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16. Abstract

Station-keeping requirements are determined for a class of high-power communication satellites in synchronous equatorial orbit. These satellites are characterized by large, Sun-oriented solar arrays. The perturbing forces on the satellite are the gravitational effects of the Sun and Moon, the triaxial mass distribution of the Earth, solar radiation pressure, and rf radiation pressure. The area-to-mass ratio of these satellites is sufficiently high that the station-keeping requirements due to solar radiation pressure must be considered. Parametric curves are presented for determining ΔV and thrust requirements.

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STATION KEEPING OF HIGH POWER COMMUNICATION SATELLITES

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SUMMARY

Station-keeping requirements were determined for a class of high-power synchronous equatorial communication satellites characterized by large, Sun-oriented solar arrays. The unkept satellite orbit was determined when the combined perturbing forces of the Sun, Moon, solar radiation pressure (including Earth shadow effects), radio-frequency radiation pressure (from transmitting antennas), and Earth's oblateness and triaxiality were acting on the satellite. The effects of radio-frequency radiation pressure and the Earth's oblateness and shadow are negligible. The remaining forces cause three appreciable and separable perturbations in the satellite orbit.

The Sun and Moon cause a nearly linear (over a 5-year period) increase in the inclination of the orbit at a rate of 0.85° per year. Orbit inclination causes the satellite to undergo an apparent daily north-south oscillation with maximum latitude equal to the inclination. For minimum propellant consumption, station correction is best effected by applying two thrust pulses (12 hr apart) with the center of the pulses occurring at the intersections of the desired orbit plane and the actual orbit plane. The velocity increment (ΔV) per year requirement for impulsive thrusting is approximately 46 meters per second.

The Earth's triaxial mass distribution causes a long period, large amplitude oscillation in the satellite longitude position. The ΔV per year requirement for impulsive thrusting is less than 1.75 meters per second.

Solar radiation pressure changes the eccentricity of the orbit. An eccentric orbit causes the satellite to have an apparent daily east-west oscillation with maximum amplitude in radians equal to twice the eccentricity. For minimum propellant consumption station correction for eccentricity is best effected by applying two thrust pulses 12 hours apart. These thrust pulses control the line of apsides such that the projection of the Earth-Sun line into the orbit plane is coincident with the Earth-perigee line. The ΔV requirement is proportional to the area-to-mass ratio of the satellite. For an area-to-mass ratio of 0.15 square meters per kilogram, an average satellite reflectivity of 0.3, and an allowable longitude error of 0.15°, the ΔV per year requirement for impulsive thrusting is 8 meters per second.

INTRODUCTION

The next generation of communication satellites will use large, lightweight, Sun-oriented solar arrays as a primary power source. The satellites will transmit narrow radio-frequency beams from synchronous equatorial orbit to high gain receiver antennas on the ground. The system engineer must make a trade-off between the gain and pointing requirements of the ground receiver antenna and the station-keeping accuracy of the satellite. This is particularly true for missions using frequencies around 12 gigahertz. For example, at 12 gigahertz, with effective receiving apertures as small as 4 square feet (0.37 m^2), ground receiver antennas will have a half power beam width of 2^o. Because of antenna mounting uncertainties, it may be mispointed by $1/2^o$. If the satellite drifts off station by an additional $1/2^o$, then the received signal strength will be reduced by a factor of two. If the satellite prime power is fixed, either the ground antenna pointing accuracy must be improved or the satellite station-keeping accuracy must be increased.

The problem of satellite station keeping is not new. The yearly ΔV requirements of 46 meters per second for north-south and 2 meters per second for east-west station keeping are documented in a number of reports. The 46-meter-per-second ΔV requirement for north-south or inclination control is based on impulsive corrections required to counter the perturbing accelerations of the Sun and Moon. The 2-meter-per-second ΔV requirement for east-west or longitude control is based on impulsive corrections required to counter accelerations arising from the Earth triaxial mass distribution. For relatively dense, rigid satellites, station-keeping requirements can be specified by considering only the previously mentioned effects. But for high-power communication satellites (HPCS's), this is not enough.

HPCS's have two characteristics that complicate the problem of determining station-keeping requirements. The first of these characteristics is the reflective properties of large Sun-tracking solar arrays. Solar pressure causes an acceleration proportional to the area-to-mass ratio of the satellite, and the resultant accelerations change the eccentricity of the satellite orbit. As a result of the large Sun-tracking solar array, the area-to-mass ratio is sufficiently high that the station-keeping requirements due to solar pressure must be considered.

Second, HPCS's behave as flexible bodies. Large roll-out solar arrays are very flexible and can tolerate only mild accelerations without structural failure. Impulsive accelerations resulting from station corrections initiate solar array panel oscillations that interact unfavorably with the fine pointing attitude control system. As a result of the flexible structure characteristics, the station-keeping impulses must be distributed over a longer period of time.

This report presents the results of an analytical study of the requirements necessary to keep a class of 24-hour synchronous equatorial communication satellites on station. The class of satellites considered can be described as Sun-tracking flat plates with area-to-mass ratios varying from 0.05 to 0.15 square meters per kilogram. This includes the HPCS series of satellites, which have a dense central body, some relatively small reflector antennas, and large Sun-oriented solar arrays.

The reader is presented with working curves and equations that show the relations between spacecraft parameters, station accuracy, and thruster and propellant requirements. The approach taken is to present the reader with a review of the unkept (or what happens if one does nothing) satellite station when the satellite is subjected to various perturbing forces. The unkept station is described in terms of the classic orbital elements. The perturbing forces considered are the Sun, Moon, solar pressure including the Earth's shadow, Earth's oblateness and triaxiality, and radio-frequency radiation pressure.

After establishing the unkept station, fundamental concepts of correcting the perturbed orbital elements are examined. Radial, tangential, and out-of-plane thrust schemes are examined to determine their applicability to controlling the orbital elements. Satellite accelerations and characteristic velocity requirements and hence thruster size and propellant weights are given for various methods of station keeping. Finally, a sample problem is presented to demonstrate the use of the design curves for a typical HPCS mission.

SYMBOLS

Symbols used for special purposes are defined where they occur in the report and are not included in the following list.

A/m	area-to-mass ratio of satellite, m^2/kg
a	semimajor axis
a_{NS}	thruster acceleration, north-south station keeping
a_{si}	thruster acceleration, east-west station keeping due to solar pressure for the i^{th} method
e	eccentricity
e*	maximum allowable eccentricity
e_p	peak eccentricity that would occur if initial orbit were circular ($e_0 = 0$) and no station keeping were applied

i	inclination
k	$(A/m)(1 + \sigma)$
ΔL	allowable longitude error
ΔL_s	allowable longitude error due to solar pressure effects
ΔL_t	allowable longitude error due to triaxiality effects
M	number of days to complete a station-keeping correction of a specified type
N	number of days between the beginnings of successive station-keeping corrections of a specified type
$\overline{P}$	unit vector from center of Earth to orbit perigee
$\overline{P}_1$	unit vector formed by projecting $\overline{P}$ into equatorial plane
p	duty cycle, equal to thruster on-time per orbit divided by orbit period
$\overline{Q}$	unit vector orthogonal to $\overline{P}$ such that $\overline{P}$ and $\overline{Q}$ lie in orbit plane
r	radial distance of the satellite from the center of the Earth
Δr	variation of r from the nominal synchronous radial distance
Δr_s	variation of r due to solar pressure effects
Δr_t	variation of r due to triaxiality effects
S	solar constant, 4.5×10^{-6} kg/(m)(sec^2)
sk	station keeping
$\overline{U}$	unit vector from center of Earth to Sun
$\overline{U}_1$	unit vector formed by projecting $\overline{U}$ into equatorial plane
V	nominal satellite velocity, 3075 m/sec
ΔV_c	velocity increment needed to make a station-keeping correction
ΔV_{NS}	velocity increment per year for north-south station keeping
ΔV_{si}	velocity increment per year for east-west station keeping (due to solar pressure) when using the i^{th} method
ΔV_t	velocity increment per year for east-west station keeping (due to triaxiality)
X, Y, Z	inertial coordinate system
β	eccentricity ratio e^*/e_p
γ	longitude of satellite measured from minor axis of Earth's equatorial section
γ_0	desired value of γ

γ_p	value of γ at the perigee of orbit just prior to an east-west correction
$\Delta\gamma$	variation of γ from γ_o where $\gamma = \gamma_o + \Delta\gamma$
$\Delta\gamma_p$	variation of γ from γ_p where $\gamma = \gamma_p + \Delta\gamma_p$
$\Delta\gamma_s$	varition of γ caused by solar pressure effects
$\Delta\gamma_t$	variation of γ caused by triaxiality effects
λ	longitude of Sun measured from X-axis
$\dot{\lambda}$	mean angular velocity of Earth's orbit about the Sun $\dot{\lambda} = 1.99\times10^{-7}$ rad/sec
$\dot{\theta}_E$	angular velocity of Earth's rotation about its axis $\dot{\theta}_E = 7.29\times10^{-5}$ rad/sec
μ	gravitational constant of Earth
σ	average reflectivity of satellite
φ	latitude of satellite
ψ	angle through which apsidal line is rotated
ω	longitude of perigee measured from X-axis

SATELLITE AND ITS ORBIT

Figure 1 shows three typical satellites of the type considered in this report. These satellites have a dense central body, some relatively small reflector antennas, and large Sun-oriented solar arrays. They can be characterized as Sun-tracking flat plates with area-to-mass ratios varying from 0.05 to 0.15 square meter per kilogram.

To facilitate continuous Earth coverage to nonsteerable ground antennas, these satellites will operate from synchronous altitude. Satellite position control to within $\pm0.2^o$ in latitude and longitude may be required. Typical missions will last for 5 years.

NATURAL PERTURBATIONS AND UNKEPT STATION

Because of the effect of various perturbing forces, a satellite in synchronous equatorial orbit will not remain stationary with respect to the rotating Earth. Perturbing forces considered in this report arise from the oblateness and triaxiality of the Earth, the Sun and Moon gravitational attraction, and solar radiation pressure. Perturbation forces arising from radio-frequency radiation pressure from communication antennas are neglected because they were found in appendix A to have negligible effect on the satellite orbit.

Before the methods and requirements of station keeping are analyzed, the resultant behavior of the unkept (no station keeping) satellite station will be determined.

The position of a satellite with respect to the rotating Earth can be completely determined by three parameters: r, distance from center of Earth to satellite; γ, satellite longitude measured from the minor axis of the Earth's equatorial section; and φ, satellite latitude. Letting r_0 be the nominal value of r for a 24-hour circular orbit and γ_0 be the initial value of γ, the variations Δr and $\Delta\gamma$ can be defined by the equations

$$r = r_0 + \Delta r \tag{1}$$

$$\gamma = \gamma_0 + \Delta\gamma \tag{2}$$

The deviations of a satellite from a nominally synchronous equatorial orbit are given by Δr, $\Delta\gamma$, and φ. An out-of-plane perturbation creates a nonzero orbit inclination which in turn causes nonzero values of φ. An in-plane perturbation causes changes in orbital period, eccentricity and orientation of the apsidal line which in turn causes changes in Δr and $\Delta\gamma$.

Out-of-Plane Perturbations (Changes in φ)

From reference 1, the effect of the Sun and Moon is to cause the orbit inclination to build sinusoidally to a peak of 14.7° after $26\frac{1}{2}$ years and then decrease to zero after 53 years. Over the first 5 years, the inclination increases at an approximately linear rate of 0.85 degree per year. The satellite latitude undergoes a sinusoidal oscillation once per orbit with amplitude in radians equal to the instantaneous orbit inclination. Figure 2 shows a plot of latitude as a function of time. The effect of the Sun and Moon is the only significant out-of-plane perturbation.

In-Plane Perturbations (Changes in Δr and $\Delta\gamma$)

The Earth's oblateness and triaxiality, the Sun and Moon, and solar radiation pressure cause in-plane perturbations. The effect of oblateness is to modify the value of r_0 from that obtained from a spherical Earth model. The effect of the Sun and Moon is to cause small oscillations in both Δr and $\Delta\gamma$. (From ref. 2, a conservative upper bound is 3000 meters for Δr and 0.06° for $\Delta\gamma$.) The effect of triaxiality is to cause a longitu-

dinal oscillation about the Earth's mionr axis. The effect of solar pressure is to change
tg
solar pressure are importantand are analyzed below

Triaxiality. - From reference 3 the Earth's equatorial cross section is approximately an ellipse with an ellipticity of $6\left|J_2^{(2)}\right|$, where $J_2^{(2)}$ has the value of -1.816×10^{-6}. The minor axis of the equatorial ellipse passes through 74.6° east longitude and 105.4° west longitude (see ref. 3). These two longitudes are stable points. Neglecting other perturbations, a satellite placed at either of these longitudes ($\gamma_0 = 0$) will tend to stay there. Hence, if a satellite is positioned at any other longitude ($\gamma_0 \neq 0$), it will undergo a longitudinal oscillation about the nearest minor axis (see ref. 4). The period of this oscillation is greater than 2.2 years and is a function of γ_0. The variation of r also undergoes an oscillation of the same period. The amplitude of this oscillation is a function of γ_0. When γ_0 is 90°, the amplitude of the Δr oscillation is a maximum (35 000 m). The variation of r and γ due to triaxiality will be denoted by Δr_t and $\Delta\gamma_t$. Figure 3 presents Δr_t as a function of time with $\gamma_0 = 45°$. Figure 4 presents $\Delta\gamma_t$ as a function of time with $\gamma_0 = 45°$.

Solar pressure. - The effect of solar pressure is to change the orbit eccentricity and orientation of the apsidal line. It has a negligible effect on the orbit period. The induced eccentricity causes a daily longitudinal oscillation with amplitude 2e radians. The period of this longitudinal oscillation (24 hr) is much smaller than the period of longitudinal oscillation due to triaxiality. Solar pressure also causes a daily oscillation in orbit radius with amplitude er_0. Unlike the other perturbations, the effect of solar pressure is dependent on satellite parameters. With the assumptions that the satellite is a flat plate whose surface is perpendicular to the Earth-Sun line and that the front side thermal radiation is the same as the back side thermal radiation, the perturbing acceleration of the satellite due to solar pressure is

$$\overline{\alpha} = -Sk\overline{U} \tag{3}$$

where U is a unit vector from the center of the Earth to the Sun, S is the solar constant at 1 AU, and

$$k = (1 + \sigma)\frac{A}{m} \tag{4}$$

or approximately 1.3 times the area-to-mass ratio for silicon cell solar arrays (assuming average satellite reflectivity $\sigma = 0.3$).

There is no solar pressure when the satellite is in the Earth's shadow. For 275 days of the year, the satellite does not enter shadow at all. Over a 1-year period, the satellite is in shadow 1 percent of the time. The satellite is in shadow no more than 5 percent of the orbit even for the worst case (at the vernal and autumnal equinoxes). The inclusion of Earth shadow in computer solutions indicated that the effect of shadow is negligible. The effect of shadow is ignored in the following analysis.

Before continuing with the discussion of solar radiation pressure, refer to figure 5 for definitions of the quantities $\overline{U}$, $\overline{U}_1$, $\overline{P}$, $\overline{P}_1$, λ, and ω. The XYZ reference coordinate system is an inertial system with the origin at the center of the Earth. The X-axis is toward the autumnal equinox, the X-Y plane is the equatorial plane, and the Z-axis is along the Earth's spin axis. Notice that ω is not the argument of perigee, but rather the longitude of perigee measured from the X-axis. When the initial orbit is circular, e as a function of time is given to a good degree of approximation by

$$e(t) = e_p \left| \sin \frac{1}{2} \dot{\lambda} t \right| \tag{5}$$

where e_p is given by

$$e_p = \frac{3Sk}{V\dot{\lambda}} = 0.022\ k \tag{6}$$

where k is in square meters per kilogram, V is the nominal satellite velocity, and $\dot{\lambda}$ is the mean angular velocity of the Earth's orbit about the Sun (2π rad/yr). Longitude of perigee $\omega(t)$ in radians is given approximately by

$$\omega(t) = \lambda_o + \frac{\pi}{2} + \frac{1}{2} \dot{\lambda} t; \quad 0 \leq t \leq 1 \text{ year} \tag{7}$$

Equation (7) implies that the line of apsides rotates uniformly in inertial space, making a 180^o rotation in a 1-year period. Equations (5) to (7) are derived analytically in appendix B with the assumption that the ecliptic plane, equatorial plane, and orbit plane are the same plane. Solutions for e(t) and $\omega(t)$, without the coplanar assumption, were obtained on a digital computer by using numerical integration. The computer solutions agreed well with analytic solutions. Figure 6 presents the computer solutions for e(t)/k for two cases. In the first case (starting at the vernal equinox), the maximum eccentricity is slightly less than for the second case (starting at the winter solstice) When starting at any other time of the year, the maximum eccentricity will be somewhere between the two maximums shown in figure 6. Figure 7 presents $\omega(t)$ and $\lambda(t)$ when starting at the vernal equinox. The apparent discontinuity in $\omega(t)$ at time equal to 1 year is resolved by realizing that the line of apsides is undefined when the eccentricity is zero. The relation of ω to λ plays an important role in choosing station-keeping techniques.

Appendix C gives a detailed discussion of the resulting orbit when the initial orbit is noncircular. However, one special case deserves attention here. If $e_o = \frac{1}{2} e_p$ and $\omega_o = \lambda_o$, then $e(t)$ and $\omega(t)$ are given to a good degree of approximation by

$$e(t) = \frac{1}{2} e_p \tag{8}$$

$$\omega(t) = \lambda_o + \dot{\lambda} t = \lambda(t) \tag{9}$$

Thus, the eccentricity remains constant, and the line of apsides rotates uniformly in synchronization with the Earth-Sun line. Equation (9) implies that the orbit perigee is Sun-oriented. The analytic derivation of equations (8) and (9), using the coplanar assumption, is included in appendix B. A 24-hour orbit having an eccentricity and longitude of perigee as given by equations (8) and (9) will be called a Sun-oriented orbit. The importance of the Sun-oriented orbit is that its eccentricity remains constant at a value of only half the maximum eccentricity obtained from an initially circular orbit. The Sun-oriented orbit is analogous to a Sun-synchronous orbit. In a Sun-synchronous orbit, a proper combination of orbital radius and inclination will cause the line of nodes to rotate with an angular velocity of 360° per year. In the Sun-oriented orbit, the proper combination of eccentricity and initial longitude of perigee causes the line of apsides to rotate with an angular velocity of 360° per year. Figure 8 presents plots of $e(t)/k$ obtained from computer solutions using numerical integration. Figure 9 presents $\omega(t)$ and $\lambda(t)$. The computer solutions did not use the coplanar assumption. Both figures show close agreement to equations (8) and (9).

Summary of Perturbations

Three significant and distinct orbital motions result from the perturbation forces acting on a synchronous equatorial communication satellite. They are summarized as follows:

(1) The Sun and Moon cause the orbit to develop an inclination at the rate of 0.85° per year. The induced inclination causes the satellite latitude to undergo a daily sinusoidal oscillation with amplitude equal to the instantaneous inclination.

(2) The Earth's equatorial section is approximately an ellipse with minor axes at 75° east and 105° west longitude. A satellite positioned at any other longitude will tend to

drift toward and oscillate about the nearest minor axis. The period of this oscillation is greater than 2.2 years and is a function of the initial longitude.

(3) Solar pressure changes the orbit eccentricity and longitude of perigee. The induced eccentricity causes the satellite longitude to undergo a daily longitudinal oscillation with amplitude $2e$ radians. The perturbing acceleration of the satellite due to solar pressure is proportional to the area-to-mass ratio.

STATION KEEPING METHODS AND REQUIREMENTS

In the previous sections, the orbit perturbations were presented in terms of Δr, $\Delta\gamma$ and φ. Changes in Δr, $\Delta\gamma$ or φ will cause changes in one or more of the orbital elements a, e, i, and ω. Table I presents a summary of the perturbations, their effects on the orbital elements, their effects on Δr, $\Delta\gamma$, φ, and some comments.

