

DYNAMIC INTERACTION BETWEEN STRUCTURE AND LIQUID PROPELLANTS IN A SPACE SHUTTLE VEHICLE MODEL

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Daniel D. Kana

William L. Ko

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FINAL REPORT, PART I

Contract No. NAS1-9890

Control No. L17-826

SwRI Project No. 02-2820

Prepared for
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia

October 15, 1970



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H. Norman Abramson, Director
Department of Mechanical Sciences

PREFACE

This report constitutes the first of two volumes which summarize the work accomplished under Contract NAS1-9890. It contains the design of an experimental space shuttle vehicle model, supporting experimental data, the design of a corresponding analytical model, comparisons of results, and a listing of the digital computer program designed for predicting natural frequencies of a typical shuttle vehicle parallel-stage configuration which includes liquid propellants. The second part of the work, which deals with other liquid dynamics problems associated with space shuttle operation, is summarized in Final Report, Part II entitled "Propellant Dynamic Problems in Space Shuttle Vehicles."

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NOMENCLATURE

Symbol	Definition
a	distance between subsystems 2 and 11
$\tilde{a}$	coefficient matrix of constraint coordinates appearing in equations of motion
b	distance between subsystem 11 and the centerline of the Orbiter
c	distance between subsystems 12 and 13
$\tilde{c}$	constraint matrix
D_k	distance between the neutral axis of a reinforcing beam section and a cylinder shell ($k = 1, 2, 3, 4$)
d	distance between subsystems 15 and 16
$\tilde{d}$	coefficient matrix of mass coordinates appearing in equations of constraint
E_s	modulus of elasticity of cylinder shells ($s = 1, 2, 3, 5, 6, 7$)
$E^{(m)}$	modulus of elasticity of a reinforcing beam section ($m = 1, 2, 3, 4$)
e	distance between subsystem 5 and the rod $K^{(5)}$
$F_i = \frac{4}{\pi} \frac{I_1 \left(\frac{\pi R_i}{2h_i} \right)}{\frac{\pi R_i}{2h_i} I_0 \left(\frac{\pi R_i}{2h_i} \right)}$	
G_s	shear modulus of cylinder shell s
g	gravitational constant
H	total number of dynamic and constraint equations of the mechanical system

NOMENCLATURE (Cont'd)

Symbol	Definition
h_i	height of liquid in cylinder i
$\tilde{h}_i$	height of horizontal sloshing mass in cylinder i
h_i^*	height of rigid mass in cylinder i
$\mathcal{I}_s$	moment of inertia of a cylinder cross section
I_0	modified Bessel function of order zero
I_1	modified Bessel function of order one
i	index
J_n	moment of inertia of mass element M_n ($n = 1, 2, \dots, 8$)
$J^{(m)}$	moment of inertia of a reinforcing beam cross section
J_i^*	moment of inertia of rigid mass m_i^*
$J_{\text{rigid}}^{(i)}$	moment of inertia of frozen liquid in cylinder i
j	index
K_i	equivalent longitudinal spring constant of cylinder i accounting for liquid effect
$K_j = \frac{2\pi R_j \delta_j E_j}{\ell_j}$	longitudinal spring constant of cylinder j
$\bar{K}_i = \frac{2\pi R_i \delta_i E_i}{\ell_i - \nu_i^2 h_i}$	longitudinal spring constant of cylinder i with liquid effect
$K^{(m)}, \bar{K}^{(3)}, \bar{K}^{(4)}$	longitudinal spring constants of the reinforcing beam sections
$K^{(6)}, K^{(7)}$	vertical and horizontal spring constants of the coupling spring system
k_i	vertical sloshing spring constant

NOMENCLATURE (Cont'd)

Symbol	Definition
$\bar{k}_i$	vertical sloshing spring constant due to Poissons effect of the cylinder
$k_i' = \frac{4\pi R_i \delta_i E_i}{h_i} \left[\frac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_0\left(\frac{\pi R_i}{2h_i}\right)} \right]^2$	
$\tilde{k}_i$	lateral sloshing spring constant
k	index
L	number of dynamic equations
L'	number of constraint equations
ℓ_s	length of cylinder section
M_n	structure mass element
$M^{()}, \bar{M}^{()}$	end moments of a beam section
$M_{[]}^{()}, \bar{M}_{[]}^{()}$	net end moments of a shell-beam section
m_i	vertical sloshing mass in cylinder i
$\tilde{m}_i$	horizontal sloshing mass in cylinder i
m_i^*	rigid mass in cylinder i
$m_T^{(i)} = \pi R_i^2 h_i \rho_i$	total mass of the liquid in cylinder i
m	index
n	index
$P()()$	mass matrix of the total system of dynamic and constraint equations

NOMENCLATURE (Cont'd)

Symbol	Definition
$\underline{p}$	mass matrix of the dynamic equations only
$\underline{Q}(\)(\)$	stiffness matrix of the total system of dynamic and constraint equations
$\underline{q}$	stiffness matrix of the dynamic equations only
r	index
R_s	R_B (Booster radius) for $s = 1, 2, 3$; R_O (Orbiter radius) for $s = 5, 6, 7$
S_r	torsional spring constant ($r = 1, 2, \dots, 7$)
s	index
t	time
$V(\)$	lateral shearing force on an end cross section of a beam section
$V \begin{Bmatrix} \\ \end{Bmatrix}$	net lateral shearing force on an end cross section of a shell-beam section
$W(\)$	vibration mode
$\underline{\tilde{x}}$	coordinates associated with mass elements (see Table III)
$\underline{\tilde{x}}$	coordinates associated with constraint conditions (see Table III)
$x(\)$	lateral displacement of a subsystem
$Y(\)$	amplitude of a vibration mode
$y(\)$	axis of rotation of a subsystem
$z(\)$	vertical displacement of a subsystem
$\alpha(\)$	rotation of a cross section of a shell beam where a horizontal sloshing spring or a rigid mass is attached

NOMENCLATURE (Cont'd)

Symbol	Definition
γ	phase angle
δ_s	cylinder shell thickness
$\zeta()$	vertical displacement of a vertical sloshing mass
$\eta()$	horizontal displacement of a lateral sloshing mass
ν_s	Poissons ratio of cylinder shell
$\xi()$	horizontal displacement of a shell beam cross section where the lateral sloshing spring or a rigid mass is attached
ρ_i	mass density of liquid in cylinder i
$\phi()$	rotation of a structure subsystem
Ω	eigenvalue of a vibration mode
ω	frequency

DYNAMIC INTERACTION BETWEEN STRUCTURE AND LIQUID PROPELLANTS IN A SPACE SHUTTLE VEHICLE MODEL

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INTRODUCTION

Currently specified design requirements of a space shuttle vehicle are anticipated to present many new problems heretofore not encountered in aerospace systems. The dynamic interaction between elastic structure and liquid propellants has always been an important design criteria for launch vehicles and aircraft; however, it is surmised that potential problems posed by this interaction will become even more critical in presently envisioned space shuttle systems (ref. 1). Therefore, the purpose of this study is to examine the applicability of existing analytical techniques for studying the coupled liquid-structural dynamics of a typical space shuttle configuration--a parallel-stage design.

The program objective has been accomplished by developing a suitable experimental model that is capable of experiencing at least the most fundamental structural dynamics of a prototype system, measuring its natural frequencies of vibration for a range of various parameters, and comparing the results with those predicted from a corresponding analytically derived model. In selecting the model details from the outset, considerable effort was exercised to utilize components which were already available from previous research programs, in order to minimize fabrication costs. Further, existing concepts of spring-mass fluid models, which have been derived to simulate liquid reactions for decoupled lateral and longitudinal motions, are employed in a straightforward manner in a system which experiences strong coupling along these axes. The results of this study of a rather fundamental model will point toward the path to follow for more complex representations of a shuttle system. We begin with a description of the physical model, then outline the analysis, and finally present results and conclusions from the study.

DESCRIPTION OF PHYSICAL MODEL

A model consisting of a parallel-stage Booster and Orbiter, each consisting of two propellant tanks and appropriate intermediate skirts and

rigid masses, was considered feasible to carry out the program objective. The major portion of the Booster was already on hand from a previous study (ref. 2) of longitudinal dynamics in axisymmetrical launch vehicles. As a result, as will be seen, it also included some components, such as stiffeners and baffles, which were not strictly essential to the present study. Nevertheless, the presence of these extra components did not alter the conclusions of the study.

A photograph of the completely assembled and suspended system is shown in Figure 1a, as it was used during most of the experiments. In order to provide a quick overall indication for model size and typical rigid masses, a schematic is shown in Figure 1b. Further details will be given now, as well as in later sections of the report.

The Booster comprises the major part of the model, and it consists of the following main components:

- (1) Upper tank
- (2) Lower tank
- (3) Skirt
- (4) Bulkheads

Both tanks in the Booster model were fabricated from 0.005-inch thick, type 302 Stainless Steel sheets, which were rolled and butt-welded along longitudinal seams. A flat steel disk was spot-welded to one of the tanks which serves as a top mass for the upper tank. The lower end of this upper tank and both ends of the lower tank were spot-welded to identical steel flanges, which can be bolted to the bulkheads and the skirt as required. Two rows of spot welds, each spot weld having a test strength of 25 pounds, were used at each end of the tanks. The spots were spaced 1/8-inch apart with 1/8-inch spacing between the rows. To provide for ullage pressure integrity, the ends of both tanks were sealed with epoxy cement.

The third main component in the Booster is the skirt. It was fabricated by rolling 0.025-inch thick, 6061-T6 aluminum sheet to the desired diameter and butt-welding it along a longitudinal seam. Two aluminum flanges were welded to the end of this cylinder with the same hole pattern as on the flanges of the tanks to provide for bolted assembly of these parts. Two small ports on the side of the skirt served as a pressure port for introducing ullage pressure to the lower tank and for filling the tank with liquid.

Flat, rigid bulkheads were machined from mild steel and 6061-T6 aluminum plates for the lower and upper tanks, respectively, with a shoulder



FIGURE 1a. --SwRI SPACE SHUTTLE VEHICLE DYNAMIC MODEL

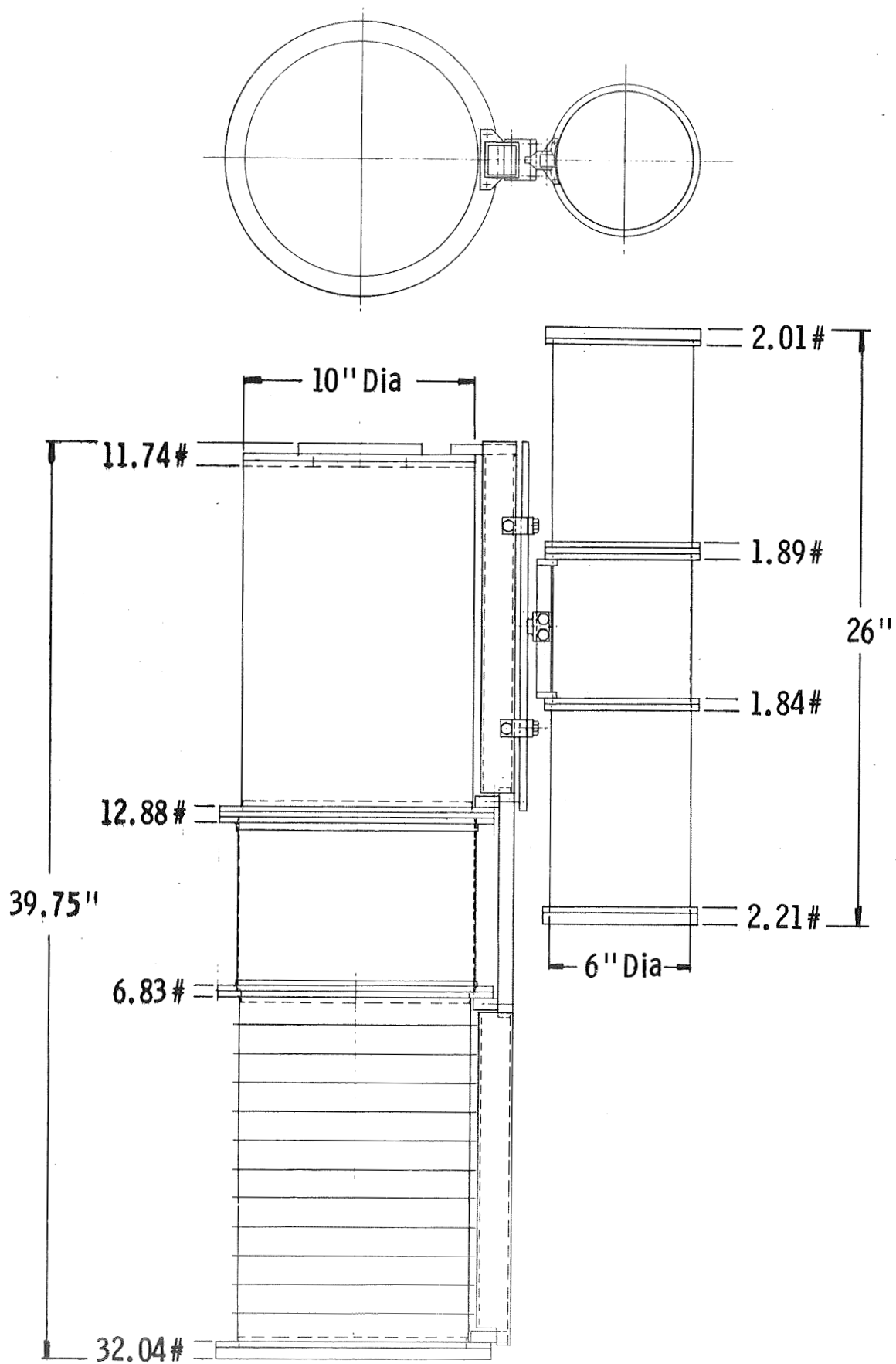


FIGURE 1b. --SCHEMATIC OF SPACE SHUTTLE VEHICLE MODEL

on them so they could partially fit inside the tanks. The shoulders and their hole patterns match the flanges on the tanks. The lower ends of the tanks were sealed by these flat, rigid bulkheads by bolting them to the corresponding flanges. The tanks in turn were joined by bolting them together using the skirt as a coupling.

The lower tank had stiffener rings, stringers, and baffles installed on it. Eleven 6061-T6 aluminum rings, 0.032-inch thick were cemented on the outside of the tank 1.25 inches apart, symmetrically about midspan. Twelve 6061-T6 aluminum stringers, $1/8 \times 1/8 \times 14$ inches in dimensions, were fastened to the inside of the tank by use of epoxy. The stringers were equally spaced around the circumference of the tank and symmetrical about midspan. Tapped holes along the length of these stringers provided the means for installing eleven yellow brass ring baffles into the tank. Data pertaining to the Booster model are given in Table I.

The Orbiter model, similar to the Booster, consists of two tanks with flat, rigid bulkheads and a skirt. The tanks and the skirt were fabricated from 0.012-inch-thick and 0.020-inch-thick 1100-H14 aluminum sheets, respectively, which were rolled and butt-welded along a longitudinal seam in the same manner as the Booster. Identical flanges with 16-hole bolt patterns were welded to each end of both tanks and skirt by a continuous weld.

Flat, rigid bulkheads were machined from 6061-T6 aluminum plates with hole patterns matching the pattern on the flanges. These bulkheads, however, did not have shoulders on them as was the case for the Booster. Plates identical to the bulkheads were used for capping both of the tanks on the Orbiter with provisions for introduction of ullage pressure and modeling liquid.

The skirt, as in the case of the Booster, was used to join the two tanks. Data pertaining to the Orbiter model are given in Table II.

Coupling between the Booster and Orbiter was achieved by the strongback assembly. As can be seen in Figure 1, the strongback spans the full length of the Booster and is attached to it at four locations, namely, at each flange and the top. This part of the strongback was fabricated from a $1-1/2 \times 1-1/2 \times 1/8$ -inch, 6063-T5 aluminum square tube with appropriate altering to be attachable to the Booster.

A short backstrap was attached to the Orbiter spanning between the two flanges of the skirt. To this backstrap, a $1/4$ -inch square steel rod was fastened which fits into two guides on the strongback allowing adjustment of the relative position between the Booster and Orbiter.

TABLE I. -- MATERIAL PROPERTIES AND GEOMETRY OF
STRUCTURAL COMPONENTS OF BOOSTER MODEL

Structural Element	Effective Length(l) (in.)	Inside Dia. (in.)	Wall Thickness (in.)	Material Density (#/in ³)	E $\times 10^6$ psi
Upper Tank	14.5	10.0	0.005	0.29	29
Lower Tank	14.5	10.0	0.005	0.29	29
Skirt	7.5	10.3	0.025	0.098	10

	Material Density (#/in ³)	E $\times 10^6$ psi	Dimensions (in.)
Flat Rigid Bulkhead Lower Tank	0.29	30	9.875 Dia. , 1/2 Height, 12 Dia. shoulder, 1/2 Height
Flat Rigid Bulkhead Upper Tank	0.098	10	9.875 Dia. , 1/2 Height, 12 Dia. shoulder, 1/2 Height

	Number Used on Tank	Material Density (#/in ³)	E $\times 10^6$ psi	Dimensions (in.)	Location	Spacing (in.)
Stiffener Ring	11	0.098	10	10.0 I. D. 10.5 O. D. 0.032 thick	Symmetrical about midspan	1.24
Stringer	12	0.098	10	0.125 $\times$ 0.125 $\times$ 14.0	Symmetrical about midspan	Equally spaced on inner circum- ference
Baffle	11	0.306	16	0.25 I. D. 9.68 O. D. 0.0125 thick	Symmetrical about midspan	1.25

TABLE II. -- MATERIAL PROPERTIES AND GEOMETRY OF
STRUCTURAL COMPONENTS OF ORBITER MODEL

<u>Structural Element</u>	<u>Effective Length(l) (in.)</u>	<u>Inside Dia. (in.)</u>	<u>Wall Thickness (in.)</u>	<u>Material Density (#/in³)</u>	<u>E $\times 10^6$ psi</u>
Upper Tank	8.5	6.0	0.012	0.098	10
Lower Tank	8.5	6.0	0.012	0.098	10
Skirt	6.0	6.0	0.020	0.098	10

	<u>Diameter (in.)</u>	<u>Thickness (in.)</u>	<u>Material Density (#/in³)</u>	<u>E $\times 10^6$ psi</u>
Bulkheads	6.7	0.25	0.098	10
Caps	6.7	0.25	0.098	10

ANALYTICAL MODEL

Mechanical Model

In the modal analysis of free vibration of the model Shuttle Vehicle, the system is represented by the equivalent mechanical model shown in Figure 2a. The motion of the system will be limited to translations in x and z directions and pitching about an axis perpendicular to xz -plane. The cylindrical shells between any two neighboring mass elements will be represented by thin-walled beamlike tubes, or shell-beams, as shown in Figure 2b. Additional details are given in Figures 2c and 2d. Masses of the cylindrical shell sections, Booster and Orbiter strongback beam sections, and between any two neighboring mass elements are divided equally into two parts, each of which is lumped into each of the two mass elements. Thus, the inertia effect of the shells and the strongback beams will be otherwise neglected. The Booster and the Orbiter strongback reinforcing beams are pin-jointed, respectively, to the Booster and the Orbiter mass elements with one torsional spring attached to each joint. The two beams are then connected together through a coupling compound spring system which permits relative displacements in x and z directions, and one relative rotation about an axis perpendicular to the xz plane. The lower end of the Orbiter is connected to the Booster strongback beam through a rod $K^{(5)}$ both of whose ends are pin-jointed.*

The vertical and lateral sloshing motions of the liquid in a cylinder are represented independently by two sloshing models. The vertical sloshing model (ref. 3) consists of one vertical sloshing mass m_i ($i = 1, 3, 5, 7$) connected to the neighboring mass elements through two springs† k_i and $\bar{k}_i$. It will be assumed that the bending of the shell-beam does not interfere with the motion of m_i . The lateral sloshing model‡ (refs. 4, 5, 6) consists of one rigid mass $\tilde{m}_i^*$ rigidly attached to the shell-beam, and one lateral sloshing mass $\tilde{m}_i$ connected to the shell-beam through two springs of spring constant $\tilde{k}_i/2$. For the vertical sloshing, quantities associated with the horizontal sloshing (i.e., m_i^* , $\tilde{m}_i$, and $\tilde{k}_i$) will be set to zero, and vice versa.

*This rod was removed from the experimental apparatus and, consequently, its effect was nullified in the numerical program by setting to zero its cross-sectional area.

†All springs in the model are massless and linear.

‡This model was originally developed for a rigid container.

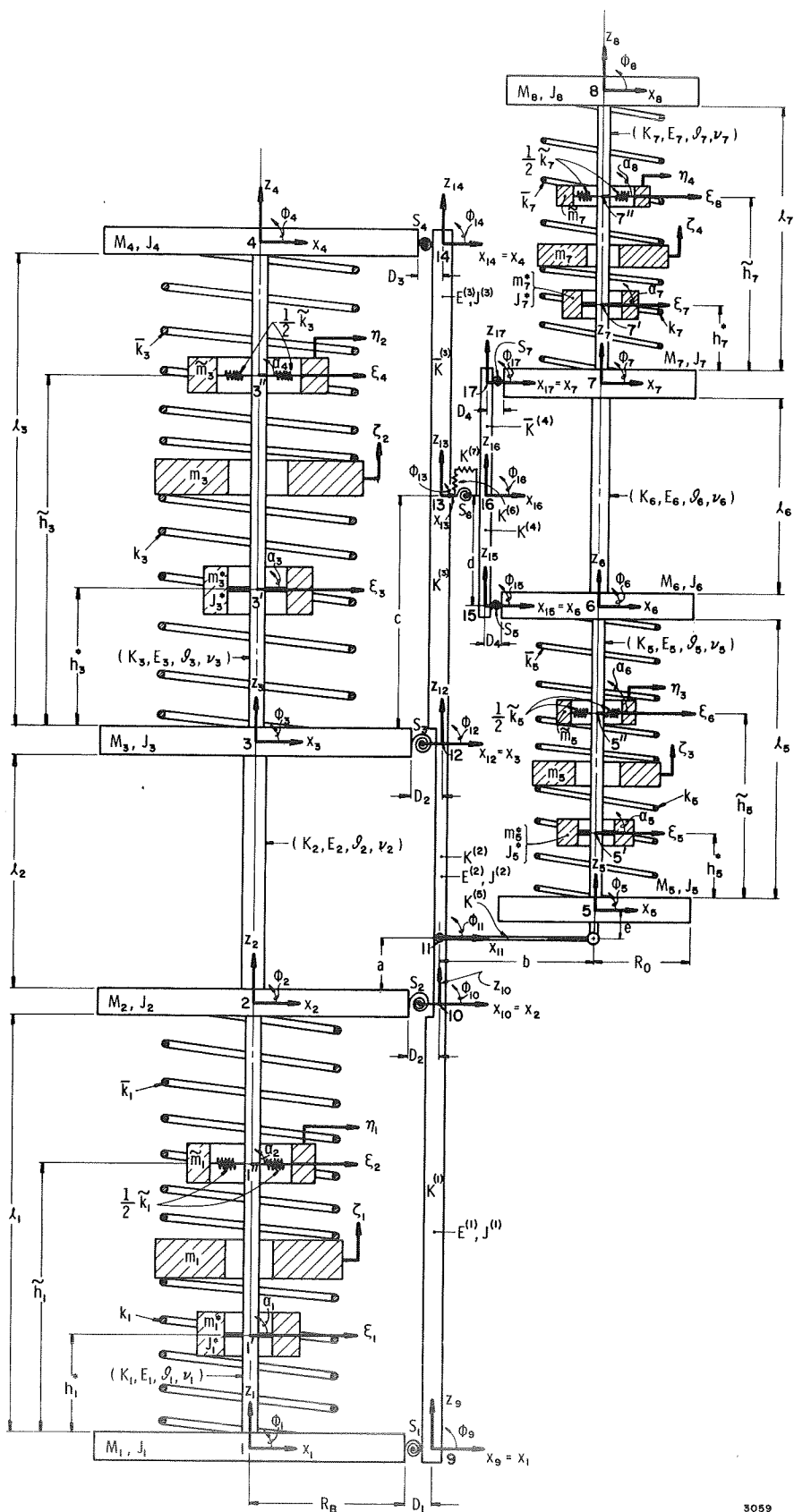
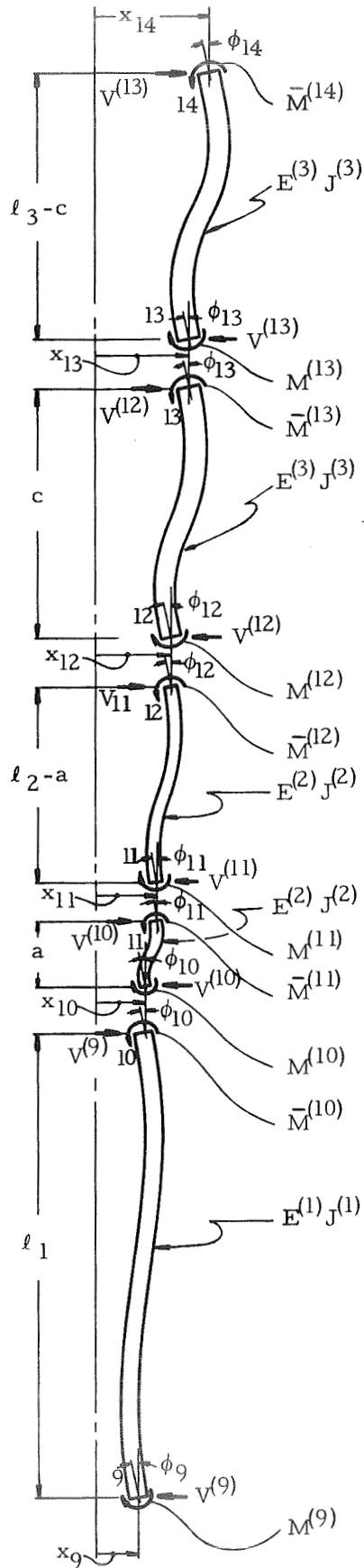


