{A}_2)} \\ \overline{(\underline{a}\cdot\underline{A}_3)} \end{bmatrix}$$
- $$\begin{aligned}\int_t^{t+\Delta t}(\underline{\omega}\cdot\underline{G}_k)dt = {} & [\overline{(\underline{\omega}\cdot\underline{G}_k)\Delta t}] - [(B_I)_k\overline{(\underline{a}\cdot\underline{G}_k)} + (B_O)_k\overline{(\underline{a}\cdot\underline{O}_k)} + (B_S)_k\overline{(\underline{a}\cdot\underline{S}_k)}]\Delta t \\ & - [(C_{II})_k\overline{(\underline{a}\cdot\underline{G}_k)}^2 + (C_{SS})_k\overline{(\underline{a}\cdot\underline{S}_k)}^2]\Delta t \\ & - [(C_{IS})_k\overline{(\underline{a}\cdot\underline{G}_k)}\,\overline{(\underline{a}\cdot\underline{S}_k)} + (C_{OS})_k\overline{(\underline{a}\cdot\underline{O}_k)}\,\overline{(\underline{a}\cdot\underline{S}_k)} + (C_{IO})_k\overline{(\underline{a}\cdot\underline{G}_k)}\,\overline{(\underline{a}\cdot\underline{O}_k)}]\Delta t \\ & - [(Q_{II})_k\overline{(\underline{\omega}\cdot\underline{G}_k)}^2 + (Q_{IS})_k\overline{(\underline{\omega}\cdot\underline{G}_k)}\,\overline{(\underline{\omega}\cdot\underline{S}_k)}]\Delta t\end{aligned}$$
- $$\int_t^{t+\Delta t}(\underline{a}\cdot\underline{A}_k)dt = [\overline{(\underline{a}\cdot\underline{A}_k)\Delta t}] - (D_2)_k\overline{(\underline{a}\cdot\underline{A}_k)}^2\Delta t - (D_3)_k\overline{(\underline{a}\cdot\underline{A}_k)}^3\Delta t$$
- $$\int_t^{t+\Delta t}(\underline{\omega}\cdot\underline{B}_k)dt = \sum_{\ell} Q_{k\ell}^G\int_t^{t+\Delta t}(\underline{\omega}\cdot\underline{G}_\ell)dt$$
- $$\int_t^{t+\Delta t}(\underline{a}\cdot\underline{B}_k)dt = \sum_{\ell} Q_{k\ell}^A\int_t^{t+\Delta t}(\underline{a}\cdot\underline{A}_\ell)dt$$

where

- $$Q^G = \begin{bmatrix} 1 & -(\underline{G}_1\cdot\underline{B}_2) & -(\underline{G}_1\cdot\underline{B}_3) \\ -(\underline{G}_2\cdot\underline{B}_1) & 1 & -(\underline{G}_2\cdot\underline{B}_3) \\ -(\underline{G}_3\cdot\underline{B}_1) & -(\underline{G}_3\cdot\underline{B}_2) & 1 \end{bmatrix}$$
- $$Q^A = \begin{bmatrix} 1 & -(\underline{A}_1\cdot\underline{B}_2) & -(\underline{A}_1\cdot\underline{B}_3) \\ -(\underline{A}_2\cdot\underline{B}_1) & 1 & -(\underline{A}_2\cdot\underline{B}_3) \\ -(\underline{A}_3\cdot\underline{B}_1) & -(\underline{A}_3\cdot\underline{B}_2) & 1 \end{bmatrix}$$

ESTIMATION ROUTINE COMPUTATIONS – GYROCOMPASS

Inputs: Preprocessed accelerometer measurements, $\underline{X}$, estimation matrix, M, and vector, $\underline{b}$

Outputs: Estimates of gravity and earth rate components in body frame, $\widehat{\underline{g} \cdot \underline{B}_i}$ and $\widehat{\underline{\omega}^E \cdot \underline{B}_i}$, $i = 1, 2, 3$

The basic estimation computation is

$$\begin{bmatrix} \widehat{\underline{g} \cdot \underline{B}_1} \\ \widehat{\underline{g} \cdot \underline{B}_2} \\ \widehat{\underline{g} \cdot \underline{B}_3} \\ \widehat{\underline{\omega}^E \cdot \underline{B}_1} \\ \widehat{\underline{\omega}^E \cdot \underline{B}_2} \\ \widehat{\underline{\omega}^E \cdot \underline{B}_3} \end{bmatrix} = M\underline{X} + \underline{b}$$

where

$$\underline{X}^T = \Bigg[\int_0^{\Delta t} \underline{a} \cdot \underline{B}_1 \, dt, \cdots, \int_{(K-1)\Delta t}^{K\Delta t} \underline{a} \cdot \underline{B}_1 \, dt, \int_0^{\Delta t} \underline{a} \cdot \underline{B}_2 \, dt, \cdots, \int_{(K-1)\Delta t}^{K\Delta t} \underline{a} \cdot \underline{B}_3 \, dt,$$

$$\int_0^{\Delta t} \underline{\omega} \cdot \underline{B}_1 \, dt, \cdots, \int_{(K-1)\Delta t}^{K\Delta t} \underline{\omega} \cdot \underline{B}_1 \, dt, \cdots, \int_{(K-1)\Delta t}^{K\Delta t} \underline{\omega} \cdot \underline{B}_3 \, dt \Bigg]$$

Δt = Intersample time

K = Number of samples

CONTINUATION OF CHART III-10

- Simple Average Technique:

$\underline{b} = \underline{0}$

$$M = \left[\begin{array}{cccccc} (K\Delta t)^{-1}\cdots(K\Delta t)^{-1} & & & & & 0 \\ & (K\Delta t)^{-1}\cdots(K\Delta t)^{-1} & & & & \\ & & (K\Delta t)^{-1}\cdots(K\Delta t)^{-1} & & & \\ & & & (K\Delta t)^{-1}\cdots(K\Delta t)^{-1} & & \\ & & & & (K\Delta t)^{-1}\cdots(K\Delta t)^{-1} & \\ 0 & & & & & (K\Delta t)^{-1}\cdots(K\Delta t)^{-1} \\ \underbrace{\hphantom{(K\Delta t)^{-1}\cdots(K\Delta t)^{-1}}}_{K} & \underbrace{\hphantom{(K\Delta t)^{-1}\cdots(K\Delta t)^{-1}}}_{K} & \underbrace{\hphantom{(K\Delta t)^{-1}\cdots(K\Delta t)^{-1}}}_{K} & \underbrace{\hphantom{(K\Delta t)^{-1}\cdots(K\Delta t)^{-1}}}_{K} & \underbrace{\hphantom{(K\Delta t)^{-1}\cdots(K\Delta t)^{-1}}}_{K} & \underbrace{\hphantom{(K\Delta t)^{-1}\cdots(K\Delta t)^{-1}}}_{K} \end{array}\right]$$

Diagonal 6 x 6K

- Hybrid Technique: Use posterior-mean estimate of gravity as given for Level Alignment and simple average for earth rate.

CHART III-11

GYROCOMPASS MATRIX

Inputs $(\underline{g}\circ \underline{B}_1)$, $(\underline{g}\cdot \underline{B}_2)$, $(\underline{g}\cdot \underline{B}_3)$, $(\underline{\omega}^E\cdot \underline{B}_1)$ $(\underline{\omega}^E\cdot \underline{B}_2)$, and $(\underline{\omega}^E\circ \underline{B}_3)$

