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ADVANCED FEED SYSTEM
COMPONENT INTERACTION ANALYSIS

NAS7-749

FINAL REPORT



Rocketdyne
North American Rockwell

6633 Canoga Avenue
Canoga Park, California 91304



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PREPARED BY

Rocketdyne Engineering
Dynamics Unit
Canoga Park, California

APPROVED BY

J. Friedman
Program Manager

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FOREWORD

This is the final report in compliance with Article II, paragraph D, of Contract NAS7-749. This work was carried out at the Rocketdyne, Canoga Park Facility during the period from 30 June 1969 to 30 June 1970. The objective of this work was to mathematically model the dynamic operation of the pressurization system and the pneumatic pressure regulator in various small propulsion systems and in test systems. Through the use of this model, the interactions between the regulator and the feed system are predicted. The contract was issued by the National Aeronautics and Space Administration, NASA, Pasadena Office, and was administered under the technical direction of Mr. Hartwell Long of Jet Propulsion Laboratory. The program manager was Mr. J. Friedman of the Rocketdyne Small Engine Division, and the principal engineer was Mr. L. E. Sack of the Dynamics Unit, Design and Development Engineering, Rocketdyne. Contributions of Mr. A. A. Liebman and Mr. E. F. Sowell, also of the Rocketdyne Dynamics Unit, are acknowledged.

ABSTRACT

A generalized method for the dynamic modeling of the pressurization system and the pneumatic pressure regulator in a pressure fed liquid propulsion system is developed herein. The entire pressurization system is represented by a series of compressible volumes with intermediate restrictions representing valves, orifices, regulators and other pressurization system components. The fluid, thermal and mechanical equations describing the physical system are developed. The system equations are normalized and scaled for solution on the analog computer. The normalization method employed results in the system being completely determined by a set of dimensionless coefficients. Thus, once the physical model is established, the analog computer model represents not a single physical system but the family of all geometrically similar physical systems. A particular solution generated by this analog model thus is applicable to all physically similar systems. It is notable that by this method, system variables such as absolute pressure levels, and thrust levels cease to be important parameters.

The generalized modeling technique developed herein is applied to two specific systems for the purpose of verifying the techniques employed. The Mariner Mars '69 midcourse-correction propulsion system was selected as the first example case, being representative of mono-propellant propulsion systems in the 50 pound thrust class. The model was successful in duplicating system performance of the Mariner '69 system, as represented in both accumulator flow bench test results and water expulsion system test results. In particular, certain abnormal regulator performances were reflected in model results exactly as observed in the test results. As a direct outcome of these model studies with the Mariner '69 system, it was possible to make recommendations for improvement of this regulator design to

eliminate the abnormal performance characteristics of the regulator. The Mariner Mars '71 midcourse-correction propulsion system was selected as the second example case, being representative of bi-propellant pressure fed propulsion systems in the three hundred pound class. Again, model results were shown to agree closely with test results for this system.

The application of models developed according to the suggested techniques to the parametric study of future propulsion systems, with emphases on the pneumatic pressure regulator and pressurization systems, is also demonstrated herein. In these example applications, again, the Mariner Mars '69 and the Mariner Mars '71 midcourse propulsion systems were used as baseline cases. Important parameters which determine system dynamic performance were varied about this baseline case for each of these systems, over ranges sufficient to cover a range of possible geometrically-similar designs. It is anticipated that these parametric results will be helpful for designers of future propulsion systems and in selecting system parameters such as pressure losses and regulator characteristics. In particular, the importance of the slope of the regulator reference spring force versus deflection curve is shown. Another sensitive parameter revealed by the parametric studies is the pressure ratio between the regulator and the propellant tanks being pressurized. Both the spring slope and this downstream pressure can effect the dynamic stability of the pressurization system.

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INTRODUCTION

SYSTEM DESCRIPTION

This study is concerned with propulsion systems similar to the one shown in Fig. (1). The upper portion of this schematic shows the pressurization system, consisting of the high pressure storage tank, some type of an isolation valve, filters, the pressure regulator, and necessary fittings to connect the gas flow system to the propellant tank. The pressurization system provides a flow of pressurizing gas to the propellant tank, or tanks, depending upon whether it is a monopropellant or bipropellant system. The liquid flow branch or branches of the system is of secondary importance here, and need be considered only to the extent necessary to provide a complete system in which to analyze the pressurization system. Typically, the liquid flow portion consists of propellant isolation valves, filters and orifices and finally the thrust chamber, in which combustion or reaction takes place.

The purpose of this entire system on a space craft is to provide a predictable thrust over the specified period of time. In order to achieve the desired predictability in system thrust, these systems are typically designed to operate at a constant chamber pressure throughout the burn period. This is achieved by insuring a constant pressure in the propellant tank during system operation. The propellant tank pressure is controlled by the pressurization system, and in particular by the pressure regulator. The pressure regulator is designed so as to regulate the pressurization gas flow in such a way as to maintain the desired pressure in the flow system downstream of the regulator. It must do this in the presence of a constantly-decaying pressure level at the input, due to outflow from the storage tank.

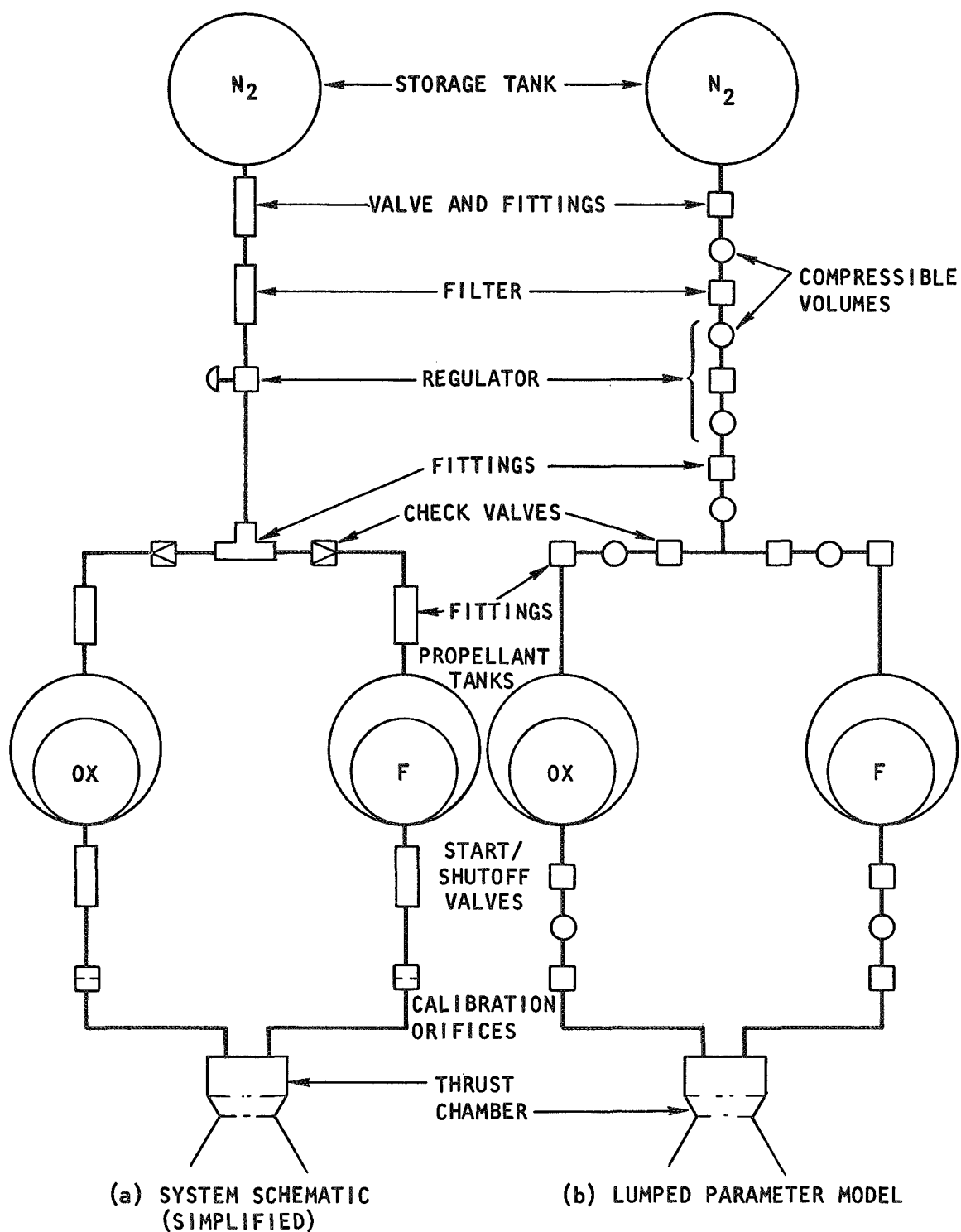


Figure 1. Typical Bipropellant Pressure-Fed Propulsion System

SYSTEM OPERATION

Systems such as shown in Fig. (1) have two easily distinguishable modes of operation. One of these modes may be called the pressurization mode, in which the propellant tank is brought from its initial low pressure level to the nominal design pressure level. This mode is characterized by higher-than-nominal gas flow rates, and the regulator is generally open further than is necessary at the nominal operating level. The second mode of operation is that during which the regulator is maintaining the tank pressure at the design level. This may be considered to be a quasi-steady state mode in that, at least for an ideal pressure regulator, the system downstream pressures remain constant. During this mode the entire system operates closest to its nominal conditions, and thus thrust is most accurately maintained. Certain non-ideal characteristics of the regulator (which will be discussed later) result in slight deviations of system pressures from their nominal value during this mode.

During the second mode the pressure at points upstream of the pressure regulator are constantly falling, making it necessary for the regulator to open wider and wider to accommodate the necessary flow to maintain the propellant tank pressure at its desired value. If the pressurization storage tank is sufficiently large, this second mode continues until all propellant is expelled from the propellant tank or tanks. A smaller pressurant storage tank introduces the possibility of a third mode. As the upstream pressure continues to fall, the regulator in this case reaches its maximum flow area, while attempting to maintain the desired propellant tank pressure. As the upstream pressure drops beyond this point, it becomes impossible for the regulator to maintain the required flow and thus the downstream pressure begins to fall below

the desired regulated values. The point at which the regulator reaches its maximum flow area may be referred to as the breakpoint, and the third mode occurring thereafter is essentially an uncontrolled blow-down mode.

OBJECTIVES OF INVESTIGATION

Figure 1-b is a model of a typical system which accounts for compressibility of the pressurization fluid. The flow system is divided at points of local restriction, and the volumes of the flow system between these points of restriction are represented by localized lumped compressible volumes. The pressure within each of these compressible volumes is designated P_i and the incoming flowrate is designated $\dot{w}_i$. This model lends itself to mathematical analysis.

With this model it is immediately recognized that such a system has many degrees of freedom. That is, there is not a single flowrate at any instant in time in the system; rather, the flowrate through each point of restriction is an independent, time variable. This large number of degrees of freedom makes it possible for there to be an equal number of modes of dynamic operation, some of which are desirable and others perhaps not. That is, there exists the possibility of undesirable dynamic interactions between the system components, such as the regulator and the propellant tank ullage volume. It is the objective of this investigation to develop a generalized technique of analysis for determining such undesirable interaction of system components in advance of hardware development for future propulsion systems. The underlying motivation is that such analyses may make it possible to select system design parameters so as to yield the desired dynamic performance of the pressurization system, and to avoid unexpected interaction phenomena.

METHOD OF INVESTIGATION

There are many possible methods of carrying out such analyses. Among these methods are simplified linear analyses, simplified non-linear analyses and complete simulation of the entire system. Each of these analytical methods has certain desirable features, certain undesirable features, certain areas of application and certain areas where they would not produce the desired results. Simplified linear analyses are often attractive because of their inherent ease, and because of the large body of existing techniques in classical control system analysis (Ref. 1). However, the simplified linear analysis will not always produce the desired results since a) the linearization makes the analysis valid only for small amplitude disturbances, b) a single operating point of the system is assumed at which the linearization is carried out. If there is interest in other operating points of the system, separate analysis must be done for each point, c) the simplified linear analysis, as in any simplified analysis, creates the possibility of omitting a significant factor due to lack of initial insight into the system characteristics.

Simplified non-linear analysis avoids the limitation of small amplitude disturbances. Also, there is a large body of classical control system analysis techniques for non-linear systems of certain characteristics (Ref. 2). However, the method still has the limitation of being applicable only to a single operating point and again leaves the possibility of omitting significant factors.

The method of analysis selected for this investigation is a simulation of the entire system. Since the equations are not linearized in this method, there is no limitation on amplitude. The analysis encompasses the entire range of system operation; furthermore, many factors

commonly omitted in simplified analyses are included. It must be noted, however, that any analytical model is to some degree a departure from reality, and therefore there still exists the possibility of omitting some significant factor, although this possibility is lessened in this technique as compared to simplified analyses. Also, it must be pointed out that this inclusion of additional detail can possibly obscure certain causative factors, sometimes making it difficult to interpret results. However, a complete system simulation will in most cases reveal much more information about the dynamic performance of the system than either of the simplified analyses.

The simulation approach employed in this investigation entails definition of a generalized system in terms of typical components, development of the equations of these components and development of an analog model representing the simultaneous solution of these equations.

Since any analytical model is a departure from reality, and there exists the possibility of error due to omission of significant factors, it is always necessary at some point to compare the results of the model with test data. However, this need not be done for every analysis. Confidence in the method of analysis and techniques employed can be obtained by the careful analysis and subsequent comparison with actual test data for a particular system. Then future systems may be analyzed employing the same techniques of analysis and these analyses will produce similar levels of confidence in the results obtained. In this investigation, this is done by performing an analysis of two specific propulsion systems, the Mariner '69 midcourse-correction propulsion system and the Mariner '71 midcourse-correction propulsion system. The results of models of these two systems are compared to test data obtained from test systems representing these two propulsion systems.

Once the analytical techniques and methods of analysis of systems such as shown in Fig. (1), have been established, they will have many applications. One possible application is the use of such a model of a particular system to carry out parametric investigations during the preliminary design phase of a planned propulsion system. Such parametric investigations will serve to identify sensitive system parameters, to show the effects of these sensitive parameters on system dynamic operation, and to permit tradeoffs among conflicting system requirements. This particular application is demonstrated herein. Other applications are suggested or may occur to the reader.

DEVELOPMENT OF THE GENERALIZED MODEL

In this section a system of subscripting and schematic representation is developed which makes it possible to represent a large number of pressure-fed propulsion systems in a uniform and generalized way. First, it is shown that these propulsion systems can be described in terms of composite parts connected in a way indicated by the system schematic for any particular system. Then the fluid, thermal and mechanical equations which describe the physical operation of each of these components are developed. This set of equations, repeated as often as is indicated by the schematic of a particular system, constitutes the mathematical model of the generalized pressure-fed propulsion system. These general equations are then normalized and scaled for solution by means of an analog computer. It is shown that the process of normalization results in the definition of certain dimensionless parameters which characterize the system.

SYSTEM DESCRIPTION

Figure (1) indicates how a complete system represented by a schematic can be divided in such a way that it may be described by the serial connection of a number of discrete parts. Furthermore, many of these parts have similarities with respect to the governing equations. The two major classifications of these components are a) tanks and other compressible volumes, and b) orifices and other types of flow resistance elements.

It is convenient to distinguish among three different types of flow resistance elements. One type is the fixed-area or scheduled-area orifice-type restriction. In this category fits valves, fittings, calibration orifices, and other elements of this general type. A different kind of flow element often found in pressurization systems is the laminar filter. Due to the small flow areas necessary for the filter to perform its function in the system, the flow is laminar, and therefore the pressure loss through these devices is a linear function of flow, and varies inversely with the fluid density. This is in contrast to the orifice-type flow device, which obeys different laws. Another type of flow device which is basically different than either of those mentioned previously is the pressure regulator. This device is different in that the flow area is neither constant nor a predetermined function of time, but is a function of local flow variables such as pressure.

Figure (2) shows a set of generalized system elements. Any complete system such as shown in Figure (1) can be represented by the proper interconnection in series of elements as indicated in Figure (2). Two basic elements shown in Figure (2) are the lumped compressible volumes and the localized restrictions.

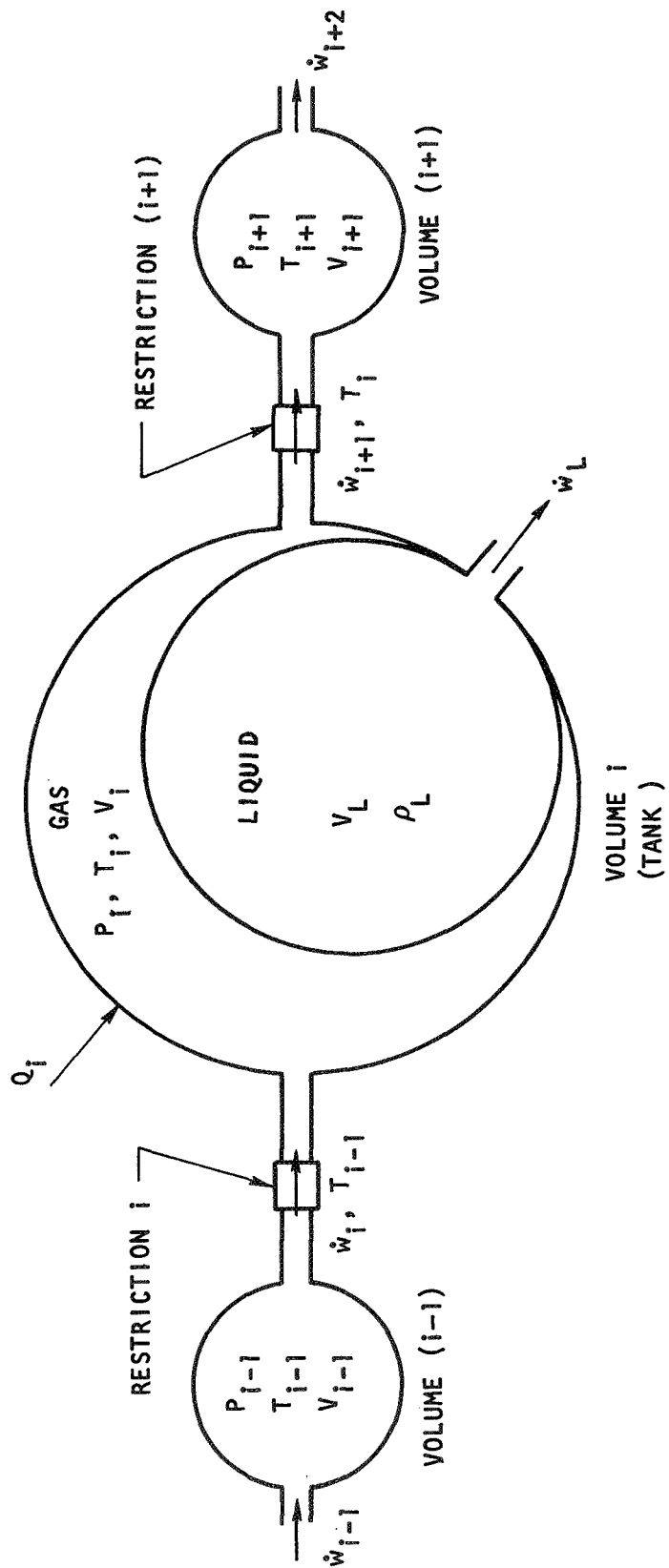


Figure 2. Schematic Showing Generalized Tank (Compressible Volume) and Subscript Scheme.

A convenient subscripting system is necessary to facilitate analyses. In order that this method be applicable to any system, it is best to use a subscript i with a range determined by the particular system. The convention adopted here, therefore, is that each compressible volume, whether it is a tank or simply the lumped volume of a length of the flow system, is designated by V_i . The smallest value of i is that corresponding to the pressurant storage tank, and the subscripts of volumes downstream of that tank have ascending values.

Certain system variables have values which can be associated with these local compressible volumes. As indicated in Fig. (2) these are; pressure, temperature, volume, and local heat flux across the boundary. These variables are all designated by the subscript i when referring to their local values in the vicinity of the i -th compressible volume. System flow rate differs from variables such as pressure and temperature in that it is a "through" variable. That is, the flow rate must be thought of as a quantity which occurs between two points in the system. In the convention indicated in Fig. (2) the flow rate is therefore associated with a particular restriction. Also by convention, the restriction immediately upstream of the i -th compressible volume is designated by the subscript i .

It will be noted in Fig. (2) that the central volume is composed of two regions; one occupied by the gas, and the other occupied by a liquid. This is the most general case of the localized volume and permits the representation of propellant tanks in a manner consistent with the established conventions. Thus, in the most general case, the localized volume has associated with it a value of liquid volume, liquid density, and liquid outflow rate, in addition to the previously mentioned variables of pressure, temperature and volume of the gaseous medium.

While the schematic shown in Fig. (2) is the most general case, in that it allows for the compressible volume with gas inflow, gas outflow, and liquid outflow, this particular combination may not occur in an actual system. A more likely arrangement is the common case where the gas flow system terminates at the propellant tank, in which case the last gaseous compressible volume has gas in-flow and liquid out-flow. This case is easily handled with the terminology established by Fig. (2) simply by eliminating the gas outflow system and appending the liquid flow system where the liquid outflow is indicated.

The possibility of a branching of the system to permit representation of a bipropellant system suggests a slight modification of the subscripting system, as indicated in Fig. (3). In the gaseous portion of the system below the branch point, the subscript consists of two elements. The first of these elements is simply a continuation of the numerical index established in the gaseous system upstream of the branch. The second subscript element is a letter designating the branch. For example, since the branch is generally due to the bipropellant system, the letter subscript can be an abbreviation for the particular propellant.

In order to avoid ambiguity in the subscripting systems for the propellant tanks themselves, it is necessary to establish a new index system for the liquid portion of the systems. The convention adopted herein is again a two part index in which the first part is a letter representing the particular propellant. The second part is a numerical index which increments in the downstream direction.

In order to complete the nomenclature and subscripting conventions, a generalized pressure regulator must also be described. The hypothetical regulator shown in Fig. (4) contains all of the elements accounted for

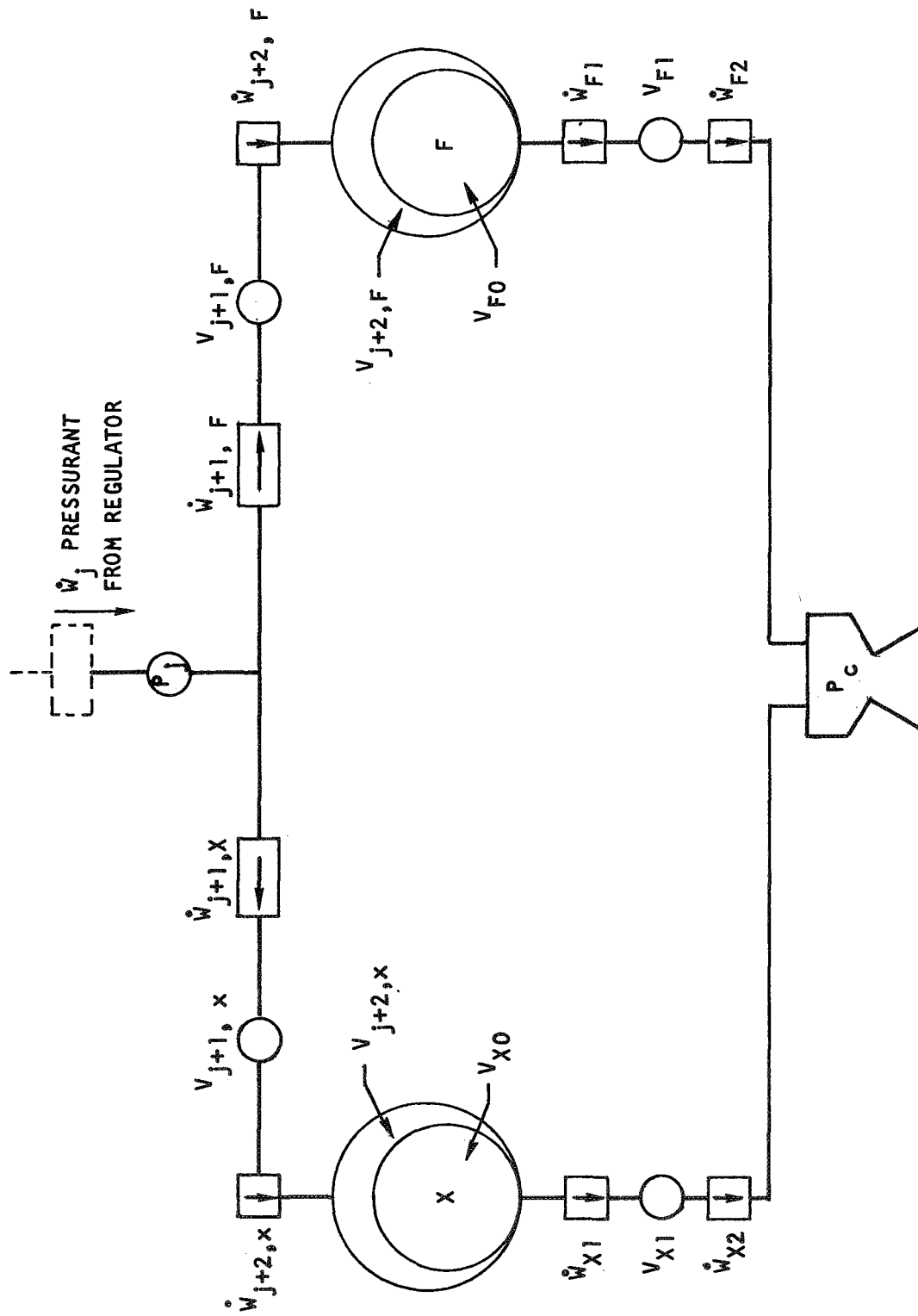


Figure 3. Subscripting Convention for Branching in Bipropellant System

in regulators to be considered in this investigation. Perhaps the most distinguishing feature of this regulator is that it is a single-stage design. That is, it has a single flow control element, namely the seat, and a single pressure sensing element which of course may be either a bellows, a diaphragm, a piston or any other device which allows a change in position in response to a pressure. In this regulator, the flow enters at the left, passes through a filter device and then through the seat at which flow regulation occurs, and then to a cavity downstream of the seat. The nomenclature convention employed herein for this flow path is a simple continuation of the numerical indexing technique established above. If the subscript for the compressible volume immediately upstream of the regulator is $k-2$ then the pressure between the filter and the seat is P_{k-1} , the pressure downstream of the seat is P_k , and the pressure in the compressible volume immediately downstream of the regulator is P_{k+1} . In some cases, the pressure sensing element is isolated from the flow by means of a special cavity, connected to the downstream pressure P_k by means of a small orifice. This cavity, in which the pressure sensing element operates, is designated by the subscript r , and is called the reference pressure cavity. The pressure on the opposite side of the pressure sensing element is usually ambient, since this region is usually vented. This pressure is therefore designated by the subscript a .

It will be noted that the hypothetical regulator shown in Fig. (4) is characterized by a radial flow filter at the inlet, a lever arm between the ball actuator and the pressure sensing element and poppet, and by a helical reference spring. The analytical techniques developed herein are by no means restricted to regulator components such as these. The equations developed will be equally applicable to any type of inlet filter employed, to designs without a mechanical ratio between the

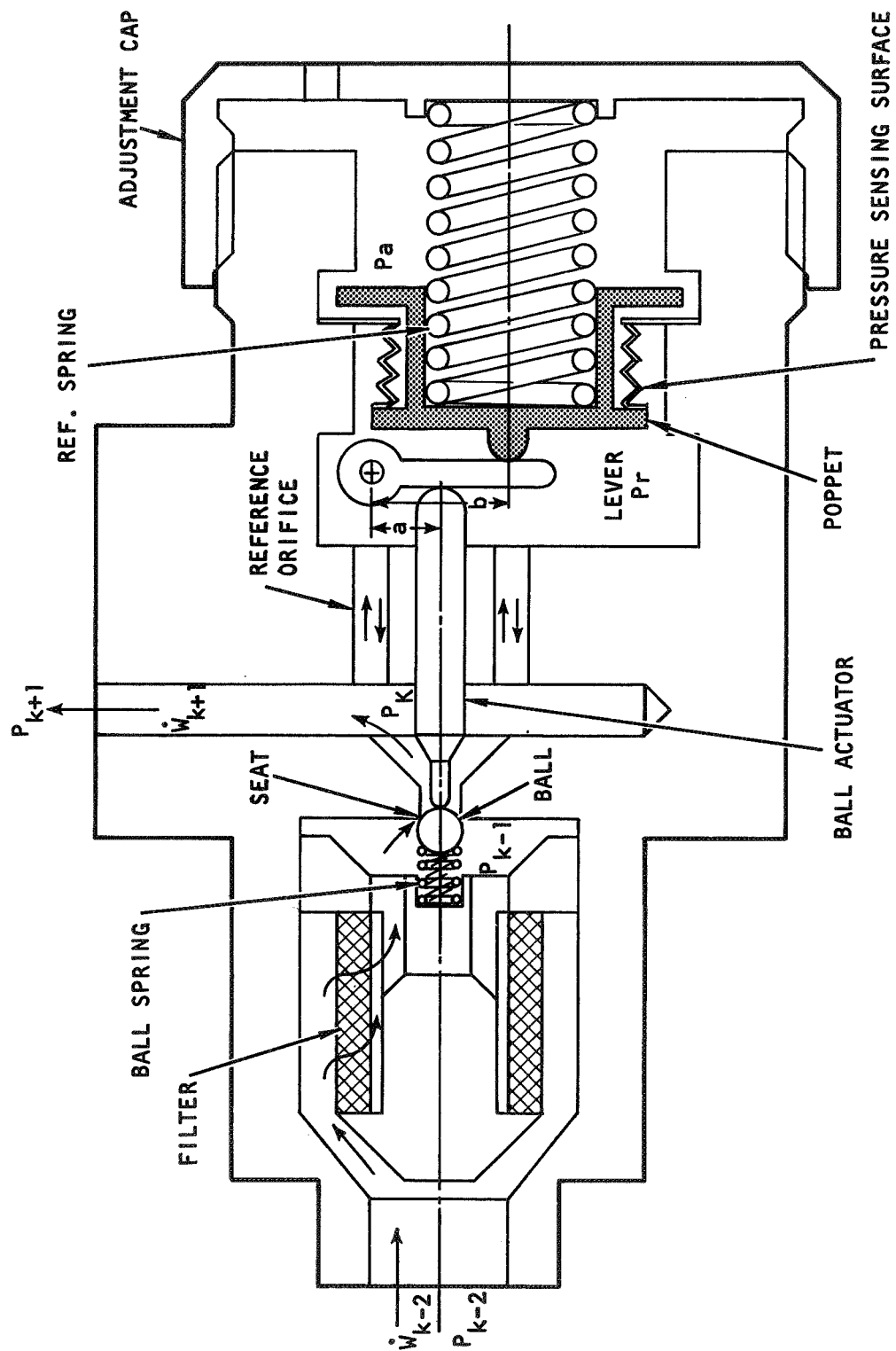


Figure 4. Hypothetical Pressure Regulator

poppet and the ball actuator, and to regulators with different kinds of springs and different kinds of pressure sensing elements. In particular, note that a regulator without a lever is in every way equivalent to a regulator with a lever having a mechanical advantage of one.

The generalized system is completed by the thrust chamber. This is a device which accepts the liquid flow and produces thrust. While this is of course a very important element of the propulsion system, it is of only secondary importance in the analysis of the pressurization system. Insofar as the analysis of the pressurization system is concerned, its only function is to provide a realistic loading of the pressurization system. That is, the thrust chamber serves to establish downstream pressure as a function of liquid flow rate. In the case of a monopropellant system, this pressure is a function of a single flow rate, whereas for a bipropellant system it is a function of both the oxidizer and the fuel flow rate. Both of these cases are considered herein. The pressure, temperature and volume, and any other local variables associated with the thrust chamber, are designated simply by the subscript c.

The components described above, and the nomenclature and indexing system presented, are sufficient to describe any simple pressure-fed propulsion system to the degree necessary to analyze the pressurization system, and in particular the pressure regulator. These elements may be connected in any order whatever corresponding to a particular system schematic. Note that test systems can also be represented. This can generally be done simply by the omission of certain elements, in particular, the thrust chamber, in which case the model represented might be that of a water expulsion test system. A flow bench accumulator test system

could be represented by simply eliminating the liquid outflow and downstream liquid flow system from the propellant tank, and instead, representing a downstream system composed of gas flow elements.

SYSTEM EQUATIONS

In this section, equations are developed for each of the system components described above. The fluid, thermal and mechanical equations of operation are developed for tanks and other compressible volumes, flow restriction devices, and the mechanical elements of the regulator. The equations describing the operation of monopropellant and bipropellant thrust chambers are also developed. All equations developed in this section are summarized in Table I.

Tanks and Other Compressible Volumes

Figure (2) shows a typical tank with gas flow in and out, liquid outflow and heat addition. Application of mass conservation, energy conservation, and the equation of state relationships will yield the desired thermodynamic conditions within such a tank at any instant.

By conservation of mass for the gas system,

(1)

$$\frac{dW_i}{dt} = \dot{W}_i - \dot{W}_{i+1}$$

where W_i is the stored weight, and $\dot{W}_i$ and $\dot{W}_{i+1}$ are gas inflow and outflow respectively. Integration of this equation yields the instantaneous stored weight of gas in the i -th tank or compressible volume.

By conservation of energy with respect to the gas system in this tank (Ref. 3), neglecting potential and kinetic energy,

$$\frac{d}{dt} [U_{TOT}] = -\Delta [H\dot{W}] + Q_i - WK_i \quad (2)$$

where U_{TOT} is the total internal energy of the gas mass. H represents the enthalpy per unit weight, and the Δ represents the change from inlet to outlet. Q_i is heat added, including convection from the walls or liquid interface, or mass transfer from the liquid interface. WK_i is the external work, i.e., the work done by the gas on the liquid surface.

The external work is

$$WK_i = P_i \frac{dV_i}{dt} = P_i \dot{w}_{Li} / \rho_L \quad (3)$$

where it is noted that the rate of change of gas volume, V_i , is simply the negative of the rate of change of liquid volume.

Substituting Eq. (3) into (2) and applying the relationships

$$U_{TOT} = C_v W T$$

$$H = C_p T$$

$$\gamma = C_p / C_v$$

it is found that,

$$\frac{dT_i}{dt} = \frac{1}{W_i} [(\gamma T_{i-1} - T_i) \dot{w}_i - (\gamma - 1) T_i \dot{w}_{i+1} - P_i \dot{w}_L / \rho_L C_v + Q_i / C_v] \quad (4)$$

This equation can be integrated to yield the instantaneous temperature of the gas within the i-th tank. Note that the absence of liquid outflow or inflow, or heat addition, can be represented simply by striking out the associated term in Eq. (4).

The instantaneous volume occupied by the gas is found from

$$\frac{dV_i}{dt} = \dot{w}_L / \rho_L \quad (5)$$

Equations (1) and (5) are sufficient to determine the density of the gas in the tank, which is a thermodynamic property. A second thermodynamic property, temperature, is obtained from Eq. (4). Assuming a single phase, these two properties are sufficient to determine all other properties, specifically pressure. That is

$$P_i = P_i(W_i/V_i, T_i) \quad (6)$$

which is an equation of state.

If it is assumed at this point that the gas is at a temperature sufficiently above its critical temperature, the equation of state can be approximated by an ideal gas form. That is

$$P_i = Z \frac{W_i}{V_i} R T_i \quad (7)$$

where Z is a compressibility factor. The compressibility factor can be taken as constant for a sufficiently small change in temperature and pressure. The validity of this approximation must be examined in each case, making reference to reduced properties charts, or other representations of actual gas behavior. The alternative would be to include a curve-fit expression for the equation of state. The remainder of this development will employ Eq. (7).

Gas-Flow Device Equations

Many flow devices can be represented by the compressible-flow orifice equation. A convenient form of this equation is, for the flow rate through the i -th device

$$\dot{W}_i = C_i P_{i-1} A_i \Phi (P_i/P_{i-1})/\sqrt{T_{i-1}} \quad (8)$$

where Φ is the compressible flow function, Fig. (5). Letting the pressure ratio P_i/P_{i-1} be α , Φ can be expressed as,

$$\left. \begin{aligned} \Phi &= \left[\frac{2\gamma g_c}{R(\gamma-1)} \left(\alpha^{2/\gamma} - \alpha^{\gamma+1/\gamma} \right) \right]^{1/2} ; \alpha \geq \alpha_c \\ \Phi &= \Phi_{\max} = \Phi(\alpha_c) ; \alpha < \alpha_c \end{aligned} \right\} \quad (9)$$

$$\alpha_c = \left(\frac{2}{\gamma+1} \right)^{\gamma/\gamma-1} \quad (10)$$

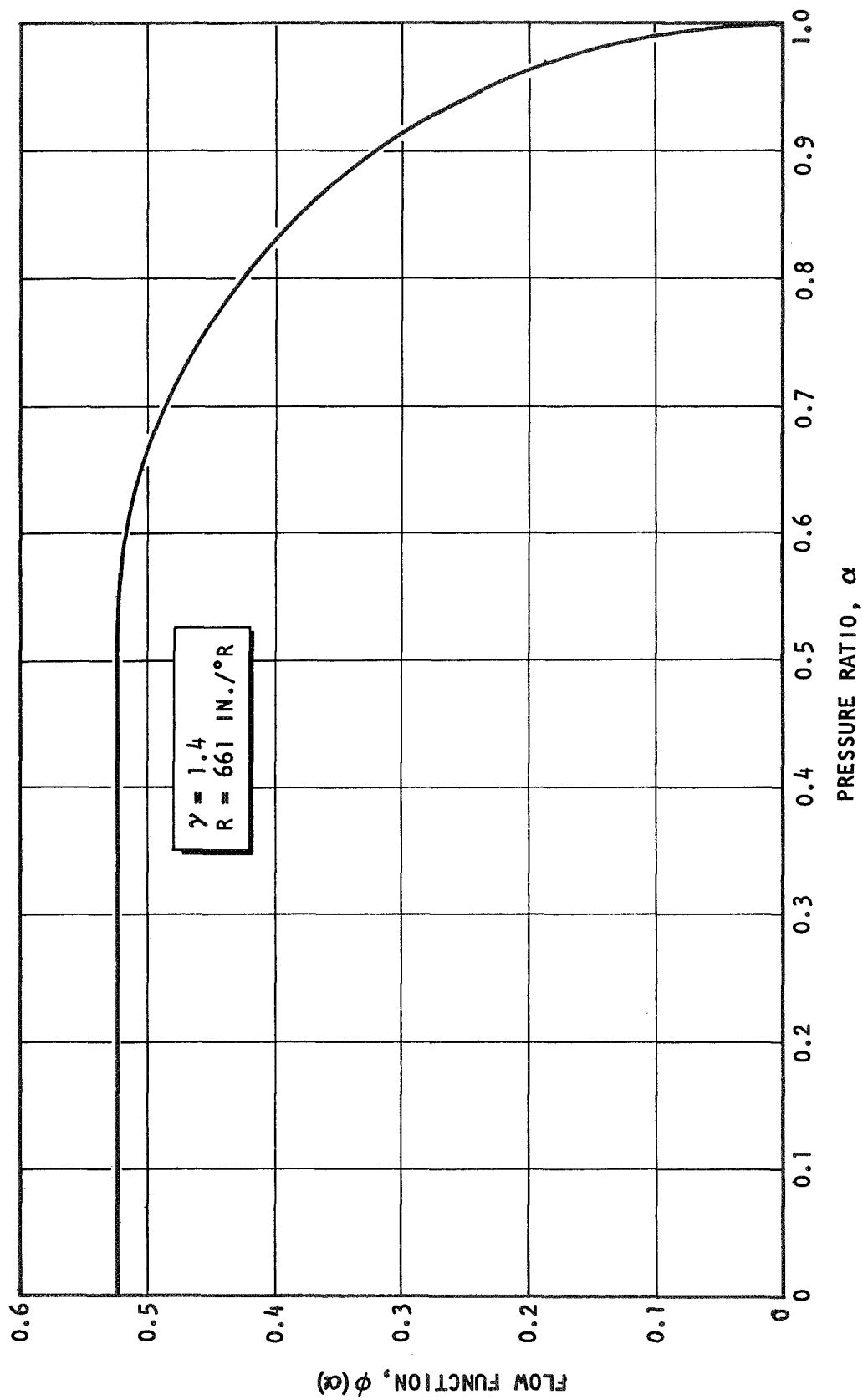


Figure 5. Compressible Flow Function

In this equation γ and R are the specific heat ratio and gas constant for the gas being used in the system.

Equation (9) can be represented within 2% by the curve-fit expression,

$$\begin{aligned}\Phi &\cong 2\Phi_{\max} (\alpha(1-\alpha))^{1/2} ; \alpha \geq .5 \\ &= \Phi_{\max} ; \alpha < .5\end{aligned}\quad (11)$$

The value of $\Phi_{\max}$ can be determined by combining Eqs. (9) and (10). This gives

$$\Phi_{\max} = \left[\frac{\gamma g_c}{R} \right]^{1/2} \left[\frac{2}{\gamma+1} \right]^{1/2} (\gamma+1/\gamma-1) \quad (12)$$

$$g_c = 396.04 \text{ in./sec}^2$$

$$R = R_0/\text{MW in./}^\circ\text{R}$$

This is a function of γ , and is plotted in Fig. (6) for nitrogen ($R = 661 \text{ in./}^\circ\text{R}$).

Filters in the gas flow system are generally laminar flow devices, and therefore obey a linear flow-pressure loss relationship. This can be expressed as

$$\dot{w}_i = K_i \rho (P_{i-1} - P_i)$$

Assuming an ideal gas density

$$\rho = P/RT$$

results in

$$\dot{w}_i = K_i P_{i-1} (P_{i-1} - P_i) / RT_{i-1} \quad (13)$$

when K_i is a proportionality constant.

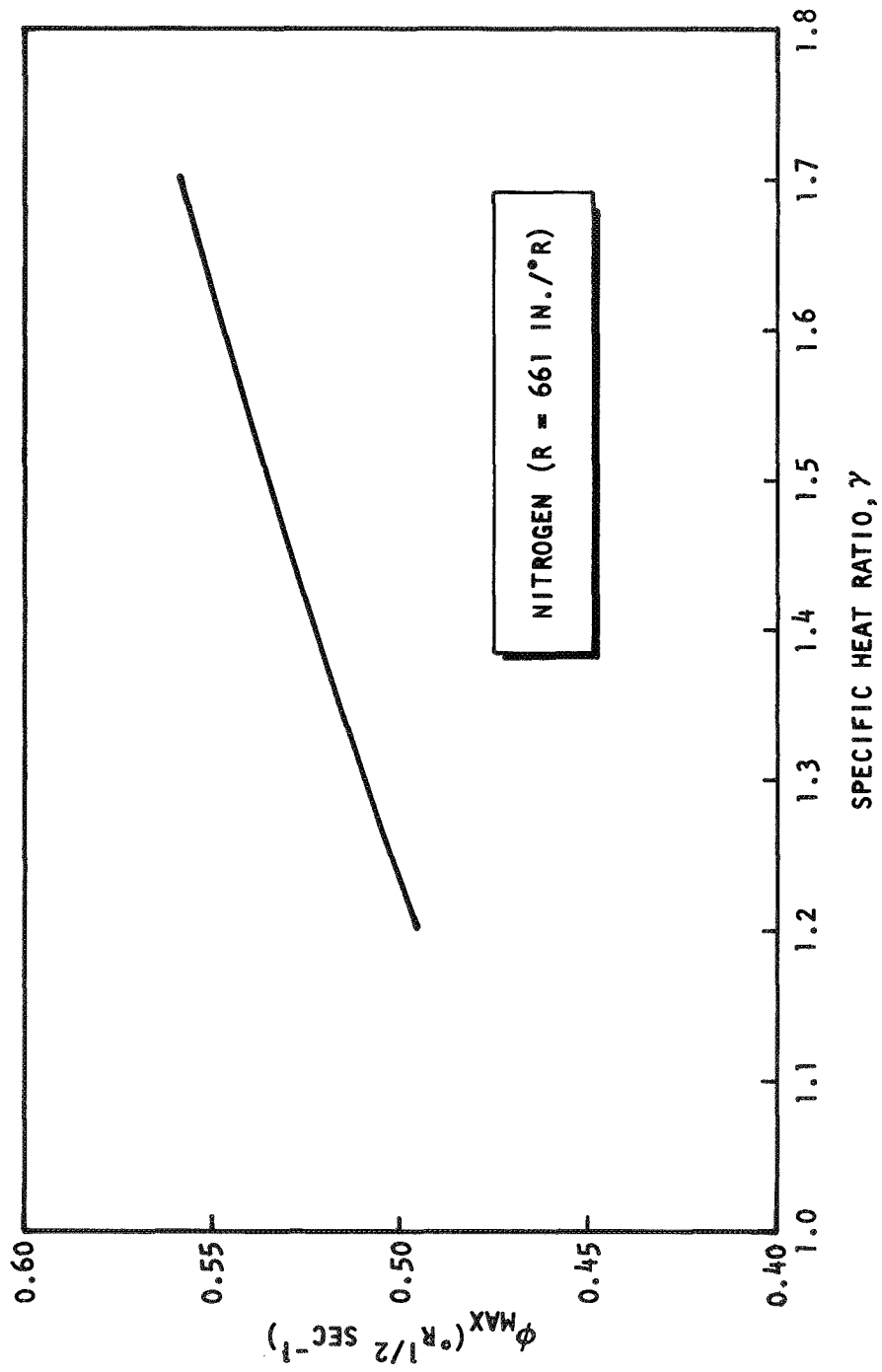


Figure 6. Variation of Compressible Flow Function Maximum With Specific Heat Ratio

Regulator Mechanical Equations

The gas flow rate through the pressure regulator can be determined by an equation such as Eq. (8). In this equation however, there appears the quantity A_i which is the flow area of the regulator, i.e., the area between the ball and the seat. By examination of Fig. (4), it is apparent that this seat area is a function of the position of the moving assembly within the regulator. Indeed, this is the way in which the regulator performs its function in the system, that is, the poppet assembly moves in response to the reference pressure thus modulating the seat flow area and hence the regulator flow. It is, therefore, apparent that the position of the moving parts within the regulator must be determined. This can be done by considering the moving mass within the regulator to be a rigid body and writing its equation of motion. It will be found that the position of the regulator and its velocity are unique functions of the initial conditions and the time-history of the sum of all forces acting on the moving assembly.

Examination of Fig. (4) shows that the moving mass consists of the poppet assembly, including some portion of the pressure sensing element or diaphragm, the mechanical linkage between the poppet and the ball actuator, the ball actuator itself, and the ball. A number of forces act at different points on this moving assembly. The most important forces are, of course, that force exerted by the reference spring, and that force opposing the reference spring force which is a result of the reference pressure acting over the effective area of the pressure sensing element. An ideal regulator would have only these forces acting on the moving assembly. However, in any regulator there are a number of other forces at play. All of these must be accounted for when writing the equation of motion on the moving assembly. For example, there are fictional forces at all points of sliding contact, such as

between the ball actuator and the housing. Also, there are pressure forces acting on the ball actuator due to the difference in pressure between the volume downstream of the seat and the reference cavity. There are also pressure forces acting on the ball due to the difference between ball upstream pressure and ball downstream pressure. Finally, the force exerted by the ball retaining spring also acts on the moving assembly.

The equation of motion of the regulator moving assembly can be written by equating the sum of all forces acting on the moving assembly to the mass-acceleration product. This is done in Eq. (14). In this equation it has been assumed that all elements of the moving assembly move in unison. That is, unless the ball is actually resting on the seat, the ball, actuator, lever and poppet assembly all move together. The implications of this assumption will be discussed later. With this assumption, there is a single position variable X , which is measured from the position at which the ball is seated.

$$m\ddot{X} = F_s - A_e (P_r - P_a) - F_{c1} \operatorname{sgn} \dot{X} - B_1 \dot{X} - \left(\frac{a}{b}\right) [F_{bs} + F_{pr} + F_{c2} \operatorname{sgn} \dot{X} + B_2 \dot{X}] U(X) \quad (14)$$

Equation (14) will now be discussed term by term. The first term on the right hand side, F_s , is the force exerted by the reference spring. As indicated by Eq. (15), this reference spring force can be a function of both poppet position and poppet velocity. The position dependent term is an expression of the spring characteristic curve. The velocity dependent term is a means of accounting for hysteresis of the reference spring.

$$F_s = F'_s(X) + F_h(\dot{X}) \quad (15)$$

The second term on the right hand side of Eq. (14) is the force exerted on the pressure sensing element. Here A_e is the effective area of the pressure sensing element. The third and fourth terms of Eq. (14) represent friction terms. The third term is Coulomb type friction, i.e., a frictional force which is always constant and acting in a direction opposite to the motion. The fourth term is a viscous friction term, i.e., ordinary damping.

The bracketed terms in Eq. (14) represent all forces which act on the floating portion of the assembly, that is, the ball actuator, ball, and lever arm. It will be noted that these forces are diminished by the mechanical ratio of the lever arm. Also, this entire bracketed expression is multiplied by the unit step function as defined in Eq. (16).

$$\begin{aligned} U(X) &= 1, X > 0 \\ &= 0, X \leq 0 \end{aligned} \quad (16)$$

This has the effect of applying these forces when the poppet is actually supporting the ball actuator and ball, i.e., in normal operation, and removing these forces when the ball comes to rest on the seat and the ball actuator and poppet assembly are further retracted. The first term within the bracket, F_{bs} , is the ball spring force. Due to the usual small travel of the ball, it is possible to represent this ball spring force by a linear equation as expressed in Eq. (17).

$$F_{bs} = F_{bso} + k_{bs} X \quad (17)$$

The second term in the bracket F_{pr} represents pressure forces which act upon the ball and upon the actuator, as indicated by Eq. (18).

$$F_{pr} = A_b (P_{k-1} - P_k) + A_p (P_k - P_r) \quad (18)$$

The third and fourth terms within the bracket are friction terms which are completely analogous to those discussed previously.

The equation of motion as indicated in Eq. (14) considers the entire moving assembly to be a rigid body except when the poppet is retracted past the point where the ball rests on its seat. This is usually a very good assumption, since common design practice must be to make the ball-retaining spring of sufficient strength to always maintain the ball in hard-contact with the ball actuator. A regulator designed otherwise would give unsatisfactory performance, due to the ball floating, i.e., assuming a position bearing no relation to the pressure sensing element. It will be noted that the ball pressure forces, during most operational conditions, also tend to hold the ball firmly against the ball actuator.

The reference spring force, F_s , as a function of the poppet position and velocity, is among the most important parameters of a pressure regulator. It is therefore important to give special attention to the representation of this force. One method of representing this force is as indicated in Eq. (15), where it is seen that the force is divided into two additive components, one representing the function of position and the other representing the function of velocity. The function of position is simply the force-deflection characteristic of the spring, which can be quite non-linear as is the case of a Bellville-type reference spring, or can be linear. For any linear spring, or for any small motion in the vicinity of linear regions of a non-linear spring, the function of position can be represented as indicated in Eq. (19). The velocity dependent term F_h in Eq.(15) is included to account for internal energy losses within the spring, commonly referred to as hysteresis losses.

$$F'_s = F'_{s0} - k_s X \quad (19)$$

A typical spring characteristic curve showing the effect of hysteresis is shown in Fig. (7). Several things may be observed from curves of this type. First, it is notable that a closed figure is defined. The area within this figure represents a conversion of mechanical energy to heat, strain, or some other form of energy. That is, not all work done in the compression of the spring is recovered during spring expansion. It is also observed that at the points of maximum compression, i.e., where expansion begins, the exerted force drops to the lower return curve. It is this characteristic that supports the assumption of the hysteresis force as a function of velocity, since obviously the force exerted by the reference spring changes as the velocity changes sign. For this reason the characteristic curve, as shown in Fig. (7), is conceived as the summation of a position dependent term and a velocity dependent term as indicated in Eq. (15).

The position dependent term in Eq. (15) may be thought of as simply the average of the compression and expansion force values at each point of deflection. Then the hysteresis induced force is simply a quantity to be added during compression of the spring and subtracted during expansion of the spring. Figure (8) shows a possible form for the hysteresis portion of the reference spring force. The exact form of such a curve could be determined by experiments in which the force exerted by the spring during repeated expansion and compression was plotted as a function of the velocity of the head of the spring testing machine. In the absence of data from such an experiment, it is only possible to somehow approximate this hysteresis curve, such as by a linear function.

It is notable that with this representation the hysteresis has the effect of adding damping to the system. That is, the velocity dependent term of the spring force equation is simply another linear term in $\dot{X}$, the coefficient of which is directly additive to other viscous damping of moving assembly.

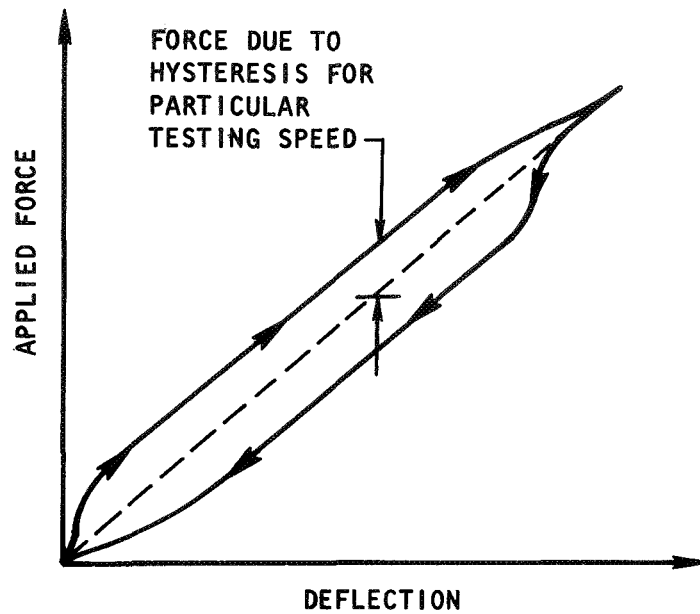


Figure 7. Typical Linear Spring Force vs Deflection Characteristic Showing Hysteresis Effect

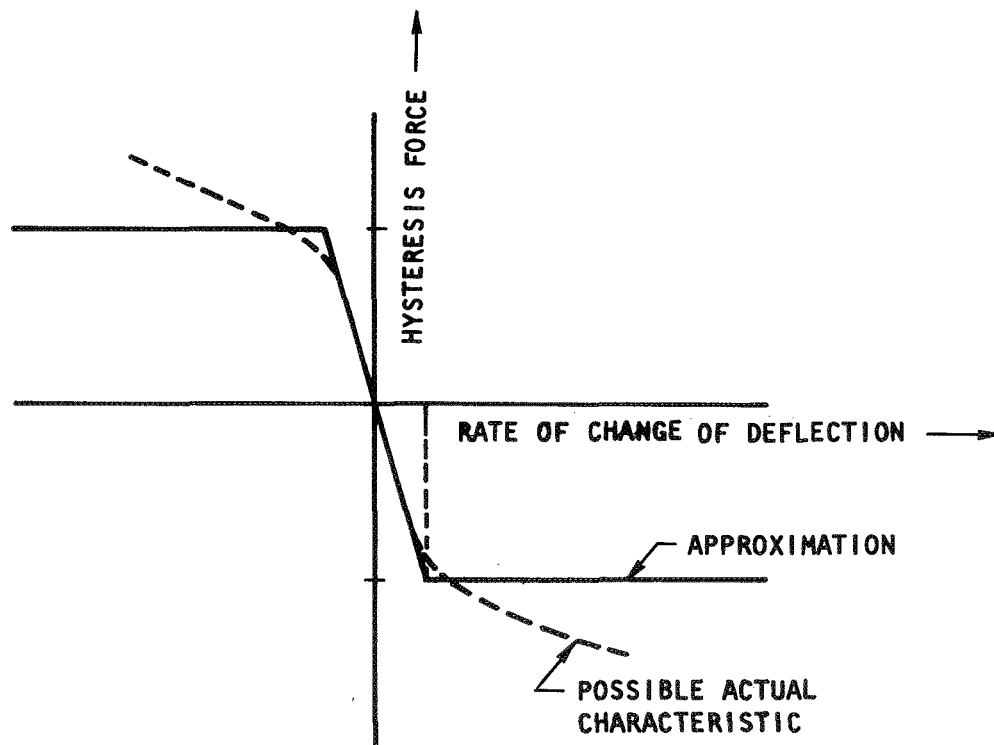


Figure 8. Portion of Reference Spring Force Due to Velocity (Hysteresis Effect)

It will be noted that the effect of the mechanical linkage or lever arm in the regulator in Eq. (14) is simply a multiplying factor on the forces which act upon the portion of the moving assembly not rigidly attached to the poppet assembly. In regulators which do not have such a lever, Eq. (14) still applies and it is necessary to simply set the mechanical ratio of the lever equal to one.

Equation (14) is versatile enough to reflect most regulator designs (single state regulator). For example, in a particular regulator, certain terms may be thought insignificant, in which case the terms are simply eliminated from Eq. (14). As discussed above, any reference spring characteristic can be included. Also, extraneous pressure forces acting for example on the ball and the ball actuator can be accounted for by modifications to Eq. (18).

Regulator Reference Cavity Equations

The reference cavity greatly influences the dynamic operation of the pressure regulator. It is only through the pressure in this cavity that the regulator moving assembly responds. The pressure within this cavity is effected by the system pressure to which its connecting orifice is exposed, the size and geometry of this connecting orifice, and the motion of the diaphragm or pressure sensing element itself.

