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A GENERAL FORMULA FOR ARTIFICIAL ELECTRON DECAY LIFE-TIMES

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Abstract

A treatment based on the analysis of data from Satellite 1963 38C has been developed to calculate the decay lifetimes of the artificial electron belt produced by the Starfish nuclear explosion. The treatment makes use of experimental data and describes the decay life time τ in the form of a three-dimensional continuum in B, L and energy space. The results are valid over the entire inner zone of the trapped radiation belt. Rèference maps of constant τ contours in B-L, B-E, and L-E space have been constructed and are discussed.

Introduction

Seven artificial radiation belts have been made by the explosion of high-altitude nuclear devices since 1958. Of these the Starfish event of July 9, 1962, a low latitude nuclear explosion, produced an intense region of trapped artificial electrons with very long lifetimes.

The spatial and temporal characteristics of the artificial radiation zone have received considerable treatment in the literature, and the decay lifetimes for the injected electrons have been discussed by several authors [Van Allen, 1964; Cladis et al., 1965; Bostrom and Williams, 1965; Brown, 1966; McIlwain, 1966; Paulikas et al., 1967; Beall et al., 1967; Pfitzer, 1968; Bostrom et al., 1970]. The picture that emerges is that of a fast decay at very low L values ($L \leq 1.2$), which is consistent with the prediction of atmospheric scattering theory [Walt, 1964]. Short lifetimes also occur at the outer edge of the inner radiation zone in the slot region. For the intermediate L values, the decay is slower with longest lifetimes occurring at $L \sim 1.4$. At low L values ($1.15 \leq L \leq 1.4$), a B dependence is also observed [Beall et al., 1967].

During the years since 1962, computer programs were developed at Goddard Space Flight Center to estimate the flux decay of artificially injected electrons and the radiation hazards to which satellites would be exposed [Stassinopoulos, 1965, 1967]. The decay lifetimes used in the calculation of orbit-integrated, vehicle-encountered particle fluxes were based on fragmentary published data, and as such they were necessarily restricted and not very accurate [Stassinopoulos, 1965]. In this paper we present a unified treatment for the decay of the artificial electrons of the inner zone by expressing the decay lifetime τ as a continuous function of B, L and energy E. The data used in the analysis is essentially taken from a single satellite, 1963 38C, and covers the time span from launch, September 28, 1963, to December 1968 when, it is felt, the energy channels of the satellite measured essentially the radiation of the natural background [Bostrom et al., 1970]. The satellite description and instrumentation can be found in the literature [Williams and Smith, 1965; Bostrom et al., 1967; Beall et al., 1967].

Discussion

In order to facilitate the handling of an otherwise difficult and complex problem, the following simplifying assumptions are made:

(a) That τ remains constant in time.

(b) That solar cycle, diurnal, and secular-change effects can be neglected.

(c) That the decay of the Starfish electrons can be approximated by an exponential form.

(d) That the B-dependence of τ does not extend beyond $L \geq 1.40$.

Assumption (c) is justified by the 1963 38C data; assumption (d) is supported by comparison with OGO data which measures a wider range of equatorial pitch angles than 38C [Pfitzer, 1968; Pfitzer and Winckler, 1968].

When ordering the data in terms of the variable L, three distinct regions emerge (see Fig. 1), each identified by a characteristic functional dependence of τ on the parameters B, L, and E. We denote the three regions as I, II, and III. In region I, which extends from $L = 1.15$ to $L = 1.40$ (for all electron energies), τ is a function of all the variables: the field strength B, the magnetic shell parameter L, and the energy E. The B dependence of the decay constant has been previously discussed by

Beall et al., (1967), and it appears to become indeterminate for $L \geq 1.4$. Consequently, in our treatment, we will take τ in region II to depend only on L and E. Region II extends from $L > 1.4$ to approximately $L \sim 2.3$, with the latter boundary being energy dependent. Curves, obtained for region II from the APL data by plotting τ vs E for all three channels at given L-values, suggest that the high energy electrons exhibit the most rapid decay and that at the time of the 1963 38C measurements almost no artificials with energies $E \geq 2$ Mev remained in regions of $L \geq 1.8$. This is in general agreement with currently prevailing opinion [Pfitzer, 1968; Vette, private communication, 1970]. Thus, in generating τ vs E plots for region II ($2.3 \geq L > 1.4$) we have not made use of the data from the 1963 38C channel 3 ($E \geq 2.4$ Mev) and have instead extended the dependence linearly to the higher energies, using only data obtained from channels 1 ($E \geq 0.28$ Mev) and 2 ($E \geq 1.2$ Mev). This is consistent with the derivation of the decay lifetimes obtained by Beall et al., (1967) and reflects the uncertainty in the channel 3 data due to background proton contamination. At the higher L values ($L > 1.8$) in this region, the data were adjusted to fit reported measurements [Van Allen, 1966; McIlwain, 1966; Brown, 1966].

