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PARTICLES AND FIELDS NEAR JUPITER

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16. Abstract

Current data on particles and fields near Jupiter are based on interpretations of Earth-based observations of radio emissions in the higher frequency ranges. The emphasis in this report is on elementary physical principles and on probable uncertainties in the results. Jupiter's magnetic field appears to be dipolar, with a moment of value 4.2×10^{30} c.g.s., directed at $7^{o}.7$ to the rotation axis. There is evidence for a small quadrupole moment, about 0.06 R_J in units of the dipole moment.

Typical energies for the relativistic electron fluxes are estimated to be 10 MeV, and represent a range of about 3 to 30 MeV. Lower energy electrons are much more difficult to estimate, and proton fluxes are virtually unknown on the basis of available data. However, sensitive upper limits to the thermal plasma density in the magnetosphere have been established empirically.

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ABSTRACT

Ground-based observations of radio emissions in the high-frequency and ultra-high-frequency ranges provide the only current data on particles and fields near Jupiter. Their interpretation is largely phenomenological but none-the-less definitive of major features of Jupiter's spatial environment.

Jupiter's magnetic field appears to be dipolar, with a moment of value 4.2×10^{30} c.g.s., directed at 7°.7 to the rotation axis. The zenographic north pole of the magnetic field lies in or parallels the central meridian longitude plane $\lambda_{III}(1957.0) = 193°$ (in 1963; in 1968 the value is 202°). This pole is probably north seeking. There is evidence for a small quadrupole moment, about 0.06 R_J in units of the dipole moment. There is also evidence for strong north-south and also lesser east-west displacements of the dipole away from the mass centroid of the planet.

On the basis of these conclusions on field strength taken from observed details of the UHF flux, we can today accurately estimate relativistic electron fluxes near Jupiter. Typical energies are 10 MeV, and represent a range from about 3 to 30 MeV.

Lower energy electrons are much more difficult to estimate and inferences even of their mere presence depend entirely on interpretations of the high-frequency emissions. Proton fluxes are virtually unknown on the basis of current observational data. On the other hand, sensitive upper limits to the thermal plasma density throughout Jupiter's magnetosphere have been established empirically.

Throughout, the emphasis is on elementary physical principles, rather than details of formulas, and on probable uncertainties in the results.

1. STATIC MAGNETIC FIELD

1.1 *Estimates of Jupiter's magnetic moment*

1.1.1 Moroz (1968) suggests that the ratio of the angular momenta of two planets equals the ratio of their magnetic dipole moments. For Jupiter, (see Appendix) as we shall shortly see, this estimate is very nearly equal to a best estimate on other bases. The angular momentum of Jupiter is not observed directly, but depends on the distribution of mass and angular velocity within the planet. From Allen's tables (1963), we learn that the moment of inertia of Jupiter, $C_J = 0.241\ M_J R_J^{\ 2}$ (equatorial); the angular velocity, Ω_J, of Jupiter's radio sources corresponds to a period of about $9^h 55^m$, i.e., 0.413 days. M_J = mass of Jupiter; R_J = its radius, subscript E refers to the earth. Setting the angular momentum of Jupiter, $L_J = C_J \Omega_J$ we find

$$L_J = L_E \frac{0.241}{0.3335} \times 317.8 \times (11.19)^2 \div 0.413$$

$$= 7.25 \times 10^4\ L_E$$

where $0.3335 = C_E/M_E R_E^{\ 2}$, $317.8 = M_J/M_E$, and $11.19 = R_J/R_E$. Since the earth's magnetic dipole moment is $M_E = 8.07\text{x}10^{25}$ gauss cm^3, we estimate, along with Moroz, that Jupiter's magnetic dipole moment is $M_J = 5.86\text{x}10^{30}$ gauss cm^3.

A physical basis for this estimate is, of course, similarity of terrestrial and Jovian internal conductivity, fluid motion, and perhaps initial condition of magnetism at the time of the formation of the solar system. In particular, if we suppose that magnetic-active parts of the two planets occupy the same fraction of the total volume of each, and if the volume and size of the individual rotors of each dynamo are the same, and if the angular velocities of the rotors vary as the angular velocity of the surface of each planet, then we might predict Moroz' result. But dynamo theory remains an extremely difficult subject today and we have no great confidence in its ability to produce deductive theories of magnetism in rotating bodies (see Gibson and Roberts, 1968).

Small additional comfort may be derived from this technique of estimation on the basis of its results for the sun. The general field of the sun is of the order of one gauss. From its angular momentum, $L_S = 1.7\times10^{48}$ c.g.s. compared with $L_E = 5.86\times10^{40}$ c.g.s., we derive $M_S = (1.7\times10^{48}/5.86\times10^{40})\times8.07\times10^{25}$ gauss $cm^3 = 2.34\times10^{33}$ c.g.s. The corresponding surface field $\simeq 2.34\times10^{33}/R_S{}^3 = 6.7$ gauss, comparable to observed values. In fact if, unlike the case for the earth, only non-central regions of the sun are involved in the field-sustaining processes, this estimate might be theoretically as well as observationally toc large. However, this estimate may break down badly in the context of other stars.

1.1.2 A range of values of the magnetic moment can be derived from certain aspects of decimetric radiation from synchrotron emission from the trapped radiation belts of Jupiter (at 2.5 R_J from its center, which is a rough centroid distance for the emission on each side of the planet). Basically the estimates depend on the stable properties in position and time of the belts. So far as the integrated flux across the belts is concerned, there appears to have been no large change since the first observations in the late 1950's. The observations of structural details by interferometers also present a picture of stable, well-trapped emission, very different from what might be expected on the basis of phenomena at relativistic energies near the earth (Hess, 1968). Omnidirectional flux variations over a factor of 100 occur in the outer belts in time scales of a few weeks.

Observationally we therefore believe that the energy density in the field at 2.5 R_J must exceed the relativistic particle energy density, and by a considerable amount, say 100-fold, to account for the probability that protons also will be stably trapped in the same spatial region.

Furthermore, it seems clear that variations of the emission on a time scale shorter than one year will, if they are present, force us to consider radiative lifetimes that short. However, variations such as they are, are weak; then, either the radiative lifetime is short and the source of energetic electrons is fairly stable despite the observations in the earth's belts to the contrary, or simply the radiative lifetime is long compared to one year. The first hypothesis seems unattractive. The second hypothesis leads to an upper limit on the field strength.

Minimum field strength, containment estimate. The energy density of the belt electrons is $N_e E$, where N_e = number of electrons of energy E, c.g.s. Then $B(\text{min}) = (8\pi \times 10^2\ N_e E)^{\frac{1}{2}}$ gauss. The radio emission directly establishes a good estimate of the value of $N_E EB$, since this determines the brightness of the source per unit path length along the sight-line (see page 61). Take this length to be $= 4\times10^{10}$cm through the region of peak brightness at $2.5R_J$. The observed brightness at, say 500 MHz, is 2×10^{-18} watts meter^{-2} cps^{-1} sr^{-1}. This depends on the other parameters as $2\times10^{-18} = 2.6\times10^{-23}\ N_e(E/E_o)\ B\ L\ B$, where $E_o = m_e c^2$, $B = B(\text{min})$, and $L = 4\times10^{10}$. $B(\text{min}) = 2\times10^{-3}$ gauss. (See page 58 for a discussion of these formulas.).

Maximum field strength, lifetime estimate. The maximum of intensity as a function of wave frequency lies at about 500 MHz (Dickel, et al., 1970). This frequency is given fairly closely by the product of field strength and the square of electron energy in the form $500 = (E/E_o)^2 B(\text{max})$. The rate, P, of radiation is quadratic in energy and field strength. At relativistic energies, $P = 4\times10^{-9} E(1 + [1/2]E/E_o)B^2(\text{max})$. Now P/E is the logarithmic gradient of electron energy, so that the characteristic lifetime of an electron is $T = 2.5\times10^8(1 + E/2E_o)^{-1} B^{-2}(\text{max})$. Letting $E/E_o >> 1$, and T = 1 year = 3×10^7 sec, we find $5\times10^8 E_o/[E\ B^2(\text{max})] = 3\times10^7$. Therefore, $B^{-3/2}(\text{max}) = 1.5$, or $B(\text{max}) = 0.76$ gauss.

At $2.5R_J$, $2\times10^{-3} \le B_o \le 0.76$ gauss. This range translates into an equivalent range of values for Jupiter's magnetic moment $.01\times10^{30} \le M_J \le 4\times10^{30}$ c.g.s.

1.1.3 By combining the observed size of Jupiter's synchrotron source with known properties of the solar wind, including its radial variation with distance away from the sun (albeit measured at one astronomical unit rather than Jupiter's distance, which is about five AU), we find a lower limit to the magnetic moment. A convenient way to derive this is in terms of the earth's field strength at the outer boundary of the stably trapped particles and quiescent magnetic fields in the magnetosphere. The best fit dipole field strength there is about 60 γ (one gamma = 10^{-5} gauss), about one-half the measured field strength just inside the interface (Hess, 1968, see Chapter Seven). Now the plasma density varies radially out from the sun as the inverse square of the distance (Dessler, 1967) so that near Jupiter it is $(5.2)^{-2}$ as dense as near the earth. We assume that its velocity remains essentially constant over the distance from the earth's orbit to Jupiter. Since the energy density in the planetary field essentially balances the kinetic energy density of the solar wind at either the earth or Jupiter, we find that the interface region of stably trapped particles at Jupiter lies at a field strength of $60\gamma \div 5.2 = 11.5\gamma$.

The problem lies in establishing where in space near Jupiter this field strength occurs. The observed maximum dimensions of synchrotron emission from Jupiter lie at about 6 R_J (McAdam, 1966; Gulkis, 1970). However, this region of obviously stable trapping may lie far inside Jupiter's magnetosphere. On the other hand rapid rotation of Jupiter may create an interface region of radically different physical nature from that of the earth. In either case, however, if we assume that the field strength at 6 R_J is 11.5γ we obtain a lower limit on the magnetic moment, $9\times10^{27} \leq M_J$, about the same as the one based on containment.

1.1.4 Many properties of Jupiter's decametric radiation suggest that it originates close to the planet and is defined by extremely stable plasma or field properties there. This point will be discussed in more detail later, but for the moment we shall simply assume that the wave frequency is close to the electron gyrofrequency. An estimate of the source position follows from the narrowband, stable radio frequency of emission that occurs at a definite longitude. For example, the emission of 39.5 MHz radiation lies in a narrowband strip within less than 1 MHz of this value, and occurs only at a certain 5° range of orientations of Jupiter and the satellite Io. Supposing that 39.5 MHz equals the electron gyrofrequency (somewhere), our task is to locate the source on the basis of its stability.

A range of only 1 MHz out of 39.5 MHz implies that the source lies within an essentially radial shell of thickness $(\Delta R/R) \leq (\Delta B/3B) = (3 \times 39.5)^{-1} = 0.008$. When $R = 6\ R_J$, this is $\Delta R \lesssim 3540$ km (which is about the diameter of Io).

Along a line of force the variation of direction of the line of force is most rapid in the equatorial plane of the field at the maximum distance from the dipole. At this point the line of force lies at right angles to the equatorial plane. Suppose that the 5° range of longitude of beaming corresponds to the change in dipole line of force direction across the dipole latitude range from -0°.5 to +0°.5. (The example is extreme because to understand the 5° beaming in this way we have to consider the dipole as lying parallel to the rotational plane of Jupiter !). The relative change in radius vector corresponding to a change of latitude from - 2°.5 to +2°.5 is only 0.0013; this is smaller than the change in radius inferred above from the bandwidth.

The beaming and the bandwidth demonstrate that no greater range in thickness than a few thousandths of the distance from Jupiter is involved in the emission source at any time; this extremely thin shell is well-defined in space by the mechanisms creating the emission.

For example, the relative thickness of Io is 0.007R. If Io creates a shell of emission this thick surrounding itself, its bandwidth will almost, but not quite, exceed the required upper limit of thickness. A disturbance near Io that covers as much as 5° of longitude along Io's orbit, with the thickness of Io, does not exceed the bandwidth requirement.

However, the lines of force undoubtedly are parallel to the axis of rotation of Jupiter, not perpendicular as in this illustration. The 5° longitude range cannot be defined as I have assumed above, but instead covers a very large range of shells, much thicker than permitted by the bandwidth.

If the source lies between Io and Jupiter, there appear to be much less favorable circumstances for this sharply-beamed, nearly monochromatic radiation. At intermediate points along any line of force, the range of directions of the line of force lying between shells of relative thickness 0.008R is much smaller than 5°. The bandwidth is too narrow to be consistent with the beaming provided naturally by the curving lines of force.

A specific selection of a radially thin region of excitation must be automatically made by physical circumstances of Jupiter's environment. The only alternative to Io is Jupiter's atmosphere, including the ionosphere. Here the scale length is small enough so that the observed upper limit of thickness, i.e., $\Delta R \lesssim 0.008 R_J$ = 600 kilometers, is indeed greater than the probable thickness of the atmospheric source region.

We then equate $(M_J/R_J^3)(1 + 3\cos^2\theta)^{1/2}$ on Jupiter's surface to (39.5/2.80) gauss (the gyro-frequency equals 2.80B MHz, where B is in gauss). Unfortunately, R and θ, the polar coordinates with respect to Jupiter's dipole moment, remain as unknowns of the problem. A conservative (upper limit) estimate of the moment however, results from taking R as large as it can be, and $\theta = 90°$. For example, this would approximately represent emission at the point where the extended magnetic dipole intersects the surface when the dipole lies at the opposite side. Then $M_J \lesssim (39.5/2.80)(8R_J^3) = 4 \times 10^{31}$ gauss cm^3. This result is less sensitive than the previous one based on electron lifetimes in belts.

1.1.5 The best values of the moment come from a detailed model of emission that combines both decimetric and decametric data. The decimetric data provide a strong indication that Jupiter's field is dipolar, with only a small quadrupole moment (see 2 below). Before these data were in hand, the symmetry of decametric drifts about a longitude lying between early and main sources of decametric emission suggested that *large* scale features of the field established the basic drift properties of the emission. Over considerably more than half a planet rotation, the slow drifting of decametric storms with frequency is consistent with this hypothesis.

The great puzzle in this hypothesis is why, if only a dipole field in involved, as would seem to be the case, does the emission occur in such a lopsided pattern in longitude, and furthermore, why is in effect only one sense of polarization observed in the decametric radiation?

I proposed to answer this question by the hypothesis that Jupiter's dipole lies asymmetrically within the planet, as well as tipped to the axis of rotation. Note that this is not a first-order approximation based on a zero-order model consisting of a centroid dipole. The model is better called a zero-order model, since the asymmetry in decametric polarization and rate of occurrence are dramatic.

Other researchers introduce several strong local irregularities in the field (Ellis, 1965) or other, perhaps more benign, departures from a dipole.

The "best" rough model for the dipole moment at this stage of the analysis, appears to be to assume that the emission involves polar lines of force at about one R_J from the dipole. Then $M_J \simeq 2.5 \times 10^{30}$ gauss cm^3, slightly less than the upper limit based on electron lifetimes.

These various upper and lower limits appear along with other material regarding the magnetic field, as discussed below, in Section 1.5, a summary of Jupiter's magnetic field.

1.2 *Shape of the magnetic field*

1.2.1 The gross appearance of Jupiter's decimetric radiation suggests an underlying dipole field. Occultation data, aperture synthesis and supersynthesis data, and interferometers show that the source is double, two sources split by 3 to $6R_J$ in the east-west direction. Each of these halves is roughly the size of the planet. The gross appearance is, alternatively, described as three-fold elongated in the east-west (equatorial) direction relative to the north-south (polar) direction (Berge, 1966).

The accepted explanation of this emission today is in terms of relativistic electrons trapped in Jupiter's magnetic field. These electrons have energies of tens of MeV, and in general terms exhibit no significant variations in time (see 1.2 above).

In general, electron trapping for long periods, say years or longer in the earth's belts, suggests that the magnetic field is longitudinally, i.e., axially, symmetric. Strong local deviations from rotational symmetry do not absolutely rule out longevity, but for the earth represent major electron sinks.

Electrons spend most of their time near the mirror points of their orbit, where the electron motion essentially is a nearly flat spiral around its field line. For this reason, most of the observed synchrotron radiation originates from the mirror points of the field, and all the more so, because the magnetic field is stronger there than at any other part of the electron's orbit. As a result we need think of the synchrotron radiation only from directions nearly at right angles to the field lines. The main sources of synchrotron emission therefore lie along the surface within the radiation belts that connect regions where the lines of force are at right angles to the sightline.

The simplest case is when the dipole axis is in the plane of the sky, at right angles to the sightline. Then all of the lines of force in the equatorial plane are orthogonal to the sightline, and therefore are sources of observable synchrotron radiation. The brightness is distributed along a straight line perpendicular to the dipole. In addition, the lines of force that lie in the plane of the sky also obviously satisfy the required condition. The first condition is the more important so far as brightness in the equatorial plane is concerned, since all electrons, no matter what L-shell they are on, contribute to it. The second condition is the more important so far as brightness out of the plane is concerned, since electrons that mirror out of the equatorial plane (i.e., that have pitch angles other than 90°) can contribute their mirror point radiation only as a result of it.

Berge's (1966) and Branson's (1968) observed brightness distributions clearly suggest the validity of this sort of model, and suggest the importance of electrons mirroring near the equatorial plane. They also strongly suggest that the field is poloidal, or nearly so, although quantitative data on higher-order moments of the field are not easy to obtain from these brightness distributions.

It is important to note that the center of gravity of these distributions is near the center of Jupiter. To determine possible differences of location of the two centroids to a precision much better than 1 R_J is very difficult by radio astronomical methods. 1 $R_J \approx 20''$, and should be compared with typical decimetric antenna beamwidths of 600", 30-fold larger. Interferometry at the required arc-second resolution also is difficult.

Interferometry and occultations (see Gulkis, 1970, with further references) do show brightness asymmetries between east and west portions of the emission that can be used to infer a rough value for E-W displacement of the centroids. Corresponding N-S displacements, which as we shall see may be much larger, have been observed accurately only once by the pencil beam technique, used differentially with respect to a well-established radio source coincidentally then within one-half a degree of Jupiter.

To infer an E - W displacement we note when the Jupiter meridian that contains the zenographically north pole of Jupiter's magnetic field lies at the east side of Jupiter, that portion of the emission to the east of Jupiter is about 4.4 per cent brighter than the western portion of the emission (Branson, 1968). Represent the Jupiter source as a ring of brightness in the equatorial plane of the dipole. As its edges, $3R_J$ from the planet's center, this ring will appear much brighter (under observations made with limited resolving power) than it does at the center. The ring is there viewed tangentially, for one thing. And, at the center, the far-side emission is occulted by the planet itself. If we entirely ignore emission from high latitude mirror points, we can set the total ring emission proportional to $2\pi(3R_J)$, of which approximately $2R_J$ is occulted by the planet. This emission is split into eastern and western portions, each amounting to $(3\pi-1)R_J$ units of emission. Suppose that the center of the ring lies KR_J units of distance from the center of the planet, to the east of the planet when the magnetic dipole tip in the zenographic northern hemisphere is tipped toward the east. Then that portion of the emission will be increased to $(3\pi-1)R_J + KR_J$ units, and the opposite will be decreased to $(3\pi-1)R_J - KR_J$ units; Branson's observations show that $2\ KR_J/(3\pi-1)R_J = 0.044$. Therefore $K = 0.044(3\pi-1)/2 = 0.21\pm0.1$ (the uncertainty here is my guess as to the stability of the contours Branson used to construct his east-west brightness distribution). The fractional amount of occulted radiation is $2R_J/(6\pi-2)R_J = 1/(3\pi-1) = 12$ per cent.

1.2.2 The total belt emission is linearly polarized, at a level of about 20 to 30 per cent. The exact amount is less important than the orientation, which is approximately orthogonal to Jupiter's rotation axis. There also may be some slight degree of circularly polarized radiation at some longitudes (Berge, 1966). The rotation of Jupiter's magnetic field was early inferred from rotational effects of the decametric emission. Likewise, the integrated flux from all parts of Jupiter's belts varies in amount, in degree of polarization, and in direction of the plane of polarization. These variations must be represented by a Fourier series of at least three terms whose base period is the period of both decametric and decimetric radio sources. It is fundamental to understand that these two quite distinct kinds of radio emission are connected integrally and to high precision through this common periodicity. The magnetic field geometry therefore establishes the properties of both types of emission.

A priori, the plane polarization of synchrotron emission from the belts might have paralleled the rotation instead of equatorial plane. The distribution of mirror points in latitude is the determining factor; the polarization direction in itself suggests a preponderance of low-latitude mirror points. Each longitude contributes to the total polarization; its direction is therefore an average over all the belts.

The distribution of the polarized intensity over an inclined dipole radiation belt system is not familiar, despite the amount of model-building and interferometry that ultimately should be based on it. As suggested above, observations made precisely in the equatorial plane have a singular viewpoint. This restriction softens somewhat as a result of the slightly broadened emission pattern of the typical electrons at, say, 10 MeV which covers roughly a cone of slightly more than one degree semiangle. Still, the smallness of this angle shows that the brightness distribution for dipole tilt angles much greater than one degree no longer can be represented properly by maps appropriate to a purely equatorial perspective.

Maps appropriate to the general problem of synchrotron emission from dipole fields can be constructed from the computations tabulated by Ortwein, et al. (1966). I also constructed the geometric projection illustrating the dipole-field orthogonal surfaces for tilt angle of 10° (Warwick, 1964). By plotting the direction at right angles to the lines of force satisfying the orthogonalicity condition I exhibited maps of the E-vector distribution over a tilted radiation belt system.

This distribution has several unfamiliar but important properties that follow easily from the geometry of the field-orthogonal points. In a tipped dipole, each line of force is in general orthogonal to the sight-line of an infinitely distant observer at two points. This property is an obvious consequence of the fact that a dipole line of force is a finite, continuous loop. Planes parallel to the plane of the sky lie tangent to this line of force at two points, one near the dipole, and the other far from it (for small dipole tilt angles). There is a conjugate pair of lines of force, consisting of the two lines lying in a single plane but separated by 180° in longitude around the dipole.

The basis for the map of a synchrotron radiation belt consists of two conjugate maps, one containing only pairs of points near the observer relative to the dipole; the points of a given pair lie respectively on meridian planes 180° apart in longitude. The other map contains point-pairs similarly on the far side of the dipole.

Each point-pair defines a point of tangency of the plane of the sky to a given line of force and its conjugate. There are two intersection lines of the plane of the sky on the plane containing the line of force, and these lines are parallel.

The direction orthogonal to these intersection lines is the direction of the E-vector of the total observed synchrotron radiation emitted by electrons along this line of force. (The intensity of the radiation depends on other factors, such as the pitch-angle distribution. It should perhaps be again emphasized that the present discussion is not rigorous, but nonetheless should be a highly accurate representation.).

There is a two-dimensional surface within the radiation belt on which the lines of force are everywhere orthogonal to the sight-line. This surface opens up conically with the dipole at its vertex; that is, its line elements all originate at the dipole.

The maximum latitude reached by electrons with a given equatorial pitch angle is represented by a symmetrical pair of these line elements above the equatorial direction and on either side of the pole. The other mirror point latitude lies on the far side of the dipole, and is similarly represented by a pair below the equatorial direction.

At a fixed radius from the dipole, a spherical shell intersects the orthogonal cone in a simple closed curve that maps into an approximate circle on the plane of the sky. Furthermore, the E-vector emitted by the shell approximately parallels this circle.

As viewed from the side, the central line of force, that lies directly between the dipole and the observer, is tipped downward through δ with respect to the sightline. $\delta(=10°)$ is the angle between the dipole direction and the plane of the sky. The center of curvature of the extremity of the line of force lies 2/3 of the radius vector to the extremity away from the dipole. The emission as seen in projection therefore lies $(2/3)R_m \sin\delta$ below the dipole, where R_m is the radius vector to the extremity (see Figure 1). From the observer's point view, the emission lies in a circular arc, limited on the polar side by two lines (labeled $\sin\alpha_L \leq 0.6$) representing the high latitude limit reached by mirroring electrons, with an equatorial pitch angle α_L arbitrarily chosen so that $\sin\alpha_L = 0.6$.

$2R_m$ here clearly corresponds to the synchrotron belt east-west dimension which we shall set equal to $4R_J$. Figure 2 schematically illustrates the situation. The large arcs are the emission loci, and direction of the E-vector. The dipole moment lies at the center; Jupiter appears to scale, and is arbitrarily set down in the center of the belt structures. These are shown at a tip of 10°, with the zenographic north pole tipped toward the earth. But the curvature characteristics are schematically always similar.