From these quantities the alignment matrix is given by:

$$\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \dfrac{1}{|\underline{W}\times\underline{U}|} \\ \dfrac{1}{|\underline{W}\times\underline{U}|} & -\dfrac{(\underline{W}\cdot\underline{U})}{|\underline{W}\times\underline{U}|} & 0 \end{bmatrix} \begin{bmatrix} (\underline{W}\cdot\underline{B}_1) & (\underline{W}\cdot\underline{B}_2) & (\underline{W}\cdot\underline{B}_3) \\ (\underline{U}\cdot\underline{B}_1) & (\underline{U}\cdot\underline{B}_2) & (\underline{U}\cdot\underline{B}_3) \\ (\underline{W}\times\underline{U})\cdot(\underline{B}_2\times\underline{B}_3) & (\underline{W}\times\underline{U})\cdot(\underline{B}_3\times\underline{B}_1) & (\underline{W}\times\underline{U})\cdot(\underline{B}_1\times\underline{B}_2) \end{bmatrix}$$

where

- $(\underline{W}\circ\underline{U}) = (\underline{W}\cdot\underline{B}_1)(\underline{U}\cdot\underline{B}_1) + (\underline{W}\cdot\underline{B}_2)(\underline{U}\circ\underline{B}_2) + (\underline{W}\circ\underline{B}_3)(\underline{U}\cdot\underline{B}_3)$
- $|\underline{W}\times\underline{U}| = [1 - (\underline{W}\cdot\underline{U})^2]^{1/2}$
- $(\underline{W}\cdot\underline{B}_k) = (\underline{\omega}^E\cdot\underline{B}_k)/\omega^E$
- $(\underline{U}\circ\underline{B}_k) = (\underline{g}\cdot\underline{B}_k)/g$
- $\omega^E = [(\underline{\omega}^E\circ\underline{B}_1)^2 + (\underline{\omega}^E\cdot\underline{B}_2)^2 + (\underline{\omega}^E\cdot\underline{B}_3)^2]^{1/2}$
- $g = [(\underline{g}\cdot\underline{B}_1)^2 + (\underline{g}\cdot\underline{B}_2)^2 + (\underline{g}\cdot\underline{B}_3)^2]^{1/2}$

An optional technique might utilize any of the following additional inputs:

- The local latitude (λ) might be utilized to find $(\underline{W}\cdot\underline{U})$ from

$$(\underline{W}\cdot\underline{U}) = \cos\lambda$$

- The magnitude of gravity (g) might be supplied from a local survey.
- The magnitude of earth rate (ω^E) might be supplied from a local survey.

A use of all additional inputs could reduce the number of necessary instruments to three (either two accelerometers and one gyro, or one accelerometer and two gyros).

III-3.3 Gyrocompass Fundamental Modes

The steps required to accomplish Gyrocompass Alignment are illustrated in Laboratory Flow Diagrams 18, 19 and 20. The procedures illustrated utilize gyro and accelerometer measurements to determine the alignment matrix. The gyros sense components of the earth rate vector ($\underline{\omega}^E$) while the accelerometers sense components of the gravity vector ($\underline{g}$). The Estimation Routine computes a best estimate of the body components of $\underline{\omega}^E$ and $\underline{g}$ which may then be used directly in the computation of the T Matrix.

The set up required for Gyrocompass Alignment is Shown on Flow Diagram 18. The master oscillator, accelerometers and gyros must be connected to the IEU, and the IEU sampling rate set. The navigation and alignment programs are loaded. The current calibration constants and the intersample time (Δt) are entered into the computer.

The operator must select one of the two programmed estimation techniques, for processing of accelerometer data the posterior mean or simple average.

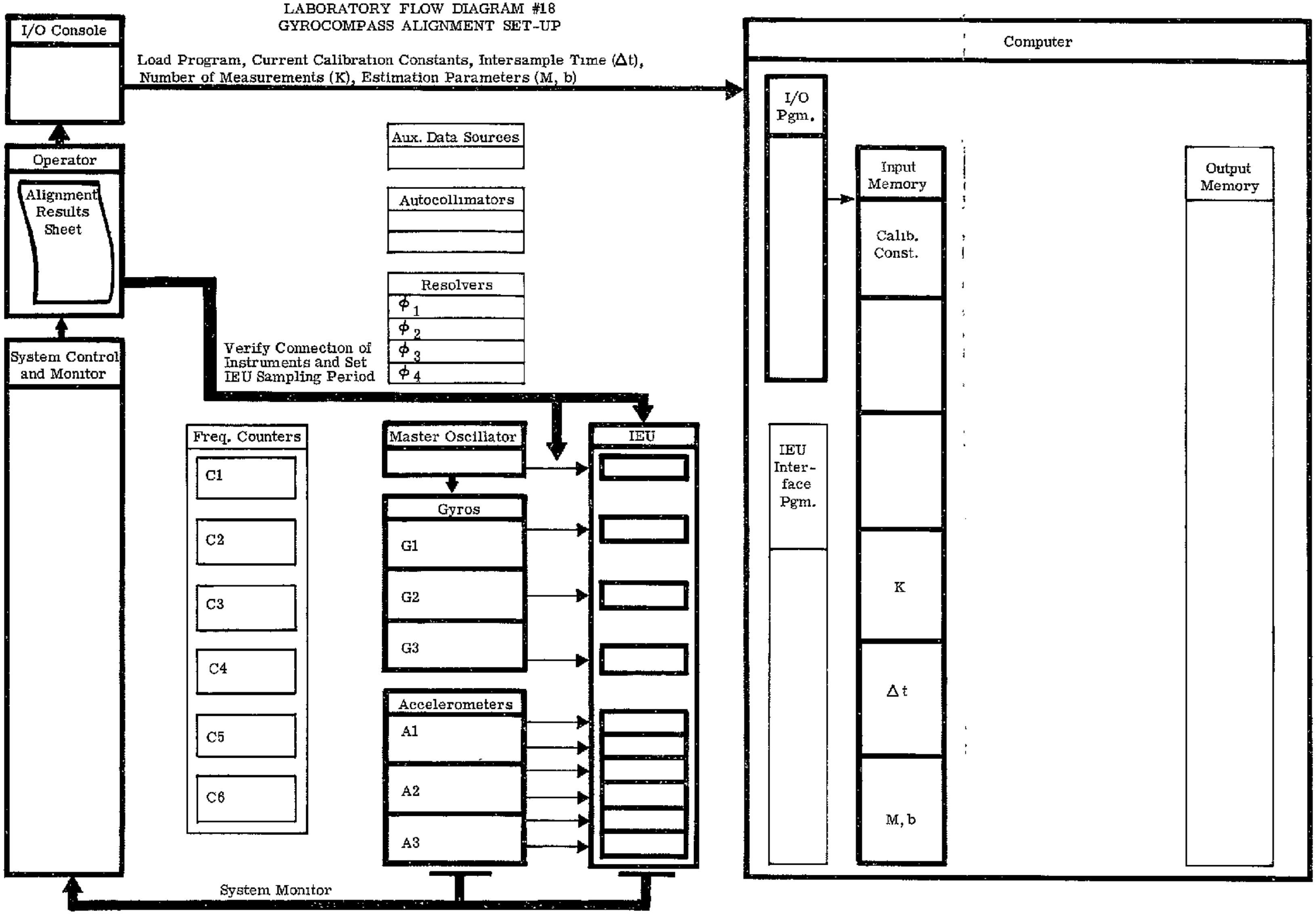
Selection of the posterior mean requires an input to the computer of previously computed elements of the M matrix and $\underline{b}$ vector via magnetic or paper tape. The required tape will be accompanied by an Alignment Parameter Evaluation Sheet (see Chart I-8). The operator must verify that the input parameters on this sheet agree with those specified for the subject experiment. If a tape with accompanying sheet cannot be found which does so agree, then the operator should produce one by following the procedures of Section I-3.

If the simple average technique is selected, the M matrix and $\underline{b}$ vector do not have to be fed into the computer. In this case, the M matrix is only a function (as shown in Chart III-6) of the chosen K and Δt while the $\underline{b}$ vector is identically zero. The computer can therefore generate M and $\underline{b}$ from the K and Δt which are input earlier in the procedure.