The pressure within the reference cavity can be determined from an equation such as Eq. (7). However, special conditions here indicate the use of a modified version of this equation. Unlike other compressible volumes within the system, this cavity does not have a thru-flow, and hence the temperature within the cavity may generally be considered constant and equal to the regulator housing temperature. Thus temperature ceases to be a significant variable in the pressure equation for the reference

cavity. However, the volume of this cavity is a variable, being linearly related to the position of the pressure sensing element. By considering temperature to be constant, and stored mass and cavity volume to be variables, Eq. (7) can be linearized to yield Eq. (20):

$$P_r = P_{rn} \left[\frac{W_r}{W_{rn}} + \frac{\Delta V_{rn}}{\Delta V_{rn}} \frac{\Delta X}{\Delta X_n} \right] \quad (20)$$

where

$$\Delta V_{rn} = A_e \Delta X_n \quad (21)$$

In this linearization, the pressure is the sum of its nominal value plus increments due to changes in the stored mass and changes in the poppet position. Equation (20) is derived from Eq. (7) by performing a Taylor series expansion and keeping only the linear terms. Here ΔX represent changes in the poppet position from its nominal value. Subscript n refers to the nominal values of each of these parameters. ΔV represents the maximum change in the volume of the reference cavity due to motion of the poppet, as indicated in Eq. (21). ΔX_n represents the difference between the maximum value of the poppet position and the nominal value of the poppet position. For convenience the change in poppet position has been normalized by ΔX_n .

In Eq. (20), W_r is simply the integral of the flow rate thru the orifice which connects the cavity to some point in the flow system. It is determined by integration of Eq. (1), recognizing that there is no outlet port, and that the flow through the inlet port is bidirectional. This flow rate could be determined by an equation such as Eq. (8).

However, by design, this orifice is generally sized such that the pressure ratio across the orifice is nearly one. That is, the pressure within the reference cavity is never largely different from the pressure outside of that sensing port. In this case a simple flow rate equation, as indicated in Eq. (22), can be employed.

$$\dot{w}_r = K_r \sqrt{|P_k - P_r|} \text{sgn}(P_k - P_r) \quad (22)$$

where

$$K_r = 2C_r A_r \phi_m \sqrt{P_{rn}/T_n} \quad (23)$$

In this equation, the compressibility of the medium is ignored and the flow rate is simply proportional to the square root of the pressure difference across the orifice. In order to make this flow rate bi-directional it is necessary to take the absolute value of this pressure difference before extracting the square root, and then use an algebraic sign the same as that of the pressure difference. The constant of proportionality is given by Eq. (23), which can be derived from Eqs. (8), (9), (10) and (11). C_r is the discharge coefficient for the reference orifice, and ϕ_m is plotted in Fig. (6). A_r is the area of the reference orifice.

Regulator Flow Area

Equation (14) defines the position of the regulator moving assembly as a function of its initial position in the time history of all forces acting upon this assembly. It is obvious from Fig. (4), that the flow area of the regulator is a function of this position. This relation between regulator flow area and poppet position is an easily-derivable function

of the geometry of the ball in the seat in which it rests. This geometry for a typical design is shown in Fig. (9). The throat area between the ball and the seat as a function of the height of the ball off of the seat, X , is given by Eq. (24).

$$A_s = 2\pi r (a/b) \sqrt{1 - (r/R)^2} X ; \quad A_s \leq A_{s \text{ max}} \quad (24)$$

The usual small stroke of most regulator designs permits the use of this linear equation. The mechanical ratio between the ball position and the poppet position has been accounted for by the factor $\frac{a}{b}$ in this expression. R is the radius of the ball and r is the radius of the bore in which the ball rests.

For large opening of the regular seat, the primary restriction may occur at some point other than the seat itself. The most likely place for this to happen is in the annular area caused by the ball actuator extending through the bore. For this reason, the actual relationship between effective regulator flow area and the position of the poppet might be as indicated in Fig. (10). In the linear portion, the controlling area is that between the ball and the seat and is given by Eq. (24). However, when the area predicted by this relationship approaches the annular area between the ball actuator and the bore diameter, the effective flow area reaches a limit. This may be approximated by a two segment curve as indicated in Fig. (10).

Liquid Flow Elements

Liquid-flow portions of the propulsion system require different mathematical representations than the gaseous portions. The differences arise due to the large difference in compressibility between the two mediums. The two equations required are for flow rate through a liquid line, and for pressure within a lumped volume.

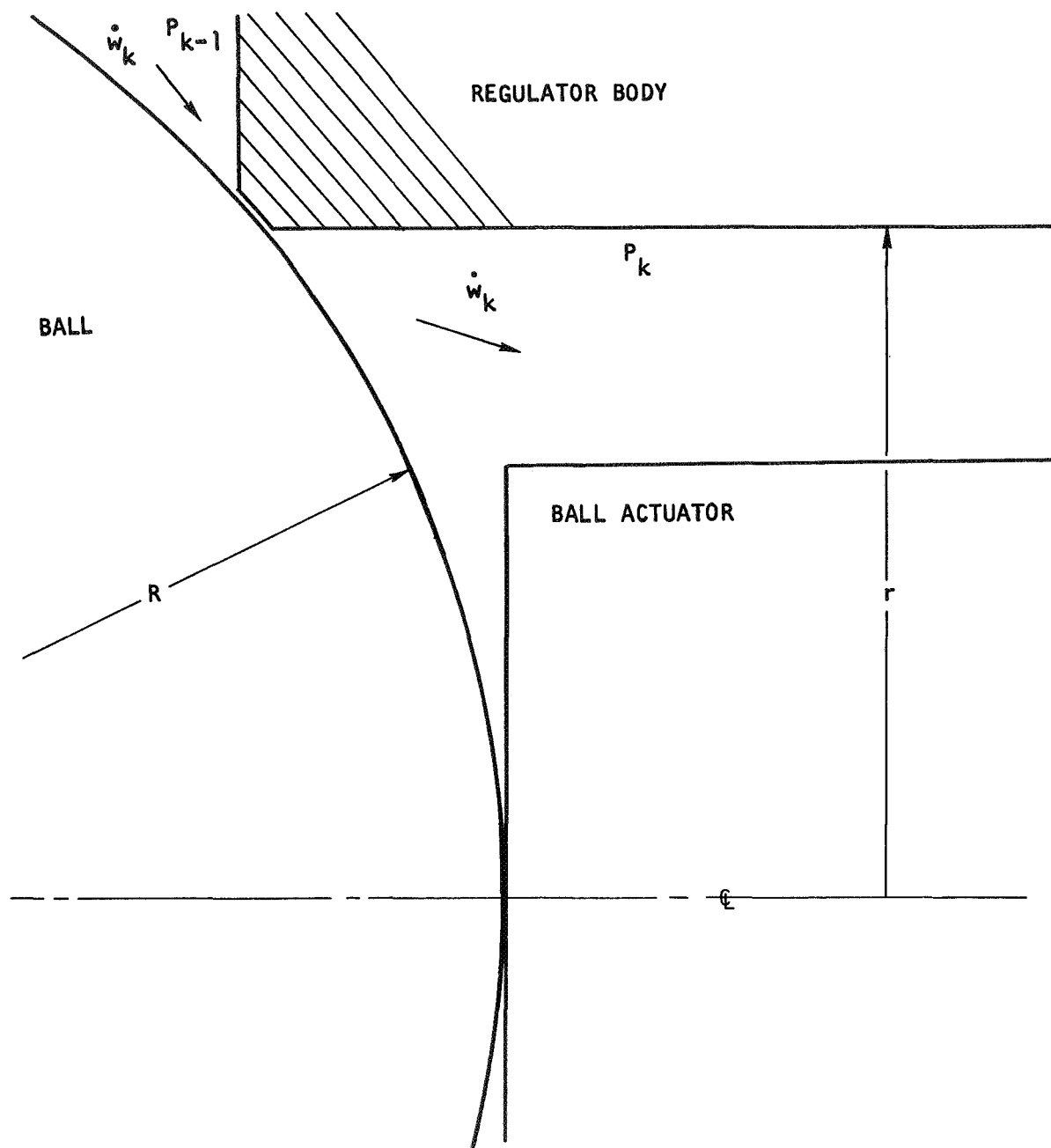


Figure 9. Geometry of Ball - Seat Flow Constriction
(100 Times Actual Size)

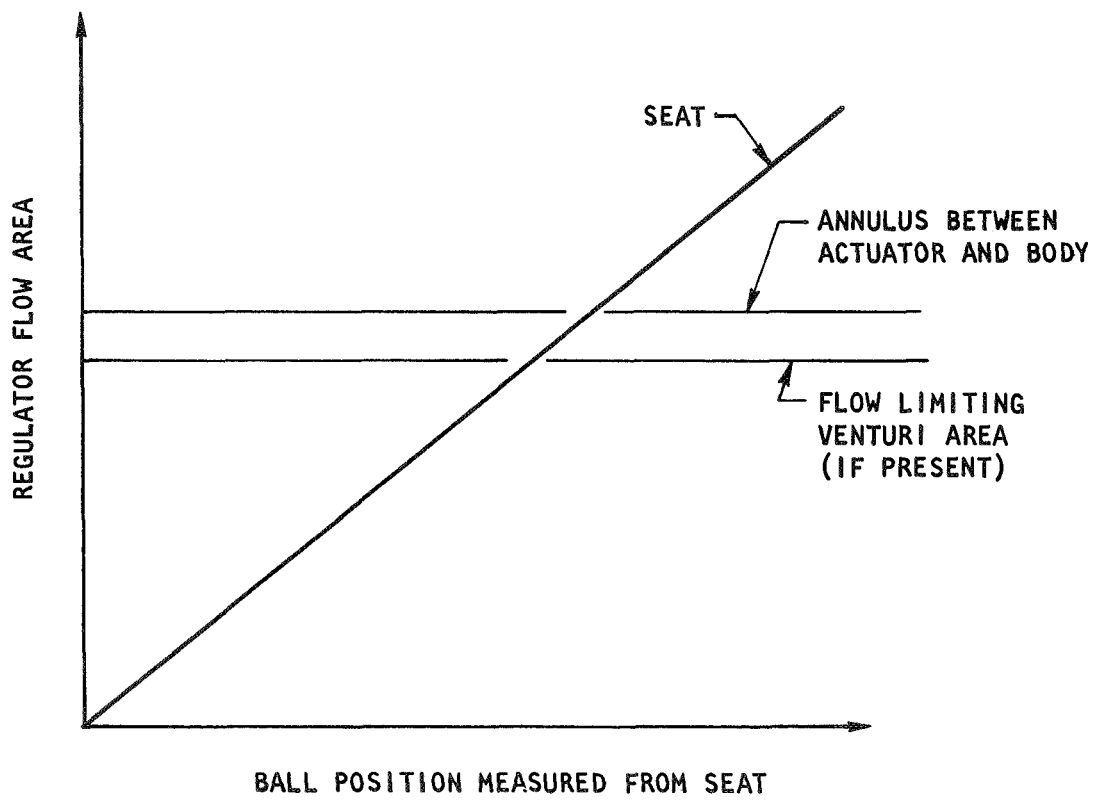


Figure 10. Regulator Flow Area

The dynamic equation for incompressible liquid flow can be found by making a force balance on the fluid column. For example, flow between a tank and a flexline storage volume is denoted by $\dot{w}_{Li}$, and the fluid column between these two points is the line length times its cross-sectional area. The mass is therefore,

$$m = \rho LA / g_c \quad (25)$$

The acceleration of the fluid column is simply the time derivative of the velocity. Forces acting on the column are the area times the pressure difference and the frictional resistance, the latter depending on the square of the velocity. From these considerations,

$$\frac{\rho LA}{g_c} \frac{dv}{dt} = A (\Delta P - R\dot{w}^2) \quad (26)$$

Noting that

$$v = \dot{w} / \rho A$$

it is found that

$$\dot{w} = \dot{w}(0) + A g_c / L \int_0^t (\Delta P - R\dot{w}^2) dt \quad (27)$$

This equation is applicable between any two points where there is no storage. Note that in the steady state the integrand is zero and the familiar equation

$$\Delta P = R\dot{w}^2 \quad (28)$$

is enforced.

System storage can result from liquid compressibility and from flexibility of the line walls. The model accounts for this at discrete points corresponding to lumped volumes in the actual system. The pressure within these lumped volumes can be found by noting that

$$\frac{dP_{Li}}{dt} = \frac{B'_i}{W_{nLi}} \frac{dW_{Li}}{dt} \quad (29)$$

where P_{Li} and W_{Li} are the pressure and stored weight of liquid respectively in the i -th volume. The factor B'_i is the effective bulk modulus of the line filled with liquid. It is found from

$$\frac{1}{B'_i} = \frac{1}{B} + \frac{1.25}{(t/D)_i E_i} \quad (30)$$

where

- B = bulk modulus of liquid
- t = line wall thickness
- D = line diameter
- E = Young's modulus of line material

The derivation of this relationship is presented in Appendix II of Ref. (4).

Noting that the stored weight is the integral of inflow minus outflow, the final equation for pressure is

$$P_{Li} = \frac{B'_i}{W_{ni}} \int_0^t (\dot{W}_{Li} - \dot{W}_{Li+1}) dt \quad (31)$$

where W_{ni} is the nominal stored weight in the line.

It should be noted that there are other definitions of effective modulus than that given by Eq. (30). The form used here is based upon an assumption of longitudinal deformation being independent of circumferential deformation. Reference (4) gives a more complete discussion of this formulation.

It is felt that the secondary importance of the liquid flow system on regulator performance does not warrant further improvements in this area.

Monopropellant Thrust Chamber Equations

In a monopropellant propulsion system the single propellant enters a catalytic bed within the thrust chamber and decomposes exothermically. This process is shown in the schematic diagram of the monopropellant thrust chamber in Fig. (11).

In order to construct a model for study of the pressurization system of such a propulsion system, it is necessary to develop equations for the thrust chamber pressure as a function of propellant flow rate. Considerable simplification is in order, since the thrust chamber is only of secondary importance to the performance of the pressurization system.

The dynamic equations of the thrust chamber can be found by making mass and energy balances on the system shown in Fig. (11). It will be assumed that:

- (a) There is no mass storage in the catalyst bed.
- (b) All heat of reaction is released in the catalyst bed.
- (c) Gas leaves the bed at the temperature of the bed.

Under the first assumption the gas flow rate out of the bed is always equal to the liquid inflow, i.e.,

$$\dot{w}_i = \dot{w}_{L2} \quad (32)$$

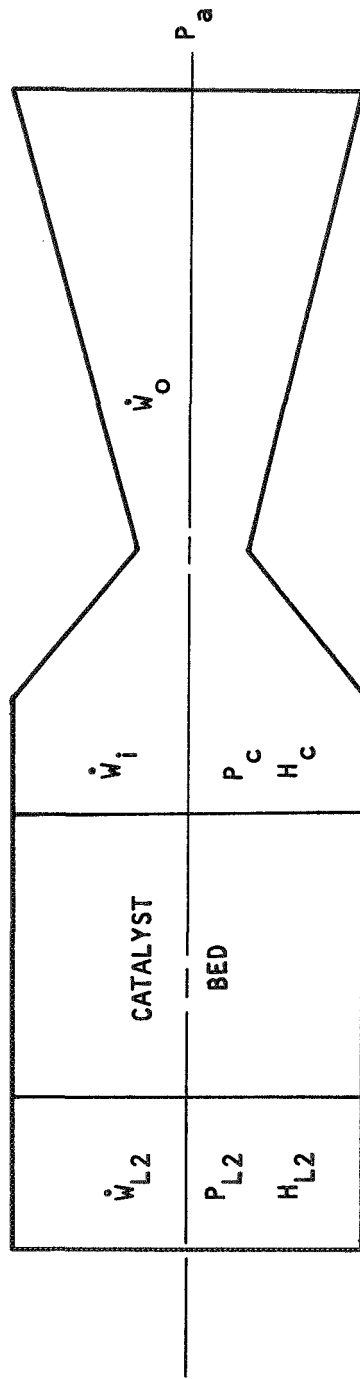


Figure 11. Monopropellant Catalytic Decomposition Thrust Chamber

where $\dot{w}_{Li}$ is the feed system flow rate. However, there is mass storage in the chamber, and it is necessary to write

$$\dot{w}_o = P_c A_t \phi_{\max} \sqrt{T_c} \quad (33)$$

where $\phi_{\max}$ is given by Eq. (12).

It is recognized that γ and MW, in Eq. (12), are strong functions of percent decomposition and weak functions of temperature (Ref. 6). However, for the purpose of this study, these will be considered as secondary factors and $\phi_{\max}$ will therefore be taken as a constant.

The instantaneous stored mass in the reaction chamber is then

$$w_c = w_c(0) + \int_0^t (\dot{w}_i - \dot{w}_o) dt \quad (34)$$

and the pressure, by the ideal gas law, is

$$P_c = R_c T_c w_c / V_c \quad (35)$$

It is now necessary to calculate the temperature, T_c . By assumptions (b) and (c) above, all heat of decomposition is released in the bed. An energy balance across the bed gives

$$m_b c_b \frac{dT_b}{dt} = \dot{w}_{L2} H_{L2} - \dot{w}_i H_c + \dot{w}_i H_r \quad (36)$$

where H_{L2} and H_c are the enthalpies of the liquid inflow and gas outflow respectively, and H_r is the heat of reaction. Note that this equation also assumes a uniform temperature profile in the bed. Otherwise it would be necessary to divide the bed into a number of axial elements and write a series of equations similar to Eq. (36)*. It is felt that Eq. (36) is sufficient for the purpose here.

Equation (36) can be further simplified by noting that in the steady state,

$$\dot{w}_{L2} H_{L2} - \dot{w}_i H_c + \dot{w}_i H_r = 0 \quad (37)$$

Further, by Eq. (32),

$$H_{L2} - H + H_r = 0 \quad (38)$$

Since it is the temperature which is desired, substituting

$$H_c = C_c T_c \quad (39)$$

into Eq. (33) gives

$$T_{c(ss)} = \frac{H_{L2} + H_r}{C_c} \quad (40)$$

where $T_{c(ss)}$ is the steady-state exit gas temperature with the nominal (design) flow rate and C_c is the steady-state specific heat of the decomposition products. Involving assumption (c) and substituting this relationship into Eq. (36) gives

$$\frac{dT_c}{dt} = \frac{\dot{w}C_c}{m_b C_b} (T_{c(ss)} - T_c) \quad (41)$$

*The steady-state axial temperature profile problem is discussed in Ref. (3), Pg. 279.

The steady-state temperature and specific heat of the decomposition products can be obtained from Ref. (5). The bed mass and specific heat capacity are characteristics of the design.

Bipropellant Thrust Chamber Equations

A bipropellant thrust chamber is a chamber in which the two propellants mix and reaction takes place, developing a chamber pressure and temperature which are functions of the propellant combination, mixture ratio, propellant flow rates and chamber geometry. Insofar as the modeling of the pressurization system is concerned, the bipropellant thrust chamber is an element in which the downstream pressure of the propellant tank outflow system is determined as a function of the propellant flow rates. The equations developed here are simplified in accordance with this limited requirement. It is not intended that these equations accurately reflect all dynamic phenomena within an actual thrust chamber.

The fundamental equation for the pressure within the combustion chamber is again Eq. (7). The stored mass in this equation, W , is now the integral of the inflow rate of both propellants minus the outflow rate of the combustion products. The temperature is the combustion temperature of the propellants at the instantaneous mixture ratio, and the volume is the volume of the combustion chamber.

In determining the instantaneous value of stored mass in the combustion chamber for use in Eq. (7), the inlet flow rate is simply the sum of the flow rates of both propellants. This is expressed in Eq. (42). The instantaneous value of each propellant flow rate is determined by the flow equations for the feed system discussed previously

$$\dot{w}_{\text{tot}} = \dot{w}_{\text{ox}} + \dot{w}_f \quad (42)$$

The chamber outflow is determined by an equation similar to Eq. (8). The formulation of this equation as commonly found in discussions of rocket thrust chambers is presented as Eq. (43).

$$\dot{w}_{out} = P_c A_t g_c / C^* \quad (43)$$

In this equation A_t is the throat area, P_c is the chamber pressure, and C^* is a function of the propellant combination, the mixture ratio, and the chamber pressure. For the purpose here, the dependence of C^* on these factors will be ignored, and this quantity will be considered as a constant. With these assumptions, the equation for chamber pressure given as Eq. (44) results.

$$P_c = \frac{RT}{V} \int (\dot{w}_{tot} - P_c A_t g_c / C^*) dt \quad (44)$$

SYSTEM MODELING

The equations developed above, in combination with the schematic for a particular system, form the basis for a mathematical model of any pressure fed propulsion system, which is adequate for study of the pressurization system and all of its components, including the pressure regulator. These equations could be used directly for creating an analog or digital computer model of any system. However, there is something to be gained by further operations on these generalized equations. In going from the equations to the analog model it is always necessary to scale the equations in order to achieve voltage amplitudes compatible with the actual analog computer. In some instances time scaling is also required in order to achieve reasonable solution time and adequate representation of the dynamics of the system giving consideration to the dynamic capabilities of the computer employed. In this section, it is intended to take these generalized equations and perform a generalized scaling, which will be applicable for any system and any analog computer. This scaling method is equation normalization, in which each system variable is normalized (divided by) its nominal value. The nominal value of each parameter thus becomes a scaling factor for that variable.

The concept of a nominal value of each system variable is based upon the existence of a steady state solution to the entire set of equations at some point in time. The nominal point at which this steady state solution or system balance is performed is arbitrarily chosen at the start of the burn period, when the propellant tanks are at their design pressures.* Thus, in terms of these normalized variables the "design conditions" of system operation are characterized by a value of unity for all normalized system variables. The machine variables are then conveniently taken as even multiple of these normalized variables.

* Actually, this is quasi-steady state, since conditions in the tanks are slowly changing with time.

It will be seen that the normalization results in the appearance of certain dimensionless groups* or parameters which then characterize the system. This characterization of the system by these dimensionless group has two distinct advantages. First, it generalizes the results. That is, the results obtained by the solution of the equations for a particular set of these dimensionless parameters represents the solution of an extremely large number of geometrically similar physical systems, with the only requirement being that they have equal values of the dimensionless groups. Also, this system removes dimensional parameters such as, system pressures and thrust levels. A second advantage of the normalization and the resulting dimensionless parameters is that it is not necessary to explicitly specify the system details such as the flow areas of restrictions. Instead of specifying actual flow areas, it is simply necessary to specify the desired pressure schedule at points in the system represented by the discrete compressible volume lumps.

Equation Normalization and Scaling

A normalized variable is defined as the ratio of the actual instantaneous value of that variable to its nominal value. The nominal value of each variable is defined by the simultaneous solution of the steady state equations of the system at the beginning of the burn period. As an example of equation normalization, consider the equation for the pressure within a compressible volume as given by Eq. (7). Assume that the compressibility factor and the gas constant for the medium are constant. Then, denoting the normalized variable by an overbar, we have that

$$\left. \begin{aligned} \bar{P}_i &= P_i/P_{in} \\ \bar{W}_i &= W_i/W_{in} \\ \bar{V}_i &= V_i/V_{in} \\ \bar{T}_i &= T_i/T_{in} \end{aligned} \right\} \quad (45)$$

* These groups are dimensionless if in an algebraic equation, or have units of reciprocal-time if in a differential equation.

By substituting the normalized variables defined in Eq. (45) directly into Eq. (7) we have the normalized equation for the normalized pressure in the i-th compressible volume:

$$\bar{P}_i = \bar{W}_i \bar{T}_i / \bar{V}_i \quad (46)$$

Note that if any of these quantities are constant, the corresponding normalized variables are always equal to one and can be left out of the normalized equation.

It will be noted in the above development that the constant terms cancel one another due to the definition of the nominal or normalizing values. So long as this set of nominal values represent a solution of the steady state equations this procedure, applied to all equations of the system, will produce a completely analogous set of dimensionless equations in which all variables have a nominal value of one.

As another example, consider the equation for the instantaneous mass within a compressible volume as given by Eq. (1). Note that the nominal value of mass outflow is equal to the nominal value of the mass inflow due to the definition of the nominal point as being a steady state solution to the equations, i.e., the rate of change of W_i is equal to 0. Thus we have for the normalized version of Eq. (1),

$$\frac{d\bar{W}_i}{dt} = \left(\frac{\dot{W}_n}{\bar{W}_{in}} \right) (\bar{W}_i - \bar{W}_{i+1}) \quad (47)$$

In Eq. (47) we see the appearance of the parameter $\frac{\dot{W}_n}{\bar{W}_{in}}$. This parameter has units of reciprocal seconds and is therefore similar to the inverse

of a time constant. Thus, the typical compressible volume has a time constant which is equal to the ratio of the nominal value of the stored weight within the volume to the nominal flow rate through the volume.

In some equations with additive terms, the normalizing quantity is different for each term. In this case each additive term in the normalized equation has a different coefficient. An example is the equation for the total flow rate in a bipropellant system, Eq. (42). When each of the variables in this equation is normalized by its own nominal value there results Eq. (48).

$$\bar{w}_{\text{tot}} = \left(\frac{\mu_n}{1+\mu_n} \right) \bar{w}_{\text{ox}} + \left(\frac{1}{1+\mu_n} \right) \bar{w}_f \quad (48)$$

In deriving Eq. (48) from Eq. (42), use was made of the fact that the nominal total flow rate is equal to the sum of the nominal values of the oxidizer and fuel flow rate. Also, use was made of the definition of the nominal mixture ratio

$$\mu_n = \dot{w}_{\text{oxn}} / \dot{w}_{\text{fn}} \quad (49)$$

In some cases, the normalization of a variable is not desirable. An example of this is in the case of Eqs. (8) thru (11). The compressible flow function, Φ , is a function of pressure ratio which is identical for all points of restriction in the gaseous flow portion of the system. Since each of these points of restriction has a different nominal pressure ratio, normalization of the Φ function at all points would produce a similar but different equation for Φ in each case. In building an analog model of the pressurization system, it is more desirable to have a single Φ -function which is applicable to all points of restriction. This introduces the possibility of time-sharing

of a single generation of the function. In order to achieve this benefit, rather than a direct normalization of Eq. (8), it is preferable to leave it in the form shown in Eq. (50).

$$\dot{w}_i = \frac{\bar{P}_{i-1} \bar{A}_i}{\sqrt{\bar{T}_{i-1}}} \frac{\Phi(\alpha_i)}{\Phi(\alpha_{in})} \quad (50)$$

In this equation the pressure, area, and temperature are normalized variables, but Φ is an unnormalized quantity, and a constant appears in the denominator which is equal to the nominal value of the compressible flow function - Φ .

In order to use a common generation of the compressible flow function for all points of restriction, it is also necessary to work in terms of absolute pressure ratios rather than normalized pressure ratios. However, since other equations involving the pressures require the use of the normalized pressure variables, it is necessary to convert to the absolute pressure ratio according to Eq. (51).

$$\alpha_i = P_i/P_{i-1} = \alpha_{in} \bar{P}_i/\bar{P}_{i-1} \quad (51)$$

Here the absolute pressure ratio is denoted by α_i which is equal to the i -th pressure divided by the pressure immediately upstream of the i -th pressure. This absolute pressure ratio is found by multiplying the nominal pressure ratio by the ratio of the normalized pressure variables as indicated in Eq. (51).

In some cases the method of normalization described above, i.e., simply dividing the actual variable by its nominal value, is not well suited for problem simulation. One example of this is the case when the nominal value of the system parameter is zero, such as in the case of the flow

rate into the reference cavity, Eq. (21). In these cases, a convenient alternative is to use the local value of a similar variable as the normalizing variable. In the case of the reference orifice flow rate, the normalizing parameter is thus chosen as the nominal value of the system pressurant flow, $\dot{w}_n$. Thus Eq. (21) normalizes to give

$$\bar{w}_r = \frac{\dot{w}_r}{\dot{w}_n} = \left(\frac{K_r \sqrt{P_{kn}}}{\dot{w}_n} \right) \sqrt{|\bar{P}_k - \bar{P}_r|} \operatorname{sgn} (\bar{P}_k - \bar{P}_r) \quad (52)$$

Another example of a case where it is more convenient to choose a different normalizing parameter is in the case of the poppet position. The equation of motion of the poppet takes on a more convenient form if the normalizing method for the position variable is as indicated in Eq. (53).

$$\bar{X} = \frac{X - X_n}{X_m - X_n} \quad (53)$$

Here it is seen that the normalized position variable is the ratio of the difference between X and its nominal value to the maximum value of this difference. Thus, when the poppet is against its forward stop $\bar{X}$ has the value of 1, and when the regulator is operating at the nominal point, the position variable $\bar{X}$ is the value of 0. For poppet positions less than the nominal value, for example, when the poppet is closed, $\bar{X}$ has a negative value.

The normalization techniques described above can be applied to all system equations previously developed. This results in a new set of equations which are completely analogous to the basic equations which define the system. Table I summarizes all of these equations as developed in their basic

form, and also in their normalized form. The basic equation in each case is denoted by the letter "a" and its number, and the normalized version of the equation is denoted by the letter "b". The third form of each equation shown there, denoted by the letter "c", represents the scaled normalized equation suitable for analog computer mechanization. The analog scaling is discussed in the next section. Scale factors for the normalized variables are presented in Table II. Miscellaneous formulæ for the constants appearing in the equations of Table I are presented in Table III.

Analog Scaling

If the normalized system equations are to be solved by means of an analog computer, consideration must be given to the range expected for each normalized variable and the allowable voltage range for the particular analog computer. If the analog computer had an unlimited voltage range, the normalized equations could be used directly and no further amplitude scaling would be necessary. However, most analog computers have a voltage range of ± 100 V or ± 10 V, depending upon the machine. It is therefore necessary to estimate the maximum value of each of the normalized variables and multiply it by an appropriate factor so that the voltage range will not be exceeded.

In pressurization systems it is found that system pressures seldom exceed their nominal values by more than the factor of 1.5, but that system flowrates can often reach values as large as 7 or 8 times their nominal values. Temperatures seldom exceed their nominal values. This establishes a basis for amplitude scaling all of the variables in most pressurization systems, as indicated in Table II. Normalized pressures are scaled by a factor of .5, normalized flow rates are scaled by a factor of .1, and normalized temperatures are scaled by a factor of .5. Thus an analog computer with a range of ± 100 V would have nominal values of pressure voltages equal to 50 V, which would therefore allow pressures to reach a maximum of twice their

nominal values without exceeding the machine voltage limits. Similarly, flow rate voltages would be nominally 10 V with a possibility of going to ten times this value without exceeding the machine capability. For analog computers with a range of ± 10 V these nominal values would be 5 V and 1 V respectively. This scaling method should be universally applicable for any pressurization system and any analog computer. Table I shows the amplitude-scaled, normalized equations, along with the basic and normalized forms.

Depending upon the particular system under consideration and the dynamic capabilities of the particular analog computer employed, it may be necessary to time scale the model in addition to the amplitude scaling discussed above. The subject of time scaling in analog modeling can be approached from at least two different viewpoints, the relative merits of each being evaluated differently by each different programmer. These methods are discussed in Ref. (6). One of these methods is to introduce a nondimensional time variable, defined as the ratio of real time to a convenient time scale factor. The introduction of this new time variable changes the numerical value of the coefficients in all differential equations of the problem. The second method is to approach the analog computer with the equations and coefficients as derived and simply acknowledge the desired time scale by the gains patched for each integrator in the problem. The difference in the two methods is purely in the viewpoint assumed by the programmer. There is however a slight advantage for the second method, which is commonly called "machine time scaling," in that it is completely compatible with the automatic time scale feature of many analog computers. This advantage is appreciated to the fullest degree when it becomes necessary to change the time scaling at some point during the analysis. In the first method, a change in time scale requires a change in the formulas used for calculation of potentiometer settings, whereas with the second

method it is simply necessary to change the capacitors of the integrators on the computer, which on many machines is possible with a single control relay. For this reason, the machine time scaling method is recommended.

It will be noted in Tabel I that certain groups of parameters, consisting of the nominal values of the system variables and properties, occur in many of the formulas. For example, the time constants of compressible volume, $\dot{w}_n / W_{Ln}$, occurs in Eqs.(1) and (2) of Table I. In some cases the method of calculation of these parameters is evident from the equations as shown in Table I. In other instances it was found cumbersome to include a more detailed formulae for the coefficients of the equations. For this reason, Table III presents a summary of miscellaneous formuli that will be found useful in calculations of the constants in the equations of Table I.

Determination of System Parameters

The coefficients of the scaled, normalized equations as shown in Table I correspond directly to the coefficient potentiometer settings for the analog computer model. Thus numerical values for each of these coefficients for the system under consideration must be calculated before the analog model can be completed. It is only the value of these coefficients which distinguish the model for one physical system from that of another geometrically similar system.

The system coefficients or potentiometer settings can be grouped into three basic areas. These are system pressures, i.e., the pressure schedule through the actual system, flow system time constants, and regulator characteristics. Pressure ratios are required for coefficients whenever the flow equations are applied. It will be noted that it is the pressure ratio across each flow device which is important rather than the actual pressure level. The time constants for each

compressible volume in the system are determined by the nominal pressure and flow rate and that portion of the system divided by the nominal stored weight of pressurant in the particular volume. By use of the ideal gas law, Eq. (7), this time constant can be expressed as

$$\frac{\dot{w}}{\dot{w}_{in}} = \dot{w}_n R T_n g_c Z/P_{in} V_{in} \quad (54)$$

It is seen that these coefficients have units of reciprocal seconds.

It is thus seen that the flow rate through flow restriction devices and the pressure within the lumped compressible volumes are completely determined by specification of system pressures, nominal flow rate and pressurant properties, and that system geometry enters only in the volume factor in the denominator of the time constant expression. This gives some insight into the way in which systems of this type can be scaled from one thrust level to another. First, if two systems of different thrust levels are compared it is seen that if pressure ratios are the same for both systems, the representation of the flow devices remains unchanged. The time constants vary as the ratio of nominal pressurant flow rate to the volume of the localized compressibilities. It is notable, however, that if the velocity of the pressurant in the flow lines is maintained as a constant design parameter between the two systems of different thrusts, then the ratio of pressurant flow rate to volume varies only with the length of the pressurization lines. It is, therefore, to be expected that system pressures (actually system pressure ratios across flow devices) are much more important design variables than system thrust.

TABLE I

SUMMARY OF GENERALIZED EQUATIONS

a) Basic equation, b) Normalized form, c) Scaled normalized form

Stored mass within i-th volume

$$W_i = W_i(o) + \int (\dot{W}_i - \dot{W}_{i+1}) dt \quad (1a)$$

$$\bar{W}_i = \bar{W}_i(o) + \left(\frac{\dot{W}_n}{\dot{W}_{in}} \right) \int (\bar{W}_i - \bar{W}_{i+1}) dt \quad (1b)$$

$$[.5 \bar{W}_i] = [.5 \bar{W}_i(o)] + 5 \left(\frac{\dot{W}_n}{\dot{W}_{in}} \right) \int ([.1 \bar{W}_i] - [.1 \bar{W}_{i+1}]) dt \quad (1c)$$

Temperature within i-th volume

$$T_i = T_i(o) + \int \{ (\gamma T_{i-1} - T_i) \dot{W}_i - (\gamma - 1) T_i \dot{W}_{i+1} - P_i \dot{W}_{Li} / \rho_L C_v + Q_i / C_v \} / W_i dt \quad (2a)$$

$$\begin{aligned} \bar{T}_i = \bar{T}_i(o) + \left(\frac{\dot{W}_n}{\dot{W}_{in}} \right) \int \{ (\gamma \bar{T}_{i-1} - \bar{T}_i) \bar{W}_i - (\gamma - 1) \bar{T}_i \bar{W}_{i+1} - (\gamma - 1) \bar{P}_i \bar{W}_{Li} \\ + \left(\frac{(\gamma - 1) Q_{in}}{\dot{W}_n R T_n} \right) \bar{Q}_i \} / \bar{W}_i dt \end{aligned} \quad (2b)$$

$$\begin{aligned} [.5 \bar{T}_i] = [.5 \bar{T}_i(o)] + \frac{1}{4} \left(\frac{\dot{W}_n}{\dot{W}_{in}} \right) \int [10 \{ (2\gamma [.5 \bar{T}_{i-1}] - 2 [.5 \bar{T}_i]) [.1 \bar{W}_i] \\ - 2(\gamma - 1) [.5 \bar{T}_i] [.1 \bar{W}_{i+1}] - 2(\gamma - 1) [.5 \bar{P}_i] [.1 \bar{W}_{Li}] \\ + \left(\frac{.2(\gamma - 1) Q_{in}}{\dot{W}_n R T_n} \right) [.5 \bar{Q}_i] \}] / [.5 \bar{W}_i] dt \end{aligned} \quad (2c)$$

Instantaneous compressible volume

$$V_i = V_i(o) + \int \dot{W}_{Li} / \rho_L dt \quad (3a)$$

$$\bar{V}_i = \bar{V}_i(o) + \left(\frac{\dot{W}_{Lin}}{\rho_L \dot{W}_{in}} \right) \int \bar{W}_{Li} dt = \bar{V}_i(o) + \left(\frac{\dot{W}_n}{\dot{W}_{in}} \right) \int \bar{W}_{Li} dt \quad (3b)$$

$$[.05 \bar{V}_i] = [.05 \bar{V}_i(o)] + \frac{1}{2} \left(\frac{\dot{W}_n}{\dot{W}_{in}} \right) \int [.1 \bar{W}_{Li}] dt \quad (3c)$$

TABLE I (Continued)

Pressure within i-th volume

$$P_i = Z R W_i T_i / V_i \quad (4a)$$

$$\bar{P}_i = \bar{W}_i \bar{T}_i / \bar{V}_i \quad (4b)$$

$$[.5\bar{P}_i] = \left[[.05\bar{W}_i] / [.05\bar{V}_i] \right] [.5\bar{T}_i] \quad (4c)$$

Gas flow rate through i-th orifice device

$$\dot{W}_i = C_i P_{i-1} A_i \phi(\alpha_i) / \sqrt{T_{i-1}} \quad (5a)$$

$$\bar{W}_i = \bar{P}_{i-1} \bar{A}_i \phi(\alpha_i) / (\phi(\alpha_{in}) \sqrt{\bar{T}_{i-1}}) \quad (5b)$$

$$[.1\bar{W}_i] = \{ [.5\bar{P}_{i-1}] [.1\bar{A}_i] / [.5\sqrt{\bar{T}_{i-1}}] \} \phi(\alpha_i) / \phi(\alpha_{in}) \quad (5c)$$

Pressure ratio

$$\alpha_i = P_i / P_{i-1} \quad (6a)$$

$$\alpha_i = \alpha_{in} \bar{P}_i / \bar{P}_{i-1} \quad (6b)$$

$$[.5\alpha_i] = (.5\alpha_{in}) [.5\bar{P}_i] / [.5\bar{P}_{i-1}] \quad (6c)$$

Compressible flow function

$$\left. \begin{aligned} \phi &= 2 \phi_m (\alpha (1-\alpha))^{1/2} & ; \quad \alpha \geq .5 \\ &= \phi_m & ; \quad \alpha < .5 \end{aligned} \right\} \quad (7a)$$

$$\left. \begin{aligned} \phi &= 4 \phi_m ([.5\alpha] (.5-[.5\alpha]))^{1/2} & ; \quad [.5\alpha] \geq .25 \\ &= \phi_m & ; \quad [.5\alpha] < .25 \end{aligned} \right\} \quad (7c)$$

TABLE I (Continued)

Laminar-flow filter flow rate

$$\dot{w}_i = K_i P_{i-1} (P_{i-1} - P_i) / RT_{i-1} \quad (8a)$$

$$\bar{w}_i = \bar{P}_{i-1} \left(\frac{1}{1-\alpha_{in}} \bar{P}_{i-1} - \frac{\alpha_{in}}{1-\alpha_{in}} \bar{P}_i \right) / \bar{T}_{i-1} \quad (8b)$$

$$[.1\bar{w}_i] = \left\{ .2[.5\bar{P}_{i-1}] \left(\frac{1}{1-\alpha_{in}} [.5\bar{P}_{i-1}] - \frac{\alpha_{in}}{1-\alpha_{in}} [.5\bar{P}_i] \right) \right\} / [.5\bar{T}_{i-1}] \quad (8c)$$

Reference cavity pressure

$$P_r = P_{rn} (W_r/W_{rn} + \left(\frac{\Delta V_{rn}}{V_{rn}} \right) \Delta X / \Delta X_n) \quad (9a)$$

$$\bar{P}_r = \bar{W}_r + \left(\frac{\Delta V_{rn}}{V_{rn}} \right) \bar{X} \quad (9b)$$

$$[.5\bar{P}_r] = [.5\bar{W}_r] + \left(\frac{\Delta V_{rn}}{V_{rn}} \right) [.5\bar{X}] \quad (9c)$$

Reference orifice flow rate

$$\dot{w}_r = K_r \sqrt{|P_k - P_r|} \operatorname{sgn} (P_k - P_r) \quad (10a)$$

$$\bar{w}_r = (K_r \sqrt{P_{kn}} / \dot{w}_n) \sqrt{|\bar{P}_k - \bar{P}_r|} \operatorname{sgn} (\bar{P}_k - \bar{P}_r) \quad (10b)$$

$$[.5\bar{w}_r] = \sqrt{K_r' [.5\bar{P}_k] - K_r' [.5\bar{P}_r]} \operatorname{sgn} ([.5\bar{P}_k] - [.5\bar{P}_r]) \quad (10c)$$

(K_r' is defined in Table III)

Regulator flow area

$$A_s = 2\pi r (a/b) \sqrt{1 - (r/R)^2} X \quad ; \quad A_s \leq A_{smax} \quad (11a)$$

$$\bar{A}_s = 1 + (\Delta X_m / X_n) \bar{X} \quad ; \quad \bar{A}_s \leq A_{smax} / A_{sn} \quad (11b)$$

$$[.1\bar{A}_s] = .1 + (.2 \Delta X_m / X_n) [.5\bar{X}] \quad ; \quad [.1\bar{A}_s] \leq .1A_{smax} / A_{sn} \quad (11c)$$

TABLE I (Continued)

Regulator equation of motion

$$\begin{aligned} m\ddot{X} = & F_s - A_e (P_r - P_a) - F_{C1} \operatorname{sgn} \dot{X} - B_1 \dot{X} \\ & - \left(\frac{a}{b}\right) [F_{bs} + F_{Pr} + F_{C2} \operatorname{sgn} \dot{X} + B_2 \dot{X}] U(X) \end{aligned} \quad (12a)$$

$$\begin{aligned} \ddot{\bar{X}} = & \frac{F_{sn}}{m\Delta X_m} \bar{F}_s - \frac{A_e P_{rn}}{m\Delta X_m} \bar{P}_r + \frac{A_e P_a}{m\Delta X_m} - \frac{F_{C1}}{m\Delta X_m} \operatorname{sgn} \dot{\bar{X}} - \frac{B_1}{m} \dot{\bar{X}} \\ & - \left(\frac{a}{b}\right) \left[\frac{F_{bsn}}{m\Delta X_m} \bar{F}_{bs} + \frac{F_{Prn}}{m\Delta X_m} \bar{F}_{Pr} + \frac{F_{C2}}{m\Delta X_m} \operatorname{sgn} \dot{\bar{X}} + \frac{B_2}{m} \dot{\bar{X}} \right] U\left(\bar{X} + \frac{X_n}{\Delta X}\right) \end{aligned} \quad (12b)$$

$$\begin{aligned} [.5\ddot{\bar{X}}] = & \left(\frac{F_{sn}}{m\Delta X}\right) [.5\bar{F}_s] - \left(\frac{A_e P_{rn}}{m\Delta X}\right) [.5\bar{P}_r] + \frac{1}{2} \left(\frac{A_e P_a}{m\Delta X}\right) - \frac{1}{2} \left(\frac{F_{C1}}{m\Delta X}\right) \operatorname{sgn} [.5\dot{\bar{X}}] \\ & - \left(\frac{B_1}{m}\right) [.5\dot{\bar{X}}] - \left(\frac{a}{b}\right) \left\{ \left(\frac{F_{bsn}}{m\Delta X}\right) [.5\bar{F}_{bs}] + \left(\frac{F_{Prn}}{m\Delta X}\right) [.5\bar{F}_{Pr}] \right. \\ & \left. + \frac{1}{2} \left(\frac{F_{C2}}{m\Delta X}\right) \operatorname{sgn} [.5\dot{\bar{X}}] + \left(\frac{B_2}{m}\right) [.5\dot{\bar{X}}] \right\} \cdot U\left([.5\bar{X}] + \frac{1}{2} \left(\frac{X_n}{\Delta X}\right)\right) \end{aligned} \quad (12c)$$

Linear reference spring

$$F_s = F_{so} - k_s X \quad (13a)$$

$$\bar{F}_s = 1 - \left(\frac{k_s \Delta X}{F_{sn}}\right) \bar{X} \quad (13b)$$

$$[.5\bar{F}_s] = .5 - \left(\frac{k_s \Delta X}{F_{sn}}\right) [.5\bar{X}] \quad (13c)$$

Linear Ball-retainer spring

$$F_{bs} = F_{bso} + k_b X \quad (14a)$$

$$\bar{F}_{bs} = 1 + \left(\frac{k_b \Delta X}{F_{bsn}}\right) \bar{X} \quad (14b)$$

$$[.5\bar{F}_{bs}] = .5 + \left(\frac{k_b \Delta X}{F_{bsn}}\right) [.5\bar{X}] \quad (14c)$$

TABLE I (Continued)

Regulator pressure forces

$$F_{Pr} = A_b (P_{k-1} - P_k) + A_p (P_k - P_r) \quad (15a)$$

$$\bar{F}_{Pr} = \left(\frac{1}{1-\alpha_k} \right) \bar{P}_{k-1} + \left(\frac{(\epsilon-1)\alpha_k}{1-\alpha_k} \right) \bar{P}_k - \left(\frac{\epsilon\alpha_k}{1-\alpha_k} \right) \bar{P}_r \quad (15b)$$

$$[.5\bar{F}_{Pr}] = \left(\frac{1}{1-\alpha_k} \right) [.5\bar{P}_{k-1}] + \left(\frac{(\epsilon-1)\alpha_k}{1-\alpha_k} \right) [.5\bar{P}_k] - \left(\frac{\epsilon\alpha_k}{1-\alpha_k} \right) [.5\bar{P}_r] \quad (15c)$$

Total pressurant flow rate for bipropellant system

$$\dot{w}_j = \dot{w}_{j+1,X} + \dot{w}_{j+1,F} \quad (16a)$$

$$\bar{w}_j = \left(\frac{\mu_v}{1+\mu_v} \right) \bar{w}_{j+1,X} + \left(\frac{1}{1+\mu_v} \right) \bar{w}_{j+1,F} \quad (16b)$$

$$[.1\bar{w}_j] = \left(\frac{\mu_v}{1+\mu_v} \right) [.1\bar{w}_{j+1,X}] + \left(\frac{1}{1+\mu_r} \right) [.1\bar{w}_{j+1,F}] \quad (16c)$$

Pressure in liquid line

$$P_{Li} = P_{Li(0)} + \frac{B_i}{w_{Lin}} \int (\dot{w}_{Li} - \dot{w}_{Li+1}) dt \quad (17a)$$

$$\bar{P}_{Li} = \bar{P}_{Li(0)} + \frac{B_i \dot{w}_{Ln}}{w_{Lin} \bar{P}_{Lin}} \int (\bar{w}_{Li} - \bar{w}_{Li+1}) dt \quad (17b)$$

$$[.5\bar{P}_{Li}] = [.5\bar{P}_{Li(0)}] + \left(\frac{5B_i \dot{w}_{Ln}}{w_{Lin} \bar{P}_{Lin}} \right) \int [.1\bar{w}_{Li}] - [.1\bar{w}_{Li+1}] dt \quad (17c)$$

Flow rate through liquid line

$$\dot{w}_{Li} = \dot{w}_{Li(0)} + \frac{A_i g_c}{L_i} \int (P_{iL} - P_{i+1L} - R\dot{w}_{Li}^2) dt \quad (18a)$$

$$\bar{w}_{Li} = \bar{w}_{Li(0)} + \left(\frac{A_i g_c \Delta P_i}{L_i \dot{w}_{Ln}} \right) \int \left\{ \left(\frac{1}{1-\alpha_{Li}} \right) \bar{P}_{iL} - \left(\frac{\alpha_{Li}}{1-\alpha_{Li}} \right) \bar{P}_{i+1L} - \bar{w}_{Li}^2 \right\} dt \quad (18b)$$

$$[.5\bar{w}_{Li}] = [.5\bar{w}_{Li(0)}] + \left(\frac{A_i g_c \Delta P_i}{L_i \dot{w}_{Ln}} \right) \int \left\{ \left(\frac{1}{1-\alpha_{Li}} \right) [.5\bar{P}_{iL}] - \left(\frac{\alpha_{Li}}{1-\alpha_{Li}} \right) [.5\bar{P}_{i+1L}] - 2 [.5\bar{w}_{Li}]^2 \right\} dt \quad (18c)$$

TABLE I (CONTINUED)

Monopropellant thrust chamber out-flow

$$\dot{w}_o = P_c A_t \phi_m / \sqrt{T_c} \quad (19a)$$

$$\bar{w}_o = \bar{P}_c \sqrt{\bar{T}_c} \quad (19b)$$

$$[.5\bar{w}_o] = .5 [.5\bar{P}_c] / [.5\sqrt{\bar{T}_c}] \quad (19c)$$

Stored mass in monopropellant thrust chamber

$$W_c = W_c(o) + \int (\dot{w}_{in} - \dot{w}_o) dt \quad (20a)$$

$$\bar{W}_c = \bar{W}_c(o) + \left(\frac{\dot{w}_{Ln}}{\bar{w}_{cn}} \right) \int (\bar{w}_{in} - \bar{w}_o) dt \quad (20b)$$

$$[.5\bar{W}_c] = [.5\bar{W}_c(o)] + \left(\frac{\dot{w}_{Ln}}{\bar{w}_{cn}} \right) \int ([.5\bar{w}_{in}] - [.5\bar{w}_o]) dt \quad (20c)$$

Monopropellant thrust chamber pressure

$$P_c = R_c T_c W_c / V_c \quad (21a)$$

$$\bar{P}_c = \bar{T}_c \bar{W}_c \quad (21b)$$

$$[.5\bar{P}_c] = 2 [.5\bar{T}_c] [.5\bar{W}_c] \quad (21c)$$

Monopropellant thrust chamber temperature

$$T_c = T_c(o) + \left(\frac{\dot{w}_{Tn} C_c}{m_b C_b} \right) \int (T_{ss} - T_c) dt \quad (22a)$$

$$\bar{T}_c = \bar{T}_c(o) + \left(\frac{\dot{w}_{Tn} C_p}{m_b C_b} \right) \int (1 - \bar{T}_c) dt \quad (22b)$$

$$[.5\bar{T}_c] = [.5\bar{T}_c(o)] + \left(\frac{\dot{w}_{Tn} C_p}{m_b C_b} \right) \int (.5 - [.5\bar{T}_c]) dt \quad (22c)$$

TABLE I (CONCLUDED)

Bipropellant thrust chamber in-flow

$$\dot{w}_{tot} = \dot{w}_{ox} + \dot{w}_f \quad (23a)$$

$$\bar{w}_{tot} = \left(\frac{\mu_n}{1+\mu_n} \right) \bar{w}_{ox} + \left(\frac{1}{1+\mu_n} \right) \bar{w}_f \quad (23b)$$

$$[.5\bar{w}_{tot}] = \left(\frac{\mu_n}{1+\mu_n} \right) [.5\bar{w}_{ox}] + \left(\frac{1}{1+\mu_n} \right) [.5\bar{w}_f] \quad (23c)$$

Bipropellant thrust chamber pressure

$$P_c = \frac{R_c T_c}{V_c} \int (\dot{w}_{tot} - \frac{A_t g_c}{c^*} P_c) dt \quad (24a)$$

$$\bar{P}_c = \frac{R_c T_c \dot{w}_{tot,n}}{P_{cn} V_c} \int (\bar{w}_{tot} - \frac{A_t g_c}{c^*} \bar{P}_c) dt \quad (24b)$$

$$[.5\bar{P}_c] = \frac{R_c T_c \dot{w}_{tot,n}}{P_{cn} V_c} \int ([.5\bar{w}_{tot}] - \frac{A_t g_c}{c^*} [.5\bar{P}_c]) dt \quad (24c)$$

TABLE II

SCALE FACTORS FOR NORMALIZED VARIABLES

<u>Symbol</u>	<u>Quantity</u>	<u>Scale Factor</u>
$\bar{A}$	Area	.1
$\bar{F}$	Force	.5
$\bar{P}$	Pressure	.5
$\bar{Q}$	Heat flow rate	.5
$\bar{T}$	Temperature	.5
$\bar{V}$	Volume	.05
$\bar{W}$	Stored mass	.5
$\dot{\bar{W}}$	Mass flow rate	.1
$\bar{X}$	Poppet position	.5

TABLE III

MISCELLANEOUS FORMULI FOR PROBLEM CONSTANTS

Time constants for compressible volumes (Eqs. (1), (23), Table I)

$$\frac{\dot{w}_n}{w_{in}} = \frac{\dot{w}_n R T_n Z g_c}{P_{in} V_{in}} \text{ sec}^{-1}$$

Gas constant:

$$R = R_o / MW \text{ in}/^\circ R$$

Rate-of-Expulsion time constants (Eq. (3), Table I)

$$\frac{\dot{w}_{Lin}}{\rho_L V_{in}} = \frac{\dot{w}_n}{w_{in}} \text{ sec}^{-1}$$

This relationship derives from the requirement that volumetric gas flow must equal volumetric liquid flow during nominal conditions.

Maximum value of compressible flow function (Eq. (7), Table I)

$$\phi_m = \left[\frac{\gamma g_c}{R} \right]^{1/2} \left[\frac{2}{(\gamma + 1)} \right]^{1/2 (\gamma + 1 / \gamma - 1)} {}^\circ R^{1/2} \text{ sec}^{-1}$$

Plotted vs γ in Fig. (6).

TABLE III (cont'd)

Volume coefficient for reference pressure (Eq. (9), Table I)

$$\frac{\Delta V_{rm}}{V_{rn}} = \frac{V_{rm} - V_{rn}}{V_{rn}} = \frac{A_e (X_m - X_n)}{V_{rn}}$$

V_{rn} = nominal reference cavity volume

Proportionality constant for reference orifice (Eq. (10), Table I)

$$K'_r = 2\phi_m^2 \left(\frac{C_r A_r P_{kn}}{\dot{w}_n \sqrt{T_n}} \right)^2 \quad (\text{dimensionless})$$

Seat flow area vs poppet position (Eq. (11), Table I)

$$\frac{\Delta X_m}{X_n} = \frac{X_{max} - X_n}{X_n}$$

Nominal seat flow area (Eq. (11), Table I)

$$A_{en} = \frac{\dot{w}_n \sqrt{T_n}}{C_s P_{k-1} \phi_m} \text{ in.}^2$$

Nominal pressurant flow rate (all Eqs.)

$$\dot{w}_n = \frac{P_{oxn} \dot{w}_{oxn}}{R T_{en} \rho_{ox}} + \frac{P_{fn} \dot{w}_{fn}}{R T_n \rho_f} \quad \text{lb/sec}$$

P_{oxn}, P_{fn} = tank nominal pressures

$\dot{w}_{oxn}, \dot{w}_{fn}$ = propellant nominal flowrates

ρ_{ox}, ρ_f = propellant densities

Nominal Poppet Position (Eq. 11, Table 1)

$$X_n = \frac{A_{sn}}{2\pi r (a/b) \sqrt{1 - (r/R)^2}} \text{ in.}$$

TABLE III (cont'd)

Poppet - ball pressure force (Eq.(15), Table I)

$$\epsilon = A_P/A_B$$

A_P = Poppet area

A_B = Ball seat (bore) hole area

Volumetric mixture ratio (Eq. (16), Table I)

$$\mu_v = \rho_f \dot{w}_{ox} / \rho_{ox} \dot{w}_f$$

Liquid Effective compressibility (Eq. (17), Table I)

$$\frac{1}{B'} = \frac{1}{B} + \frac{1.25}{(t/D)E}$$

B = liquid bulk modulus

t = wall thickness of line

D = diameter of line

E = Young's modulus for line material

APPLICATION AND VERIFICATION

The generalized methods developed in the previous section find application wherever it is necessary to perform dynamic analysis of pressure-fed propulsion systems. There are many instances when such an analysis is desirable or necessary. One of these is the case where it is desired to perform preliminary analyses of propulsion systems which are yet to be designed. Due to the normalization of the equations and the resulting description of the system in terms of dimensionless groups, it is possible to proceed with such a preliminary analysis of a planned propulsion system well in advance of firm hardware designs. Once the general schematic of the desired system is specified, the model can be constructed and the various constants required in the model can be treated parametrically. For this type of an application, the approach is to vary each of these constants in a systematic way so as to determine the value of each parameter required to produce the desired dynamic and steady-state performance of the final system. Once the desired performance has been achieved, these dimensionless parameters can be used to calculate actual hardware sizes necessary to achieve that performance in the real system.

Another application of the method is in the case when it is desired to provide analytical support for the design effort during the development of a new propulsion system. In this case the model is developed for some base-line schematic and set of parameters, with the thought that as the design advances the effects of each and every design change can be readily evaluated by slight modifications to the model.

Still another application is the case where an existing system exhibits performance which is abnormal or otherwise undesirable, and it is desired to determine the causative factors. In this case the schematic of the system is well known, and also since all physical dimensions are

established, all of the model parameters can be calculated. Assuming that there have been no gross oversights in the construction of the lumped-parameter model from the system schematic, the model will exhibit the same abnormal performance characteristics as the real system. It is then possible to vary the parameters which are suspect and determine the changes which are required in order to achieve the desired performance. Once the factors have been determined, it is a simple matter to make corresponding changes in the actual system schematic or parameters in order to achieve the desired performance in the actual system.