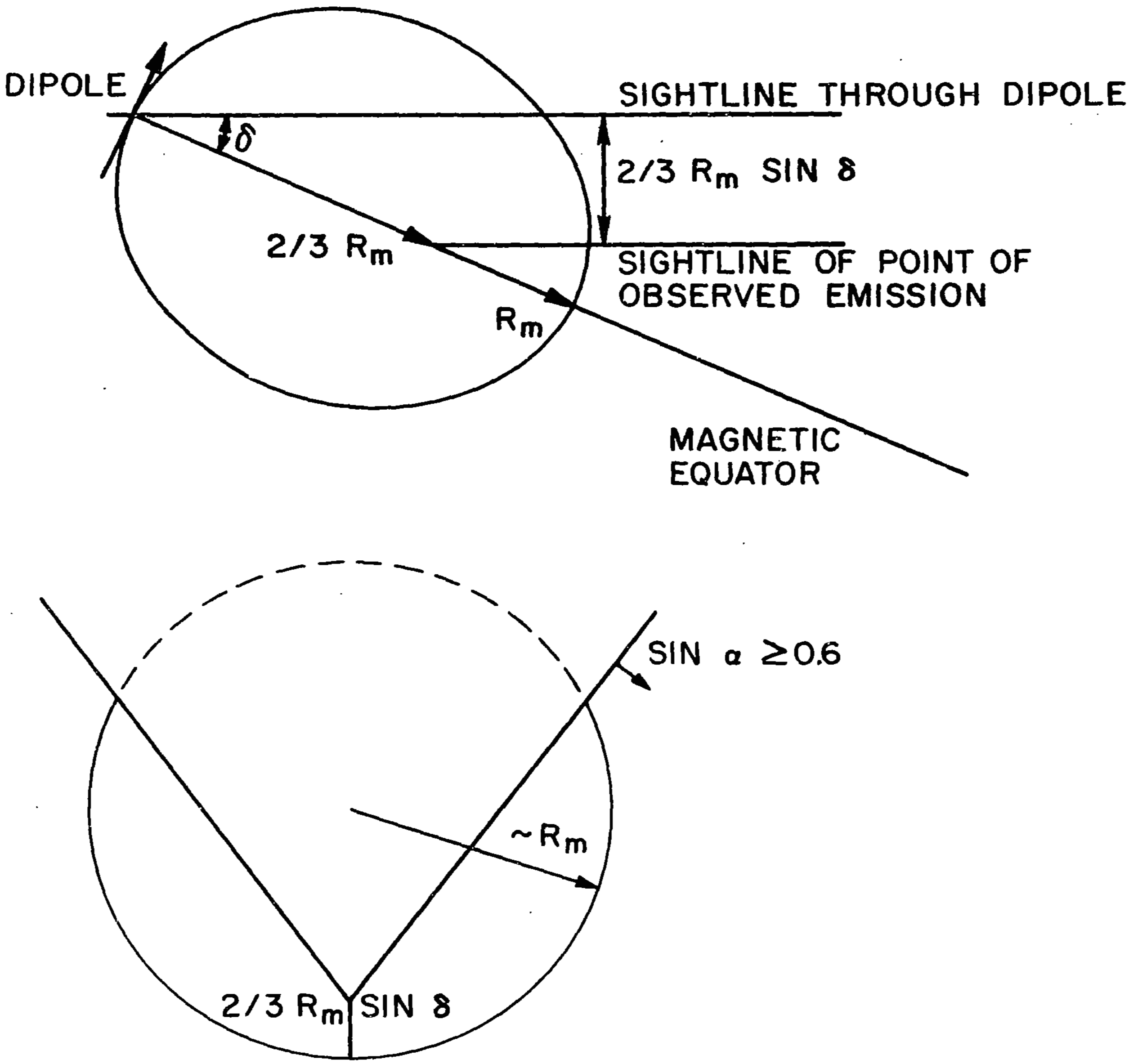

Figure 1. (Top) The dipole line of force. Its radius of curvature is $(1/3)R_m$ in the equatorial plane.

(Bottom) The shape of the locus of near-side emission from points lying on a spherical shell centered on the dipole (schematic).

Some remarks should be made about the observational checks of this model. Different portions along these curved arcs will have different brightnesses, that depend, for example, on the pitch angle distribution in the equatorial plane. The central region obviously will be reduced in brightness because far-side emission is there occulted. The extreme portions of the belts should be split into north and south portions corresponding to near-side and far-side emission, respectively. The N and S extensions should be polarized with E-vector roughly parallel to the rotation axis.

As Jupiter rotates, the tilt angle changes; the curvature of these various loci changes as well. So far all of the variations of polarization that have been considered correspond to the tilt. We now consider asymmetries corresponding to the displacement of the dipole with respect to the center of Jupiter.

A modification of Figure 2 can illustrate how an offset position of the dipole introduces asymmetries into observations of the total flux from the belts. The most interesting of several of these that has been observed is an asinusoidal variation in the direction of total polarization as Jupiter rotates. These data were originally observed at a wavelength of 20 cm by Roberts and Komesaroff (1965), and have recently been in essence completely verified at the extremely short synchrotron wavelength of 6 cm by Whiteoak, et al. (1969) as well as the long wavelength of 50 cm (Roberts and Ekers, 1968).

These verifications are all the more important insofar as Jupiter's rotation axis is now virtually perpendicular to the sightline. Figure 3 represents schematically the significant aspects (in my opinion) of these results for indicating the field structure near Jupiter. They are:

(a) The down slope at 180° longitude is much steeper than the up slope at 60° longitude, measured rates being -0.156 ±0.008 degrees of position angle per degree of longitude compared with 0.133 ±0.007

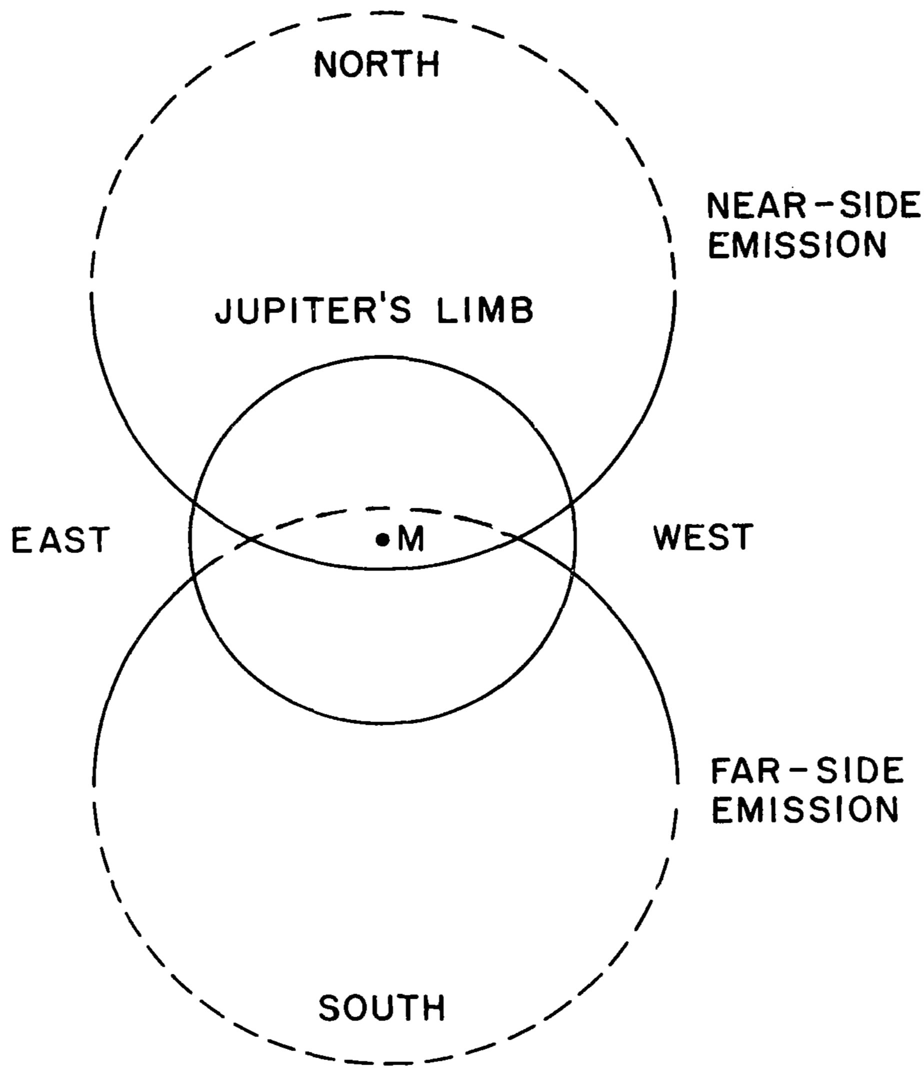

Figure 2. The loci of near-side, and far-side emission from a spherical shell. The dipole tip in the north is tilted toward the observer. The labeled directions are as viewed from the earth (schematic).

degrees per degree respectively. The uncertainties assigned here are my own, based on $\pm5^\circ$ uncertainty over 90° of planetary rotation. The recent 6 cm-data for the gentler-sloping portion at the counter-clockwise cross-over (the electric vector is rotating counterclockwise on this portion of the curve), is virtually identical in the two sets of data taken five years apart and at radically different wavelengths. The clockwise cross-over data are -0.156 degree per degree in 1963 at 20 cm and -0.178 degree per degree in 1968 at 6 cm. This difference is conceivably real, but in my opinion refers instead to the absence of data over 50° longitude on the most rapidly portion of the curve as observed recently.

(b) The high-position-angle portions of the curve are much sharper than its low position angle portions; these latter are close to flat. This feature appears to be identical in the two available sets of data.

Before entering upon a discussion of offsets, I emphasize that the authors of these beautiful decimetric data consider that a discrete anomaly in the magnetic field structure is responsible for the asymmetries.

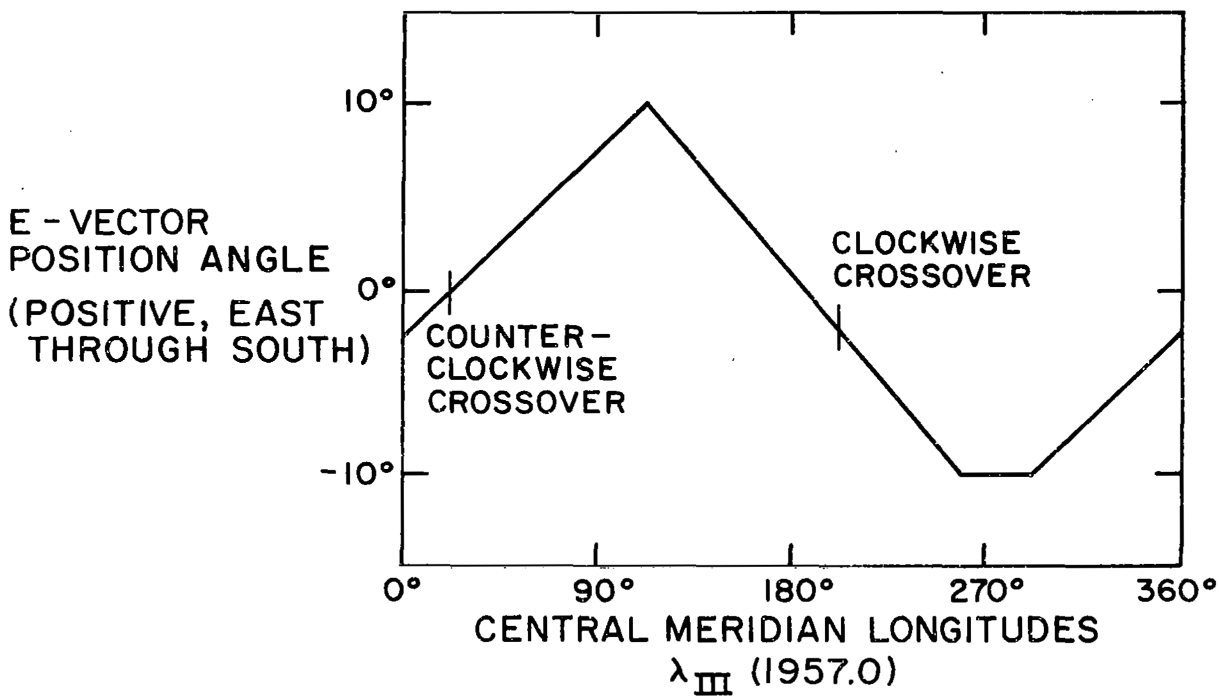

Figure 3. Schematic depiction of the decimetric polarization's position-angle variations as Jupiter rotates [see Roberts and Komesaroff (1965) for detailed data]. The dipole tilt, δ , follows from the rate of change of position angle at the counter-clockwise cross-over. The formula is $\delta = (360^{\circ}/2\pi)\ d(\mathrm{P.A.})/d\ \lambda_{III}$.

There appear to be at least two reasons why they come to this conclusion:

(a) pencil-beam measures of N-S location of the radiation belts appear to establish that they lie at the center of the planet to within $0.3R_J$ (Roberts and Ekers, 1965).

(b) a dipole offset can be no larger than one planetary radius, and this may not be only insufficient to produce the observed asymmetries but also seems unappealing on the purely intuitive grounds of planetary symmetry.

Whiteoak et al. (1969) suggest that the E-W brightness enhancement observed by Branson (1968) on the east limb at a longitude of about 135° is the required anomaly; this places the feature at central meridian passage near the clockwise cross-over, and behind the planet at the counterclockwise cross-over. Therefore, we can infer they should deduce the rate of change of position angle at the counterclockwise cross-over and infer therefrom the true inclination angle of the basic magnetic moment. (They used a different procedure however, see below).

This result agrees with mine, which is based on entirely different reasoning (Warwick, 1964, and see below). I derived a tilt of the dipole to axis of rotation = 7°.7 ±0°.1; the tip of the dipole in the northern hemisphere inclines toward the earth at a central meridian longitude $\lambda_{III}(1957.0) = 202° \pm 2°$, at the epoch 1 July 1968. The rotation period of the source is $9^h55^m29^s.70 \pm 0^s.05$ slightly longer than $P_{III}(1957.0) = 9^h55^m29^s.37$. (Current radio observations are being presented sometimes on one, or the other basis, so that care needs to be taken in reading the literature.).

Whiteoak et al. (1969) conclude on a tilt of 9°.2 ±0°.3 between dipole and rotation axis, not 7°.7 as I have above. Their value is the amplitude of the leading term in the polarization curve represented by a three-term Fourier series. Other authors quote values of 10° or even more for the same angle.

If the field isn't dipole, the tilt angle isn't uniquely defined in any case. If there exists a single anomaly superposed on a dipole, then the procedure I suggest above seems to lead to a more rational result. If the field is a nearly pure dipole, the dipole tilt should be taken at the small value I previously quoted.

In any case I believe that the intensity variations of 11.4 cm radiation observed by Roberts (1965) show conclusively that the magnetic field is very nearly a pure dipole (Warwick, 1967 and Wilson, 1968). The problem is then to demonstrate how it is possible that an offset dipole can account for the observed asymmetries. The magnitude of the necessary offset turns out to be about one planetary radius; the observations identify a large shift into the southern hemisphere of Jupiter, and a small shift toward $\lambda_{III}(1957.0) = 230°$.

To demonstrate these results I assume the model for the emission shown on Figure 2 but with the planet's position arbitrarily located; the moment only must lie within the limb of the planet (see Figure 4). The arc defining the location and polarization of far-side emission rotates in space as the magnetic moment M rotates. On the figure, imagine that this arc is fixed in space and does not change in position or shape. On the other hand, imagine that the disk of the planet swings back and forth along this arc. Two positions of the disk are shown. When the centers of the disk, of the emission arc, and the magnetic moment line up, the drawing represents the clockwise cross-over. When the disk is rotated by δ relative to M, with the location of M defining the center of rotation, the disk occults the far side emission more on the east side of the arc than on its west side. This position represents the dipole moment pointing to the west of the north position on the disk. It therefore occurs after clockwise cross-over, by about 90° in longitude.

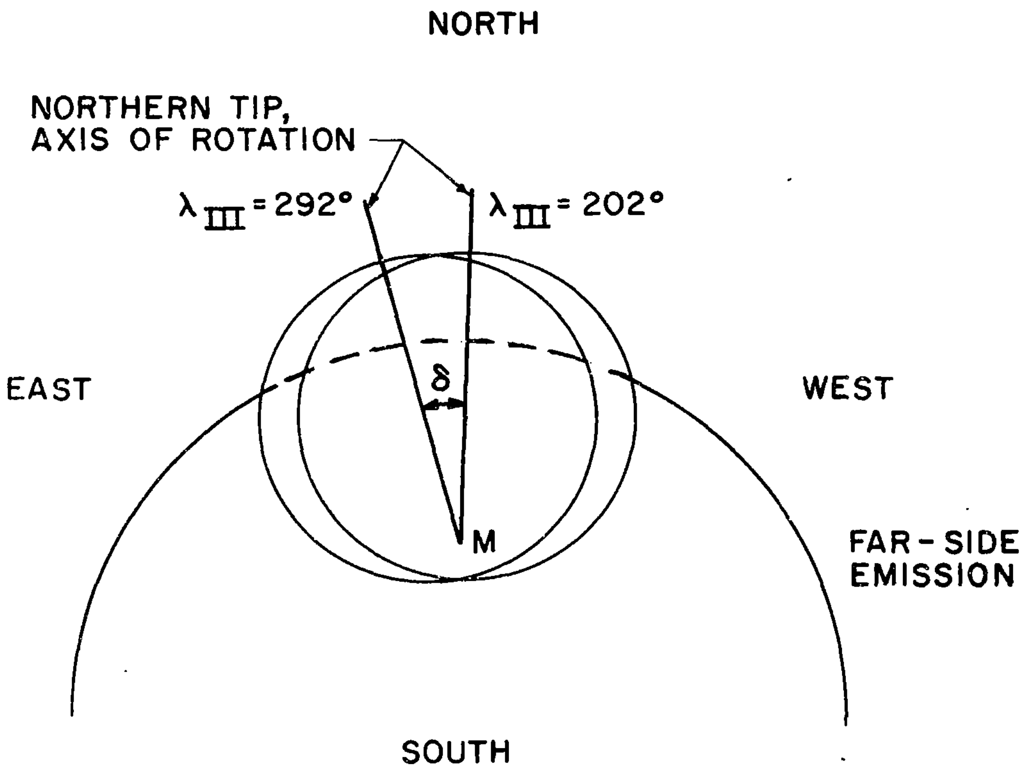

Figure 4. The arc of representative far-side emission is shown with the planet in its position at $\lambda_{III}(1957.0) = 202°$ and also with the planet in the later position $\lambda_{III}(1957.0) = 292°$; δ is the tilt of the dipole to the rotation axis of Jupiter and measures the amount and position of occulted portions of the far-side emission (schematic).

The total far side emission is polarized with E-vector parallel to Jupiter's equator; the blocked emission shown on Figure 4 to the east of the emission arc is polarized not in the plane of the equator, but at an angle rotated slightly counterclockwise out of the plane. This is the direction of the *blocked* emission. The remaining emission, which is not occulted, obviously is polarized at an angle rotated slightly clockwise out of the plane of Jupiter's equator.

Throughout the clockwise cross-over, the plane of polarization rotates clockwise not only as a result of the gross rotation of the magnetic moment, but also, as Figure 4 shows, as a result of the asymmetric and changing occulatation of the far-side emission. Exactly at the cross-over the E-vector defines the direction orthogonal to the dipole if M lies on the axis of rotation. Earlier than the cross-over, the plane of polarization is rotated counterclockwise to the equatorial plane, and afterwards, it is rotated clockwise. The observed rate of change of the position angle of the total polarization therefore is more rapid at the clockwise cross-over than the rate of change of the position angle of the dipole.

At the opposite cross-over, i.e., the counterclockwise cross-over, the asymmetry tends to produce exactly the opposite effect, that is it decreases the apparent rate of rotation of the dipole. However, note that the presumably most intense far-side emission at the counterclockwise cross-over lies to the south of the dipole M, and the body of the planet is largely to the north of the dipole. Therefore, not as much of it is occulted as at clockwise cross-over, thus reducing the strength of the opposite effect.

The magnitude of these effects is obviously critical to understanding whether they play a role in the observed asymmetries. Qualitatively, a southward shift of M agrees with the data. Quantitatively we need to make closer examination.

The far-side emission is covered along an arc segment of about one to two planetary radii by the disk. This occulted segment of the total far-side arc translates a distance that is measured by $\pm(\delta/57.^{\circ}3)S$ planetary radii, where S is the distance from the center of Jupiter to M in planetary radii. This assumes central occultation of the far-side arc; other regions will produce greater or smaller effects.

Suppose that the total belt emission is represented by these two arcs uniformly illuminated, one unocculted, the other partially occulted in a manner that varies with time. The total near-side emission is proportional to the arc lengths, $2\pi R_J$ as shown in Figure 4. Because of the symmetry of the polarization along the arcs, the belts as shown would be unpolarized. To introduce polarization requires that they be shorter, say, for example, along a 90° arc, which then corresponds to a total arc length on the near side of πR_J.

On the far side, something like $(\pi - 2)R_J$ is unblocked. The total emission is then $(2\pi - 2)R_J$. Assume this emission is 20 per cent polarized. The blocked radiation is of intensity $2R_J$, which is 100 per cent polarized. The direction of the E-vector of this blocked radiation swings through an angle of $\pm(\delta/57.^{\circ}3)(S/2)$ radians, or $(\pm)5^{\circ}S$ degrees, when $\delta = 10^{\circ}$.

A small intensity plane-polarized radiation, I_1, lying with its E-vector at a small angle, ε, from the E-vector of a large intensity plane-polarized radiation field, I_2, rotates the net E-vector through an angle $\theta \sim I_1/(I_1 + I_2)$. Now let $I_1 = 2R_J$ and $I_2 = (2\pi - 2)R_J$. Letting $\varepsilon = \pm 5^{\circ}\ S$, I find $\theta = \pm(5^{\circ}/\pi)S$.

This displacement S required to make the total range $\theta = (10^{\circ}/\pi)S \leq 4^{\circ}$ (which is the difference in dipole inclination angles determined from clockwise and counterclockwise cross-overs) is $S \sim$ one planetary radius.

At best this is a semi-quantitative estimate of the displacement. Much more careful theoretical study of the brightness distributions in the belts, and much higher resolution observations will be required to confirm this possibility.

The difference in shape of the maximum of the position angle curve and its minimum follow, in this model, from a slight difference in timing between the occultation effect and the cross-overs. In other words when the northern end of the dipole is tipped toward the earth, the projected position of M on the disk lies to one side of the axis of rotation. For example, suppose the longitude plane physically containing the dipole transits the west limb while the northern end of the dipole is still tipped toward the earth. Then, the occultation of far-side emission tends to produce a counterclockwise rotation of the plane of polarization while the dipole itself still is rotating clockwise. These effects tend to cancel one another over a range of central meridian longitudes, thus explaining the flat minimum of the position angle curve, from 240° to 330° (in Figure 3 after clockwise cross-over). Similarly, the same kind of effect, but reversed, occurs as the dipole longitude transits the east limb. There the occultation effect is less as explained above. But the passage onto the disk tends in any case to sustain the general counterclockwise rotation slightly longer, before the forward tipping of the dipole converts the occultation effect into a clockwise rotation.

Since the occultation effect has the least influence during counterclockwise rotation just before the maximum position angle is reached, the longitude of passage through this maximum determines essentially the orientation of the dipole. This value $\lambda_{III}(1957.0) = 112°$ (on 1 July 1968) CML, but corresponds to a dipole tipping meridian of $112° + 90° = 202°$. 180° later, i.e., when $\lambda_{III}(1957.0) = 292°$ CML the dipole is again in the plane of the sky. However, the position angle curve has been at that time already distorted from its peak shape since about 260°, since the occultation effect is rotating the polarization in the opposite sense from the dipole rotation, as discussed above.

The physical location of the dipole must lie in the meridian about 30° to the east along the planet's surface from the meridian plane parallel to the dipole. In this position it is occulted sooner at west limb passage. This value of 30° follows from the difference 292° - 260°, and probably should be given the uncertainties $30°^{+20°}_{-10°}$. Putting this together with my interpretation (see page 11) of Branson's asymmetry, I locate the dipole $0.2R_J$ from the axis of rotation in the meridian plane $\lambda_{III}(1957.0) = 232°$ (on 1 July 1968).

In summation, then, the dipole appears to lie physically in a far southerly position at $X_N = 0.2R_J$, $Y_N = + 0.1R_J$, $Z_N = -1R_J$. The uncertainties in X_N and Y_N are relatively small, say $0.1R_J$, compared with that of Z_N, which is surely large.[1] Hopefully not fortuitously, the values X_N, Y_N, Z_N, derived above entirely from decimetric data are extraordinarily close to the values deduced in the early analysis entirely from decametric data. It is clear, for example, that both X_N and Y_N are positive and small, while Z_N is large and negative. It is also quite probable that $X_N > Y_N$.

1. *The coordinate system here is one I introduced (Warwick 1963a); the axis of Z lies in the rotation axis and is measured positive northward. The X-axis and the Z-axis define a plane parallel to the magnetic dipole, fortuitously taken in 1963 to be $\lambda_{III}(1957.0) = 200°$, nearly the correct value 202°, for 1 July 1968.*

1.2.3 The existence of an axisymmetric quadrupole moment for Jupiter's field follows from decimetric data on the variation of the total intensity as Jupiter rotates. This has to do with the pitch-angle distribution of the trapped electrons, which determines their mirror points. A distribution clustered around flat (90°) pitch angles defines the point of minimum magnetic field strength along the line of force encircled by the electrons. The direction of emission from these electrons lies at right angles to the line of force at the point of minimum field strength.

Define two elongations, east or west, when the northern end of Jupiter's dipole lies east or west of the rotation axis and lies parallel to the plane of the sky. Then, emission emerges very nearly along the magnetic equatorial plane (except for the two sets of dipole lines of force in the plane of the sky). At other aspects of the planet's rotation the sightline from the dipole center to the observer emerges at somewhat higher or lower magnetic latitudes. The typical beaming pattern of the relativistic electrons at Jupiter is (as stated below, see page 62) only a little more than one degree. Therefore, when the sightline passes more than one or two degrees above or below the inferred magnetic equator of Jupiter it takes in radiation from electrons definitely out of the plane. In crossing through the magnetic equatorial plane the sightline passes through a maximum of radiation intensity. This maximum corresponds to a summation of radiation from contribution points extending over several radii along the plane of the belts.

If we knew exactly where was the elongation longitude, we could examine the variation of intensity in its neighborhood to establish whether the maximum emission actually does come at the point where the sightline lies in the magnetic equatorial plane. If we could measure a difference between these two directions, i.e., between the direction of maximum emission, and the direction of the magnetic equator, we would be in a position to discuss the deviations of Jupiter's field from that of a pure dipole.

It is important to understand that this deviation represents averages over large ranges in longitude. The presence of a large scale systematic small deviation therefore is heavily favored over extremely localized irregularities. We should expect that in Jupiter's synchrotron emission, only the lowest order moments play important roles, inasmuch as the belts lie typically one or two radii above Jupiter's surface.

Recently Roberts and Ekers (1968) re-addressed themselves to the question of asymmetry in Jupiter's magnetic field based on the beaming properties of decimetric emission. They show that these properties persist over all observations, covering a frequency range at least five-to-one. They also partially explain an early result (Bash et al., 1964) which concludes that the angle between Jupiter's dipole and rotation axes is considerably larger than the 7°.7 established above, or for that matter, the more commonly quoted 10°.

It is especially important for me to clarify these disagreements since they are at the basis for my deductions concerning Jupiter's quadrupole moment. Furthermore, Roberts and Ekers (1968, page 162) misinterpret my explanation (Warwick, 1967). In turn *mea culpa* is that what precise longitudes are adopted to represent the elongation points determines the result and I did not mention this sensitivity.