FIGURE 2a. -- EQUIVALENT MECHANICAL MODEL



$$\bar{M}^{(14)} = \frac{2E^{(3)}J^{(3)}}{(l_3 - c)^2} [3(x_4 - x_{13}) + (l_3 - c)(2\phi_{14} + \phi_{13})]$$

$$V^{(13)} = \frac{6E^{(3)}J^{(3)}}{(l_3 - c)^3} [2(x_4 - x_{13}) + (l_3 - c)(\phi_{13} + \phi_{14})]$$

$$M^{(13)} = \frac{-2E^{(3)}J^{(3)}}{(l_3 - c)^2} [3(x_4 - x_{13}) + (l_3 - c)(2\phi_{13} + \phi_{14})]$$

$$\bar{M}^{(13)} = \frac{2E^{(3)}J^{(3)}}{c^2} [3(x_{13} - x_3) + c(2\phi_{13} + \phi_{12})]$$

$$V^{(12)} = \frac{6E^{(3)}J^{(3)}}{c^3} [2(x_{13} - x_3) + c(\phi_{12} + \phi_{13})]$$

$$M^{(12)} = \frac{-2E^{(3)}J^{(3)}}{c^2} [3(x_{13} - x_3) + c(2\phi_{12} + \phi_{13})]$$

$$\bar{M}^{(12)} = \frac{2E^{(2)}J^{(2)}}{(l_2 - a)^2} [3(x_3 - x_{11}) + (l_2 - a)(2\phi_{12} + \phi_{11})]$$

$$V^{(11)} = \frac{6E^{(2)}J^{(2)}}{(l_2 - a)^3} [2(x_3 - x_{11}) + (l_2 - a)(\phi_{11} + \phi_{12})]$$

$$M^{(11)} = \frac{-2E^{(2)}J^{(2)}}{(l_2 - a)^2} [3(x_3 - x_{11}) + (l_2 - a)(2\phi_{11} + \phi_{12})]$$

$$\bar{M}^{(11)} = \frac{2E^{(2)}J^{(2)}}{a^2} [3(x_{11} - x_2) + a(2\phi_{11} + \phi_{10})]$$

$$V^{(10)} = \frac{6E^{(2)}J^{(2)}}{a^3} [2(x_{11} - x_2) + a(\phi_{10} + \phi_{11})]$$

$$M^{(10)} = \frac{-2E^{(2)}J^{(2)}}{a^2} [3(x_{11} - x_2) + a(\phi_{11} + 2\phi_{10})]$$

$$\bar{M}^{(10)} = \frac{2E^{(1)}J^{(1)}}{l_1^2} [3(x_2 - x_1) + l_1(2\phi_{10} + \phi_9)]$$

$$V^{(9)} = \frac{6E^{(1)}J^{(1)}}{l_1^3} [2(x_2 - x_1) + l_1(\phi_9 + \phi_{10})]$$

$$M^{(9)} = \frac{-2E^{(1)}J^{(1)}}{l_1^2} [3(x_2 - x_1) + l_1(\phi_{10} + 2\phi_9)]$$

FIGURE 2d. --DETAIL OF BOOSTER REINFORCING BEAM

The vertical sloshing mass m_i and the sloshing spring constants k_i and $\bar{k}_i$ are defined as follows (ref. 3)

$$m_i = \frac{16}{\pi} \frac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_0\left(\frac{\pi R_i}{2h_i}\right)} m_T^{(i)} \quad ; \quad (i = 1, 3, 5, 7) \quad (1)$$

$$k_i = k_i' - \nu_i \bar{K}_i F_i + \nu_i^2 \bar{K}_i F_i^2 \quad ; \quad (\text{no summation}) \quad (2)$$

$$\bar{k}_i = \nu_i \bar{K}_i F_i \quad (3)$$

where

I_1, I_0 = modified Bessel functions of the first kind

R_i = R_B (Booster radius) for $i = 1, 3$, = R_O (Orbiter radius) for $i = 5, 7$

h_i = height of the liquid in cylinder i

$m_T^{(i)}$ = $\pi R_i^2 h_i \rho_i$ = total mass of the liquid in the cylinder

ρ_i = mass density of the liquid

$$k_i' = \frac{4\pi R_i \delta_i E_i}{h_i} \left[\frac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_0\left(\frac{\pi R_i}{2h_i}\right)} \right]^2 \quad (4)$$

ν_i = Poissons ratio of the cylindrical shell

$$\bar{K}_i = \frac{2\pi R_i \delta_i E_i}{\ell_i - \nu_i^2 h_i} \quad (5)$$

$$F_i = \frac{4}{\pi} \frac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_0\left(\frac{\pi R_i}{2h_i}\right)} \quad (6)$$

ℓ_i = length of the cylinder

δ_i = cylinder wall thickness

E_i = Young's modulus of the cylinder shell

The lateral sloshing mass for the first mode or fundamental slosh mass $\tilde{m}_i$, and its height $\tilde{h}_i$, the rigid mass m_i^* and its height h_i^* and the sloshing spring constant are defined in the following (refs. 4, 5, 6)

$$\tilde{m}_i = m_T^{(i)} \left(\frac{R_i}{2.2h_i} \right) \tanh 1.84 \frac{h_i}{R_i} \quad (7)$$

$$m_i^* = m_T^{(i)} - \tilde{m}_i \quad (8)$$

$$\tilde{k}_i = m_T^{(i)} \left(\frac{g}{1.19h_i} \right) \left(\tanh 1.84 \frac{h_i}{R_i} \right)^2 \quad (9)$$

$$\tilde{h}_i = h_i - \frac{R_i}{0.92} \tanh 0.92 \frac{h_i}{R_i} \quad (10)$$

= height of $\tilde{m}_i$

$$h_i^* = h_i - \frac{m_T^{(i)} h_i}{2m_i^*} + (h_i - \tilde{h}_i) \frac{\tilde{m}_i}{m_i^*} \quad (11)$$

= the height of m_i^*

g = gravitational constant, and the polar moment of inertia of m_i^* is defined as

$$J_i^* = J_{\text{rigid}}^{(i)} + m_T^{(i)} \frac{h_i^2}{4} - \frac{m_T^{(i)} R_i^2}{2} \left[1.995 - \frac{2.14 R_i}{h_i} \tanh 0.92 \frac{h_i}{R_i} \right] \\ - m_i^* (h_i - h_i^*)^2 - \tilde{m}_i (h_i - \tilde{h}_i)^2 \quad (12)$$

where

$$J_{\text{rigid}} = \frac{m_T^{(i)} R_i^2}{4} \left[\frac{1}{3} \frac{h_i^2}{R_i^2} + 1 \right] \quad (13)$$

Figures 2a and 2b show that $\tilde{m}_i$ is situated above m_i^* ; however, as the depth of the liquid h_i decreases, $\tilde{m}_i$ will shift to a position below m_i^* .

Mathematical Formulation

The free dynamic behavior of the mechanical model can be described completely in mathematical terms by writing the equations of motion and equations of constraint for the system. In order to do this, the mechanical model is divided into subsystems for which equations of dynamic and static equilibrium for the free oscillation can easily be set up. The coordinate system at each subsystem is shown in Figure 2a.

In the model, the structure mass elements M_n ($n = 1, 2, \dots, 8$) will have three degrees of freedom in motion (two translations and one rotation) the rigid mass m_i^* ($i = 1, 3, 5, 7$) two degrees of freedom (one horizontal translation and one rotation), the lateral sloshing mass $\tilde{m}_i$ one degree of freedom (horizontal translation), and the vertical sloshing mass m_i one degree of freedom (vertical translation). Thus, the entire system will have forty degrees of freedom and therefore forty equations of motion. However, only equations associated with subsystem 1 will be shown for the purpose of illustration.

Equations of motion. -- At subsystem 1 there are two equations of motion for translation and one for rotation. They are

$$M_1 \ddot{x}_1 - V_1^{(1)} - V^{(9)} = 0 \quad (14)$$

$$\begin{aligned} M_1 \ddot{z}_1 - K^{(1)} [(z_2 - z_1) + (R_B + D_1)(\phi_2 - \phi_1)] - K_1(z_2 - z_1) \\ - k_1(\xi_1 - z_1) = 0 \end{aligned} \quad (15)$$

$$\begin{aligned} J_1 \ddot{\phi}_1 - K^{(1)}(R_B + D_1) [(z_2 - z_1) + (R_B + D_1)(\phi_2 - \phi_1)] \\ + S_1(\phi_1 - \phi_9) - M^{(1)} = 0 \end{aligned} \quad (16)$$

where

D_1 = distance between the neutral axis of the Booster strongback beam section 9-10 and the Booster shell

K_1 = longitudinal spring constant of cylinder 1

$K^{(1)}$ = longitudinal spring constant of beam section 9-10

J_1 = polar moment of inertia of mass M_1 about y_1 axis

S_1 = torsional spring constant at joint 9

$$M_1^{(1)} = - \frac{2E_1 \mathcal{J}_1}{(h_1^*)^2 N_{11}} [3(\xi_1 - x_1) + h_1^* (2A_{11}\phi_1 + B_{11}a_1)] \quad (17)$$

= net moment at lower end of shell-beam 1

$$V^{(9)} = \frac{6E^{(1)}J^{(1)}}{\ell_1^3} [2(x_2 - x_1) + \ell_1(\phi_9 + \phi_{10})] \quad (18)$$

= end shearing force of the Booster reinforcing beam section at end 9 (ref. 7)

$$V_1^{(1)} = \frac{6E_1 \mathcal{J}_1}{(h_1^*)^3 N_{11}} [2(\xi_1 - x_1) + h_1^* (\phi_1 + a_1)] \quad (19)$$

= net shearing force at lower end of shell-beam 1 (see Appendix A)

$E^{(1)}$ = Young's modulus of the reinforcing beam section 9-10

$J^{(1)}$ = moment of inertia of the cross section of the reinforcing beam section 9-10

$\mathcal{J}_1$ = moment of inertia of the cylinder cross section

and

$$N_{11} = 1 + \frac{12E_1 \mathcal{J}_1}{(h_1^*)^2 \pi R_B \delta_1 G_1}, \quad A_{11} = 1 + \frac{3E_1 \mathcal{J}_1}{(h_1^*)^2 \pi R_B \delta_1 G_1},$$

$$B = 1 - \frac{6E_1 \mathcal{J}_1}{(h_1^*)^2 \pi R_B \delta_1 G_1} \quad (20)$$

in which

G_1 = shear modulus

Equations (20) provide correction factors which arise from the analysis based on linear membrane theory of the shell. These factors are unity for an ordinary beam (see Appendix A). Similar equations of motion may be written for the rest of subsystems.

Equations of constraint. --As shown in Figure 2a, at each massless subsystem of 9, 10, 12, 14, 15, and 17 there will be one constraint condition in the ϕ direction; at subsystem 11 there are two constraint conditions (x and ϕ directions); at subsystems 13 and 16 there are three constraint conditions (x , z , ϕ directions) for each subsystem; at subsystem i ($i = 1, 3, 5, 7$) there are two constraint conditions (α , ξ directions). Thus there will be twenty-two constraint conditions associated with the model. For the purpose of illustration only the constraint condition at the massless subsystem 9 will be shown below.

$$0 \times \ddot{\phi}_9 - S_1(\phi_1 - \phi_9) - M^{(9)} = 0$$

where

$$M^{(9)} = - \frac{2E^{(1)} J^{(1)}}{\ell_1^2} [3(x_2 - x_1) + \ell_1(2\phi_9 + \phi_{10})] \quad (21)$$

= end moment of the Booster reinforcing beam
section at end 9

Similar constraint equations may be written for the rest of massless nodes.

Eigenvalue problem. -- Since only free oscillations are being considered, the aforementioned system of H equations of motion and constraint is homogeneous. Taken together, the total system of dynamic and constraint equations can be written in compact notation (employing the summation convention) as

$$P_{rs}\ddot{W}_s + Q_{rs}W_s = 0 \quad r, s = 1, 2, \dots, H \quad (22)$$

It is assumed that the vibration modes are harmonic, i.e.,

$$W_s = Y_s \cos(\omega t + \gamma) \quad (23)$$

where each frequency ω is real. On substituting Eq. (23) into Eq. (22) one finds

$$(Q_{rs} - \omega^2 P_{rs})(Y_s) = 0 \quad (24)$$

There is a well-known principle (ref. 7) in vibration theory which states that a system of H equations consisting of L dynamic and L' constraint equations ($L + L' = H$) can always be reduced to a problem involving L equations. This can be accomplished by incorporating the constraint equations into the equations of motion. In the present problem, this can be done by segregating the set of generalized coordinates $\underline{W}$ into two nonintersecting sets:

$$\underline{W} = \underline{X} \cup \underline{x}$$

where

$\underline{X}$ = set of coordinates associated with mass elements

$\underline{x}$ = set of coordinates associated with constraint conditions

For the present problem, $\underline{X}$ and $\underline{x}$ are defined in Table III. Equations (24) can now be replaced by two sets of equations, the first of which gives the L equations of motion expressed in terms of the constraint coordinates $\underline{x}$:

$$(q_{ij} - \Omega^2 p_{ij})(X_j) = -a_{in}x_n \quad i, j = 1, 2, \dots, L$$

$$n = 1, 2, \dots, L' \quad (25)$$

The second set of conditions relates the $\underline{x}$ coordinates to the $\underline{X}$ coordinates:

$$c_{lm}x_m = d_{lj}X_j \quad l, m = 1, 2, \dots, L' \quad (26)$$

TABLE III. --DEFINITION OF COORDINATES

<u>Coordinates Associated with Mass Elements</u>		<u>Coordinates Associated with Constraint Conditions</u>	
$X_1 = z_1$	$X_{21} = x_5$	$x_1 = z_{13}$	$x_{12} = x_{11}$
$X_2 = z_2$	$X_{22} = x_6$	$x_2 = z_{16}$	$x_{13} = x_{13}$
$X_3 = z_3$	$X_{23} = x_7$	$x_3 = \phi_9$	$x_{14} = x_{16}$
$X_4 = z_4$	$X_{24} = x_8$	$x_4 = \phi_{10}$	$x_{15} = a_6$
$X_5 = z_5$	$X_{25} = \zeta_3$	$x_5 = \phi_{11}$	$x_{16} = a_8$
$X_6 = z_6$	$X_{26} = \zeta_4$	$x_6 = \phi_{12}$	$x_{17} = \xi_6$
$X_7 = z_7$	$X_{27} = a_5$	$x_7 = \phi_{13}$	$x_{18} = \xi_8$
$X_8 = z_8$	$X_{28} = a_7$	$x_8 = \phi_{14}$	$x_{19} = a_2$
$X_9 = \phi_1$	$X_{29} = \xi_5$	$x_9 = \phi_{15}$	$x_{20} = a_4$
$X_{10} = \phi_2$	$X_{30} = \xi_7$	$x_{10} = \phi_{16}$	$x_{21} = \xi_2$
$X_{11} = \phi_3$	$X_{31} = \eta_3$	$x_{11} = \phi_{17}$	$x_{22} = \xi_4$
$X_{12} = \phi_4$	$X_{32} = \eta_4$		
$X_{13} = \phi_5$	$X_{33} = \zeta_1$		
$X_{14} = \phi_6$	$X_{34} = \zeta_2$		
$X_{15} = \phi_7$	$X_{35} = a_1$		
$X_{16} = \phi_8$	$X_{36} = a_3$		
$X_{17} = x_1$	$X_{37} = \xi_1$		
$X_{18} = x_2$	$X_{38} = \xi_3$		
$X_{19} = x_3$	$X_{39} = \eta_1$		
$X_{20} = x_4$	$X_{40} = \eta_2$		

This system of equations can be written such that the square matrix $\underline{c}$ is symmetric and that $\underline{d} = -\underline{a}^T$. Solving for $\underline{x}$:

$$x_n = c_{nl}^{-1} d_{lj} X_j \quad (27)$$

and substituting into Eq. (25):

$$\begin{aligned} (q_{ij} - \Omega^2 p_{ij})(X_j) &= -a_{in} c_{nl}^{-1} d_{lj} X_j \\ &= a_{in} c_{nl}^{-1} a_{lj}^T X_j \end{aligned} \quad (28)$$

Thus:

$$[(q_{ij} - a_{in} c_{nl}^{-1} a_{lj}^T) - \Omega^2 p_{ij}](X_j) = 0 \quad (29)$$

By premultiplying by p^{-1} this equation takes the form of a standard eigenvalue problem

$$[p_{ki}^{-1}(q_{ij} - a_{in} c_{nl}^{-1} a_{lj}^T) - \Omega^2 \delta_{kj}](X_j) = 0, \quad k = 1, 2, \dots, L \quad (30)$$

Nontrivial solutions for the eigenvector $\underline{X}$ exist if, and only if, the determinant of the coefficient matrix vanishes:

$$\left| p_{ki}^{-1}(q_{ij} - a_{in} c_{nl}^{-1} a_{lj}^T) - \Omega^2 \delta_{kj} \right| = 0 \quad (31)$$

Thus, the problem reduces to finding the eigenvalues of the $L \times L$ matrix $p^{-1}(q - \underline{a} \underline{c}^{-1} \underline{a}^T)$. The eigenvalues found by this process are the natural frequencies of the Booster/Orbiter system, expressed in radians/second. Included in this set of frequencies are the zero frequencies identified with translation and rotation of the system as a rigid body.

In the present problem, the matrix $\underline{q}$ and the matrix $\underline{a} \underline{c}^{-1} \underline{a}^T$ both are symmetric, and the matrix $\underline{p}$ is diagonal (and, hence, $\underline{p}^{-1}$ is diagonal). However, the product matrix $\underline{p}^{-1}(\underline{q} - \underline{a} \underline{c}^{-1} \underline{a}^T)$ is not symmetric. Mathematically, the eigenvalues of a real, nonsymmetric matrix may be complex, all or in part. On physical grounds, however, one knows that the mathematical model being solved represents a linear conservative system, and therefore a correct solution must result in real eigenvalues. Complex eigenvalues introduce growth and decay characteristics in the modal response which are inadmissible for the free vibrations of the problem under consideration. In the numerical solution for the present problem an eigenvalue routine was used

which computes the complex eigenvalues of a nonsymmetric matrix. All non-trivial eigenvalues, however, were found to be real.

The nonzero elements of the matrices $\underline{q}$, $\underline{a}$, and $\underline{c}$ are tabulated in Appendix B for three different cases: $\tilde{h}_i > h_i^*$, $\tilde{h}_i < h_i^*$ and the empty case.

Numerical Aspects

The physical quantities which are involved in the equations of motion and constraint for the analytical model consist of parameters which can be computed directly (masses, moments of inertia, etc.) and of terms which do not lend themselves to direct computation (effective* spring constants). These latter terms can be estimated from force-displacement calculations based on idealized models, but these estimates by no means serve as valid input data for computational purposes. Effective spring constants, in some cases, also can be determined experimentally by impedance techniques. The general approach used in this program for determining the input parameters for the analytical models was first to calculate by some means (quite approximately in some cases) all of the input data required of the model. Following this, certain of the parameters, least amenable to accurate calculation, were adjusted within certain narrow bounds in an attempt to match the frequencies computed theoretically with the experimental values for the empty tank case. This procedure is not the same as "curve fitting" where one takes much greater liberty with the number of parameters varied and disregards theoretical estimates on their magnitudes. The approach adopted here limits the amount of empiricism to a practical minimum in fixing the input data.

In the case of the Booster K_1 and K_2 , the effective axial spring constants of the thin-walled shells were found experimentally by an axial vibration test, as described in the next section. The axial spring constants characterising the strongback were adjusted from their calculated values to match the empty, decoupled booster data. For the Orbiter, the axial spring constant of cylinder 5 was calculated, and a correction factor (a corrected shell thickness δ_5) introduced to force agreement with the experimental result for cylinder 5 alone in both axial and bending motion. This procedure was repeated using cylinders 5 and 6 together, to determine K_6 . For cylinder 7, δ_7 was taken equal to δ_5 . The parameters finally arrived at are listed. Once so determined, they were held constant throughout all computational work.

*Note that effective spring constants include effects difficult to predict, such as bolted joint compliances and longitudinal stiffeners.

$$\begin{aligned}
K_1 &= 3.48 \times 10^5 && \text{lb/in.} \\
K_2 &= 7.36 \times 10^5 && \text{lb/in.} \\
K^{(1)} &= 1.8333 \times 10^5 && \text{lb/in.} \\
K^{(2)} &= 5.3571 \times 10^5 && \text{lb/in.} \\
K^{(3)} &= 3.4921 \times 10^5 && \text{lb/in.} \\
K^{(4)} &= 1.2083 \times 10^6 && \text{lb/in.} \\
\overline{K}^{(3)} &= 3.8596 \times 10^5 && \text{lb/in.} \\
\overline{K}^{(4)} &= 1.2083 \times 10^6 && \text{lb/in.} \\
\delta_5 &= \delta_7 = 9.84 \times 10^{-3} && \text{in.} \\
K_6 &= 1.885 \times 10^5 && \text{lb/in.} \\
S_5 &= S_7 = 2.60 \times 10^4 && \text{in. -lb/rad}
\end{aligned}$$

In the case of the coupled Booster/Orbiter system three additional spring constants were available for adjustment, within limits, to match the theoretical and experimental coupled empty tank condition. These three constants represented the torsional coupling spring S_6 , the vertical coupling spring $K^{(6)}$, and the horizontal coupling spring $K^{(7)}$. All three of these spring constants were first calculated on the basis of idealized models, to serve as nominal values in the adjustment process. The values finally chosen were:

$$\begin{aligned}
S_6 &= 3.9027 \times 10^4 && \text{in. -lb/rad} \\
K^{(6)} &= 2.677 \times 10^5 && \text{lb/in.} \\
K^{(7)} &= 9.9672 \times 10^3 && \text{lb/in.}
\end{aligned}$$

The calculations for the empty-tank condition omit the slosh models from the system of equations, and do not represent simply a degenerate case of vanishingly small liquid levels. The general system of equations, which does include the sloshing models, must predict frequencies compatible with the empty-tank results for small, but nonzero liquid levels. This criterion serves as a checkpoint on the accuracy of the numerical program. Also, a transition point occurs at a liquid level of 1.043 times the tank diameter, below which $\tilde{h}_1^*$, the location of the rigid mass, reverses its relative position with $\tilde{h}_1$, the location of the sloshing mass. On either side of this transition

certain equations must be rewritten in a different form, with the result that the program differs according to whether one is considering a "near full" or a "near empty" tank. Continuity in the computed frequencies, of course, must be maintained across this transition point, and this criterion serves as an additional check on the programming accuracy.

The matrix eigenvalue problem was solved on a CDC-6400 computer using a standard eigenvalue routine (modified Jacobi method) for finding the eigenvalues of a real, nonsymmetric matrix. A listing of the computer program and instructions for use are given in Appendix C.

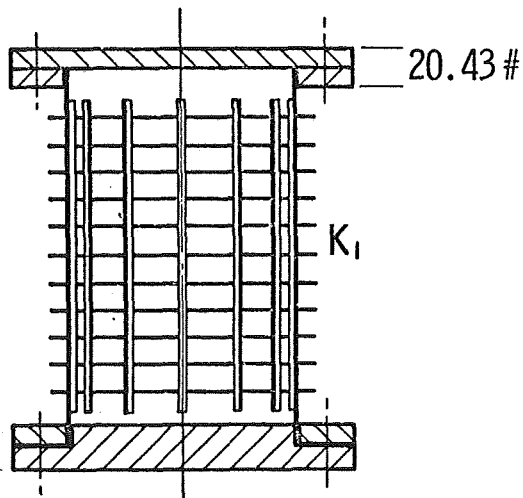
EXPERIMENTAL PROCEDURE

The test program performed on the previously described physical model can be divided into distinctive phases and may be listed as the determination of:

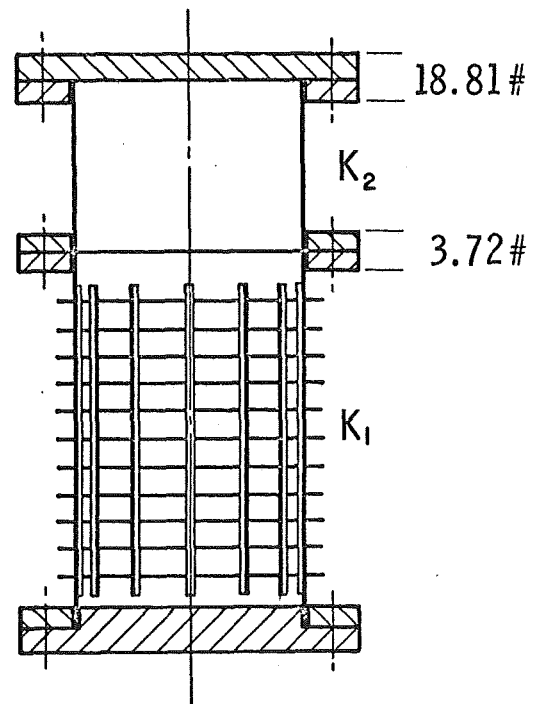
- (1) Effective spring constants for subassemblies
- (2) Natural bending frequencies of decoupled Booster and Orbiter models
- (3) Natural frequencies of decoupled Booster and Orbiter models with longitudinal excitation
- (4) Natural frequencies of Shuttle Vehicle Model.

The first part of the experimental program was to determine the effective spring constants of the Booster model components. This was accomplished by mounting the intermediate empty configurations illustrated in Figure 3 on an electrodynamic shaker and determining the natural frequencies of the components. Frequencies obtained by this test were used to calculate the effective spring constants. Similar procedure was used with the Orbiter model components, both for axial and lateral excitation. The resulting spring constants were tabulated in the previous section for both models.

