17618-H017-R0-00

PROJECT TECHNICAL REPORT

TASK E-9H

OPTIMUM PREEMPHASIS - DEEMPHASIS FILTER CHARACTERISTICS FOR LCRU TELEVISION

NAS 9-8166 18 SEPTEMBER 1970

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

Prepared by
Communication and Sensor Systems Department
Electronic Systems Laboratory

[redacted]

(C.R. Candidate)

TRW
SYSTEMS GROUP

CR 108676

70-7251-WBW-1286
18 September 1970
Re: Task E-9H

National Aeronautics and Space Administration
Manned Spacecraft Center
Houston, Texas 77058

Attention: Dr. G. D. Arndt
Systems Analysis Branch, TCSD
Building 440, Room 265

Subject: Optimum Preemphasis-Deemphasis Filter Characteristics for LCRU Television

Gentlemen:

Attached are ten copies of the subject report, TRW No. 17618-H017-R0-00, dated 18 September 1970. This report determines the optimum preemphasis-deemphasis filter pair to improve the performance of the Lunar Communications Relay Unit (LCRU) television system.

The report derives the optimum preemphasis-deemphasis filter pair for FM transmission of baseband video signals, such as the LCRU color television signal, and predicts the performance of the optimum filter pair. The optimum filter pair is designed to maximize the video output signal-to-noise ratio for a given average modulating signal power and thus improve the performance of the television system. The analysis is based upon an envelope model of the video power spectral density and an FM channel which has a constraint on maximum modulating signal power. It is further assumed that the signal-to-noise ratio on the channel is above FM threshold, and that the network first breakpoint frequency equals the video envelope -3 dB frequency.

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Attachment

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Prepared for
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CONTENTS

1. INTRODUCTION

During extended lunar explorations on future Apollo missions, there will be times when a loss-of-signal will occur between the lunar module and the lunar surface astronauts. The primary purpose of the Lunar Communications Relay Unit (LCRU) subsystem is to provide a communications relay link between the Manned Space Flight Network (MSFN) and the lunar astronauts when these loss-of-signal conditions occur. A secondary purpose of the LCRU subsystem is to provide downlink transmission of the color television signal during lunar exploration, and also during ascent of the lunar module.

The purpose of this report is to determine the optimum preemphasis-deemphasis filter pair for FM transmission of baseband video signals (or any baseband signal where the power spectrum is concentrated near the lower end of the band) such as the LCRU color television signal. The optimum filter pair is designed to maximize the video output signal-to-noise ratio for a given average modulating signal power and thus improve the performance of the television system. The theoretical performance of the optimum filter pair is compared with the theoretical performance of a suboptimum filter pair experimentally found to be very effective in enhancing the signal-to-noise ratio of the LCRU color television signal (Reference 1).

1.1 LUNAR COMMUNICATIONS RELAY UNIT SUBSYSTEM

The primary purpose of the Lunar Communications Relay Unit subsystem is to provide a communications relay link between the Manned Space Flight Network and the astronauts on the lunar surface when there is a loss-of-signal between the lunar module (LM) and the lunar surface astronauts. In particular, the LCRU subsystem will provide a communications relay for uplink (MSFN to LCRU) and downlink (LCRU to MSFN) voice and downlink EMU (Extravehicular Mobility Unit) data after loss-of-signal between the LM and the lunar surface astronauts during extended lunar exploration on the Apollo J-Missions (Apollo 15-18). A secondary purpose of the LCRU subsystem is to provide downlink transmission of the color television signal from the Ground Commanded Television Assembly (GCTA) and to provide uplink relay of the 70 kHz subcarrier for GCTA commands during lunar exploration sorties and also during ascent of the lunar module.

Figure 1-1 is a sketch of the LCRU installed on the lunar roving vehicle (LRV) which is shown in mobile operation on the lunar surface. The LCRU S-band downlink carrier frequency which will transmit voice, data, and TV is 2272.5 MHz, while the S-band uplink received carrier frequency containing voice and commands is at 2101.8 MHz. A LCRU VHF carrier with frequency 296.8 MHz is used to relay voice information to the astronauts from the earth. The VHF carrier at 259.7 MHz is used to transmit voice information from EVA-1 (Extra-vehicular Astronaut No. 1) to EVA-2 (Extra-vehicular Astronaut No. 2) and also to transmit voice/data to the LCRU for subsequent relay to the MSFN.

The LCRU subsystem itself is to be small, lightweight, and self-contained so that it can be stowed in the LM modularized equipment storage area (MESA) and later installed by the crew on the lunar roving vehicle or on the mobile equipment transporter (MET). The LCRU will be portable so that in an emergency situation requiring a walk-back to the LM the astronauts can hand-carry the LCRU while continuing to maintain voice/data communications. A hardline connection between the LCRU and the GCTA will supply power and the 70 kHz command subcarrier as well as to receive