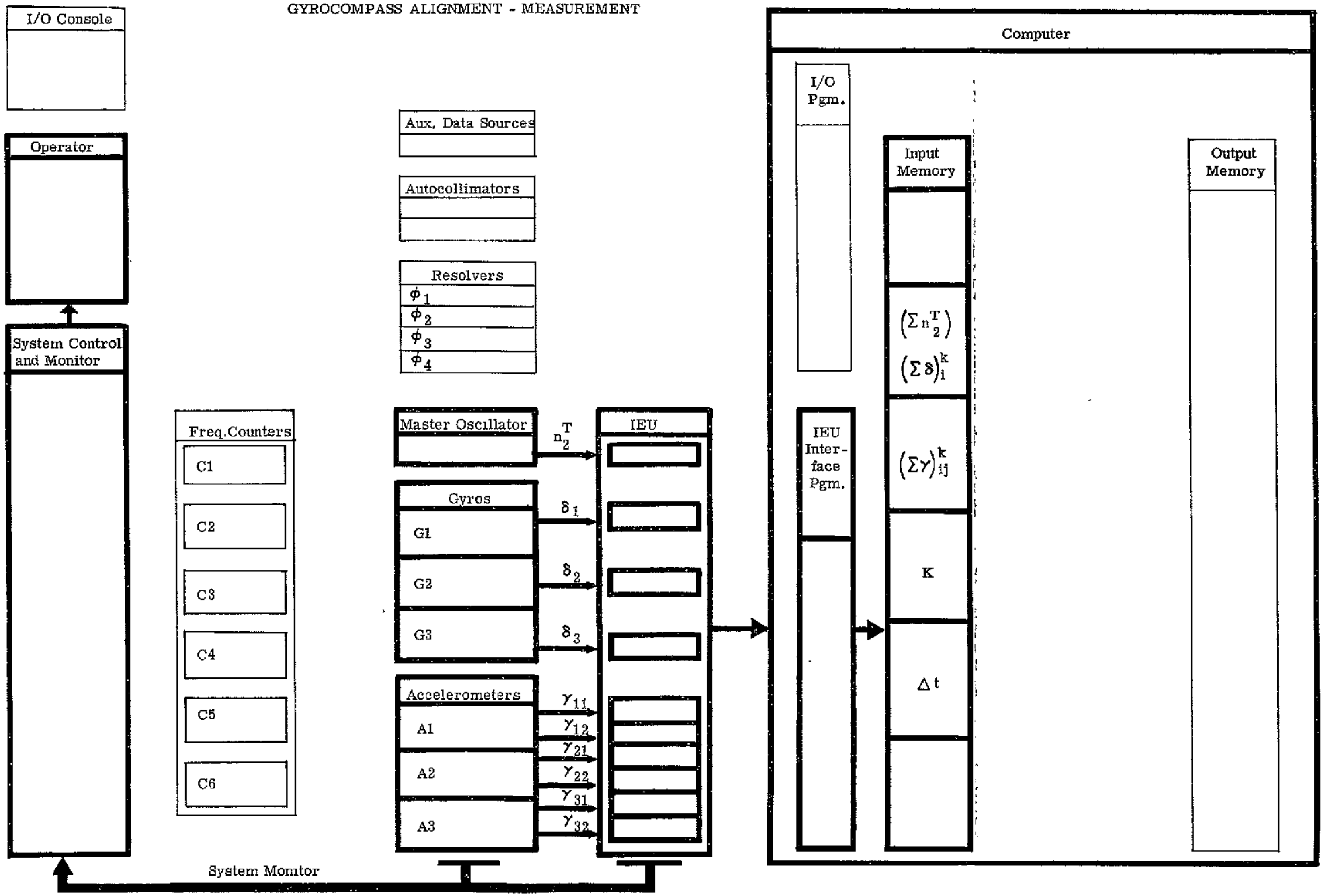
The measurement mode for Gyrocompass Alignment is shown in Flow Diagram 19. Master oscillator timing, gyro and accelerometer data are collected in the IEU registers until the IEU time register reaches a value equal to the IEU sampling period. It is then dumped into computer memory and summed under computer control with previous samples until a total time Δt has elapsed. If $K \neq 1$ the process is repeated until the required number of samples are obtained.

The computations performed are shown in Flow Diagram 20. The routine converts the measurement data into integrals of acceleration and angular velocity in body coordinates. This same function is required for a navigation program. Thus, alignment may use the

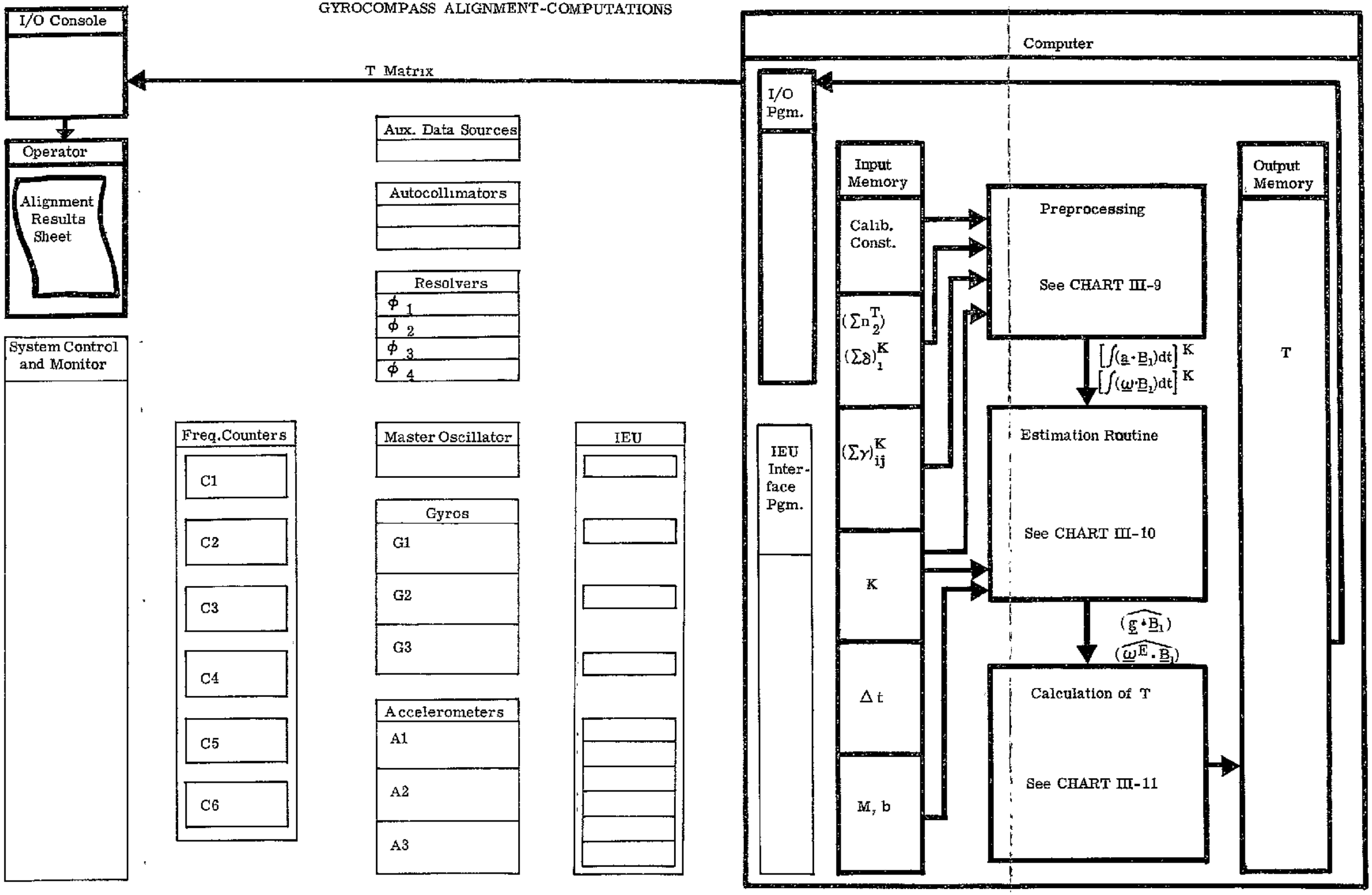
LABORATORY FLOW DIAGRAM #18
GYROCOMPASS ALIGNMENT SET-UP



LABORATORY FLOW DIAGRAM #19
GYROCOMPASS ALIGNMENT - MEASUREMENT



LABORATORY FLOW DIAGRAM #20
GYROCOMPASS ALIGNMENT-COMPUTATIONS



same program as navigation. The preprocessing routine most probably will be repeatedly performed while data is being accumulated. The outputs of this routine are K sets of $\int(\underline{g} \cdot \underline{B}_i)\, dt$ and $\int(\underline{\omega}^E \cdot \underline{B}_i)\, dt$, for the K consecutive measurement periods.

The estimation routine receives the outputs of the processing routine and determines an estimate of the $\underline{g}$ and $\underline{\omega}^E$ vectors in the body axes frame by application of the M matrix and $\underline{b}$ vector.

The final routine computes the T matrix from the estimate of the body axes components of $\underline{g}$ and $\underline{\omega}^E$. This matrix is output by the computer program and must be recorded by the operator on the Alignment Results Sheet. This T matrix also initializes the navigation program.

The operator will record the test table resolver settings and readings from the bubble levels on the Alignment Results Sheet to complete the record of the alignment.

