NASA CONTRACTOR REPORT

NASA CR-1666

AN IMPROVED ULTRAVIOLET INTERFEROMETER

by Richard A. Day

Prepared by
UNIVERSITY OF OKLAHOMA
Norman, Okla. 73069
for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1970

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-1666		

4. Title and Subtitle	5. Report Date
AN IMPROVED ULTRAVIOLET INTERFEROMETER	October 1970
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Richard A. Day	
9. Performing Organization Name and Address	10. Work Unit No.
University of Oklahoma, Univ. Res. Inst., ~~OU Research Institute~~ Department of Physics and Astronomy Norman, Oklahoma 73069	11. Contract or Grant No. NGR-37-003-047
	13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

This work was supported in part by NASA Grant NGR-37-003-026

16. Abstract

The performance of a new ultraviolet interferometer described in Applied Optics 9, 31 (1970) can be improved by the tilting of the plates. We describe the conditions of optimum performance for such a configuration of the interferometer.

17. Key Words (Selected by Author(s))	18. Distribution Statement
Ultraviolet, interferometer, spectroscopy	Unclassified-Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	11	$3.00

AN IMPROVED UV INTERFEROMETER

A recent publication has described a new interferometer which derived its coherent beam from a set of grazing incidence reflections.(1) The interferometer called a GIMBI, derived these beams from reflections from two plane parallel plates.

We discuss a modification of this original GIMBI by considering the effect of a tilt between the two plates. Again the interferometer is illuminated by a pinhole (or, a line source) placed coaxially with the plates. A wedge angle Θ_o is introduced between the plates by requiring the separation at the exit plane of the interferometer "t_2" to be larger than the separation at the exit plane of the interferometer "t_1". The plates if extended would meet at a point (or line) a distance R from the entrance aperture. Under these circumstances, the problem of finding the path lengths of the various reflections contributing to the interference pattern can be replaced by the problem of finding the interference pattern created by the contributions from a set of virtual sources positioned on the circumference of a circle separated from one another and the original source by the angle Θ_o. Due to the π phase change at reflection each of these virtual sources emits with a phase shift of π relative to its two neighbors. The amplitude at a point "L" distant axially and "a" distant from the axis is then:

$$\phi = b_o \sum_{n=-N}^{+N} \rho^{|n|} \frac{\exp i(k\,d_n + n\pi)}{d_n} \qquad (1)$$

where ρ is the amplitude reflectivity which causes each virtual source to decrease in amplitude due to the reflections from the mirrors, d_n is the distance from the nth source (n counted positive above the original source and negative below), b_o is the amplitude of the original source, k is the wave number of the assumed monochromatic radiation ($k\lambda = 2\pi$) and N is the last ray which contributes to the interference pattern and will normally be determined by adjacent optics.

Hence forward, we will ignore the variation in d_n in the denominator and will also neglect the loss of intensity due to reflections (i.e. $\rho \simeq 1$); then (1) becomes

$$\phi = \frac{b_o}{d_o} \sum_{n=-N}^{+N} (-1)^n \exp\{ikd_n\} , \tag{2}$$

where d_n is the crucial term. And from the geometry of the situation shown in Fig. 1.

$$d_n^2 = L^2 + (2LR+2R^2)(1-\cos\Theta) - 2aR\sin\Theta + a^2 \tag{3}$$

where Θ is the angle between the axis and the nth virtual source,

$$\Theta_n = n\Theta_o \tag{4}$$

Then if $L'^2 = L^2 + a^2$

$$d = L'\left\{1 + 4\,\frac{LR+R^2}{L'^2}\sin^2\frac{\Theta}{2} - \frac{2aR\sin\Theta}{L'^2}\right\}^{\frac{1}{2}} \tag{5}$$

and a double expansion will be attempted, first of the square root and then of the angle. First, define

$$\frac{R}{L'} \equiv r \; ; \qquad \frac{a}{L'} \equiv \alpha \; ; \qquad \frac{L}{L'} \equiv h \tag{6}$$

giving

$$\frac{d}{L'} = 1 + 2(hr+r^2)\sin^2\frac{\Theta}{2} - \alpha r \sin\Theta - \frac{1}{8}[4(hr+r^2)\sin^2\frac{\Theta}{2} - 2\alpha r \sin\Theta]^2 + \ldots \tag{7}$$