The second phase of the test program was devoted to determining the natural bending frequencies of the decoupled models. The Booster model was vertically suspended by a nylon-rope, pulley, and spring combination which was designed to simulate a free-free condition. The rope was attached to the model at its bottom flange on the lower tank and was guided at the top of the skirt. A small electrodynamic shaker connected to the model at the



a) LOWER TANK



b) LOWER TANK and SKIRT

FIGURE 3. --INTERMEDIATE EMPTY CONFIGURATIONS OF BOOSTER MODEL

upper flange of the skirt was used to excite the model in a lateral direction. Four piezoelectric accelerometers were mounted on the model, one on each flange and one on top with their axes in line with the direction of excitation, and monitored simultaneously. The model was tested with empty, full, and intermediate liquid conditions and its natural frequencies were recorded. The information so obtained served a dual purpose; it provided data for comparison with the frequencies obtained by an analytical model discussed in the preceding section, and also allowed for the identification of bending modes when the model was later tested in its coupled configuration. This test procedure was also repeated using the Orbiter model.

To determine the natural frequencies of the Booster and Orbiter models with longitudinal excitation, the models were suspended as previously described and excited along their vertical axes by a small electrodynamic shaker. Four piezoelectric accelerometers, mounted on the flanges and the top of the models with their axes along the direction of excitation, were monitored together with pressure transducers installed in the center of each bulkhead. As in the preceding phase, each model was tested with empty, full, and intermediate liquid conditions.

The final step in the test program was the determination of natural frequencies of the complete shuttle vehicle model shown in Figure 1. The system was suspended in such a manner so that the driving force introduced to the model by the electrodynamic shaker always acted through the gravitational center of the model. Four piezoelectric accelerometers, located on the bottom flanges and the top of the models, were monitored measuring acceleration along the axes of the models, while two others were located at the tops recording acceleration in the lateral direction. In addition, four pressure transducers, one located in each bulkhead, were monitored. All tests were performed with the Orbiter tanks full. The liquid level in the Booster tanks was varied from empty to full with intermediate conditions. Thus, a normal operational sequence was simulated.

Throughout the entire test program, distilled water was used as a modeling liquid propellant. Ullage pressure was provided in all tanks to raise the natural frequencies of nonsymmetric shell modes above the frequency range used during the tests.

As a conclusion to the experimental program, the modeling liquid was replaced in the tanks by a granular substance with bulk density very closely equal to the modeling liquid and the model was tested at full and half-full levels in the Booster tanks, and similar levels in the Orbiter tanks. This substitution was implemented to facilitate identification of liquid and structural modes in the data obtained from tests completed on the coupled system, as well as to show more vividly the effects of liquid propellants.

COMPARISON OF RESULTS

The output from the theoretical model was in the form of natural frequencies representing:

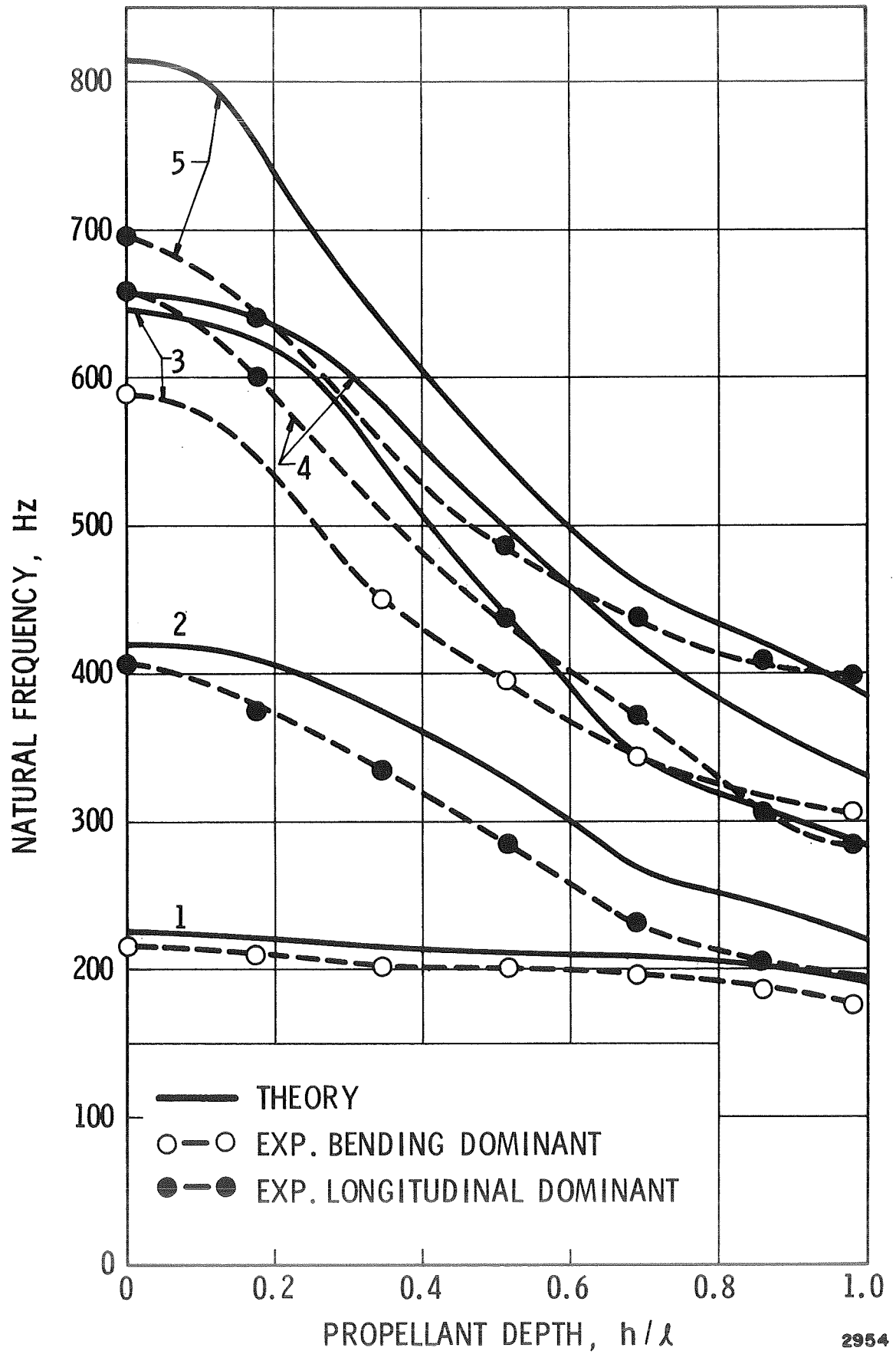
- (1) Rigid body motion of the system (zero frequencies)
- (2) Sloshing frequencies of the four liquid-containing tanks
- (3) Frequencies of the system.

Regarding the rigid body modes, the analytical model predicts three zero frequencies for the coupled Booster/Orbiter system, and six when the Booster/Orbiter coupling is set to zero. This prediction is consistent with rigid body motion in a plane. The four calculated sloshing frequencies were in the range of 1 to 3 Hz and are quite small when compared with system frequencies. Thus, the sloshing modes are essentially decoupled from the natural frequencies of the total system.

Figures 4, 5, and 6 present a comparison of the theoretically-predicted with the experimentally-determined results. Figure 4 shows the first five frequencies of the Booster alone, and Figure 5 shows the first three frequencies of the Orbiter alone. The analytical and experimental frequency values are quantitatively compared in Table IV. This agreement index was based upon the maximum (absolute value) percentage error between the theoretical and the experimental values, with the experimental values taken as the basis. The agreement is considered good if this error is within 10%, fair if between 10% and 20%, and poor if greater than 20%.

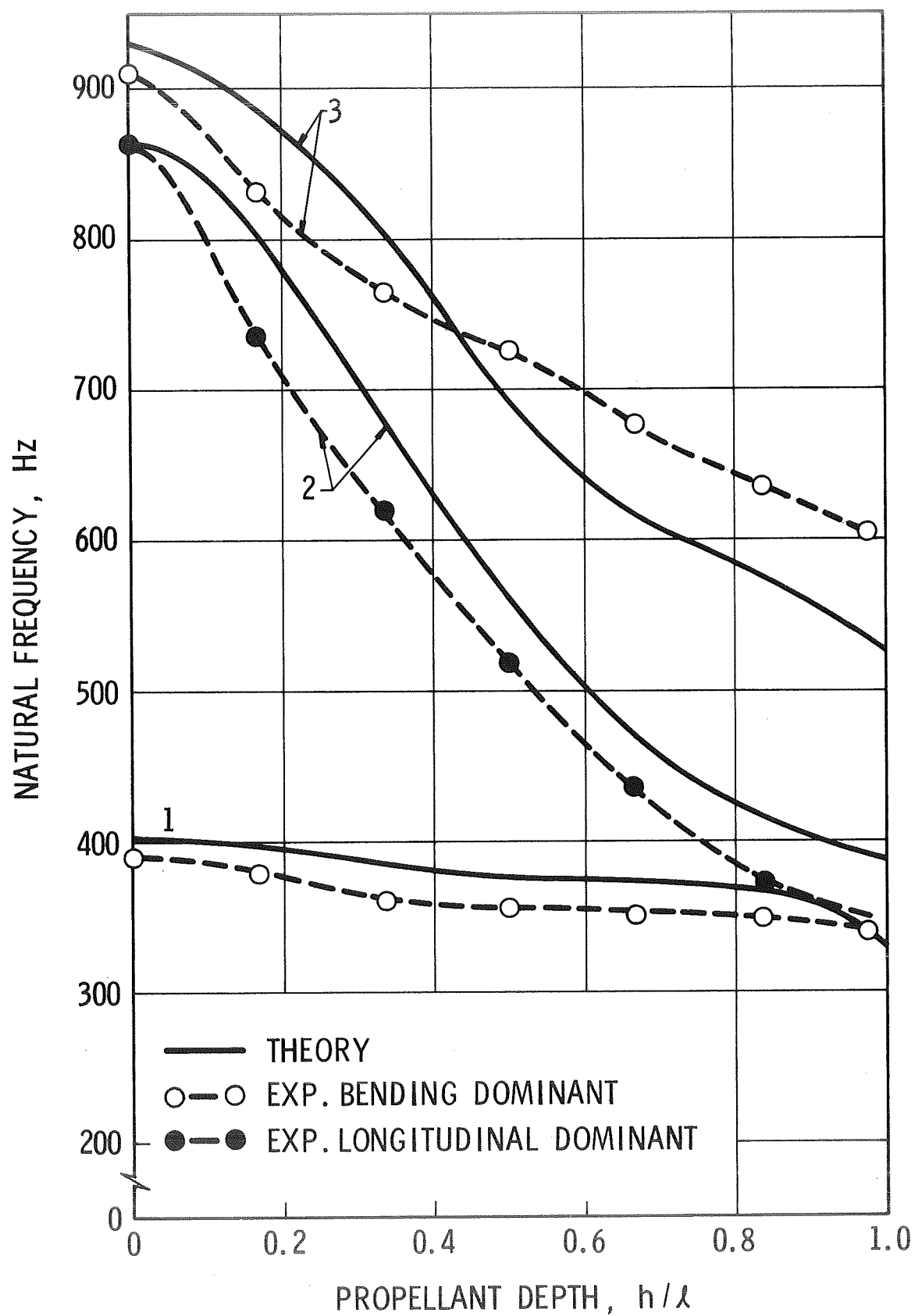
It is seen from this comparison, and from Figures 4 and 5, that the agreement between theory and experiment is generally better in the case of bending modes than in the longitudinal modes. There is a particularly significant lack of agreement in the second and third longitudinal Booster modes. Also, the theoretical frequencies tend to be somewhat higher than the experimental values (except in the case of the second Orbiter bending mode, where the theoretical and experimental curves cross each other).

Table V.A compares the theoretical and experimental frequencies for the coupled empty-tank condition. In terms of the comparison index mentioned above, the agreement is good for the first six modes, and poor for the next three higher modes. It is obvious that above the sixth mode, the relatively simple analytical model is no longer adequate to describe the motion of the system.



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FIGURE 4. --UNCOUPLD BOOSTER NATURAL FREQUENCIES



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FIGURE 5. --UNCOUPLD ORBITER NATURAL FREQUENCIES

TABLE IV. --AGREEMENT INDEX FOR DECOUPLED MODELS

<u>BOOSTER</u>		
<u>Mode Number</u>	<u>Mode Form</u>	<u>Agreement</u>
1	1st bending	Good
2	1st longitudinal	Fair
3	2nd bending	Good
4	2nd longitudinal	Poor
5	3rd longitudinal	Poor

<u>ORBITER</u>		
<u>Mode Number</u>	<u>Mode Form</u>	<u>Agreement</u>
1	1st bending	Good
2	1st longitudinal	Good
3	2nd bending	Good

Figure 6 compares the theoretical and experimental frequencies for the coupled Booster/Orbiter system for full Orbiter and various Booster propellant levels. The two lowest modes, approximately constant at 15 Hz and 70 Hz, represent modes in which the Booster and Orbiter act essentially as rigid bodies, but vibrate relative to each other through the torsional coupling spring and the lateral coupling spring, respectively. The third mode is dominantly Booster bending, while the fourth mode is the remaining rigid body mode, with relative Booster/Orbiter motion resisted through the longitudinal coupling spring. The fifth and sixth modes exchange motions of dominantly Orbiter bending and Booster longitudinal motion. Above this, the discrepancies become quite large.

Additional results are shown in Tables V.B and V.C where frequencies are given for the case of a solid-like fluid. A mixture of soil and flour was used to produce a substance having a bulk specific gravity of 1.0. Only the first two modes remained relatively unaltered, while most higher modes disappeared. Apparently, considerably more damping was displayed by this mixture than experienced with water. Thus, in this case a better study of the effects of liquid rather than solid propellant simulation could have been obtained from the analytical model.

In Tables V.D and V.E, results are given for alternate positions of the Orbiter on the Booster, when all tanks were full.* In these cases the difference was surprisingly small. Other such results at various liquid levels would be highly desirable.

It appears reasonable to conclude that the liquid propellant models, as derived, provide only a fair overall prediction of frequencies for the decoupled models and a somewhat better prediction for the coupled case. However, possibilities for refinement of the models can immediately be considered. For example, the use of additional modes for the longitudinal liquid model could very likely improve the results at the higher frequencies. A better estimation of joint compliances in the structural model would also help.

Finally, Figure 7 shows an instability that occurs in the system for both Orbiter and Booster full and excitation through the system center of gravity at 453 Hz. The oscilloscope traces show only the pulsating envelope of the high frequency responses. The origin or cause of this type of instability remains to be investigated. However, inspection of the liquid surfaces showed no apparent slosh coupling with the low frequency pulsation, even though it was near the frequency for those modes.

*Because of the coupling design, it was subsequently determined that these results include the effects of the indicated amount of Orbiter position change, as well as effects of an undetermined amount of variation in the coupling springs $K^{(6)}$, $K^{(7)}$, and S_6 .

TABLE V. --NATURAL FREQUENCIES FOR SPACE
SHUTTLE VEHICLE MODEL

A. All Tanks Empty
(Orbiter Position $c = 7.87$ in.)

<u>Experimental (Hz)</u>	<u>Theoretical (Hz)</u>		<u>Mode</u>
23.0	23.2	Good	Torsional Coupling
109	105	Good	Lateral Coupling
212	221	Good	Booster Bending
343	355	Good	Longitudinal Coupling
402	417	Good	Orbiter Bending
431	463	Good	Booster-Orbiter Bending
471	649	Poor	
546	659	Poor	
597	815	Poor	
708	863	Poor	

B. Booster and Orbiter $h/\ell = 0.983$ - Granular Propellant
(Orbiter Position $c = 7.87$ in.)

<u>Experimental (Hz)</u>	<u>Mode</u>
14.7	Torsional Coupling
60.0	Lateral Coupling
436	
845	

C. Booster and Orbiter $h/\ell = 0.517$ - Granular Propellant
(Orbiter Position $c = 7.87$ in.)

<u>Experimental (Hz)</u>	<u>Mode</u>
14.7	Torsional Coupling
64.0	Lateral Coupling
420	
869	

TABLE V. --NATURAL FREQUENCIES FOR SPACE
SHUTTLE VEHICLE MODEL (Cont'd)

D. All Tanks Full
(Orbiter Position $c = 4.06$ in.)

<u>Experimental (Hz)</u>	<u>Mode</u>
15.5	Torsional Coupling
121	Lateral Coupling
171	Booster Bending
191	Booster Longitudinal
216	
300	
310	
416	
458	
476	

E. All Tanks Full
(Orbiter Position $c = 11.69$ in.)

<u>Experimental (Hz)</u>	<u>Mode</u>
15.2	Torsional Coupling
124	Lateral Coupling
173	Booster Bending
190	Booster Longitudinal
221	
299	
312	

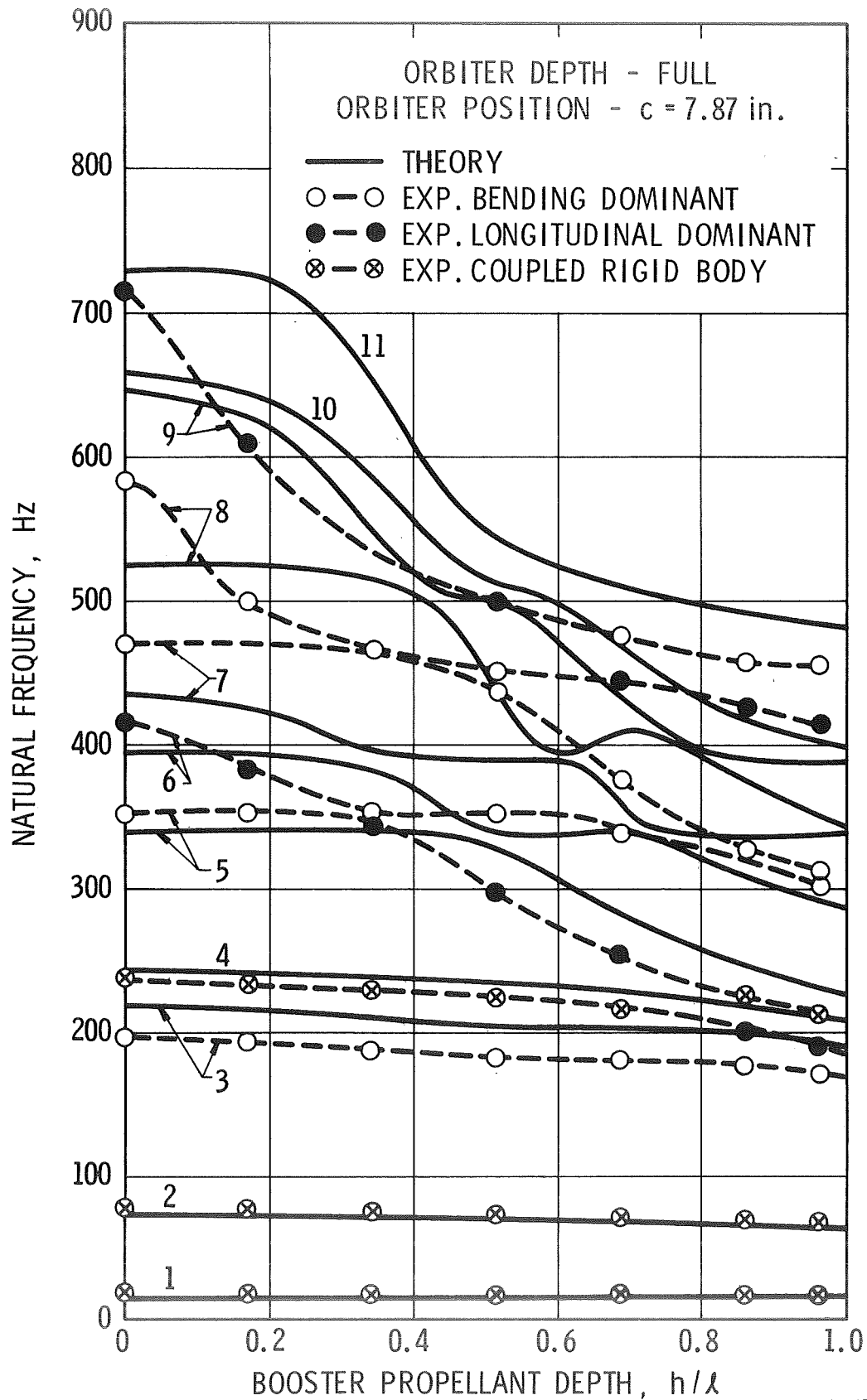


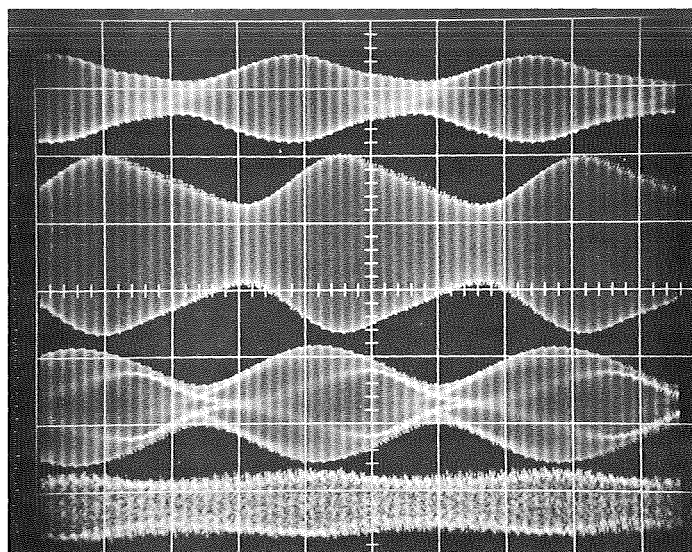
FIGURE 6. --NATURAL FREQUENCIES FOR SPACE SHUTTLE VEHICLE MODEL

Top Booster
Longitudinal
Acceleration

Top Booster
Lateral
Acceleration

Bottom Booster
Longitudinal
Acceleration

Bottom Booster
Pressure



1.4 g

2.8 g

1.8 g

0.1 psig

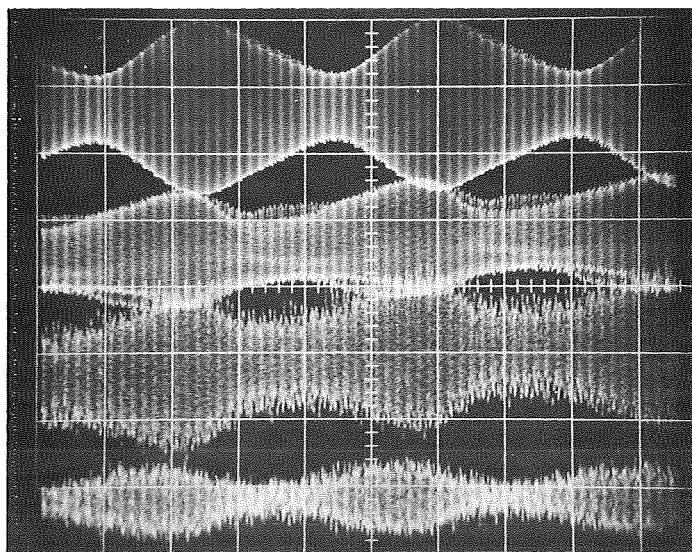
0.1 sec.

Top Booster
Lateral
Acceleration

Top Orbiter
Longitudinal
Acceleration

Top Orbiter
Lateral
Acceleration

Lower Orbiter
Pressure



2.8 g

0.5 g

0.5 g

0.1 psig

FIGURE 7. --SPACE SHUTTLE VEHICLE MODEL INSTABILITY

SUGGESTED FURTHER STUDIES

The results of this study indicate that a very effective, yet rather simple model of a typical space shuttle system has been developed, whereby many potential problems can be studied. Desirable steps to follow in this process are as follows:

- (1) Minor refinement of the analytical model is in order. This can be done by incorporating higher longitudinal liquid modes and getting better estimates of effective spring constants.
- (2) The analytical model should be used to compute results for a wide variety of parameters including other Booster-Orbiter liquid depth combinations, Orbiter positions, coupling springs, etc. The assumption of solid propellants can easily be made and results determined from the analytical model. Some experiments should be performed to verify select cases of the results.
- (3) Experimental transfer functions should be run between various response points and the excitation. A better description of coupling between longitudinal and lateral motions would result. Various gimbal angles on the excitation should be used along with this.
- (4) The present analytical model incorporates motion only in the plane of symmetry or system pitch plane. Experiments should be conducted to determine system response for both yaw and roll excitation. The derivation of an analytical model for these types of motion is also appropriate.
- (5) The origin of the pulsating instability should be identified and its significance explored.

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ACKNOWLEDGEMENTS

The authors wish to express their sincere appreciation to several individuals who contributed significant efforts toward the conduct of this program. Dr. H. Norman Abramson provided overall consultation, Mr. Luis Garza did much of the experimental apparatus design, Mr. Herbert G. Pennick performed the computer programming and computations, and Mr. Dennis C. Scheidt and Mr. George W. Downey performed the experiments.

