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KUNITOMO SAKURAI

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THE STABILITY OF SUNSPOT MAGNETIC FIELDS AND THE ORIGIN OF SOLAR FLARES

by

Kunitomo Sakurai*
Radio Astronomy Branch
Laboratory for Extraterrestrial Physics
NASA, Goddard Space Flight Center
Greenbelt, Maryland 20771

ABSTRACT

The steady motion in sunspot magnetic regions is considered for both current-free and force-free configurations. The sufficient condition for stability is obtained in the presence of both external current-free and force-free magnetic fields and a steady motion. It is shown that the pattern of such steady motion is most important in triggering an instability of sunspot magnetic fields, both for the current-free and force-free configuration. When there is no steady motion, the current-free configuration of sunspot magnetic fields is always stable, whereas the stability in the case of force-free magnetic fields is connected with the configuration. The onset of a solar flare seems to be associated with an instability connected to the steady motion within the sunspot magnetic regions.

*NAS-NRC Associate with NASA

INTRODUCTION

It has recently been shown that most solar proton flares take place within the δ-type sunspot groups (e.g., Sawyer, 1965). Furthermore, Sakurai (1967, 1969) suggests that the twisting motion of sunspot magnetic lines of force is related to the origin of solar proton flares. He also discusses the relation of the δ-type configuration with the stability of sunspot magnetic fields.

The configuration of sunspot magnetic fields has been thought to be force-free (e.g., Parker, 1957; Gold and Hoyle, 1960; Longmire, 1963; Gold, 1964, 1968), but there is no valid supporting observational evidence. However, it is estimated that the magnitude of the electromagnetic force is much greater than that of the pressure and gravitational forces within sunspot magnetic fields (Lüst and Schlüter, 1955; Gold and Hoyle, 1960). Accordingly, the hypothesis that the configuration of sunspot magnetic fields is force-free appears reasonable. As has been shown by Sakurai (1967), the steady fluid motion within sunspot groups, which is connected with the convection near the photosphere, must be important in generating an instability related to the origin of solar flares.

In this paper, we will first discuss steady motion within sunspot groups and some problems related to the configuration of sunspot magnetic fields. Then, the stability of a force-free configuration of sunspot magnetic fields will be discussed. A brief discussion will also be given of the mechanism of solar proton flares.

1. THE STEADY MOTION IN SUNSPOT MAGNETIC FIELDS AND THEIR FORCE-FREE CONFIGURATION

The types of steady motion in sunspot magnetic fields will be considered. Fundamental equations appropriate to this problem are

$$n_c m_e (\vec{v}_{e,o} \cdot \nabla) \vec{v}_{e,o} = - \nabla P_{e,o} - n_o e \left(\vec{E}_o + \frac{1}{c} \vec{v}_{e,o} \times \vec{H}_o\right) + n_o m_e \vec{g} \quad (1)$$

$$n_o m_i (\vec{v}_{i,o} \cdot \nabla) \vec{v}_{i,o} = - \nabla P_{i,o} + n_o e \left(\vec{E}_o + \frac{1}{c} \vec{v}_{i,o} \times \vec{H}_o\right) + n_o m_i \vec{g} \quad (2)$$

$$\nabla \cdot n_o \vec{v}_{e,o} = 0 \quad (3)$$

$$\nabla \cdot n_o \vec{v}_{i,o} = 0 \quad (4)$$

and

$$\nabla \times \vec{H}_o = \frac{4\pi}{c} n_o e (\vec{v}_{i,o} - \vec{v}_{e,o}), \quad (5)$$

where $\vec{v}$, m, n, $\vec{E}$, $\vec{H}$, $\vec{P}$, $\vec{g}$ and c are the velocity, mass, number density, electric and magnetic fields, pressure, gravity force and the speed of light, respectively. The subscripts, i, e, and o denote ion, electron and steady state. The collision term is neglected because the mean free path of particles is so large that the medium is highly conductive.

