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AN APPROXIMATE DENSITY-EFFECT CORRECTION FOR THE IONIZATION LOSS OF CHARGED PARTICLES*

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Abstract

An approximate density-effect correction for determining the stopping power and range of high-energy charged particles is discussed and evaluated. For a variety of materials and all particle energies, using the approximate density-effect correction, which can be easily computed for both elements and compounds, overestimates the stopping power by $\lesssim 6\%$ and underestimates the range by $\lesssim 4\%$. This error should be acceptable in many practical problems that involve materials for which the correct density effect has not been evaluated.

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To determine the ionization produced by charged particles at high energies ($p/m_o c \gtrsim 1$), the Bethe-Bloch stopping-power formula[1] must be modified to account for the reduction in ionization caused by the polarization of the medium. This density-effect correction has been evaluated by Sternheimer for various elements and compounds.[2-6] Sternheimer fits the results of his calculations by the expressions

$$\delta = 0 , \qquad x < x_o \tag{1a}$$

$$\delta = 2\ln \eta^2 + C + a(x_1 - x)^m , \qquad x_o < x < x_1 \tag{1b}$$

$$\delta = 2\ln \eta^2 + C , \qquad x > x_1 \tag{1c}$$

where δ is the density-effect correction which enters the stopping-power formula, $x \equiv (\log_{10} e)\, 2\ln \eta = 0.43429\, 2\ln \eta$, $\eta \equiv p/m_o c$, and p is the momentum and m_o is the rest mass of the charged particle. The quantities a, m, x_o, and x_1 are constants which must be evaluated for each material, and C is given by

$$C = -2\, 2\ln(I/h\nu_p) - 1 , \tag{2}$$

where I is the mean ionization potential and $h\nu_p$ is the plasma energy of the material;

$$h\nu_p = h(ne^2/\pi m_e)^{\frac{1}{2}} , \tag{3}$$

where n is the electron density (electrons per cm^3), and m_e is the rest mass and e the charge of an electron.

The density-effect correction is often needed for materials not considered by Sternheimer.* Although the computation of δ for other materials

*It should be pointed out that although Sternheimer has evaluated the density effect for many elements these results in general cannot be utilized to determine the density effect for compounds containing these elements. This is because the density effect for compounds depends upon the electron configuration and density of each constituent element in such a manner that additivity does not hold.

is, conceptually, rather straightforward, the calculations are rather lengthy because of the large amount of data needed for the oscillator strengths, transition energies, etc. for each material. This is particularly true for some materials of practical interest (e.g., concrete) which contain numerous elements.

Essentially all of the effort in determining δ is involved in evaluating the quantities a, m, x_0, and x_1. However, the asymptotic portion of δ (Eq. 1c) can be easily evaluated since it depends only on the mean ionization potential and electron density of the medium. Combining Eqs. 1c, 2, and 3, the asymptotic form for the density-effect correction can be written as

$$\delta' = \ln(an\eta^2/I^2) - 1 , \qquad (4)$$

where

$$a \equiv h^2 e^2/\pi m_e .$$

For compounds (or mixtures*) the effective mean ionization potential can be determined by[4]

$$\ln I = \frac{1}{n} \sum_i n_i \ln I_i ,$$

where n_i is the electron density for the ith element and I_i is the corresponding atomic ionization potential. Thus the asymptotic form is as easily evaluated for compounds as for single-element materials.

Sternheimer has suggested that for some practical applications it may be adequate to use only the asymptotic density-effect correction for all charged-particle energies.[7] To determine the error introduced by this

*No distinction is made here between compounds and mixtures; that is, molecular binding effects are assumed negligible.

approximation, we have computed the stopping power and range using both the correct and asymptotic density-effect corrections for several elements and compounds for which the material constants (a, m, x_0, and x_1) needed for the correct density effect are available.