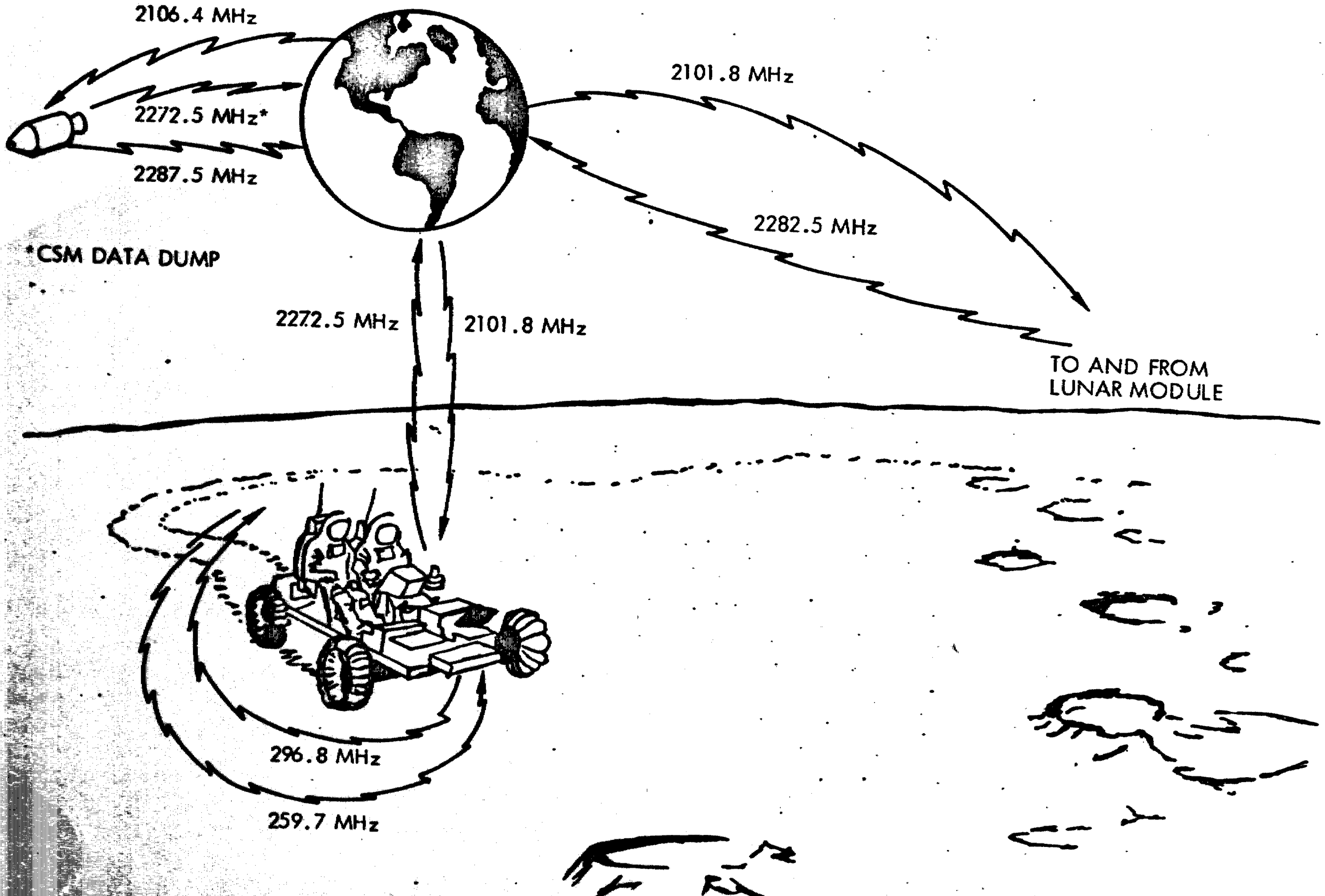


Figure 1-1. LCRU in Mobile Operation on the LRV

the TV video signal. The LCRU will furnish voice/data/TV transmission for as long as three 6-hour sorties. Good voice/data communications is required from the LCRU subsystem to the 85-foot MSFN ground station during mobile operation of the LRV or MET.

Figure 1-2 is a sketch of the LCRU installed on the lunar roving vehicle which is shown in fixed base operation on the lunar surface. Television/voice/data communications between the LCRU and MSFN is obtained with a 23 dB deployable dish antenna and the S-band uplink and downlink carriers, 2101.8 MHz and 2272.5 MHz, respectively. Voice and data information from EVA-2 is transmitted to EVA-1 on a 279.0 MHz VHF carrier. Voice and/or data information from EVA-1 is transmitted to EVA-2 and the LCRU on a 259.7 MHz VHF carrier. Voice information is relayed from the MSFN to the astronauts exploring the lunar surface through the use of a LCRU VHF carrier with frequency 296.8 MHz. The voice/data communication links between the LCRU and MSFN 85-foot dish employ an adjustable helical antenna with 6.5 dB minimum antenna gain over a pattern $\pm 30^{\circ}$ off boresight. The color TV downlink capability is provided from fixed base operations (LRV or MET in stationary position). Good television communications performance is required between the LCRU dish and the MSFN 210-foot dish. However, minimally acceptable TV communications is sufficient when using the LCRU dish and the MSFN 85-foot station.