TABLE I. - SUMMARY OF PERTURBATIONS

Perturbation	Effect on the orbital elements a, e, i, ω	Effect on Δr, $\Delta\gamma$, φ	Comments
Sun and Moon	$i = (0.86^o/\mathrm{yr})t$ Small oscillations in a	$\varphi = i \sin \dot{\theta}_E t$ Small oscillations in Δr, $\Delta\gamma$	Station keeping needed to control i No station keeping needed to control a
Oblateness	---------------	---------------	Oblateness modifies r_o No station keeping needed as long as orbit is nominally equatorial
Triaxiality	Long-period oscillation in a	Long-period oscillation in Δr_t and $\Delta\gamma_t$	Station keeping needed to control a

Solar radiation pressure	If $e_o = 0$, then
	$e = e_p \left\| \sin \frac{1}{2} \dot{\lambda} t \right\|$
	$\omega = \lambda_o + \frac{\pi}{2} + \frac{1}{2} \dot{\lambda} t$
	If $e_o = \frac{1}{2} e_p$ and $\omega_o = \lambda_o$, then
	$e = \frac{1}{2} e_p$
	$\omega = \dot{\lambda} t + \lambda_o$
		Short-period oscillation in Δr_s, $\Delta\gamma_s$	Station keeping may be needed to control e and/or ω

In discussing station-keeping methods and their associated requirements, it is convenient to use equations that give the time rate of change of orbital parameters as a function of the station-keeping acceleration. Integrating these equations over the time interval of thrusting will yield the change in the orbital parameters. The station-keeping acceleration vector can be resolved into a component R along the radius vector (measured positive away from the Earth), a transverse component T in the instantaneous orbital plane (measured positive when in the same direction as the orbital velocity vector), and a component W normal to the instantaneous orbital plane (measured positive to the north). From reference 5, the equations for the time rate of change of the orbital elements are

$$\frac{da}{dt} = \frac{2e \sin \theta}{n\sqrt{1 - e^2}} R + \frac{2a\sqrt{1 - e^2}}{nr} T \tag{10}$$

$$\frac{de}{dt} = \frac{\sqrt{1 - e^2} \sin \theta}{na} R + \frac{\sqrt{1 - e^2}}{na^2 e}\left[\frac{a^2(1 - e^2)}{r} - r\right] T \tag{11}$$

$$\frac{d\omega}{dt} = -\frac{\sqrt{1 - e^2} \cos \theta}{nae} R + \frac{\sqrt{1 - e^2}}{nae}\left(1 + \frac{1}{1 + e \cos \theta}\right)(\sin \theta) T \tag{12}$$

$$\frac{di}{dt} = \frac{r \cos \varphi}{na^2\sqrt{1 - e^2}} W \tag{13}$$

where n is the orbital angular velocity, r is the orbital radius, θ is the true anomaly, and φ is the angle from the ascending node to the instantaneous position of the satellite. Retaining only first-order perturbations in e and assuming $e \cos \theta << 2$, equations (10) to (13) can be reduced to

$$\frac{da}{dt} = \frac{2ae \sin \theta}{V} R + \frac{2a}{V} T \tag{14}$$

$$\frac{de}{dt} = \frac{\sin \theta}{V} R + \frac{2 \cos \theta}{V} T \tag{15}$$

$$\frac{d\omega}{dt} = \frac{-\cos \theta}{eV} R + \frac{2 \sin \theta}{eV} T \tag{16}$$

$$\frac{di}{dt} = \frac{\cos \varphi}{V} W \tag{17}$$

In all of the station-keeping methods to be described, there are two thrusting periods per orbit of 12 p hours each where p is the thruster on-time per orbit divided by the orbit period (24 hr). The first thrusting period is centered about a position in the orbit to be called A_1. The second thrusting period is centered about A_2. In all cases, A_2 is separated from A_1 by an angular distance of 180°. Each station-keeping correction is assumed to take place over M consecutive orbits. Thus one station-keeping correction consists of 2M thrusting periods and a total thrusting time of Mp days.

In the following sections, the station-keeping requirements are given in terms of the ΔV per year and the thruster acceleration.

North-South Station-Keeping Methods and Requirements Due to the Sun and Moon

In the previous sections, it was shown that the Sun and the Moon cause the orbit to develop an inclination. To control the effects of the Sun and the Moon, one must control the inclination of the orbit. To change the orbit inclination an amount Δi, it is necessary to thrust normal to the orbital plane. Figure 10 presents a sketch of this maneuver for M = 1 and for a duty cycle close to zero (impulsive thrusting). The correction is made by thrusting in the south direction in the vicinity of A_1 and thrusting in the north direction in the vicinity of A_2 where A_1 and A_2 are the ascending and desending nodes, respectively. The change in inclination Δi as a function of duty cycle p can be found by integrating equation (17).

$$\Delta i = 2M \int_{-\pi p/2\dot{\theta}_E}^{\pi p/2\dot{\theta}_E} \frac{\cos(\dot{\theta}_E t)}{V} W \, dt \tag{18}$$

The result is

$$\Delta i = \frac{2 \, \Delta V_c \sin \frac{p\pi}{2}}{V p\pi} \tag{19}$$

Solving for ΔV_c gives

$$\Delta V_c = V\,\Delta i \,\frac{p\pi}{2\sin\frac{p\pi}{2}} \tag{20}$$

Let a_{NS} be the north-south acceleration caused by the station-keeping thruster. Then $a_{NS} = W$ and the change in inclination for a complete correction (firing near each node for M consecutive orbits) is given by

$$\Delta i = \frac{4Ma_{NS}}{\dot{\theta}_E V}\sin\frac{p\pi}{2} \tag{21}$$

Recalling that the Sun and Moon cause i to increase at the rate 0.85 degree per year, Δi can be expressed in radians as

$$\Delta i = \frac{0.85}{57.3}\,\frac{N}{365} \tag{22}$$

where N equals the number of days between the beginnings of successive inclination corrections. Equations (21) and (22) can be combined to yield

$$a_{NS}\left(\frac{m}{sec^2}\right) = (2.3\times10^{-6})\,\frac{N/M}{\sin\frac{p\pi}{2}} \tag{23}$$

Using equation (20), the ΔV per year (ΔV_{NS}) is given by

$$\Delta V_{NS}\left(\frac{m}{sec}\right) = 46\,\frac{p\pi}{2\sin\frac{p\pi}{2}} \tag{24}$$

The ΔV_{NS} is plotted as a function of p in figure 11. In figure 12, Ma_{NS} is plotted as a function of p for $N = 1$, 7, and 60.

East-West Station-Keeping Methods and Requirements Due to Triaxiality

Triaxiality causes a long period east-west or longitude oscillation. Accompanying the longitudinal drift is a change Δa in the semimajor axis. The method used to control against triaxiality is based on changing the semimajor axis.

The semimajor axis of the orbit can be increased by some small amount ($\Delta a > 0$) by thrusting eastward in the vicinity of both A_1 and A_2. Position A_1 can be either the apogee or the perigee of the orbit. If Δa is negative, then the direction of the thrust is westward. Figure 13 presents a sketch of this maneuver for impulsive thrusting when Δa is negative. For impulsive thrusting, equation (14) can be integrated to yield

$$\Delta a = \frac{2a}{V} \Delta V_c \tag{25}$$

Solving for ΔV_c gives

$$\Delta V_c = \frac{V}{2a} \Delta a \tag{26}$$

For nonimpulsive thrusting,

$$\Delta V_c = \frac{V}{2a} \Delta a \left(\frac{p\pi}{2 \sin \frac{p\pi}{2}} \right) \tag{27}$$

Figure 14 presents a plot of Δa against $\Delta\gamma_t$. The dashed portion of the curve represents the motion when station keeping is not used. When station keeping is used, the satellite will be initially positioned so that $\Delta\gamma_t = -\Delta L_t$, where ΔL_t is the maximum allowable excursion due to triaxiality. The initial semimajor axis will be $a_o + \Delta a_c$ (see point A, fig. 14). The satellite will drift eastward until $\Delta\gamma_t = +\Delta L_t$ (point B, fig. 14), and then begin drifting westward. When the satellite has drifted to a point where $\Delta\gamma_t = -\Delta L_t$ (point C, fig. 14), the semimajor axis will be $a_o - \Delta a_c$. Station keeping is then used to increase the semimajor axis an amount $2\Delta a_c$, bringing the satellite back to point A again. The process is then repeated. The station-keeping acceleration level is small enough so that the corrections can be done impulsively. The ΔV per year (ΔV_t) for correcting triaxiality is then only a function of the off-longitude γ_o (see ref. 4).

$$\Delta V_t\left(\frac{m}{sec}\right) = 1.75\left|\sin 2\gamma_o\right| \tag{28}$$

Figure 15 presents a plot of ΔV_t as a function of γ_o.

East-West Station-Keeping Methods and Requirements Due to Solar Pressure

In the previous sections, it was shown that solar pressure changes the eccentricity and rotates the line of apsides of the orbit. It will be shown in this section that the effects of solar pressure can be controlled by continuously thrusting directly against the Sun, or by controlling the eccentricity, or by controlling the line of apsides. Four methods are presented. Appendix D presents station-keeping techniques for changing orbital eccentricity and rotating the line of apsides. Derivations of the formulas for ΔV and acceleration requirements for each method are presented in appendix E.

Method 1. - In this method the effect of solar pressure is canceled by continuously thrusting toward the Sun. The acceleration of the satellite due to the thrust is equal but opposite to the acceleration caused by solar pressure. The ΔV per year is

$$\Delta V_{s1} = \frac{3Sk\pi}{2\dot{\lambda}}\left(\frac{4}{3}\right) \tag{29}$$

The acceleration level is

$$a_{s1} = Sk \tag{30}$$

Method 2. - In this method each time the eccentricity becomes equal to a predetermined maximum allowable eccentricity e^* the orbit is circularized. Appendix C shows that an eccentric orbit can be circularized by thrusting either collinearly with the orbit velocity vector or collinearly with the orbit radius vector. Appendix C also shows that tangential thrusting requires only one-half as much ΔV as radial thrusting. For that reason, the requirements for this method are given for tangential thrusting only. Let ΔL_s denote the maximum allowable longitude excursion due to solar pressure consistent with the station accuracy requirement. Since eccentricity causes a daily longitude oscillation of amplitude $2e$ radians, $e^* = \frac{1}{2}\Delta L_s$ (where ΔL_s is in radians). The parameter β is defined as the ratio of the maximum allowable eccentricity to the maximum eccentricity that would result from an initially circular orbit (assuming no station keeping), that is,

$$\beta = \frac{e^*}{e_p} \qquad (31)$$

Clearly, if $\beta > 1$, no station keeping is required. If ΔL_s is in degrees and k is in m^2/kg, β can be expressed as a simple function of ΔL_s and k.

$$\beta = 0.4 \frac{\Delta L_s}{k} \qquad (32)$$

Figure 16 presents a plot of e/e_p, ω, and λ as functions of time when method 2 is used. The ΔV per year is

$$\Delta V_{s2} = \frac{3Sk\pi}{2\dot{\lambda}} \left(\frac{p\pi}{2 \sin \frac{p\pi}{2}} \right) \left(\frac{\beta}{\sin^{-1}\beta} \right) \qquad (33)$$

The acceleration level is

$$a_{s4} = \left(\frac{Sk}{M} \right) \left(\frac{3\dot{\theta}_E}{8\dot{\lambda}} \right) \left(\frac{\beta}{\sin \frac{p\pi}{2}} \right) \qquad (34)$$

Figure 17 presents a plot of $\Delta V_{s2}/k$ as a function of p with β as a family parameter. Figure 18 presents the corresponding plot for $(M/k)a_{s2}$.

Method 3. - In method 3, the line of apsides is rotated each time the eccentricity becomes equal to the maximum allowable eccentricity e^*. The line of apsides is rotated in such a manner that the solar pressure will cause the eccentricity to decrease to zero before increasing again. Figure 19 gives curves of e/e_p, ω, and λ when station keeping is not used. As seen from figure 19 the eccentricity is increasing when the apsidal line leads the Earth-Sun line $(\omega - \lambda > 0)$. The eccentricity is decreasing when the apsidal line lags the Earth-Sun line $(\omega - \lambda < 0)$. For method 3, the apsidal line is rotated so that $\omega - \lambda$ changes from the lead angle $\omega_1 - \lambda_1$ to the lag angle $\omega_2 - \lambda_2$. If the apsidal line is rotated through an angle $\Delta\omega = 2(\omega_1 - \lambda_1)$, at times t_c and $3t_c$, then e/e_p, ω, and λ curves as given in figure 20 are obtained. Appendix C shows that tangential thrusting requires only one-half as much ΔV to rotate the line of apsides as radial thrusting. For that reason, the requirements for this method are given for tangential thrusting only. The ΔV per year for method 3 is

$$\Delta V_{s3} = \frac{3Sk\pi}{2\dot{\lambda}}\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)\left(\frac{\beta\sqrt{1 - \beta^2}}{\sin^{-1}\beta}\right) \tag{35}$$

Notice that

$$\Delta V_{s3} = \sqrt{1 - \beta^2}\ \Delta V_{s2}$$

The acceleration level is

$$a_{s3} = \frac{Sk}{M}\left(\frac{3\dot{\theta}_E}{4\dot{\lambda}}\right)\left(\frac{\beta\sqrt{1 - \beta^2}}{\sin \frac{p\pi}{2}}\right) \tag{36}$$

Figure 21 presents a plot of $\Delta V_{s3}/k$ as a function of p with β as a family parameter. Figure 22 presents the corresponding plot for $(M/k)a_{s3}$.

Method 4. - In method 4, the orientation of the apsidal line with respect to the Earth-Sun line is controlled such that the eccentricity is maintained at or slightly below the maximum allowable eccentricity e^*. Figure 23 presents plots of e/e_p, ω, and λ when method 4 is used. Initially the eccentricity is e^*, and the apsidal line lags the Earth-Sun line by a small amount ($\omega - \lambda$ slightly less than 0). Due to the lag, eccentricity will decrease slightly. When the apsidal line coincides with the Earth-Sun line ($\omega = \lambda$), the eccentricity reaches a minimum. When the apsidal line leads the Earth-Sun line ($\omega - \lambda > 0$), eccentricity increases and eventually becomes equal to e^* again. At this time, the apsidal line is rotated so that $\omega - \lambda$ has the same lag value it had at time equal to zero. The process is then repeated. Each time the eccentricity becomes equal to e^*, the apsidal line is rotated through the same angle $\Delta\omega$.

For very frequent corrections, the angle $\Delta\omega$ is very small. As a result, $\omega \approx \lambda$ and $e \approx e^*$. Frequent corrections are desirable because the ΔV per year decreases as the frequency of correction increases.

The requirements for this method are given for tangential thrusting only. The ΔV per year for frequent corrections is

$$\Delta V_{s4} = \left(\frac{3Sk\pi}{2\dot{\lambda}}\right)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)(1 - 2\beta) \tag{37}$$

If corrections are made every N days, then the acceleration level is

$$a_{s4} = \frac{Sk}{M}\left(\frac{3N}{4}\right)\left(\frac{1 - 2\beta}{\sin \frac{p\pi}{2}}\right) \tag{38}$$

Figure 24 presents a plot of $\Delta V_{s4}/k$ as a function of p with β as a family parameter. Figures 25(a) and (b) present the corresponding plot for $(M/k)a_{s4}$ with N = 7 and N = 30.

Comparison of methods. - To explicitly compare the different methods of controlling eccentricity, figure 26 plots $\Delta V_{si}/k$ (for i = 2, 3, and 4) as a function of β with a duty cycle of 0.01. When β is 0, $\Delta V_{si}/k$ is 106 kilograms per meter per second (m/sec)/(m^2/kg) for methods 2 to 4. Equation (29) shows that $\Delta V_{s1}/k$ is independent of β and has a value of 142 kilograms per meter per second.

Several factors must be considered in choosing one of these methods of controlling eccentricity. If a cold gas system is used for station keeping, it is desirable to choose a method having a small ΔV per year. If β is large, method 4 requires significantly less ΔV per year (see fig. 26). However, this method requires more complex or more frequent station-keeping maneuvers than methods 2 or 3. If β is close to 0, then the ΔV per year is nearly the same for methods 2 to 4. In this case, method 2 might be chosen because it is the simplest. If a low-thrust, high-specific-impulse system is used for station keeping, then it is not so critical to minimize the ΔV per year. In this case, either method 1 or 2 may be best.

Interactions Between Station-Keeping Methods

To this point, the station-keeping requirements for correcting effects due to solar pressure and triaxiality have been discussed separately. Since longitude excursions are due to both solar pressure and triaxiality, two allowable longitude excursions ΔL_s and ΔL_t can be specified in such a way that

$$\Delta L_s + \Delta L_t = \Delta L \tag{39}$$

where ΔL is the maximum allowable longitude excursion due to both solar pressure and triaxiality. How ΔL_s and ΔL_t are chosen depends on the methods of eccentricity control and triaxiality control to be used. Method 1 for controlling eccentricity is a special case because the eccentricity is always zero. ΔL_s is not an applicable parameter. Therefore, ΔL_t may be set equal to any number less than or equal to ΔL. When using method 2, 3, or 4, the veolicity increment ΔV_{si}, for $i = 2$, 3, or 4, is dependent on ΔL_s through the parameter β. For each of these three methods, ΔV_{si} becomes smaller as ΔL_s becomes larger. On the other hand, ΔV_t is independent of ΔL_t. A general rule can thus be stated that ΔV requirements are minimized by choosing ΔL_s only slightly smaller than ΔL.

To demonstrate the interaction of station keeping for triaxiality and station keeping for solar pressure, assume that method 2 was chosen for controlling eccentricity. The spacecraft is assumed to have an area-to-mass ratio of 0.15 square meter per kilogram and a reflectivity of 0.3, so that $k = 0.195$ square meter per kilogram. Choose γ_o to be 45°. For $\Delta L = 0.30^{\circ}$, choose $\Delta L_s = 0.26^{\circ}$ and $\Delta L_t = 0.04^{\circ}$. With these assumed parameters, triaxiality corrections would be made every 20 days, and eccentricity corrections would be made every 65 days. Figure 27 presents a plot of $\Delta\gamma_t$ as a function of time when radial thrust is used for eccentricity control. The solid line represents $\Delta\gamma_t$. In any one orbit, $\Delta\gamma$ (in deg) is given approximately by

$$\Delta\gamma = \Delta\gamma_t + \Delta\gamma_s = \Delta\gamma_t + 2(57.3e)\sin \dot{\theta}_E t \tag{40}$$

where e is the instantaneous eccentricity of the orbit. Thus, $\Delta\gamma$ oscillates between $\Delta\gamma_t - 2(57.3e)$ and $\Delta\gamma_t + 2(57.3e)$. The dashed lines in figure 27 represent $\Delta\gamma_t + 2(57.3e)$ and $\Delta\gamma_t - 2(57.3e)$. For example, on the 50^{th} orbit ($t = 50$ in fig. 27), $\Delta\gamma$ oscillates between -0.16° and $+0.24^{\circ}$.