The central point is two-fold:

(a) what are the precise elongation longitudes? (in the context of answering (b), below, the previous values 112° and 292° may not be good enough).

(b) what latitude scale should be given to correspond to longitudes near elongation? The zenomagnetic latitude of the sightline varies most rapidly exactly at the elongations, which means that total flux also should fall away most rapidly on either side of the elongation points. It is inconceivable that the flux fails to achieve a symmetric maximum about *some* zenomagnetic latitude. Asymmetry north and south of the equator is therefore outside the question. What needs to be determined is where the point of symmetry in intensity lies, with respect to the equator.

Roberts and Ekers (1968) and others deduce a beaming parameter n which is the exponent of the cosine of the zenomagnetic latitude, ϕ, required to make $\cos^n \phi$ represent the observations. What they have shown is that the choice of n is very sensitive to the variations assumed for ϕ. By spreading ϕ over a wide range, variations of $\cos \phi$ can be introduced that are large enough so that $\cos^n \phi$ varies sufficiently in both north and south zenomagnetic latitudes. With only a single n value used to represent fluxes in both northern and southern latitudes, the best-fit solutions are therefore driven toward a large enough dipole tilt angle to effectively spread ϕ over a sufficiently wide range as Jupiter rotates. Roberts and Ekers (1968) have performed a significant service by pointing out the relevance of this fact to the large tilt angle deduced by Bash, et al. (1964).

No matter how more simply this technique represents the data, it does not answer the central question of north-south symmetry; quite the contrary, it conceals it. This fact is shown most clearly by Figure 2, graphs (c) and (d) of Roberts and Ekers (1968). In 620 MHz data, it appears that n must be relatively greater (in southern zenomagnetic latitudes) than it is in 2650 MHz data. That is the drop-off of the flux into the southern hemisphere is much more rapid than the drop-off into the northern hemisphere. When the 620 MHz data are replotted on the basis of a very large inclination angle, 15°7, they approximately fit the law $(\cos \phi)^{1.0}$. Nevertheless at southern latitudes the flux falls still more steeply than this curve.

The basic question is whether these curves (whatever is the value of the tilt angle assumed to construct them) are symmetrical around any latitude. I answered this question strongly affirmatively for essentially the same 2650 MHz data analyzed by Roberts and Ekers (1968); the data are accurately symmetric around north zenomagnetic latitude + 1°.2 (with elongation prescribed as above) and n (determined with respect to this zero latitude) = 4(now 3.6 according to the paper before us). The higher n value follows entirely from the fact that the symmetry is not around $\phi = 0°$. The 620 MHz data now published are symmetric around a slightly greater northern latitude, + 2°.5 to 3°.0, but there are fewer points and their scatter is greater; I conclude that the significance of the difference from the earlier value of + 1°.2 is probably not high.

Since the intensity drops off monotonically either side of the maximum, conveivably one could deduce the maximum, and its latitude, by looking just for the highest points. This isn't a tautology. The points are scattered about, and a statistical estimate is required. With the observed large spread of points, there seems no way to justify determination of the maximum in this way.

On the other hand, if the intensity drop-off is symmetrical either side of the maximum, we can combine the data over a wide range of latitudes, north and south, to establish precisely not only the latitude of the maximum, but also the fact that the emission drops off similarly both to the north and to the south. In fact, this clearly is the case at both 2650 MHz and 620 MHz. This justifies calling the radiation symmetric about its maximum point, as is required by simple trapping theory.

If we inferred that elongation occurs at a longitude that is slightly in error, then displacement of the maximum intensity in latitude may conceivably result. We can deduce from the latitude of maximum a correction to the longitude of cross-over, say, that permits us to assume that the beaming maximizes precisely in the equatorial plane. To observe the + 1° maximum we used data based on a 10° tilt angle of the north end of the dipole into the longitude $\lambda_{III}(1957.0) = 198°$. The tilt of Jupiter's rotation axis to the sightline is 3°.3 (north end toward the earth), at the time of these data. The question is what is the difference between longitudes corresponding to zenomagnetic latitude 0° and zenomagnetic latitude -1°.2? These are given by λ_1 and λ_2 in the expressions

$$\text{(a)} \quad 0° = (\cos 10°)(\sin 3°.3)+(\sin 10°)(\cos 3°.3)\cos (\lambda_1 -198°)$$

and

$$\text{(b)} \quad \sin(-1°.2) = (\cos 10°)(\sin 3°.3)+(\sin 10°)(\cos 3°.3) \cos(\lambda_2-198°)$$

A quick calculation shows that $(\lambda_1 - \lambda_2) = -8°.3$. According to formula (a) eastern elongation occurs when $\lambda_1 = 89°.0$, but the observations say that $\lambda_1 = 89°.0$ is really a time when the latitude is -1°.2. To make the latitude farther south near eastern elongation requires that we push the longitude of the dipole to greater values, i.e., that the 198° figure be *increased* by 8°.3, to 206°.3. At western elongation, the formula (a) as it is written requires $\lambda_1 = 306°.4$. Again, however, this value actually occurs at -1°.2 latitude rather than zero. To achieve this requires that the 198° figure be *decreased* by 8°.3 to 189°.7.

I shall rule out the simultaneous occurrence of both possibilities. In fact, the data curves show that 89° is rather too small a longitude for the eastern elongation insofar as the intensity curve describes it. 96° appears to fit the peak better. 306° probably defines the western elongation approximately but some scatter of the data points at this maximum renders matters uncertain, and makes possible a smaller value. 314° probably falls too late with respect to western elongation. A better peak value actually would be 300° instead of 306°.

In other words, both elongations tend to exhibit a peak flux at a longitude not correctly given by the system in which the dipole lies at 198°. The eastern longitude fits better with 206°, and the western with 190°.

These corrections cannot *both* be proper, with only a simple model. The possibility is ruled out that an error in the choice of dipole longitude can correct the asymmetry effect.

The simplest correction derives from giving up the assumption that the field is purely dipolar. Since the emission represents large regions of space remote from the surface of Jupiter, it is reasonable to consider the distortions in terms of the next multipole expansion term above the dipole. This is the quadrupole term.

Unfortunately, quadrupoles come in three forms, corresponding to the three associated Legendre polynominals that appear in potential function expansions to the second degree. The various terms are necessary to represent asymmetries within different sectors in longitude.

Obviously the data do not require all the higher-order polynominals for a good fit. Their gross appearance is too nearly sinusoidal to suggest either the necessity or desirability of going to this extreme.

In fact, the symmetry of the error in the dipole fit at the two elongations suggests that a much simpler model will do, namely, an axisymmetric quadrupole oriented roughly parallel to the dipole. *Both* eastern and western elongations must correspond to southerly latitudes for their maxima to appear, respectively, at later and earlier, longitudes. The elongations represent two independent pieces of evidence, spread 155° in longitude, that a one degree correction in the direction of the lines of force is necessary. In both elongations the correction not only has the same magnitude, but also the same sense, in which the lines of force are bent so that their minima lie north of Jupiter's equator (see below).

An axisymmetric quadrupole has uniform radial equatorial and radial polar magnetic field. The sense of the equatorial field opposes the polar fields, i.e., is radially either inward or outward, while they are both outward or inward. When a small quadrupole is added to a parallel dipole field, the resultant field tends to have one pole weakened and the other strengthened. In the equatorial plane, the resultant deviates toward the weaker pole.

The lines of force of the combined fields are displaced from the equatorial plane toward a symmetry latitude that lies in the same hemisphere as the stronger pole. Jupiter's quadrupole strengthens the northern pole.[2] The strength of the quadrupole can easily be estimated. The lines of force at $2.5R_J$ are deviated by $1^\circ\!.2$ as a result of the quadrupole component of the field. Let M_1 = dipole moment and M_2 = quadrupole moment. The ratio of their field strengths is as $[M_2/(2.5R_J)^4]/[M_1/(2.5R_J)^3] = 1^\circ\!.2/57^\circ\!.3$. Therefore, $M_2/M_1 \sim (3/57.3)R_J = 0.05R_J$. In a more careful estimate Wilson (1968) derives $M_2/M_1 = 0.057R_J$.

This same type of discussion applies as a general boundary condition on any field irregularity that may from time to time be proposed to account for Jupiter's peculiar radio properties. As an extreme, suppose that the quadrupole lies on Jupiter's surface. Then $[M_2/(1.5R_J)^4]/[M_1/(2.5R_J)^3] = (1^\circ\!.2/57^\circ\!.3)$ or $M_2/M_1 \sim (1.8/57.3)(1.5/2.5)^3R_J$. This value for M_2 is one order of magnitude smaller than the previous one, but it should be noted, explains the effect at only one elongation. In other words, at least a second, and probably several, quadrupole of this same strength would have to be arranged around Jupiter to explain the wide longitude range over which the field distortion appears.

The surface field strength ratio between quadrupole and dipole fields is measured by the ratio $M_2/M_1R_J \sim 0.05$. This value is quite comparable to the ratio of the terrestrial dipole and quadrupole fields. The difference in polar field strengths is very small, if the magnetic poles are at the center of Jupiter. The quadrupole field strength is about 5 per cent of the dipole field strength. Since it adds to one pole and subtracts from the other, the quadrupole results in a difference in polar field strength, therefore, of about 10 per cent of the average polar field strength.

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2. *This conclusion is contrary to the one stated by me (Warwick, 1967). A correct analysis appears in Wilson 1968.*

1.2.4 Most students of Jupiter's radio emissions have concluded that its high frequency (i.e., decametric) radiation corresponds closely to the electron gyrofrequency at the point of origin. If this is so, then the HF radiation shows that as Jupiter rotates starting at about 0° longitude, the magnetic field in the sources slowly increases to a maximum value of 14.1 gauss (corresponding to equating the electron cyclotron frequency to the precise upper limit of recorded emission at 39.5 MHz). After this maximum, which occurs stably at about 140° to 150° in longitude λ_{III}(1957.0) in 1962, there is a hiatus of emission at all frequencies in a 20° range very close to 180°, followed by a slow decrease in magnetic field over the longitude range from 200° to 360°.

This planetary-scale variation appears without exception in all data. It suggests a smooth planetary distribution of the field, rather than a distribution strongly peaked at various points distributed either in or close to the surface of Jupiter. This interpretation appears still to be controversial (see, e.g. Ellis, 1965). However, since I first suggested it in 1960, I have seen the discovery of a succession of phenomena that only strengthened my belief. Again, the smooth variation of HF emission on a planetary scale suggests that the field is dipolar.

The sense of the variations reverses following the longitude range 150° to 190° (see Warwick, 1961). It appears highly significant that this region is virtually identical with the longitude region in which Jupiter's northern dipole tip is presented to the earth.

The evidence against this conclusion on the poloidal character of Jupiter's field starts with the fact that the HF radiation is strongly polarized with the right-hand sense document. In itself evidence that the magnetic field plays a vital role in the emission, this fact is a puzzle from the point of view that only one magnetic pole (i.e., only one sense of polarization) seems to be manifest in the data. The conclusion softens toward the lower frequencies of the HF range, but is not invalidated by any data so far in hand.

Further evidence against the general poloidal nature of the field as manifest in HF data is the fact that the emission seems to concentrate in a few "sources" that appear only within limited longitude ranges. In particular, two sources predominate, one at eastern elongation, and another about 45° after clockwise cross-over. (These are variously called source one and source two, source B and source A, or early source and main source, respectively). The total range covered by these two sources, plus a less distinct third source (source C or late source), is less than one rotation, by about 90 degrees. In fact, most data concentrate within about 100°, at source two (source A or the main source). In other words, the distribution of emission is highly lopsided, quite unlike a symmetric double-lobed pattern as perhaps might be expected from the alternate presentation of two magnetic poles.

This pattern suggests to many students that a magnetic anomaly is superposed on the poloidal field generally conceded to be responsible for the most outstanding characteristics of the decimetric radiation. This anomaly, according to wide opinion, governs HF frequency, polarization, and longitude properties.

While it seems clear to me that an anomaly can be devised *ad hoc* to solve many outstanding properties of the emission, this procedure serves no purpose before a thorough working-out of the simpler dipole field model. It may have the real disadvantage of overlooking the common threads that connect decimetric and decametric emissions. These, it appears, are clearly recognizable in both sets of data. If nothing else, then we should insist that decametric theories are consistent with observations not only in that frequency range but also at the higher radiation belt frequencies.

The next section 1.3.1, provides an outline for a consistent decametric theory.

1.3 *Location of current sources within Jupiter*

1.3.1 The sources of the field, as inferred externally from radio emission, must lie within the planet. At standard temperature and pressure, the energy density of the planetary atmosphere is 10^6 ergs cm^{-3}. The field energy density only amounts to about 5 ergs cm^{-3}, but equals the atmosphere's energy density within a few hundred kilometers above the clouds. This range of Jupiter's atmosphere may not be appreciably ionized so far as field dynamics are concerned. A possibility still exists that appreciable fields are generated by the satellites, but certainly not comparable to the main field. The conclusion is inevitable that the field sources lie below the cloud tops, within Jupiter where both ionization and gas kinetic energies are favorable to the development of natural dynamos.

With so much said, there is little more that can be. Dynamo theory remains too primitive to provide a deductive basis for predicting magnetic fields in even well-known objects, such as the insides of stars. For example, the earth's general field makes a poor analogue on which to discuss the topology of the sun's general field. Perhaps the order of magnitude of the solar field strength can be guessed at from the case of the earth, as suggested earlier in this report. Nevertheless, the shape of the solar field, and especially its variability, do not appear to resemble the terrestrial field.

The interior density, pressure, and conductivity of the sun are known, along with the properties of its surface magnetic field. Where the field currents lie is unknown, however, within a wide range of possibilities. Present-day theory (Starr and Gilman, 1968) is beginning to provide insights into the reversibility of the solar field, and suggests a dynamo near the surface.

For Jupiter a wide range of possibilities has been discussed in both internal structure and electrical conductivity. It appears to me that if anything is self-evident, it is that an intuitive assumption of poloidal symmetry based on currents in the deep interior is no less speculative than an assumption that the current sources lie in the surface of Jupiter. There is no theoretical basis on which to exclude even what may be outrageous field models from other points of view.

1.3.2 Some features of Jupiter's atmosphere, particularly the Great Red Spot, seem to possess the kind of permanence we should like to ascribe to the global planetary magnetic field. At the outset of HF studies of Jupiter, a chance coincidence between central meridian passage of the GRS and of the times of observation of radio bursts seemed to suggest a connection there. Indeed the radio rotation period clearly belongs within the family of System II, the nominal rotation period appropriate to Jupiter's temperate zone atmospheric phenomena.

The stability of rotation period of HF radiation suggested to the early observers that it corresponds to the rotation of the solid planet. Perhaps it represents a superficial volcano, which not only rotates stably but also singles out a narrow longitude range like the main source. That point of view appears today essentially irrelevant, in consideration of the strong role played by the magnetic field.

The possibility of intrinsic variations in the rate of rotation of the magnetic field is very real, and Jupiter observers are alert to examining their data from this point of view. In fact, the nomenclature chosen to describe the currently conventional radio period, $P_{III}(1957.0)$, emphasizes its epochal character. The first careful measurement of the period, over a sufficiently long interval to establish its value to within $0\overset{s}{.}1$, determined the value $9^h55^m29\overset{s}{.}37$; this period now is the basis of ephemeral longitudes $\lambda_{III}(1957.0)$ in tabulations by the U. S. Naval Observatory. The base interval extends from 1950 to

1960, and contains pre-discovery observations only in 1950 and 1951, then a gap until 1954. Positions since 1960 but based on $\lambda_{III}(1957.0)$ in a real sense are extrapolations.

Despite the fact that since 1961 data depart systematically from System III(1957.0) it still provides a thoroughly adequate basis for presentation of data over a short interval, not exceeding, say, one or two years. The departure relates, in all probability to effects on the longitudes of the radio sources of the changing inclination of Jupiter's axis to the sightline. The gross rate of occurrence of HF emission varies with the same period as the axial inclination, that is, in 11.9 years, the orbital revolution period of Jupiter. In a classic coincidence, this value approximates the sun-spot cycle. Therefore, Jupiter HF data vary in an 11-year period not only because the earth's ionosphere makes observations at maximum more difficult, but also because intrinsically less radiation is then beamed toward the earth.

Recent treatments of the problem of determining Jupiter's magnetic rotation period therefore emphasize the importance of taking rotation axis tilt variations into account and have resulted in a new value for the period, $9^h55^m29\overset{s}{.}70 \pm 0\overset{s}{.}05$. This period also provides the best fit to decimetric data based on cross-over longitude determinations from 1963 through 1968.

Throughout the twelve-year revolution of Jupiter the sources maintain a recognizable and similar pattern as a function of Jupiters's rotational longitude. But data become strikingly less frequent at times of sunspot maxima, which (in the mid-20th century) coincide with times when the south end of Jupiter's rotational axis points toward the earth. The occurrence probability is therefore a sine wave whose period is the orbital period of Jupiter (presumably its sidereal value, since the invariant (non-precessing) rotation axis is involved).

If the magnetic field were symmetrical around the rotational equator, no difference in occurrence statistics could appear during successive halves of one revolution. Polarization phenomena would, on the other hand, reverse in these phases. The "sunspot cycle" effect, a single-peaked curve of rate of occurrence during Jupiter's revolution, becomes further evidence for asymmetry of the field between the north-south hemispheres.

1.3.3 The phemomenon called "beaming" refers to the strong rotational effects observed in synopses of decametric data either at single or multiple frequencies. The rate of occurrence of emission is much higher at some longitudes than at others. At its maximum the change of occurrence rate is at least two per cent per degree of rotation in single frequency data analyzed by means of composite dynamic spectra (see Warwick, 1963a).

Along with this sharply defined beaming into prescribed longitude ranges, there goes an equivalently sharp "beaming" into narrow spectral regions (see Section 1.1.4 above); the bandwidth frequently is only a few hundred kHz, or less. As I suggested above, this narrow a frequency range implies that an extremely narrow range of radial distances is involved in the source. Because of the rapid decrease of field strength as a function of distance from the planet, no more than a few hundred kilometers can simultaneously create radiation in one of these narrow band events.

The narrow bandwidth emission occurring repeatedly at given longitude argues simply and directly that the region of emission is the planet's atmosphere.

The direction of propagation obviously relates to the direction of the lines of force. Occultation of radiation by the planet itself cannot provide a narrow band source in any reasonable model. Sources appear and disappear as the lines of force come into appropriate geometrical relations to the sightline. Other factors, for example, even including external stimulus by the Galilean satellite Io, do not play a more fundamental role.

The range of directions into which radiation propagates must be narrow in some sense.

For example, suppose that a source emits at right angles to the local magnetic field at a point where the field parallels the rotation axis. The resulting radiation could be viewed over a large range of longitudes. As a way of coping with this problem, we can imagine the field inclined to the rotation axis. The sightline lies at right angles to the magnetic field at only two longitudes spaced 180° where emission can be viewed.

This model for the emission is suggested by some of the features of Io-controlled decametric emission (Davis, 1965 or Dulk, 1965; also see below). Emission controlled by Io propagates simultaneously into *two* directions spaced about 120 degrees in the ecliptic plane. Suppose that the line of force at the emission point is inclined 48 degrees to the rotation axis (itself inclined 3 degrees to the normal to the ecliptic plane). Suppose that this line of force passes through Io. Emission goes into a conical, cuff-shaped beam in space. The angle between the sightline and the line of force on which emission is assumed to lie is 66°, and is less than 25° from the direction at right angles to the magnetic field in the source.

The principal features of this model are that, on the one hand, only the line of force through Io (Goldreich and Lynden-Bell, 1969) is involved in emission, and on the other, that radiation goes into all azimuthal directions around the line of force and lies at a large angle from it. A further virtue of the model is that only a very small "point" source of emission is created, and this may well be required to satisfy decametric source size observations.

I have on other occasions emphasized the possible interpretive advantages of a model where emission originates along the lines of force. Only a pencil of radiation originates from a given point on the planet. Whether we record that radiation or not depends on whether the earth lies within the pencil after it leaves the vicinity of Jupiter. That two directions of emission are excited simultaneously is a result of the (postulated) great extent in longitude along which Io's influence on Jupiter's atmosphere extends at any moment. Whether the earth receives any radiation depends on whether Jupiter's aspect is arranged so that the lines of force in the source latitudes point at the earth or bear a simple relation to the earth.[3]

In other models, less detailed properties of the emission appear to be emphasized. In many models deviations of the field from dipolar are introduced *ad hoc*. At one time I considered constructing a model in which the fine structure of the field is so complicated that each point on the surface creates emission at a different characteristic frequency and direction. The datum would be the permanent dynamic spectrum; the problem of uniqueness of the resulting field configuration appears to me unwieldy. I rejected this model when it became clear that beaming properties of the radiation define directions no *more* precisely than to within a few degrees. This becomes a severe limitation on models, because it implies integration over the convolutions of the field covering a considerable area of the planet.

The same objection holds for the wide range that is implied by the model of an equatorially beamed radiation pattern around the foot of the line of force through Io. This line of force has to be singled out of the entire polar capful of lines of force as that one which emits toward the earth when Io lies on it. Lines of force in the general vicinity of this one also may be the Io line of force at other aspects of Jupiter. When the longitude relations are correct they should produce Io-related emission also and it will be observed on earth. To be more specific,

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3. *Such as being the reflection of the sightline. That is, radiation beamed inward along the lines of force may reflect off the planet, e.g., its ionosphere, clouds, or surface, enroute to the earth.*

consider a centered dipole model.[4] The family of lines of force that pass through Io at some time during Jupiter's rotation lies along small circles, one in the northern, the other in the southern hemisphere. Around each line of force we construct a "cuff" (a cone) flaring out to an angle of 66° from the line of force. The earth cuts across one of these cuffs at two points spaced 120° in longitude. It also, however, intersects every other cuff as well, in both hemispheres, north and south, unless there are very large non-dipolar perturbations of the field on a planetary scale. The small segment of almost any line of force near the atmosphere will lie in the plane of the sky at two longitudes of Jupiter separated by 180°. Only those very near the poles escape this condition. Io-related emission should then appear at all longitudes, but it does not. The emission processes appear to single out more sensitively than this the line of force that is favored at any moment. We therefore reconsider the basic implications of beaming.

If the direction of the line of force is involved then the pattern depends on how close to the field direction the emission actually lies. Only one line points toward the earth in the ecliptic plane from each hemisphere at any moment. This line of force stands in no particular relation to Io. Obviously north-south asymmetry plays a role, since emission from both hemisphere does not appear simultaneously.[5]

It is important to take into account the fraction of Jupiter's emission that does not correlate with Io's position, or any other satellite's. This lack of correlation maintains itself despite the fact that the Io-independent radiation exhibits very close dependence on rotation just as the Io-related emission does. Even the longitude profile of Io-independent radiation resembles the Io-dependent radiation, particularly the rapid increase of radiation probability on the early side of the main peak. This rapid increase has precisely the same longitude

4. *For simplicity; but any other field model can be similarly analyzed.*

5. *That is true in any case, for example, also in the case of equatorially beamed radiation.*

and the entire main peak has the same range in longitude, in both Io-connected and Io-independent emissions (Wilson, et al., 1968a).

This Io-independent "fifth source" differs from Io-related main source emission in the important sense that its frequency range is essentially restricted below about 28 MHz; Io-related emission in the main peak extends to about 34 MHz in the most favorable longitudes. Also, Io-independent fifth source emission does not occur at all above a very low frequency, 15 to 20 MHz, in the early source.

The fact that Io-independent emission occurs abundantly in the sharply defined main peak means that a wide range of longitudes in the magnetosphere contains phenomena like those due to Io itself. It is even a possibility that Io's disturbance excites the plasma within essentially 360° of longitude around Jupiter.

One natural basis for discussing the position of HF sources is to suppose that an L-shell of some particular value is singled out as the source for all of the radio emission we see in the ecliptic. Obviously Io's L-shell is highly eligible. It intersects the surface of Jupiter at zenomagnetic latitude 64°, in a planet-centered dipole; at that point on the surface, the line of force lies about 12° to the equatorial side of the zenith. The field therefore is pointed 52° above the ecliptic plane, plus or minus the inclination angles of the dipole axis and the rotation axis.

Consider a centered dipole model again. The portion of dipole lines of force that parallel the magnetic equatorial plane lie at latitude 35°, north or south. From the surface at this latitude the lines of force cross the equatorial plane at $1.49R_J$.

My conclusion is that the obvious candidate lines of force involved in the emission fail to qualify under geometrical scrutiny. There is nothing special about either the Io-L-shell, or the L-shell whose line of force at Jupiter's surface points toward the earth. Furthermore, the Io-independent emission suggests that even Io's longitude is not an essential component.

Under these circumstances, it is still useful to retain the idea that a special L-shell is involved, and to determine where it is by means of the highly specialized geometry shown in decametric emission. Since a pencil of radiation is involved, rather than a conical sheet, the direction parallel to the lines of force is really the only special direction defined by the field that is natural to the problem. This implies that for us to observe emission, the field must either be non-poloidal, or that the emission does not propagate directly out along the lines of force. The only surface field line that points to the earth in a dipole is an uninteresting one from the point of view of its L-shell; by distorting the field Jupiter may produce emission that comes from a more satisfying L-shell. This possibility is, however, severely limited by the observed purity of the Jupiter dipole.

We are left with just one paradoxical possibility; that the emission does not propagate out along the lines of force *even though* it is generated along them.

This combination of circumstances led me to conclude that the emission reflects from the surface of Jupiter, and that the dipole is not centered within the planet. I have shown that a detailed model built on this premise can be constructed with the L-shells between $2R_J$ and $3R_J$ as the active regions of the magnetosphere. I proposed the model before any asymmetry in decimetric emission had been discovered, and before Io's effects were known. It is important to understand the extent to which this model is today verified.

I showed above that a strong southward displacement of the dipole and a small displacement away from the axis of rotation, and toward the surface in radio longitude $\lambda_{III}(1957.0) \sim 200°$ is required to interpret the decimetric asymmetries. This is an exact description of the location I inferred to interpret decametric emission phenomena in the L-shell range $2R_J$ to $3R_J$, although my reasons were more restrictive.

