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LOW DEGREE RESONANT GEOPOTENTIAL COEFFICIENTS FROM EIGHT 24-HOUR SATELLITES

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ABSTRACT

The most extensive tracking data base on 24 hour satellites has been analyzed for resonant geopotential effects in the period 1963-1969. The observations used consisted of 279 sets of mean Kepler elements in 21 free drift arcs of eight satellites, each arc between 7 weeks and 3-1/2 years in length.

Current geopotential solutions from the Smithsonian Astrophysical Observatory (SAO) which do not use the 24-hour data are shown to recover most of these observations almost as well as in solutions using this data. However, since the longest arcs are poorly recovered with the "best" Smithsonian field, an improved resonant geopotential is desirable. Therefore, a new solution is derived which combines the best available low degree Smithsonian coefficients with the 24 hour data. This solution is virtually indistinguishable from the "best fitting" solution from only 24 hour data. A comparison of this combined solution with the other current solutions shows that: 1. the east-west equilibrium points for geostationary satellites are now known accurately to better than 1/2 degree, and 2. the resonant acceleration on a 24 hour satellite can now be predicted accurately to better than 10^{-6} radians/day^2.

The computational efficiency of this resonant solution should be noted. The solution essentially uses only 279 pieces of data compared to over 50,000 in the SAO geopotentials: the resonant coefficients themselves agreeing remarkably well. The total processing time for the resonant solution (involving five iterations) was less than 4 minutes on an IBM 360/91 computer.

LOW DEGREE RESONANT GEOPOTENTIAL

COEFFICIENTS FROM EIGHT

24-HOUR SATELLITES

INTRODUCTION

Since 1963 more than forty 24-hour satellites have been placed in orbit by the National Aeronautics and Space Administration and the Department of Defense.[1] Of these, only about twenty have been close enough to exact commensurability with the earth's rotation to be especially useful for the purposes of dynamic satellite geodesy; determining the geopotential from measuring the perturbations of satellite trajectories. The tracking record of the eight satellites discussed here represents well all the geographic longitudes covered by all the nearly commensurate objects for which data have been available (i.e. unclassified) from 1963 to 1969.

The theory of the greatly amplified long term perturbations of these synchronous, or resonant, orbits has been much discussed in the literature.[2] The analysis presented here is based on the rapid numerical evaluation of perturbations to mean Kepler elements over long tracking arcs[3] (7 weeks to 3-1/2 years). The observations used are themselves mean Kepler elements determined from "raw tracking data" over short (generally 1-2 week) arcs by outside orbit programs of varying degrees of sophistication. The ultimate "raw" tracking data in these arcs are: range and range rate and minitrack (radio interferometric angles for the

Synchronous Communication Satellites (SYNCOM's); radar range, azimuth and altitude for the International Telecommunication Satellites (INTELSATS); and range and range rate for the Applications Technology Satellites (ATS's).

The use of Kepler element data from deeply resonant satellites to aid in the definition of the longitude dependent geopotential has also been extensively discussed in the literature.[2-6] Previous results for 24 hour satellites have depended on much less data than is available now.[7-9]

The methods of analysis used previously have also been somewhat limited so that the full resonant information content in the Kepler element evolution has not been used. For example Allan and Piggott[7] in 1966, used the most sensitive longitude information (the equator crossings) directly, fitting this data to a numerically integrated trajectory. But the sensitive information in the semimajor axis was ignored in their results. On the other hand Wagner[10], in 1968, used both satellite position and orbit energy but indirectly, to derive accelerations which were fitted to a purely analytic model. However, by far the greatest deficiency in all previous results has been due to the sparse longitude coverage of the data. This deficiency has been greatly aleviated by the addition of ATS and INTELSAT data filling large gaps in the previous record. Continued tracking of SYNCOM 2 and 3 has also made valuable extensions to the longitudes covered previously.

ANALYSIS

The analysis here is an application to these orbits of the geopotential recovery method described in Wagner (1969).[3] Two points of departure should be mentioned.

1. In this analysis, the partial derivatives of Kepler elements with respect to the initial arc parameters and geopotential coefficients are calculated by numerical integration of variation equations derived from the Lagrange planetary equations.

The Lagrange equations (for the evolution of a perturbed trajectory) are abbreviated by:

$$\dot{e}_i = f_i\left(e_j, c_k\right), \qquad i, j = 1, 2, \cdots 6.$$

$$k = 1, 2, \cdots n.$$

where

n = number of gravity constants solved for.

e_i = Kepler element.

Then the "variation equations" for the c_k constants are found by taking the partial derivatives of these Lagrange equations with respect to the c_k's (using the chain rule) and interchanging total with partial differentiation:

$$\frac{d}{dt}\left[\frac{\partial e_i}{\partial c_k}\right] = \sum_{j=1}^{6} \frac{\partial f_i}{\partial e_j}\left[\frac{\partial e_j}{\partial c_k}\right] + \frac{\partial f_i}{\partial c_k}. \qquad i = 1, 2, \cdots 6.$$

The bracketed quantities are the required "partials" necessary for the first order differential correction of the c_k's with respect to observations of the e_i. The Kepler elements analyzed in this fashion are the classical set: semi-major axis, eccentricity, inclination, argument of pericenter, right ascension of the ascending node and mean anomaly.

The so-called "state transition" equations (for the $6 \times 6 = 36$ partials of the set of e with respect to the set of e_0, the initial Kepler elements in the arc) are easier to write down because there is no functional dependence here of the gravity coefficients with respect to the initial Kepler elements. Strictly speaking, of course, there is such a dependence because the data used are mean elements. But the observations here are given as fixed (i.e. not corrected for gravity model changes inside or outside this long arc orbit determination process). Thus there is no formal dependence. In order to introduce such a dependence, it would be necessary to go back to the original observations and use the corrected gravity field to produce new mean elements. The changes so produced would be entirely insignificant compared to the changes in the long arc reflecting the resonance. The 36 "state transition" equations, numerically integrated, are:

$$\frac{d}{dt}\left[\frac{\partial e_i}{\partial e_{0,j}}\right] = \sum_{m=1}^{6} \frac{\partial f_i}{\partial e_m}\left[\frac{\partial e_m}{\partial e_{0,j}}\right], \qquad i, j = 1, 2, \cdots 6 .$$

The initial conditions for the complete set of variation equations are simply:

$$\left[\frac{\partial e_i}{\partial e_{0,j}}\right]_{t=0} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$$

$$\left[\frac{\partial e_i}{\partial c_k}\right]_{t=0} = 0$$

From the simultaneous integration of these "variation equations" along with the trajectory (Lagrange) equations, the observation partials and residuals are found and normal equations accumulated for a standard weighted least squares process.[2] For example, the observation equations formed for the first order differential correction process, using these partials, are:

$$\left[e_j\,(\text{observed}) - e_j\,(\text{computed})\right] = \sum_{i=1}^{6}\left[\frac{\partial e_j}{\partial e_{0,i}}\right]\Delta e_{0,i} + \sum^{k}\left[\frac{\partial e_j}{\partial c_k}\right]\Delta c_k$$

An estimate of the accuracy of each observation quantity is used as the weight for the corresponding equation of condition (or observation equation) (see Chapter 5 in Kaula's text[2]). The programming system which does the differential correction process is called <u>ROAD</u> (for <u>R</u>apid <u>O</u>rbit <u>A</u>nalysis and <u>D</u>etermination).[11] <u>ROAD</u> can also accept a-priori information in the form of initial diagonal covariance matrices for the reference gravity field and the starting arc elements.

The inverse of these initial covariance matrices forms the initial estimate of the normal matrix. The initial normal vector of residuals, the right hand side of the normal equations, is zero. It is recalculated at each subsequent iteration as the product of the a-priori normal matrix and the difference between the new and a-priori solution vectors.

2. In the present 24 hour solution, the only data actually fit-to was the mean longitude of the ascending equator crossing (also called the mean longitude) which is physically the most meaningful and sensitive resonant orbit parameter.[12] This quantity, for 24 hour satellites, is the Greenwich hour angle subtracted from the sum of the Kepler elements mean anomaly, argument of perigee and right ascension of the ascending node. The semimajor axes, though available were not used in the analysis because their definitions vary from arc to arc. For example, in some arcs the luni-solar perturbations in the semimajor axis have been removed analytically or by smoothing (where mean Kepler elements have been originally reported). In other arcs these perturbations have been left in (where osculating elements have been originally reported). The present ROAD preprocessor converts osculating Kepler to mean Kepler elements, but only accounting for the effects of geopotential zonal harmonics using Brouwer's theory.[13] This conversion is accomplished by subtracting from the given osculating elements, the short period perturbations (due to the zonal harmonics) calculated from Brouwer's theory.

It is difficult to find a definition of mean semimajor axis completely consistent with the ROAD integration even in a single arc. The real problem is in the long term evolution of the mean anomaly which is extremely sensitive both to small semimajor axis changes and to the resonance effects that are to be determined. The most convenient solution to this problem would appear to be to allow the ROAD program to correct automatically for a small constant shift in the semimajor axis observations for each long arc. In this way the semimajor axis data could be utilized simultaneously with the longitude data without the need for manually correcting the observations. The statistics (or weight) of the present solution should be improved considerably when the semimajor axis data are used. However, major improvement in the results are only expected when additional longitude gaps are filled.

DATA AND PREPROCESSING

The actual data used (for fitting purposes) in this solution was the mean longitude of the ascending equator crossing (lambda, λ) as described previously. However, in Table 1 I list the full set of mean Kepler elements available for the 21 arcs. The Table 1 column headings are: Time is the epoch in modified Julian days (Julian days - 2400000.5), A is the orbits semimajor axis in earth radii, E is the orbits eccentricity, Incl is the orbits inclination in degrees, Omega is the orbit's argument of perigee in degrees, Node is the orbit's right ascension of the ascending node in degrees, Lambda is the orbit's mean geographic longitude in degrees. The reference and solution trajectories duplicated these elements quite faithfully although they were not specifically designed to do so. In future solutions specific use of the other Kepler element data in Table 1 will be made, to improve this solution.

The original orbit elements given from short arc solutions varied from satellite to satellite. The source of this original data and it's subsequent processing to the mean elements in Table 1, is given below.

SYNCOM 2 Arc 1

The original data were mean elements derived from Goddard range and range rate and minitrack data, as determined (at NASA-Goddard) from an analytic program using Brouwer's orbit theory.[13,14] These "Brouwer" mean elements were used directly by ROAD in this solution.

SYNCOM 2 Arcs - 2, 4, 5 and 8; SYNCOM 3 Arc 6

The same kind of information applies to this arc as that above except that essentially no minitrack information is contained in the mean elements.

SYNCOM 3 Arc 7

This arc contains the same kind of elements as those above. However there was an orbit maneuver 2 weeks after the first set of elements which reduced the inclination by about 1° but did not substantially change the orbit energy or longitude elements sensitive to the resonance effect. Nevertheless there has been some difficulty getting good individual arc solutions with this data. In the future the first two data sets will probably be dropped when many arcs are processed simultaneously.