PART IV
DIAGRAMS AND FORMS

This part of the document contains supplementary information on the ISU, the laboratory, and terminology used in this presentation. In Section 1, some of the charts and figures from the Development Document are repeated so that they are readily available for use in the laboratory. The Master Laboratory Flow Diagram is also contained in Section 1, with descriptions of each of the elements which are on the diagram. In Section 2, there are masters of the forms to be used during the calibration, alignment, or related procedures.

PART IV
SECTION 1
SYSTEM DIAGRAMS

This section contains the master flow diagram, along with several figures and charts selected from the Development Document.

LABORATORY FLOW DIAGRAM

The Master Laboratory Flow Diagram is shown on the following page. This diagram has been used throughout this manual to describe procedures. The elements of this diagram are defined as follows:

Input/Output Console – The input/output console consists of the equipment that provides a manual computer interface. Included in the input/output console are: Computer control panel, keyboard and typewriter, paper tape reader and punch and the display panel.

Operator – The operator in this system must perform many of the tasks of control and data transfer. The box "operator" includes not only the person(s) directing the laboratory but his work sheets, instructions, and notes.

Systems Control and Monitor – This box represents the equipment, capability, and activity used to monitor and control the system during calibration and alignment.

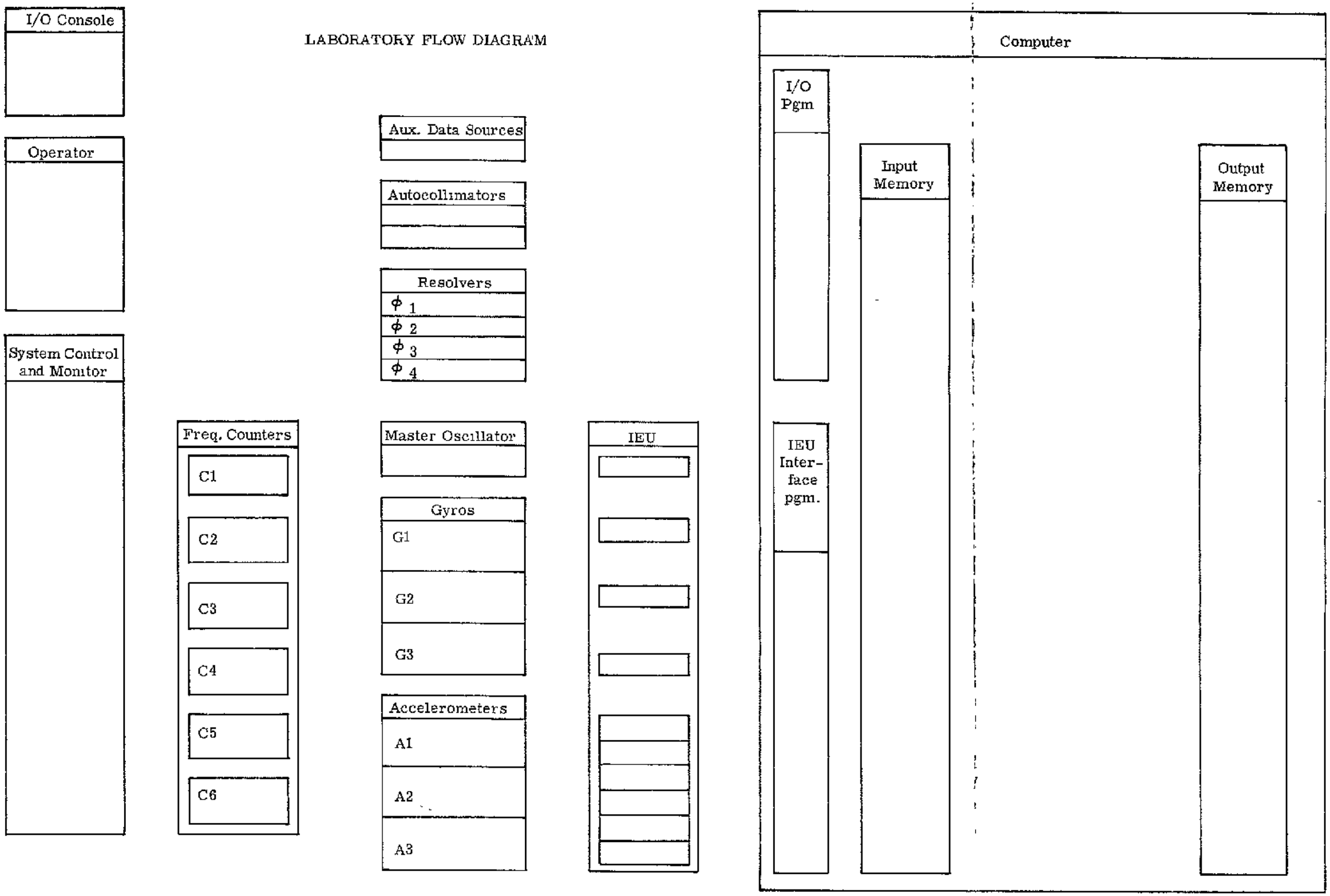
Frequency Counters – Six frequency counters are available for use in calibration to measure instrument output. These counters measure the number of counts on one pulse train for a fixed number of counts on another. One of the two trains may be a difference train formed from two inputs. The frequency counters are used in calibration because they can read the leading edge of one pulse train, and thus substantially reduce the quantization error relative to the use of the computer registers.

Auxiliary Data Sources – These include data sources available to the operator but not sufficiently well defined as equipment or measuring devices to be represented individually on the system flow diagram. Examples of these sources are bubble levels, and survey information on the magnitude of $\underline{g}$ and $\underline{\omega}^E$.

Autocollimators – The two-degree-of-freedom autocollimators are available to measure the orientation of the ISU mirror normals relative to earth fixed space.

Resolvers – These resolvers measure the orientation of the test table. The angles ϕ_1, ϕ_3 and ϕ_4 are static resolver readouts on the trunnion, outer and inner axes of the test table. The angle ϕ_2 is a rotary axis readout which can be used in either a static or dynamic mode.

LABORATORY FLOW DIAGRAM



IV

Master Oscillator – This is the central timing source of the system. The master oscillator includes countdown circuitry.

Gyros, Accelerometers – These are the instruments contained within the strapdown ISU.

Interface Electronics Unit (IEU) – The IEU allows the computer to sample outputs of the inertial instruments and timer. The IEU contains accumulating registers for each of the inputs shown in the diagram, and the capability to periodically interrupt the computer to allow for sampling and resetting (without loss of data) of each of the registers.

Computer – The computer schematically indicated in the system diagram is the laboratory computer Honeywell DDP-124. Other portions of the data processing shown may however be performed on other computers at the discretion of the programmers and operators. Blocks shown within the computer represent functions used in both calibration and alignment. Shown are programs to input and output data from and to the console; a program to input data from the IEU; and memory buffers for input data and output data (the results of computations). Space has been left within the computer block to allow representation of the various data processing tasks.