The southward displacement of the dipole is a consequence of the slowly drifting frequency of emission in the early and main sources, coupled with the reflection condition. The emission arises from a region not far north of Jupiter's rotational equator.

The sense of polarization of the emission in the direction of its generation before reflection follows from the observations. Right-handed after reflection, the emission is left-handed before reflection.

The reflection itself undoubtedly occurs in a magneto-active medium; if the waves originate in just one of the two characteristic modes for the medium, it can be expected that after reflection, both characteristic modes are excited. That is, mode coupling occurs at reflection. Since the observed radiation is often, or usually, elliptically polarized there might seem to be a basis for expecting a Jupiter Faraday effect like that in the earth's ionosphere. The observations show that the Jupiter polarization ellipse rotates, but solely due to the terrestrial effect; observationally there is no "room" for a Jupiter rotation even remotely comparable to that in the earth's atmosphere. This suggests that the observed elliptical radiation is a base mode of the emission.

In fact, a Jupiter Faraday effect has been observed on one occasion and in a way that confirms both the existence of a reflection effect, and of elliptical base modes. This Faraday effect results from the interaction of elliptically polarized base modes. What happens is that the *sense* of rotation of the E-vector reverses as a function of frequency. The required base modes are elliptical, not orthogonal linear, because the one circular sense (left-handed) is never at its maxima, as intense as the other (right-handed) at *its* maxima.

This Faraday effect confirms the existence of a reflection, or at least a mode coupling similar to a reflection. In general, a very sharp reflecting layer reproduces in the reflected wave the mode of the incident wave. For Jupiter, normally only one mode exists, right-handed elliptical, and this suggests that the incident wave before experiencing clean reflection is left-handed elliptical propagating southward down along a northern hemisphere line of force. The implication is that the sense of rotation of the wave polarization vector in the generating region is right-handed elliptical in the direction away from Jupiter, and left-handed elliptical toward the planet. This implies a sense for the dipole, as we shall see in a moment.

But the Jupiter Faraday effect, peculiar because of its elliptical base modes, shows that two modes are present on this occasion. The natural way to assume they originate is in a reflection mode-coupling process. The values for ionospheric field strength and electron density derived by Gordon and Warwick (1967) represent values after reflection, but very near the source region.

The existence of elliptically polarized radiation implies with great persuasiveness that the emission frequency lies near the electron gyro-frequency. The reason is a result of magneto-ionic theory (Ratcliffe, 1959). Characteristic modes for almost all directions of propagation are circular. Within a very small angle off the perpendicular direction to the lines of force, they become linear. The thin transition region is where elliptical modes occur under most conditions for radio propagation. The crucial parameter in this discussion is Y, the quotient of the electron gyro-frequency divided by the wave frequency. When $Y \sim 1$, the modes are elliptical, with *slowly* varying axial ratio as a function of direction of propagation. If $Y \neq O(1)$, the modes are either circular, for almost all directions of propagation, or linear, in a sheet of directions orthogonal to the line of force. Even when the plasma density is very small, the wave modes are circular for almost all directions. These facts lead naturally to the introduction of the classical QL ("quasi-longitudinal", wave modes circularly polarized) and QT ("quasi-transverse", wave modes linearly polarized) regimes of magnetoionic theory.

The interpretation of Jupiter radio emission as lying near the gyro-frequency then comes about because of its characteristic *elliptical* (*not* circular) polarization. Only if $Y = O(1)$ will the set of directions for which the base modes are elliptical become for practical purposes a set of non-zero measure.

Furthermore, the Faraday effect in Jupiter's ionosphere is quite distinct in its phenomemology from the standard Faraday effect of the earth's ionosphere. To make this distinction sharper, I call the Jupiter effect the Y-one Faraday effect.

The existence of the Y-one Faraday effect shows that the radiation lies in two elliptical base modes on the (perhaps rare) occasions when the effect occurs. Then the reflection mechanism proposed in general for Jupiter's decametric emission appears to be confirmed observationally.

The sign of Jupiter's dipole moment follows from the sense of rotation of the characteristic mode *if* we know which mode the radiation lies in. All of the existing theories of decametric emission predict that it lies in the extraordinary mode. In this mode the electrons spiral in the same sense that the electric vector rotates. Probably it makes physical sense to speak of this mode as maximizing the interactions between wave and electrons. But there is only one weak observational verification of this hypothesis. It follows from the "unwinding" of the (QL) Faraday rotation observed on decametric emission. The major axis of the final ellipse probably, though with a large uncertainty, is orthogonal to the Jupiter field line at the point of emission (Parker, et al., 1969). At small plasma densities the extraordinary mode has the property that its major axis lies at right angles to the projected field direction. We conclude that physically and observationally the radiation is probably in the extraordinary mode.

Before reflection, extraordinary mode, left-handed radiation propagates southward toward Jupiter's dipole. The extraordinary mode away from the dipole, right-handed, northward, is right-handed. This implies that the line of force points northward away from the dipole. In other words, the magnetic pole in Jupiter's northern hemisphere is a north-seeking pole. The conventional sign for the pole in the northern hemisphere is positive in this case. This is an orientation opposite the dipole field of the earth (Warwick, 1963b).

It is perhaps important to observe than *any* theory for which extraordinary mode emission comes from Jupiter's northern hemisphere will predict this same result. The question that is therefore basic to the determination is whether that theory really establishes the zenomagnetic hemisphere of emission. Dowden (1963), for example, states that the northern hemispheric pole is north-seeking, in agreement with my conclusion. However, he equates the geometrical position of the right-handed emission to the northern hemispheric pole merely on the basis of their close (but not exact) longitude coincidence. This is not necessarily correct, although his conclusion is the same as mine.

There is one fragile decimetric observation that may confirm this conclusion from decametric observations. Berge (1966) believes to have seen a very small left-handed circularly polarized component at the clockwise cross-over. This implies that the equatorial field at $2R_J$ to $3R_J$ has a component that points away from us, that is, southward. This sense confirms the conclusion made from decametric data. A confirmation of the sense (but not the magnitude) of Berge's result has recently been carried out by Gulkis (1969, private communication).

The decametric emissions provide, in a sense, a magnified view of the shape and amplitude of Jupiter's surface field. They are complementary to the decimetric emissions, which inform us generally only about the low order magnetic moments of Jupiter. The conclusions from decametric emissions are that the field is essentially dipolar and oriented very much the way implied by decimetric data. They also establish that the sense of the dipole is parallel (rather than antiparallel) to Jupiter's rotation axis.

1.4 *The magnitude of the magnetic dipole moment*

The magnetic moment follows from decametric data with rather high precision. The magnetic co-latitude of emission is in the range about 45°, and the distance from the dipole is about $1.8R_J$. The resulting moment is $M = 1.3\times10^{31}$ gauss cm^3. This value depends sensitively on the details of where the dipole is located within Jupiter. On the basis of a detailed calculation of dynamic spectra, Warwick (1963a) concluded that $M = 4.2\times10^{30}$ gauss cm^3. The values for the position of the dipole for this value of M are tabulated in the Summary Table, Section 1.5.

To assign a valid uncertainty to this figure requires an external confirmation of the validity of the interpretations that underlie it. One may choose to regard the good agreement of the decimetric and decametric data on asymmetry as strong support for the interpretations. On the other hand, a critical test is to observe the north-south shifts of decimetric emission that seem to be implied not only in decametric emission, but also in the decimetric polarization asymmetry. These notably have *not* been confirmed by direct observation; indeed, the most accurate data in hand (Roberts and Ekers, 1965) suggest a symmetric north-south geometry. This is also true of Branson's (1968) data. In neither case is the claimed uncertainty in the measurements much less than the predicted north-south displacements. A measured uncertainty in the north-south displacement would then perhaps be close to the claimed uncertainty by Roberts and Ekers, $\pm0.3R_J$ about the center of Jupiter. Taking this much from my "measured" southerly displacement would leave a net southerly displacement of $0.44R_J$, just outside the observed southerly limits imposed by Roberts and Ekers. In the east-west direction they measured $0.1R_J$ displacement, essentially in agreement with decametric data. These results seem to suggest that the decametric uncertainties are such that the north-south position is $Z_N = 0.73R_J{}^{+0.4}_{-0.1}R_J$, and $X_N = +(0.15\pm0.1)R_J$, $Y_N = +(0.1\pm0.1)R_J$ with $X_N > Y_N$ in any event.

Jupiter's Magnetic Field

Property	Reference
Shape:	
Basically dipolar	General decimetric phenomena General decametric phenomena
plus	
Small axisymmetric quadrupole	Decimetric flux asymmetry in zenomagnetic latitude
Dipole moment magnitude:	
$\geq .01\text{x}10^{30}$ c.g.s.	Trapping of relativistic electrons resp. for decimetric phenomena.
$\leq 4\text{x}10^{30}$ c.g.s.	Estimated minimum lifetime, decimetric phenomena.
$\simeq 2\text{x}10^{30}$ c.g.s.	Equating electron gyrofrequency at surface to maximum observed decametric frequency.
$= (4.2 \pm 0.4) \times 10^{30}$ gauss cm^3	Detailed decametric observations and theory.
Dipole orientation:	
Tilt to rotation axis $7\overset{\circ}{.}7 \pm 0\overset{\circ}{.}1$	Rate of change of decimetric total polarization position angle at counter-clockwise cross-over.
Longitude of Jupiter meridian plane parallel to dipole axis:	
$\lambda_{III}(1957.0) = 202° \pm 2°$	1. Time of maximum position angle of decimetric polarization.
on 1 July 1968.	2. Symmetry position of decametric frequency drifts.
Longitude of Jupiter meridian plane physically containing dipole:	
$\lambda_{III}(1957.0) = 232°^{+20°}_{-10°}$	1. Shape of decametric dynamic spectrum. 2. Difference in shapes of decimetric curves of polarization position angle, at eastern and western elongation. 3. East-west asymmetry of decimetric emissions at $\lambda_{III}(1957.0) \sim 135°$.

Property	Reference
Distance from center of Jupiter.	
$R_o = 0.75R\{^{+0.1R_J}_{-0.4R_J}$	1. Decametric observation and theory.
Distance from axis:	
$= (0.18 \pm 0.1)R_J$	2. Decimetric asymmetry in position angle of polarization at the two cross-overs.
Zenocentric latitude of dipole:	
$= -77° \, ^{+49°}_{-01°}$(south)	3. Pencil beam and occultation observations of decimetric source location.
Sense of dipole:	
Tip of dipole in northern hemisphere of Jupiter is north-seeking.	1. Decametric observation and theory. 2. Decimetric circular polarization at clockwise cross-over.
Quadrupole moment magnitude:	
$= (0.06 \pm 0.01) \times$ dipole moment $\times 1R_J$	Decimetric flux asymmetry in zenomagnetic latitude.
Sense of Quadrupole:	
a. Strengthens northern zenographic dipole field. b. Displaces minimum field strength northward of magnetic equator at all longitudes.	Decimetric flux symmetry is centered at a northern zenomagnetic latitude.

2. PARTICLES

2.1 *Relativistic electrons*

For electrons at relativistic energies, the particle fluxes at Jupiter are firmly based on observations. The general formulas for synchrotron radiation appear to provide good interpretation of decimetric emissions from wavelengths of a few centimeters out to a few meters.

An exceedingly complicated summation over a vast array of parameters is involved in the interpretation of these data, especially the total flux observations. There is a three dimensional volume integration over which field strength and orientation varies widely; integration over different electron energies; and integrations over electron pitch angle and L-shell distributions. All of these parameters must be put together in computations separately worked out for each of a large range of dipole tilt angles, and probably also for different amounts of vignetting by the planet. In addition, the most detailed observations may provide information concerning departures of the magnetic field from dipole symmetry. There will then be required introduction of the distribution over the flux invariant, as well as over the L-shell and magnetic moment invariants.

All of this leaves detailed interpretations at the present time less desirable than semi-quantitative overviews of the data where they are sufficiently well-defined to establish one or more parameters in isolation from the remainder.

The basis today for interpreting decimetric fluxes is synchrotron emission theory. This is classical, extending back nearly to the origins of special relativity. In radio astronomy, interpretations of broad-band linearly-polarized emissions usually are given in terms of synchrotron theory (very often called "magnetic bremsstrahlung" in the literature).

The modern analysis of synchrotron emission, viz, its spectrum, polarization, and intensity have recently been re-scrutinized. Much of the work within the last two decades was based on a Fourier-series expansion of the time variations of the classical expression for the E-vector (for example) in terms of $\omega_B E_o/E_1$, the relativistic gyro-frequency in a field B gauss, of a particle whose energy is E_1 including the rest energy E_o. Here $\omega_B = eB/mc$ where e, m, c are electron charge (4.80×10^{-10} e.s.u.), mass (9×10^{-28} grams), and the velocity of light (3×10^{10} cm sec^{-1}), respectively. There appear to be some serious questions about both the derivations of the fundamental results of the theory (although possibly its results are correct) in terms of this basic frequency and also its generality. Currently under examination are the assumption that the observation point is at infinity, and that there is physical doppler shift involved. Further problems that are of deep concern derive from the plasma surrounding the emitting electrons. In the final analysis, the plasma of these electrons, itself, and the existence of collective effects among the electrons is involved.

All of this seems to be less important than an overview of the fundamentals of synchrotron emission to provide semi-quantitative formulas to permit easy analysis of the data. I have derived such a set of formulas (1963) and will now present them briefly.

The first formula concerns the total emitted power. Classically, the total emitted power by a charge accelerated $\ddot{x}$ cm sec^{-2} is

$$\frac{dW}{dt} = \frac{2}{3} \frac{e^2}{c^3} (\ddot{x})^2 .$$

Now, classically still, $$\frac{dW}{dt} = \frac{2}{3} \frac{e^2}{m^2 c^3} p^2 \omega_B{}^2 \sin^2\alpha$$

where p is electron momentum and α is the angle between its velocity vector and the field direction. To modify this expression to represent the case of high-energy electrons, we substitute for p^2, $p^2 = (E_1{}^2 - E_o{}^2)/c^2$, the expression for the square of the relativistic momentum. Note that we do not substitute relativistic expressions for ω_B, nor for t. There results

$$\frac{dW}{dt} = \frac{2}{3}\frac{e^2}{m^2c^3}\frac{\omega_B^2}{c^2}\sin^2\alpha\,(E_1^2 - E_o^2);$$

if we let $E_1 = E_o + E$, we obtain $(E_1^2 - E_o^2) = 2E\,E_o\,(1 + E/2E_o)$,

and $\frac{dW}{dt}$ (synchrotron) = $\frac{dW}{dt}$ (classical) $(1 + E/2E_o)$.

Numerically the classical (non-relativistic) radiation from a gyrating electron is

$$\frac{dW}{dt}(\text{classical}) = 4\text{x}10^{-9}\,E\,B^2\,\sin^2\alpha\ ,$$

and finally $\frac{dW}{dt}$(synchrotron) = $4\text{x}10^{-9}B^2\,\sin^2\alpha\,E(1 + \frac{E}{2E_o})$ (E -units)/sec.

The radiation increases quadratically above the classical limit at $E \sim E_o$.

The bandwidth of the emission follows from the narrow beaming we observe for a single electron. Moving at v, a frame which to the electron appears orthogonal to the direction of motion appears to us to be folded down around the direction v. The angle between axes is about $\Delta\theta = E_o/E$ radians. The spectrum follows from the time it takes this beam to sweep across the sightline. It might seem that the correct angular velocity for this beam would be the reduced gyro-frequency $\omega_B(\sin\alpha)E_o/E$. A simple computation shows that at this rate the beam sweeps through the sightline in just $(\omega_B \sin\alpha)^{-1}$ seconds, independently of the particle energy. The correct answer follows from computing with the accelerated gyro-frequency $\omega_B(\sin\alpha)E/E_o$. This is the proper procedure physically because of the apparent time contraction in the moving system. (The reduced gyro-frequency is required in problems of dynamics, where the time for the entire orbit around the line of force is involved). The observed pulse width is $\Delta t = \Delta\theta/\ (\omega_B[\sin\alpha]E/E_o) = (E_o/E)^2/(\omega_B \sin\alpha)$.

We assume the pulse to be Gaussian with the e-fold whole-width, Δt. Its Fourier transform is Gaussian, with maximum at zero frequency and e-fold point at $\omega_e = 4/\Delta t = 4\omega_B(\sin\alpha)(E/E_o)^2$.

The maximum intensity does not occur at zero frequency as this model implies. Rather the spectrum drops abruptly to zero beginning at and below a frequency much less than ω_e. Physically, this occurs because of the fine structure of the beam of emission. Actually this "beam" describes the momentary occurrence at our observatory of an electric field. This electric field varies in an unsymmetric way as a function of time through the pulse (see Westfold, 1959; equation [17]). We previously described the pulse as a (symmetric) Gaussian. If the asymmetry has an amplitude equal to roughly the pulse height, and if the field varies only once through the pulse, then there will be a zero at zero frequency. The peak frequency lies at a very small, but fixed fraction of the bandwidth. Numerically, the bandwidth is $\Delta f = 10(E/E_o)^2\, B\sin\alpha$ MHz, and the spectral peak frequency is $f = 0.1\Delta f$. The emission lies in a cone of width $\Delta\theta = E_o/E$ radians, and creates a total power into all directions,

$$\frac{dW}{dt} = 4\times10^{-9}\, E\left(1 + \frac{E}{2E_o}\right)B^2 \sin^2\alpha \qquad \text{(E-units)/sec.}$$

This last formula is ostensibly complete. The others are approximate, but sufficiently accurate for any realistic calculation in the relativistic limit $E \sim E_1 \gg E_o$.

One often reads in the literature that the spectrum of synchrotron emission consists of a series of spikes closely spaced throughout the range defined by Δf. Fourier analysis of a sequence of identical pulses yields this result rather than a continuous spectrum, if the emitting electron moves strictly periodically. This point of view is not necessary to derive the spectrum and tends to cloud the physical description. In many cases, where the source lies at finite distance,or when the field varies appreciably along the electron's orbit, successive pulses will not be identical. Since very high harmonics are involved, the criterion for periodicity is correspondingly sharp if the successive harmonics are not to mix incoherently.

If the base frequency is $\omega_B E_o/E$ (the relativistic gyro-frequency) then harmonics of order $4\omega_B \sin\alpha(E/E_o)^2/(\omega_B E_o/E) = 4(E/E_o)^3 \sin\alpha$ are involved. A change in magnetic field strength by $(1/[4 \sin\alpha])(E_o/E)^3$ in one orbit destroys coherence essential to the origin of individual harmonics. With the required electron energies at Jupiter, it is possible to show that individual harmonics are still meaningful.

If an estimate can be made of the intensity of emission from a limited portion of the Jupiter source, the inferred particles and fields will surely be more accurate than inferences based on total flux. To evaluate the numbers quoted in the literature requires a sorting out proceeding along these lines. Before then, however, we need formulas for the intensity. These formulas require assumptions on the directional distribution of emitting electrons.

Suppose that the distribution of electron velocity is isotropic. The individual beamwidth of a given electron is essentially $\Delta\theta = E_o/E$. The total solid angle into which electrons moving at angle α_1 to the lines of force emit is $2\pi \sin\alpha_1 \, d\alpha_1 = 2\pi \sin\alpha_1 \, \Delta\theta$. The number of electrons moving within this range also is proportional to this solid angle. The power emitted into this range is proportional to the number of electrons moving per unit solid angle, times the solid angle; the intensity, which is the power per unit solid angle, is just the number of electrons moving per unit solid angle times the total power emitted per electron moving at α_1. That is, for an isotropic distribution of velocities, the intensity at α to the lines of force is

$$\frac{I_o}{N} = \frac{1}{4\pi}\frac{dW}{dt} = 3.2\text{x}10^{-10}\ E(1 + \frac{E}{2E_o})B^2 \sin^2\alpha$$

per electron in E-units sr^{-1} sec^{-1}. Since this formula assumes that a given electron emits into a range of directions $\Delta\theta$ much smaller than the range of directions containing the electron velocity vectors, it applies only when $E >> E_o$, or

$$\frac{I_o}{N} = 1\text{x}10^{-10} E_o \left(\frac{E}{E_o}\right)^2 B^2 \sin^2 \alpha$$

E-units sr^{-1} sec^{-1}, per electron of an isotropic distribution, consisting of N electrons cm^{-3} or $N/4\pi$ electrons cm^{-3} sr^{-1}. If α is translated into $\phi = 90° - \alpha$, then $I \propto \cos^2 \phi$. This should not be compared with the function giving the flux variation as an observed function of zenomagnetic latitude.

The opposite extreme type of distribution is when electron velocities lie entirely at a certain pitch angle α_1. In this case the number of electrons, $N(\alpha_1)$, moving within a great circle arc $\Delta\theta$ tangent to a small circle at α_1 is $N(\alpha_1)\, \Delta\theta / (2\pi \sin \alpha_1)$ [for α_1 such that $N(\alpha_1) > 1$]. Each of these electrons emits into a solid angle $(\Delta\theta)^2$ steradians. Because we observe emission only when $\alpha_1 \sim \alpha$

$$\frac{I_\alpha}{N(\alpha)} = \frac{1}{2\pi \sin \alpha\, \Delta\theta} \frac{dW}{dt} = 6.4\text{x}10^{-10}\, E \left(1 + \frac{E}{2E_o}\right) \left(\frac{E}{E_o}\right) B^2 \sin \alpha$$

per electron in E-units sr^{-1} sec^{-1}. Again, reducing this formula for $E >> E_o$,

$$\frac{I_\alpha}{N(\alpha)} = 3.2\text{x}10^{-10}\, E \left(\frac{E}{E_o}\right)^2 B^2 \sin \alpha$$

per electron in E-units sr^{-1} sec^{-1} where the formula applies to the total number of electrons moving at α_1. This expression reduces to the one giving I_o when $N(\alpha_1) = \frac{1}{4\pi}\, 2\pi (\sin \alpha_1)(\Delta\theta) N$. Summarizing, then, for N electrons cm^{-3}, moving isotropically at $N/4\pi$ electrons cm^{-3} sr^{-1}, the observed intensity will be

$$I_o = \frac{N}{4\pi} \frac{dW}{dt} = 1.3\text{x}10^{-16}\, N \left(\frac{E}{E_o}\right)^2 B^2 \sin^2 \alpha$$

ergs sec^{-1} sr^{-1} cm^{-3}. For $N(\alpha)$ electrons cm^{-3}, moving uniformly at all azimuthal angles around the zone at α, the observed intensity will be

$$I_\alpha = \frac{N(\alpha)}{2\pi \sin \alpha \, \Delta\theta} \frac{dW}{dt} = 2.6\text{x}10^{-16} \, N(\alpha) \left(\frac{E}{E_o}\right)^3 B^2 \sin \alpha$$

ergs sec^{-1} sr^{-1} cm^{3}. The intensity per unit frequency interval comes from the quotient of these expressions Δf. That is

$$I_o(f) = 1.3\text{x}10^{-23} \, NB \sin \alpha \quad \text{ergs cm}^{-3} \text{ sec}^{-1} \text{ sr}^{-1} \text{ Hz}^{-1}$$

for an isotropic distribution of total density N electrons cm^{-3}, and

$$I_\alpha(f) = 2.6\text{x}10^{-23} \, N(\alpha) \left(\frac{E}{E_o}\right) B \quad \text{ergs cm}^{-3} \text{ sec}^{-1} \text{ Hz}^{-1},$$

for a distribution of $N(\alpha)$, the total number of electrons moving at α to the lines of force. For intermediate distributions, say $N_G(\alpha_1)$ per steradian, the formula $I_\alpha(f)$ suffices when by $N(\alpha)$ is understood the number of electrons moving within a range $\Delta\theta$ about α. That is,

$$N(\alpha) = 2\pi \int_{\Delta\theta} N_G(\alpha_1) \sin \alpha_1 d\,\alpha_1 \,.$$

To use these intensity formulas depends on a knowledge of the detailed two dimensional brightness distribution across Jupiter, ideally for all frequencies over more than the synchrotron bandwidth ten to one. Data on this rich a scale do not exist, and only hints suggest to us the probable answers.

The earliest hint came from the observations of total flux at various wavelengths. The suggestion appears quite reasonable that the flux density (e.g., the intensity integrated over the entire source) is nearly constant in frequency between a value of about 400 to 4,000 MHz, at $S = 7\text{x}10^{-26}$ W m^{-2} Hz^{-1}. 400 to 500 MHz represents a nominal peak frequency; by 4,000 to 5,000 MHz the non-thermal emission begins to fall off in the face of thermal disk radiation (Dickel, et al. 1970).

In early literature, this flat flux is used to indicate that the spectrum of particles is flat, much more like E^{-1} than the E^{-4} or E^{-5} values of the Earth's belts. This conclusion does not appear to be essential in the face of the observations that the outer regions of the belts appear relatively more prominently at low frequencies than at high (Gulkis, 1970; Branson, 1968; McAdam, 1966). Apparently a better hypothesis is that to each L-shell there corresponds a particular relativistic energy, which increases as L decreases. This strongly suggests that the electrons receive their energies through L-shell crossing (see Hess, 1968), and that the flatness of the spectrum results from the changing mix of the energy-field-particle distributions rather than a locally-flat spectrum. In any case, the spectrum of a monoenergetic flux is so broad that considerable smoothing from all parts of the particle spectrum results.

The size of the source depends basically on the pitch angle distribution and the L-shell distribution. The only way that inferences on pitch-angle distributions have been made is via polarization, on the one hand, and the variation of total flux with zenomagnetic latitude on the other. The fact that the source is narrow, north-south, implies a limited range of pitch angles, say $\sin \alpha_L \geq 0.6$. With this flat pitch angle distribution, any L-shell distribution will tend to be polarized, E-vector parallel to the equator, as is observed. This range of α_L values is also consistent with the variation of flux with latitude. As we shall see, radiation beaming probably is characteristically only a few degrees in total width. However, the flux density falls rapidly over 10 or 15 degrees of latitude. A much greater range of latitudes is covered than can be reached by a single electron beaming over a wide angular range. The consequence is that we essentially observe the electrons mirroring at a latitude given by the geometry shown in Figure 1.