SYNCOM 2 Arc DoD, SYNCOM 3 Arcs 11, 13 and 14

The original data for these arcs were osculating polar coordinates of the spacecraft: Right ascension, declination, distance from the earth's center; azimuth, elevation and speed of the velocity. These elements were reported by the Air Force Systems Command, Sunnyvale California, from range and range rate data. The numerical orbit theory used to derive these elements included sun and moon gravity effects and radiation pressure as well as a fairly accurate set of fixed geopotential constants including resonant ones. However in no case did the data span for each set of elements extend beyond 3 weeks. Thus the orbit programs resonant constants served mainly to define more accurate individual elements for the spacecraft, for prediction purposes.

These osculating polar elements were converted to osculating Kepler elements[15] and then to mean Kepler elements using Brouwer's theory.[13] Thus they do not have the short (less than 1 day) period gravity effects of the sun and moon removed from them. The most serious result of this deficiency is the introduction of a significant constant bias in the semimajor axis which makes that data incompatible with the evolution of the longitude data when both are used in ROAD.

ATS 1 and ATS 3 Arc 1

The original tracking data in these arcs were Goddard range and range rate observations. The preprocessing consisted of fitting data in these separate long arcs to a trajectory computed numerically by the GEOSTAR program.[16] This program found individual best fitting values of the harmonics C_{22} and S_{22} according to the data in these arcs. (The program also calculated a full set of fixed geopotential as well as the usual luni-solar effects.) 12 sets of osculating Kepler elements from the GEOSTAR solution trajectories within the data periods, were chosen as representative of the evolution for each long arc. The osculating data was then converted to mean data through Brouwers theory. Smoothing tests of this orbit data were made with the ROAD (mean element) program. They revealed that the short period perturbations not removed by the Brouwer theory (due to the sun and moon for example) provided "pseudo noise" to the GEOSTAR elements of about the same magnitude (0.002° in longitude) as that indicated by the original GEOSTAR solution residuals. Thus, in spite of the fact that the GEOSTAR solution elements for these arcs can not be proven to be strictly uncorrelated, they appeared to be so because of the short period perturbations left in them. By this device I was able to produce a set of "noisy" elements representative of the excellent range and range rate data over these long arcs. It would have been difficult or impossible to obtain equally good "short arc" elements (correcting for the resonance effect) in strictly independent data spans. There were few periods in ATS tracking when the simultaneous coverage from 2 or more ground stations was good enough to obtain such independent orbits.

ATS 3, Arcs 2 and 3; and ATS 5

These arcs were treated in the same way as the ATS arcs above except for the introduction of artificial noise into the GEOSTAR solution elements. Here it was found that the "noise" introduced because the short period perturbations were not removed by the Brouwer theory was too great and too biased to adequately represent the original GEOSTAR results for these arcs. Therefore an additional smoothing was performed followed by a controlled "noising" up of the data. The procedure was as follows: ROAD (mean element) trajectories were fitted to the representative osculating elements in the GEOSTAR solution trajectories (using the resonant coefficients calculated in the GEOSTAR long arc solutions). Longitudes of the equator crossings from these mean element trajectories were then put through a random noise generator and noised up to a ± 0.002° (1σ) level (representative of the excellent GEOSTAR results for these arcs). The weight actually assigned (in the multi-arc ROAD solutions) for all the ATS longitude data was 0.005°, to compensate further for any correlation remaining from the original GEOSTAR solutions.

Early Bird, Arcs 1 and 2

The original tracking data in these arcs consists of radar ranges, elevations and azimuths from the Communication Satellite Corporations (COMSAT) facility in Andover Maine. The large amount of these data were reduced by COMSAT to a small set of independent mean Kepler elements with the use of a trajectory computation program based on Brouwers 1959 theory but including sun and moon gravity effects as well. This mean Kepler data was used directly (without further processing) by ROAD in the solutions reported here.

INTESAT 2F3, Arcs 1 and 2, and INTELSAT 2F4

For these arcs, COMSAT employed a number of transportable antennas to provide Pacific ocean coverage (for INTELSAT 2F4) and assist the Andover radar for tracking over the Atlantic ocean. A "Cowell" type osculating element orbit determination program (with a limited low degree geopotential field) was used to fit the radar data for these arcs. The resulting independent short arc "best fitting" osculating Kepler elements reported by COMSAT were converted to mean elements by Brouwer's 1959 theory. Once again the short period sun and moon effects are not accounted for in this conversion.

NEW 24-HOUR SATELLITE GEOPOTENTIAL SOLUTIONS

Many trajectories were determined by the ROAD program for the individual arcs in Table 1. These preliminary solutions, including the determination of a few critical resonant terms to absorb the residual acceleration in each long arc, established the quality or weight of the longitude (λ) data. The standard deviation of this data type in these individual arc solutions, rounded up to the nearest 0.005° to be as conservative as possible, served as the arc weight when the multi-arc solutions were made. These conservative standard deviations and other summary data from the multi-arc solutions are shown in Table 2. It should be noted in Table 2 that many of the arc root mean square (RMS) residuals in the multi-arc solutions are indeed smaller than the estimate of the standard deviation from the preliminary best fitting trajectories. This is not surprising considering the rounding-up of the estimate as well as the greater number of degrees of freedom lost in determining individual best fitting resonance constants for each arc with limited data. Indeed, as Table 3 shows the weighted RMS deviation of an observation in the best multi-arc solutions is actually less than 1.

Table 2 presents the root mean square residuals (observed minus computed values of the mean longitude) in the 21 arcs from 3 of the 4 multi-arc solutions whose geopotential coefficients and overall statistics are found in Table 3. The multi-arc processing with ROAD had three objectives; illustrated by the 3 solutions in Table 2:

1. To see how well the best nonresonant satellite geopotentials could recover the longitude data in fixed field solutions for only the 6 initial Kepler elements for each arc. The best such solution was with the 1969 Smithsonian Astrophysical Observatory's (SAO) COSPAR field.[17] The RMS residuals in the longitude (λ) for the arcs in this solution are in the last column of Table 2. The COSPAR field coefficients used and the overall weighted RMS residual of this solution is shown in Table 3. Except for the especially long arcs SYNCOM 2 DoD and SYNCOM 3-14, the COSPAR solution compares satisfactorily with the adjusted coefficient solutions. In a number of other arcs however (ATS 5 and INTELSAT 2F4 for example) this best nonresonant fixed field solution showed significantly degraded residuals (revealing clear systematic errors arising from the geopotential). On the other hand, in the excellent ATS 1 and 3 arcs the COSPAR solutions were fully competitive with the adjusted solutions (perhaps because these arcs were relatively short). It is remarkable that the otherwise excellent SAO 1970 (standard earth II) field,[18]

which is very close to the COSPAR field in the resonant coefficients sensitive for 24-hour satellites, gives a solution through this data considerably degraded from the COSPAR. (See Table 3.) Yet the accelerations on a geostationary satellite calculated from the SAO 1970 field are well within 0.1×10^{-5} radians/day^2, or almost indistinguishable from those of the preferred adjusted field (see Figure 1). One may fairly conclude from these fixed field solutions that only minor adjustments to them are necessary for use with 24 hour satellites. However one should also conclude that even such minor adjustment will produce significantly better results with very long 24 hour trajectories (3 months to years in length). For this reason I had as my next objective:

2. To see how much change would be necessary in the latest SAO nonresonant fields to recover the longitude data in the 21 arcs to as high an accuracy as possible (i.e. in a weighted least squares sense). The first and best such test of the 24 hour data was made by solving (in ROAD) without constraint for the 3 dominating harmonics (2, 2), (3, 3) and (3, 1). The arc results of this solution are in column 8 of Table 2. The field coefficients and the overall statistic of the solution (which included fixed SAO 1970 values of (4, 2) and (4, 4)) are in Table 3. The covariance statistics of this solution are displayed in Table 4. The Table 4 statistics can be said to be the minimum data necessary to rigorously combine the 24 hour tracking results with other gravity data for improved geodetic solutions.[19]

The weak value for (3, 1) and the relatively large number of high correlations in this unconstrained solution suggested a third objective:

3. To see how much improvement in the correlations and the (3, 1) harmonic would be possible with a constrained solution through the 24-hour data using the 1970 SAO field as a-priori information. For this purpose I allowed the following 1σ variations in the resonant SAO field coefficients (unnormalized) through (4, 4) (based on the differences among all the 1969 and 1970 SAO geopotentials): (2,2) and (3, 1), 15×10^{-9}; (3, 3) and (4, 2), 10×10^{-9} and (4, 4), 10×10^{-10}. With this constraint, the weighted least squares multi-arc ROAD solution for all these resonant coefficients, using the longitude data, gave the field having the statistics shown in Tables 2 (column 9), 3 and 5. Evidently this is a considerably smoother and better solution than the unconstrained one as revealed by the lower correlation coefficients, and the more representative value of (3, 1). In addition, the overall arc residuals (Tables 2 and 3) are only slightly degraded over the unconstrained solution.

In conclusion, this constrained solution should be preferred for work with 24 hour satellites. In Figure 1 is plotted the east-west longitude acceleration on a geostationary (E = 0, I = 0°, Period = 24 hrs.) satellite as calculated from this constrained field and the resonant part of the 1970 SAO geopotential. The formula used to compute this acceleration is found in Wagner (1968).[10] Also in Figure 1 is a histogram giving the number of 24 hour satellite observations used in the present solution per 10° of longitude (total = 279) compared to that used in a 1966 solution by Allan and Piggott.[7] This histogram shows the significant improvement in the data base for 24 hour satellite results in the past 4 years. The biggest gap in data is now between 0° and 60° which will soon be partly filled by the tracking of Britain's skynet satellites launched in 1970.

Table 6 gives a comparison of east-west equilibrium points found from the longitude acceleration zero's on the geostationary satellite. The NWL-8D (classified) field was determined (by the Naval Weapons Laboratory) from doppler measurements on the Navy's "transit" (high inclination, low altitude) navigation satellites. It is recalled that the SAO COSPAR (1969) field (which is used here in the constrained solution) resulted from only satellite optical data while the 1970 SAO field is relatively independent from it having significant contributions from surface gravity and deep space probe measurements. It is clear from this table that the equilibrium points can now be said to be known to better than 0.5°. Similarly it can be seen from Figure 1 that the geostationary satellite's accelerations are known to better than 0.1×10^{-5} rad/day^2.

Finally, in Figure 2 are displayed the observations and ROAD computed values of the mean longitude (lambda or λ in degrees) in the most revealing arcs of the constrained resonant solution. (Only those arcs whose "raw" mean longitude histories show strong accelerations are displayed. The arcs not shown reveal equally significant accelerations when the data plotted is the actual mean longitude minus the longitude in a trajectory drifting at the steady average rate for the arc.) If no significant resonant perturbations acted on these satellites, their longitude histories would be linear.

CONCLUSIONS

Tests of long arcs of 24-hour satellite tracking data (mean elements) in the period 1963-1969 reveal:

1. For most of the arcs less than a year in length, the use of recent Smithsonian gravity fields (derived without the use of 24-hour data) can recover the data satisfactorily.