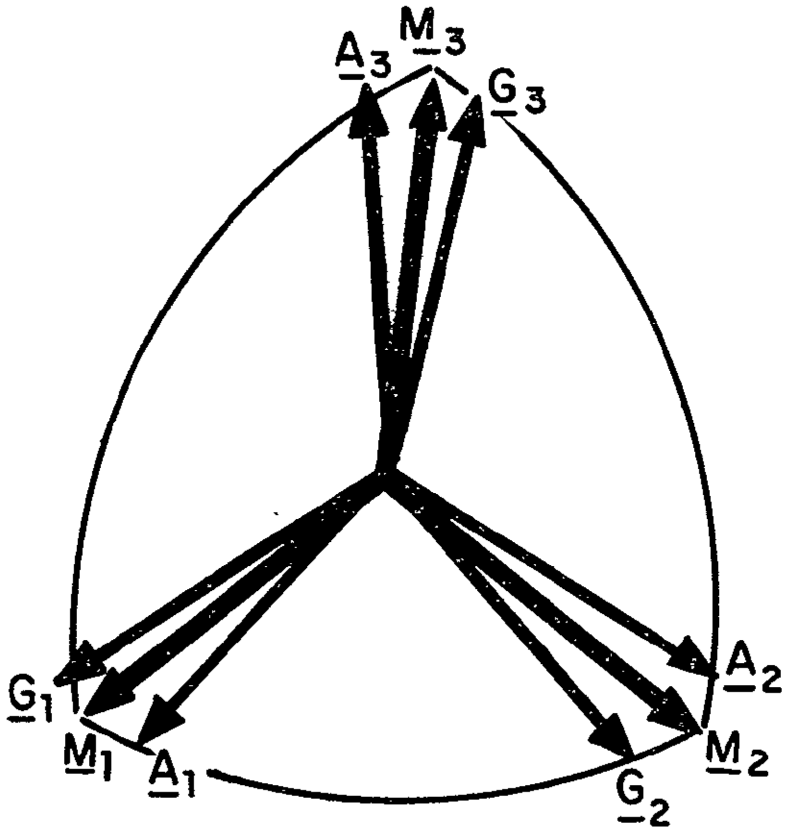


Figure IV-1. Instrument and Mirror Axes

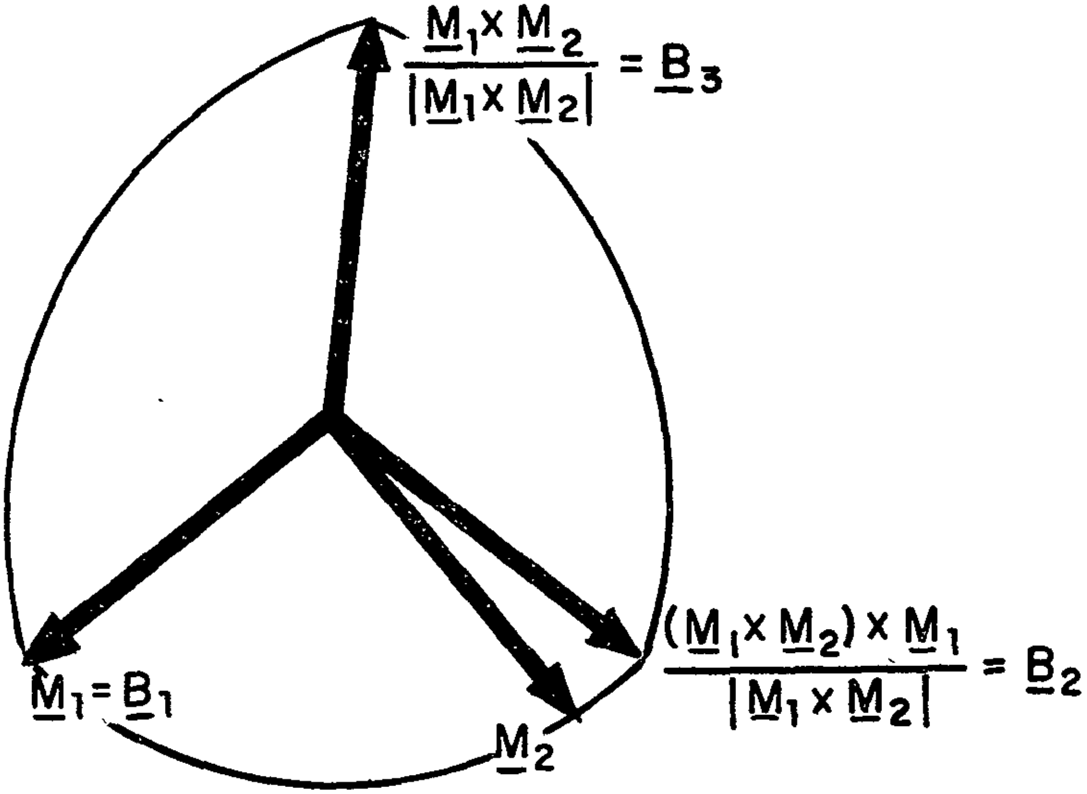
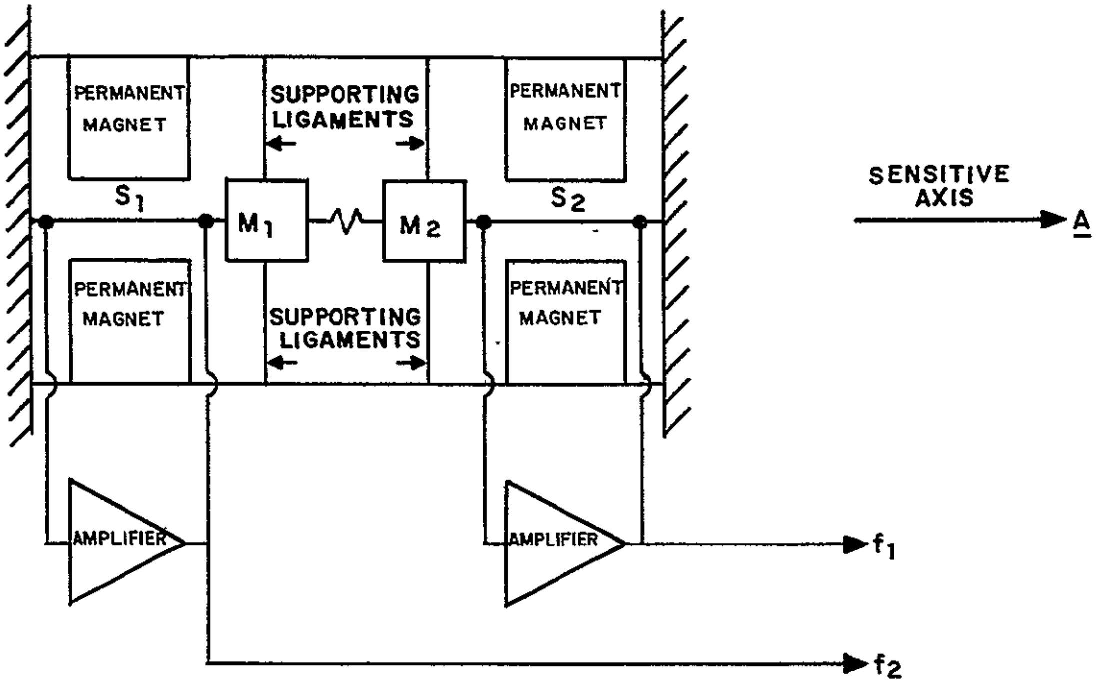


Figure IV-2. Body and Mirror Axes



Model: ARMA D4E Vibrating String Accelerometer

Axis: $\underline{A}$ is a unit vector directed along strings S_1 and S_2
(the sensitive axis)

Figure IV-3. A Schematic Diagram of the Accelerometer

THE FUNDAMENTAL ACCELEROMETER MODEL

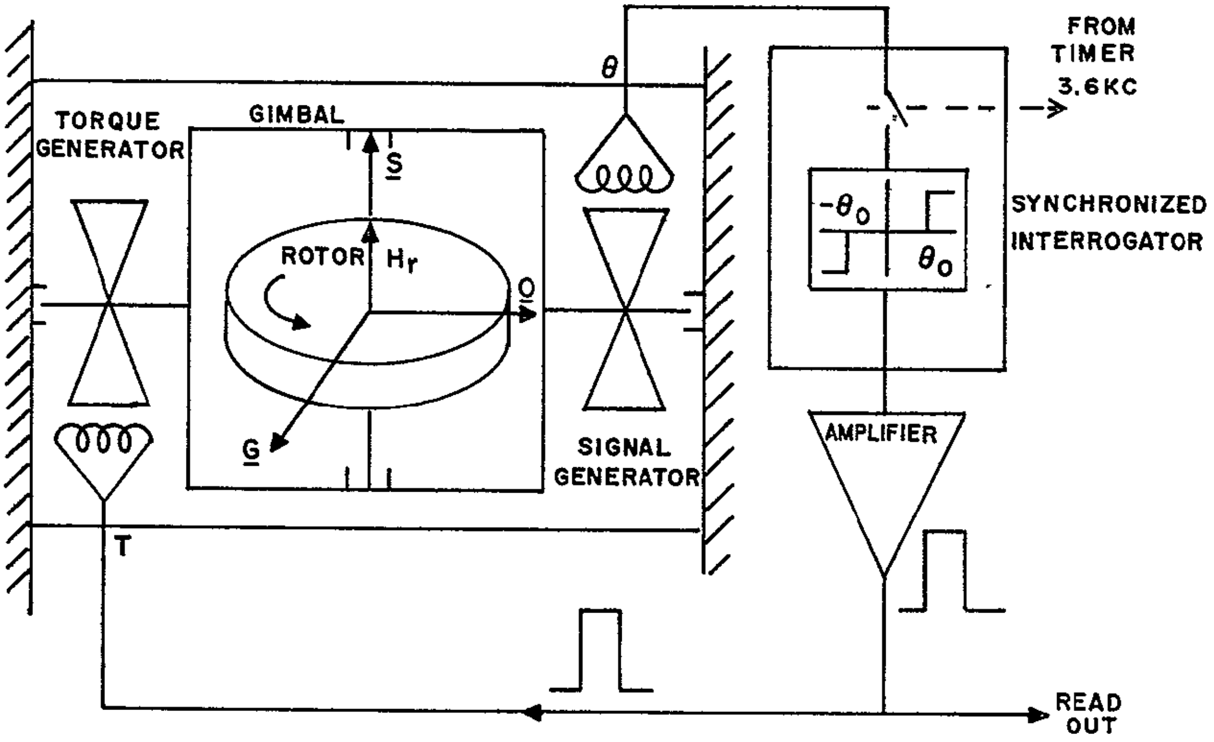
THE ACCELEROMETER MODEL IS:

$$\int_{t_a}^{t_b} f_2\,dt - \int_{t_a}^{t_b} f_1\,dt = (N_2 - N_1) + Eq = D_1 \int_{t_a}^{t_b} (\underline{a} \cdot \underline{A})dt$$

$$+ D_1 \left\{ \int_{t_a}^{t_b} [D_0 + D_2(\underline{a} \cdot \underline{A})^2 + D_3(\underline{a} \cdot \underline{A})^3]dt \right\}$$

WHERE:

- $\underline{a}$ is the acceleration applied to the accelerometer
- $t_a \le t \le t_b$ is the time interval over which $\underline{a}$ is measured
- $\underline{A}$ is a unit vector directed along the input axis of the accelerometer
- N_1 and N_2 are the number of zero crossings detected in $t_a \le t \le t_b$ from both strings of the accelerometer
- Eq is the instrument quantization error due to the fact that t_a and t_b do not correspond to zero crossings
- D_1 is the accelerometer scale factor
- D_0 is the accelerometer bias
- D_2 is the second order coefficient
- D_3 is the third order coefficient
- f_2 and f_1 are string frequencies in pulses/second



Model: Honeywell GG 334A single-degree-of-freedom, pulse rebalance gyroscope.

Axes: $\underline{S}$ is a unit vector along the spin axis of the rotor.
$\underline{O}$ is a unit vector along the output axis as defined by the gimbal.
$\underline{G} = \underline{O} \times \underline{S}$ is the sensitive axis of the gyro.

Figure IV-4. A Schematic Diagram of the Gyro

CHART IV-2

THE FUNDAMENTAL GYRO MODEL

THE GYRO MODEL IS:

$$\Delta\Phi\left[\sum_{k=1}^{N}\delta_k\right] = \int_{t_0}^{t_N}(\underline{\omega}\cdot\underline{G})\,dt + \int_{t_0}^{t_N}\Big[R + B_I(\underline{a}\cdot\underline{G}) + B_O(\underline{a}\cdot\underline{O}) + B_S(\underline{a}\cdot\underline{S}) + C_{II}(\underline{a}\cdot\underline{G})^2 + C_{SS}(\underline{a}\cdot\underline{S})^2$$

$$+ C_{IS}(\underline{a}\cdot\underline{G})(\underline{a}\cdot\underline{S}) + C_{OS}(\underline{a}\cdot\underline{O})(\underline{a}\cdot\underline{S}) + C_{IO}(\underline{a}\cdot\underline{G})(\underline{a}\cdot\underline{O})$$

$$+ Q_{II}(\underline{\omega}\cdot\underline{G})^2 + Q_{IS}(\underline{\omega}\cdot\underline{G})(\underline{\omega}\cdot\underline{S}) + J\frac{d}{dt}(\underline{\omega}\cdot\underline{O})\Big]dt + \Delta n + Eq$$

WHERE

- $\underline{\omega}$ is the angular velocity applied to the gyro
- $\underline{a}$ is the acceleration applied to the gyro
- $t_0 \leq t \leq t_N$ is the time interval over which $\underline{a}$ and $\underline{\omega}$ are measured
- $t_N - t_0 = N\tau$, where N is an integer, and τ is the gyro sampling period
- $\underline{S}$ is a unit vector along the spin axis of the rotor
- $\underline{O}$ is a unit vector directed along the output axis as defined by the gimbal
- $\underline{G}$ is a unit vector along $\underline{O} \times \underline{S}$ (that is, the sensitive axis of the gyro)
- δ_k is the kth gyro pulse, equal to +1, -1, or 0 for positive, negative, or no pulse
- $\Delta\Phi$ is the gyro scale factor
- R is the gyro bias
- B_I B_O and B_S are the gyro unbalance coefficients
- C_{II} C_{SS} C_{IS} C_{OS} and C_{IO} are the gyro compliance coefficients
- Q_{IS} and Q_{II} are dynamic coupling coefficients due to gimbal deflection and scale factor nonlinearity, respectively
- J is the angular rate coefficient
- Δn is the effect of gyro noise over the $[t_0, t_N]$ interval
- Eq is the gyro quantization error

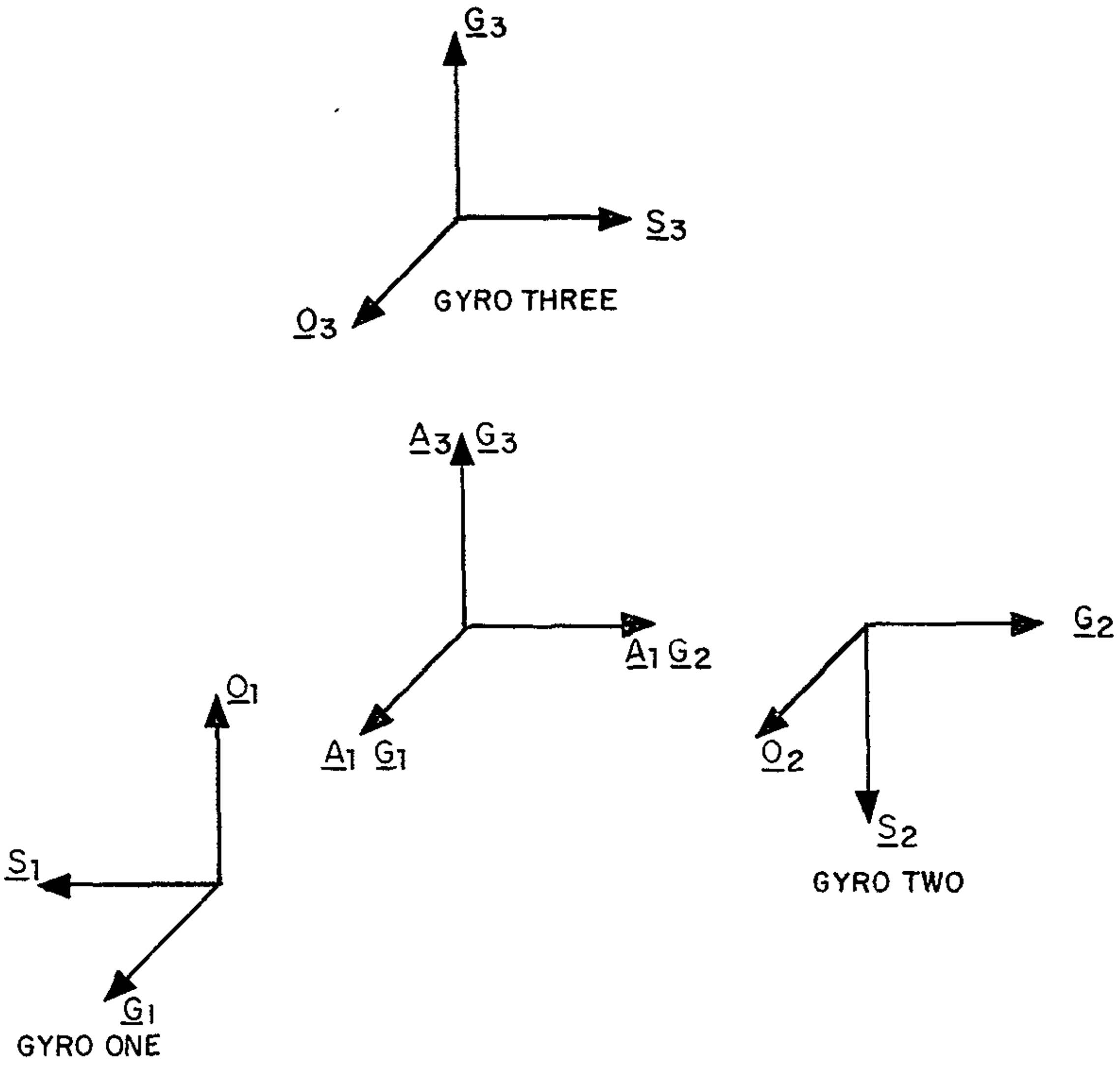


Figure IV-5. Nominal Orientation Between Gyros

CHART IV-3

DEFINITION OF FRAMES

LABORATORY FIXED FRAMES

• $\underline{E}_1\ \underline{E}_2\ \underline{E}_3$	A triad of unit vectors directed up, east, and north, respectively
• $\underline{S}_1\ \underline{S}_2\ \underline{S}_3$	A triad of unit vectors defined by the two optical lines of the autocollimators
• $\underline{F}_1\ \underline{F}_2\ \underline{F}_3$	A triad of unit vectors fixed to the base of the test table