Tangential thrust for eccentricity control is more desirable than radial thrust because only half as much ΔV is required (see eqs. (D2) and (D5)). However, unlike the radial-thrust case, tangential thrust for circularizing the orbit causes a change of $3\pi e/4$ radians in $\Delta\gamma_t$ (see appendix D). Figure 28 presents a plot of $\Delta\gamma_t$ as a function of time when tangential thrust is used. At $t = 62$, the first orbit circularization is made, causing an increase of 0.31° in $\Delta\gamma_t$. The triaxiality effect is used to advantage here because it causes $\Delta\gamma_t$ to decrease steadily until $\Delta\gamma_t = -0.04^{\circ}$ on the 82nd orbit, at which time the triaxiality correction is made. It should be pointed out that figures 27 and 28 are idealized curves used only to give a feel for how triaxiality, solar pressure, and east-west station keeping affect $\Delta\gamma$. In an actual situation, other factors must be considered which might alter these curves. In particular, one must consider the effect

TABLE II. - STATION-KEEPING METHODS AND REQUIREMENTS

Perturbation	Method	Requirements
Sun and Moon	Change i every N days	$\Delta V_{NS}\left(\frac{m}{sec}\right) \quad (46)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)$ $a_{NS}\left(\frac{m}{sec^2}\right) = (2.3\times10^{-6})\left(\frac{N/M}{\sin \frac{p\pi}{2}}\right)$
Solar pressure	1 - Continuous thrust	$\Delta V_{s1} = \left(\frac{3Sk\pi}{2\dot{\lambda}}\right)\left(\frac{4}{3}\right)$ $a_{s1} = Sk$
	2 - Circularize whenever $e = e^*$	$\Delta V_{s2} = \left(\frac{3Sk\pi}{2\dot{\lambda}}\right)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)\left(\frac{\beta}{\sin^{-1}\beta}\right)$ $a_{s2} = \left(\frac{Sk}{M}\right)\left(\frac{3\dot{\theta}_E}{8\dot{\lambda}}\right)\left(\frac{\beta}{\sin \frac{p\pi}{2}}\right)$
	3 - Change ω whenever $e = e^*$	$\Delta V_{s3} = \left(\frac{3Sk\pi}{2\dot{\lambda}}\right)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)\left(\frac{\beta\sqrt{1 - \beta^2}}{\sin^{-1}\beta}\right)$ $a_{s3} = \left(\frac{Sk}{M}\right)\left(\frac{3\dot{\theta}_E}{4\dot{\lambda}}\right)\left(\frac{\beta\sqrt{1 - \beta^2}}{\sin \frac{p\pi}{2}}\right)$
	4 - Change ω slightly whenever $e = e^*$	$\Delta V_{s4} = \left(\frac{3Sk\pi}{2\dot{\lambda}}\right)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)(1 - 2\beta)$ $a_{s4} = \left(\frac{Sk}{M}\right)\left(\frac{3N}{4}\right)\left(\frac{1 - 2\beta}{\sin \frac{p\pi}{2}}\right)$
Triaxiality	**Change a whenever $\|\Delta\gamma_t\| = \Delta I_t$**	$\Delta V_t\left(\frac{m}{sec}\right) = 1.75\|\sin 2\gamma_o\|$

of errors in north-south corrections. Misalinement of the north-south thrust vector may cause an in-plane error. From reference 2, the ΔV per year to correct these in-plane errors is of the order of three meters per second.

The station keeping methods and requirements discussed in this section are summarized in table II. The equations given in table II are available in curve form in the report. Their use is best demonstrated by considering the sample problem given in appendix F.

CONCLUDING REMARKS

Station-keeping requirements have been determined for a class of high-power synchronous equatorial communication satellites characterized by large Sun-tracking solar arrays. The requirements for north-south control and for east-west control due to the Earth's triaxiality are the same as for previous communication satellites. However, because of the larger solar arrays, the effect of solar radiation pressure must also be considered for this new class of satellites. Solar radiation pressure produces an acceleration proportional to the area-to-mass ratio of the satellite, and the resultant accelerations change the eccentricity of the satellite orbit. The eccentricity causes an apparent daily east-west oscillation in the position of the satellite. For high area-to-mass ratio satellites, the east-west drift would be nearly 1° longitude which would require station keeping for some missions.

Equations and curves are given based on the assumption that all station-keeping corrections are carried out over a specified number of consecutive orbits with two thrusting periods per orbit. For north-south corrections, the thrust is directed toward the north in one thrusting period and directed toward the south in the other. Alternatively, north-south corrections could be made with only one thrusting period per orbit, the thrust always being in the same direction. The curves giving ΔV for north-south corrections can be modified to handle the case of one thrusting period per orbit. The ΔV curves for east-west corrections, however, are usable only for the case of two thrusting periods per orbit.

This report covers the station keeping problem with emphasis on nonimpulsive low-thrust methods of station keeping. Parametric equations and curves giving ΔV and thruster acceleration requirements for the various methods of station keeping as a function of duty cycle and frequency of correction are presented.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 14, 1970,
164-21.

APPENDIX A

RADIO-FREQUENCY RADIATION PRESSURE

The momentum of a quantum of energy is given by

$$H = \frac{E}{C} = \frac{h\nu}{C} \tag{A1}$$

where E is the energy of the quantum, C is the velocity of light, h is Planck's constant and ν is the frequency of radiation. If a plane source is emitting electromagnetic radiation, the force F associated with the radiation is given by

$$F = \frac{d}{dt}(H) = \frac{d}{dt}\left(\frac{E}{C}\right) = \frac{W}{C} \tag{A2}$$

where W is the total radiated power. The acceleration of a satellite due to radio-frequency radiation pressure is given by

$$a_{rf} = \frac{F}{m} = \frac{W/m}{C} \tag{A3}$$

Based on a solar array packing density of 100 watts per square meter, a maximum area-to-mass ratio of 0.15 square meter per kilogram, and assuming that all of the collected power is radiated as radio-frequency power, then the maximum power-to-mass ratio W/m = 15 watts per kilogram. Hence,

$$\left(a_{rf}\right)_{max} = \frac{(W/m)_{max}}{C} = \frac{15 \text{ watts/kg}}{3\times10^{8} \text{ m/sec}} = 5\times10^{-8} \text{ m/sec}^2 \tag{A4}$$

A focused beam of radio-frequency energy emanating from an equatorial synchronous satellite may be deliberately pointed off the local vertical by as much as 8.7°. The resultant acceleration vector $\bar{a}_{rf}$ of such a beam may be resolved into in-plane and out-of-plane components. The out-of-plane component, being nearly constant, will have a negligible effect on the orbit. The in-plane component of $\bar{a}_{rf}$ may be further resolved into radial and tangential components.

The radial component of $\bar{a}_{rf}$ has the same kind of effect as the Earth's oblateness. If an adjustment in the orbit radius is made, no station keeping is necessary. The adjustment in radius for oblateness effects was 520 meters. For a radial component of $\bar{a}_{rf}$ equal to 5×10^{-8} meter per second squared, the adjustment in radius for radio-frequency effects is only 3 meters.

The effect of the tangential component of $\bar{a}_{rf}$ is similar to that of triaxiality in that a steady longitudinal drift of the satellite is produced. Let $a_{t_{rf}}$ denote the tangential component of $\bar{a}_{rf}$. The calculation of ΔV due to $a_{t_{rf}}$ is the same as the calculation of ΔV due to triaxiality. It can be shown that the ΔV is proportional to the magnitude of the perturbing acceleration. The maximum value of $a_{t_{rf}}$ would occur if the satellite antenna is directed toward the east or west horizon at the equator. For this case, the maximum value of $a_{t_{rf}}$ is 7.5×10^{-9} meter per second squared. The maximum tangential acceleration due to triaxiality is seven times greater. The ΔV due to $a_{t_{rf}}$ is no more than one-seventh as much as the ΔV due to worst-case triaxiality. The total ΔV per year due to radio-frequency effects does not exceed 0.3 meter per second.

APPENDIX B

ANALYSIS OF e(t) AND ω(t) WHEN $e_0 = 0$

In this appendix equations are derived for e(t) and ω(t) when the initial orbit is circular ($e_0 = 0$). As mentioned previously, solar radiation pressure is the only perturbation having an appreciable effect on e and ω. From reference 6, the time rate of change of e and ω is given by

$$\frac{de}{dt} = \frac{-3\sqrt{a(1 - e^2)}}{2\sqrt{\mu}} Sk(\overline{Q} \cdot \overline{U}) \tag{B1}$$

$$\frac{d\omega}{dt} = \frac{3\sqrt{a(1 - e^2)}}{2\sqrt{\mu}e} Sk(\overline{P} \cdot \overline{U}) \tag{B2}$$

For small e, the equations can be simplified to yield

$$\frac{de}{dt} = \frac{-3Sk}{2V} (\overline{Q} \cdot \overline{U}) \tag{B3}$$

$$\frac{d\omega}{dt} = \frac{3Sk}{2Ve} (\overline{P} \cdot \overline{U}) \tag{B4}$$

To simplify the analysis, assume the orbit plane, equatorial plane, and ecliptic plane are one and the same. Then, as shown in figure 29, the unit vectors $\overline{U}$, $\overline{P}$, $\overline{Q}$ lie in this plane. Further, assume that the Sun is initially along the X-axis so that $\lambda = \dot{\lambda}t$. Expanding the dot products in equations (B3) and (B4) gives

$$\frac{de}{dt} = \frac{3Sk}{2V} \sin(\omega - \dot{\lambda}t) \tag{B5}$$

$$\frac{d\omega}{dt} = \frac{3Sk}{2Ve} \cos(\omega - \dot{\lambda}t) \tag{B6}$$

Figure 30 shows the process by which solar pressure causes an initially circular orbit to become eccentric. When the satellite is in the vicinity of the positive Y-axis, solar pressure accelerates its motion and causes it to seek a higher altitude. When the satellite is in the vicinity of the negative Y-axis, solar pressure decelerates the satellite's motion and causes it to seek a lower altitude. The result is a shifting of the orbit, creating a perigee on the positive Y-axis and an apogee on the negative Y-axis. Therefore, if e_0 is 0, then ω_0 must be $\pi/2$ radians. The solution to equations (B5) and (B6) with the initial conditions $e_0 = 0$, $\omega_0 = \pi/2$ is found by assuming a linear variation of ω with time. The solution is

$$e = \frac{3Sk}{V\dot{\lambda}} \sin \frac{\dot{\lambda}}{2} t \tag{B7}$$

$$\omega = \frac{\dot{\lambda}t}{2} + \frac{\pi}{2}, \quad 0 \leq t \leq 1 \text{ year} \tag{B8}$$

APPENDIX C

ANALYSIS OF e(t) AND ω(t) WHEN $e_0 \neq 0$

In appendix B, analytic solutions for e(t) and ω(t) were derived for the case $e_0 = 0$. Consider the initial conditions $e_0 = \frac{1}{2} e_p$ and $\omega_0 = 0$. Using the same assumptions and definitions as in appendix B, analytic solutions for e(t) and ω(t) can be found. The equations for e and ω are

$$\dot{e} = \frac{-3Sk}{2V} (\overline{Q} \cdot \overline{U}) \tag{C1}$$

$$\dot{\omega} = \frac{+3Sk}{2Ve} (\overline{P} \cdot \overline{U}) \tag{C2}$$

Assuming that the vectors $\overline{U}$, $\overline{P}$, and $\overline{Q}$ lie in the same plane allows equations (C1) and (C2) to be written as

$$\dot{e} = \frac{+3Sk}{2V} \sin(\omega - \dot{\lambda}t) \tag{C3}$$

$$\dot{\omega} = \frac{+3Sk}{2Ve} \cos(\omega - \dot{\lambda}t) \tag{C4}$$

The solution to equations (C3) and (C4) with the initial conditions $e_0 = \frac{1}{2} e_p$ and $\omega_0 = 0$ is found by assuming a linear variation of ω with time. The solution is

$$e = \frac{3Sk}{2V\dot{\lambda}} = \frac{1}{2} e_p \tag{C5}$$

$$\omega = \dot{\lambda}t \tag{C6}$$

This solution for e(t) and ω(t) corresponds to the Sun-oriented orbit. When e_0 has a value other than 0 or $\frac{1}{2} e_p$, equations (C3) and (C4) are not amenable to closed-form solutions. Computer solutions were obtained for e(t) and ω(t) by numerically integrating equations (C1) and (C2). The assumption of the planar problem was not used when obtaining computer solutions. In all cases, it was assumed that $\lambda_0 = 0$ (starting at autumnal equinox). For any values of e_0 and ω_0, e(t) and ω(t) were found to be periodic functions with period of 1 year. In all cases, the apsidal line made either 0 or 1 net revolution per year. Figure 31 presents a plot of e/k as a function of time for the initial conditions $e_o = 0.1\, e_p$ and $\omega_o = 0$. Notice that the minimum eccentricity is greater than zero and the maximum eccentricity is less than e_p. For a given e_o, define two functions of ω_o. Let $e\ \max(\omega_o)$ be the maximum value of eccentricity obtained when (e_o, ω_o) are the initial conditions, and let $e\ \min(\omega_o)$ be the minimum value of eccentricity when (e_o, ω_o) are the initial conditions. For $e_o = 0.1\, e_p$, figure 31 shows that $e\ \min(0) = 0.002\ k$ and $e\ \max(0) = 0.018\ k$. Figures 32 to 34 present $e\ \max(\omega_o)/k$ and $e\ \min(\omega_o)/k$ for three different values of e_o. In all cases, $e\ \max$ is smallest when $\omega_o = 0$. Of particular interest is the case $e_o = \frac{1}{2} e_p$ (fig. 34). $e\ \max(0)$ and $e\ \min(0)$ are nearly equal, implying that the eccentricity is nearly constant. The reason for this is that the initial conditions $e_o = \frac{1}{2} e_p$, $\omega_o = 0$ correspond to the Sun-oriented orbit.

APPENDIX D

STATION-KEEPING TECHNIQUES FOR CHANGING ORBITAL ECCENTRICITY AND ROTATING THE LINE OF APSIDES

Circularizing a Slightly Eccentric Orbit

In this appendix, it is assumed that all station-keeping corrections are completed in a 24-hour period ($M = 1$). A slightly eccentric orbit can be circularized by radial thrusting (thrusting in a direction collinear with the radius vector) or by tangential thrusting (thrusting in a direction collinear with the orbit velocity vector). Figure 35 presents a sketch of the radial thrusting maneuver when impulsive thrusting is used. Position A_1 corresponds to a true anomaly of 90^o. This correction is made by first thrusting inward (toward the Earth) in the vicinity of A_1 and then thrusting outward in the vicinity of A_2. The transfer orbit has an eccentricity of $\frac{1}{2}$ e. The ΔV_c for impulsive thrusting is

$$\Delta V_c = eV \tag{D1}$$

When nonimpulsive thrusting is used,

$$\Delta V_c = eV \frac{p\pi}{2 \sin \frac{p\pi}{2}} \tag{D2}$$

Let γ_p be the value of γ at the perigee of the orbit just prior to the circularization maneuver. Then define $\Delta\gamma_p$ by the equation

$$\Delta\gamma_p = \gamma - \gamma_p \tag{D3}$$

where γ_p is the mean longitude in the sense that the satellite longitude (before the station-keeping correction) oscillates about γ_p with an amplitude of 2e radians and a period of 24 hours. Figure 36 presents a plot of $\Delta\gamma_p$ as a function of time when the duty cycle of the station-keeping correction is 0.01. Notice that $\Delta\gamma_p$ is zero after the completion of the station-keeping maneuver.

Figure 37 presents a sketch of the tangential thrusting maneuver when impulsive thrusting is used. Position A_1 is the orbit perigee for tangential thrusting. The correction is made by first thrusting westward in the vicinity of A_1 and then thrusting eastward in the vicinity of A_2. The transfer orbit has eccentricity $\frac{1}{2}$ e.

The ΔV_c for impulsive thrusting is

$$\Delta V_c = \frac{1}{2} eV \tag{D4}$$

When nonimpulsive thrusting is used,

$$\Delta V_c = \frac{1}{2} eV \frac{p\pi}{2 \sin \frac{p\pi}{2}} \tag{D5}$$

which is one-half as much ΔV_c as required with the radial correction scheme.

Figure 38 presents a plot of $\Delta\gamma_p$ as a function of time when tangential thrusting is used. Notice that a residual longitude error of $3\pi e/4$ radians is left after the completion of this station-keeping maneuver. It can be shown that an additional ΔV of approximately $eV/4d$ is required to remove the residual longitude error if d is the time in days to make the correction.

Rotating Line of Apsides

Now consider radial and tangential thrust maneuvers which will rotate the line of apsides of an orbit having a small eccentricity e. Let ψ be the angle through which the apsidal line is to be rotated. Figure 39 is a sketch of a radial correction maneuver when impulsive thrusting is used. The correction is made by thrusting outward in the vicinity of A_1 and thrusting inward in the vicinity of A_2. With radial thrusting, A_1 corresponds to a true anomaly of $\frac{1}{2}\psi$ in the final orbit. When impulsive thrusting is used, ΔV_c is given by

$$\Delta V_c = 2eV \sin \frac{\psi}{2} \tag{D6}$$

For nonimpulsive thrusting, ΔV_c is given by

$$\Delta V_c = 2eV \sin \frac{\psi}{2} \left(\frac{p\pi}{2 \sin \frac{p\pi}{2}} \right) \tag{D7}$$

Figure 40 presents a plot of $\Delta\gamma_p$ as a function of time when rotating the apsidal line 20^0 with radial thrusting. Notice that $\Delta\gamma_p$ is zero after the completion of the station-keeping maneuver.

When rotating the line of apsides with tangential thrusting, A_1 corresponds to a true anomaly of $\frac{1}{2}\psi + 90^0$ in the final orbit. The correction is made by first thrusting westward in the vicinity of A_1 and then thrusting eastward in the vicinity of A_2. Figure 41 is a sketch of this maneuver when impulsive thrusting is used.

When impulsive thrusting is used, ΔV_c is given by

$$\Delta V_c = eV \sin\frac{\psi}{2} \tag{D8}$$

For nonimpulsive thrusting, ΔV_c is given by

$$\Delta V_c = eV \sin\frac{\psi}{2}\left(\frac{p\pi}{2\sin\frac{p\pi}{2}}\right) \tag{D9}$$

Only half as much ΔV_c is required with tangential thrusting as with radial thrusting. Figure 42 is a plot of $\Delta\gamma_p$ as a function of time when rotating the apsidal line 20^0 with tangential thrusting. A residual longitude error of $(3\pi e/2)\sin\frac{\psi}{2}$ radians is left after the completion of the maneuver.