The fall off of intensity with latitude indicates the mirror point distribution, and therefore the pitch angle distribution. The mirror point latitude ϕ_L, <<1, follows from setting the mirror point field strength,

$B_m = B_o/\sin^2 \alpha_L$. Since for $\alpha_L \sim \pi/2$,

$$\frac{B_o}{B_m} = (1 + \frac{9}{2}\phi_L{}^2)^{-1} = \sin^2 \alpha_L \sim 1 - (\frac{\pi}{2} - \alpha_L)^2,$$

there results

$$\phi_L = \frac{\sqrt{2}}{3}(\frac{\pi}{2} - \alpha_L).$$

Figure 1 shows that the electrons observed at right angles to the line of force (upper part of Figure 1) lie at a mirror point latitude $R_m \sin \phi/3R_m$ (where $\phi = 10°$ in Figure 1). Since ϕ, the inclination angle between the dipole and the plane of the sky, is <<1 radian, it appears that $\phi_L \sim \phi/3$. ϕ is also the zenomagnetic latitude of the observation. Therefore $\phi = \sqrt{2}[(\pi/2) - \alpha_L]$ is the zenomagnetic latitude at which we observe electrons with pitch angles that are α_L in the equatorial plane. Writing the intensity as $\cos^n \phi d\phi = \cos^n \sqrt{2}([\pi/2] - \alpha_L)\, d\, \alpha_L$, I find the variation proportional to $[1 - n\,([\pi/2] - \alpha_L)^2]$. By extrapolation it appears that when $([\pi/2] - \alpha_L) \sim \frac{1}{n}$, the distribution changes character. With the observed $n = 3.5$, $(\pi/2) - \alpha_L \sim 0.53$. A pitch-angle distribution limited to about $\sin \alpha_L \geq 0.87$ will cut off in about the same way the observations do. A satisfactory model of the pitch-angle distribution then should be $\exp[-n\,([\pi/2] - \alpha_L)^2]$, where e-fold points of the distribution lie at $([\pi/2] - \alpha_L) = \pm 0.53$, or within a range $\Delta\alpha_L = 61°$, centered at 90°.

Considerable simplification results when this model is applied to the data. For example, Branson's 21-cm brightness distributions (1968), allow an estimate of the intensity at the very center of the synchrotron source. I read a brightness of 433° K for the emission at the center at elongation. The values at the east and west peaks of the symmetrical sources are 564° K respectively. Branson takes 250° K of brightness temperature for the disk, which leaves 183° K for the equatorial portion of the belts. This temperature parameter converts to an intensity via the Rayleigh-Jeans law, viz., $I_{\alpha}(f) = 2\ k\ T\ f^2/c^2$, where $k = 1.38 \times 10^{-16}$ ergs per degree. The result of combining Branson's observation with the formula on page 58 is

$$(2\ k\ f^2/c^2)\ 183°\ K = 2.6 \times 10^{-23}\ L\ N(0)(\frac{E}{E_o})\ B.$$

The value L stands for the path length in centimeters; $N(0)$, $(\frac{E}{E_o})$, and B are also all unknowns so far as this particular datum is concerned.

The combination of four factors, N(0), E, B, and L is involved, and other relations are required to separate them. For example, Branson's data show that each side of the source extends laterally over about $2R_J$; I assume this measures the radial thickness at the disk center. Therefore, $L = 2R_J$. (E/E_o) occurs in the formula for peak frequency, or for bandwidth. If $\Delta f = 3600$ MHz, then this relation permits us to eliminate either E or B from the computations, but does not identify uniquely N(0).

At this point in the analysis, most computations of Jupiter's synchrotron radiation either assume a field value for the belts, or compute for a range of field values that appear to satisfy other apparent conditions of the problem. These might be lifetimes for radiating electrons, which, as we saw, lead to upper limits of the field, or to minimum energy trapping fields, which lead to very low lower limits. The value of the field enters the problem relatively insensitively, since $E \propto B^{-1/2}$. Estimates of field strength derived from decametric emission at the planet seem valid indications of the magnetic moment, perhaps, as I have indicated, the most accurate of all. For these reasons, the appropriate procedure appears to be to use $M = 4.2 \times 10^{30}$ gauss cm^3 to compute the field in the peak of Branson's observed belt distribution. $R = 1.5R_J$ in his measures, which corresponds to a field of 3.4 gauss. As a result of the uncertainty in the E - W location of the maximum position, this figure may vary; 3.4 ±2 gauss is probably a good range, where the assigned values are really cut-offs, not probable errors. As a consequence of the bandwidth equations, we learn that 3600 MHz = $10(E/E_o)^2$ B where E = 4.2 MeV or 8.2 MeV for, respectively, the upper and lower field strengths quoted a moment ago. I emphasize that these values are larger in gauss, and lower in energy than former estimates, because Branson's brightness curves peak relatively close to the disk. Berge's values (1966) at 10.5 cm lie at $R = 1.8R_J$. Correspondingly, the field strength is 2.0 gauss.

Substituting in the intensity formula I find $N(0) = 7\text{x}10^{-6}\ \text{cm}^{-3}$ ($B = 5.4$ gauss, $E = 4.2$ MeV) or $N(0) = 1.4\text{x}10^{-5}\ \text{cm}^{-3}$ ($B = 1.4$ gauss; $E = 8.2$ MeV). The most likely values appear to be at $R = 1.8R_J$; $B_o = 2.0$ gauss; $E = 6.2$ MeV; $N(0) = 1.3\text{x}10^{-5}\ \text{cm}^{-3}$ where $N(0)$ is the total density in the equatorial zone of width $E_o/E = 0.082$ radians = 4.8 degrees. This value of $N(0)$ converts to either an equatorial ($\alpha = 90°$) density of $dN/d\Omega = 2.5\text{x}10^{-5}\ \text{cm}^{-3}\ \text{sr}^{-1}$ distributed over a zone $\pm 61°$, or an omnidirectional density $\int (dN/d\Omega) d\Omega = 1.6\text{x}10^{-4}\ \text{cm}^{-3}$. The corresponding fluxes at $1.8R_J$ are $dF/d\Omega = 8\text{x}10^{5}\ \text{cm}^{-2}\ \text{sec}^{-1}$ or $\int (dF/d\Omega) = 4.8\text{x}10^{6}\ \text{cm}^{-2}\ \text{sec}^{-1}$ omnidirectional. The summary table tabulates these values and as well, a set of values based on an L-shell distribution formula presented on page 66.

As a check on these data, note that at 10.4 cm Berge's (1966) central intensity is 80° K blackbody equivalent, after subtraction of a disk component of 260° K. This value increases the product $LN(0)(E/E_o)$ by a factor $(21\ \text{cm}/10.4\ \text{cm})^2\ (80°/183°) = 1.78$, which is satisfactorily close to unity. Furthermore, Berge's peak brightness corresponds to 160° K, e.g., 2.0 x the central brightness; in Branson's data, the corresponding ratio is (564°/183°) = 3.1 x the central brightness.

The distribution of these electrons in L-shell is assumed to be uniform in the shell of thickness $2R_J$, extending from $R \simeq 1.2R_J$ out to $3.2R_J$. With this thickness, the ratio of peak to central thickness (= brightness) is $6.4R_J\ [1 - (1.2/3.2)^2]^{1/2}/2.0R_J = 3.0$, about the right value.

If we suppose that this is a correct description of the 10 cm and 20 cm source, the question arises how to fit the longer wavelength data into the picture. Clearly resolved data taken at as low a frequency as possible are necessary. As mentioned above these seem to be McAdam's (1966), Branson's (1968), and Gulkis' (1970). McAdam states his result in terms of a two-belt distribution, the first very much like the above, but the second lying at 6 radii and producing 16 - 20 percent of the flux at 408 MHz (74 cm). Branson plots his 21 cm data for strip brightness on the same graphs as his 75 cm data. In outer parts of the belts, from about $3R_J$ on, the strip brightness falls about 10 per cent lower at 21 cm than at 75 cm. Gulkis presents his data much the same way, relative to Branson's 21 cm distributions; Gulkis shows emission far outside of Branson's 21 cm curve, both at 74 cm and 128 cm. These data suggest a more dramatic contrast with the high frequency data than do Branson's results.

McAdam's result essentially requires sources of one flux unit (10^{-26} W m^{-2} Hz^{-1}) at $6R_J$ either side of the planet. These sources may be as large as one or two R_J. I assume that they subtend an angle the size of the disk of Jupiter, $3x10^{-8}$ sr, with a flux density of $1x10^{-23}$ ergs cm^2 sec^{-1} Hz^{-1}. The intensity is then $3x10^{-16}$ ergs cm^{-2} sec^{-1} Hz^{-1} sr^{-1}, on either side of the planet. Now we have $3x10^{-16} = 2.6x10^{-23}$ L $N(0)$ (E/E_o) B, the same formula as before, but with new values for all of the parameters. The most difficult to estimate proves to be the path length L which depends on the model for the radial distribution of electrons. A thin shell, of outer and inner radii P_2 R_J and P_1 R_J respectively, has maximum path length at P_1 R_J of

$$2 \sqrt{1 - \left(\frac{P_1}{P_2} \right)^2} \; P_2 \, R_J .$$

Outwards the path length falls to zero within distance $(P_2 - P_1)$ R_J. Inward it falls with radial distance P R_J as

$$2 \left(\sqrt{P_2{}^2 - P_1{}^2} - \sqrt{P_1{}^2 - P^2} \right) R_J .$$

It has fallen to half its peak value at P_o where

$$\sqrt{P_2^{\,2} - P_1^{\,2}} = 2 \left\{ \sqrt{P_2^{\,2} - P_o^{\,2}} - \sqrt{P_1^{\,2} - P_o^{\,2}} \right\} .$$

Representative solutions are $P_1 = 6$, $P_2 = 7$, $P_o = 5.25$ and $P_1 = 6$, $P_2 = 6.84$, $P_o = 5.80$, for example. The thickness must be less than one R_J in order that the size appear to be only $1\ R_J$. Take, therefore, $(P_2 - P_1) = R_J/2$ with $P_1 = 6\ R_J$; the maximum thickness

$$2\sqrt{1 - \left(\frac{P_1}{P_2}\right)^2}\ P_2 R_J = 5\ R_J .$$

(E/E_o) follows if we know the peak frequency or the bandwidth of the outer parts of the belts; this is certainly somewhat arbitrary. However, it appears that they do not appear above 1000 MHz; a bandwidth of 600 MHz seems appropriate. At $6\ R_J$ the field is $B = 0.054$ gauss. Then the electron energy is 17 MeV. This gives $N(0) = 1.9 \times 10^{-4}\ cm^{-3}$. It seems that McAdam's results require higher energy electrons than the inner belts, more of them cm^{-3}, and (of course) more in total as well.

The result depends on a very "hard" fact, that the field strength between the inner and outer belts drops as inverse cube of the distance; neither the detailed model of field asymmetry, nor the value of the magnetic moment is involved. The mere detection of these outer belts at $6\ R_J$ implies strongly that greater numbers of more energetic electrons are present there than in the inner belts. Gulkis' result is of the same nature as McAdam's and in fact may require an even greater particle contrast with respect to the inner belts.

On the other hand, Branson's data are much subtler in defining these differences. Roughly speaking, he finds an outward displacement of about $0.2\ R_J$ and between 21 cm and 75 cm results with about the same brightness in each case. Taking the same bandwidths as before, i.e., 600 MHz; in a belt at R = 2.0 R_J, of L = 2 R_J, produces the values B ≈ 1.5 gauss, E = 3.3 MeV, N(0) = 3.4×10^{-5}. The energy per particle, and the energy density, are less than in the inner belt. The main contrast is, of course, in the particle energy, which is conditioned by the contrast in the observed frequency range.

The crucial distinction among these results is the measured distance from the center of the planet in each case. The detection of belts at large distances may in fact have been achieved; that this is a possibility needs to be considered in the design of experiments to be flown to Jupiter.

It seems to me probable, nevertheless, that outlying belts have not been detected. My reasons are several and essentially philosophical:

(a) Branson's data show only a relatively weak extension at 75 cm; the obviously high quality and experienced techniques of the Cambridge group require that we weight their data heavily.

(b) An equipment with limited resolving power tends to make small things seem larger, never to make larger things seem smaller. Purely internal measures of precision tend always to make a source near the actual resolution limit of an experiment seem larger than it is, and to weight the results more heavily than it merits.

The evidence for the L-shell distribution of relativistic electrons is very limited. Since for many reasons some further statements regarding this point may be useful it is necessary to turn toward theory and what is known about the earth's belts to suggest what the situation at Jupiter may be. The earth's electron belts at relativistic electron energies are fortunately for this purpose more stable than at lower energies. Furthermore, the terrestrial electrons are more stable in nearby parts of the magnetosphere than at large distances from the earth. Since the relativistic electrons at Jupiter which are "observed", are near the planet with respect to estimates of Jupiter's magnetopause and appear to be quite stable, it seems reasonable to consider the steady-state quiet-time inward diffusion of electrons through Jupiter's magnetosphere as their source. The solution to this problem posed in terms of a Fokker-Planck equation as given for the earth's magnetosphere by Davis and Chang (1962) seems appropriate under the circumstances: the electron density N(R) at distance R (where R_o and R_1 are the inner and outer limits, respectively, of the region of diffusion [Hess, 1968, equation 6.64]) is given by

$$\frac{N(R)}{N(R_1)} = K\left(\frac{R_1}{R}\right)^{11}\left\{(R/R_o)^7 - 1\right\} .$$

Here $K = [(R_1/R_o)^7 - 1]^{-1}$, a constant; also, in the formula quoted by Hess, g = 8, and a = 9. This equation can be regarded, if one wishes, as an interpolation formula between the magnetopause and the earth. It also connects observations of relativisitc electrons at or near the inner boundary of Jupiter's magnetosphere, with the properties of the sources of these electrons at the magnetopause. Inasmuch as R_1 and R_o are estimable parameters, while N(R) is observed, this provides a rational way to connect our estimates of the magnetic field of Jupiter to the source density $N(R_1)$. This is presumably some small fraction of the solar wind density at the magnetopause. Furthermore, the energy acquired by these electrons, as they diffuse inward to R, is given by $E(R) = (R_1/R)^3(R_1)$.

The most important property of the Fokker-Planck solution given above is that N(R) maximizes at $R = 1.16\ R_o$, and falls rapidly, as $(R_1/R)^4 N(R_1)$, outwards to its value $N(R_1)$. From the "best" estimate of Jupiter magnetic moment, 4.2×10^{30} c.g.s., and the strength of the solar wind at the orbit of Jupiter, 0.2 protons cm^{-3} moving at 400 km sec^{-1}, I find Jupiter's magnetopause at about $59R_J$. Its stand-off shock may be larger; in any case take $R_1 = 59R_J = 4.2\times10^{11}$ cm. Now we need an estimate of the inner boundary of the diffusing region. I take, somewhat arbitrarily, but based in part on the $0.2R_J$ displacement of Jupiter's magnetic field from the rotation axis, as well as the possibly observed maximum of these electrons the value as $R_o = 1.6R_J$. Take $E(R) = 6.2$ MeV for $R = 1.8R_J$. Then, $E(R_1) = 6.2 \text{ MeV} \times (1.8/59)^3 = 177$ eV. In the earth's belts, this scaling law for electrons is empirically a quadratic law instead; applied to Jupiter this yields $E(R_1) = 6.2 \text{ MeV} \times (1.8/59)^2 = 6670$ eV, which is a greater energy than carried by solar wind protons; perhaps the L-shell diffusion model for Jupiter electrons works better than it does for the earth. N(R) is a maximum at $R = 1.1555\ R_o = 1.8R_J$. Also, $K = 1.1\times10^{-11}$. If the formulae for the intensity (page 58) are replaced by appropriate integrals over an electron distribution of the shape just discussed, the peak electron density required to match Branson's (1968) synchrotron brightness temperature (183° K) above the disk is $N(R) = 6.3\times10^{-4}\ \text{cm}^{-3}$ at $R = 1.8R_J$. The distribution then predicts $N(R_1) = 7\times10^{-10}\ \text{cm}^{-3}$ at $R_1 = 59R_J$, which suggests that a fraction about 10^{-9} of the total number density of solar wind electrons incident on Jupiter's magnetosphere are trapped and accelerated (see below, for a comparison with the earth). The density of energetic electrons therefore is $N(R) = 2.2\times10^{-3}R^{-4}$, where R is in units of the radius of Jupiter, and $1.8R_J \le R \le 59R_J$. The maximum of the distribution is at $R = 1.8R_J$; N(R) falls rapidly to zero, at $R = 1.6R_J$. The suggested typical electron energy distribution in radius is $E(R) = 3.6\times10^7\ R^{-3}$ eV where R is again expressed in units of Jupiter's radius, and $R \ge 1.8R_J$. I shall not attempt an estimate of the spectrum at this time.

The pitch-angle distribution suggests a crude estimate of the distribution of electrons in latitude. The cut-off latitude is about 45°. It is important to know how this value changes with L-shell, but no relevant data apparently exist. The average height of a cylindrical shell equivalent to this mirror latitude is $4.2R_J$, between an inner radius of $1.8R_J$ and an outer radius of $3.8R_J$. There may be computational advantages to representing this distribution analytically. For example, this could be the exponential, $\exp[-(\phi/\phi_o)^2]$, where ϕ_o = "cutoff" latitude = 45°.

2.2 *Low energy electrons*

Low-energy components of Jupiter's belts, the analogue of the keV electrons in the earth's belts, like Jupiter's MeV electrons, have not been observed there. Several theories suggest the existence of these particles, and there is modest indirect observational support for them.

The most direct (of this essentially indirect) evidence for keV electrons is the existence of drifting millisecond bursts in the decametric frequency range. This possibility appears in work by Ellis (1965). The phenomenon is the presence of decametric fine structure, lasting no more than a few milliseconds within the passband of a receiver operating at a fixed frequency. Ellis predicted that as electrons spiral through Jupiter's magnetic field, they emit the doppler-shifted cyclotron frequency appropriate to their position along the lines of force, which changes as a result of their motion. The expected kilovolt electrons move with speeds within an order of magnitude or so of the velocity of light, that is, $v \sim 0.1c$. In the radial part of a dipole field, the radial logrithmic derivative of the field is $(1/B)(dN/dR) = -3/R$. If an electron moves outwards along this field at speed $v_{\parallel}$, then the time-rate of change of the field is $(1/B)(dB/dt) = -3v_{\parallel}/R$. Therefore $(1/f)(df/dt) = -0.3c/R$.

Warwick and Gordon (1965; 1967) describe such bursts, which are observed spectrographically in the range 24 to 28 MHz. Their drifts invariably are in the sense $df/dt<0$, and the rates are (numerically) extremely high, -25 to -35 MHz $\sec^{-1}$. Normalized to 25 MHz, the rate is $-1 \sec^{-1}$. Let $dR/dt = v_{\parallel} = -(R/3f)(df/dt)$; at $R = R_J = 7.1 \times 10^9$ cm, $v_{\parallel} = 0.079c$ outbound.

These bursts have bandwidths no greater than 100 kHz (a conservative upper limit); the rate at which this band sweeps through the spectrum is 25,000 kHz $\sec^{-1}$. In a fixed frequency receiver whose bandwidth is considerably less than 100 kHz, this burst will appear therefore for less than 100/25,000 seconds = 4 milliseconds. In view of the fact that most receivers in use for decametric studies are of fixed frequency, with bandwidths this narrow, or narrower, it seems appropriate to call these drifting emissions simply "millisecond bursts".

Their actual duration is greater than 4MHz/25 MHz sec^{-1} = 160 milliseconds. This implies a very considerable distance through which the electron moves from the time of its first detection. At a velocity 0.1 c, the electron moves at least (0.70)(0.1)c = 5100 kilometers while we have it under "observation". Near Jupiter's surface, the field strength varies non-linearly over this large a distance. The burst drifts actually seem to be quite linear. If nothing further were said about the matter, this fact would seem to rule out the attractive hypothesis of their origin in rapidly moving electrons.

A way to rescue the hypothesis is possible, however, if the electron's adiabatic motion is taken into account. Electrons move away from the planet more rapidly, the farther they are above their mirror point.[6] This effect tends to compensate for the highly non-linear variation of field strength with radius. It is possible to find a combination of a mirror point and a height at which outward motion produces a linear value df/dt.

The hypothesis is that the local magnetic field B defines the observed emission frequency. Adiabatic theory says that $v_{\parallel} = v(1 - B/B_m)^{1/2}$. Here v is the constant total velocity, and B_m the mirror-point magnetic field. For motion along a radius, we find the drift rate $(dB/dt) = (dB/dR)(dR/dt) = (dB/dR)v$; in the dipole, $(1/B)(dB/dR) = 3/R$, and $dB/dt = (3B/R)v_{\parallel}$. Therefore

$$\frac{1}{f}\frac{df}{dt} = \frac{3v}{R}\sqrt{1 - (R_o/R)^3}$$

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6. *It seems quite fair to assume these electrons* are *moving away from Jupiter; the radiation they generate still may originate in a backwards mode that satisfies the reflection condition discussed earlier.*

where R_o is the *mirror-point radius*. *The observation is that df/dt is* independent of frequency, between f_2 = 24 MHz and f_1 = 28 MHz. We conclude that

$$\left(\frac{f_2}{f_1}\right)^2 = \left(\frac{R_{24}}{R_{28}}\right)^2 \frac{1 - (R_o/R_{28})^3}{1 - (R_o/R_{24})^3} \quad .$$

Putting $X_2 = R_o/R_{24}$, and $X_1 = R_o/R_{28}$, we have

$$\left(\frac{f_2}{f_1}\right)^2 = \left(\frac{X_1}{X_2}\right)^2 \frac{1 - X_1^3}{1 - X_2^3}$$

Also $f_2/f_1 = (X_2/X_1)^3$. The problem is to find X_1 and X_2 when $f_2/f_1 = 6/7$. The solution is $R_{24} = 1.235\ R_o$, and $R_{28} = 1.176\ R_o$. Since the minimum R_o is presumably (for a centered dipole) $1R_J$, the minimum value of $v_{\parallel}$, according to this model, is

$$\frac{1}{f_{24}} \frac{df}{dt} \frac{R_J}{3} \left[1 - (R_o/R_{24})^3 \right]^{-\frac{1}{2}} = 3.6\mathrm{x}10^9 \text{ cm sec}^{-1} = 0.12c.$$

Alternatively, if we assume that the dipole moment, $M_J = 4.2\mathrm{x}10^{30}$ gauss cm^3, we find $R_{24} = [\ 2M_J/(24/2.80)\ ]^{1/3} = 9.9\mathrm{x}10^9$ cm; we use the previous formula, with R_J replaced by this value, R_{24}. Then $v_{\parallel} = 5.0\mathrm{x}10^9$ cm sec^{-1} = 0.17c. The electron energy is about 4 keV or 8 keV respectively.

There are several objections to this interpretation of drifting millisecond bursts.

(a) The explanation just given for their linearity of frequency drift is *ad hoc*; in other words, why does the electron start emitting only at that point on the line of force where the just-described cancellation of effects occurs? There is no easy explanation, all the more because the lowest frequency we observed, 24 MHz, lies far above the ionosphere.

(b) Some electrons must be moving downward, while others move upward, and precipitation should be statistical. Nevertheless, millisecond bursts clearly appear as distinct events. There must be another physical effect present that organizes the times of occurrence of these motions. While this phenomenon may well exist, it has no obvious source so far as the present interpretation is concerned.

(c) The intensity of emission in these events is extraordinarily high, greater than in normal Jupiter emissions by a large factor. In any case (see below) the wavelets emitted by individual electrons must add coherently to produce the observed intense emission. In the context of the present discussion, we must conclude that electron bunching takes place; this is a highly likely phenomenon no matter what the details of origin of the emission may be. But for drifting millisecond bursts, the implication is strong that electrons occur in very compact (D<<10 meters, the free space wavelength of the emission) bunches, which are widely separated in space. What bunching implies for drift rates, electron energies, radiation beaming, and so forth, is unclear.

(d) The combination of (c) and (b) asserts that electron bunches are highly separated in *both* space and time.

(e) The observations strongly favor *outward* moving bunches, for which the interpretation provides no easy explanation.

There may be no answers given to these problems. It remains as a nevertheless real coincidence that the frequency drifts are what kilovolt electrons could produce. Whether we should be impressed by this coincidence depends on whether we feel confident that electrons of this energy are present near Jupiter. They are omnipresent in the earth's belts, and play important roles within the auroral zones. I feel that provisionally the basic elements of Ellis' interpretation of drifting millisecond bursts ought to be accepted, but some aspects of their phenomenology may well be difficult to fit into that theory. Better data, especially concerning the spatial and angular properties of millisecond emissions, ought to help sharpen up the interpretation.

A number of X-ray detectors recently have been flown to discover whether Jupiter produces observable emission. Only upper limits have been set, and these are too high to be of practical value in decametric theories (see below). Nevertheless, these data can be used to estimate the upper limit of low-energy electron fluxes in the belts. The interaction of these fluxes with the atmosphere, and their bremsstrahlung against the upper atmosphere is what we should observe if Jupiter's magnetosphere bears any resemblance to the earth's. The energy ranges, upper limits, and sources (authors) of these values are tabulated below; all observers fail to detect Jupiter.