$$= 1 + 2(hr+r^2)\sin^2\frac{\Theta}{2} - \alpha r \sin\Theta - 2(hr+r^2)^2\sin^4\frac{\Theta}{2} + 2\,\alpha r(hr+r^2)\sin^2\frac{\Theta}{2}\sin\Theta - \frac{1}{2}\alpha^2 r^2 \sin^2\Theta + \ldots \tag{8}$$

then, the usual expansion of the sine is used

$$\begin{aligned} \sin\Theta &= \Theta - \frac{\Theta^3}{3!} + \frac{\Theta^5}{5!} - \frac{\Theta^7}{7!} + \ldots \\ \sin^2\Theta &= \Theta^2 - \frac{\Theta^4}{3} \end{aligned} \tag{9}$$

to obtain

$$\frac{d}{L'} = 1 + 2(hr+r^2)(\frac{\Theta^2}{4} - \frac{\Theta^4}{48}) - \alpha r(\Theta - \frac{\Theta^3}{3!}) - 2(hr+r^2)^2(\frac{\Theta^4}{16} - \frac{\Theta^6}{96}) + $$

$$+ 2\alpha r(hr+r^2)(\frac{\Theta^3}{4} - \frac{3\Theta^5}{48}) - \frac{1}{2}\alpha^2 r^2(\Theta^2 - \frac{\Theta^4}{3!}) \quad .$$

Collecting powers of Θ

$$\begin{aligned} \frac{d}{L'} = 1 & \\ &+ \Theta[-\alpha r] \\ &+ \Theta^2[\tfrac{1}{2}(hr+r^2) - \tfrac{1}{2}\alpha^2 r^2] \\ &+ \Theta^3[\frac{\alpha r}{3!} + \frac{\alpha r}{2}(hr+r^2)] \end{aligned} \tag{10}$$

$$+ \Theta^4[- \frac{(hr+r^2)}{24} - \frac{(hr+r^2)^2}{8} + \frac{\alpha^2 r^2}{2\cdot 3!}]$$

$$+ \Theta^5[- \frac{\alpha r}{5!} - \frac{\alpha r(hr+r^2)}{8}] \tag{10}$$

$$+ \Theta^6[\frac{4(hr+r^2)^3}{64} + \frac{2(hr+r^2)^2}{96} + \ldots] \ .$$

Approximations

As was the case in the GIMBI, the even powers of Θ answer the question - is there a resonance? The odd powers ask the question - where is the resonance? To simplify the subsequent calculations we will deal only with axial modes (i.e. $\alpha=0$), and furthermore we will keep only those terms whose contributions are greater than 10^{-6}. Recall that the largest angle that could be accepted by an $f/10$ instrument placed at the exit aperture of the interferometer is $X_{max} = \frac{1}{20}$, so

$$\frac{y_{max}}{L} = \frac{R}{L} \sin\Theta_{max} = r\ \Theta_{max} \leq .05\ .$$

Furthermore, we expect $\Theta_{max} \leq .01$, i.e. the plates are only slightly inclined. Then some typical orders of magnitude are:

$$r\Theta \sim .1 \qquad r\Theta^2 < 10^{-3}$$

$$(r\Theta)^2 \sim .01 \tag{11}$$

$$(r\Theta)^4 \sim 10^{-4} \qquad h = 1$$

and the terms which are to be retained are

$$\frac{d}{L'} = 1 + \tfrac{1}{2}\,(r+r^2)\Theta^2 - \frac{(r+r^2)^2}{8}\,\Theta^4\ . \tag{12}$$

Phase Relationship

We require then that

$$kd_n = kL\{1 + \tfrac{1}{2}(r+r^2)\Theta^2 - \frac{(r+r^2)^2}{8}\Theta^4\} \tag{13}$$

be an odd multiple of π for all possible values of Θ. Actually, the first term is a constant and can be ignored; we direct our attention to the second term

$$\frac{kL}{2}(r+r^2)n^2\Theta_o^2 = Z\pi \tag{14}$$

where Z is an odd integer. This can be satisfied by having

$$\pi\frac{L}{\lambda}r\Theta_o^2 = p\pi \tag{15}$$

$$\pi\frac{L}{\lambda}r^2\Theta_o^2 = q\pi \tag{16}$$

and p+q = odd number.

