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GREASE LUBRICATED SPIRAL GROOVE GYRO BEARINGS

by

J. T. McCabe

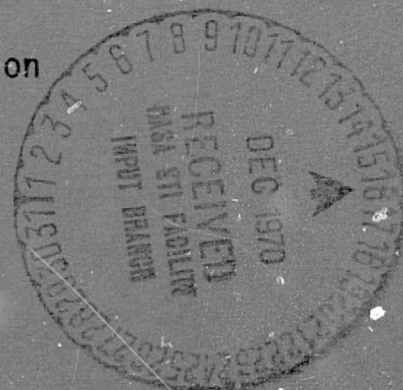
T. Y. Chu

October 1969

Prepared for

GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama
National Aeronautics and Space Administration

Contract No. NAS8-20741



THE FRANKLIN INSTITUTE RESEARCH LABORATORIES
BENJAMIN FRANKLIN PARKWAY • PHILADELPHIA, PENNA. 19103

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ABSTRACT

Grease lubricated spiral groove conical bearings were investigated theoretically by the finite element method and experimentally in several types of instrumented test rigs. A special class of lubricants developed and tested specifically for application to the spin axis bearings of a 24,000 rpm gyro rotor.



FOREWORD

The work described in this report was performed under Contract NAS8-20741, Marshall Space Flight Center, Mr. J. L. Burch, Principal, and Mr. P. H. Broussard, Alternate.

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1. INTRODUCTION

A grooved shaft rotating in a close-fitting sleeve can develop high pressure when designed as a viscous pump. The same principle can be used to design fluid film load supports called spiral groove bearings. When grease is employed as the lubricant, small bearing components develop substantial pressures at low surface velocities making possible a high film thickness to power loss ratio. Self-sealing and dependable boundary lubrication broadens the application potential of the grease/spiral groove combination.

The object of the present study was to determine by analysis and testing if grease lubricated spiral groove bearings are suitable for the spin axis of a gyro such as might be used for an auto pilot or a platform stabilizing momentum wheel.

Previous investigators (Ref. 1, 6) have analyzed, designed and tested grease lubricated spiral groove bearings for similar applications but in general, design information is available for the concentric clearance case only and no attempt has been made to formulate a lubricant specifically for the application. In this report, a general analysis is developed and correlated, analytical and test data are presented and properties of a class of lubricants suitable for high speed gyro spin axis bearings are tabulated.

2. SUMMARY

Bearings fitted with spiral grooves and lubricated with grease will develop a full fluid film, provide high load capacity, excellent tolerance to overload and start-stop rubbing and have the potential for a long operating life, provided:

- the bearing design is such that the net lubricant flow out of the clearance space is zero.
- the lubricant physical and chemical properties remain relatively constant over the operating life.

During the course of this program a numerical implemented analysis based on the Finite Element method was developed for the general class of spiral groove bearings lubricated with an isoviscous, Newtonian fluid. A test facility was constructed specifically for the purpose of validating the analysis and the results of this test program showed the analysis to be accurate. The latter tests, performed with an instrumented ten centimeter diameter journal, also provided data on film pressure and flow pattern history and thus gave an insight for the requirements necessary to insure a successful bearing design.

A dead-ended bearing with a spiral groove conical journal was analyzed and general parametric data was generated. In addition, optimization and manufacturing tolerance data such as the influence of cone axis misalignment is also presented.

Experiments were conducted with air turbine and motor driven rotors supported by conical bearings with a base diameter of 3 mm. These tests with rotors of angular momentum about $3 \times 10^6 \text{ g cm}^2/\text{sec}$ indicated the feasibility of employing spiral groove bearings in small gyros or large momentum wheels provided the bearings meet the two requirements listed above.



With the aid of the Air Force Materials Laboratory and Philips Research of the Netherlands, a parallel program was completed in which greases demonstrating shear stability and having predictable and reproducible properties were developed and tested. The properties of the base fluid, thickener and grease formulation are presented and the results of viscosity and vapor pressure measurements are also given. At the high shear rates of the present application these tests showed the lubricants to be very slightly pseudo-plastic and therefore treatable as Newtonian fluids. About 40 of the lubricants were evaluated in a 24,000 rpm spin axis bearing application and ranked with respect to maximizing the ratio of film thickness to power loss. The ranking indicated the existence of optimum constituent and formulation properties for each class of grease.

The theoretical and experimental results obtained from this program for the development of a spiral groove grease lubricated gyro bearing system indicate additional study, concentrating on the entrance region of the bearing, is needed and that the effort is warranted by virtue of the net advantages of this type of bearing system.

3. BEARING ANALYTICAL STUDIES

Two types of analyses were employed in treating dead-ended conical spiral groove bearings. Both were concerned with obtaining steady-state characteristics only. The first was an approximate analysis (Ref. 1) used during the feasibility study. The second was an exact solution using numerical methods (Ref. 2). The approximate analysis assumed a linear pressure distribution, required a uniform film thickness, assumed an infinite number of grooves and did not satisfy the boundary condition of constant pressure at the film entrance. Nevertheless, for the case of constant film thickness, the results of the approximate analysis showed good agreement with the exact analysis because 1), the pressure distribution was essentially linear, 2) a finite number of grooves (as few as 7) did not substantially reduce the load capacity and 3) deviations from the constant boundary pressure in the approximate analyses were small.

The approximate solution had the advantage of requiring little computational time and was therefore ideal for sizing the bearing during the feasibility study as well as for determining the influence of geometric characteristics such as groove angle and groove depth on bearing performance characteristics. The exact analysis was contrived to eliminate the deficiencies of the approximate analysis. This analysis solved the appropriate fluid film equation with the proper boundary conditions. It was not restricted to an infinite number of grooves nor a uniform film thickness. Thus, radial load capacity, radial stiffness and the relation between radial displacement and the bearing geometric parameters could be determined, and the majority of the design data presented here is based on this analysis.



3.1 Approximate Analysis

3.1.1 General Comments

The approximate analysis was based on the parallel groove pattern and pressure profile shown on Figures 3-1 and 3-2. Note that Figure 3-2 implies that a boundary condition of constant pressure at the groove termination can not be satisfied without the addition of "correction" terms to the basic mathematical formulation.

A comparison with the exact analysis (described in Section 3.3) can be found in Appendix A. The comparison showed good agreement for load capacity and fair agreement for friction power loss.

3.1.2 Mathematical Basis

For an element of a dead-ended conical bearing (no transverse flow), the pressure rise Δp and the magnitude of the friction force $|F|$ are given by (Ref. 1)

$$\frac{\Delta p h_o^2}{6\eta U d} = \frac{h_o^2}{h_2^2} g_1(H, \gamma, \alpha) \quad (3.1)$$

and

$$\frac{|F| h_o}{\eta U d a_1} = \frac{h_o}{h_2} g_2(H, \gamma, \alpha) \quad (3.2)$$

where

$$g_1(H, \gamma, \alpha) = \frac{\gamma H^2 \cot \alpha (1 - H)(1 - H^3)}{(1 + \gamma H^3)(\gamma + H^3) + H^3 \cot^2 \alpha (1 + \gamma)^2}$$

$$g_2(H, \gamma, \alpha) = \gamma + H + g_1(h, \gamma, \alpha)$$

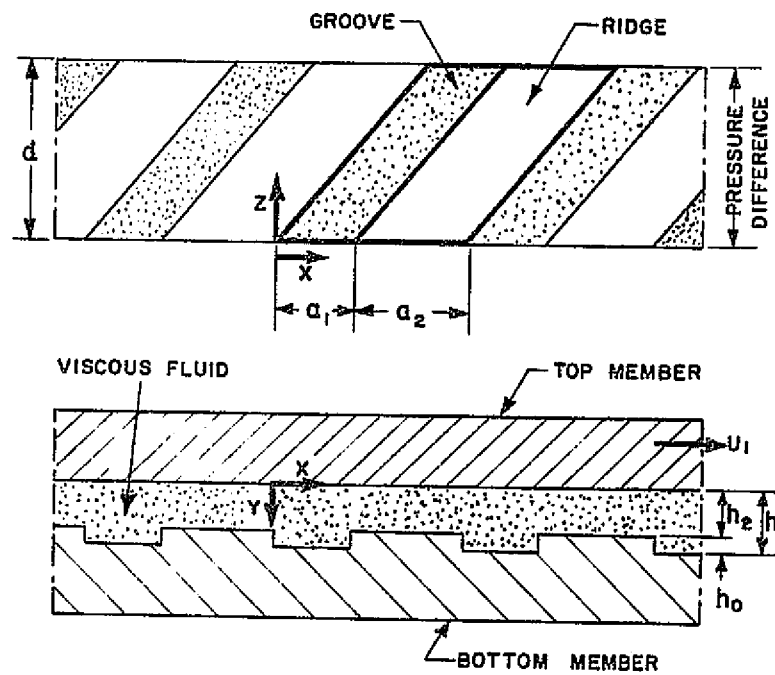


FIG. 3-1 MODEL OF PARALLEL SPIRAL GROOVE PATTERN

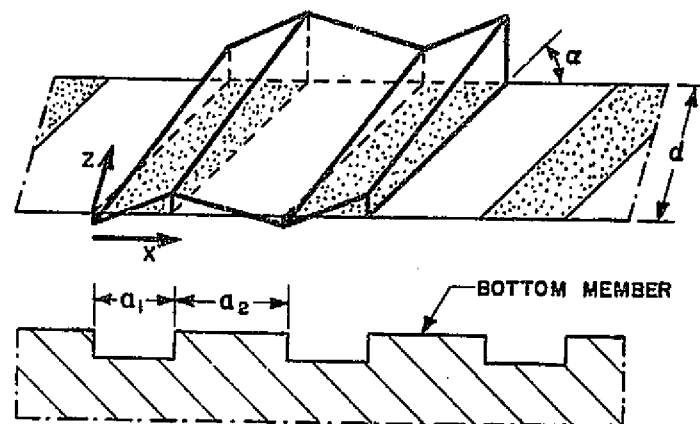


FIG. 3-2 LINEAR PRESSURE PROFILE

a_1, a_2 = groove width, land width

$$H = \frac{h_2}{h_1} = \frac{\delta}{1 + \delta}$$

$$\delta = h_2/h_o$$

$$\gamma = a_2/a_1$$

U = velocity of bearing surface

η = dynamic viscosity

α = angle between direction of U_1 and the groove tangent

d = width of the basic element

The width of the basic element for the conical geometry shown on Figure 3-3 is

$$d = \frac{dy}{\cos t} \quad (3.3)$$

where dy is a differential element.

The surface velocity is

$$U = (r_2 - y \tan t)\omega \quad (3.4)$$

where ω is the angular velocity.

The pressure rise is

$$\frac{(p - p_a) h_o^2}{6\eta\omega} = \left(\frac{h_o}{h_2}\right)^2 \frac{g_1}{\cos t} \left(r_2 y - \frac{y^2}{2} \tan t\right) \quad (3.5)$$

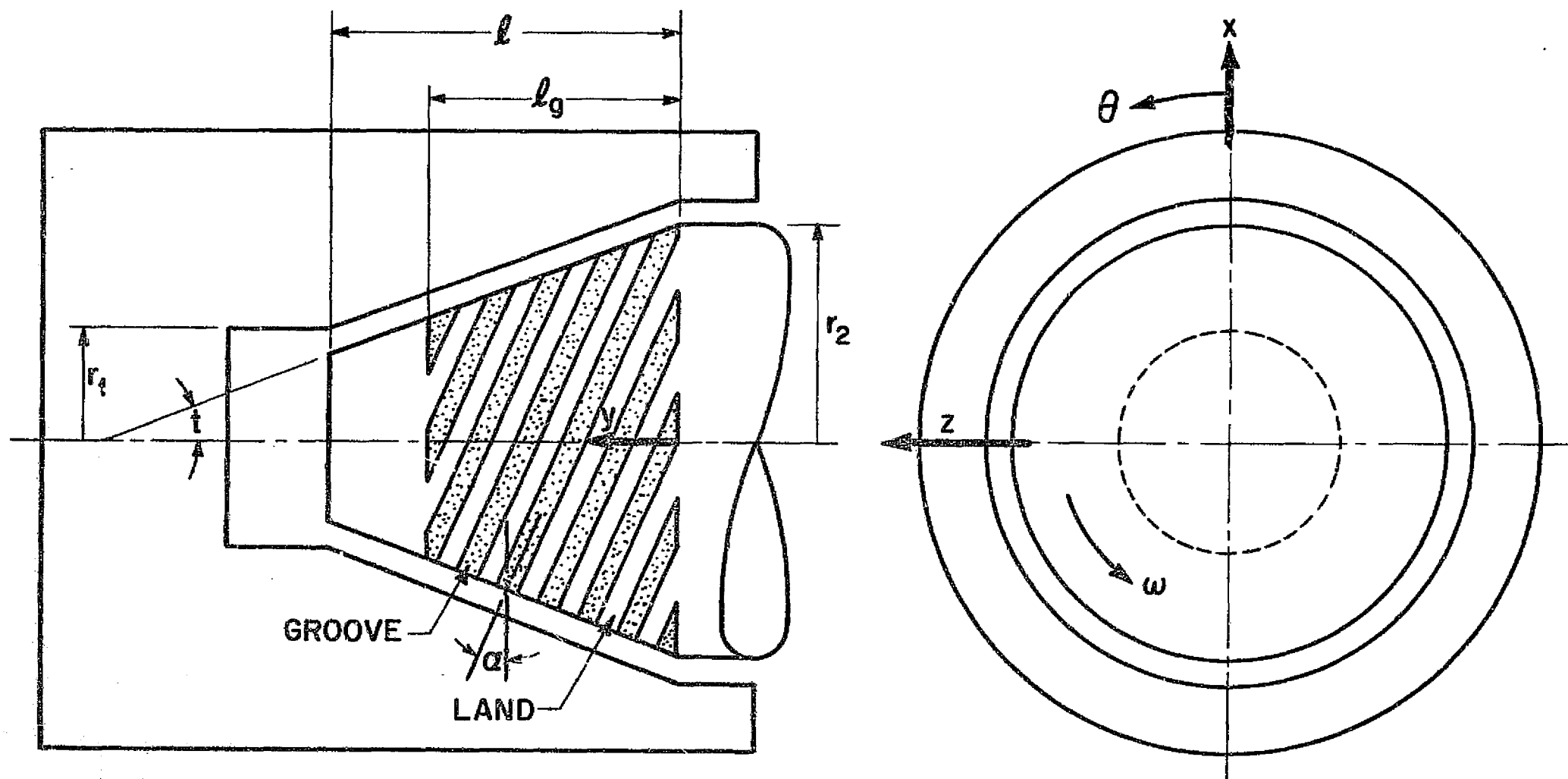
The dimensionless thrust load W_T and friction moment M are given by

$$W_T = \frac{w_t h_o^2}{6\eta\omega} = \pi \left(\frac{h_o}{h_2}\right)^2 g_1 C_1 \left(\frac{\tan t}{\cos t} C_1 + C_2\right) \quad (3.6)$$

where

$$C_1 = \frac{l}{r_2} \left(1 - \frac{l}{2r_2} \tan t\right)$$

$$C_2 = 1 - \frac{l}{r_2} \tan t$$



Bearing Specifications

$$r_2 = 4.76 \text{ mm}$$

$$t = 15^\circ$$

$$\alpha = 15^\circ$$

$$l = 4.165 \text{ mm}$$

$$l_g = 2.388 \text{ mm}$$

Number of grooves = 7

Land to groove width ratio = 1

Groove depth = 0.025 mm

FIG. 3-3 GEOMETRIC PARAMETERS OF CONICAL BEARING

and

$$M = \frac{m h_o}{\eta \omega r_2^4} = \left(\frac{h_o}{h_2} \right)^2 g_2 \left(\frac{l}{r_2} \right) \frac{1}{\cos t (1 + \gamma)} \left\{ 1 - \frac{3}{2} l \tan t + \left(\frac{l}{r_2} \right)^2 \tan^2 t - \frac{1}{4} \left(\frac{l}{r_2} \right)^3 \tan^3 t \right\} \quad (3.7)$$

The dimensional load and friction moment are designated by w_t and m respectively.

3.1.3 Results of Approximate Analysis

Although the approximate analysis was used extensively in the initial feasibility study, the results presented here cover only variations in those parameters not treated by the exact analysis. Other results using the approximate analysis may be found in Ref. 3.

Variation In Land To Groove Ratio (γ)

Figure 3-4 shows the relation between the axial load and friction moment for a fully grooved conical bearing. The insert on this figure shows the fall-off in friction moment as γ is reduced holding the load constant. Figure 3-5 shows the sensitivity of the axial load and stiffness to γ for constant concentric clearance. Between $\gamma = 0.8$ and 1.2, the variation of both load and stiffness is negligible. Thus, grooves of constant width, cut by one pass of a single point tool or grinding wheel can sometimes be employed with little deviation from optimum performance.

Variation In Length To Radius Ratio (L/r_2)

Figures 3-6 and 3-7 show the change in axial load and friction moment with L/r_2 . Note that although the pressure rise increases linearly as the groove length is increased, the groove length is not linear in L/r_2 . Thus, there is a rapid slow-down in the rate of increase of W_T with respect to L/r_2 because of the conical geometry. The same is true of the friction moment.



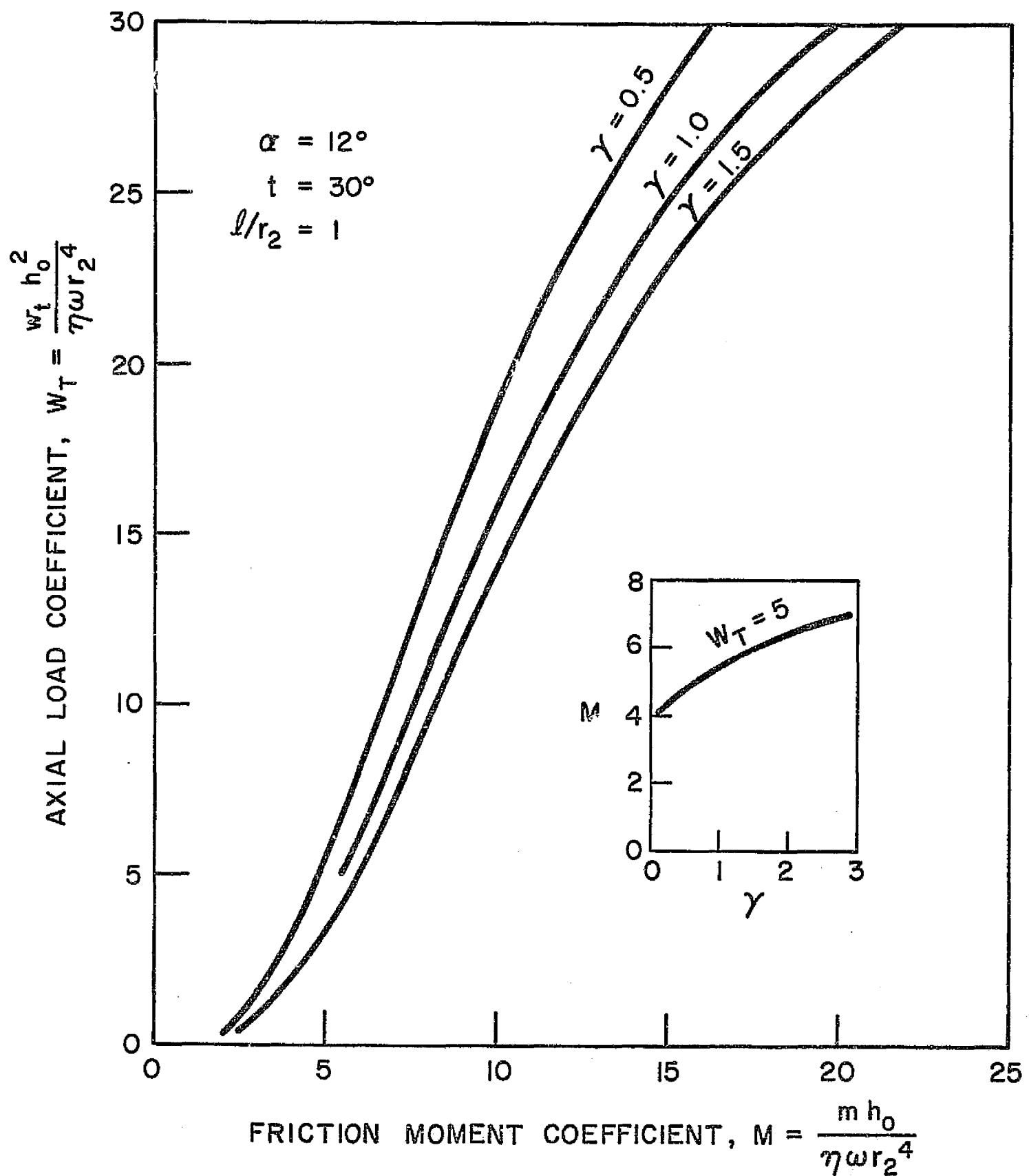


FIG. 3-4 INFLUENCE OF LAND TO GROOVE WIDTH RATIO ON AXIAL LOAD AND FRICTION MOMENT COEFFICIENTS

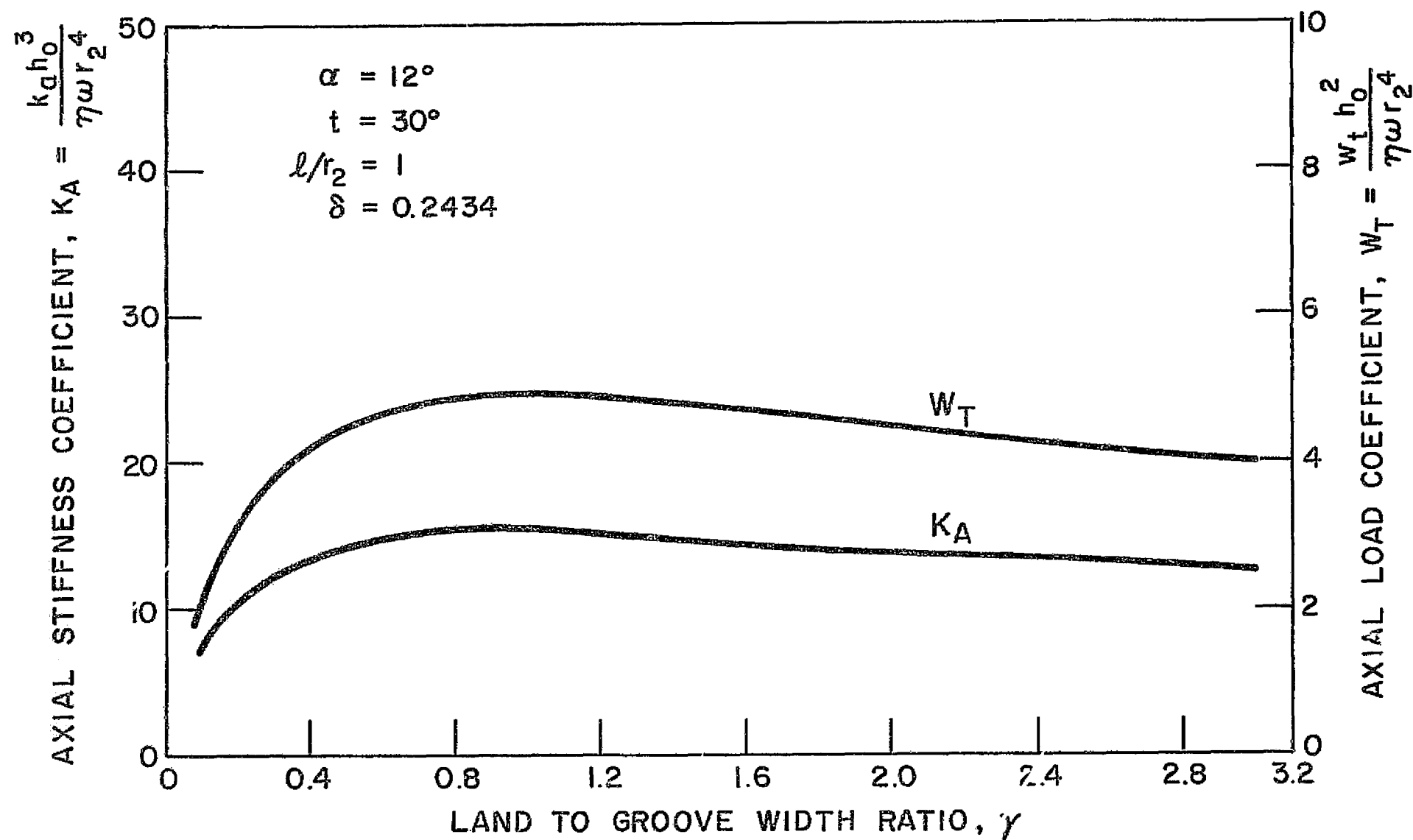


FIG. 3-5 OPTIMIZATION OF AXIAL LOAD OR AXIAL STIFFNESS COEFFICIENTS WITH RESPECT TO LAND-TO-GROOVE WIDTH RATIO

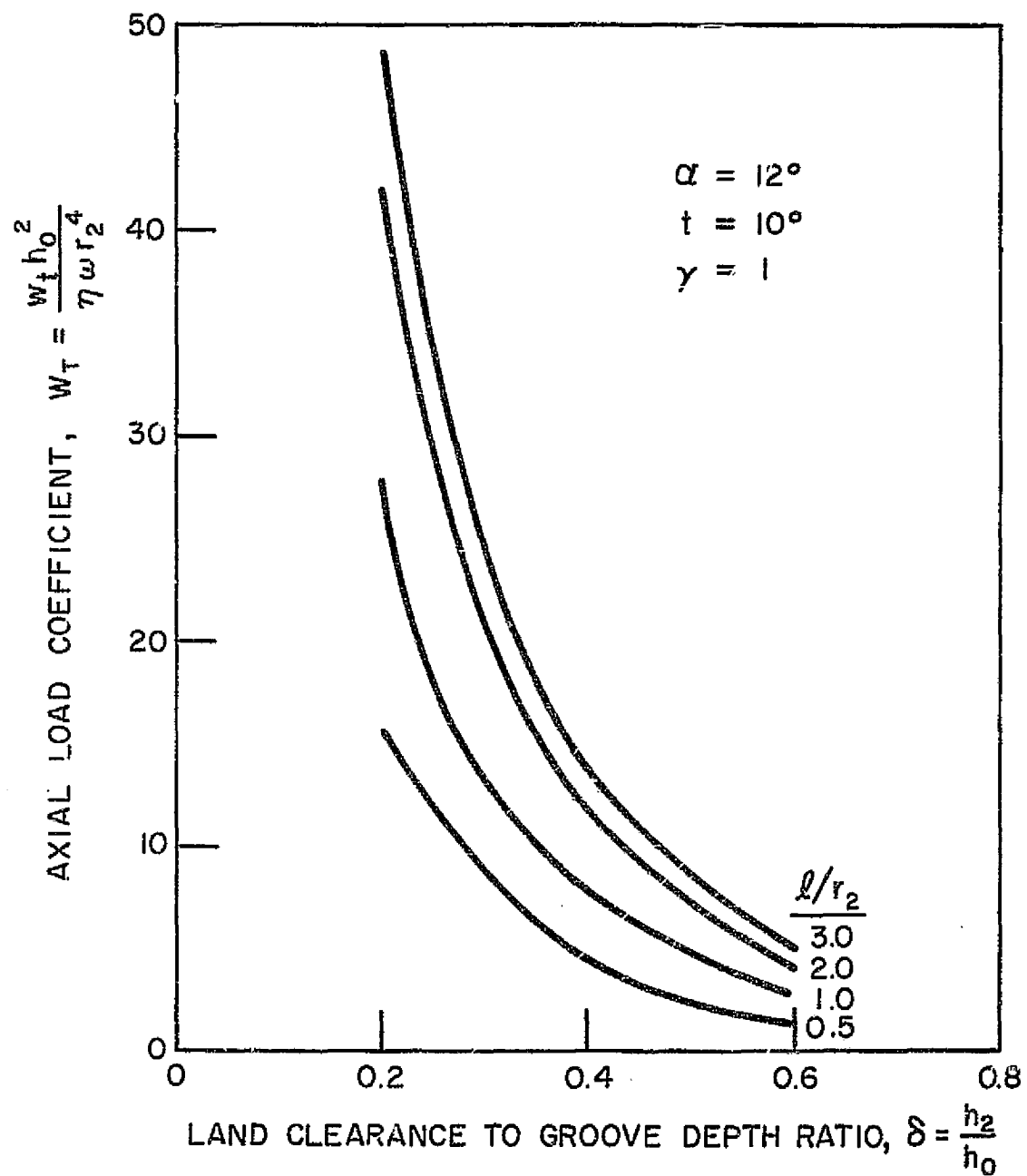


FIG. 3-6 INFLUENCE OF CONE LENGTH TO BASE RADIUS RATIO ON AXIAL LOAD COEFFICIENT

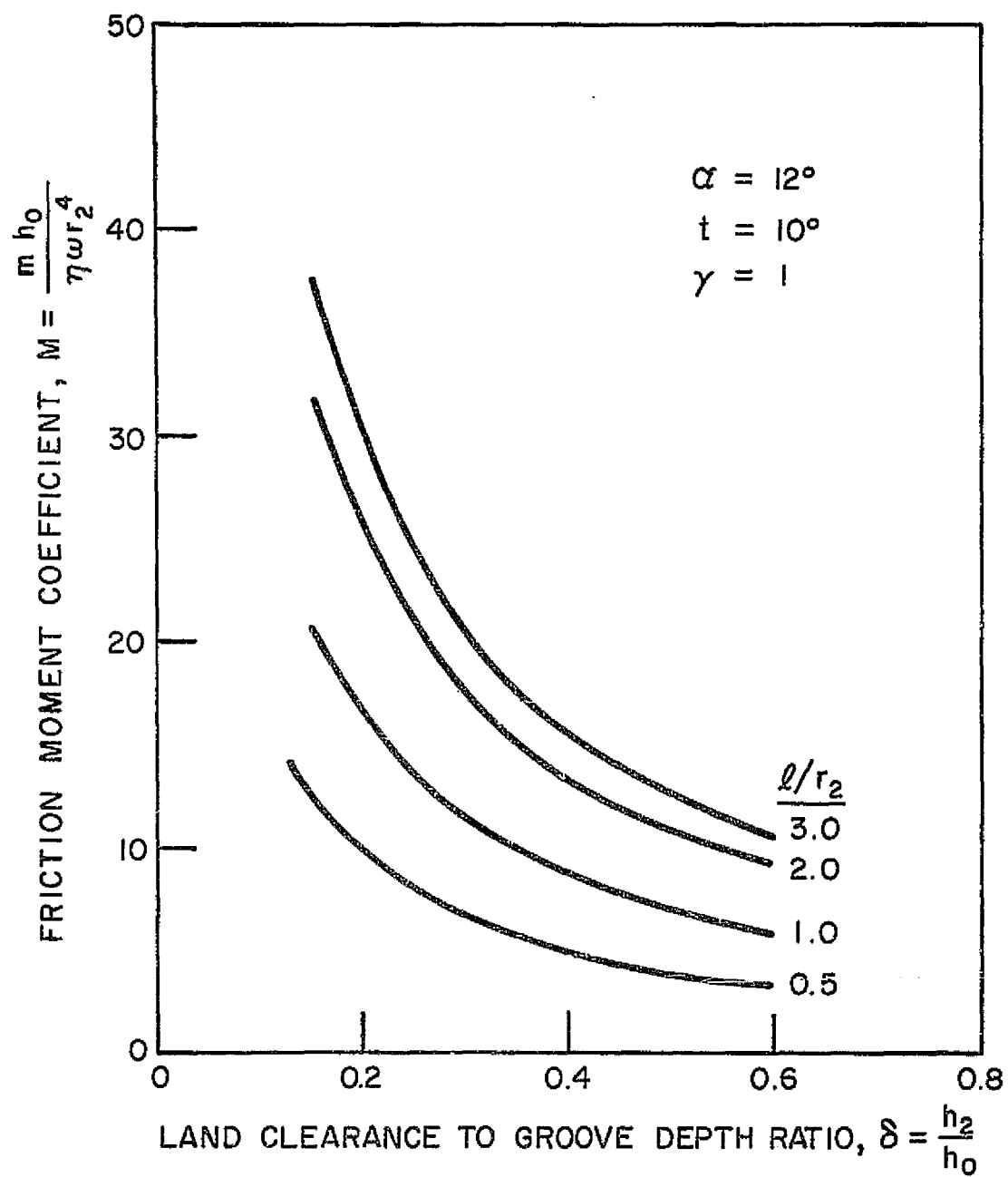


FIG. 3-7 INFLUENCE OF CONE LENGTH TO BASE RADIUS RATIO ON FRICTION MOMENT COEFFICIENT

3.2 Exact Analysis

3.2.1 General Comments

The term "exact analysis" was applied to the numerical procedure called the Finite Element method which was used to obtain general steady-state performance characteristics of the spiral groove bearing.