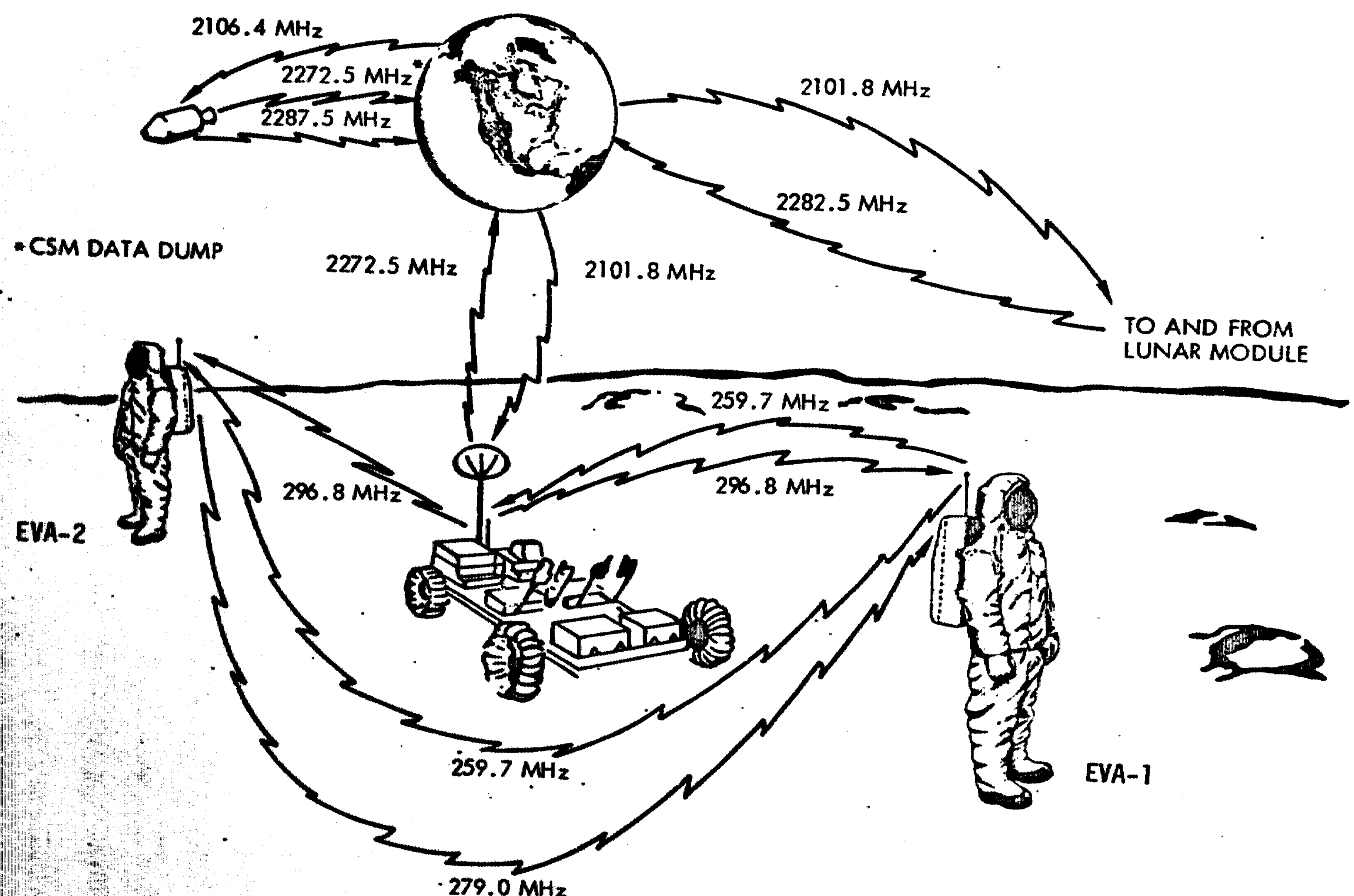


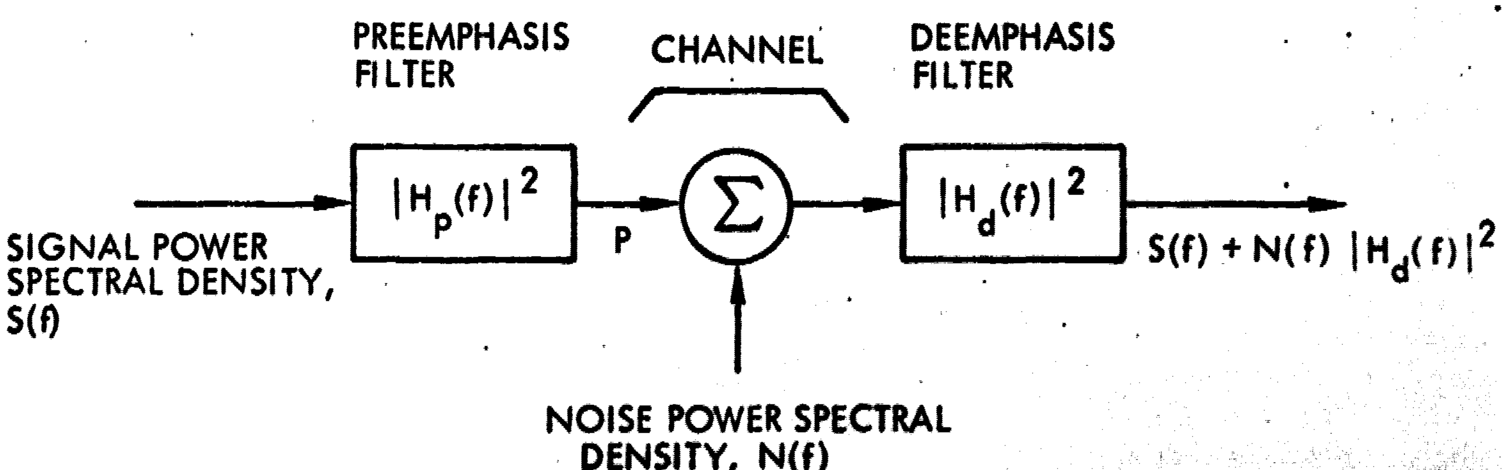
Figure 1-2. LCRU in Fixed Base Operation on the LRV

2. OPTIMUM PREEMPHASIS-DEEMPHASIS THEORY

The use of preemphasis-deemphasis to improve the performance of the Lunar Communications Relay Unit television system has been extensively analyzed in Reference 1. It was found that simple suboptimum preemphasis and deemphasis networks could provide significant improvement in LCRU television performance at a modest cost in terms of circuitry. However, no theoretical optimization analysis was performed to determine the optimum filter pair. In Section 2, a simple model for the power spectral density of a random video signal is assumed in order to derive a practical optimum filter pair which will maximize the video signal-to-noise ratio at the output of the baseband video filter.

2.1 OPTIMUM PREEMPHASIS-DEEMPHASIS FILTER PAIR

Consider the design of optimum preemphasis and deemphasis filters for transmission over a noisy FM channel above threshold as shown in Figure 2-1. Assume that the channel has a constraint on maximum modulating signal power and that it is desired to maximize the signal-to-noise ratio at the deemphasis filter output.



$$P = \int_{-\infty}^{\infty} S(f)|H_p(f)|^2\, df = \text{Modulating Signal Power on Channel}$$

Figure 2-1. Spectrum Shaping for Transmission Over Noisy Channel

A quantity that enters into most transmission systems performance criteria is the signal-to-noise ratio at the receiving filter output. The ratio of average signal power to average noise power at the output of the baseband video filter is

$$\rho_0 = \frac{\int_0^{f_M} S(f)\, df}{\int_0^{f_M} N(f)\,|H_d(f)|^2\, df} \tag{2-1}$$

where f_M is the bandwidth of the video signal and also the bandwidth of the baseband filter.

This ratio is to be maximized for a given average modulating signal power, P, on the channel. Alternately, P is to be minimized for a fixed ratio ρ_0. In either case, the optimum condition is attained for the maximum value of the ratio

$$\frac{\rho_0}{P} = \frac{\int_0^{f_M} S(f)\, df}{\int_0^{f_M} 2\, S(f)\,|H_p(f)|^2\, df \int_0^{f_M} N(f)\,|H_d(f)|^2\, df} \tag{2-2}$$

$$= \frac{P_s/2}{\int_0^{f_M} \frac{S(f)\, df}{|H_d(f)|^2} \int_0^{f_M} N(f)\,|H_d(f)|^2\, df} \tag{2-3}$$

where P_s is the average signal power and the two filters are essentially inverse.

As shown in Appendix A, the maximum value of the ratio ρ_0/P is obtained when the two integrals in the denominator are equal, which is the case for the following optimum value of $|H_d(f)|^2$

$$|H_d^o(f)|^2 = c \left[\frac{S(f)}{N(f)}\right]^{1/2} \tag{2-4}$$

where c is a constant. The corresponding optimum preemphasis filter characteristic is obtained from the inverse relation, $|H_d(f)|^2 = |H_p(f)|^{-2}$, and is

$$|H_p^o(f)|^2 = \frac{1}{c} \left[\frac{N(f)}{S(f)}\right]^{1/2} \tag{2-5}$$

where c is the same constant as in Equation (2-4) and is adjusted to meet the modulating signal power constraint.