APPENDIX A
REVISED BEAM EQUATIONS

END MOMENT AND SHEAR EQUATIONS FOR BEAM-LIKE THIN-WALLED CIRCULAR TUBES

Based on the linear membrane theory, the governing equations for beam-like, thin-walled circular cylinder following the sign convention shown in Figure A-I(a) are given by (ref. 8)

$$\frac{dV}{dz} = 0 \quad (\text{A.1})$$

$$\frac{dM}{dz} = V \quad (\text{A.2})$$

$$\frac{d\phi}{dz} = \frac{M}{E_i \mathcal{I}_i} \quad (\text{A.3})$$

$$\frac{dx}{dz} = -\phi + \frac{V}{\pi R_i \delta_i G_i} \quad (\text{A.4})$$

where

V = net shearing force on a cross section

M = net moment on a cross section

ϕ = measure of net rotation

E_i = modulus of elasticity

G_i = shear modulus

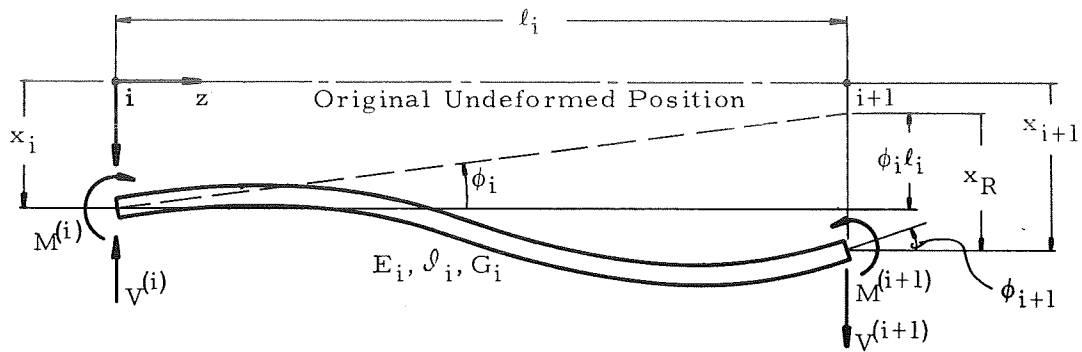
$\mathcal{I}_i$ = moment of inertia of the cylinder cross section

R_i = radius of the cylinder

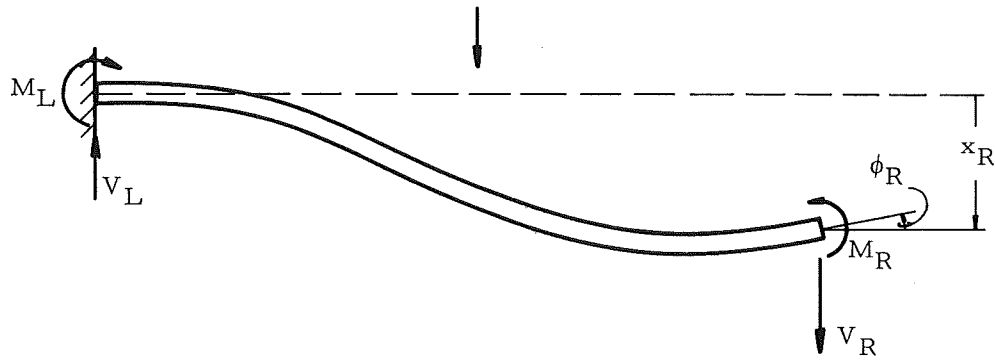
δ_i = thickness of cylinder wall

x = displacement in x-direction

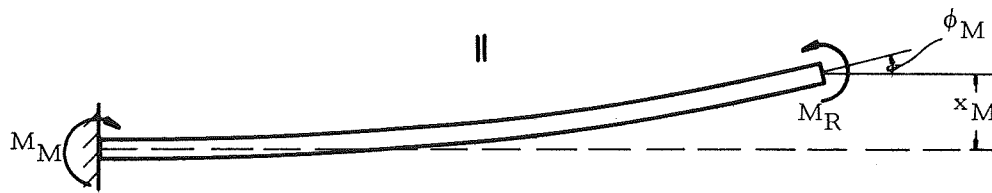
The general deformation of a beam-like thin-walled cylinder subjected to end moments $M^{(i)}$, $M^{(i+1)}$ and end shear $V^{(i)}$, $V^{(i+1)}$ [see Figure A-I(a)] may be represented by an equivalent cantilever beam subjected to equivalent end moment M_R and end shear V_R [see Figure A-I(b)], with the conditions:



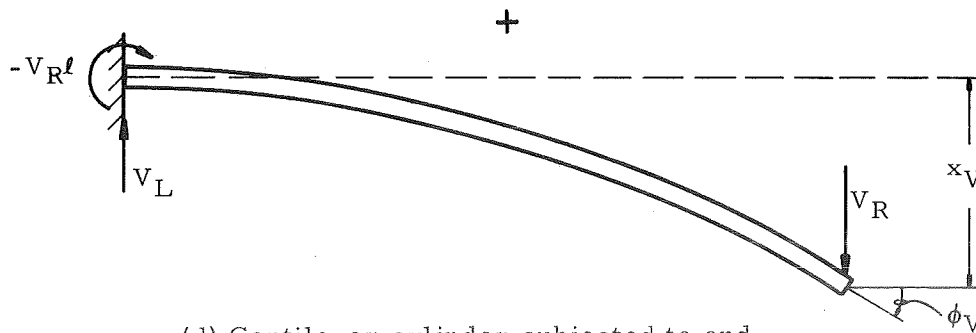
(a) Free body diagram of deformed (net) cylindrical shell section between stations (i-1) and i.



(b) Equivalent cantilever cylinder subjected to end moment and shear



(c) Cantilever cylinder subjected to end moment



(d) Cantilever cylinder subjected to end shearing force

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FIGURE A.1. --SIGN CONVENTION FOR BENDING LOADS

$$x_R = x_{i+1} - x_i + \ell_i \phi_i \quad (\text{A.5})$$

$$\phi_R = \phi_{i+1} - \phi_i \quad (\text{A.6})$$

If we decompose the deformation of the equivalent cantilever beam into that due to end moment M_R only and that due to end shear V_R only [see Figure A-I(d)], the governing equations (A.1 to A.4) written for the two cases are

(1) Due to M_R alone

$$V' = 0 \quad (\text{A.7})$$

$$\frac{dM'}{dz} = 0 \quad (\text{A.8})$$

$$\frac{d\phi'}{dz} = \frac{M'}{E_i \mathcal{J}_i} \quad (\text{A.9})$$

$$\frac{dx'}{dz} = -\phi' \quad (\text{A.10})$$

(2) Due to V_R alone

$$\frac{dV''}{dz} = 0 \quad (\text{A.11})$$

$$\frac{dM''}{dz} = V_R \quad (\text{A.12})$$

$$\frac{d\phi''}{dz} = \frac{M''}{E_i \mathcal{J}_i} \quad (\text{A.13})$$

$$\frac{dx''}{dz} = -\phi'' + \frac{V_R}{\pi R_i \delta_i G_i} \quad (\text{A.14})$$

Solving the above two sets of equations with proper boundary conditions one obtains

$$x_M = -\frac{M_R \ell_i^2}{2 E_i \mathcal{J}_i} \quad (\text{A.15})$$

$$\phi_M = \frac{M_R \ell_i}{E_i \mathcal{J}_i} \quad (\text{A.16})$$

$$x_V = \frac{V_R \ell_i^3}{3 E_i \mathcal{J}_i} + \frac{V_R \ell_i}{\pi R_i \delta_i G_i} \quad (\text{A.17})$$

$$\phi_V = - \frac{V_R \ell_i^2}{2 E_i \mathcal{J}_i} \quad (\text{A.18})$$

Substitution of Equations (A.15) to (A.18) into the following equations

$$x_R = x_M + x_V \quad (\text{A.19})$$

$$\phi_R = \phi_M + \phi_V \quad (\text{A.20})$$

and solving for M_R and V_R , there results

$$M_R = \frac{2 E_i \mathcal{J}_i}{\ell_i^2 N_i} [3 x_R + 2 \ell_i A_i \phi_R] \quad (\text{A.21})$$

$$V_R = \frac{6 E_i \mathcal{J}_i}{\ell_i^3 N_i} [2 x_R + \ell_i \phi_R] \quad (\text{A.22})$$

where

$$N_i \equiv 1 + \frac{12 E_i \mathcal{J}_i}{\ell_i^2 \pi R_i \delta_i G_i} \quad (\text{A.23})$$

$$A_i \equiv 1 + \frac{3 E_i \mathcal{J}_i}{\ell_i^2 \pi R_i \delta_i G_i} \quad (\text{A.24})$$

Using Equations (A.21) and (A.22), one obtains

$$M_L = - \frac{2 E_i \mathcal{J}_i}{\ell_i^2 N_i} [3 x_R + \ell_i B_i \phi_R] \quad (\text{A.25})$$

$$V_L = V_R \quad (\text{A.26})$$

where

$$B_i \equiv 1 - \frac{6E_i \mathcal{J}_i}{\ell_i^2 \pi R_i \delta_i G_i} \quad (\text{A.27})$$

Now writing $M_L = M^{(i)}$, $M_R = M^{(i+1)}$, $V_R = V^{(i)} = V^{(i+1)}$, and using Equations (A.5) and (A.6), the end moments and end shear of the original beam section are obtained

$$M^{(i)} = - \frac{2E_i \mathcal{J}_i}{\ell_i^2 N_i} [3(x_{i+1} - x_i) + \ell_i (2A_i \phi_i + B_i \phi_{i+1})] \quad (\text{A.28})$$

$$M^{(i+1)} = \frac{2E_i \mathcal{J}_i}{\ell_i^2 N_i} [3(x_{i+1} - x_i) + \ell_i (2A_i \phi_{i+1} + B_i \phi_i)] \quad (\text{A.29})$$

$$V^{(i)} = V^{(i+1)} = \frac{6E_i \mathcal{J}_i}{\ell_i^3 N_i} [2(x_{i+1} - x_i) + \ell_i (\phi_i + \phi_{i+1})] \quad (\text{A.30})$$

For ordinary beams one simply sets A_i , B_i , and N_i to unity.

APPENDIX B
MATRIX ELEMENTS

ADDITIONAL NOMENCLATURE

<u>Symbol</u>	<u>Definition</u>
$A_{i1} = 1 + \frac{3E_i \mathcal{J}_i}{(h_i^*)^2 \pi R_i \delta_i G_i}$	= Correction factor in modified beam theory for thin-walled beam-like cylinders ($i = 1, 3, 5, 7$, no summation)
$A_{i2} = 1 + \frac{3E_i \mathcal{J}_i}{(\tilde{h}_i - h_i^*)^2 \pi R_i \delta_i G_i}$	"
$A_{i3} = 1 + \frac{3E_i \mathcal{J}_i}{(\ell_i - \tilde{h}_i)^2 \pi R_i \delta_i G_i}$	"
$A_j = 1 + \frac{3E_j \mathcal{J}_j}{\ell_j^2 \pi R_j \delta_j G_j}$	" ($j = 2, 6$)
$\bar{A}_{i1} = A_{i1} \Big _{h_j^* \rightarrow \tilde{h}_i}$	"
$\bar{A}_{i2} = A_{i2}$	"
$\bar{A}_{i3} = A_{i3} \Big _{\tilde{h}_i \rightarrow h_i^*}$	"
$A'_{i1} = A_{i1} \Big _{h_i^* \rightarrow \ell_i}$	"
$B_{i1} = 1 - \frac{6E_i \mathcal{J}_i}{(h_i^*)^2 \pi R_i \delta_i G_i}$	= Correction factor in modified beam theory for thin-walled beam-like cylinders
$B_{i2} = 1 - \frac{6E_i \mathcal{J}_i}{(\tilde{h}_i - h_i^*)^2 \pi R_i \delta_i G_i}$	"

ADDITIONAL NOMENCLATURE (Cont'd)

<u>Symbol</u>	<u>Definition</u>
$B_{i3} = 1 - \frac{6E_i \mathcal{J}_i}{(\ell_i - \tilde{h}_i)^2 \pi R_i \delta_i G_i}$	= Correction factor in modified beam theory for thin-walled beam-like cylinders
$B_j = 1 - \frac{6E_j \mathcal{J}_j}{\ell_j^2 \pi R_j \delta_j G_j}$	"
$\overline{B}_{i1} = B_{i1} \big _{h_i^* \rightarrow \tilde{h}_i}$	"
$\overline{B}_{i2} = B_{i2}$	"
$\overline{B}_{i3} = B_{i3} \big _{\tilde{h}_i \rightarrow h_i^*}$	"
$B'_{i1} = B_{i1} \big _{h_i^* \rightarrow \ell_i}$	"
$N_{i1} = 1 + \frac{3E_i \mathcal{J}_i}{(h_i^*)^2 \pi R_i \delta_i G_i}$	= Correction factor arising from modified beam theory
$N_{i2} = 1 + \frac{3E_i \mathcal{J}_i}{(\tilde{h}_i - h_i^*)^2 \pi R_i \delta_i G_i}$	"
$N_{i3} = 1 + \frac{3E_i \mathcal{J}_i}{(\ell_i - \tilde{h}_i)^2 \pi R_i \delta_i G_i}$	"
$N_j = 1 + \frac{3E_j \mathcal{J}_j}{\ell_j^2 \pi R_j \delta_j G_j}$	"
$\overline{N}_{i1} = N_{i1} \big _{h_i^* \rightarrow \tilde{h}_i}$	"

ADDITIONAL NOMENCLATURE (Cont'd)

<u>Symbol</u>	<u>Definition</u>
$\overline{N}_{i2} = N_{i2}$	= Correction factor arising from modified beam theory
$\overline{N}_{i3} = N_{i3} \Big _{\tilde{h}_i \rightarrow h_i^*}$	"
$N'_{i1} = N_{i1} \Big _{h_i^* \rightarrow l_i}$	"

NONZERO ELEMENTS OF $\mathcal{I}$ MATRIX (SYMMETRIC)

q_{ij}	q'_{ij}	q_{ij}	q'_{ij}	q_{ij}	q'_{ij}
$q_{0-2} = K^{(1)}(R_0 + D_1) + K^{(1)}(R_0 + D_2)$	SAME AS q_{05}	$q_{112} = K^{(1)}(R_0 + D_3)^2 + S_4 + \frac{4E_3 D_3 A_{33}}{(I_3 - I_5^2)N_{53}}$	$q'_{112} = K^{(1)}(R_0 + D_3)^2 + S_4 + \frac{4E_3 D_3 A_{33}}{(I_3 - I_5^2)N_{53}}$	$q_{112} = K^{(1)}(R_0 + D_3)^2 + S_4 + \frac{4E_3 D_3 A_{33}}{(I_3 - I_5^2)N_{53}}$	$q'_{112} = K^{(1)}(R_0 + D_3)^2 + S_4 + \frac{4E_3 D_3 A_{33}}{(I_3 - I_5^2)N_{53}}$
$q_{10-3} = -K^{(1)}(R_0 + D_2)$		$q_{119} = 0$	$q'_{119} = 0$	$q_{119} = 0$	$q'_{119} = 0$
$q_{10-9} = -K^{(1)}(R_0 + D_1)^2$		$q_{120} = 0$	$q'_{120} = 0$	$q_{120} = 0$	$q'_{120} = 0$
$q_{1010} = K^{(1)}(R_0 + D_1)^2 + K^{(1)}(R_0 + D_2)^2 + S_2 + \frac{4E_3 D_3 A_{33}}{(I_3 - I_5^2)N_{53}} + \frac{4E_3 D_3 A_{33}}{(I_3 - I_5^2)N_{53}}$		$q_{121} = 0$	$q'_{121} = 0$	$q_{121} = 0$	$q'_{121} = 0$
$q_{1011} = -K^{(1)}(R_0 + D_2)^2 + \frac{2E_3 D_3 B_{31}}{I_3^2 N_{51}}$		$q_{122} = 0$	$q'_{122} = 0$	$q_{122} = 0$	$q'_{122} = 0$
$q_{1017} = 0$		$q_{123} = 0$	$q'_{123} = 0$	$q_{123} = 0$	$q'_{123} = 0$
$q_{1018} = 6 \left[\frac{E_3 D_3}{(I_3 - I_5^2)N_{53}} - \frac{E_3 D_3}{I_3^2 N_{51}} \right]$		$q_{124} = 0$	$q'_{124} = 0$	$q_{124} = 0$	$q'_{124} = 0$
$q_{1019} = \frac{6E_3 D_3}{I_3^2 N_{51}}$		$q_{125} = 0$	$q'_{125} = 0$	$q_{125} = 0$	$q'_{125} = 0$
$q_{11-2} = -K^{(1)}(R_0 + D_2)$		$q_{126} = 0$	$q'_{126} = 0$	$q_{126} = 0$	$q'_{126} = 0$
$q_{11-3} = K^{(1)}(R_0 + D_1) + K^{(1)}(R_0 + D_2)$		$q_{127} = 0$	$q'_{127} = 0$	$q_{127} = 0$	$q'_{127} = 0$
$q_{1110} = -K^{(1)}(R_0 + D_3)^2 + \frac{2E_3 D_3 B_{31}}{I_3^2 N_{51}}$		$q_{128} = 0$	$q'_{128} = 0$	$q_{128} = 0$	$q'_{128} = 0$
$q_{1111} = K^{(1)}(R_0 + D_1)^2 + K^{(1)}(R_0 + D_2)^2 + S_3 + \frac{4E_3 D_3 A_{33}}{(I_3 - I_5^2)N_{53}} + \frac{4E_3 D_3 A_{33}}{(I_3 - I_5^2)N_{53}}$		$q_{129} = 0$	$q'_{129} = 0$	$q_{129} = 0$	$q'_{129} = 0$
$q_{1118} = -\frac{6E_3 D_3}{I_3^2 N_{51}}$		$q_{130} = 0$	$q'_{130} = 0$	$q_{130} = 0$	$q'_{130} = 0$
$q_{1119} = 6 \left[\frac{E_3 D_3}{(I_3 - I_5^2)N_{53}} - \frac{E_3 D_3}{I_3^2 N_{51}} \right]$		$q_{131} = 0$	$q'_{131} = 0$	$q_{131} = 0$	$q'_{131} = 0$
$q_{1126} = \frac{2E_3 D_3 B_{31}}{I_3^2 N_{51}}$		$q_{132} = 0$	$q'_{132} = 0$	$q_{132} = 0$	$q'_{132} = 0$
$q_{1138} = \frac{6E_3 D_3}{(I_3 - I_5^2)N_{53}}$		$q_{133} = 0$	$q'_{133} = 0$	$q_{133} = 0$	$q'_{133} = 0$
$q_{12-4} = K^{(1)}(R_0 + D_3)$		$q_{134} = 0$	$q'_{134} = 0$	$q_{134} = 0$	$q'_{134} = 0$
$q_{1211} = 0$		$q_{135} = 0$	$q'_{135} = 0$	$q_{135} = 0$	$q'_{135} = 0$

[illegible]

NONZERO ELEMENTS OF $\bar{Q}$ MATRIX (SYMMETRIC)

$\bar{q}_{ij}$	$\bar{q}_{ji}$
$\bar{q}_{35-9} = \frac{2E_3 \beta_1 B_{11}}{\bar{q}_1^3 N_{11}}$	$\bar{q}_{35-10} = \frac{2E_3 \beta_1 \bar{B}_{13}}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{10}}$
$\bar{q}_{36-17} = -\frac{6E_3 \beta_1}{(\bar{q}_1^*)^3 N_{11}}$	$\bar{q}_{35-12} = \frac{6E_3 \beta_1}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{12}}$
$\bar{q}_{35-25} = 4E_3 \beta_1 \left[\frac{A_{11}}{\bar{q}_1^3 N_{11}} + \frac{A_{12}}{(\bar{q}_1^* - \bar{q}_1^*)^2 N_{12}} \right]$	$\bar{q}_{35-25} = 4E_3 \beta_1 \left[\frac{A_{12}}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{12}} + \frac{A_{11}}{(\bar{q}_1^* - \bar{q}_1^*)^2 N_{11}} \right]$
$\bar{q}_{35-27} = 6E_3 \beta_1 \left[\frac{1}{(\bar{q}_1^*)^3 N_{11}} - \frac{1}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{12}} \right]$	$\bar{q}_{35-27} = 6E_3 \beta_1 \left[\frac{1}{(\bar{q}_1^* - \bar{q}_1^*)^2 N_{12}} - \frac{1}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{11}} \right]$
$\bar{q}_{36-11} = \frac{2E_3 \beta_3 B_{33}}{\bar{q}_3^3 N_{31}}$	$\bar{q}_{36-12} = \frac{2E_3 \beta_3 \bar{B}_{33}}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{32}}$
$\bar{q}_{36-19} = -\frac{6E_3 \beta_3}{(\bar{q}_3^*)^3 N_{31}}$	$\bar{q}_{36-20} = \frac{6E_3 \beta_3}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{32}}$
$\bar{q}_{36-26} = 4E_3 \beta_3 \left[\frac{A_{31}}{\bar{q}_3^3 N_{31}} + \frac{A_{32}}{(\bar{q}_3^* - \bar{q}_3^*)^2 N_{32}} \right]$	$\bar{q}_{36-26} = 4E_3 \beta_3 \left[\frac{A_{32}}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{32}} + \frac{A_{31}}{(\bar{q}_3^* - \bar{q}_3^*)^2 N_{31}} \right]$
$\bar{q}_{36-28} = 6E_3 \beta_3 \left[\frac{1}{(\bar{q}_3^*)^3 N_{31}} - \frac{1}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{32}} \right]$	$\bar{q}_{36-28} = 6E_3 \beta_3 \left[\frac{1}{(\bar{q}_3^* - \bar{q}_3^*)^2 N_{32}} - \frac{1}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{31}} \right]$
$\bar{q}_{37-9} = \frac{6E_3 \beta_1}{(\bar{q}_1^*)^3 N_{11}}$	$\bar{q}_{37-10} = -\frac{6E_3 \beta_1}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{13}}$
$\bar{q}_{37-17} = -\frac{12E_3 \beta_1}{(\bar{q}_1^*)^3 N_{11}}$	$\bar{q}_{37-18} = -\frac{12E_3 \beta_1}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{13}}$
$\bar{q}_{37-25} = 6E_3 \beta_1 \left[\frac{1}{(\bar{q}_1^*)^3 N_{11}} - \frac{1}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{12}} \right]$	$\bar{q}_{37-25} = 6E_3 \beta_1 \left[\frac{1}{(\bar{q}_1^* - \bar{q}_1^*)^2 N_{12}} - \frac{1}{(\bar{q}_1 - \bar{q}_1^*)^2 N_{11}} \right]$
$\bar{q}_{37-27} = 12E_3 \beta_1 \left[\frac{1}{(\bar{q}_1^* - \bar{q}_1^*)^3 N_{12}} + \frac{1}{(\bar{q}_1^*)^3 N_{11}} \right]$	$\bar{q}_{37-27} = 12E_3 \beta_1 \left[\frac{1}{(\bar{q}_1^* - \bar{q}_1^*)^3 N_{12}} + \frac{1}{(\bar{q}_1 - \bar{q}_1^*)^3 N_{11}} \right]$
$\bar{q}_{38-11} = \frac{6E_3 \beta_3}{(\bar{q}_3^*)^3 N_{31}}$	$\bar{q}_{38-12} = -\frac{6E_3 \beta_3}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{33}}$
$\bar{q}_{38-19} = -\frac{12E_3 \beta_3}{(\bar{q}_3^*)^3 N_{31}}$	$\bar{q}_{38-20} = -\frac{12E_3 \beta_3}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{33}}$
$\bar{q}_{38-26} = 6E_3 \beta_3 \left[\frac{1}{(\bar{q}_3^*)^3 N_{31}} - \frac{1}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{32}} \right]$	$\bar{q}_{38-26} = 6E_3 \beta_3 \left[\frac{1}{(\bar{q}_3^* - \bar{q}_3^*)^2 N_{32}} - \frac{1}{(\bar{q}_3 - \bar{q}_3^*)^2 N_{31}} \right]$
$\bar{q}_{38-28} = 12E_3 \beta_3 \left[\frac{1}{(\bar{q}_3^* - \bar{q}_3^*)^3 N_{32}} + \frac{1}{(\bar{q}_3^*)^3 N_{31}} \right]$	$\bar{q}_{38-28} = 12E_3 \beta_3 \left[\frac{1}{(\bar{q}_3^* - \bar{q}_3^*)^3 N_{32}} + \frac{1}{(\bar{q}_3 - \bar{q}_3^*)^3 N_{31}} \right]$
$\bar{q}_{39-29} = \bar{q}_1$	$\bar{q}_{39-29} = \bar{q}_1$
$\bar{q}_{40-30} = \bar{q}_3$	$\bar{q}_{40-30} = \bar{q}_3$

NONZERO ELEMENTS OF $\bar{Q}$ MATRIX (NONSYMMETRIC)