Photon Energy, $h\nu$	Upper Limit of Observable Flux Φ	Source
4 - 8 keV	2.4×10^{-8} erg cm^{-2} sec^{-1}	Fisher, et al. (1964)
20 - 150 keV	1.4×10^{-9} erg cm^{-2} sec^{-1} (at 40 keV, point of greatest sensitivity)	Edwards and McCracken (1967)
40 - 100 keV	2×10^{-9} erg cm^{-2} sec^{-1} (for nominal 70 keV photons)	Haymes, et al. (1968)

The relationship describing bremsstrahlung is $\Delta E/E_1 = 6.4\times10^{-4}(E_1/E_o)$ (Chamberlain, 1961) where ΔE is the total energy radiated by an electron of kinetic energy E_1 which lies in a broad continuum down in energy from $E_{max} \sim E_1$. E_o is the rest energy. Suppose that the region on Jupiter's atmosphere into which electrons precipitate extends one degree in latitude and 60 degrees in longitude, and covers (60/41,253) of the entire surface of Jupiter. Suppose that E_1 is the photon energy, $h\nu$. Assume that energy ΔE is radiated omnidirectionally and corresponds in wavelength to photons with energy equal to the electron energy. Then the observed upper limits on electron flux averaged over the area $A = 9.2 \times 10^{17}\ cm^2$ are as follows:

Nominal Electron Energy, E_1	Upper Limits on Flux cm^{-2}, Nv	Upper Limits on Electron Density, N	Upper Limits on Total Electron Flux Over "A", ANv
6 keV	$1.7 \times 10^{16}\ cm^{-2}\ sec^{-1}$	$3.7 \times 10^{6}\ cm^{-3}$	$1.8 \times 10^{34}\ sec^{-1}$
40 keV	$2.0 \times 10^{13}\ cm^{-2}\ sec^{-1}$	$1.7 \times 10^{3}\ cm^{-3}$	$1.9 \times 10^{31}\ sec^{-1}$
70 keV	$6.7 \times 10^{12}\ cm^{-2}\ sec^{-1}$	$4.3 \times 10^{2}\ cm^{-3}$	$6 \times 10^{30}\ sec^{-1}$

The relations among the column entries in the tables are $E_1 = h\nu$ and $ANv = 4\pi D_J^2 \Phi/\Delta E$. Here, N is the electron number density, v, the electron speed, and D_J the distance to Jupiter. Fluxes greater than these can exist in the belts if electrons are essentially permanently trapped or if the few observations so far attempted by happenstance failed to be made during a particle "event" in Jupiter's atmosphere. This seems unlikely however; it appears that these limits are very safe upper values.

Another type of analysis that can yield estimates depends on detailed interpretation of decametric emission levels. This interpretation in many ways parallels the interpretation of synchrotron radiation from relativistic particles in Jupiter's belts. It suffers a severe disadvantage that alternate interpretations of the data may be possible, or philosophically speaking, are even probable.

The starting point for interpretations of decametric emission is invariably the assumption -- really a conclusion, broadly speaking -- that the observed radio frequency lies close to the gyrofrequency in the source (see above for an example of this assumption as made by Ellis).

Whether the frequency is precisely the gyrofrequency, or rather, below or above it by some constant small (even large) factor, varies from one interpretation to another. In the case -- emission at the gyrofrequency -- it seems reasonable to equate the observed power to the power emitted by low-energy electrons in the form of gyro radiation. A single low-energy electron produces power at the (previously described) rate

$$dW/dt = 4 \times 10^{-9}\ EB^2 \sin^2 \alpha \qquad \text{E-units sec}^{-1};$$

B is in gauss. This goes into a broadly-beamed pattern, essentially producing power $4 \times 10^{-9}\ EB^2 \sin^2\alpha\ (3/16\pi)(1 + \cos^2\Theta)$ E-units $\sec^{-1}$ sr^{-1} at Θ to the line of force; α is the pitch angle of the electron's spiral trajectory. This emission is circularly polarized at $\Theta = 0°$, and 180°, and linearly polarized at $\Theta = 90°$. The radiation parallel to the magnetic field is RH, and antiparallel, LH polarized in the radio sense. The observed decametric emission in, for example, millisecond bursts, is close enough to the circular state for us to set $\Theta = 0°$. Then, the intensity is $I = 4.8 \times 10^{-10}\ EB^2 \sin^2$ E-units $\sec^{-1}$ sr^{-1}.

Questions now arise of the phase relations and of physical separations between the radiating electrons. Clearly, these relations reorganize the power in the electromagnetic field into a different pattern than it would have had from a single accelerated electron. If the motion of each of the electrons in the fields produced by all the other electrons remains essentially what it would be if the other electrons were not present, then this gyro formula is still useful. In this case, we need to know observationally just one more parameter; this is the beaming pattern resulting from the total distribution of electrons. Values of this beaming are discussed widely in the literature (see Warwick, 1967) and will not be reviewed again here. I take the value 3° as the semi-angle of the beaming cone, based on the observed decametric effects of the variable tilt of the rotation axis during Jupiter's orbital revolution. The total power emitted during, say a millisecond burst at 25 MHz follows from a flux density of 1×10^{-19} W m^{-2} Hz^{-1}. This value is intense, as is appropriate to these very strong emissions. The bandwidth is less than, say 100 kHz. The spectrally-integrated total flux observed at the earth is then $1x10^{-11}$ erg cm^{-2} sec^{-1}. A cone of semi-angle 3° subtends a solid angle of $8.63x10^{-3}$ sr. At 4.04 AU (the standard opposition distance of Jupiter), this corresponds to $8.62x10^{-3}x(4.04\ AU)^2$ = $3.7x10^{27}$ cm^2; the total power in a millisecond burst is therefore about $4x10^{16}$ erg sec^{-1} (which is to say $4x10^9$ watts; I have taken a strong burst, which accounts for why this estimate is more powerful than I have quoted elsewhere). The power per electron is ($4x10^9$ watts)$EB^2 \sin^2 \alpha$. Suppose that E = 10 keV, and B = 8.9 gauss (25 MHz = gyrofrequency). Then the total number of electrons is

$$N = 4x10^{16}/(4x10^{-9}\ EB^2 \sin^2\alpha) = 7.9x10^{30} \sin^2 \alpha \ .$$

We convert this to a density value, and then to a flux estimate. The duration of 4 milliseconds at a single frequency implies a source that is nowhere more than 4x300 = 1200 kilometers from a sphere centered on us, the observers. If the source lies at some general angle it may be that the surface is ∿ 1200 kilometers across; direct interferometry (but not of millisecond bursts) shows emission sources less than 400 kilometers across (Dulk, 1969). I take a nominal transverse dimension of 10^8 cm. Along the magnetic field, the depth can be no thicker than the narrow frequency bandwidth permits. This value at the surface of Jupiter is

$$\Delta R = R(\Delta B/3B) = R(\Delta f/3f) = 95\ km\ !$$

For 100 kHz radiation at 25 MHz the resulting volume is 10^8 cm x 10^8 cm x $9x10^6$ cm = $9x10^{22}$ cm^3; the density of 10 keV electrons is $9x10^7/\sin^2 \alpha$ cm^{-3}; their flux is (0.1)cN = $2.7x10^{17}/\sin^2 \alpha$ cm^{-2} sec^{-1}.

The total number of electrons required on this simple hypothesis to generate the decametric emission is only one order of magnitude less than the upper limit set by the attempts to observe Jupiter by 6 keV X-ray photons. The energy density of these electrons is 16 erg cm^{-3}, greater than the field energy density for the field at 25 MHz gyrofrequency (= 3.2 erg cm^{-3}). The value is much less than one atmosphere, but may exceed the pressure at the relevant heights in Jupiter's ionosphere. On this model there appears no way out of the dilemma posed by these energetics, other than to assume a larger source; only its transverse dimensions are alterable, and only by a factor of about 10^2 in area. This decreases the particle energy density to a level about one or two orders of magnitude less than the ionospheric pressure in the Jupiter equivalent of our E-region. It appears to increase the dimensions of the source beyond values consistent either with the burst durations, or source interferometry.

The entire thrust of this analysis depends on the individual electron's total contribution to the emission being much less than the electron energy. The radiated power divided by the kinetic energy is much less than the gyrofrequency, in fact

$$(dW/dt)/E\omega_B = 2.25\text{x}10^{-16}\ B \sin^2 \alpha \ll 1 \ .$$

In the duration of a burst at one frequency, say $4\text{x}10^{-3}$ seconds, the total emitted energy from an electron is

$$[(dW/dt)/E] \text{ x } 4\text{x}10^{-3} = 1.6\text{x}10^{-11}\ B^2 \sin^2 \alpha \ .$$

For fields of 10 gauss, the total radiated energy is much less than the particle energy, as required.

Other mechanisms, if they are not so conservative of electron energy, will require fewer electrons of a given energy and will therefore lead in the direction of more reasonable total particle energetics. In other words, a more efficient mechanism may convert more of each electron's energy into radio radiation, and thus require fewer electrons to begin with.

Such mechanisms can rely on strong interaction between energetic electrons and their resultant wave field. To estimate the size of this effect I compare the force on an electron as a result of the total wave field from all electrons, with the force on the electron from the Lorentz force of the static field. The total wave field is in a real sense observable; that is the Poynting flux over the source follows from the total emitted power if we know the source area. Again, the power is 4×10^{16} ergs sec^{-1}, and the area[7] of the source 10^{16} cm^2 = 1000 km x 1000 km. The Poynting flux is $c\overline{\varepsilon^2}/4\pi = 4$ ergs cm^{-2} sec^{-1}, where $\overline{\varepsilon^2}$ is the mean square electric vector (in e.s.u.) at the surface of the source. Let $\overline{\varepsilon^2} = \overline{\varepsilon_o^2}/2$. The wave energy density is 1.3×10^{-10} ergs cm^{-3}, and $\varepsilon_o = 5.8 \times 10^{-5}$ e.s.u. Now the comparison is between $e\varepsilon_o$ and $evB(\sin\alpha)/c$. Their ratio is $6.5 \times 10^{-5}/\sin\alpha$. I conclude that the wave field is not of itself strong enough to perturb electron motions perceptibly from their helical motions. The wave field is quite comparable to the field near a one-watt transmitter in the ionosphere where waves in various modes generally appear as a result.

We have learned two things: to create the emission by direct gyro radiation even from coherently oscillating electrons requires too many electrons of the 10 keV variety, from the point of view of either magnetic confinement, atmospheric energetics, and possibly X-ray observations, and the electric wave field within the source leads to forces on the electrons much weaker than the Lorentz force of the static magnetic field.

The conclusion appears inevitable that the electric fields, which organize the electron motions into coherent patterns of the required amplitudes and in proper relation to the Lorentz forces, are not the wave electric fields radiated by streaming particles. The wave fields may not even relate to electron fluxes from Jupiter's belts, although this may be a very reasonable hypothesis. The streaming, if it is to be relevant, must create electrostatic wave fields that somehow couple into EM radiation.

7. *I (Warwick, 1967) previously estimated energy density for a source 10^4 km x 10^4 km; those results, allowing for the difference in assumed source size, are quite similar to the present ones.*

By "electric wave fields" I really intend "electric fields oscillating near the electron gyrofrequency", not necessarily propagating waves.

The role of the plasma near Jupiter therefore seems vital to the generation mechanisms of decametric emission. If particles from the magnetosphere also are vital, then it suggests we consider the interaction of a fast electron with a magnetic plasma. If, on the other hand, waves are vital, we ought to consider the interaction of, say hydromagnetic waves with a magnetic plasma.

The first type of process involves the stimulation, by incident electrons, of waves whose phase velocity corresponds in a simple way to the longitudinal component of the incident electron velocity. In cases where there is only one electron, or just a few electrons, the amount of energy transferred to plasma waves in this way is very small, only comparable to the energy lost, for example, by a single gyrating electron through gyro radiation. This process is called incoherent Cerenkov emission; its amplitude can be estimated (Warwick, 1961, 1963a, 1963c) from an expression given initially by Pines and Bohm (1952). This is

$$\frac{dE}{ds} = \frac{1}{4} \frac{\omega_p^2}{E} me^2 \log_e \left(1 + \frac{E}{kT}\right),$$

where dE/ds is particle energy loss per unit distance traveled in a plasma, whose characteristic frequency is $\omega_p = (4\pi Ne^2/m)^{1/2}$ radians sec^{-1}. Here kT is the thermal energy of the ambient plasma, per particle. The formula requires that E>>kT. Numerically, dE/ds $\sim 1.7 \times 10^{-22}$ ergs cm^{-1} for a 10 keV electron in a 140° K plasma whose characteristic frequency is 10 MHz. Assume that the distance over which the interaction occurs is one ionospheric scale height, 10^7 cm. This value incidentally insures that the gyrofrequency range lies within the observed bandwidth. Then, the total energy lost is 1.7×10^{-15} ergs, which is the fraction 1×10^{-7} of the initial energy. For gyro radiation, the equivalent calculation (p. 74) shows only the fraction $1.3 \times 10^{-9} \sin^2 \alpha$. In other words, incoherent Cerenkov emission is more favorable in terms of the energy resources of an individual electron than is gyro radiation by two orders of magnitude.

This isn't very favorable, however, inasmuch as far too many particles still are required. Because incoherent Cerenkov emission is more efficient than gyro radiation, the total number of electrons involved is about three orders of magnitude less than the upper limits set by X-ray observations at 40 keV. Their energy density is still comparable to, albeit less than, the field energy density, and probably exceeds the atmospheric pressure at the assumed heights of origin.

The physical situation in which a beam of electron moves as though each electron were independent into the planetary ionosphere is unrealistic. However, the ionospheric electrons are a low-energy plasma cloud and the external electrons are, relative to the ionosphere, a plasma cloud. These two clouds interpenetrate as two streams of particles. The velocity distribution of the sum of the two is bimodal, the thermal velocity distribution within each cloud representing a small spread compared with the large difference in velocity resulting from streaming. It is a famous problem in plasma physics to describe the dispersion relation for waves in this bimodal distribution. For us, it is important to understand that in the analysis, the gross velocity and density distribution remains essentially unmodified. However (in this perturbation theory) waves can exist in the medium which have a growing amplitude as a function of time. This means that the proposed velocity distribution is unstable.

The two-stream instability results in a kind of coherent Cerenkov emission. Individual electrons of the incident stream and of the ambient plasma bunch around points along the direction of motion because the ambient plasma contains a wave mode whose velocity matches that of the stream. It is clear that an upper limit to the amplitude of the resulting plasma wave, or a lower limit of the required fluxes, is given by the complete conversion of the incident particle stream into a plasma wave.

Whether this upper limit is also an approximation to the actual wave generated is another matter entirely. Furthermore, this wave of course need not, or perhaps will not, be a wave that propagates freely into the empty space surrounding Jupiter, and to which the observational estimates of wave electric field strength apply. But on purely intuitive grounds, these estimates may be valid, as applied to the actual circumstances at Jupiter.

All of the dozens of physical phenomena that are neglected in this analysis must in our final definitive understanding of Jupiter decametric emission appear as factors that *decrease* the efficiency of the Cerenkov process somewhat, without invalidating the strict lower limit on particle fluxes that results. A comparison of these results with the incoherent Cerenkov process will show that about seven orders of magnitude greater efficiency per electron is gained in this way. Suppose intuitively therefore, that 99 percent of the available single particle energy goes into other processes than escaping EM radiation. The gain in the coherence process is about 10^5 x the single electron Cerenkov theory, and 10^7 x the gyro radiation theory.

I estimate the wave amplitude of the essentially electrostatic wave in the medium on the hypothesis that the disturbance is sinusoidal and that the flux of escaping radiant energy equals the flux of energy in the particle stream that is its source. Where s lies in the direction of particle streaming, and $\mathcal{E}$ is the magnitude of the electric vector, the divergence equation states that $d\mathcal{E}/ds = -4\pi e N_1 \sin ks$ e.s.u. Here N_1 is the density amplitude of the wave created in the medium by the particle stream. For a wave freely propagating in empty space the wave vectors $\mathcal{E}$ and $\mathcal{B}$ are equal in magnitude, and the flux of energy is the Poynting value $(c/4\pi)\mathcal{E}\mathcal{B}$. In the present case, the divergence equation represents a disturbance, $\mathcal{E}$, along the stream direction, and not a complete EM wave including a value for $\mathcal{B}$. The mechanisms that produce $\mathcal{B}$, a vector that together with $\mathcal{E}$ represents a propagating EM wave, are essentially irrelevant at this point. Rather, the electric vector will, in any of the mode coupling mechanisms, generate the B value required to fit the boundary conditions in the coupling region. It is therefore plausible to equate the Poynting flux to the value of $(c/4\pi)\, E^2$ as estimated from the divergence equation. That is, the EM flux eventually arising from an electrostatic wave of amplitude $E_o = 4\pi e N_1/k$ is $c\, E_o^{\,2}/8\pi$. The incident flux of energy in the form of particles is $(mv^2/2)N_s v$ where N_s is the particle stream density, m is the particle mass, and v is its velocity. Therefore, by hypothesis

$$\frac{cE_o^2}{8\pi} = \frac{1}{2}\, N_s\, mv^3 = NEv.$$

In general, there will be a relation between k and v, and between k and the total electron density of the ambience. Therefore, a relation exists also between the total electron density and N_1 which must be consistent with the total electron density.

For a given value of v there are several frequencies than usually satisfy the Cerenkov condition $v(\text{phase}) \sim v$. These are frequencies at which the index of refraction of the excited wave mode is greater than unity. One of these frequencies is near the electron gyro frequency which is one of the features that turned my attention toward Cerenkov effect as a source of decametric emission. In these circumstances, k, the magnitude of the wave vector, is greater than its free space value, and essentially can take on any value for a given value of v, with small variation of the plasma frequency bringing this about.

The frequency of the generated wave depends on which (or which ones of the) singularity(ities) is(are) excited by the particle stream. The stream entering the planetary ionosphere externally encounters the plasma instability regions in a definite sequence. If the electron gyrofrequency exceeds the plasma frequency, the first singularity encountered will be near, and slightly below, the gyrofrequency. If a significant fraction of stream energy goes into exciting this instability (as I shall argue is the case) then no further singularities such as the plasma frequency really need be considered since there won't be incident particles to excite them.

As an example, I find that an escaping wave corresponding to observed emission from Jupiter requires $N_s = 4 \times 10^{-2}$ cm^{-3} electrons at $E \approx 10$ keV. Their flux is $N_s v = 2 \times 10^8$ cm^{-2} sec^{-1}, depositing a total of 2×10^{24} electrons sec^{-1} over the source. The total power deposited is (by hypothesis, and, as a check, by computation) 4×10^{16} ergs sec^{-1}. The density of plasma electrons forming the electrostatic wave is $N_1 = 10^2$ cm^{-3}, where k, the propagation constant, is taken corresponding to the stream electron energy (velocity) 10 keV (6×10^9 cm sec^{-1}). That is $k = \omega/v$, with $\omega = 2\pi \times 10$ MHz; the corresponding wavelength is $2\pi v/\omega = 6$ meters. The index of refraction $n = c/v = 5$; v is of course far higher than the expected thermal electron velocities, in Jupiter's atmosphere or magnetosphere.

The required flux of electrons for incoherent Cerenkov radiation is 2.5×10^{15} cm^{-2} sec^{-1}; the lower limit of flux required for coherent radiation must exceed 2.5×10^8 cm^{-2} sec^{-1}. The required flux for gyro radiation is 2.7×10^{17} $(\sin \alpha)^{-2}$ cm^{-2} sec^{-1}. With an arbitrary efficiency factor of 10^2 included, the coherent Cerenkov process appears to require a particle flux of 2.5×10^{10} cm^{-2} sec^{-1} 10 keV electrons. We have excluded direct gyro radiation on the basis of particle energy densities and of a possible observational limitation based on X-ray flux limits.

Plasmas of the densities undoubtedly found near the upper side of Jupiter's ionosphere can interact with incident streams of electrons and will produce large-enough amplitude oscillations to account for decametric emissions with only relatively modest electron fluxes. For these reasons, the two-stream instability mechanism, driven to the highly non-linear limit in which significant fractions of incident stream energy flux lie in electrostatic modes of the ambient plasma, appears to be the most plausible explanation of decametric emission.

However, a significant qualifier needs to be added. The assumption is vital that there be particle streaming. The reason that we infer that particles are involved may be no more sophisticated than the observed presence of negatively-drifting millisecond bursts with the "correct" drift rates. That is, of course, no small support for the theory, but nevertheless cooperative phenomena of Jupiter's magnetosphere may also play a role. For example, low-frequency waves certainly must move through Jupiter's magnetosphere and may act as large amplitude sources of magnetic disturbances in Jupiter's upper atmosphere. The current sheet required by Goldreich and Lynden-Bell (1969) in their theory toward an explanation of Io's modulation of decametric emission in a sense is such a large amplitude source of field variation. It happens that I am not particularly satisfied by several aspects of their theory. In any case, however, the way in which a low-frequency HM wave can induce radio emission from regions close to Jupiter is of interest.

At a first glance, the case for this mechanism appears formidably difficult. HM wave frequencies typically lie in the neighborhood of cycles per second, not megacycles. The direct modal coupling mechanism such as proposed above for the Cerenkov process therefore seems an unlikely one because of the gross difference in propagation vectors for the two waves. However, in solar radio emission, apparently just this sort of coupling effectively goes on, although it may be that complex intermediate processes bridge the gap. For example, Tidman et al. (1966) suggest that plasma turbulence develops behind a magnetic disturbance moving through the corona; the observed radio emission originates in the coupling of plasma waves in this turbulent region. At the sun, however, these disturbances propagate through a dense plasma (relative to ionosphere or magnetosphere of Jupiter). Plasma waves behind the front emit very small amounts of their energy into observed radio emission. The phenomenon obviously is extravagant of plasma energy in that a small amount of observed radio energy implies a large amount of plasma kinetic energy. That situation, is, of course, precisely what Jupiter theories cannot tolerate, as a result of the enormous strength of the radiated electric fields.

If the presence of a magnetic disturbance can be taken to imply a large electrical current flowing through Jupiter's ionosphere or inner magnetosphere, the current itself may be a direct source of instability having to do with the decametric emission.

Goldreich and Lynden-Bell suggest that such current sheets may develop instabilities leading to cyclotron emission. Their explanation involves coherent gyro emission, as discussed above, in the (unrealistic) limit of very dilute streams of low-energy electrons. Their deduction of an amplification factor = 10^{83} over 100 kilometers of stream distance should be taken to imply nothing more than the existence of an instability; the number itself is (literally) highly irrelevant.

My earlier discussion of coherent gyro radiation seeks to by-pass the very deep theoretical difficulties blocking a realistic discussion in which the fluxes and wave amplitudes are not infinitesimally small. This procedure is central to an understanding of the physics of the problem; by emphasizing the observed beaming angle and the observed power levels, the computation eliminates the problem of *whether* there is an instability. There must be coherence, else the radiated power would be omnidirectional, in contradiction to the data. The weakness in my discussion is its emphasis on the observed properties of the emission, which may indeed not prove to be consistent with the higher quality data, or data easier to interpret, that may forthcome in future years.

Jupiter's decametric emission has the property of being so complicated that observers or theorists are able, often, to read it in support of almost any hypothesis whatsoever. The problem is to extract the relevant phenomena from the tangled mess that lies before us. My impression is that bandwidth, power levels, and beaming are the strongest points to emphasize, as I have done above. The mere existence of an instability at the gyro resonance is almost the only feature common to all theories, and should not be considered a success of a new theory. In the future the finite amplitude problem should be tackled directly to improve our present inadequate knowledge of Jupiter's decametric emission.

The existence of energetic electron streams is inferred by Goldreich and Lynden-Bell in their discussion of induction electric fields created by the motion of Io through Jupiter's magnetosphere. A current of electrons moving at $v = 0.1c$, a density of $0.5\ cm^{-3}$ and in total carrying 1.1×10^6 amperes, is driven by the induction. The large electron speeds are required to maintain the current at the level pre-established theoretically in a region that is low in electron density. By "theoretically" I mean "as a result of currents inferred to be flowing through Jupiter's ionospheric equivalent electrical resistance".

The flux of these electrons is $1.5 \times 10^9\ cm^{-2}\ sec^{-1}$, one order of magnitude greater than the minimum value inferred earlier from an analysis of coherent Cerenkov emission. The comparison is actually quite close, since the area through which this flux passes at Io is about $10^{16}\ cm^2$, the same size as the area I assumed for the decametric source *at the surface of Jupiter*. The total *number* of streaming electrons is virtually identical but from these two radically different points of view. The flux per unit area of electrons lying on the boundary of Io's force tube will be larger at the surface of Jupiter roughly by one hundred fold than it is at Io. The "observed" field strength E_o in Jupiter's decametric source must be taken larger than previously assumed, by one order of magnitude. Since the density of streaming electrons goes as E_o^2, the value of N_s and of the incident flux all increase by just the correct amounts to maintain the total number of electrons at the value required by Goldreich and Lynden-Bell.

It may be a matter of taste, then, whether an electron flux estimate derived from a theory of Io's influence, or an estimate based on interpretation of decametric power levels is preferred. The electron fluxes appear to be quite similar (except very near Jupiter).

The difference in the methods may imply a different phenomenology within Jupiter's magnetosphere. On the one hand, Io (and the other satellites) serves as a source of disturbance of an extremely localised nature. On the other hand, the omnipresence of decametric emission at low frequencies, and especially, at all times when the main source is at the central meridian, suggests sources of wave excitation other than Io or even the other satellites.

To generalize, then, I conclude that Jupiter's magnetosphere is filled with hydromagnetic disturbances that can be described in terms of electrical currents and electron fluxes of the order of magnitude of 10^8 to 10^{10} cm^{-2} sec^{-1} at 10 keV energy. They may be required on the one hand by the presence of moving satellites within the conducting plasma of Jupiter's magnetosphere, and, on the other, to explain the physical generation of intense decametric emission near the surface of Jupiter.

2.3 *Energetic protons*

The evidence for proton belts at Jupiter is non-existent. We must therefore infer their properties entirely by analogy and what is known theoretically about the earth's proton belts. As is described above for the energetic electrons, for energetic protons also, even more emphatically, the earth's belts are relatively stable. Furthermore, the L-shell diffusion model fits their distribution in space and energy to very good approximation. Nakada and Mead (1965) quite successfully fit the experimentally determined proton fluxes with a Fokker-Planck diffusion model in which 10^{-6} of the incident solar wind protons are trapped and accelerated to megavolt energy. This model includes detailed loss factors that are difficult to estimate for Jupiter; it also includes details on the efficacy of sudden variations in the magnetic field which perturb the adiabatic orbits of the protons. For Jupiter, unlike the earth, it seems quite clear that the cosmic ray albedo neutron decay mechanism is small, and therefore this feature of the earth's belts does not need to be discussed (Chang and Davis, 1962).