The gyro-frequency of ions is denoted as $\Omega_{i,o}$ ($= eH_o/m_i c$) and the characteristic length of the sunspot group is designated as L. If the condition

$$|\vec{v}_{i,o}|, \ |\vec{v}_{e,o}| << \Omega_{i,o} L \tag{6}$$

is satisfied, the non-linear terms of eqs. (1) and (2) can be neglected (Namikawa and Matsushita, 1970). With the values of $H_o = 10^2 \Gamma$ and $L = 10^9$ cm, the order of magnitude of $\Omega_{i,o} L$ is 8.8×10^{13} cm sec^{-1} which is much larger than c. Thus, we can always neglect the non-linear terms in eqs. (1) and (2).

Thus,

$$-\nabla P_{e,o} - n_o e(\vec{E}_o + \frac{1}{c} \vec{v}_{e,o} \times \vec{H}_o) + n_o m_e \vec{g} = 0 \tag{7}$$

$$-\nabla P_{i,o} + n_o e(\vec{E}_o + \frac{1}{c} \vec{v}_{i,o} \times \vec{H}_o) + n_o m_i \vec{g} = 0 \tag{8}$$

By adding these two equations, we obtain

$$- \nabla P_o + \frac{1}{c} \vec{j}_o \times \vec{H}_o + \rho_o \vec{g} = 0, \tag{9}$$

where the total pressure, $P_o = P_{i,o} + P_{e,o}$, the mass density, $\rho_o = n_o (m_i + m_e)$ and the electric current density, $\vec{j}_o = n_o e(\vec{v}_{i,o} - \vec{v}_{e,o})$. The above equation gives the hydrostatic equilibrium condition.

When we solve eqs. (7) and (8) with respect to $\vec{v}_{e,o}$ and $\vec{v}_{i,o}$, respectively, we have the following equations:

$$\vec{v}_{e,o} = \frac{c}{H_o^2} (\vec{E}_o \times \vec{H}_o) + \frac{c}{H_o^2} \frac{1}{n_o e} \nabla P_{e,o} \times \vec{H}_o - \frac{cm_e}{eH_o^2} \vec{g} \times \vec{H}_o \times \alpha_e \vec{H}_o \tag{10}$$

and

$$\vec{v}_{i,o} = \frac{c}{H_o^2} (\vec{E}_o \times \vec{H}_o) - \frac{c}{H_o^2} \frac{1}{n_o e} \nabla P_{i,o} \times \vec{H}_o + \frac{cm_i}{eH_o^2} \vec{g} \times \vec{H}_o + \alpha_i \vec{H}_o, \tag{11}$$

where α_i and α_e are arbitrary scalars. Thus, the electric current density is given by

$$\vec{j}_o = n_o\, e(\vec{v}_{i,o} - \vec{v}_{e,o}) = - \frac{c}{H_o^2} (\nabla P_o \times \vec{H}_o) + \frac{\rho_o c}{H_o^2} (\vec{g} \times \vec{H}_o) + n_o\, e(\alpha_i - \alpha_e)\, \vec{H}_o. \quad (12)$$

On the right hand side of this equation, the first two terms give the electric current perpendicular to the external magnetic field $\vec{H}_o$, whereas the third term gives the electric current along $\vec{H}_o$ and is recognized as a force-free electric current. It is clear that this electric current $\vec{j}_o$ is independent of the electric conductivity of the medium treated here (Cowling, 1953; Sakurai, 1968).

If we define the electric currents perpendicular to and along $\vec{H}_o$ as $\vec{j}_{o,\perp}$ and $\vec{j}_{o,\parallel}$, respectively, eq. (5) can be expressed as

$$\nabla \times \vec{H}_o = \frac{4\pi}{c} (\vec{j}_{o,\perp} + \vec{j}_{o,\parallel}). \quad (13)$$

If the contribution from the pressure and gravitational forces is negligible as has been numerically estimated (e.g., Lüst and Schlüter, 1955; Gold, 1968), $\vec{j}_{o,\perp}$ will be negligible. This leads to the conclusion that the general steady motion in sunspot magnetic fields is a force-free motion:

namely,

$$\nabla \times \vec{H}_o = \frac{4\pi}{c} n_o e(\alpha_i - \alpha_e) \vec{H}_o. \tag{14}$$