2. For the arcs over one year long, the Smithsonian fields cannot adequately recover the data due to small deficiencies in low degree resonant geopotential terms.

3. A combined Smithsonian-24 hour data field can recover all the 24 hour data with satisfactory precision and is recommended for use with all synchronous satellites at any longitude or inclination.

4. This recommended field should predict the long term geopotential acceleration on a 24 hour satellite to better than 10^{-6} radians/day^2, and the equilibrium points for a geostationary satellite to better than 0.5°.

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TABLE 1

OBSERVATIONS

SYNCOM 2,1 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38253.25542590	6.61077489	.0001800	33.0830	26.2850	317.5690	24.1260	305.1070
2	38267.70833330	6.61076549	.0001600	33.0900	26.0990	317.4540	190.8410	305.1683
3	38272.00000000	6.61100066	.0001800	33.0620	30.0730	317.4750	296.1250	305.2172
4	38277.00000000	6.61105868	.0001200	33.0640	357.7550	317.3620	333.5210	305.2550
5	38281.00000000	6.61105240	.0001500	33.0480	9.0770	317.2720	326.2070	305.2294
6	38284.08333330	6.61108376	.0001500	33.0790	4.6970	317.2240	3.6570	305.2123
7	38289.08333330	6.61110728	.0001900	33.0430	0.5810	317.1650	12.6940	305.1461
8	38292.08333330	6.61122016	.0001800	33.0100	16.2820	317.0980	359.9700	305.0991
9	38299.08333330	6.61123427	.0002200	33.0460	344.1620	316.9960	38.9220	304.9296
10	38310.08333330	6.61148983	.0002000	33.0130	350.8660	316.7800	42.8890	304.5425
11	38316.08333330	6.61159018	.0002800	32.9820	2.6730	316.8130	36.7270	304.3066
12	38324.08333330	6.61186141	.0002600	32.9930	344.2460	316.6030	62.8330	303.8904
13	38332.00000000	6.61179556	.0002800	32.9480	354.5480	316.5700	29.8650	303.3884
14	38339.00000000	6.61196176	.0002500	32.9520	354.3130	316.3280	36.6990	302.8458
15	38345.20833330	6.61201820	.0003100	32.9200	3.4250	316.3080	108.2390	302.3586
16	38351.54166700	6.61200760	.0002640	32.9240	0.8780	316.1280	236.6200	301.7701

SYNCOM 2,2 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38361.04166670	6.61098028	.0000500	32.9200	203.9010	315.9760	222.1700	300.8275
2	38367.00000000	6.61118567	.0000900	32.8920	17.5640	315.9190	39.4350	300.8257
3	38373.00000000	6.61118254	.0001000	32.8810	10.9580	315.8770	51.9420	300.7708
4	38379.70833330	6.61133775	.0000700	32.8720	24.5050	315.7350	300.0000	300.6218
5	38400.70833330	6.61131267	.0001300	32.8670	11.6250	315.5440	332.9970	299.8492
6	38403.25000000	6.61160899	.0001500	32.8570	16.9920	315.4690	165.0310	299.6700
7	38414.87500000	6.61184897	.0001200	32.8260	28.8420	315.3000	29.0980	298.9599
8	38423.83333330	6.61192413	.0001900	32.8590	13.1710	315.2120	37.9560	298.2291
9	38430.66666670	6.61207778	.0001900	32.8000	36.2750	315.0750	321.1680	297.6728
10	38435.79166670	6.61214363	.0001400	32.8330	14.5530	314.9820	32.5170	297.1554
11	38442.70833330	6.61239135	.0001900	32.7620	35.5510	314.8830	347.7740	296.4940

NOTE: TIME is the epoch in modified Julian days (Julian days - 2400000.5), A is the orbits semimajor axis in earth radii, E is the orbits eccentricity, INCL is the orbits inclination in degrees, OMEGA is the orbit's argument of perigee in degrees, NODE is the orbit's right ascension of the ascending node in degrees, LAMBDA is the orbit's mean geographic longitude in degrees.

TABLE 1 (Continued)

SYNCOM 2,4 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38510.08333000	6.62103334	.0011900	32.6030	198.7160	313.8790	333.7520	243.2262
2	38520.66667000	6.62084520	.0012300	32.5620	199.1050	313.7390	185.3310	234.6204
3	38527.66667000	6.62072447	.0012700	32.6250	201.9590	313.5920	183.8850	228.9818
4	38534.58333000	6.62068684	.0011900	32.5770	199.8590	313.5480	157.3420	223.4799
5	38540.62500000	6.62066333	.0012800	32.5590	200.2700	313.4010	173.2790	218.7247
6	38548.87500000	6.62051752	.0013000	32.5780	201.1360	313.3310	264.1600	212.2701
7	38555.37500000	6.62036387	.0012300	32.5790	199.6400	313.2470	267.2150	206.8456
8	38562.62500000	6.62015534	.0012200	32.5670	200.8240	313.1410	177.6550	201.7105
9	38569.62500000	6.62011771	.0012900	32.5610	200.3440	313.0200	179.9090	196.4639

SYNCOM 2,5 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38580.08333000	6.61679074	.0008600	32.5410	211.7230	312.8900	336.4340	188.9309
2	38583.12500000	6.61693341	.0008900	32.5370	210.4870	312.8440	354.2750	187.4907
3	38589.70833000	6.61680014	.0008600	32.5020	212.1510	312.8010	205.9720	184.3210
4	38597.87500000	6.61672959	.0009500	32.5160	211.6910	312.6440	270.7840	180.4654
5	38603.66667000	6.61666688	.0009400	32.5020	215.5600	312.5890	194.9680	177.7536
6	38610.70833000	6.61663238	.0009200	32.4760	214.8330	312.4550	214.5280	174.5145
7	38618.04167000	6.61654302	.0009000	32.4460	216.0210	312.4020	337.2650	171.1560
8	38624.79167000	6.61656967	.0009700	32.4090	215.3420	312.3080	251.6560	168.1208
9	38632.41667000	6.61651009	.0009900	32.4430	223.0450	312.2620	113.0490	164.6553
10	38639.41667000	6.61655085	.0008700	32.3970	218.8320	312.0670	121.2450	161.5438
11	38647.58333300	6.61642072	.0009300	32.3730	218.0100	311.9160	186.5570	157.8346
12	38653.50000000	6.61656497	.0009300	32.3620	221.2440	311.8630	156.5710	155.1978
13	38660.41666670	6.61652420	.0009600	32.3020	220.5810	311.7440	131.0100	152.0372
14	38667.25000000	6.61670294	.0008800	32.3270	221.8140	311.6580	73.4560	148.8951
15	38674.20833000	6.61672645	.0009200	32.3200	228.3690	311.5890	55.6630	145.7308
16	38681.00000000	6.61680485	.0009400	32.3120	225.4950	311.4400	347.1710	142.5205
17	38688.66667000	6.61681425	.0009100	32.2890	226.8770	311.3520	229.7850	138.8706
18	38694.66667000	6.61709960	.0008300	32.2510	226.9840	311.1930	232.9030	136.0227
19	38701.20833000	6.61676095	.0009000	32.2480	225.0150	311.1380	73.0810	132.7314
20	38710.08333000	6.61714193	.0008400	32.2870	232.4660	311.0120	24.9560	128.1838
21	38770.25000000	6.61752919	.0007500	32.1310	240.9110	310.1860	103.1450	94.6874
22	38773.65667000	6.61845893	.0007300	32.1630	228.4690	310.0660	267.1630	92.7746
23	38780.50000000	6.61848088	.0007300	32.1200	226.5520	309.9050	211.8020	88.6015
24	38807.17014000	6.61833350	.0006900	32.1110	232.1210	309.5950	97.7240	72.2448

TABLE 1 (Continued)

SYNCOM 2,8 GSFC DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38815.81250000	6.61133462	.0007600	31.9820	327.9680	309.4710	237.2230	67.6989
2	38816.00000000	6.61162780	.0007700	31.9560	328.6930	309.4640	304.2010	67.7101
3	38822.00000000	6.61114647	.0007500	31.9310	327.5000	309.4630	310.9640	67.3852
4	38825.00000000	6.61128470	.0007600	31.9390	333.0480	309.3700	308.3590	67.2582
5	38832.00000000	6.61134290	.0006900	31.9120	321.4470	309.0950	326.8450	66.9687
6	38855.00000000	6.61114450	.0006900	31.8300	340.6340	308.0960	329.6120	66.0538
7	38862.00000000	6.61113350	.0007300	31.8430	348.5850	308.9470	328.2690	65.8133
8	38869.00000000	6.61105850	.0006500	31.8670	330.8380	308.5860	353.1460	65.6820
9	38876.00000000	6.61102670	.0006300	31.7870	337.0050	308.5800	353.7170	65.5152
10	38883.00000000	6.61100350	.0006100	31.8070	337.7570	308.4170	359.8970	65.3647
11	38890.00000000	6.61094070	.0005900	31.7310	344.5150	308.3350	0.0120	65.2762
12	38897.00000000	6.61113550	.0006300	31.8200	336.2140	308.3120	15.1790	65.2196
13	38904.00000000	6.61105240	.0005300	31.7170	345.1510	308.1920	13.1960	65.1441
14	38911.00000000	6.61101321	.0005100	31.7250	334.7680	307.9540	30.7300	65.1676
15	38918.00000000	6.61096460	.0005600	31.7270	337.3120	307.8820	35.1820	65.1920

TABLE 1 (Continued)

