NASA TECH BRIEF

Marshall Space Flight Center

Digital Simulation Error Curves for a Spring-Mass-Damper System

The problem:

Reduce the excessive number of samples per cycle and the unnecessary iterations which increase cost and computation time on digital simulations of continuous data systems.

$A = \omega\left(\frac{1}{S}\right)$ is the normalized integration gain.
ω_D is the effective angular resonance frequency.
ω_P is the programmed angular resonance frequency.
ζ_P is the programmed damping ratio.
T is the sampling interval in seconds.

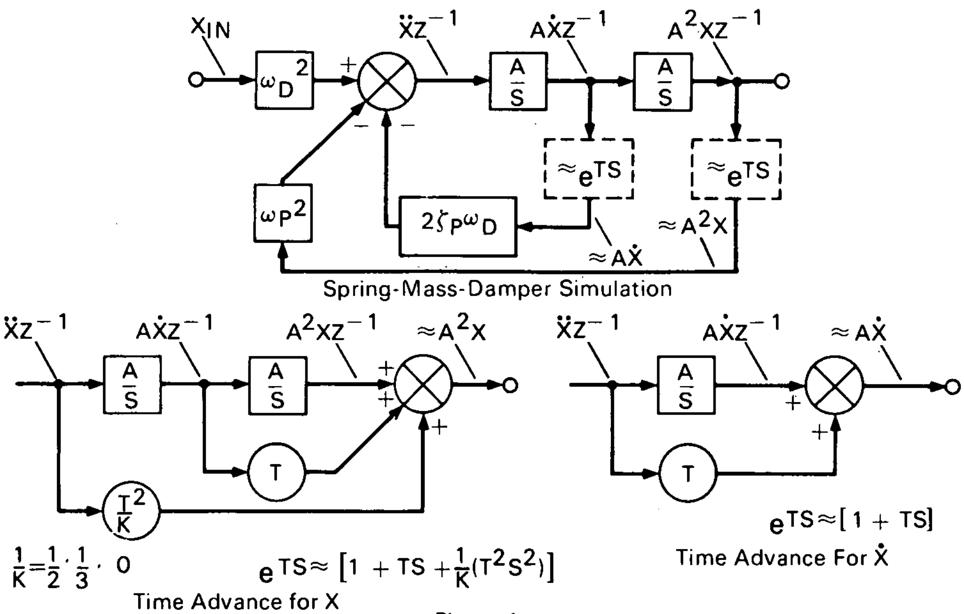

Figure 1

The solution:

Plot the digital simulation errors for a spring-mass-damper system. Using these error curves, select the type of integration, the feedback update method, and the number of samples per cycle at resonance.

How it's done:

Block diagrams of a spring-mass-damper simulation and the preferred time advance update method are shown in Figure 1, where:

Z^{-1} is a transfer function for T seconds time delay.
e^{TS} is a transfer function for T seconds time advance.
$\frac{1}{K}$ is a constant for time advance update.
I is the number of iterations for iteration update.
X, $\dot{X}$, and $\ddot{X}$ represent the mass displacement, velocity, and acceleration, respectively.
S is the Laplace operator.
N_R is the number of samples per cycle at resonance.
The effective damping ratio is: $\zeta_D \approx \zeta_P + \xi_R$

(continued overleaf)