Finally in region III, which includes all L-space beyond $L \sim 2.3$, the decay lifetime was taken to be only a function of energy. The τ's in this latter region are typically of the order of one month or less and, as stated previously, will be neglected in this treatment. Figure 1 outlines the domains of the regions discussed above for electrons with $E \geq 0.28$ Mev, and indicates the dependence of τ on the variables appropriate to each domain.

In formulating the problem, a four dimensional space is considered in which the decay lifetime τ is a continuous function of the variables B, L and energy E. The boundaries beyond which computation of τ is discontinued are:

(i) The equatorial envelope.

(ii) The radiation belt terminus.

(iii) The $\tau = 0$ surface

where the equatorial envelope is as defined by McIlwain (1961); and where the radiation belt terminus is designated as the constant electron intensity contour of one particle per square centimeter per second as given for trapped electrons with energies $E \geq 0.5$ Mev by Vette et al., 1966.

It is assumed that the terminus is time and energy independent in B, L space, since any slight changes in its position due to energy dependence or secular variations do not affect the derivations and conclusions presented in this treatment. The four dimensional space is then subdivided into volume elements such that within each volume, the parameter τ is a linear function of all its variables (i.e., all partial derivatives of τ are constant).

Since τ depends mainly on the magnetic shell parameter L, appropriate increments in L have to be taken so as to preserve the linearity requirement within each volume element. Thus, region I is subdivided into 11 L-intervals and region II in 19 L-intervals. Figure 2 shows the dependence of τ on the variables in region I for the three energy channels of satellite 1963 38C. In the next two figures, several cross sections of the τ domain in region I are plotted. Figure 3 shows four B-L cross sections for E ≥ 0.25, 0.5, 1, and 3 Mev; and Figure 4 shows several B-E cross sections at discrete L-values. In the latter figure, the B dependence of τ at these low L values is clearly evident. Finally, the τ dependence on L and E in region II is shown in Figure 5. The slowest decay in this model occurs at L ~ 1.6 for E ~ 0.3 Mev and gradually drifts toward L ~ 1.4 at the higher integral energies. Curves for τ are not plotted for E < 0.2 Mev because there is evidence that the low energy component (E ~ 0.1 Mev) of the integral electron flux does not manifest a perceptible decay [Pfitzer, 1968; Pfitzer and Winckler, 1968].

Data Processing

A generalized treatment for all three variables, such as pertaining to region I, will be given here. The same treatment, appropriately modified to deal with only two variables, applies to region II. In general,

$$\tau = \tau(B, L, E) \tag{1}$$

The following relations then hold within each volume element:

$$\begin{aligned} &\tau = K(B,L)E + \lambda(B,L) \\ &K = \frac{\partial\tau}{\partial E} = \mu(L)B + \nu(L) \quad ; \quad \nu = \gamma L + \eta \\ &\mu = \frac{\partial^2\tau}{\partial B\,\partial E} = \alpha L + \delta \quad ; \quad \alpha = \frac{\partial^3\tau}{\partial L\,\partial B\,\partial E} \end{aligned} \tag{2}$$

and

$$\lambda = \pi(L)B + \sigma(L) \qquad ; \quad \sigma = \zeta L + \phi$$

$$\pi = \frac{\partial\lambda}{\partial B} = \beta L + \epsilon \qquad ; \quad \beta = \frac{\partial^2\lambda}{\partial L\,\partial B} \tag{3}$$

Combining the expressions we obtain a general function for the mean lifetime within the volume element

$$\tau(B,L,E) = \alpha BLE + \beta BL + \gamma LE + \delta BE + \epsilon B + \zeta L + \eta E + \phi \tag{4}$$

The coefficients of the variables, B,L and E form a matrix and are found experimentally, for a particular volume element, by a linear fit to the 1963 38C data. For those intermediate volume elements for which coefficients cannot be determined due to sparse or insufficient data points, the known coefficient values from adjacent bracketing volume elements are linearly extrapolated, so as to match all slopes at the respective boundaries. Thus, a continuous functional relationship of the decay lifetime τ on its variables B,L and E is generated for the inner zone. Table 1 gives the values of the 30 sets of coefficients in the respective L-ranges.