The density-effect correction enters the stopping-power formula in the following way:*

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{4\pi e^4 z^2 n}{m_e c^2 \beta^2} \left\{ \ln\left[\frac{2m_e c^2 \beta^2}{I(1-\beta^2)} \right] - \beta^2 - \delta/2 \right\} ,$$

where $-\langle dE/dx \rangle$ is the stopping power, e is the charge of an electron, z is the charge (in units of e) of the moving particle, and β is the particle speed (in units of c). As an example of the influence of the density effect on the stopping power, Fig. 1 shows the stopping power of protons in aluminum obtained by neglecting the density effect ($\delta = 0$), by using the asymptotic density effect ($\delta = \delta'$), and by using the correct density effect (Eqs. 1a-1c).† In obtaining these and subsequent results, the material constants for the correct density effect have been taken from ref. 4, and the electron densities and ionization potentials used to compute $-\langle dE/dx \rangle$ and δ' are the same as used in ref. 4. It is evident from Fig. 1 that the largest difference between δ and δ' occurs in the energy range where the density effect is small.

*Since the interest here is for high energies, the shell-correction term, which is important when the particle speed is not large compared with the speed of the atomic electrons, has not been included in the stopping-power formula.

†In applying the asymptotic expression at all energies, we set $\delta' = 0$ whenever Eq. 4 gives a negative value for δ'.

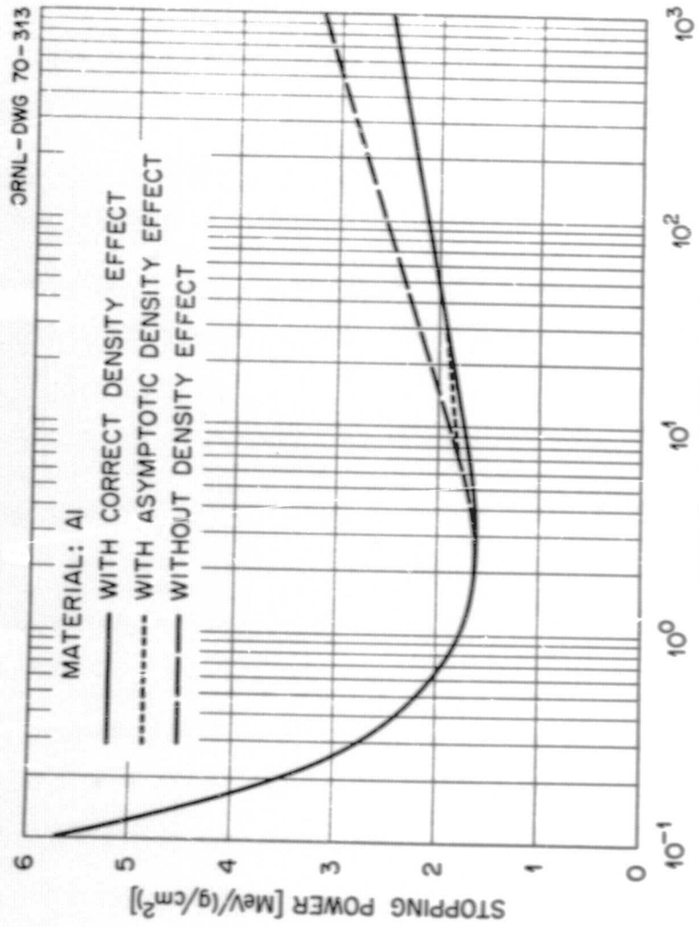

Fig. 1. Stopping Power of Protons in Aluminum.

Differences in the stopping power and range of protons obtained using δ and δ' are shown in Figs. 2 and 3, respectively, for several materials. For a variety of materials and all energies, using only the asymptotic density-effect correction at all energies overestimates the stopping power by $\gtrsim 6\%$ and underestimates the range by $\gtrsim 4\%$. Errors of this magnitude should be acceptable in many practical problems that involve materials for which the correct density effect has not been evaluated.