SYNCOM 2, DoD DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38816.00000000	6.61107053	.0006461	31.9874	332.5182	309.4749	300.3415	67.6867
2	38819.99000000	6.61129993	.0006812	31.9536	337.8505	309.4288	297.8877	67.5621
3	38829.00000000	6.61147511	.0007091	31.9044	347.0738	309.3855	298.0381	67.0360
4	38848.00000000	6.61114288	.0006379	31.8762	345.2041	309.0267	318.2442	66.2864
5	38943.42361110	6.61040583	.0005913	31.7076	342.4342	307.4889	213.3832	65.6354
6	38955.00000000	6.61029105	.0005659	31.6938	329.3020	307.4157	80.7468	65.8116
7	38962.12222220	6.61043629	.0005554	31.6381	331.4867	307.3034	129.9194	66.0366
8	38976.12222220	6.60276188	.0010945	31.6654	320.8936	307.2129	154.8824	66.5170
9	38985.82980110	6.61043656	.0006127	31.6306	338.5150	307.0796	42.1337	66.9380
10	39005.14236110	6.61014741	.0005807	31.5839	336.5661	306.5485	177.1346	67.9235
11	39028.12222220	6.60990432	.0007038	31.5170	328.8083	306.3422	201.8947	69.3196
12	39047.49291480	6.61000288	.0006044	31.4424	343.9560	306.1570	340.6963	70.5417
13	39064.50113880	6.60996924	.0007691	31.4784	327.2277	305.7833	18.8822	71.9009
14	39074.41533500	6.60993507	.0008331	31.4709	326.1351	305.6436	359.7789	72.6827
15	39087.34317460	6.60991993	.0008233	31.4435	328.5226	305.4406	359.7870	73.6713
16	39116.31150850	6.61002124	.0008449	31.3760	336.6173	304.9878	357.1909	75.9677
17	39131.27737440	6.61016283	.0008745	31.2879	347.0668	304.8862	351.1496	77.1256
18	39145.26334840	6.61005964	.0008535	31.2745	346.9571	304.6665	0.6219	78.2545
19	39160.22635850	6.61010062	.0008719	31.1724	357.8365	304.5104	352.5100	79.3619
20	39188.15257450	6.61056052	.0009867	31.1828	354.0836	304.1809	359.6291	81.5077
21	39202.10319860	6.61010564	.0008721	31.1393	353.1938	303.9889	359.4121	82.4289
22	39215.07517310	6.61023933	.0008332	31.1149	356.4382	303.5875	358.1956	83.1536
23	39226.04199070	6.61020809	.0007287	31.0897	354.7400	303.3917	359.5405	83.7508
24	39248.98761120	6.61011992	.0007219	31.0375	358.8828	303.0777	359.8217	84.8211
25	39255.95873190	6.61047464	.0007671	31.0253	356.1331	303.0644	359.3083	85.0810
26	39262.14316960	6.61025671	.0006658	31.0134	356.6372	302.9077	0.4920	85.3196
27	39277.59913700	6.61038164	.0006549	31.0065	355.6192	302.6039	1.0809	85.6971
28	39307.80109040	6.61089168	.0006560	30.9714	350.4983	302.1977	1.1621	86.0652
29	39355.26477180	6.61120846	.0007131	30.8342	359.3097	301.5453	62.8229	85.5959
30	39385.51942690	6.61121070	.0006804	30.7425	346.6628	301.1813	12.4702	84.4777
31	39491.00069440	6.61178938	.0007711	30.4914	357.5159	299.7468	240.5873	77.6394
32	39598.09580950	6.61169930	.0008579	30.3369	4.2531	298.1918	358.1066	70.0270
33	39674.24236110	6.61172210	.0013533	30.1011	13.6712	298.0426	121.8205	65.7121
34	39689.19861110	6.61134310	.0007404	30.3386	4.4585	296.5716	131.1257	65.3419
35	39711.15694440	6.61085152	.0006832	30.0137	357.2455	296.3032	145.0761	65.1678
36	39711.15694440	6.61084507	.0006693	30.0840	349.5686	296.4420	152.6227	65.1763
37	39748.16506670	6.61083753	.0005804	29.9072	354.9456	295.7352	188.1666	65.4119
38	39748.16656670	6.61081545	.0006908	30.0119	358.0934	295.7584	185.0401	65.4564
39	39767.97430560	6.61056444	.0007463	29.9673	1.7120	295.4214	132.4594	65.8839
40	39927.04106820	6.61006897	.0006507	28.6999	329.1322	293.2338	358.5033	76.3421
41	39976.63074440	6.61014474	.0006499	29.5069	347.1968	291.9420	336.9825	80.1864
42	40103.69944240	6.61089089	.0006468	29.7228	340.0201	290.0658	51.4573	85.8784

TABLE 1 (Continued)

SYNCOM 3,6 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38699.08333000	6.61187396	.0002200	0.0970	305.9490	210.9590	92.7660	180.2659
2	38702.55417000	6.61207778	.0001100	0.0660	301.4880	135.7200	345.1590	180.0354
3	38706.33333000	6.61191629	.0001300	0.0580	335.1540	167.5320	203.6410	179.7729
4	38715.12500000	6.61184260	.0000800	359.9620	276.0880	287.3830	75.9880	179.2382
5	38723.12500000	6.61175637	.0000700	0.0780	142.5650	112.6080	31.7590	178.8261
6	38729.44097200	6.61174696	.0	0.0900	24.0800	68.9270	313.5880	178.5138
7	38737.53125000	6.61158077	.0001300	0.2590	221.5720	54.1430	171.0270	178.1866
8	38744.50000000	6.61146161	.0000900	0.1250	186.8390	74.9390	180.3890	177.9929
9	38750.37500000	6.61113080	.0	359.7960	347.8500	246.0320	168.8940	177.8112

SYNCOM 3,7 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38774.97916700	6.61256695	.0018100	358.8620	82.5480	145.8910	58.8860	180.6090
2	38782.08333000	6.61280683	.0023800	1.4090	260.8590	22.2780	47.8140	179.7341
3	38790.54861000	6.61254970	.0002500	0.1750	273.4870	298.3830	293.7490	178.5576
4	38793.25000000	6.61274255	.0001300	0.1030	261.3670	323.9320	175.1520	178.2265
5	38800.45833000	6.61261241	.0002100	0.1670	283.7060	277.8360	280.1170	177.3309
6	38807.50000000	6.61259733	.0001100	0.0600	298.3700	248.4140	315.9630	176.4771
7	38814.00000000	6.61244152	.0002000	0.1400	260.6660	258.9630	168.7950	175.7474
8	38821.00000000	6.61259360	.0002800	0.2050	276.0170	241.6250	176.9940	175.0598
9	38828.00000000	6.61248699	.0001900	0.0770	268.6000	287.3390	144.7490	174.2123
10	38835.00000000	6.61280212	.0006000	0.3550	267.1970	255.0850	184.6300	173.5368

SYNCOM 3,11 DoD DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	39075.00086660	6.61211801	.0003280	0.2846	105.4203	117.1014	359.6835	171.9617
2	39097.98076090	6.61201798	.0002874	0.3291	124.1968	103.8597	358.7775	171.0350
3	39103.97576880	6.06194926	.0431297	0.4357	317.3238	102.3735	180.0947	170.0247
4	39105.97576830	6.61186712	.0002849	0.4333	155.1239	100.6390	344.0273	170.0231
5	39115.95543350	6.57278143	.0056881	0.4488	326.0161	98.5565	180.0901	169.3254
6	39116.96650720	6.61179840	.0002356	0.4608	152.8526	99.7637	355.9137	169.2927
7	39130.95793870	6.61175112	.0004699	0.2327	168.3998	76.4761	13.6687	168.6013
8	39131.96148110	6.61171846	.0004557	0.2379	169.4612	76.9838	14.3184	168.5556
9	39144.87241510	6.61150537	.0003619	0.2887	171.9469	75.2528	353.6614	167.9915
10	39160.86412900	6.61136028	.0002474	0.6017	190.4381	92.5952	329.9671	167.3517
11	39187.91752610	6.61148804	.0002517	0.7220	215.3542	93.5163	349.1806	166.5143
12	39201.94059050	6.61143075	.0001777	0.7830	234.8028	95.6955	349.3504	166.1508
13	39214.89194170	6.61135245	.0002084	0.7491	222.5306	89.5571	2.6942	165.8682
14	39225.00905520	6.61116906	.0000693	0.7239	298.0397	93.0557	335.6286	165.6774
15	39247.97628260	6.61135216	.0001290	0.7706	295.8686	97.6107	344.4611	165.3325
16	39262.00121120	6.61127456	.0001654	0.9463	292.3467	97.1756	10.3626	165.2010

TABLE 1 (Continued)

SYNCOM 3, 13 DoD DATA

NO.	TIM	A	e	INCL	OMEGA	NODE	MEAN	LAMBDA
1	39384.70344280	6.60445276	.0008663	1.2357	142.5956	101.9846	183.2321	161.1904
2	39412.49025890	6.61127299	.0001616	1.2795	344.8335	96.7796	278.4063	160.9493
3	39472.65302280	6.61101518	.0000715	1.4351	296.7595	93.7620	86.7869	160.3440
4	39491.00064440	6.61106936	.0000780	1.4976	275.3430	93.7807	251.1823	160.0954

SYNCOM 3, 14 DoD DATA

NO.	TIM	A	e	INCL	OMEGA	NODE	MEAN	LAMBDA
1	39663.13611110	6.60601685	.0003382	1.9529	227.4926	95.3559	153.7691	157.9922
2	39689.13263890	6.61076456	.0005979	2.1302	350.1640	98.8084	52.6567	158.6303
3	39713.13125000	6.61097791	.0003699	1.9549	28.1253	93.1863	43.9130	159.0718
4	39747.08955560	6.61098118	.0003649	1.8089	60.1388	86.4486	34.2538	159.4763
5	39797.17222220	6.61116897	.0001868	2.2125	93.3977	83.0627	86.9726	159.6953
6	39860.45833330	6.61128307	.0000640	2.4006	322.9791	88.2501	204.0037	159.7106
7	39870.48924050	6.61123957	.0001392	2.3631	1.1293	88.5010	0.1281	159.7293
8	39894.30208330	6.61121841	.0001744	2.4814	330.2135	88.4259	347.1144	159.5304
9	39926.35607550	6.61124423	.0001615	2.4417	357.5418	89.0464	13.5052	159.2290
10	39983.18850970	6.61127827	.0003207	2.5974	7.6921	88.5836	354.8210	158.1490
11	40055.74387720	6.61164384	.0004574	2.8966	0.0304	87.1931	272.7751	155.6546
12	40091.90723190	6.61175372	.0003629	2.9620	44.7728	85.4619	322.1267	153.5154
13	40102.90068680	6.61212545	.0005407	2.9656	32.3375	87.9949	1.3781	152.7260
14	40140.52893410	6.61226612	.0004736	3.0481	37.4076	83.4051	239.2246	149.4546
15	40174.78690870	6.61264308	.0002949	3.2601	41.8882	86.403	355.3186	145.6687

ATS 1 DATA

NO.	TIM	A	e	INCL	OMEGA	NODE	MEAN	LAMBDA
1	40204.44883110	6.61161468	.0001202	1.1450	218.5719	83.5646	153.2151	210.6032
2	40208.44883110	6.61153634	.0001203	1.1582	218.6622	83.5482	156.8909	210.4114
3	40212.44883110	6.61145789	.0001203	1.1714	218.7365	83.5527	160.5876	210.2430
4	40216.44883110	6.61137937	.0001203	1.1846	218.7955	83.5771	164.3055	210.1020
5	40220.44883110	6.61130077	.0001204	1.1976	218.8402	83.6200	168.0447	209.9861
6	40224.44883110	6.61122212	.0001204	1.2105	218.8718	83.6798	171.8057	209.8958
7	40228.44883110	6.61114344	.0001204	1.2231	218.8918	83.7547	175.5889	209.8318
8	40232.44883110	6.61106472	.0001204	1.2355	218.9019	83.8426	179.3946	209.7932
9	40236.44883110	6.61098599	.0001204	1.2475	218.9036	83.9411	183.2236	209.7803
10	40240.44883110	6.61090725	.0001204	1.2593	218.8989	84.0479	187.0763	209.7912
11	40244.44883110	6.61082853	.0001204	1.2706	218.8896	84.1603	190.9533	209.8293
12	40248.44883110	6.61074983	.0001204	1.2816	218.8776	84.2760	194.8553	209.8925