In this case, it follows from eqs. (10) and (11) that

$$\vec{v}_{e,o} = \frac{c}{H_o^2} (\vec{E}_o \times \vec{H}_o) + \alpha_e \vec{H}_o \tag{15}$$

and

$$\vec{v}_{i,o} = \frac{c}{H_o^2} (\vec{E}_o \times \vec{H}_o) + \alpha_i \vec{H}_o. \tag{16}$$

We furthermore obtain, from the Maxwell equation $\nabla \times \vec{E}_o = 0$,

$$\nabla \times (\vec{v}_o \times \vec{H}_o) = 0, \tag{17}$$

where $\vec{v}_o$ is the mean plasma velocity perpendicular to $\vec{H}_o$, which is easily calculated from eqs. (15) or (16) and equal to (c/H_o^2) $(\vec{E}_o \times \vec{H}_o)$ in the present case. The continuity equations (3) and (4) reduce to

$$\nabla \cdot (n_o \vec{v}_o + \alpha_i n_o \vec{H}_o) = 0 \tag{18}$$

$$\nabla \cdot (n_o \vec{v}_o + \alpha_e n_o \vec{H}_o) = 0 \tag{19}$$

The equations obtained above ((14) - (19)) are applicable to the steady motion associated with the force-free configuration of sunspot magnetic fields.

If $\alpha_i = \alpha_e$ ($= \alpha$), we have a current-free motion determined by the following equations:

$$\nabla \times \vec{H}_o = 0 \tag{20}$$

$$\nabla \times (\vec{v}_o \times \vec{H}_o) = 0 \tag{17}$$

and

$$\nabla \cdot (n_o \vec{v}_o + \alpha n_o \vec{H}_o) = 0 \tag{21}$$

In this case, the configuration of sunspot magnetic fields is derived from a scalar potential.

It is likely, however, that the configuration of sunspot magnetic fields is of the force-free type (e.g., Anzer, 1968; Zwann, 1968). In order to deal with the stability problem of both current-free and force-free magnetic fields, we must consider the results obtained above for the steady state and their perturbation.

2. HYDROMAGNETIC STABILITY

2-1 General

The hydromagnetic stability of the steady motion of a plasma will be considered under the influence of an external magnetic field. The equations of motions appropriate to this problem are

$$\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_{e,o} \cdot \nabla)\, \vec{v}_e + (\vec{v}_e \cdot \nabla)\, \vec{v}_{e,o} = - \frac{e}{m_e} (\vec{e} + \frac{1}{c} \vec{v}_{e,o} \times \vec{h} + \frac{1}{c} \vec{v}_e \times \vec{H}_o), \tag{22}$$

$$\frac{\partial \vec{v}_i}{\partial t} + (\vec{v}_{i,o} \cdot \nabla)\, \vec{v}_i + (\vec{v}_i \cdot \nabla)\, \vec{v}_{i,o} = \frac{e}{m_i} (\vec{e} + \frac{1}{c} \vec{v}_{i,o} \times \vec{h} + \frac{1}{c} \vec{v}_i \times \vec{H}_o), \tag{23}$$

where $\vec{v}_i$, $\vec{v}_e$, $\vec{h}$ and $\vec{e}$ are respectively ion and electron perturbed velocity, magnetic and electric field of the perturbation. The second and third terms on the left hand side of eqs. (22) and (23) can be neglected in comparison with the second and third terms on the right hand side, when

$$|\vec{v}_{e,o}|,\ |\vec{v}_{i,o}| \ll L\, \Omega_{i,o} \tag{6}$$

$$|\vec{v}_e|,\ |\vec{v}_i| \ll L\, \Omega_i \tag{24}$$

and

$$\Omega_i = \frac{e\, h}{m_i\, c}.$$

We assume here that the change of magnetic field strength associated with a solar flare is of the order of 1ι as a lower limit. Even in this case, $\Omega_i = 1.8 \times 10^7$ rad/sec and then $\Omega_i\, L \simeq 1.8 \times 10^{16}$ cm/sec. Since both $|v_e|$ and $|v_i|$ must be smaller than c, the condition (24) is always satisfied.