In this section it is shown how the previously developed generalized methods are applied to actual systems, with objectives similar to those described above. The Mariner Mars '69 and Mariner Mars '71 midcourse propulsion systems were chosen as candidate systems in order to demonstrate the techniques employed. Using these systems as examples also permits verification of the analytical techniques employed. This verification is accomplished by comparison of the model results with actual test data previously obtained for these propulsion systems, and various test systems which were devised to simulate the actual propulsion systems. The correlation obtained as the result of these comparisons validates the approach taken, and lends considerable confidence in analytical results which might be generated in the analysis of future propulsion systems using these same techniques.

MARINER MARS '69 SYSTEM APPLICATION

Systems Considered

The Mariner Mars '69 midcourse correction propulsion system is a mono-propellant system using hydrazine as the propellant, and nitrogen as the pressurant. A model schematic representing this propulsion system is shown in Fig. (12a). The pressure regulator employed in this system is shown in Fig. (13). Figures (12b) and (12c) represent test systems in which the regulator of Fig. (13) has been tested; all three of these systems have been modeled. The two test systems were modeled in order to permit correlation between model and test results. The propulsion system itself was modeled in order to permit parametric studies to be carried out.

Development of Equations

Regulator. The MM'69 regulator shown in Fig. (13) consists of a gas flow system and an interacting mechanical system, i.e., the poppet. The dynamic model of the regulator can be obtained by developing a model of each of these systems separately, and joining them by coupling variables.

The gas flow system can be represented by a series of compressible volumes separated by localized restrictions. In the MM'69 regulator, the only significant points of restriction are the filter and the seat, and therefore the models shown in Fig. (12) are representative.* Here the volume V_3 represents the volume between the filter and seat. The Volume V_4 represents the volume within the regulator downstream of the seat, plus that of the outlet tube to the next point of restriction.

* At points other than the seat, gas velocities in the regulator are low. The annular region downstream of the seat represents the next most important point of restriction, and amounts to about 1.3 psi at nominal flow conditions.

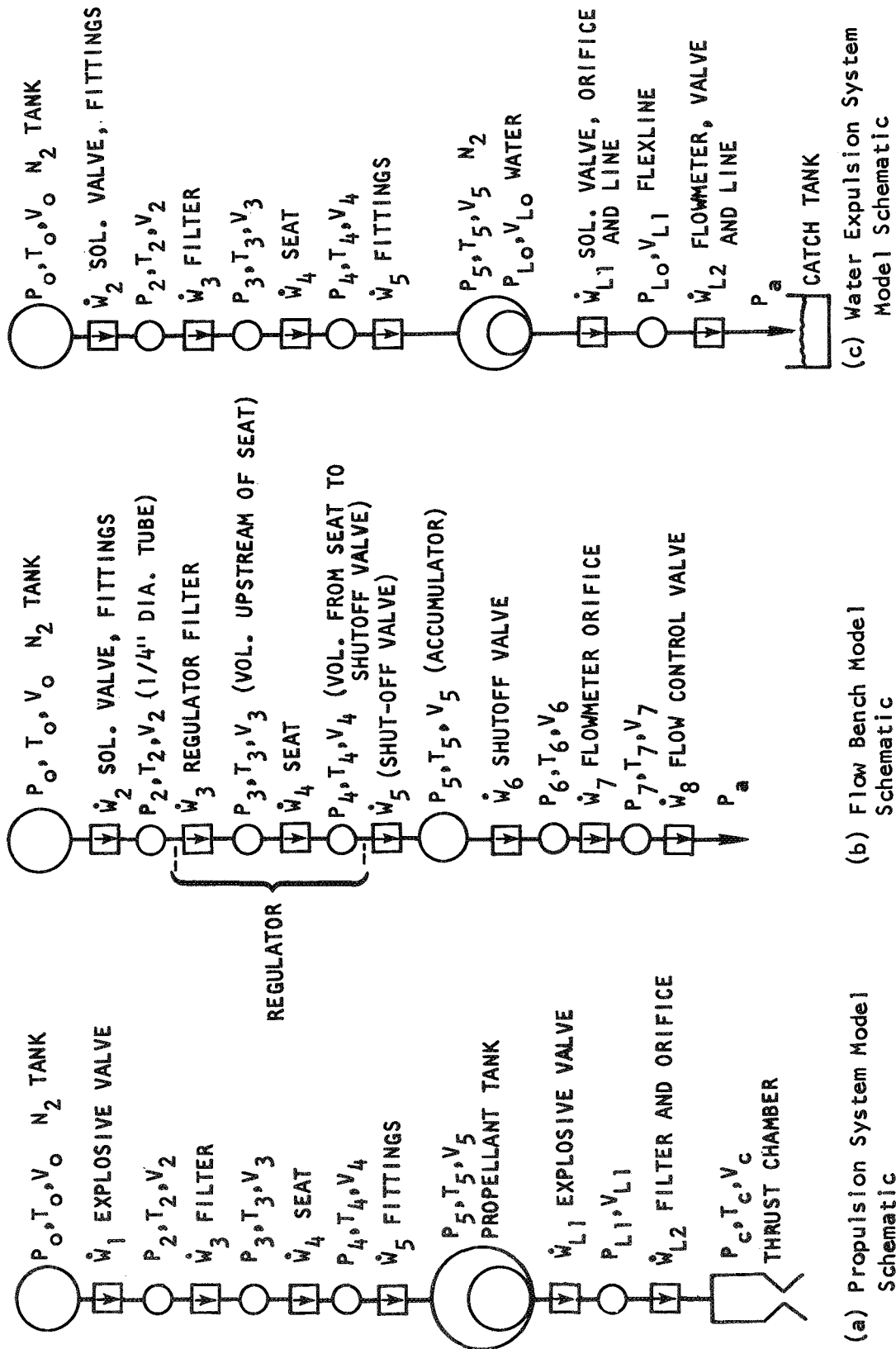


Figure 12. Mariner Mars '69 Systems

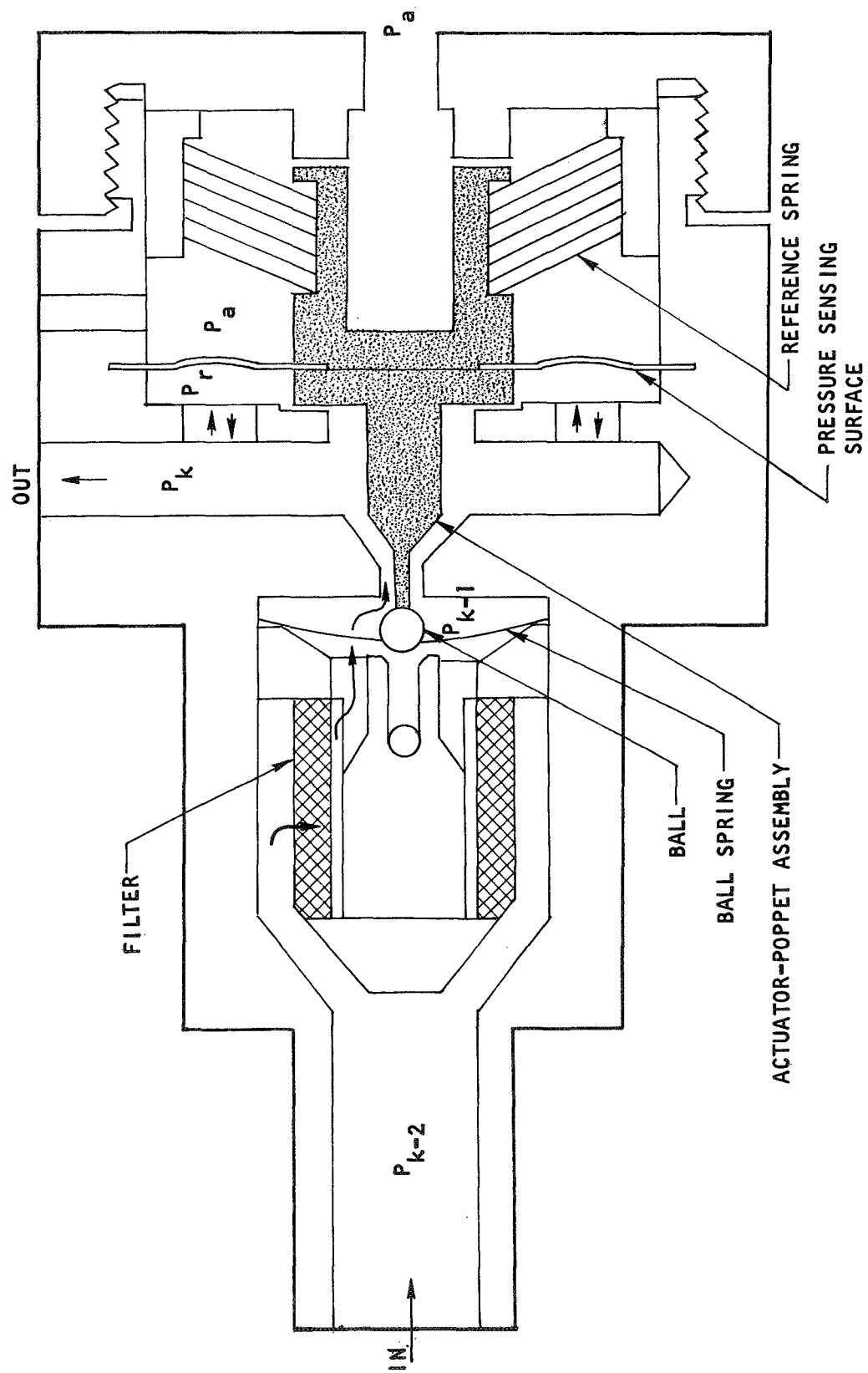


Figure 13. Simplified Cross-Section of MM '69 Pressure Regulator

The pressures and temperatures at the lumped volume or nodal points can be determined from the equations of conservation of mass and energy. The flow rate between nodal points can be determined by a flow equation as indicated by experimental data. Data for the filter, for example, indicate that laminar flow exists and the flow rate is related to pressure by Eq. (8) of Table I.

Flow rate through the seat follows the compressible flow orifice equation, Eq. (5) of Table I. In this equation C_d is the discharge coefficient, which can be assumed to be constant due to the small changes in ball position. The factor ϕ is a function of upstream and downstream pressure given approximately by Eq. (7) of Table I.

The stored weight of gas in the lumped volumes is found from the conservation of mass equation, Eq. (1) of Table I. The same equation is applicable for the stored weight downstream of the seat, W_4 . The temperature within each lumped volume can be calculated from the conservation of energy. Assuming perfect gas behavior and an adiabatic process, this is represented as Eq. (2) of Table I. Once the stored weight and temperature within each volume have been determined, the local pressure can be calculated from the perfect gas law, Eq. (4) of Table I.

The lumped parameter model of the gas flow equations for the MM'69 regulator does not have a separate volume representing the diaphragm cavity. An early model included the diaphragm cavity and accounted for flow in and out through the provided passages. However, due to the large flow areas of the passages, the pressure difference between the cavity and the reference volume was found to be negligible (less than 1 psi) even during high-frequency oscillation of the diaphragm. The model was therefore simplified to that shown in Fig. (12) by elimination of the diaphragm cavity. In this model the diaphragm pressure is assumed to be the pressure

immediately downstream of the seat, P_r . It is notable that if the passages to the diaphragm cavity were smaller, this simplification would not be possible.

The mechanical system consists of the poppet diaphragm, spring assembly, and ball. The forces acting on the poppet are the diaphragm pressure force, the balancing spring force, and the force transmitted through the ball. The sum of these forces determine the instantaneous acceleration, integration of which yields the velocity and position. Position of the poppet is limited by mechanical stops, and that of the ball by the seat.

If it is assumed that the ball follows the poppet perfectly, there is only one equation of motion. Assuming a position variable X which is positive as the ball is lifted from its seat, the equation of motion is given by Eq. (12) of Table I. In this equation F_s is the reference spring force, which is a function of spring deflection and rate of change of deflection. The dependence on rate of change is indicated by the hysteresis of the experimental force-deflection curve. The second term is the diaphragm pressure force, which is the effective area times the pressure difference across the diaphragm. A damping term, $B\dot{X}$, is included, although the coefficient is not easily determined and will most likely be small. The force transmitted through the ball is the sum of the ball retaining spring force and the pressure forces which act on the ball, F_{bs} and F_{pr} respectively. The function $U(X)$ is a unit step function which removes the ball forces from the poppet when the ball is resting on the seat.

A typical poppet spring force characteristic is shown in Fig. (14). The non-linear characteristic derives from the fact that the assembly contains a number of Belleville springs. The hysteresis exhibited in the

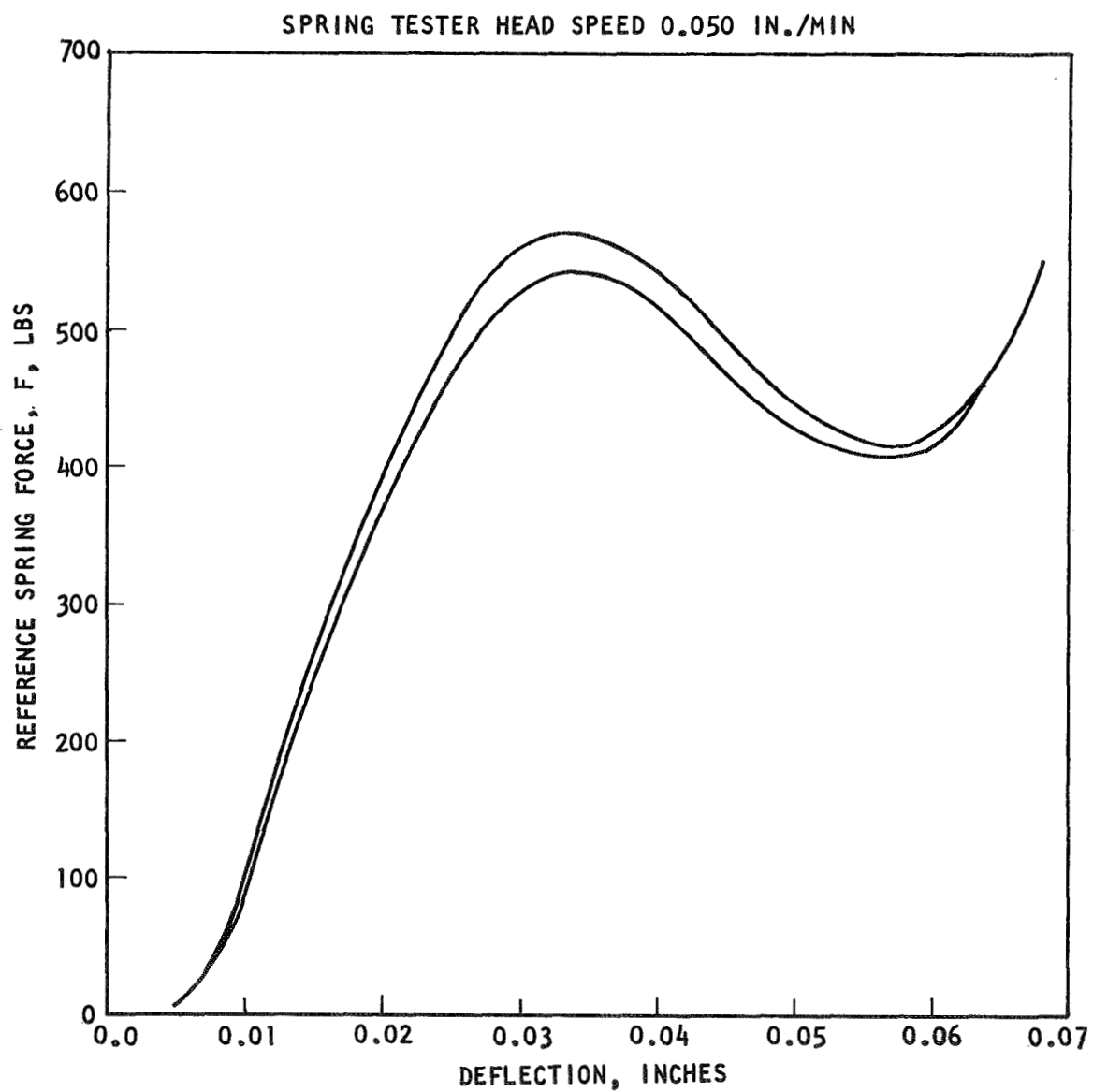


Figure 14. Typical Reference Spring Characteristics for MM #69 Regulator
(S/N 001 Spring Assembly with S/N 6 Spring Removed)

characteristic represents an irreversible conversion of mechanical energy to internal energy in the spring material. The characteristic can be approximated, including the hysteresis, by considering the force to be the sum of two terms, one dependent only on deflection and the other dependent only on velocity, is indicated by Eq. (15) developed in Section I. The deflection function must be assumed to be the mean curve in the absence of contrary evidence. Ideally, one could generate $F_s'(X)$ by measuring steady-state forces, as opposed to the curves in Fig. (14) which were generated at a finite velocity. Using the mean curve for $F_s'(X)$, the velocity-dependent term $F_h(\dot{X})$ is then an odd function. A function which fits the available data is shown in Fig. (8). This function is based upon the observed height of the hysteresis of ± 15 pounds and the given head speed of the tester of 0.05 in/min. It assumes that the height of the hysteresis loop has reached its maximum at this velocity. It is notable that for small changes in poppet velocity, this representation of hysteresis amounts to extremely high damping. Also, if poppet motion is reduced to zero gradually, as in the case of a spring tester, there is a smooth convergence to the steady-state curve, as is suggested in Fig. (14).

The spring deflection, δ , is the sum of the deflection of each end. Noting that the poppet position coordinate, x , is positive in the opening direction, the total spring deflection is then

$$\delta = \delta_0 - x$$

where δ_0 is the deflection effected by rotation of the adjustment cap. This equation permits the set point to be variable parameter in the model.

The effective diaphragm area is less than the total area since the outer edge is constrained. The effective area of the diaphragm to be used in Eq. (12) of Table I can be determined by equating the work done by the gas moving into the elemental volume caused by an elemental change in X to the work done moving the resisting force through the elemental change in X .

That is,

$$(P_L - P_a) dV = A_e (P_L - P_a) dX$$

It is seen then that

$$A_e = \frac{dV}{dX}$$

By careful calculations using the regulator dimensions and assuming the diaphragm to deflect in the shape of a cone, it can be shown that $A_e = 1.665 \text{ in}^2$.

The ball retaining spring is a Belleville-type spring with deep cut-outs around the outer circumference, making it star-shaped. In its nominal position it is compressed beyond its flat position. In the absence of actual force-deflection data for this spring, and in view of the small range in ball movement, the ball spring force is approximated by the linear equation given as Eq. (14) of Table I.

Pressure forces acting on the ball are also transmitted to the poppet when the ball is not on the seat. The pressure force on the ball can be estimated by considering the geometry of the constriction of flow between the ball and the seat. This is shown at 100 times actual size in Fig. (9). Due to the small size of the flow passage relative to the ball

and bore diameters, it is apparent that the downstream static pressure, P_4 , acts on the ball over the entire bore diameter. The corresponding area on the upstream side of the ball is acted upon by the upstream static pressure, P_s .

Therefore the ball pressure force is

$$F_{pr} = A_b (P_s - P_4)$$

This equation is obtained by elimination of the second term of Eq. (15) of Table I. At nominal conditions, i.e., with $P_s = 3570$ and $P_r = 308$ psia, $F_{pr} = 15.65$ pounds. This is approximately 3 percent of the nominal force applied to the poppet by the poppet spring.

Equations discussed above are sufficient to compute ball and poppet position as a function of time, provided that there is always contact pressure between the poppet and ball. Due to the high pressure force on the ball and the low mass of the ball, this will most likely be the case when upstream pressure is high. When upstream pressure drops, the pressure force falls to nearly zero, leaving the ball with only its retaining spring to hold it firmly against the poppet. Under these conditions the ball might "float" at high frequencies of poppet motion. The model as presented herein will not reflect this phenomenon, so further investigation may be required if indicated by comparisons with the test data.

Flow Bench System. The flow bench test system for the Mariner Mars '69 pressure regulator, as shown in Fig. (12a), was modeled by repeated application of the elemental equations developed in the previous section and summarized in Table I. For example, the pressure and temperature within the pressurant tank are described by Eqs (1), (2), and (4) of Table I. The pressure in each of the lumped volumes, which represent the volume of the feed system between the lumped restrictions, are all represented by Eq. (4).

It is notable that in this test system there is no propellant tank and, therefore, the expulsion term in Eq. (1) and the work term in Eq. (2) of Table I were not employed in this model.

Flow-through valves and orifices were represented by Eqs. (5) and (7) of Table I. The system filter for the flow bench set-up was found to result in insignificant pressure drop, and was, therefore, eliminated from the model.

A summary of the equations and constants utilized for the simulation of the low bench system is presented in appendix A, the derivation of the dynamic temperature equation [Eq. (2) of Table I] and analog computer diagrams are also included.

Water Expulsion System. The second test system in which the Mariner Mars '69 pressure regulator was tested is shown schematically in Fig. (12c). A model of this system was also developed. Due to the similarities in the schematics upstream of the accumulator, the model of the water expulsion test system was based upon the flow bench model discussed above. It was only necessary to include the expulsion and work terms in equations for temperature and stored mass in order to account for the variable volume of the propellant tank as opposed to the constant volume accumulator in the flow bench model. It was also necessary to replace the gas flow elements downstream of the accumulator by a liquid flow system as indicated in Fig. (12c).

The water expulsion test system, liquid portion, consists physically of a start valve, calibration orifice, and line within an environmental chamber. This system was connected to a point outside of the environmental chamber by means of a flex line. The flow then passed through a flowmeter, a valve and an additional length of line outside of the test chamber. The model, shown in schematic form in Fig. (12c), represents this system as a single lumped resistance immediately downstream of the tank, a compressible volume representing the flex line, and another

lumped resistance downstream of the flex line. The lumped resistances to the flow are represented by Eq. (18) of Table I. Pressure within the flex line is represented by Eq. (17) of Table I.

The equations and constants used to simulate the water expulsion system are summarized in appendix A, along with the deviation of the effective bulk modulus relationship and the analog computer diagrams.

Mariner Mars '69 Propulsion System. The propulsion system represented by the schematic in Fig. (12a) was also modeled. It is apparent by comparison with the water expulsion system model in Fig. (12c) that the propulsion system can be represented by essentially the same model as the water expulsion system. It is only necessary to add the equations for the thrust chamber. This similarity is to be expected, since the water expulsion model was designed to simulate the actual propulsion system.

The thrust chamber affects the rest of the propulsion system by creating a relationship between pressure and flow rate at the terminal of the liquid system. In the water expulsion system, the exit of the liquid flow system was always exposed to ambient pressure, whereas in the thrust chamber the feed system is exposed to chamber pressure, which is a function of flow rate and temperature of the thrust chamber. The model must therefore include equations representing the pressure, within the thrust chamber. The temperature within the thrust chamber is an intermediate quantity affecting this pressure.

Eq. (22) of Table I was used to calculate the temperature of the decomposition products within the thrust chamber. Eq. (21) was then used to represent the actual pressure within the thrust chamber. These equations were incorporated with the previously developed water expulsion system, yielding the model of the propulsion system. The details of this model are summarized in appendix A.

Analog Model. Analog models were constructed for each of the three Mariner Mars '69 systems described above. These models were constructed

on the Applied Dynamics AD-256 analog computer in the Analog Simulation Facility at the Rocketdyne Canoga Park plant. These computers employ bipolar amplifiers, solid-state quarter-square multipliers, and offer a large number of logical functions. This computer is tied into a Digital Equipment Corporation PDP-8 digital computer, which is employed for automatic coefficient setting and automatic read-out.

Amplitude scaling for the analog model was according to the method discussed in Section I and shown in Table II. The computer has an automatic time scaling feature, so that the problem solution could be easily changed from real time to 10 or 100 to 1 time scaling.

The analog diagrams for these models are given in appendix A.

The only difficulty encountered in the development of the analog model from the previously developed equations was due to the large difference in the time constants of several of the dynamic elements of the system. For example, the pressurant storage tank is large enough to operate the system for a 90 second period, while the poppet assembly of the regulator is extremely light and is acted upon by large forces, causing the regulator poppet to move stop-to-stop in far less than a millisecond. When dynamic elements with responses as widely separated as this are incorporated in the same analog model, the integrator gain for the faster element must be near the maximum permitted by the machine, while that for the slower element must have the minimum gain. The slowing-down of the problem solution time reduces the required gain for the fast integrator, but at the same time reduces the required gain at the slow integrator. When the slow element is so slow that the required gain is much less than 1, errors are introduced due to the inability to set coefficient potentiometers accurately beyond the third decimal place. In the Mariner '69 system, the differences in these responses were such that the full range could just barely be accommodated.

One solution to the problem caused by wide differences in element responses is to consider two modes of model operation. For example, if the transients associated with regulator poppet motion are of interest, the problem should be time scaled 10 or perhaps 100 to 1 so that the dynamics of this element can be accurately represented. However, at this timescale the gain of the integrators for the larger volume elements are so small that the upstream pressures are essentially constant. This time scale is entirely unsuitable for simulation of an entire burn-period, since the inability to set the required small gains introduces inaccuracies, and also at this timescaling a solution time would be too long. Therefore, the second mode of operation of the model would be when it is desired to observe the entire burn period. In this case the model should be run in real time, and if necessary sacrifice dynamic accuracy of the faster elements. This sacrifice of this dynamic accuracy should not introduce any inaccuracies in the slower dynamics of the system. However, in this mode it is not possible to make any judgments about the dynamics of the faster elements. For example, it is not possible to predict stability of the higher frequency modes from data obtained from the model in which the high frequency accuracy has been sacrificed.

In the Mariner '69 system a time scale of 10 to 1 was a compromise which permitted a reasonable solution time and a sufficiently accurate representation of both low and high frequency system dynamics. For this reason most of the data generated for the Mariner '69 system was at 10 to 1 time scaling.

Another difficult encounter in the analog mechanization of the system model was due to the extremely low pressure drop across one of the system flow elements. It is evident from the compressible flow function

shown in Fig. (5) that when the pressure ratio across a flow element is nearly 1, the rate of change of flow with respect to pressure ratio approaches infinity. On the analog computer, this situation translates into dynamic loops having extremely high gains, and for pressure ratios close enough to 1, can challenge the stability limits of the analog computer itself. This is particularly true when the compressible flow function is timeshared at several points in the model by means of the multiplexing methods. The multiplexing method (see appendix A) works well only so long as the required value of the generated function does not change greatly between sampling intervals. Operation too close to the pressure ratio of 1 on the curve of Fig. (5) can invalidate this assumption. It is, therefore, necessary to use extreme caution in representing flow devices of extremely low pressure drops. Devices in the physical system which have extremely low pressure drops are best omitted entirely rather than attempting to operate at pressure ratios above .995.

In the Mariner '69 system the start valves at the exit of the pressurant tank have extremely low pressure ratios due to the large flow area, low flow rate and high pressure in this region. Yet they could not be omitted since it was desired to be able to control flow at this point in the model, as it is in the real system. In order to include this element in the model and at the same time avoid the problems discussed above, the pressure ratio across these valves was arbitrarily taken to be .99. This represents a pressure loss through the fitting of approximately 1% which amounts to approximately 30 psi, which is considerably higher than the actual pressure drop across such a valve. This departure from reality, however, has no substantial effect on the dynamic performance of the system.

It was also found in the development of the analog model of the Mariner '69 propulsion system that the dynamics of the liquid system compressibility are several orders of magnitude greater than those of the gas system. For the same reasons as discussed previously, it was impossible to accurately represent these dynamics at the same time scaling which permits simulation of the entire burn period with a reasonable solution time. In view of the secondary effect that the liquid system has on the pressurization system, which is of primary importance here, the accuracy of the liquid line dynamics were sacrificed. The gains of the integrators in this portion of the analog model were simply made as high as the analog computer permitted. It is felt that no significant reduction in the accuracy of the results occurred because of this simplification.

Results and Correlations for the Mariner '69 Systems

Actual test data for the Mariner '69 pressure regulator, in both the flow bench test setup and the water expulsion test setup, revealed both normal and abnormal modes of operation. In the abnormal modes of operation, large-amplitude cycling of the downstream pressure was observed, instead of the desired constant pressure. Results obtained with the analog model of both the flow bench and the water expulsion system revealed both of these modes of operation. A comparison between these results is provided in Fig. (15).

In each of the four plots of Fig. (15), the downstream (regulated) pressure is plotted as a function of the upstream or storage tank pressure. These curves are called "regulation curves" or "characteristic curves" for the regulator. Obviously, for a perfect regulator the characteristic curve would be perfectly flat, i.e., the downstream pressure would not

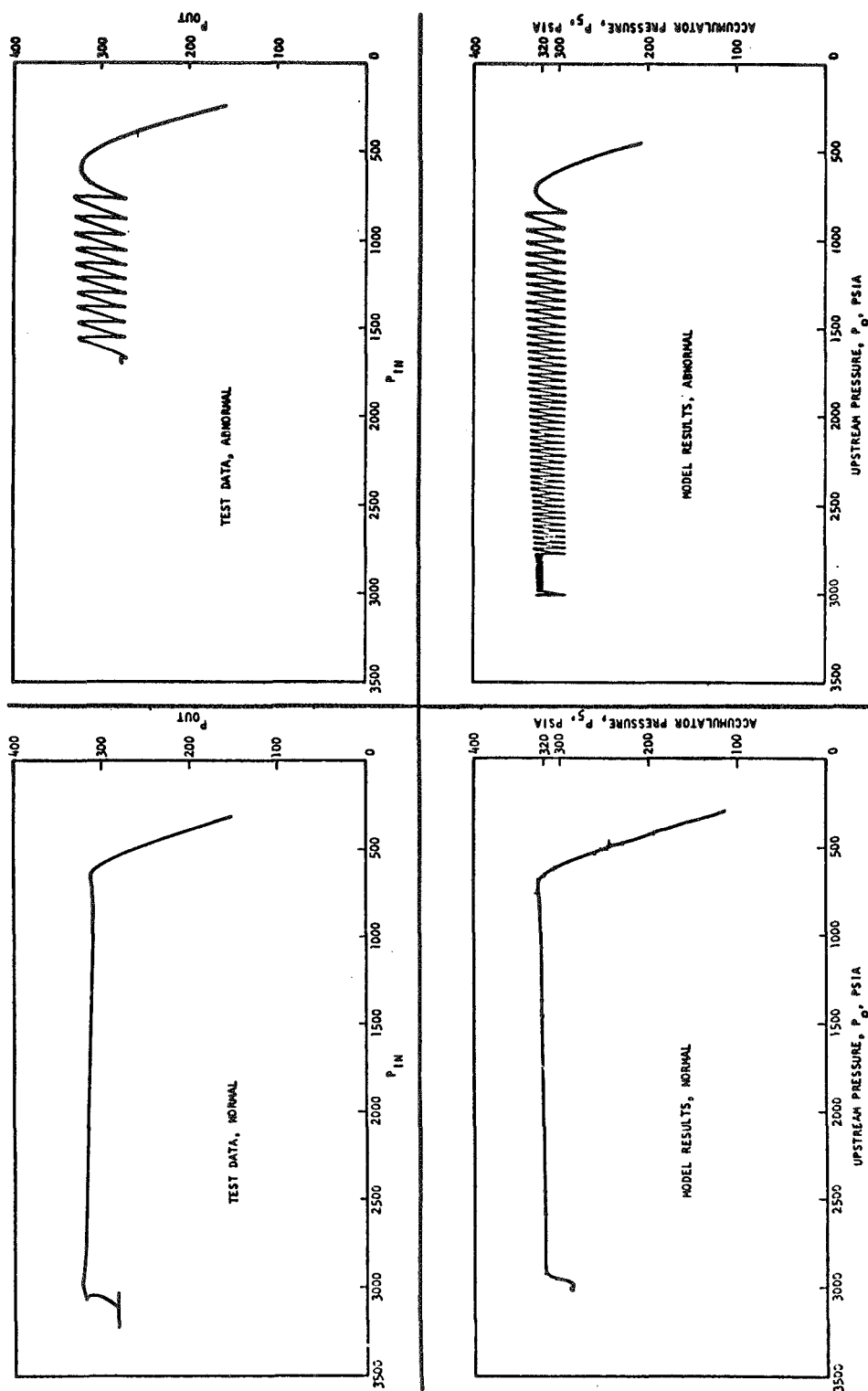


Figure 15. Comparison of Model Results With Test Data for MM '69 Flow Bench

vary as the upstream pressure changed, since this is the function of the pressure regulator. However, in any real regulator some departure from constancy in the downstream pressure is always observed, due to changing conditions within the regulator.

The two left hand plots in Fig. (15) are normal characteristic curves for the Mariner '69 regulator. The starting point on these curves is at the left where upstream pressure is high and downstream pressure is lower than the design value. As the valves are opened the upstream pressure begins to decay and at the same time the downstream pressure begins to approach the nominal value. Throughout the longest period of operation, the regulation pressure is flat as it should be. However, eventually the breakpoint is reached where the upstream pressure has fallen so low that the widest opening of the regulator will not pass sufficient flow rate of pressurant to maintain the design pressure in the downstream system. At this point the regulated pressure begins an uncontrolled decay, i.e., the blowdown mode is entered.

The upper left hand curve in Fig. (15) is an actual test of the Mariner '69 regulator during normal operation. The lower left hand curve was produced by the flow bench system model developed according to the techniques developed herein. The similarity in these curves is a measure of the accuracy with which the model reflects the actual system; in most respects it is seen the agreement is quite good.

In the upper right hand curve in Fig. (15) is presented actual test data for the Mariner '69 pressure regulator in the flow bench system while it is experiencing abnormal operation. It is noted that the regulation pressure cycles about a set point. The lower right hand curve was produced by the analog model developed in this study. Similarity is evident.

It is on the basis of the information presented in Fig. (15) that it can be aserted that the modeling techniques presented herein produce a model which closely predicts the operation of the actual system.

The analog models developed in this study were also used to investigate the causes of the abnormal behavior exhibited by the Mariner '69 pressure regulator in its test systems. It was found that this abnormal mode is a direct result of the negative slope of the spring characteristic curve for this regulator, Fig. (14). It was found that while a certain degree of negative slope on this curve was permissable, an excessively negative slope results in this instability. The degree of negative slope which can be tolerated is a function of the hysteresis characteristic of the spring, since the hysteresis mechanism is the only physical damping present in the system.

Further investigations performed with these models revealed the importance of the pressure ratio between the regulator outlet and the ullage volume of the propellant tank or the accumulator. It was found that the frequency of the abnormal oscillations corresponds with the ullage volume if this pressure ratio is extremely close to unity. It is in this case that low frequency, high amplitude, abnormal oscillations are encountered. When there is a large pressure drop between the regulator and the ullage volume, the regulator pressure sensing surface responds to the pressure in the compressible volume between the regulator and the point of restriction, instead of to the pressure in the ullage volume itself. In this case any oscillations which occur are therefore of a higher frequency and lower in amplitude.

On the basis of these findings it was recommended that if the Mariner '69 regulator were to be used in other applications, the reference spring

design should be changed to give a positive slope at the operating point. It was also recommended that there be a large pressure drop designed into the flow system between the regulator and the ullage volume.

Prior to development of the non-linear model of the MM '69 regulator, a linear model of the water expulsion system was formulated and analyzed with respect to the test results. The purpose of this effort was to determine the cause for regulator discharge pressure fluctuations observed in the test data. The results would then be used to guide the development of the detailed regulator model and to indicate the degree of sophistication required to accurately predict the performance of future regulators. The details and results of the linear analysis are presented in appendix B.

MARINER MARS '71 SYSTEM APPLICATION

The Mariner Mars '71 midcourse correction propulsion system was selected as the second example case in order to further verify the methods of analysis developed herein, and also to further demonstrate the application of these methods. This system is a bipropellant propulsion system as opposed to the Mariner '69 monopropellant system. It operates on the propellant combination N_2O_4/MMH operating at a mixture ratio of about 1.5. It is designed to provide 300 lbs of thrust and to operate at a chamber pressure of 120 psia. The propellants are pressurized by nitrogen.

In this section the Mariner '71 propulsion system, and the corresponding lumped parameter analytical model are described in detail. It is shown how the equations representing the model are developed from the generalized equations presented in Section II. The parameters which determine this system are developed, and an analog model is devised from these equations and parameters. It is shown that results obtained with this analog model agree closely with test data obtained from a test system which simulates the Mariner '71 propulsion system.

System Description

The Mariner Mars '71 propulsion system is a bipropellant pressure fed propulsion system. In this system, there is a common pressurization system which supplied pressurant and maintains pressure in both propellant tanks. Thus there is a single pressure regulator. Fig. (16) shows a lumped parameter model schematic which represent the essential elements of the Mariner '71 system. It is seen that this model consists of a storage tank, a series of lumped restrictions and compressibilities representing the regulator and downstream flow system, the propellant tanks and liquid feed system in parallel, and finally the thrust chamber.

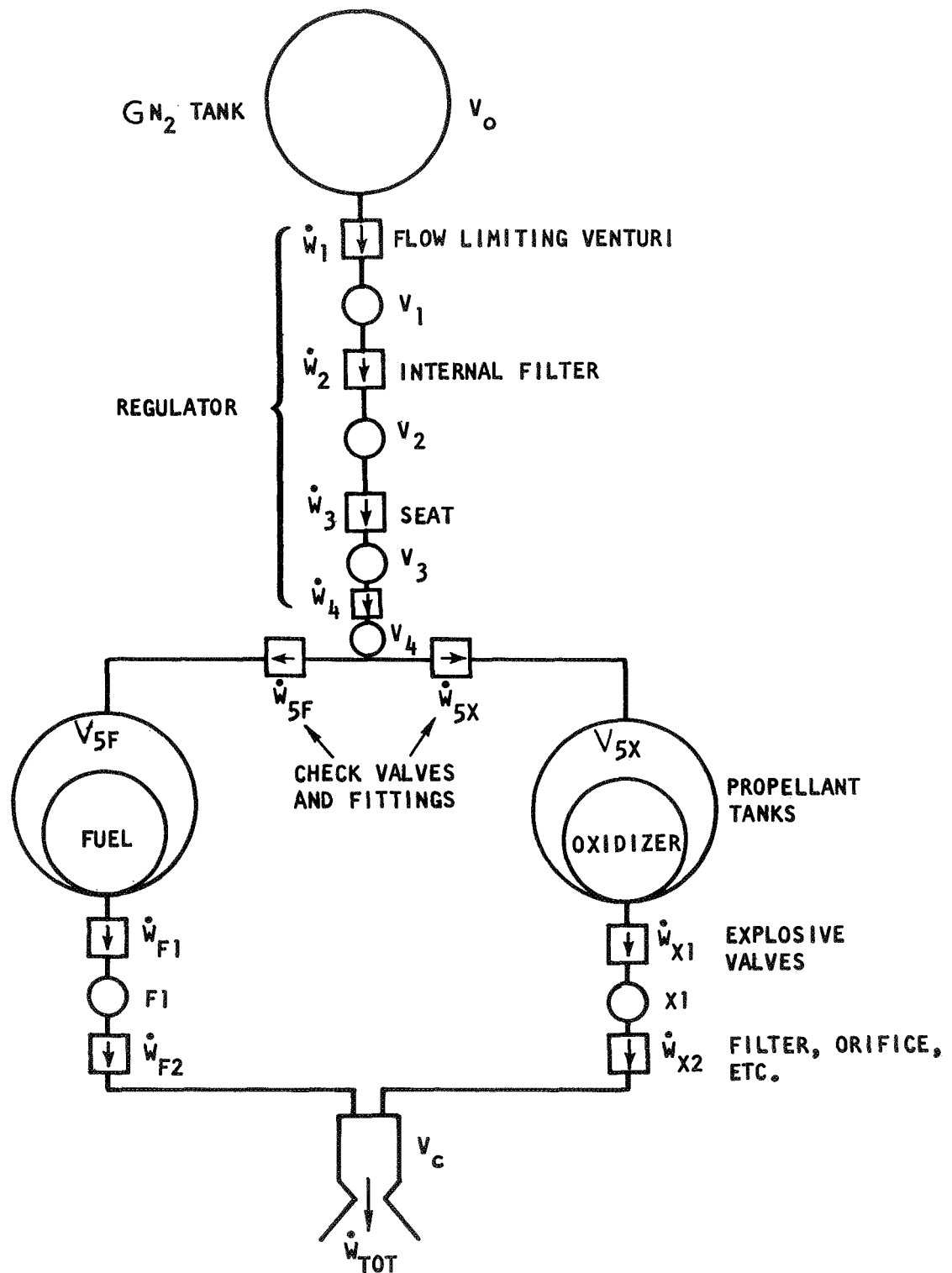


Figure 16. Lumped Parameter Model Schematic for MM '71 System

It is notable that Fig. (16) does not show explicit points (valves, fittings, volumes, etc.) in the pressurization system upstream of the regulator. This simplification was made in order to achieve a model of manageable portions. A justification for this simplification comes from several findings of the analysis of the Mariner '69 system. There, it was found that system parameters upstream of the regulator did not have a significant effect on the dynamics and operation of the low-pressure portion of the system, the latter being the area of primary interest in this analysis. This model will yield valid information about pressure and flows through the regulator and in the low pressure portions of the system, so long as the tank pressure of this model is interpreted as the pressure immediately upstream of the regulator entrance.

The Mariner '71 pressure regulator is shown in diagrammatic form in Fig. (17). It is obvious that this regulator resembles closely the generalized model shown previously as Fig. (4). This particular regulator is characterized by a flow limiting venturi upstream of the seat which is built into the regulator, and an isolated pressure sensing surface. Regulation occurs through flow modulation at the seat formed by a spring-retained ball resting in a hole. The ball is actuated through a lever mechanism and a ball actuator. The pressure sensing surface consists of a bellows assembly. The poppet to which the bellows is attached is loaded by a pair of helical springs operating in parallel. Flow into the reference cavity is through a small orifice which connects it to the cavity of the main pressurant flow path immediately downstream of the seat. This regulator is a standard design supplied by the National Water Lift Company, adapted for the Mariner '71 propulsion system.

The model of the gas flow system of this regulator may be seen in Fig. (16). It is seen that the first restriction represents the flow limiting venturi.

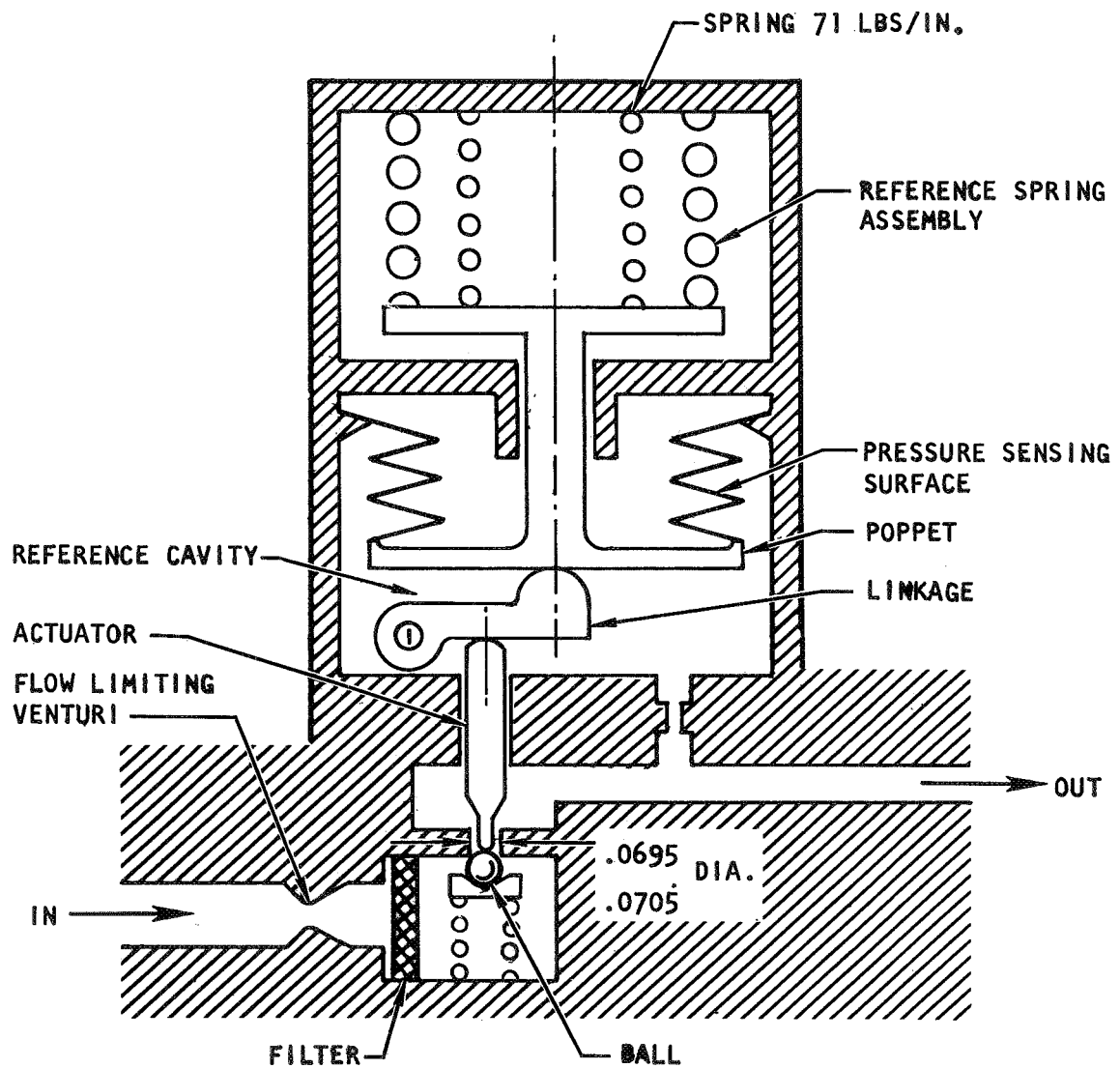


Figure 17. Schematic Mariner '71 Regulator NWL Model 6894

The next restriction is the internal filter, which is separated from the flow limiting venturi by a small compressible volume. The regulator seat constitutes the third restriction, and the fourth restriction represents that of the outlet fittings and other constrictions of the regulator.

Another important and distinguishing feature of the Mariner '71 pressure regulator is the regulator flow area as a function of poppet position. Due to the relatively long stroke of the ball and actuator, the point of minimum area, i.e., maximum restriction, can change from the seat itself to the area of the upstream flow limiting venturi. Figure (18) shows the flow area of the seat as a function of ball position off of the seat. It is seen that for small openings, the seat is indeed the point of minimum restriction and thus the point at which the flow will be controlled. However, when the seat opens beyond .014 inches, it is seen that the upstream venturi represents the smaller flow area. Therefore, beyond this opening the upstream venturi will be the point of flow control rather than the seat. Furthermore, even if the upstream venturi were not present, opening of the seat beyond .016 inches would produce no further increase in effective flow area of the regulator, since the annular area between the body and the ball actuator would then become the minimum flow area in the system, and thus the point at which the flow was limited.

The pneumatic system downstream of the regulator consists of the compressible volume at the point where the pressurant flow is divided, a check valve in the line going to each propellant tank, and the ullage volumes of each propellant tank. The check valves in this system are designed to prevent mixing of the two propellants in the feed system during certain periods of abnormal operation or handling. They are of a spring loaded poppet design, designed to open at a 2 psi pressure drop.

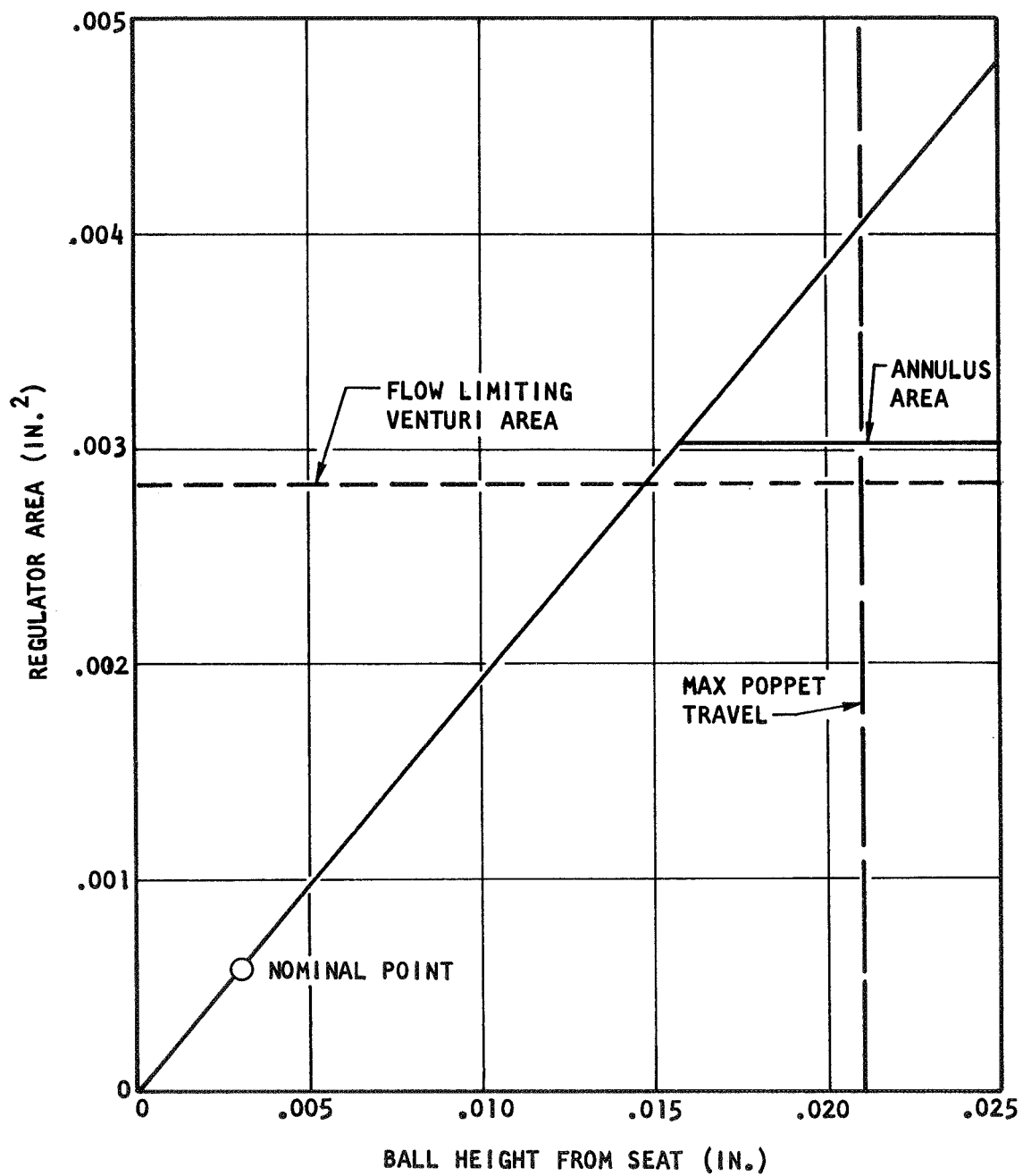


Figure 18. MM '71 Regulator Flow Area

The schematic shown in Fig. (16) is intended to represent not only the Mariner '71 propulsion system, but also the test systems which were designed to simulate the actual propulsion system. These test systems are similar schematically, and also are designed to have similar flow geometry. Of course, the regulator shown in Fig. (17) is employed in all of these test systems.

The liquid portion of the flow system as shown in Fig. (16) is intended to be typical of the Mariner '71 propulsion system, and hence typical for its test systems. It was found during the analysis of the Mariner '69 system that the characteristics of the liquid flow system have negligible effect on the results for the pressurization system. This justifies the use of the simplified model of the liquid portion of the system as shown in Fig. (16). The first restriction represents the system start valves and the second represents the pressure loss due to filters, calibration orifices and the injector. Line losses are lumped along with these local restrictions. The two restrictions are separated by a compressible volume, although the compressibility in the liquid system has no significant effect on the analysis of the pressurization system. They are included only because it permits maximum use of analog circuitry developed during the analysis of the Mariner '69 system.

The thrust chamber is also shown in the schematic in Fig. (16). As in the case of the monopropellant propulsion system, the thrust chamber in this analysis is significant only insofar as it permits determination of liquid system exit pressure as a function of propellant flow rates.

Development of Equations

All equations required for the modeling of the system shown in Fig. (16) were developed in generalized form in Section II, and are presented in Table I. Also, the subscripting system employed in Fig. (16) follows the conventions established in Figs. (2) and (3).

The gas flow portion of the system of Fig. (16) can be represented by repeated application of Eqs. (1) through (8) of Table I. The flow restriction devices representing the flow limiting venturi, the regulator seat, the regulator outlet restriction, and the downstream check valves and fittings can all be represented by Eqs. (5), (6) and (7). The flow rate through the filter, assuming it to be a laminar flow device, can be represented by Eq. (8). The pressure within each of the compressible volumes can be represented by Eq. (4), with the instantaneous mass, temperature, and volume to be determined by Eqs. (1), (2), and (3) respectively. This completely defines the flows, pressures and temperatures at each point in the lumped parameter model.

The check valves in the Mariner '71 system are treated in this analysis simply as points of flow restriction at which the flow is limited to the downstream direction only. The dynamics of the poppet motion within the check valves is not simulated. This is an expedient simplification of this analysis, and may not be justified in a more detailed analysis. The only justification offered here is that the period of primary interest during this analysis is the initial pressurization of the propellant tanks, during which the pressurant flow rates are extremely high. Thus the check valves will always be open during this period. If it was desired to analyze this system in all possible modes of operation, it would most likely be necessary to develop equations of motion for the poppets and model these valves in detail.

The equations for the main pressure flow through the regulator were discussed above. The complete description of the regulator, however, requires the mechanical equation of motion for the poppet assembly and other mechanical elements, and also the equations for the conditions within the reference pressure cavity.

The equation of motion for the mechanical portion of the Mariner^{'71} pressure regulator is presented as Eq. (12) of Table I. Due to the sliding contact of both the poppet and the ball actuator (see Fig. 17) both coulomb friction terms of Eq. (12) are required. The viscous damping term, B, in Eq. (12) need only be included for the poppet portion of the assembly. Thus the only term which is neglected in Eq. (12) is that of the viscous friction on the actuator. This is justified since no estimates are available for viscous friction acting on either the poppet or the actuator. By including a viscous damping term on the poppet/bellows assembly, the effect of this form of damping can be determined during operation of the model.

The spring force term in Eq. (12) accounts for both of the reference springs in the Mariner^{'71} regulator, and also the effective spring rate of the bellows. Assuming all of these spring forces to be essentially linear over the range of poppet travel, the spring force can be represented by Eq. (13) of Table I. In this equation k_s is the total effective spring rate of the entire assembly, i.e., including both springs and the bellows. In the normalized, scaled equations, Eq. (13c) of Table I, F_{sn} is then the total force exerted by all three of these elements when the poppet assembly is at its nominal position. By looking at the steady-state conditions of the poppet at the nominal point, it can be shown that F_{sn} is found from the equation

$$F_{sn} = (P_{3n} - P_a)A_e + (a/b) (F_{bsn} + (P_{2n} - P_{3n})A_b) \quad (55)$$

Due to the assumption of a linear reference spring function, Eq. (12) can be simplified for the case of the Mariner⁷¹ regulator. By substituting Eq. (13) in Eq. (12) and multiplying through by the coefficient of the spring force term, it is found that the constant portion of the spring force term is directly additive to the constant term due to the ambient pressure acting on the reverse of the pressure sensing surface. The spring force term in the equation of motion then becomes a term in the first power of the position variable, the coefficient of which is the spring rate over the mass of the moving assembly, k_s/m . This coefficient is recognized as the square of the natural frequency of the second-order system represented by the poppet moving assembly.

The ball retaining spring may also be assumed linear. In this case Eq. (14) of Table I represents the force of this spring. By looking at the normalized equation, Eq. (14c), it is seen that only the effective spring rate of the retaining spring, the maximum excursion of the ball, and the force exerted by the retaining spring at the nominal ball position are required for the representation of this force in the normalized system. It is notable that when the ball is off of the seat, the unit step function is unity in Eq. (12c), in which case the ball spring force equation adds directly to the reference spring force equation.

The pressure forces acting on the ball and poppet of the Mariner⁷¹ regulator are represented in Eq. (15) of Table I. In Eq. (15a), it is seen that these pressure forces include the pressure exerted on the ball and also the pressure exerted on the poppet. When this equation is normalized as in Eq. (15b) it is seen that the coefficients depend only upon the pressure ratio across the ball, and the ratio of the poppet cross-sectional area to the cross sectional area of the bore which forms the seat. This area ratio is designated by ϵ .

The relation between the poppet position and the flow area at the seat of the regulator is given by Eq. (11) in Table I. In the basic equation, Eq. (11a), it is seen that this area is directly proportional to the position of the poppet, but is limited to a maximum area, which is determined in the case of the Mariner '71 regulator by the annular area between the ball actuator and the regulator body.

From Fig. (17) it is evident that the pressure sensing surface of the Mariner '71 regulator is isolated from the flow system proper. The reference pressure cavity communicates with the flow system pressure only through a small reference orifice. Due to the small net flow of this cavity, the temperature in the cavity may be assumed essentially constant and equal to the body temperature of the regulator. The volume of this cavity is linearly related to the position of the poppet assembly. Since the volume change is very small due to the small travel of the poppet assembly, Eq. (4a) can be linearized yielding the linear relationship between pressure, the stored mass with the cavity, and the position of the poppet, as indicated in Eq. (9) of Table I. The position of the poppet is obtained from the integration of Eq. (12) of Table I. The amount of stored mass in the reference pressure cavity is given by Eq. (1) of Table I, which is a linearized version of Eq. (5).