Although only protons have been considered here, the conclusions obtained are also valid for other charged particles since the stopping power and range for other charged particles can be determined from the proton results by using scaling relations.[5]

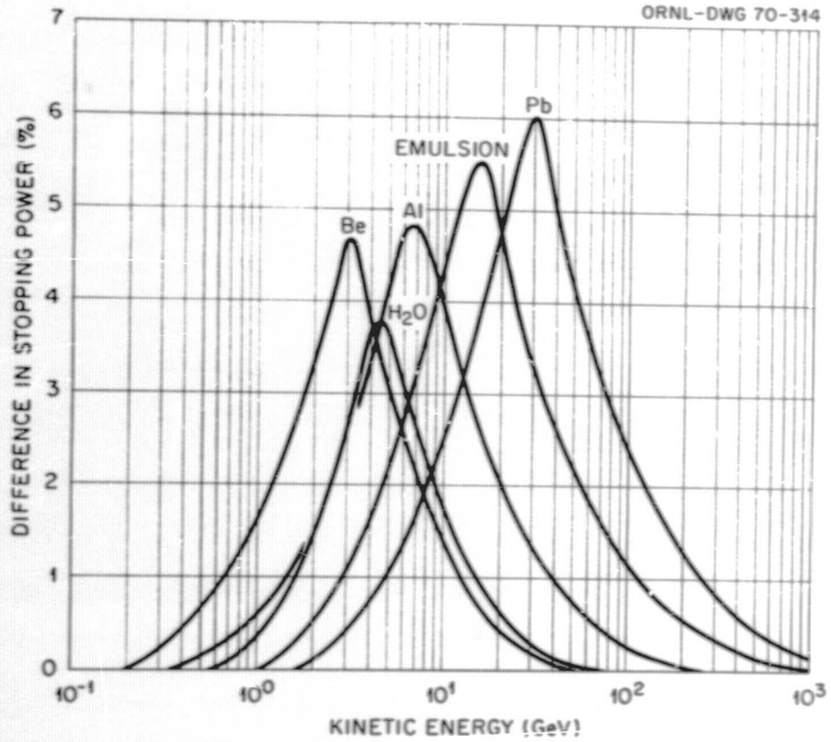

Fig. 2. Difference in Proton Stopping Power for Various Materials Obtained Using Correct and Asymptotic Density-Effect Corrections.

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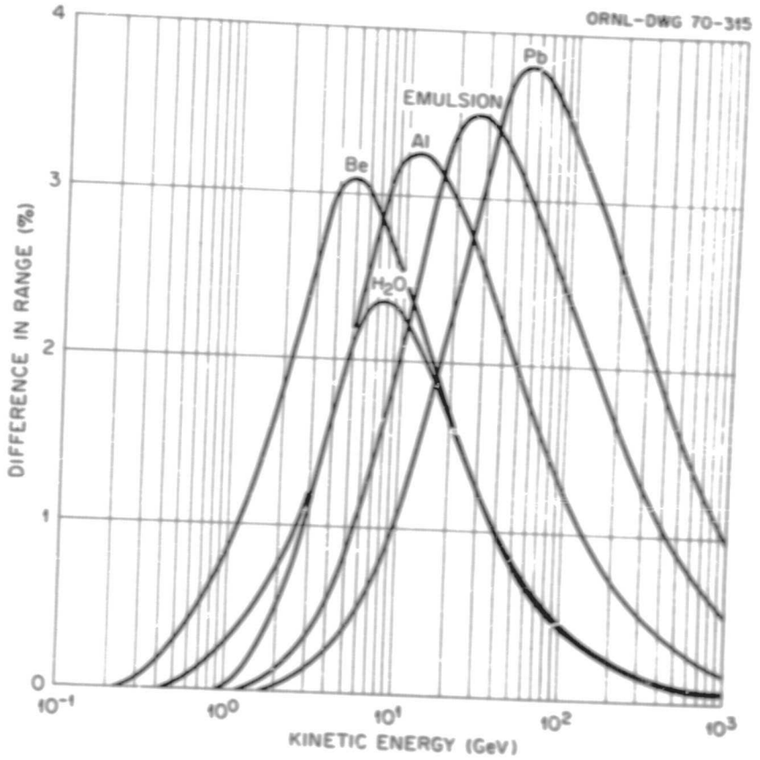

Fig. 3. Difference in Proton Range for Various Materials Obtained Using Correct and Asymptotic Density-Effect Corrections.

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