TABLE 1 (Continued)

ATS 3-1 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	40197.17725710	6.61737143	.0009488	0.6735	305.8965	270.3626	235.8261	312.2694
2	40201.17725710	6.61743932	.0009484	0.6605	305.9256	270.4134	237.5884	310.1702
3	40205.17725710	6.61750828	.0009480	0.6474	305.9814	270.4325	239.3335	308.0479
4	40209.17725710	6.61757799	.0009476	0.6341	306.0677	270.4159	241.0610	305.9024
5	40213.17725710	6.61764813	.0009472	0.6208	306.1883	270.3602	242.7707	303.7351
6	40217.17725710	6.61771835	.0009468	0.6075	306.3462	270.2623	244.4621	301.5456
7	40221.17725710	6.61778832	.0009465	0.5942	306.5443	270.1197	246.1350	299.3287
8	40225.17725710	6.61785770	.0009462	0.5811	306.7848	269.9306	247.7891	297.0923
9	40229.17725710	6.61792613	.0009460	0.5681	307.0691	269.6940	249.4241	294.8317
10	40233.17725710	6.61799327	.0009458	0.5553	307.3980	269.4097	251.0399	292.5509
11	40237.17725710	6.61805875	.0009457	0.5428	307.7712	269.0785	252.6362	290.2465
12	40241.17725710	6.61812224	.0009457	0.5307	308.1876	268.7023	254.2131	287.9198

INTELSAT 2 F3-1 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	39607.02430560	6.61102299	.0002459	1.3270	286.7815	287.0465	357.8790	348.6381
2	39621.54180550	6.61115304	.0001358	1.2890	313.8516	287.6700	187.1493	348.7486
3	39643.66666670	6.61105090	.0002560	1.2350	284.5675	288.1574	271.4931	348.8544
4	39648.66666670	6.61085889	.0002933	1.2190	280.4352	287.9384	275.8003	348.8104
5	39684.04027780	6.61096899	.0002826	1.1000	272.3587	288.0572	111.5111	349.1482
6	39719.00000000	6.61106906	.0004205	1.0160	264.8617	287.4722	121.6912	349.3375
7	39781.16666670	6.61077119	.0005701	0.8940	290.2674	287.1439	218.5057	349.9551
8	39814.04166670	6.61099826	.0005517	0.8260	292.8240	289.9911	200.9500	350.3999
9	39845.00000000	6.61076524	.0005678	0.7290	302.5942	292.7730	204.4508	350.9390
10	39864.00000000	6.61077218	.0004933	0.6730	313.0696	293.4055	212.4459	351.3147
11	39881.00000000	6.61077707	.0003222	0.6260	308.2134	294.5369	233.3107	351.6987
12	39905.00000000	6.61056405	.0003249	0.5570	319.7986	294.1462	246.4002	352.3271

INTELSAT 2 F4 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	40323.00000000	6.61268084	.0003727	0.5260	303.7305	95.6957	347.4348	186.8425
2	40398.00000000	6.61151578	.0004162	0.7270	339.2008	90.1279	24.6763	180.0630
3	40434.00000000	6.61119486	.0003176	0.8600	2.0015	91.3600	35.1345	179.0707
4	40493.00000000	6.61049804	.0001221	0.9650	35.3841	91.1202	61.1927	180.1185
5	40537.00000000	6.60994196	.0000494	1.0580	220.3501	87.6700	286.3809	183.4540
6	40540.00000000	6.60966492	.0000640	1.0760	282.0962	87.5959	227.9724	183.7611
7	40566.00000000	6.60951587	.0001569	1.1580	248.8397	86.8510	290.6423	186.8022
8	40608.00000000	6.60874269	.0002899	1.2970	257.9097	87.0369	329.6863	193.7050

TABLE 1 (Continued)

ATS 3-2 DATA

NO.	TIM	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	40257.00015050	6.61038101	.0000749	0.4272	82.3547	265.5134	83.7967	286.7892
2	40269.00015050	6.61041088	.0000749	0.4226	82.7063	265.2167	85.8081	286.8843
3	40293.00015050	6.61077258	.0000750	0.3743	86.6169	262.0180	109.5030	287.6309
4	40297.00015050	6.61083324	.0000750	0.3669	87.1360	261.6215	113.3781	287.6931
5	40299.00015050	6.61086360	.0000750	0.3633	87.3746	261.4440	115.3084	287.7119
6	40303.00015050	6.61092433	.0000749	0.3560	87.8066	261.1333	119.1548	287.7336
7	40309.00015050	6.61101544	.0000749	0.3449	88.3350	260.7836	124.8906	287.7352
8	40311.00015050	6.61104581	.0000749	0.3412	88.4779	260.6990	126.7938	287.7263
9	40315.00015050	6.61110652	.0000748	0.3336	88.7130	260.5783	130.5876	287.6902
10	40317.00015050	6.61113696	.0000748	0.3297	88.8051	260.5421	132.4783	287.6674
11	40323.00015050	6.61122777	.0000747	0.3177	88.9809	260.5282	138.1269	287.5630
12	40335.00015050	6.61140888	.0000746	0.2915	89.9087	260.8937	149.3223	287.2259
13	40337.00015050	6.61143894	.0000746	0.2868	88.8475	260.9994	151.1755	287.1501

ATS 5 DATA

NO.	TIM	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	40475.95857640	6.61050800	.0018000	2.7170	282.5173	275.6023	30.3853	252.6364
2	40477.95857640	6.61050647	.0019006	2.7131	282.6120	275.5606	32.3792	252.7143
3	40479.95857640	6.61050502	.0018011	2.7093	282.7057	275.5209	34.3723	252.7871
4	40481.95857640	6.61050365	.0018017	2.7054	282.7983	275.4833	36.3646	252.8631
5	40483.95857640	6.61050238	.0018022	2.7017	282.8897	275.4479	38.3562	252.9391
6	40485.95857640	6.61050118	.0018027	2.6979	282.9798	275.4146	40.3471	253.0165
7	40497.95857640	6.61049586	.0018048	2.6758	283.4852	275.2638	52.2820	253.4803
8	40499.95857640	6.61049527	.0018051	2.6720	283.5627	275.2469	54.2701	253.5563
9	40525.95857640	6.61049564	.0018048	2.6192	284.3725	275.2154	80.1318	254.5696
10	40527.95857640	6.61049628	.0018046	2.6146	284.4200	275.2242	82.1250	254.6413
11	40539.95857640	6.61050195	.0018024	2.5850	284.6689	275.2915	94.1042	255.1127
12	40541.95857640	6.61050320	.0016020	2.5797	284.7054	275.3[illegible]37	96.1043	255.1918
13	40553.95857640	6.61051251	.0017993	2.5459	284.9071	275.3678	108.1290	255.6503
14	40555.95857640	6.61051436	.0017988	2.5400	284.9392	275.3755	110.1370	255.7286

TABLE 1 (Continued)

INTELSAT 2 F3-2 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	40405.77083330	6.61114706	.0001313	0.7980	358.9549	72.7948	116.0613	348.7097
2	40412.50000000	6.61115195	.0001561	0.8040	285.2277	71.9092	279.2271	348.6229
3	40439.00000000	6.61106397	.0000652	0.8730	236.5199	74.0109	352.2141	348.3914
4	40468.00000000	6.61108504	.0001308	0.9350	181.3160	75.6925	74.0376	348.1087
5	40503.00000000	6.61129305	.0001941	1.0030	201.5150	76.6686	86.9993	347.7480
6	40530.00000000	6.61120705	.0003596	1.0640	206.0583	75.5901	109.8416	347.4425
7	40573.00000000	6.61117406	.0004427	1.1960	220.0291	75.0781	138.1758	346.8527
8	40598.25000000	6.61125118	.0003828	1.2800	230.4830	75.9287	241.3823	346.4761
9	40630.00000000	6.61117226	.0004725	1.3590	242.2044	76.9230	169.3286	345.8438
10	40641.66666670	6.61131687	.0003921	1.4410	273.5268	69.7090	36.4511	345.5756

EARLY BIRD 1 DATA

NO.	TIME	A	E	INCL	OMEGA	NODE	MEAN	LAMBDA
1	38897.00000000	6.61056400	.0003490	0.1850	320.8860	82.6300	162.0800	331.1106
2	38918.00000000	6.61054800	.0004250	0.2610	324.9450	77.9230	184.0050	331.6890
3	38932.54583330	6.61090000	.0003300	0.3320	328.9920	76.7850	32.1620	331.9180
4	38942.00000000	6.61090000	.0003300	0.3320	329.2460	76.6580	204.9230	331.9875
5	38947.00000000	6.61095000	.0003300	0.3320	329.3800	86.5910	199.7270	331.9303
6	38953.51944440	6.61123290	.0004280	0.2410	348.6340	103.8940	356.5650	331.8994
7	38996.41111260	6.61155800	.0003370	0.4730	335.8330	93.5710	21.7060	330.6398
8	39005.54131930	6.61064000	.0003770	0.5200	339.9640	93.4870	73.0530	330.1602
9	39036.83420140	6.61207800	.0002020	0.5620	331.8070	96.5060	212.3320	328.0199
10	39054.50277780	6.61230100	.0002280	0.5910	346.0190	96.2160	98.4020	325.9669
11	39079.53680550	6.61264200	.0001460	0.6020	334.3490	92.2830	162.5710	323.5014

TABLE 1 (Continued)

EARLY BIRD 2 DATA

NO.	TIM	A		INCL	OMEGA	NODE	MEAN	LAMBDA
1	39090.0[illegible]833330	6.60918700	.0003430	0.6710	149.5330	90.7170	172.6740	321.2373
2	3909[illegible].05833330	6.60917100	.0003220	0.6460	145.4300	90.2040	177.2980	321.2453
3	39124.[illegible]472220	6.60954800	.0003750	0.7620	154.3330	90.6890	29.1730	324.8914
4	39160.03594449	6.61026000	.0002950	0.8700	151.9770	92.0790	220.3330	328.1715
5	39194.37[illegible]69440	6.61036400	.0002640	0.9360	178.1880	93.2750	182.0690	330.1962
6	39219.40277780	6.61061800	.0001380	0.9840	213.9850	93.3110	0.9380	330.9732

ATS 3-3 DATA

NO.	TIM	A		INCL	OMEGA	NODE	MEAN	LAMBDA
1	40524.00000000	6.61108300	.0005000	0.2940	219.1900	82.7900	51.5300	315.3768
2	40530.00000000	6.61117712	.0005000	0.3094	219.9479	82.0787	57.3240	315.3019
3	40536.00000000	6.61127436	.0005000	0.3257	220.6346	81.4426	63.0673	315.1864
4	40542.00000000	6.61137176	.0005002	0.3429	221.2242	80.9092	68.7576	315.0168
5	40549.00000000	6.61146938	.0005004	0.3610	221.7000	80.4974	74.3930	314.7994
6	40554.00000000	6.61156726	.0005007	0.3797	222.0533	80.2172	79.9720	314.5425
7	40560.00000000	6.61166546	.0005010	0.3989	222.2826	80.0713	85.4930	314.2281
8	40566.00000000	6.61176401	.0005013	0.4185	222.3925	80.0557	90.9552	313.8694
9	40572.00000000	6.61186295	.0005016	0.4383	222.3923	80.1614	96.3579	313.4656
10	40577.00000000	6.61194572	.0005019	0.4547	222.3169	80.3330	100.8147	313.0941