Equations are also required for the conditions and properties within the propellant tanks. It is necessary to determine the pressure, temperature and ullage volume in these tanks as a function of time during the operation of the system.

Introducing the usual assumptions of ideal gas behavior, the pressure within either of the propellant tanks is given by Eq. (4) of Table I. In this equation, it is seen that the instantaneous stored mass, temperature and volume of the ullage space are all required in the calculation

of this pressure. Physically, the ullage volume is simply the tank volume minus the volume of the remaining propellant in the tank. Thus, the volume of the ullage space grows with time as the propellant flows out of the tank. This is expressed in Eq. (3) in Table I. This states that the instantaneous ullage volume is the sum of the initial value plus the integral of the liquid outflow from the tank. The instantaneous stored mass within the ullage space is simply the integral of the gas flow through the last flow restriction device in the gas flow system. This is obtained from Eq. (1) of Table I by striking out the term representing gas outflow from the tank, assuming that there is no gas leakage from the tank. Finally, the temperature of the gas in the ullage volume is given by Eq. (2) of Table I. This equation results from performing an energy balance on the open system represented by the gas mass within the tank, assuming ideal gas behavior.

The liquid outflow of each propellant tank is determined by the liquid flow system in each branch of the propulsion system. The tank outflow, i.e., the propellant flow rate, is determined by Eq. (18) of Table I, applied to the first restriction downstream of the tank. This same equation can be applied to the second restriction in the liquid flow system, which represents the system calibration orifices, filters and injector. The intermediate pressure in the liquid line is determined from Eq. (17) of Table I. It is notable, however, that the coefficient in Eq. (17) is extremely large when evaluated for a liquid flow system which is rigid. That is to say, there is very little compressibility in the liquid flow system. This means that the liquid flow rate throughout the liquid portion of the system can be considered constant for the low frequency dynamics which are of interest in a pressurization system study. As mentioned previously, the liquid flow model shown in

Fig. (16) was used in the analysis of the Mariner '71 system primarily to make maximum use of the existing model developed for the Mariner '69 system. An algebraic representation of the liquid flow system would serve equally well for the dynamics of interest here.

The thrust chamber represents a termination for both paths of the bi-propellant propulsion system. Equations (23) and (24), Table I, show the relationship between the propellant flow rates and the pressure at the termination of these lines, i.e., chamber pressure. Equation (23) states simply that the total flow rate is the sum of the oxidizer and fuel flow rates. In the normalized version of this equation, Eq. (23c), it is seen that the coefficients are functions of the nominal mixture ratio at which the system operates. Equation (24) is a simplified expression for the chamber pressure. Equation (24) assumes a constant C^* , i.e., constant temperature and properties of the combustion products. The latter is an assumption which is justified only because this portion of the system is of secondary importance to the dynamics of the pressurization system.

The above equations completely define a mathematical model for the Mariner '71 propulsion system, which is well suited to the study of the dynamic operation of the pressurization system and the regulator.

Determination of Parameters.

As can be seen from the equations in Table I, the normalized equations which represent the propulsion system have coefficients which are determined by certain ratios or dimensionless groups of the physical descriptors of the system. For example, the gas flow rate equation which is applied to every point of restriction in the gas flow system, Eq. (5c) of Table I, involves a constant which has the value of the compressible flow function, Fig. (5), at the nominal pressure ratio for

the particular device. Also, the equation which determines the stored mass within each compressible volume, Eq. (1c) has a constant coefficient $\dot{w}_n/w_{in}$, which has the units of reciprocal seconds and characterizes the rate at which the particular compressible volume changes pressure with time. When each of the coefficients have been determined for all equations which are required to define the system indicated by the schematic, the system model is completely defined.

The determination of these parameters can proceed in two different ways. If the system to be modeled is not completely defined physically, then reasonable values of the coefficients themselves, i.e., the dimensionless ratios, can be assumed. However, in the case of the Mariner '71 propulsion system, the physical parameters, i.e., pressures, geometry, etc., themselves are to some degree specified or known. In this case, the ratios and dimensionless groups are calculated from the known values of the physical parameters. In these calculations, the formulas given in Table III are helpful. The physical parameters which define the Mariner '71 propulsion system according to the schematic of Fig. (16) are presented in Table IV. This data was obtained from examination of the steady state values obtained in certain tests which have been conducted with the Mariner '71 system. The pressure at each of the points of lumped compressibility in the model were selected as the pressure at the closest physical point in the actual test system at which there was instrumentation. The regulator parameters were kindly provided by the regulator manufacturer. Nominal pressurant flow rates were calculated according to formula provided in Table III, as was the nominal regulator area. System volumes were estimated from line sizes given on the system schematic and from the regulator schematic. The ullage volumes were taken to correspond to those employed in certain tests for which model correlation was desired.

TABLE IV

MARINER MARS '71 SYSTEM CONSTANTS

STATION	PRESSURE (psia)	VOLUME (in ³)	TEMPERATURE (°R)
0	2688	5000	530
1	2646	.2	"
2	2625	.2	"
3	275	.3	"
4	267	3.0	"
5X	263	1100	"
5F	263	600	"
X1	245	1.0	"
F1	245	1.0	"
c	122	10.0	5280
a	15	-	530
r	275	0.6	"

Flow rates

pressurant flow, oxidizer	=	.00948	lb/sec
pressurant flow, fuel	=	.01037	lb/sec
pressurant flow, total	=	.01985	lb/sec
oxidizer flow	=	0.66	lb/sec
fuel flow	=	0.44	lb/sec
thrust chamber flow	=	1.10	lb/sec

Pressurant Properties

$$R = 661 \text{ in}^3/\text{°R}$$

$$\gamma = 1.4$$

Combustion Properties

$$MW_c = 19.6 \text{ lb/lb-mole}$$

$$R_c = 946 \text{ in}^3/\text{°R}$$

TABLE IV (cont'd)

Regulator Parameters

X_n	=	0.0063 in.
$X_{\max}$	=	0.0462 in.
$X_{\min}$	=	0.050 in. (assumed)
k_s	=	317 lb/in.
k_{bs}	=	10 lb/in.
m	=	0.518×10^{-3} lb-sec ² /in.
F_{bsn}	=	5.1b (assumed)
F_{sn}	=	272.8 lb.
F_{c1}	=	0.4 lbs.
F_{c2}	=	0.1 lbs.
A_e	=	1.02 in ²
A_b	=	0.00385 in ²
A_p	=	0.0207 in ²
b/a	=	2.2
r	=	0.035 in.
R	=	0.0625 in.
A_r	=	0.000254 in ² (reference orifice)
A_{sn}	=	0.000521 in ² (seat, nominal)

It is important to note that the system parameters necessary for the analytical model cannot always be determined directly from available test data in a straightforward manner as indicated above. The primary difficulty is that breaking the physical system into discrete parts, to produce the lumped parameter model, may introduce the requirement for local pressures at points in the system at which there was no pressure instrumentation. In the analysis of the Mariner '71 system, for example, it was found necessary to adjust the pressure ratios across the flow restrictions representing the check valve and the regulator out-flow restriction. Thus the pressure distribution in this portion of the system shown in Table IV represents values determined partially during the analysis, as required to show correct dynamic trends. This problem can be minimized by using a lumped parameter model with a sufficient number of discrete points to adequately represent all significant points of flow restriction, and whenever necessary using analytical means to determine local pressures at intermediate points based on the available pressure data. If no test data is available, it is necessary to calculate the pressure at each point in the physical system corresponding to the discrete pressure nodes in the analytical model.

Analog Model

An analog model was developed for the mathematical model defined above. This model was developed for the Applied Dynamics AD256 computer at the Analog Simulation Facility, at the Rocketdyne Canoga Plant. The analog diagrams representing this model are presented in Appendix C of this report.

Results and Correlations for the Mariner '71 System

The analog model of the Mariner '71 system was used to show correlation with test data obtained from the test conducted on a simulated

Mariner⁷¹ pressurization system on 23 January 1970. This test was concerned only with the initial pressurization of the system, and the data obtained are indicated in Fig. (19). Only four seconds of the test were available for comparison as indicated by the solid lines in Fig. (19). The dashed portion represent extrapolation to assumed steady state values.

The most distinguishing feature of the test data shown in Fig. (19) is the momentary overshoot in the pressure measured immediately downstream from the pressure regulator, as seen in the uppermost curve in Fig. (19). It is also characterized by the oxidizer tank pressure building up more slowly than that of the fuel tank. The latter is a result of the ullage volume in the oxidizer tank being 1100 cu. in. as opposed to 700 cu. in. in the fuel tank. It is also notable in these results that there was no liquid outflow from the tanks, i.e., the downstream valves were closed.

Figure (20) shows results obtained from the analog model of the same test represented by Fig. (19). The left most portion of Fig. (20) shows the full pressurization transient, i.e., from the time of opening the pressurant valves upstream of the regulator till the time that the tank pressures approached their nominal values. Note that all of the variables in this figure are normalized to their nominal, i.e., steady state values. The first six strip records represent pressures at various points in the system, moving from upstream to downstream. In particular, note that $\bar{P}_{41}$ is the pressure between the regulator and check valve and $\bar{P}_{5x}$ and $\bar{P}_{5f}$ are the pressures in the oxidizer and fuel tanks respectively. The regulator area is shown in the seventh strip record, and system pressurant flows are shown in the last three records.

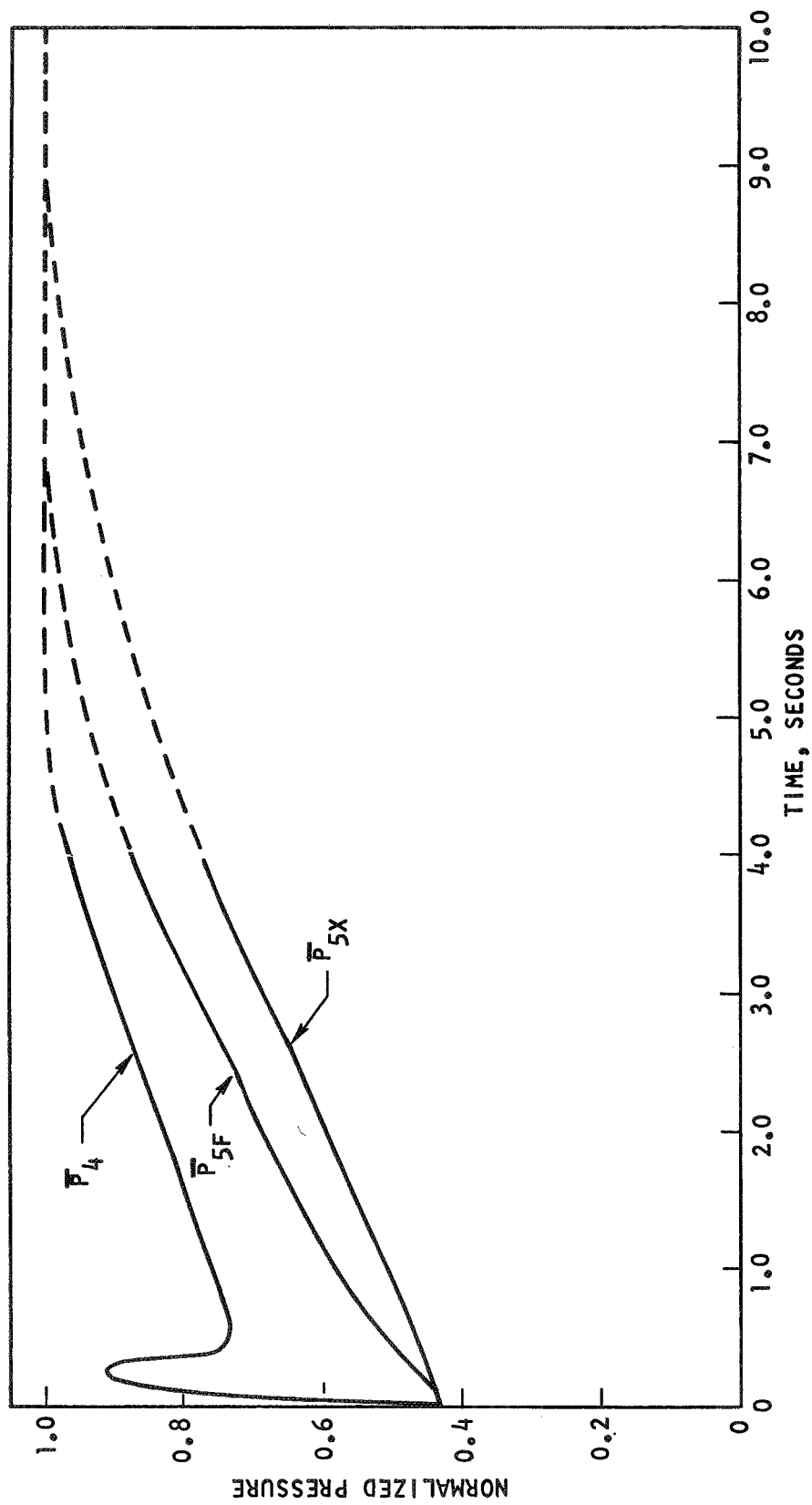


Figure 19. Test Results Showing MM '71 Pressurization Transient

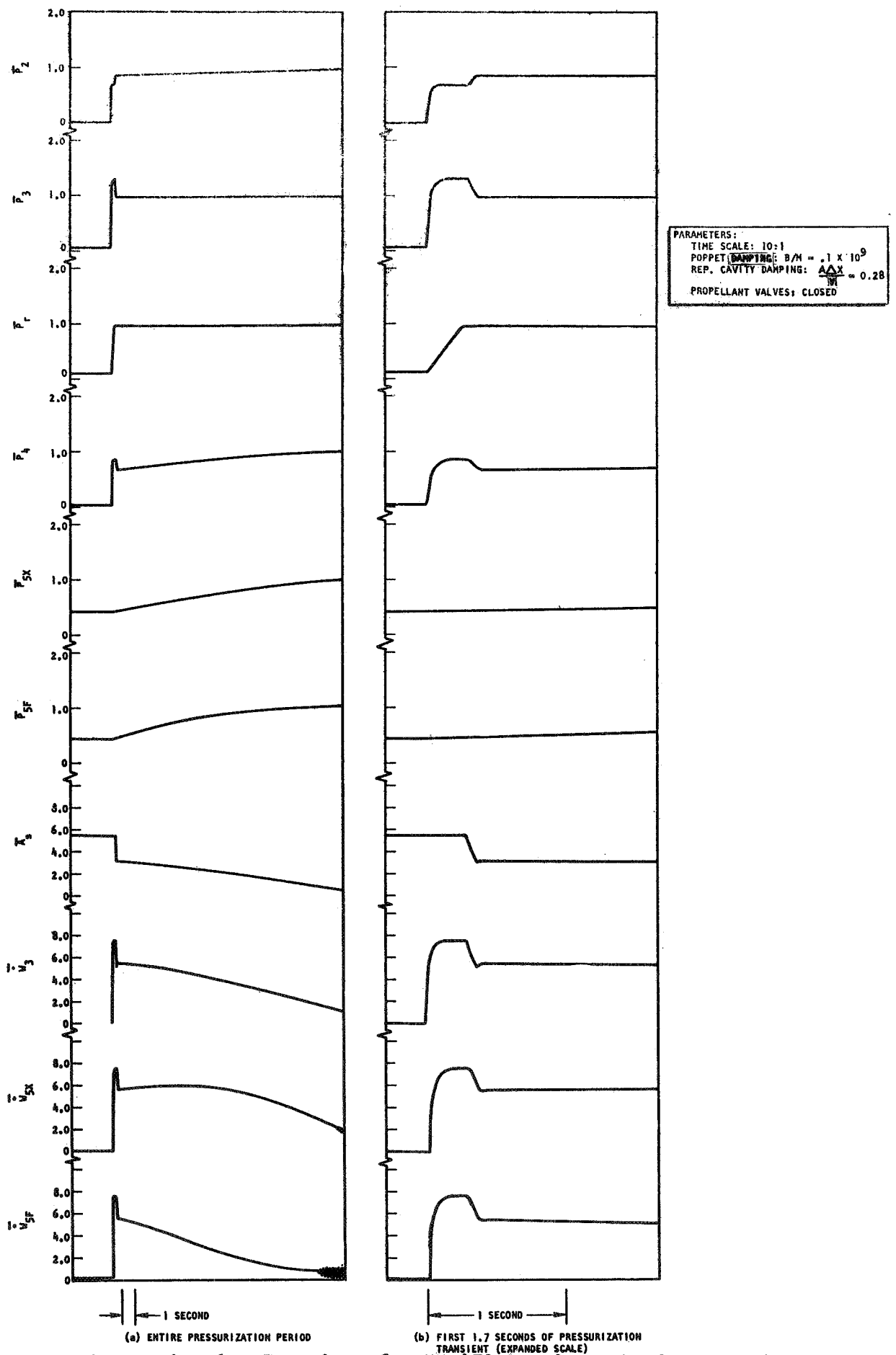


Figure 20. Pressurization Transient for MM '71 Regulator During Slam Start Test

It is seen in Fig. (20) that the characteristic overshoot in the pressure upstream from the propellant tank is reproduced by the model. The magnitude of this overshoot was adjusted by variation of the nominal pressure ratios across the check valve and regulator outlet restrictions, in order to match the overshoot indicated in the test data.

Figure (20b) shows the first 1.7 seconds of the pressurization transients with an expanded time scale so that the form of the overshoot can be examined. The form of this overshoot transient can be compared with the test data from another test as shown in Fig. (21). It is seen that agreement is satisfactory with respect to shapes and magnitude.

At the onset of the analysis of the Mariner⁷¹ system, the reason for the overshoot in the pressure upstream of the propellant tanks was unknown. Operation of the model immediately revealed that this overshoot was a result of flow restrictions downstream of the regulator combined with the charging characteristics of the regulator reference cavity. At the onset of pressurant flow, the pressure at all points in the downstream portion of the pressurization system are low, i.e., 40% of their nominal values, including the pressure in the reference cavity. Thus the pressure on the pressure sensing surface of the regulator is low, causing the pressure regulator to be wide open. This results in high flow rates thru the regulator, as can be seen in the eighth strip chart of Fig. (20). As can be seen in the third strip chart of Fig. (20), the pressure within the reference cavity builds up over a period of approximately 250 msec. The rate at which this volume charges is of course a function of the size of the orifice and of the pressure in the regulator downstream of the seat. When this volume does fill and the pressure within it comes very close to its nominal value, the regulator closes to a smaller seat area due to the increased pressure

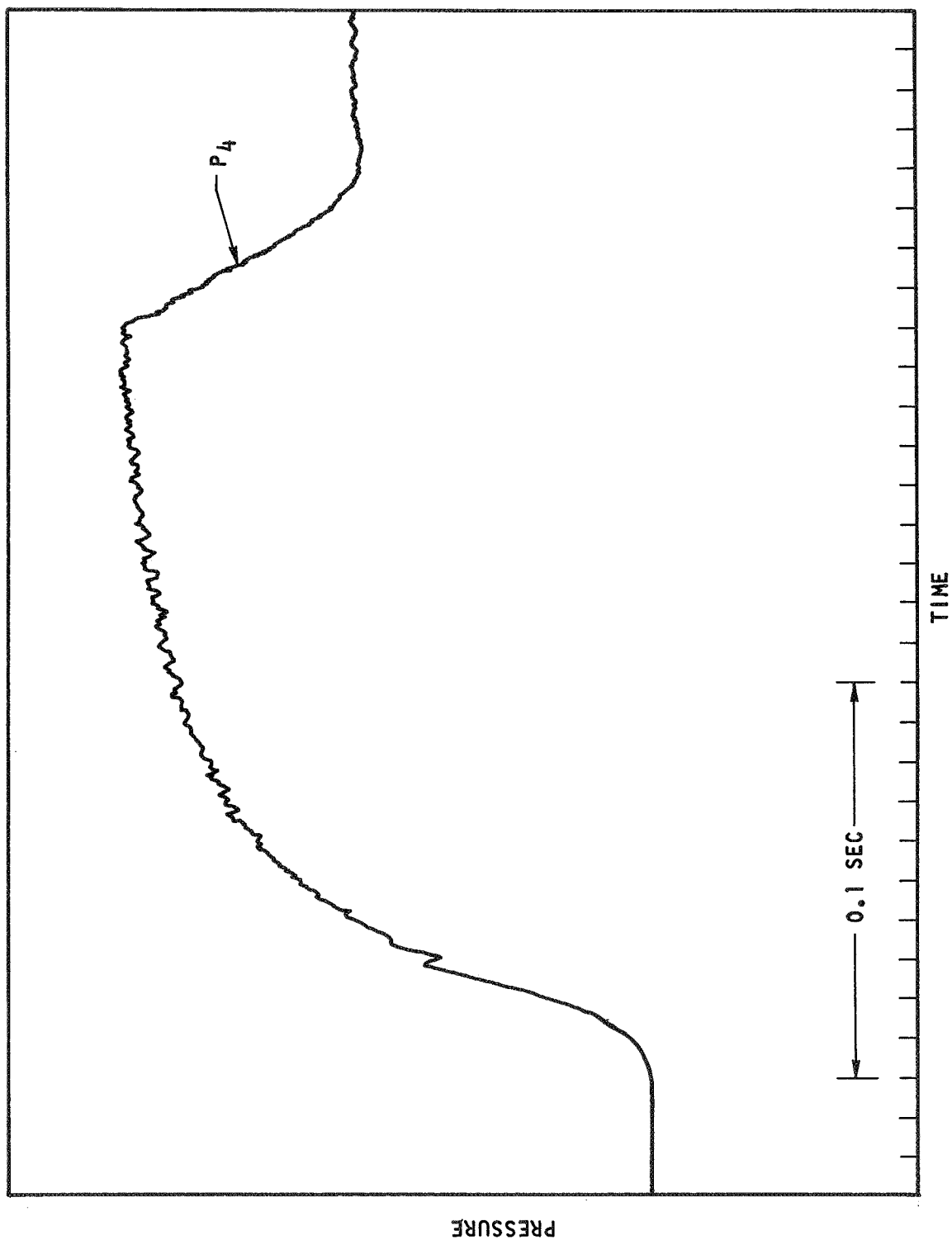


Figure 21. Test Data Showing Initial Overshoot for MM '71 Regulator

on the pressure sensing surface. This reduces the pressure at points intermediate to the propellant tank and the regulator, since the pressure at these points is the sum of the tank pressure plus losses through the points of restriction, which are functions of flow rate. Once this phenomena was understood, it was a simple matter to adjust the nominal pressure ratios, which in effect sets the physical restriction or effective flow area at these points of restriction, so as to make the model pressure transients agree with the test data.

An attendant benefit of the model of this system is that it yields pressures and flow rates at all points in the system which are adequately represented by the lumped parameter model. Thus, in the second strip chart of Fig. (20), it may be seen that the pressure immediately downstream of the seat within the regulator first overshoots, and then within 300 msec of the beginning of pressurization reaches its final value. The regulator is, in effect, adjusting the flow rate through the the seat to maintain this local pressure in the presence of varying upstream pressures and varying pressure drop between the regulator and the tank. In order to observe this phenomena in actual tests, the regulator itself would have to be tapped to permit instrumentation of this local pressure.

It is also apparent from the above that the model would provide an efficient means of adjusting regulator parameters to achieve any desired performance of the system. Some of these variations are considered in the next section of this report.

PARAMETRIC RESULTS

A parametric study was defined so as to encompass a wide range of monopropellant system designs. This study was carried out using the monopropellant model, and results are presented in this section. These results should permit the designer to estimate dynamic performance of pressurization systems with regulators of this type, and to determine the effects of several important parameters normally at his disposal.

It should be noted that these results, although obtained from the monopropellant model, are equally applicable to bipropellant systems which employ a single pressure regulator. Furthermore, the results are independent of thrust level so long as geometric similarity of the regulator is maintained.

Some parametric variations are also shown for the bipropellant model.

DEFINITION OF PARAMETRIC STUDY

Selection of Parameters

In order for the results to be broadly applicable, the parameters which determine system dynamic performance must be varied over ranges which include many possible designs. Since the number of parameters is large, however, it is not feasible to vary each individual parameter. Some method must be sought to reduce the independent parameters to a short list of important ones.

An important first step in reducing the number of independent parameters is to employ the normalized form of the equations in Table I. It is this step which eliminates parameters such as pressures, flow rates,

and physical dimensions of the system.

Normalization of the equations provides a reduction in the number of parameters, but the number of dimensionless coefficients is still large, due to the large number of system elements. Further reductions can be obtained by making assumptions about the range and importance of certain parameters. These assumptions are:

1. Storage tank is sized to provide sufficient pressure for regulation during full burn period.
2. The only important compressible volumes are the storage tank and the ullage volume.
3. Thrust chamber and liquid system time constants have insignificant effect on gas system dynamics.
4. Regulator poppet forces other than diaphragm force are a constant fraction of the nominal reference spring force.

The first assumption requires that the storage tank be large enough and at a high enough initial pressure such that the design pressurant flow can pass through the maximum regulator area after all propellant has been expelled. This is reasonable since larger tanks result in weight penalties, and smaller tanks result in uncontrolled thrust during the latter portions of the burn period. This design criteria establishes a unique value of the parameter $\dot{w}_n / \dot{w}_{on}$, which is the inverse of the pressurant tank time constant. In effect, this establishes the $P \times V$ product for the storage tank for a system if other physical parameters are fixed.

The second assumption can also be justified. Note that

$$\frac{\dot{w}_n}{\dot{w}_{in}} = \frac{A_i v_i}{V_i}$$

is the inverse time constant of all localized compressibilities. Here, A_i and v_i are the local cross-sectional area and local pressurant

velocity respectively. However, the lumped volume, V_i , is simply the line length between restrictions times the local area. Therefore, the inverse time constants can be written as

$$\frac{\dot{w}_n}{\dot{w}_{in}} = v_i / \ell_i$$

Gas velocity is a factor which is commonly held constant among different propulsion system designs. That is, line sizes are frequently selected so as to give a desired gas velocity. As a result, it is to be expected that the compressible volumes associated with the gas flow system will not vary greatly from system to system even though thrust or pressure levels may be different. Thus, parametric variation of compressibility time constants can be limited to that associated with the ullage volume.

The third assumption derives from the fact that the large compressibility of the ullage volume effectively isolates the pressurization system from the liquid system. Therefore, parametric variations of the liquid system would have no effect on the pressurization system.

The fourth assumption allows attention to be concentrated on the more important of the many regulator parameters. From Eq. (12) of Table I, it is seen that the equation of motion of the mechanical elements involves 9 coefficients. Varying each of these in a systematic way looking for interaction effects would require 18 parametric cases. The size of this task can be reduced by neglecting the less important factors, such as the pressure and retaining spring forces on the ball and actuator. However, due to the steady-state balance requirement, Eq. (55), these secondary forces cannot simply be left constant while varying the other parameters. Assumption four is a reasonable method of maintaining the necessary steady-state balance while varying the important regulator

parameters. Physically, this is justifiable on the grounds that
a) by proper regulator design, these forces are small relative to the major forces and therefore have small influence, b) these forces vary somewhat according to regulator size, as do the major forces.

With these assumptions, the important parameters are reduced to those shown in Table V. The range of each of these parameters was determined by estimation of reasonable values which might be encountered in propulsion system designs.

Physical Significance of Parameters

Most of these parameters are self-explanatory. Particular attention is called to certain ones which pertain specifically to the regulator. The regulator range, $\Delta X/X_n$, is defined by

$$\frac{\Delta X}{X_n} = \frac{X_{pmax} - X_{pn}}{X_{pn}} \quad (56)$$

where

X_{pmax} = maximum poppet position

X_{pn} = nominal poppet position

All positions are measured from the closed position, and the nominal point, n, refers to conditions which prevail at the beginning of the first burn.

TABLE V

SIGNIFICANT PARAMETERS FOR THE MONOPROPELLANT PROPULSION SYSTEM

<u>PARAMETER</u>	<u>FIRST VALUE</u>	<u>SECOND VALUE</u>
Burn Time - t_B (sec)	50	500
Ullage Fraction - V_{5n}/V_{tank}	.05	.15
Regulator Range - $\Delta X/X_n$	5	15
Upstream Pressure Ratio - P_{3n}/P_{0n}	.970	.945
Regulator Pressure Ratio - P_{4n}/P_{3n}	.080	.800
Downstream Pressure Ratio - P_{5n}/P_{4n}	.980	.995
Liquid Pressure Ratio - P_c/P_5	.200	.800
Regulator Gain - $P_{\text{ref}}/\Delta P_{\text{ref}}$	10	100
Regulator Frequency - $\sqrt{k/m}$ (rad/sec)	750	7500

The regulator gain, $P_{ref}/\Delta P_{ref}$, is an expression of the spring slope. This parameter results from consideration of the poppet transfer function, which can be reduced to, in the absence of damping,

$$\frac{\bar{X}}{\bar{P}_L}(\omega) = \frac{P_{Ln} A_D / m_p \Delta X}{(k/m_p) - \omega^2} \quad (57)$$

It is seen that the D.C. gain (G_{dc}), i.e., $\omega = 0$, is

$$G_{dc} = \frac{P_{Ln} A_D}{m_p \Delta X} \div \frac{k}{m_p} = \frac{P_{Ln} A_D}{k \Delta X} \quad (58)$$

Now, P_{Ln} is the reference (diaphragm) pressure and $k \Delta X$ is the change in spring force as the poppet moves through its range. Dividing the numerator and denominator of the right side of Eq. (58) by the diaphragm area, A_D , shows that the regulator gain (G_{dc}) is therefore the ratio of nominal reference pressure to the change in reference spring force per unit diaphragm area. More simply,

$$G_{dc} = \frac{P_{Ln}}{k \Delta X / A_D} = \frac{P_{ref}}{\Delta P_{ref}} \quad (59)$$

Note that changes in either spring rate, k , diaphragm area A_D , or poppet range, ΔX , result in changes in the gain. Thus many regulator designs are represented by a single value of this parameter.

The regulator natural frequency, $\sqrt{k/m}$, also can be seen in Eq. (57). It is the square root of the ratio of the spring rate to the moving mass of the poppet-diaphragm assembly, and is seen to be the resonance frequency of the transfer function.

The burn time, t_b , is a parameter which, in effect, specifies the time constant of the ullage volume. This is seen from the easily-derivable relationship,

$$\frac{\dot{w}_n}{w_{5n}} = \frac{(V_{\text{tank}}/V_{5n} - 1)}{t_b} \quad (60)$$

Thus, once the ullage fraction, V_{5n}/V_{tank} , has been specified, one has the option of arbitrary selection of either burn time or $\dot{w}_n/w_{5n}$. It is felt that t_b is more meaningful to the designer. The physical interpretation is simply that for any given ullage fraction, longer burn time implies larger actual ullage volume, since the tank itself must be larger. This larger ullage volume means longer pressurization time, and more compressibility in the system.

While not done in this analysis, it is conceivable that the results could be further generalized by employing the burn time to non-dimensionalize system operation time. This would eliminate burn time as an explicit parameter.

Definition of Cases

A systematic variation of these parameters is indicated in Table VI. Each column represents a case, i.e., a set of coefficients for which a computer solution is desired. Each parameter is represented by a row. An entry of "1" in a particular element indicates the first value, i.e., the first column-value from Table V, of that particular parameter for that case. For example, the base-line case, case P.1, is shown in column 1 of Table VI to consist of all parameters at their first values. An entry of "2" similarly indicates the second-column value from Table V. Case P.2 shows that the parameter P_{3n}/P_{0n} has its second value and all others are the first value.

TABLE VI

DEFINITION OF PARAMETRIC CASES

<u>CASE</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>	<u>12</u>	<u>13</u>	<u>14</u>	<u>15</u>	<u>16</u>	<u>17</u>
t_B	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	1
V_{5n}/V_{tank}	1	1	1	1	1	1	1	2	2	2	2	1	1	1	1	2	2
$\Delta X/X_{pn}$	1	1	1	1	1	1	1	1	1	1	1	2	2	2	1	1	1
P_{3n}/P_{0n}	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
P_{4n}/P_{3n}	1	1	2	1	1	1	1	1	2	1	1	1	2	1	1	1	1
P_{5n}/P_{4n}	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	2
P_{cn}/P_{5n}	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1
$P_{\text{ref}}/\Delta P_{\text{ref}}$	1	1	1	1	1	2	1	1	1	2	1	1	1	2	1	1	1
$\sqrt{k/m_p}$	1	1	1	1	1	1	2	1	1	1	2	1	1	1	1	1	1

KEY: (1) indicates first-column value from Table V.

(2) indicates second-column value from Table V.

It will be noted that cases 2 - 8, 12, and 15 represent first-order effects, i.e., effects of single parameters changed to their second value. Case 9 - 11, 13, 14, 16, and 17 represent combined effects of changing two parameters to their second values. This set of 17 cases represents the first-order effects of all selected parameters, and shows how these effects change when certain parameters are simultaneously changed.

Performance Criteria

The term "effects" used in the above discussion refers to all changes in system performance relative to the base-line case due to parametric changes. While practically all system variables are affected by some parametric changes, not all of these effects are of interest or importance. In order to employ the results obtained from this set of cases, it is necessary to establish a method of comparing each case to the base-line case in a quantitative manner. One method of comparison can be obtained by considering the basic purpose of the system, i.e., to deliver a pre-determined impulse during a burn period of known duration. Impulse is defined by

$$I = \int_0^{t_B} F(t) dt \quad (61)$$

where $F(t)$ is the system thrust. Now, let the subscript n refer to the nominal state, then define

$$\begin{aligned}\bar{F} &= F/F_n \\ F &= P_c A_t C_f \\ \bar{P} &= P_c/P_{cn}\end{aligned}$$

Assuming constant throat area A_t and thrust coefficient C_f ,

$$\bar{F}_c = \bar{P}_c$$

Then Eq. (61) reduces to

$$I = F_n \int_0^{t_B} \bar{P}_c dt \quad (62)$$

Ideal impulse is defined as

$$I_i = F_n t_B \quad (63)$$

Combining Eq. (62) and (63), the impulse error is then

$$\begin{aligned}I - I_i &= F_n \int_0^{t_B} \bar{P}_c(t) dt - F_n t_B \\ &= F_n \int_0^{t_B} (\bar{P}_c(t) - 1) dt \quad (64)\end{aligned}$$

The nominal thrust level, F_n , is not a model parameter, but the quantity

$$E_1 = \int_0^{t_B} (\bar{P}_c(t) - 1) dt \quad (65)$$

is easily generated. This is then a single quantity which reflects overall system performance. From it, the total impulse error can be calculated directly once nominal thrust is specified, Eq. (64). It is obvious that poor regulator performance will result in $\bar{P}_c(t)$ deviating from unity, resulting in changes in this quantity.

While E_1 as defined above gives a direct measure of departure from desired total impulse, it has the disadvantage of positive errors canceling the effect of negative errors. That is, a system in which $\bar{P}_c(t)$ was low over the first portion of the burn and high over the latter portion could have a low or zero value of E_1 . In some cases, this may be desirable, and the regulator should not be penalized in the evaluation process. This gives rise to another performance parameter.

$$E_2 = \int_0^{t_B} \left| \bar{P}_c(t) - 1 \right| dt \quad (66)$$

in which the absolute value of instantaneous error is integrated over the burn period. Only regulator/system combinations which maintain $\bar{P}_c$ within a narrow band around 1.0 throughout the burn will have a low value of this parameter. The designer must decide which of these performance parameters is most important in each case.

DISCUSSION OF RESULTS

The monopropellant propulsion system model was used to run the 17 parametric cases described above. System coefficients not determined by the set of parameters shown in Table V were taken to be the values for the MM '69 system.

The initial conditions for all cases were made to correspond to an initial propellant tank pressure 80 percent of its nominal value. At this condition, the regulator is full open, so all points between the propellant tank and the valve upstream of the regulator are at the same absolute pressure. In the liquid portion of the system, points upstream of the start valve were taken to be tank pressure, and downstream, zero.

Regulation curves for all parametric cases are presented in Fig. (22) through Fig. (44). These curves show the variation of propellant tank pressure, $\bar{P}_5$, as a function of storage tank pressure, $\bar{P}_0$. The over-bar indicates normalized quantities. That is

$$\bar{P} = P_5/P_{5n}$$

so that these curves are applicable to any system pressure levels, so long as the parameters in Table V are the same.

In each figure, the parameter whose value differs from the base-line case is indicated on the chart. All other parameters are base-line values.

Corresponding transient plots of several important system variables (normalized) are shown in Fig. (45) through Fig. (48). The two top channels show storage tank pressure and temperature respectively. Regulator seat area is shown on the third channel. The pressure and ullage

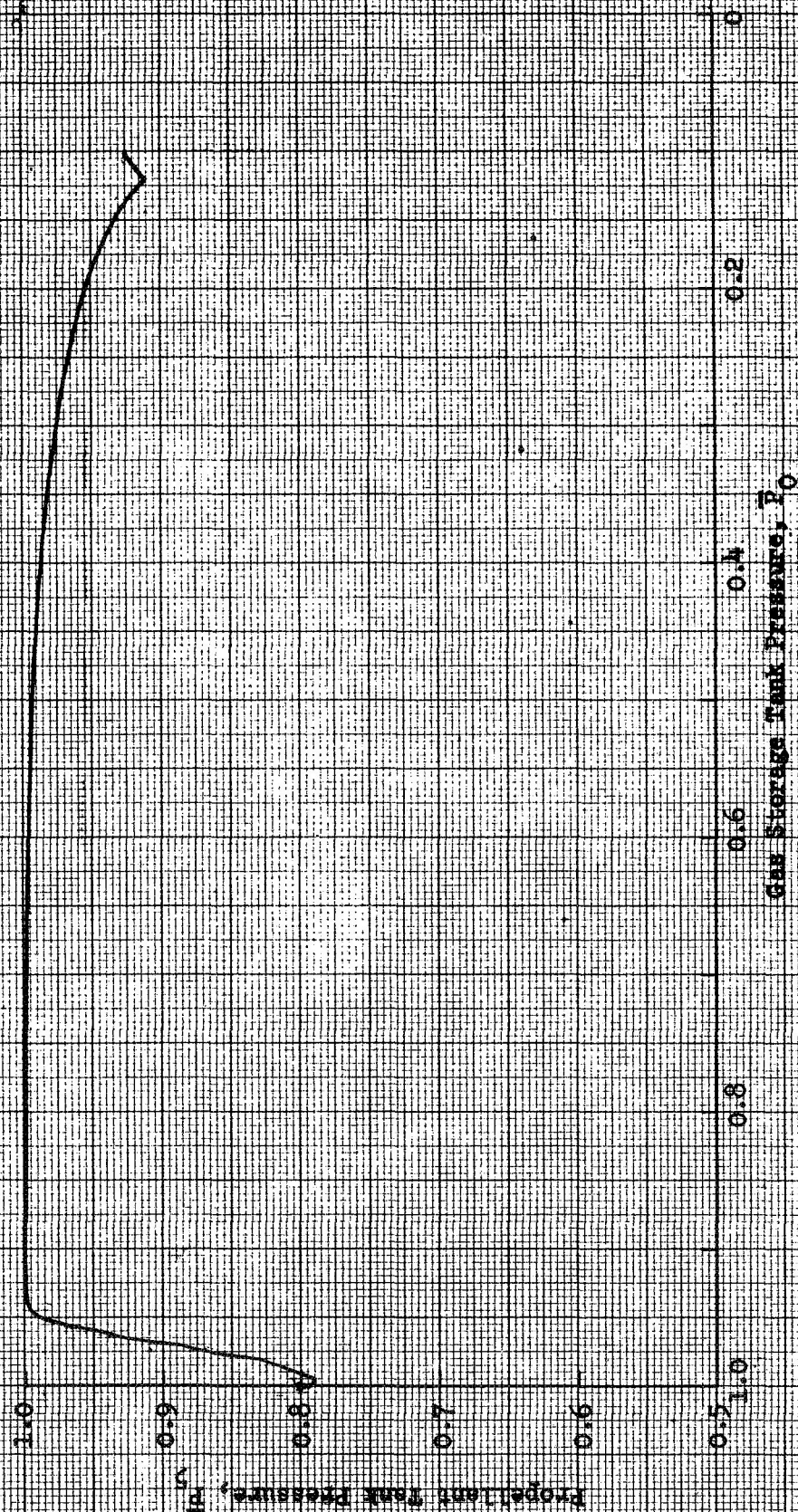


FIGURE 22 Base-line Regulation Curve for Monopropellant Parametric Study,
Case P.1 (25 March 1970)

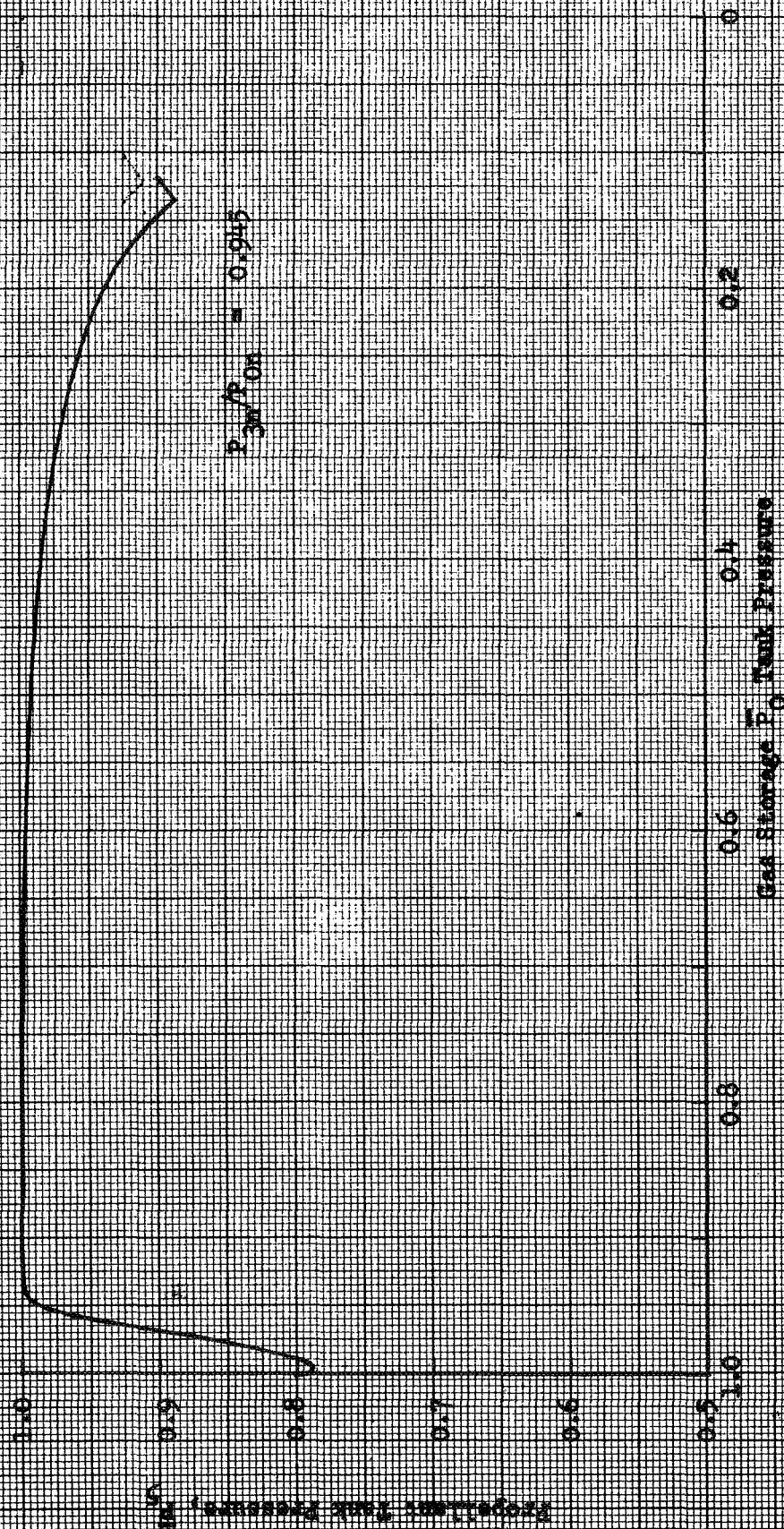


FIGURE 23 Case P.2, Effect of Upstream Pressure Losses

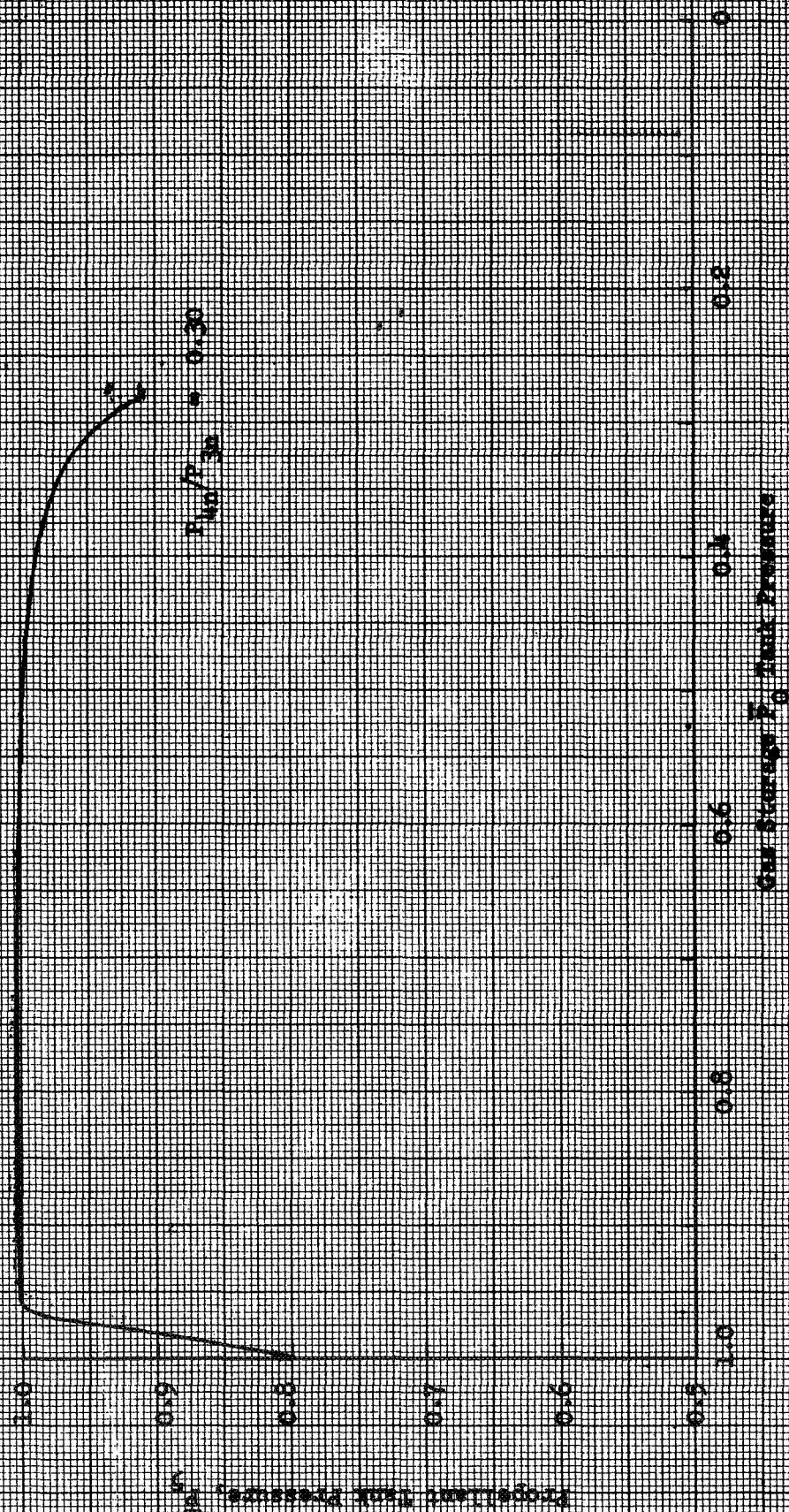


FIGURE 24 Case P.3, Effect of Regulator Seat Pressure Ratio

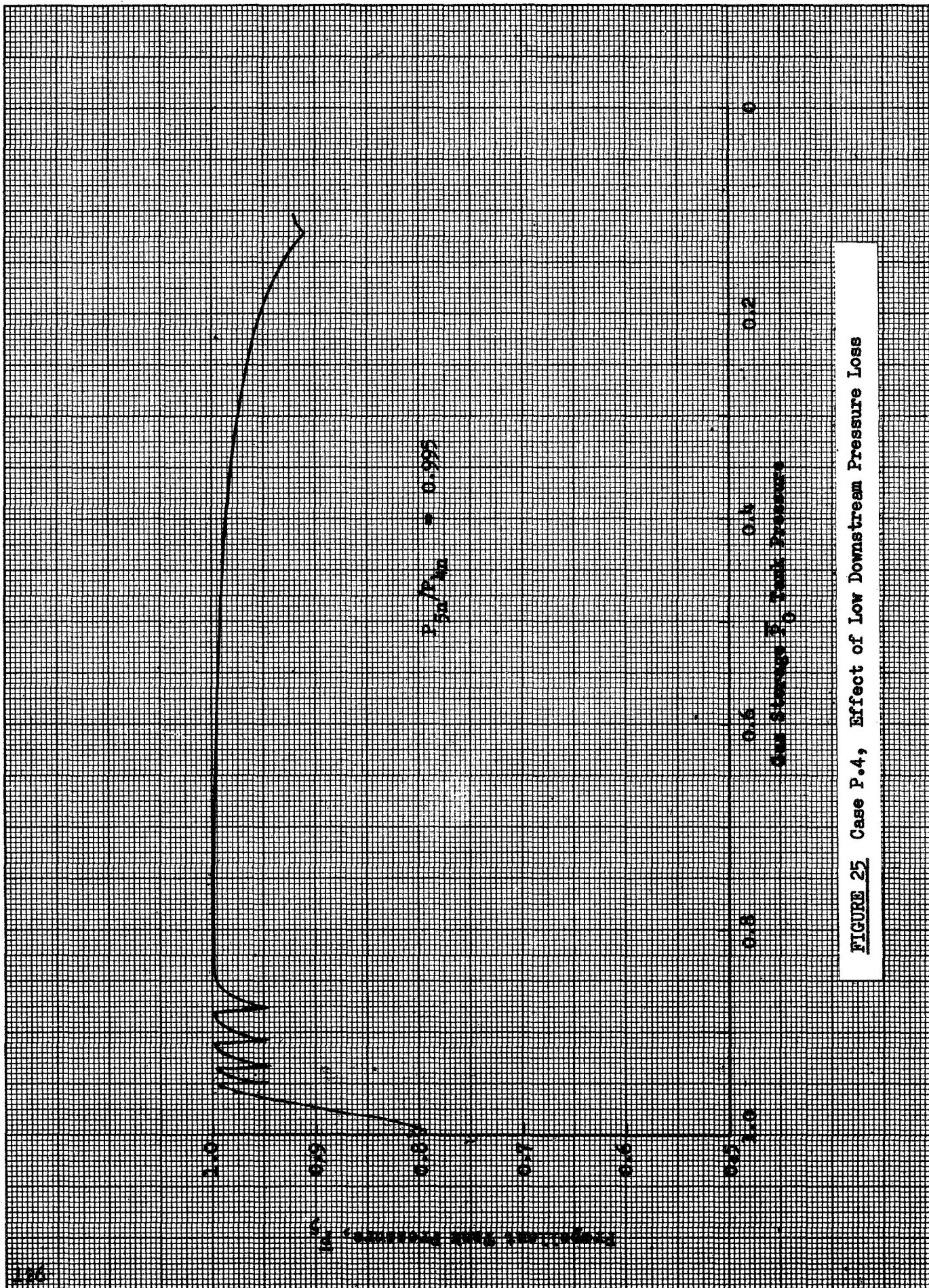


FIGURE 25 Case P.4, Effect of Low Downstream Pressure Loss

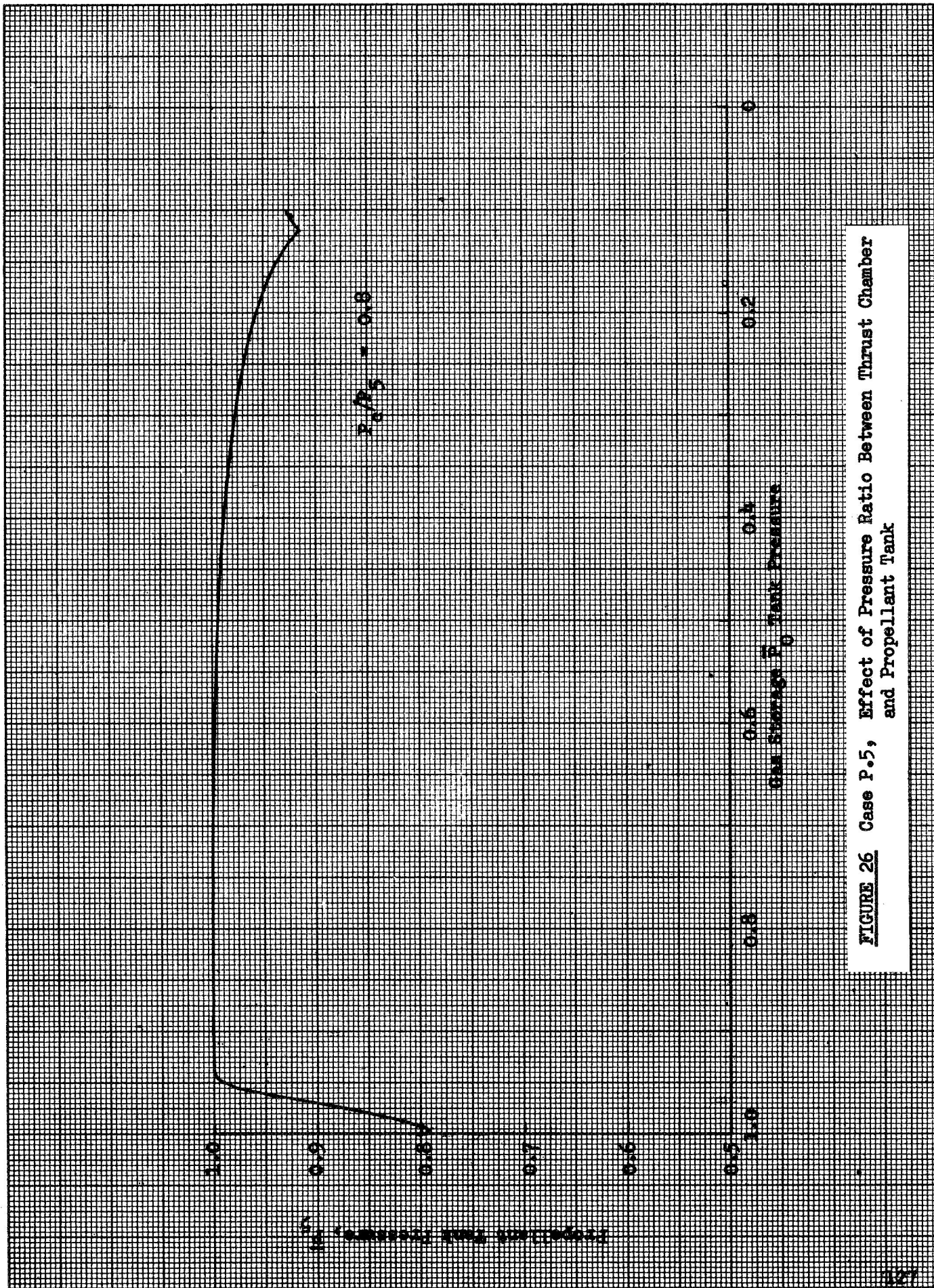


FIGURE 26 Case P.5, Effect of Pressure Ratio Between Thrust Chamber and Propellant Tank