Or these two expressions can be reduced to

$$R\,\Theta_o^2 = p\lambda \tag{17a}$$

$$\frac{R^2}{L}\Theta_o^2 = q\lambda \quad . \tag{17b}$$

The requirement is

$$\frac{R}{L} = \frac{q}{p} \quad , \tag{18}$$

the ratio of the radius of the circle to the length is the ratio of two whole numbers.

Of course, this is not a very restrictive condition. The other condition to be fulfilled is that

$$N \frac{R\Theta_o}{L} \simeq \frac{1}{20} \qquad \text{and if } N \simeq 25$$

$$\frac{R}{L} \Theta_o = \frac{1}{500} \quad .$$

At this point, we still have a great deal of flexibility in the choice of arrangements to fulfill these conditions. For example, $R = 100$ cm, $L = 10$ cm, $\frac{R}{L} = 10$, $\Theta_o = 2\text{x}10^{-4}$. Then $q = 2.5\text{x}10^2$, $p = 2.5\text{x}10^3$ if $\lambda = 10^{-5}$ cm and the phase relationship of the beams is ensured.

Quartic Terms

The present discussion has given a set of fringes caused by beams which are in phase up to terms of order n^4 and, as such represents no improvement over the plane parallel GIMBI. But, now, with the additinnal degree of freedom, it should be possible to improve the approximation so that terms of order n^4 either do not contribute or else contribute completely in phase with the n^2 terms. Such indeed proves to the case.

Our requirement will be to find terms such that for n=1 the first term is equal to π or 3π etc. so that all terms of order n^2 are in phase; however, the dephasing of terms of order n^4 is less or equal to $\pi/4$ for $n = N$. Then again,

$$kd_n = kL\{1 + \frac{1}{2}\left(\frac{R\Theta_o^2}{L} + \frac{R^2\Theta_o^2}{L^2}\right)n^2 - \frac{1}{8}\left(\frac{R\Theta_o^2}{L} + \frac{R^2\Theta_o^2}{L^2}\right)^2 n^4 + \ldots\} \qquad (19)$$

and the coefficient of n^2 must be π, 3π etc. and the coefficient of n^4 must be so small that when multiplied by N^4, it is less than $\pi/4$. We have the subsidiary condition

$$N\frac{R}{L}\Theta_o \leq \delta' \ .$$

where δ' is the tangent of $\frac{1}{2}$ of the maximum angle accepted by the optics, typical values would be $N = 20$, $\delta' = 1/20$. Using the equality

$$\frac{R}{L}\Theta_o = \frac{\delta'}{N} \ ,$$

and $kd = kL +$

$$\frac{kL}{2}\left(\frac{\Theta_o\delta'}{N} + \frac{\delta'^2}{N^2}\right)n^2 - \frac{kL}{8}\left(\frac{\Theta_o\delta'}{N} + \frac{\delta'^2}{N^2}\right)^2 n^4 \qquad (20)$$

the requirements are

$$\frac{kL}{2}(\Theta_o\delta' N + \delta'^2)\ \frac{1}{N^2} = \pi,\ 3\pi,\ \ldots \qquad (21)$$

and

$$\frac{kL}{8}(\Theta_o\delta' N + \delta'^2)^2 \leq \pi/4 \ . \qquad (22)$$

This last equation clearly cannot be satisfied for $\Theta_o\delta' N$ positive and $\delta' \simeq \frac{1}{20}$. We have here more than an option, we have a requirement to make one of these terms negative; this is, fortunately, possible by making R negative. Now the radius of curvature is on the same side as the exit slit (cf. Fig. 2).