TABLE 2. 24 HOUR SATELLITE ARCS IN 1970 RESONANT GEOPOTENTIAL SOLUTIONS

SATELLITE	ARC	NUMBER OF KEPLER ELEMENT SETS USED	SPAN OF DATA (modified Julian Days)	GEOGRAPHIC LONGITUDE SPAN IN ARC (degrees)	ORBIT INCLINATION (degrees)	ESTIMATE OF QUALITY OF DETERMINATION OF THE GEOGRAPHIC LONGITUDE, σ (λ), IN ARC, FROM INDIVIDUAL ARC ANALYSES (degrees)	RMS RESIDUALS IN LONGITUDE FROM COMBINED ARC SOLUTIONS (degrees)		
							WITH 24 HOUR SATELLITES ALONE, SOLVING FREELY FOR 6 RESONANT COEFFICIENTS	WITH 24 HOUR SATELLITES COMBINED WITH SAO 1969 COSPAR DATA, IN CONSTRAINED SOLUTION FOR 10 RESONANT COEFFICIENTS	WITH SAO 1969 COSPAR FIELD ALONE
SYNCOM 2	1	16	38263-38351	302-305	33.0	0.025	0.013	0.013	0.013
SYNCOM 2	2	11	38361-38443	296-301	32.8	0.025	0.034	0.034	0.035
SYNCOM 2	4	9	38510-38570	196-243	32.5	0.020	0.015	0.014	0.014
SYNCOM 2	5	24	38590-38807	72-189	32.3	0.035	0.039	0.038	0.035
SYNCOM 2	8	15	38816-3891[illegible]	65-68	31.8	0.020	0.018	0.018	0.018
SYNCOM 2	DoD	42	38816-401[illegible]4	65-86	29-32.0	0.040	0.037	0.038	0.103
SYNCOM 3	6	9	38689-38750	178-180	0.0	0.015	0.014	0.015	0.013
SYNCOM 3	7	10	38775-38835	174-181	0.0	0.055	0.054	0.055	0.052
SYNCOM 3	11	16	39075-39262	165-172	0.5	0.020	0.012	0.014	0.013
SYNCOM 3	13	4	39386-39401	160-161	1.3	0.015	0.005	0.006	0.010
SYNCOM 3	14	15	39663-40175	146-160	2-3.0	0.045	0.061	0.061	0.110
ATS 1	1	12	40204-42248	210-211	1.2	0.005	0.001	0.001	0.001
ATS 3	1	12	40197-40241	288-312	0.6	0.005	0.002	0.003	0.003
ATS 3	2	13	40267-40337	287-288	0.3	0.005	0.004	0.003	0.002
ATS 3	3	10	40524-40577	313 315	0.3	0.005	0.002	0.003	0.003
INTELSAT 2-F3	1	12	39697-39985	349-352	1.0	0.025	0.028	0.028	0.030
INTELSAT 2-F3	2	10	40486-40642	346-348	1.0	0.020	0.011	0.012	0.015
INTELSAT 2-F4	1	8	40323-40608	179-194	1.0	0.040	0.043	0.046	0.062
EARLY BIRD	1	11	38897-39080	324-332	0.5	0.025	0.025	0.026	0.026
EARLY BIRD	2	6	39096-39219	321-331	0.8	0.010	0.007	0.009	0.009
ATS 5	1	14	40476-40556	253-256	2.[illegible]	0.005	0.004	0.006	0.009

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TABLE 3

RESONANT GEOPOTENTIAL COEFFICIENTS FOR 24 HOUR SATELLITES
AND RMS FITS TO 21 ARCS

:UNNORMALIZED COEFFICIENTS IN UNITS OF 10^{-6}:

Field	Overall Weighted RMS Residual in 21 Arc Solution	2,2		3,1		3,3		4,2		4,4	
		C	S	C	S	C	S	C	S	C	S
SAO STANDARD EARTH II (1970)	2.46	1.558	-0.881	2.128	0.281	0.096	0.199	0.074	0.158	-0.0017	0.0072
SAO COSPAR (1969)	1.38	1.566	-0.896	2.040	0.262	0.096	0.198	0.073	0.148	-0.0028	0.0078
WAGNER (UN-CONSTRAINED) 2,2–3,1–3,3 (1970)	0.849	1.568	-0.907	1.687	0.483	0.103	0.204	0.074	0.158	-0.0017	0.0072
WAGNER–SAO COSPAR (CON-STRAINED) (1970)	0.896	1.570	-0.908	2.029	0.267	0.098	0.205	0.075	0.150	-0.0029	0.0079

TABLE 4

CORRELATION COEFFICIENTS AND STANDARD DEVIATIONS IN UNCONSTRAINED 24 HOUR SATELLITE GEOPOTENTIAL SOLUTION

	(0.20) C(02, 02)	(0.15) S(02, 02)	(7.50) C(03, 01)	(5.76) S(03, 01)	(0.16) C(03, 03)	(0.14) S(03, 03)
C(02,02)	1.000D 00					
S(02,02)	-5.633D-01	1.000D 00				
C(03,01)	1.191D-01	-8.771D-02	1.000D 00			
S(03,01)	-6.032D-01	4.812D-01	-4.698D-01	1.000D 00		
C(03,03)	-4.278D-01	7.391D-01	-5.364D-01	7.332D-01	1.000D 00	
S(03,03)	-5.206D-01	-3.399D-02	4.948D-01	1.461D-01	-1.613D-01	1.000D 00

( ) = STANDARD DEVIATION OF UNNORMALIZED COEFFICIENT IN UNITS OF 10^{-8}.

TABLE 5

CORRELATION COEFFICIENTS AND STANDARD DEVIATIONS IN CONSTRAINED 24 HOUR SATELLITE GEOPOTENTIAL SOLUTION

	(0.18) C(02, 02)	(0.15) S(02, 02)	(1.47) C(03, 01)	(1.45) S(03, 01)	(0.13) C(03, 03)	(0.13) S(03, 03)	(0.99) C(04, 02)	(0.99) S(04, 02)
C(02,02)	1.000D 00							
S(02,02)	-3.752D-01	1.000D 00						
C(03,01)	-2.485D-02	4.752D-02	1.000D 00					
S(03,01)	-1.525D-01	1.117D-01	-2.632D-02	1.000D 00				
C(03,03)	9.260D-02	6.065D-01	-1.914D-02	2.312D-01	1.000D 00			
S(03,03)	-3.800D-01	-3.966D-01	1.013D-01	1.482D-01	-3.319D-01	1.000D 00		
C(04,02)	1.839D-01	3.891D-02	-2.265D-03	-1.347D-02	4.008D-02	-8.725D-02	1.000D 00	
S(04,02)	4.936D-02	2.544D-01	1.417D-02	1.003D-02	1.119D-01	-8.644D-02	-4.259D-03	1.000D 00
C(04,04)	1.285D-01	-2.875D-01	-5.815D-02	6.044D-02	-2.229D-01	5.582D-01	8.029D-02	9.336D-02
S(04,04)	-3.615D-01	1.924D-01	-4.614D-02	-1.170D-01	-4.437D-01	-4.709D-02	-7.151D-03	7.454D-02

	(0.060) C(04, 04)	(0.070) S(04, 04)
C(04,04)	1.000D 00	
S(04,04)	-1.703D-01	1.000D 00

( ) = STANDARD DEVIATION OF UNNORMALIZED COEFFICIENT IN UNITS OF 10^{-8}.

TABLE 6

EAST-WEST EQUILIBRIUM POINTS FOR

GEOSTATIONARY SATELLITES

Gravity Field	Indian Ocean Stable Pt.	West Pacific Unstable Pt.	East Pacific Stable Pt.	Atlantic Stable Pt.
WAGNER-SAO COSPAR (CON-STRAINED) (1970)	75.0°	161.8°	-105.5°	-11.5°
SAO STANDARD EARTH II (1970)	75.1	162.1	-105.1	-11.2
NWL 8-D (1967)	74.9	161.7	-105.4	-11.7