Conclusions

In using the results presented in this treatment, the following considerations should be taken into account:

(a) Decay lifetimes should not be extrapolated backward too close to the epoch of the nuclear event because the initial decay (first few weeks) is considerably faster than at later times, which are considered in the present analysis.

(b) The estimated maximum value of the error for the calculated τ is ~ 15 days at intermediate L values ($1.3 \leq L \leq 2.0$). In this region the decay lifetime is of the order of several hundred days. At low and high L values ($L \leq 1.15$ and $L \geq 2.2$), where the τ's are typically of the order of a few weeks, the estimated error is ≤ 5 days.

(c) The functional dependence of τ on energy has been derived using integral energy channels, and may differ from decay constants obtained from differential energy measurements.

(d) At low energies ($E \leq 0,15$ Mev), the validity of the energy dependence of τ becomes marginal. There is evidence that electrons of these energies do not manifest a decay [Pfitzer, 1968; Pfitzer and Winckler, 1968] but are mainly part of the natural component of the radiation belt. At the higher integral energies, the treatment should not be extrapolated beyond $E \geq 6$ Mev.

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TABLE 1

Coefficients of the Lifetime Function

Volume Element	L-Range	α	β	γ	δ	ε	ζ	η	φ
1	1.15 - 1.16	6710.0	24520.0	- 1236.0	- 8529.46	- 4439.40	- 31067.7	1532.58	5778.45
2	1.16 - 1.17	6322.0	19470.0	- 1150.0	- 8074.70	- 2174.50	- 24849.9	1433.12	3151.15
3	1.17 - 1.18	6323.0	15000.0	- 1579.0	- 10415.91	- 1500.00	- 19620.0	1935.05	2362.00
4	1.18 - 1.20	6564.5	12080.0	- 1176.5	- 8364.48	- 963.10	- 16174.4	1460.10	1729.36
5	1.20 - 1.25	4203.6	11854.0	- 776.2	- 5507.40	- 1070.74	- 15903.2	979.74	1858.29
6	1.25 - 1.28	3520.67	10383.3	- 657.13	- 4653.73	- 878.33	- 14064.8	830.91	1617.78
7	1.28 - 1.30	2770.0	9020.0	- 520.8	- 3692.88	- 563.25	- 12319.8	656.40	1578.48
8	1.30 - 1.32	1812.0	8030.0	- 406.3	- 2447.48	- 820.80	- 11032.8	507.55	1543.29
9	1.32 - 1.34	1485.5	7045.0	- 341.7	- 2016.50	- 506.20	- 9732.6	422.28	1128.02
10	1.34 - 1.38	578.0	5634.15	- 139.25	- 800.45	- 484.325	- 7842.06	150.995	1098.708
11	1.38 - 1.40	140.5	3346.7	- 18.0	- 196.70	- 216.95	- 4685.38	- 16.33	729.73
12	1.40 - 1.425	0	0	- 258.88	0	560.0	0	320.[illegible]	- 358.0
13	1.425 - 1.45	0	0	- 292.00	0	400.0	0	368.10	- 130.0
14	1.45 - 1.475	0	0	- 276.00	0	320.0	0	344.90	- 14.0
15	1.475 - 1.51	0	0	- 297.143	0	200.0	0	376.086	163.0
16	1.51 - 1.55	0	0	- 337.75	0	175.0	0	437.403	200.75
17	1.55 - 1.60	0	0	- 422.22	0	160.0	0	568.33	224.0
18	1.60 - 1.625	0	0	- 425.20	0	40.0	0	573.098	416.0
19	1.625 - 1.65	0	0	- 419.3[illegible]	0	- 80.0	0	563.545	611.0
20	1.65 - 1.70	0	0	- 358.34	0	- 220.0	0	462.928	842.0
21	1.70 - 1.75	0	0	- 321.38	0	- 400.0	0	400.096	1148.0
22	1.75 - 1.775	0	0	- 241.24	0	- 520.0	0	259.857	1358.0
23	1.775 - 1.82	0	0	- 147.778	0	- 644.444	0	93.956	1578.888
24	1.82 - 1.85	0	0	0	0	- 766.667	0	- 175.00	1801.334
25	1.85 - 1.87	0	0	75.00	0	- 850.0	0	- 213.75	1955.5
26	1.87 - 1.95	0	0	198.413	0	- 962.5	0	- 544.581	2165.875
27	1.95 - 2.00	0	0	222.60	0	- 980.0	0	- 591.697	2200.0
28	2.00 - 2.05	0	0	217.76	0	- 900.0	0	- 582.296	2040.0
29	2.05 - 2.10	0	0	212.18	0	- 740.0	0	- 570.857	1710.0
30	2.10 - 2.50	0	0	69.28	0	- 600.0	0	- 270.488	1418.0