APPENDIX E

DERIVATION OF EQUATIONS FOR ΔV_{si} AND a_{si} FOR THE FOUR METHODS OF CONTROLLING ECCENTRICITY

METHOD 1

Method 1 is to continuously cancel the effect of solar pressure by thrusting in a direction toward the Sun. The acceleration level is given simply by

$$a_{s1} = Sk \tag{E1}$$

The ΔV per year is given by

$$\Delta V_{s1} = Sk\left(\frac{2\pi}{\dot{\lambda}}\right) \tag{E2}$$

It will be found convenient to express ΔV_{s1} as

$$\Delta V_{s1} = \left(\frac{3Sk\pi}{2\dot{\lambda}}\right)\left(\frac{4}{3}\right) \tag{E3}$$

METHOD 2

Method 2 is to circularize the orbit each time the eccentricity becomes equal to e^*. For this method, ΔV_c (ΔV per correction) can be found by using equation (D5).

$$\Delta V_c = \frac{e^* V}{2}\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right) \tag{E4}$$

Recalling that $e^* = \beta e_p$ and from equation (C5)

$$e_p = \frac{3Sk}{V\dot{\lambda}}$$

allows equation (E4) to be written as

$$\Delta V_c = \frac{3Sk\beta}{2\dot{\lambda}}\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right) \tag{E5}$$

From appendix B equation (B7), the unkept eccentricity is

$$e = e_p \sin \frac{\dot{\lambda}}{2} t \tag{E6}$$

The time t_c between corrections can then be found from the equation

$$e_p\beta = e^* = e_p \sin \frac{\dot{\lambda}}{2} t_c \tag{E7}$$

Solving for t_c yields

$$t_c = \frac{2 \sin^{-1}\beta}{\dot{\lambda}} \tag{E8}$$

The number of corrections per year K is given by

$$K = \frac{1 \text{ year}}{t_c} = \left(\frac{2\pi}{\dot{\lambda}}\right)\bigg/\left(\frac{2 \sin^{-1}\beta}{\dot{\lambda}}\right) = \frac{\pi}{\sin^{-1}\beta} \tag{E9}$$

ΔV_{s2} is then given by $(K)(\Delta V_c)$.

$$\Delta V_{s2} = \left(\frac{3\pi Sk}{2\dot{\lambda}}\right)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)\left(\frac{\beta}{\sin^{-1}\beta}\right) \tag{E10}$$

The acceleration level can be found from the equation

$$\Delta V_c = a_{s2}\left(\frac{2\pi pM}{\dot{\theta}_E}\right) \tag{E11}$$

Solving for a_{s2} yields

$$a_{s2} = \left(\frac{Sk}{M}\right)\left(\frac{3\dot{\theta}_E}{8\dot{\lambda}}\right)\left(\frac{\beta}{\sin\frac{p\pi}{2}}\right) \tag{E12}$$

METHOD 3

Method 3 is to rotate the line of apsides in such a manner that the solar pressure will cause the eccentricity to decrease to zero before increasing again. Let $2t_c$ be the time between corrections. Using the same derivation as in method 2,

$$t_c = \frac{2 \sin^{-1}\beta}{\dot{\lambda}} \tag{E13}$$

ΔV_c can be found by using equation (D9) to be

$$\Delta V_c = \left(e^{*}V \sin\frac{\Delta\omega}{2}\right)\left(\frac{p\pi}{2 \sin\frac{p\pi}{2}}\right) \tag{E14}$$

The values of ω and λ at time t_c are

$$\omega = \frac{\dot{\lambda}}{2} t_c + \frac{\pi}{2} = \sin^{-1}\beta + \frac{\pi}{2} \tag{E15}$$

$$\lambda = \dot{\lambda} t_c = 2 \sin^{-1}\beta \tag{E16}$$

The angle $\Delta\omega$ is then given by

$$\Delta\omega = 2(\omega - \lambda) = \pi - 2 \sin^{-1}\beta \tag{E17}$$

Now ΔV_c may be expressed as

$$\Delta V_c = \frac{3Sk\beta}{\dot{\lambda}} \sin\left(\frac{\pi}{2} - \sin^{-1}\beta\right)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right) \tag{E18}$$

It may be shown that

$$\sin\left(\frac{\pi}{2} - \sin^{-1}\beta\right) = \sqrt{1 - \beta^2}$$

Hence,

$$\Delta V_c = \frac{3Sk\beta\sqrt{1 - \beta^2}}{\dot{\lambda}}\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right) \tag{E19}$$

The number of corrections per year K is given by

$$K = \frac{1 \text{ year}}{2t_c} = \frac{\pi}{2 \sin^{-1}\beta} \tag{E20}$$

ΔV_{s3} is then given by $(K)(\Delta V_c)$.

$$\Delta V_{s3} = \left(\frac{3\pi Sk}{2\dot{\lambda}}\right)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)\left(\frac{\beta\sqrt{1 - \beta^2}}{\sin^{-1}\beta}\right) \tag{E21}$$

The acceleration level can be found from the equation

$$\Delta V_c = a_{s3}\left(\frac{2\pi pM}{\dot{\theta}_E}\right) \tag{E22}$$

Solving for a_{s3} gives

$$a_{s3} = \left(\frac{Sk}{M}\right)\left(\frac{3\dot{\theta}_E}{4\dot{\lambda}}\right)\left(\frac{\beta\sqrt{1 - \beta^2}}{\sin\frac{p\pi}{2}}\right) \tag{E23}$$

METHOD 4

Method 4 maintains the eccentricity nearly equal to e^* by frequently rotating the apsidal line in such a manner that $\omega \approx \lambda$. To derive formulas for ΔV_{s4} and a_{s4}, return to the planar problem where it was assumed that the Earth, the Sun, and the satellite are in the same plane. Further assume that the Sun and the orbit perigee initially are along the X-axis and that $\beta < \frac{1}{2}$. Thus, $\lambda = \dot{\lambda}t$ and $\omega_o = 0$. In equations (B5) and (B6), $\dot{e}$ and $\dot{\omega}$ are given by

$$\dot{e} = \frac{3Sk}{2V}\sin(\omega - \dot{\lambda}t) \tag{E24}$$

$$\dot{\omega} = \frac{3Sk}{2Ve}\cos(\omega - \dot{\lambda}t) \tag{E25}$$

With method 4, the apsidal line is rotated a small amount each time the eccentricity becomes equal to e^*. This small rotation would cause the eccentricity to decrease only slightly before increasing again. Eccentricity as a function of time for this method is shown in figure 43. In this curve, $2t_c$ is the time between corrections, and αe^* is the minimum eccentricity. The time t_c will be relatively small, and α will be only slightly less than 1. Since e is kept nearly constant, then de/dt must be approximately zero. From equation (E24), it follows that ω must be kept nearly equal to $\dot{\lambda}t$. Thus the apsidal line must be controlled so that perigee remains directed toward the Sun. Assume now that at $t = 0$, $e = \alpha e^*$. From equation (E25), for small t,

$$\dot{\omega} = \frac{3Sk}{2Ve_o} \tag{E26}$$

By assumption $e_o < \frac{1}{2} e_p$. Using the equation $\frac{1}{2} e_p = 3Sk/2V\dot{\lambda}$, we obtain $\dot{\omega} > \dot{\lambda}$. The apsidal line, if uncorrected, will rotate faster than the Sun. The control of the apsidal line must then be as given in figure 43.

The times of correction are t_c, $3t_c$, $5t_c$, ... With the assumption that t_c is small, ΔV requirements can be determined by linearizing and solving equations (E24) and (E25) in the time interval $0 \leqq t \leqq t_c$. The initial conditions are $\omega_o = 0$, $\lambda_o = 0$, and $e_o = \alpha e^*$. The linearized equations are

$$\frac{de}{dt} = \frac{3Sk}{2V} (\omega - \dot{\lambda} t) \tag{E27}$$

$$\frac{d\omega}{dt} = \frac{3Sk}{2Ve_o} \tag{F28}$$

Equation (E28) can be further simplified by substituting for e_o:

$$e_o = \frac{3Sk\alpha\beta}{V\dot{\lambda}} \tag{E29}$$

Using the assumption that α is approximately equal to 1, equation (E28) becomes

$$\frac{d\omega}{dt} = \frac{\dot{\lambda}}{2\beta} \tag{E30}$$

Solving for ω,

$$\omega = \frac{\dot{\lambda}}{2\beta} t \tag{E31}$$

Substituting this solution for ω into equation (E27) gives

$$\frac{de}{dt} = \frac{3Sk\dot{\lambda}}{2V} \left(\frac{1}{2\beta} - 1\right) t \tag{E32}$$

which can be integrated to give

$$e = e_o + \frac{3Sk\dot{\lambda}}{4V} \left(\frac{1}{2\beta} - 1\right) t^2 \tag{E33}$$

From equation (E33), t_c is calculated to be

$$t_c = \frac{2\beta}{\dot{\lambda}} \sqrt{\frac{2(1 - \alpha)}{1 - 2\beta}} \tag{E34}$$

The rotation angle $\Delta\omega$ of the apsidal line is

$$\Delta\omega = 2(\dot{\omega} t_c - \dot{\lambda} t_c) = 2\dot{\lambda} t_c \left(\frac{1 - 2\beta}{2\beta}\right) = 2\sqrt{2(1 - \alpha)(1 - 2\beta)} \tag{E35}$$

Assuming $\Delta\omega$ is small and α is approximately 1, ΔV_c can be found by using equation (D9)

$$\Delta V_c = e^* V \frac{\Delta\omega}{2} \left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right) \tag{E36}$$

$$\Delta V_c = \left(\frac{3Sk\beta}{\dot{\lambda}}\right)(\dot{\lambda} t_c)\left(\frac{1 - 2\beta}{2\beta}\right)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right) \tag{E37}$$

Rearranging terms,

$$\Delta V_c = \frac{3Skt_c}{2} (1 - 2\beta)\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right) \tag{E38}$$

The number of corrections per year K is given by

$$K = \frac{1 \text{ year}}{2t_c} = \frac{\pi}{\dot{\lambda} t_c} \tag{E39}$$

ΔV_{s4} is then given by $(K)(\Delta V_c)$.

$$\Delta V_{s4} = \frac{3Sk\pi}{2\dot{\lambda}}\left(\frac{p\pi}{2 \sin \frac{p\pi}{2}}\right)(1 - 2\beta) \tag{E40}$$

The acceleration level a_{s4} can be found from the equation

$$\Delta V_c = a_{s4}\left(\frac{2\pi pM}{\dot{\theta}_E}\right) \tag{E41}$$

Solving for a_{s4} yields

$$a_{s4} = \frac{3Sk\dot{\theta}_E(1 - 2\beta)t_c}{8M \sin \frac{p\pi}{2}} \tag{E42}$$

If corrections are made every N days, then

$$2t_c = \frac{2\pi N}{\dot{\theta}_E}$$

and

$$a_{s4} = \frac{3Sk\pi N(1 - 2\beta)}{8M \sin \frac{p\pi}{2}} \tag{E43}$$

APPENDIX F

SAMPLE PROBLEM

INTRODUCTION

To demonstrate the use of the curves in this report, an example of a typical high-power communication satellite mission is presented. The satellite parameters are

Mass, kg	1000
Area-to-mass ratio, m^2/kg	0.154
Average reflectivity	0.3

The mission requirements are

Allowable longitudinal error, deg	0.2
Longitude position, deg west longitude	95
Mission life, yr (does not include 1 yr reserve)	5

Thruster systems available are

(1) Thrust, N	9.8
Specific impulse, I_{sp}, sec	100
(2) Thrust, N	4.9×10^{-3}
Specific impulse, I_{sp}, sec	2000

The high-thrust system yields an acceleration of 10^{-3} g on the satellite; the low-thrust system yields an acceleration of 5×10^{-5} g on the satellite.

NORTH-SOUTH STATION KEEPING

High-Thrust System

Figures 11 and 12 are used to determine ΔV_{NS}. The first step is to determine from figure 12 the duty cycle p for given values of M, N, and a_{NS}. Assume that corrections are made once every 60 days (N = 60) and that the correction is carried out in 1 day (M = 1). The ordinate Ma_{NS} of figure 12 is 10^{-3} g, and the corresponding abscissa p is approximately 0.01. So the correction is accomplished by thrusting in the northerly direction for approximately 0.12 hour, and a half orbit later thrusting in the southerly direction for approximately 0.12 hour. Knowing p from figure 12, the ΔV per year, ΔV_{NS}, is then determined from figure 11 to be 46 meters per second.

Low-Thrust System

For the low-thrust case, assume first that corrections are made daily (M = 1, N = 1). The ordinate, Ma_{NS}, of figure 12 is 5×10^{-7} g, and the corresponding abscissa p is 0.3. From figure 11, ΔV_{NS} is 48 meters per second. If it is now assumed that corrections are made weekly (N = 7), the smallest possible value of the ordinate Ma_{NS} in figure 12 is approximately 1.5×10^{-6} g, corresponding to a unity duty cycle p. Since a_{NS} is 5×10^{-7} g, M must be 3 for this case. Thus the correction scheme consists of continuous thrusting for 3 days and no thrusting for the next 4 days. Since the duty cycle p is unity, the ΔV per year from figure 11 is 72 meters per second.

Figures 11 and 12 can be modified to handle the case of one thrusting period per orbit instead of two. The modifications are:

(1) The thrusting time per orbit divided by the orbit period is 2p (instead of p).

(2) The number of days between the beginnings of successive inclination corrections is N/2 (instead of N).

EAST-WEST STATION KEEPING DUE TO TRIAXIALITY

The ΔV required for triaxiality is a function only of γ_0. For a desired satellite position of 95° west longitude, $\gamma_0 = 105^\circ - 95^\circ = 10^\circ$. The ΔV per year is given in figure 15 to be 0.5 meters per second.

EAST-WEST STATION KEEPING DUE TO SOLAR PRESSURE

Method 1

For continuous thrusting against the Sun, the ΔV per year, ΔV_{s1}, is given by equation (29) to be

$$\Delta V_{s1} = \frac{3\pi Sk}{2\dot{\lambda}}\left(\frac{4}{3}\right) \tag{F1}$$

For this satellite, the ΔV per year is 28.4 meters per second.

Method 2

High-thrust system. - In determining ΔV requirements for the remaining station-keeping methods, ΔL_s and ΔL_t must first be chosen. Let $\Delta L_s = 0.15^o$ and $\Delta L_t = 0.05^o$. The parameters k and β can be calculated from equations (4) and (32), respectively.

$$k = (1 + \sigma) \frac{A}{m} = 0.20 \frac{m^2}{kg} \tag{F2}$$

$$\beta = 0.4 \frac{\Delta L_s}{k} = 0.30 \tag{F3}$$

To determine ΔV_{s2}, refer to figures 17 and 18. The first step is to determine from figure 18 the duty cycle p for given values of M and k. Assuming M = 1, the ordinate $(M/k)\, a_{s2}$ of figure 18 is 5×10^{-3} g's per square meter per kilogram. The corresponding abscissa p, with the family parameter $\beta = 0.30$, is seen to be much less than 0.01. Thus the orbit is circularized by two thrust pulses (12 hr apart) of duration much less than 0.24 hour. Knowing p and β, figure 17 shows $\Delta V_{s2}/k$ is 104 (m/sec)/(m^2/kg), so that the ΔV per year, ΔV_{s2}, is 20.8 meters per second.

Low-thrust system. - From figure 18, with the family parameter $\beta = 0.30$, the smallest possible value of the ordinate $(M/k)a_{s2}$ is approximately 2.0×10^{-5} g's per square meter per kilogram, corresponding to an abscissa p of 1.0. Since $(1/k)a_{s2}$ is 2.5×10^{-6} g's per square meter per kilogram, M is 8 in this particular case. Thus, for this low-thrust case, 8 days of continuous thrust is required to circularize the orbit. From equation (E8), corrections are made once every 35 days. For a unity duty cycle and $\beta = 0.30$, figure 17 shows $\Delta V_{s2}/k$ is 162 (m/sec)/(m^2/kg), so that the ΔV per year, ΔV_{s2}, is 32.4 meters per second.

Since ΔV_{s2} is largest when p = 1.0, it is desirable to choose a smaller p. If p is chosen to be 0.3, then from figure 18, $(M/k)a_{s2}$ is 3×10^{-5} g's per square meter per kilogram, implying that M is 12. For this case, the orbit is circularized by thrusting 7.2 hours per orbit for 12 consecutive orbits. From figure 17, the ΔV per year, ΔV_{s2}, is 21.2 meters per second, considerably less than the 32.4 meters per second for a unity duty cycle.

Method 3

High-thrust system. - To determine ΔV_{s3}, refer to figures 21 and 22. Assuming $M = 1$, the ordinate $(M/k)a_{s3}$ of figure 22 is 5×10^{-3} g's per square meter per kilogram. The corresponding abscissa p, with the family parameter $\beta = 0.30$, is seen to be less than 0.01. Thus the apsidal line is rotated by two thrust pulses (12 hr apart) of duration less than 0.24 hours. Knowing p and β, figure 21 shows $\Delta V_{s3}/k$ is 100 (m/sec)/ (m^2/kg), so that the ΔV per year, ΔV_{s3}, is 20.0 meters per second.

Low-thrust system. - If p is chosen to be 0.4, then from figure 22 $(M/k)a_{s3}$ is 6×10^{-5} g's per square meter per kilogram, implying that M is 24. For this case, the apsidal line is rotated by thrusting 9.6 hours per orbit for 24 consecutive orbits. From equation (E13), corrections are made once every 70 days. From figure 21, $\Delta V_{s3}/k$ is 105 (m/sec)/(m^2/kg), so that the ΔV per year, ΔV_{s3}, is 21.0 meters per second.

Method 4

High-thrust system. - Unlike methods 2 and 3, the parameter N in method 4 is not a function of β. The only restriction on N in method 4 is that it be small enough to justify linearizing the differential equations for e and ω. For the range of parameters considered, the linearized equations can be justified for $N \leq 30$. To determine ΔV_{s4} for the high-thrust case, refer to figures 24 and 25. Assuming $N = 30$ and $M = 1$, the ordinate $(M/k)a_{s4}$ of figure 25(b) is 5×10^{-3} g's per square meter per kilogram. The corresponding abscissa p, with the family parameter $\beta = 0.30$, is seen to be much less than 0.01. Thus the apsidal line is rotated by two thrust pulses (12 hr apart) of duration much less than 0.24 hour. Knowing p and β, figure 24 shows $\Delta V_{s4}/k$ is 42 (m/sec)/ (m^2/kg), so that the ΔV per year, ΔV_{s4}, is 8.4 meters per second.

Low-thrust system. - If N is now chosen to be 7, and a duty cycle p of 0.2 is desired, then from figure 25(a), $(M/k)a_{s4}$ is approximately 5×10^{-6} g's per square meter per kilogram, implying that M is 2. For this case, the apsidal line is rotated by thrusting 4.8 hours per orbit for 2 consecutive orbits. From figure 24, $\Delta V_{s4}/k$ is 43 (m/sec)/(m^2/kg), so that the ΔV per year, ΔV_{s4}, is 8.6 meters per second.

SUMMARY OF REQUIREMENTS

Assuming the ratio of propellant mass to spacecraft mass is small, the propellant mass as function of ΔV is given by

$$m_p = \frac{m \, \Delta V}{g I_{sp}} \qquad \text{(F4)}$$

where m_p is the propellant mass, m is the spacecraft mass, and g is the acceleration of gravity (9.8 m/sec^2). Equation (F4) can be used to calculate the propellant mass once the ΔV is known. Table III summarizes the station-keeping requirements.

TABLE III. - SAMPLE PROBLEM STATION-KEEPING REQUIREMENTS FOR 5 + 1 YEAR MISSION

	High thrust				Low thrust			
	Propellant mass, kg	Velocity increment, ΔV, m/sec	Duty cycle, p	Time between corrections, days	Propellant mass, kg	Velocity increment, ΔV, m/sec	Duty cycle, p	Time between corrections, days
North-south	282	276	0.01	60	14.7	288	0.30	1
Solar pressure method 4	51	50	<0.01	30	2.7	52	0.20	7
Triaxiality	3	3	-----	------	0.2	3	----	------

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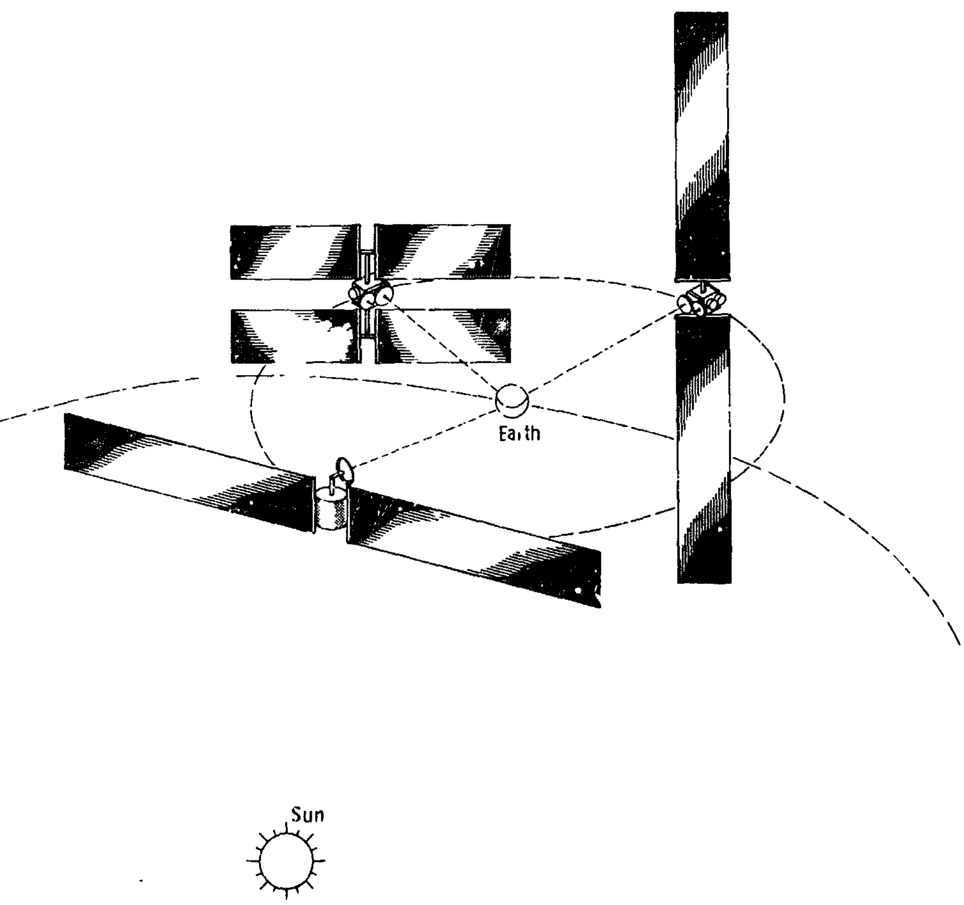


Figure 1. - Three typical configurations of a high-power communication satellite.