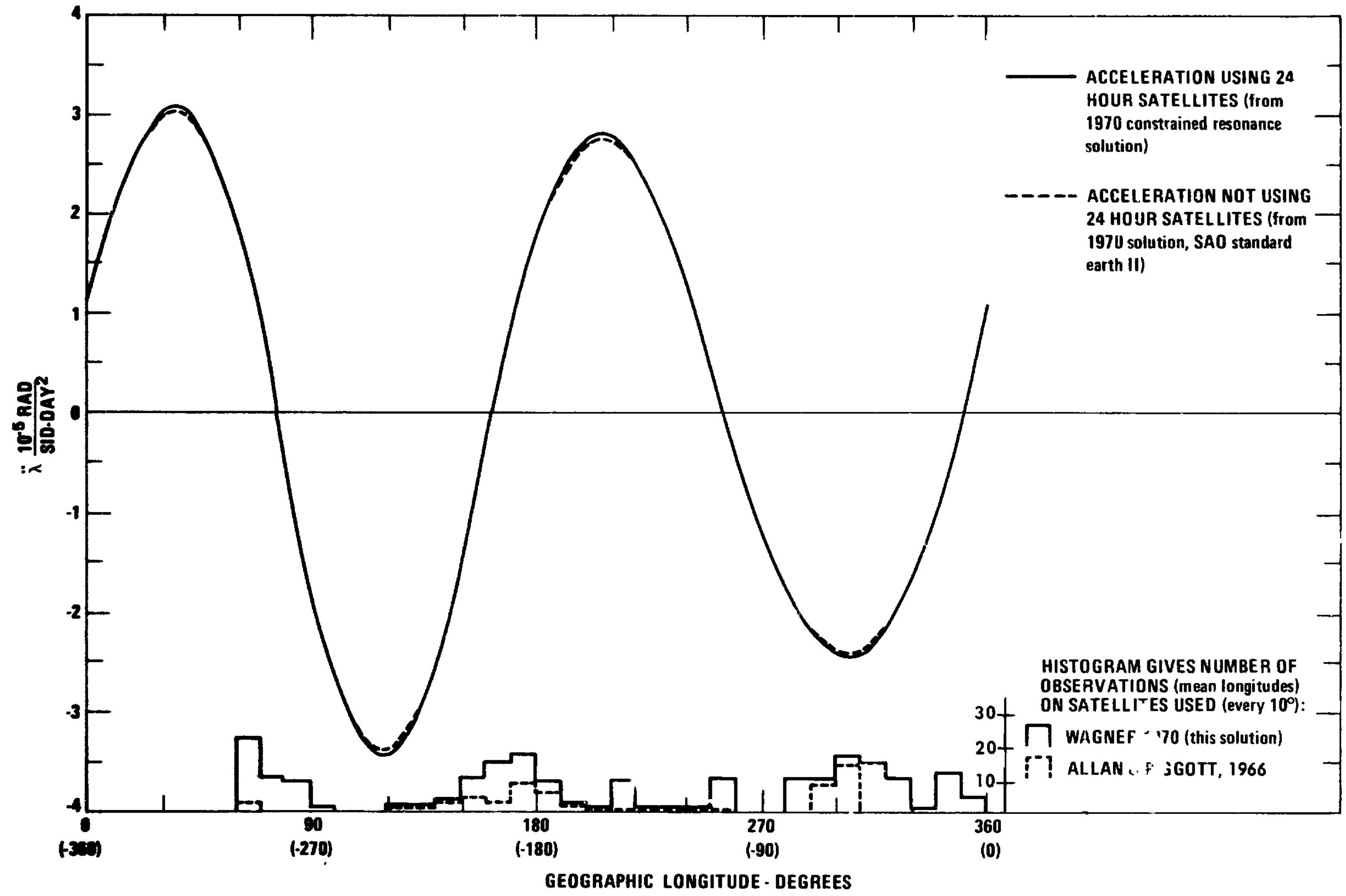
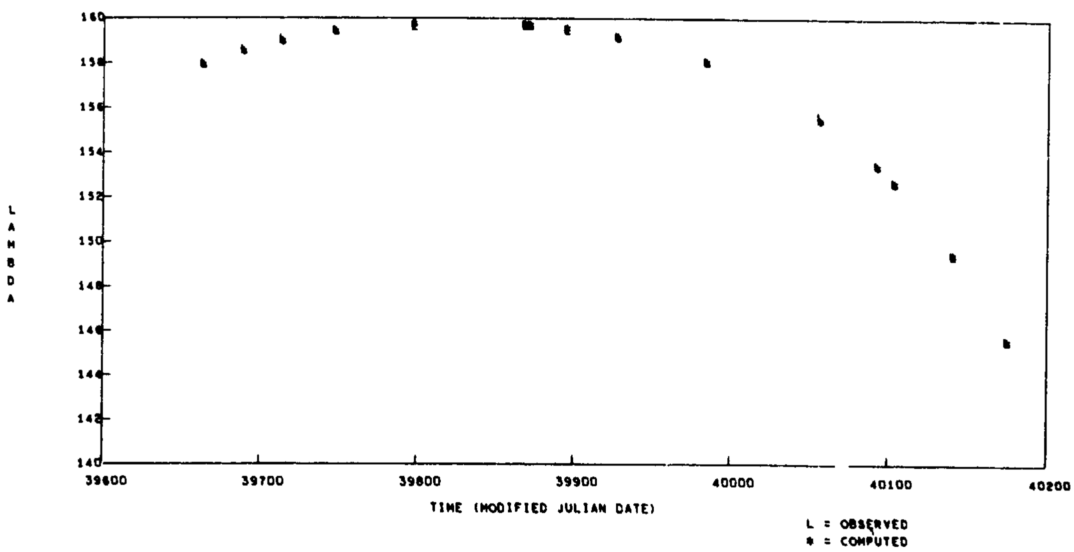
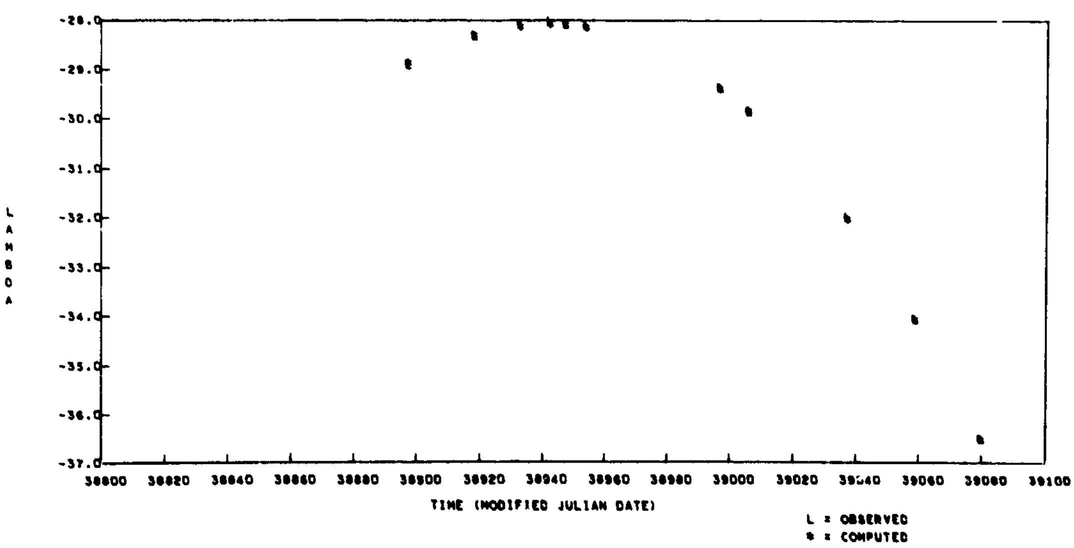


Figure 1. Longitude Acceleration on a Geostationary Satellite (and Observation Histogram)