Then the replacement of R by -R gives the condition

$$\frac{kL}{8}\,(\delta'^2 - \Theta_0\delta' N)^2 \leq \pi/4 \quad , \tag{23}$$

and for $\frac{L}{\lambda} = 10^6$, $\delta' = \frac{1}{20}$ we have

$$\left(\frac{1}{400} - \frac{\Theta_0 N}{20}\right)^2 \leq 10^{-6} \tag{24}$$

with the equality sign satisfied by either

$N\Theta_o = 3x10^{-2}$ or $7x10^{-2}$ and the inequality satisfied for intermediate points. If N is taken as 30 then condition (21) is fulfilled, and $N\Theta_o$ is taken most usefully to be $3x10^{-2}$. Then a suitable set of conditions for the tilted GIMBI is

$$\Theta_o = \frac{3x10^{-2}}{N} = 10^{-3}$$

$$\frac{R}{L} = \frac{\delta}{N\Theta_o} = \frac{1}{20x3x10^{-2}} = 1.66$$

i.e. if L = 10 cm, R = 16.6 cm, λ = 1000 Å. This completes the set of conditions for the complete constructive interference on axis of 60 beams in a tilted plate GIMBI arrangement. The true interference condition still to be determined is then

$$\frac{kL}{2}\,(\Theta_o \delta' N + \delta'^2)\,\frac{1}{N^2} = p\pi$$

where p is an odd number. Using (24) as an exact relation one can choose

$$\pi \frac{L}{\lambda}\,\frac{10^{-3}}{900} = p\pi$$

or

$$\lambda = 1.1\text{x}10^{-5}\ L\ p$$

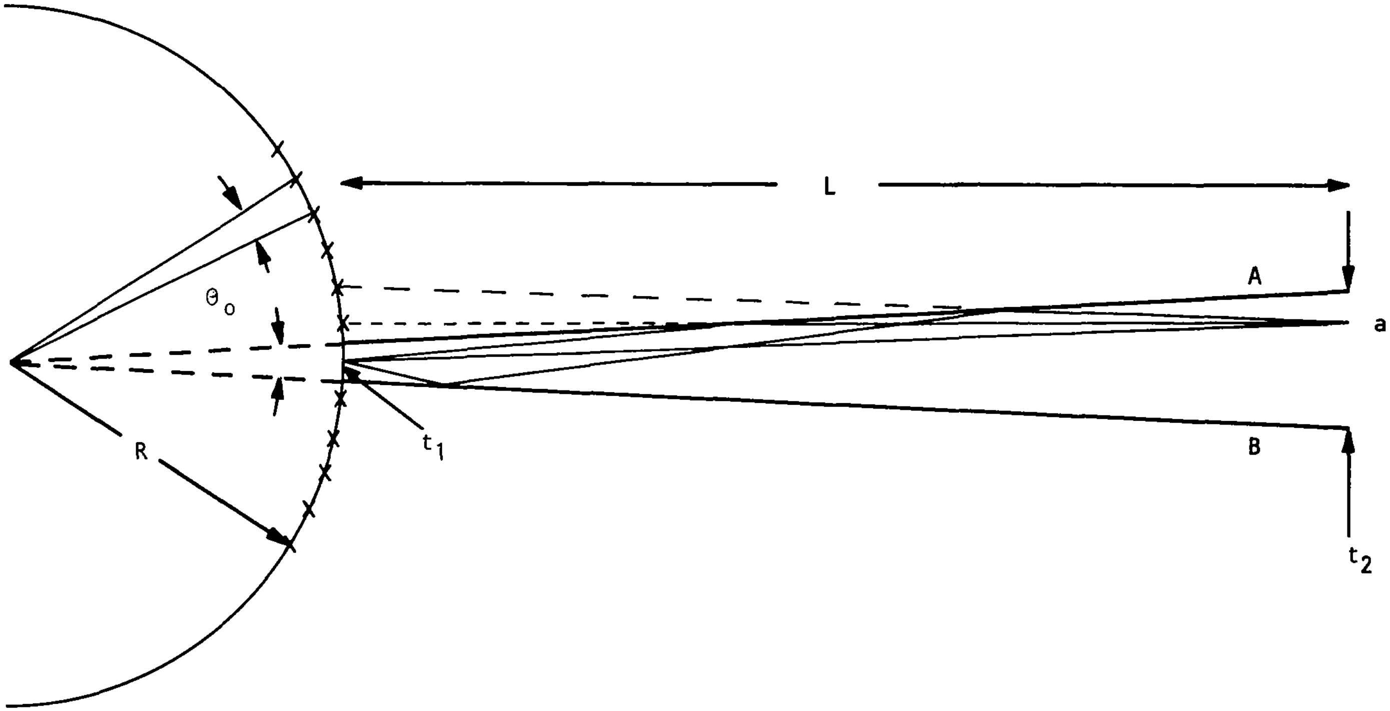
if p is equal to one, λ varies linearly with L or with the distance from the real point source.