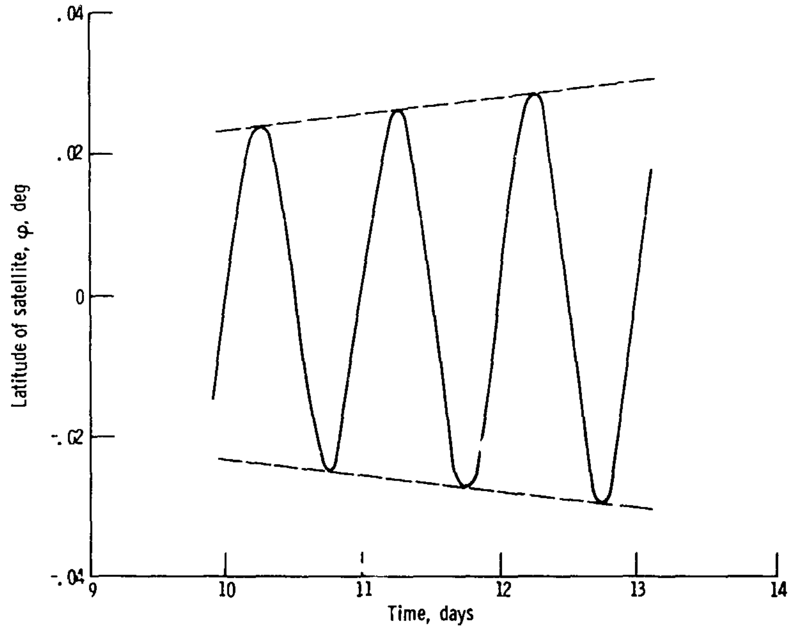


Figure 2. - Satellite latitude as function of time. Perturbations, Sun and Moon.

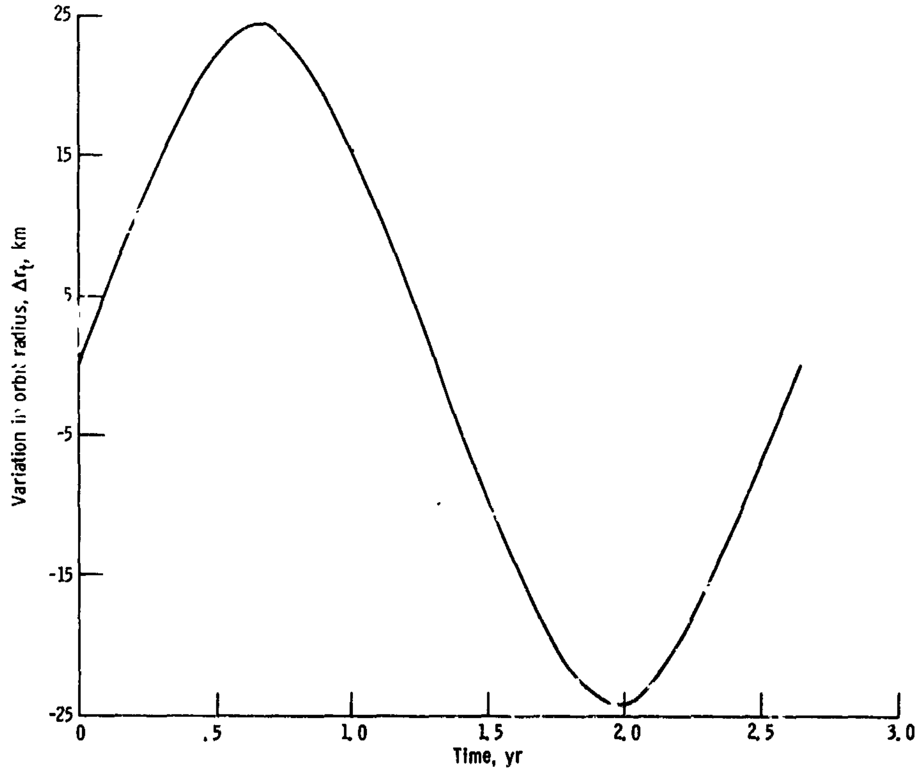


Figure 3. - Variation in orbit radius as function of time. Perturbation, triaxiality; desired satellite longitude, 45°.

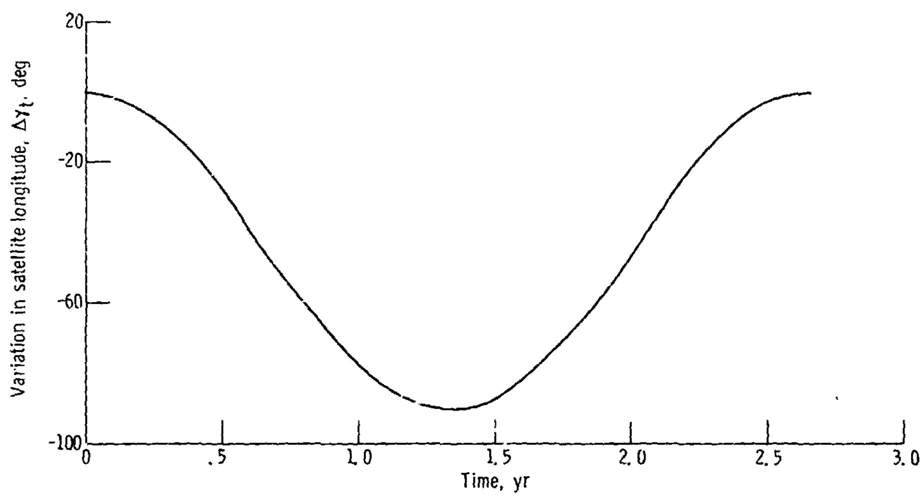


Figure 4. - Variation in satellite longitude as function of time. Perturbation, triaxiality; desired satellite longitude measured from Earth's minor axis, 45°.

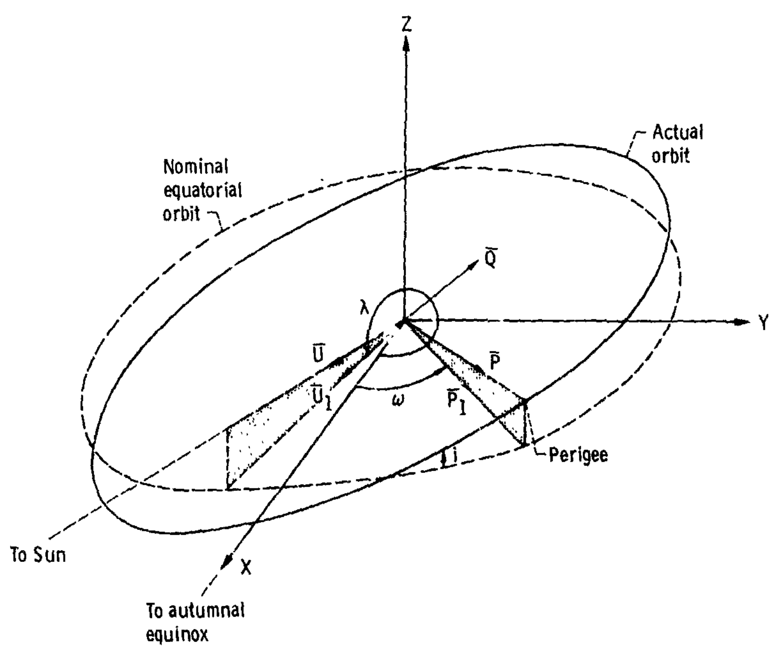


Figure 5. - Inertial coordinate system.

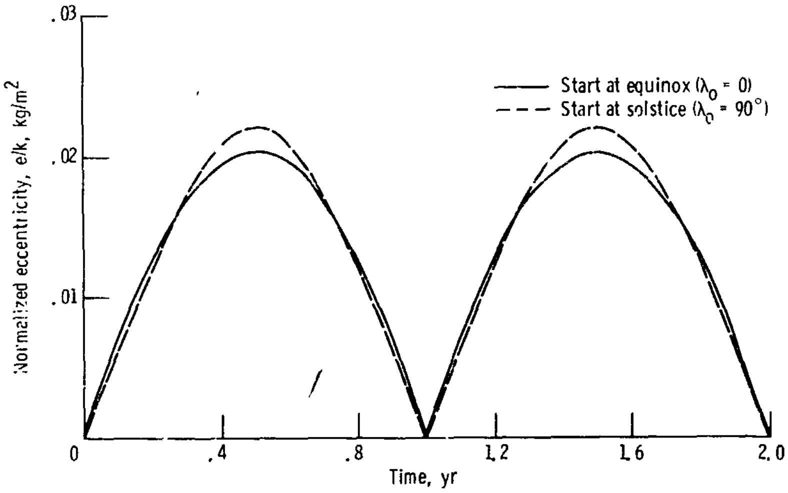


Figure 6. - Normalized eccentricity as a function of time. Initial conditions: eccentricity, 0.

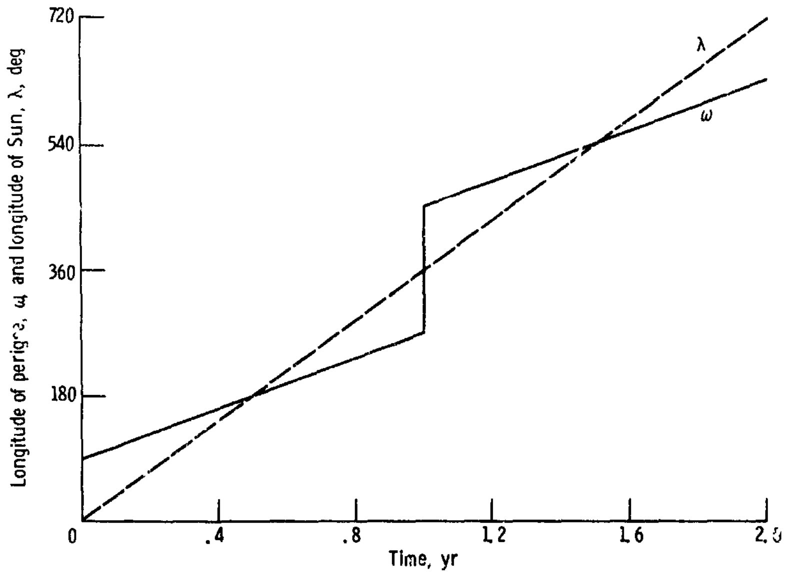


Figure 7. - Longitude of perigee and longitude of Sun as functions of time. Initial conditions: eccentricity, 0; longitude of Sun, 0.

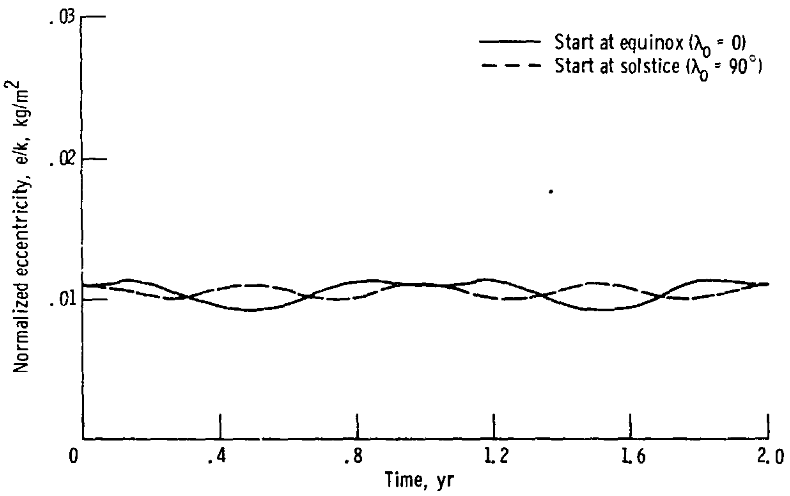


Figure 8. - Normalized eccentricity as a function of time. Initial conditions: eccentricity, 1/2 e_p; longitude of perigee, 0.

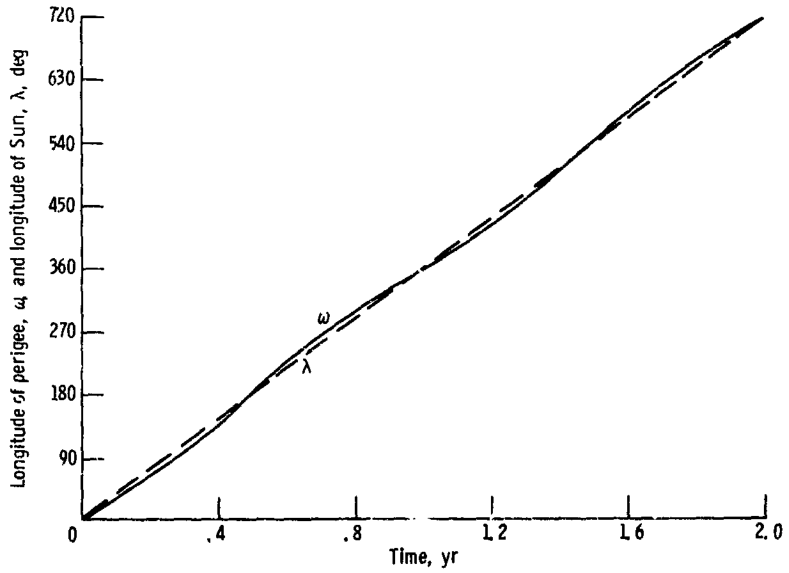


Figure 9. - Longitude of perigee and longitude of Sun as functions of time. Initial conditions: eccentricity, 1/2 e_p; longitude of perigee, 0; longitude of Sun, 0.

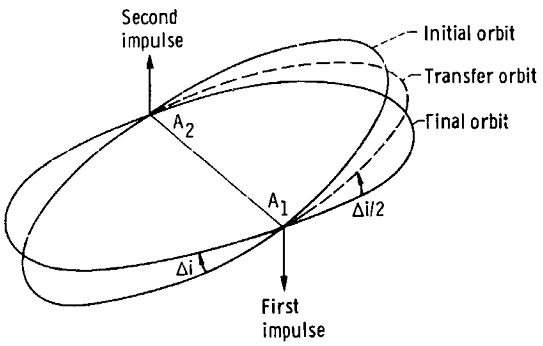


Figure 10. - Changing orbit inclination using two normal impulses.

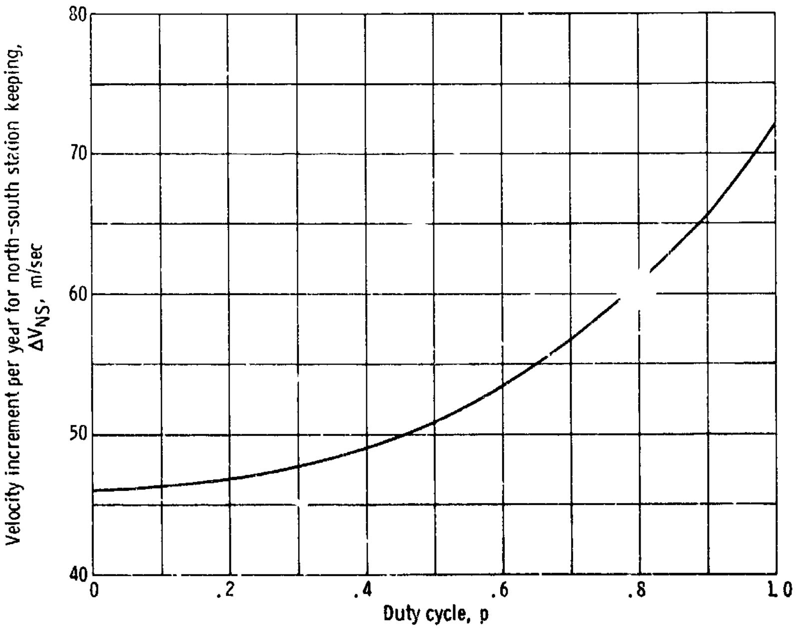


Figure 11. - Velocity increment per year for north-south station keeping as function of duty cycle.

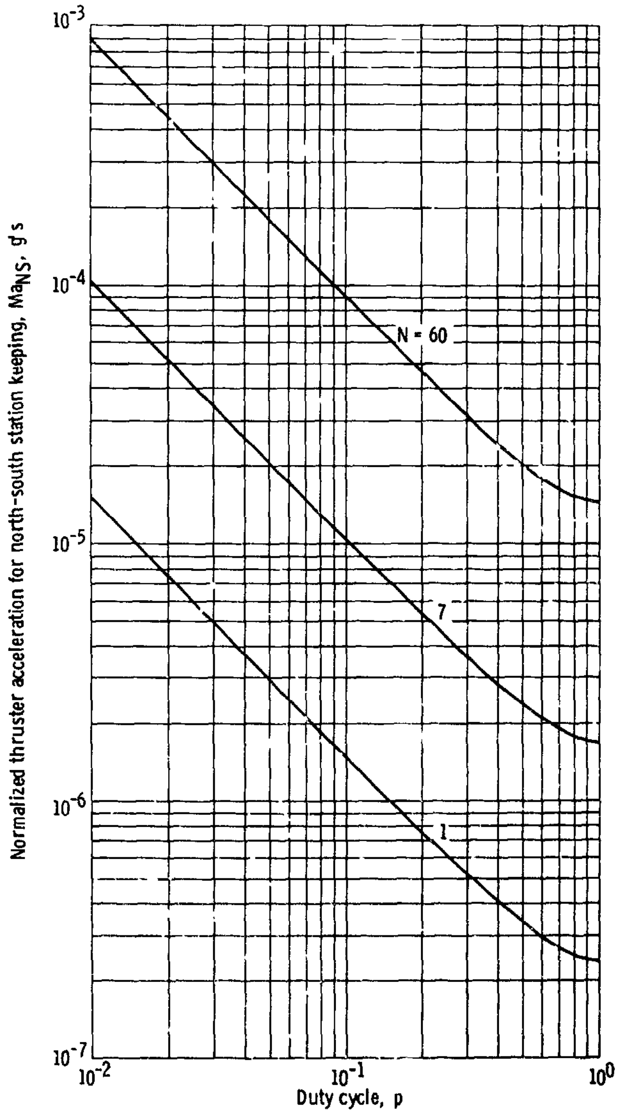


Figure 12. - Thruster acceleration for north-south station keeping as function of duty cycle.

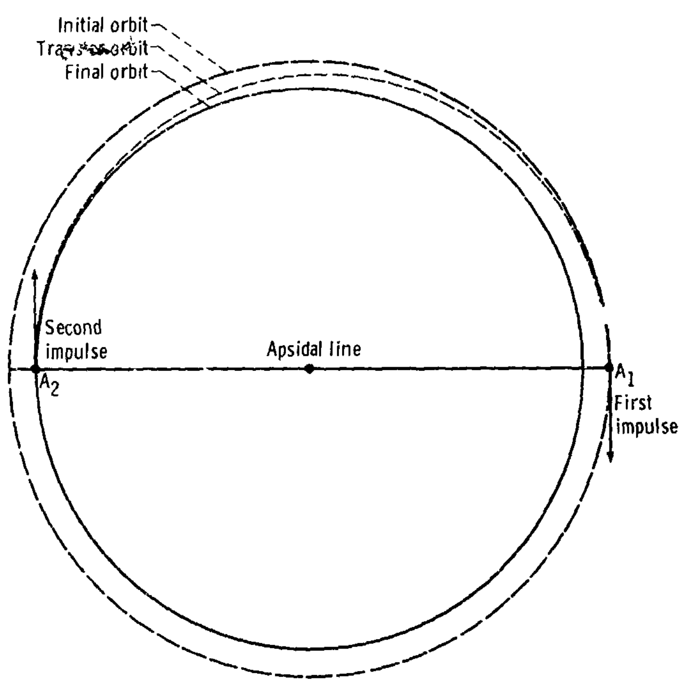


Figure 13. - Changing semimajor axis using two tangential impulses.