Figure Captions

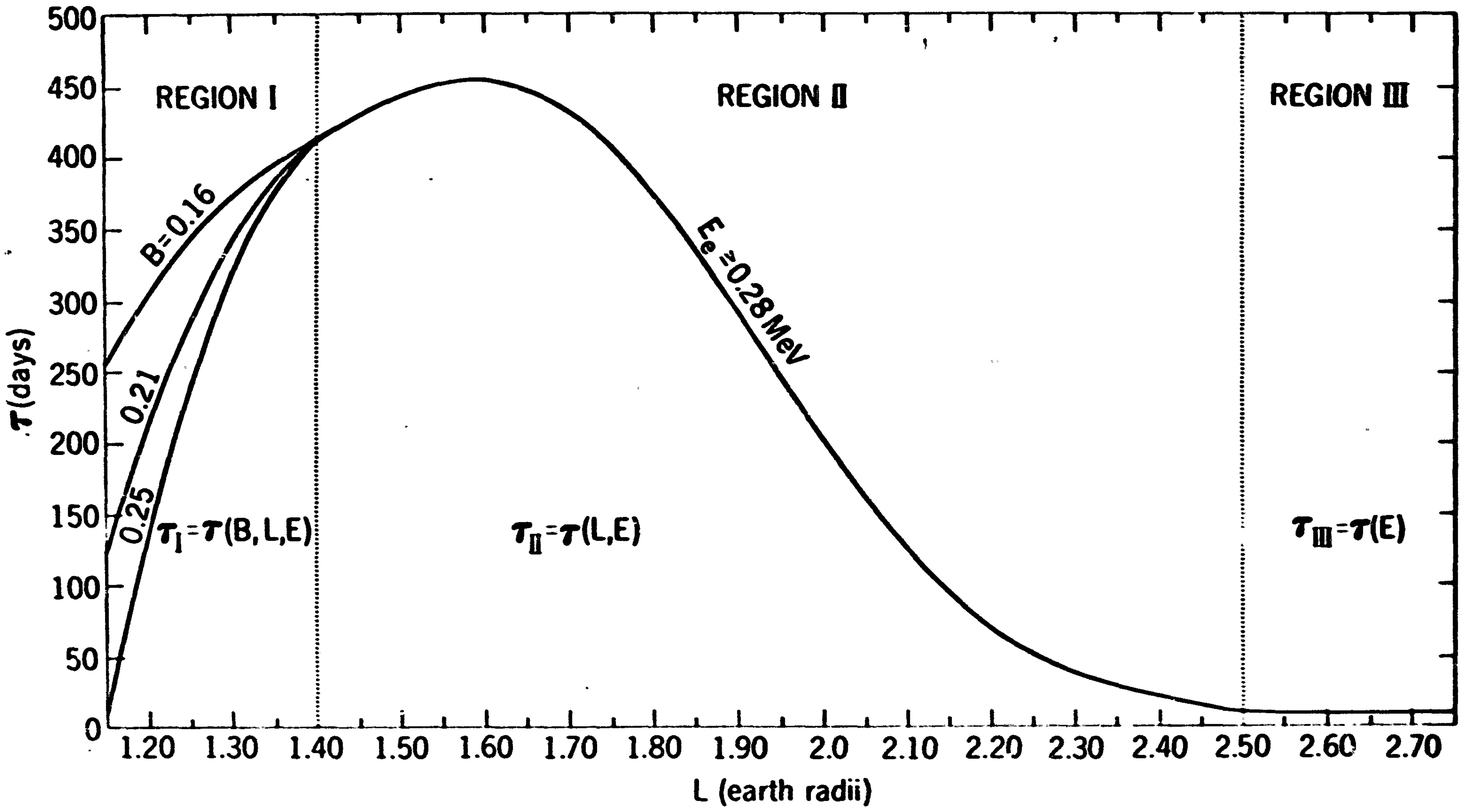
Figure 1. Domains of functional dependence of the decay lifetime on B, L and E for E ≥ 0.28 Mev.