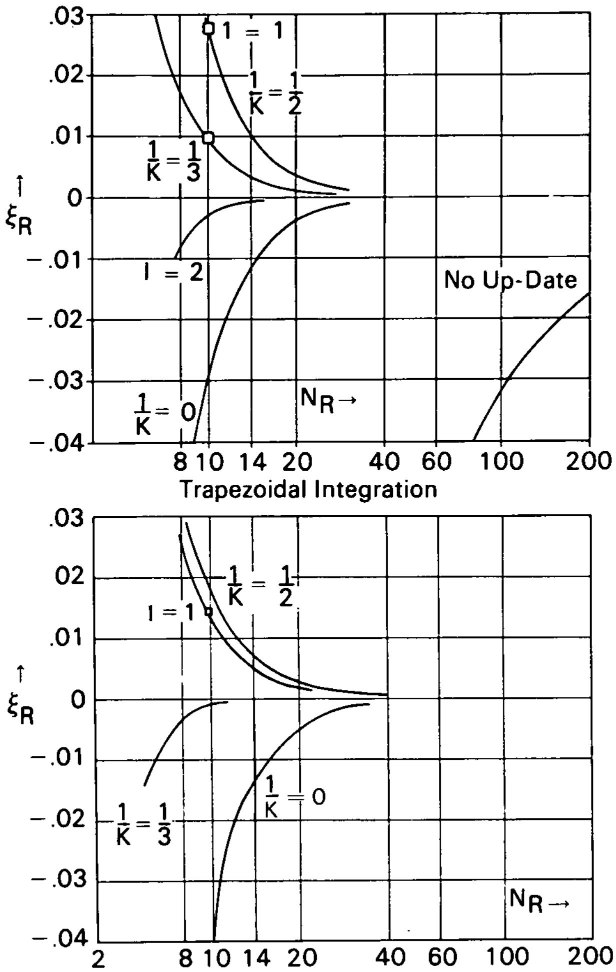

Figure 2 Simpson's Integration

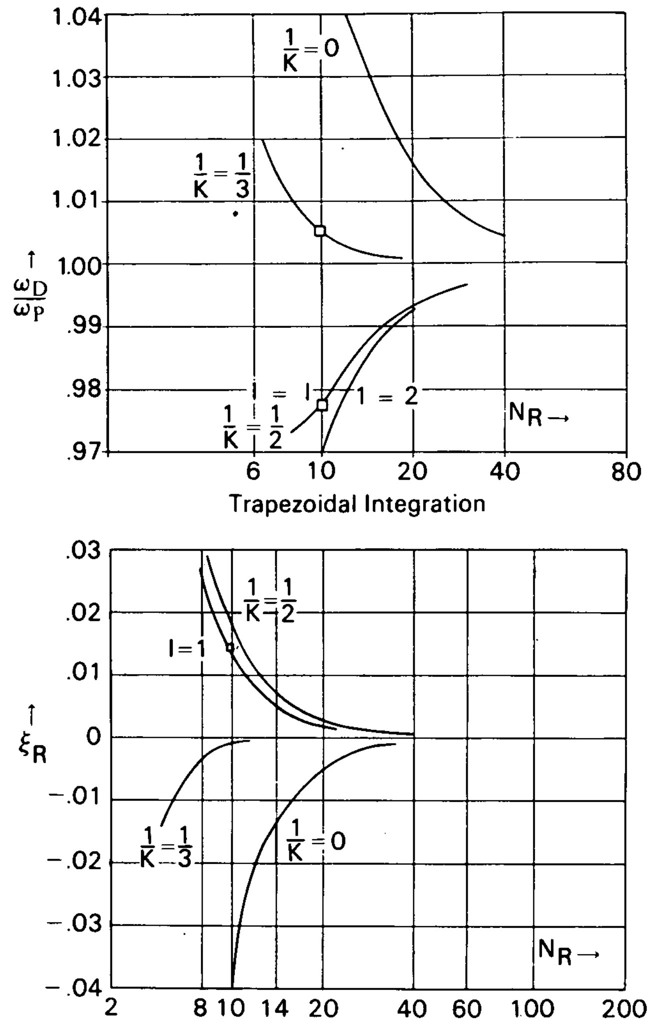

Figure 3 Simpson's Integration

where ξ_R is the damping bias error. This error is plotted (Figure 2) for trapezoidal integration and Simpson's 1/3 formula, where N_R is the number of samples per cycle at resonance. Curves are shown for the iteration update method where I = 1, and for the time advance update method where 1/K = 0, 1/3, and 1/2.

The normalized resonance frequency, ω_D/ω_P, shown in Figure 3, is for trapezoidal integration and Simpson's 1/3 formula.

For the curves for trapezoidal integration, the time advance update method with 1/K = 1/3 has smaller errors at 20 samples per cycle than the one-iteration update method at 30 samples per cycle. Thus, for the same accuracy, the time advance update method will solve the simulated problem in half the time of the one-iteration update method. This is based on an added time per iteration of T/4, i.e., one-quarter of the computation time without iteration.

Note:

Requests for further information may be directed to:

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Patent status:

No patent action is contemplated by NASA.

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