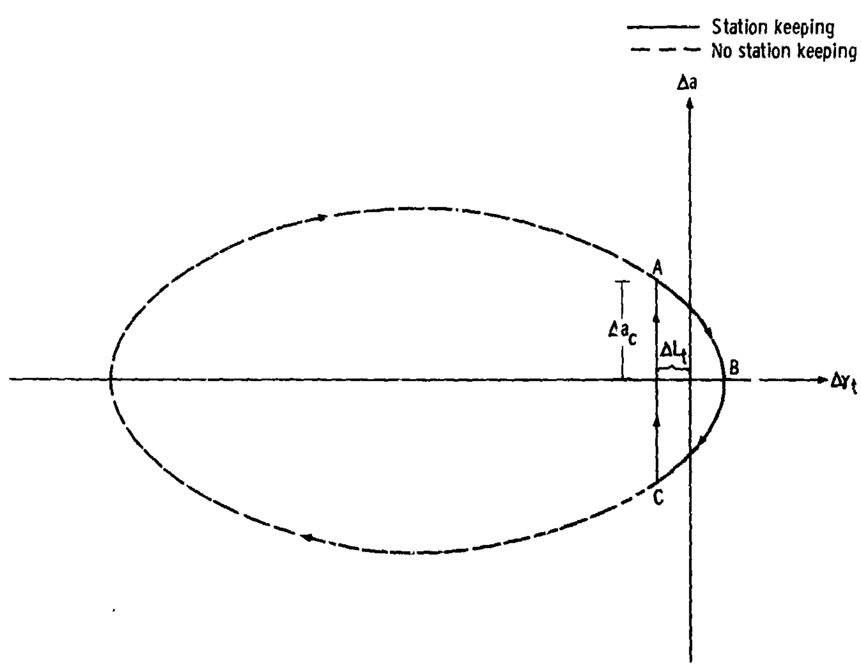


Figure 14. - Variation in semimajor axis as function of variation of satellite longitude. Perturbation, triaxiality.

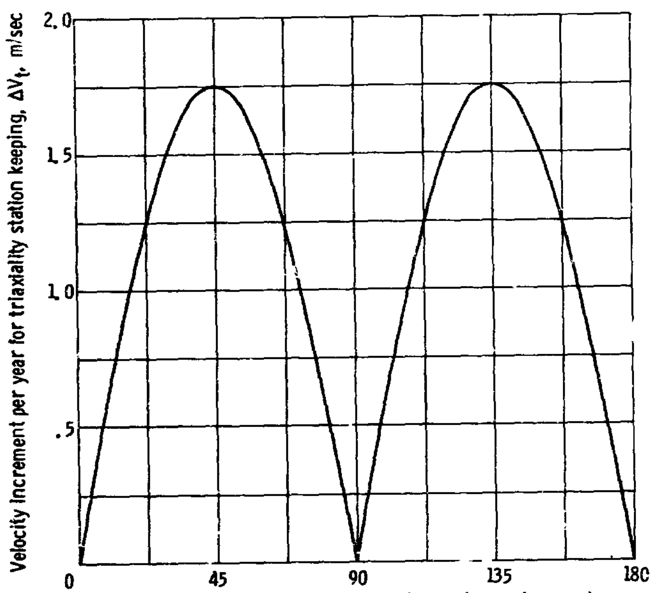


Figure 15. - Velocity increment per year for triaxiality station keeping as function of desired longitude of satellite.

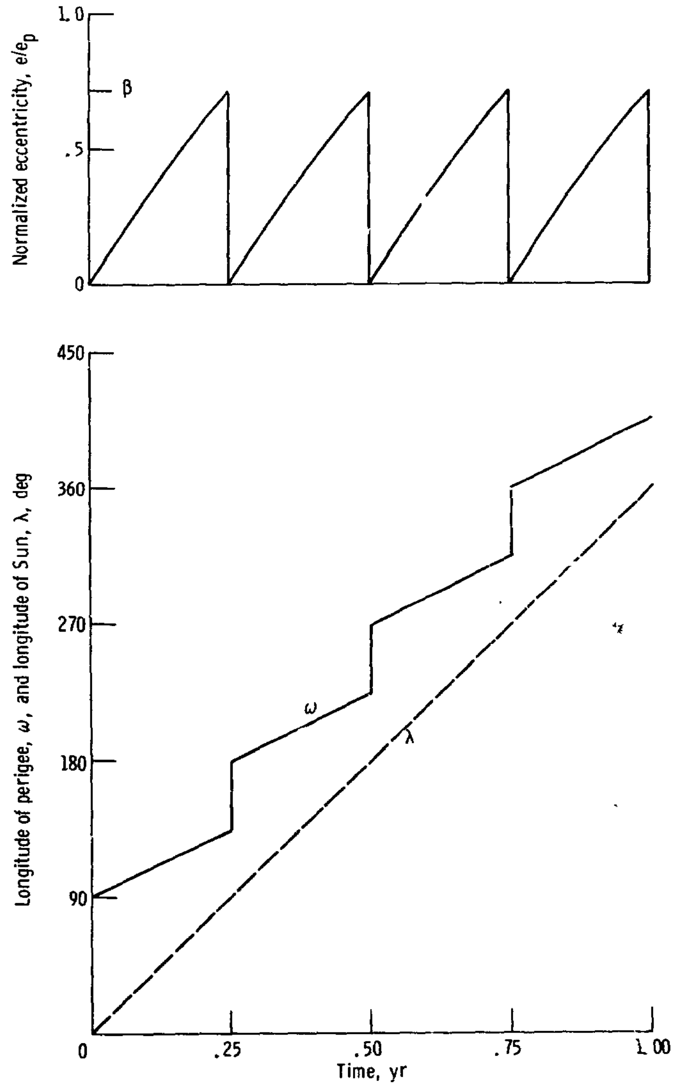


Figure 16. - Station-keeping parameters as functions of time when method 2 is used. Eccentricity ratio, 0.707. Initial conditions: eccentricity, 0; longitude of Sun, 0.

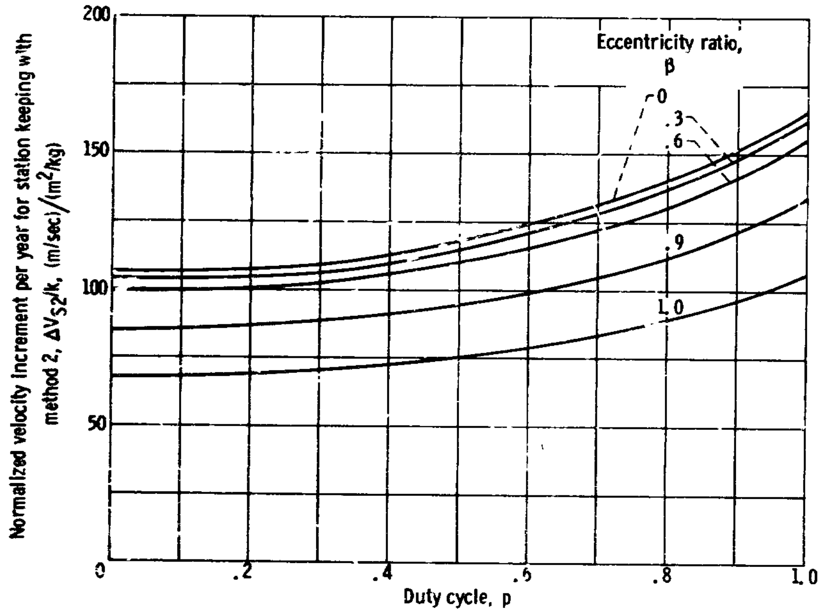


Figure 17. - Normalized velocity increment per year for station keeping with method 2 as function of duty cycle with eccentricity ratio as a parameter.

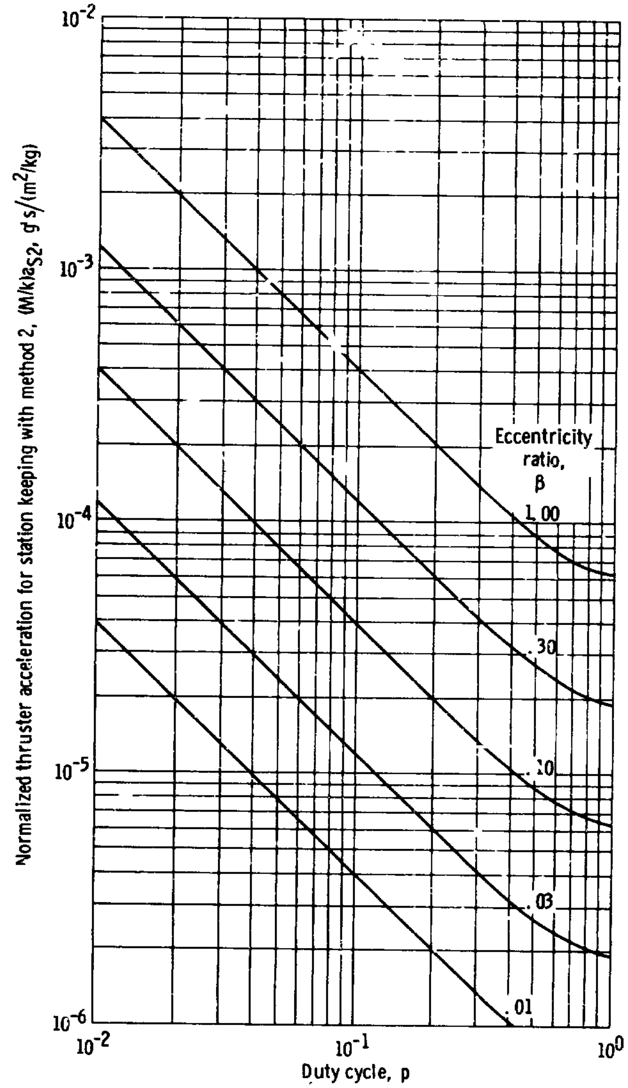


Figure 18. - Normalized thruster acceleration for station keeping with method 2 as function of duty cycle with eccentricity ratio as a parameter.

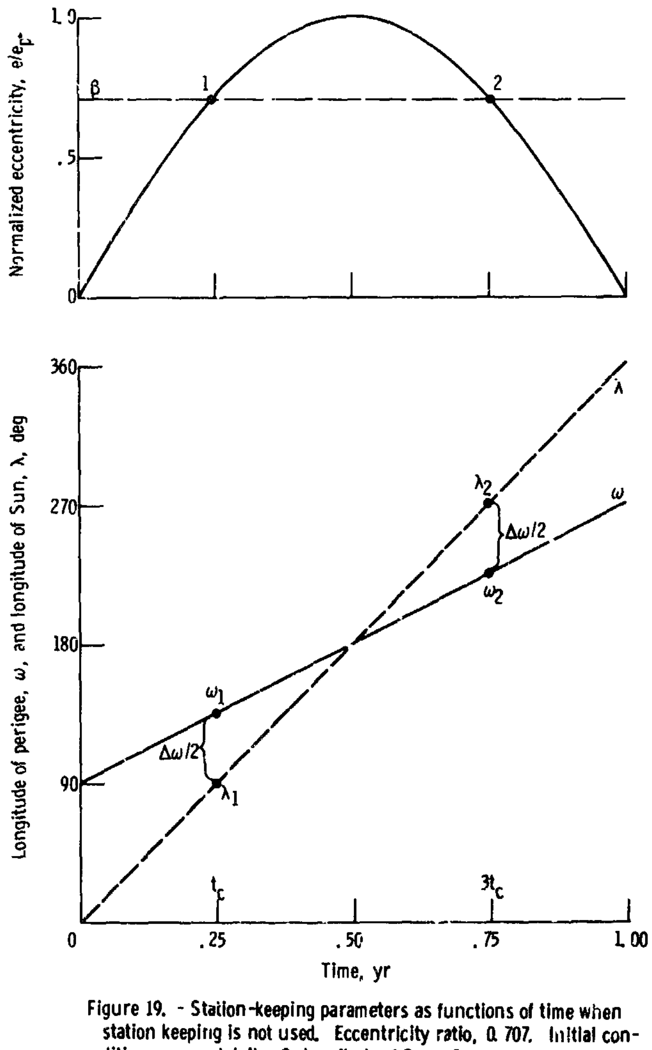


Figure 19. - Station-keeping parameters as functions of time when station keeping is not used. Eccentricity ratio, 0.707. Initial conditions: eccentricity, 0; longitude of Sun, 0.

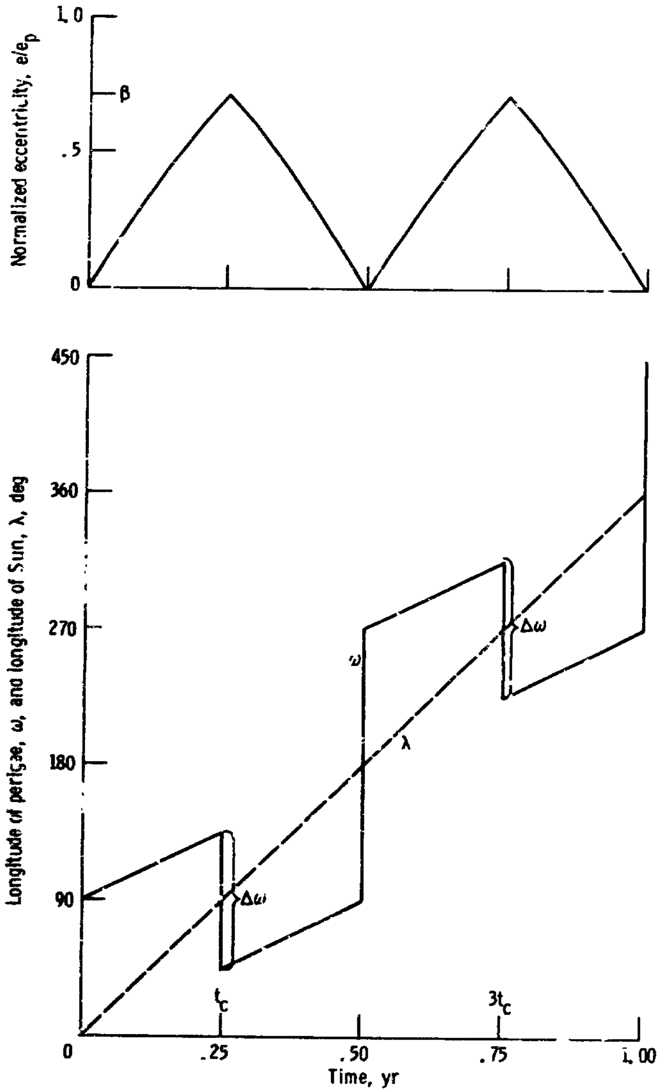


Figure 20. - Station-keeping parameters as functions of time when method 3 is used. Eccentricity ratio, 0.707. Initial conditions: eccentricity, 0; longitude of Sun, 0.

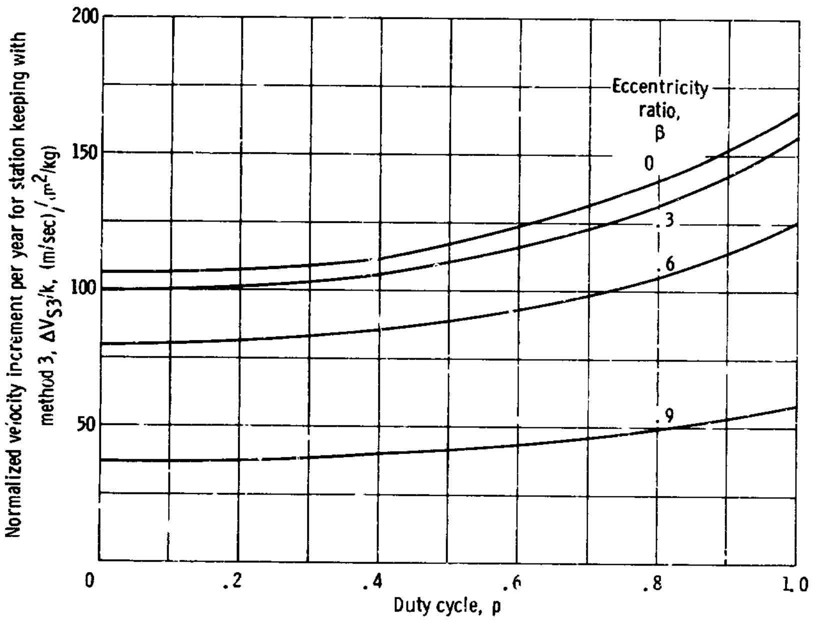


Figure 21. - Normalized velocity increment per year for station keeping with method 3 as function of duty cycle with eccentricity ratio as a parameter.

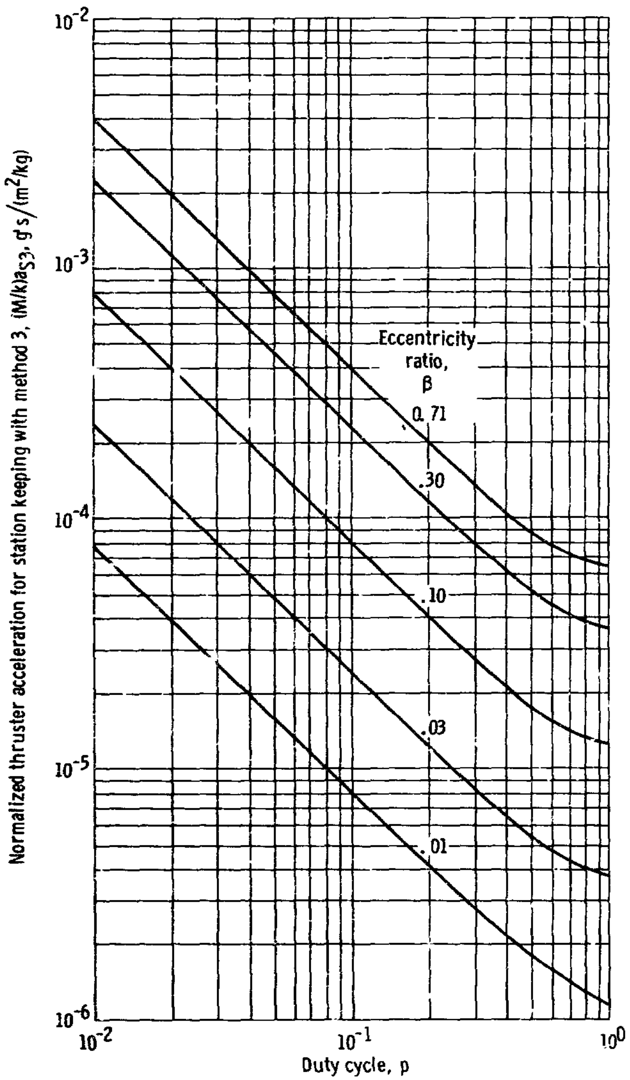


Figure 22. - Normalized thruster acceleration for station keeping with method 3 as function of duty cycle with eccentricity ratio as a parameter.