Using the Maxwell equation

$$\nabla \times \vec{e} = -\frac{1}{c}\frac{\partial \vec{h}}{\partial t}, \qquad (25)$$

we obtain hydromagnetic equations from eqs. (22), (23), (5) and (14) where small terms proportional to $(\partial/\partial t)\ \nabla \times \vec{h}$ and $(\nabla \times \vec{h}) \times \vec{h}$ are neglected (e.g., Spitzer, 1962; Alfvén and Fälthammer, 1963) as follows:

$$\rho_o \frac{\partial \vec{v}}{\partial t} = \frac{1}{4\pi}(\nabla \times \vec{h}) \times \vec{H}_o + \frac{n_o e}{c}(\alpha_i - \alpha_e)\ \vec{H}_o \times \vec{h} \qquad (26)$$

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{v} \times \vec{H}_o) + \nabla \times (\vec{v}_o \times \vec{h}) \qquad (27)$$

or

$$\vec{e} + \vec{v}_o \times \vec{h} + \vec{v} \times \vec{H}_o = 0, \qquad (27)'$$

where

$$\vec{v} = \frac{n_o}{\rho_o}(m_i\ \vec{v}_i + m_e\ \vec{v}_e) \qquad (28)$$

The second term of the right hand side of eq. (26) describes the effect due to the force-free magnetic field.

By differentiating eq. (27) with respect to time t, and making use of eq. (26), we have

$$\frac{\partial^2 \vec{h}}{\partial t^2} = \frac{1}{4\pi\ \rho_o}\ \nabla \times \{[(\nabla \times \vec{h}) \times \vec{H}_o] \times \vec{H}_o + \frac{4\pi\ n_o e}{c}(\alpha_i - \alpha_e)(\vec{H}_o \times \vec{h}) \times \vec{H}_o\} + \nabla \times (\vec{v}_o \times \frac{\partial \vec{h}}{\partial t}). \qquad (29)$$

Let us assume that $\vec{v}$ and $\vec{h}$ are proportional to exp (ωt). Scalar multiplication of eq. (29) with $\vec{h}$ and integrating over a volume V enclosed by a surface S, gives

$$\omega^2 \int_V \vec{h}^2 \, dV = - \omega \int_S \{\vec{h} \times (\vec{v} \times \vec{H}_o)\} \cdot d\vec{S}$$

$$- \int_V \frac{(\nabla \times \vec{h})^2 \vec{H}_o{}^2}{4\pi \rho_o} dV + \omega \int_V \vec{v}_o \cdot \{\vec{h} \times (\nabla \times \vec{h})\} \, dV$$

$$- \omega \int_S \vec{h} \times (\vec{v}_o \times \vec{h}) \cdot d\vec{S}$$

$$+ \frac{n_o e}{\rho_o c} (\alpha_i - \alpha_e) \int_V \vec{h} \cdot \nabla \times \{(\vec{H}_o \times \vec{h}) \times \vec{H}_o\} \, dV, \tag{30}$$

where we have used the identities

$$\nabla \cdot \left[\vec{h} \times \left\{\left(\frac{\nabla \times \vec{h}}{4\pi \rho_o}\right) \times \vec{H}_o) \times \vec{H}_o\right\}\right.$$

$$= \left[\left\{\frac{(\nabla \times \vec{h})}{4\pi \rho_o} \times \vec{H}_o\right\} \times \vec{H}_o\right] \cdot (\nabla \times \vec{h}) - \vec{h} \cdot \nabla \times$$

$$\left\{\left(\frac{\nabla \times \vec{h}}{4\pi \rho_o}\right) \times \vec{H}_o) \times \vec{H}_o\right\}$$

and

$$\nabla \cdot \{\vec{h} \times (\vec{v}_o \times \vec{h})\} = (\vec{v}_o \times \vec{h}) \cdot \nabla \times \vec{h} - \vec{h} \cdot \nabla \times (\vec{v}_o \times \vec{h})$$