The exact analysis was formulated to treat the case of a conical bearing with a finite number of grooves and non-uniform film thickness. A non-uniform film thickness analysis was necessary for computing radial load capacity and stiffness because design data for this purpose was not available in current literature. General parametric data for the spiral groove conical bearing was also not available.

The data obtained by the exact analysis and presented below covers variations in the cone angle, groove angle, concentric land clearance and radial eccentricity ratio. Axial and radial load capacity and stiffness, friction moment and optimizations of load and stiffness with respect to power loss are plotted.

3.2.2 Mathematical Basis

It has been shown (Ref. 2) that the general incompressible problem of determining the relation between pressure and clearance is equivalent to the problem of minimizing the functional $\Phi\{p\}$ given by

$$\begin{aligned} \Phi\{p\} = \int_R & - \left(-\frac{h^3}{12\eta} \nabla p \cdot \nabla p + h \vec{U} \cdot \nabla p - 2\dot{h}p \right) dA \\ & + 2 \int_{C_2} q p ds \end{aligned} \quad (3.8)$$

where q is the normal flow prescribed across the boundary segment C_2 of the region R and

$h, \dot{h}$ = clearance and its time derivative

∇p = grad p (pressure)

$\vec{U}$ = velocity vector



The procedure for minimizing the functional and computing the unknown pressure field was called the Finite Element Method and consisted of the following steps:

- The grooved conical bearing surface was developed onto a plane, and was divided into quadrilateral areas called finite elements.
- Each element was formed by straight lines connecting the nodes placed at the intersection of circular arcs and logarithmic spiral lines.
- During computation, each quadrilateral was further subdivided into four triangular elements.
- Within each triangle the pressure distribution was assumed to be of the form

$$p^i(x,y) = \sum_{k=1}^3 f_k(x,y) p_k \quad (3.9)$$

where p_k denoted pressures at the nodes of the i^{th} element and the functions f_k were expressed explicitly in terms of the nodal coordinates of the developed surface.

- Substitution of 3.9 into 3.8 yielded an expression for the functional $\Phi\{p\}$ in terms of the nodal pressures.
- A system of linear algebraic equations for the unknown nodal pressures was obtained by using the equation

$$\delta \Phi\{p\} = 0$$

where $\delta \Phi\{p\}$ denotes the first variation of $\Phi\{p\}$.

- The boundary conditions were taken as

$$p = p_a \text{ at } r = r_2 \quad (\text{Figure 3-3})$$

$$\left. \begin{array}{l} p = p_e \text{ (unknown)} \\ \text{net flow} = 0 \end{array} \right\} \text{ at } r = r_1$$

- The system of equations was solved by Gaussian elimination for the unknown nodal pressures.
- Referring to Figure 3-3, the net film force components acting on the rotor were computed as

$$\begin{aligned}
 F_x &= \sum_{i=1}^N \int_{A_i} -p_i \cos t \cos \theta \, dA \\
 F_z &= \sum_{i=1}^N \int_{A_i} -p_i \cos t \sin \theta \, dA \\
 F_y &= \sum_{i=1}^N \int_{A_i} p_i \sin t \, dA + \pi r_1^2 p_e
 \end{aligned}
 \tag{3.10}$$

where the integration was taken over the area of the i^{th} element and the results summed for the N elements that composed the complete pressure field. The second term in the expression for F_y accounts for the force acting on the truncated end of the cone due to the "boundary" pressure p_e .

- The friction moment M was computed as

$$\begin{aligned}
 M &= -\omega \sin^2 t \sum_{i=1}^N \int \frac{\eta}{h} (x^2 + y^2) \, dA \\
 &\quad - \frac{\sin t}{2} \int h \left(-y \frac{\partial p}{\partial x} + x \frac{\partial p}{\partial y} \right) \, dA
 \end{aligned}
 \tag{3.11}$$

Appendix B gives more detail on the application of the finite element method to spiral groove conical bearings.

3.2.3 Results of Exact Analysis

The results of the exact analysis are presented both in tabular form and in graphs. The performance data of all cases that have been investigated are given in Table 3-1. The influences of various parameters on the bearing performance are shown in Figure 3-8 thru Figure 3-33. Optimum curves of load, friction moment and stiffness obtained through manipulation of the basic data are presented in Figure 3-34 thru Figure 3-40.

TABLE 3-1

COMPUTED PARAMETRIC PERFORMANCE DATA

$$\gamma = 1, \ell/r_2 = 1.75, N_g = 7$$

$$t = 10^\circ \quad \epsilon_r = 0 \quad \ell_g/r_2 = 1.02$$

δ	$\alpha = 10^\circ$		$\alpha = 20^\circ$		$\alpha = 30^\circ$	
	W_T	M	W_T	M	W_T	M
0.1	53.4	52.1	26.5	52.2	16.5	52.2
0.2	28.3	27.1	18.3	28.0	12.0	28.2
0.3	13.7	18.2	11.9	19.2	8.59	19.5
0.5	3.94	11.0	4.91	11.6	4.28	12.0
0.7	1.52	8.04	2.23	8.37	2.21	8.64
1.0	.513	5.80	.841	5.94	.927	6.09

$$t = 15^\circ \quad \epsilon_r = 0 \quad \ell_g/r_2 = 1$$

δ	$\alpha = 10^\circ$		$\alpha = 15^\circ$		$\alpha = 20^\circ$		$\alpha = 30^\circ$	
	W_T	M	W_T	M	W_T	M	W_T	M
0.1	46.5	40.3	31.5	40.4	23.1	40.4	14.3	40.4
0.2	24.3	21.1	20.1	21.6	15.9	21.9	10.5	22.0
0.3	11.7	14.1	11.8	14.7	10.4	15.0	7.47	15.3
0.5	3.35	8.60	4.15	8.89	4.28	9.13	3.72	9.46
0.7	1.29	6.29	1.74	6.43	1.94	6.57	1.92	6.42
1.0	.435	4.55	.619	4.61	.733	4.68	.808	4.81

$$t = 20^\circ \quad \epsilon_r = 0 \quad \ell_g/r_2 = .973$$

δ	$\alpha = 10^\circ$		$\alpha = 20^\circ$		$\alpha = 30^\circ$	
	W_T	M	W_T	M	W_T	M
0.1	39.8	31.4	20.0	31.6	12.4	31.5
0.2	20.6	16.5	13.8	17.2	9.06	17.3
0.3	9.81	11.1	8.97	11.9	6.47	12.1
0.5	2.78	6.76	3.71	7.23	3.23	7.51
0.7	1.07	4.96	1.69	5.21	1.67	5.41
1.0	.361	3.60	.639	3.71	.703	3.83



TABLE 3-1 (CONT.)

COMPUTED PARAMETRIC PERFORMANCE DATA

$$\gamma = 1, \ell/r_2 = 1.75, N_g = 7$$

 $t = 10^\circ, \alpha = 10^\circ, \ell_g/r_2 = 1.02$

δ	ϵ_r	W_T	W_R	ϕ	M
0.3	.25	12.9	2.73	72.2	18.7
	.50	10.8	6.26	75.4	20.9
0.5	.25	3.66	1.49	72.7	11.4
	.50	3.01	3.31	74.8	12.9

 $t = 10^\circ, \alpha = 30^\circ, \ell_g/r_2 = 1.02$

δ	ϵ_r	W_T	W_R	ϕ	M
0.3	.25	8.12	3.04	68.4	20.1
	.50	7.02	6.96	71.3	22.4
0.5	.25	3.98	1.86	64.0	12.5
	.50	3.30	4.11	65.7	14.0

 $t = 20^\circ, \alpha = 10^\circ, \ell_g/r_2 = 0.973$

δ	ϵ_r	W_T	W_R	ϕ	M
0.3	.25	9.29	1.66	74.4	11.4
	.50	8.04	3.77	77.6	12.6
0.5	.25	2.64	.877	73.3	6.99
	.50	2.31	1.93	76.1	7.83

 $t = 20^\circ, \alpha = 30^\circ, \ell_g/r_2 = 0.973$

δ	ϵ_r	W_T	W_R	ϕ	M
0.3	.25	6.25	2.03	69.2	12.5
	.50	5.70	4.54	72.0	13.8
0.5	.25	3.09	1.22	64.3	7.77
	.50	2.75	2.64	66.3	8.71

 $t = 15^\circ, \alpha = 10^\circ, \ell_g/r_2 = 1$

δ	ϵ_r	W_T	W_R	ϕ	M
0.1	.25	45.03	11.52	87.8	41.5
	.50	43.21	28.88	88.4	46.2
0.3	.25	11.1	1.88	71.7	14.5
	.40	10.2	3.12	73.2	15.3
	.50	9.51	4.06	74.5	16.1
0.5	.25	3.16	1.05	72.0	8.89
	.50	2.70	2.22	72.3	9.95
0.7	.25	1.22	0.75	75.9	6.52
	.50	1.07	1.65	77.4	7.38

 $t = 15^\circ, \alpha = 15^\circ, \ell_g/r_2 = 1$

δ	ϵ_r	W_T	W_R	ϕ	M
0.1	.25	31.1	11.2	88.3	41.7
	.50	30.2	27.3	88.3	46.4
0.3	.25	11.2	2.39	69.5	15.1
	.40	10.3	4.02	71.0	15.9
	.50	9.65	5.34	71.9	16.7
0.5	.25	3.89	1.40	67.9	9.20
	.50	3.30	3.01	70.2	10.3
0.7	.25	1.64	.923	71.2	6.68
	.50	1.41	1.98	72.3	7.58

Legend:

 γ = land to groove width ratio ℓ = axial length of cone r_2 = base radius of cone N_g = no. of grooves t = half cone angle α = groove angle δ = concentric land clearance to groove depth ratio

$$W_T = \frac{w_t h_o^2}{\eta \omega r_2^4}$$

TABLE 3-1 (CONT.)

COMPUTED PARAMETRIC PERFORMANCE DATA

$$\gamma = 1, \quad \ell/r_2 = 1.75, \quad N_g = 7$$

 $t = 15^\circ, \quad \alpha = 20^\circ \quad \ell_g/r_2 = 1$

δ	ϵ_r	W_T	W_R	ϕ	M
0.1	.25	22.73	12.00	87.6	41.7
	.50	21.95	30.23	88.1	46.4
0.3	.25	9.85	2.56	68.4	15.5
	.50	8.63	5.73	71.4	17.2
0.5	.25	4.03	1.52	65.5	9.45
	.50	3.44	3.27	67.3	10.6
0.7	.25	1.83	1.00	68.3	6.8
	.50	1.57	2.18	69.4	7.8

 $t = 15^\circ, \quad \alpha = 30^\circ \quad \ell_g/r_2 = 1$

δ	ϵ_r	W_T	W_R	ϕ	M
0.1	.25	14.09	12.34	87.4	41.7
	.50	13.61	31.29	87.9	46.5
0.3	.25	7.14	2.52	69.0	15.8
	.50	6.36	5.73	71.7	17.5
0.5	.25	3.52	1.53	64.2	9.8
	.50	3.03	3.35	66.1	11.0
0.7	.25	1.81	1.02	66.0	7.1
	.50	1.57	2.25	66.7	8.1

 $t = 10^\circ, \quad \alpha = 20^\circ \quad \ell_g/r_2 = 1.02$

δ	ϵ_r	W_T	W_R	ϕ	M
0.1	.25	26.04	13.95	87.2	53.85
	.50	24.89	35.81	87.8	60.03
0.3	.25	11.2	3.22	67.7	19.8
	.50	9.52	7.27	70.8	22.0
0.5	.25	4.56	1.88	65.1	12.05
	.50	3.74	4.14	67.2	13.6
0.7	.25	2.07	1.22	68.2	8.7
	.50	1.71	2.69	68.9	9.9

 $t = 20^\circ, \quad \alpha = 20^\circ \quad \ell_g/r_2 = 0.973$

δ	ϵ_r	W_T	W_R	ϕ	M
0.1	.25	19.78	9.98	88.0	32.5
	.50	19.26	24.38	88.4	36.2
0.3	.25	8.62	2.02	69.0	12.2
	.50	7.71	4.50	72.0	13.5
0.5	.25	3.53	1.19	65.7	7.48
	.50	3.09	2.56	68.3	8.38
0.7	.25	1.61	0.79	68.4	5.4
	.50	1.43	1.70	69.7	6.1

 $t = 15^\circ, \quad \alpha = 18^\circ \quad \ell_g/r_2 = 1$

δ	ϵ_r	W_T	W_R	ϵ	M
0.5	.25	4.02	1.50	66.4	9.36

 $t = 15^\circ, \quad \alpha = 25^\circ \quad \ell_g/r_2 = 1$

δ	ϵ_r	W_T	W_R	ϕ	M
0.5	.25	3.84	1.54	64.3	9.65

 ϵ_r = radial eccentricity

 ℓ_g = axial length of grooved section of cone

 ϕ = attitude angle, deg.

 w_t = thrust load, N

 h_o = groove depth, m

 ω = speed, rad/sec

$$M = \frac{m h_o}{4 \eta \omega r_2}$$

$$W_R = \frac{w_r h_o^2}{4 \eta \omega r_2}$$

 η = viscosity, N sec/m²
 m = friction moment, N m

Deviations From Idealized Geometry

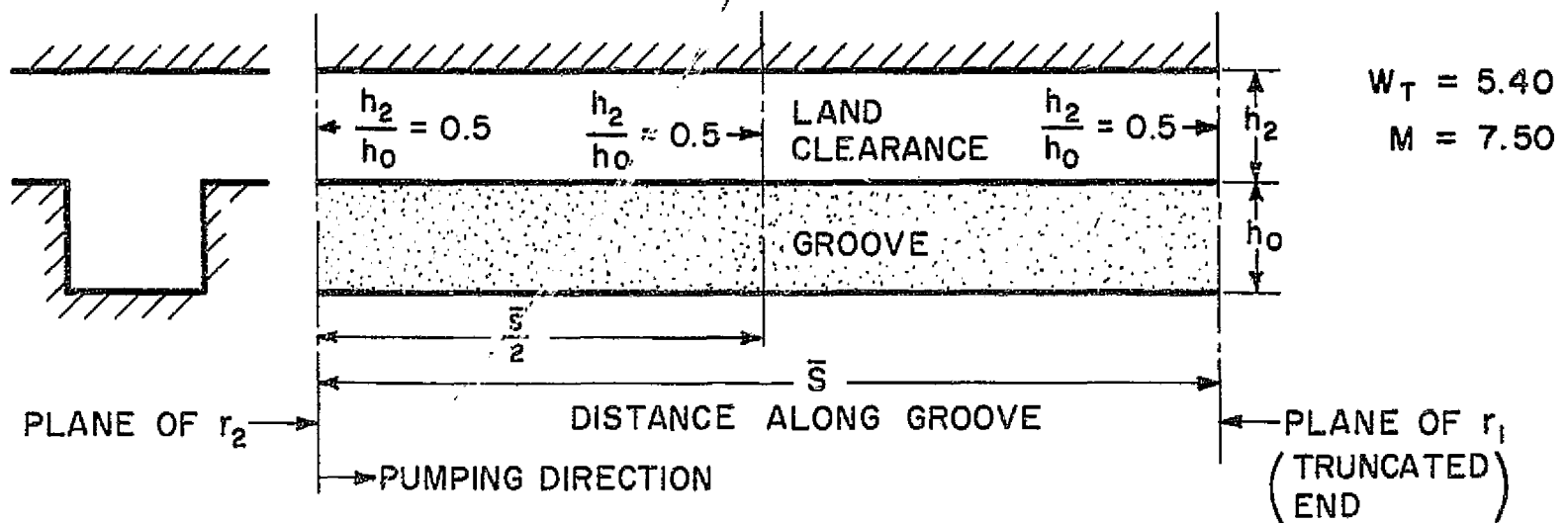
Groove depth - Figure 3-8 gives the axial load and friction moment coefficients (W_T and M) for 3 cases in which the groove depth was allowed to vary linearly along its length. The highest load coefficient was obtained with a uniform groove depth. A decreasing groove depth (as shown in Case b) decreases the load coefficient by 1% and increases the friction moment coefficient by 4%. An increasing groove depth (as shown in Case c) decreases the load coefficient by 11% and decreases the friction moment coefficient by 1%.

Figure 3-8 shows it is desirable to dimension the groove so that a deviation from uniform depth results in a decrease in groove depth in the direction of pumping.

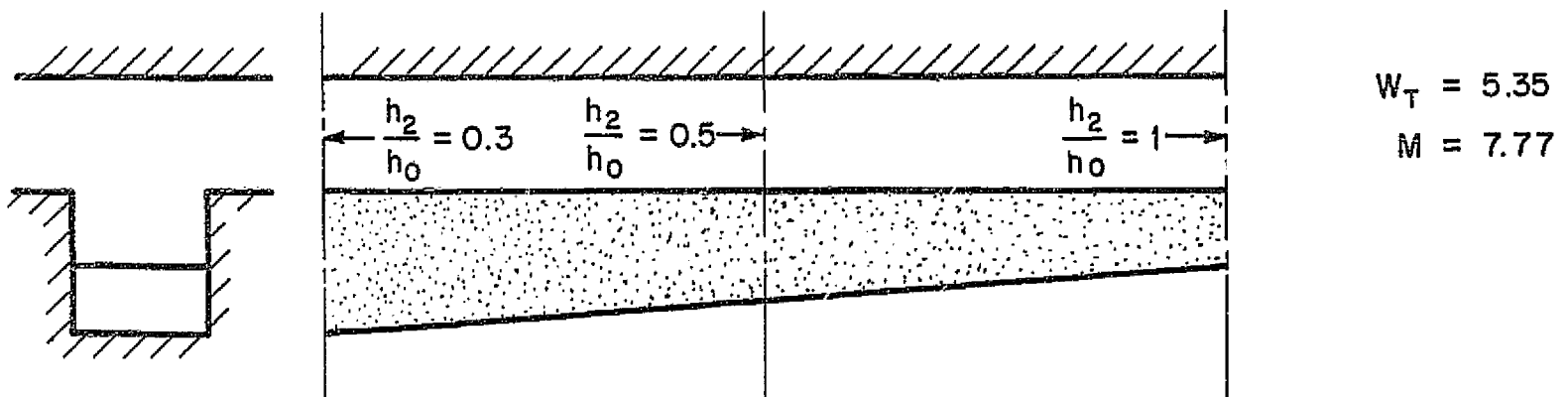
Groove cross section - Figure 3-9 shows the highest load and lowest friction power loss were obtained with a square cross-sectioned groove. Case d indicated small deviations in the squareness of the groove side walls with the bottom had little influence on the load capacity. Thus, side wall tolerances for manufacturing the groove need not be too stringent.

Mismatched cone angles - The effect of small variations in cone angle between matching components on the bearing performance is illustrated in Figure 3-10 through Figure 3-12. Two combinations of male and female cones were investigated. Case b represents the combination in which the cone angle of the female is smaller than that of the male and Case c the opposite. The deviations from perfectly matched cones (Case a) in both cases is about 13%.

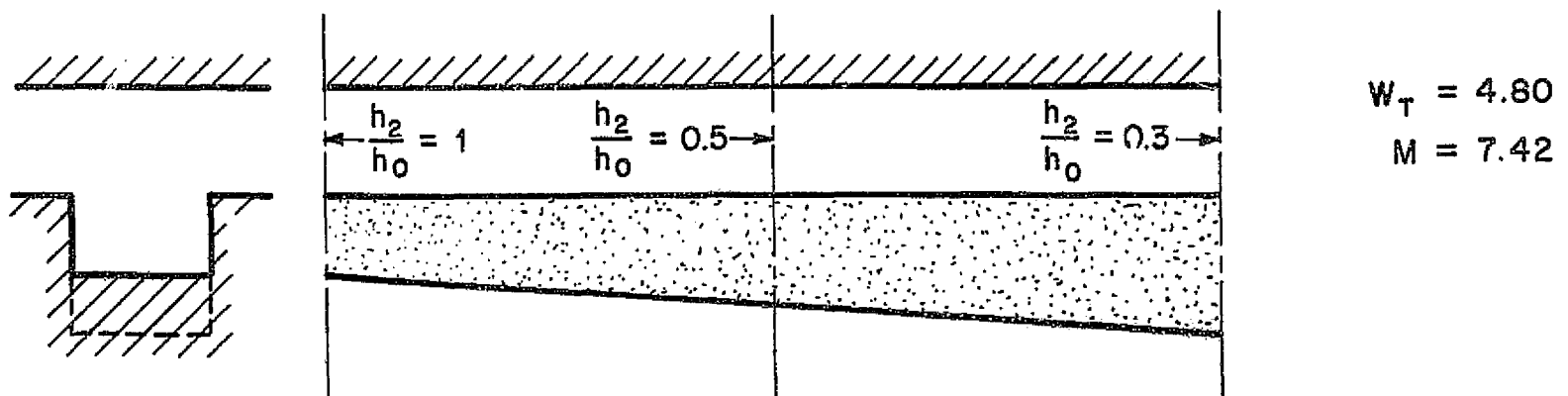
Figure 3-10 through Figure 3-12 show that the value of axial load and stiffness coefficients for (b) nearly double that of (a) while friction moment coefficient is increased only by 33%. On the other hand, the axial load and stiffness coefficients for (c) are reduced by