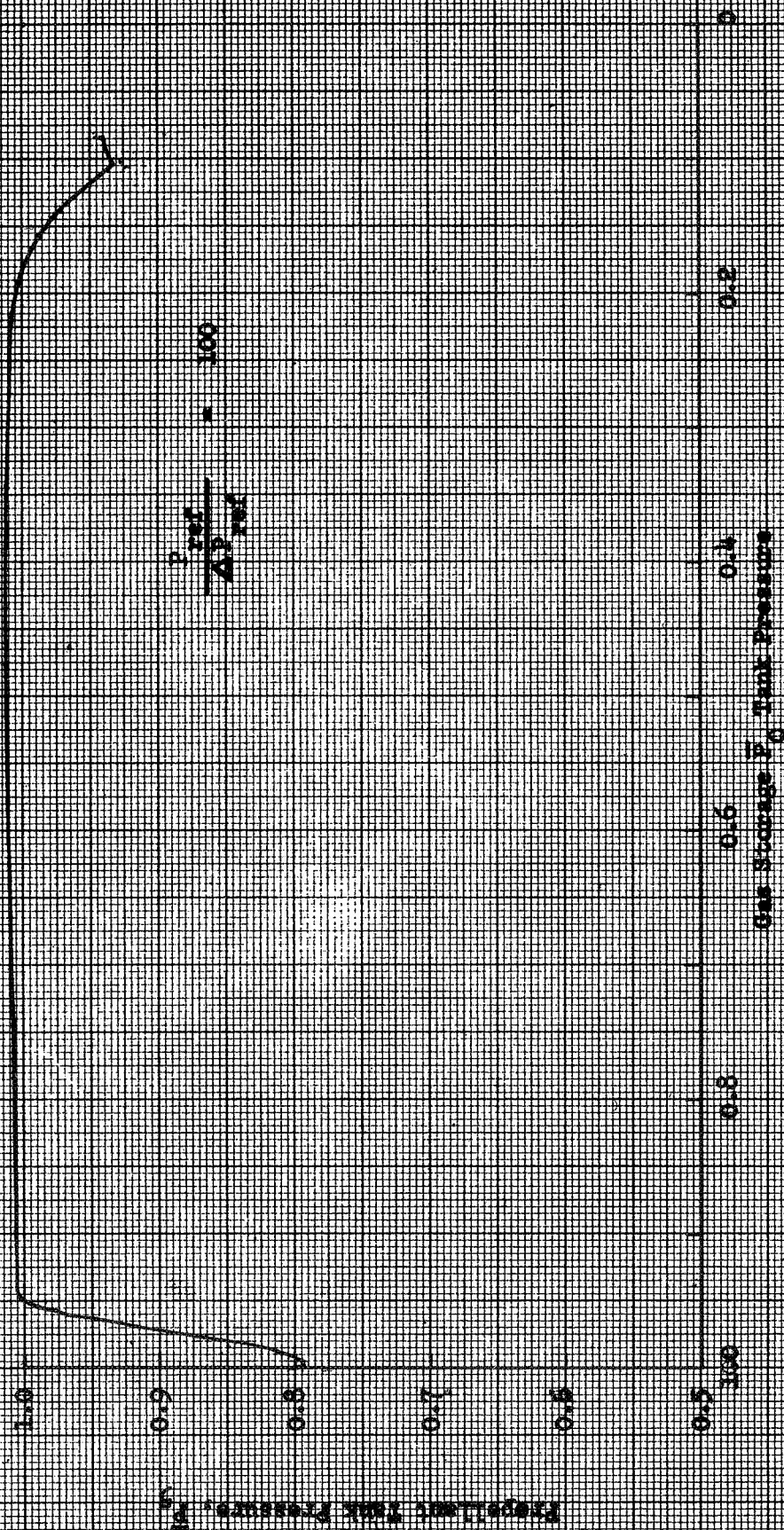


FIGURE 27 Case P.6, Effect of Regulator Gain

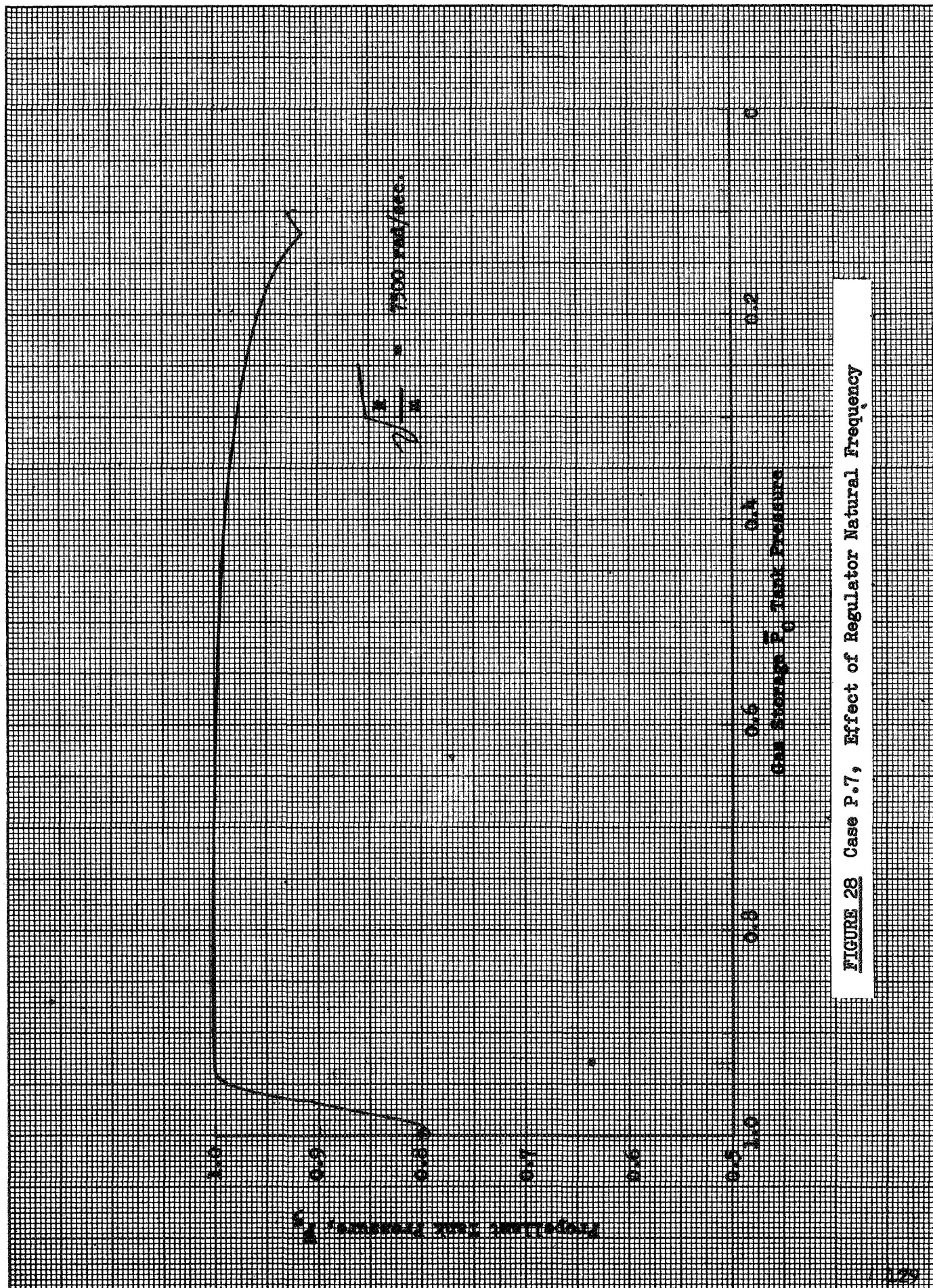


FIGURE 28 Case P.7, Effect of Regulator Natural Frequency

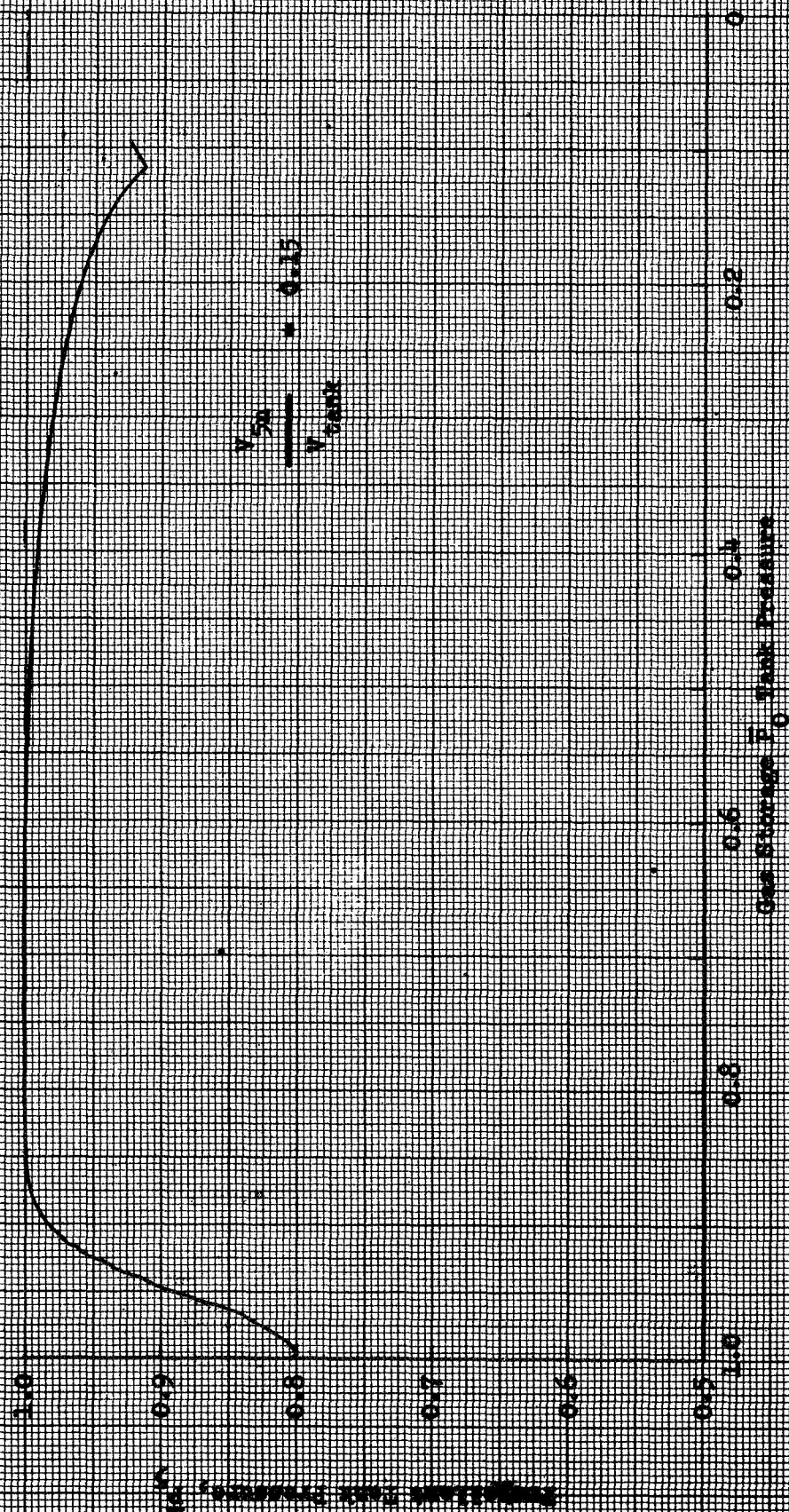


FIGURE 29 Case P.8, Effect of Ullage Volume

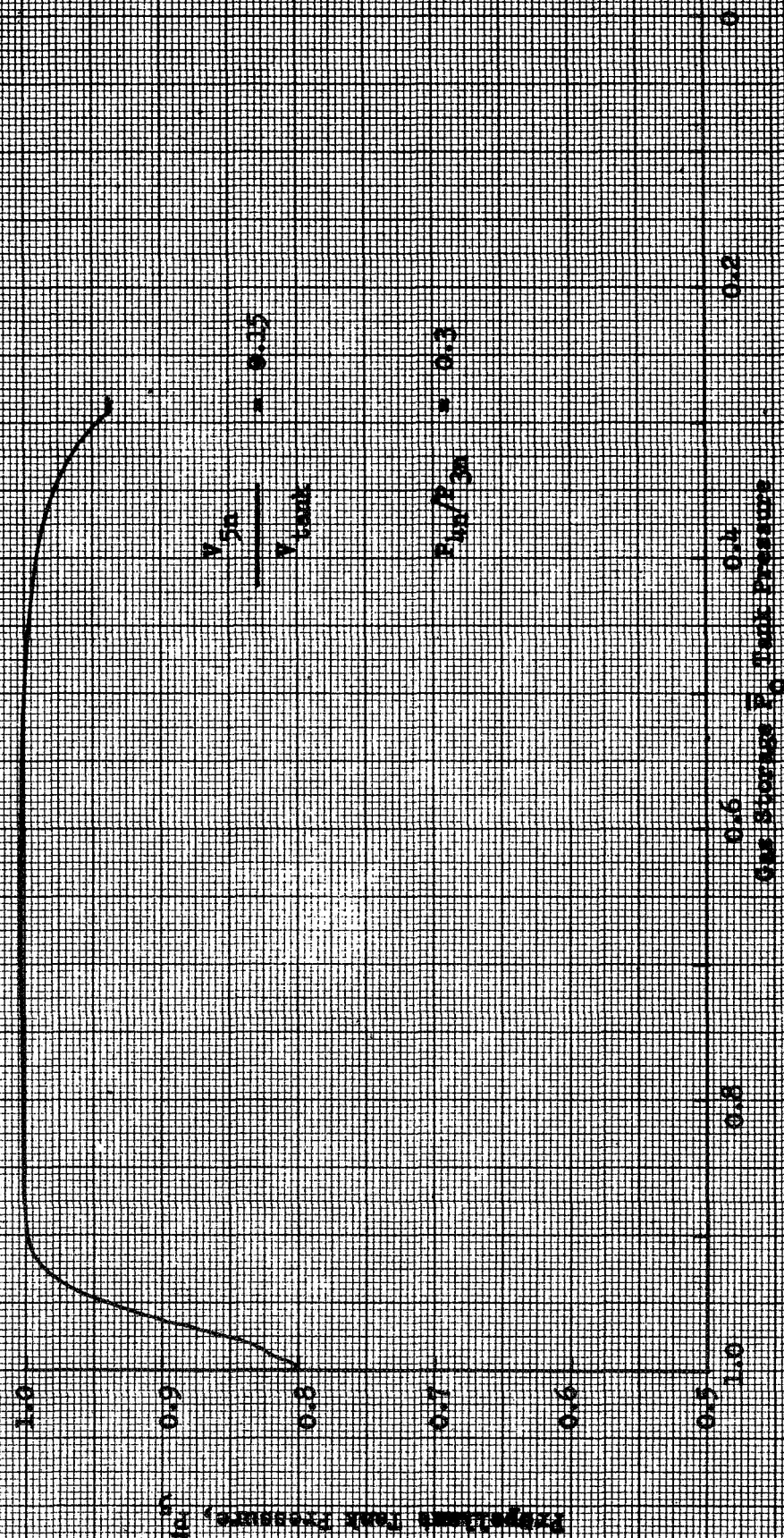


FIGURE 30 Case P.9, Combined Effect of Ullage Volume and Regulator Seat Pressure Ratio

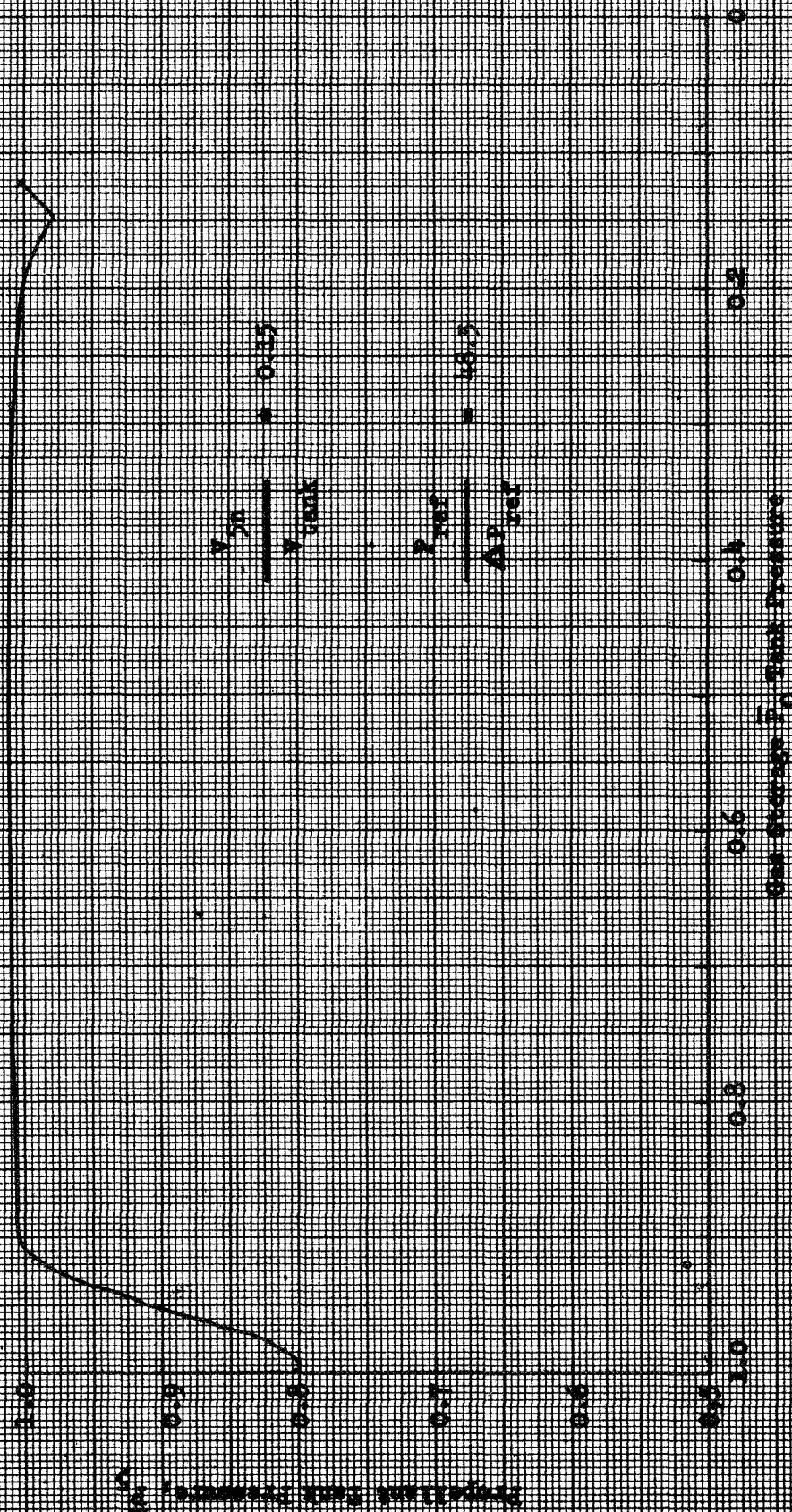


FIGURE 31 Case P.10, Combined Effect of Ullage Volume and Regulator Gain

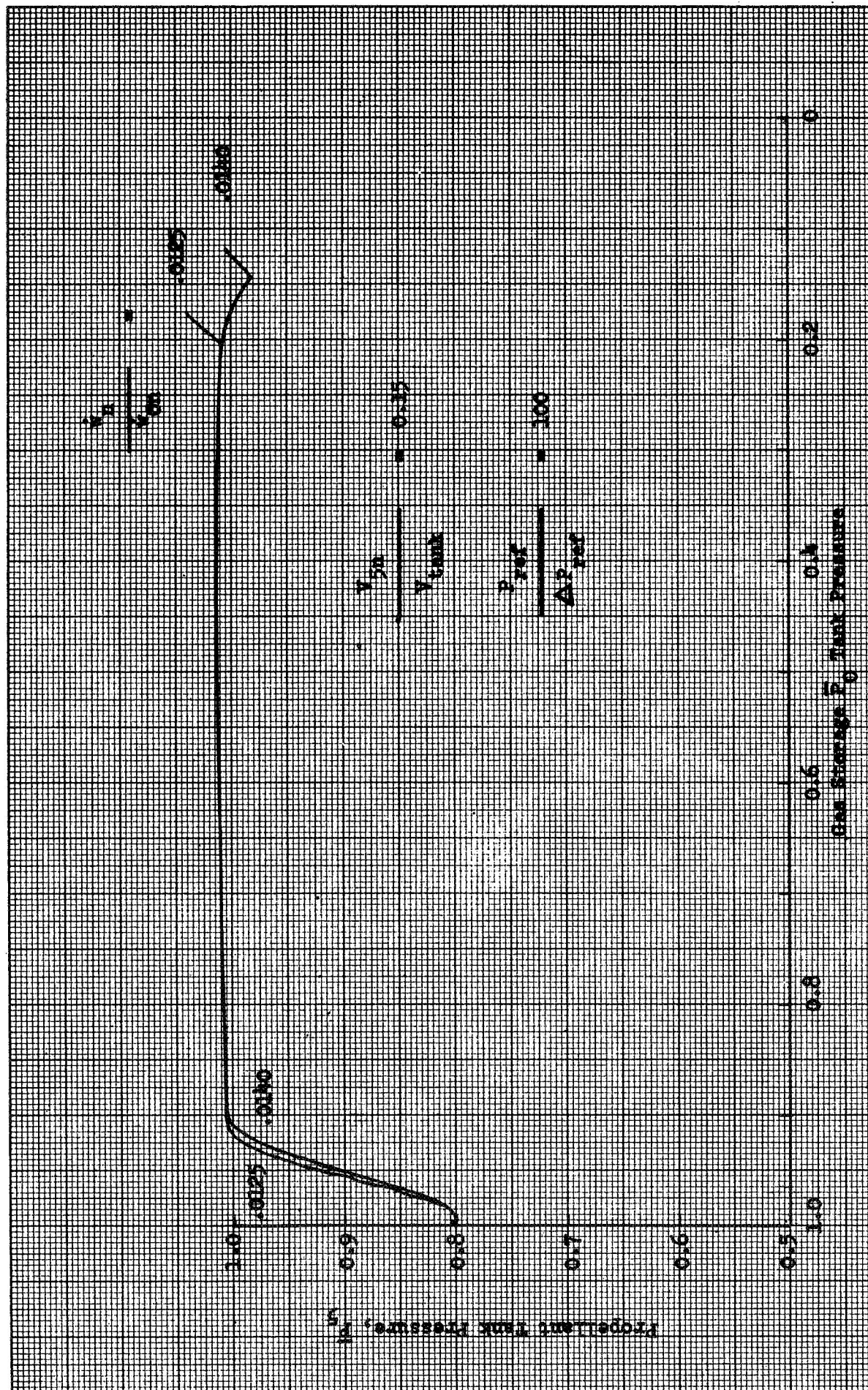


FIGURE 32 Case P.10, Combined Effect of Ullage Volume and Regulator Gain

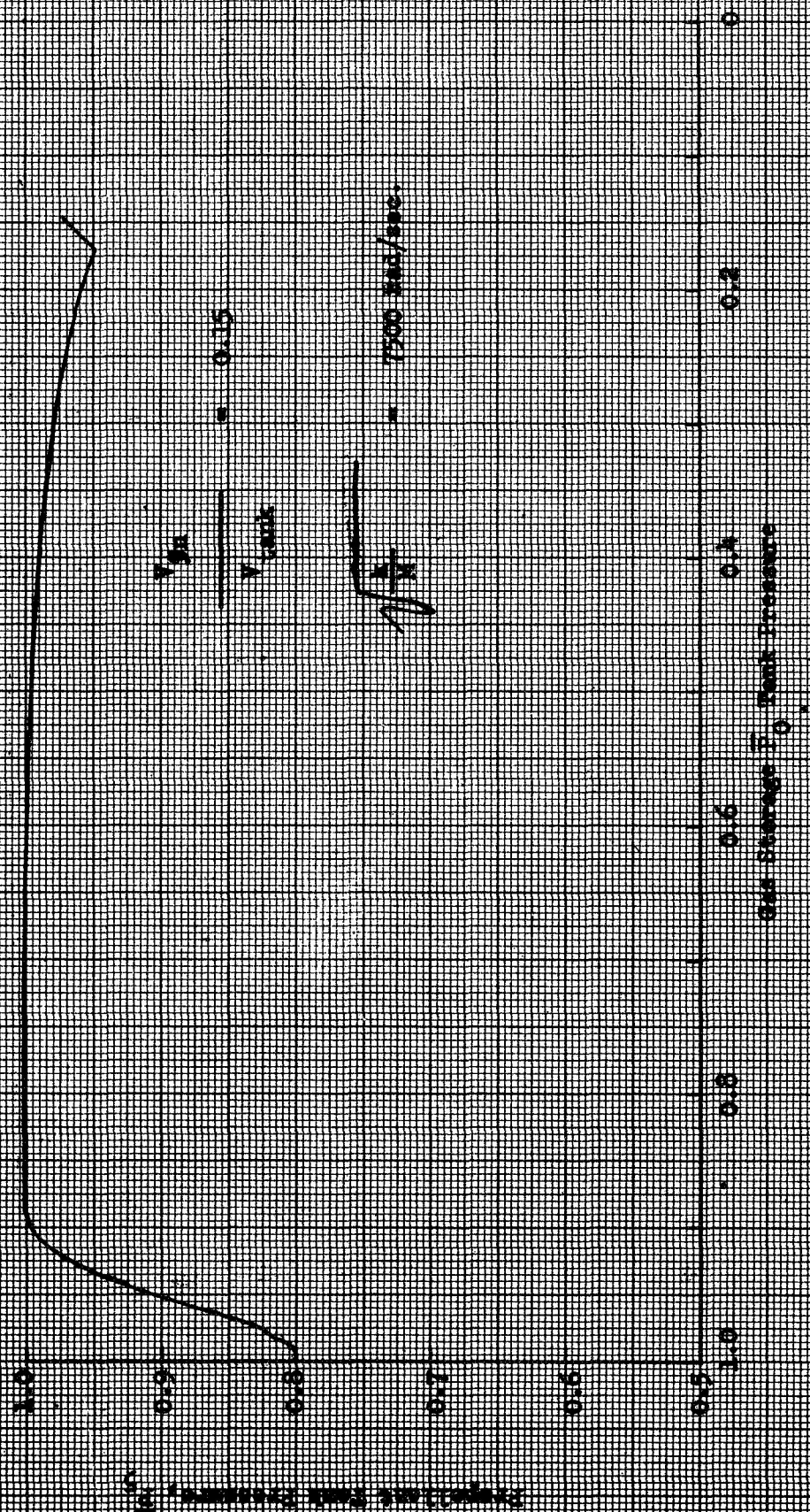


FIGURE 32 Case P.11, Combined Effect of Ullage Volume and Regulator Natural Frequency

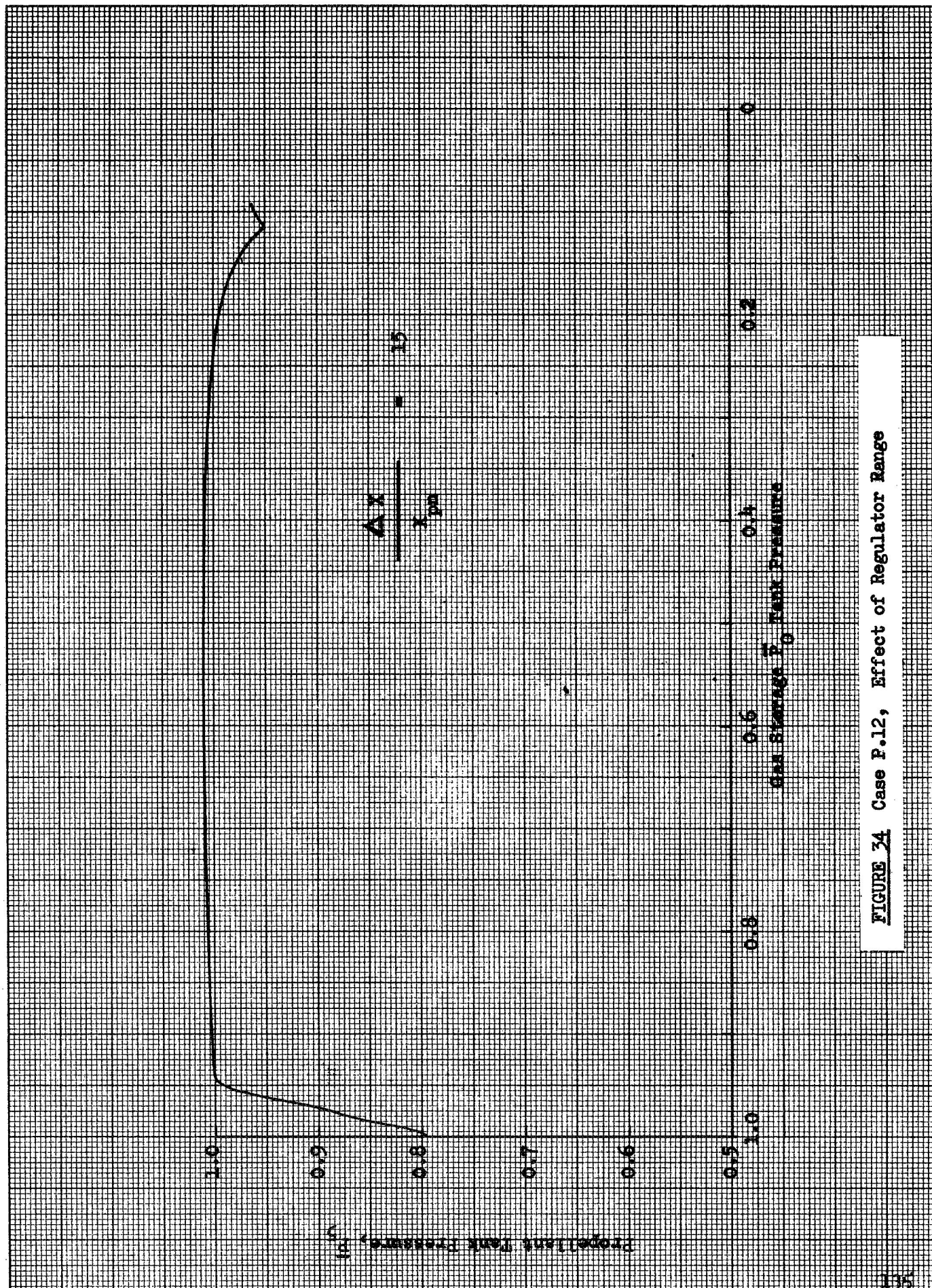


FIGURE 34 Case P.12, Effect of Regulator Range

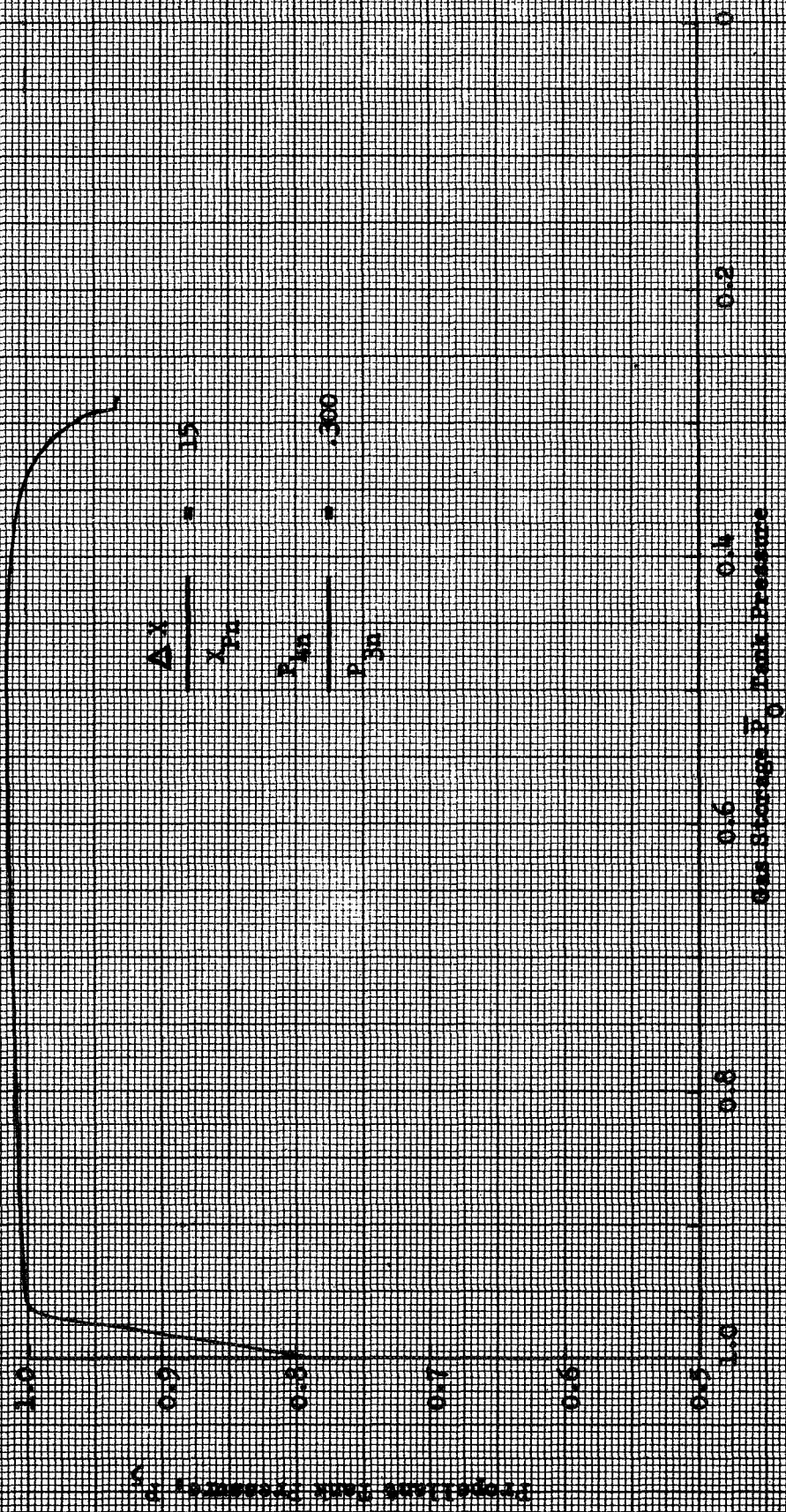


FIGURE 32 Case P.13, Combined Effect of Regulator Range and Regulator Seat Pressure Ratio

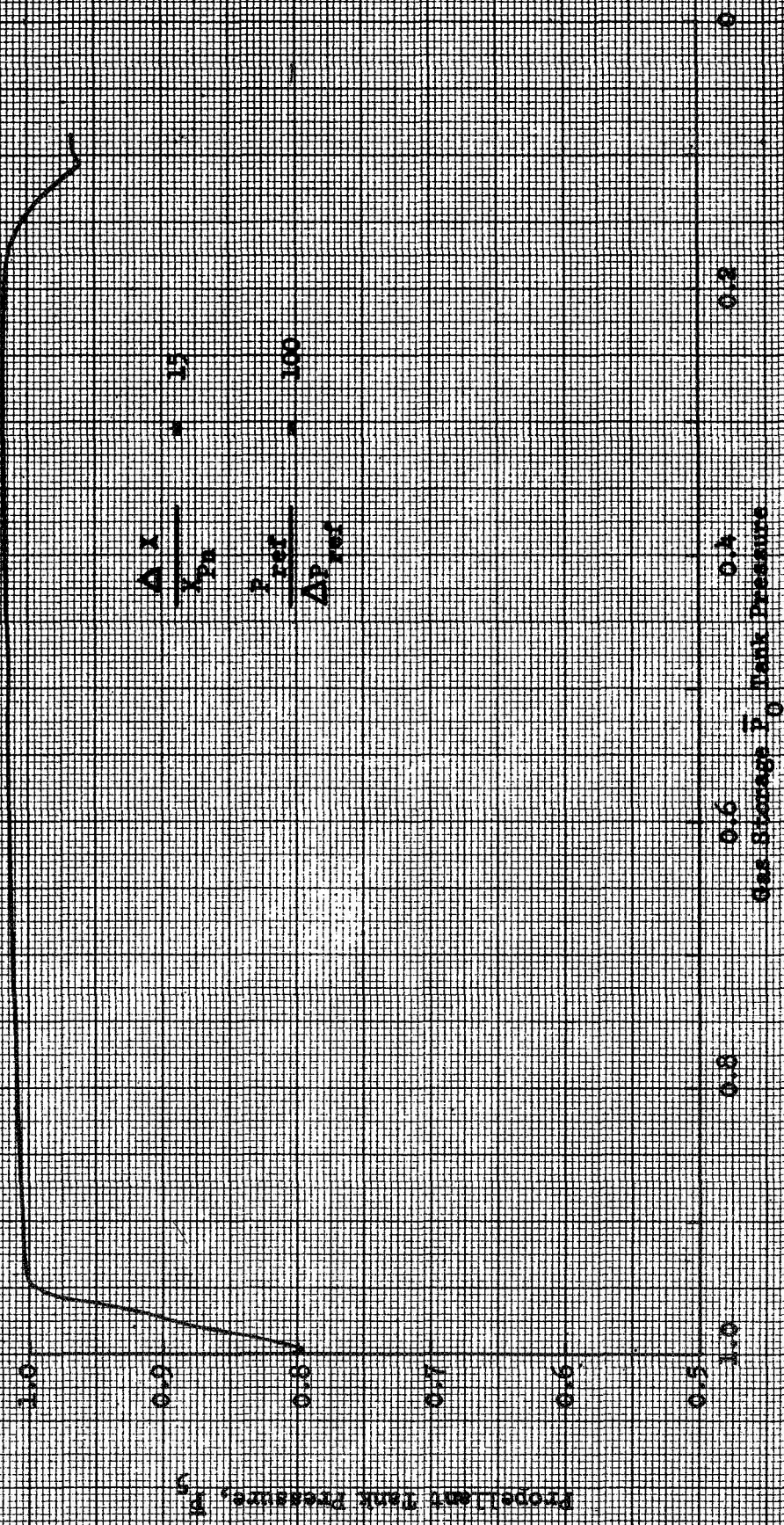


FIGURE 36 Case P.14, Combined Effect of Regulator Gain and Range

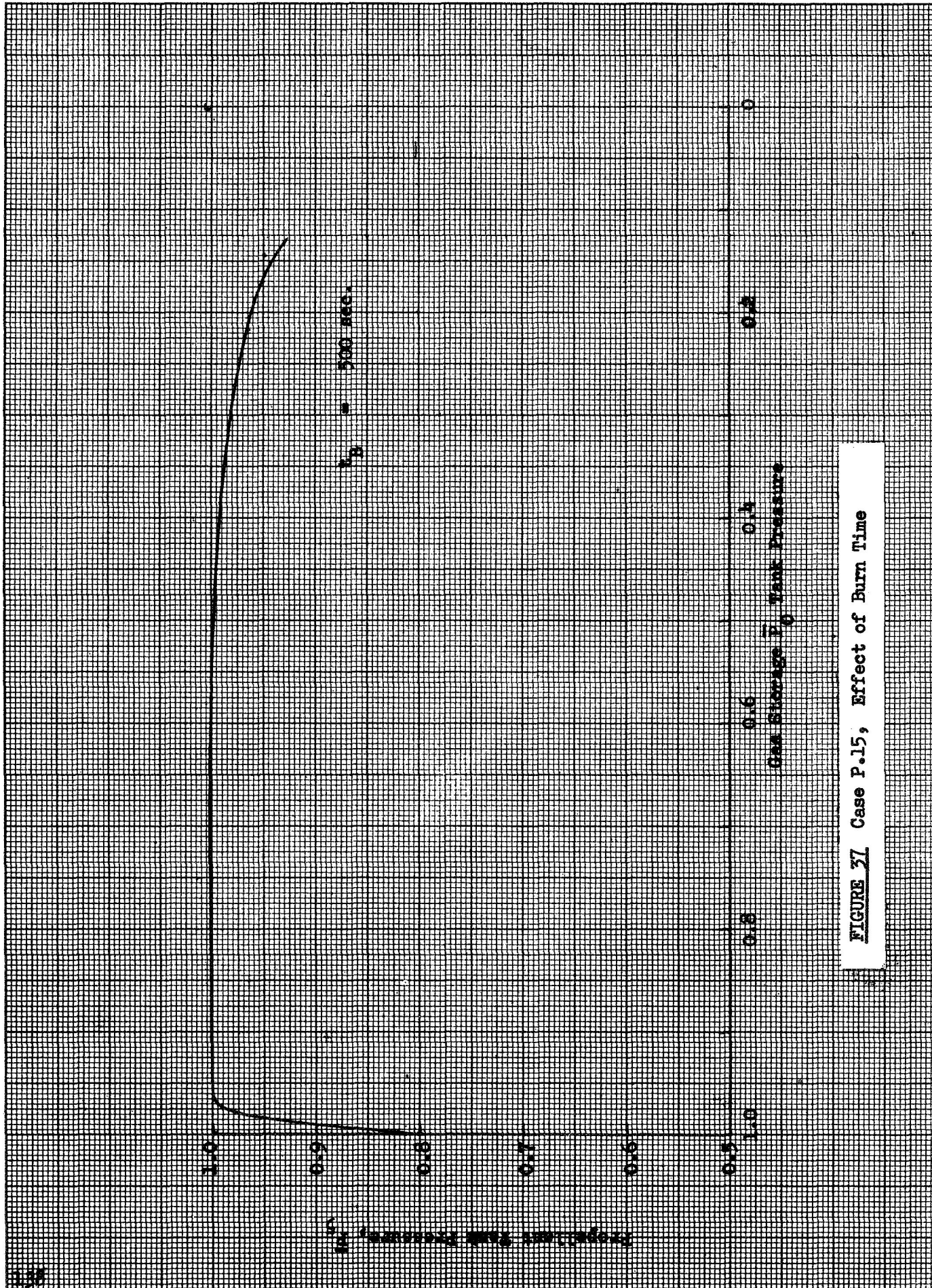


FIGURE 37 Case P.15, Effect of Burn Time

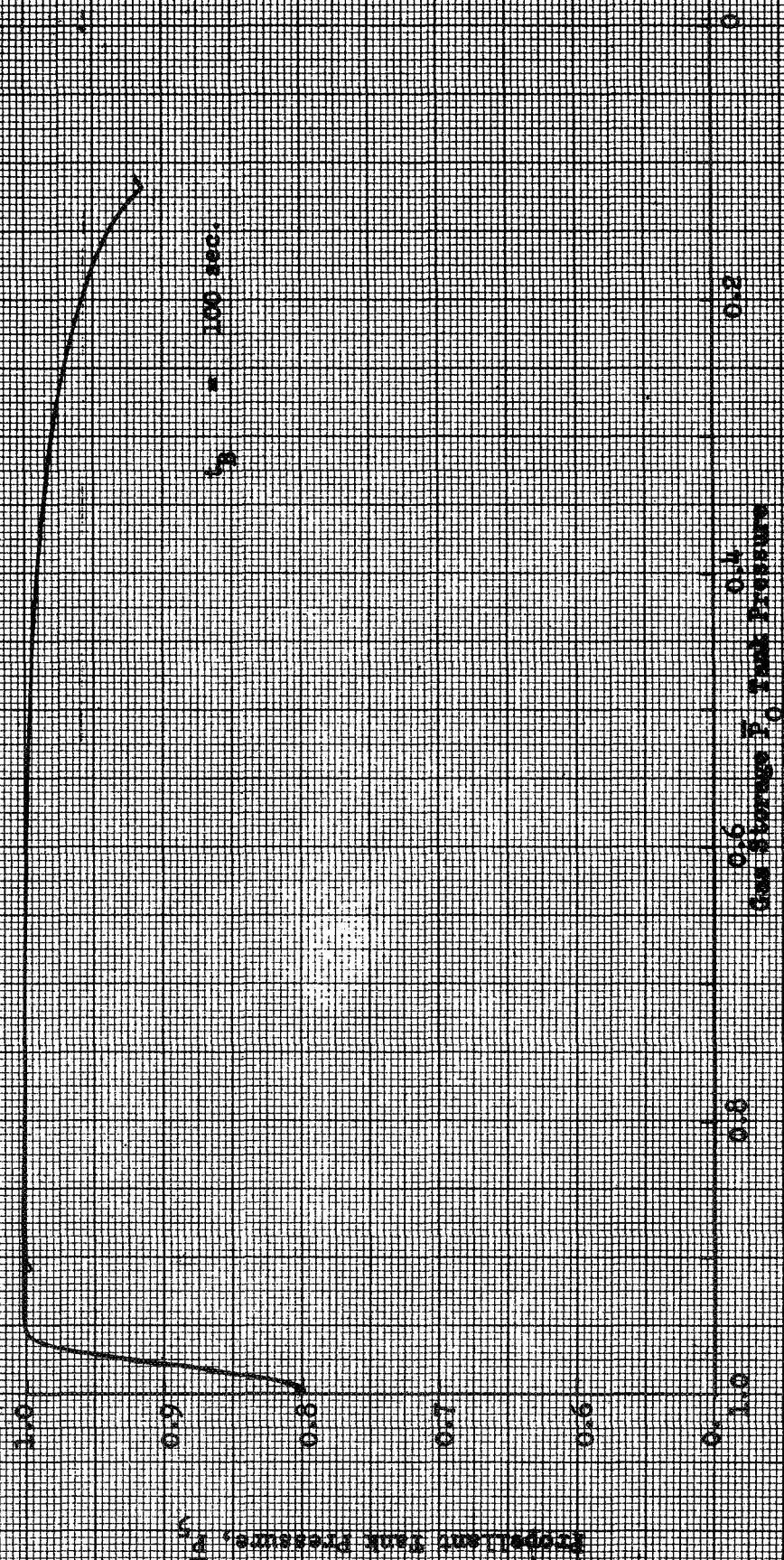


FIGURE 38 Case P.15 (modified), Effect of Burn Time

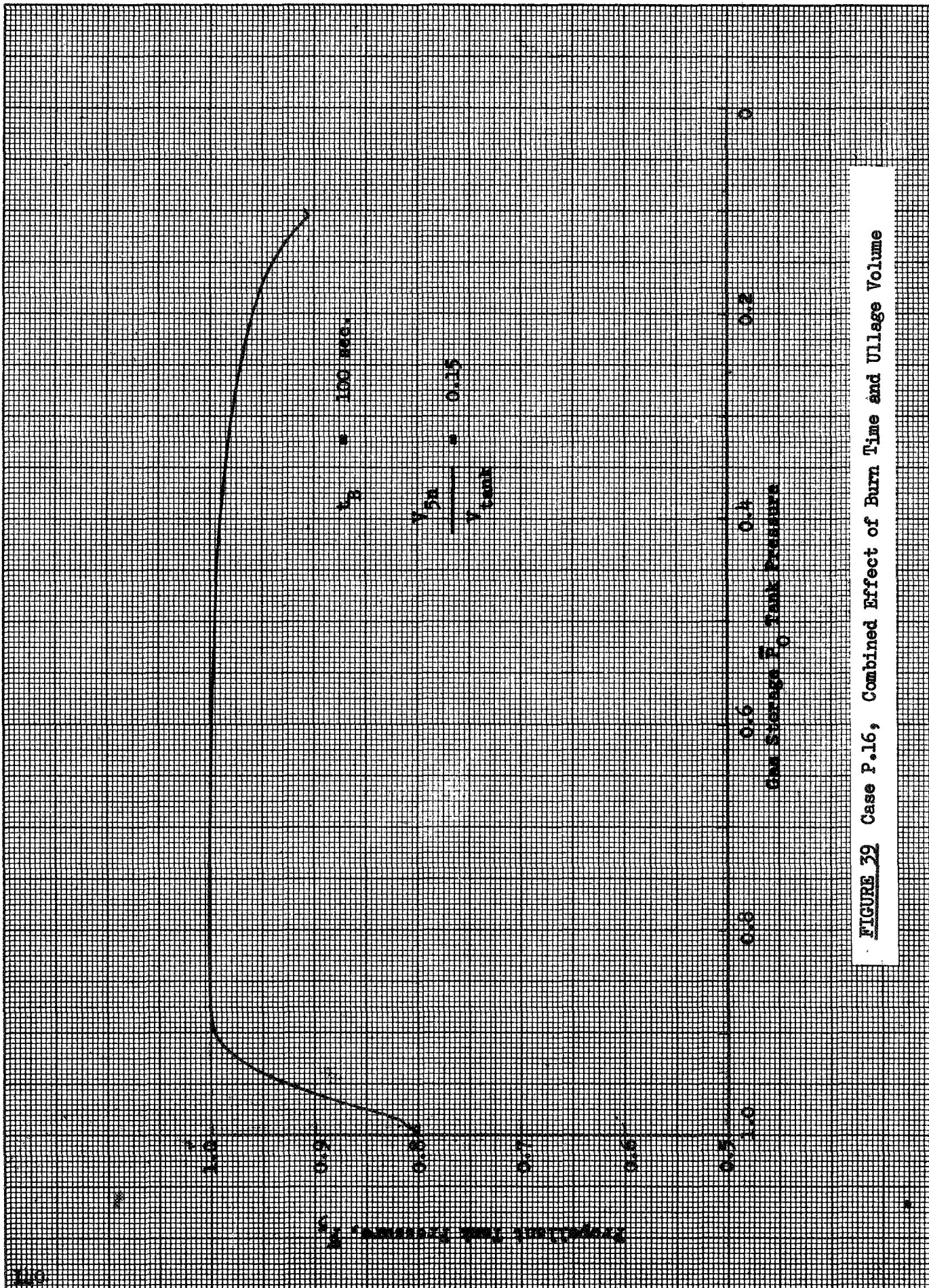


FIGURE 39 Case P.16, Combined Effect of Burn Time and Ullage Volume

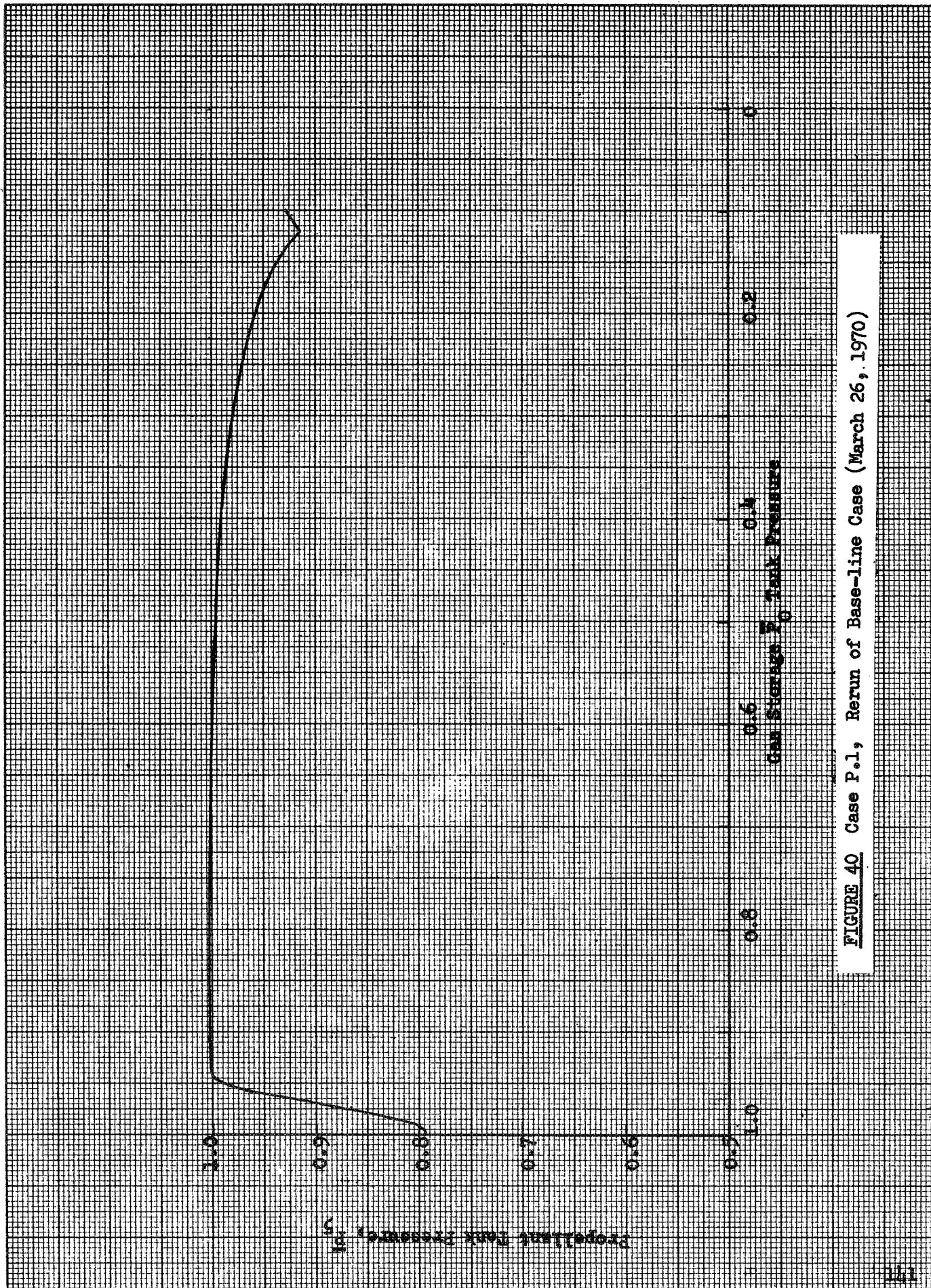


FIGURE 40 Case P.1, Rerun of Base-line Case (March 26, 1970)

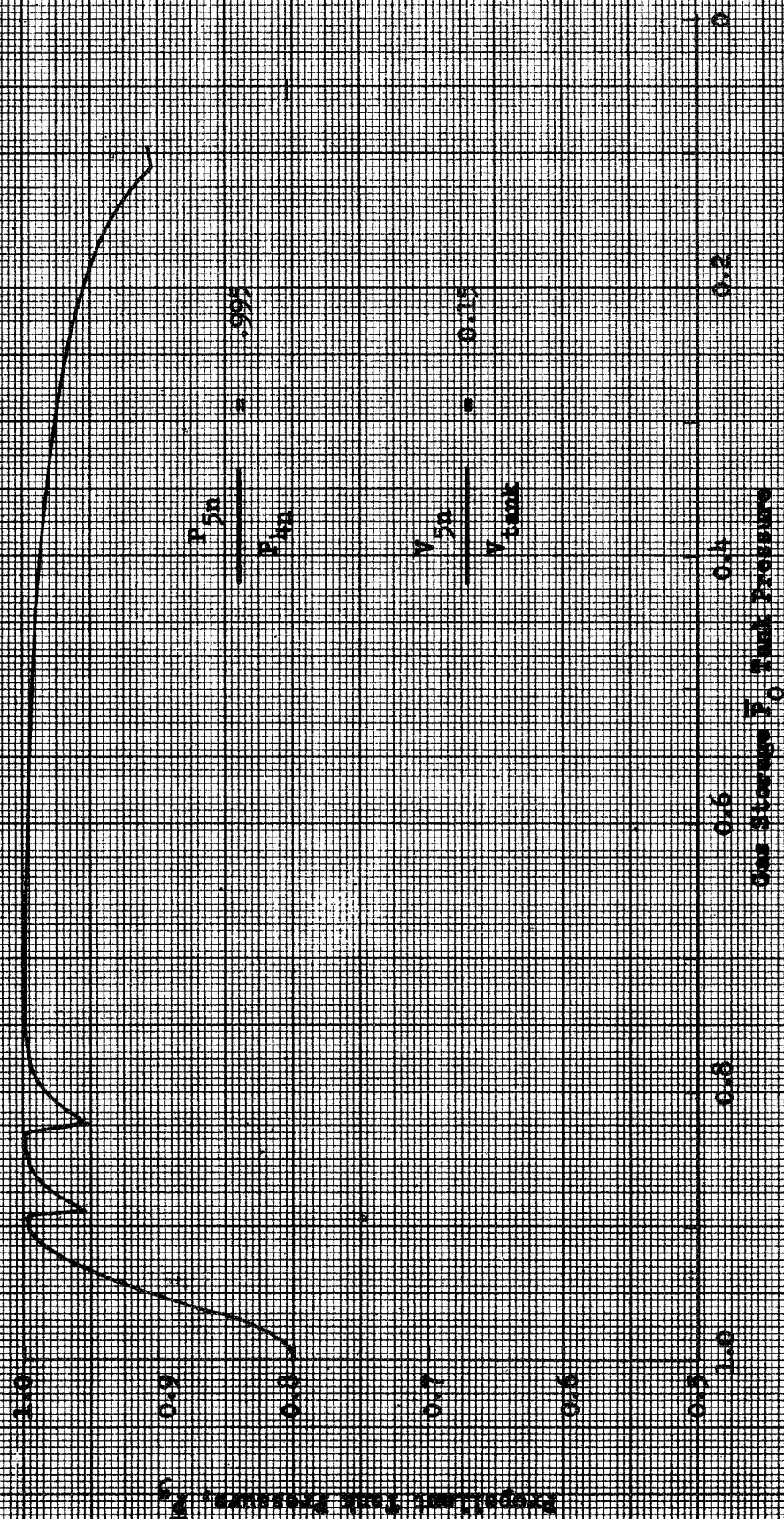


FIGURE 41 Case P.18, Combined Effect of Downstream Pressure Loss and Ullage Volume