$Q_{in} (\bar{r} > \bar{r}')$ ($i=1, \dots, 24$)	$\bar{Q}_{in} (\bar{r} < \bar{r}')$	Q_{in}	$\bar{Q}_{in}$	Q'_{in}	$\bar{Q}_{in}$	Q_{in}	Q'_{in}
$Q_{3-1} = -K^{(12)}$		$Q_{14-2} = K^{(4)}(R_0 + D_4)$	$\bar{Q}_{14-2} = Q_{14-2}$			$Q_{14-5} = \frac{6E^{(12)}J_1}{(I_2 - a)^2}$	
$Q_{4-1} = -K^{(12)}$		$Q_{14-9} = -F_5$	$\bar{Q}_{14-9} = Q_{14-9}$			$Q_{14-6} = 6 \left[\frac{E^{(12)}J_1}{(I_2 - a)^2} - \frac{E^{(12)}J_1}{C^2} \right]$	
$Q_{6-2} = -K^{(4)}$		$Q_{14-15} = \frac{2E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{53}$	$\bar{Q}_{14-15} = 0$			$Q_{14-7} = -\frac{6E^{(12)}J_1}{C^2}$	
$Q_{7-2} = -K^{(4)}$		$Q_{14-17} = -\frac{6E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{53}$	$\bar{Q}_{14-17} = 0$			$Q_{14-12} = -\frac{12E^{(12)}J_1}{(I_2 - a)^2}$	
$Q_{9-3} = -\bar{F}_1$		$Q_{15-2} = K^{(4)}(R_0 + D_4)$	$\bar{Q}_{15-2} = Q_{15-2}$			$Q_{14-13} = -\frac{12E^{(12)}J_1}{C^2}$	
$Q_{9-19} = 0$		$Q_{15-11} = -F_7$	$\bar{Q}_{15-11} = Q_{15-11}$			$Q_{14-20} = -\frac{6E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{51}$	
$Q_{9-21} = 0$		$Q_{15-16} = 0$	$\bar{Q}_{15-16} = 0$			$Q_{14-22} = -\frac{12E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{51}$	
$Q_{10-4} = -\bar{F}_2$		$Q_{15-18} = 0$	$\bar{Q}_{15-18} = 0$			$Q_{14-20} = 0$	
$Q_{10-19} = \frac{2E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{13}$		$Q_{16-16} = \frac{2E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{73}$	$\bar{Q}_{16-16} = 0$			$Q_{14-22} = 0$	
$Q_{10-21} = -\frac{6E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{13}$		$Q_{16-18} = -\frac{6E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{73}$	$\bar{Q}_{16-18} = 0$			$Q_{14-20} = 0$	
$Q_{11-1} = -K^{(12)}(R_0 + D_2)$		$Q_{17-3} = -\frac{6E^{(12)}J_1}{I_1^2}$	$\bar{Q}_{17-3} = Q_{17-3}$			$Q_{14-22} = 0$	
$Q_{11-6} = -\bar{F}_3$		$Q_{17-4} = -\frac{6E^{(12)}J_1}{I_1^2}$	$\bar{Q}_{17-4} = Q_{17-4}$			$Q_{14-20} = 0$	
$Q_{11-20} = 0$		$Q_{17-19} = 0$	$\bar{Q}_{17-19} = -\frac{6E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{14}$			$Q_{14-22} = 0$	
$Q_{11-22} = 0$		$Q_{17-21} = 0$	$\bar{Q}_{17-21} = -\frac{12E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{14}$			$Q_{14-20} = 0$	
$Q_{12-1} = -K^{(12)}(R_0 + D_3)$		$Q_{18-3} = \frac{6E^{(12)}J_1}{I_1^2}$	$\bar{Q}_{18-3} = 0$			$Q_{14-22} = 0$	
$Q_{12-8} = -\bar{F}_4$		$Q_{18-4} = 6 \left[\frac{E^{(12)}J_1}{I_1^2} - \frac{E^{(12)}J_1}{C^2} \right]$	$\bar{Q}_{18-4} = 0$			$Q_{14-20} = 0$	
$Q_{12-20} = \frac{2E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{13}$		$Q_{18-5} = -\frac{6E^{(12)}J_1}{I_1^2}$	$\bar{Q}_{18-5} = 0$			$Q_{14-22} = 0$	
$Q_{12-22} = -\frac{6E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{13}$		$Q_{18-12} = -\frac{12E^{(12)}J_1}{C^2}$	$\bar{Q}_{18-12} = 0$			$Q_{14-20} = 0$	
$Q_{13-12} = -K^{(12)}$		$Q_{18-14} = \frac{6E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{13}$	$\bar{Q}_{18-14} = 0$			$Q_{14-22} = 0$	
$Q_{13-15} = 0$		$Q_{18-21} = -\frac{12E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{13}$	$\bar{Q}_{18-21} = 0$			$Q_{14-20} = 0$	
$Q_{13-17} = 0$		$Q_{19-12} = \frac{6E^{(12)}J_1}{(I_2 - \bar{r}_5)^2} N_{51}$	$\bar{Q}_{19-12} = 0$			$Q_{14-22} = 0$	

NONZERO ELEMENTS OF $\bar{Q}_2$ MATRIX (NONSYMMETRIC)

Q_{ik}	Q'_{ik}	$\bar{Q}_{ik}$	Q_{ik}	$\bar{Q}_{ik}$
$Q_{2314} = -\frac{12E_7 d^2}{(d_6 - d)^3}$	$\downarrow$ End of Q'_{ik}	$\bar{Q}_{2314} = Q_{2314}$	$Q_{3602} = -\frac{6E_3 d_3}{(d_3^* - d_3^*)^2 N_{32}}$	$\bar{Q}_{3602} = -\frac{6E_3 d_3}{(d_3^* - d_3^*)^2 N_{32}}$
$Q_{2316} = 0$		$\bar{Q}_{2316} = -\frac{6E_7 d_7}{(d_7^* - d_7^*)^2 N_{12}}$	$Q_{3719} = -\frac{6E_1 d_1}{(d_1^* - d_1^*)^2 N_{12}}$	$\bar{Q}_{3719} = -\frac{6E_1 d_1}{(d_1^* - d_1^*)^2 N_{12}}$
$Q_{2318} = 0$		$\bar{Q}_{2318} = -\frac{12E_3 d_3}{(d_3^* - d_3^*)^2 N_{12}}$	$Q_{3721} = -\frac{12E_1 d_1}{(d_1^* - d_1^*)^2 N_{12}}$	$\bar{Q}_{3721} = -\frac{12E_1 d_1}{(d_1^* - d_1^*)^2 N_{12}}$
$Q_{2416} = \frac{6E_7 d_7}{(d_7 - d_7^*)^2 N_{73}}$	$Q'_{2416} = 0$	$\bar{Q}_{2416} = 0$	$Q_{3820} = -\frac{6E_3 d_3}{(d_3^* - d_3^*)^2 N_{32}}$	$\bar{Q}_{3820} = -\frac{6E_3 d_3}{(d_3^* - d_3^*)^2 N_{32}}$
$Q_{2418} = -\frac{12E_7 d_7}{(d_7 - d_7^*)^2 N_{73}}$	$Q'_{2418} = 0$	$\bar{Q}_{2418} = 0$	$Q_{3822} = -\frac{12E_3 d_3}{(d_3^* - d_3^*)^2 N_{32}}$	$\bar{Q}_{3822} = -\frac{12E_3 d_3}{(d_3^* - d_3^*)^2 N_{32}}$
$Q_{2715} = \frac{2E_5 d_5}{(d_5 - d_5^*)^2 N_{52}}$		$\bar{Q}_{2715} = \frac{2E_5 d_5}{(d_5^* - d_5^*)^2 N_{52}}$	$Q_{3921} = -\tilde{d}_1$	$\bar{Q}_{3921} = Q_{3921}$
$Q_{2717} = \frac{6E_5 d_5}{(d_5 - d_5^*)^2 N_{52}}$		$\bar{Q}_{2717} = -\frac{6E_5 d_5}{(d_5^* - d_5^*)^2 N_{52}}$	$Q_{4022} = -\tilde{d}_3$	$\bar{Q}_{4022} = Q_{4022}$
$Q_{2719} = \frac{2E_7 d_7}{(d_7 - d_7^*)^2 N_{72}}$		$\bar{Q}_{2719} = \frac{2E_7 d_7}{(d_7^* - d_7^*)^2 N_{72}}$		
$Q_{2818} = \frac{6E_7 d_7}{(d_7 - d_7^*)^2 N_{72}}$		$\bar{Q}_{2818} = -\frac{6E_7 d_7}{(d_7^* - d_7^*)^2 N_{72}}$		
$Q_{2915} = -\frac{6E_5 d_5}{(d_5 - d_5^*)^2 N_{52}}$		$\bar{Q}_{2915} = \frac{6E_5 d_5}{(d_5^* - d_5^*)^2 N_{52}}$		
$Q_{2917} = -\frac{12E_5 d_5}{(d_5 - d_5^*)^2 N_{52}}$		$\bar{Q}_{2917} = -\frac{12E_5 d_5}{(d_5^* - d_5^*)^2 N_{52}}$		
$Q_{3016} = -\frac{6E_7 d_7}{(d_7 - d_7^*)^2 N_{72}}$		$\bar{Q}_{3016} = \frac{6E_7 d_7}{(d_7^* - d_7^*)^2 N_{72}}$		
$Q_{3018} = -\frac{12E_7 d_7}{(d_7 - d_7^*)^2 N_{72}}$		$\bar{Q}_{3018} = -\frac{12E_7 d_7}{(d_7^* - d_7^*)^2 N_{72}}$		
$Q_{3117} = -\tilde{d}_5$		$\bar{Q}_{3117} = Q_{3117}$		
$Q_{3218} = -\tilde{d}_7$		$\bar{Q}_{3218} = Q_{3218}$		
$Q_{3519} = \frac{2E_1 d_1}{(d_1 - d_1^*)^2 N_{12}}$		$\bar{Q}_{3519} = \frac{2E_1 d_1}{(d_1^* - d_1^*)^2 N_{12}}$		
$Q_{3521} = \frac{6E_1 d_1}{(d_1 - d_1^*)^2 N_{12}}$		$\bar{Q}_{3521} = -\frac{6E_1 d_1}{(d_1^* - d_1^*)^2 N_{12}}$		
$Q_{3620} = \frac{2E_3 d_3}{(d_3 - d_3^*)^2 N_{32}}$		$\bar{Q}_{3620} = \frac{2E_3 d_3}{(d_3^* - d_3^*)^2 N_{32}}$		

IONIZED ELEMENTS OF ζ MATRIX (SYMMETRIC)

C_{nl}	C_{nl}	C_{nl}
$C_{12-4} = \frac{6E^{(7)}J^{(7)}}{Q^2}$	$C_{1715} = 6E_5J_5 \left[\frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} - \frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} \right]$	$\bar{C}_{1715} = 6E_5J_5 \left[\frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} - \frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} \right]$
$C_{12-5} = 6E^{(11)}J^{(11)} \left[\frac{1}{Q^2} - \frac{1}{(I_2 - a)^2} \right]$	$C_{1717} = \tilde{\alpha}_5^* + 12E_5J_5 \left[\frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{51}} + \frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} \right]$	$\bar{C}_{1717} = \tilde{\alpha}_5^* + 12E_5J_5 \left[\frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{51}} + \frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} \right]$
$C_{12-6} = -\frac{6E^{(13)}J^{(13)}}{(I_2 - a)^2}$	$C_{1816} = 6E_7J_7 \left[\frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{71}} - \frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{72}} \right]$	$\bar{C}_{1816} = 6E_7J_7 \left[\frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{71}} - \frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{72}} \right]$
$C_{1212} = K^{(6)} + 12E^{(13)}J^{(13)} \left[\frac{1}{Q^2} + \frac{1}{(I_2 - a)^2} \right]$	$C_{1818} = \tilde{\alpha}_7^* + 12E_7J_7 \left[\frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{71}} + \frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{72}} \right]$	$\bar{C}_{1818} = \tilde{\alpha}_7^* + 12E_7J_7 \left[\frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{71}} + \frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{72}} \right]$
$C_{13-6} = \frac{6E^{(13)}J^{(13)}}{Q^2}$	$C_{1919} = 4E_9J_9 \left[\frac{A_{12}}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{91}} + \frac{A_{13}}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{92}} \right]$	$\bar{C}_{1919} = 4E_9J_9 \left[\frac{A_{12}}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{91}} + \frac{A_{13}}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{92}} \right]$
$C_{13-7} = 6E^{(13)}J^{(13)} \left[\frac{1}{Q^2} - \frac{1}{(I_2 - c)^2} \right]$	$C_{1921} = 6E_9J_9 \left[\frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{91}} - \frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{92}} \right]$	$\bar{C}_{1921} = 6E_9J_9 \left[\frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{91}} - \frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{92}} \right]$
$C_{13-8} = -\frac{6E^{(13)}J^{(13)}}{(I_2 - c)^2}$	$C_{2020} = 4E_3J_3 \left[\frac{A_{32}}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{31}} + \frac{A_{33}}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{32}} \right]$	$\bar{C}_{2020} = 4E_3J_3 \left[\frac{A_{32}}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{31}} + \frac{A_{33}}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{32}} \right]$
$C_{1413} = K^{(7)} + 12E^{(13)}J^{(13)} \left[\frac{1}{Q^2} + \frac{1}{(I_2 - c)^2} \right]$	$C_{2022} = 6E_3J_3 \left[\frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{31}} - \frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{32}} \right]$	$\bar{C}_{2022} = 6E_3J_3 \left[\frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{31}} - \frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{32}} \right]$
$C_{1314} = -K^{(7)}$	$C_{2119} = 6E_9J_9 \left[\frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{91}} - \frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{92}} \right]$	$\bar{C}_{2119} = 6E_9J_9 \left[\frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{91}} - \frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{92}} \right]$
$C_{14-9} = \frac{6E^{(4)}J^{(4)}}{Q^2}$	$C_{2121} = \tilde{\alpha}_9^* + 12E_9J_9 \left[\frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{91}} + \frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{92}} \right]$	$\bar{C}_{2121} = \tilde{\alpha}_9^* + 12E_9J_9 \left[\frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{91}} + \frac{1}{(\bar{\alpha}_9^* - \bar{\alpha}_9^*)N_{92}} \right]$
$C_{1410} = 6E^{(4)}J^{(4)} \left[\frac{1}{Q^2} - \frac{1}{(I_2 - d)^2} \right]$	$C_{2220} = 6E_3J_3 \left[\frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{31}} - \frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{32}} \right]$	$\bar{C}_{2220} = 6E_3J_3 \left[\frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{31}} - \frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{32}} \right]$
$C_{1411} = -\frac{6E^{(4)}J^{(4)}}{(I_2 - d)^2}$	$C_{2222} = \tilde{\alpha}_3^* + 12E_3J_3 \left[\frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{31}} + \frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{32}} \right]$	$\bar{C}_{2222} = \tilde{\alpha}_3^* + 12E_3J_3 \left[\frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{31}} + \frac{1}{(\bar{\alpha}_3^* - \bar{\alpha}_3^*)N_{32}} \right]$
$C_{1413} = -K^{(7)}$		
$C_{1414} = K^{(7)} + 12E^{(4)}J^{(4)} \left[\frac{1}{Q^2} + \frac{1}{(I_2 - d)^2} \right]$		
$C_{1515} = 4E_5J_5 \left[\frac{A_{52}}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} + \frac{A_{53}}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{53}} \right]$		$\bar{C}_{1515} = 4E_5J_5 \left[\frac{A_{52}}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} + \frac{A_{53}}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{53}} \right]$
$C_{1517} = 6E_5J_5 \left[\frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{51}} - \frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} \right]$		$\bar{C}_{1517} = 6E_5J_5 \left[\frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{51}} - \frac{1}{(\bar{\alpha}_5^* - \bar{\alpha}_5^*)N_{52}} \right]$
$C_{1616} = 4E_7J_7 \left[\frac{A_{72}}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{72}} + \frac{A_{71}}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{71}} \right]$		$\bar{C}_{1616} = 4E_7J_7 \left[\frac{A_{72}}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{72}} + \frac{A_{71}}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{71}} \right]$
$C_{1618} = 6E_7J_7 \left[\frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{71}} - \frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{72}} \right]$		$\bar{C}_{1618} = 6E_7J_7 \left[\frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{71}} - \frac{1}{(\bar{\alpha}_7^* - \bar{\alpha}_7^*)N_{72}} \right]$

SAME AS
 C_{nl}

APPENDIX C

COMPUTER PROGRAM

The computer program consists of a main program SHUTTLE together with eight function and subroutine subprograms. The purpose of SHUTTLE is to accept input data, compute certain constant data, generate and manipulate the requisite matrices and set up the final matrix for the computation of eigenvalues and ordered printout of frequencies. SHUTTLE has one basic option, viz., whether the Booster and Orbiter tanks are all empty or whether all tanks contain liquid. The input data required of SHUTTLE are listed below:

Mathematical Symbol	FORTTRAN Symbol	Meaning
ρ_i , $i = 1, 3, 5, 7$	RHO(I) , $I = 1, 3, 5, 7$	Density of liquids in each of the four tanks
a_s	AS	Constant, = 4. 375 (see footnote)
h_i , $i = 1, 3, 5, 7$	HI(I) , $I = 1, 3, 5, 7$	Height of liquids in each of the four tanks
K_1	KCAP1	Longitudinal spring constant of lower Booster system, = 3.48E+05
--	TANK	Tank code, = 0.0 if all tanks empty; = 1.0 if all tanks contain liquid
E_s	ES	Coupling parameter, = 0.0 for uncoupled Booster/Orbiter; = 3.0E+07 for coupled Booster/Orbiter

footnote: a_s is the distance from the lower support of the strongback to the location of the coupling spring.

The input order is RHO(I), AS, HI(I), KCAP1, TANK, ES. The format is (8F10.0), so that two cards per case are required.

The names and purposes of the eight subprograms are given below:

FORTRAN Name	Purpose
SINCH	Computes $\sinh(x)$
COSCH	Computes $\cosh(x)$
MMATRX	Multiplies two matrices AA and BB and stores result in CC
MPRINT	Prints matrices (call to MPRINT optional)
BESNIS	Computes $I_0(x)$ and $I_1(x)$
SRTCYC	Sorts eigenvalues in ascending order and converts to Hz
EIGEN	Computes eigenvalues (and, optionally, eigenvectors) of real nonsymmetric matrix
MATINV	Computes inverse of square matrix

```

PROGRAM SHUTTLE (INPUT,OUTPUT,TAPE6=INPUT)
00003 REAL NNN,NNN2,NNN6
00003 REAL I1,I2,I3,I5,I6,I7,L1,L2,L3,L5,L6,L7,LS,JUP1,JUP2,JUP3,JUP4,
*IS,LL,MM,MT,MTDA,JSTR,JRIG,MSTR,KCAP1,KCAP2,KCAP3,
3 KCAP5,KCAP6,KCAP7,KUP1,KUP2,KUP3,KUP4,KUP5,KUP6,KUP7,KBUP3,
4 KBUP4,K1,K3,K5,K7,KB1,KB3,KB5,KB7,KT1,KT3,KT5,KT7,NU,KP,KCB,
5 NUST,NUAL,MASS,
* LW,KR,KW
C****
00003 DIMENSION AAA(7,3),BETA(7,3),NNN(7,3),BBB(7,3)
00003 DIMENSION CYC(40,2)
00003 DIMENSION LL(7),DEL(7),NU(7),EE(7),RHO(7),H1(7),GARR(40,40),
1 MASS(40,40),F(7),ANS(50),T1(40),T(40),PDUCT(40,40),R(7),
*KCB(7),MSTR(7),MTDA(7),JSTR(7),JRIG(7),MT(7),MM(7),KP(7),
* DARR(40,40),ARR(40,40),GARR(40,40)
00003 900 FORMAT(3F10,0)
00003 500 FORMAT(1H039X,10HINPUT DATA//5X,
* * RH01 ==E13,4,4X,* RH03 ==E13,4,4X,* RH05 ==E13,4,4X,
* * RH07 ==E13,4,4X,
* * H1 ==E13,4,4X,* H3 ==E13,4,4X,* H5 ==E13,4,4X,
* * H7 ==E13,4,4X,
* * AS ==E13,4,4X,* KCAP1==E13,4,4X,* ES ==E13,4)
00003 550 FORMAT(1H1)
00003 600 FORMAT(1H038X,14HROW FREQUENCY//
* 40(39X,F2,0,1X,E10,2,/))
00003 50 CONTINUE
C**** ZERO ALL ELEMENTS
00003 JC=40
00004 JR=40
00005 DO 5 I=1,JR
00007 DO 5 J=1,JC
00010 ARR(I,J)=0.0
00013 CARR(I,J)=0.0
00016 DARR(I,J)=0.0
00021 MASS(I,J)=0.0
00024 PDUCT(I,J)=0.0
00027 GARR(I,J)=0.0
00032 5 CONTINUE
C
C**** OPTIONS DESIRED FROM EIGENVALUE SUBROUTINE
C
00037 MXX=40
00040 IJ=JI=22
00042 TOL=1.E-5
00043 PI = 3.1415927
00045 N1OPT= 0
00046 N2OPT= 0
00047 MAXIT=50
00050 NDEC = 2
C
C DATA STATEMENTS FOR PROGRAM CONSTANTS
C****
00051 DATA GST,GAL/1.2E7,4.E6/
00051 DATA AW,TO,EW,LW/0.0,1.0,2.9E7,7./
00051 DATA A,B,C,D,E/2.25,5.0,7.875,3.0,1.0/
00051 DATA RO,RB,D1,D2,D3,D4/3.0,5.0,1.125,1.5,1.125,0.375/
00051 DATA E1,E2,E3,E5,E6,E7/2.9E7,1.E7,2.9E7,3*1.E7/

```

```

00051 DATA EUP1,EUP2,EUP3,EUP4,EUP5/3*1,E7,2*2,9E7/
00051 DATA I1,I2,I3,I5,I6,I7/3,487,10,59,1,963,,84577,1,40934,0,84577/
00051 DATA L1,L2,L3,L5,L6,L7,LS/15,,7,,15,,8,5,6,,8,5,8,75/
00051 DATA JUP1,JUP2,JUP3,JUP4/2,175E-1,3,060E-2,2,175E-1,1,020E-2/
00051 DATA AUP1,AUP2,AUP3/,6875,,9375,,6875/
00051 DATA AUP4,AUP5,ACAPS/,3125,0.000,,0976/
00051 DATA NUAL,NUST /,36,,26/
00051 DATA IS,G,S1,S2,S3/7,94E-4,386,4,3*8,67E4/
00051 DATA S4,S5,S7/8,67E4,2*2,60E4/
00051 DATA (R(I),I=1,7,2)/2*5,0,2*3,0/
00051 DATA (LL(I),I=1,7)/15,,7,,15,,0,,8,5,6,,8,5/
00051 DATA (NU(I),I=1,7)/0,,0,,26,0,,36,0,,36/
00051 DATA (DEL(I),I=1,7)/5,E-3,2,5E-2,5,E-3,0,0,9,84E-3,2,E-2,9,84E-3/
00051 DATA (EE(I),I=1,7)/0,,0,,2,9E7,0,,1,E7,0,,1,E7/