Case a. Uniform Groove Depth

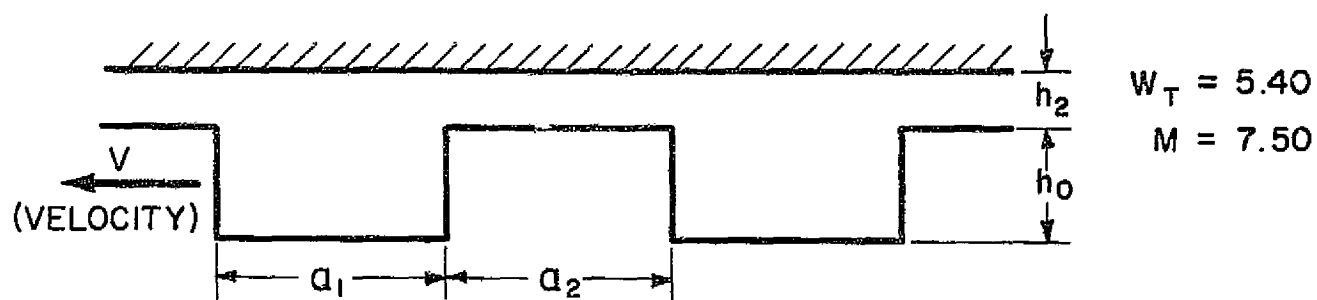


Case b. Decreasing Groove Depth

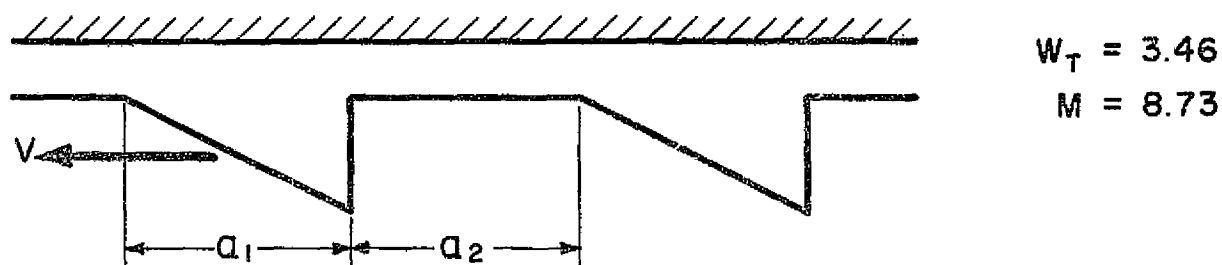


Case c. Increasing Groove Depth

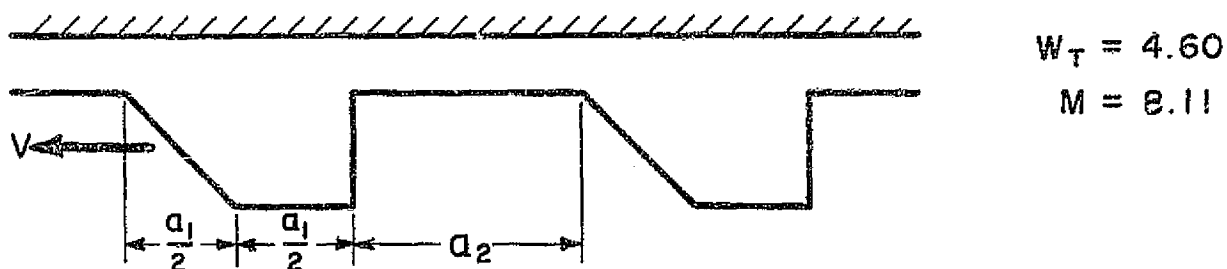
FIG. 3-8 VARIATIONS IN DEPTH ALONG A GROOVE



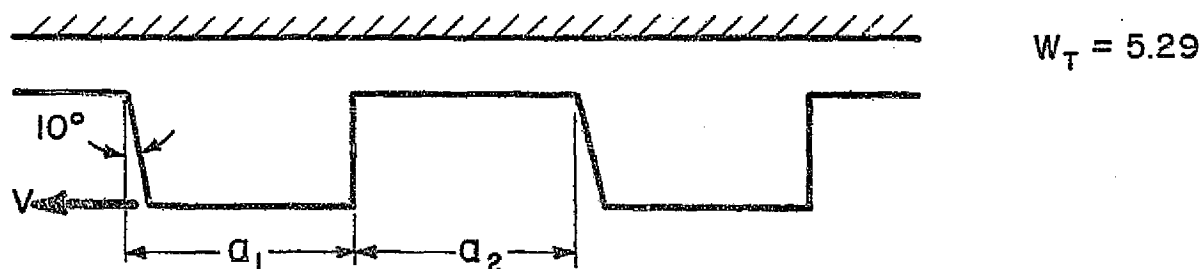
Case a. Square Groove Cross Section



Case b. Full Ramp Cross Section



Case c. Half Ramp Cross Section



Case d. Steep Sided Cross Section

FIG. 3-9 VARIATIONS IN GROOVE CROSS-SECTION

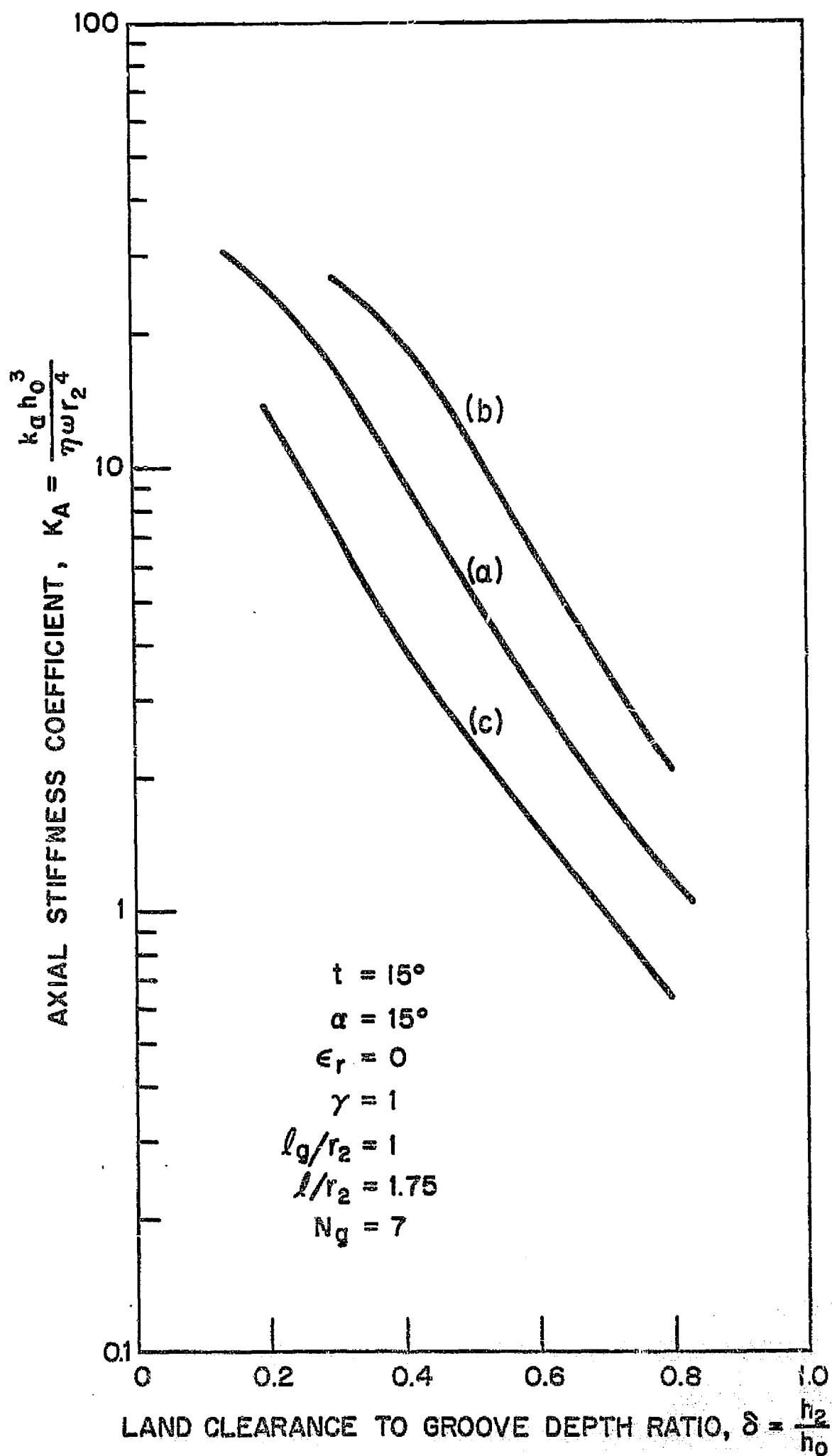


FIG. 3-10 INFLUENCE OF MISMATCHED CONE ANGLES ON AXIAL LOAD COEFFICIENT



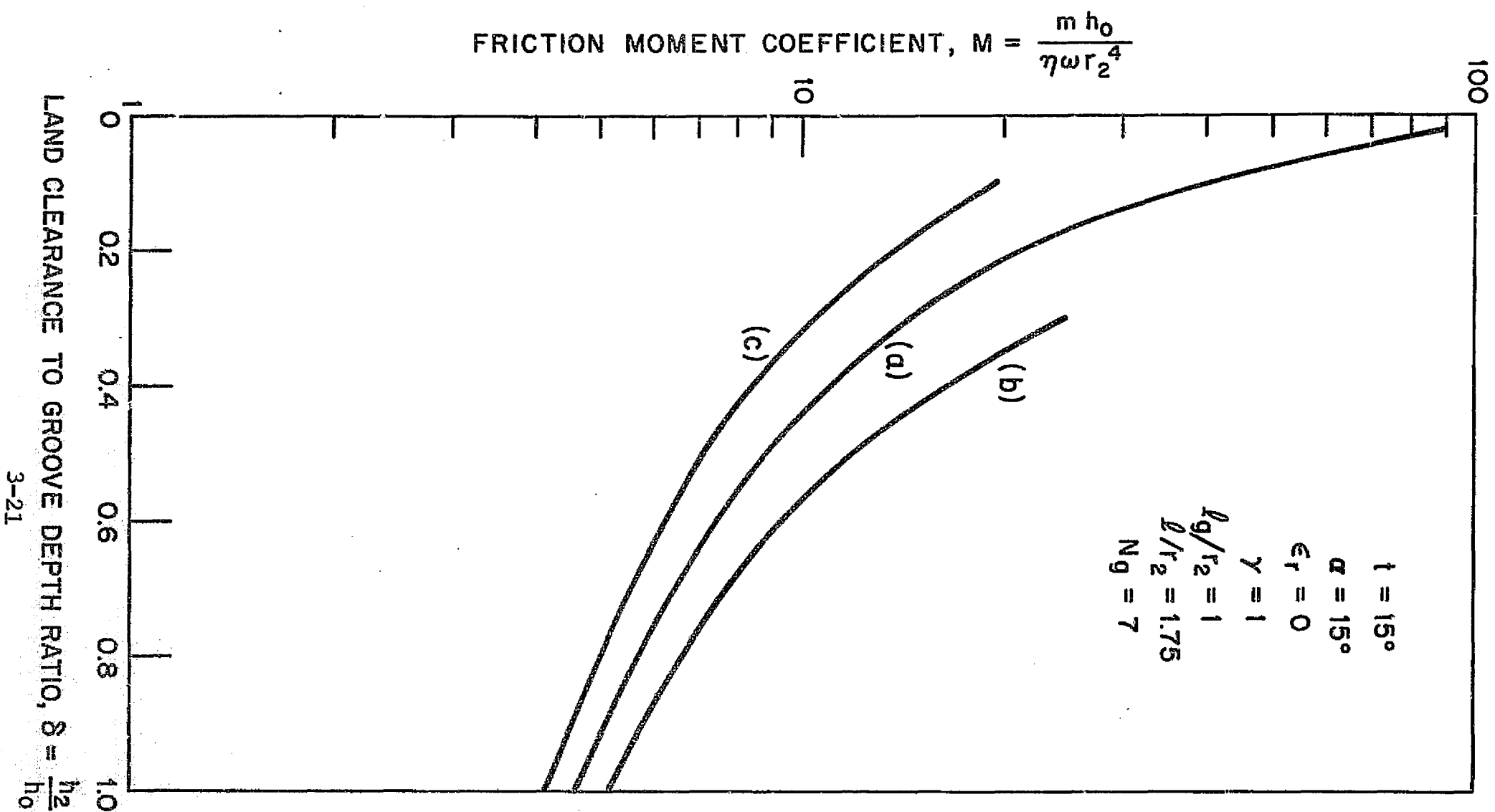


FIG. 3-11 INFLUENCE OF MISMATCHED CONE ANGLES ON FRICTION MOMENT COEFFICIENT

$$\text{AXIAL LOAD COEFFICIENT, } W_T = \frac{w_t h_0^2}{\eta \omega r_2^4}$$

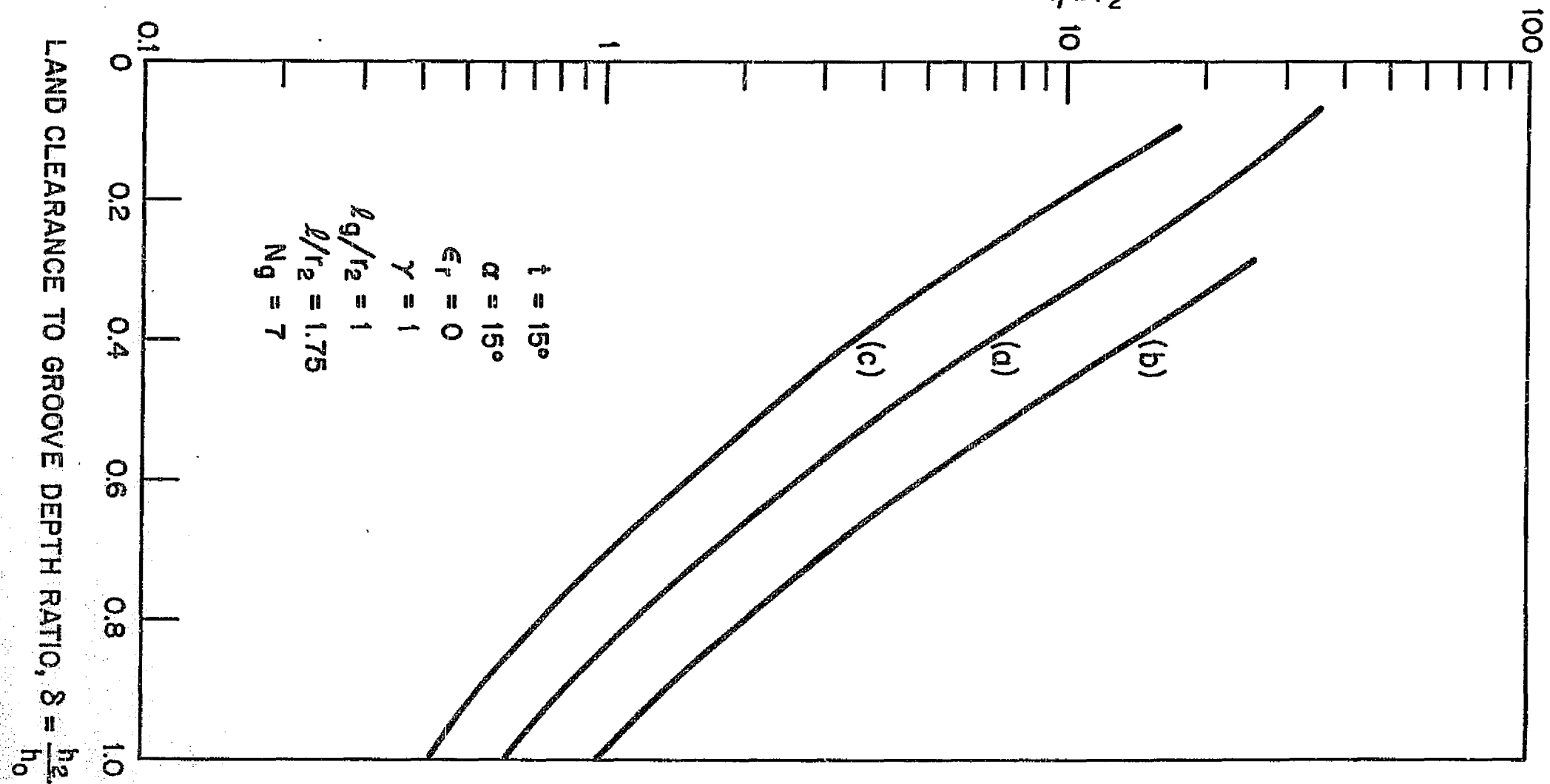


FIG. 3-12 INFLUENCE OF MISMATCHED CONE ANGLES ON AXIAL STIFFNESS COEFFICIENT

50% and the friction moment coefficient for (c) is lowered by 20% as compared to (a). Thus, these plots indicate that if mismatch is to be tolerated it is preferable to have the combination (b).

Misaligned cone axes - The effect of misalignment was investigated by considering the case in which the bearing axis is rotated with respect to the rotor axis in such a manner that the film thickness at both ends of the truncated cone is changed by 10% from the concentric case. The result shows that the axial load and friction moment coefficient differ from the concentric case only by 0.2%. Thus, to the degree of accuracy of the plots they can not be distinguished from case (a) in Figure 3-10 and Figure 3-12.

Influence of Geometric Parameters

Surface grooving - Figure 3-13 shows that the maximum load capacity and minimum power loss were attained for a fully grooved bearing (i.e., $l_g = l$). An experimental comparison showed a vibratory condition existed for the fully grooved bearing that was not present for the partially grooved bearing. Therefore, the degree of surface grooving employed should be determined from dynamical as well as static consideration. All subsequent steady-state performance data was based on l_g/r_2 of about 1.

Cone angle - Figure 3-14 illustrates the effect of the half cone angle (t) on the axial load coefficient. It can be seen that W_T decreases monotonically as t increases. This trend is typical for the radial load and friction moment coefficient as well.

Groove angle (α) - Figure 3-15 shows that the axial load coefficient can be maximized by varying α and the optimum groove angle increases with increasing δ . The radial load coefficient can also be maximized by varying α as shown in Figure 3-16. However, the value of optimum α is not critical in this case since W_R changes very little from its maximum value for a wide range of groove angles. Figure 3-17 indicates that the friction moment coefficient is insensitive to variations in the groove angle.

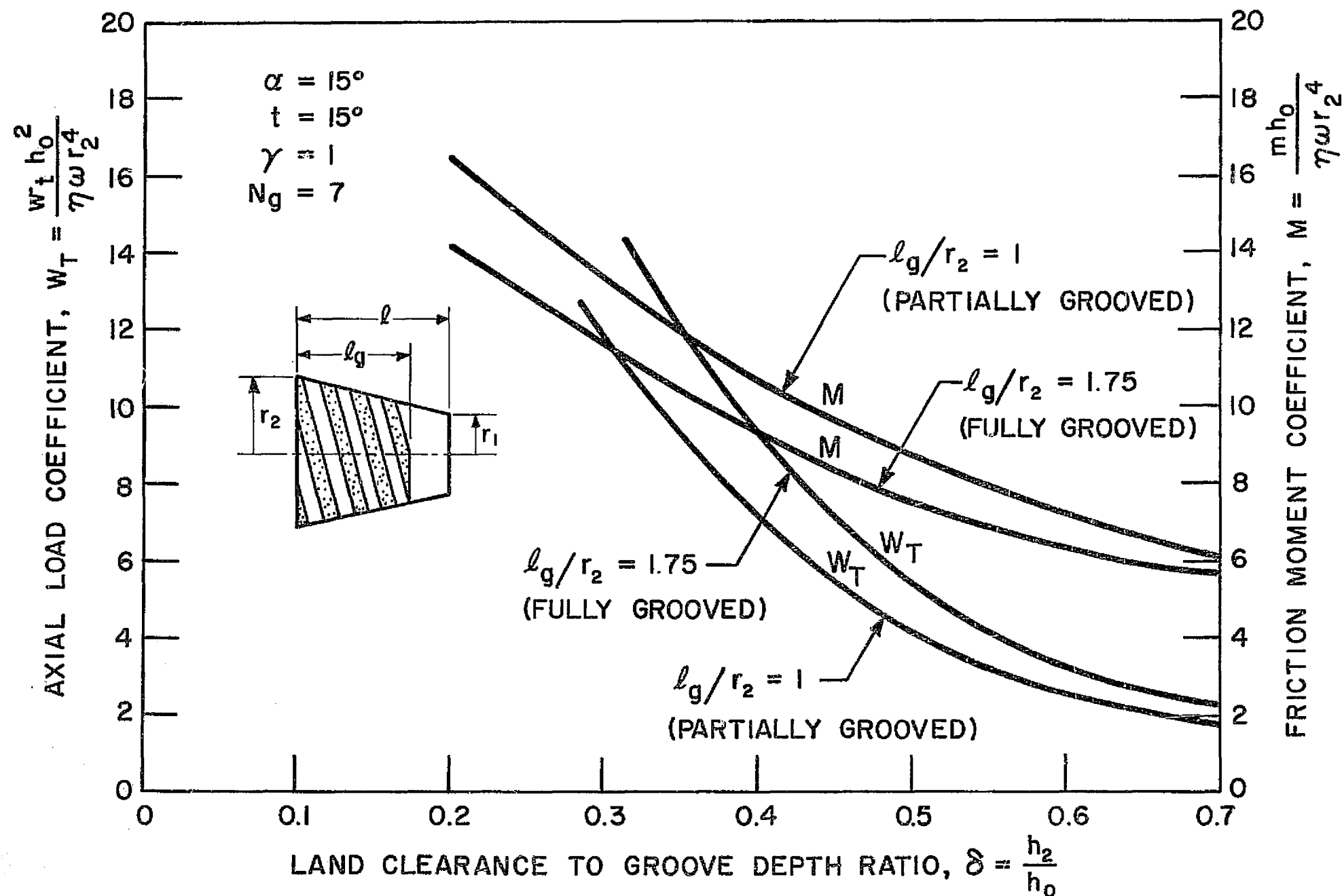


FIG. 3-13 AXIAL LOAD AND FRICTION MOMENT COEFFICIENTS FOR FULLY AND PARTIALLY GROOVED BEARING SURFACE

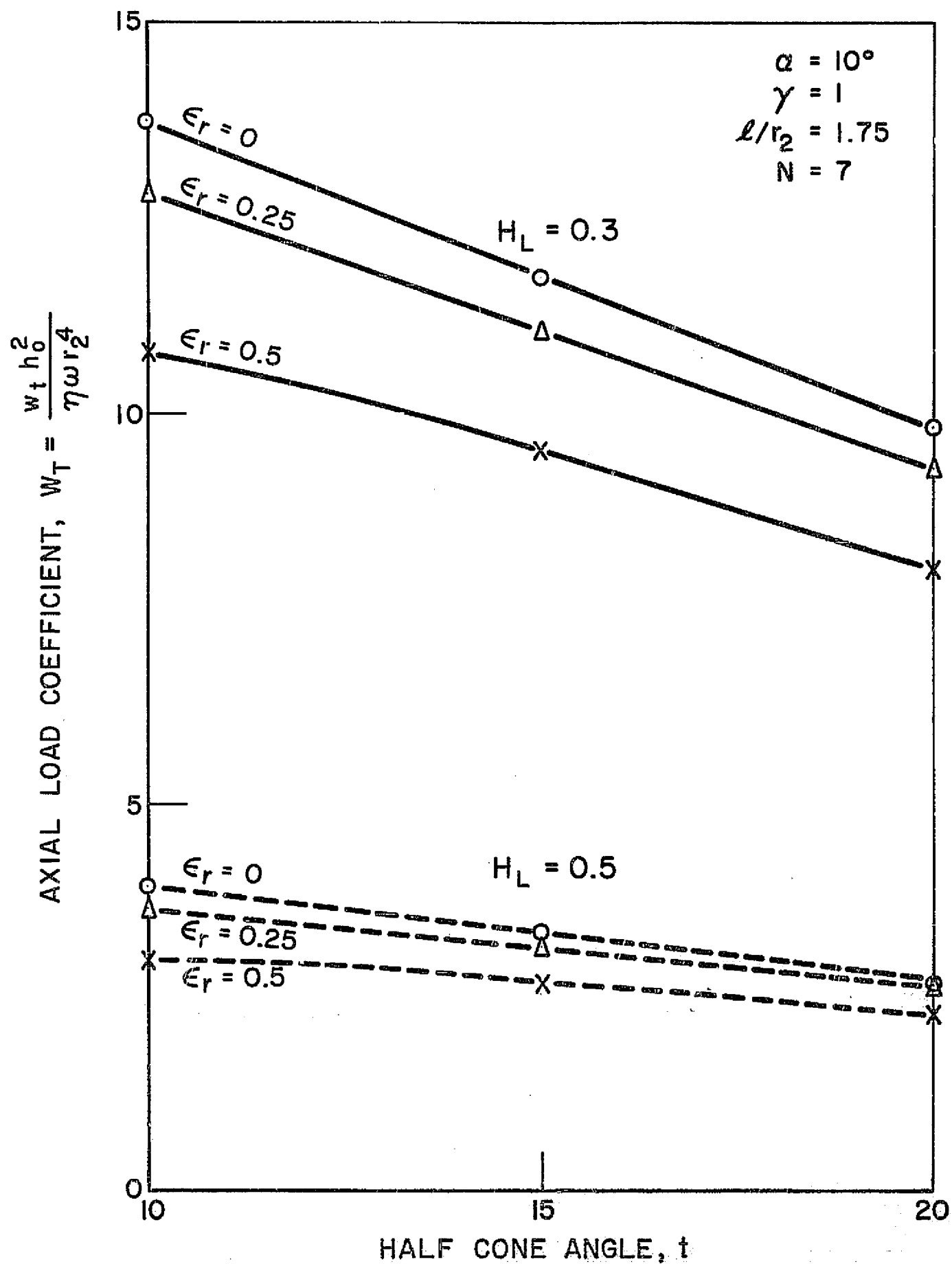


FIG. 3-14 INFLUENCE OF CONE ANGLES ON AXIAL LOAD COEFFICIENT

$$\text{AXIAL LOAD COEFFICIENT, } W_T = \frac{w_t h_o^2}{\eta \omega r_2^4}$$

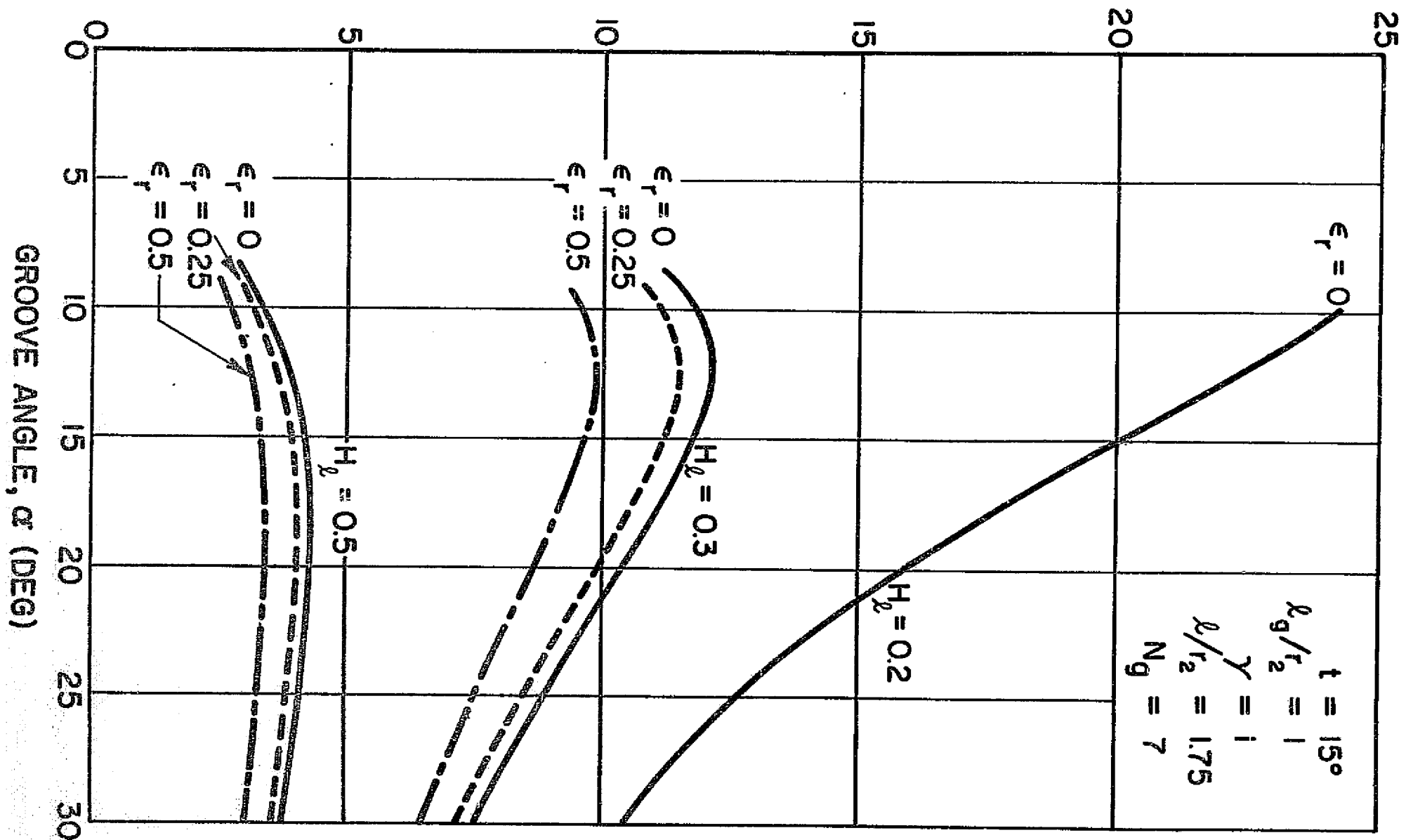


FIG. 3-15 INFLUENCE OF GROOVE ANGLES ON AXIAL LOAD COEFFICIENT

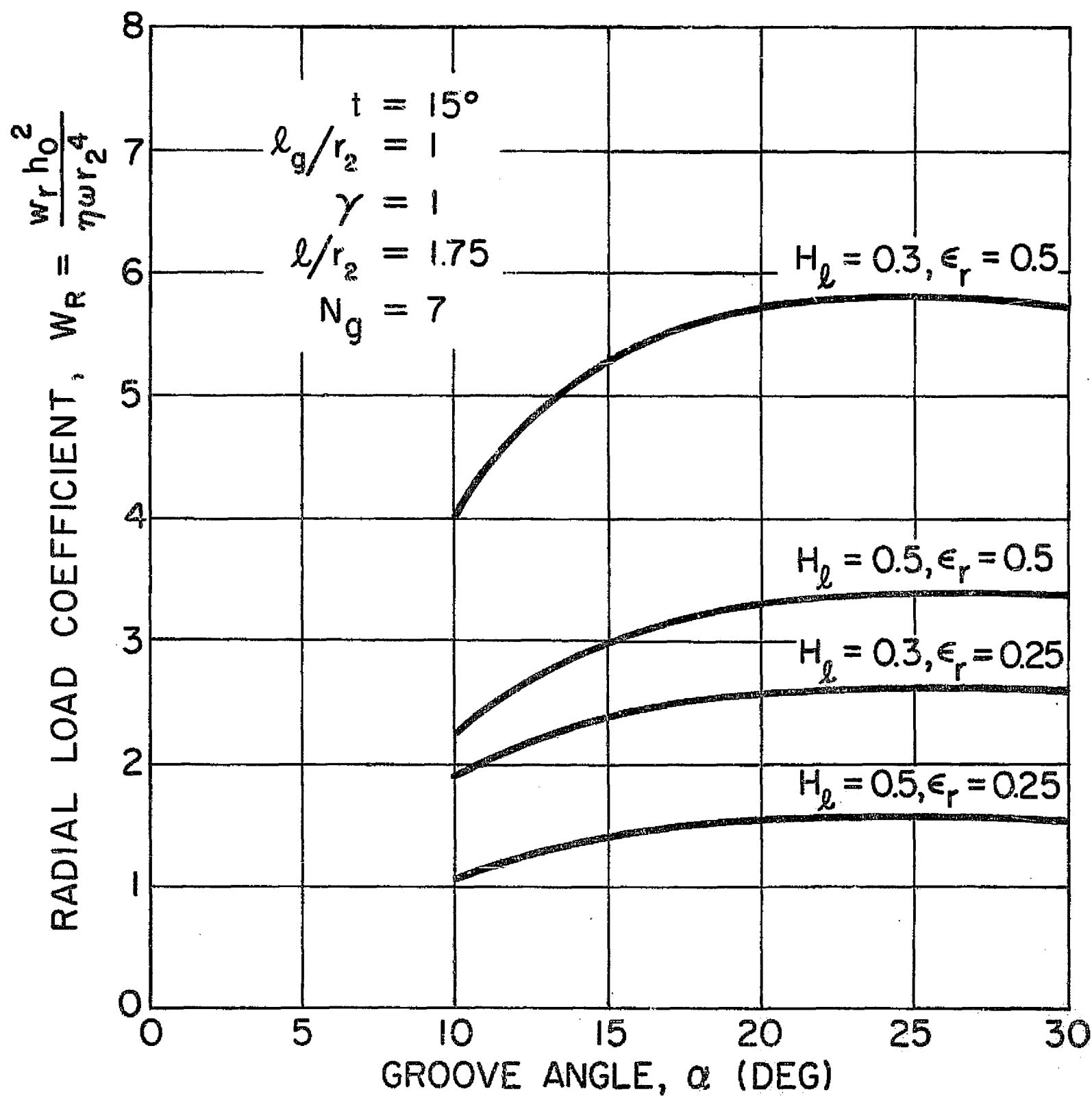


FIG. 3-16 INFLUENCE OF GROOVE ANGLES ON RADIAL LOAD COEFFICIENT

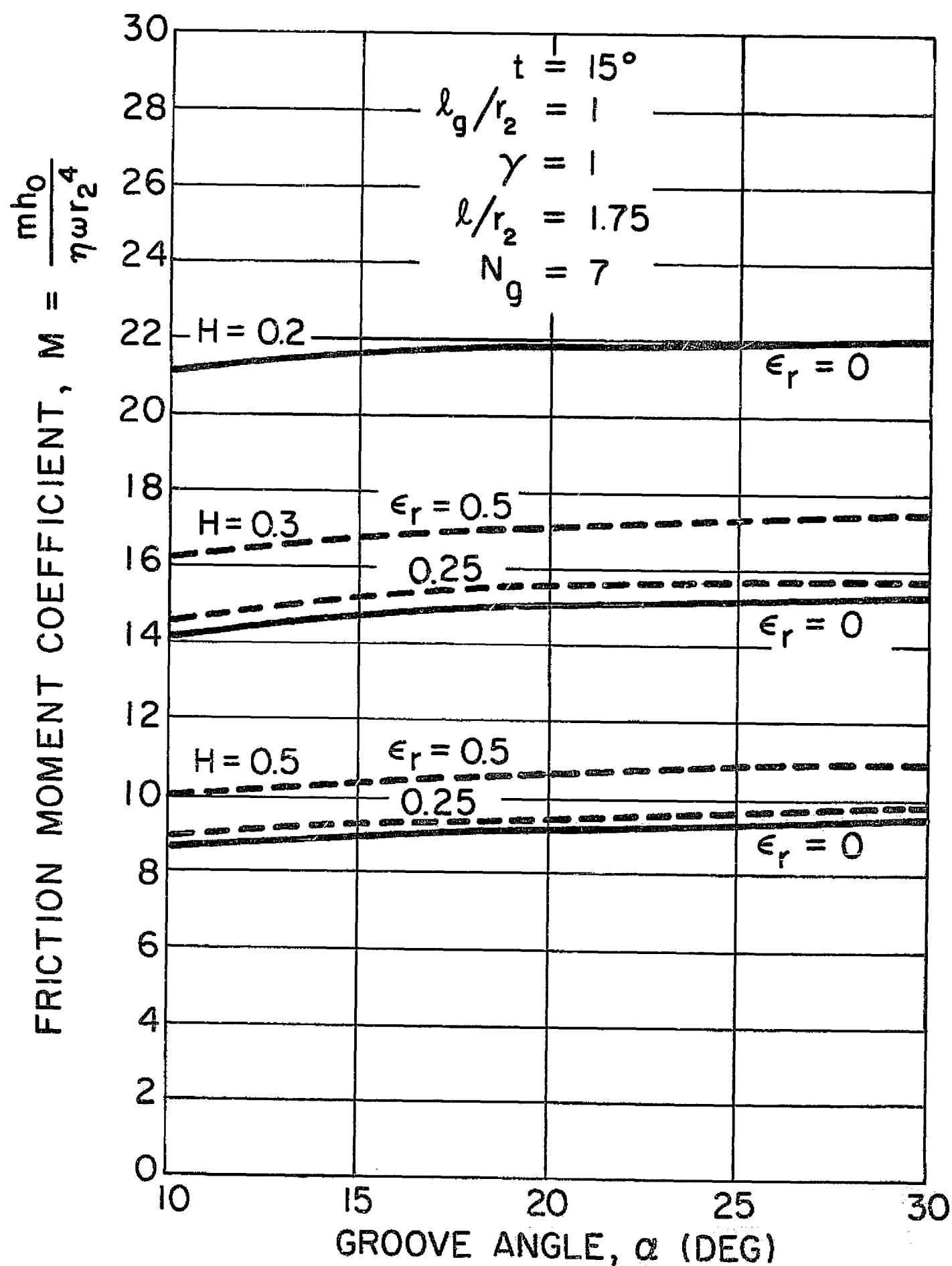


FIG. 3-17 INFLUENCE OF GROOVE ANGLES ON FRICTION MOMENT COEFFICIENT

Concentric land clearance (h_2) - The variations in radial load coefficient with δ are presented in Figure 3-18 through Figure 3-23 for several combinations of t and α . Each plot contains curves for two eccentricity ratios ($\epsilon_r = 0.25$ and 0.5). Since the ordinate and abscissa both contain h_0 , the curves of this section essentially reflect the influence of h_2 . Figure 3-18 through 3-23 show that W_R increases rapidly as δ decreases and the change in W_R with ϵ_r is approximately linear.

Figure 3-24 through Figure 3-26 show the variation in radial stiffness coefficient with δ . It should be noted that the radial stiffness is defined as

$$k_r = \frac{d W_R}{d e}$$

where e is the eccentricity. Thus, the radial stiffness coefficient is given by

$$K_R = \frac{d W_R}{d \epsilon_r} \left(\frac{\cos t}{\delta} \right)$$

where

$$\epsilon_r = \frac{e \cos t}{h_0}$$

The quantity $\frac{d W_R}{d \epsilon_r}$ is computed from plots of W_R vs. ϵ_r by the central difference method.