TEST TABLE FRAMES

• $\underline{F}_1\ \underline{F}_2\ \underline{F}_3$	A triad of unit vectors fixed to the base of the test table
• $\underline{T}_1\ \underline{T}_2\ \underline{T}_3$	A triad of unit vectors fixed to the body containing the trunnion axis
• $\underline{R}_1\ \underline{R}_2\ \underline{R}_3$	A triad of unit vectors fixed to the body containing the rotary axis
• $\underline{O}_1\ \underline{O}_2\ \underline{O}_3$	A triad of unit vectors fixed to the body containing the outer axis
• $\underline{I}_1\ \underline{I}_2\ \underline{I}_3$	A triad of unit vectors fixed to the body containing the inner axis

ISU FIXED FRAMES

• $\underline{I}_1\ \underline{I}_2\ \underline{I}_3$	A triad of unit vectors fixed to the body containing the inner axis
• $\underline{B}_1\ \underline{B}_2\ \underline{B}_3$	A triad of unit vectors defining the body axes as defined by the mirror normals (see Section 2. 2. 1)

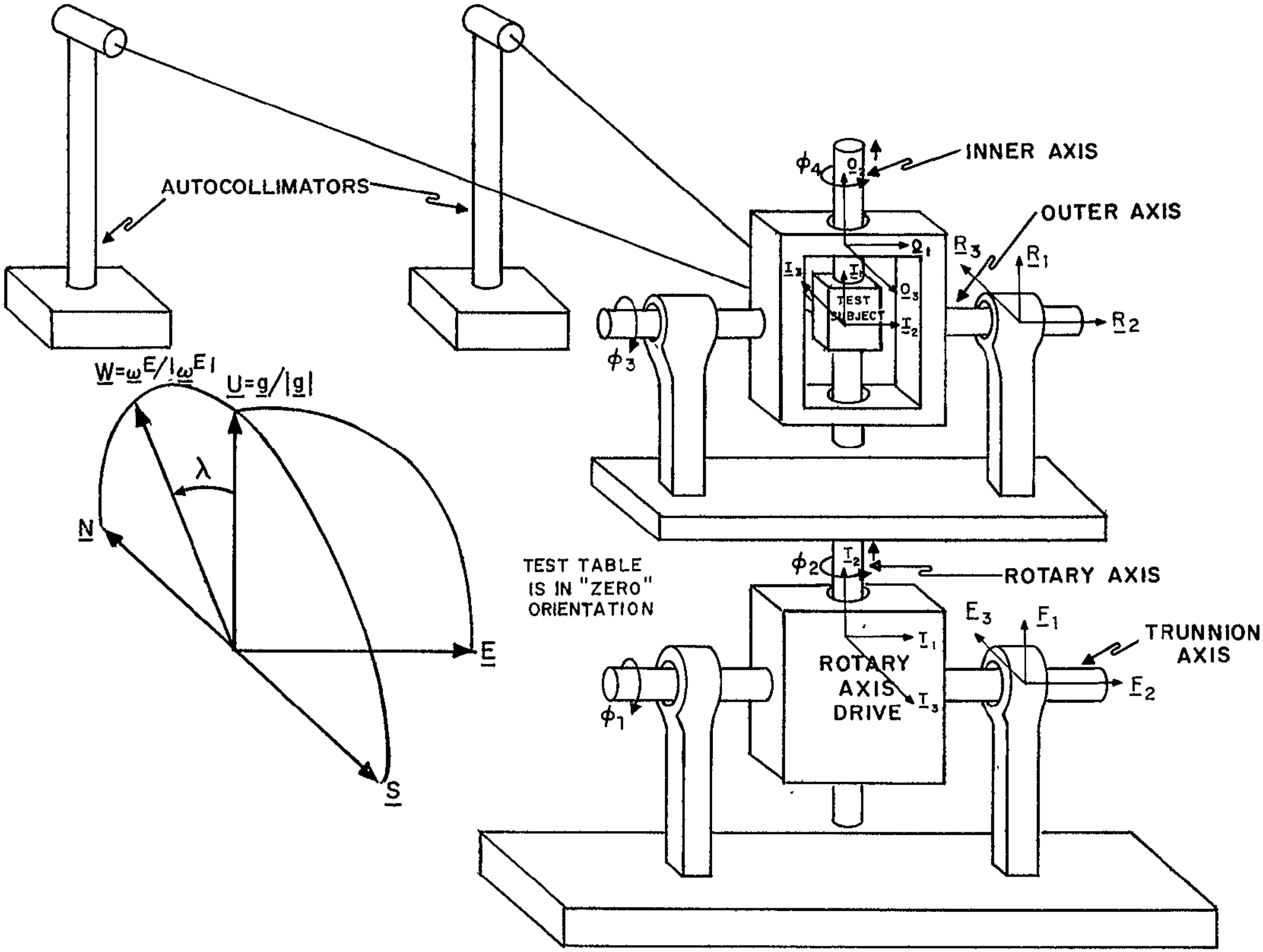


Figure IV-6. Laboratory Coordinate Frames

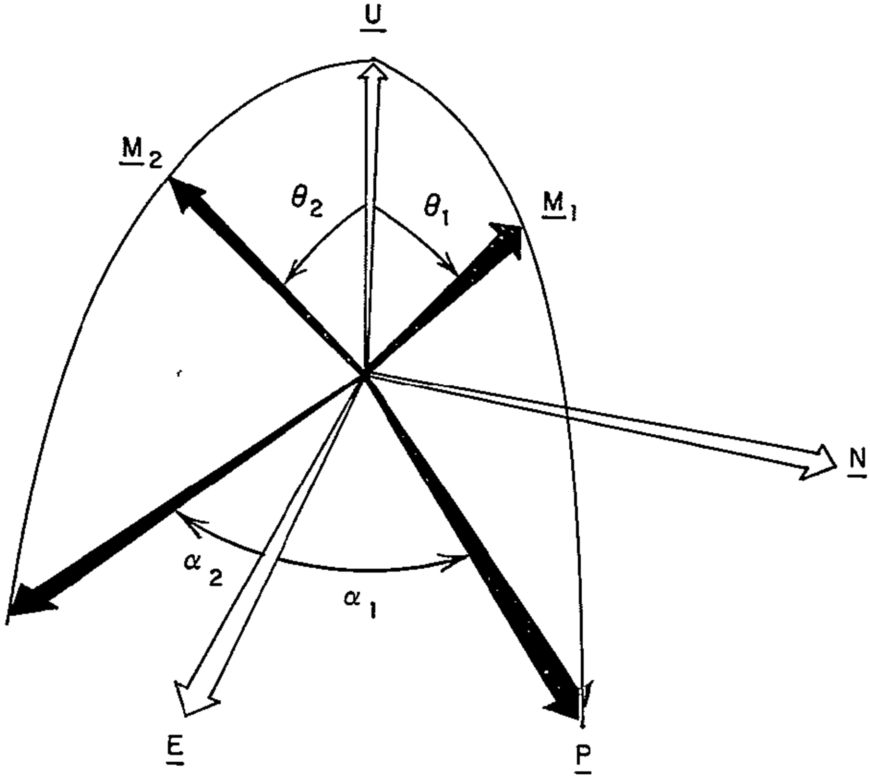


Figure IV-7. Earth and Mirror Axes

PART IV
SECTION 2
MASTER FORMS

The master forms to be duplicated and used in the procedures are contained in this section.