```

C

C**** READ IN THE CASE CONSTANTS

```

00051 READ 900,(RHO(I),I=1,7,2),AS,(HI(I),I=1,7,2),KCAP1,TANK,ES
00100 IF(EOF,6)702,55
00103 55 CONTINUE
00103 CALL SECOND(X)
00105 IF(TANK.NE.0.0)GO TO 3001
00106 DO 905 I=3,7,2
00110 905 KCB(I)=2.*PI*R(I)*EE(I)*DEL(I)/LL(I)
00120 KCB(1)=KCAP1
00122 KCAP2=7,36E5
00123 KCAP3=KCB(3)
00123 KCAP5=KCB(5)
00126 KCAP6 = 2.*PI*R0*E6*DEL(6)/L6*.30
00134 KCAP7=KCB(7)
00136 K1=K3=K5=K7=0,0
00142 KT1=KT3=KT5=KT7=0,0
00146 KB1=KB3=KB5=KB7=0,0
00152 HT1=HT3=HT5=HT7=0,0
00156 HST1=L1
00157 HST3=L3
00161 HST5=L5
00162 HST7=L7
00164 JR=JC=24
00166 JI=14
00167 3001 CONTINUE

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C****

COMPUTED DATA

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00167 KR = AUP5*EUP5/B
00172 KW = (T0*AW*EW)/LW
00176 KUP1 = (AUP1*EUP1)/L1 *.4
00201 KUP2 = (AUP2*EUP2)/L2 *.4
00204 KUP3 = (AUP3*EUP3)/C *.4
00207 KBUP3 = (AUP3*EUP3)/(L3=C) *.4
00213 KUP4 = (AUP4*EUP4)/D *.4
00216 KBUP4 = (AUP4*EUP4)/(L6=D) *.4
00221 KUP5 = KR*KW/(KR*KW)
00224 KUP6 = ACAPS*ES*LS/(AS*(LS-AS)) *.2
00232 KUP7 = 3.*ES*IS*LS**3/(AS**3*(LS-AS)**3) *1.46
00243 S6 = 4.*ES*IS*LS/(AS*(LS-AS)) *.896
00252 IF(TANK.EQ.0.0)GO TO 4001
00253 DO 100 I=3,7,2
00255 ARG = PI*R(I)/(2.*HI(I))
00261 CALL BESNIS(ARG,2,ANS)
00263 F(I)=((4./PI)*ANS(2))/((PI*R(I)*ANS(1))/(2.*HI(I)))

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00275      KCB(I)=2.*PI*R(I)*EE(I)*DEL(I)/(LL(I)-NU(I)**2*HI(I))
00306      KP(I)=(4.*PI*R(I)*EE(I)*DEL(I)/HI(I))*((ANS(2)/((PI*R(I)*ANS(1))/
1(2.*HI(I))))**2
00324      100 CONTINUE
C****      CALCULATIONS FOR F(1), KCB(1), KB1, KP(1), K1
00325      ARG = PI * RB/(2.*HI(1))
00331      CALL BESNIS(ARG,2,ANS)
00333      F(1)= ((4./PI)*ANS(2))/((PI*RB*ANS(1))/(2.*HI(1)))
00344      KCB(1)=KCAP1/(1.-NUST*F(1))
00350      KB1=NUST*KCB(1)*F(1)
00351      KP(1)=(2.*KCB(1)*(L1-NUST**2*HI(1))/HI(1))
1 *((ANS(2)/((PI*RB*ANS(1))/(2.*HI(1))))**2
00365      K1=KP(1)-NUST*KCB(1)*F(1)+NUST**2*KCB(1)*F(1)**2
00375      DO 200 J=1,7,2
00376      ARG = PI*R(J)/(2.*HI(J))
00402      CALL BESNIS(ARG,2,ANS)
00404      MT(J)= PI*R(J)**2*HI(J)*RHO(J)
00411      MM(J)= 16./(PI**2)*((ANS(2)*MT(J))/(PI*R(J)*ANS(1)/(2.*HI(J))))
00422      200 CONTINUE
C****      CALCULATE THE KCAP
00424      KCAP2=7.36E5
00425      KCAP3 = (1.-NU(3)*F(3))*KCB(3)
00431      KCAP5 = (1.-NU(5)*F(5))*KCB(5)
00433      KCAP6 = 2.*PI*RQ*E6*DEL(6)/L6*.30
00442      KCAP7 = (1.-NU(7)*F(7))*KCB(7)
C****      CALCULATE THE SMALL KS AND KBS
00446      K3 = KP(3)-NU(3)*KCB(3)*F(3)+NU(3)**2*KCB(3)*F(3)**2
00457      K5 = KP(5)-NU(5)*KCB(5)*F(5)+NU(5)**2*KCB(5)*F(5)**2
00467      K7 = KP(7)-NU(7)*KCB(7)*F(7)+NU(7)**2*KCB(7)*F(7)**2
00500      KB3= NU(3)*KCB(3)*F(3)
00502      KB5= NU(5)*KCB(5)*F(5)
00505      KB7= NU(7)*KCB(7)*F(7)
C****      CALCULATE DATA FOR MASS MATRIX
00507      DO 300 I=1,7,2
00511      ARG =1.84*HI(I)/R(I)
00514      MTDA(I)=MT(I)*R(I)/(2.2*HI(I))*TANH(ARG)
00522      MSTR(I)=MT(I)-MTDA(I)
00525      JRIG(I)=MT(I)*R(I)**2*(1./3.*(HI(I)/R(I))**2+1.)/4.
00535      300 CONTINUE
00537      ARG=1.84*HI(1)/R(1)
00541      HT1=HI(1)-R(1)/.92*TANH(.92*HI(1)/R(1))
00550      KT1=MT(1)*G/(1.19*HI(1))*(TANH(ARG))**2
00557      HST1=HI(1)-MT(1)* HI(1)/(2.*MSTR(1))
1 +(HI(1)-HT1)*MTDA(1)/MSTR(1)
00567      JSTR(1)=JRIG(1)+MT(1)*(HI(1)/2.)***2 -MT(1)*R(1)**2/2.*(1.995=
1 2.*R(1)/HI(1)*((1.07*COSCH(ARG)-1.07)/SINCH(ARG)))-MSTR(1)*(HI(1)
2-HST1)**2 = MTDA(1)*(HI(1)-HT1)**2
00625      ARG=1.84*HI(3)/R(3)
00627      HT3=HI(3)-R(3)/.92*TANH(.92*HI(3)/R(3))
00637      KT3=MT(3)*G/(1.19*HI(3))*(TANH(ARG))**2
00646      HST3=HI(3)-MT(3)* HI(3)/(2.*MSTR(3))
1 +(HI(3)-HT3)*MTDA(3)/MSTR(3)
00656      JSTR(3)=JRIG(3)+MT(3)*(HI(3)/2.)***2 -MT(3)*R(3)**2/2.*(1.995=
1 2.*R(3)/HI(3)*((1.07*COSCH(ARG)-1.07)/SINCH(ARG)))-MSTR(3)*(HI(3)
2-HST3)**2 = MTDA(3)*(HI(3)-HT3)**2
00714      ARG=1.84*HI(5)/R(5)
00716      HT5=HI(5)-R(5)/.92*TANH(.92*HI(5)/R(5))
00726      KT5=MT(5)*G/(1.19*HI(5))*(TANH(ARG))**2

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00735      HST5=HI(5)-MT(5)* HI(5) /((2.*MSTR(5))
1 +(HI(5)-HT5)*MTDA(5)/MSTR(5)
00745      JSTR(5)=JRIG(5)+MT(5)*(HI(5)/2.)*2-MT(5)*R(5)**2/2.*(1.995-
1 2.*R(5)/HI(5))*((1.07*COSCH(ARG)-1.07)/SINCH(ARG))-MSTR(5)*(HI(5)
2-HST5)**2 - MTD(5)*(HI(5)-HT5)**2
01003      ARG=1.84*HI(7)/R(7)
01005      HT7=HI(7)-R(7)/.92*TANH(.92*HI(7)/R(7))
01015      KT7=HT(7)*G/(1.19*HI(7))*(TANH(ARG))**2
01024      HST7=HI(7)-MT(7)* HI(7) /((2.*MSTR(7))
1 +(HI(7)-HT7)*MTDA(7)/MSTR(7)
01034      JSTR(7)=JRIG(7)+MT(7)*(HI(7)/2.)*2-MT(7)*R(7)**2/2.*(1.995-
1 2.*R(7)/HI(7))*((1.07*COSCH(ARG)-1.07)/SINCH(ARG))-MSTR(7)*(HI(7)
2-HST7)**2 - MTD(7)*(HI(7)-HT7)**2
01072      4001 CONTINUE
C
C      COMPILED DATA FOR FACTORS OF MULTIPLICATION
01072      IF(TANK.EQ.0.0)GO TO 2999
01073      IF(HT1.GE.HST1)GO TO 2999
C
C      INVERTED CASE HT1 LT HST1
C      INVERTED BOOSTER
C
01076      BETA(1,1)= 3.*EE(3)*I1/( HT1**2*PI*R(1)*DEL(1)*GST)
01105      BETA(3,1)= 3.*EE(3)*I3/( HT3**2*PI*R(3)*DEL(3)*GST)
01114      BETA(1,2)= 3.*EE(3)*I1/((HST1-HT1)**2*PI*R(1)*DEL(1)*GST)
01124      BETA(3,2)= 3.*EE(3)*I3/((HST3-HT3)**2*PI*R(3)*DEL(3)*GST)
01135      BETA(1,3)= 3.*EE(3)*I1/((L1-HST1)**2*PI*R(1)*DEL(1)*GST)
01145      BETA(3,3)= 3.*EE(3)*I3/((L3-HST3)**2*PI*R(3)*DEL(3)*GST)
C
C      INVERTED ORBITER
C
01156      IF(HT5.GE.HST5)GO TO 4002
01160      BETA(5,1)= 3.*EE(5)*I5/( HT5**2*PI*R(5)*DEL(5)*GAL)
01167      BETA(7,1)= 3.*EE(7)*I7/( HT7**2*PI*R(7)*DEL(7)*GAL)
01176      BETA(5,2)= 3.*EE(5)*I5/((HST5-HT5)**2*PI*R(5)*DEL(5)*GAL)
01207      BETA(7,2)= 3.*EE(7)*I7/((HST7-HT7)**2*PI*R(7)*DEL(7)*GAL)
01217      BETA(5,3)= 3.*EE(5)*I5/((L5-HST5)**2*PI*R(5)*DEL(5)*GAL)
01230      BETA(7,3)= 3.*EE(7)*I7/((L7-HST7)**2*PI*R(7)*DEL(7)*GAL)
01240      GO TO 4003
C
01241      2999 CONTINUE
C      REGULAR CASE HT1 GT HST1
C
01241      BETA(1,1)= 3.*EE(3)*I1/(HST1**2*PI*R(1)*DEL(1)*GST)
01251      BETA(3,1)= 3.*EE(3)*I3/(HST3**2*PI*R(3)*DEL(3)*GST)
01260      BETA(1,2)= 3.*EE(3)*I1/((HT1-HST1)**2*PI*R(1)*DEL(1)*GST)
01270      BETA(3,2)= 3.*EE(3)*I3/((HT3-HST3)**2*PI*R(3)*DEL(3)*GST)
01301      BETA(1,3)= 3.*EE(3)*I1/((L1-HT1)**2*PI*R(1)*DEL(1)*GST)
01311      BETA(3,3)= 3.*EE(3)*I3/((L3-HT3)**2*PI*R(3)*DEL(3)*GST)
01322      4002 CONTINUE
01322      BETA(5,1)= 3.*EE(5)*I5/(HST5**2*PI*R(5)*DEL(5)*GAL)
01332      BETA(7,1)= 3.*EE(7)*I7/(HST7**2*PI*R(7)*DEL(7)*GAL)
01341      BETA(5,2)= 3.*EE(5)*I5/((HT5-HST5)**2*PI*R(5)*DEL(5)*GAL)
01351      BETA(7,2)= 3.*EE(7)*I7/((HT7-HST7)**2*PI*R(7)*DEL(7)*GAL)
01362      BETA(5,3)= 3.*EE(5)*I5/((L5-HT5)**2*PI*R(5)*DEL(5)*GAL)
01372      BETA(7,3)= 3.*EE(7)*I7/((L7-HT7)**2*PI*R(7)*DEL(7)*GAL)
01403      4003 CONTINUE
01403      BETA(2,2) = 3.*EE(7)*I2/(L2**2*PI*RB*DEL(2)*GAL)

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01413      BETAN6 = 3.*EE(7)*I6/(L6**2*PI*RO*DEL(6)*GAL)
01422      DO 90 I=1,7,2
01423      DO 90 J=1,3
01424      NNN(I,J) = 1. + 4.*BETA(I,J)
01433      AAA(I,J) = 1. + BETA(I,J)
01436      RBR(I,J) = 1. - 2.*BETA(I,J)
01442      90 CONTINUE
01446      NNN2 = 1. + 4. * BETAN2
01451      NNN6 = 1. + 4. * BETAN6
01453      AAA2 = 1. + BETAN2
01454      AAA6 = 1. + BETAN6
01455      RBR2 = 1. - 2. * BETAN2
01457      RBB6 = 1. - 2. * BETAN6

C
C****      FILL THE P-MATRIX (MASS) WITH ITS NON-ZERO ELEMENTS
C
01461      MASS(1,1)=MASS(17,17)=8.291E-2
01464      MASS(2,2)=MASS(18,18)=1.768E-2
01467      MASS(3,3)=MASS(19,19)=3.334E-2
01472      MASS(4,4)=MASS(20,20)=3.037E-2
01475      MASS(5,5)=MASS(21,21)=5.715E-3
01500      MASS(6,6)=MASS(22,22)=4.755E-3
01503      MASS(7,7)=MASS(23,23)=4.885E-3
01506      MASS(8,8)=MASS(24,24)=5.208E-3
01511      MASS(33,33)= MM(1)
01513      MASS(34,34)= MM(3)
01514      MASS(25,25)= MM(5)
01516      MASS(26,26)= MM(7)
01517      MASS( 9, 9)=7.65E-1
01521      MASS(10,10)=2.09E-1
01522      MASS(11,11)=3.255E-1
01524      MASS(12,12)=1.721E-1
01525      MASS(13,13)=1.782E-2
01527      MASS(14,14)=2.546E-2
01530      MASS(15,15)=2.582E-2
01532      MASS(16,16)=1.643E-2
01533      MASS(35,35)= JSTR(1)
01535      MASS(36,36)= JSTR(3)
01536      MASS(27,27)= JSTR(5)
01540      MASS(28,28)= JSTR(7)
01541      MASS(37,37)= MSTR(1)
01543      MASS(38,38)= MSTR(3)
01544      MASS(29,29)= MSTR(5)
01546      MASS(30,30)= MSTR(7)
01547      MASS(39,39)= MTDA(1)
01551      MASS(40,40)= MTDA(3)
01552      MASS(31,31)= MTDA(5)
01554      MASS(32,32)= MTDA(7)

C****      INVERT THE DIAGONAL P-MATRIX (MASS)
C****
01555      DO 75 I=1,JC
01557      72 MASS(I,I)=1./MASS(I,I)
01563      75 CONTINUE
C      THE BEGINNING OF PROGRAM CALCULATIONS FOR THE NON-ZERO
C      ELEMENTS OF THE SYMMETRIC Q-MATRIX ( QARR )
C****      ASSIGNING VARIABLE NAMES TO OFTEN USED CONSTANTS
C****
01565      RBD1 = RB+01

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01567      RBD2 = RB+D2
01571      RBD3 = RB+D3
01572      ROD4 = RO+D4
01574      EIL2  = E2*I2/L2
01577      E1I1  = E1*I1
01601      E2I2  = E2*I2
01602      E3I3  = E3*I3
01604      E5I5  = E5*I5
01606      E6I6  = E6*I6
01610      E7I7  = E7*I7
01612      EIHST1 = E1*I1/HST1
01614      EIHST3 = E3*I3/HST3
01617      EIHST5 = E5*I5/HST5
01621      EIHST7 = E7*I7/HST7
01624      EILHT3 = E3*I3/(L3-HT3)
01627      EILHT5 = E5*I5/(L5-HT5)
01633      EILHT7 = E7*I7/(L7-HT7)
01636      EUJUP1 = EUP1*JUP1
01640      EUJUP2 = EUP2*JUP2
01642      EUJUP3 = EUP3*JUP3
01644      EUJUP4 = EUP4*JUP4
01646      QARR( 1, 1) = KCAP1+KUP1+K1
01651      QARR( 1, 2) = -KCAP1-KUP1
01653      QARR( 1, 9) = KUP1*RBD1
01654      QARR( 1,10) = -KUP1*RBD1
01655      QARR( 1,33) = -K1
01656      QARR( 2, 2) = KCAP1+KCAP2+KUP1+KUP2+KB1
01662      QARR( 2, 3) = -KCAP2-KUP2
01663      QARR( 2, 9) = QARR(1,10)
01665      QARR( 2,10) = QARR(1,9)+KUP2*RBD2
01667      QARR( 2,11) = -KUP2*RBD2
01670      QARR( 2,33) = -KB1
01671      QARR( 3, 3) = KCAP2+KCAP3+KUP2+KUP3+K3
01676      QARR( 3, 4) = -KCAP3
01677      QARR( 3,10) = QARR(2,11)
01700      QARR( 3,11) = -QARR(2,11)+KUP3*RBD3
01702      QARR( 3,34) = -K3
01703      QARR( 4, 4) = KCAP3+KBUP3+KB3
01706      QARR( 4,12) = KBUP3*RBD3
01707      QARR( 4,34) = -KB3
01710      QARR( 5, 5) = KCAP5+K5
01712      QARR( 5, 6) = -KCAP5
01713      QARR( 5,25) = -K5
01714      QARR( 6, 6) = KCAP5+KCAP6+KUP4+KB5
01717      QARR( 6, 7) = -KCAP6
01720      QARR( 6,14) = -KUP4*ROD4
01722      QARR( 6,25) = -KB5
01723      QARR( 7, 7) = KCAP6+KCAP7+KBUP4+K7
01726      QARR( 7, 8) = -KCAP7
01727      QARR( 7,15) = -KBUP4*ROD4
01730      QARR( 7,26) = -K7
01731      QARR( 8, 8) = KCAP7+KB7
01732      QARR( 8,26) = -KB7
01733      QARR( 9, 9) = QARR(1,9)*RBD1*S1+4,*EIHST1*AAA(1,1)/NNN(1,1)
01742      QARR( 9,10) = -QARR(1,9)*RBD1
01744      QARR( 9,17) = -6,*E1I1/HST1**2*1./NNN(1,1)
01751      QARR( 9,35) = 2,*EIHST1*BBB(1,1)/NNN(1,1)
01754      QARR( 9,37) = -QARR(9,17)

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01755      QARR(10,10) =-QARR(9,10)+KUP2 *RBD2**2*S2+4.*(E1I1/(L1-HT1)*AAA
*(1,3)/NNN(1,3)+EIL2*AAA2/NNN2)
01773      QARR(10,11) =-KUP2 *RBD2**2+2.*EIL2*BBB2/NNN2
02000      QARR(10,18) = 6.*(E1I1/(L1-HT1)**2*1./NNN(1,3)-EIL2/(L2*NNN2))
02011      QARR(10,19) = 6.*EIL2/L2*1./NNN2
02014      QARR(11,11) = KUP2*RBD2**2+KUP3*RBD3**2+S3+4.*(EIL2*AAA2/NNN2+
*EIHST3*AAA(3,1)/NNN(3,1))
02030      QARR(11,18) =-QARR(10,19)
02031      QARR(11,19) = QARR(10,19)-6.*E3I3/HST3**2*1./NNN(3,1)
02040      QARR(11,36) = 2.*EIHST3*BBB(3,1)/NNN(3,1)
02043      QARR(11,38) = 6.*EIHST3/HST3*1./NNN(3,1)
02047      QARR(12,12) = QARR(4,12)*RBD3+S4+4.*EILHT3*AAA(3,3)/NNN(3,3)
02055      QARR(12,20) = 6.*EILHT3/(L3-HT3)*1./NNN(3,3)
02062      QARR(13,13) = 4.*EIHST5*AAA(5,1)/NNN(5,1)
02066      QARR(13,21) = KUP5*E=6.*EIHST5/HST5*1./NNN(5,1)
02074      QARR(13,27) = 2.*EIHST5*BBB(5,1)/NNN(5,1)
02077      QARR(13,29) = 6.*EIHST5/HST5*1./NNN(5,1)
02102      QARR(14,14) = KUP4*ROD4**2+S5+4.*EILHT5*AAA(5,3)/NNN(5,3)+4.*E6I6*
*AAA6/(L6*NNN6)
02117      QARR(14,15) = 2.*E6I6/L6*BBB6/NNN6
02123      QARR(14,22) = 6.*(EILHT3/(L5-HT5)*1./NNN(5,3)-E6I6/(L6**2*NNN6))
02134      QARR(14,23) = 6.*E6I6/(L6**2*NNN6)
02137      QARR(15,15) = KBUP4*ROD4**2+S7+4.*(E6I6/L6*AAA6/NNN6+E7I7/HST7*
*AAA(7,1)/NNN(7,1))
02153      QARR(15,22) =-QARR(14,23)
02154      QARR(15,23) = QARR(14,23)-EIHST7/HST7*6.*1./NNN(7,1)
02160      QARR(15,28) = 2.*EIHST7*BBB(7,1)/NNN(7,1)
02163      QARR(15,30) = 6.*EIHST7/HST7*1./NNN(7,1)
02166      QARR(16,16) = 4.*EILHT7*AAA(7,3)/NNN(7,3)
02172      QARR(16,24) = 6.*EILHT7/(L7-HT7)*1./NNN(7,3)
02177      QARR(17,17) = 12.*(E1I1/HST1**3*1./NNN(1,1)+EUJUP1/L1**3)
02207      QARR(17,18) =-12.*EUJUP1/L1**3
02211      QARR(17,35) =-QARR(9,37)
02213      QARR(17,37) =-12.*EIHST1/HST1**2*1./NNN(1,1)
02217      QARR(18,18) = 12.*(E1I1/((L1-HT1)**3*NNN(1,3))+EUJUP1/L1**3 +
* E2I2/(L2**3*NNN2) + EUJUP2/A**3)
02236      QARR(18,19) =-12.*EIL2/L2**2*1./NNN2
02243      QARR(19,19) =-QARR(18,19)+12.*(EUJUP2/(L2=A)**3+EIHST3/HST3**2
**1./NNN(3,1)+ EUJUP3/C**3)
02257      QARR(19,36) =-6.*EIHST3/HST3*1./NNN(3,1)
02263      QARR(19,38) =-12.*EIHST3/HST3**2*1./NNN(3,1)
02266      QARR(20,20) = 12.*(EILHT3/(L3-HT3)**2*1./NNN(3,3)+EUJUP3/(L3=C)
**3)
02301      QARR(21,21) = KUP5+ 12.*EIHST3/HST5**2*1./NNN(5,1)
02306      QARR(21,27) =-QARR(13,29)
02307      QARR(21,29) =-12.*EIHST5/HST5**2*1./NNN(5,1)
02314      QARR(22,22) = 12.*(E5I5/((L5-HT5)**3*NNN(5,3))+E6I6/(L6**3*NNN6)
+ EUJUP4/D**3)
02331      QARR(22,23) =-12.*E6I6/L6**3*1./NNN6
02335      QARR(23,23) = 12.*(E6I6/(L6**3*NNN6)+E7I7/(HST7**3*NNN(7,1))+
* EUJUP4/(L6=D)**3)
02353      QARR(23,28) =-QARR(15,30)
02354      QARR(23,30) = 2.*QARR(23,28)/HST7
02357      QARR(24,24) = 2.*QARR(16,24)/(L7-HT7)
02362      QARR(25,25) = K5+KB5
02364      QARR(26,26) = K7+KB7
02366      QARR(27,27) = 4.*E5I5*(AAA(5,1)/(HST5*NNN(5,1))+AAA(5,2)/((HT5=HST
5)*NNN(5,2)))

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02377      QARR(27,29) = QARR(13,29)-6.*E5I5/(HT5-HST5)**2*1./NNN(5,2)
02406      QARR(28,28) = 4.*E7I7*(AAA(7,1)/(HST7*NNN(7,1)) + AAA(7,2)/((HT7-
*      HST7)*NNN(7,2)))
02417      QARR(28,30) = QARR(15,30)-6.*E7I7/(HT7-HST7)**2*1./NNN(7,2)
02426      QARR(29,29) = 12.*E5I5*(1./((HT5-HST5)**3*NNN(5,2))+1./((HST5**3*
*      NNN(5,1))) )
02440      QARR(30,30) = 12.*E7I7*(1./((HT7-HST7)**3*NNN(7,2))+1./((HST7**3*
*      NNN(7,1))) )
02452      QARR(31,31) = KT5
02453      QARR(32,32) = KT7
02455      QARR(33,33) = K1+KB1
02457      QARR(34,34) = K3+KB3
02461      QARR(35,35) = 4.*E1I1*(AAA(1,1)/(HST1*NNN(1,1))+AAA(1,2)/
*      ((HT1-HST1)*NNN(1,2)))
02472      QARR(35,37) = 6.*E1I1*(1./((HST1**2*NNN(1,1))-1./((HT1-HST1)**2*
*      NNN(1,2))) )
02503      QARR(36,36) = 4.*E3I3*(AAA(3,1)/(HST3*NNN(3,1)) + AAA(3,2)/
*      ((HT3-HST3)*NNN(3,2)))
02514      QARR(36,38) = 6.*E3I3*(1./((HST3**2*NNN(3,1))-1./((HT3-HST3)**2*
*      NNN(3,2))) )
02525      QARR(37,37) = 12.*E1I1*(1./((HT1-HST1)**3*NNN(1,2))+1./((HST1**3*
*      NNN(1,1))) )
02537      QARR(38,38) = 12.*E3I3*(1./((HT3-HST3)**3*NNN(3,2))+1./((HST3**3*NNN
*      (3,1))) )
02551      QARR(39,39) = KT1
02552      QARR(40,40) = KT3

```

```

C
C      GENERATE REMAINDER OF QARR MATRIX FROM SYMMETRY
C

```

```

02554      K=1
02555      DO 352 I=2,40
02556      DO 351 J=1,K
02557      351 QARR(I,J)=QARR(J,I)
02571      K=K+1
02572      352 CONTINUE

```

```

C
02574      IF(TANK,NE,0.0)GO TO 2001
C      ZERO CASE
C

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02575      QARR( 9,10) = QARR(9,10)+2.*E1I1/L1*BBB(1,1)/NNN(1,1)
02602      QARR( 9,18) = QARR(9,37)
02604      QARR( 9,35) = 0.0
02605      QARR(10,17) =-6.*E1I1/L1**2*1./NNN(1,1)
02610      QARR(11,12) = QARR(11,36)
02612      QARR(11,20) = QARR(11,38)
02613      QARR(12,19) =-6.*E3I3/L3**2*1./NNN(3,1)
02620      QARR(13,14) = QARR(13,27)
02622      QARR(13,22) = QARR(13,29)
02623      QARR(14,21) =-6.*E5I5/L5**2*1./NNN(5,1)
02630      QARR(15,16) = QARR(15,28)
02632      QARR(15,24) = QARR(15,30)
02633      QARR(16,23) =-6.*E7I7/L7**2*1./NNN(7,1)
02640      QARR(17,18) = QARR(17,18)-12.*E1I1/L1**3*1./NNN(1,1)
02646      QARR(17,37) = 0.0
02647      QARR(19,20) = QARR (19,38)
02650      QARR(21,22) = QARR(21,29)
02652      QARR(23,24) = QARR(23,30)
02653      2001 CONTINUE

```

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02653      C      IF(TANK, EQ, 0, 0) GO TO 2009
02654      C      IF(HT1, GE, HST1) GO TO 2011
      C      INVERTED CASE
      C      INVERTED BOOSTER