Values of attitude angle (ϕ) for the case $t = 15^\circ$ and $\alpha = 15^\circ$ are shown in Figure 3-27. The insert in this figure shows the definition of the attitude angle. Note that attitude angles are fairly high for all values of δ .

Figure 3-28 shows the variation of axial load coefficients (W_T) with δ for several groove angles with $t = 15^\circ$ and $\epsilon_r = 0$. Figure 3-29 shows the variation of W_T with δ for several values of ϵ_r with t and α held constant. Note that curves for constant α cross each other while curves for constant ϵ_r are nearly parallel. This is typical for all cases investigated.

$$\text{RADIAL LOAD COEFFICIENT, } W_R = \frac{w_r h_0^2}{\eta \omega r_2^4}$$

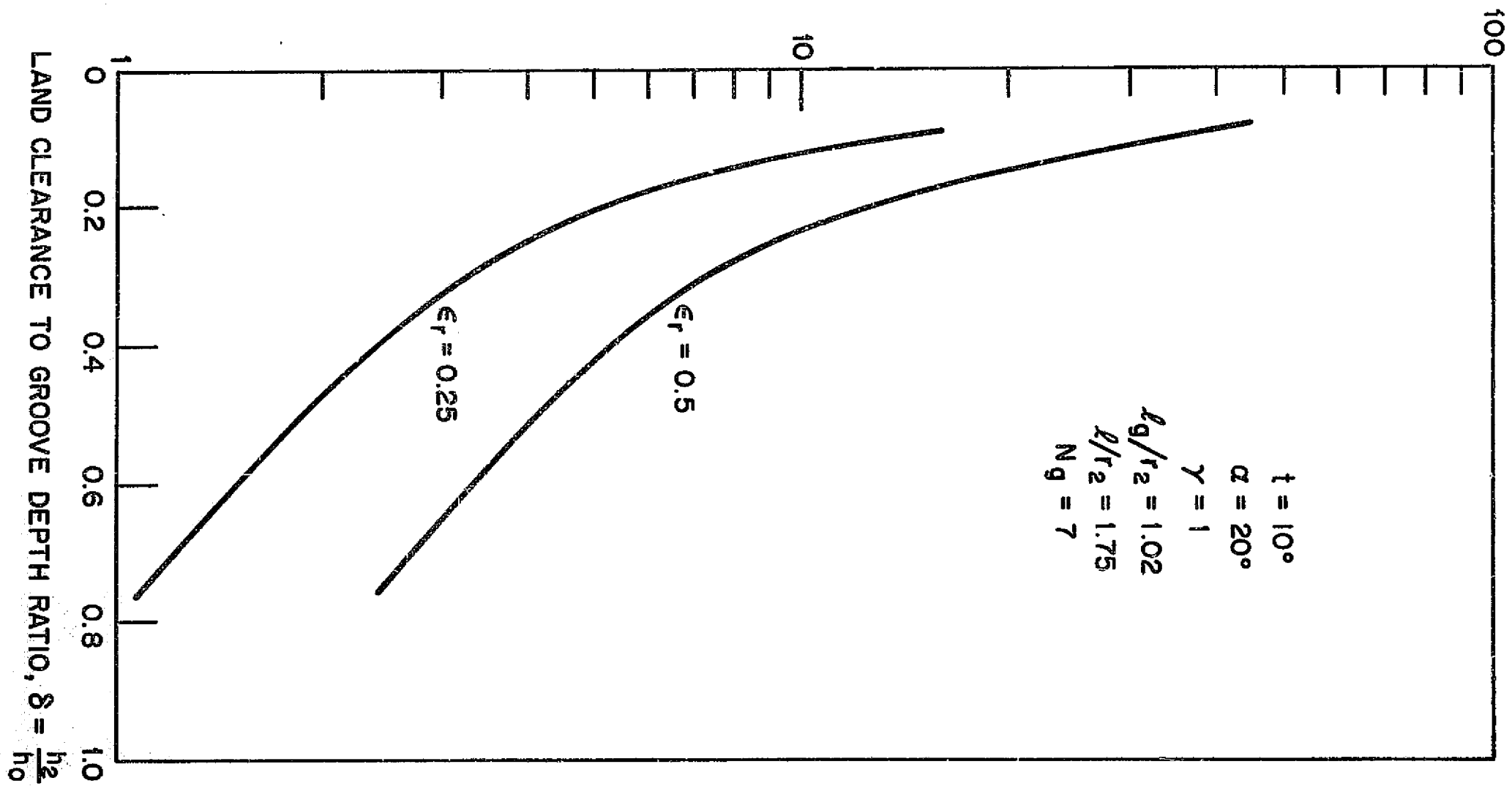


FIG. 3-18 RADIAL LOAD COEFFICIENT VS. δ FOR $t = 10^\circ$, $\alpha = 20^\circ$

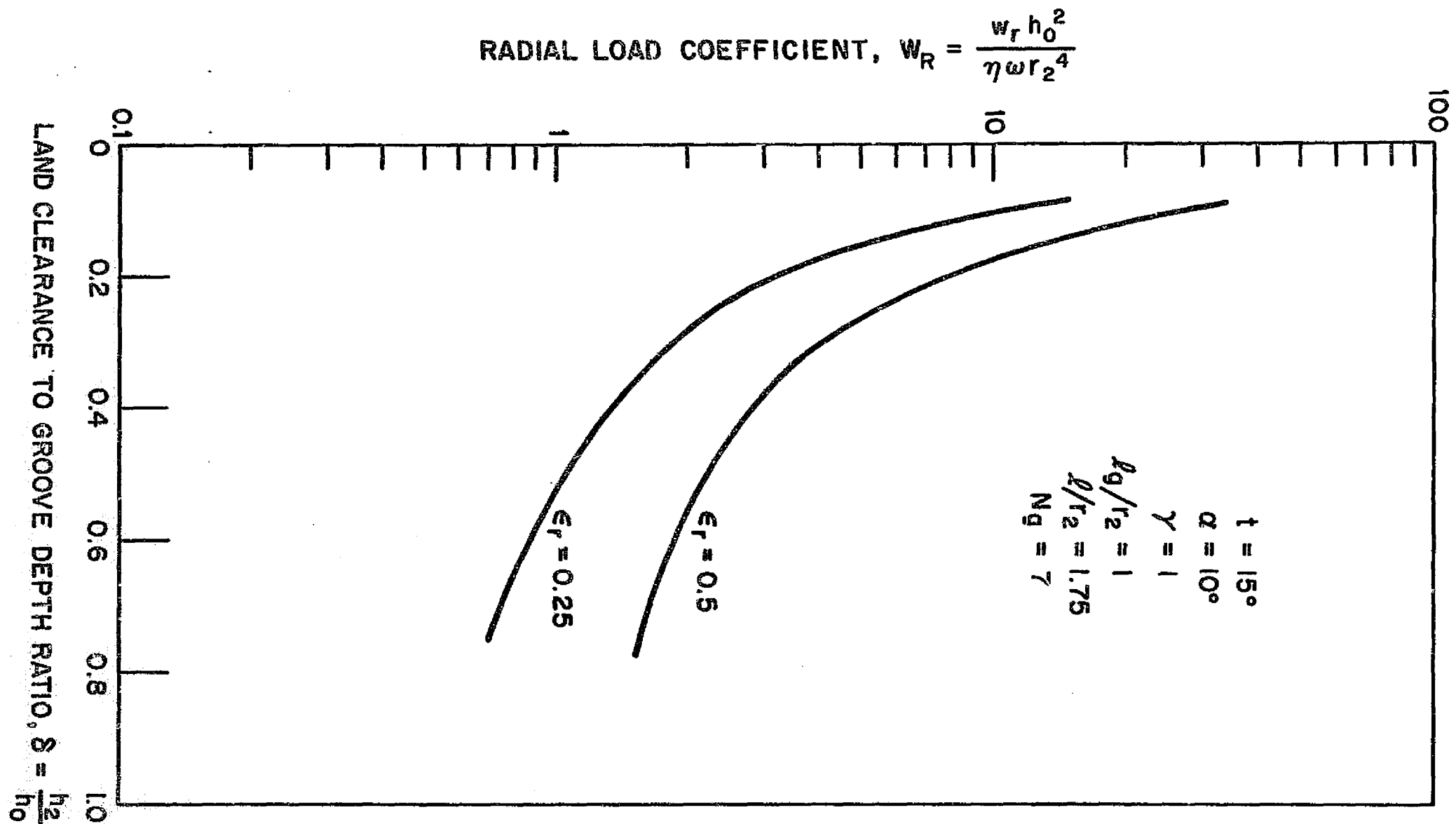


FIG. 3-19 RADIAL LOAD COEFFICIENT VS. δ FOR $t = 15^\circ$, $\alpha = 10^\circ$

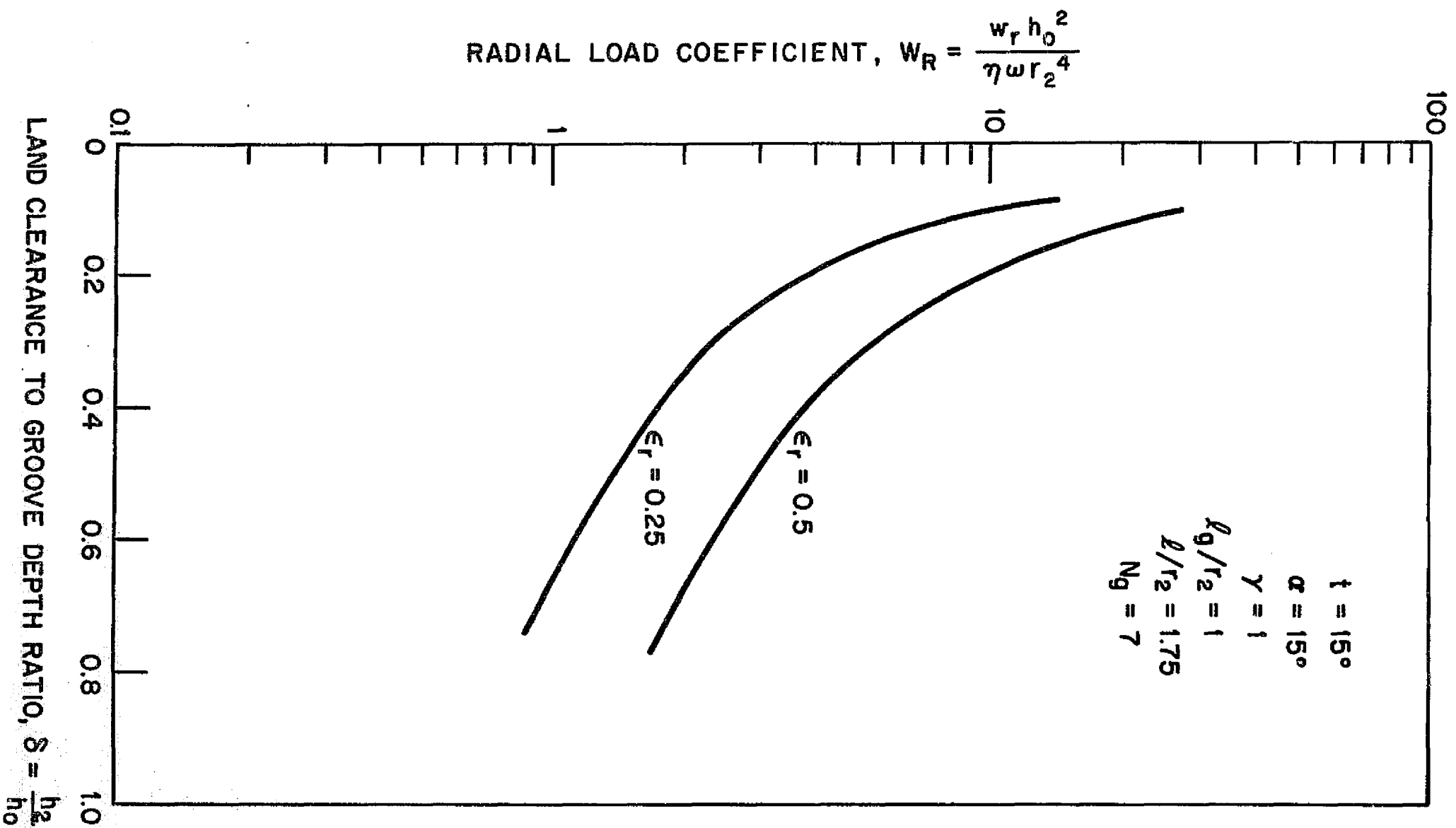


FIG. 3-20 RADIAL LOAD COEFFICIENT VS. δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

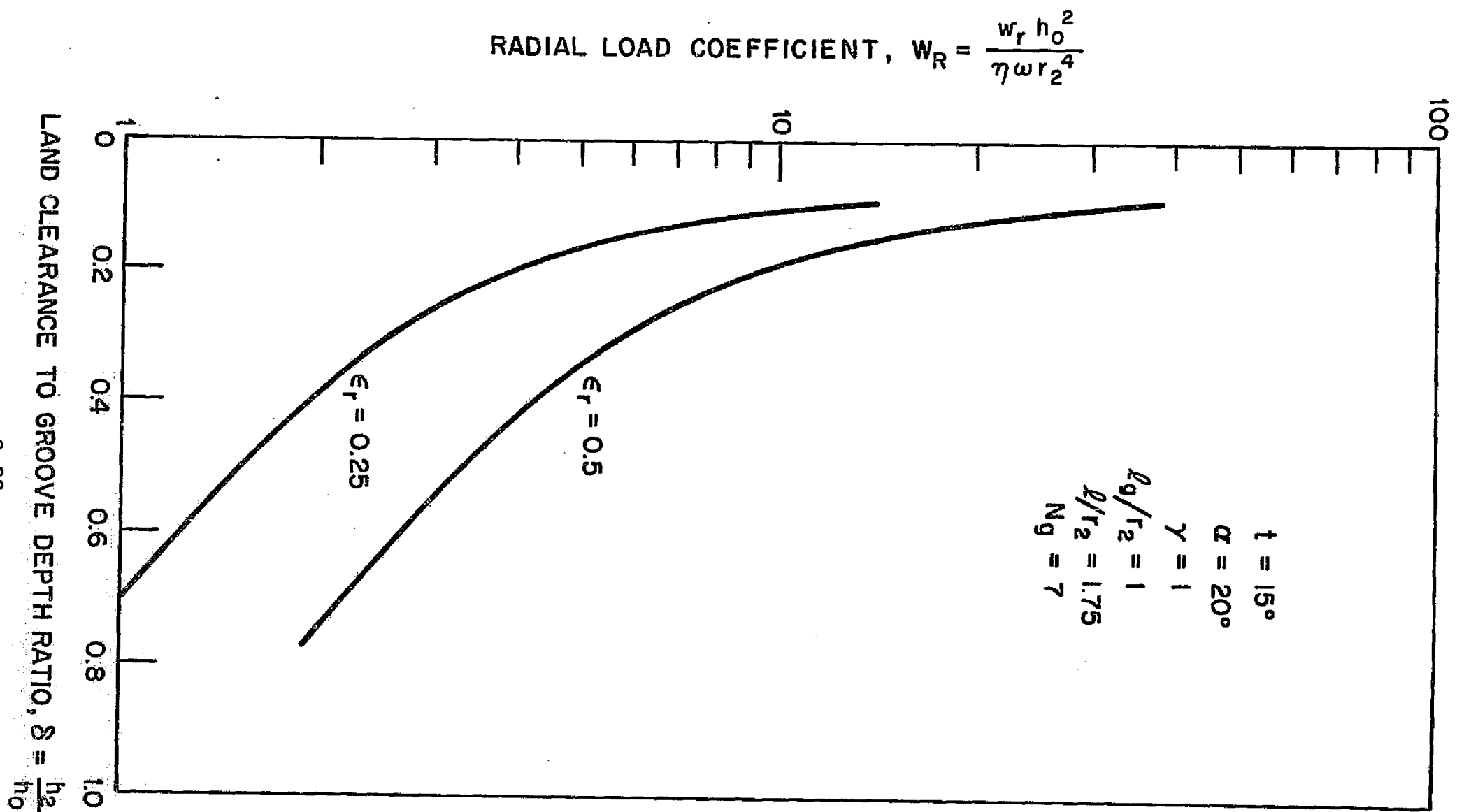


FIG. 3-21 RADIAL LOAD COEFFICIENT VS. δ FOR $t = 15^\circ$, $\alpha = 20^\circ$

$$\text{RADIAL LOAD COEFFICIENT, } W_R = \frac{w_r h_0^2}{\eta \omega r_2^4}$$

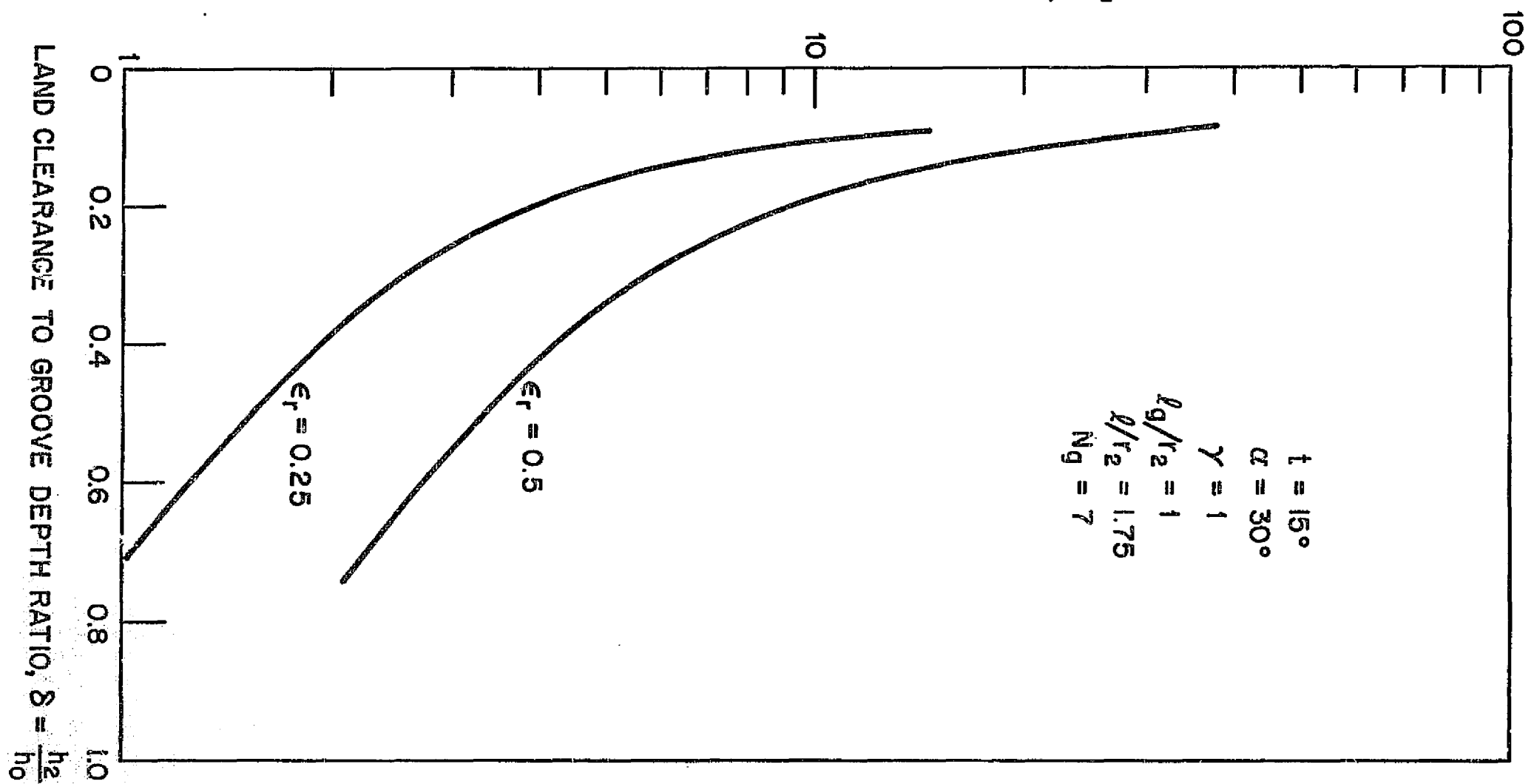


FIG. 3-22 RADIAL LOAD COEFFICIENT VS. δ FOR $t = 15^\circ$, $\alpha = 30^\circ$

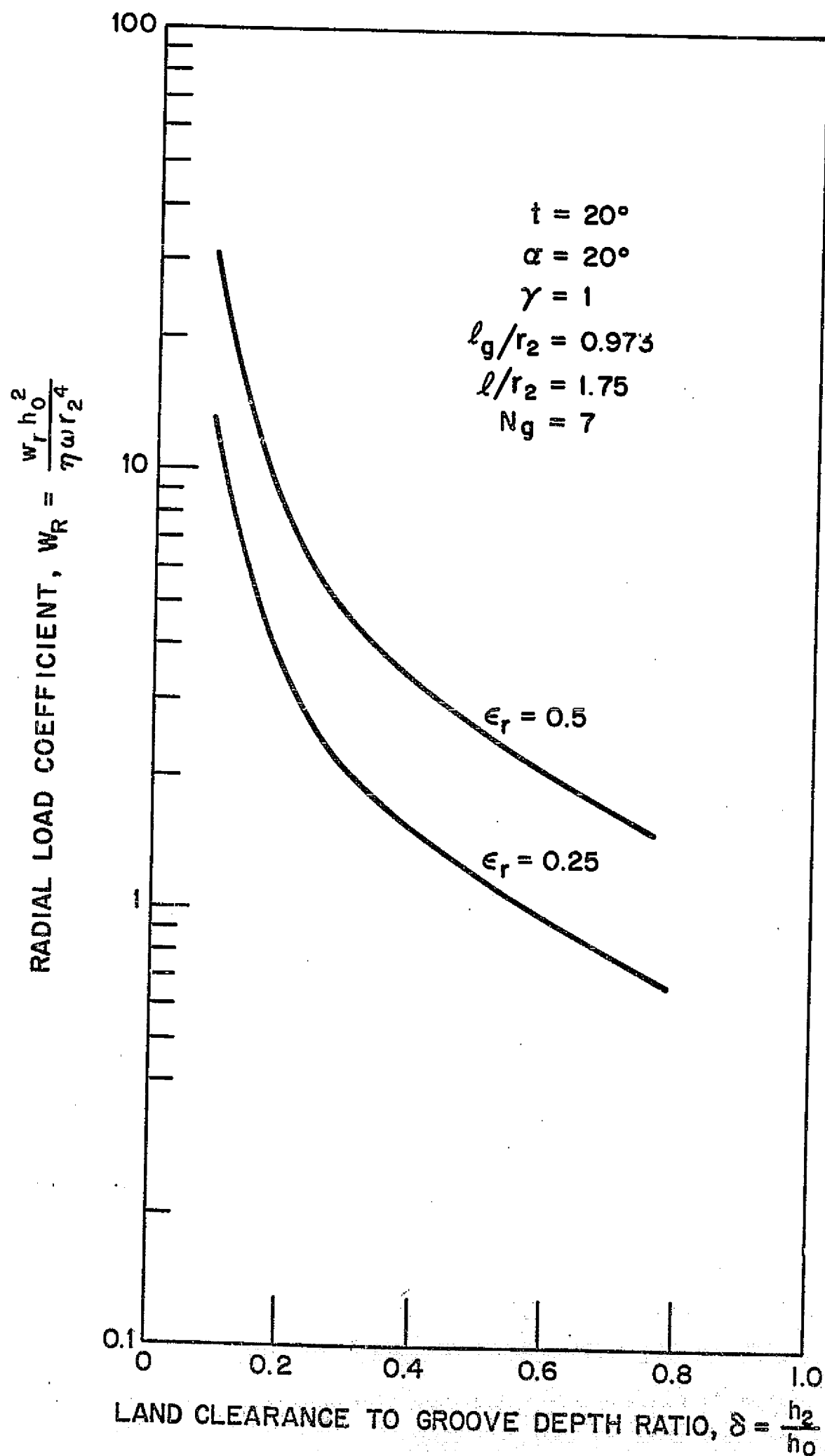


FIG. 3-23 RADIAL LOAD COEFFICIENT VS. δ FOR $t = 20^\circ$, $\alpha = 20^\circ$

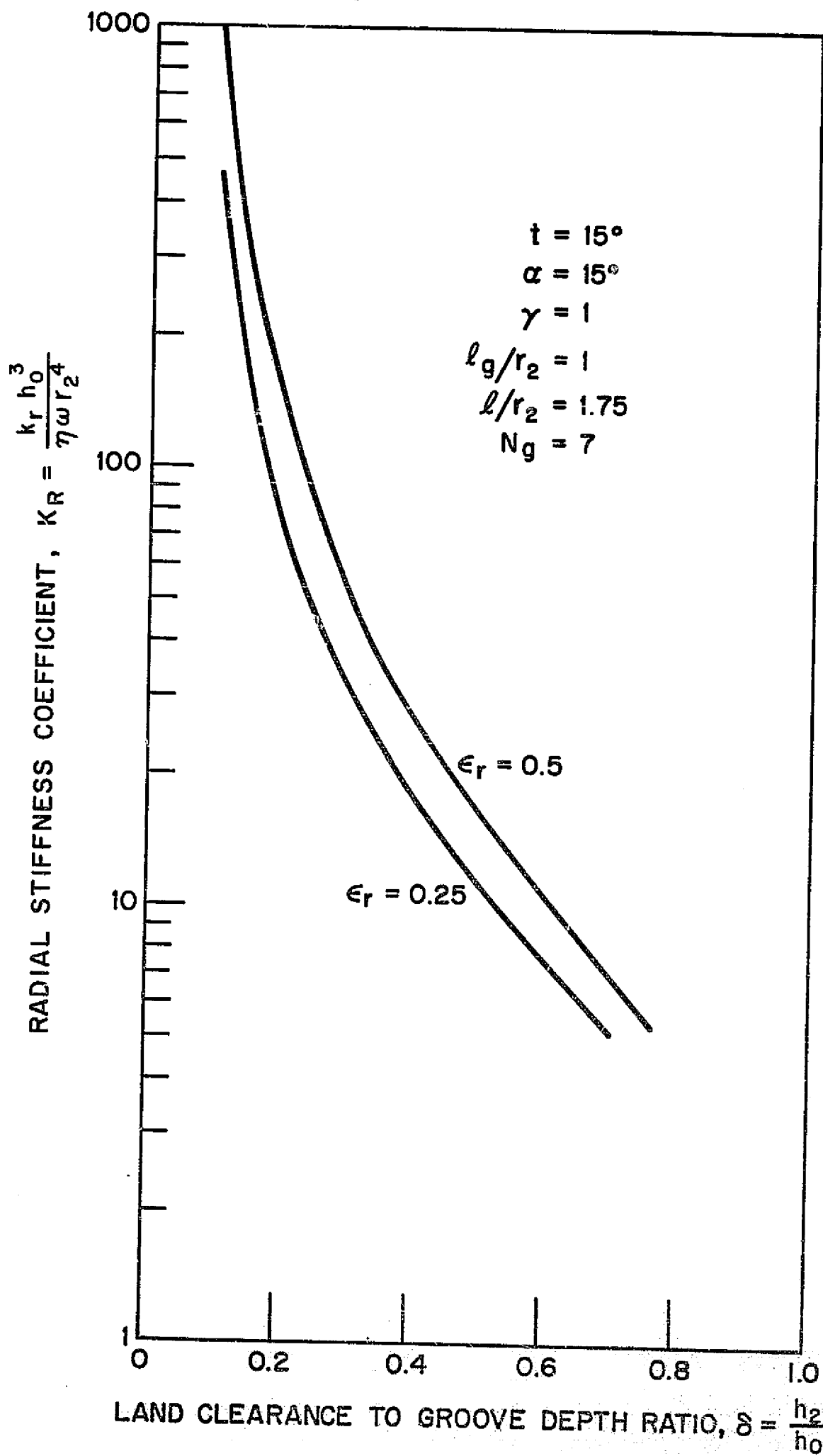


FIG. 3-24 RADIAL STIFFNESS COEFFICIENT VS. δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

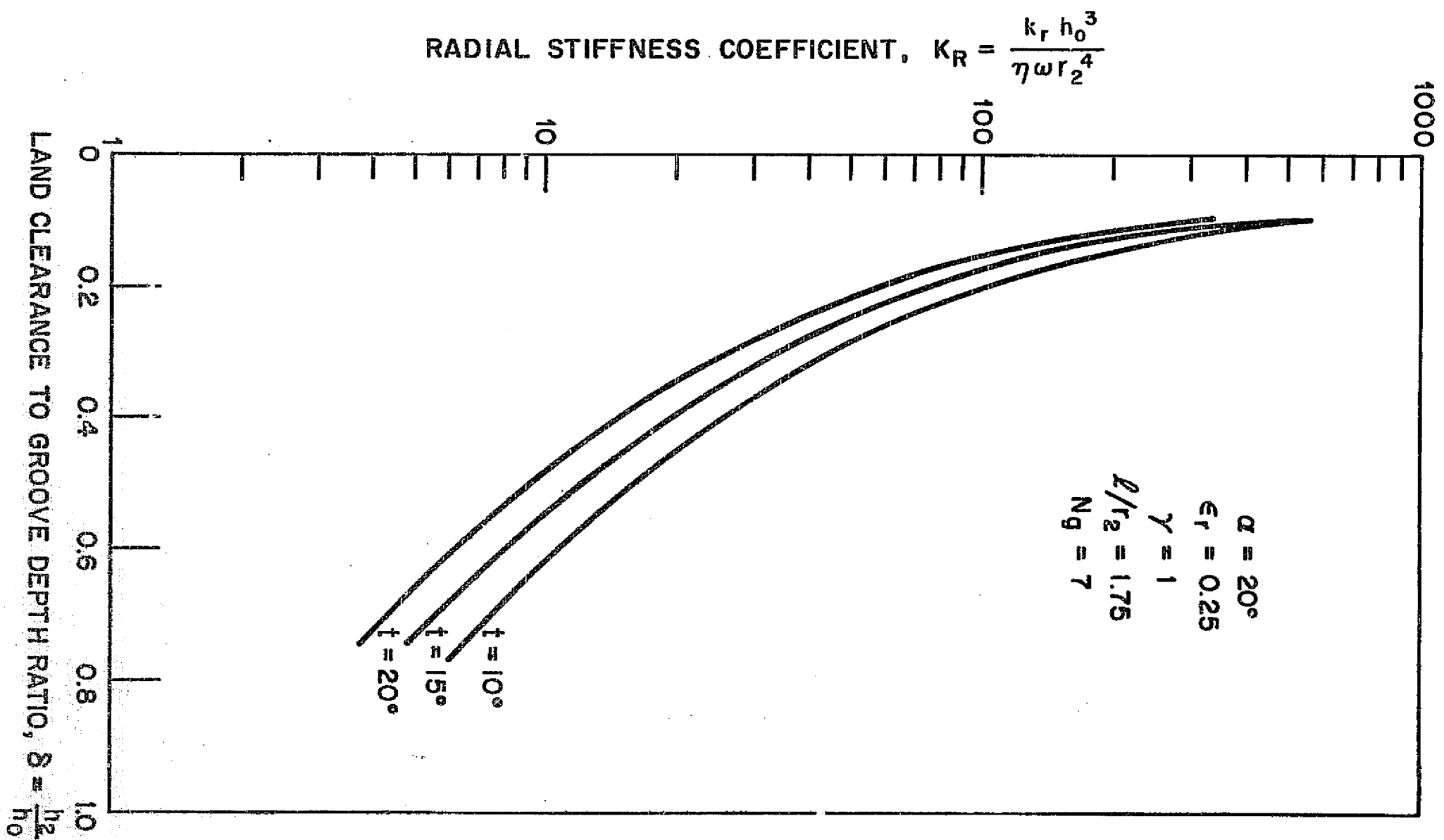


FIG. 3-25 RADIAL STIFFNESS COEFFICIENT VS. δ FOR $\alpha = 20^\circ$, $\epsilon_r = 0.25$