LABORATORY PROCEDURES

Identification ____________

Date ____________

INSTRUCTIONS	√	COMMENTS

LABORATORY PROCEDURES

INSTRUCTIONS	✓	COMMENTS

SURVEY RESULTS SHEET

Identification:

Date:

Operator:

Current Laboratory Survey Information

$|g| =$ ________

$\omega^E \sin\lambda =$ ________

$\omega^E \cos\lambda =$ ________

AUTOCOLLIMATOR READINGS

Position 0 Readings

$\phi_1^0 =$ ________

$\phi_2^0 =$ ________

$\phi_3^0 =$ ________

$\phi_4^0 =$ ________

Calculated Results

$$T^{BI} = \begin{pmatrix} \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} \\ \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} \\ \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} & \rule{2cm}{0.4pt} \end{pmatrix}$$

Calibration Positions

m	ϕ_3^m	ϕ_4^m
1		
3		
5		
13		
14		
15		

m	T_{12}^{BRm}	T_{22}^{BRm}	T_{32}^{BRm}
1			
3			
5			
13			
14			
15			

CALIBRATION RESULTS SHEET

Experiment:
Principal Operator:
Date:

PROCEDURE DESCRIPTION

Survey Results Sheet Used
Resolver Settings Sheet Used
Counter Settings Sheet Used
Counter Outputs Sheet Used
Special Procedures

RESULTS

Parameter	Units	Instrument 1	Instrument 2	Instrument 3
Gyros				
$\Delta\ \Phi$	Deg/Pulse			
$(\underline{G}_i \cdot \underline{B}_1)$				
$(\underline{G}_i \cdot \underline{B}_2)$				
$(\underline{G}_i \cdot \underline{B}_3)$				
R	Deg/Hr.			
B_I	Deg/Hr. -g			
B_O	Deg/Hr. -g			
B_S	Deg/Hr. -g			
C_{II}	Deg/Hr. -g^2			
C_{SS}	Deg/Hr. -g^2			
C_{IS}	Deg/Hr. -g^2			
C_{OS}	Deg/Hr. -g^2			
C_{IO}	Deg/Hr. -g^2			
Q_{II}	Hr. /Deg.			
Q_{IS}	Hr. /Deg.			
J	Hours			
Accelerometers				
D_1	(Pulses/Sec)/g			
$(\underline{A}_i \cdot \underline{B}_1)$				
$(\underline{A}_i \cdot \underline{B}_2)$				
$(\underline{A}_i \cdot \underline{B}_3)$				
D_0	g			
D_2	g/g^2			
D_3	g/g^3			

TEST TABLE RESOLVER SETTINGS

	Position 1	Position 2	Position 3	Position 4	Position 5	Position 6
ϕ_1	90°	90°	90°	90°	90°	90°
ϕ_2	Rotating	Rotating	Rotating	Rotating	Rotating	Rotating
ϕ_3		$\phi_3^1 + 180^\circ$		$\phi_3^3 + 180^\circ$		$\phi_3^5 + 180^\circ$
ϕ_4		ϕ_4^1		ϕ_4^3		ϕ_4^5

	Position 7	Position 8	Position 9	Position 10	Position 11	Position 12
ϕ_1	0° *	0° *	0° *	0° *	0° *	0° *
ϕ_2	90°	90°	90°	90°	90°	90°
ϕ_3	ϕ_3^1 *	ϕ_3^2 *	ϕ_3^3 *	ϕ_3^4 *	ϕ_3^5 *	ϕ_3^6 *
ϕ_4	ϕ_4^1	ϕ_4^1	ϕ_4^3	ϕ_4^3	ϕ_4^5	ϕ_4^5

	Position 13	Position 14	Position 15
ϕ_1	0° *	0° *	0° *
ϕ_2	90°	90°	90°
ϕ_3	*	*	*
ϕ_4			

* Requires bubble level correction.

COUNTER SETTINGS

Experiment:

Date:

Positions		Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6
GYRO		$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$	$(\Sigma n\phi)^m$		
	1					Not Required	Not Required
	2						
	3						
	4						
	5						
	6						
GYRO		Not Required	$(\Sigma n_1^T)_1^{Gm}$	$(\Sigma n_1^T)_2^{Gm}$	$(\Sigma n_1^T)_3^{Gm}$		
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						
ACCELEROMETER		$(\Sigma\gamma)_{11}^m$	$(\Sigma\gamma)_{12}^m$	$(\Sigma\gamma)_{21}^m$	$(\Sigma\gamma)_{22}^m$	$(\Sigma\gamma)_{31}^m$	$(\Sigma\gamma)_{32}^m$
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						

COUNTER OUTPUTS

Experiment:
Date:

Positions		Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6
GYRO		$(\Sigma n_1^T)^{Gm}$	$(\Sigma\delta)_1^m$	$(\Sigma\delta)_2^m$	$(\Sigma\delta)_3^m$		
	1					Not Required	Not Required
	2						
	3						
	4						
	5						
	6						
GYRO		Not Required	$(\Sigma\delta)_1^m$	$(\Sigma\delta)_2^m$	$(\Sigma\delta)_3^m$		
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						
ACCELEROMETER		$(\Sigma n_1^T)_{11}^{Am}$	$(\Sigma n_1^T)_{12}^{Am}$	$(\Sigma n_1^T)_{21}^{Am}$	$(\Sigma n_1^T)_{22}^{Am}$	$(\Sigma n_1^T)_{31}^{Am}$	$(\Sigma n_1^T)_{32}^{Am}$
	7						
	8						
	9						
	10						
	11						
	12						
	13						
	14						
	15						

COUNTER SETTINGS

Experiment: Gyro Scale Factor Nonlinearity

Date.

Positions	Counter 1	Counter 2	Counter 3	Counter 4		Angular Velocity
					Not Required	

COUNTER OUTPUTS

Experiment: Gyro Scale Factor Nonlinearity

Date:

Positions	Counter 1	Counter 2	Counter 3	Counter 4	ω	P

COUNTER CONNECTIONS

Experiment: Date:

Positions	Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6
X-Y Connection						
Z Connection						
Select Mode						
X-Y Connection						
Z Connection						
Select Mode						
X-Y Connection						
Z Connection						
Select Mode						

COUNTER SETTINGS

Experiment:

Date:

Positions	Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter 6

COUNTER OUTPUTS

Experiment:

Date:

Positions	Counter 1	Counter 2	Counter 3	Counter 4	Counter 5	Counter6

ALIGNMENT PARAMETER EVALUATION

Operator. ______________ Output Reel No ______

Date ______________

INPUT PARAMETERS

K ______________

$|g|$ ______________ Ft./Sec^2

Δt ______________ Sec

σ_g ______________ Ft /Sec^2

σ_θ ______________ Rad.

T^{EB}(EST.) = ______ ______ ______
______ ______ ______
______ ______ ______

NOISE COVARIANCE FUNCTIONS

Time (Seconds)	Accelerometer Noise ϕ_n	Environmental Acceleration ϕ_α	Environmental Rotation ϕ_θ

ALIGNMENT RESULTS SHEET

Experiment:

Principal Operator:

Date:

Procedure (Check one)· - Accel. Level ☐ Gyrocompass ☐ Mirror Align. ☐

GYROCOMPASS INPUT

Calibration Constants Identification ____________________

Filtering Parameter Identification ____________________

Intersample Time, Δt ____________________

Number of Measurements, K ____________________

ACCELEROMETER LEVEL INPUT

Calibration Constants Identification ____________________

Filtering Parameter Identification ____________________

Intersample Time, Δt ____________________

Number of Measurements, K ____________________

Azimuth Angle, α_1 ____________________

MIRROR ALIGNMENT INPUT

Autocollimator 1	Autocollimator 2
Azimuth, α_1 ____________	Azimuth, α_2 ____________
Elevation,. θ_1 ____________	Elevation, θ_2 ____________

TABLE POSITION

ϕ_1 ____________	ϕ_2 ____________
ϕ_3 ____________	ϕ_4 ____________

Bubble Level 1 ____________

Bubble Level 2 ____________

RESULTS

$$T = \begin{pmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{pmatrix}$$