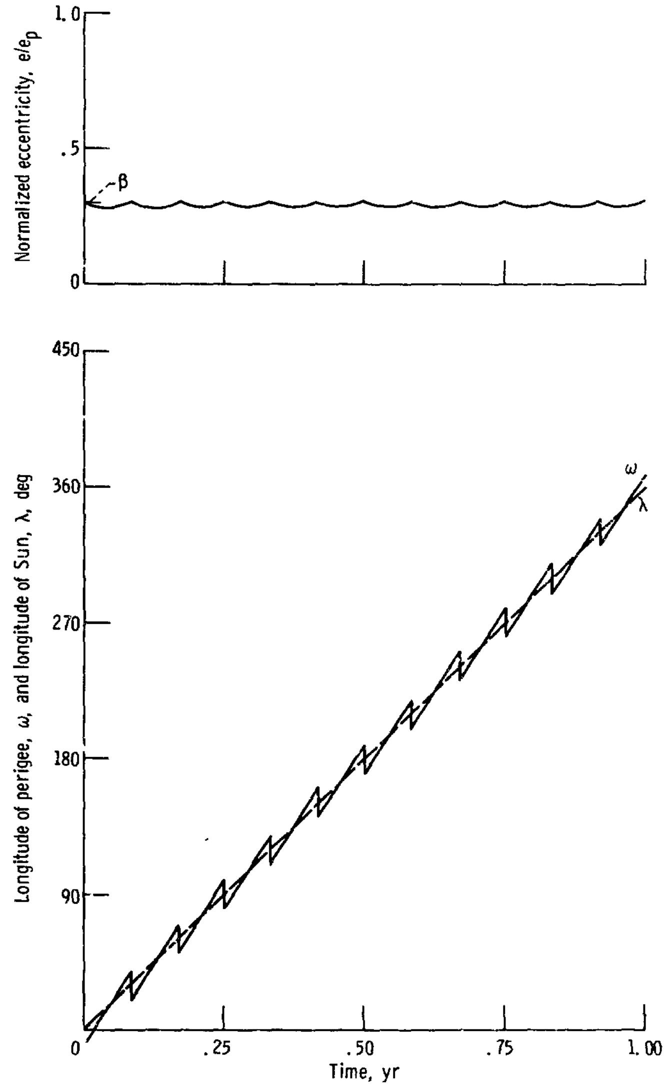


Figure 23. - Station-keeping parameters as functions of time when method 4 is used. Eccentricity ratio, 0.3. Initial conditions: eccentricity, βe_p; longitude of perigee, -10°; longitude of Sun, 0.

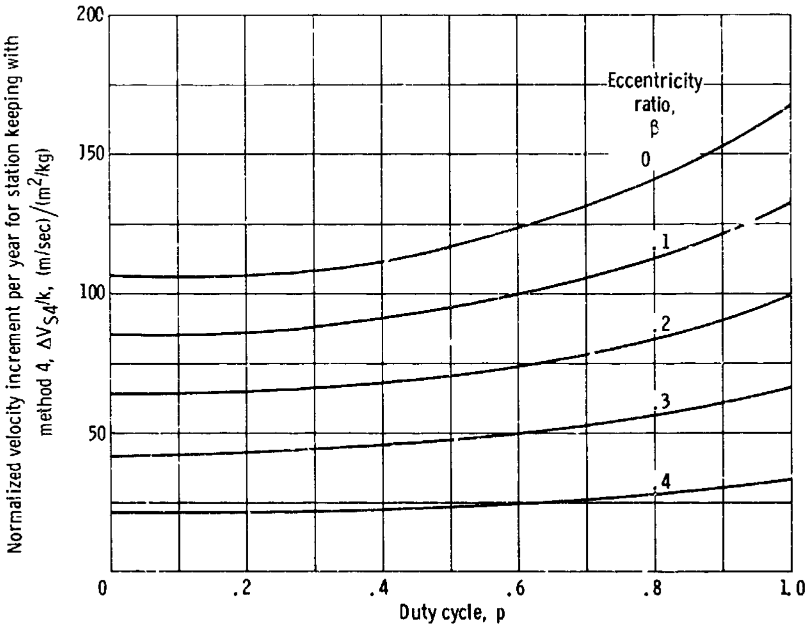
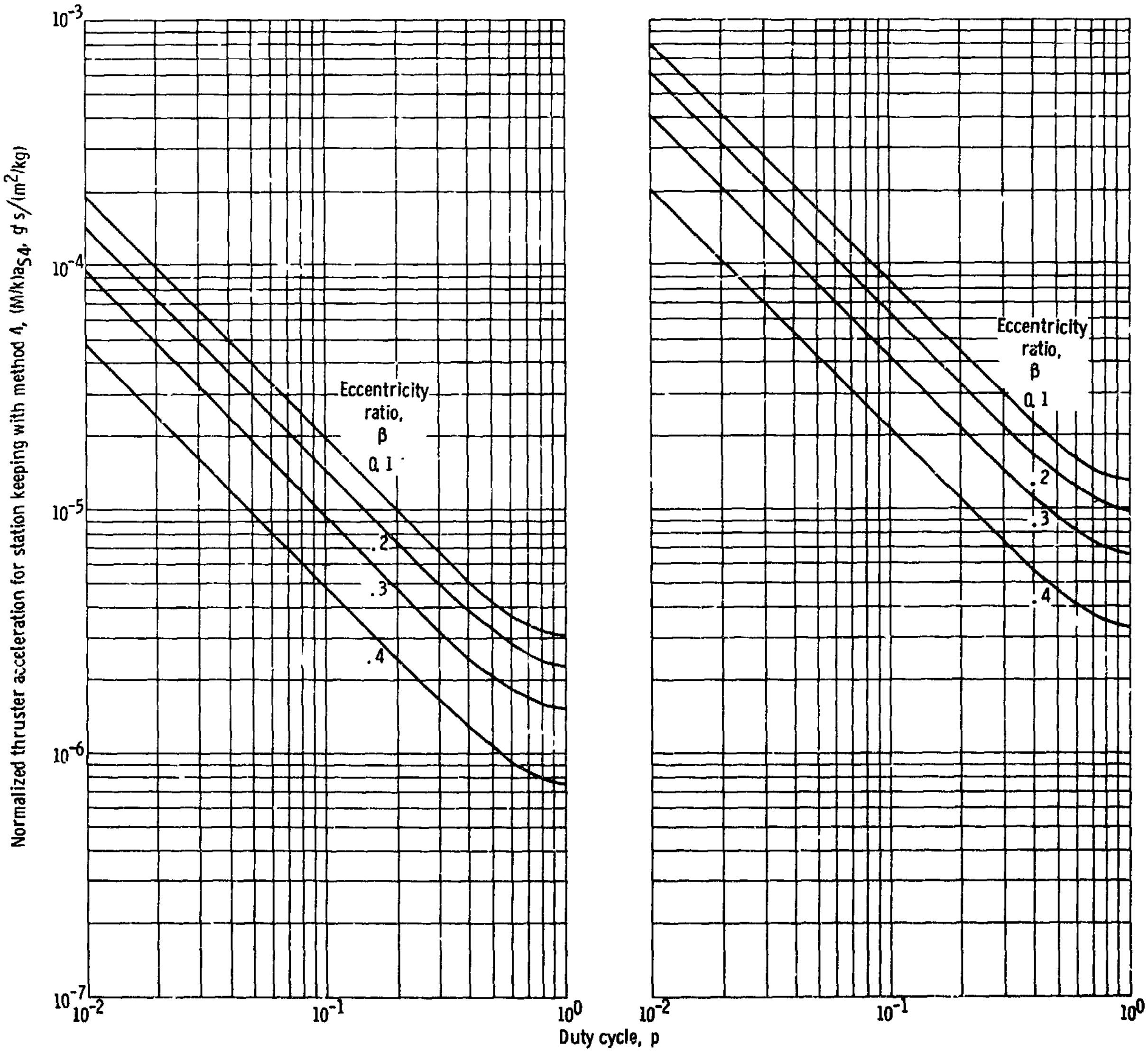


Figure 24. - Normalized velocity increment per year for station keeping with method 4 as function of duty cycle with eccentricity ratio as a parameter.



(a) Corrections every 7 days.

(b) Corrections every 30 days.

Figure 25. - Normalized thruster acceleration for station keeping with method 4 as function of duty cycle with eccentricity ratio as a parameter.

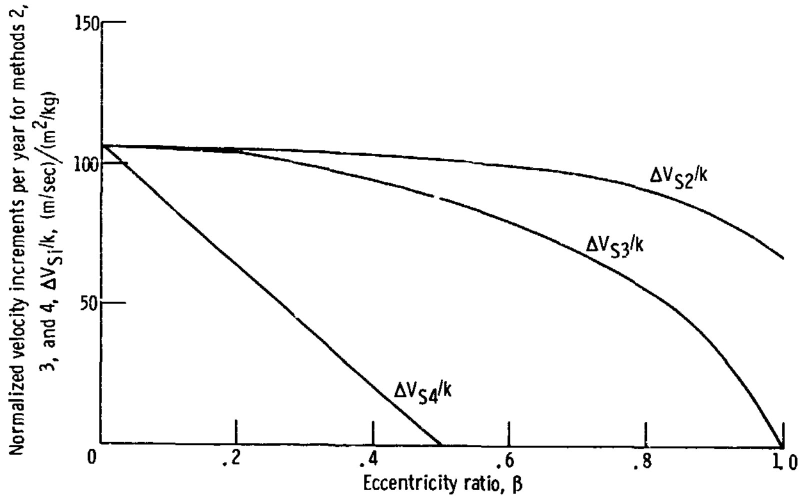


Figure 26. - Normalized velocity increments per year for methods 2, 3, and 4 as functions of eccentricity ratio. Duty cycle, 0.01.

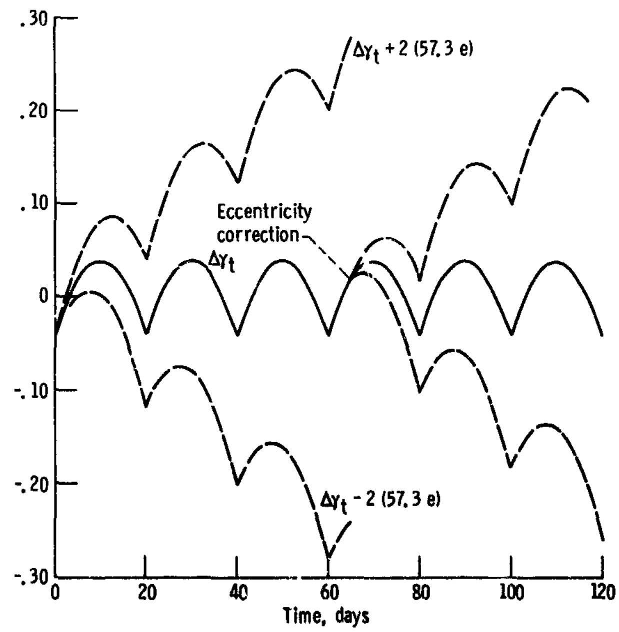


Figure 27. - Variation in satellite longitude as function of time when station-keeping method 2 with radial thrusting is used.

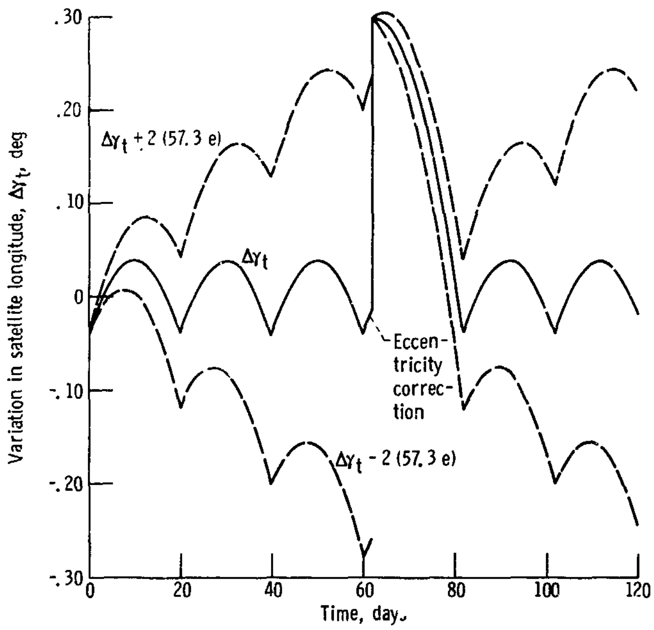


Figure 28. - Variation in satellite longitude as function of time when station-keeping method 2 with tangential thrusting is used.

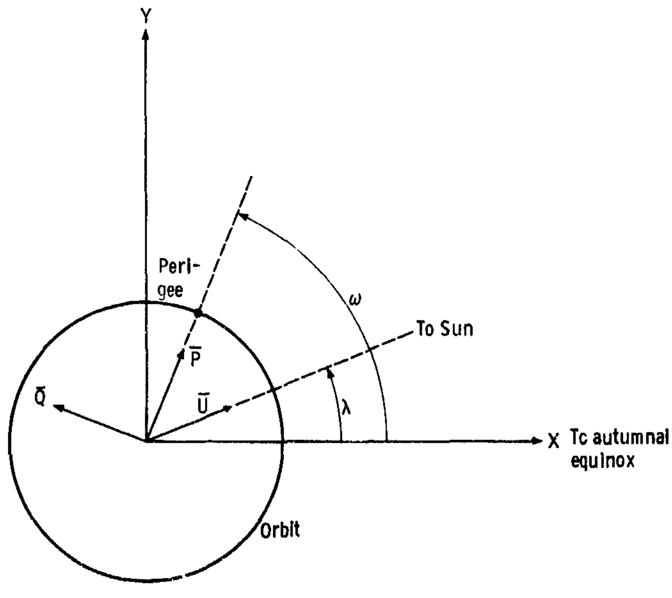


Figure 29. - Inertial coordinate system for planar problem.

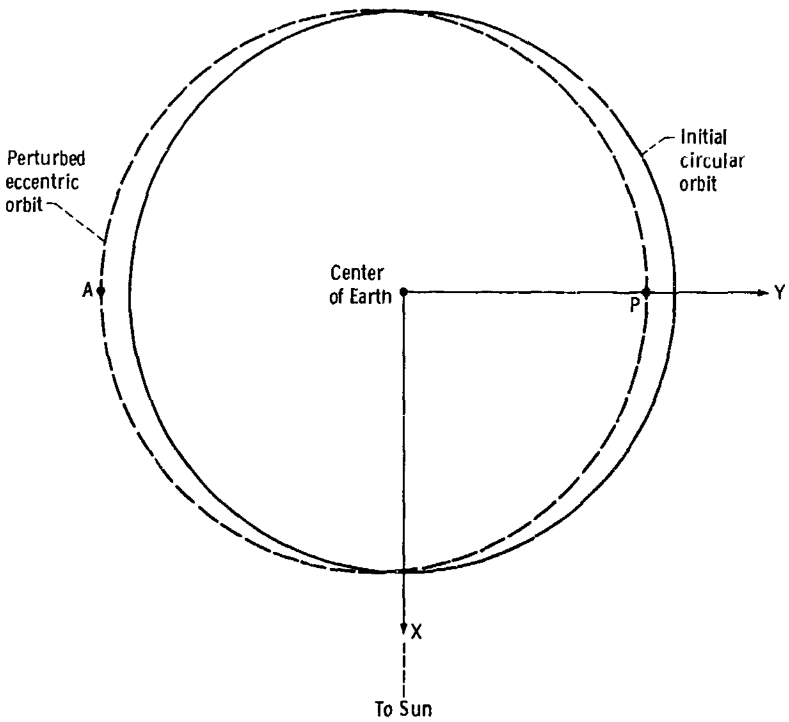


Figure 30. - Change in eccentricity when initial orbit is circular.

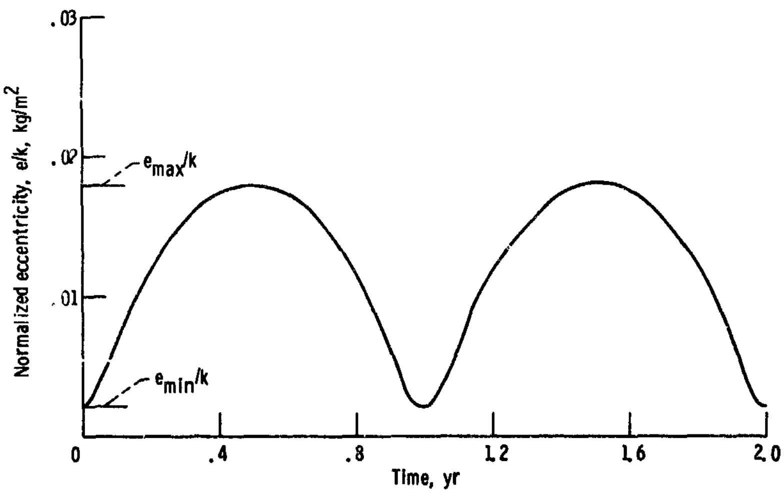


Figure 31. - Normalized eccentricity as function of time. Initial conditions: eccentricity, 0.1 e_p; longitude of perigee, 0; longitude of Sun, 0.

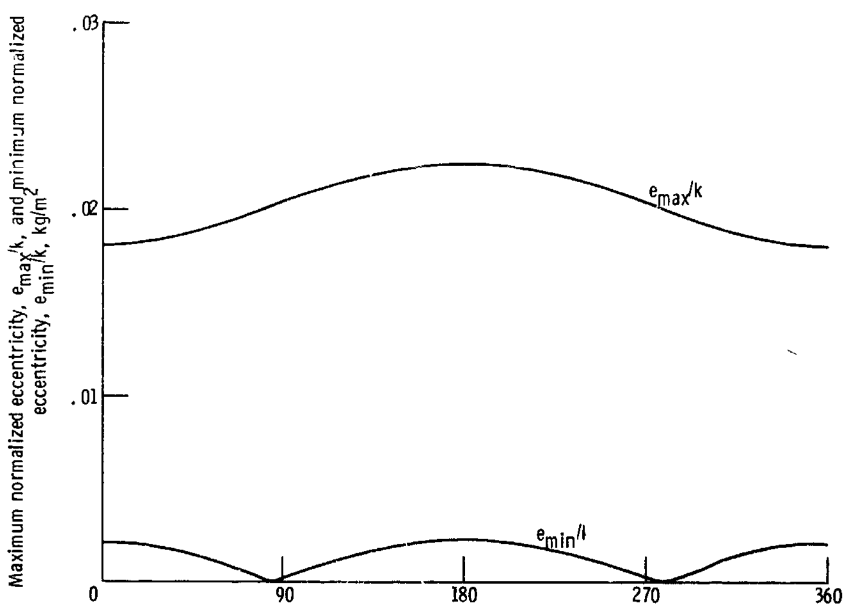


Figure 32. - Maximum and minimum normalized eccentricity as function of initial perigee longitude. Initial conditions: eccentricity, 0.1 e_p; longitude of Sun, 0.

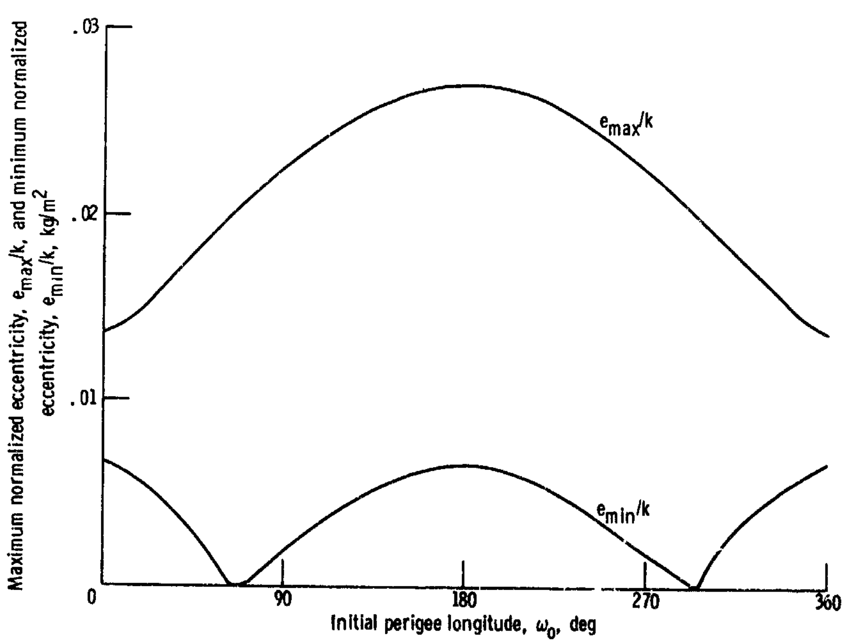


Figure 33. - Maximum and minimum normalized eccentricity as function of initial perigee longitude. Initial conditions: eccentricity, 0.3 e_p; longitude of Sun, 0.