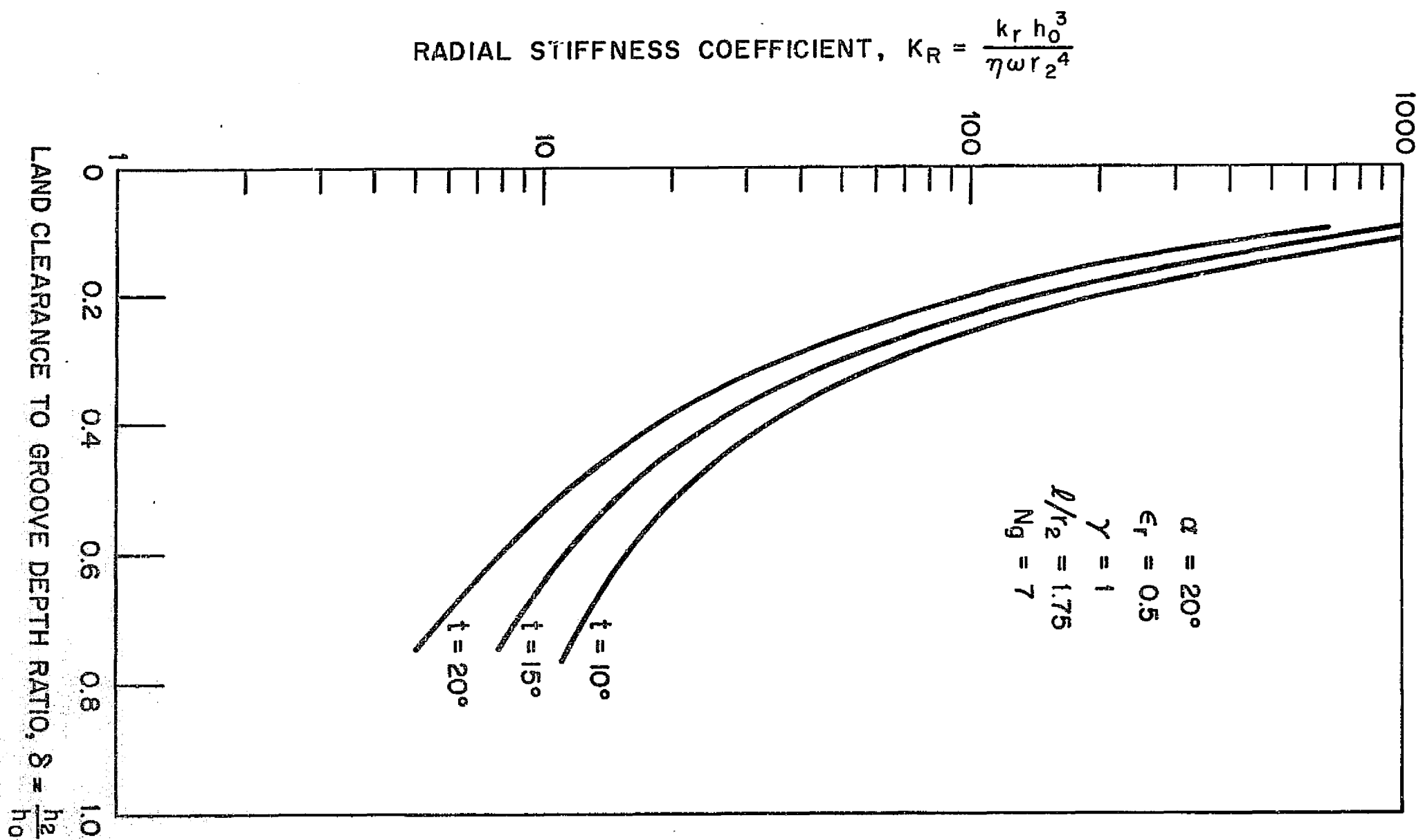


FIG. 3-26 RADIAL STIFFNESS COEFFICIENT VS. δ FOR $\alpha = 20^\circ$, $\epsilon_r = 0.50$

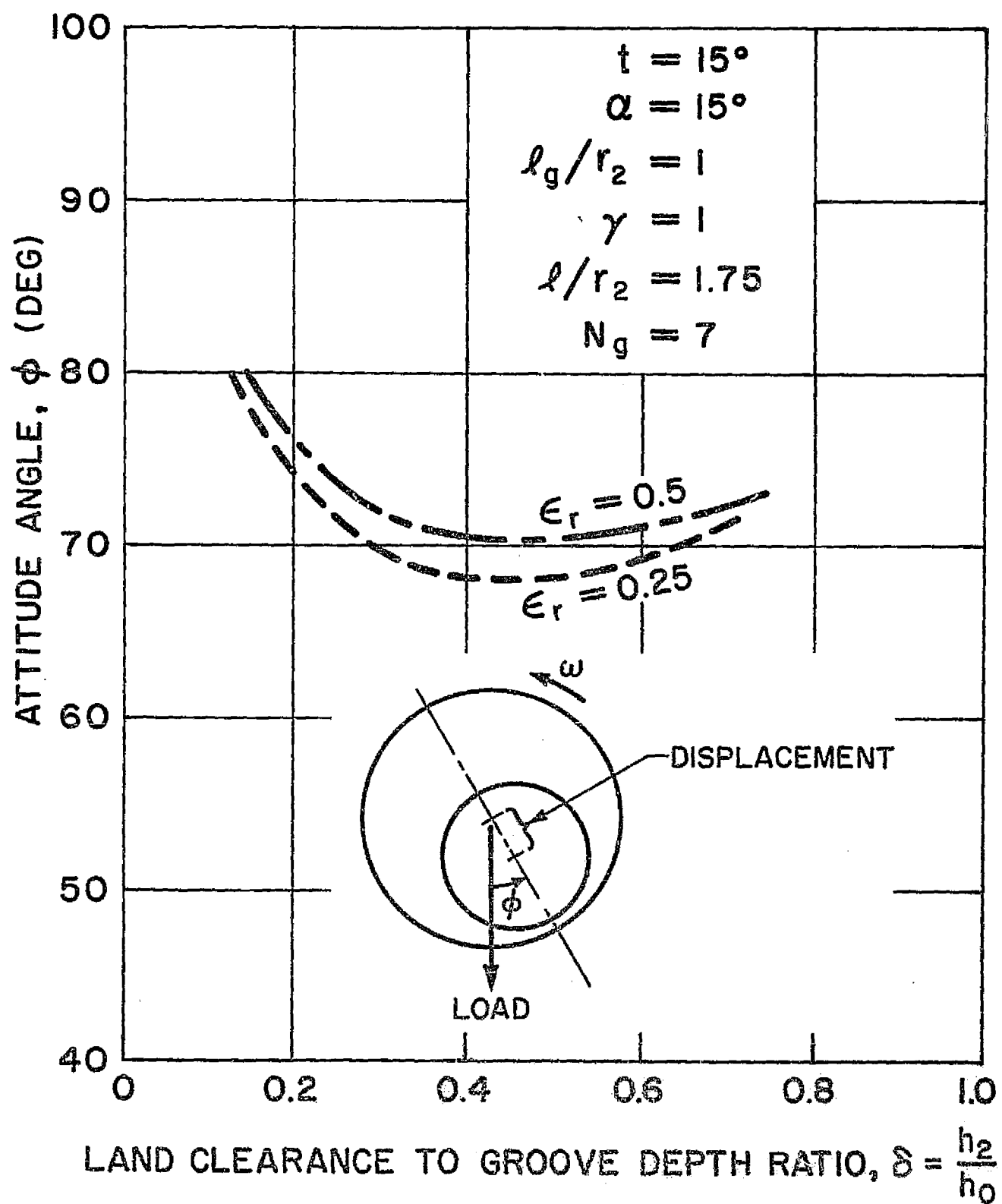
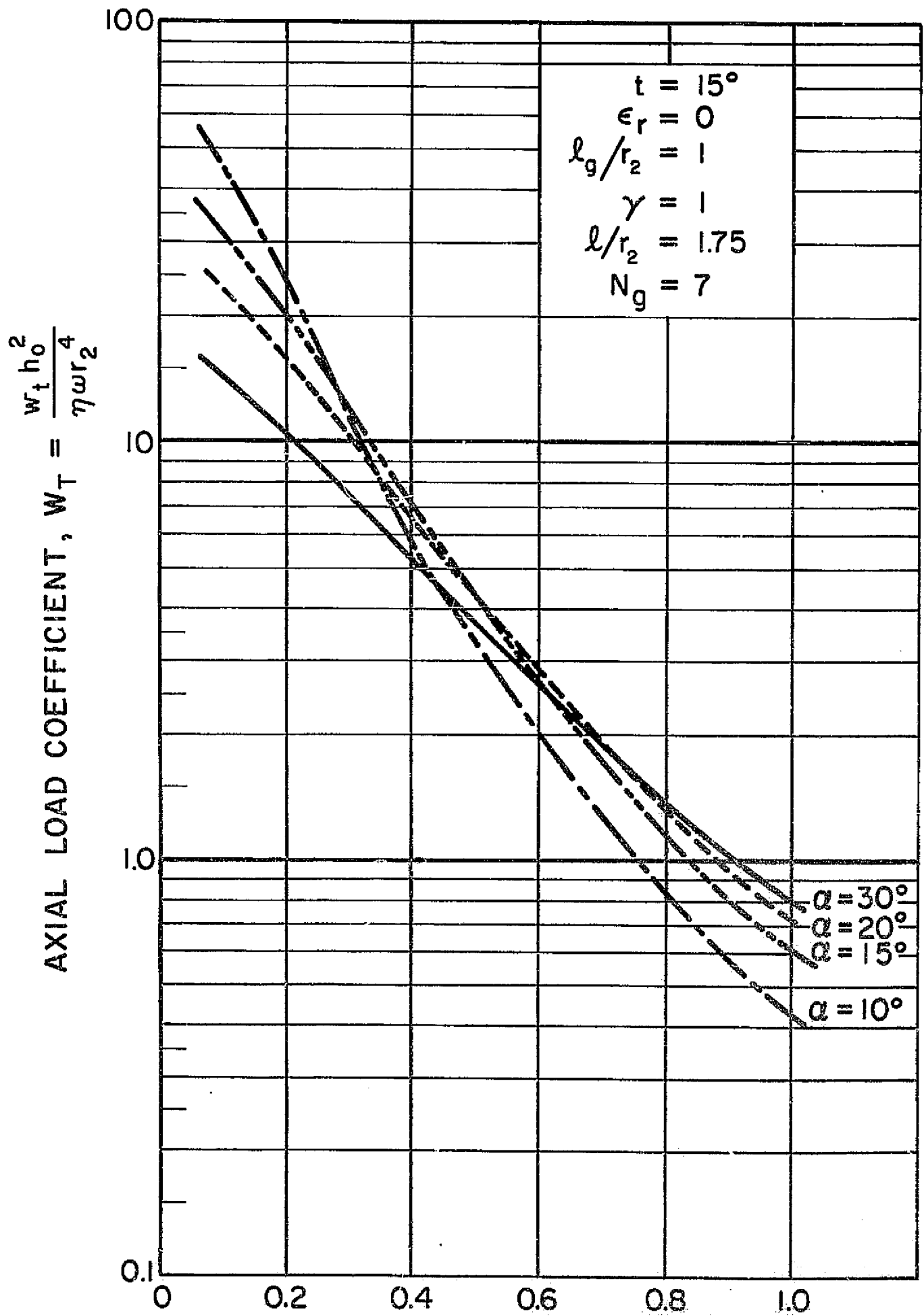


FIG. 3-27 ATTITUDE ANGLE VS. δ FOR $t = 15^\circ$, $\alpha = 15^\circ$



LAND CLEARANCE TO GROOVE DEPTH RATIO, $\delta = \frac{h_2}{h_0}$

FIG. 3-28 AXIAL LOAD COEFFICIENT VS. δ FOR $t = 15^\circ$, $\epsilon_r = 0$



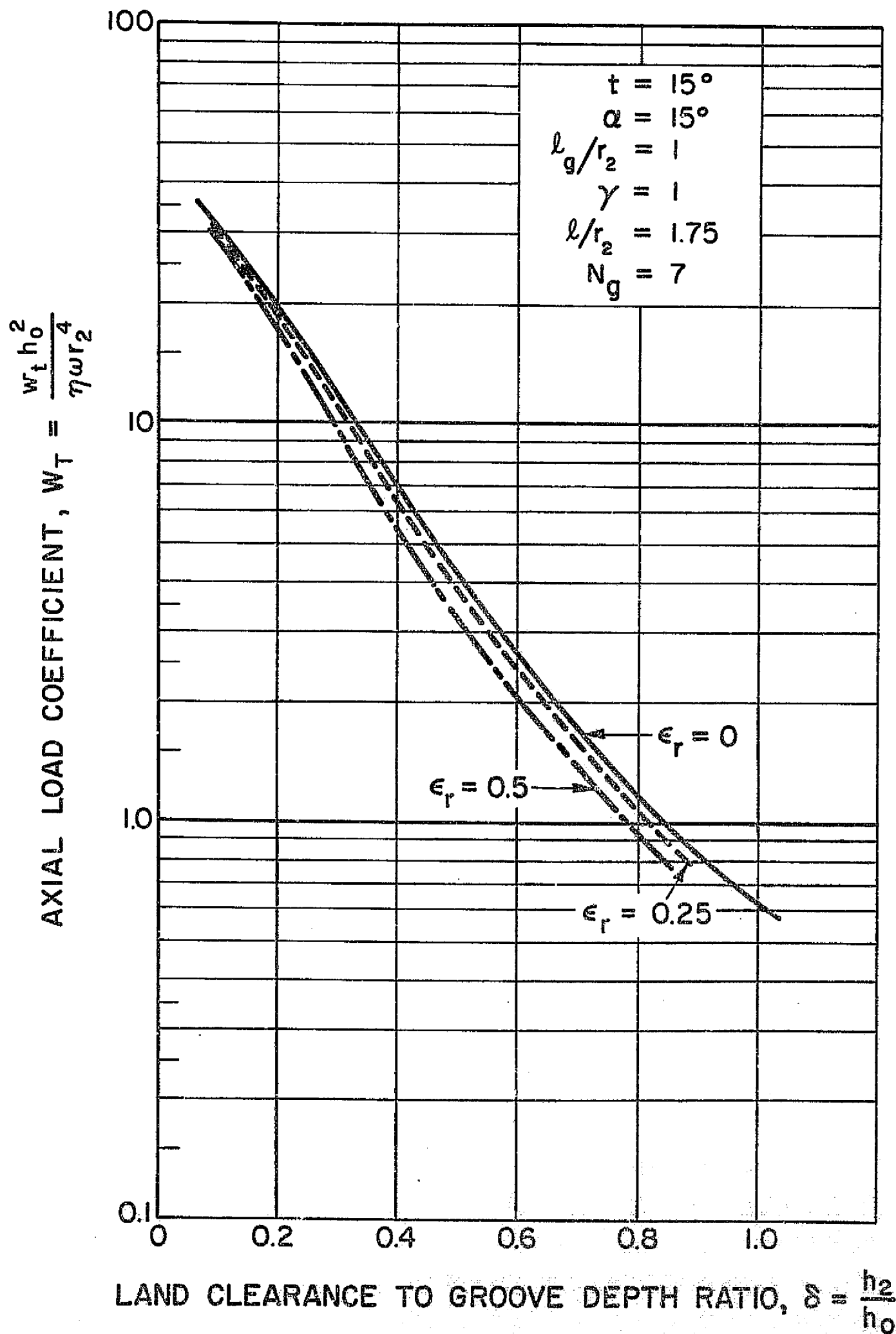


FIG. 3-29 AXIAL LOAD COEFFICIENT VS. δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

The axial stiffness is defined by

$$k_a = \frac{d w_t}{d y}$$

where y is the axial coordinates (Figure 3-3). Since

$$\Delta y = \frac{\Delta h_2}{\sin t}$$

the axial stiffness can be written as

$$k_a = \frac{d w_t}{d h_2} (\sin t)$$

Thus, the axial stiffness coefficient becomes

$$K_A = \frac{k_a h_o^3}{\eta \omega r_2^4} = \frac{d W_T}{d \delta} (\sin t)$$

The derivative $\frac{d W_T}{d \delta}$ are computed from plots of W_T vs. δ such as Figures 3-28 and 3-29 using the central difference method. The resulting stiffness coefficients are plotted in Figure 3-30 and 3-31. It is interesting to note that although small cone angles possess higher axial loads as shown in Figure 3-14, the opposite is true with regard to axial stiffnesses (Figure 3-31). This is because in computing the stiffness, the derivative $\frac{d W_T}{d \delta}$ is multiplied by $\sin t$ which decreases with lower values of t .

Figure 3-32 and 3-33 illustrate the variation of the friction moment coefficient for $t = 15^\circ$. Plots of M for other cone angles are similar to those presented.

Optimization

Axial load - Figure 3-34 and Figure 3-35 show the variation of modified axial load coefficient (W_T') with δ . Note that W_T' is defined as

$$W_T' = \frac{w_t h_2^2}{\eta \omega r_2^4}$$

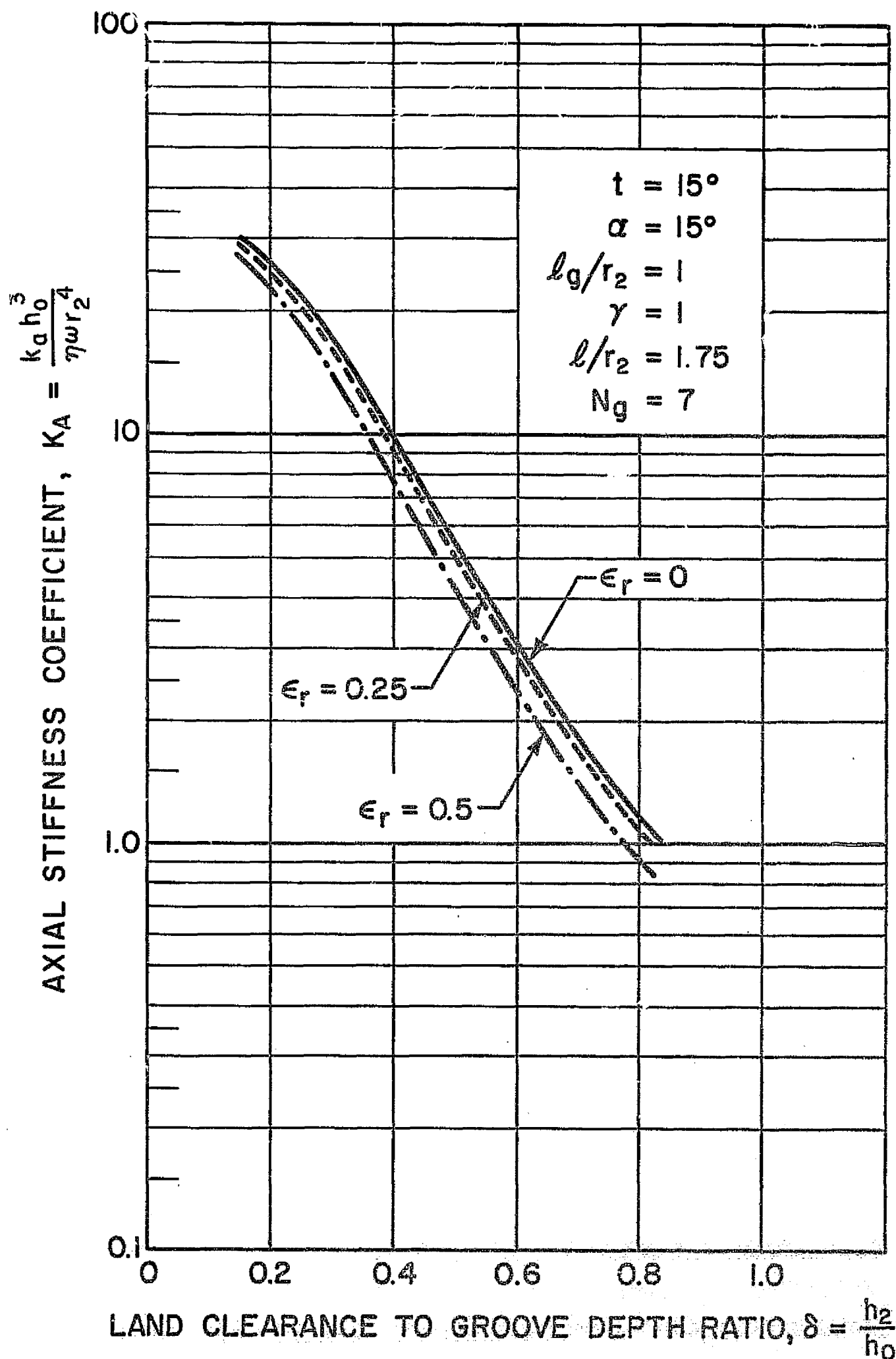


FIG. 3-30 AXIAL STIFFNESS COEFFICIENT VS. δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

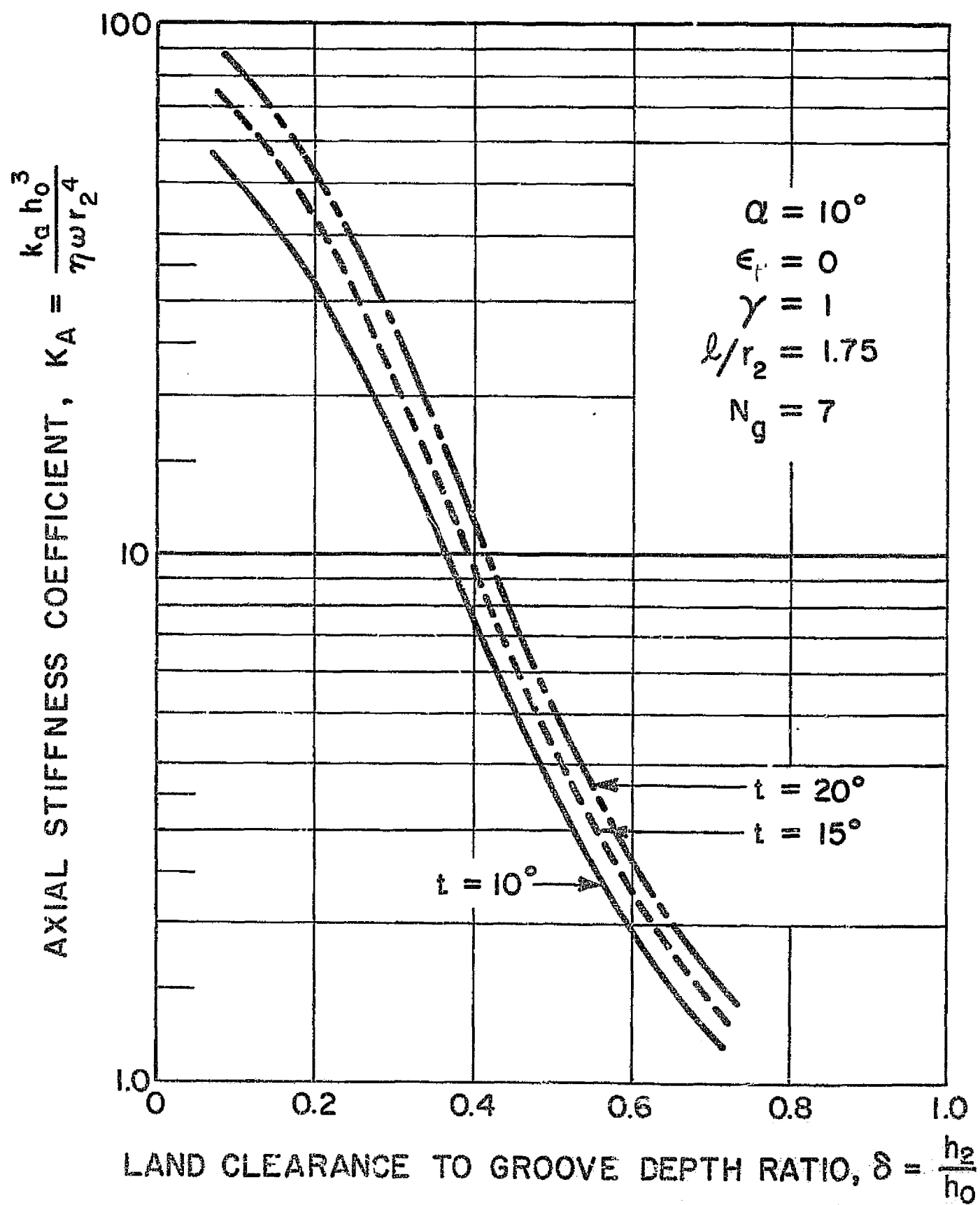


FIG. 3-31 AXIAL STIFFNESS COEFFICIENT VS. δ FOR $\alpha = 10^\circ$; $\epsilon_r = 0$

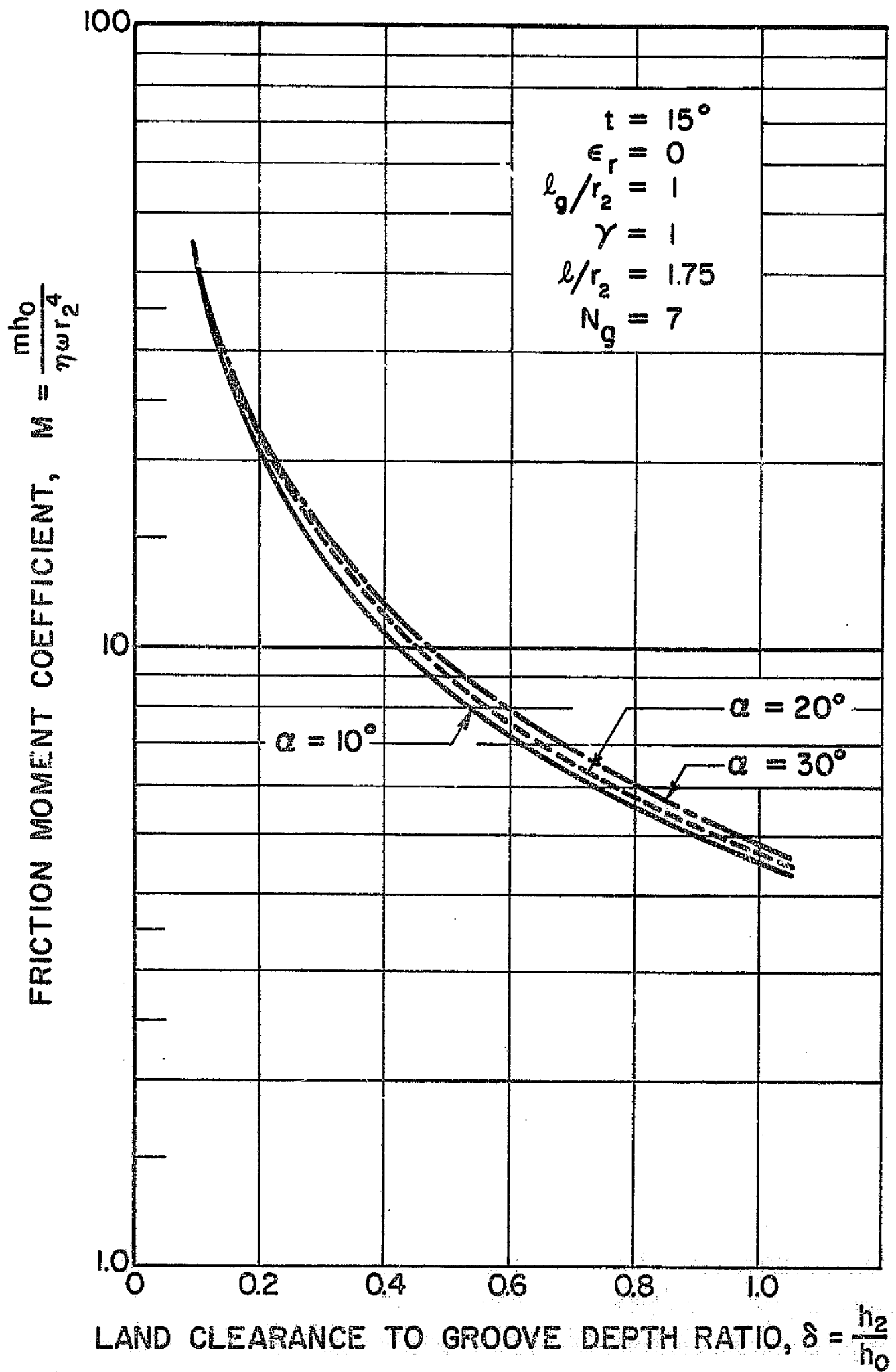


FIG. 3-32 FRICTIONAL MOMENT COEFFICIENT VS. δ FOR $t = 15^\circ$, $\epsilon_r = 0$