02657      QARR( 9, 9) = KUP1*RBD1**2+S1+4, *E1I1*AAA(1,1)/(HT1*NNN(1,1))
02667      QARR( 9,17) = -6, *E1I1/(HT1**2*NNN(1,1))
02673      QARR( 9,35) = 0, 0
02674      QARR( 9,37) = 0, 0
02675      QARR(10,10) = KUP1*RBD1**2+KUP2*RBD2**2+S2+4, *E1I1*AAA(1,3)/
      * ((L1-HST1)*NNN(1,3))+4, *E2I2*AAA2/(L2*NNN2)
02716      QARR(10,18) = 6, *(E1I1/((L1-HST1)**2*NNN(1,3))-E2I2/(L2**2
      * NNN2))
02726      QARR(10,35) = 2, *E1I1*BBB(1,3)/((L1-HST1)*NNN(1,3))
02734      QARR(10,37) = -6, *E1I1/((L1-HST1)**2*NNN(1,3))
02741      QARR(11,11) = KUP2*RBD2**2+KUP3*RBD3**2+S3+4, *E1L2*AAA2/NNN2
      * + 4, *E3I3*AAA(3,1)/(HT3*NNN(3,1))
02757      QARR(11,19) = 6, *(E1L2/(L2*NNN2)-E3I3/(HT3**2*NNN(3,1)))
02766      QARR(11,36) = 0, 0
02767      QARR(11,38) = 0, 0
02770      QARR(12,12) = KBUP3*RBD3**2+S4+4, *E3I3*AAA(3,3)/((L3-HST3)
      * NNN(3,3))
03002      QARR(12,20) = 6, *E3I3/((L3-HST3)**2*NNN(3,3))
03007      QARR(12,36) = 2, *E3I3*BBB(3,3)/((L3-HST3)*NNN(3,3))
03014      QARR(12,38) = -6, *E3I3/((L3-HST3)**2*NNN(3,3))
03022      QARR(17,17) = 12, *(E1I1/(HT1**3*NNN(1,1))+EUJUP1/L1**3)
03030      QARR(17,35) = 0, 0
03031      QARR(17,37) = 0, 0
03032      QARR(18,18) = 12, *(E1I1/((L1-HST1)**3*NNN(1,3))+EUJUP1/L1**3+
      * E2I2/(L2**3*NNN2)+EUJUP2/A**3)
03051      QARR(18,35) = 6, *E1I1/((L1-HST1)**2*NNN(1,3))
03055      QARR(18,37) = -12, *E1I1/((L1-HST1)**3*NNN(1,3))
03062      QARR(19,19) = 12, *(E2I2/(L2**3*NNN2)+EUJUP2/(L2-A)**3 +
      * E3I3/(HT3**3*NNN(3,1))+EUJUP3/C**3)
03103      QARR(19,36) = 0, 0
03104      QARR(19,38) = 0, 0
03105      QARR(20,20) = 12, *(E3I3/((L3-HST3)**3*NNN(3,3))+EUJUP3/(L3-C)**3)
03117      QARR(20,36) = 6, *E3I3/((L3-HST3)**2*NNN(3,3))
03124      QARR(20,38) = -12, *E3I3/((L3-HST3)**3*NNN(3,3))
03130      QARR(35,35) = 4, *E1I1*(AAA(1,3)/((L1-HST1)*NNN(1,3))+
      * AAA(1,2)/((HST1-HT1)*NNN(1,2)))
03142      QARR(35,37) = 6, *E1I1*(1, /((HST1-HT1)**2*NNN(1,2))-1, /
      * ((L1-HST1)**2*NNN(1,3)))
03154      QARR(36,36) = 4, *E3I3*(AAA(3,3)/((L3-HST3)*NNN(3,3))+
      * AAA(3,2)/((HST3-HT3)*NNN(3,2)))
03166      QARR(36,38) = 6, *E3I3*(1, /((HST3-HT3)**2*NNN(3,2))-1, /
      * ((L3-HST3)**2*NNN(3,3)))
03200      QARR(37,37) = 12, *E1I1*(1, /((HST1-HT1)**3*NNN(1,2))+1, /
      * ((L1-HST1)**3*NNN(1,3)))
03212      QARR(38,38) = 12, *E3I3*(1, /((HST3-HT3)**3*NNN(3,2))+1, /
      * ((L3-HST3)**3*NNN(3,3)))
03224      C      IF(HT5, GE, HST5) GO TO 2009
      C      INVERTED ORBITER

03226      QARR(13,13) = 4, *E5I5*AAA(5,1)/(HT5*NNN(5,1))
03232      QARR(13,21) = KUP5*E - 6, *E5I5/( HT5**2*NNN(5,1))

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```

03240      QARR(13,27) = 0.0
03241      QARR(13,29) = 0.0
03242      QARR(14,14) = KUP4*ROD4**2+S5+4.*E5I5*AAA(5,3)/
*          ((L5-HST5)*NNN(5,3)) + 4.*E6I6*AAA6/(L6*NNN6)
03261      QARR(14,22) = 6.*(E5I5/((L5-HST5)**2*NNN(5,3))-
*          E6I6/(L6**2*NNN6))
03271      QARR(14,27) = 2.*E5I5*BBB(5,3)/((L5-HST5)*NNN(5,3))
03277      QARR(14,29) = -6.*E5I5/((L5-HST5)**2*NNN(5,3))
03304      QARR(15,15) = KBUP4*ROD4**2+S7+4.*E6I6*AAA6/(L6*NNN6)+
*          4.*E7I7*AAA(7,1)/(HT7*NNN(7,1))
03322      QARR(15,23) = 6.*(E6I6/(L6**2*NNN6)-E7I7/(HT7**2*NNN(7,1)))
03331      QARR(15,28) = 0.0
03332      QARR(15,30) = 0.0
03333      QARR(16,16) = 4.*E7I7*AAA(7,3)/((L7-HST7)*NNN(7,3))
03340      QARR(16,24) = 6.*E7I7/((L7-HST7)**2*NNN(7,3))
03346      QARR(16,28) = 2.*E7I7*BBB(7,3)/((L7-HST7)*NNN(7,3))
03353      QARR(16,30) = -6.*E7I7/((L7-HST7)**2*NNN(7,3))
03361      QARR(21,21) = KUP5 + 12.*E5I5/(HT5**3*NNN(5,1))
03366      QARR(21,27) = 0.0
03367      QARR(21,29) = 0.0
03370      QARR(22,22) = 12.*(E5I5/((L5-HST5)**3*NNN(5,3))+E6I6/(L6**3*NNN6)
*          + EUJUP4/D**3)
03405      QARR(22,27) = 6.*E5I5/((L5-HST5)**2*NNN(5,3))
03411      QARR(22,29) = -12.*E5I5/((L5-HST5)**3*NNN(5,3))
03416      QARR(23,23) = 12.*(E6I6/(L6**3*NNN6)+E7I7/(HT7**3*NNN(7,1))
*          + EUJUP4/(L6=D)**3)
03434      QARR(23,28) = 0.0
03435      QARR(23,30) = 0.0
03436      QARR(24,24) = 12.*E7I7/((L7-HST7)**3*NNN(7,3))
03442      QARR(24,28) = 6.*E7I7/((L7-HST7)**2*NNN(7,3))
03447      QARR(24,30) = -12.*E7I7/((L7-HST7)**3*NNN(7,3))
03454      QARR(27,27) = 4.*E5I5*(AAA(5,3)/((L5-HST5)*NNN(5,3))
*          + AAA(5,2)/((HST5-HT5)*NNN(5,2)))
03466      QARR(27,29) = 6.*E5I5*(1./((HST5-HT5)**2*NNN(5,2))-
*          1./((L5-HST5)**2*NNN(5,3)))
03500      QARR(28,28) = 4.*E7I7*(AAA(7,3)/((L7-HST7)*NNN(7,3))
*          + AAA(7,2)/((HST7-HT7)*NNN(7,2)))
03512      QARR(28,30) = 6.*E7I7*(1./((HST7-HT7)**2*NNN(7,2))-
*          1./((L7-HST7)**2*NNN(7,3)))
03524      QARR(29,29) = 12.*E5I5*(1./((HST5-HT5)**3*NNN(5,2))+
*          1./((L5-HST5)**3*NNN(5,3)))
03536      QARR(30,30) = 12.*E7I7*(1./((HST7-HT7)**3*NNN(7,2))+
*          1./((L7-HST7)**3*NNN(7,3)))
03550      2009 CONTINUE
03550      K=1
03551      DO 354 I=2,JC
03553      DO 353 J=1,K
03554      353 QARR(I,J)=QARR(J,I)
03566      K=K+1
03567      354 CONTINUE
03571      2011 CONTINUE
03571      ARR( 3, 1) ==KUP3
03573      ARR( 4, 1) ==KBUP3
03574      ARR( 6, 2) ==KUP4
03576      ARR( 7, 2) ==KBUP4
03577      ARR( 9, 3) ==S1
03601      ARR(10, 4) ==S2
03602      ARR(10,19) = 2.*E1I1/(L1-HT1)*BBB(1,3)/NNN(1,3)

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03610 ARR(10,21) ==6,*E1I1/((L1-HT1)**2*NNN(1,3))
03614 ARR(11,1) ==KUP3*RB03
03616 ARR(11,6) ==S3
03620 ARR(12,1) ==KBUP3*RR03
03621 ARR(12,6) ==S4
03623 ARR(12,20) = 2,*EILHT3*BBB(3,3)/NNN(3,3)
03626 ARR(12,22) ==6,*EILHT3/(L3-HT3)*1./NNN(3,3)
03633 ARR(13,12) ==KUP5*E
03635 ARR(14,2) = KUP4*ROD4
03637 ARR(14,9) ==S5
03641 ARR(14,15) = 2,*EILHT5*BBB(5,3)/NNN(5,3)
03644 ARR(14,17) ==6,*EILHT5/(L5-HT5)*1./NNN(5,3)
03651 ARR(15,2) = KBUP4*ROD4
03653 ARR(15,11) ==S7
03655 ARR(16,16) = 2,*EILHT7*BBB(7,3)/NNN(7,3)
03660 ARR(16,18) ==6,*EILHT7/(L7-HT7)*1./NNN(7,3)
03665 ARR(17,3) ==-6,*EUJUP1/L1**2
03670 ARR(17,4) = ARR(17,3)
03671 ARR(18,3) = 6,*EUJUP1/L1**2
03673 ARR(18,4) = 6,*EUJUP1/L1**2-EUJUP2/A**2)
03677 ARR(18,5) ==6,*EUJUP2/A**2
03701 ARR(18,12) ==12,*EUJUP2/A**3
03703 ARR(18,19) = 6,*E1I1/(L1-HT1)**2*1./NNN(1,3)
03711 ARR(18,21) ==-12,*E1I1/(L1-HT1)**3*1./NNN(1,3)
03717 ARR(19,5) = 6,*EUJUP2/(L2-A)**2
03723 ARR(19,6) = ARR(19,5)-6,*EUJUP3/C**2
03727 ARR(19,7) ==6,*EUJUP3/C**2
03731 ARR(19,12) ==-2,*ARR(19,5)/(L2-A)
03734 ARR(19,13) ==12,*EUJUP3/C**3
03737 ARR(20,7) = 6,*EUJUP3/(L3-C)**2
03742 ARR(20,8) = ARR(20,7)
03743 ARR(20,13) ==-2,*ARR(20,7)/(L3-C)
03746 ARR(20,20) = 6,*EILHT3/(L3-HT3)*1./NNN(3,3)
03753 ARR(20,22) ==-2,*ARR(20,20)/(L3-HT3)
03756 ARR(21,12) ==KUP5
03760 ARR(22,9) ==-6,*EUJUP4/D**2
03763 ARR(22,10) = ARR(22,9)
03764 ARR(22,14) = -12,*EUJUP4/D**3
03766 ARR(22,15) = 6,*EILHT5/(L5-HT5)*1./NNN(5,3)
03773 ARR(22,17) ==-12,*EILHT5/(L5-HT5)**2*1./NNN(5,3)
04001 ARR(23,10) = 6,*EUJUP4/(L6-D)**2
04005 ARR(23,11) = ARR(23,10)
04006 ARR(23,14) ==-2,*ARR(23,10)/(L6-D)
04011 ARR(24,16) ==ARR(16,18)
04013 ARR(24,18) = 2,*ARR(16,18)/(L7-HT7)
04017 ARR(27,15) = 2,*E5I5/(HT5-HST5)*BBB(5,2)/NNN(5,2)
04024 ARR(27,17) = 6,*E5I5/(HT5-HST5)**2*1./NNN(5,2)
04032 ARR(28,16) = 2,*E7I7/(HT7-HST7)*BBB(7,2)/NNN(7,2)
04037 ARR(28,18) = 6,*E7I7/(HT7-HST7)**2*1./NNN(7,2)
04045 ARR(29,15) ==ARR(27,17)
04047 ARR(29,17) = 2,*ARR(29,15)/(HT5-HST5)
04052 ARR(30,16) ==-6,*E7I7/(HT7-HST7)**2*1./NNN(7,2)
04060 ARR(30,18) = 2,*ARR(30,16)/(HT7-HST7)
04063 ARR(31,17) ==KT5
04065 ARR(32,18) ==KT7
04066 ARR(35,19) = 2,*E1I1/(HT1-HST1)*BBB(1,2)/NNN(1,2)
04074 ARR(35,21) = 6,*E1I1/(HT1-HST1)**2*1./NNN(1,2)
04102 ARR(36,20) = 2,*E3I3/(HT3-HST3)*BBB(3,2)/NNN(3,2)

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04107 ARR(36,22) = 6.*E3I3/(HT3-HST3)**2*1./NNN(3,2)
04115 ARR(37,19) = -ARR(35,21)
04117 ARR(37,21) = 2.*ARR(37,19)/(HT1-HST1)
04122 ARR(38,20) = -6.*E3I3/(HT3-HST3)**2*1./NNN(3,2)
04130 ARR(38,22) = 2.*ARR(38,20)/(HT3-HST3)
04133 ARR(39,21) = -KT1
04135 ARR(40,22) = -KT3
04136 IF(TANK,NE,0,0)GO TO 3002
04137 ARR(10,19)=0,0
04140 ARR(10,21)=ARR(12,20)=ARR(12,22)=ARR(14,15)= ARR(14,17)=0,0
04146 ARR(16,16)=ARR(16,18)=ARR(18,19)=ARR(18,21)=ARR(20,20)=0,0
04154 ARR(20,22)=ARR(22,15)=ARR(22,17)=ARR(24,16)=ARR(24,18)=0,0
04162 GO TO 3006
04163 3002 CONTINUE
04163 IF(HT1,GE,HST1)GO TO 3006

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INVERTED BOOSTER

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04166 ARR( 9,19) = 2.*E1I1*BBB(1,1)/(HT1*NNN(1,1))
04172 ARR( 9,21) = 6.*E1I1/(HT1**2*NNN(1,1))
04175 ARR(10,19) = 0,0
04176 ARR(10,21) = 0,0
04177 ARR(11,20) = 2.*E3I3*BBB(3,1)/(HT3*NNN(3,1))
04203 ARR(11,22) = 6.*E3I3/(HT3**2*NNN(3,1))
04206 ARR(12,20) = 0,0
04207 ARR(12,22) = 0,0
04210 ARR(17,19) = -6.*E1I1/(HT1**2*NNN(1,1))
04213 ARR(17,21) = -12.*E1I1/(HT1**3*NNN(1,1))
04217 ARR(18,19) = 0,0
04220 ARR(18,21) = 0,0
04221 ARR(19,20) = -6.*E3I3/(HT3**2*NNN(3,1))
04224 ARR(19,22) = -12.*E3I3/(HT3**3*NNN(3,1))
04230 ARR(20,20) = 0,0
04231 ARR(20,22) = 0,0
04232 ARR(35,19) = 2.*E1I1*BBB(1,2)/((HST1-HT1)*NNN(1,2))
04237 ARR(35,21) = -6.*E1I1/((HST1-HT1)**2*NNN(1,2))
04245 ARR(36,20) = 2.*E3I3*BBB(3,2)/((HST3-HT3)*NNN(3,2))
04252 ARR(36,22) = -6.*E3I3/((HST3-HT3)**2*NNN(3,2))
04260 ARR(37,19) = 6.*E1I1/((HST1-HT1)**2*NNN(1,2))
04264 ARR(37,21) = -12.*E1I1/((HST1-HT1)**3*NNN(1,2))
04271 ARR(38,20) = 6.*E3I3/((HST3-HT3)**2*NNN(3,2))
04276 ARR(38,22) = -12.*E3I3/((HST3-HT3)**3*NNN(3,2))
04303 IF(HT5,GE,HST5)GO TO 3006

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04306 ARR(13,15) = 2.*E5I5*BBB(5,1)/(HT5*NNN(5,1))
04312 ARR(13,17) = 6.*E5I5/(HT5**2*NNN(5,1))
04315 ARR(14,15) = 0,0
04316 ARR(14,17) = 0,0
04317 ARR(15,16) = 2.*E7I7*BBB(7,1)/(HT7*NNN(7,1))
04323 ARR(15,18) = 6.*E7I7/(HT7**2*NNN(7,1))
04326 ARR(16,16) = 0,0
04327 ARR(16,18) = 0,0
04330 ARR(21,15) = -6.*E5I5/(HT5**2*NNN(5,1))
04333 ARR(21,17) = -12.*E5I5/(HT5**3*NNN(5,1))
04337 ARR(22,15) = 0,0
04340 ARR(22,17) = 0,0

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04717      CARR(19,19) = 4.*E111*(AAA(1,2)/((HT1-HST1)*NNN(1,2))+AAA(1,3)/
*          ((L1-HT1)*NNN(1,3)))
04731      CARR(19,21) = 6.*E111*(1./((HT1-HST1)**2*NNN(1,2))-1./((L1-HT1)
*          **2*NNN(1,3)))
04743      CARR(20,20) = 4.*E313*(AAA(3,2)/((HT3-HST3)*NNN(3,2))+AAA(3,3)/
*          ((L3-HT3)*NNN(3,3)))
04755      CARR(20,22) = 6.*E313*(1./((HT3-HST3)**2*NNN(3,2))-1./((L3-HT3)**2
*          *NNN(3,3)))
04767      CARR(21,21) = KT1+12.*E111*(1./((L1-HT1)**3*NNN(1,3))+1./((HT1-HST
*          1)**3*NNN(1,2)))
05003      CARR(22,22) = KT3+12.*E313*(1./((L3-HT3)**3*NNN(3,3))+1./((HT3-HST
*          3)**3*NNN(3,2)))
05017      IF(TANK.EQ.0.0)GO TO 3010
05020      IF(HT1.GE.HST1)GO TO 3010

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INVERTED BOOSTER

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05023      CARR(19,19) = 4.*E111*(AAA(1,2)/((HST1-HT1)*NNN(1,2))+
*          AAA(1,1)/(HT1*NNN(1,1)))
05033      CARR(19,21) = 6.*E111*(1./((HT1**2*NNN(1,1))-1./
*          ((HST1-HT1)**2*NNN(1,2)))
05044      CARR(20,20) = 4.*E313*(AAA(3,2)/((HST3-HT3)*NNN(3,2))+
*          AAA(3,1)/(HT3*NNN(3,1)))
05055      CARR(20,22) = 6.*E313*(1./((HT3**2*NNN(3,1))-1./
*          ((HST3-HT3)**2*NNN(3,2)))
05066      CARR(21,21) = KT1+12.*E111*(1./((HT1**3*NNN(1,1))+1./
*          ((HST1-HT1)**3*NNN(1,2)))
05100      CARR(22,22) = KT3+12.*E313*(1./((HT3**3*NNN(3,1))+1./
*          ((HST3-HT3)**3*NNN(3,2)))
05113      IF(HT5.GE.HST5)GO TO 3010

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INVERTED ORBITER

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05116      CARR(15,15) = 4.*E515*(AAA(5,2)/((HST5-HT5)*NNN(5,2))+
*          AAA(5,1)/(HT5*NNN(5,1)))
05126      CARR(15,17) = 6.*E515*(1./((HT5**2*NNN(5,1))-1./
*          ((HST5-HT5)**2*NNN(5,2)))
05137      CARR(16,16) = 4.*E717*(AAA(7,2)/((HST7-HT7)*NNN(7,2))+
*          AAA(7,1)/(HT7*NNN(7,1)))
05150      CARR(16,18) = 6.*E717*(1./((HT7**2*NNN(7,1))-1./
*          ((HST7-HT7)**2*NNN(7,2)))
05161      CARR(17,17) = KT5+12.*E515*(1./((HT5**3*NNN(5,1))+1./
*          ((HST5-HT5)**3*NNN(5,2)))
05173      CARR(19,18) = KT7 + 12.*E717*(1./((HT7**3*NNN(7,1))+1./
*          ((HST7-HT7)**3*NNN(7,2)))

```

```

05206      3010 CONTINUE
C          GENERATE SYMMETRY FOR THE C-MATRIX
05206          K=1
05207          DO 375 I=2,JI
05211          DO 374 J=1,K
05212          374 CARR(I,J)=CARR(J,I)
05224          K=K+1
05225          375 CONTINUE
C          GENERATE THE D-MATRIX AS THE NEGATIVE TRANSPOSE OF A-MATRIX
C
05227          DO 95 I=1,JI
05231          DO 95 J=1,JC
05232          95 DARR(I,J)=-ARR(J,I)

```

```

04341 ARR(23,16) =-6.*E7I7/(HT7**2*NNN(7,1))
04344 ARR(23,18) =-12.*E7I7/(HT7**3*NNN(7,1))
04350 ARR(24,16) = 0.0
04351 ARR(24,18) = 0.0
04352 ARR(27,16) = 2.*E5I5*B8B(5,2)/((HST5-HT5)*NNN(5,2))
04357 ARR(27,17) =-6.*E5I5/((HST5-HT5)**2*NNN(5,2))
04365 ARR(28,16) = 2.*E7I7*B8B(7,2)/((HST7-HT7)*NNN(7,2))
04372 ARR(28,18) =-6.*E7I7/((HST7-HT7)**2*NNN(7,2))
04400 ARR(29,16) = 6.*E5I5/((HST5-HT5)**2*NNN(5,2))
04404 ARR(29,17) =-12.*E5I5/((HST5-HT5)**3*NNN(5,2))
04411 ARR(30,16) = 6.*E7I7/((HST7-HT7)**2*NNN(7,2))
04416 ARR(30,18) =-12.*E7I7/((HST7-HT7)**3*NNN(7,2))
04423 3006 CONTINUE
04423 CARR( 1, 1) = KUP3+KBUP3+KUP6
04426 CARR( 1, 2) =-KUP6
04427 CARR( 2, 2) = (KUP4+KBUP4+KUP6)
04432 CARR( 3, 3) = S1+4.*EUJUP1/L1
04436 CARR( 3, 4) = 2.*EUJUP1/L1
04437 CARR( 4, 4) = S2+4.*(EUJUP1/L1+EUJUP2/A)
04444 CARR( 4, 5) = 2.*EUJUP2/A
04446 CARR( 4,12) = 6.*EUJUP2/ A**2
04450 CARR( 5, 5) = 4.*EUJUP2*L2/(A*(L2-A))
04454 CARR( 5, 6) = 2.*EUJUP2/(L2-A)
04456 CARR( 5,12) = 6.*EUJUP2*(1./A**2-1./(L2-A)**2)
04463 CARR( 6, 6) = S3+4.*(EUJUP2/(L2-A)+EUJUP3/C)
04472 CARR( 6, 7) = 2.*EUJUP3/C
04474 CARR( 6,12) =-6.*EUJUP2/(L2-A)**2
04500 CARR( 6,13) = 6.*EUJUP3/ C**2
04503 CARR( 7, 7) = S6+4.*EUJUP3*L3/(C*(L3-C))
04511 CARR( 7, 8) = 2.*EUJUP3/(L3-C)
04513 CARR( 7,10) =-S6
04515 CARR( 7,13) = 6.*EUJUP3*(1./C**2-1./(L3-C)**2)
04522 CARR( 8, 8) = S4+4.*EUJUP3/(L3-C)
04527 CARR( 8,13) =-6.*EUJUP3/(L3-C)**2
04532 CARR( 9, 9) = S5+4.*EUJUP4/D
04535 CARR( 9,10) = 2.*EUJUP4/D
04537 CARR( 9,14) = 6.*EUJUP4/D**2
04541 CARR(10,10) = S6+4.*L6*EUJUP4/(D*(L6-D))
04546 CARR(10,11) = 2.*EUJUP4/(L6-D)
04551 CARR(10,14) = 6.*EUJUP4*(1./D**2-1./(L6-D)**2)
04556 CARR(11,11) = S7+4.*EUJUP4/(L6-D)
04563 CARR(11,14) =-6.*EUJUP4/(L6-D)**2
04566 CARR(12,12) = KUP5+12.*EUJUP2*(1./A**3+1./(L2-A)**3)
04576 CARR(13,13) = KUP7+12.*EUJUP3*(1./C**3+1./(L3-C)**3)
04606 CARR(13,14) =-KUP7
04607 CARR(14,14) = KUP7+12.*EUJUP4*(1./D**3+1./(L6-D)**3)
04617 CARR(15,15) = 4.*E5I5*(AAA(5,2)/((HT5-HST5)*NNN(5,2)) +AAA(5,3)/
* ((L5-HT5)*NNN(5,3)))
04631 CARR(15,17) = 6.*E5I5*(1./((HT5-HST5)**2*NNN(5,2))-1./((L5-HT5)**2
* NNN(5,3)))
04643 CARR(16,16) = 4.*E7I7*(AAA(7,2)/((HT7-HST7)*NNN(7,2))+AAA(7,3)/
* ((L7-HT7)*NNN(7,3)))
04655 CARR(16,18) = 6.*E7I7*(1./((HT7-HST7)**2*NNN(7,2))-1./((L7-HT7)**2
* NNN(7,3)))
04667 CARR(17,17) = KT5+12.*E5I5*(1./((L5-HT5)**3*NNN(5,3))+1./((HT5-HST
* 5)**3*NNN(5,2)))
04703 CARR(18,18) = KT7+12.*E7I7*(1./((L7-HT7)**3*NNN(7,3))+1./((HT7-HST
* 7)**3*NNN(7,2)))

```