Summary

We have described an interferometer which is suitable for the ultraviolet region of the spectrum. This device utilizes the same principle for forming its interference pattern which was previously described in the GIMBI; however, tilting of the plates has decreased the dephasing problem evidenced in the forerunner. We have described one specific instrumental set-up which optimizes the interference pattern for λ = 1000 Å.

Reference

R. A. Day, Applied Optics 9, 31 (1970).



TILTED GIMBI

Figure 1. Schematic drawing of interferometer showing interferometer illuminated by a point (or line) source. The virtual sources separated by an angle Θ_o, the inclination of the interferometer plates, are indicated.

NASA-Langley, 1970 — 14 CR-1666

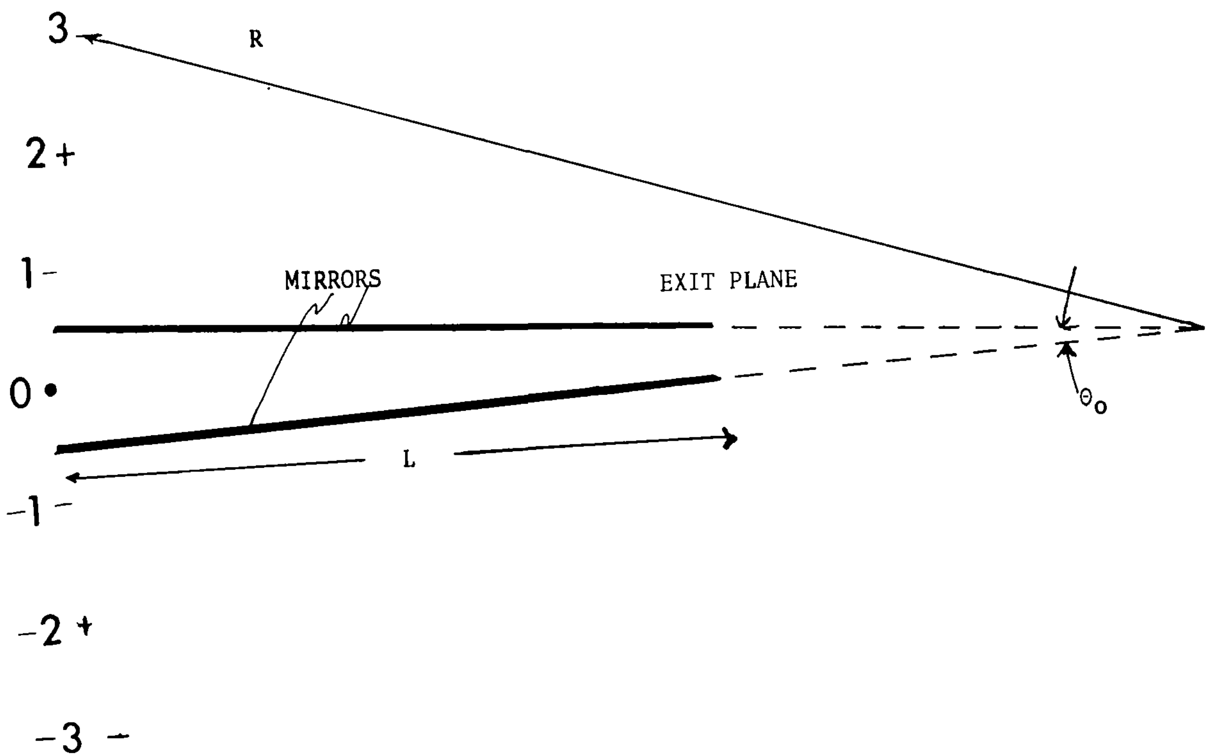


Figure 2. Arrangement of the interferometer plates for optimum performance; i.e., tilt is away from the source toward the plane of interference. Phase shifts between the virtual sources are indicated by + and −.