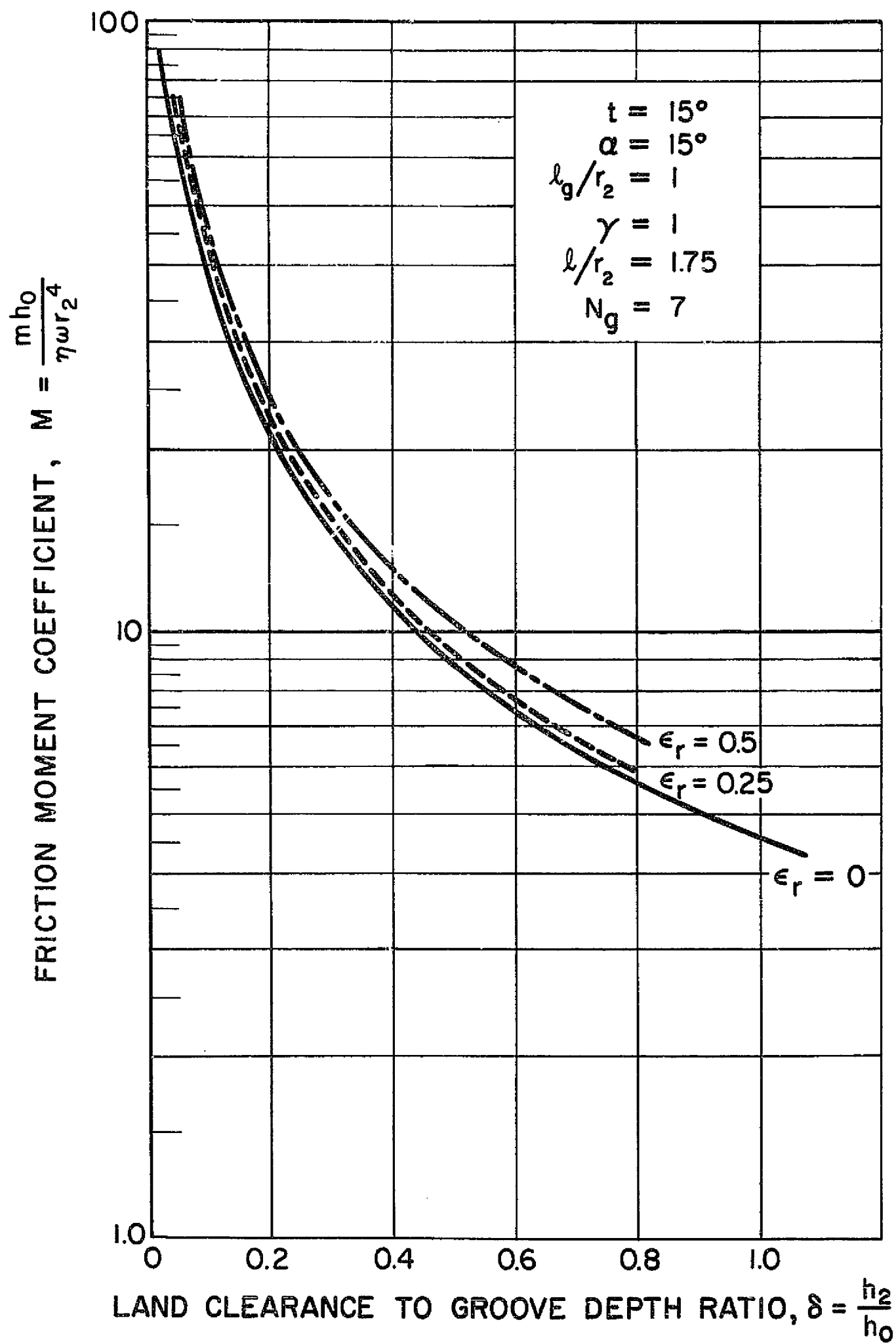


FIG. 3-33 FRICTIONAL MOMENT COEFFICIENT VS. δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

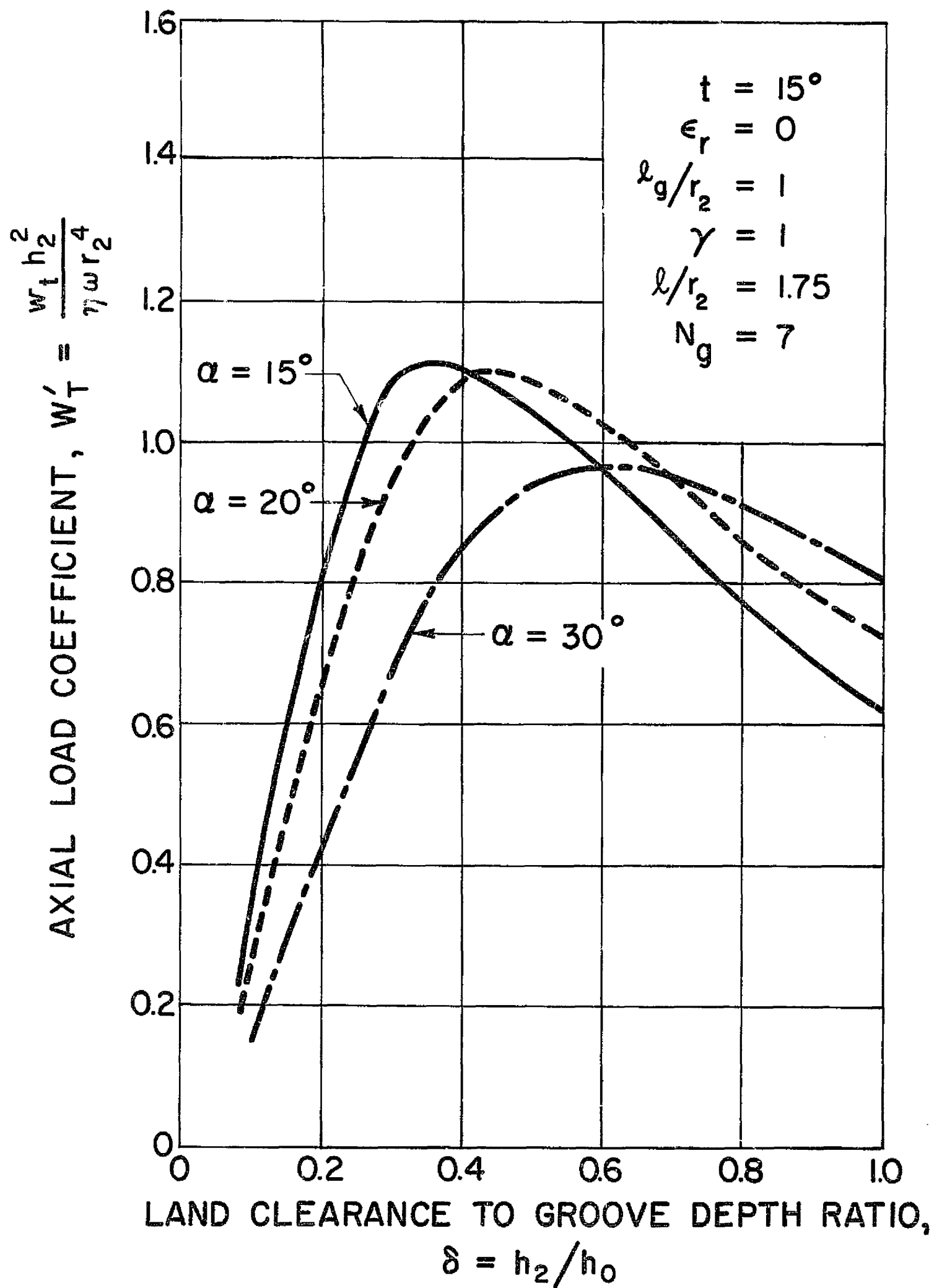


FIG. 3-34 OPTIMIZATION OF AXIAL LOAD COEFFICIENT WITH RESPECT TO δ FOR $t = 15^\circ$, $\epsilon_r = 0$

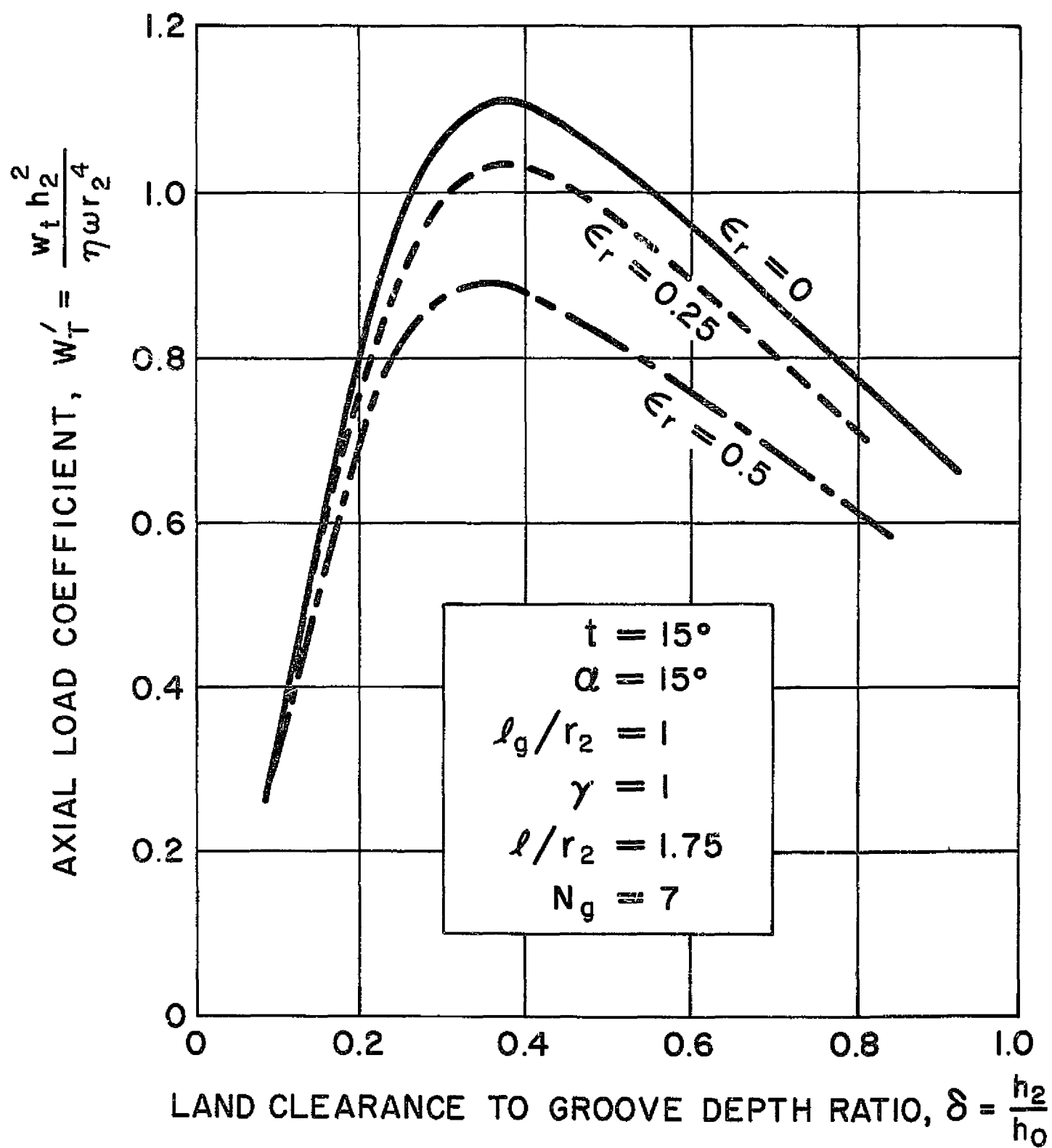


FIG. 3-35 OPTIMIZATION OF AXIAL LOAD COEFFICIENT WITH RESPECT TO δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

Thus, these plots can be used to illustrate the influence of groove depth on axial load capacity with h_2 held constant. Figure 3-34 shows that the optimum groove depth decreases as α increases. Figure 3-35 indicates that the optimum groove depth is insensitive to changes in ϵ_r .

Friction moment to axial load ratio - Figure 3-36 shows that a minimum in friction moment to axial load ratio can be found by varying groove depth while holding h_2 constant.

Axial stiffness to friction moment ratio - Values of axial stiffness to friction moment ratio are plotted in Figure 3-37 and 3-38. The maximum values shown in Figure 3-38 are plotted in Figure 3-39. It is interesting to note that the optimum δ in all cases is about 0.2.

Stiffness ratio - Figure 3-40 shows that the ratio of radial to axial stiffness is always above 1. However, the radial stiffness coefficient decreases with increasing cone angle (Figure 3-25 and Figure 3-26) while the opposite is true for the axial stiffness coefficient (Figure 3-31) and a reduction in the stiffness ratio can be achieved by increasing the cone angle.

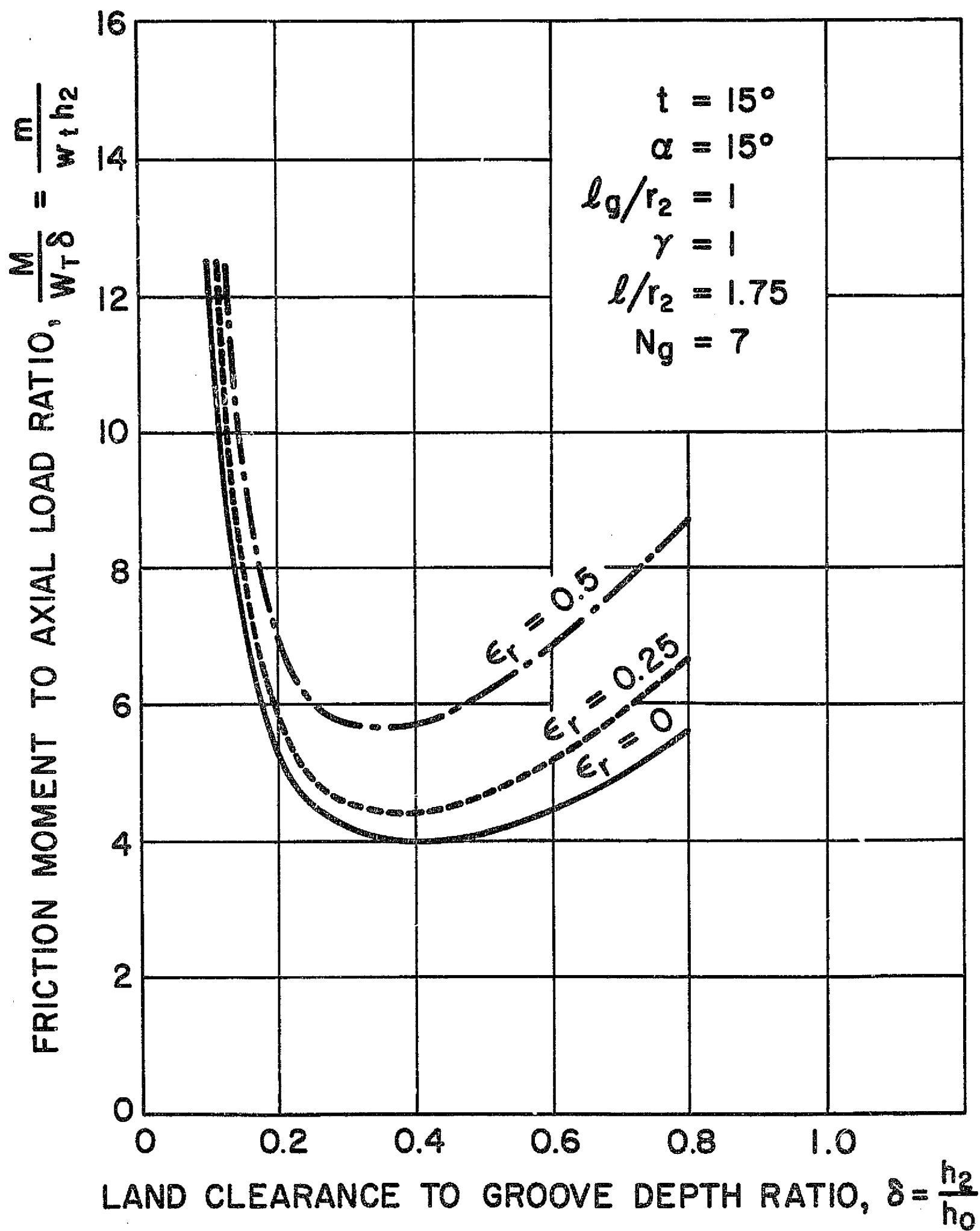


FIG. 3-36 OPTIMIZATION OF FRICTION MOMENT TO AXIAL LOAD RATIO WITH RESPECT TO δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

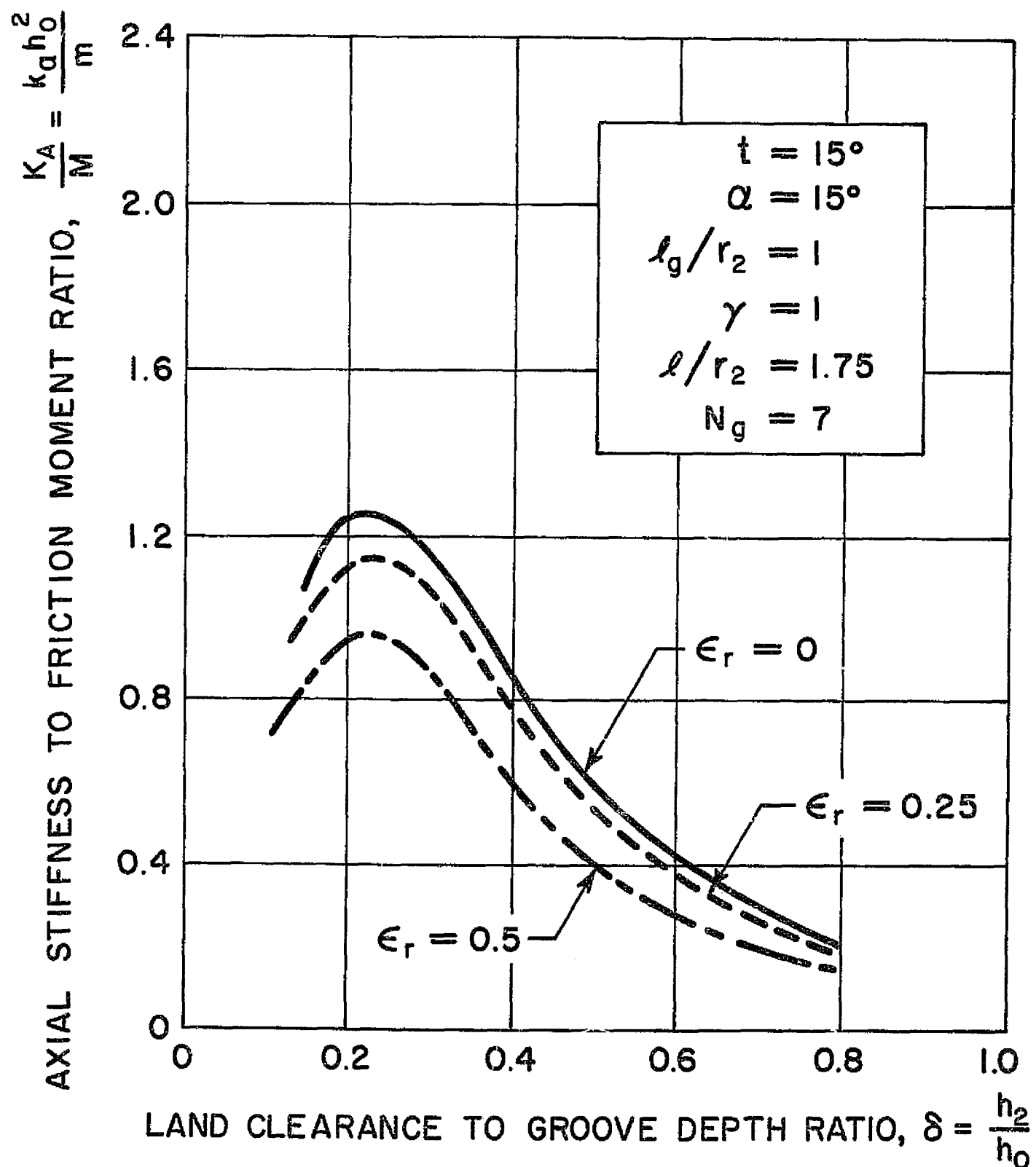


FIG. 3-37 OPTIMIZATION OF AXIAL STIFFNESS TO FRICTION MOMENT RATIO WITH RESPECT TO δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

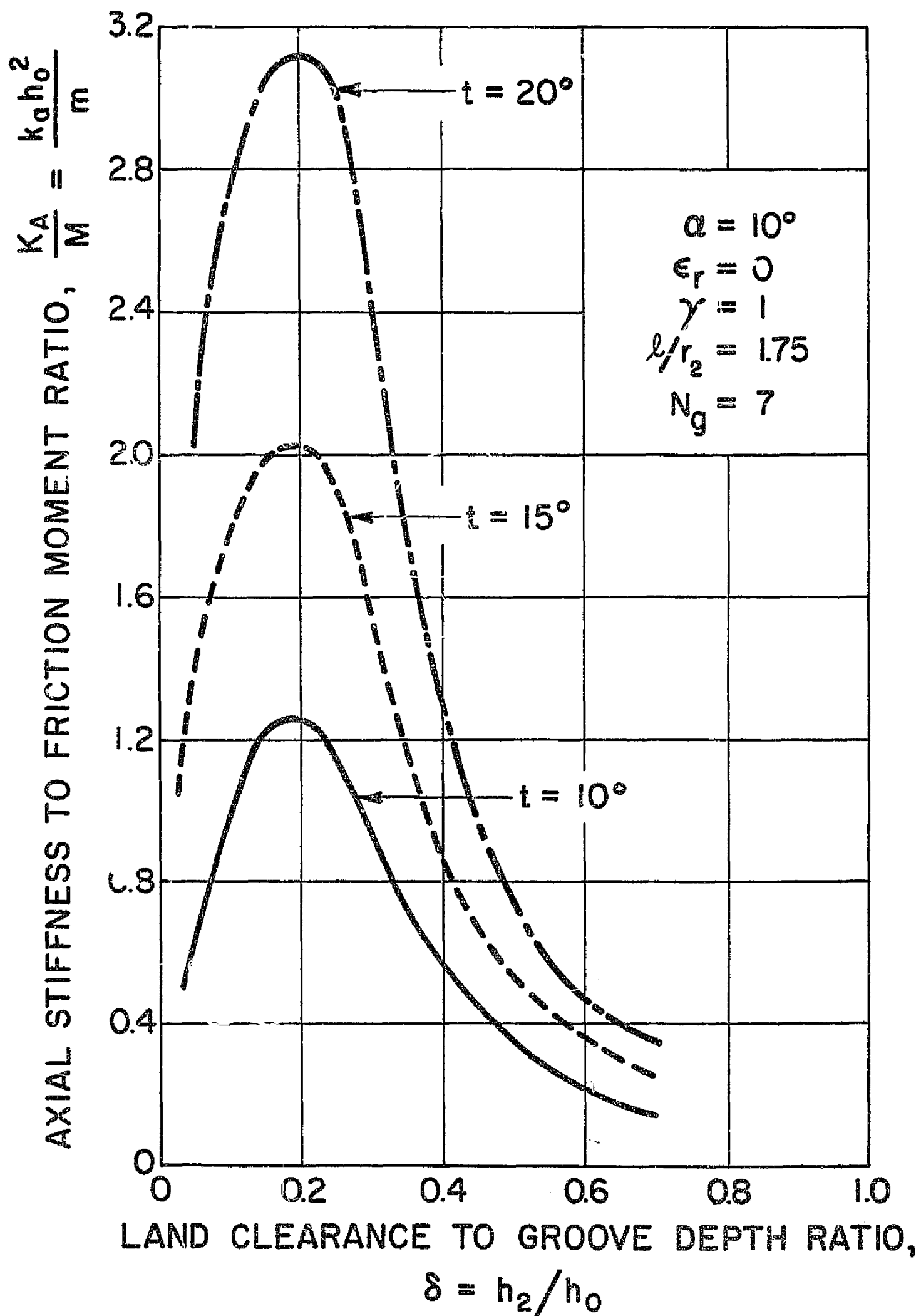


FIG. 3-38 OPTIMIZATION OF AXIAL STIFFNESS TO FRICTION MOMENT RATIO WITH RESPECT TO δ FOR $\alpha = 10^\circ$, $\epsilon_r = 0$

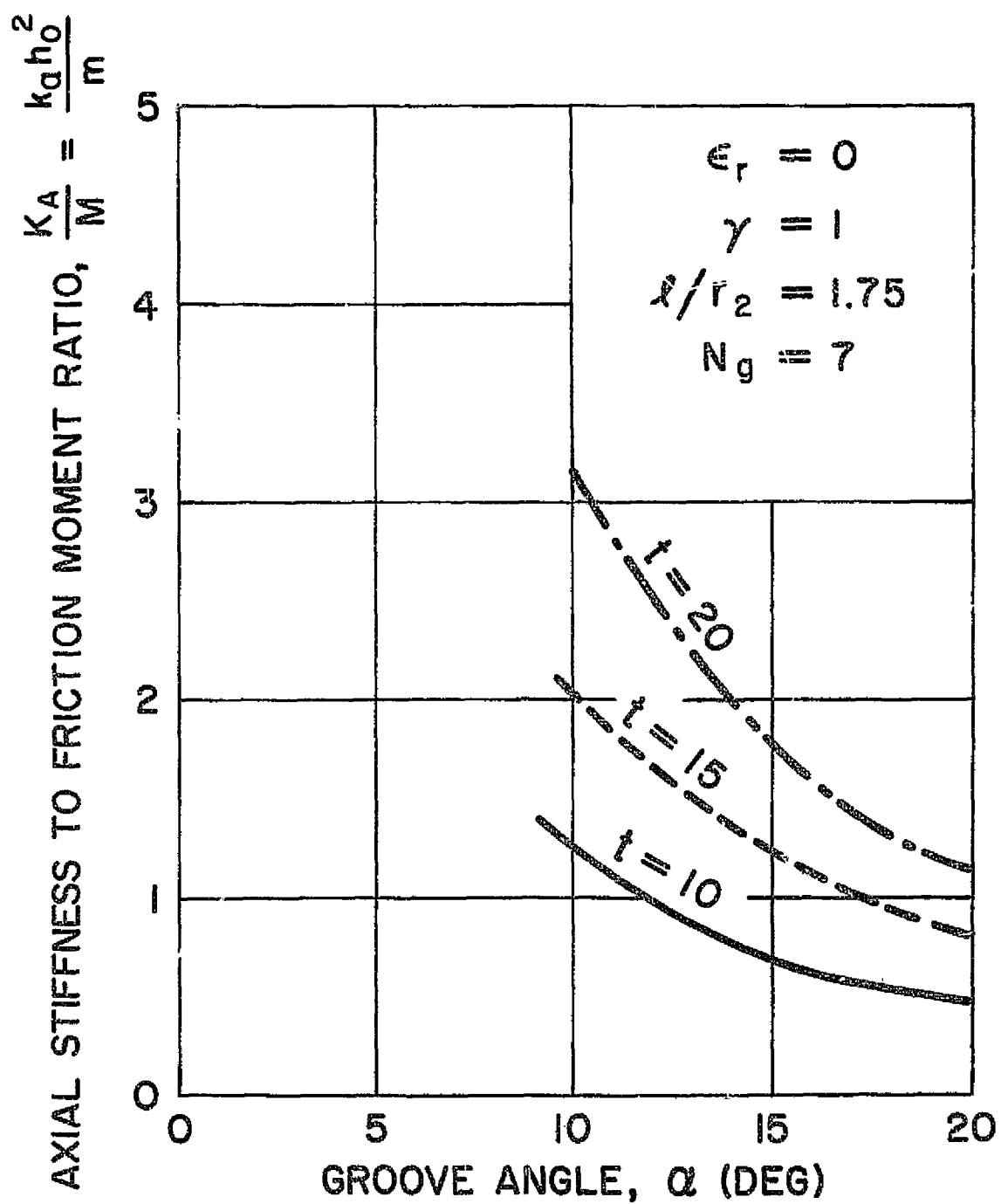


FIG. 3-39 MAXIMUM AXIAL STIFFNESS TO FRICTION MOMENT RATIO VS. α FOR $\epsilon_r = 0$

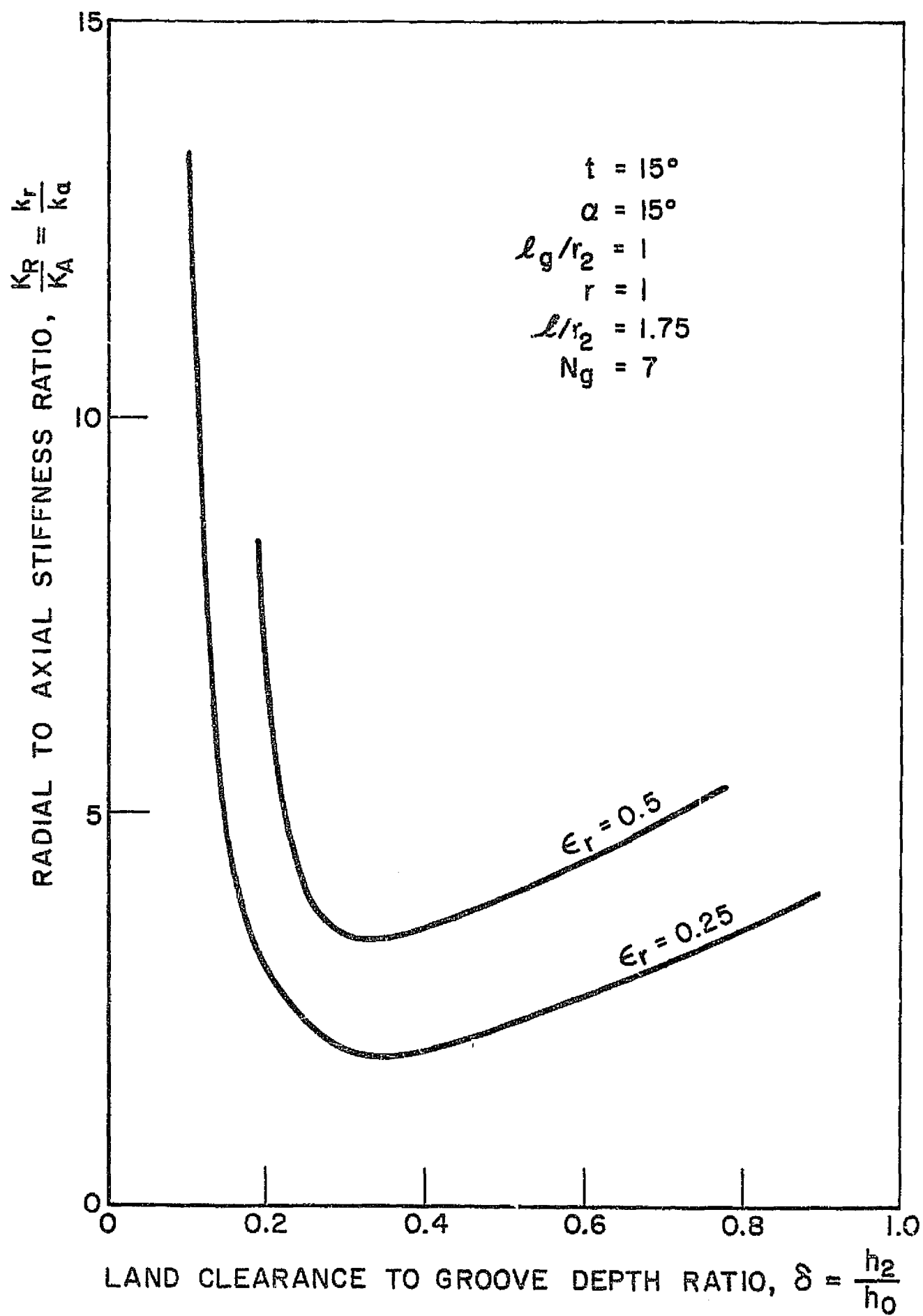


FIG. 3-40 RADIAL TO AXIAL STIFFNESS RATIO VS. δ FOR $t = 15^\circ$, $\alpha = 15^\circ$

4. BEARING EXPERIMENTAL STUDIES

4.1 Typical Test Rotor Specifications

All rotors, except the retrofitted LEV-3 gyros, had the following specifications:

- weight = 2.81 Newtons
- speed = 24,000 rpm
- available motor power = 8 watts
- angular momentum 2.6×10^6 g cm²/sec
- slew rate = 1 radian per second
- maximum design load = 6 g
- working life = 2 years
- number of start-stops = 1000

The retrofitted LEV-3 had a weight of 5.4 Newtons and an angular momentum of 9.2×10^6 g cm²/sec.