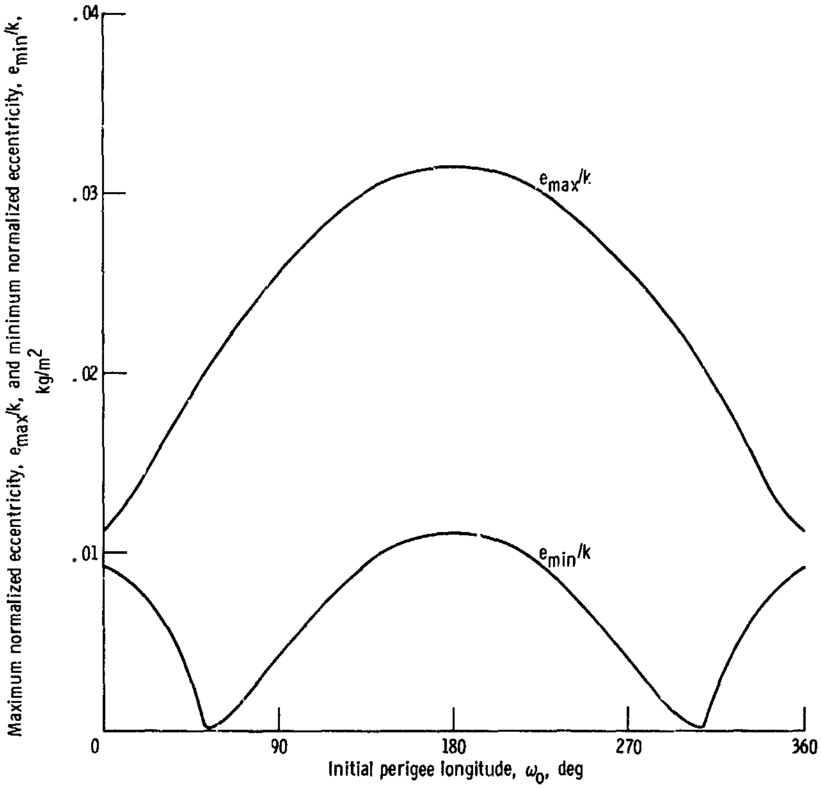


Figure 34. - Maximum and minimum normalized eccentricity as function of initial perigee longitude. Initial conditions: eccentricity, 0.5 e_p; longitude of Sun, 0.

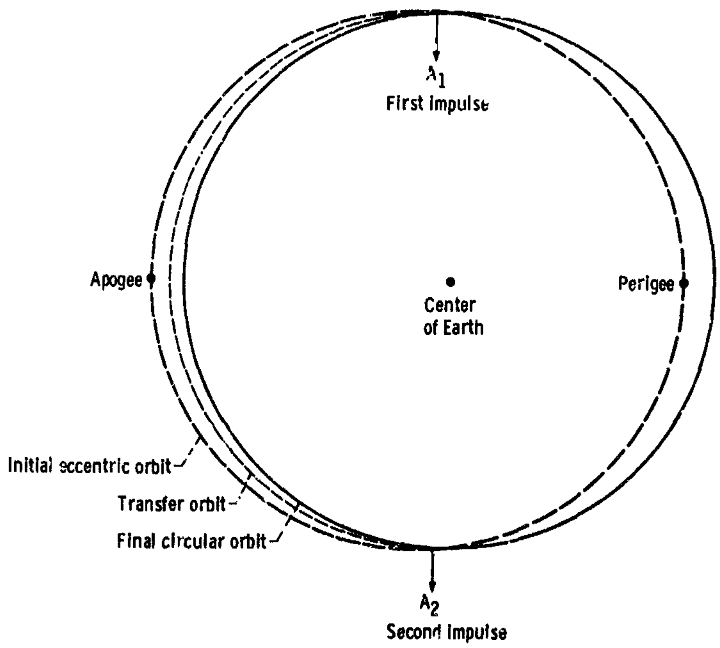


Figure 35. - Circularizing the orbit using two radial impulses.

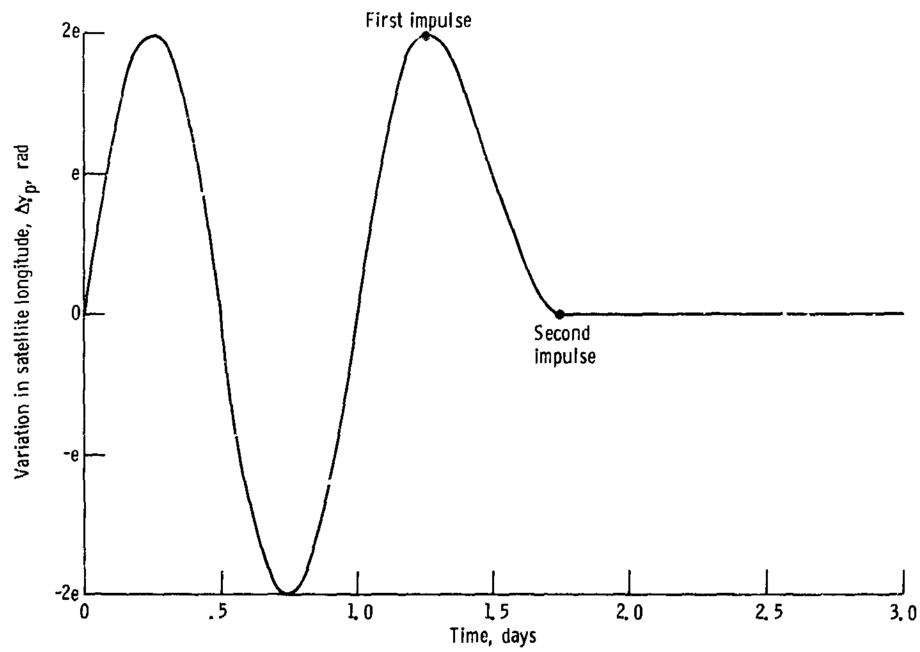


Figure 36. - Variation in satellite longitude as function of time when circularizing orbit with radial thrust. Eccentricity of original orbit is e.

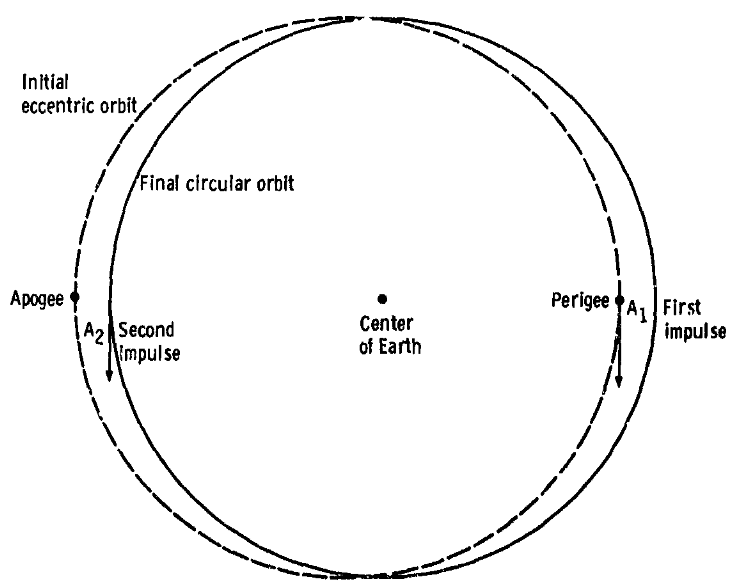


Figure 37. - Circularizing the orbit using two tangential impulses.

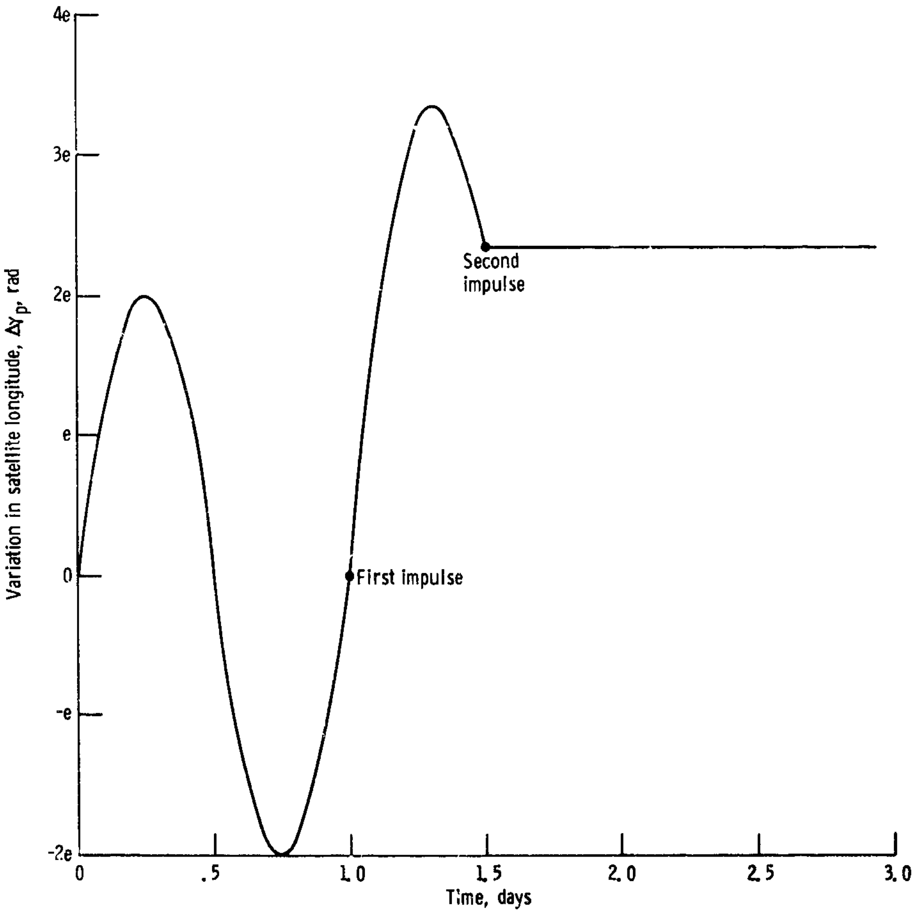


Figure 38. - Variation in satellite longitude as function of time when circularizing orbit with tangential thrust. Eccentricity of original orbit is e.

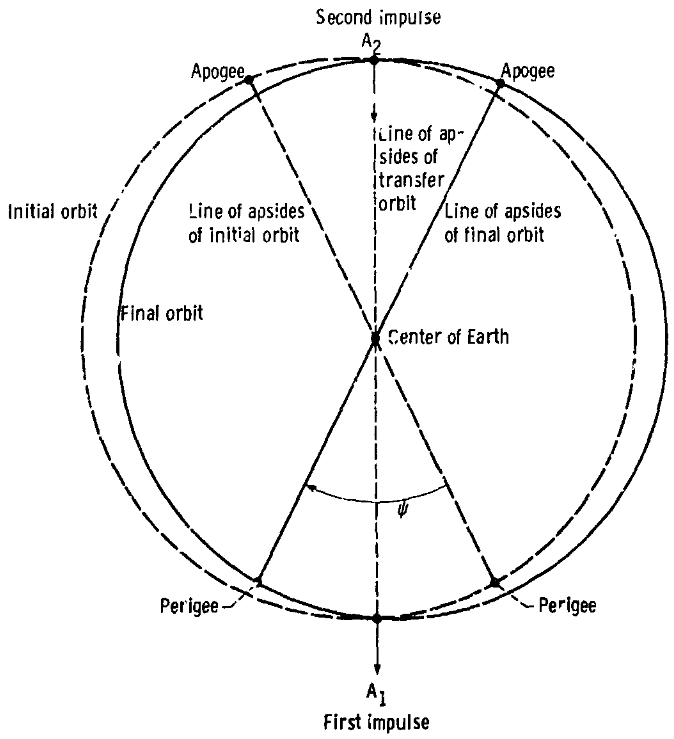


Figure 39. - Rotating the apsidal line through an angle ψ using two radial impulses.

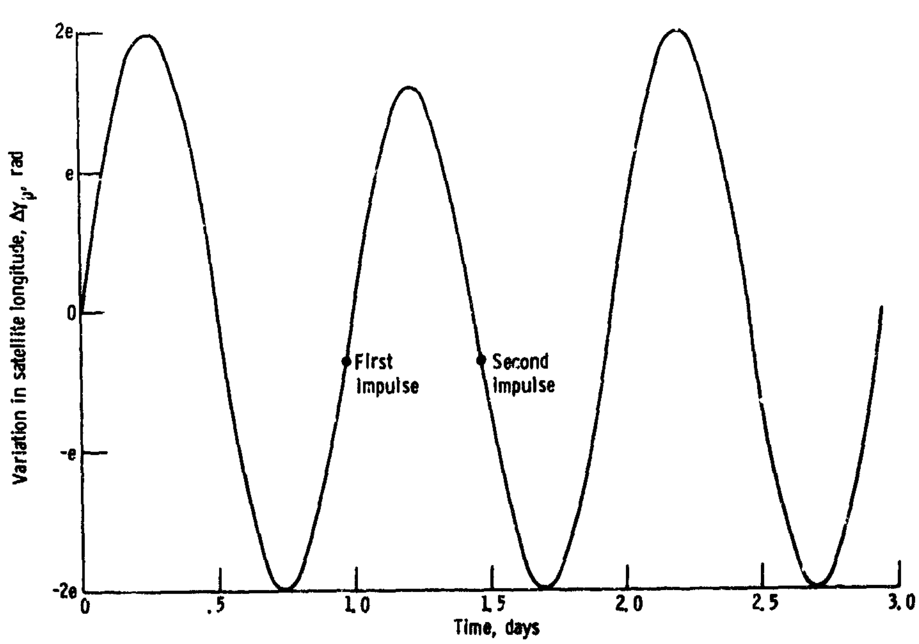


Figure 40. - Variation in satellite longitude as function of time when rotating apsidal line through 20° using radial thrust. Eccentricity is e.

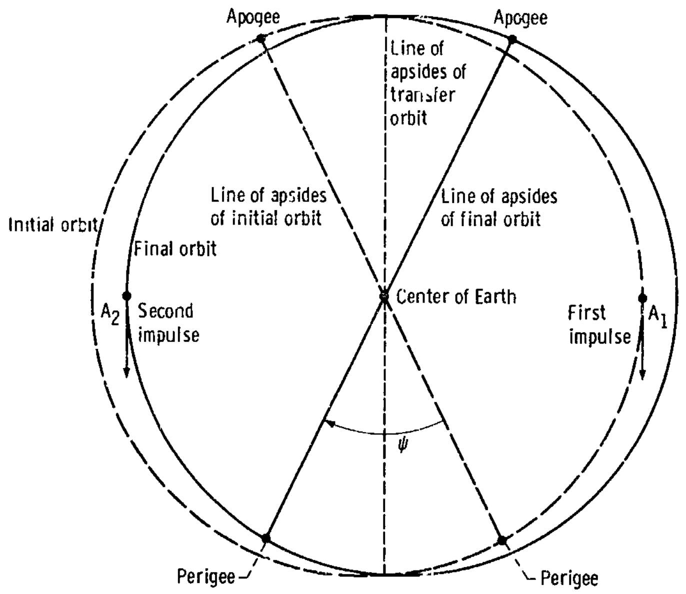


Figure 41. - Rotating the apsidal line through an angle ψ using two tangential impulses.

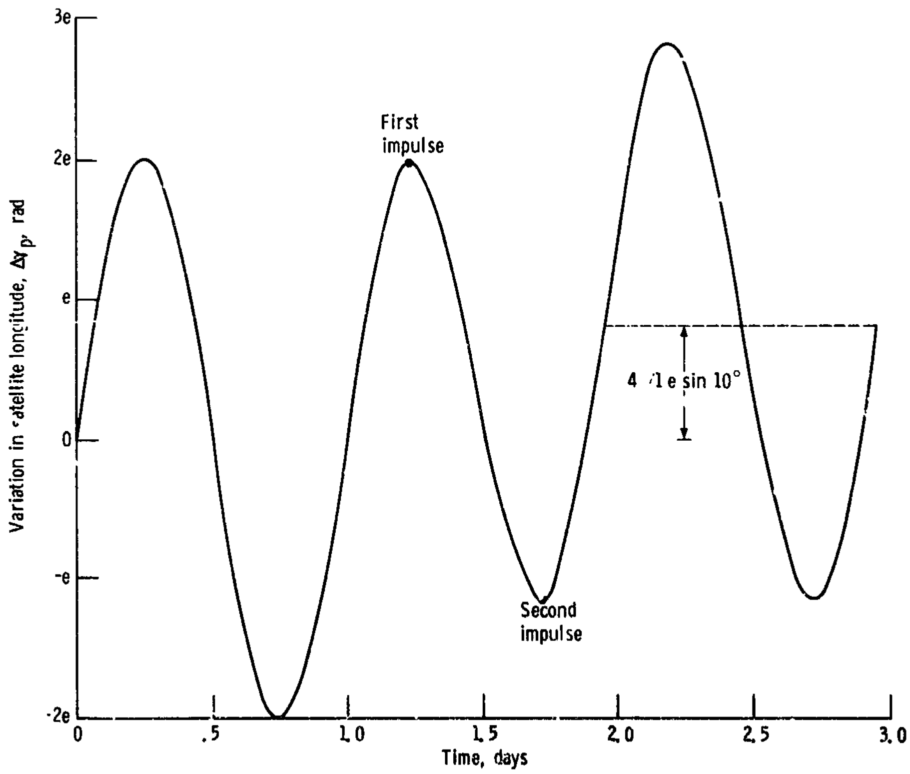


Figure 42. - Variation in satellite longitude as function of time when rotating apsidal line through 20° using tangential thrust. Eccentricity is e.

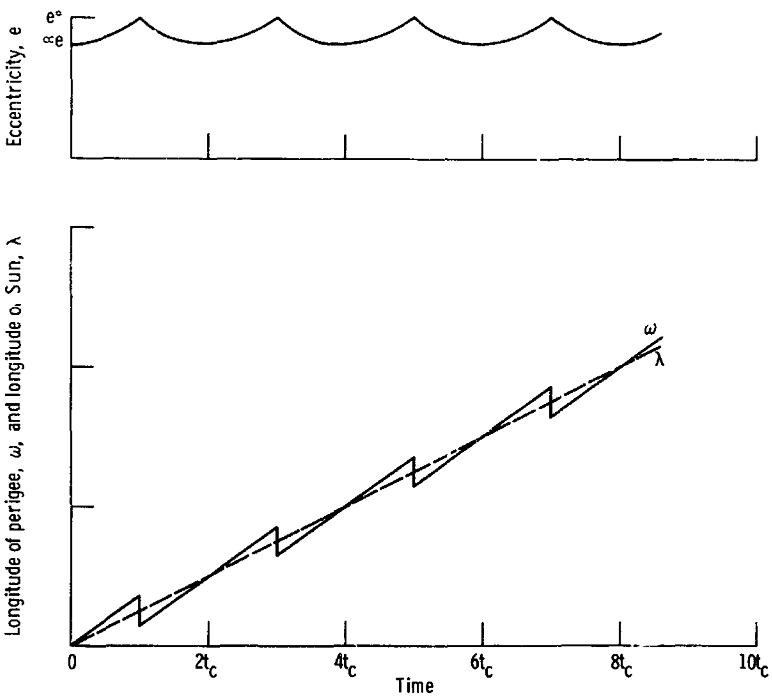


Figure 43. - Station-keeping parameters as functions of time when method 4 is used. Initial conditions: eccentricity, ≈e°; longitude of perigee, 0; longitude of Sun, 0.