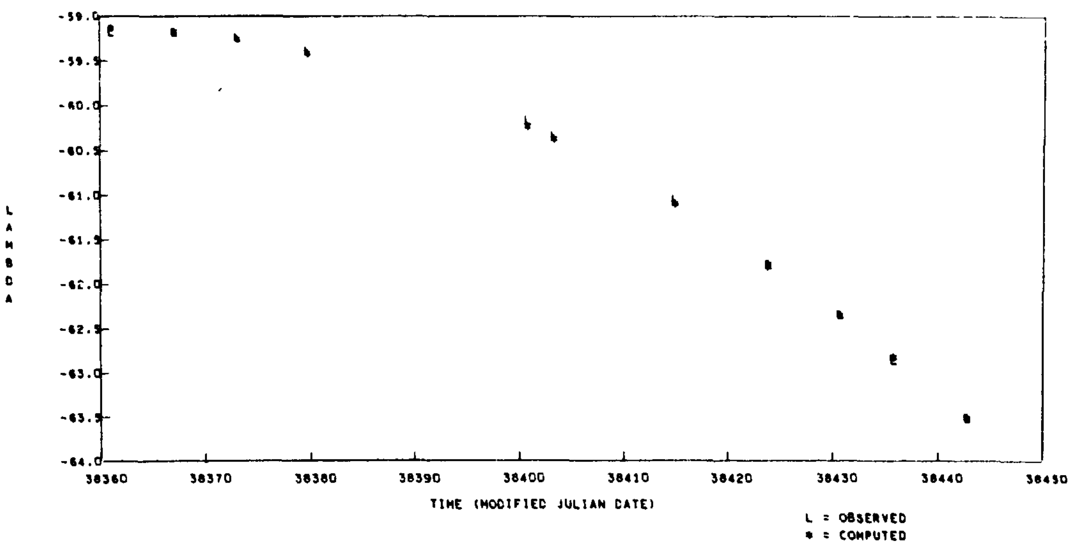


(a) SYNCOM 3, 14 DoD Data

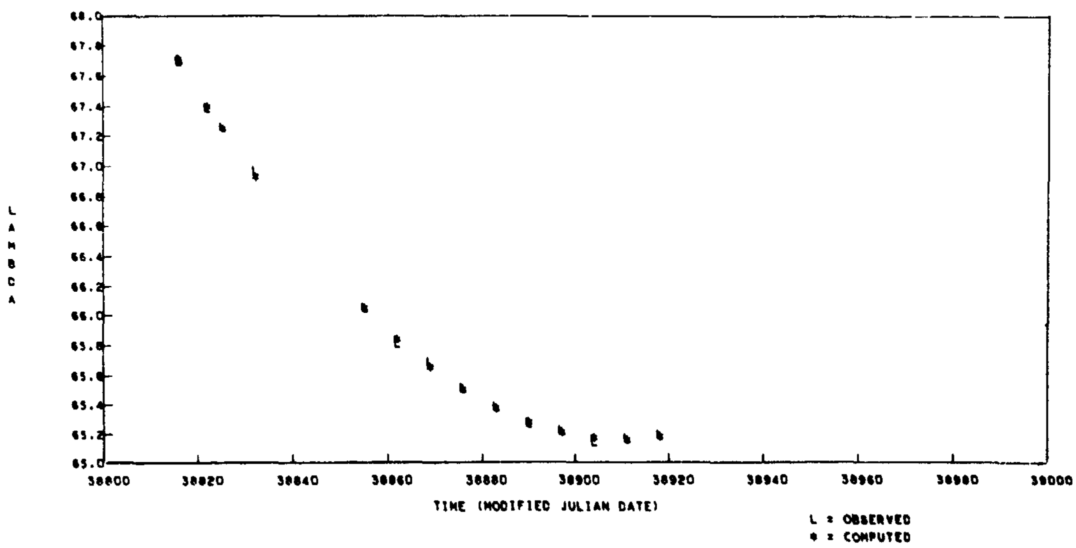


(b) Early Bird 1 Data

Figure 2. Observed and Computed Measurements - Lambda

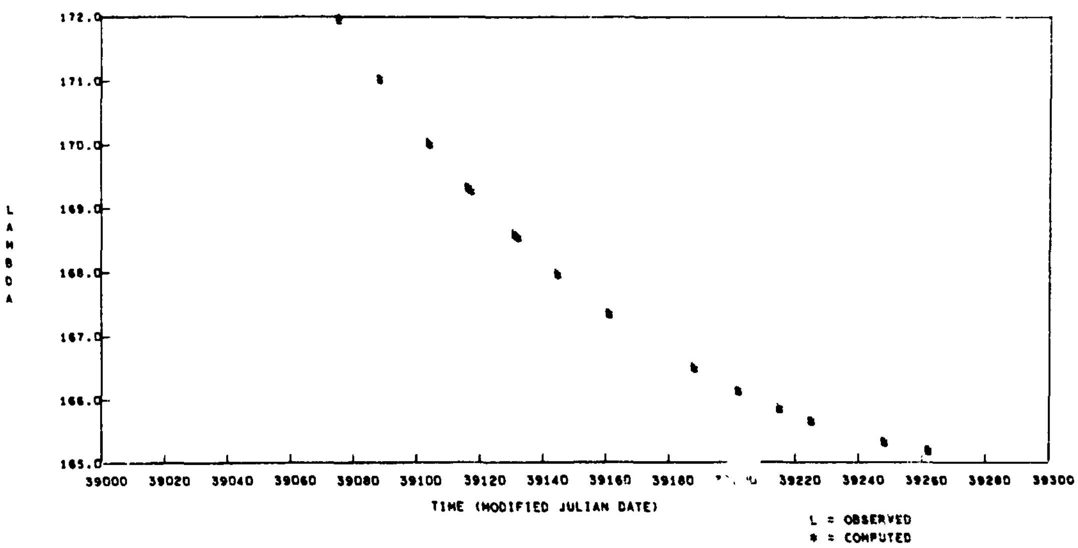


(c) SYNCOM 2, 2 Data

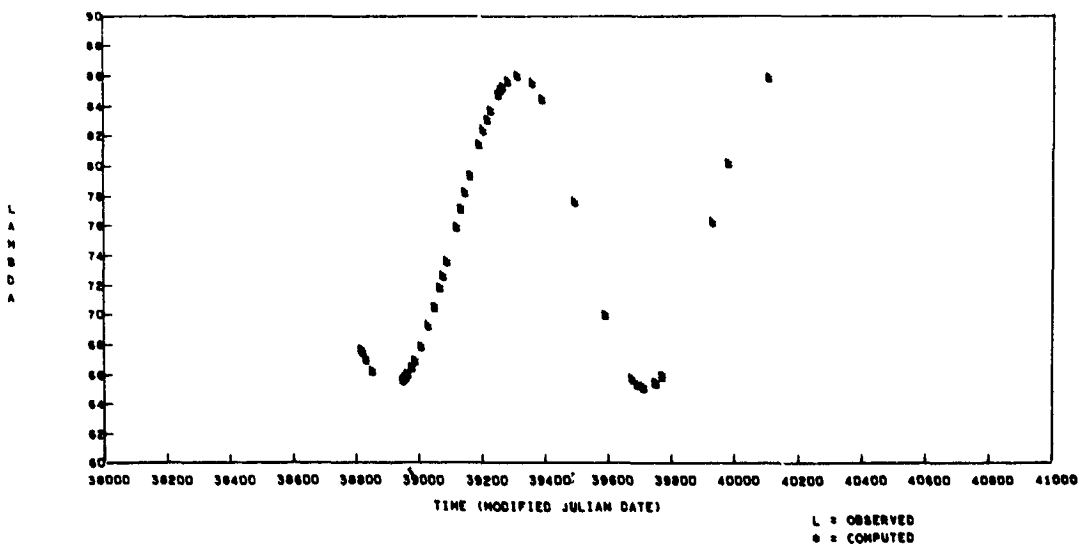


(d) SYNCOM 2, 8 GSFC Data

Figure 2 (Continued). Observed and Computed Measurements - Lambda

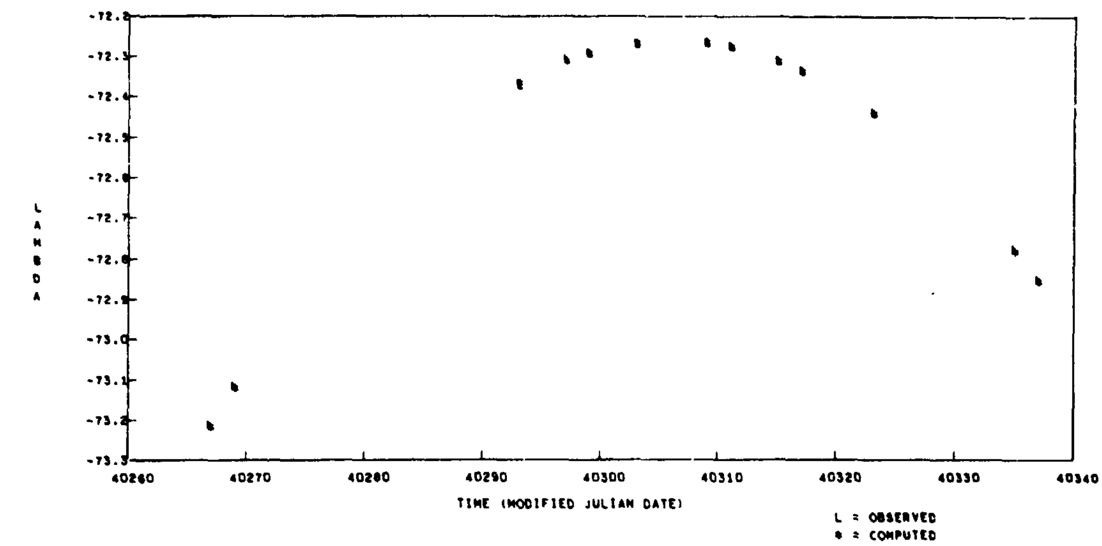


(e) SYNCOM 3, 11 DoD Data

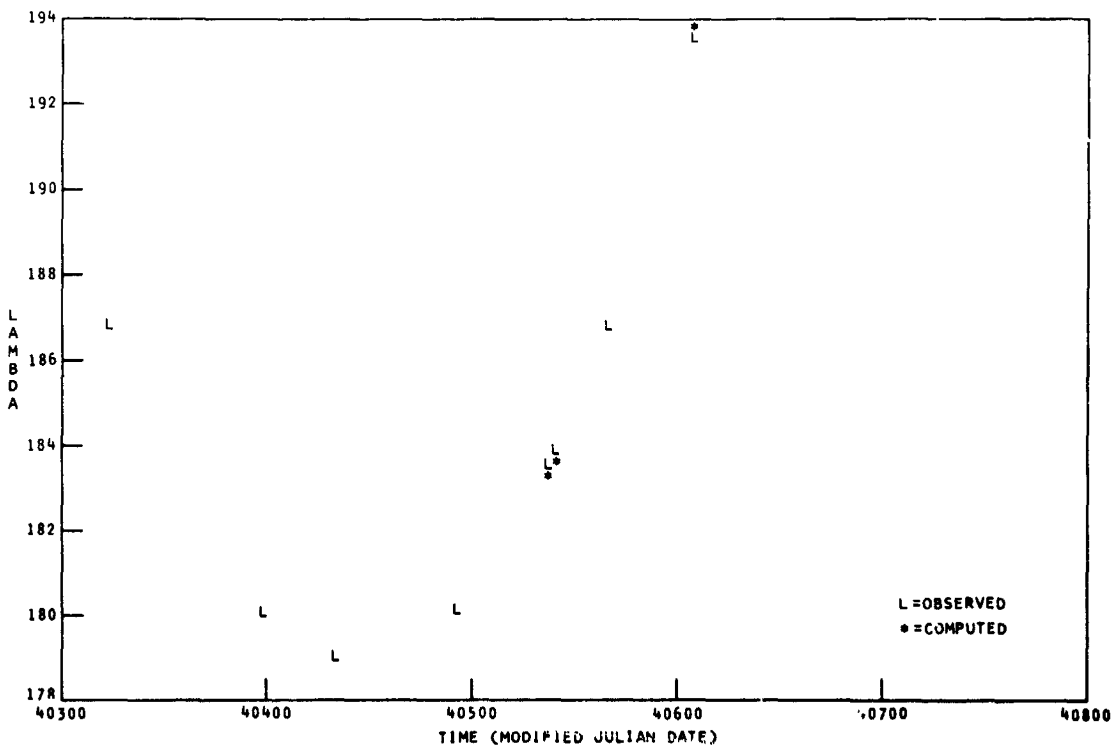


(f) SYNCOM 2, DoD Data

Figure 2 (Continued). Observed and Computed Measurements - Lambda

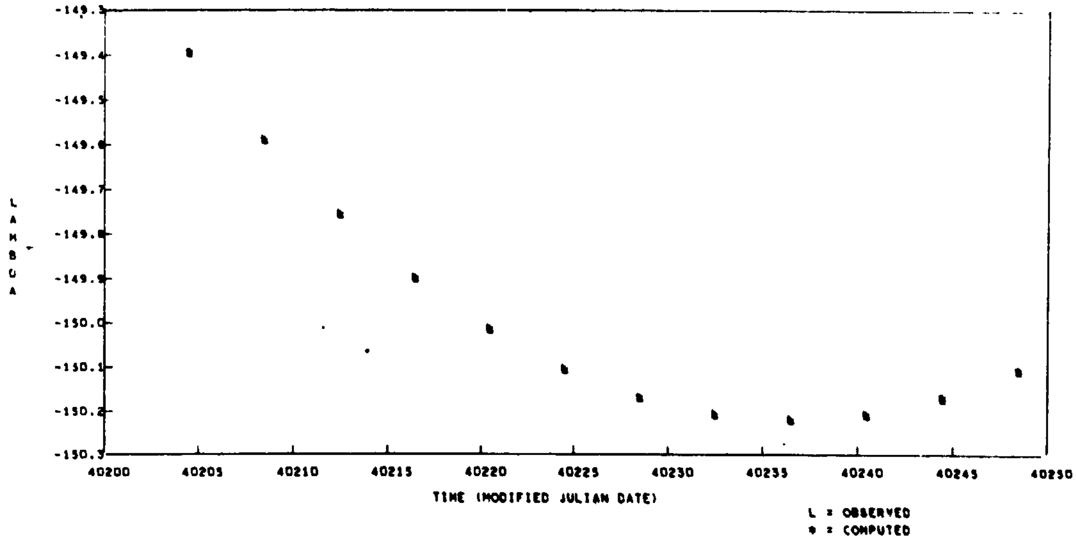


(g) ATS 3 Data

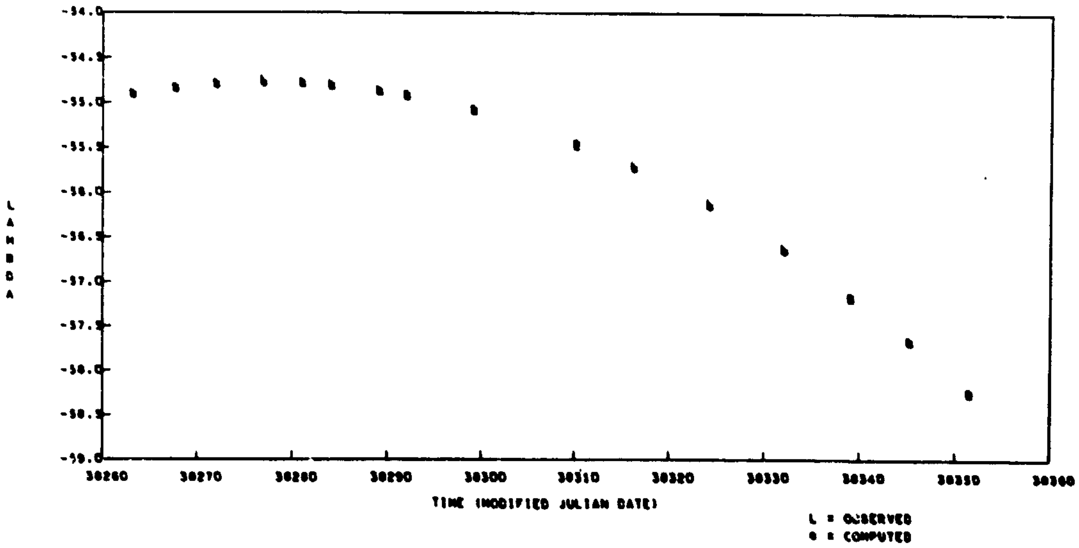


(h) SYNCOM 3, 14 Data

Figure 2 (Continued). Observed and Computed Measurements - Lambda



(i) ATS 1 Data



(j) SYNCOM 2, 1 Data

Figure 2 (Continued). Observed and Computed Measurements - Lambda