together with eq. (26) and Gauss's theorem. According to eq. (27)',the first and forth terms on the right hand side of eq. (30) are reduced to

$$\omega \int_S (\vec{h} \times \vec{e}) \cdot d\vec{S} \tag{31}$$

In the outer space W - V of the sunspot magnetic regions, where W denotes the whole space, we obtain

$$\begin{aligned} \int_{W-V} \nabla \cdot (\vec{h} \times \vec{e}) \cdot dV &= - \int_S (\vec{h} \times \vec{e}) \cdot d\vec{S} \\ &= \int_{W-V} \vec{e} \cdot \nabla \times \vec{h} \, dV - \int_{W-V} \vec{h} \cdot \nabla \times \vec{e} \, dV \\ &= \omega \int_{W-V} \vec{h}^2 \, dV \end{aligned} \tag{32}$$

where we have used eq. (25) and the condition

$$\nabla \times \vec{h} = 0 \text{ in } W - V \tag{33}$$

This indicates that perturbation current flows only in the volume V.

Eq. (30) is thus reduced to

$$I_1 \omega^2 + I_2 \omega + I_3 = 0, \tag{34}$$

where

$$I_1 = \int_V \vec{h}^2 \, dV \tag{35}$$

$$I_2 = \int_V \vec{v}_o \cdot \{(\nabla \times \vec{h}) \times \vec{h}\} \, dV \tag{36}$$

and

$$I_3 = \int_V \frac{1}{4\pi \rho_o} (\nabla \times \vec{h})^2 \cdot \vec{H}_o^2 \, dV +$$

$$\int_V \frac{n_o e}{\rho_o c} (\alpha_i - \alpha_e) \vec{H}_o^2 \, \vec{h} \cdot \nabla \times \vec{h} \, dV, \qquad (37)$$

where we have used the identity

$$\vec{h} \cdot \nabla \times \{(\vec{H}_o \times \vec{h}) \times \vec{H}_o\} = \vec{H}_o^2 \, \vec{h} \cdot \nabla \times \vec{h} \qquad (38)$$

The first term of I_3 is always positive, but the sign of the second term is dependent on $(\alpha_i - \alpha_e) \, \vec{h} \cdot \nabla \times \vec{h}$.

In order to study the necessary and sufficient condition for stability, we solve eq. (34) with respect to ω:

$$\omega = \frac{1}{2I_1} (-I_2 \pm \sqrt{I_2^2 - 4 I_1 I_3}) \qquad (39)$$

Thus, the necessary condition for stability is

$$I_2 > 0 \text{ and } I_2^2 - 4 I_1 I_3 \leq 0 \qquad (40)$$

or

$$I_2 = 0 \text{ and } I_3 > 0. \qquad (41)$$

Under the condition (41), we have a pure oscillation of frequency ω given by

$$\omega = \sqrt{- \frac{I_3}{I_1}} \qquad (42)$$

2-2 Current-free fields

In this case, the second term of eq. (37) is neglected since

$$\nabla \times \vec{H}_o = 0. \qquad (20)$$

It thus follows that I_3 is always positive. Consequently, the condition for stability is given by eqs. (40) and (41). In order that the current-free magnetic configuration is unstable, the following condition must be satisfied:

$$I_2 = \int_V \vec{v}_o \{(\nabla \times \vec{h}) \times \vec{h}\} \, dV < 0. \quad (43)$$

If we use a dyadic notation (Jackson, 1962), this result can be written as

$$I_2 = \int_V \vec{v}_o \cdot (\nabla \cdot \overleftrightarrow{T}) \, dV, \quad (44)$$

where

$$\overleftrightarrow{T} = \vec{h}\,\vec{h} - \frac{1}{2} \overleftrightarrow{I} h^2 \quad (45)$$

and

$$\overleftrightarrow{I} = \vec{e}_1\vec{e}_1 + \vec{e}_2\vec{e}_2 + \vec{e}_3\vec{e}_3.$$

$\vec{e}_1$, $\vec{e}_2$ and $\vec{e}_3$ denote the unit vectors of Cartesian coordinate. In general, the equation of momentum change in a magnetic field is given by

$$\frac{d\vec{P}}{dt} = -\nabla \cdot \overleftrightarrow{T}$$

Therefore, the term $\vec{v}_o(d\vec{P}/dt)$ $(-\vec{v}_o \cdot (\nabla \cdot \overleftrightarrow{T}))$expresses the rate of work due to the motion $\vec{v}_o$. Thus, I_2 gives the rate of the work done by the steady motion $\vec{v}_o$ for the whole space where this motion is observed.