Finally, the trapping factor 10^{-6} determined by Nakada and Mead may be in error for Jupiter as a result of the grossly different configuration of the solar wind magnetic field at the interface region between the planetary and interplanetary fields. Rather than estimate from the physics of the problem how serious this difference may be, I take an upper limit to the proton factor to be the trapping factor prviously estimated for energetic electrons, viz. 10^{-9}. This is a region in space in which both the protons and electrons transfer from a charge-neutral plasma environment into individual orbits defined to a large extent by the adiabatic invariants. The protons react relatively sluggishly to the ambient magnetic and electric fields, compared with the electrons. Whether this should result in their being more efficiently trapped than the electrons isn't clear. I shall take the value as 10^{-9}. This corresponds to a trapped proton density (at the magnetopause) of $n_p(R_1) = 2 \times 10^{-10}$ cm^{-3}. With the same R_o and R_1, as before, I find from the Davis and Chang solution that the proton density has exactly the same values as the electron density, viz. 1.6×10^{-4} cm^{-3} at $1.8R_J$. The proton energy starts at 820 eV(400 km sec^{-1} solar wind speed), and accelerates to 29 MeV at a distance of $1.8R_J$. The speed of this proton is subrelativistic, 5.9×10^9 cm sec^{-1}. As a result, the flux of protons is less than electrons, only 1.1×10^6 cm^{-2} sec^{-1}.

The main thrust of this analysis is toward much higher energies than occur in the earth's belts. This is a result of the much greater range in L-shell covered by Jupiter's belts, which is in turn a result of the much greater magnetic moment of Jupiter. These higher energies seem very difficult to escape, since both the theory and observation (of synchrotron emission) point to the same qualitative result.

2.4 *Relaxation times*

The rate of particle loss should balance the rate of particle trapping at the magnetopause. The only way we have to estimate losses is via the estimates made just above for trapping factors. Assume that Jupiter's magnetosphere presents a cross-sectional area of $\pi(59R_J)^2$. Then the rate of trapping is in the whole 4.4×10^{21} sec^{-1} (which assumes a trapping factor of 10^{-9}). In a steady state the corresponding loss per unit area over the surface of a sphere the size of Jupiter is 7 cm^{-2} sec^{-1}.

This figure is identical for both protons and electrons. Inasmuch as the detailed loss mechanisms for these two kinds of particles are undoubtedly quite different, their equal loss rates describe the crudeness of our estimate rather than physical reality. The guess is probably better for electrons than for protons, because we have evidence for their existence.

But continuing this kind of estimate still further, we can come to a guess for emptying times; the guess is again likely to be better for electrons than for protons. The total number of electrons in the belts is their density times the volume. In the region of the synchrotron belts, their equatorial density is 1.6×10^{-4} cm^{-3}. With the pitch angle distribution as estimated on page 60 of this report, I estimate that the volume occupied by electrons at this density lies in a cylindrical annulus between $r = 1.8R_J$ and $r = 3.8R_J$, extending $2.1R_J$ on either side of the equatorial plane. This is 5.3×10^{31} cm^3; the total number of electrons is then $1.6\times10^{-4} \times 5.3\times10^{31} = 8.5\times10^{27}$. The total loss rate of electrons is, as calculated before, equal to the rate of trapping, 4.4×10^{21} sec^{-1}. The lifetime of a synchrotron radiation (but note -- not the radiative lifetime) electron is therefore $8.5\times10^{27} \div 4.4\times10^{21} = 1.9\times10^{6}$ sec = 0.06 years (only 22 days!). This is somewhat less than the radiative estimate of electron lifetime, 0.3 years for a typical electron, and suggests that loss mechanisms other than radiation are important.

Observational evidence supporting variability of the synchrotron source also supports this short lifetime. It recently has been reported by Gerard (1970) that with allowance for geometrical variations, such as polarization and zenomagnetic latitude, there remains synchrotron variability related to variability in solar activity. This lifetime is much less than lifetimes of energetic electrons in the earth's belts, which leads to a suspicion that the estimate has gone awry at some point. The obvious candidate for error is the analytic Fokker-Planck solution used to connect density at the magnetopause with density at the synchrotron source. It would have to be changed in the sense of increasing the density contrast between the inner belts and the trapped electrons at the magnetopause.

That is, the electron density varies with a higher-than-inverse-fourth power of the radius. That has the effect of making it empirically necessary that fewer electrons (than 10^{-9}), out of the total solar wind flux impinging on the magnetopause, be trapped in a way that makes them eligible for L-shell diffusion ("L"-igible electrons!). Finally, the steady-state balance of loss-versus-gain leads to a longer average dwell time of the electrons at a given point within Jupiter's magnetosphere.

At the present time this speculation about possible improvements to the diffusion solution seems so remote from observed reality that I won't develop it further. In short, the best estimate for the belts at present is that the electrons have short radiative, and still shorter alternative, lifetimes and are of low energy, relativistic, but only 10 or $20E_o$.

Witting (1966; and, in an unpublished work, Rather, also) proposes that the satellite Amalthea (Jupiter V) removes relativistic electrons from the synchrotron source in a time scale less than two years. He also suggests that large amplitude Alfvén waves created by Amalthea may accelerate electrons. These suggestions on the other hand may provide the "observed" rapid loss mechanism of the relativistic electrons. On the other hand, they may render moot the discussion of L-shell diffusion presented above. As for the rapid loss, the satellite plays an "acceptable" role in the estimates given above. Its role in acceleration mechanisms seems much more speculative.

2.5 *Summary table*

Energetic Particles

Relativistic Electrons	Reference
Energy ∿ 6.2 ± 2 MeV	Field is given by decametric emission; emission originates at 1.8 R_J from Jupiter's center, in B ∿ 2.0 gauss.
Mean density 1.6×10^{-4} cm^{3} and Omnidirectional flux = 4.8×10^{6} cm^{-2} sec^{-1}; an alternative value is quoted below under "L-shell distribution"	Branson's brightness in the belts (21 cm data).
Electron lifetime against: radiation = 0.3 years;	The above-quoted electron energy and field.
other loss mechanisms = 22 days	This report's estimate of trapping rates.
L-shell distribution: Boundaries at 1.56 and 59 R_J Maximum density 6.3×10^{-4} cm^{-3} and flux 1.9×10^{7} cm^{-2} sec^{-1} at $1.8R_J$, proportional to R^{-4} elsewhere	Davis-Change solution of Fokker-Planck equation. The assumed L-shell distribution and magentic dipole moment imply this omnidirectional flux. The previous omnidirectional flux is based on uniform field strength and spatial distribution.
Beaming of radiation into 5 degree (full angle) cone	The above-quoted electron energy

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Energy ∿ 3.3 MeV	Field is given by decametric emission; emission originates at $2.02R_J$ from Jupiter's center, in B ∿ 1.5 gauss.
Omnidirectional flux = 1.0×10^{6} cm^{-2} sec^{-1}	Branson's brightness in the belts, 75 cm data.
Electron lifetime against radiation = 1 year	The above-quoted electron energy and field.
Beaming of radiation into 9° (full angle) cone	The above-quoted electron energy.
Pitch angles lie within $\alpha_L = 90° \pm 30°$	1. Latitude variation of decametric flux. 2. N - S source extent. 3. Polarization.
Energy spectrum: energies less than synchrotron bandwidth requirements this allows a range $\lesssim$ 10:1	

Non-Thermal Protons	Reference
Energy: 29 MeV at peak, $1.8R_J$	Solar wind energy at $59R_J$ increases by factor $(\frac{59}{1.8})^3$ at $1.8R_J$.
Density: $1.6x10^{-4}$ cm^{-3} at peak, $1.8R_J$	Protons follow same density distribution as electrons.
L-shell distribution: inner boundary at $1.6R_J$	
Fall-off from maximum proportional to R^{-4}.	
Flux: $1x10^6$ cm^{-2} sec^{-1}	Proton speed $\sim 6x10^9$ cm sec^{-1}

Kilovolt Electrons	References
Energy 6 keV Flux $\leq 1.7x10^{16}$ cm^{-2} sec^{-1} Total number $\leq 1.6x10^{34}$ sec^{-1}	Fisher, et al. (1964).
Energy 40 keV Flux $\leq 2.1x10^{13}$ cm^{-2} sec^{-1} Total number $\leq 1.9x10^{31}$ sec^{-1}	Edwards and McCracken (1967)
Energy 70 keV Flux $\leq 6.7x10^{12}$ cm^{-2} sec^{-1} Total number $\leq 6x10^{30}$ sec^{-1}	Haymes, et al. (1968)
Energy 10 keV	Drifting millisceond bursts (Ellis, 1965; Gordon and Warwick, 1967)
Flux $\sim 10^{10}$ cm^{-2} sec^{-1} Total $\sim 10^{26}$ sec^{-1}	Coherent Cerenkov emission as emission source (this memo) or currents required by induction theory of Io's effect (Goldreich and Lynden-Bell, 1969)
Position:	Decametric emission originates close to Jupiter's surface.
Observations are essentially made made at $1R_J$ Other locations are unknown	

3. PLASMAS

3.1 Plasmasphere

Some models for the distribution of Jupiter's magnetospheric plasma, my own not included, imply large densities, say much greater than 10^3 cm^{-3} at distances of several Jupiter's radii from the planet. There are two deductive paths leading to this kind of result. The first, and strongest from a theoretical point of view, depends on Jupiter's large magnetic field in rapid rotation to produce large centrifugal forces that throw plasma out to great planetocentric distances. The second argument is that Io's strong control of decametric radiation implies that emission is generated in the immediate vicinity of the satellite, $6R_J$ from Jupiter. As a result, the plasma frequency is comparable to the wave frequency.

Fortunately there are observed data that relate to this point, e.g., the electron density in Jupiter's magnetosphere. Decametric emission is often, perhaps usually, elliptically polarized (see page 45, et seq.). Generally only one base mode of the magnetoionic medium near the point of generation is involved. That is, no mode coupling has occurred in the region of generation, and the observed elliptical polarization is a base mode.

That is a strong conclusion, which depends on the fact that as a function of frequency or of time the observed polarization is constantly elliptical during most events (the important kind of exception was discussed on page 45, et seq.). In a region of space with strong magnetic field and large plasma density the observed elliptical polarization could not long remain invariant with changes in source geometry, and would not be invariant as a function of frequency, unless it is a base mode.

This elliptically-polarized radiation impinges on the earth's ionosphere and finally into our receivers. In the earth's ionosphere, waves of 30 MHz frequency have base modes that are nearly purely circular. Enroute through the earth's ionosphere, its polarization is therefore *not* in a base mode. The incident Jupiter radiation lies partly in both of the *two* base modes. Since it originated in just one base mode, it has undergone mode coupling between us and its source. In a broad sense this coupling occurs near Jupiter. The plasma density of interplanetary space is so low that base modes through this region are almost exactly circular for almost all directions of propagation. The mode coupling must therefore occur near Jupiter, either within its magnetosphere, or perhaps in the source region; several possibilities exist for the latter.

Suppose that the magnetic moment for Jupiter suggested on page 49, 4.2×10^{30} cgs, is an upper limit. Then nowhere within the magnetosphere, or outside a radius of about $1.5R_J$ does the gyrofrequency exceed 3 MHz. Suppose the electron density is less than 10^7 cm^{-3} everywhere in this region. Then conditions of propagation are everywhere quasi-longitudinal, which means that the base modes are almost circular. But elliptically-polarized radiation is observed. If this radiation is created within a sphere less than $1.5R_J$ in diameter centered on the planet, then it will suffer the Faraday effects, whatever they may be, that are imposed by quasi-longitudinal propagation through the magnetosphere. Furthermore, since the radiation was created in a base mode, at some place *within* the $1.5R_J$ it ceased to lie uniquely in one base mode. This is exactly what is meant by "mode coupling": the escaping radiation is clearly (this is an observational result) not in base modes through most of Jupiter's magnetosphere although it began in one; therefore, mode coupling must have occurred. This is a strong conclusion, and not a hypothesis.

But it is barely conceivable that either of the original conditions that the magnetic moment has a value less than 4.2×10^{30} cgs, or that the density is everywhere less than 1×10^7 cm^{-3}, is incorrect. This is not very likely, however, and even if the density or the field is larger they must decrease regularly to interplanetary values. The results of the preceding paragraph can be translated from planet-centered geometry to a sphere surrounding whatever point in the magnetosphere is the radiation source, and then the same statement regarding mode coupling applies to this sphere.

The importance of this discussion lies in the fact that the elliptically-polarized radiation we observe undergoes its mode coupling before it arrives in the magnetosphere outside of a few tenths of a radius above the surface of Jupiter. It then is capable of establishing the quasi-longitudinal Faraday rotation that occurs in sum along the ray path through the magnetosphere, and this in turn measures a field-weighted mean of the ambient electrons.

The quasi-longitudinal Faraday rotation expected for a 30 MHz wave moving at an angle of 80° to the lines of force above Jupiter, is

$$\Omega = 4.37\times10^{11} \int_{R_i}^{\text{Near Earth}} N \left(\frac{R_J}{R} \right)^3 ds$$

where Ω is measured in radians, N is in electrons cm^{-3}, and the integral is taken from the initial point R_i where the field is 10 gauss, to a receiver just outside the earth (see Warwick and Dulk, 1964). This integral, of course, could be written to include the earth. Instead, Ω stands just for Jupiter's contribution. We showed that, for virtually all observed decametric emissions, this integral amounts to no more than 10 per cent of the terrestrial rotation, that is, to one rotation in a total of 10 rotations. The value of the integral through Jupiter's magnetosphere down to the point where Faraday rotation begins (this is the region of mode coupling, and almost certainly lies near the surface of Jupiter), is therefore no greater than 2π.

The range of integration can be broken into contributions of equal magnitude only if we know N(R). However, if we assume that N(R) is constant, we can investigate how large the integral's contribution will be per unit electron density in various shells about Jupiter. The integral is measured by

$$\int_{R_1}^{R_2} \left(\frac{R_J}{R}\right)^3 dR = \frac{R_J}{2} \left[\left(\frac{R_J}{R_1}\right)^2 - \left(\frac{R_J}{R_2}\right)^2\right] .$$

If the bracketed term = 1/6, with $R_1 = 1.0R_J$, then $R_2 = 1.0954R_J$. The path of integration out to $R_n = \infty$ is thus divided into six equal intervals of Faraday effect from equal densities of electrons cm^{-3}.

n	1	2	3	4	5	6
R_n/R_J	1.0954	1.2247	1.414	1.732	2.4495	∞

The total path from $1.0R_J$ to $\sqrt{2}\,R_J$ contributes as much as the remaining path from $1.414R_J$ to ∞. Or, the first tenth of a radius (from this point of view) is as important as the entire magnetosphere outside of $2.4495R_J$. The contribution from each shell in terms of the rotation from one electron cm^{-3} is $\Delta\Omega = 0.0537$, and for N electrons cm^{-3} distributed throughout the shell, $\Delta\Omega = 0.0537N$ radians. Since all we know is that the total rotation for all shells is $<2\pi$ radians, we can distribute these electrons in any way over the six shells, say according to the entries N in the table below.

R_0	R_1	R_2	R_3	R_4	R_5	R_6	Model
	243	0	0	0	0	0	I
	121.5	121.5	0	0	0	0	II
	40.5	40.5	40.5	40.5	40.5	40.5	III

There is another, alternative way to describe this stringent upper bound on the electron density of Jupiter's magnetosphere. Suppose that the electrons are concentrated into a highly non-uniform distribution, a single stratum of thickness h at radius r, where $h<<R$. Then the Faraday rotation upper limit is

$$2\pi \geq 4.37\times10^{-12} \; [N \; B \; h] \text{ radians.}$$

Suppose that the field B at the radius R is $10 \, (R_J/R)^3$ gauss. Then the condition is $2\pi \geq 4.37\times10^{-11}(R_J/R)^3 \; N \, h$; therefore

$$Nh \leq 1.44\times10^{11}(R_J/R)^3 \; .$$

What kind of a scale one wishes to choose for h depends on other factors than can be discussed here. Suppose that $h = 10^4$ km. Then the upper limit electron density at $1.4R_J$ is 430 cm^{-3} and at $2.7R_J$, 2900 cm^{-3}; at Io's orbit, this density is $28{,}000 \text{ cm}^{-3}$. Clearly, the better way to represent this density upper limit is in terms of Nh. For the same cases the following table lists maximum Nh values.

location	$R_3 = 1.414R_J$	$R_5 = 2.72R_J$	$5.9R_J$ (at Io)
maximum Nh	$4.3\times10^{11} \text{ cm}^{-2}$	$2.9\times10^{12} \text{ cm}^{-2}$	$2.8\times10^{13} \text{ cm}^{-2}$

Theories requiring large electron densities at distances from Jupiter must also require the electrons to lie within very thin strata, only a very few tenths of a Jupiter radius in thickness. At the orbit of Io, it is clearly out of the question to assume a density corresponding to the plasma frequency equalling the wave frequency. This is 10^7 cm^{-3}, and requires a lamina of thickness $\sim (2.8\times10^{13}/10^7)$ cm = 28 kilometers!

The most likely model depends on a smooth radial distribution of electrons, dropping off, perhaps rapidly, or perhaps slowly away from Jupiter, but nowhere increasing in density on a scale larger than a few tenths of one R_J. The inner parts of this distribution are distributed uniformly over a sphere extending to $R_2 = 1.2247R_J$ above the center of Jupiter and based on the ionosphere. The estimate equates the number of electrons within R_2 to the number that will produce 2π radians of Faraday rotation at 30 MHz. This value is (from page 95, Model II) just 121.5 electrons cm^{-3}. It is consistent with crude estimates of Jupiter's plasmasphere (see Warwick, 1967). These electrons might have a distribution decreasing in height exponentially, which would make their density and total number less than this upper limit. The thermal proton density and distribution are the same as the electron density and distribution.

3.2 Ionosphere

There are two recent, essentially consistent, statements about Jupiter's ionosphere in the literature. Gross and Rasool (1964) deduce temperature and ionization from a theoretical aeronomical point of view. The values depend on assumptions about the major ionospheric constituents. Almost all ionospheric estimates for Jupiter take the atmosphere as nearly pure molecular hydrogen. Gordon and Warwick (1967; see also Warwick [1967]) explain polarization diversity in millisecond decametric bursts through a special type of Faraday effect near the electron gyrofrequency. The observations permit a direct inference on the electron density in the Jupiter ionosphere essentially outwards from the source region, and close inferences on the ionospheric magnetic field and direction of propagation as well.

I argue on page 45, et seq. that the two elliptical base modes present in this "Y-one" Faraday effect are created in reflection of decametric emission. This reflection undoubtedly occurs in Jupiter's ionosphere. The Gordon and Warwick data may refer to that height in the ionosphere where the oblique propagation critcial frequency is close to 26 MHz. This is the "maximum usable frequency" of terrestrial radio engineers. For the same angle of incidence, higher frequency waves would pass through without reflection.

The angle of incidence onto the ionization layer is unknown, save by model building of the type I previously attempted (Warwick, 1963a). Combining this inferred angle of incidence, 15° to 20°, with the supposed MUF of 26 MHz, leads, via the simple reflection condition, $f_{critical}$ = (26MHz)(cos 20°) to 24.4 MHz, as the frequency of a wave that will be reflected even at vertical incidence onto Jupiter's ionosphere. Note that this frequency, derived from a simple model and based on a representative frequency, 26 MHz, is a typical value for the early source. At other longitudes of Jupiter, somewhat higher or lower frequencies might be appropriate. The value 24.4 MHz represents the plasma frequency at the point of maximum electron density in the layer. The electron density that corresponds is given by $N_e = 1.2 \times 10^4 f_p^{\,2}$, where f_p is the plasma frequency in MHz. For f_p = 24.4 MHz, the corresponding electron density is 7×10^6 cm^{-3}, quite high, but probably not completely out of the question; Gross and Rasool quote a value 20 times smaller. Gordon and Warwick quote a value of $f_{critical}$ = 1.5 MHz, corresponding to only 3×10^4 cm^{-3}, 200 times smaller than required from the MUF argument.

Our value depended sensitively on the exact angle of propagation assumed for the ray direction with respect to the line of force. While it is clear that an oblique angle is involved, its exact value does not follow from the observed polarization ellipse axial ratio (0.36) unless the plasma frequency is known. We assumed a propagation angle of 60° to 70° with respect to the lines of force. This value requires a smaller plasma density than does a propagation angle in the range, 35° to 45°, which are what the reflection model requires (Warwick, 1963a). The reflection model implies plasma frequencies of the order of magnitude of 20 MHz or slightly greater. The true value for ionospheric density at its peak may lie between the Gross and Rasool value and the higher value required by total reflection of the decametric emission. The tabulated values (see below) correspond to this range.

The ionospheric field strength must exceed the value required for the electron gyrofrequency to equal the wave frequency. In no case, that is for no direction of propagation, does the Y-one Faraday effect make sense unless the gyrofrequency lies in the range near $Y = 1.5$; for emission at 26 MHz, this corresponds to about 13 or 14 gauss.

3.3 *Summary table*

Plasmas

Plasmasphere	Reference
Spatial Distribution: within $R_J/3$ of surface	1. Low ionospheric-exospheric temperature (Gross and Rasool, 1964) 2. Constant and large acceleration of gravity (Warwick, 1967) through exosphere
Density: $\leq 1 \times 10^2$ cm^{-3}	Lack of quasi-longitudinal Faraday effect on decametric emission (Warwick and Dulk, 1964).

Magnetosphere	Reference
Some plasma must extend to Io's orbit, but with much reduced density.	A controversial subject. No direct data available.

Ionosphere	Reference
Density: 10^5 to 10^7 cm^{-3}	Reflection of decametric emission (this memo, page 98)
Scale Height: observationally unknown	Assumed to be 20 kilometers for the purpose of computing density from Y-one Faraday effect
Magnetic Field: 13 gauss	Measured in decametric "early" source near the point of reflection of 26 MHz waves
Temperature: observationally unknown	Assumed to be a few hundred degrees

4. ELECTROMAGNETIC WAVE FIELDS

4.1 *Microwave fields*

For the purpose of realistic design of space vehicles, it may be useful to translate the observed data on microwave emission from Jupiter into another form more directly applicable to communications problems. I shall use "antenna temperature", which is a measure of the power to a receiver from a matched hot load equivalent to the antenna. If an antenna is immersed in a black-body cavity at a certain temperature, T_a, this represents the power absorbed (and emitted as well) by the antenna. The crucial features of the radiation field in any real situation are its intensity, and the angle subtended by the radiation field at the antenna, in relation to the beam width of the antenna. Two further conditions are important. The vehicle's communication system must both receive signals from the earth and transmit information back to the earth.

The link to the earth from Jupiter is largely free from difficulties resulting from the additional noise produced by microwave radiation. The microwave source intensity is defined by a brightness temperature of about 100° K at 10.4 cm wavelength (Berge, 1966). It covers a solid angle that, for our present purposes, is adequately defined as a circle of radius one minute of arc. To observe the vehicle probably will require in any event large antennas. Assume a 200-foot dish with a full beamwidth of about 6 minutes of arc at 10.4 cm. Then the antenna temperature produced by Jupiter is $100° K \times (2/6)^2 = 11° K$. This is a fairly small temperature compared with common receiving system temperature in general, and remains roughly constant as a function of wavelength from 5 centimeters longwards.

The link from earth to Jupiter suffers somewhat from the environmental microwave noise. This radiation is obviously directive as viewed from the earth, but is nevertheless quite broad compared with the likely beam for any communication antennas mounted on the vehicle.

This I assume will not exceed a very few degrees. The characteristic electron radiation pattern by itself is probably comparatively large; the different perspectives of electrons that can radiate toward a given point within the radiation belts is larger still. Since at 10 centimeters the observed surface brightness of the microwave source against Jupiter corresponds to about 100° K, I shall assume this is also the surface brightness of the source as seen from within Jupiter's radiation belts, say on the surface of Jupiter. In view of the fact that the receiving antenna has a beam that is filled with this 100° K radiation, I conclude that the antenna temperature for an antenna on the "surface" of Jupiter and pointed toward the earth will be 100° K also. As the vehicle moves out through the belts in the plane of the equator, the antenna temperature should remain constant at this value, until a distance of 1.5 to $2.0R_J$ from Jupiter's center is reached. From that point outward, to about $3R_J$, the antenna temperature should fall rapidly to zero. This antenna temperature increases as the square of the wavelength for all wavelengths longwards from about 5 centimeters.

The full effects of this noise appear at vehicles looking at the earth from points near Jupiter (within 2 radii of the center) in the equatorial plane. In a polar orbit or fly-by, when the vehicle is over the poles of the planet, the signals from earth will be observed against an essentially cold sky background.

4.2 *VLF, LF, and HF wave fields*

There are no data bearing on the strengths of very low or low frequency electromagnetic waves that may be present within Jupiter's magnetosphere. It has been argued from time to time that decametric emission represents an analogue to whistler-mode waves within the earth's magnetosphere. For many reasons this point of view doesn't appear correct to me. For that reason, as well as the lack of independent observational evidence, I shall not attempt an estimate of VLF or LF field strengths for Jupiter's magnetosphere. An experiment covering wide dynamic and spectral ranges aboard fly-bys seems crucial.

Estimates of field strength in regions near Jupiter's decametric sources are important not only because they give insight into the physical nature of the generating mechanism. They also may influence the design of electronic circuitry on orbiting or fly-by vehicles near Jupiter. For this purpose an estimate of field strengths near Jupiter appears more appropriate than of equivalent blackbody antenna temperature.

Any such estimate needs to appear with great emphasis on the variability of the sources as they are observed from the earth. Furthermore, different spectral ranges (especially below 5 or 10 MHz) which cannot be observed, undoubtedly will provide many surprises. Finally, the sources are known to be highly directive, which implies that vehicles that are making approaches nearby to Jupiter, insofar as they fly through different latitude ranges than can reach the earth, may encounter very different conditions than these estimates imply.

All observers agree on a nominal intense Jupiter event at opposition reaching a flux density of the order of 10^{-20} watts m^{-2} Hz^{-1}. Many events are weaker; a few may be stronger. There is some evidence that some very intense short-lived fluctuations reach at their peaks the highest flux densities of all, but that their average over periods of many fluctuations work out not to exceed this nominal event.