The optimum output signal-to-noise ratio as obtained with Equation (2-4) in Equation (2-1) is

$$\rho_0^o = \frac{\int_0^{f_M} S(f)\, df}{c \int_0^{f_M} [S(f)\, N(f)]^{1/2}\, df} \tag{2-6}$$

For many problems concerning the transmission of video signals, it is sufficient to model the video spectrum as a slowly changing envelope function with relatively small variation over an interval of width f_1, the line scan rate. In these applications, where only the bandwidth and envelope of the video spectrum are important, the video power spectral density may be expressed (Reference 1) as

$$S(f) = \begin{cases} \dfrac{S_0}{1 + \left(\dfrac{f}{f'}\right)^2} & |f| \le f_M \\ 0 & |f| > f_M \end{cases} \tag{2-7}$$

where

S_0 = spectral density value at zero frequency

f' = the frequency at which $S(f)$ has fallen by 3 dB from S_0.

From FM noise analysis, the two-sided noise spectral density at the output of an FM demodulator can be written for large carrier-to-noise ratio (CNR) as

$$N(f) = \frac{4\pi^2 \alpha^2 \eta f^2}{A^2} \tag{2-8}$$

where α is the demodulator constant, η the input noise spectral density, A the carrier amplitude, and $A^2/2$ the mean carrier power at the demodulator input.

Insertion of Equations (2-7) and (2-8) into the optimum preemphasis filter characteristic [Equation (2-5)] yields

$$|H_p^o(f)|^2 = \frac{1}{c} \left[\frac{\dfrac{4\pi^2 \alpha^2 \eta f^2}{A^2}}{\dfrac{S_0}{1 + \left(\dfrac{f}{f'}\right)^2}} \right]^{1/2} \tag{2-9}$$

$$= \frac{2\pi\alpha}{cA} \sqrt{\frac{\eta}{S_0}}\; f \sqrt{1 + \left(\frac{f}{f'}\right)^2} \tag{2-10}$$

The resultant preemphasis network response can be considered to have the form

$$|H_p^o(f)|^2 = |H_d^o(f)|^{-2} = k^2 f \sqrt{1 + \left(\frac{f}{f_1}\right)^2} \tag{2-11}$$

where f_1 is the breakpoint frequency and k is the network coefficient with a value less than unity. A normalized logarithmic plot of $H_p^o(f)$ is given in Figure 2-2(a) along with the plot of a conventional preemphasis filter characteristic of the form

$$|H_p(f)|^2 = |H_d(f)|^{-2} = k_1^2 \left[1 + \left(\frac{f}{f_1}\right)^2\right] \qquad k_1 < 1 \tag{2-12}$$

which determines a suboptimum filter pair. A normalized logarithmic plot of $H_d(f)$ is shown in Figure 2-2(b) for both the optimum and suboptimum deemphasis filters.

2.2 SIGNAL-TO-NOISE IMPROVEMENT THROUGH EMPHASIS

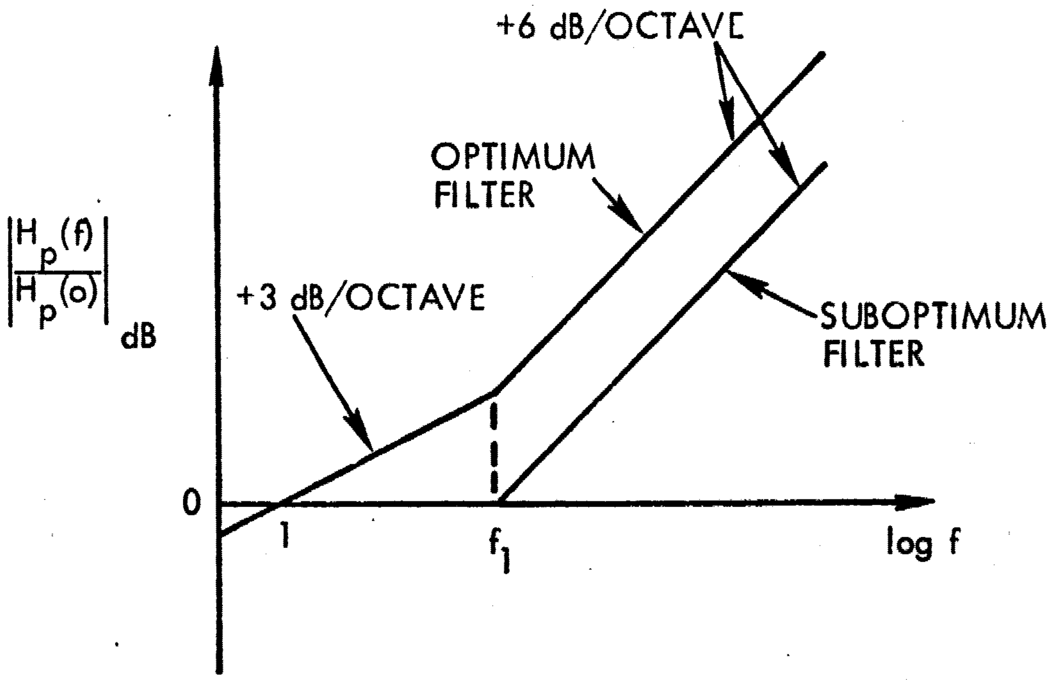
The ratio $\mathscr{R}$ by which preemphasis-deemphasis improves the signal-to-noise ratio of an FM signal (for large carrier-to-noise ratio) is given (Reference 1) by

$$\mathscr{R} = \frac{f_M^3/3}{\int_0^{f_M} \frac{f^2}{|H_p(f)|^2}\, df} \tag{2-13}$$