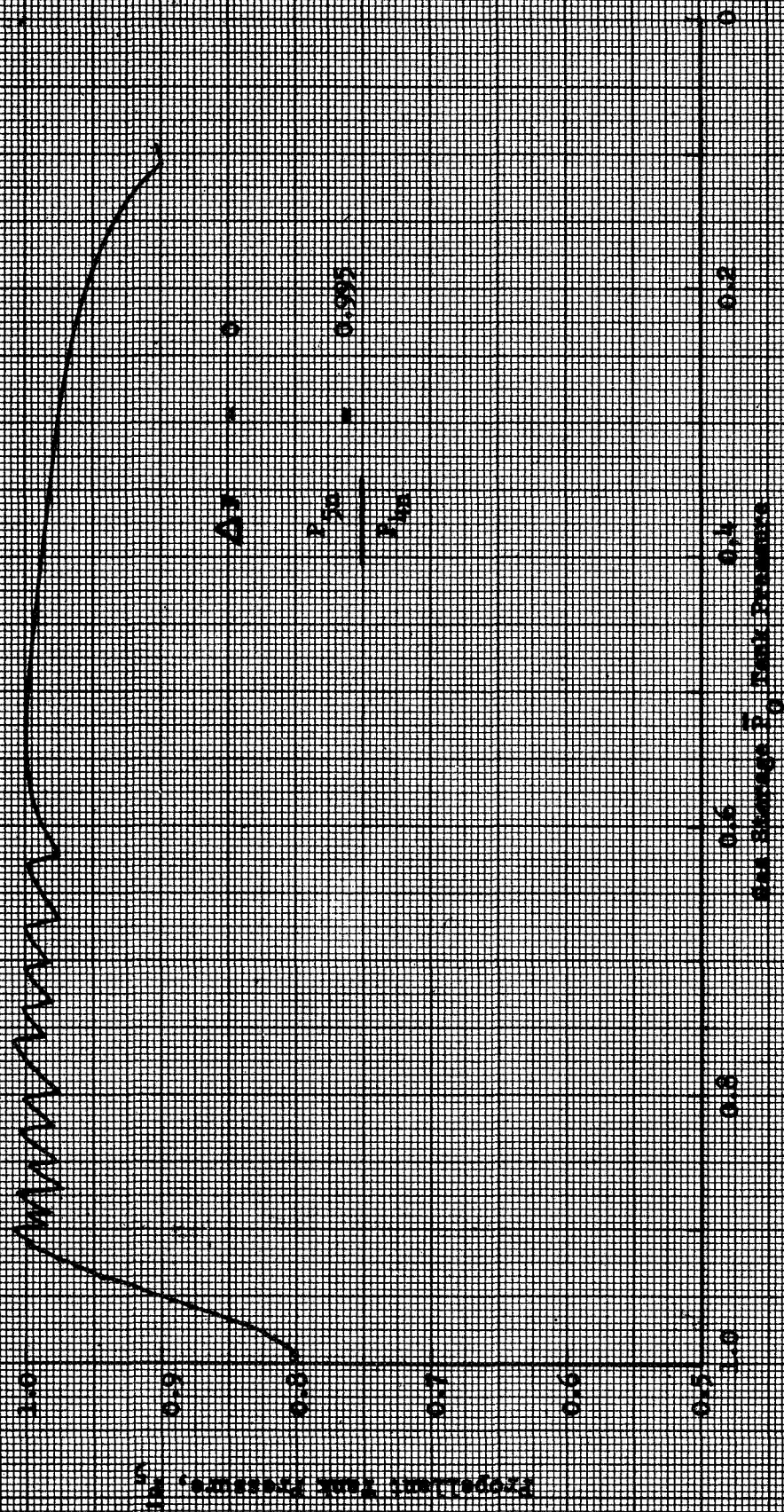


FIGURE 42 Case P.19, Effect of Downstream Pressure Loss Without Hysteresis

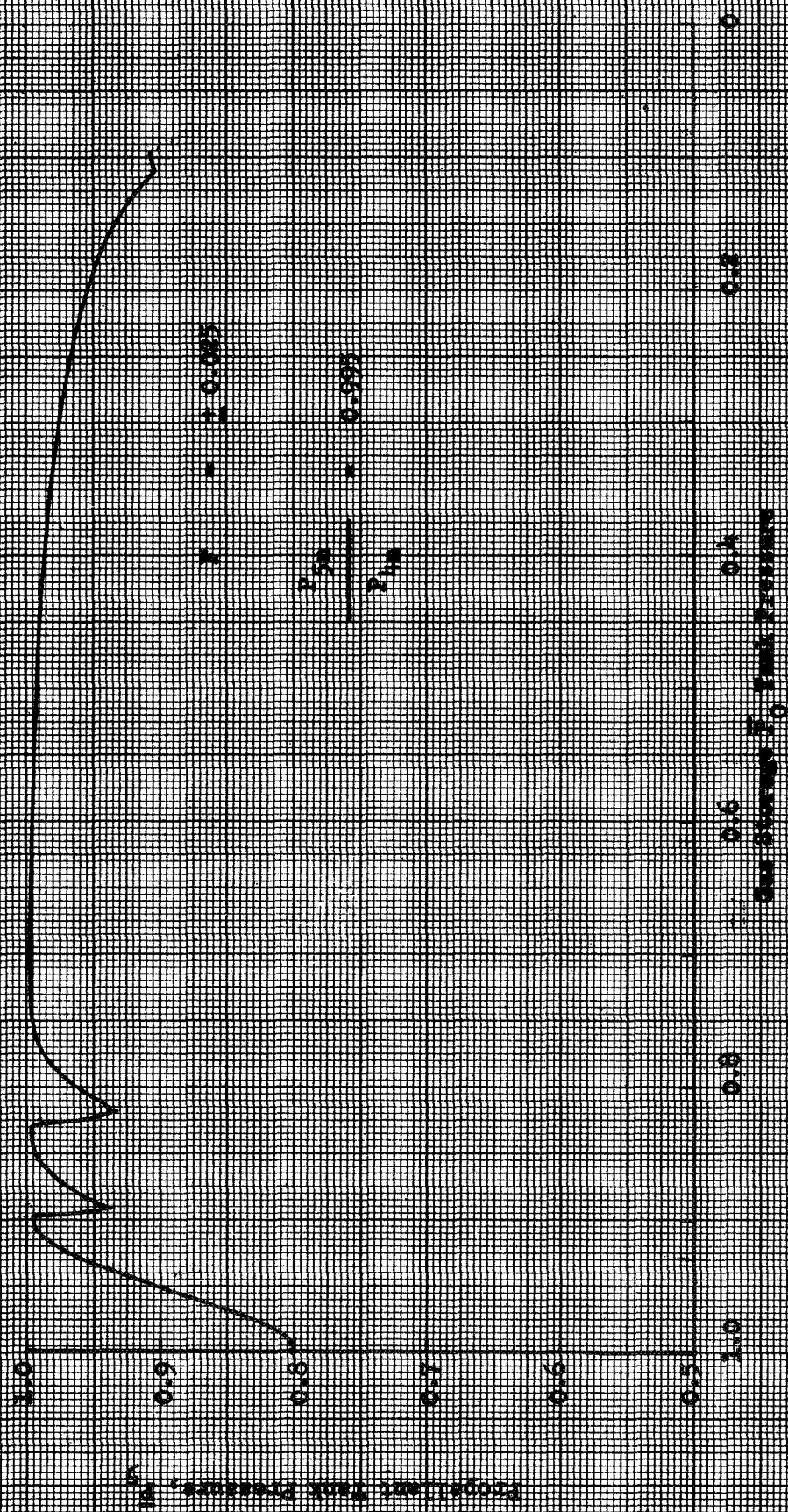


FIGURE 43 Case P.20, Effect of Downstream Pressure Loss With Increased Hysteresis

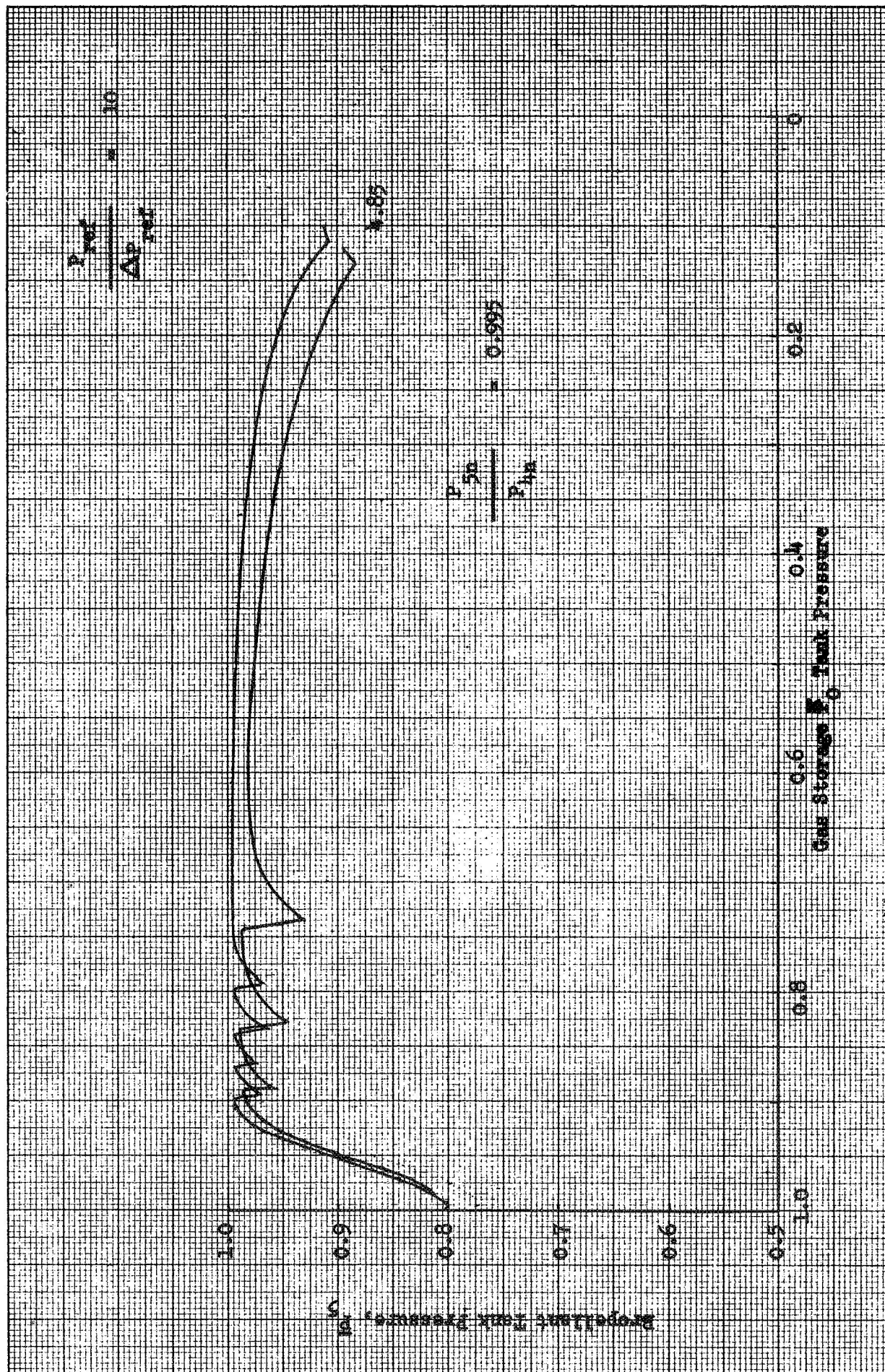


FIGURE 44 Case P.23, Effect of Downstream Pressure Loss and Regulator Gain

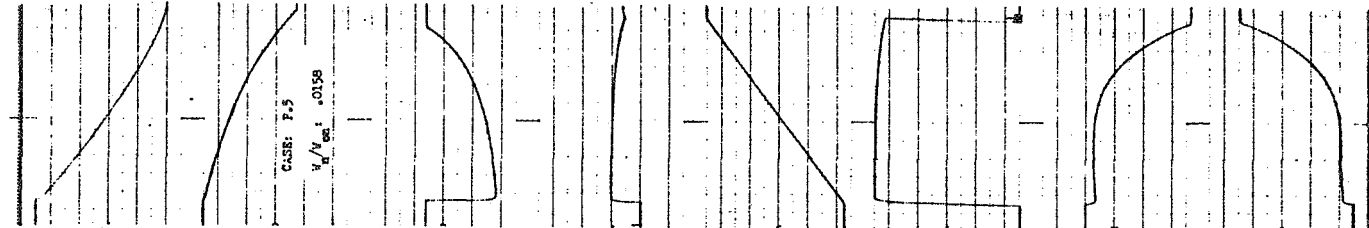
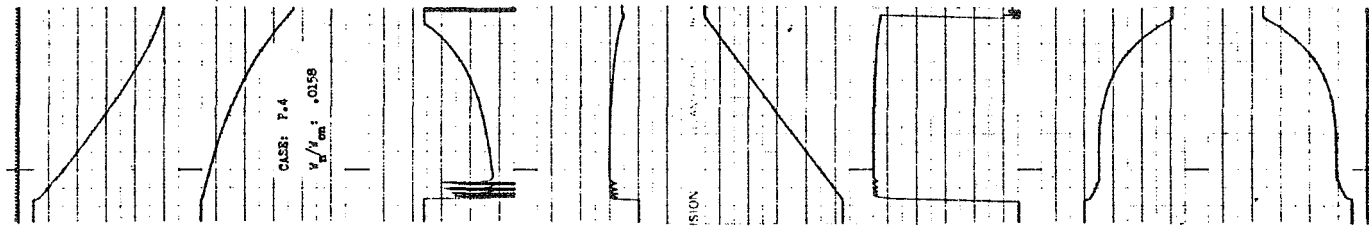
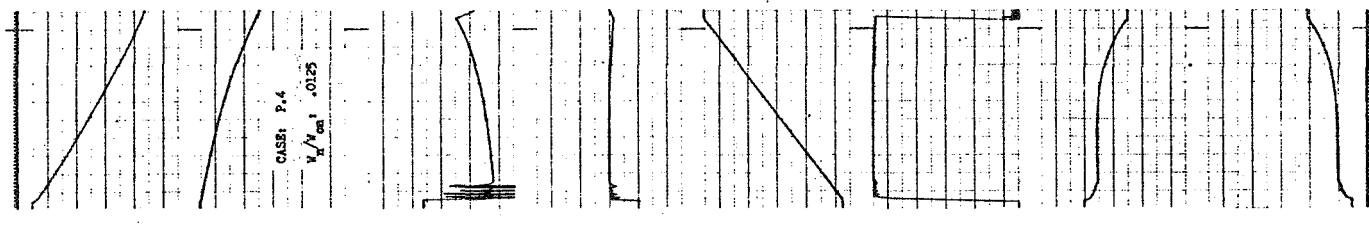
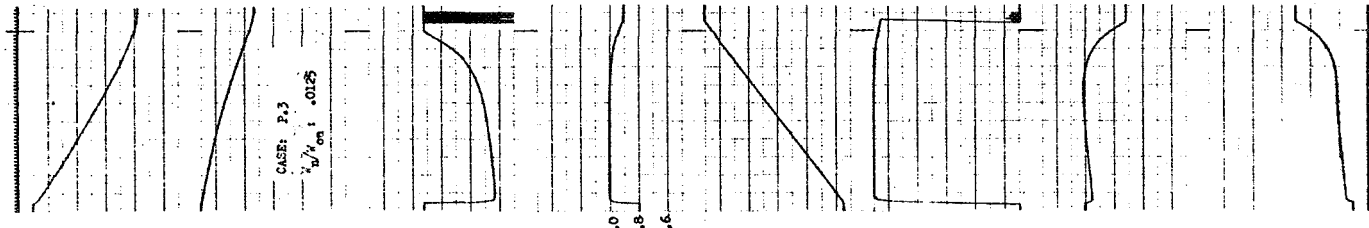
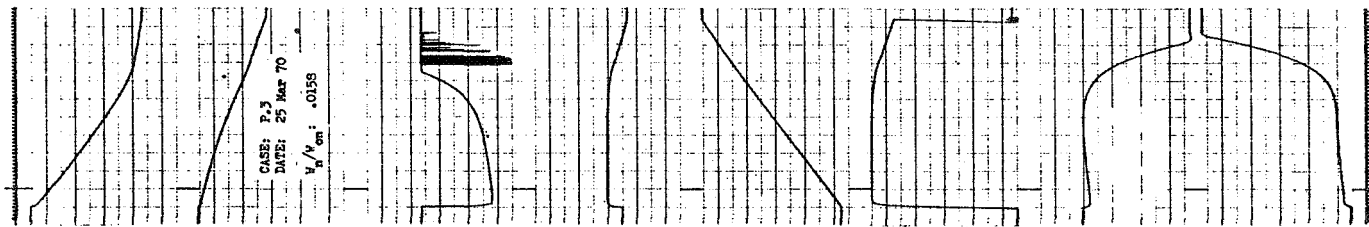
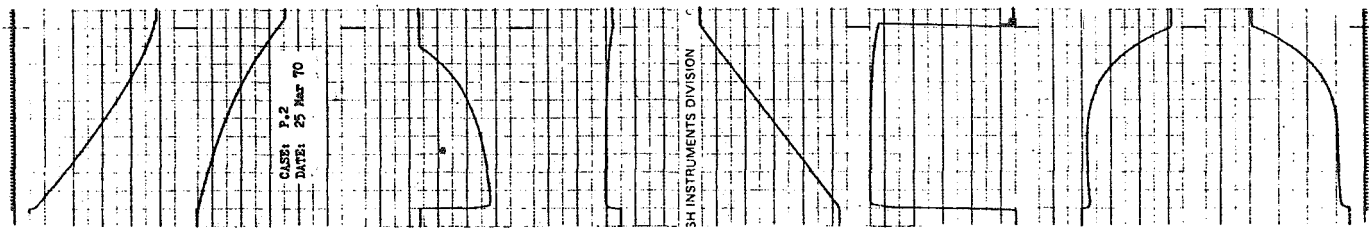
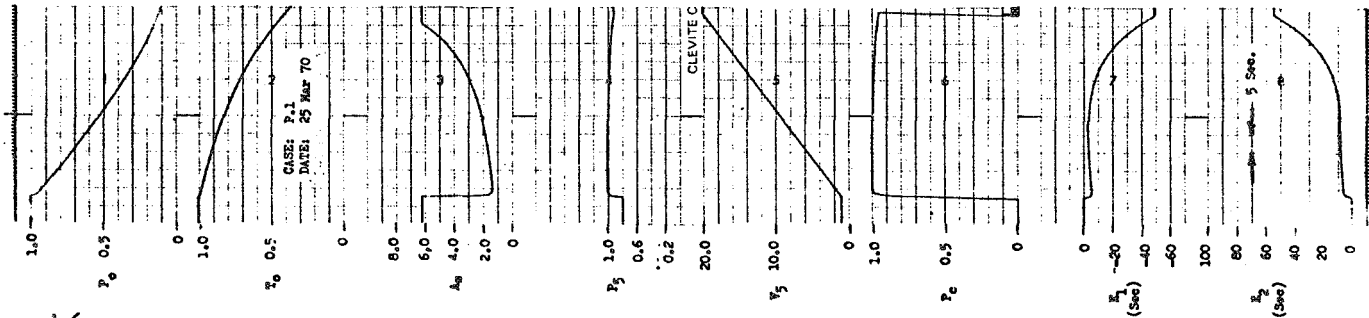


FIGURE 445 - TRANSIENT RESPONSE FOR PARAMETRIC STUDY, MONOPROPELLANT SYSTEM

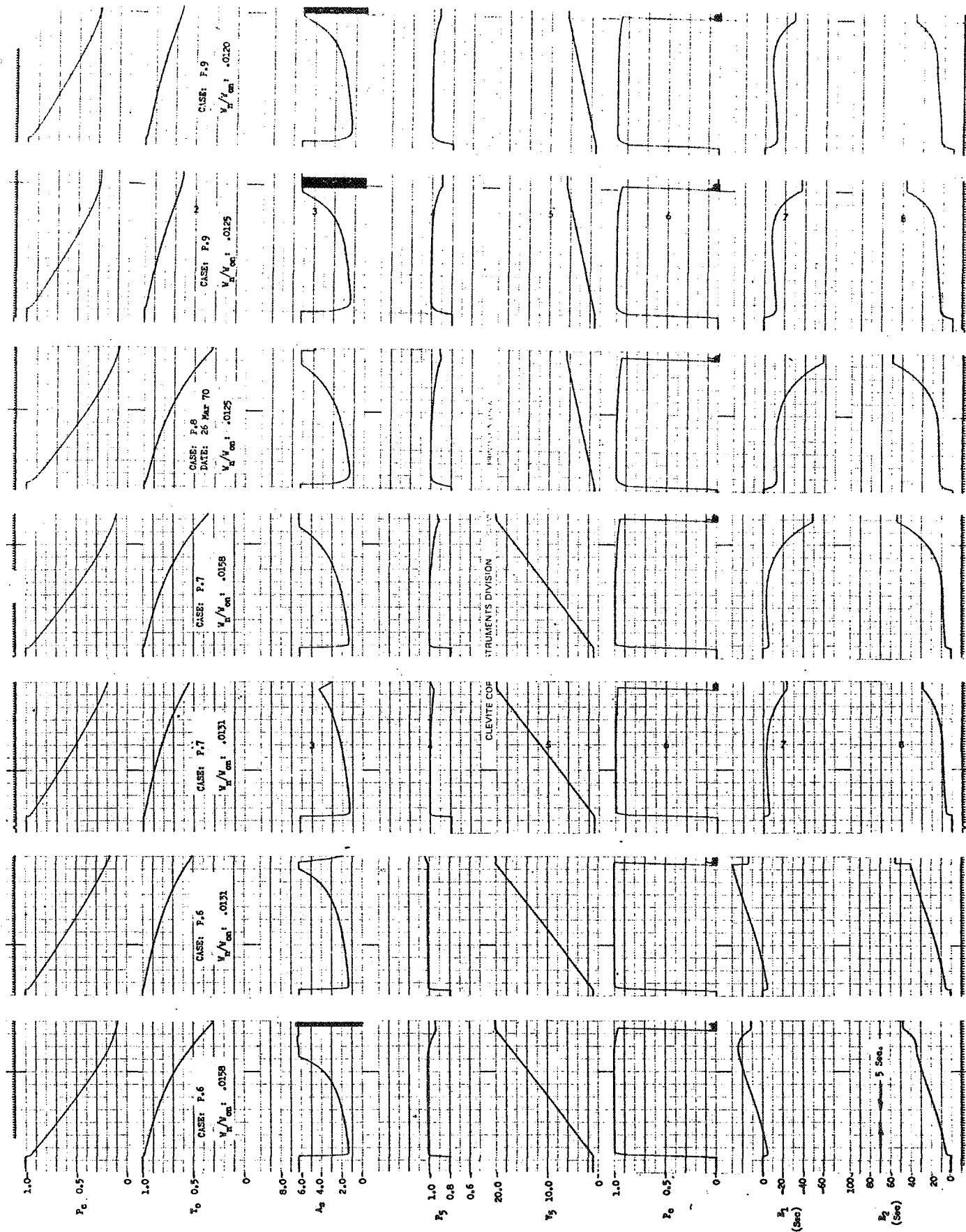


FIGURE 46 - TRANSIENT RESPONSE FOR PARAMETRIC STUDY, MONOPROPELLANT SYSTEM (CONT'D)

FIGURE 47 - TRANSIENT RESPONSE FOR PARAMETRIC STUDY, MONOPROPELLANT SYSTEM (CONT'D)

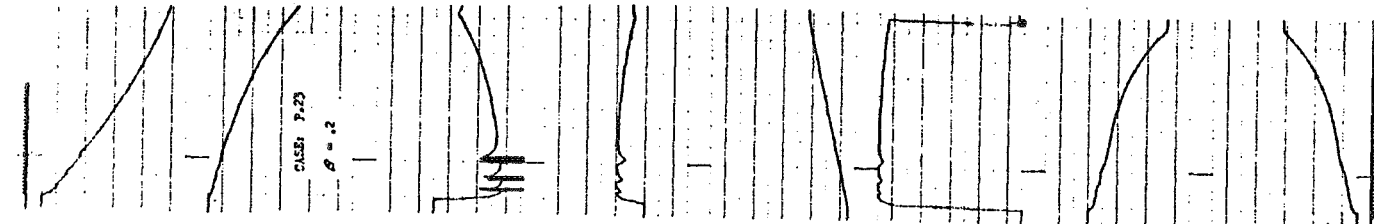
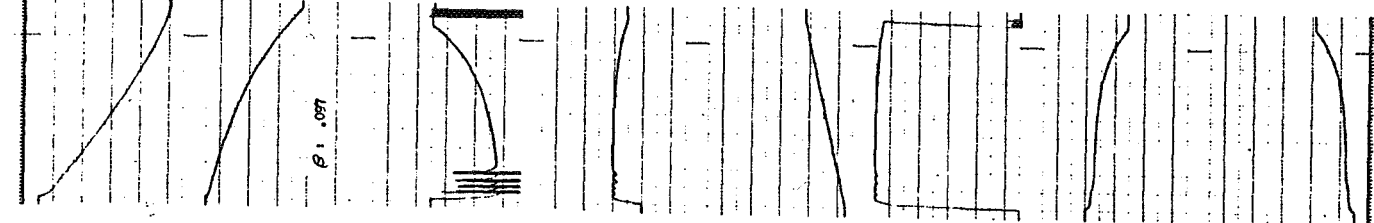
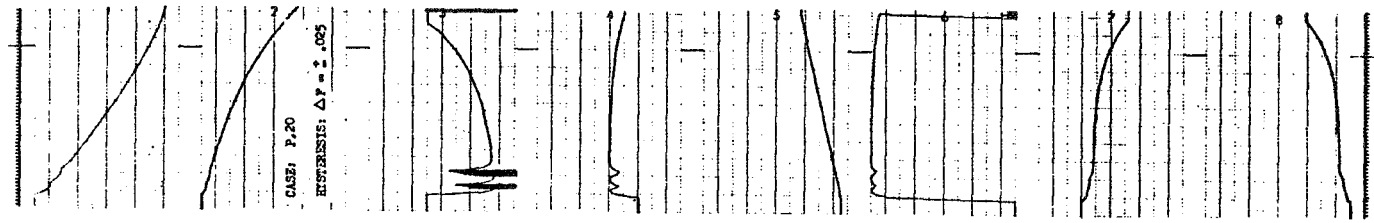
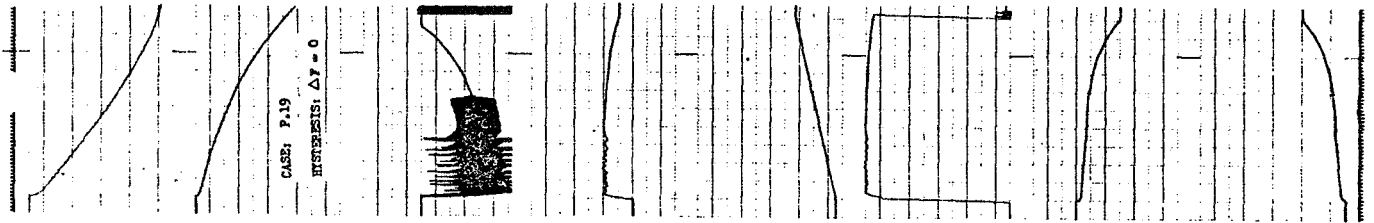
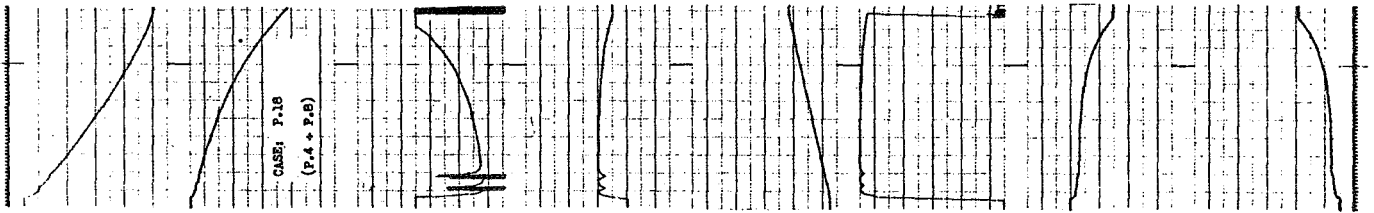
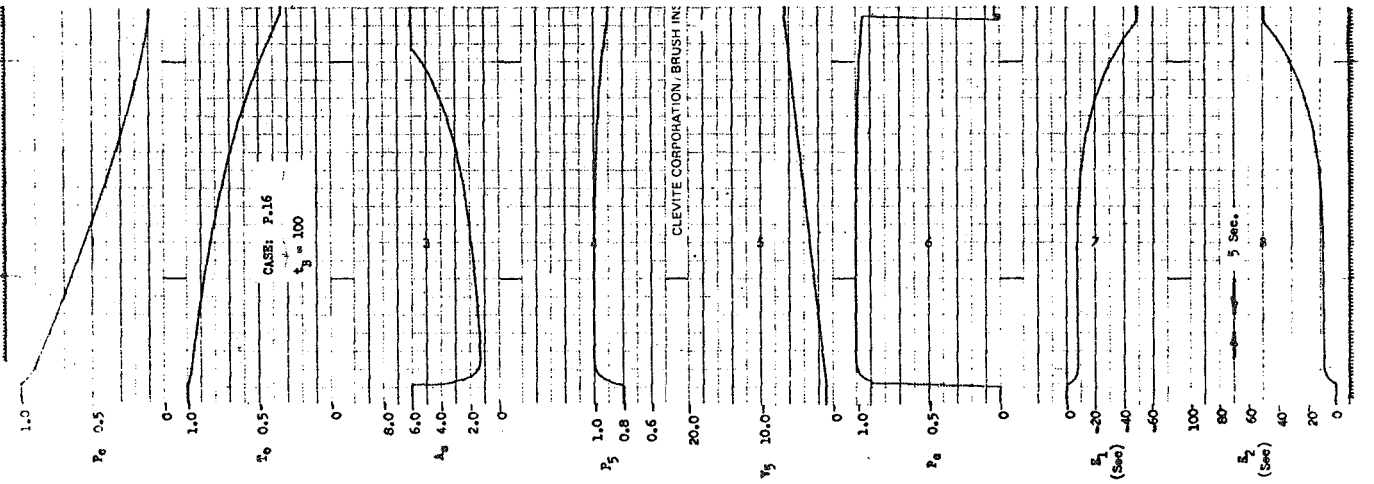


FIGURE 48 - TRANSIENT RESPONSE FOR PARAMETRIC STUDY, MONOPELLANT SYSTEM (CONT'D)

volume are on the fourth and fifth channels, and chamber pressure is shown in the sixth. The two performance functionals E_1 and E_2 are shown on the bottom two channels.* Due to normalization, all quantities are dimensionless with the exception of the performance functionals, which have units of seconds. Scales are shown at the left.

The results for the base-line case are shown in Fig. (22) and (45). In Fig. (22) it is seen that tank pressure rises to normal as the storage tank falls to about 94 percent of nominal. From Fig. (45) this is seen to occur in about 1 second. Note that the performance functional E_1 drops to about -0.06 seconds during this period, due to the error during P_c build-up.

The latter portion of blow-down is characterized by a decay in regulated pressure. This is due to the finite value of regulator gain, $P_{ref}/\Delta P_{ref}$. As the upstream pressure drops, the regulator must open further to pass the same mass-flow rate, which results in a reduction of reference spring force and hence regulation pressure. The hook on the end of the regulation curve is due to propellant depletion. At this point gas flow stops, and the disappearance of associated pressure losses in the gas flow system causes the tank pressure to rise slightly. The terminus of the line in the regulation curves is a steady-state point.

In some of the transient curves, it will be noted that the parameter W_{5n}/W_{On} is given. This is the parameter which "sizes" the storage tank. Since this is a parameter which is determined by proper design, it was sought to make it a dependent rather than independent parameter in this study. This was done by finding by trial and error the value which resulted in the regulator breakpoint occurring approximately 3 seconds

*Actually, $100 E_1$ and $100 E_2$ are plotted in order to expand the scale.

prior to total propellant depletion. This ensures that the regulator is in its range, i.e., can regulate over the largest part of the burn. This is approximately the procedure which should be followed in good design. A smaller value would result in large amounts of residual gas after burn and larger tank weight. A larger value would result in excessive impulse error due to lack of regulation near depletion. The trial and error process whereby the correct value was found is preserved in some of the parametric cases, for whatever value it may have to the designer.

Note that the same parameter (W_{5n}/W_{On}) is reflected in the parameter $\dot{w}_n/W_{On}$ in Fig. (32). This is so, since

$$\frac{W_{5n}}{W_{On}} = \frac{\dot{w}_n}{W_{On}} \cdot \frac{\dot{w}_n}{\dot{w}_{5n}}$$

The quantity $\dot{w}_n/W_{5n}$ can be determined from the parameters given in Table V, and Eq. (60).

The single most important observation from the results is the importance of regulator gain and range on the flatness of the regulation curve. As can be seen in Fig. (27), a gain of 100 results in a tank pressure less than 1.5 percent different from nominal or upstream pressures from 95 to 17 percent of nominal. Figure (32) shows this effect to also be true for larger ullage volumes ($\dot{w}_n/W_{On} = 0.0140$). The regulator range, $\Delta X/X_{pn}$ can have a similar effect, as is seen in Fig. (34).

Another observation from the results is that some abnormal oscillations occurred at the beginning of burn when the downstream pressure ratio was

very close to 1.0. This is seen in Fig. (25) and (41). This effect was further explored, Fig. (42 - 44), by variation of the spring hysteresis and slope, but could not be eliminated. No completely satisfactory explanation was found for this phenomenon, but the following observation was made. For values of pressure ratio very near 1.0, the compressible flow function is very steep, and therefore the analog mechanization is very critical. In the model, this function is supplied via the multiplexing network (Appendix A), and therefore noise is definitely present. This could have caused poppet oscillations, driven by erratic calculation of downstream flow rate and hence diaphragm pressure. This indicates that the mechanization may be inducing this, rather than any real phenomenon. However, the similarity to observed phenomenon in tests of the MM'69 regulator makes it difficult to rule out some real cause completely. This must be further investigated.

The bipropellant model introduced another important regulator parameter. The regulator employed in the bipropellant example case differed from that of the monopropellant system in that it has an isolated pressure sensing surface. Operation of this model revealed that this design introduces damping in the pneumatic system by means of the effect of poppet position on reference pressure. This relationship is shown in Eq. (9) of Table I.

Another effect of the isolated reference cavity was found to be overshoot of pressures in the flow system between the regulator and tank during the pressurization transient. This is a result of the regulator remaining open until the reference cavity charges. During this period, flow rates are higher than normal, resulting in larger pressure losses at the restrictions between the tanks and the regulator.

Figure (49) shows the pressure overshoot transients as the downstream pressure losses are varied parametrically. It is seen that lower pressure losses, i.e., pressure ratios closer to 1, result in lower overshoot peak pressures.

Figure (50) shows the effect of the reference cavity volume factor, $A_e \Delta X_m / V_{rm}$. This parameter is a measure of the degree of damping afforded by this mechanism. It is seen that small values result in rapid decay of pressures from their peak values, and considerable undershoot. Larger values cause the poppet motion to be slowed, thus decreasing the rate at which the pressures return to lower levels. Also, the undershoot is lessened. Apparently, values of this parameter greater than 0.14 are not required, since no further change in response is noted as it is increased to 0.28. The value 0.07 is that of the MM '71 regulator. Although not shown in the results presented herein, it was found that a value of zero of this parameter results in undesirable and possibly unstable oscillations in system pressures. This was not explored to any significant depth in this study, however.

It is notable that damping by the above mechanism is only possible when the pressure sensing surface is indeed isolated. If there is no restriction between the reference cavity and the main flow, the poppet position has no effect on reference pressure. It is therefore to be expected that the area of the reference orifice is also a parameter of considerable importance. However, this parameter was not varied in the analysis.

In summary, the results of the parametric study show that, in order of importance,

- a) The regulator gain, $P_{ref} / \Delta P_{ref}$, is the most important regulator parameter. A very low value gives large changes in regulated pres-

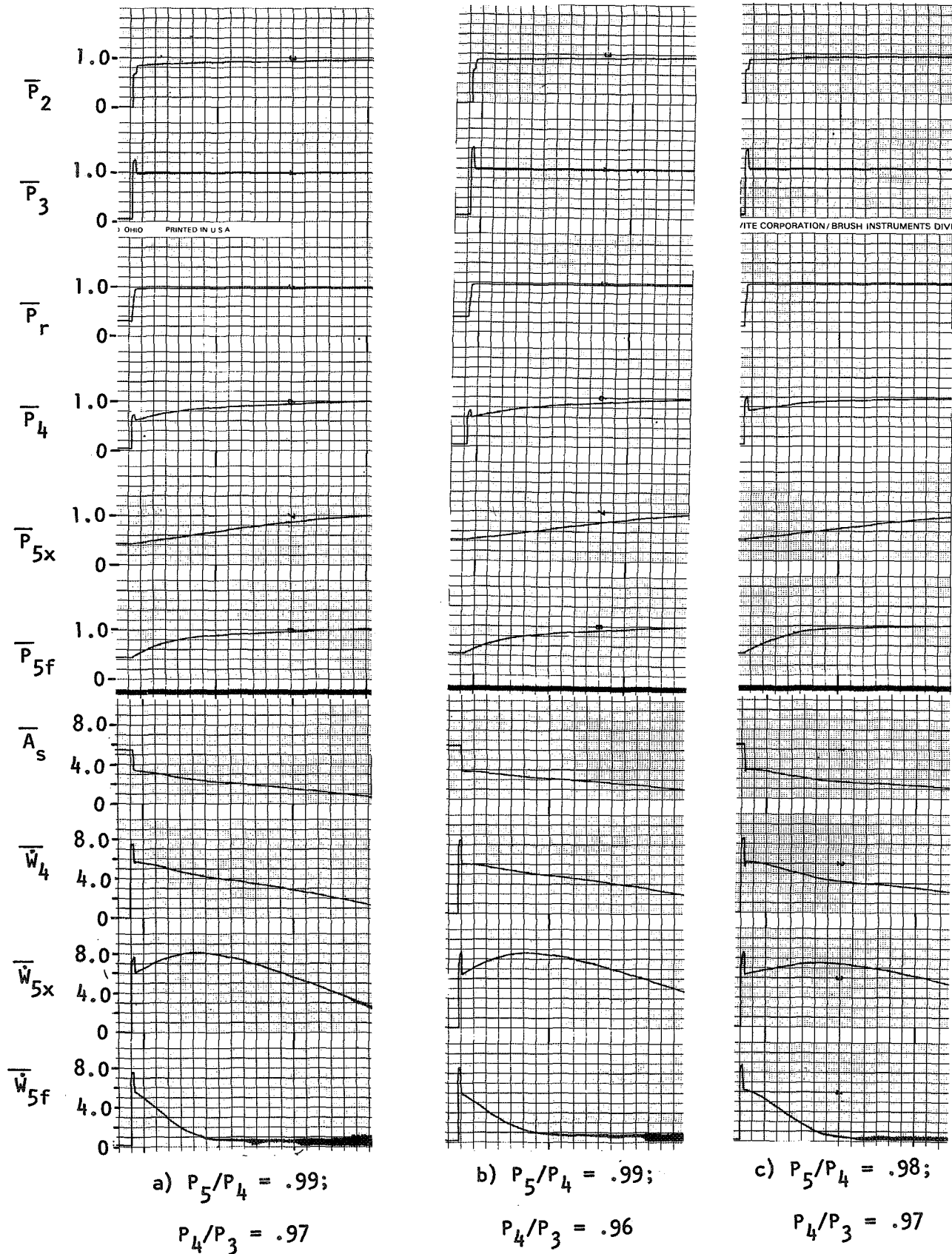


Figure 49. Effect Of Pressure Drop Across Downstream Restrictions For Bipropellant System

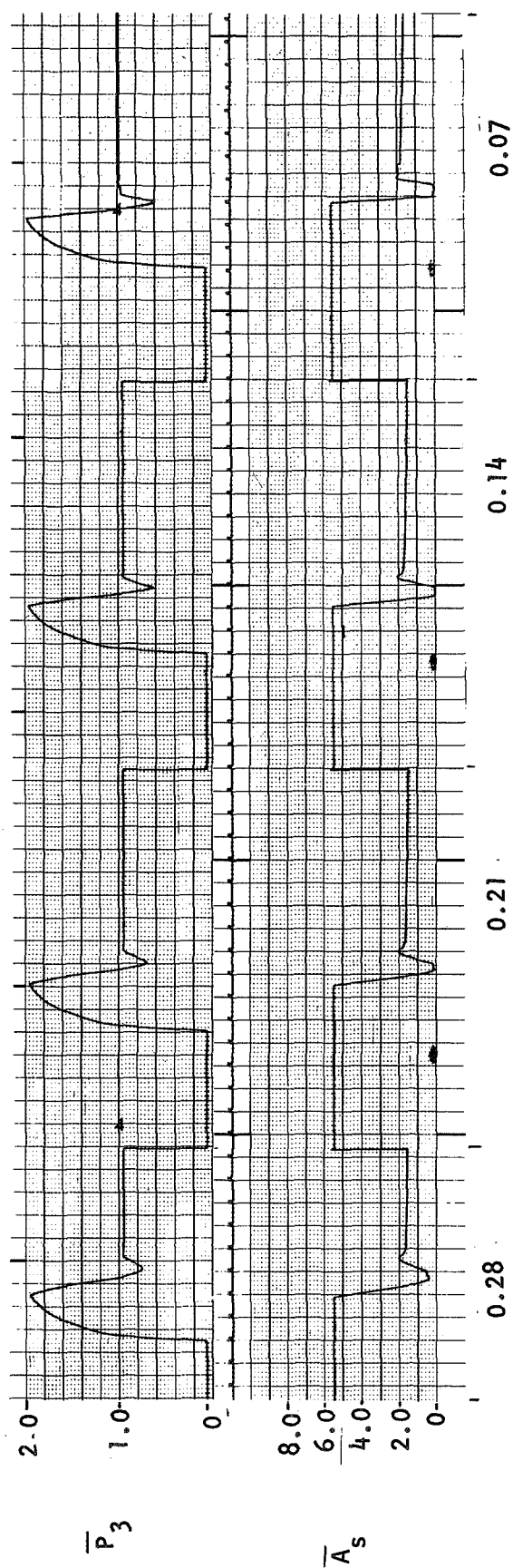


Figure 50. Effect of Reference Cavity Volume Factor, $A_e \Delta X_m / V_{rm}$

sure as the supply pressure changes. An extremely high value, such as would be obtained from an almost flat spring curve results in difficulty in achieving the desired set point. A flat spring curve results in infinite gain, in which case the regulator acts simply as an on-off device, and is, therefore, unsatisfactory.

- b) The regulator range has an effect similar to the gain, in that large values of this parameter can improve regulation.
- c) Pressure ratio between the regulator and the ullage volume can influence stability. Pressure ratios near 1, i.e., low pressure drop, can result in the regulator operating in a cycling mode.
- d) An isolated reference cavity gives damping to the pneumatic system. However, when an isolated reference cavity is present pressure overshoots occur between the regulator and ullage volume.

The above observations from the parametric results are applicable to mono- or bipropellant systems.

CONCLUSIONS AND RECOMMENDATIONS

A generalized method has been developed for analysis of the pressurization systems of pressure-fed propulsion systems. The method is based upon generalized elements from which lumped parameter models can be developed for either monopropellant or bipropellant systems. The equations of each of these elements are developed, and put into a normalized, scaled form for maximum generality and solution on an analog computer.

The method has been verified by application to two different propulsion systems for which test data were available. The essential characteristics of the test results were successfully duplicated by the models. In both cases, the models also revealed additional information about system performance which was not available from test data.

The models developed in the example cases were also used to generate parametric data for future propulsion systems. These parametric studies revealed several parameters of particular importance to system performance. These were summarized in the previous section of this report.

The study also revealed new questions which remain unanswered. It is recommended that future effort be directed toward the following areas, which are given in the order of importance:

- a) It was shown that the regulator spring slope, downstream pressure losses, reference cavity geometry, and downstream volumes all affect system stability. The precise ranges of these parameters which ensure satisfactory operation should be determined in a future effort.
- b) Certain regulator parameters which were not varied in this study should be considered in a future effort. These include reference cavity volume change ratio ($A_e \Delta X / V_n$), reference orifice diameter and secondary poppet forces.

- c) The generalized model developed herein includes equations for a single stage regulator only. Equations should also be developed for two-stage regulators.

LIST OF SYMBOLS AND SUBSCRIPTS

SYMBOLS

A	- Area, in. ²
a/b	- Mechanical ratio of regulator [see Fig. (4)]
B	- Damping coefficient (Regulator Eq. of motion), lb-sec/in.
B'	- Effective liquid compressibility (See Eq. 24), lbs/in ²
C	- Discharge coefficient
C_b	- Specific heat of catalyst bed, BTU/lb ^o R
C_c	- Specific heat of combustion or decomposition products, BTU/lb ^o R
C_v	- Pressurant specific heat at constant volume, BTU/lb ^o R
F	- Force, lb.
F_c	- Coulomb friction force, lb.
F_{pr}	- Regulator pressure forces, lb.
F_{bs}	- Regulator ball spring force, lb.
g_c	- Gravitational constant, in/sec ²
K	- Proportionality constant
K'_r	- " "
k_s	- Regulator reference spring constant, lb/in
k_b	- Regulator ball retaining spring constant, lb/in
L	- Length of liquid line, in.
m	- Mass of regulator moving assembly, lb-sec ² /in
m_b	- Mass of thrust chamber assembly, lb.
P	- Pressure, lb/in ²
ΔP	- Nominal pressure drop in liquid line, lb/in ²

LIST OF SYMBOLS AND SUBSCRIPTS - CONT'D.

SYMBOLS

Q	- Heat flow into propellant tank, BTU/sec
R	- Gas constant, in/ ^o R
r	- Radius of regulator seat, in.
r/R	- Ratio of seat radius to ball radius
T	- Temperature, ^o R
U	- Unit step function
V	- Lumped compressible volume, in. ³
W	- Stored mass, lb-sec ² /in
$\dot{w}$	- Mass flow rate, lb-sec/in
X	- Poppet position (positive for regulator opening), in
Z	- Compressibility factor

GREEK

α	- Pressure Ratio, downstream/upstream
γ	- Ratio of specific heats, pressurang
ϵ	- Ratio of actuator area to ball area
μ	- Mixture ratio, oxidizer to fuel
ρ	- Density, lb-sec ² /in ⁴
Φ	- Compressible flow function Fig. (5)

LIST OF SYMBOLS AND SUBSCRIPTS - CONT'D.

SUBSCRIPTS

B	- Regulator bore (seat hole)
b	- Regulator ball
c	- Combustion or decomposition chamber
i	- General position index
j	- Position index upstream of flow division point in bipropellant system
k	- Position index for cavity downstream of regulator seat
L	- Liquid
m	- Maximum
n	- Nominal
o	- Seated position of ball
p	- Poppet
r	- Reference cavity or spring
s	- Regulator seat
v	- Volumetric

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4. Parmakian, John, Waterhammer Analysis, Daven Publications, New York, 1963, p. 10
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APPENDIX A

MM '69 REGULATOR

- EQUATIONS
- CONSTANTS
- COMPRESSIBLE FLOW FUNCTION
- SPRING CHARACTERISTIC

MM '69 FLOW BENCH

- EQUATIONS
- CONSTANTS
- DERIVATION OF DYNAMIC TEMPERATURE EQUATIONS
- ANALOG MULTIPLEX METHOD
- COMPUTER DIAGRAMS
- PHOTOGRAPH OF COMPUTER SETUP

MM '69 WATER EXPULSION SYSTEM

- EQUATIONS
- CONSTANTS
- DERIVATION OF EFFECTIVE BULK MODULUS
- COMPUTER DIAGRAMS

MM '69 PROPULSION SYSTEM

- EQUATIONS
- THRUST CHAMBER BLOCK DIAGRAM
- COMPUTER DIAGRAMS

MM '69 REGULATOR

MM '69 - REGULATOR EQUATIONS

$$(1) \quad W_2 = K_1 P_1 (P_1 - P_3)/T_1 \quad (P_1, T_1 = \text{inputs}, K_1 = \text{const.})$$

$$(2) \quad W_3 = \int (\dot{W}_2 - \dot{W}_3) dt$$

$$(3) \quad T_3 = \int \left[\dot{W}_2 (\sigma T_1 - T_3) - \dot{W}_3 T_3 (\sigma - 1) \right] / W_3 dt \quad (\sigma = \text{const.})$$

$$(4) \quad P_3 = W_3 R T_3 / V_3 \quad (R, V_3 = \text{const.})$$

$$(5) \quad \dot{W}_3 = C_s A_s P_3 \phi_3 / \sqrt{T_3} \quad (C_s = \text{const.})$$

$$(6) \quad \phi_3 = 1.06 \left[P_4/P_3 (1 - P_4/P_3) \right]^{\frac{1}{2}} \quad P_4/P_3 \geq .5 \quad (\text{Fig. 4})$$

$$= 0.53 \quad P_4/P_3 < .5$$

$$(7) \quad A_3 = K_2 X_p \quad A_3 \geq 0 \quad (K_2 = \text{const.})$$

$$(8) \quad T_4 = \int \left[\dot{W}_x (\sigma T_3 - T_4) - \dot{W}_5 T_4 (\sigma - 1) \right] / W_4 dt \quad (\dot{W}_5 = \text{input})$$

$$(9) \quad W_4 = \int (\dot{W}_3 - \dot{W}_5) dt$$

$$(10) \quad P_4 = W_4 R T_4 / V_4 \quad (V_4 = \text{const.})$$

$$(11) \quad m_p \ddot{X}_p = F_{ps} (\sigma, \dot{X}_p) - A_d (P_4 - P_a) - B_p \dot{X}_p - (F_{bs} + F_{pr}) U(X_p)$$

$$X_{pmin} \leq X_p \leq X_{pmax} \quad \dot{X}_p = 0 \text{ for } X_p = X_{pmax}, X_{pmin}$$

$$m_p, A_d, P_a, B_p = \text{const.}$$

$$(12) \quad U(X_p) = \begin{cases} 0 & X_p \leq 0 \\ 1 & X_p > 0 \end{cases}$$

$$(13) \quad F_{ps}(\sigma, \dot{X}_p) = \text{curve fit equation of spring characteristics (JPL print No. 10000976)}$$

MM '69 - REGULATOR EQUATIONS (CONT'D)

$$(14) \quad \delta = \delta_o - x_p \quad (\delta_o = \text{input})$$

$$(15) \quad F_{bs} = F_{bso} + K_5 x_p \quad (F_{bso}, K_5 = \text{const.})$$

$$(16) \quad F_{pr} = A_b (P_3 - P_4) \quad (A_b = \text{const.})$$

MM '69 REGULATOR SYMBOLS AND CONSTANTS

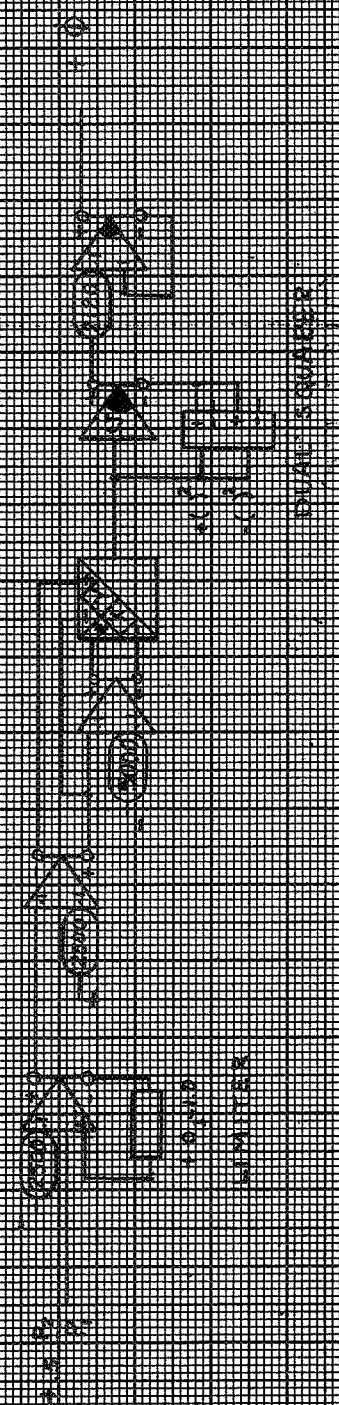
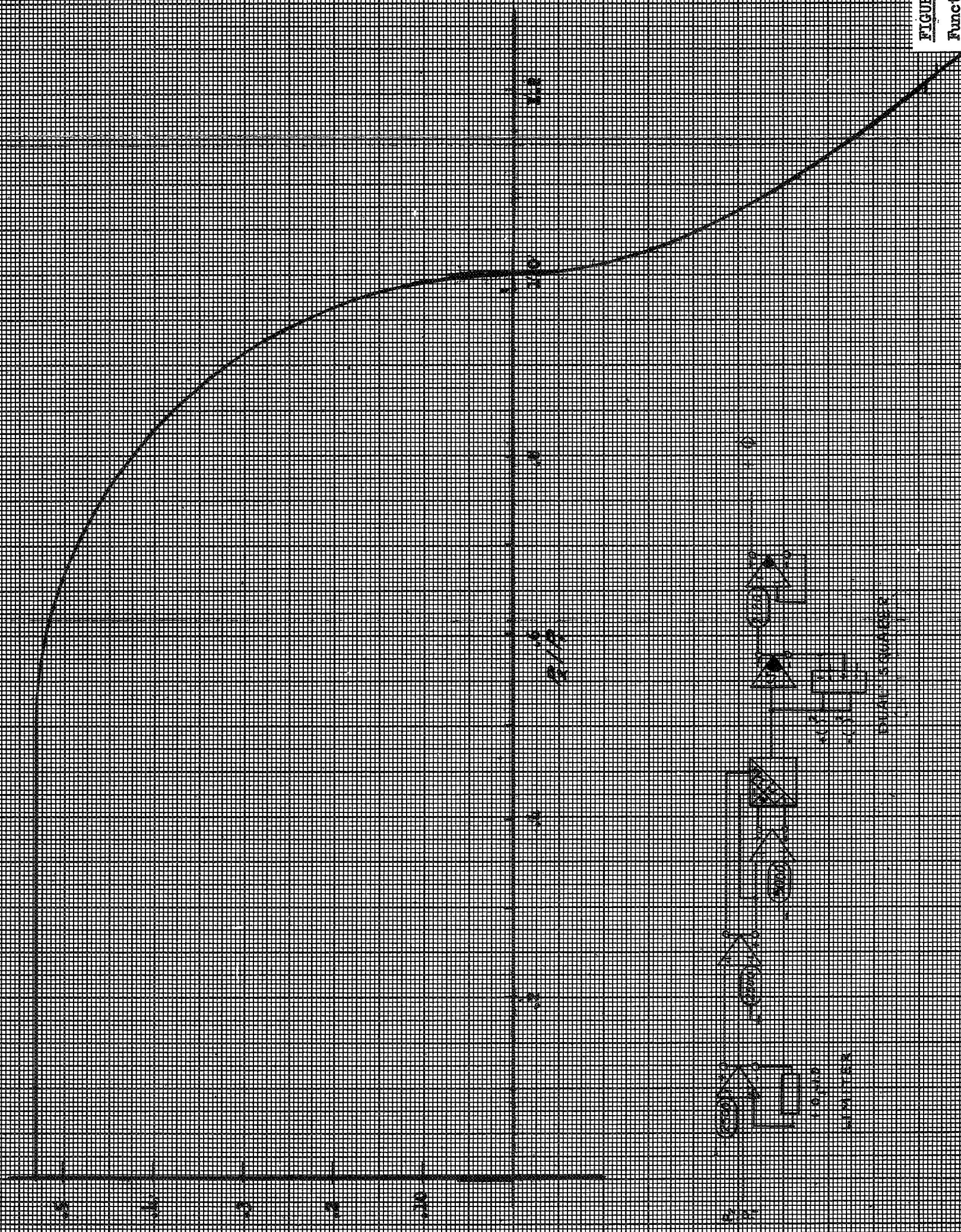
	<u>Nominal Value</u>
A_d - Effective diaphragm area (in. ²)	1.665
A_3 - Seat flow area (in. ²)	1.211×10^{-4}
A_b - Area of ball seat bore (in. ²)	4.78×10^{-3}
B_p - Damping coefficient for poppet (lb-sec/in.)	
C_s - Seat flow coefficient	0.6
F_{bs} - Ball retaining spring force (lbs)	2
F_{pr} - Pressure force on ball (lbs)	15.65
F_{ps} - Poppet spring force (lbs)	507
K_1 - Empirical constant in filter flowrate equation ($\frac{\text{in.}^4}{\text{lb-sec}} - R$)	2.945×10^{-5}
K_2 - Constant ratio of seat flow area to ball position (in)	0.1916
K_5 - Ball spring rate (lbs/in.)	4
m_p - Mass of poppet moving assembly, including 1/2 of springs ($\frac{\text{lbs-sec.}^2}{\text{in.}}$)	2.50×10^{-4}

MM '69 REGULATOR SYMBOLS AND CONSTANTS (CONT'D)

	<u>Nominal Value</u>
P_a - Ambient pressure (lbs/in. ²)	14.7
P_1 - Inlet pressure, upstream of filter (lbs/in. ²)	3600
P_3 - Pressure upstream of seat (lbs/in. ²)	3570
P_4 - Regulated pressure, downstream of seat (lbs/in. ²)	308
T_1 - Inlet temperature, upstream of filter (°R)	530
T_3 - Temperature upstream of seat (°R)	530
T_4 - Temperature downstream of seat (°R)	530
U - Unit step function	-
V_3 - Volume from filter to seat (in. ³)	0.0404
V_4 - Volume between the seat and the downstream restriction (in. ³)	1.30
W_3 - Stored weight of gas between filter and seat (lbs)	4.12×10^{-4}
W_4 - Stored weight of gas between seat and downstream restriction (lbs)	6.70×10^{-5}

MM '69 REGULATOR SYMBOLS AND CONSTANTS (CONT'D)

	<u>Nominal Value</u>
$\dot{W}_2$ - Inlet flowrate (lbs/sec)	0.006
$\dot{W}_3$ - Seat flowrate (lbs/sec)	0.006
$\dot{W}_5$ - Outlet flowrate (lbs/sec)	0.006
X_p - Poppet position, measured from the point at which the plunger contacts the seated ball. The opening direction is positive (in.)	6.33×10^{-4}
X_{pmax} - Full open position of poppet (in.)	0.005
X_{pmin} - Fully retracted position of poppet (in.)	-0.007
δ - Poppet spring deflection measured from free position (in.)	Variable
δ_o - Poppet spring deflection when poppet is contacting seated ball	Variable input
ϕ - Compressible flow function	0.53



SPRING CHARACTERISTIC CURVE
FOR ME-69 REGULATOR 8/8 008

$$\begin{aligned}\bar{\Phi}_s &= (P-400)/175 \\ \Delta &= (S-0.020)/0.050 \\ \bar{\Phi}_s &= 0.6972 + 2.932\Delta - 15.31\Delta^2 + 14.29\Delta^3 \\ \text{Max. Error} &= 6.6\%\end{aligned}$$

