November 1971

Brief 71-10423

# NASA TECH BRIEF

*Marshall Space Flight Center*



## Error Evaluation for Difference Approximations to Ordinary Differential Equations

A recently available report reviews various one-step and multistep numerical methods for deriving difference equations to approximate ordinary differential equations, and describes a new method for evaluating the errors which result from that approximation. The method is then applied to a nontrivial numerical analysis problem and to an investigation of "stiff" equations (equations with widely separated eigenvalues).

The method involves relationships between (1) the errors introduced by using finite sampling rates and (2) the parameters describing the specific numerical method used. The approach treats an analog system in terms of isolated modes, and approximates the digital algorithm for each mode by its order, p, and by certain constants, C, $A_0,\ldots,A_p$. The results show that the approximation causes a shift of each root, given by $-Ch^p\lambda^p$, and also an error associated with each forcing function $\gamma e^{\omega t}$, given by

$$-Ch^p \sum_{i=0}^{p} A_i \lambda^i \omega^{p-i}.$$

These results are valid for all small products of sampling period, h, and analog system natural frequency $\lambda$.

The method is primarily intended for use in the design and analysis of digital filters and simulators. In such applications, errors (primarily due to roundoff) are generated during the solution of the difference equation, in addition to the approximation errors discussed above. These solution errors must be evaluated through further analysis.

### Notes:

1. For related work involving the application of finite difference approximation to solving partial differential equations, see NASA Tech Brief B71-10424.
2. The following documentation may be obtained from:

   National Technical Information Service
   Springfield, Virginia 22151
   Single document price $3.00
   (or microfiche $0.95)

   Reference:

   NASA CR-103015 (N71-16573),
   Difference Equations to Approximate
   Ordinary Differential Equations

### Patent status:

No patent action is contemplated by NASA.

Source: J.L. Hammond, Jr., of
Georgia Institute of Technology
under contract to
Marshall Space Flight Center
(MFS-21610)

Category 09