Figure 2. Decay lifetimes in region I (1.15 ≤ L ≤ 1.4) showing the B, L dependence of τ for the 1963 38C satellite energy channels.

Figure 3. B-L cross sections of the τ domain in region I (1.15 ≤ L ≤ 1.4) for several integral energies.

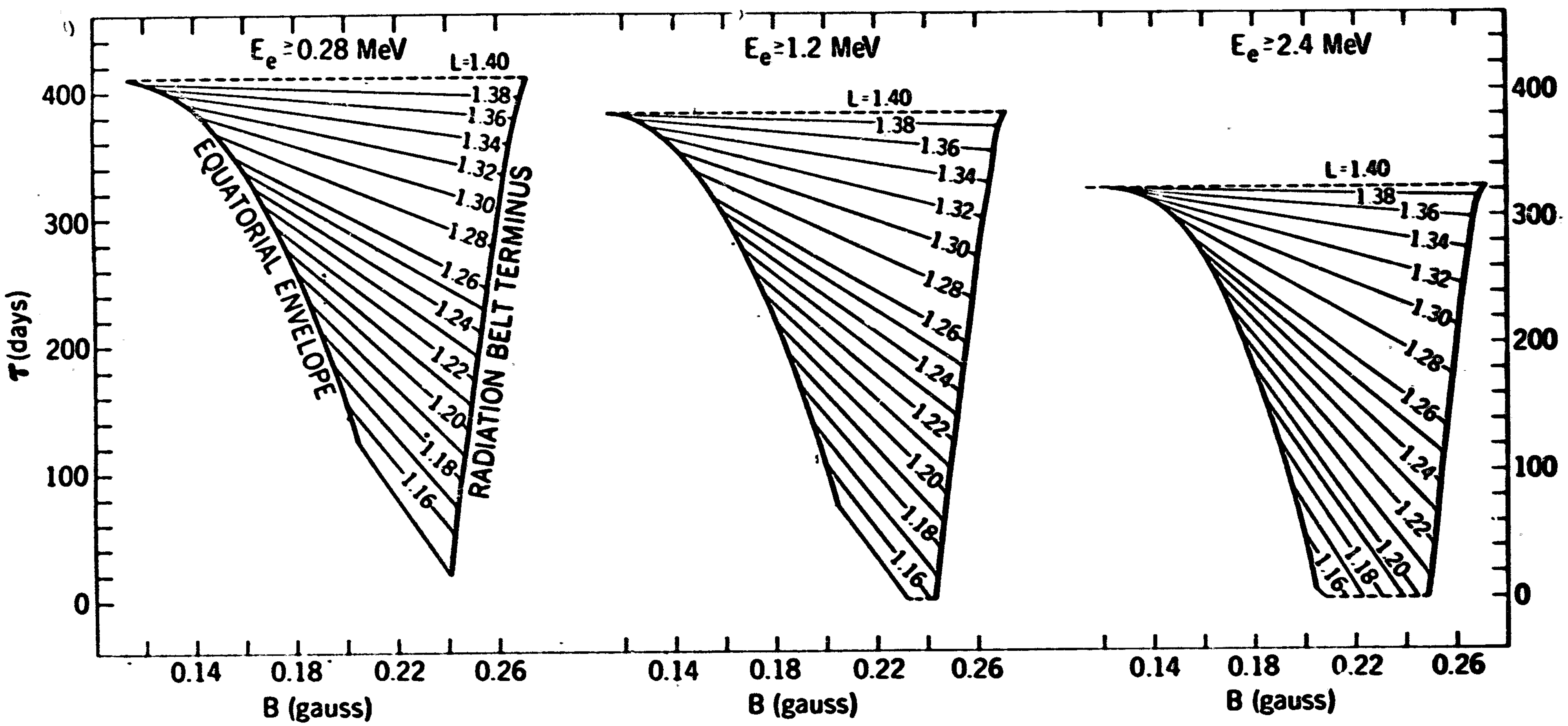
Figure 4. B-E cross sections of the τ domain in region I (1.15 ≤ L ≤ 1.4) at several L values.

Figure 5. Decay lifetimes in region II (1.4 ≤ L ≤ 2.3) for integral energies above 0.2 Mev.