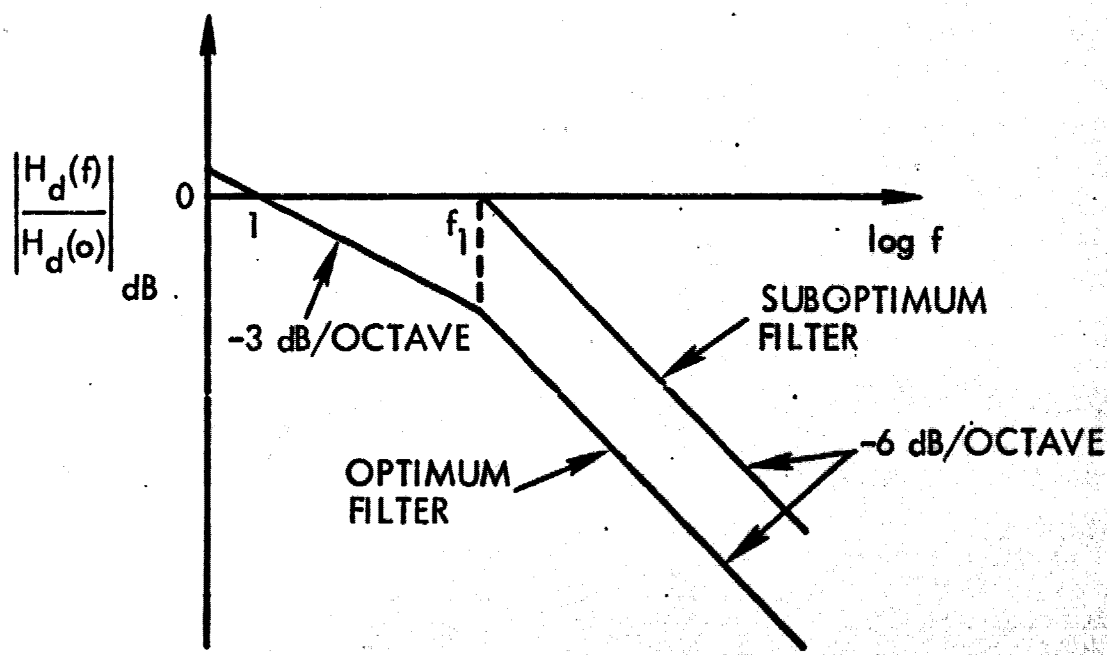
The optimum signal-to-noise ratio improvement, $\mathscr{R}_{opt}$, for an FM channel through emphasis is defined to be

$$\mathscr{R}_{opt} = \frac{f_M^3/3}{\int_0^{f_M} \frac{f^2\, df}{|H_p^o(f)|^2}} \tag{2-14}$$

The above expression applies for values of large carrier-to-noise ratio (CNR $\geq$ 10).



(a)



(b)

Figure 2-2. Approximate Logarithmic Plots of the Frequency Responses for (a) Preemphasis Networks and (b) Deemphasis Networks

Substitution of Equation (2-11) into the above equation gives

$$\mathscr{R}_{opt} = \frac{f_M^3/3}{\displaystyle\int_0^{f_M} \frac{f^2 \quad df}{k^2 f\sqrt{1 + \left(\frac{f}{f_1}\right)^2}}} \tag{2-15}$$

$$= \frac{k^2 f_M^3/3}{\displaystyle\int_0^{f_M} \frac{f \quad df}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}}} . \tag{2-16}$$

Recalling that

$$\int \frac{X}{\sqrt{X^2 + 1}} dx = \sqrt{X^2 + 1} , \tag{2-17}$$

the integral in the denominator becomes

$$\int_0^{f_M} \frac{f \quad df}{\sqrt{1 + \left(\frac{f}{f_1}\right)^2}} = f_1^2 \left\{ \left[\left(\frac{f_M}{f_1}\right)^2 + 1 \right]^{1/2} - 1 \right\} . \tag{2-18}$$

Making use of the above expression in the equation for $\mathscr{R}_{opt}$ gives the result

$$\mathscr{R}_{opt} = \frac{k^2 f_M^3}{3f_1^2} \cdot \frac{1}{\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} - 1} . \tag{2-19}$$

2.2.1 Solving for the Coefficient k

So that the bandwidth occupied by the FM modulated carrier may remain constant, the requirement is made that the average power of the preemphasized signal, P, equal the average power of the original baseband signal, P_s. This condition is described by the equality

$$\int_{-f_M}^{f_M} S(f)\,|H_p^o(f)|^2\, df = \int_{-f_M}^{f_M} S(f)\, df \quad . \tag{2-20}$$

The rms bandwidth will thus be the same with or without preemphasis.

Substitution of the signal density function [assumed in Equation (2-7)] into the above equality and making use of Equation (2-11) gives

$$\int_{-f_M}^{f_M} \frac{S_o}{1+\left(\frac{f}{f'}\right)^2} \cdot k^2 f \sqrt{1+\left(\frac{f}{f_1}\right)^2}\, df = \int_{-f_M}^{f_M} \frac{S_o\, df}{1+\left(\frac{f}{f'}\right)^2} \, . \tag{2-21}$$

The coefficient k is adjusted so that the constraint of Equation (2-21) is satisfied. For the special case where the breakpoint frequency, f_1, of the networks is located at the -3 dB point, f', of the signal density function S(f), the constraint reduces to

$$k^2 \int_0^{f_M} \frac{f\, df}{\left[1+\left(\frac{f}{f_1}\right)^2\right]^{1/2}} = \int_0^{f_M} \frac{df}{1+\left(\frac{f}{f_1}\right)^2} \, . \tag{2-22}$$

Using the results of Equation (2-18), the left hand side of Equation (2-22) becomes

$$k^2 f_1^2 \left[\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} \; - 1 \right] \quad . \tag{2-23}$$

Evaluating the integral on the right hand side of Equation (2-22),

$$\int_0^{f_M} \frac{df}{1 + \left(\frac{f}{f_1}\right)^2} = f_1 \tan^{-1}\left(\frac{f}{f_1}\right) \Big|_0^{f_M}$$

$$= f_1 \tan^{-1}\left(\frac{f_M}{f_1}\right) \quad . \tag{2-24}$$

Using Equations (2-23) and (2-24), the power constraint can now be expressed as

$$k^2 f_1^2 \left[\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} \; - 1 \right] = f_1 \tan^{-1}\left(\frac{f_M}{f_1}\right) \tag{2-25}$$

or rearranging in terms of the coefficient k,

$$k^2 = \frac{\tan^{-1}\left(\frac{f_M}{f_1}\right)}{f_1 \left[\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} \; - 1 \right]} \quad . \tag{2-26}$$

The optimum signal-to-noise ratio improvement (assuming an envelope video signal density function) resulting from preemphasis and subsequent deemphasis is determined by substituting Equation (2-26) into Equation (2-19).

$$\mathscr{R}_{opt} = \frac{\tan^{-1}\left(\frac{f_M}{f_1}\right)}{f_1\left[\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} - 1\right]} \cdot \frac{f_M^3}{3f_1^2} \cdot \frac{1}{\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} - 1} \tag{2-27}$$

$$= \frac{1}{3}\left(\frac{f_M}{f_1}\right)^3 \frac{\tan^{-1}\left(\frac{f_M}{f_1}\right)}{\left[\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} - 1\right]^2} \cdot \tag{2-28}$$