4.2 Machining Spiral Grooves on a Conical Surface

To provide a groove geometry that closely approached the idealized analytical model, a machining rather than etching process was adapted. Compared to etching, machining provided sharp groove edges and a relatively constant groove depth. Although etching is considered an acceptable process for producing grooves, good control of the groove features was desirable during experimental evaluation.

Figure 4-1 shows a Bridgeport milling machine with a Precise grinding spindle clamped to the rear-end of the ram. The grinding spindle was positioned over the table and fitted with a chuck to hold an end-mill. The cutting speed was controlled by the Variac shown on the right hand side of the milling table. For a 4.76 mm base diameter cone with 7 grooves, the end-mill was ground to a 0.178 mm cutting width and rotated at 35,000 rpm.

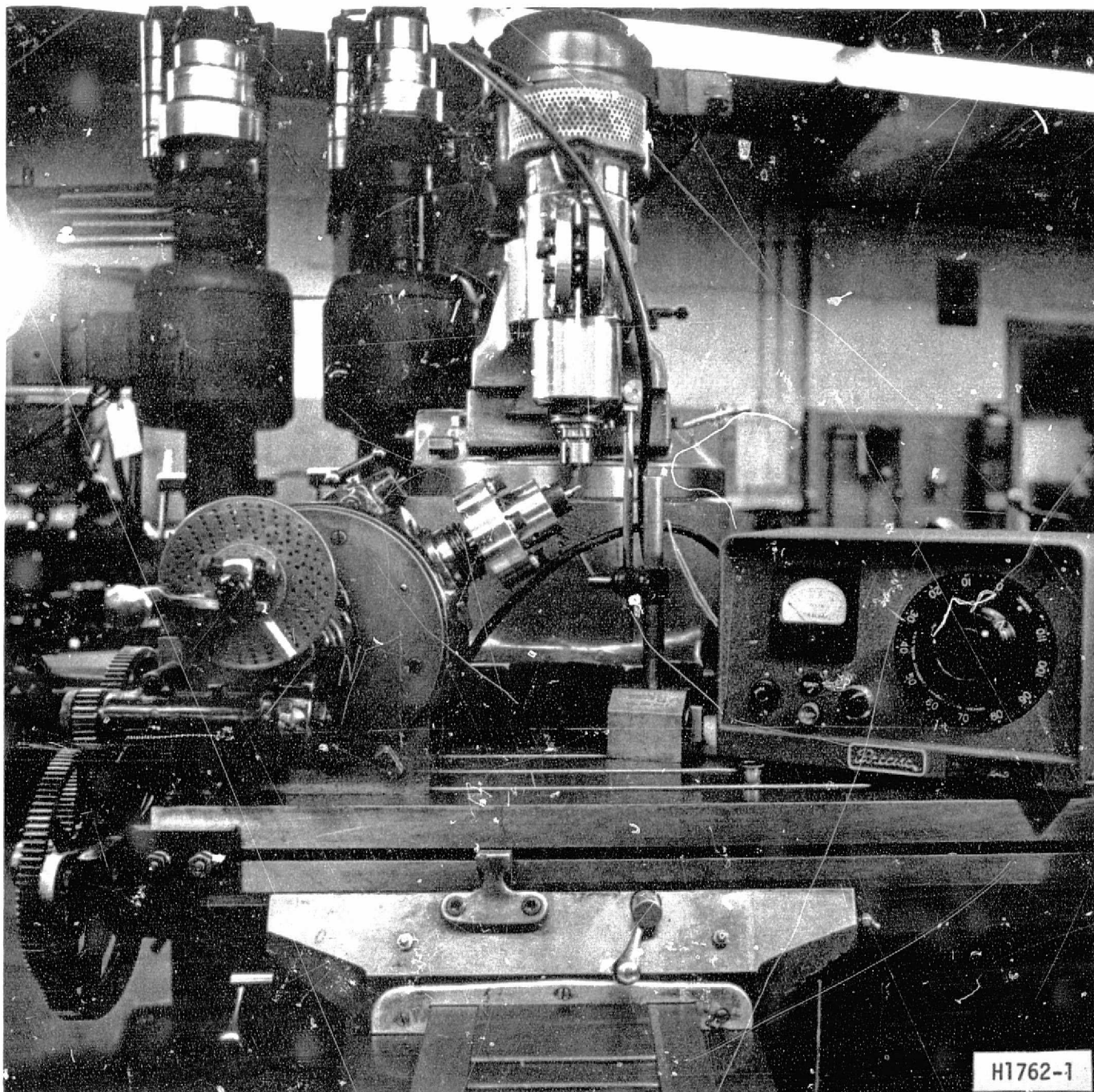


Figure 4-1. Milling Machine Set-up Used to Cut Spiral Grooves on a Conical Surface

The journal to be grooved was mounted in a tilted universal dividing head with the end-mill cutting plane tangent to the conical surface and parallel to the table.

The crank handle on the dividing head indexed the cone, thus determining the number of grooves. With the index plate locking pin removed, rotation of the crank handle caused the cone to turn about its axis and the milling table to translate. The gear arrangement (left foreground, Fig. 4-1) used to couple the dividing head to the table lead screw determined the advance of the spiral groove per rotation of the cone. Since the lead of the cutter was constant, the groove angle α varied along the cone axis with the largest α located at the end of the cone near the apex.

For the cone angles and groove lengths used in the experimental studies, the analysis showed that data based on a logarithmic spiral with the average groove angle and land to groove width ratio, would be suitable for experimental comparison.

Figure 4-2 shows a typical grooved conical journal cut by the milling method.

4.3 Description of Test Arrangements

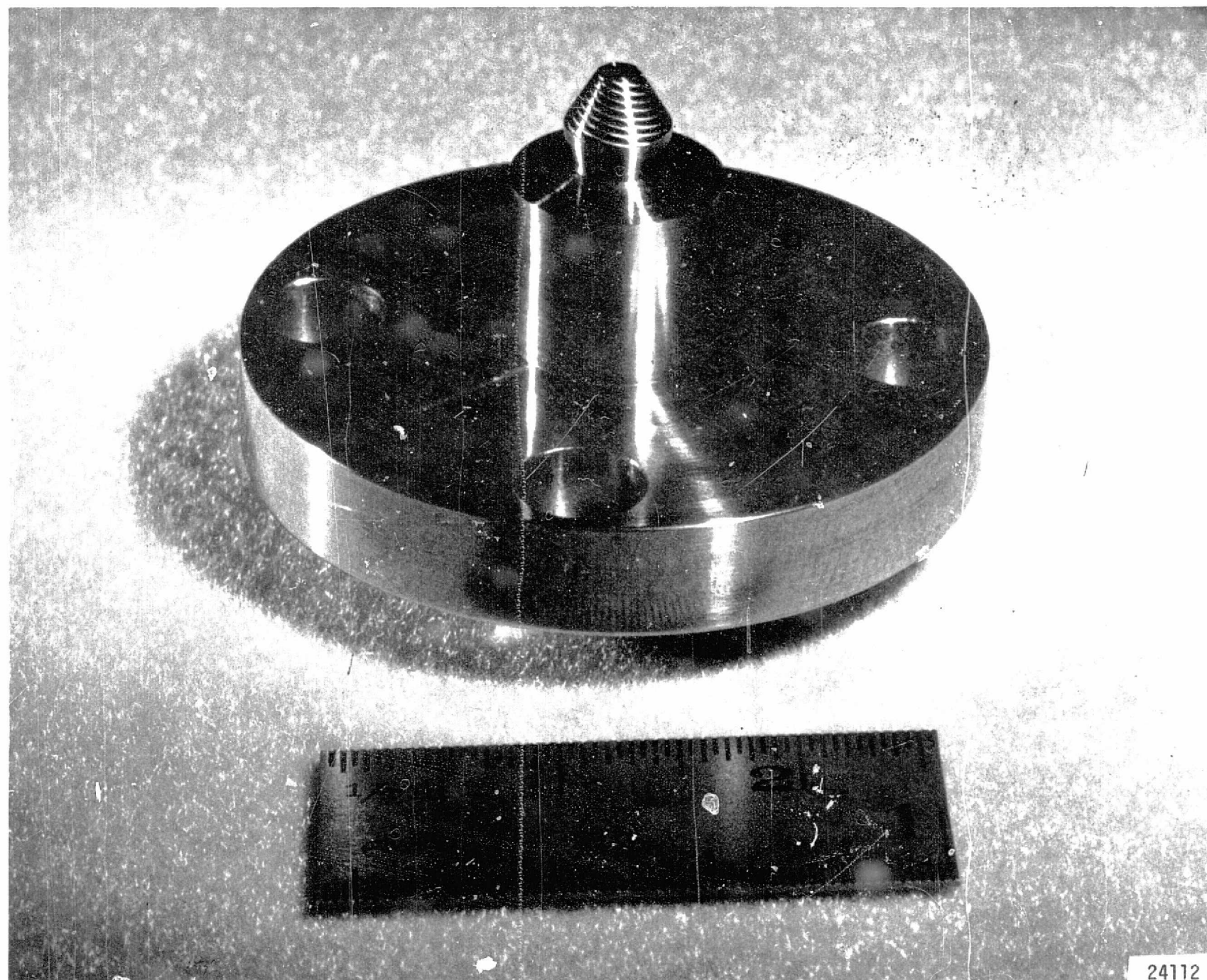
Four test rigs of different designs were employed. There were the

- belt driven single bearing rig.
- air driven gyro simulator dynamometer.
- air driven grease development rig.
- motor driven converted LEV-3 gyro rig.

4.3.1 Belt Driven Single Bearing Rig

The single bearing rig was used to evaluate the method of machining spirals on a conical surface, to screen grease lubricants and to provide data for checking the approximate analysis.





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Figure 4-2. Machined Sprial Grooves on a Conical Surface

Figure 4-3 shows a sketch of the single bearing rig. Its basic components are a belt driven rotor, a load piston and a torque disc. The test journal is threaded into the rotor and drawn tight in a taper. The test bearing is fixed to the torque disc which is supported on a hydrostatic air bearing and bearing friction torque is computed from the moment required to restrain the torque disc from rotating. Axial load is applied to the test bearing by an air-actuated piston. Bearing speed (up to 10,000 rpm) is fixed by the motor-spindle pulley ratio.

Figure 4-4 shows a photograph of the test stand. This relatively simple set-up was used in the initial testing of the conical bearing and was particularly useful in obtaining axial load and friction data. Figure 4-4 shows the test bearing with a grease fitting on the left and three thermocouples wires on the right. The thermocouples are monitored by a continuous, multi-channel temperature recorder and the friction torque, by a strip chart recorder. Two of the thermocouples are imbedded in the skin of the bearing and a third is located in the lubricant cavity below the truncated end of the cone.

4.3.2 Air Driven Gyro Simulator Dynamometer

The gyro simulator was used to determine the power requirements and dynamics of a rotor supported by grease lubricated spiral groove bearings.

Figure 4-5 shows a sketch of the test rig. Its basic components are a test rotor, conical test bearings, a dynamometer rotor and an outside housing. The test rotor is driven by two air nozzles. The test bearing friction moment is transmitted to the dynamometer rotor which is floated on a hydrostatic gas film.

Friction torque is computed from the moment required to restrain the dynamometer rotor. Loads are applied to the test rotor through gas bearing films and dynamic motions and deflections are measured by non-contacting probes.

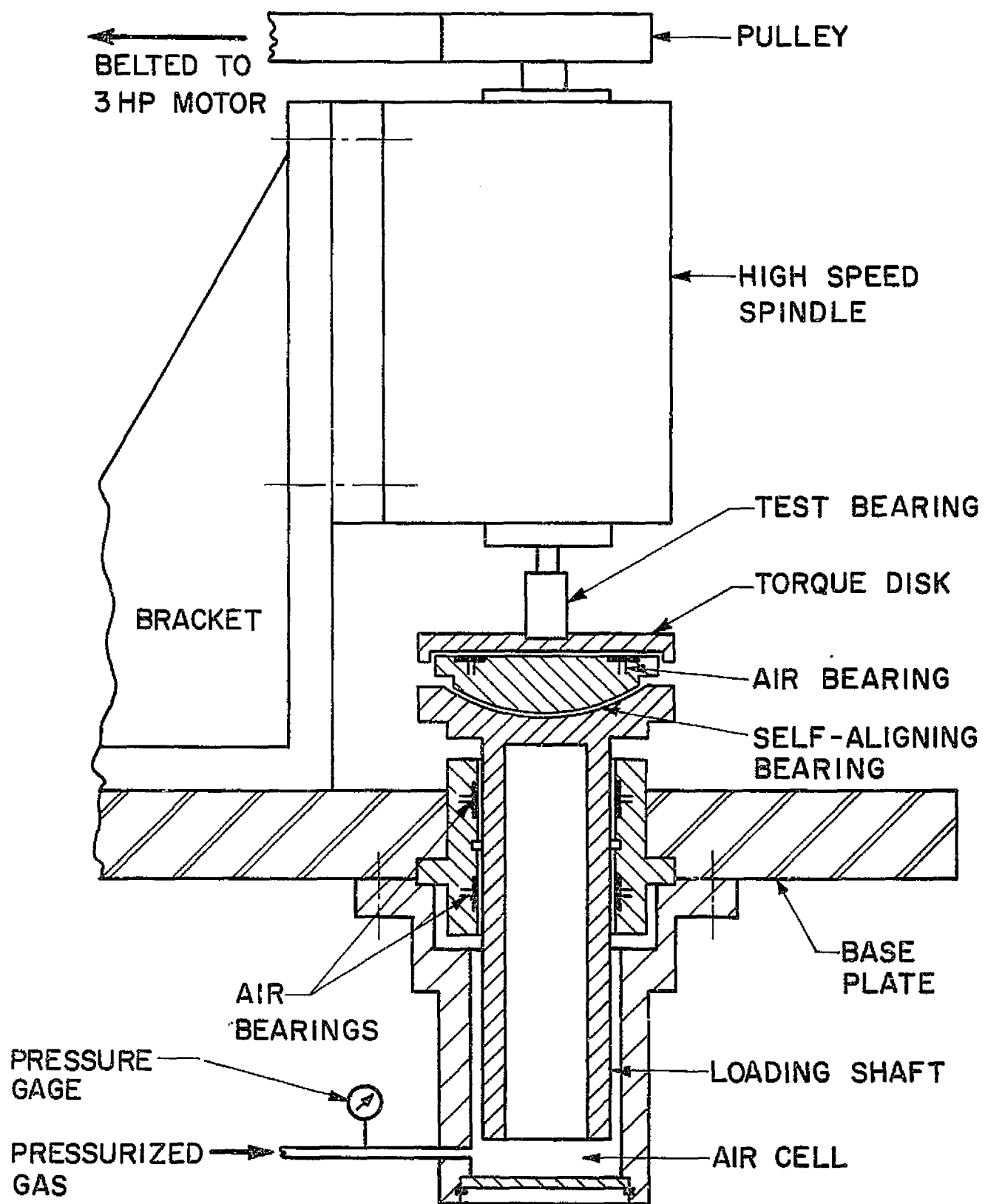


FIG. 4-3 SKETCH OF SINGLE BEARING TEST RIG



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Figure 4-4. Single Bearing Test Stand

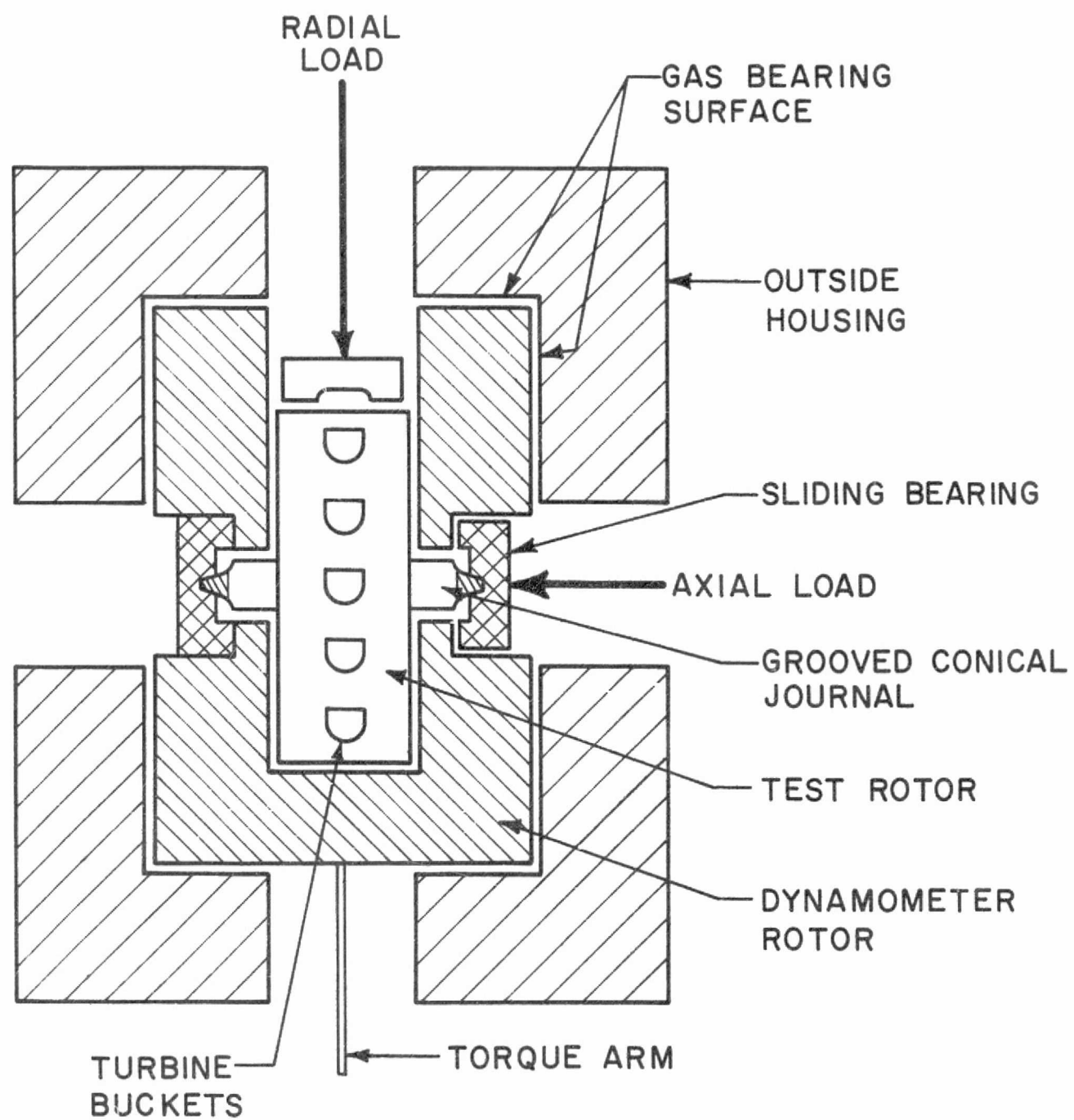


FIG. 4-5 SKETCH OF AIR DRIVEN GYRO SIMULATOR DYNAMOMETER

The rotor axis may be positioned at any angle between vertical and horizontal. Dead weight determines vertical loads; horizontal load and torque are measured by strain gage transducers. Rotor speed is set by turbine nozzle pressure and measured by a photoelectric indicator.

The simulated gyro rotor, with a disc diameter of about 2 inches is shown in Figure 4-6 near the center of the test stand base plate. To the left and right of the rotor in Figure 4-6 are the two sections of the cylindrical dynamometer body. The left bearing, (female cone) together with two orthogonal probe mounting slots, is visible at the center of the dynamometer section located to the left of the rotor. The milled "cross" in this piece provides access to the wheel for the radial load piston, the turbine nozzles and the speed counter. One of the bearings is visible in the right fore front of the test stand bed plate. To accommodate axial loadings, this component is machined to slide in the dynamometer cap. A dowel hole in the bearing flange and a pin through the cap prevents rotation with respect to the dynamometer body.

Clamped to an angle plate and mounted on the test stand column is the dynamometer housing. One of the segmented air lubricated thrust bearings can be seen at the inside rear of this cylindrical housing. The other thrust bearing is in front of the test stand. The ports in the dynamometer housing are provided for the radial load cell, the turbine nozzle and the speed indicator. The interior cylindrical surfaces on either side of the ports are the journal bearing areas for the dynamometer body. The threaded piece to the left-front of the test stand in Figure 4-4 is one of the gas manifold covers.

Figure 4-7 shows the assembly with some of the instrumentation attached. To the outside of the left manifold cover can be seen the rear of the torque arm assembly. Connected to the base of the torque arm, and behind the right angle mounting bracket is a strain gage transducer used for test bearing friction read-out.

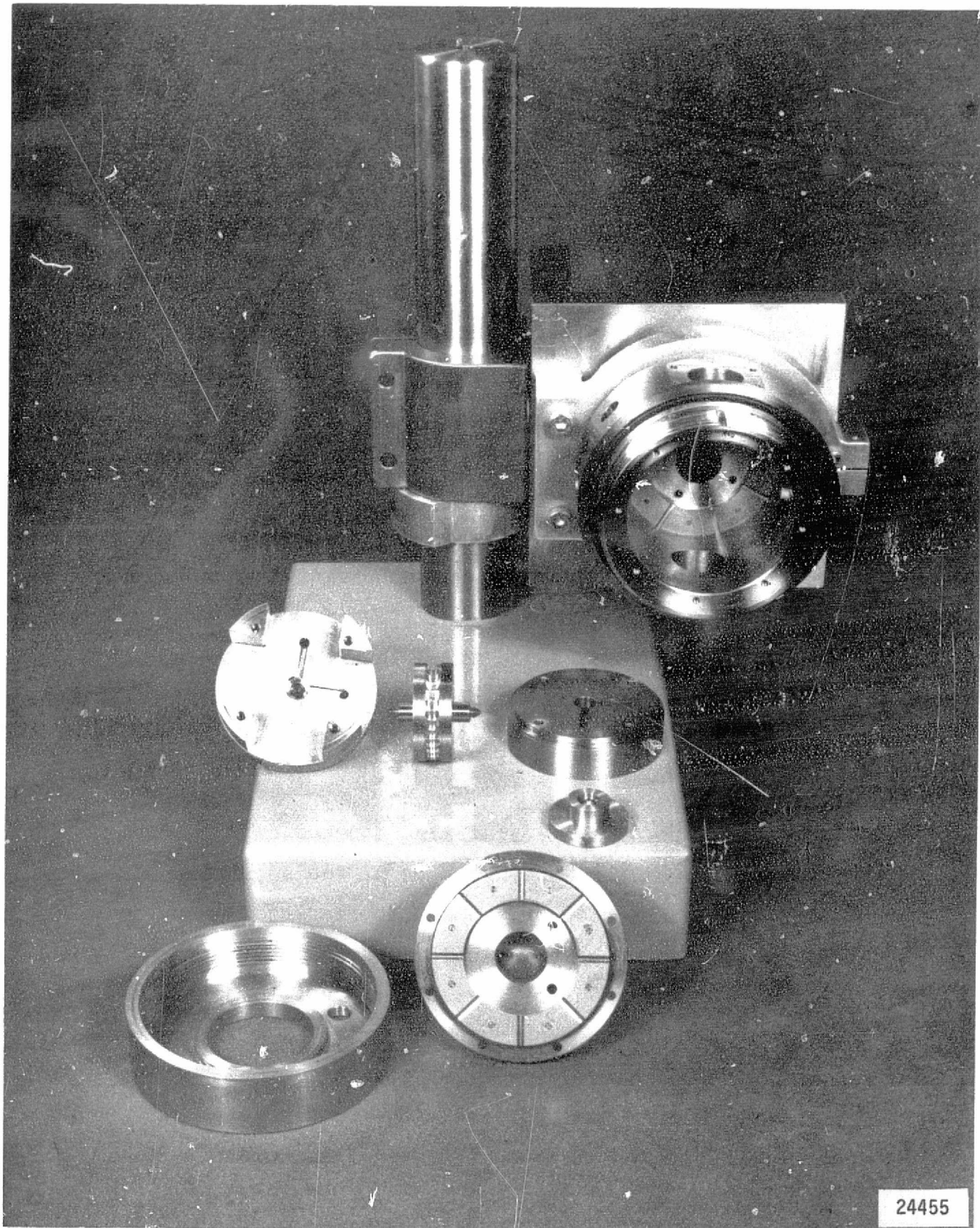


Figure 4-6. Gyro Simulator Test Rig with Rotor and Dynamometer Body Disassembled

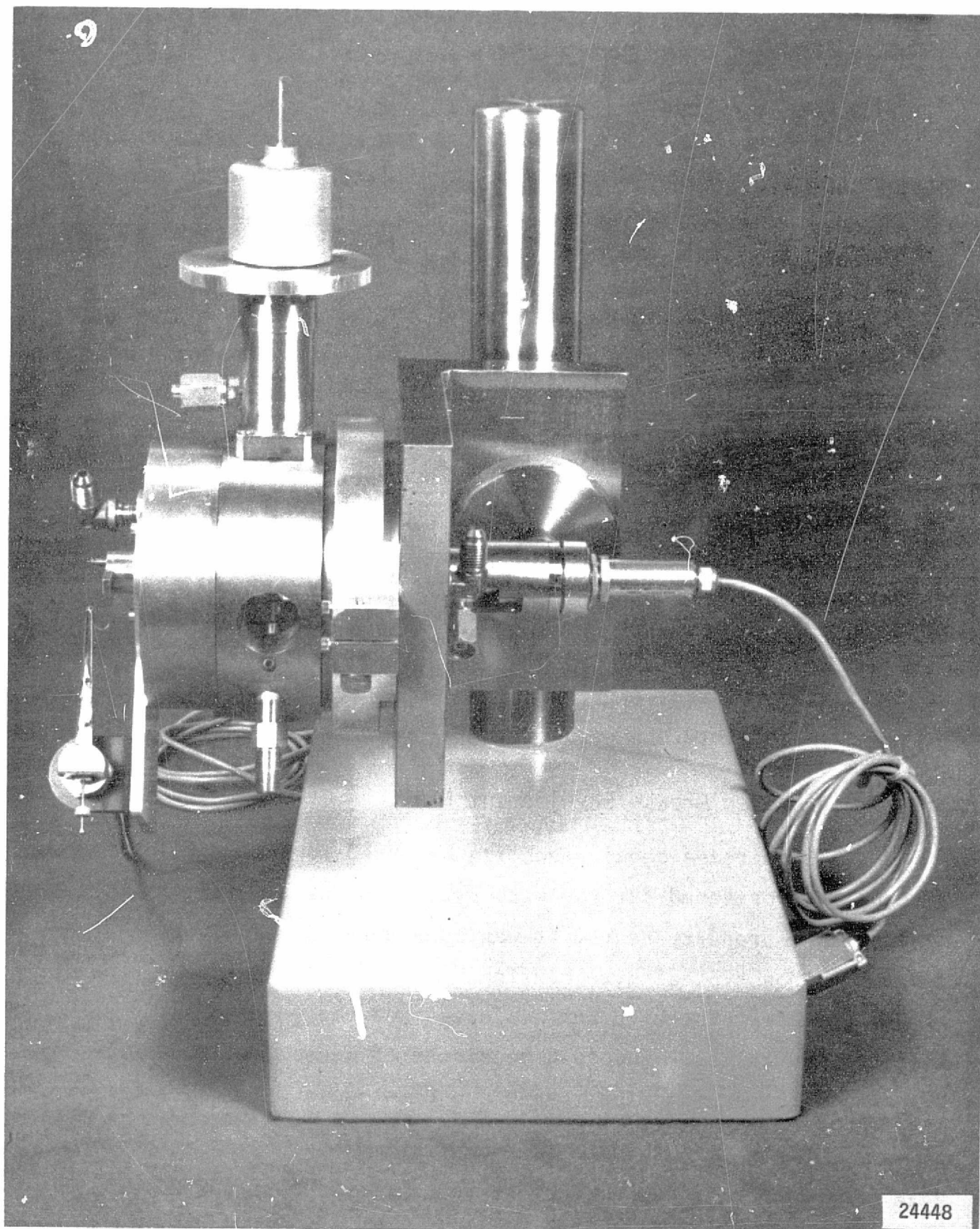


Figure 4-7. Gyro Simulator Test Rig Assembled

The radial load is applied by the weight sitting on the platform at the top left of Figure 4-7. The platform threads into the end of a load piston with a gas bearing load face which bears against the rotor disc.

Axial load is applied through the assembly that protrudes to the right of the angle bracket in Figure 4-7. This assembly consists of a strain gage transducer, a load range scaling adapter and a guide tube that bolts to the dynamometer housing. Inside the guide tube is a hollow piston whose air bearing face transmits an axial force to the test bearing. Turning a differential screw results in a small motion of the axial load piston towards the grease bearing and the imposed force on the bearing is measured by the transducer.

When the test stand angle bracket is rotated so the spin axis is vertical, the dead weight platform and the force transducer assembly are removed and interchanged.

4.3.3 Air Driven Grease Development Rig

The grease development rig was used to evaluate lubricants that were being considered for use with gyro spin axis bearings. In addition to lubricant property tests, it was considered necessary to test the lubricant in a typical gyro bearing because of the nature of the spiral groove geometry, the high surface speed and the requirement for low friction power loss. The test consisted of measuring load, film thickness, speed, friction torque and coast-down time for each grease.

Figure 4-8 shows a sketch of the grease development rig. Its basic components are an air driven rotor fitted with spiral groove conical journals, a bearing fixed to ground, a bearing floated in porous, gas lubricated journal bearings, a dead weight load platform fixed to the floated bearing and restrained from rotation by a torque arm, a capacitance probe used for computing film thickness and a photoelectric speed counter.