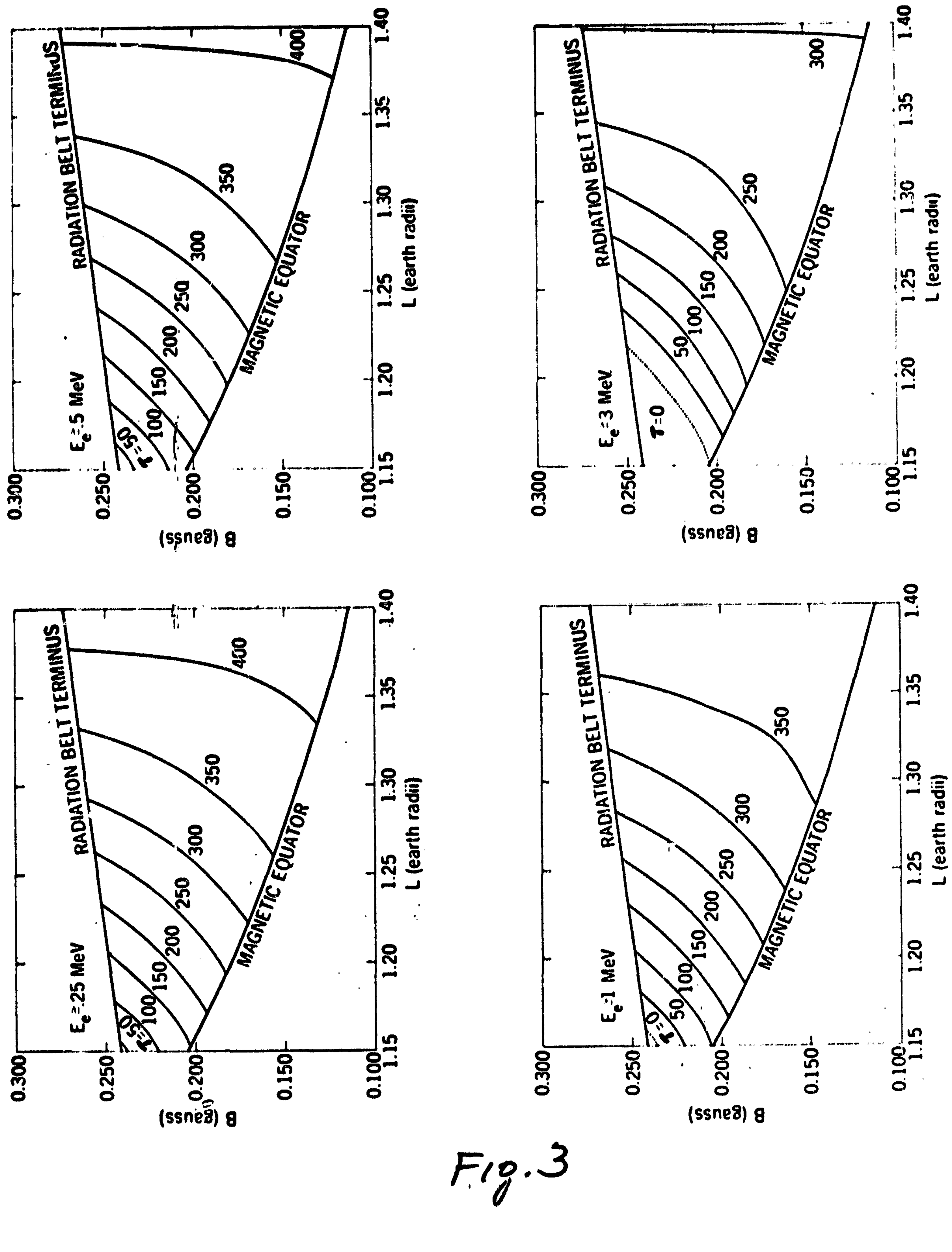
REGION I
REGION II
REGION III
B=0.16
0.21
0.25
$E_e \geq 0.28$ MeV
$\tau_I = \tau(B,L,E)$
$\tau_{II} = \tau(L,E)$
$\tau_{III} = \tau(E)$
τ (days)
L (earth radii)
500
450
400
350
300
250
200
150
100
50
0
1.20
1.30
1.40
1.50
1.60
1.70
1.80
1.90
2.0
2.10
2.20
2.30
2.40
2.50
2.60
2.70

Fig. 1



$E_e \geq 0.28$ MeV
$E_e \geq 1.2$ MeV
$E_e \geq 2.4$ MeV
L=1.40
1.38
1.36
1.34
1.32
1.30
1.28
1.26
1.24
1.22
1.20
1.18
1.16
EQUATORIAL ENVELOPE
RADIATION BELT TERMINUS
τ (days)
400
300
200
100
0
0.14
0.18
0.22
0.26
B (gauss)