If the power in the baseband signal is concentrated at low frequencies, so that $f_M/f_1 >> 1$, then $\mathscr{R}_{opt}$ will be considerably greater than one. For very large values of f_M/f_1,

$$\tan^{-1}\left(\frac{f_M}{f_1}\right) \approx \frac{\pi}{2} \tag{2-29}$$

and

$$\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} \approx \frac{f_M}{f_1} \tag{2-30}$$

so that the optimum improvement ratio given in Equation (2-28) becomes approximately

$$\mathscr{R}_{opt} \simeq \frac{1}{3}\left(\frac{f_M}{f_1}\right)^3 \frac{\pi/2}{\left(\frac{f_M}{f_1}\right)^2}$$

$$\simeq \frac{\pi}{6}\frac{f_M}{f_1}, \quad \frac{f_M}{f_1} >> 1. \tag{2-31}$$

The SNR improvement increases linearly with increasing f_M.

Increasing f_M indefinitely provides no absolute improvement in the output signal-to-noise ratio, however, for the output noise increases linearly with increasing f_M, while the signal density decreases as the square of the frequency. The output filter bandwidth f_M should thus be restricted to just the bandwidth required to pass the highest significant video frequency and no more.

For the condition $f_M/f_1 \ll 1$ (the deemphasis effect is then introduced beyond the range of the baseband filter), the limiting value of $\mathscr{R}_{opt}$ can be determined with the use of the series approximations,

$$\tan^{-1} X \approx X \qquad (|X| \ll 1) \tag{2-32}$$

and

$$\sqrt{1 + X^2} \approx 1 + \frac{X^2}{2} \quad . \tag{2-33}$$

It is found that $\mathscr{R}_{opt}$ approaches the value

$$\mathscr{R}_{opt} \approx \frac{1}{3} \left(\frac{f_M}{f_1}\right)^3 \frac{f_M/f_1}{\left[1 + \frac{1}{2}\left(\frac{f_M}{f_1}\right)^2 - 1\right]^2} \tag{2-34}$$

$$\approx \frac{4}{3} \; , \quad \frac{f_M}{f_1} \ll 1 \; . \tag{2-35}$$

Figure 2-3 contains a plot of the optimum SNR improvement $\mathscr{R}_{opt}$ in decibels versus the ratio f_M/f_1 according to Equation (2-28).

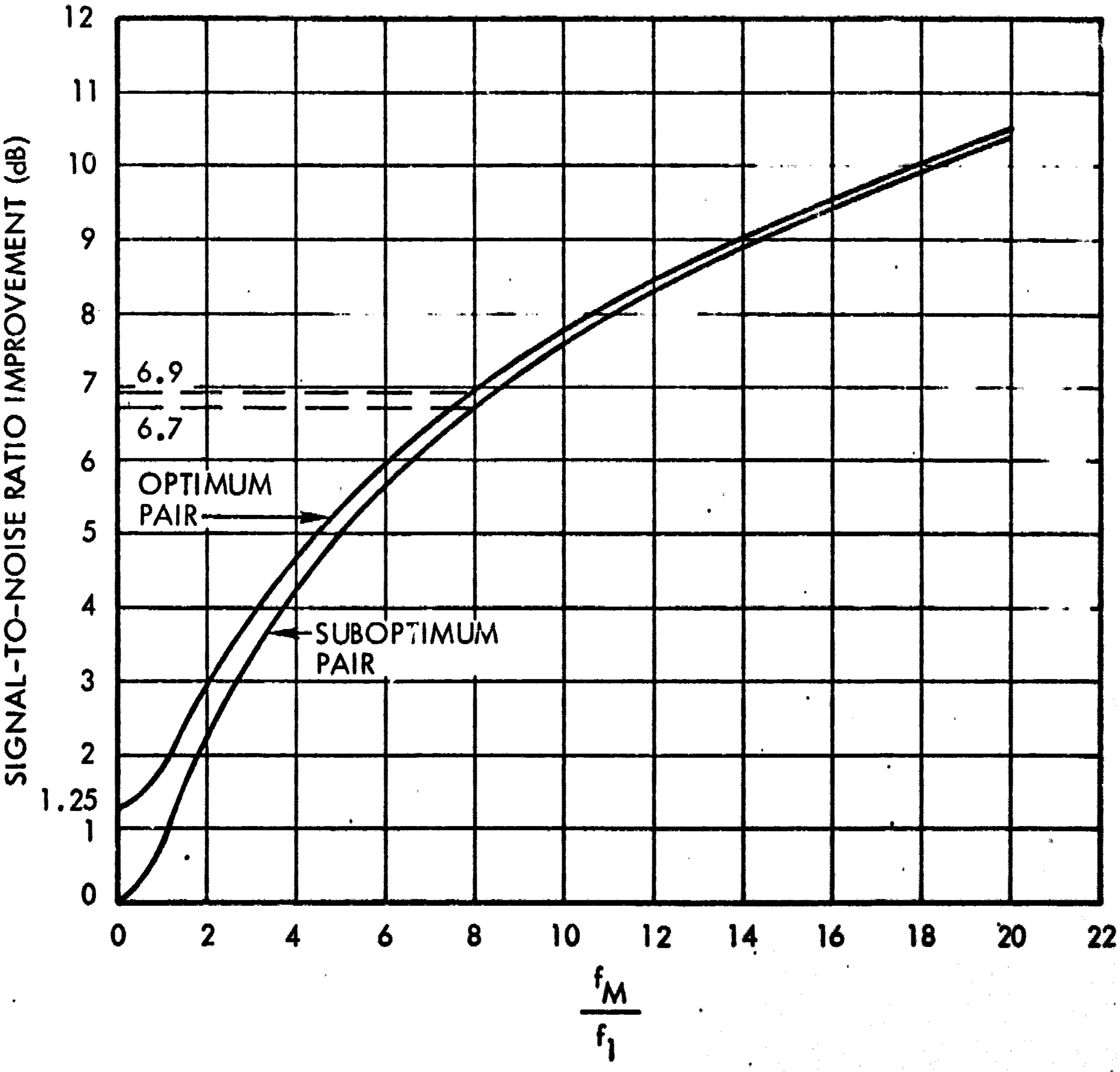


Figure 2-3. Signal-to-Noise Ratio Improvement Obtained With Optimum and Suboptimum Filter Pairs