The inequality (43) shows that the above rate must be negative. This means that the initial magnetic state is not at energy minimum, but tends to move to a lower energy state whenever a perturbation is introduced. We can estimate the sign of the integral I_2 by considering both modes of magnetic perturbation and steady motion. We can conclude that the pattern of steady motion is most important in triggering an instability of the sunspot magnetic fields.

2-3. Force-free Fields

When eq. (37) is not negative, the criterion for the stability of current free magnetic fields can be applied to that for force-free magnetic fields. In this case, when $I_2 < 0$, the configuration of force-free fields is also unstable provided there is some steady motion.

When there is no steady motion (and therefore $I_2 = 0$), we obtain another criterion for the instability of force-free magnetic fields such as

$$I_3 < 0. \tag{46}$$

Consequently, the following inequality must be satisfied:

$$\int_V \frac{1}{4\pi \rho_o} (\nabla \times \vec{h})^2 \cdot \vec{H}_o^2 \, dV + \int_V \frac{n_o e}{\rho_o c} (\alpha_i - \alpha_e) \vec{H}_o^2 \vec{h} \cdot (\nabla \times \vec{h}) \, dV < 0. \tag{47}$$

Since the first term is always positive in this equation, the second term must be negative and its absolute value must be greater than the first term.

3. IMPORTANCE OF STEADY MOTION ON THE STABILITY OF SUNSPOT MAGNETIC FIELDS

As discussed in the last section, the steady fluid motion plays an important part in generating the instability of sunspot magnetic fields, both current-free and force-free. As we know, the growth and decay of sunspot magnetic fields is well connected with the convective flow near the photospheric surface (e.g., Parker, 1955; Babcock, 1961; Sakurai, 1967). The pattern of this flow seems to be most effective in generating the instability of sunspot magnetic fields.

At present, the configuration of sunspot magnetic fields is thought to be force-free (e.g., Lüst and Schlüter, 1955; Gold and Hoyle, 1960; Sakurai, 1967; 1969; Anzer, 1968). If the induced force-free current $|n_o e(\alpha_i - \alpha_e) \vec{h}|$ is much smaller than the induction current $|c (\nabla \times \vec{h})/4\pi|$, the effect of the ambient force-free current seems to be negligible in the problem of the stability of sunspot magnetic fields even if their configuration is force-free. Therefore, the criterion for the stability given by eqs (40) and (41) can be applied to the problem of the stability of force-free magnetic fields. The fluid motion is both necessary and sufficient to produce the instability of both current-free and force-free magnetic fields as discussed.

The development of sunspot magnetic fields is associated with the convective motion near the photosphere (e.g., Parker, 1955; Babcock, 1961; Sakurai, 1967). This motion may be responsible for the generation of sunspot magnetic instability. An instability produced in this way seems to be connected with the triggering of a solar flare. As suggested by Sakurai, (1967, 1970), the ambient convective fluid motion may be associated with the rotating motion of sunspot magnetic regions and the type of sunspot groups. The fact that the onset of solar proton flares is well correlated with this rotating motion suggests that some steady ambient fluid motion is very important in triggering solar flares.

4. CONCLUDING REMARKS

It has been shown that the steady motion plays an important part in the stability of sunspot magnetic fields. Such motion may be associated with the growth and decay of sunspot groups and also related to the convective flow near the photospheric surface within sunspot magnetic regions.

The steady motion within sunspot magnetic regions and its pattern must be studied in detail when we deal with the stability problems of sunspot magnetic fields. The results obtained here may have application to other astrophysical problems.

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