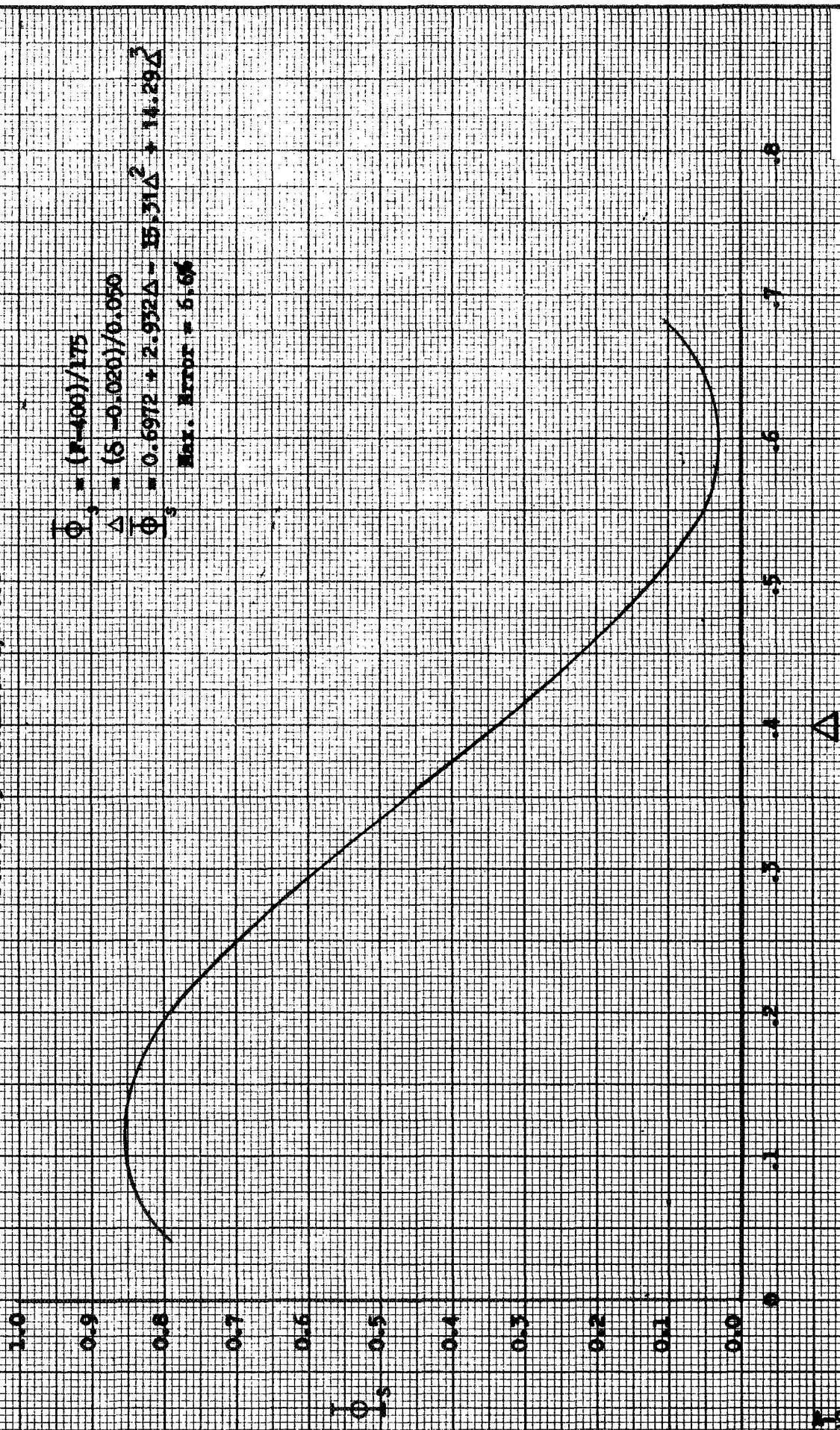


FIGURE A-2

MM '69 FLOW BENCH

MM '69 FLOW BENCH EQUATIONS

$$(1) \quad T_o = T_o(0) - \int_0^t [(\gamma-1)T_o \dot{W}_1 + UAT_a - UAT_o] / W_o \, dt$$

$$(2) \quad W_o = W_o(0) - \int_0^t \dot{W}_1 \, dt$$

$$(3) \quad P_o = W_o R T_o / V_o$$

$$(4) \quad \dot{W}_1 = CA_1 P_o \phi(P_1/P_o) / \sqrt{T_o}$$

$$(5) \quad T_1 = T_1(0) + \int_0^t [\dot{W}_1(\gamma T_o - T_1) - \dot{W}_2 T_1(\gamma-1)] / W_1 \, dt$$

$$(6) \quad W_1 = W_1(0) + \int_0^t (\dot{W}_1 - \dot{W}_2) \, dt$$

$$(7) \quad P_1 = W_1 R T_1 / V_1$$

$$(8) \quad \dot{W}_5 = CA_5 P_4 \phi(P_5/P_4) / \sqrt{T_4}$$

$$(9) \quad T_5 = T_5(0) + \frac{1}{W_{sm}} \int_0^t [\dot{W}_5(\gamma T_4 - T_5) - \dot{W}_6 T_5(\gamma-1)] \, dt$$

$$(10) \quad W_5 = W_5(0) + \int_0^t (\dot{W}_5 - \dot{W}_6) \, dt$$

$$(11) \quad P_5 = W_5 R T_5 / V_5$$

$$(12) \quad \dot{W}_6 = CA_6 P_5 \phi(P_6/P_5) / \sqrt{T_5}$$

$$(13) \quad T_6 = T_6(0) + \frac{1}{W_{6m}} \int_0^t [(\gamma-1)T_m(\dot{W}_6 - \dot{W}_7) + \gamma \dot{W}_m(T_5 - T_6)] \, dt$$

$$(14) \quad W_6 = W_6(0) + \int_0^t (\dot{W}_6 - \dot{W}_7) \, dt$$

$$(15) \quad P_6 = W_6 R T_6 / V_6$$

$$(16) \quad \dot{W}_7 = CA_7 P_6 \phi(P_7/P_6) / \sqrt{T_6}$$

$$(17) \quad T_7 = T_7(0) + \frac{1}{W_{7m}} \int_0^t [(\gamma-1)T_m(\dot{W}_7 - \dot{W}_8) + \gamma \dot{W}_m(T_6 - T_7)] \, dt$$

MM '69 FLOW BENCH EQUATIONS (CONT'D)

$$(18) \quad W_7 = W_7(0) + \int_0^t (\dot{W}_7 - \dot{W}_8) dt$$

$$(19) \quad P_7 = W_7 R T_7 / V_7$$

$$(20) \quad \dot{W}_8 = C A_8 P_7 \phi(P_8 / P_7) / \sqrt{T_7}$$

$$(21) \quad P_8 = P_a$$

$$(22) \quad \phi = 1.06 \operatorname{sgn}(1 - P_{r_i}) (P_{r_i} |1 - P_{r_i}|)^{1/2}$$

$$(23) \quad P_{r_i} = P_i / P_{i-1} \quad ; \quad 0.5 \leq P_{r_i} \leq 2.0$$

NOMENCLATURE :

A - AREA

C - DISCHARGE COEFFICIENT

P - PRESSURE

R - GAS CONSTANT

T - ABSOLUTE TEMPERATURE

t - TIME

U - OVER-ALL HEAT TRANSFER COEFFICIENT

V - VOLUME

W - STORED WEIGHT

$\dot{W}$ - WEIGHT FLOWRATE

γ - SPECIFIC HEAT RATIO

SUBSCRIPTS

0, 1, ... 8 - MODEL STATION POINTS

a - AMBIENT

m - NOMINAL POINT

MM '69 FLOW BENCH SYSTEM CONSTANTS

NOMINAL PRESSURES

$$P_0 = 3018 \text{ PSIA}$$

$$P_1 = 3015$$

$$P_2 = 3015$$

$$P_3 = 2988$$

$$P_4 = 323$$

$$P_5 = 319$$

$$P_6 = 315$$

$$P_7 = 311$$

$$P_8 = 15$$

NOMINAL FLOWRATE

$$\dot{W}_m = 0.006 \text{ LB. / SEC.}$$

NOMINAL TEMPERATURE

$$T_m = 530 \text{ }^\circ\text{R}$$

VOLUMES

$$V_0 = 100 \text{ IN.}^3$$

$$\left. \begin{array}{l} V_1 = \\ V_2 = \end{array} \right\} 1 \text{ IN.}^3 \text{ TOTAL}$$

$$V_3 = 0.0404 \text{ IN.}^3$$

$$V_4 = 1.3 \text{ IN.}^3$$

$$V_5 = 50 \text{ IN.}^3$$

$$V_6 = 1.25 \text{ IN.}^3$$

$$V_7 = 0.6 \text{ IN.}^3$$

PROPERTIES AND MISCELLANEOUS

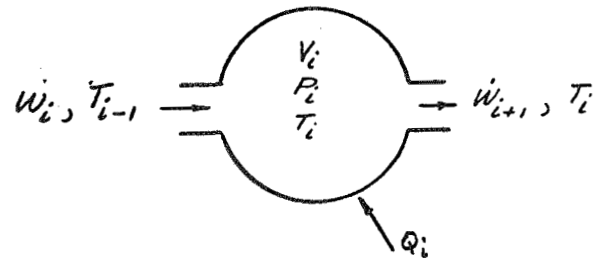
$$R = 55.16 \text{ FT-LB / LB }^\circ\text{R} = 661 \text{ IN. / }^\circ\text{R}$$

$$\gamma = 1.4$$

$$(UA)_{\text{TANK}} \approx 0.49 \text{ FT-LB / SEC }^\circ\text{R}$$

DERIVATION OF DYNAMIC TEMPERATURE EQUATION

REF : BIRD, STEWART, LIGHTFOOT, TRANSPORT PHENOMENA
Pg. 458



$$\frac{d}{dt}(U_{TOT} + K_{TOT} + \Phi_{TOT}) = -\Delta\left[(\hat{U} + p\hat{V} + \frac{1}{2}v^2 + \hat{\Phi})\dot{W}\right] + Q - W$$

WHERE

U = INTERNAL ENERGY
K = KINETIC ENERGY
 Φ = POTENTIAL ENERGY
v = VELOCITY
 $\dot{W}$ = MASS FLOWRATE
Q = HEAT TRANSFER
W = EXTERNAL WORK

NOTE THAT

$$U_{TOT} = \rho_i V_i \hat{U}_i = \rho_i V_i C_v T_i = \dot{W}_i C_v T_i$$

$$K_{TOT} = \int_{V_i} \frac{1}{2} \rho v^2 dV \approx 0$$

$$\Phi_{TOT} = \int_{V_i} \rho \hat{\Phi} dV \approx 0$$

$$\hat{U}_i = C_v T_i$$

$$\hat{V}_i = 1/\rho_i$$

$$\hat{U}_i + p_i \hat{V}_i \equiv \hat{H}_i = C_p T_i$$

SUBSTITUTING, NEGLECTING KINETIC & POTENTIAL E

$$C_v \frac{d}{dt}(\dot{W}_i T_i) = \hat{H}_{i-1} \dot{W}_i - \hat{H}_i \dot{W}_{i+1} + Q_i$$

DERIVATION OF DYNAMIC TEMPERATURE EQUATION (CONT'D)

NOTE THAT

$$\frac{d}{dt}(w_i T_i) = w_i \frac{dT_i}{dt} + T_i \frac{dw_i}{dt}$$

$$\frac{dw_i}{dt} = \dot{w}_i - \dot{w}_{i+1}$$

SO THAT

$$c_v w_i \frac{dT_i}{dt} = c_p T_{i-1} \dot{w}_i - c_p T_i \dot{w}_{i+1} - c_v T_i (\dot{w}_i - \dot{w}_{i+1}) + Q_i$$

$$\frac{dT_i}{dt} = \frac{1}{w_i} \left\{ \dot{w}_i (\gamma T_{i-1} - T_i) - \dot{w}_{i+1} T_i (\gamma - 1) + Q_i / c_v \right\}$$

$$T_i(t) = T_i(0) + \int_0^t [\dot{w}_i (\gamma T_{i-1} - T_i) - \dot{w}_{i+1} T_i (\gamma - 1) + \frac{Q_i}{c_v}] / w_i dt$$

IN ORDER TO LINEARIZE, LET

$$\dot{T}_i = f(w_i, \dot{w}_i, \dot{w}_{i+1}, T_{i-1}, T_i) \quad (\text{NEGLECT } Q_i)$$

LET x DENOTE POINT OF LINEARIZATION. THEN

$$\begin{aligned} \dot{T}_i &= f(w_{i,x}, \dot{w}_{i,x}, \dot{w}_{(i+1),x}, T_{(i-1),x}, T_{i,x}) \\ &+ \left. \frac{\partial f}{\partial w_i} \right|_x \Delta w_i + \left. \frac{\partial f}{\partial \dot{w}_i} \right|_x \Delta \dot{w}_i + \dots + \left. \frac{\partial f}{\partial T_i} \right|_x \Delta T_i \\ &+ \cancel{O(\Delta^2)} + \cancel{O(\Delta^3)} + \dots \end{aligned}$$

NOTE THAT IF x IS A STEADY STATE POINT,

$$f(w_{i,x}, \dot{w}_{i,x}, \dots) = 0$$

DERIVATION OF DYNAMIC TEMPERATURE EQUATION (CONT'D)

FROM ORIGINAL NONLINEAR EQUATION,

$$\left. \frac{\partial f}{\partial w_i} \right|_x = 0 \quad (\text{i.e., } \Delta w_i \text{ IS ONLY 2ND \& HIGHER ORDER EFFECT})$$

$$\left. \frac{\partial f}{\partial \dot{w}_i} \right|_x = \frac{T_x (\gamma - 1)}{w_{ix}} \quad (\text{ASSUMING } x \text{ IS STEADY-STATE})$$

$$\left. \frac{\partial f'}{\partial \dot{w}_{i+1}} \right|_x = - \frac{T_x (\gamma - 1)}{w_{ix}} \quad (\quad " \quad " \quad " \quad " \quad)$$

$$\left. \frac{\partial f}{\partial T_{i-1}} \right|_x = \gamma \frac{\dot{w}_{ix}}{w_{ix}} \quad (\quad " \quad " \quad " \quad " \quad)$$

$$\left. \frac{\partial f}{\partial T_i} \right|_x = - \gamma \frac{\dot{w}_{ix}}{w_{ix}} \quad (\quad " \quad " \quad " \quad " \quad)$$

SUBSTITUTING,

$$\dot{T}_i = \frac{1}{w_{ix}} [(\gamma - 1) T_x \Delta \dot{w}_i - (\gamma - 1) T_x \Delta \dot{w}_{i+1} + \gamma \dot{w}_{ix} \Delta T_{i-1} - \gamma \dot{w}_{ix} \Delta T_i]$$

$$= \frac{1}{w_{ix}} [(\gamma - 1) T_x (\dot{w}_i - \dot{w}_{i+1}) + \gamma \dot{w}_{ix} (T_{i-1} - T_i)] dt$$

$$T_i = T_i(0) + \frac{1}{w_{ix}} \int_0^t [(\gamma - 1) T_x (\dot{w}_i - \dot{w}_{i+1}) + \gamma \dot{w}_{ix} (T_{i-1} - T_i)] dt$$

ANALOG MULTIPLEX METHOD

The generation of the compressible flow function requires 5 amplifiers and 2 multipliers. Since this same function is required at 6 places in the flow bench model, considerable savings in analog equipment can be effected by sharing or "multiplexing" the same set of components at each place where the function is required. To achieve this, it is necessary to construct a switching network which sequentially shifts the inputs and outputs of the generating circuit to the several points in the problem where it is required. The mechanization of this switching network using logic elements on the AD-256 computer is shown in the computer diagram. For this method to work it is necessary for the switching rate to be considerably faster than the attempted analog solution. It was found that a total cycle time of 1.5 milliseconds was within the range of the logic switches, and that this permitted problem solution at 1/10th of real time. It is notable that this switching time corresponds to 667 cycles per second so that if a problem variable is varying at 66.7 Hz machine time, it is based on a multiplexer output which is being updated 10 times per cycle. If it is assumed that 10 samples per cycle are the minimum tolerable for convergence of the solution, this means that the model is producing good solutions for frequencies up to 667 Hz real time. Other parts of the model, such as the lumped-parameter representation of the lines, probably have a lower limit than this.

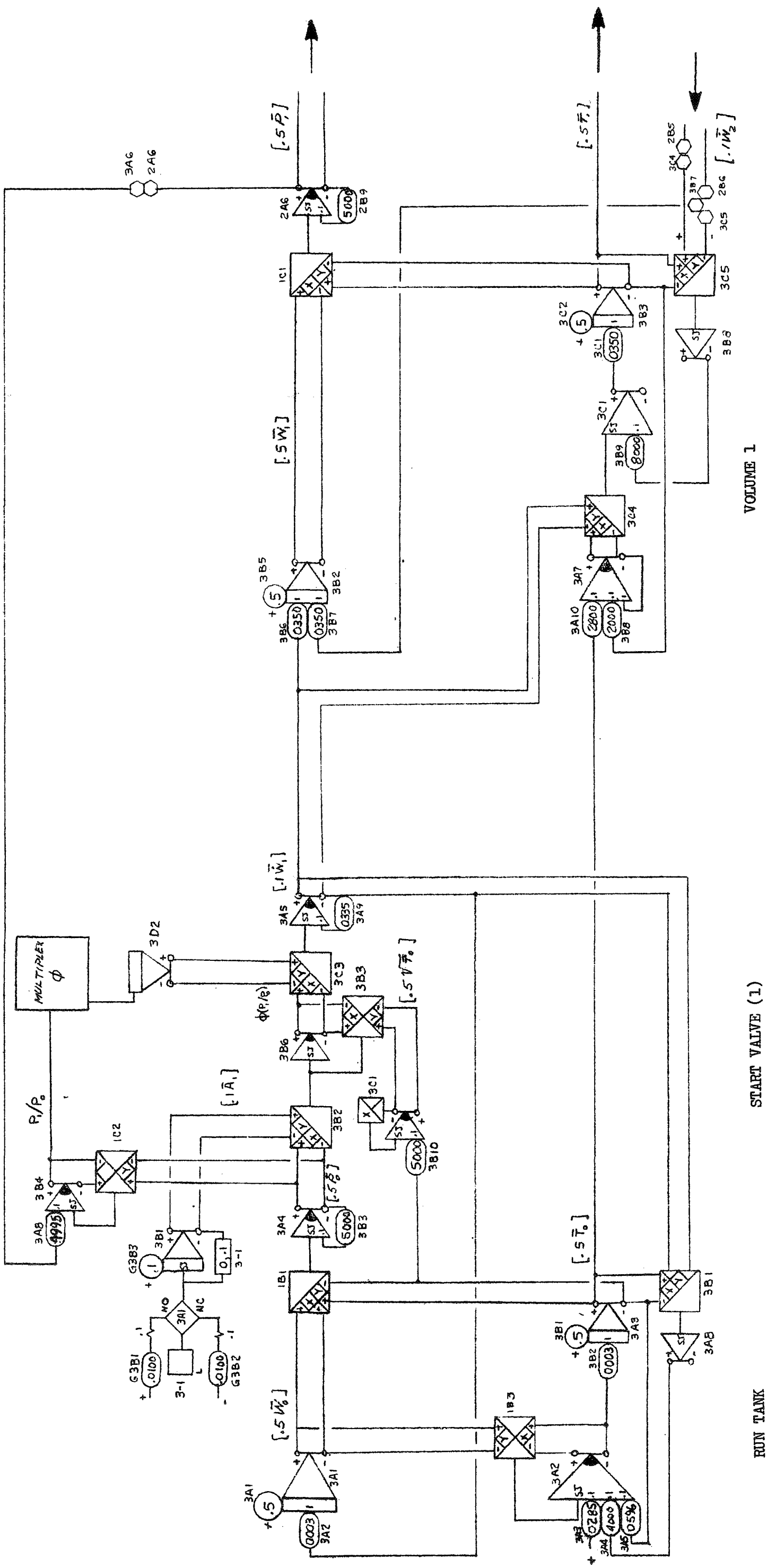


FIGURE A-3 Computer Mechanization of MW*69
Flow Bench Model

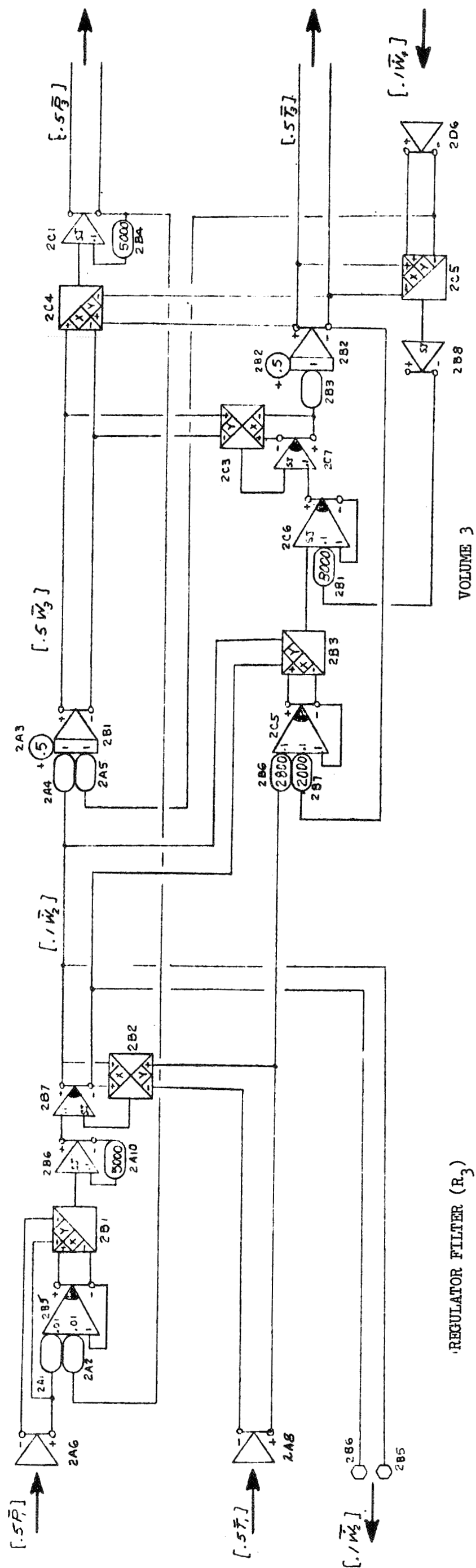


FIGURE A-4 Computer Mechanization of MM⁶⁹
Flow Bench Model (continued)

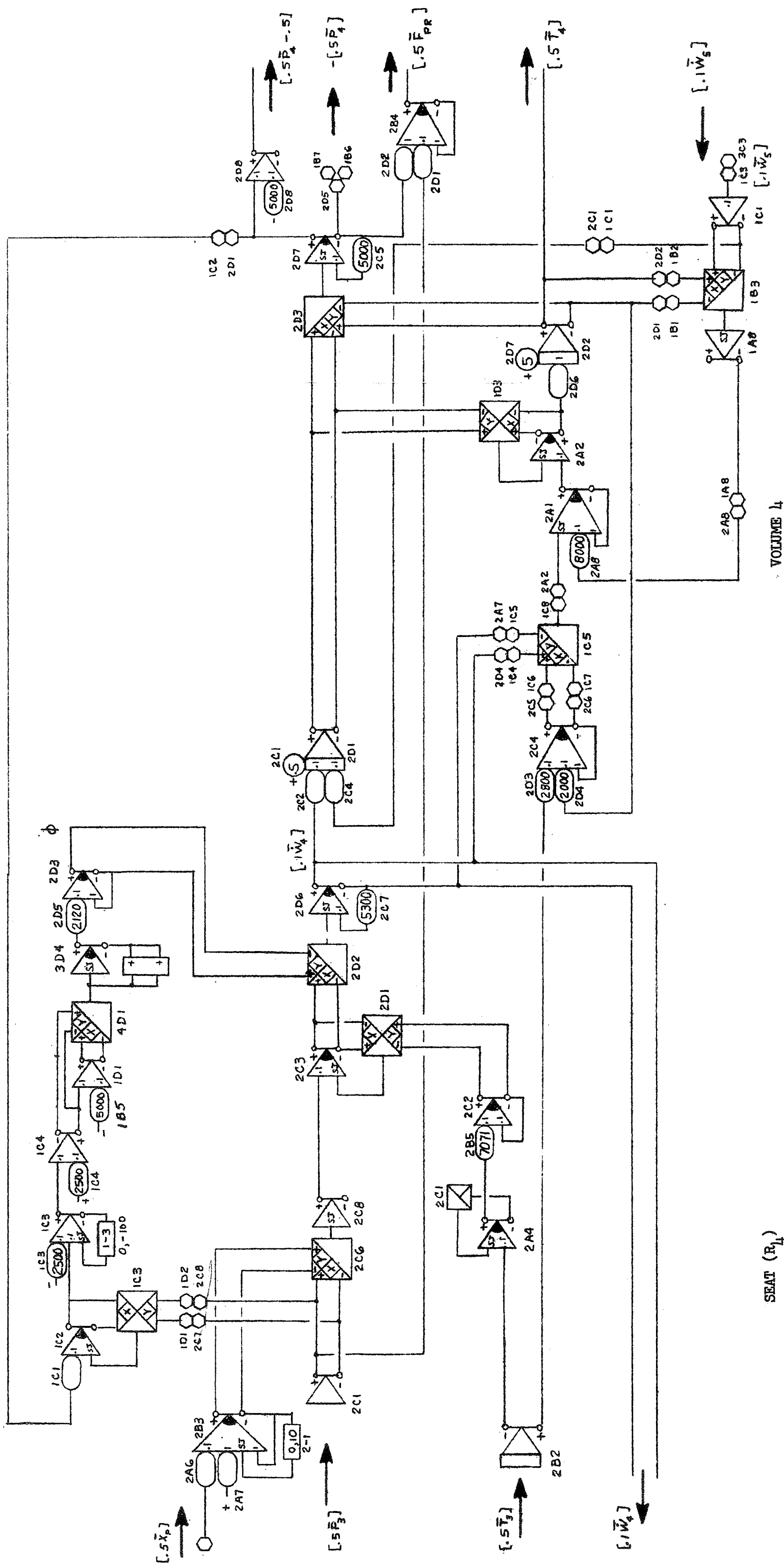
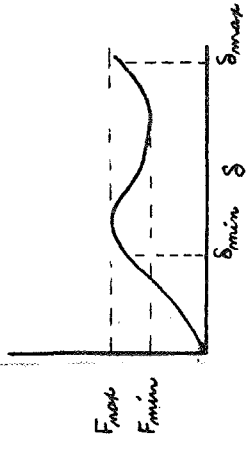


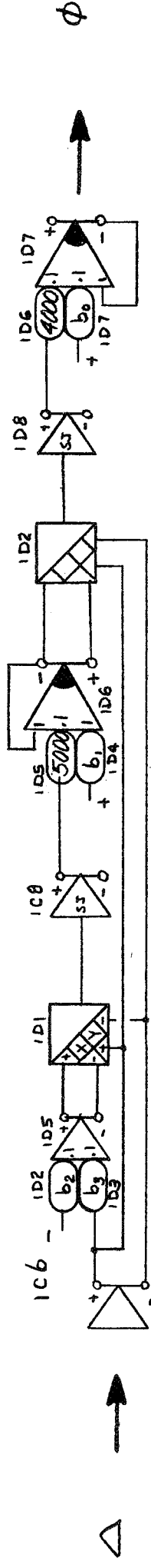
FIGURE A-5 Computer Mechanization of MM'69
Flow Bench Model (continued)



$$\Delta = (\delta - \delta_{min}) / (\delta_{max} - \delta_{min})$$

$$\phi = (F - F_{min}) / (F_{max} - F_{min})$$

$$\phi = a_0 + a_1 \Delta + a_2 \Delta^2 + a_3 \Delta^3$$



$$b_0 = a_0$$

$$b_1 = a_1 / 4$$

$$b_2 = a_2 / 20$$

$$b_3 = a_3 / 20$$

SPRING CHARACTERISTIC GENERATION

FIGURE A-7 Computer Mechanization of MM-69
Flow Bench Model (continued)

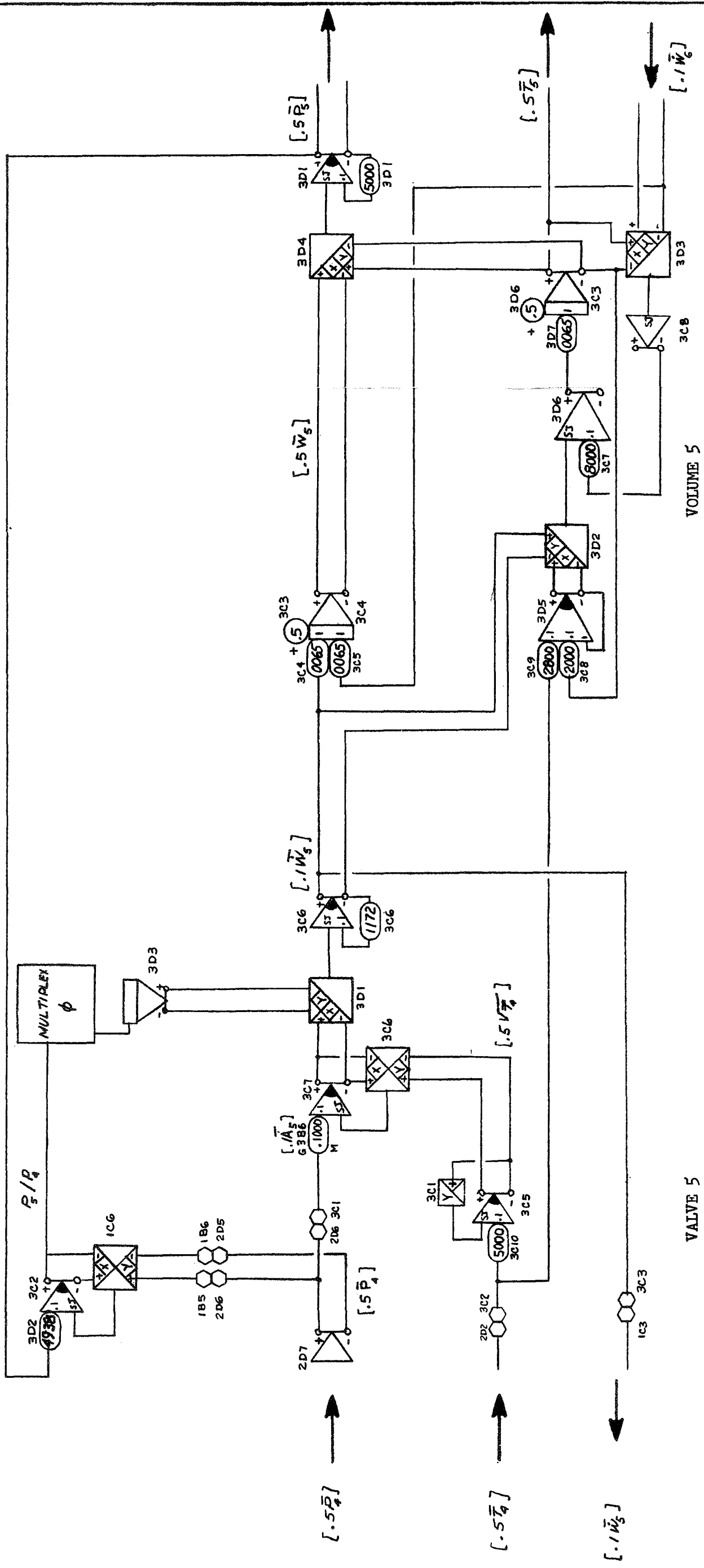


FIGURE A-8 Computer Mechanization of MM69
Flow Bench Model (continued)

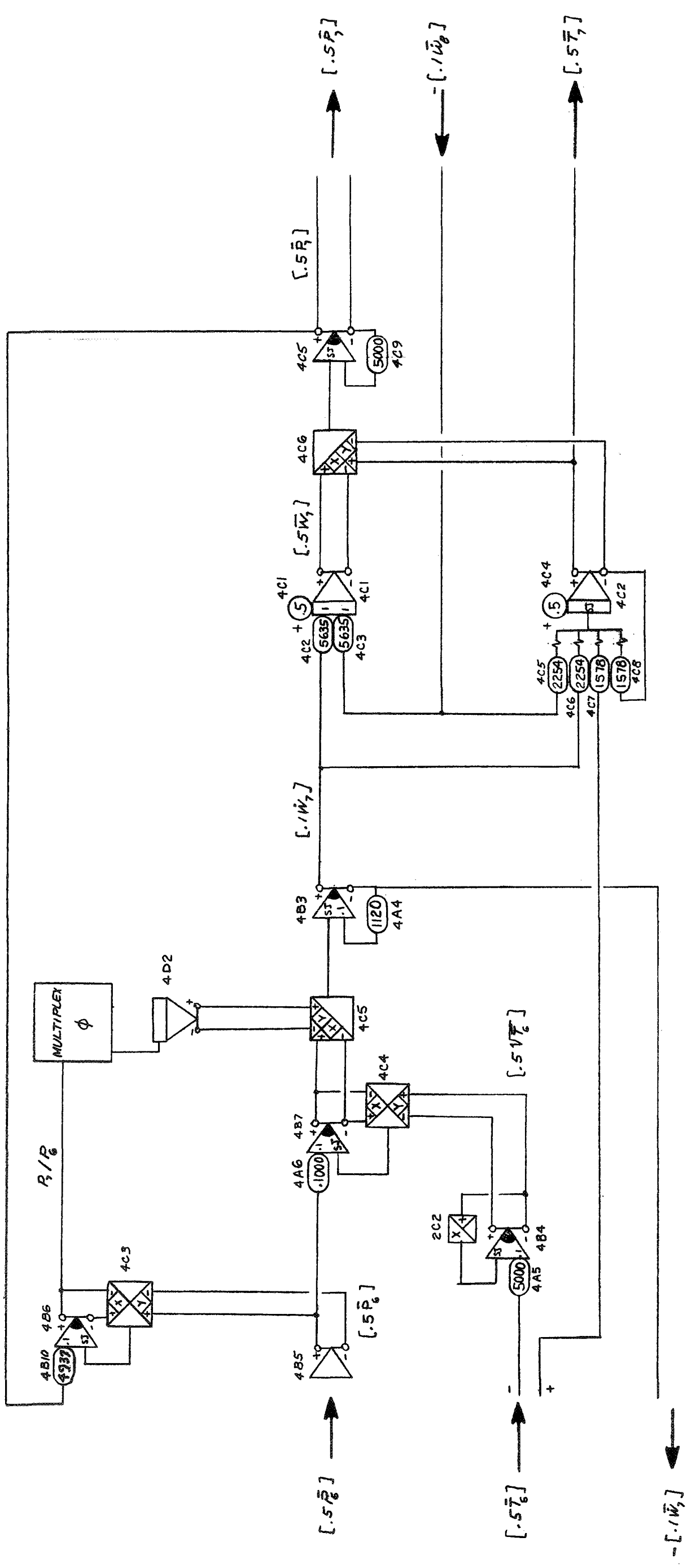


FIGURE A-10 Computer Mechanization of MM'69
Flow Bench Model (continued)
A-25

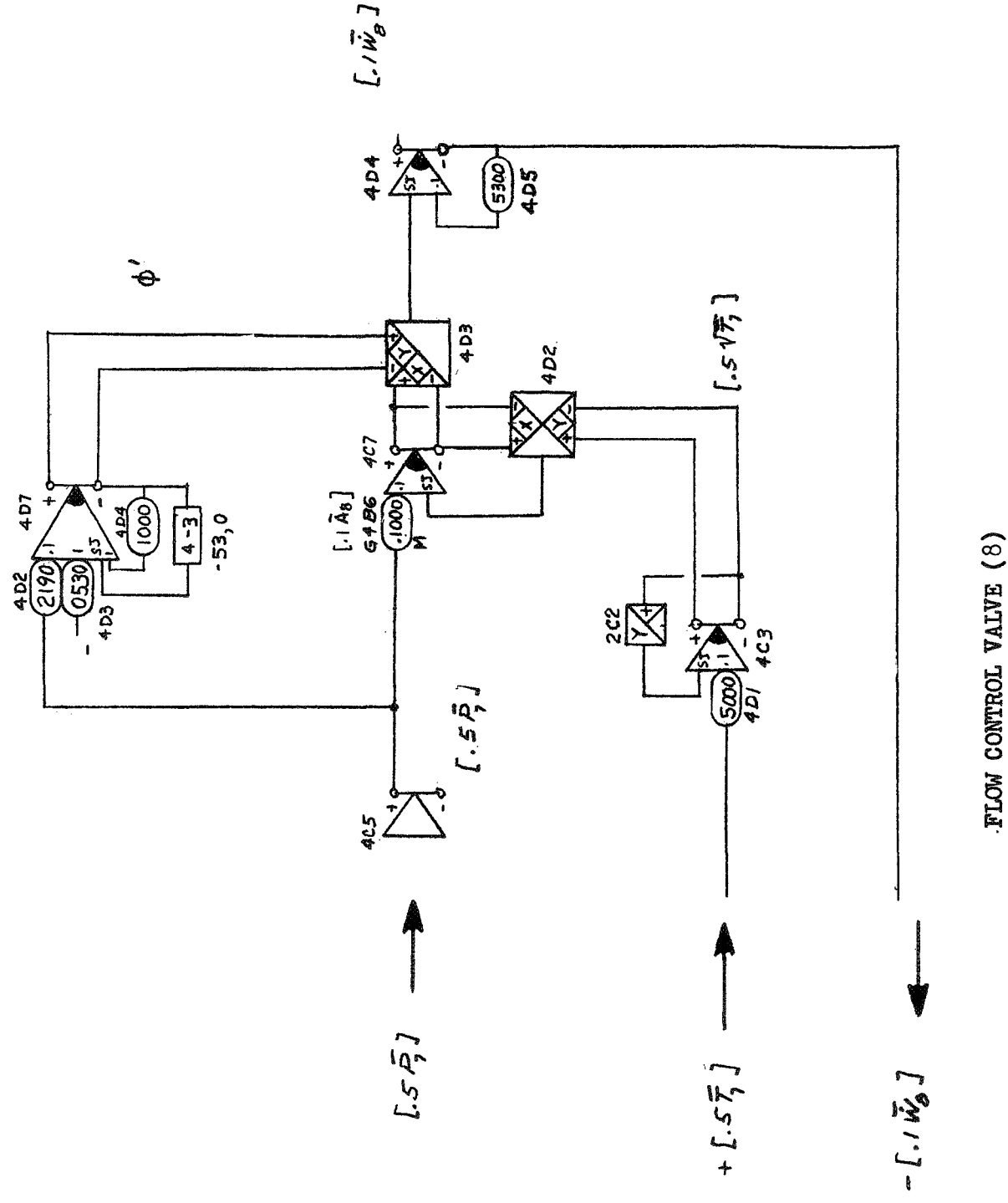


FIGURE A-11 Computer Mechanization of MM'69
Flow Bench Model (continued)

MULTIPLEX $\phi(x)$

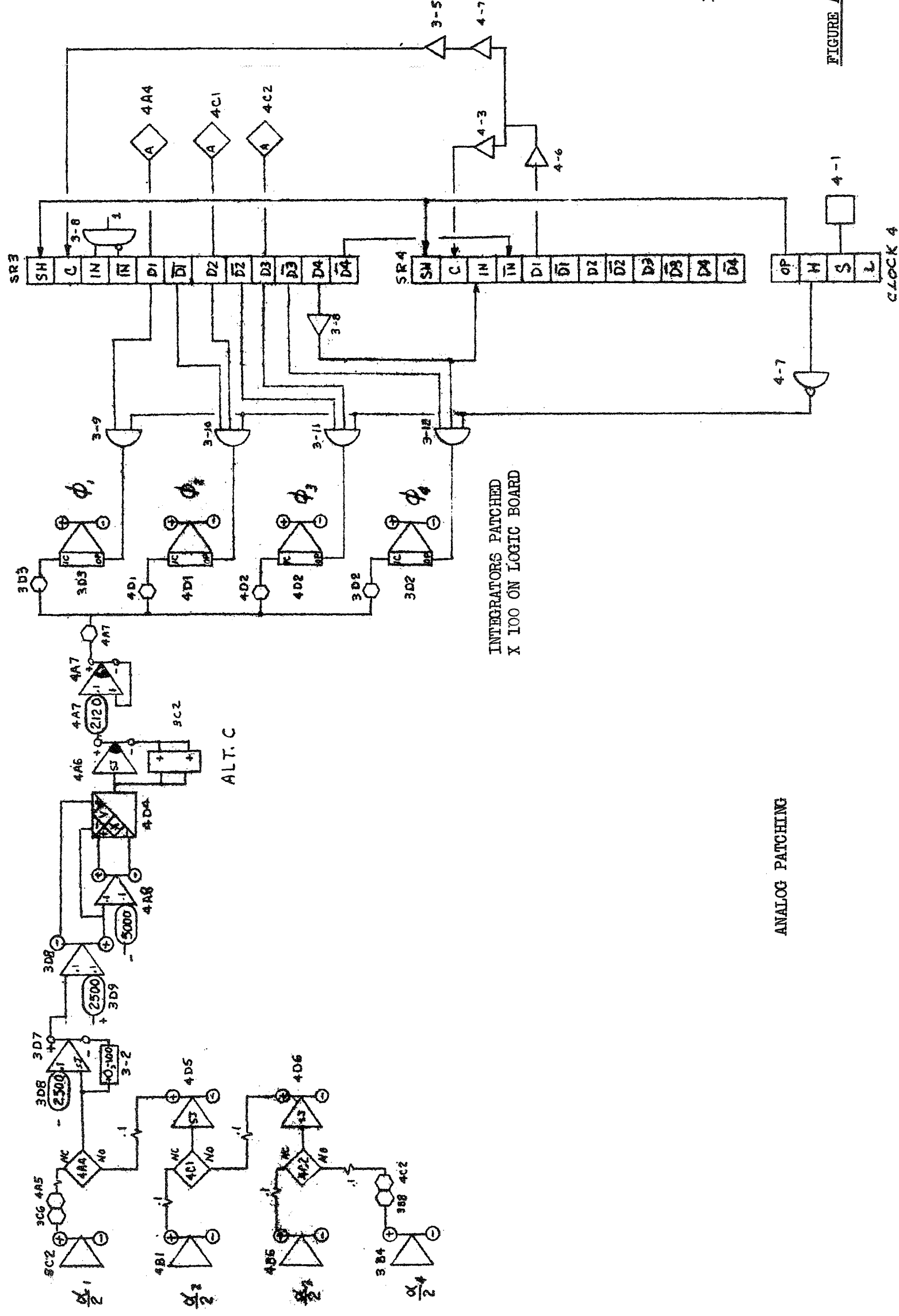
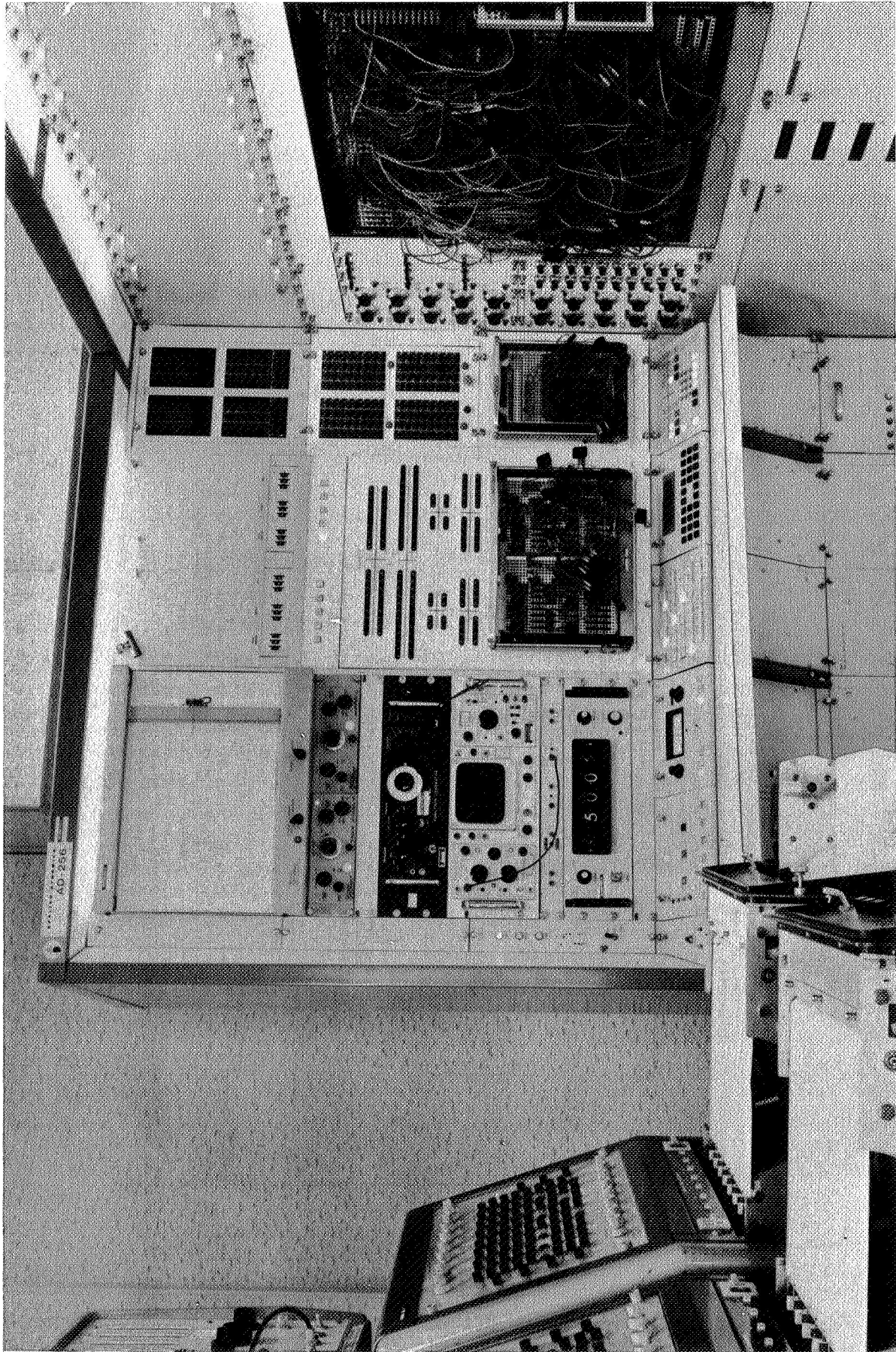


FIGURE A-12 Computer Mechanization of MM'69
Flow Bench Model (concluded)
A-27



PICTURE OF COMPUTER MECHANIZATION

MM ' 69 WATER EXPULSION SYSTEM

MM '69 WATER EXPULSION EQUATIONS

[Also see flow bench Equations (1) to (7)]

- (1) $V_5 = V_{PT} - V_{L0}(0) + \int_0^t \dot{w}_{L1} / \rho_L dt$
- (2) $\dot{w}_{L1} = \dot{w}_{L1}(0) + \left(\frac{A_1 g_0}{\rho_L} \right) \int_0^t [P_5 - P_{L1} - (\dot{w}_{L1} / K_1 A_2)^2 - R_1 \dot{w}_{L1}^2] dt$
- (3) $P_{L1} = P_{L1}(0) + \frac{B'}{W_0} \int_0^t (\dot{w}_{L1} - \dot{w}_{L2}) dt$
- (4) $\dot{w}_{L2} = \dot{w}_{L2}(0) + \left(\frac{A_3 g_0}{\rho_2} \right) \int_0^t [P_{L1} - P_a - (\dot{w}_{L2} / K_2 A_4)^2 - R_2 \dot{w}_{L2}^2] dt$
- (5) $P_5 = W_5 R T_5 / V_5$
- (6) $W_5 = W_5(0) + \int_0^t \dot{w}_5 dt$
- (7) $T_5 = T_5(0) + \frac{1}{W_{5,m}} \int_0^t \dot{w}_5 (\gamma T_4 - T_5) dt$

NOMENCLATURE :

- A_1, A_3 - LINE CROSS-SECTIONAL AREAS (CONSTANTS)
- A_2, A_4 - VALVE AREAS (VARIABLE)
- B' - EFFECTIVE BULK MODULUS OF FLEXLINE
- K - $\dot{w} / A \sqrt{\Delta P}$ FOR VALVES
- L - LINE LENGTHS
- P - PRESSURE
- R - $\Delta P / \dot{w}^2$ FOR LUMPED RESISTANCES
- T - TEMPERATURE
- V - VOLUME
- W - STORED WEIGHT
- $\dot{w}$ - WEIGHT FLOWRATE
- γ - SPECIFIC HEAT RATIO
- ρ - DENSITY

MM '69 WATER EXPULSION SYSTEM (CONT'D)

SUBSCRIPTS

L - LIQUID

4,5 - SEE FLOW BENCH NOMENCLATURE

m - NOMINAL

PRESSURES

$$P_{sm} = 323 \text{ PSIA}$$

$$P_{lim} = 56.77$$

$$P_a = 15$$

$$\Delta P_{VALVE,1} = 15$$

$$\Delta P_{LINE,1} = 251.23 \text{ (INCLUDES ORIFICE)}$$

$$\Delta P_{VALVE,2} = 15$$

$$\Delta P_{LINE,2} = 26.17$$

FLOWRATES

$$\dot{W}_{Lm} = 0.224 \text{ LB/SEC.}$$

VOLUMES, LENGTHS, & AREAS

$$V_{PT} = 697 \text{ IN.}^3 \text{ (EXPULSION TANK)}$$

$$V_{Lom} = 596 \text{ IN.}^3 \text{ (INITIAL WATER FILL)}$$

$$V_{sm} = 101 \text{ IN.}^3 \text{ (INITIAL ULLAGE)}$$

$$l_1 = 36 \text{ IN. (TANK TO CENTER OF FLEXLINE)}$$

$$l_2 = 48 \text{ IN. (CEN. OF FLEX. TO CATCH TANK)}$$

$$A_{l_1}, A_{l_2} = 0.0314 \text{ IN.}^2$$

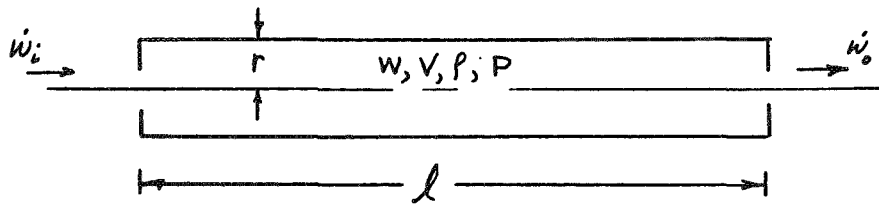
MISCELLANEOUS

$$\rho_L = 62.4 \text{ LB/FT}^3$$

$$W_o = 0.041 \text{ LBS. (STORED WEIGHT IN FLEXLINE)}$$

$$B' \approx 0.5 \cdot 10^7 \text{ LB/FT}^2$$

DERIVATION OF EFFECTIVE BULK MODULUS



$t = \text{WALL THICKNESS}$

CONSERVATION OF MASS :

$$\frac{dW}{dt} = \dot{w}_i - \dot{w}_o$$

RELATION BETWEEN STORED WEIGHT AND PRESSURE

$$W = \rho V$$

WHERE

$$\rho = \rho(P)$$

$$V = V(r, l) = \pi r^2 l$$

$$r = r(P)$$

$$l = l(P)$$

$$\frac{dW}{dP} = \frac{d(\rho V)}{dP} = V \frac{d\rho}{dP} + \rho \frac{dV}{dP}$$

$$\frac{d\rho}{dP} \equiv \rho/B \quad \text{WHERE } B = \text{FLUID BULK MODULUS}$$

$$\frac{dV}{dP} = \pi \left(r^2 \frac{dl}{dP} + 2rl \frac{dr}{dP} \right)$$

BY HOOKE'S LAW, NEGLECTING 2ND ORDER TERMS

$$\frac{dl}{dP} \approx \frac{l_0 r_0}{2tE}$$

$$\frac{dr}{dP} \approx \frac{r_0^2}{tE}$$

DERIVATION OF EFFECTIVE BULK MODULUS (CONT'D)

THEN, SUBSTITUTING

$$\begin{aligned}\frac{dV}{dP} &\approx \pi \left(r_o^2 \frac{l_o r_o}{2tE} + 2 r_o l_o \frac{r_o^2}{tE} \right) \\ &\approx \frac{2.5 V_o}{(t/r_o) E}\end{aligned}$$

THEN

$$\begin{aligned}\frac{dW}{dP} &= \frac{\rho V_o}{B} + \frac{2.5 \rho V_o}{(t/r_o) E} \\ &= \rho V_o \left(\frac{1}{B} + \frac{2.5}{(t/r_o) E} \right)\end{aligned}$$

LET

$$\frac{1}{B'} = \frac{1}{B} + \frac{2.5}{(t/r_o) E}$$

THEN

$$\frac{dW}{dP} = \frac{\rho V_o}{B'}$$

$$dP = \frac{B'}{\rho V_o} dW$$

$$\frac{dP}{dt} = \frac{B'}{\rho V_o} \frac{dW}{dt} = \frac{B'}{\rho V_o} (\dot{W}_i - \dot{W}_o)$$

$$P(t) = P(0) + \frac{B'}{\rho V_o} \int_0^t (\dot{W}_i - \dot{W}_o) dt$$

MM '69 WATER EXPULSION SYSTEM COMPUTER
DIAGRAMS

The computer diagrams for the MM '69 Water
Expulsion System are composed of figures
A-1 through A-5 in addition to figure A-13.

MM '69 PROPULSION SYSTEM

MM '69 PROPULSION SYSTEM

(Also see water expulsion system equations.)

$$(1) \quad \dot{W}_o = P_c A_t / \sqrt{T_c} \quad f(\gamma, M)$$

$$(2) \quad f(\gamma, M) = \left[\frac{\gamma M}{R_o} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \right]^{1/2}$$

$$(3) \quad \dot{W}_c = \dot{W}_c(0) + \int_0^t (\dot{W}_i - \dot{W}_o) dt$$

$$(4) \quad \dot{W}_i = \dot{W}_{L2}$$

$$(5) \quad P_c = R_o T_c \dot{W}_c / V_c M$$

$$(6) \quad T_c = T_c(0) + \frac{c_p}{m_b c_b} \int_0^t \dot{W}_o (T_{c(ss)} - T_c) dt$$

$$(7) \quad \dot{W}_{L2} = \dot{W}_{L2}(0) + \left(\frac{A_2 g_o}{g_2} \right) \int_0^t [P_{L1} - P_c - R_2 \dot{W}_{L2}^2] dt$$

NOMENCLATURE :

$\dot{W}_o$ = THRUST CHAMBER OUT-FLOW

P_c = CHAMBER PRESSURE

A_t = THROAT AREA

T_c = CHAMBER TEMPERATURE

M = MOLECULAR WEIGHT

R_o = UNIVERSAL GAS CONSTANT

γ = SPECIFIC HEAT RATIO

$\dot{W}_c$ = STORED WEIGHT IN T/C

$\dot{W}_i$ = T/C INFLOW

$\dot{W}_{L2}$ = FEED SYSTEM OUTFLOW

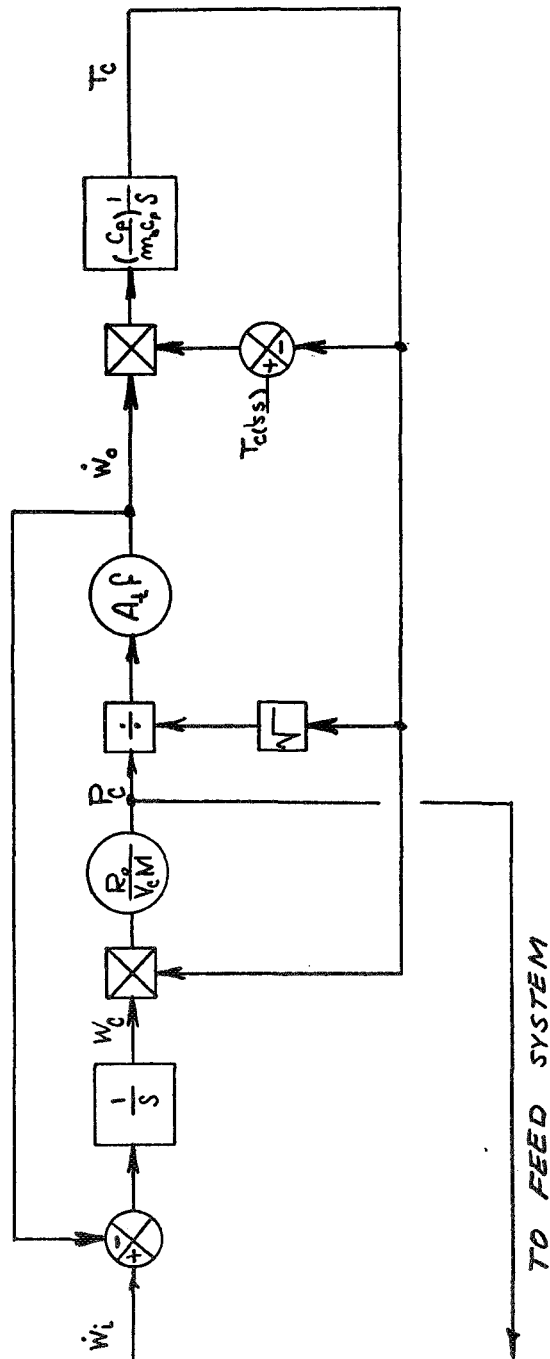
c_p = REACTION PRODUCTS SPECIFIC HEAT

c_b = CATALYST BED SPECIFIC HEAT

m_b = " " MASS

R_2 = INJECTOR RESISTANCE

V_c = CHAMBER VOLUME



THRUST CHAMBER BLOCK DIAGRAM

MM '69 PROPULSION SYSTEM COMPUTER PROGRAMS

The computer diagrams for the MM '69 propulsion system are composed of figures A-1 through A-5, A-13 and A-14.