2.3 COMPARISON OF OPTIMUM AND SUBOPTIMUM PERFORMANCE

In a previous report (Reference 1), the SNR improvement $\mathscr{R}$ for a suboptimum preemphasis-deemphasis filter pair having the filter characteristics (see Figure 2-2)

$$|H_p(f)|^2 = k_1^2 \left[1 + \left(\frac{f}{f_1}\right)^2\right] \tag{2-36}$$

$$|H_d(f)|^2 = \frac{1}{k_1^2\left[1 + \left(\frac{f}{f_1}\right)^2\right]} \tag{2-37}$$

was shown to be given by

$$\mathscr{R} = \frac{\tan^{-1}\left(\frac{f_M}{f_1}\right)}{3\left(\frac{f_1}{f_M}\right)\left[1 - \frac{f_1}{f_M}\tan^{-1}\left(\frac{f_M}{f_1}\right)\right]} \tag{2-38}$$

The values of $\mathscr{R}$ predicted for the suboptimum filter pair are also shown on Figure 2-3 and are seen to be remarkably close to the optimum values. The suboptimum filter pair, specified by Equations (2-36) and (2-37), is easy to synthesize and has been found very effective in practice.

The advantage gained by using the optimum filter pair, instead of the suboptimum filter pair cited above, can be expressed as a signal-to-noise ratio improvement factor ν which is simply the ratio of $\mathscr{R}_{opt}$ to $\mathscr{R}$. For example, from Equations (2-28) and (2-38),

$$\nu = \frac{\mathscr{R}_{opt}}{\mathscr{R}} \tag{2-39}$$

$$= \frac{1}{3}\left(\frac{f_M}{f_1}\right)^3 \cdot \frac{\tan^{-1}\left(\frac{f_M}{f_1}\right)}{\left[\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} - 1\right]^2} \cdot \frac{3\left[1 - \frac{f_1}{f_M}\tan^{-1}\left(\frac{f_M}{f_1}\right)\right]}{\frac{f_M}{f_1}\tan^{-1}\left(\frac{f_M}{f_1}\right)} \tag{2-40}$$

$$= \frac{\frac{f_M}{f_1}\left[\frac{f_M}{f_1} - \tan^{-1}\left(\frac{f_M}{f_1}\right)\right]}{\left[\sqrt{\left(\frac{f_M}{f_1}\right)^2 + 1} - 1\right]^2} \quad (2\text{-}41)$$

By applying suitable approximations or by making use of previous results, it can be shown that

$$\lim_{\frac{f_M}{f_1} \to 0} \nu \to \frac{4}{3} \ (1.25 \text{ dB}) \quad (2\text{-}42)$$

and

$$\lim_{\frac{f_M}{f_1} \to \infty} \nu \to 1 \ (0 \text{ dB}). \quad (2\text{-}43)$$

A plot of ν versus f_M/f_1 is given in Figure 2-4.

In Reference 1, experimental results are presented showing the remarkable improvement in LCRU television performance achieved when using preemphasis-deemphasis networks as described by Equations (2-36) and (2-37). A number of network pairs with different frequency breakpoints were evaluated. The most extensively tested network pair had a breakpoint at 250 kHz and a second breakpoint at 2000 kHz. This particular choice of frequencies corresponds to the ratio $(f_M/f_1) = 8$. From the curve in Figure 2-4, it is observed that an optimum filter pair, with the breakpoints cited above, should furnish an additional .2 dB of signal-to-noise ratio improvement over that expected from the suboptimum filter pair.