```

05246      CALL MATINV(CARR,T,T1,J1,40,TOL,IERR)
05255      CALL MMATRX(ARR,CARR,PDUCT,JR,J1,J1,MXX)
05264      CALL MMATRX(PDUCT,DARR,ARR,JR,J1,JR,MXX)
05273      K=1
05274      DO 8 I=2,JC
05276      DO 7 J=1,K
05277      IF(ABS(ARR(I,J)),LT.1.E-15)GO TO 6
05305      DIFF=ARR(I,J)-ARR(J,I)
05312      PERCENT=DIFF/ARR(I,J)
05316      IF(ABS(PERCENT).GT.1.E-6)GO TO 9
05322      6 CONTINUE
05322      GO TO 7
05323      9 CONTINUE
05323      PRINT 550
05327      CALL MPRINT(ARR,JR,JC,40)
05332      GO TO 700
05333      7 CONTINUE
05336      K=K+1
05337      8 CONTINUE
05341      DO 400 I=1,JR
05343      DO 400 J=1,JC
05344      QARR(I,J)= QARR(I,J)+ARR(I,J)
05354      400 CONTINUE
C****      MULTIPLY THE INVERSE OF THE MASS MATRIX AND THE
C              QARR MATRIX
05360      CALL MMATRX(MASS,QARR,PDUCT,JR,JR,JR,MXX)
05367      PRINT 2000,KR,KW,KUP1,KUP2,KUP3,KBUP3,KUP4,KBUP4,KUP5,KUP6,KUP7,S6
05423      2000 FORMAT(1X,*KR=*E11.4,*KW=*E11.4,*KUP1=*E11.4,*KUP2=*E11.4,*KUP3=*
*E11.4,*KBUP3=*E11.4,*KUP4=*E11.4,*KBUP4=*E11.4,/1X,*KUP5=*E11.4,
* *KUP6=*E11.4,*KUP7=*E11.4,*S6=*E11.4)
05423      3000 FORMAT(1X,*F1=*E11.4,*KCB1=*E11.4,*KP1=*E11.4,/
* 1X,*F3=*E11.4,*KCB3=*E11.4,*KP3=*E11.4,/
* 1X,*F5=*E11.4,*KCB5=*E11.4,*KP5=*E11.4,/
* 1X,*F7=*E11.4,*KCB7=*E11.4,*KP7=*E11.4)
05423      PRINT 3000,(F(I),KCB(I),KP(I),I=1,7,2)
05441      5000 FORMAT(1X,*MT1=*E11.4,*MM=*E11.4,*MTDA=*E11.4,*MSTR=*E11.4,/
* 1X,*MT3=*E11.4,*MM=*E11.4,*MTDA=*E11.4,*MSTR=*E11.4,/
* 1X,*MT5=*E11.4,*MM=*E11.4,*MTDA=*E11.4,*MSTR=*E11.4,/
* 1X,*MT7=*E11.4,*MM=*E11.4,*MTDA=*E11.4,*MSTR=*E11.4)
05441      PRINT 5000,(MT(I),MM(I),MTDA(I),MSTR(I),I=1,7,2)
05461      4000 FORMAT(1X,*K1=*E11.4,*KB1=*E11.4,*KCAP2=*E11.4,*KCAP6=*E11.4,/
* 1X,*K3=*E11.4,*KB3=*E11.4,*KCAP3=*E11.4,/
* 1X,*K5=*E11.4,*KB5=*E11.4,*KCAP5=*E11.4,/
* 1X,*K7=*E11.4,*KB7=*E11.4,*KCAP7=*E11.4)
05461      PRINT 4000,K1,KB1,KCAP2,KCAP6,K3,KB3,KCAP3,K5,KB5,KCAP5,K7,
* KB7,KCAP7
05517      6000 FORMAT(1X,*HT1=*E11.4,*KT1=*E11.4,*HST1=*E11.4,*JSTR1=*E11.4,
* *JRIG=*E11.4,/
* 1X,*HT3=*E11.4,*KT3=*E11.4,*HST3=*E11.4,*JSTR3=*E11.4,
* *JRIG=*E11.4,/
* 1X,*HT5=*E11.4,*KT5=*E11.4,*HST5=*E11.4,*JSTR5=*E11.4,
* *JRIG=*E11.4,/
* 1X,*HT7=*E11.4,*KT7=*E11.4,*HST7=*E11.4,*JSTR7=*E11.4,
* *JRIG=*E11.4)
05517      PRINT 6000,HT1,KT1,HST1,JSTR(1),JRIG(1),
* HT3,KT3,HST3,JSTR(3),JRIG(3),
* HT5,KT5,HST5,JSTR(5),JRIG(5),
* HT7,KT7,HST7,JSTR(7),JRIG(7)

```

```
05573      CALL EIGEN(JC,PDUCT,QARR,QARR,MAXIT,NDEC,N1OPT,N2OPT,40)
05604      PRINT 500,(RHO(I),I=1,7,2),(HI(I),I=1,7,2),AS,KCAP1,ES
05632      DO 450 I=1,JC
05634      CYC(I,1)=PDUCT(I,1)
05640      CYC(I,2)=FLOAT(I)
05641 450 CONTINUE
05644      CALL SRTCYC(CYC,JR,40)
05646      PRINT 600,(CYC(I,2),CYC(I,1),I=1,JC)
05663      CALL SECOND(Y)
05665      TIME=Y*X
05667      PRINT 551,TIME
05675 551 FORMAT(118X,5HTIME F7.2)
05675 700 CONTINUE
05675      GO TO 50
05676 702 CONTINUE
05676      STOP
05700      END
```

```
000003      FUNCTION SINCH(X)  
000015      SINCH= .5*(EXP(X)-EXP(-X))  
000015      RETURN  
000015      END
```

```
000003      FUNCTION COSCH(X)
000015      COSCH= .5*(EXP(X)+EXP(-X))
000015      RETURN
000015      END
```

```
000012      SUBROUTINE MMATRIX(AA,BB,CC,NR,JR,JC,MD)
000012      C      SUBROUTINE MULTIPLIES THE AA AND BB MATRICES AND PLACES THE ANSWER IN
000013      DIMENSION AA(MD,MD),BB(MD,MD),CC(MD,MD)
000013      DO 102 I=1,NR
000014      DO 102 J=1,JC
000014      ELEM = 0.00
000015      DO 95 K=1,JR
000017      ELEM = ELEM + AA(I,K)*BB(K,J)
000032      95 CONTINUE
000034      CC(I,J)=ELEM
000040      102 CONTINUE
000044      RETURN
000044      END
```

```

SUBROUTINE MPRINT ( A , M , N , MD )
C      MATRIX PRINT SUBROUTINE
C      THE CALL FOR THIS SUBROUTINE IS AS FOLLOWS,
C      CALL MPRINT (A,M,N,MD)
C      WHERE A IS THE MATRIX TO BE PRINTED
C      M IS THE NUMBER OF ROWS
C      N IS THE NUMBER OF COLUMNS
C      MD IS DIMENSIONED NO. OF ROWS OF MATRIX A
000007 DIMENSION A ( 1 ) , JT ( 6 ) , C ( 6 )
000007 EQUIVALENCE ( JT , C )
000007 N1 = N
000010 N2 = 6
000011 N3 = 6
000012 N4 = 1
000013 100 IF ( N3 - N1 ) 120, 120, 110
C
000016 110 N2 = N1 - N3 + 6
000021 N3 = N1
000021 120 K = 0
000022 DO 130 I = N4, N3
000024 K = K + 1
000026 130 JT ( K ) = I
000032 PRINT 1, ( JT ( I ) , I = 1 , N2 )
000045 DO 150 I = 1, M
000051 K = 0
000052 L = MD * ( N4 - 1 ) + I
000056 DO 140 J = N4, N3
000057 K = K + 1
000061 C ( K ) = A ( L )
000064 140 L = L + MD
000070 150 PRINT 2, I , ( C ( K ) , K = 1 , N2 )
000111 IF ( N3 - N1 ) 160, 170, 170
C
000113 160 N3 = N3 + 6
000115 N4 = N4 + 6
000116 GO TO 100
C
000116 170 RETURN
C
000117 1 FORMAT (1H , 4X, 6( 6X, 7HCOLUMN 114 ) / )
000117 2 FORMAT (1H 114, X, (6E 17.8) )
000117 END
C3 BKY BESNIS

```

MP
MP
MP
MP
MP
MP
MP
MP
MP

	FORTTRAN IV SUBROUTINE BESNIS (X,NMAX,FI)	BSIS
C	X- A FLOATING POINT SINGLE PRECISION VARIABLE	
C	J- IS THE NUMBER OF VALUES	
C	FI- IS A ONE-DIMENSIONAL ARRAY OF ANSWERS	
C	TO EVALUATE $\exp(-x)I(x)$ MAKE THE FOLLOWING CHANGE	
C	$A=1./(PI(1)+SUM)$	
000006	DIMENSION FI(50), PI(200)	BSIS
000006	SUM=0.	BSIS
000006	I=X	BSIS
000010	JMAX=I+21	BSIS
000012	TZ=2./X	BSIS
000013	PI(JMAX+2)=0.	BSIS
000015	PI(JMAX+1)=1.E-20	BSIS
000017	DO 1 J=1,JMAX	BSIS
000020	K=JMAX+2-J	BSIS
000022	DK=K-1	BSIS
000024	PI(K-1)=DK*TZ*PI(K)+PI(K+1)	BSIS
000032	1 SUM=SUM+PI(K)	BSIS
000037	SUM=SUM+SUM	BSIS
000040	A=EXP(X)/(PI(1)+SUM)	BSIS
000047	DO 2 N=1,NMAX	BSIS
000050	2 FI(N)=A*PI(N)	BSIS
000055	RETURN	BSIS
000056	END	BSIS

```

SUBROUTINE SRTCYC(A,NROWS,NDIM)
C  SUBROUTINE SORTS THE FIRST COLUMN OF THE IN COMING ARRAY IN
C  ASCENDING ORDER WHILE THE CORRESPONDING ROW IN THE SECOND
C  COLUMN IS MOVED ALONG WITH IT, THE SUBROUTINE ALSO DIVIDES
C  THE SORT OF THE SORTED ELEMENT BY 2*PI IF THE ELEMENT IS NOT NEGATIVE
C  IF THE ELEMENT IS NEGATIVE IT IS PLACED BACK INTO THE ARRAY.
000006  DIMENSION A(1)
000006  DO 15 I=1,NROWS
000007  DO 14 J=1,NROWS
000010  IF(A(I)-A(J))14,14,11
000014  11 L=I-NDIM
000015  LL=J-NDIM
000017  DO 12 K=1,2
000021  L=L+NDIM
000022  LL=LL+NDIM
000023  F=A(L)
000025  A(L)=A(LL)
000027  12 A(LL)=F
000034  14 CONTINUE
000037  15 CONTINUE
000041  DO 20 I=1,NROWS
000042  IF(A(I).LT.0.0)GO TO 20
000044  A(I)=SQRT(A(I))/6.2831
000054  20 CONTINUE
000057  RETURN
000057  END

```

```

SUBROUTINE EIGEN ( N , B , TI , T , MAXIT , NDEC , N1OPT , N2OPT , EI
* NDIM )
C SUBROUTINE FOR GENERATING THE EIGENVALUES AND EIGENVECTORS
C OF A REAL SYMMETRIC OR NON-SYMMETRIC MATRIX.
C THIS PROGRAM GENERATES THE EIGENVALUE MATRIX (REAL OR COMPLEX),
C AND AS OPTIONS, THE EIGENVECTOR MATRIX AND ITS INVERSE,
C THE CALL FOR THIS SUBROUTINE IS AS FOLLOWS,
C CALL EIGEN (N,B,TI,T,MAXIT,NDEC,N1OPT,N2OPT,NDIM)
C WHERE N IS THE ORDER OF THE MATRIX
C B IS THE MATRIX WHOSE EIGENVALUES ARE DESIRED
C TI IS THE INVERSE OF THE EIGENVECTOR MATRIX
C T IS THE EIGENVECTOR MATRIX
C MAXIT IS MAX NO. OF ITERATIONS FOR GENERATING EIGENVALUES
C NDEC IS THE NUMBER OF TIMES RESULT IS REFINED
C N1OPT IS 1 IF OPTION =1 IS DESIRED, OTHERWISE 0.
C OPTION =1 PROVIDES FOR GENERATING THE EIGENVECTOR MATRIX INVERSE
C N2OPT IS 1 IF OPTION =2 IS DESIRED, OTHERWISE 0.
C OPTION =2 PROVIDES FOR GENERATING THE EIGENVECTOR MATRIX
C NDIM IS DIMENSIONED NO. OF ROWS OF MATRIX (B)
C THE ORIGINAL MATRIX B IS LOST DURING THE COMPUTATIONS AND IS
C REPLACED BY THE EIGENVALUE MATRIX.
000014 DIMENSION B ( 1 ) , TI ( 1 ) , T ( 1 )
C INITIALIZE COUNTERS FOR NO. OF ITERATIONS AND YR,YS REDUCTIONS
PRINT 1, N , NDEC , MAXIT
IT = 0
000014 NTIMES = 0
000025 ANORM = 0 . 0
000026 DO 100 I = 1, N
000030 DO 100 J = 1, N
000035 IJ = ( J - 1 ) * NDIM + I
000036 100 ANORM = ANORM + B ( IJ ) * * 2
000042 ANORM = SQRTF ( ANORM )
000052 DO 110 I = 1, N
000054 DO 110 J = 1, N
000061 IJ = ( J - 1 ) * NDIM + I
000062 110 B ( IJ ) = B ( IJ ) / ANORM
000066 C FORM IDENTITY MATRIX IN TI LOCATION IF OPTION 1 IS DESIRED
000076 120 IF ( N1OPT ) 160, 160, 130
C
000100 130 DO 150 I = 1, N
000102 II = ( I - 1 ) * NDIM + I
000106 DO 140 J = 1, N
000107 IJ = ( J - 1 ) * NDIM + I
000113 140 TI ( IJ ) = 0 .
000120 150 TI ( II ) = 1 . 0
C FORM IDENTITY MATRIX IN T LOCATION IF OPTION 2 DESIRED
000124 160 IF ( N2OPT ) 200, 200, 170
C
000126 170 DO 190 I = 1, N
000130 II = ( I - 1 ) * NDIM + I
000134 DO 180 J = 1, N
000135 IJ = ( J - 1 ) * NDIM + I
000141 180 T ( IJ ) = 0 .
000146 190 T ( II ) = 1 . 0
000152 200 CONTINUE
000152 YR = 10 . 0 E = 7
000153 YS = 10 . 0 E = 7

```

```

000155      210 NO = N - 1
000156      SUM = 10 . 0 E 20
000160      220 TAU = 0 . 0
000161      EN = 0 . 0
000162      DO 230 I = 1, NO
000163      JO = I + 1
000165      DO 230 J = JO, N
000167      IJ = ( J - 1 ) * NDIM + I
000173      JI = ( I - 1 ) * NDIM + J
000176      230 TAU = TAU + B ( IJ ) * * 2 + B ( JI ) * * 2
000211      DO 240 I = 1, N
000213      II = ( I - 1 ) * NDIM + I
000216      TE = B ( II )
000221      240 EN = EN + TE * * 2
000225      250 EN = EN + TAU
000227      260 DELN = SUM - EN
000231      270 IF ( DELN ) 290, 290, 280

C
000233      280 SUM = EN
000234      IT = IT + 1
000236      IF ( MAXIT - IT ) 970, 970, 310

C
000241      290 CONTINUE
000241      IF ( NDEC - NTIMES ) 970, 970, 300

C
000243      300 YR = YR / 100 . 0
000245      YS = YS / 100 . 0
000247      PRINT 2, NTIMES , IT
000256      NTIMES = NTIMES + 1
000260      IT = IT + 1
000261      310 DO 960 K = 1, NO
000266      KK = ( K - 1 ) * NDIM + K
000271      KO = K + 1
000273      320 DO 960 M = KO, N
000275      MM = ( M - 1 ) * NDIM + M
000300      KM = ( M - 1 ) * NDIM + K
000303      MK = ( K - 1 ) * NDIM + M
000307      H = 0 . 0
000307      G = 0 . 0
000310      HJ = 0 . 0
000312      330 DO 450 I = 1, N
000314      340 IF ( I - K ) 350, 450, 350

C
000316      350 IF ( I - M ) 360, 450, 360

C
000320      360 IK = ( K - 1 ) * NDIM + I
000324      KI = ( I - 1 ) * NDIM + K
000327      IM = ( M - 1 ) * NDIM + I
000333      MI = ( I - 1 ) * NDIM + M
000335      BQ = B ( IK )
000340      370 BR = B ( KI )
000343      380 BQ = B ( IM )
000346      390 BS = B ( MI )
000351      400 H = H + BR * BS - BQ * BQ
000356      410 TEP = BQ * BQ + BS * BS
000361      420 TEM = BR * BR + BQ * BQ
000364      430 G = G + TEP + TEM
000367      440 HJ = HJ - TEP + TEM

```

```

000156      ITEMP = ICOL ( MAXC )
000160      ICOL ( MAXC ) = ICOL ( 1 )
000162      ICOL ( 1 ) = ITEMP
000164      200 TEMP = A ( ITER )
000166      ITEMP = ICOL ( 1 )
000170      DO 210 J = 2, N
000172      ITJM1 = ( J - 2 ) * NDIM + ITER
000175      ITJ = ( J - 1 ) * NDIM + ITER
000200      A ( ITJM1 ) = A ( ITJ ) / TEMP
000204      210 ICOL ( J - 1 ) = ICOL ( J )
000211      ITN = ( N - 1 ) * NDIM + ITER
000214      A ( ITN ) = 1 . 0 / TEMP
000217      ICOL ( N ) = ITEMP
000221      DO 240 I = 1, N
000223      IF ( I = ITER ) 220, 240, 220
C
000225      220 TEMP = A ( I )
000230      DO 230 J = 2, N
000231      IJM1 = ( J - 2 ) * NDIM + I
000234      IJ = ( J - 1 ) * NDIM + I
000237      ITJM1 = ( J - 2 ) * NDIM + ITER
000242      A ( IJM1 ) = A ( IJ ) = A ( ITJM1 ) * TEMP
000251      230 CONTINUE
000254      IN = ( N - 1 ) * NDIM + I
000257      ITN = ( N - 1 ) * NDIM + ITER
000261      A ( IN ) = - ( TEMP * A ( ITN ) )
000267      240 CONTINUE
000274      DO 290 I = 1, N
000275      DO 250 J = I, N
000276      IF ( IROW ( J ) = I ) 250, 260, 250
C
000301      250 CONTINUE
000304      260 IF ( I = J ) 270, 290, 270
C
000306      270 DO 280 L = 1, N
000310      LI = ( I = 1 ) * NDIM + L
000313      LJ = ( J = 1 ) * NDIM + L
000316      TEMP = A ( LI )
000321      A ( LI ) = A ( LJ )
000324      280 A ( LJ ) = TEMP
000331      IROW ( J ) = IROW ( I )
000334      290 CONTINUE
000337      DO 340 I = 1, N
000340      DO 300 J = I, N
000341      IF ( ICOL ( J ) = I ) 300, 310, 300
C
000344      300 CONTINUE
000347      310 IF ( I = J ) 320, 340, 320
C
000351      320 DO 330 L = 1, N
000353      IL = ( L = 1 ) * NDIM + I
000356      JL = ( L = 1 ) * NDIM + J
000361      TEMP = A ( IL )
000364      A ( IL ) = A ( JL )
000367      330 A ( JL ) = TEMP
000374      ICOL ( J ) = ICOL ( I )
000377      340 CONTINUE
000402      IROW ( NP1 ) = 0

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000613      JK = ( K - 1 ) * NDIM + J      EI 1
000617      JM = ( M - 1 ) * NDIM + J      EI 1
000622      860 B0 = B ( JK )              EI 1
000625      870 BR = B ( JM )              EI 1
000630      880 B ( JK ) = B0 * C2 - BR * S2 EI 1
000635      890 B ( JM ) = - B0 * S1 + BR * C1 EI 1
000643      900 IF ( N1OPT ) 930, 930, 910 EI 1
C
000645      910 DO 920 J = 1, N              EI 1
000647      KJ = ( J - 1 ) * NDIM + K      EI 1
000653      MJ = ( J - 1 ) * NDIM + M      EI 1
000656      BQ = TI ( KJ )                  EI 1
000661      BS = TI ( MJ )                  EI 1
000663      TI ( KJ ) = C1 * BQ + S1 * BS    EI 1
000670      920 TI ( MJ ) = S2 * BQ + C2 * BS EI 1
000676      930 IF ( N2OPT ) 960, 960, 940 EI 1
C
000700      940 DO 950 J = 1, N              EI 1
000702      JK = ( K - 1 ) * NDIM + J      EI 1
000706      JM = ( M - 1 ) * NDIM + J      EI 1
000711      B0 = T ( JK )                  EI 1
000713      BR = T ( JM )                  EI 1
000716      T ( JK ) = B0 * C2 - BR * S2    EI 1
000723      950 T ( JM ) = - B0 * S1 + BR * C1 EI 1
000731      960 CONTINUE                    EI 1
000736      GO TO 220                       EI 2
C
000737      970 DO 980 I = 1, N              EI 2
000741      DO 980 J = 1, N                EI 2
000742      IJ = ( J - 1 ) * NDIM + I      EI 2
000746      980 B ( IJ ) = B ( IJ ) * ANORM EI 2
000756      PRINT 3, NTIMES, IT             EI 2
000765      RETURN                           EI 2
C
000766      1 FORMAT(*EIGEN SUBROUTINE STATISTICS*/ * SIZE OF MATRIX IS*,14, EI 2
      * * MAX. NUM. OF REFN. IS *,13, * MAX. NUM. OF ITER. IS *,14,/) EI 2
000766      2 FORMAT(*OREFN,*,13,* FINISHED ON ITER,*,14) EI 2
000766      3 FORMAT(*OEXIT ON REFN,*,13,* ITER,*,14) EI 2
000766      END                               EI 2

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000012      SUBROUTINE MATINV ( A , IROW , ICOL , N , NDIM , SMLST , IERR )
C      DIMENSION A ( 1 ) , IROW ( 1 ) , ICOL ( 1 )
C      709-16065
C      709-16065  SUBROUTINE MATINV = MATRIX INVERSION ROUTINE
C
C      A = ARRAY NAME OF MATRIX
C      IROW = DIMENSIONED AT N+1 OR GREATER
C      ICOL = DIMENSIONED AT N OR GREATER
C      N = NUMBER OF EQUATIONS
C      NDIM = VALUE OF I IN DIMENSION A(I,J) , I AND J MAY DIFFER
C      SMLST = SMALLEST LEADING ELEMENT ALLOWED BEFORE CALLING THE
C      SYSTEM SINGULAR , USUALLY = 1.0 E-04 OR 1.0 E-05
C      IERR = ERROR INDICATOR
000012      IERR = 0
000012      NP1 = N + 1
000015      DO 100 I = 1, N
000016      ICOL ( I ) = I
000017      100 IROW ( I ) = I
000022      DO 240 ITER = 1, N
000024      MAXR = ITER
000025      MAXC = 1
000026      TEMP = ABSF ( A ( MAXR ) )
000031      LIMITC = NP1 - ITER
000033      DO 120 I = ITER, N
000034      DO 120 J = 1, LIMITC
000035      IJ = ( J - 1 ) * NDIM + I
000040      IF ( TEMP = ( ABSF ( A ( IJ ) ) ) ) 110, 120, 120
C
000045      110 MAXR = I
000046      MAXC = J
000050      TEMP = ABSF ( A ( IJ ) )
000053      120 CONTINUE
000060      IF ( TEMP = SMLST ) 130, 130, 140
C
000062      130 IROW ( NP1 ) = ITER
000065      PRINT 1, ITER , SMLST
000074      IERR = 1
000076      RETURN
C
000076      140 IF ( MAXR = ITER ) 150, 170, 150
C
000104      150 DO 160 J = 1, N
000106      MAXRJ = ( J - 1 ) * NDIM + MAXR
000111      ITJ = ( J - 1 ) * NDIM + ITER
000114      TEMP = A ( MAXRJ )
000117      A ( MAXRJ ) = A ( ITJ )
000122      160 A ( ITJ ) = TEMP
000127      ITEMP = IROW ( MAXR )
000131      IROW ( MAXR ) = IROW ( ITER )
000134      IROW ( ITER ) = ITEMP
000136      170 IF ( MAXC = 1 ) 180, 200, 180
C
000140      180 DO 190 I = 1, N
000142      IMAXC = ( MAXC - 1 ) * NDIM + I
000145      TEMP = A ( I )
000147      A ( I ) = A ( IMAXC )
000153      190 A ( IMAXC ) = TEMP

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000372      450 CONTINUE
000375      460 H = 2 . 0 * H
000377      D = A ( KK ) - B ( MM )
000403      TEP = D ( KM )
000405      TEM = B ( MK )
000410      C = TEP + TEM
000411      E = TEP - TEM
000413      470 IF ( ABSF ( C ) - YR ) 480, 480, 510
C
000417      480 CC = 1 . 0
000421      490 SS = 0 . 0
000422      500 GO TO 620
C
000423      510 BY = D / C
000425      520 IF ( BY ) 530, 540, 540
C
000427      530 SIG = - 1 . 0
000431      GO TO 550
C
000431      540 SIG = 1 . 0
000433      550 COT = BY + ( SIG * SQRTF ( BY * BY + 1 , 0 ) )
000442      560 SS = SIG / SQRTF ( COT * COT + 1 , 0 )
000450      570 CC = SS * COT
000452      580 TEP = CC * CC - SS * SS
000455      590 TEM = 2 . 0 * SS * CC
000460      600 D = D * TEP + C * TEM
000464      610 H = H * TEP - HJ * TEM
000470      620 CONTINUE
000470      630 ED = 2 , 0 * E * D
000473      640 EDH = ED - H
000475      650 DEN = G + 2 , 0 * ( E * E + D * D )
000502      660 TEE = EDH / ( DEN + DEN )
000505      670 CONTINUE
000505      680 IF ( ABSF ( TEE ) - YS ) 690, 690, 710
C
000514      690 CH = 1 . 0
000516      700 SH = 0 . 0
000517      GO TO 730
C
000520      710 CH = 1 . 0 / SQRTF ( 1 , 0 - TEE * TEE )
000526      720 SH = TEE * CH
000530      730 C1 = CH * CC - SH * SS
000534      740 C2 = CH * CC + SH * SS
000540      750 S1 = CH * SS + SH * CC
000544      760 S2 = - CH * SS + SH * CC
000550      770 CONTINUE
000550      780 IF ( S1 ) 800, 790, 800
C
000555      790 IF ( S2 ) 800, 960, 800
C
000556      800 DO 840 J = 1, N
000560      KJ = ( J - 1 ) * NDIM + K
000564      MJ = ( J - 1 ) * NDIM + M
000570      810 B0 = B ( KJ )
000573      820 BR = B ( MJ )
000576      830 B ( KJ ) = C1 * B0 + S1 * BR
000603      840 B ( MJ ) = S2 * B0 + C2 * BR
000611      850 DO 890 J = 1, N

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000404		RETURN	MA 1
	C		MA 1
000404		1 FORMAT (7H00N THE13.63TH ITERATION ALL THE REMAINING TERMS WERE	LMA 1
		LESS THAN OR EQUAL TO E11.4,18H IN ABSOLUTE VALUE)	MA 1
000404		END	MA 1