If we agree on this figure, our estimates of the total power involved in the waves at the source may still vary widely according to our different ideas on bandwidth and directivity of the emission. A strong event may infrequently reach bandwidths as large as 10 MHz at this flux level. Also, it may be confined to a beam as narrow as 5° total cone vertex angle, in a right circular cone. In that case, the total radio frequency power involved is 2×10^8 watts. Much larger estimates, as high as 10^{10} or 10^{11} watts, appear in the literature. They seem to be based on hemispheric directivity patterns. My own estimates on other occasions have been sometimes one order of magnitude smaller and, on page 75, 20 times larger; they were based on a narrower assumed bandwidth for these smaller values, and exceptionally intense events in the larger. The present value is designed to be conservative in the sense of representing an upper limit for commonly observed events. An absolute upper limit would be the $4\text{x}10^{16}$ ergs $\sec^{-1}$ of page 75. Admittedly, 10 MHz is a broad region over which to observe this radiation at as high a typical flux level as 10^{-20} watts m^{-2} Hz^{-1}.

To come to a field strength in the source requires a knowledge of the energy density of the radiation, U. We find this from the expression $U = W/cA$, where A is the area of the source on the surface of Jupiter. I have previously computed (Warwick, 1967) U on the basis of $A = (10^4 \text{ km})^2$, but today regard this as probably much too large in view of interferometric upper limits of 400 km on the source dimensions (Dulk, 1969); this small an area also appears better to satisfy the requirements of the very short duration millisecond bursts sometimes observed in decametric emission which, however, have not been observed interferometrically. With this value of a typical source dimension, $A = 1.6\text{x}10^{15}$ cm^2. Then $U = 2\text{x}10^{15}$ erg $\sec^{-1}/(3\text{x}10^{10}$ cm $\sec^{-1}$ x $1.6\text{x}10^{15}$ $cm^2) = 4\text{x}10^{-11}$ erg cm^{-3}. The corresponding field strength comes from $U = E_o^2/8\pi$, where E_o is in e.s.u. For the quoted value of U, $E_o = 3\text{x}10^{-5}$ e.s.u., or about one volt per meter. For the most intense events, this value should be increased to about three volts per meter.

To compute the field strength at distances from the source requires knowledge of how the energy flux varies with distance. At the earth, we infer that the radiation moves in something like a cone of 5° vertex angle. Within this cone, as we move back to Jupiter, the flux should increase as the inverse square of the distance from the source region, until we come within distances from the source comparable to the source dimensions, 400 km. This means that for virtually all mission planning purposes, it may be correct to assume HF fields varying as the inverse of $(R/R_J)-1$ where R is the vehicle distance from Jupiter's center. I find the field strength, $E_o \lesssim 1.7\text{x}10^{-2}/[(R/R_J)-1]$ volts m^{-1} for distances such that $(R - R_J) \geq 400$ kilometers; at 400 kilometers $E \sim 3$ volts m^{-1}. The spectrum over which this large field is distributed can vary widely with time, and, especially, position near Jupiter. Furthermore, the relative positions of Jupiter I (Io) and Jupiter itself are vital. Other satellites also may play roles, as yet undetected.

The upper limit of radio frequencies to which these fluxes and field strengths apply is 40 MHz. This value applies strictly only near Jupiter's equatorial plane, within a very few degrees of zero latitude. At latitudes greater than 15° or 20° north or south, the low-frequency emission has not been observed; furthermore, a prediction of its properties there is essentially unfounded today. The lower limit to frequencies that have been observed is in the neighborhood of 5 MHz. Reliable phenomenology at this frequency does not exist owing to the difficulties of observation from ground level. No space vehicle has detected Jupiter at broadcast frequencies or below, where the ionosphere cuts off propagation to the ground of astronomical sources.

Refraction and reflection effects at VHF frequencies and less will be very great (as is indicated by the high values of the frequency quoted earlier). Absorption may be small, corresponding to the long wavelength of these radiations compared with the resonant frequencies of ammonia and methane gases in microwave regions. This prediction should be valid into regions of relatively high pressure (say a few atmospheres). So far as the magnetosphere is concerned, there appear to be no significant propagation effects even at low frequencies.

5. HYDROMAGNETIC WAVE FIELDS

Low-frequency wave fields in Jupiter's magnetosphere certainly exist as a result of the interaction of Jupiter's magnetosphere with the soalr wind. They explicitly are involved in our previous estimates of L-shell diffusion characteristics within the magnetosphere. Be that as it may, Jupiter's magnetosphere contains other sources of low-frequency fields that are unique to it, and require special discussion here.

The innermost Galilean satellite Io strongly modulates decametric emission at all frequencies observed from the ground (Wilson, et al., 1968a). Similar modulations might on general principles be expected from the other Galilean satellites, especially the next one out, Europa (and the tiny satellite, Amalthea). They have, however, not been established independently of Io's strong influence (Wilson, et al., 1968b). The connection between the distant satellite and Jupiter's ionosphere lies, according to everyone, in hydromagnetic disturbances propagating from Io to Jupiter. Perhaps I should note that the exception to the preceding statement is theories that depend on decametric emission coming from the immediate vicinity of the satellite. As the previous section emphasizes, however, this seems very unlikely.

These theories of hydromagnetic connections take two forms representing extremes. On the one hand, Goldreich and Lynden-Bell (1969) and Piddington and Drake (1968) formulate direct-current models of Io's interaction based on the "unipolar dynamo" action of the conducting satellite as the rotating magnetic field of Jupiter cuts through it. The solid body of Io, and the magnetic flux tube (one half leading to the north, the other to the south) connecting it to Jupiter's ionosphere, plus the connection across the ionosphere from the north half of the flux tube to its south half constitute a current loop essentially fixed to Io; the ionospheric foot of this tube moves westward at the angular velocity corresponding to the difference in angular rates of Jupiter's rotation and Io's orbital motion. On the other hand Ellis (1965) and Warwick (1967) propose that Io creates waves just because it is an obstacle to the rotational motion of Jupiter's magnetosphere. These waves propagate through Jupiter's magnetosphere; their interaction with the ionosphere creates decametric emission. The source, Io, presents a large obstacle of mild conductivity and probably also of non-unit magnetic permeability at which the magnetospheric lines of force must be to some degree distorted. The propagation away from Io of the resulting Alfvén waves may initially lie in a low amplitude disturbance; at a distance from Io, the waves must steepen into shocks.

These two kinds of theories contrast particularly in the time dependence involved. The DC model is essentially a steady-state model except in the parts of the current loop near the ionosphere where current sheet instabilities arise at the gyrofrequency. The wave model is essentially unstable in that waves are generated moving in *all* directions away from Io; steepened by their propagation in a compressible medium, these waves interacting with the ionosphere create the emission.

On the one hand, the two feet of Io's flux tube are singled out as the source of decametric emission. On the other hand, decametric emission occurs wherever the propagation conditions in Jupiter's ionosphere, especially the orientation of the lines of force, direct the radiation into the ecliptic plane toward the earth.

Wave modes in which Io generates hydromagnetic energy lie not only in the mode traveling strictly along the lines of force, but also in the oblique modes that propagate into all directions. The magnetosonic modes do not contain freely propagating energy (Schmahl, 1970). The longitudinal mode is in effect collimated, while the oblique modes spread through space and are weakened according to an inverse-square law.

The treatment of the Io effect along the analogy of the unipolar dynamo requires that wave effects be negligible. Goldreich and Lynden-Bell observe that waves moving from Io to Jupiter's surface and back consume about one minute for the round trip. Most of the time is spent near Jupiter in the ionosphere. During this time Io moves relative to a corotating point in Jupiter's magnetosphere a distance 54 km $\sec^{-1}$ x 60 sec = 3240 km, a distance about equal to its diameter, 3340 km. One criterion to evaluate the DC analogy is whether Io's motion during the round trip is larger or smaller than its diameter. Since the criterion is clearly borderline it might be that the Io-Jupiter system represents a resonant region for longitudinal Alfven waves. In other words, the wavelike nature of the disturbance may be an essential part of the problem.

Goldreich and Lynden-Bell use a different criterion. They ask for the relative angular velocity of the lines of force past Io that results from Io's finite resistance. According to their formulae, if Io has infinite resistance, the lines of force slip by it at the angular rate of Jupiter's magnetosphere relative to Io. If Io has zero resistance, their formula predicts that the lines of force do not move relative to the satellite. They show that if Io's conductivity is larger than about 10^{-8} mho cm^{-1}, the latter condition is met. This may be smaller than the actual conductivity of Io, perhaps much smaller.

These conditions can be restated directly in terms of the dynamo model. If Io represents an open circuit with respect to the ionospheric conductivity across its flux tube, then no current flows in the flux tube, and there is no way for the satellite to excite decametric emission (in their model of the process). If Io is to create a current, it must produce a short circuit across the load resistance represented by the ionosphere.

The condition that Io's conductivity is very large carries implications for the hydromagnetic wave interactions. For example, the classic diffusion time of magnetic field into an object of Io's dimension is $\tau \sim 8\pi\, \sigma_{Io}\, R_{Io}{}^2/c^2$, where α_{Io} is the volume conductivity of Io in e.s.u. and R_{Io} is the radius of Io. Goldreich and Lynden-Bell quote $\sigma \lesssim 10^{-7}$ mho cm^{-1} or $\sigma \lesssim 0.9 \times 10^{5}$ sec^{-1} (in e.s.u.) for the moon. Assuming the same σ for Io, I find that $\tau \lesssim 68$ seconds. This implies diffusion times that may be comparable to the round trip Alfvén wave travel time to Jupiter. The Alfvén speed in Jupiter's magnetosphere is $V_A = B/\sqrt{4\pi\rho}$ where ρ is the mass density there and B is the local magnetic field in gauss. For the predicted field near Io, about 0.07 gauss, and for densities of the order of 10 protons cm^{-3}, $V_A \sim 0.1c$. At this speed a disturbance crosses to Jupiter and back in 16 seconds. If σ_{Io} is larger than this value, then τ could be much longer than the round-trip travel time.

It is misleading to state that a large enough conductivity for Io results in no slippage of the lines of force with respect to the satellite. If its conductivity is essentially infinite, the lines of force will be completely excluded from Io, and will cleave around the satellite as Io moves with respect to the magnetospheric plasma. This condition was assumed to exist by me (Warwick, 1967), in my discussion of the nature of the Io effect. In a sense, the lines of force do not slip through in this case, as they do in the case of a non-conducting satellite, but bend externally around Io.

The unipolar dynamo model requires, among other things, that the motion of the lines of force through Io create a polarization field that cancels the $v \times B$ field of the motion if Io is an open-circuited conductor.

There is, therefore, ambiguity between the non-conductive and non-permeable satellite which moves through the magnetosphere without bending the lines of force, and the perfectly conductive satellite which cleaves its way by diverting the lines of force out of its path. A satellite so sufficiently conductive that it has less resistance than the ionosphere is required to prevent open-circuiting the dynamo action. But, it cannot be so conductive that it distorts the lines of force and excludes them from its interior. The first condition is, of course, a standard one in electrical technology. The second condition is, so far as I know, never important on earth since the conductivity even of copper is small enough in comparison to typical conductor dimensions that the finite field-diffusion time doesn't enter the problem of describing a terrestrial dynamo at AC frequencies.

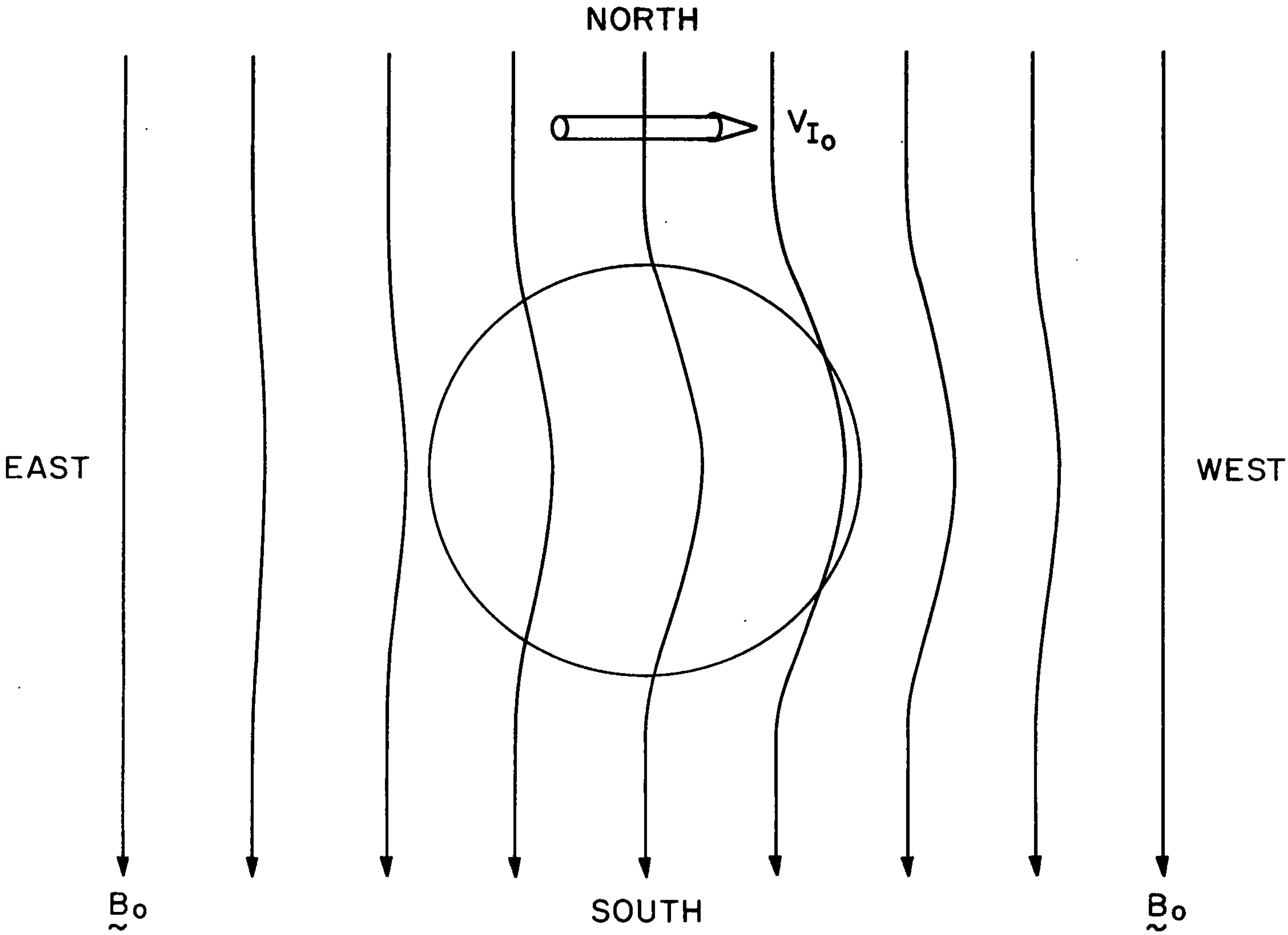
To avoid open circuiting this unipolar dynamo requires an Io conductivity greater than 10^{-8} mho cm^{-1}, but to avoid its diffusion time problems requires a conductivity less than 10^{-7} mho cm^{-1}. We should not conclude that Io's conductivity is between 10^{-7} and 10^{-8}!

A general and rigorous quantitative description of the total phenomenon is impossible at this time. However, a semi-quantitative description including both the hydromagnetic wave processes and the DC current effect seems worthwhile. The problem is to include the Piddington, Drake, Goldreich, and Lynden-Bell current loop in the same model as the Alfvén wave source, both of these under consideration of a realistic range of estimates of satellite conductivity and permeability.

First, consider the current loops created within Io by the field lines dragged through the satellite as it moves relative to the magnetosphere. Figure 5 illustrates the distortion schematically. The lines of force are compressed on Io's leading (e.g., westward) side, and are expanded on its eastward edge. The energy in the field enhancement derives from the kinetic energy of Io.

This distorted field corresponds to the presence of currents flowing within Io. Fields generated by these currents add to the field independently present to produce the distorted field. This current system within Io flows even if there is no external, conducting plasma surrounding the satellite. The current flows across Io away from Jupiter in the leading, westward portions of Io as seen from Jupiter, and also in the trailing, eastward portions. The current flows toward Jupiter across the middle of Io. The field on the leading side of Io is thus enhanced, and on the trailing edge decreased.

For a satellite with finite conductivity, this description implies that there are ohmic losses. This loss of power occurs at the expense of Io's kinetic energy through the field. Unless there is some external power source, such as gravitational energy, Io's motion cannot then be in a steady state but must always slow down. The effect is, however, for practical purposes unimportant on planetary time scales.



'igure 5. The distorted magnetic field created by the mildly conducting satellite Io moving through Jupiter's magnetic field. The view is as Io is seen from Jupiter. The magnetic flux density is greater at the western edge of Io than at its eastern edge. [The figure is similar to one published by Piddington and Drake (1968)].

Within Io, the magnetic field through the central regions essentially is the undisturbed ambient field B_o. The electric field strength corresponding to Io's motion through B_o is $v_{Io}B_o/c$, in e.s.u., through the central regions of Io. This corresponds to a current density $\sigma_{Io}v_{Io}B_o/c$.

In this order of magnitude calculation, set the cross-sectional area of Io that carries this current equal to πR_{Io}^2. The total current (flowing toward Jupiter) is

$$I_{Io} = \pi R_{Io}^2 \frac{\sigma_{Io}}{c} v_{Io}B_o.$$

This current flows in a path of total resistance measured by $(\sigma_{Io} R_{Io})^{-1}$. The ohmic losses are then $I_{Io}^2/\sigma_{Io} R_{Io}$.

We now set these losses equal to the rate at which Io sweeps up the energy density in the field at its leading edge, as though the enhanced field there were waiting in space to be caught. If U is this energy density, given by $B^2/8\pi$ where B is the actual field at the leading edge, then the rate of field sweeping is $\pi R_{Io}^2 v_{Io} U$. Setting this power equal to the ohmic losses within Io as previously estimated, we find

$$\pi R_{Io}^2 v_{Io} U = I_{Io}^2/\sigma_{Io} R_{Io}.$$

This expression reduces to

$$B^2 = (8\pi^2\sigma_{Io} v_{Io} R_{Io}/c^2) B_o^2.$$

The previous expression for a diffusion time, τ_o, follows from this equation when we set $v_{Io} = R_{Io}/\tau$. Then $\tau = (8\pi\sigma_{Io}\pi R_{Io}^2/c^2)(B_o/B)^2$. This can be rewritten $\tau/\tau_o = B_o^2/B^2$ where

$$\tau_o = \frac{8\pi\sigma_{Io} \pi R_{Io}^2}{c^2}.$$

Here τ_o is the time that would be required for B to decrease to B_o if Io were suddenly to be stopped in its motion through the field.

What happens to the magnitude of the current as a result of the presence of external, short-circuiting paths connecting one side of Io to the other through the two long paths to Jupiter's ionosphere and return? These represent two paths essentially in parallel with the previously-mentioned return paths through the leading and trailing side of Io. Which path the current follows depends on the relative conductivity of these three paths. If the ionospheric conductivity is large compared with that of Io, the current will follow the two long paths. If it is small, the current returns through Io itself. The condition that Io's conductivity be large, compared with that of the ionospheric circuits, so that the current attains the value computed by Goldreich and Lynden-Bell is identical to the condition required to make the currents close within Io itself. There is an upper limit to the current that flows through the long circuit to Jupiter and back; the limit is reached when Io's and the ionosphere's conductivities are equal. When Io's conductivity exceeds this value, the currents close within Io.

For these reasons, I shall ignore the DC path to Jupiter in favor of the DC loops within Io, and estimate the energy created by Io in the form of Alfvén waves. I have previously calculated this wave amplitude on the basis of an infinitely-conducting satellite. While this computation still appears to me correct in principle, it does not consider the very poor conductivity of Io; this correction leads to a much smaller wave amplitude. Within τ_o = 214 seconds (= 68 seconds x π), and τ = 31 seconds, $B^2/B_o^2 = \tau_o/\tau$ = 214/31. Therefore, $B = 2.6\ B_o$.

I set the minimum radius of curvature of the characteristic Alfvén sine waves created by Io equal to the size of the current-containing region responsible for the B-field. This is a natural boundary condition of the problem. The wave amplitude is $B = 2.6\ B_o$. The smallest radius of curvature of this sine-wave is B_o/kB where k is the propagation constant of the wave. The radius of curvature is to be R_{Io}. Then, $k = B_o/BR_{Io}$. The value of the wave frequency is $\omega_A = kv_A$ where v_A is the Alfvén speed. If there are 10 proton-electron pairs per cm^3, and if B_o = 0.06 gauss, then $v_A = 4 \times 10^9$ cm sec^{-1}. The Alfvén wave period is 0.7 seconds, comparable to decametric modulation times.

I shall estimate the power carried in Alfvén waves as $P_A = \pi R_{Io}{}^2 \, v_A \, B^2/8\pi = 4x10^{16}$ watts. The ratio of this power to P_J, the power dissipated in Jupiter's ionosphere, is

$$P_A/P_J = \frac{\pi^2}{4} \; \frac{v_A}{v_{Io}} \quad \frac{\sigma_{Io} \, R_{Io}}{\Sigma_J} \; .$$

Here Σ_J is Jupiter's ionospheric conductivity, for which I take the value used by Goldreich and Lynden-Bell, 0.57 mho. With $\sigma_{Io} = 10^{-7}$ mho cm^{-1}, then $P_A/P_J = 6x10^4$. This value of P_J is the one appropriate to Goldreich's and Lynden-Bell's mechanism operating in parallel to the one presented here. Dissipated at the rate $4x10^{16}$ watts, Io's orbital kinetic energy would be exhausted in about 10^7 years. Finally, the Io source of Alfvén waves undoubtedly creates disturbances into all directions. The power in Alfvén waves intercepted by an area on Jupiter the size of the decametric source, say 400 km square, is $3x10^9$ watts. This estimate assumes strictly isotropic wave emission, but is comparable to decametric energy requirements.

These waves propogate outward from Io in all directions. It is likely that similar waves are created by the other Galilean satellites, and to a lesser extent, by Jupiter V, Amalthea. This system of waves, transformed into shocks within some nominal distance of the satellites, must propagate sporadically throughout Jupiter's magnetosphere. Disturbances of this sort are in addition to the same kinds of waves that are present within the earth's magnetosphere, viewed as an analogue to Jupiter's magnetosphere.

APPENDIX

Here is a tabulation of the ratio L/M for a number of astronomical objects (angular momentum L in erg-sec, and magnetic dipole moment M in gauss-cm^3):

Object	L (c.g.s.)	M (c.g.s.)	L/M (c.g.s.)	Source for M
Moon	1.5×10^{36}	$\leq 3\times10^{21}$	$\geq 0.5\times10^{15}$	no magnetopause (surface B $\leq 50\gamma$)
Mars	1.5×10^{39}	$\leq 8\times10^{23}$	$\geq 2\times10^{15}$	Mariner 4 (no magnetopause, 13,000 km; B $\leq 30\gamma$)
Venus	$L_{\oplus}/243$	$\leq 8\times10^{23}$	$\geq 0.4\times10^{15}$	Mariner 5 (no magnetopause, 10,000 km; B $\leq 80\gamma$)
Earth	6×10^{40}	8×10^{25}	0.7×10^{15}	Allen (1963)
Jupiter	4×10^{45}	4×10^{30}	1×10^{15}	This report
Sun	1.7×10^{48}	0.35×10^{33}	5×10^{15}	Assume surface field = 1 gauss
AO_p Star	$10^2\ L_{\odot}$	$5\times10^3\ M_{\odot}$	1×10^{14}	Surface field of 2,000 gauss, $\Omega_{*} = 6\Omega_{\odot}$, $R_{*} = 2.5\ R_{\odot}$, and $(\text{mass})_{*} = 6(\text{mass})_{\odot}$
Our Galaxy	4×10^{72}	10^{60}	4×10^{12}	See below

NB: $L_{Galaxy} = (10^{10}\text{ stars}) \times (2\times10^{33}\text{ gm}) \times (10^{44}\text{ cm}^2) \times \dfrac{6.28}{3\times10^{15}\text{sec}} = 4\times10^{72}$ c.g.s.

$M_{Galaxy} = (10^{-6}\text{ gauss}) \times (10^{66}\text{ cm}^3) = 10^{60}$ c.g.s.

Note that the ratio L/M lies within a 100-fold range of 10^{15} c.g.s., for L varying over a 10^{14} -fold range. This observation is of long standing (Babcock, 1948).

One hypothesis for this limited range may be that hydromagnetics of circulation within rotating objects requires about the same convective-element size. That hypothesis seems unlikely since some of the objects on this list may be solid to their core. These same objects, i.e., the moon and Mars, may also have much smaller magnetic moments than the upper limits derived for the table. A better upper limit for each of them, and for Venus, clearly is highly to be desired.

A second hypothesis was initiated many years ago by Babcock (1948) and Blackett (1947). A fundamental property of angular momentum in massive objects would be a magnetic field. Their hypothesis appears now to have a much richer empirical basis than it could have in 1947, with not only the earth, the sun, and magnetic stars (which owing to their variability appeared to be its points of greatest disparity) on the list, but now also Venus, the Moon, Mars, Jupiter, and the Galaxy. Especially, the solar and Jupiter points are useful.

A modification of the early picture is required today. Cosmical objects contain electrically conducting fluids performing motions of great complexity. These must distort and modify any fundamental magnetic field that rotating matter might in general possess, and create variations in the L/M ratio from object to object on the list. Especially, the external topology should vary strongly from object to object.

A comment about the spinning electron may be worthwhile. It possess a magnetic moment, such that the ratio of its angular momentum, $\hbar/2$, to the Bohr magneton's magnetic moment, $\mu = \hbar e/2mc$, is mc/e. The numerical value of

$$(L/M)_{(\text{electron})} = \frac{9\text{x}10^{-28} \text{ x } 3\text{x}10^{10}}{4.8\text{x}10^{-10}} = 5\text{x}10^{-8} \text{ c.g.s.}$$

Whatever may be the merit of the hypothesis, it does not refer to spinning charges in any direct way.

A final comment concerns rotating neutron stars, probably the sources of pulsar radiation. Their angular momentum is about 10^{46} c.g.s.; the ratio apparently established above in many astronomical objects suggests a magnetic moment of 10^{31} c.g.s. The corresponding surface field on a pulsar regarded as a neutron star will be about 10^{13} gauss. Values of fields on neutron stars suggested from other points of view range almost as high as this (10^{12} gauss, Gold, 1968).

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