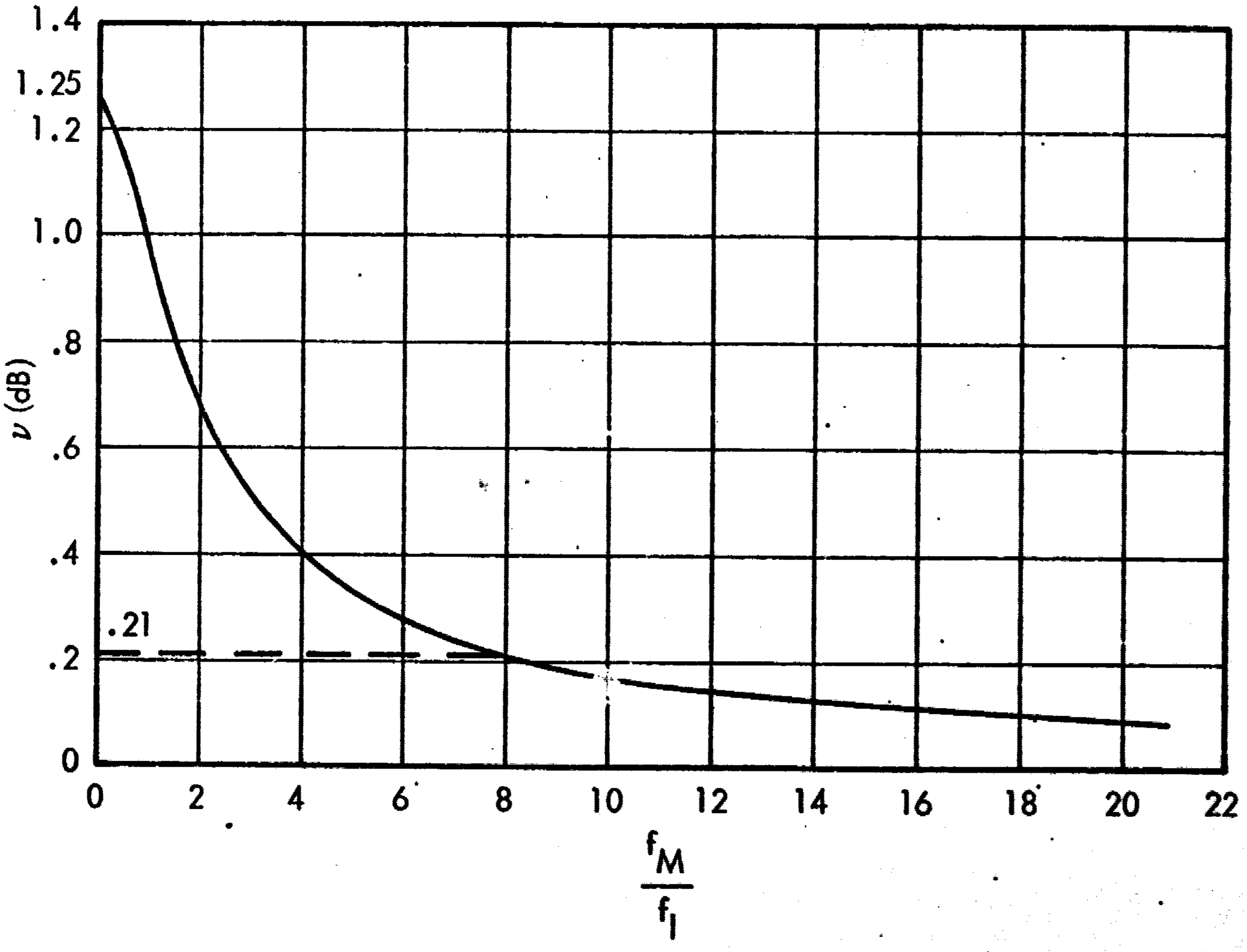


Figure 2-4. Signal-to-Noise Ratio Improvement Gained by Using Optimum Filter Pair Instead of Suboptimum Filter Pair

3. CONCLUSIONS

The results of the study show that usage of the optimum filter pair would yield only a .2 dB increase in signal-to-noise ratio improvement over that predicted for the suboptimum filter pair (250 kHz/2000 kHz) as reported in Reference 1. The values of SNR improvement using the suboptimum filter pair are found to be remarkably close to the optimum values.

The conclusion is that the preemphasis-deemphasis networks to be implemented in the LCRU television system are, for all practical purposes, near optimum networks. The above statement is based upon an envelope model of the video power spectral density and an FM channel which has a constraint on maximum modulating signal power. It is also assumed that the network first breakpoint frequency, f_1, equals the video signal density -3 dB frequency, f'. Equations (2-4) and (2-5) define a general optimum preemphasis-deemphasis filter pair (assuming the signal-to-noise ratio on the channel is moderately high) for a communication channel subject to a modulating signal power constraint. The general expressions for the optimum filter characteristics are in terms of the signal and noise power spectral densities on the channel. A particular signal and noise density together define an optimum filter pair.

APPENDIX A
DETERMINATION OF OPTIMUM FILTER CHARACTERISTICS

A.1 GENERAL

In the optimization problem considered here, the product or ratio of integrals, which contain the variable function X(f) in the integrands, is involved. The optimum variation of X(f) as a function of the variable of integration f is to be determined (Reference 2).

Let Y be the product of two such integrals

$$Y = Q_1[X(f)]\ Q_2[X(f)], \tag{A-1}$$

where

$$Q_1 = \int_{-\infty}^{\infty} H_1(f)\ H_2[X(f)]\ df, \tag{A-2}$$

and

$$Q_2 = \int_{-\infty}^{\infty} H_3(f)\ H_4[X(f)]\ df\ . \tag{A-3}$$

The functions H under consideration have even symmetry about f=0, so that the lower integration limits can be replaced by zero.

With a small variation $\delta X(f)$, the resultant variation in Y is

$$\delta Y = Q_2 \cdot \delta Q_1 + Q_1 \cdot \delta Q_2 \tag{A-4}$$

$$= Q_2 \int_{-\infty}^{\infty} H_1(f)\ H_2'[X(f)]\ \delta X(f)\ df + Q_1 \int_{-\infty}^{\infty} H_3(f)\ H_4'[X(f)]\ \delta X(f)\ df \tag{A-5}$$

$$= \int_{-\infty}^{\infty} \{Q_2 \, H_1(f) \, H_2'[X(f)] \quad + \quad Q_1 \, H_3(f) \, H_4'[X(f)]\} \, \delta X(f) \, df, \tag{A-6}$$

where H_2' and H_4' are derivatives with respect to X(f).

The minimum or maximum value of Y is obtained with $\delta Y = 0$, which is the case when

$$H_1(f) \, H_2'[X(f)] + a \, H_3(f) \, H_4'[X(f)] = 0, \tag{A-7}$$

where the constant a is

$$a = \frac{Q_1}{Q_2} \ . \tag{A-8}$$

A.2 OPTIMUM FILTER PAIR

In this case the optimum condition is attained for the maximum value of the ratio (Reference 2)

$$\frac{\rho_o}{P} = \frac{\int_0^{f_M} S(f) \, df}{2\int_0^{f_M} \frac{S(f) \, df}{|H_d(f)|^2} \int_0^{f_M} N(f)|H_d(f)|^2 df} \tag{A-9}$$

$$= \frac{P_s/2}{Q_1[X(f)] \, Q_2[X(f)]} , \tag{A-10}$$

where

$$Q_1 = \int_0^{f_M} \frac{S(f)}{|H_d(f)|^2} \, df \tag{A-11}$$

$$Q_2 = \int_0^{f_M} N(f)|H_d(f)|^2 \, df. \tag{A-12}$$

Comparison with Equations (A-2) and (A-3) shows that in this case

$$H_1(f) = S(f); \qquad H_2(f) = \frac{1}{|H_d(f)|^2} = \frac{1}{X(f)} \tag{A-13}$$

$$H_3(f) = N(f); \qquad H_4(f) = |H_d(f)|^2 = X(f). \tag{A-14}$$

The relationship given in Equation (A-7) which maximizes the ratio $\frac{\rho_0}{P}$ becomes

$$\frac{-S(f)}{|H_d(f)|^4} + a\,N(f) = 0, \tag{A-15}$$

or

$$|H_d^o(f)|^2 = \left[\frac{S(f)}{a\,N(f)}\right]^{1/2} \tag{A-16}$$

$$= c\left[\frac{S(f)}{N(f)}\right]^{1/2} \tag{A-17}$$

where $c = (1/a)^{1/2}$. The expression for the ratio $\frac{\rho_0}{P}$ as given by Equation (A-9) is independent of c, since it cancels in the denominator. That the above solution gives the maximum value for the ratio ρ_0/P, that is, the minimum value of Q_1Q_2, can be established by noting that it will always be possible to have a greater product Q_1Q_2, except in the special case where $|H_d(f)|^2$ = a constant.

The corresponding optimum preemphasis filter characteristic is obtained from the inverse relation

$$|H_d^o(f)|^2\,|H_p^o(f)|^2 = 1 \tag{A-18}$$

and is

$$|H_p^o(f)|^2 = \frac{1}{c} \left[\frac{N(f)}{S(f)} \right]^{1/2} \qquad \text{(A-19)}$$

where c is the same constant as in Equation (A-17).

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