Fig. 2



E_e ≥ 5 MeV
E_e ≥ .25 MeV
E_e ≥ 3 MeV
E_e ≥ 1 MeV
RADIATION BELT TERMINUS
MAGNETIC EQUATOR
τ = 0
50
100
150
200
250
300
350
400
B (gauss)
0.300
0.250
0.200
0.150
0.100
L (earth radii)
1.15
1.20
1.25
1.30
1.35
1.40

Fig. 3

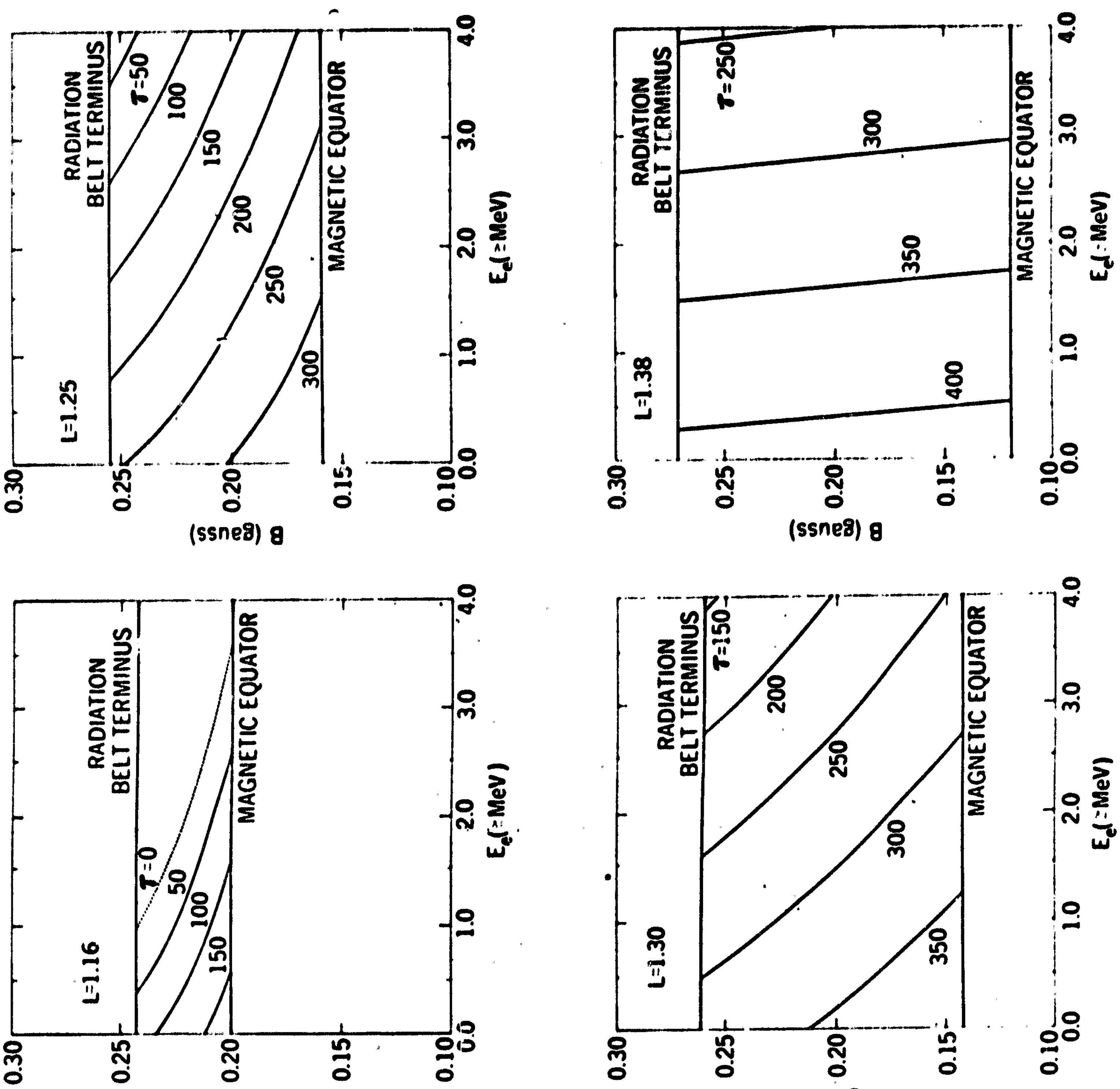
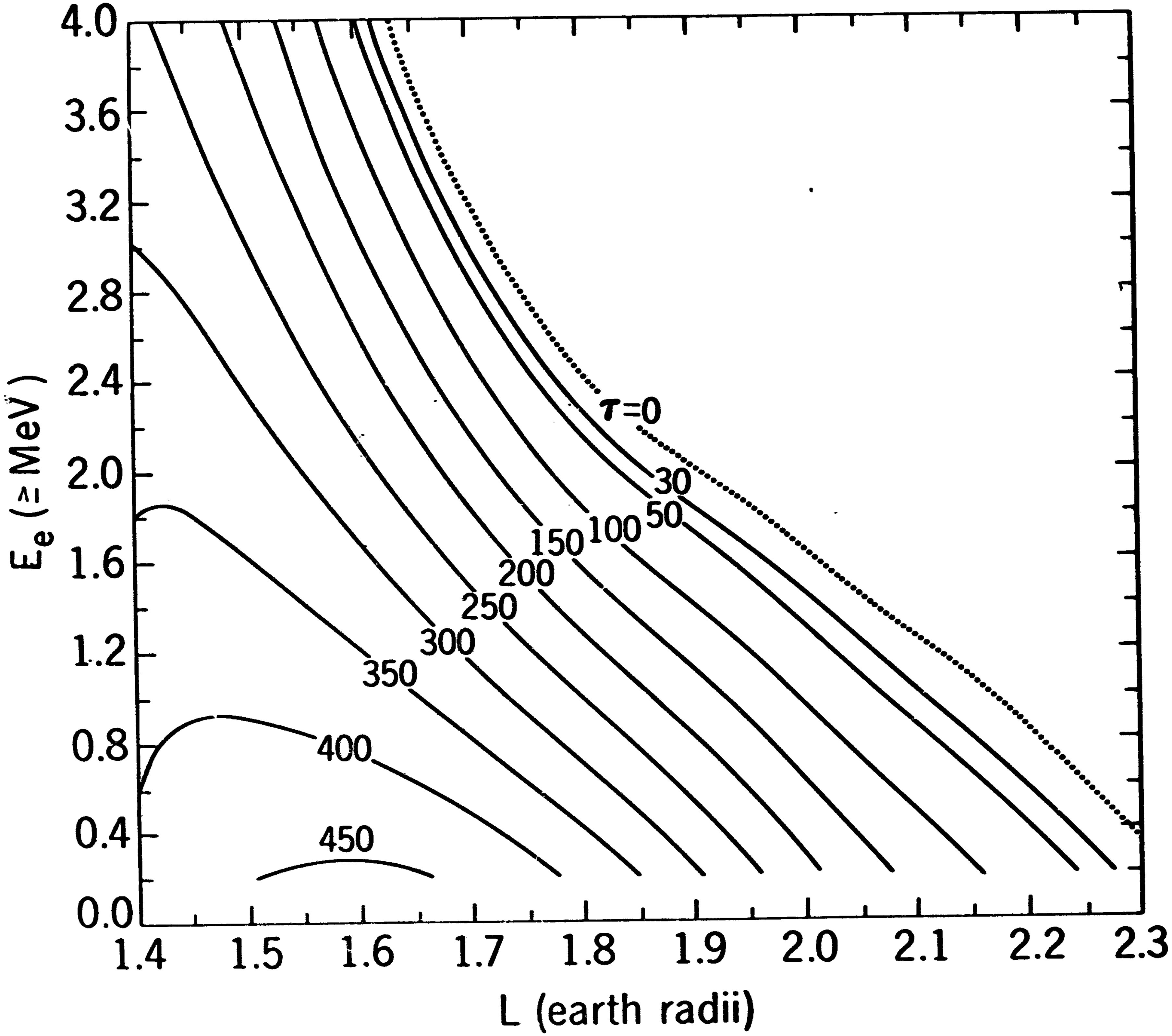

L=1.16
L=1.25
L=1.30
L=1.38
RADIATION BELT TERMINUS
MAGNETIC EQUATOR
B (gauss)
E_e (>MeV)
τ=0
τ=50
τ=150
τ=250
50
100
150
200
250
300
350
400
0.30
0.25
0.20
0.15
0.10
0.0
1.0
2.0
3.0
4.0

Fig. 4



τ=0
30
50
100
150
200
250
300
350
400
450
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0.0
E_e (≥ MeV)
1.4
1.5
1.6
1.7
1.8
1.9
2.0
2.1
2.2
2.3
L (earth radii)

Fig. 5

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