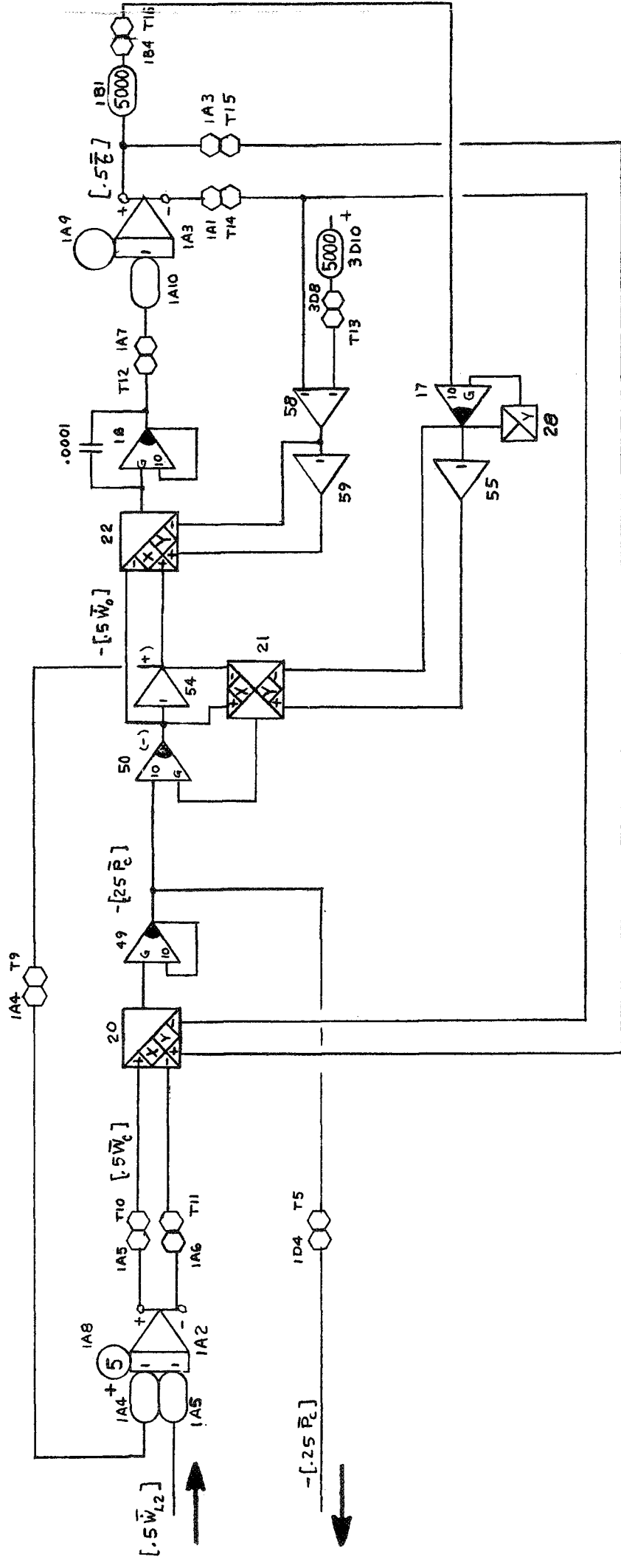


FIGURE A-14 Computer Mechanization of Thrust Chamber

APPENDIX B

SIMPLIFIED WATER EXPULSION MODEL - LINEAR ANALYSIS

The MM '69 regulator was tested as part of the propulsion system in the Water Expulsion Tests. In this test, the propellant tank was filled with water, and an engine burn simulated by expelling the water under the controlled pressure provided through the calibrated regulator. During some of these tests, particular regulators exhibited wide pressure excursions. It was felt that the cause of these fluctuations might be revealed by a simple model of this system and regulator. If so, the results would indicate where emphasis might best be placed in the development of the complete, detailed regulator model. Further, comparison of results of the simplified and detailed models might indicate the degree of sophistication required in order to accurately predict the performance of future regulators.

A simplified water expulsion system is shown schematically in Fig. B-1. Only the most significant elements are included. Further simplifications are afforded by the following assumptions:

1. The pressurization tank is sufficiently large that its pressure is constant over the time period considered.
2. Gas flow is isothermal.
3. There is no pressure drop between the regulator and the propellant tank.
4. Perfect gas behavior.
5. Poppet forces consist of diaphragm pressure and spring, i.e., ball forces are negligible.

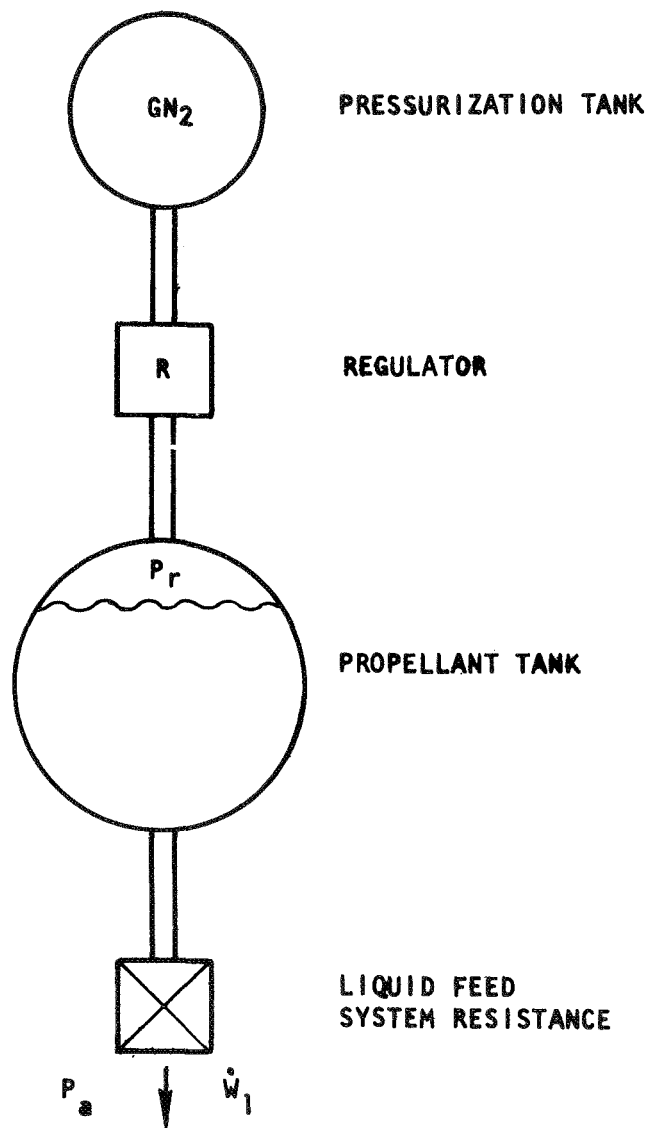


Figure B-1. Simplified Schematic of Water Expulsion System

6. Changes in downstream (regulated) pressure are small,
so that linearization is permissible.

Under assumptions 1 through 5, the following system of equations can be developed.

$$\dot{w}_1 = \left[(P_r - P_a)/R_1 \right]^{\frac{1}{2}} \quad (B1)$$

$$\dot{P}_r = RT (\dot{w}_g/V_g - \rho_g \dot{V}_g/V_g) \quad (B2)$$

$$\dot{V}_g = \dot{w}_1/\rho_1 \quad (B3)$$

$$m\ddot{X}_p = F_{ps}(X_p) - B\dot{X}_p - (P_r - P_a)A_d ; X_{p_{\max}} \leq X_p \leq X_{p_{\min}} \quad (B4)$$

$$X_b = X_p ; X_p \geq 0 \quad (B5)$$

$$= 0 ; X_p < 0$$

$$\dot{w}_g = C_s P_t A_s \phi / \sqrt{T_1} \quad (B6)$$

$$C_s \phi / \sqrt{T_1} = \text{CONST} \quad (B7)$$

$$A_s = K X_b \quad (B8)$$

In these equations, the poppet and ball positions, X_p and X_b , are measured from the seated position, with the opening direction being positive. The spring force, F_{ps} , is shown only as a function of poppet position, which implies that the free end is fixed. Changes in the set point must be accomplished by using different functions.

These equations can be normalized and linearized to permit easier analysis. After considerable simplification, there results

$$\ddot{\bar{X}}_p + \gamma \dot{\bar{X}}_p - \beta K_2 \bar{X}_p = -K_2 \bar{P}; a \leq \bar{X}_p \leq 1 \quad (B9)$$

$$\dot{\bar{X}}_p = 0 \text{ for } \bar{X}_p = a, \bar{X}_p = 1$$

$$\dot{\bar{P}} + (1/T_1) \bar{P} = K_1 \bar{X}_b \quad (B10)$$

$$\bar{X}_b = \bar{X}_p; \bar{X}_b \geq b \quad (B11)$$

Equation (B9) is the linear second order equation of poppet/ball motion, with regulated pressure as a driving function. Equation (B10) is the linear first order equation of regulated pressure with poppet position as a driving function. They are obviously coupled by $\bar{P}$ and Eq. (B11). Due to the linearization about the nominal point, the equilibrium point is $\bar{X}_p = \bar{X}_b = \bar{P} = 0$. The constants are

	<u>Approx. Mag.</u>
$\beta = \frac{\partial F_{ps}}{\partial X_p}$ (note: $\beta = -\frac{F_{ps}}{\partial \delta}$)	-0.2, +0.1
$\gamma = B/m$	(?) sec ⁻¹
$K_1 = (X_{p_{max}}/X_{pn} - 1) \dot{V}_{ln}/(1 - P_a/P_{rn}) V_{an}$	.8 sec ⁻¹
$K_2 = (P_{rn} - P_a) A_d/m(X_{p_{max}} - X_{pn})$	$0.5 \cdot 10^9$ sec ⁻²
$1/T_1 = \left[1 + \frac{1}{2} \left(\frac{1}{1 - P_a/P_{rn}} \right) \right] \dot{V}_{ln}/V_{gn}$	0.08 sec ⁻¹
$a = (X_{p_{min}} - X_{pn})/(X_{p_{max}} - X_{pn})$	-1.7
$b = -X_{pn}/(X_{p_{max}} - X_{pn})$	-0.14

As a result of normalization, the variables are dimensionless and the time derivatives have units of reciprocal seconds raised to the appropriate power.

Stability of these equations in the presence of small disturbances and be determined by examination of the characteristic equation. In order to determine the characteristic equation, define the state variables

$$x_1 = \bar{X}_p$$

$$x_2 = \dot{\bar{X}}_p$$

$$x_3 = \bar{P}$$

Then Eq. (B9) through (B11) can be written

$$\dot{\underline{x}} = A \underline{x} \quad (B12)$$

where $\underline{x}$ is the vector of state variables and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ K_2 & - & -K_2 \\ K_1 & 0 & -1/T_1 \end{bmatrix} \quad (B13)$$

The characteristic equation is then

$$|A - \lambda I| = 0 \quad (B14)$$

Upon expansion this results in

$$\lambda + (\gamma + 1/T_1) \lambda^2 + (\gamma/T_1 - \beta K_2) \lambda + (K_1 K_2 - \beta K_2 / T_1) = 0 \quad (B15)$$

By application of Routh's stability criterion (Ref. 1), the following necessary and sufficient conditions for stability can be determined:

$$\gamma > K_2 T_1 \quad (B16)$$

$$K_1 > 1/T_1 \quad (B17)$$

$$(\gamma/T_1 + \beta K_2) (\gamma + 1/T_1) > (K_1 - \beta/T_1) K_2 \quad (B18)$$

A slight rearrangement of Eq. (B18) gives

$$K_2 < \frac{\gamma}{T_1} \frac{(\gamma + 1/T_1)}{(K_1 + \beta \gamma)} \quad (B19)$$

From these inequalities it is apparent that stability exists for at least some values of K_2 when the spring slope, β , is negative, since this would satisfy Eq. (B16) and (B17) automatically. However, when β is positive, as it is when the reference setting is in the negative-slope region of the spring characteristic, stability depends upon damping and downstream volume. Eq. (B16) shows the required damping. The required downstream volume is indicated by Eq. (B17) since K_1 varies inversely with volume.

By considering the approximate values of the constants, given above, further insight is possible. Assuming that $\beta = +0.1$

$$\begin{aligned} \gamma &> 0.63 \cdot 10^9 \text{ sec}^{-1} \\ K_1 &> 0.008 \text{ sec}^{-1} \\ K_2 &< \frac{\gamma}{\beta T_1} \text{ sec}^{-2} \end{aligned}$$

It is noted that for large values of damping, γ , the inequality Eq. (B18) reduces to Eq. (B16). Also, Eq. (B17) is satisfied even for the

large volume of the propellant tank ullage, 100 cu. in. It therefore appears that large damping is the only requirement for stability in the $+\beta$ region.

The only indication of the actual damping present in the mechanical system is the hysteresis in the spring characteristic, Fig. (B-2). If it is assumed that the hysteresis loop is a result of a force proportional to velocity, then the damping coefficient can be calculated from Fig. (B-2). This method gives

$$\gamma \approx 0.715 \cdot 10^8 \text{ sec}^{-1}$$

which is approximately one order of magnitude less than that required for stability by Eq. (B16).

The above stability analysis assumed small changes in the state variables from their nominal value, zero. An indication of instability automatically ensures that any small disturbance will immediately begin to grow, and in time will result in violation of the small amplitude assumption. Large amplitude oscillations drive the system into non-linear regions, and there may be convergence to a stable limit cycle, of either large or small amplitude. This is best studied with the detailed model.

It must also be noted that some degree of positive β is permitted by the above development. For example, Eq. (B16) can be re-arranged to give

$$\beta < \frac{\gamma}{K_2 T_1}$$

Thus for

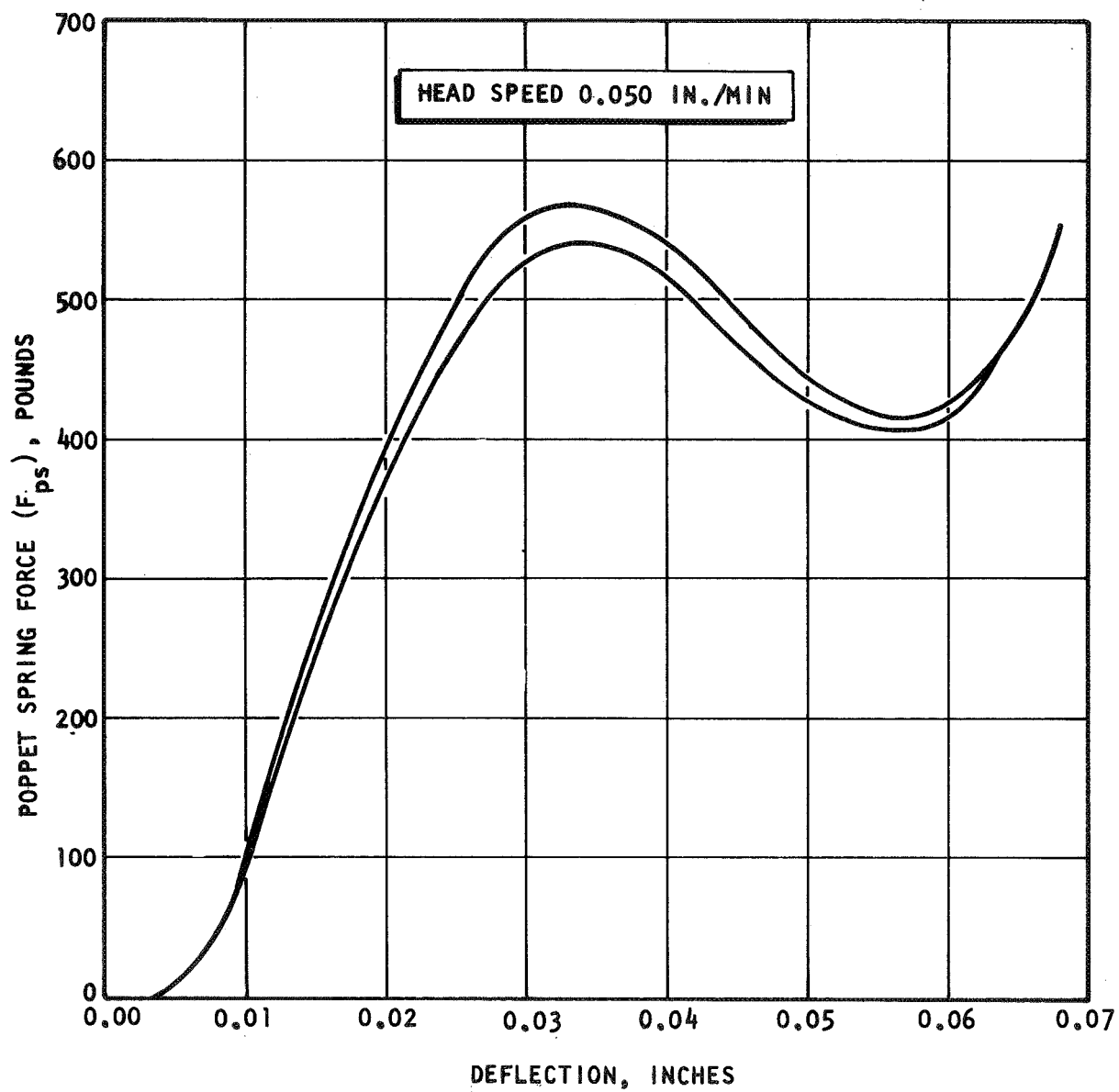


Figure B-2. Typical Poppet Spring Characteristics (S/N 001 Spring Assembly With S/N 6 Spring Removed)

$$\gamma = 0.7 \cdot 10^8 \text{ sec}^{-1}$$

$$K_2 = 0.5 \cdot 10^9 \text{ sec}^{-2}$$

$$1/T_1 = 0.08 \text{ sec}^{-1}$$

the maximum tolerable slope is given by

$$\beta < 1.12 \cdot 10^{-2}$$

Further, if the downstream volume is smaller, even a larger slope could be tolerated.

In order to show the effect of the non-linear spring characteristic, the equations in state-variable form, Eq. (B12), can be examined graphically. The case most easily examined is when the mechanical system is lightly damped and the downstream volume is large. In this case the poppet moves extremely fast relative to regulated pressure, and the system can be represented in two-dimensional state space, i.e., the phase plane.

Figure (B-3) shows the phase plane of system downstream pressure versus poppet position. In this plane, the spring characteristic appears as a switching line. That is, above this line the poppet moves to the left, and below it the poppet moves to the right. The forward poppet stop, full open, appears as boundary at $\bar{X}_p = 1.0$, and the retraction stop appears as a boundary at $\bar{X}_p = -1.7$. When $\bar{X}_p = -0.14$, the ball is on the seat. The spring characteristic is shown for a nominal setting with a small positive β , i.e., just to the left of the minimum force point on the deflection curve, Fig. (B-4).

The path marked with arrows shows a typical pressurization transient. The starting point shown corresponds to an initial reference pressure

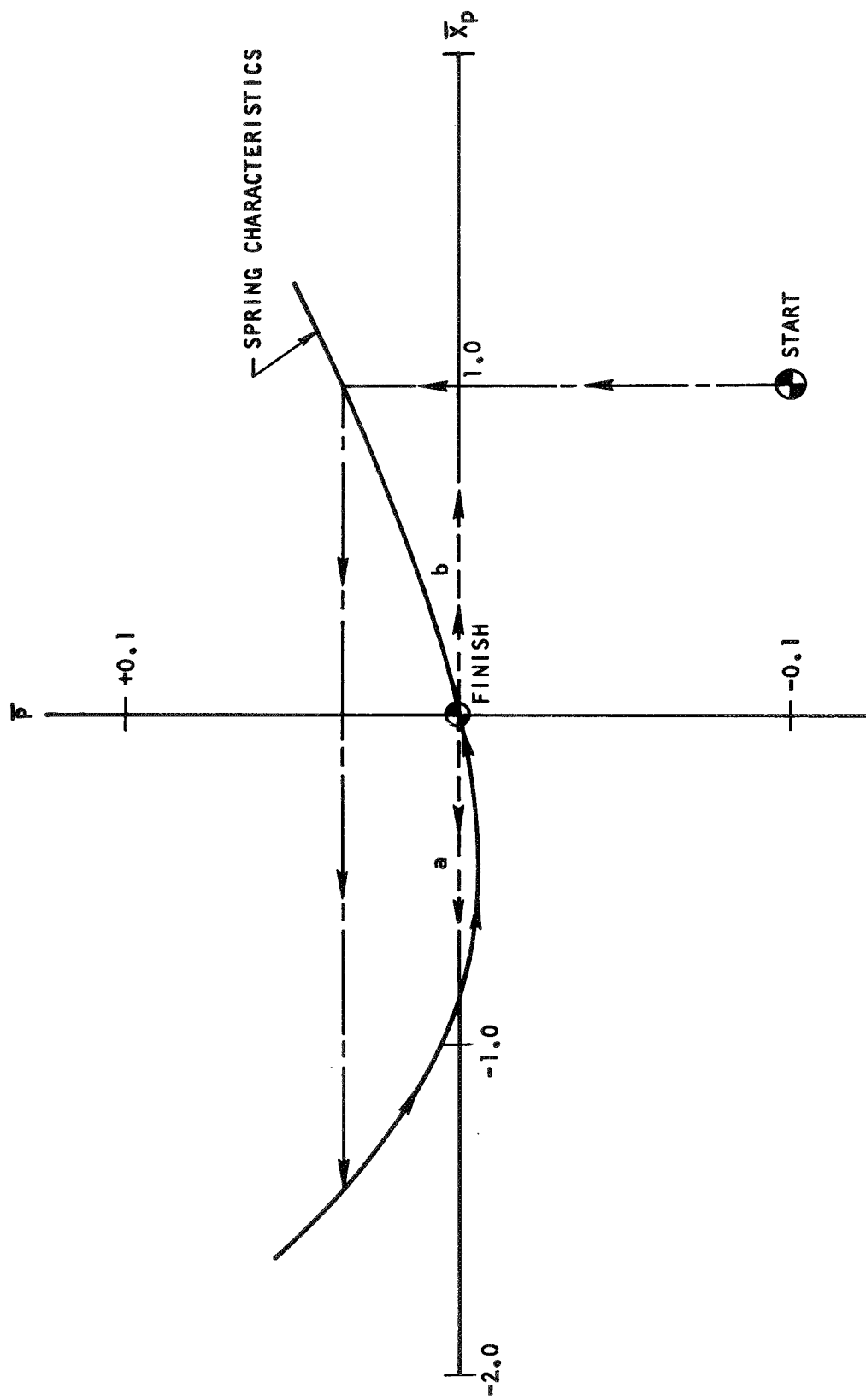


Figure B-3. Phase Plane of Pressurization of Simplified Water Expulsion System

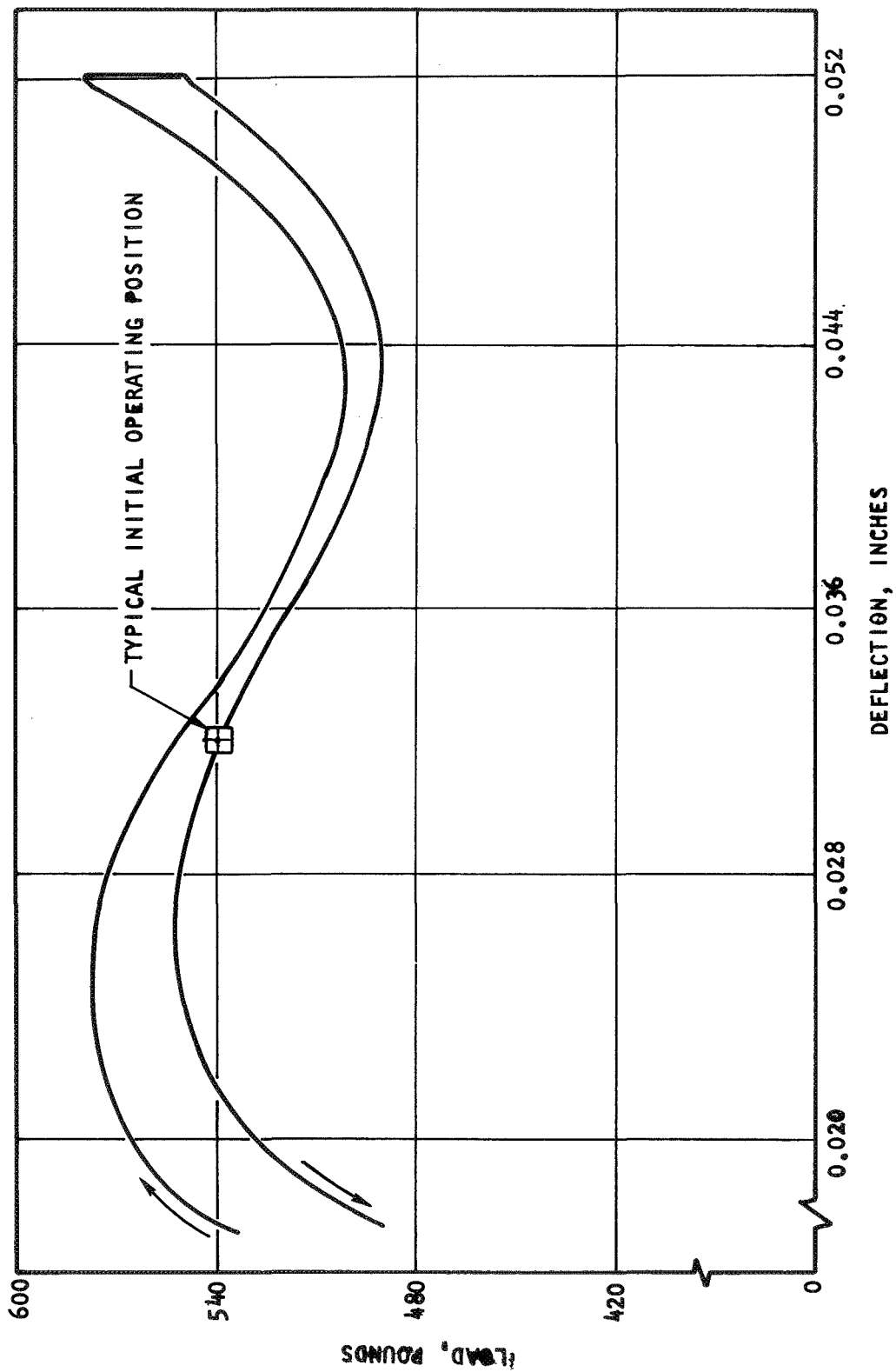


Figure B-4. Typical Load vs Deflection Curve of Belleville Spring Package for 308 psig Regulation (JPL Drawing No. 10000976)

of about 277 psia, which is 10 percent low. Since the pressure is low, the poppet is in the fully opened position. As the upstream valve is opened, flow begins through the seat and downstream pressure begins to rise. The poppet remains on the forward stop until the pressure force on the diaphragm equals the spring force, i.e., until the spring curve is reached. At this point the rate of exchange of pressure, $\dot{\bar{P}}$, is still large, as can be seen from Eq. (B10), which results in the pressure rising above the spring curve. Once the trajectory enters the region above the spring curve, the poppet is driven in the negative direction. In the absence of significant damping, the trajectory will be almost parallel to the $\bar{X}_p$ axis due to the extremely fast motion of the poppet relative to the pressure change. This trajectory intersects the spring curve in the second quadrant, where the pressure derivative is negative, resulting in a tendency to cross the spring curve. Crossing the spring curve introduces positive poppet velocity, which drives the trajectory back to the curve. It will be observed that β is negative in this region, making the poppet equation of motion stable, so that the trajectory follows the spring curve. The system comes to rest at the origin since all time derivatives are zero at this point. If the β at this point is sufficiently small, i.e., if Eq. (B16) is satisfied locally, this is a stable equilibrium point and satisfactory operation is achieved.

In the event that the set-point on the spring characteristic curve is such that Eq. (B16) is not met at the origin, the origin is an unstable equilibrium point. In this event, a small disturbance in either pressure or position which causes the operating point to shift up or to the left will result in trajectory "a". Similarly, a shift down or to the right will result in trajectory "b". The excursion along "a" would result in an extremely small pressure excursion, less than 1 percent. The excursion along "b" would cause a complete re-cycle of the start-up transient. It is conceivable that, with a certain set

point, either of these excursions could repeat indefinitely.

The above arguments could be extended to include the effects of damping and hysteresis. Large damping would make the upper portion of the trajectory curve upward. Hysteresis represented as discussed previously would have similar effects, since it is a manifestation of internal damping.

A smaller downstream volume would have the effect of increasing the rate of change of pressure. This would also cause the upper trajectory to curve upward.

The findings above, i.e., the stability criteria and the phase trajectories, indicate the importance of certain parameters on system performance. First, required damping is a function of the magnitude and sign of the spring characteristic slope, and of the size of the downstream volume. This suggests that the detailed model must reflect the actual regulator precisely in these areas. The importance of β , and the shape of the spring curve in the region of the set point indicate that the set point must be a variable parameter in the detailed model.

SIMPLIFIED WATER EXPULSION MODEL SYMBOLS

A_d	-	diaphragm area
A_s	-	seat area
B	-	damping coefficient
C_s	-	seat discharge coefficient
F_{ps}	-	spring force, a function of spring deflection
K	-	ratio of seat flow area to ball position
m	-	moving mass of poppet
P_a	-	ambient pressure
P_r	-	liquid tank pressure
P_t	-	pressurization tank pressure
R_l	-	liquid system resistance
T	-	uniform gas temperature throughout
V_g	-	gas volume in liquid tank
$\dot{w}_g$	-	gas weight flowrate
$\dot{w}_e$	-	liquid out flow
X_b	-	ball position
X_p	-	poppet position (positive in opening direction)
$X_{p_{max}}$	-	maximum poppet position
$X_{p_{min}}$	-	minimum poppet position
β	-	slope of normalized spring force-deflection curve

ρ_g	-	gas density in liquid tank
ρ_l	-	liquid density
ϕ	-	compressible flow function

APPENDIX C

ANALOG COMPUTER PROGRAMS FOR MM '70 PROPULSION
SYSTEM

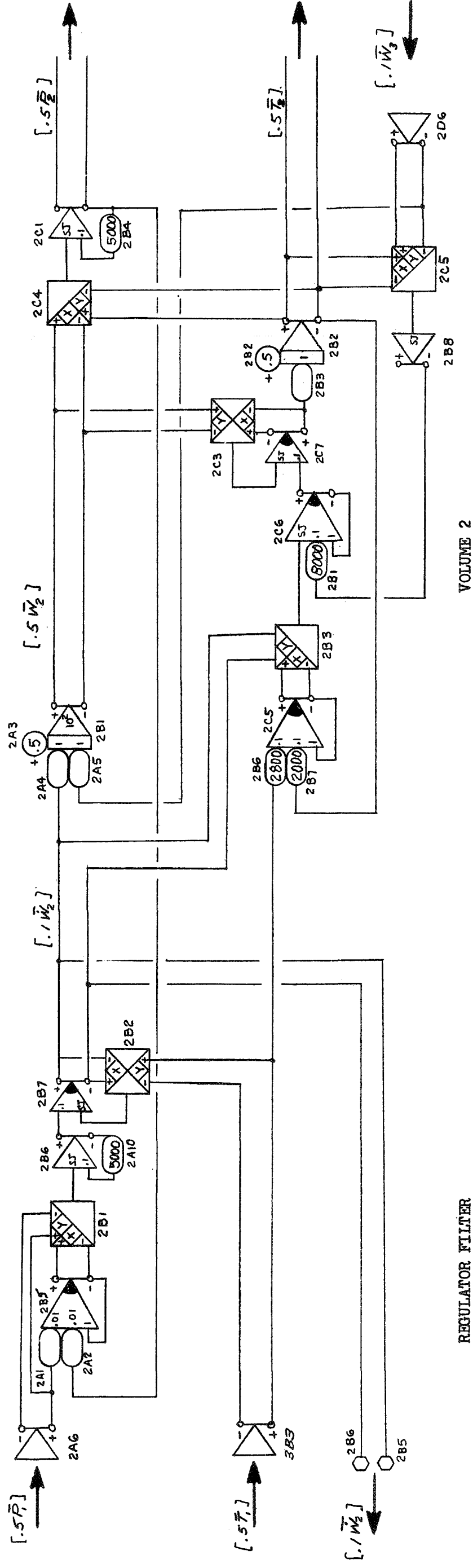


FIGURE C-2 Computer Mechanization For MM70
Propulsion System (continued)

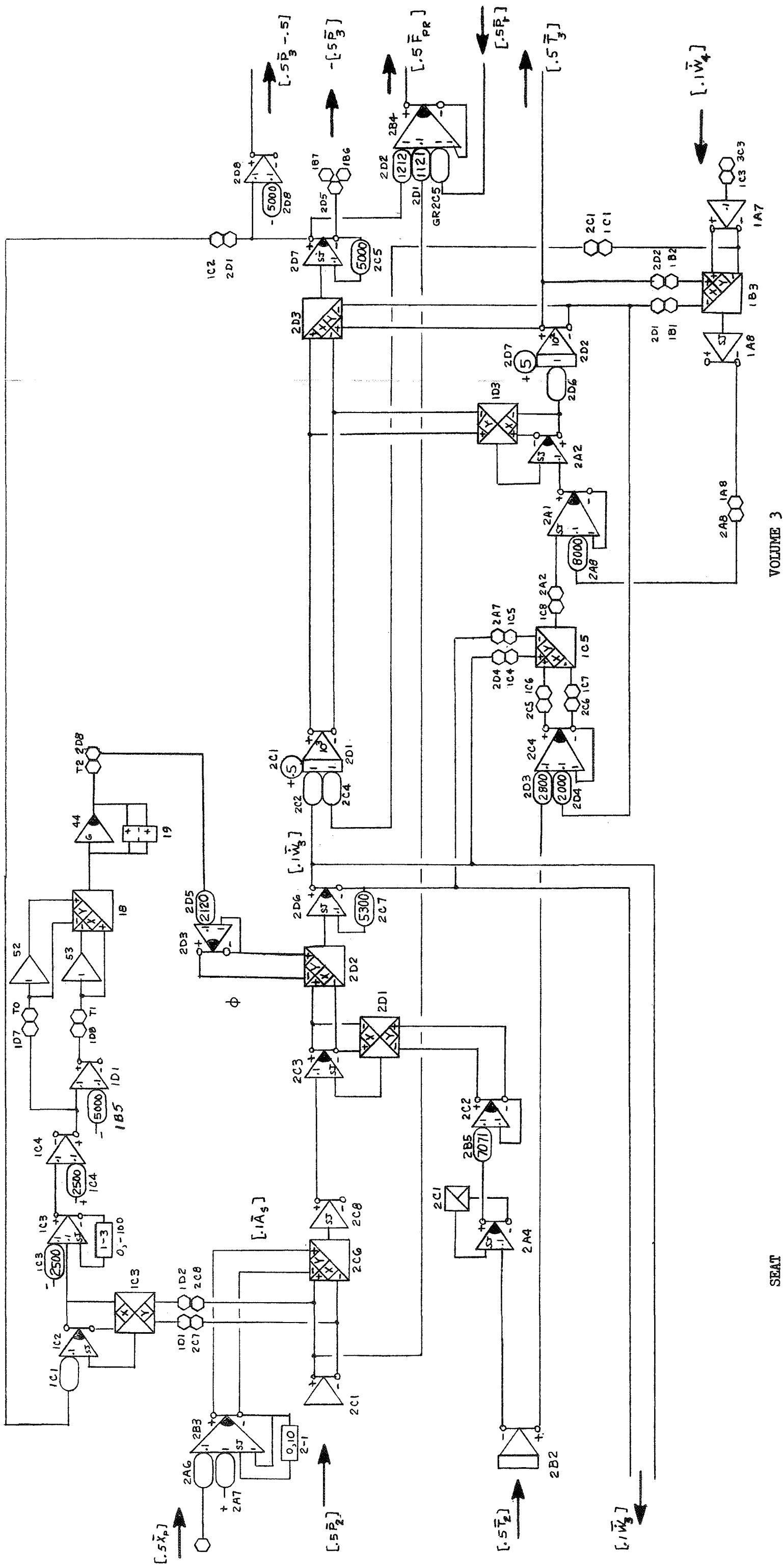
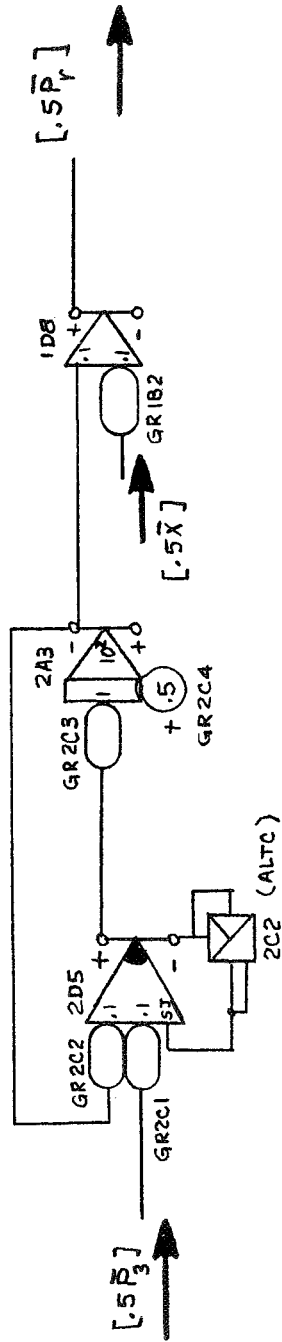


FIGURE C-3 Computer Mechanization For MM*70
Propulsion System (continued)



REF. CAVITY PRESSURE

FIGURE C-4 COMPUTER MECHANIZATION FOR MM '70 PROPULSION SYSTEM (CONTINUED)

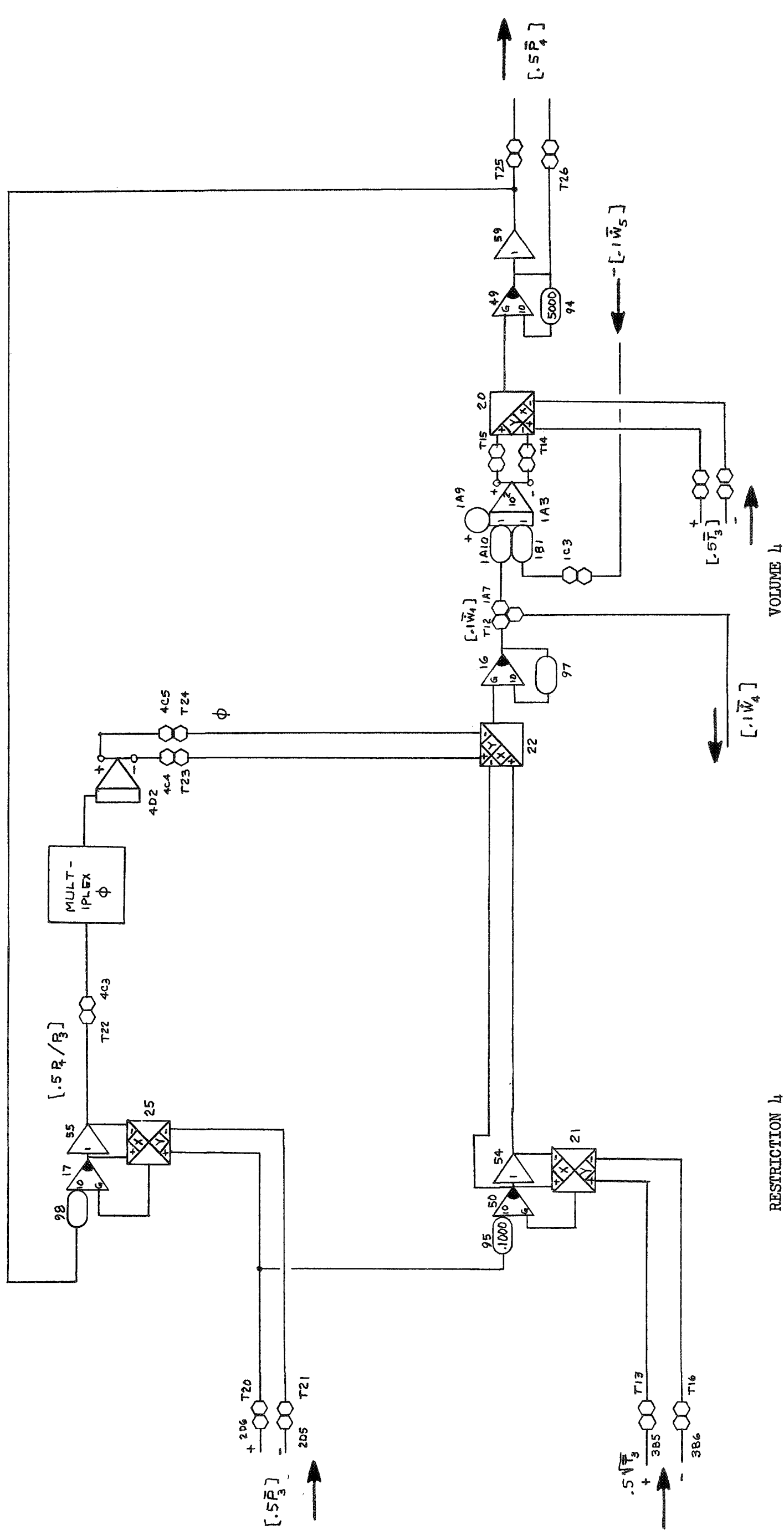


FIGURE C-5 Computer Mechanization For MM-70
Propulsion System (continued)

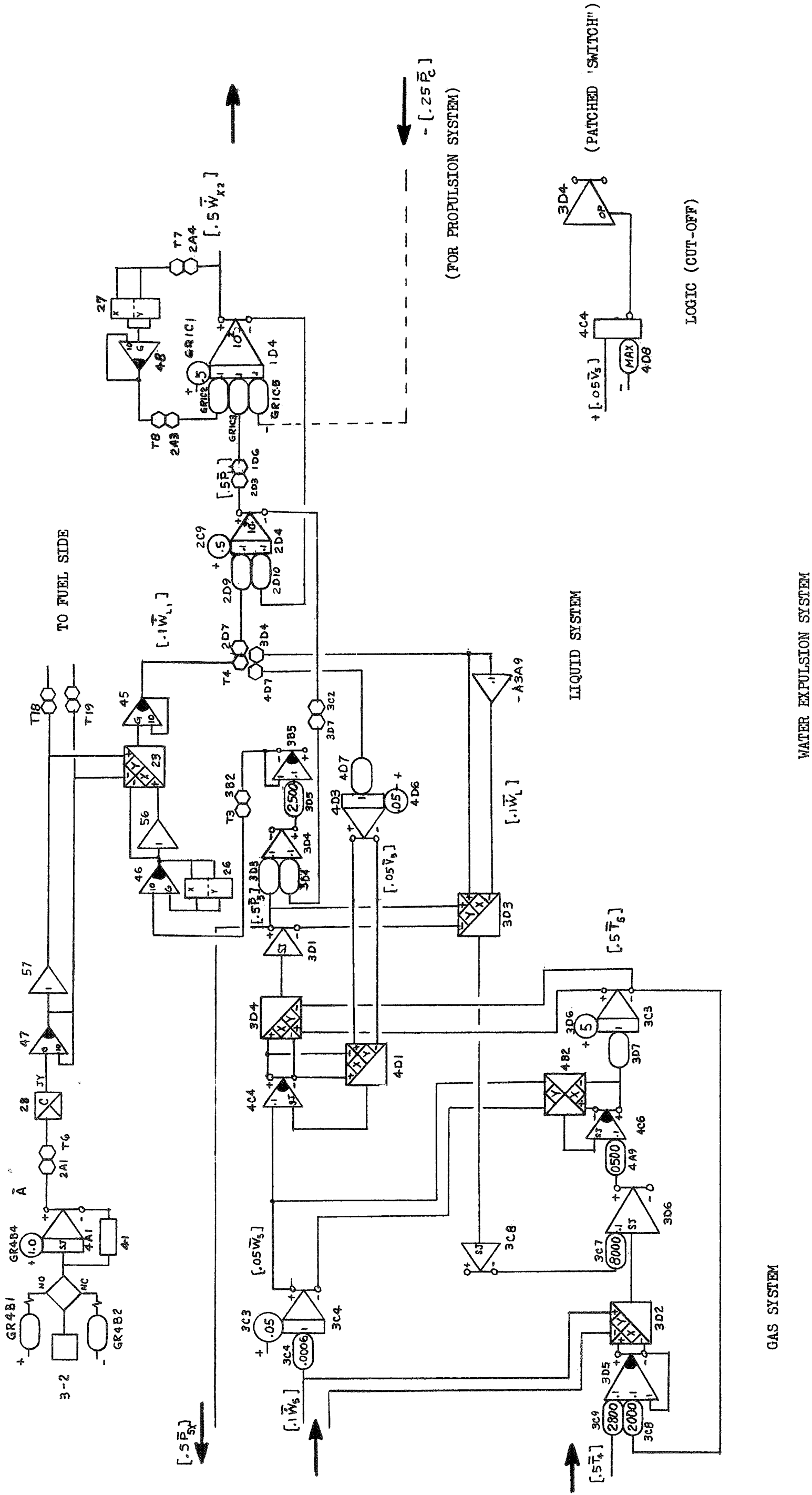


FIGURE C-7 Computer Mechanization For MM-70
Propulsion System (continued)

MULTIPLEX $\phi(ac)$

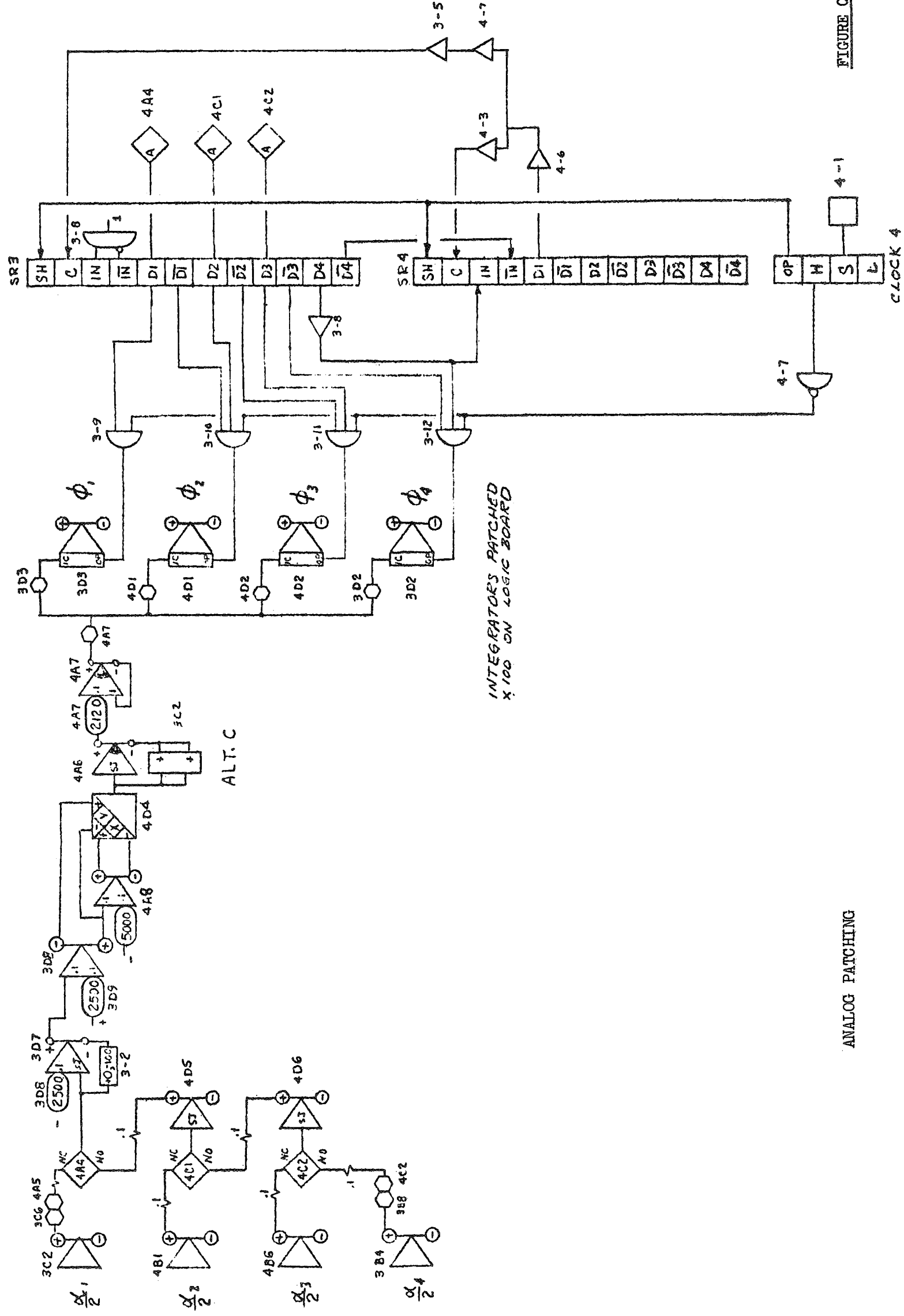
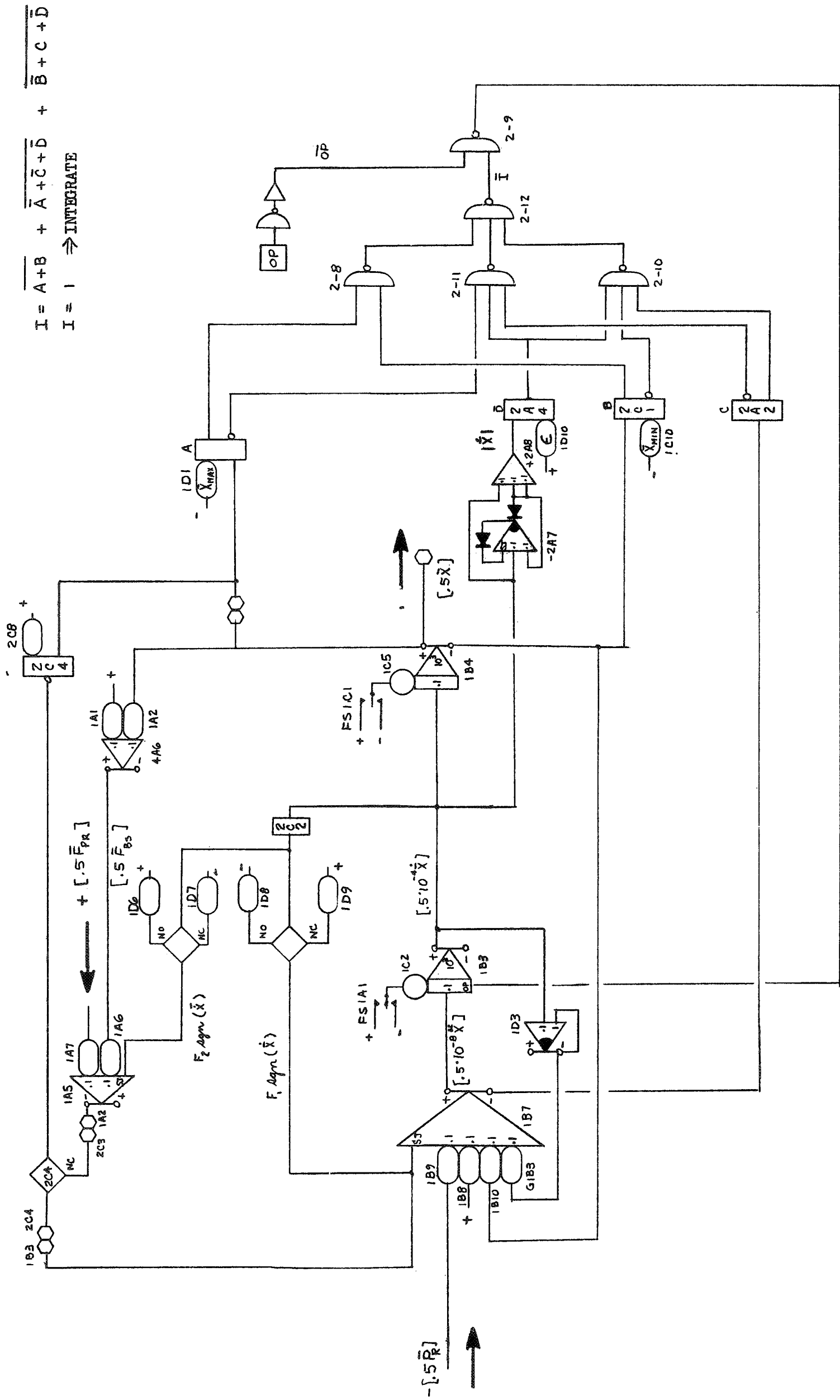
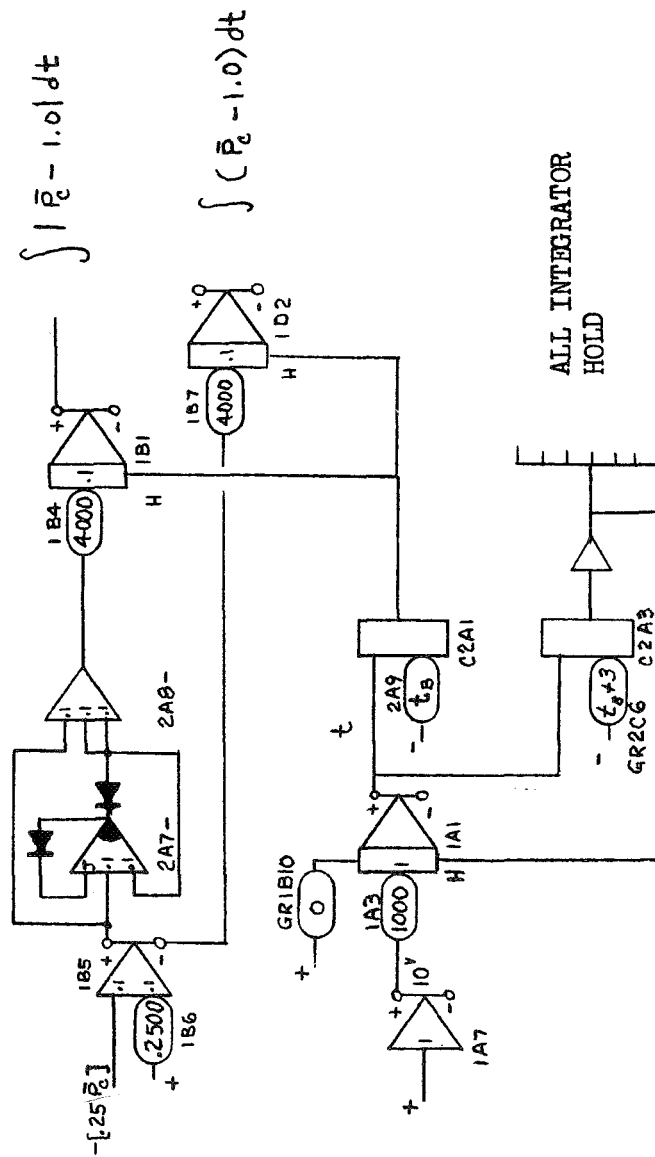


FIGURE C-2 Computer Mechanization For MM70
Propulsion System (continued)
C-10



MM '71 REGULATOR POPPET



P_c ERROR INTEGRATION AND CUT-OFF HOLD

FIGURE C-11 COMPUTER MECHANIZATION FOR MM '70 PROPULSION SYSTEM (CONTINUED)

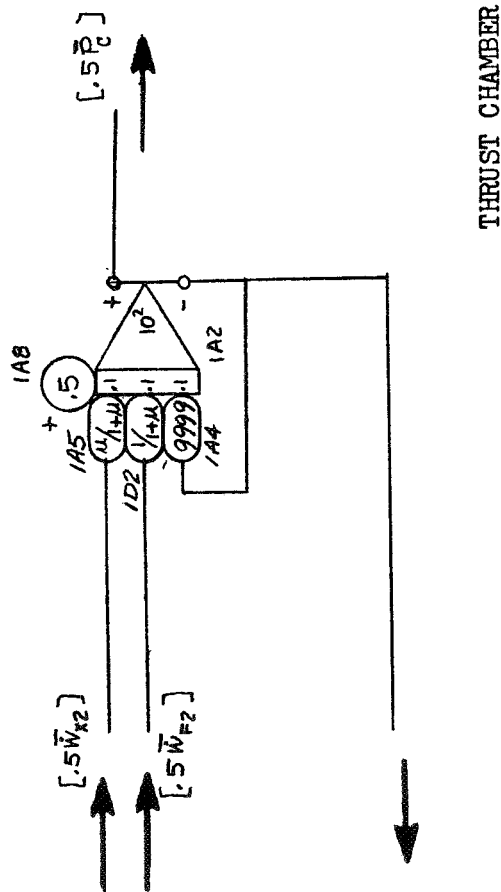


FIGURE C-12 COMPUTER MECHANIZATION FOR MM '70 PROPULSION SYSTEM (CONCLUDED)

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