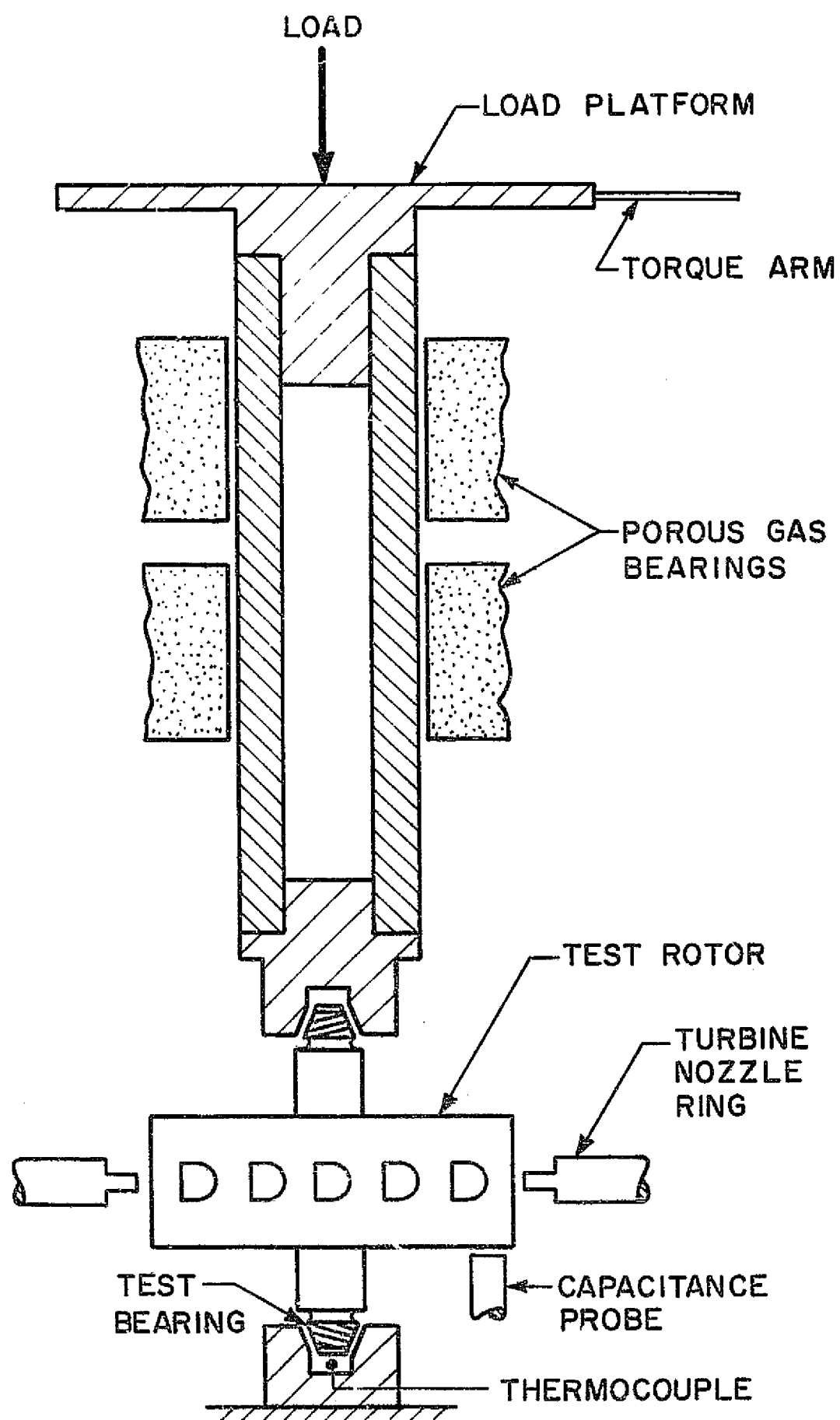


FIG. 4-8 SKETCH OF GREASE DEVELOPMENT RIG

Figure 4-9 shows a photograph of the test stand where the rotor is shrouded by a nozzle ring drive. The force transducer, shown mounted at the top center of the stand, is coupled to the lever assembly and connected to the torque arm of the load platform by a thin wire that is not discernable in the photograph.

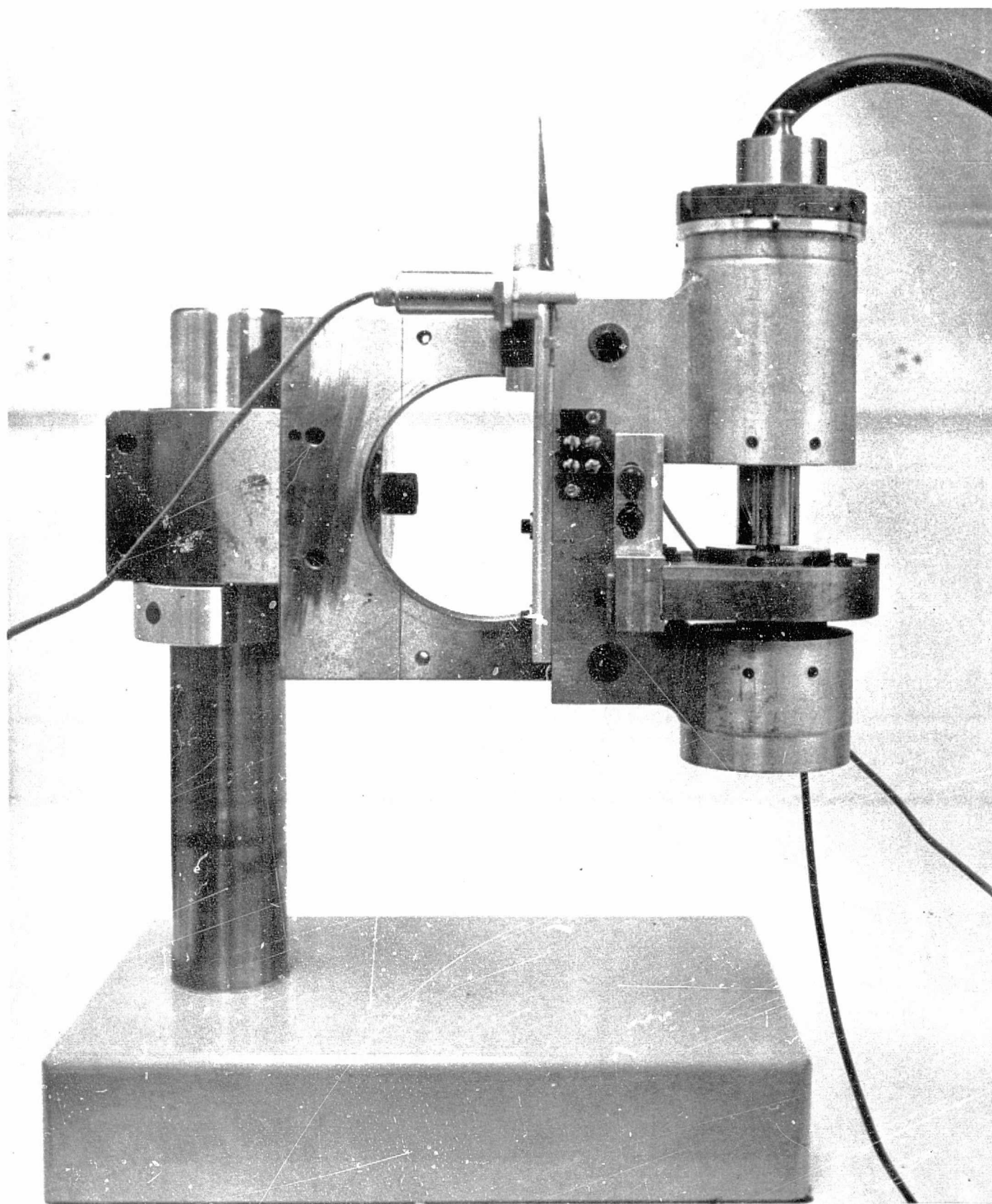
4.3.4 Motor Driven Retrofitted LEV-3 Gyro Rig

The motor driven gyro rig was constructed from an LEV-3 Mod. II Auto-Pilot Assembly. Only the motor and wheel were retained. The original ball bearing shaft was shortened and bored. A hardened steel shaft with grooved conical ends was pressed into the bore and the rotor was balanced to within 5×10^{-3} g cm.

A split rotor housing was machined to accommodate assembly of the wheel and motor stator. The bearings (female cones) were machined into cylinders and located in line-bored bosses in the rotor housing. All critical dimensions were held to within 10^{-3} mm.

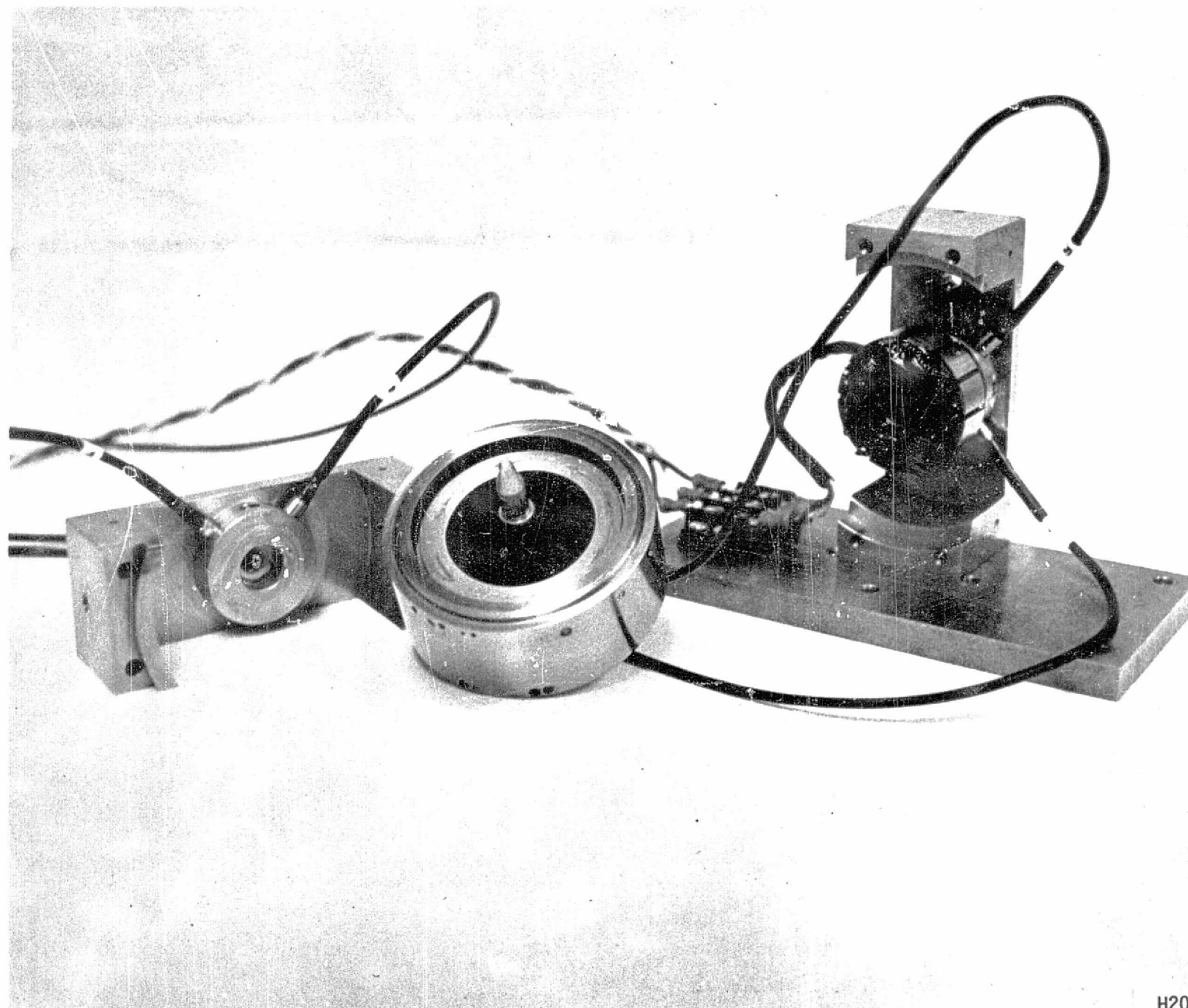
Figure 4-10 shows the rotor with one of its spiral grooved conical journals in the center of the photograph. To the left, is half of the split housing containing two capacitance probes and one of the bearings. To the right, is the other half of the housing and the base plate. Two capacitance probes and the stator are attached to the motor end of the housing. A third probe, for monitoring wheel axial motion (not shown) is also located at the motor end.

Figure 4-11 shows the arrangement used in the gyro bearing tests. The five capacitance probes and motor current are monitored by oscilloscopes, the rotor speed with the photoelectric counter shown to the right of the test stand and the vibration level by an accelerometer fixed to the test stand the angle bracket. The rotor is enclosed by transparent covers bolted to the sides of the split rotor housing and the assembly can be operated with the spin axis vertical or horizontal.



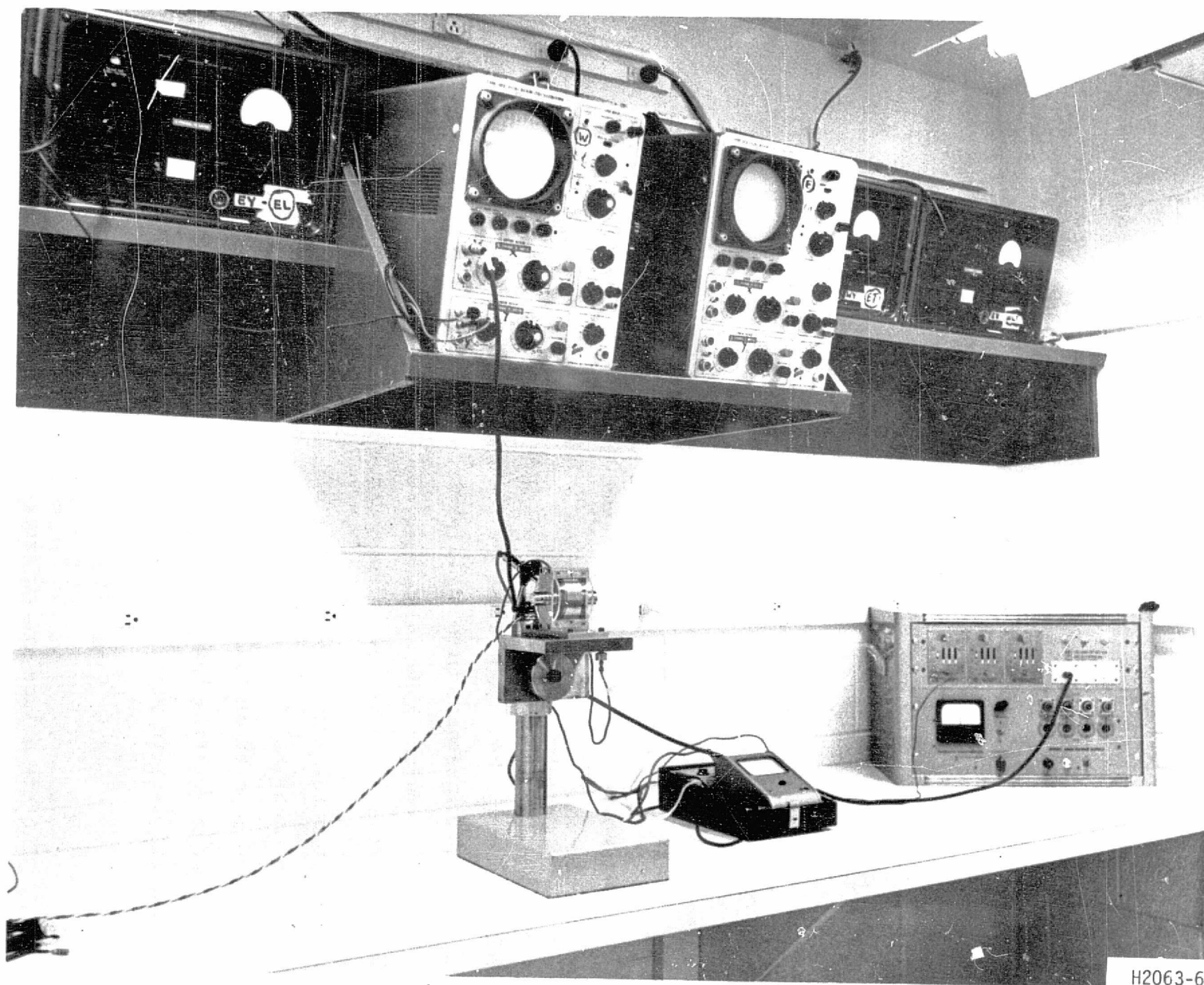
H2063-3

Figure 4-9. Grease Development Test Stand



H2063-2

Figure 4-10. Rotor, Stator and Housing of Retrofitted LEV-3 Gyro



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Figure 4-11. Test Arrangement for Retrofitted LEV-3 Gyro

Two of these test rigs were built for the purpose of testing start-stop characteristics, power requirements and rotor dynamics.

4.4 Comparison Between Theory and Experiment

Figure 4-12 shows good agreement between data from the Gyro Simulator Rig (Section 4.3.2) and the approximate analysis for load vs. concentric clearance. Under the circumstances of the test comparison, Appendix B shows that good agreement between the approximate and exact analyses can also be expected.

Figure 4-13 shows a comparison between test data from the Single Bearing Rig and the approximate analysis for load vs. friction moment. In this case, the grooved journal was relatively long and the groove angle α varied from 18.8° at the base of the cone to 29.6° at the nose of the cone. Figure 4-13 indicates two features of the bearing's character. First, the load to friction moment ratio decreases for higher loads. Second, when the α and γ at the end of the cone are used in the analysis, the agreement with experiment is best indicating the geometry at the high pressure zone is more influential in determining performance characteristics than is the geometry at the low pressure zone.

Special experimental tests are currently being conducted to provide an accurate check for the the exact analysis. Essentially, the experiments consist of fixing the clearance between a stationary grooved journal and a rotating bearing and measuring the film pressure and temperature at discrete points in both the lands and grooves.

4.5 Time History of Bearing Start-Up

The air driven grease development rig was used to study the sequence of events that took place when torque was applied about the spin axis of a rotor supported on spiral groove grease lubricated conical bearings. The grooved conical journals had the following specifications:

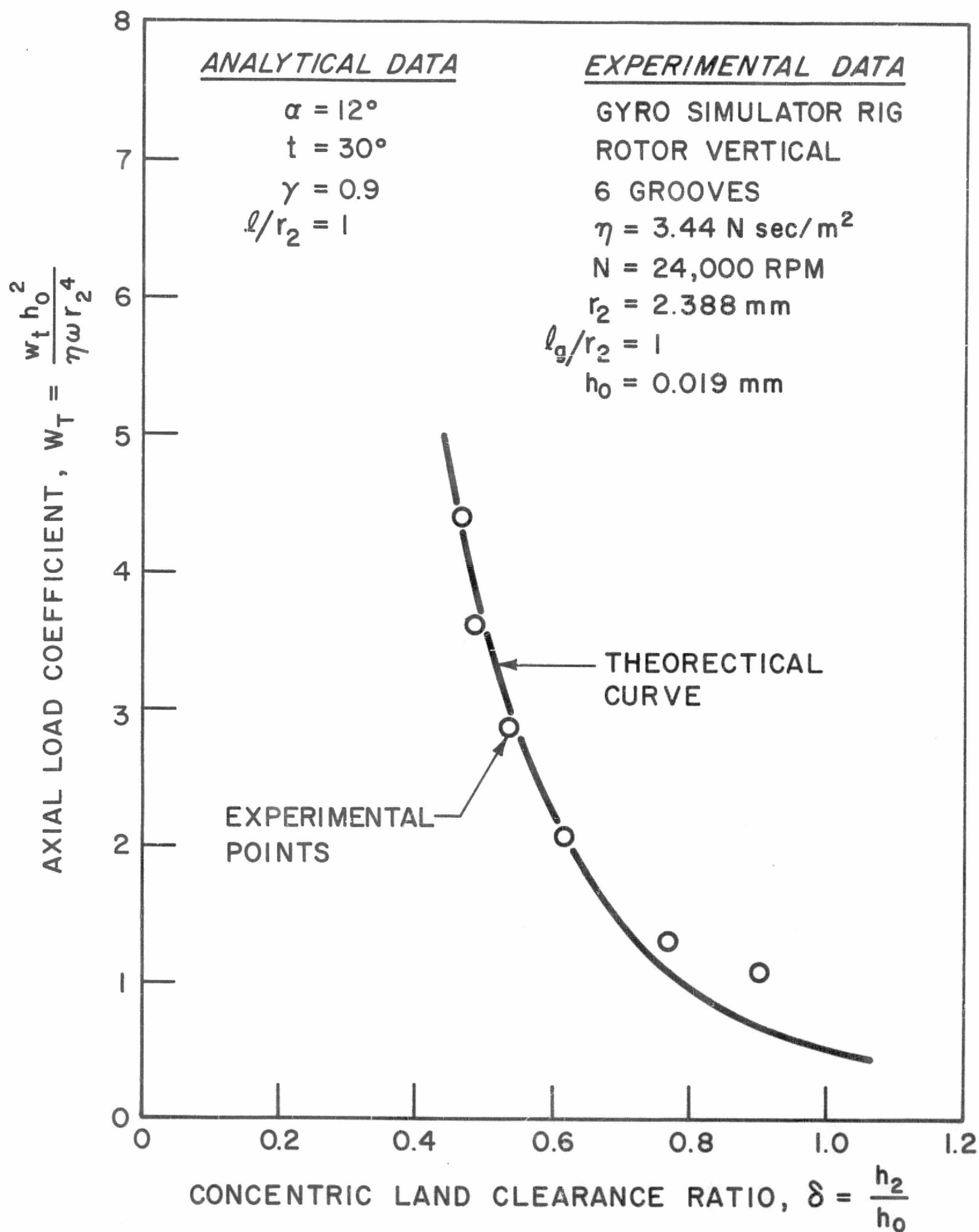


FIG. 4-12 COMPARISON BETWEEN APPROXIMATE ANALYSIS AND
EXPERIMENTAL DATA: DATA FOR W_T VS. δ

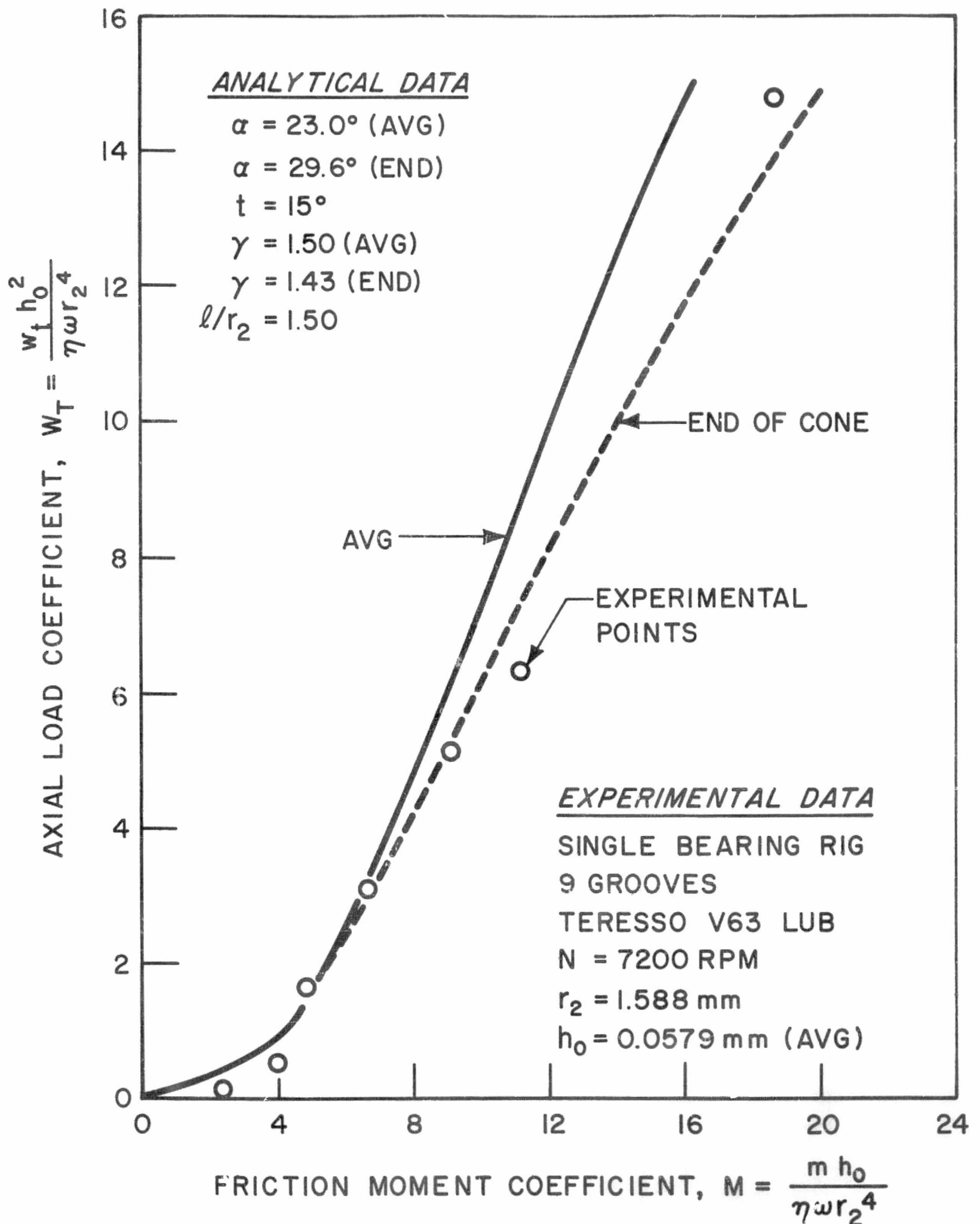


FIG. 4-13 COMPARISON BETWEEN APPROXIMATE ANALYSIS AND EXPERIMENTAL DATA FOR W_T VS. M

Base diameter of cone = 4.76 mm.

Half cone angle, $t = 15^\circ$

Number of grooves = 7

Groove depth, $h_o = 0.025$ mm

Axial length of cone, = 4.165 mm

Axial length of grooved section, $l_g = 2.388$ mm

Land to groove width ratio = 1

Groove angle, $\alpha = 15^\circ$

Surface area = $49.3 \times 10^{-6} \text{ m}^2$

The grease lubricant was designated MCG-68-11 and its specifications are given in Section 5.

Figure 4-14 shows a typical friction moment history during start-up of a grease lubricated spiral groove bearing. Various stages of the start-up sequence can be identified and these are discussed below.

Depending on past loading or storage conditions, the lubricant film thickness on the bearing surface has been measured to be as small as 0.1 microns. Consequently, at least one of the rotor bearings often begins operation with boundary lubrication. The existence of a grease layer sufficient for boundary lubrication is essentially assured since tests have shown that all the grease studied adhered tenaciously to the bearing surface and could not be removed except by deliberate cleaning.

Break-away on the boundary lubricant required only the application of a certain torque; rotor acceleration required grease to be forced into the clearance space by the pumping action of the spiral grooves. If the quantity of grease available at the groove entrance was very small, the rotor motion under the application of spin torque was sporadic. If the quantity of grease available to the groove entrance was insufficient to establish a full film over the entire bearing surface, some of the grease adhered to the rear surfaces of the bearing and the remainder was pumped to the front of the bearing. Thus, for a given load,

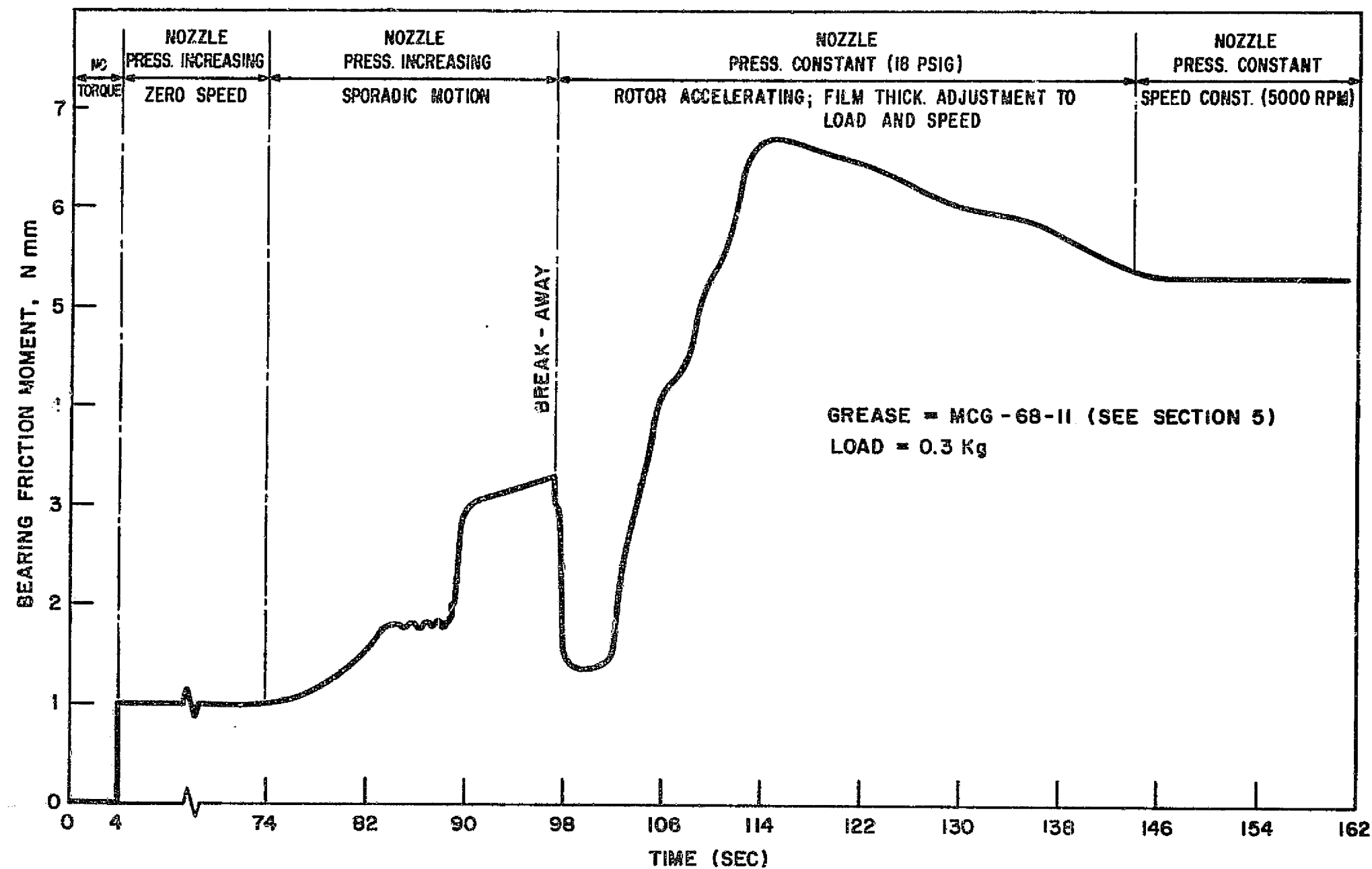


FIG. 4-14 TIME HISTORY OF BEARING START-UP

the equilibrium film thickness corresponded to a bearing with a smaller surface area than the total surface area of the actual cone. These conditions led to a higher power loss, a reduction in film damping and increased susceptibility to rotor instability.

In addition to the possibility of a lack of lubricant available for establishing a full fluid film, another occasional problem, concerned with gas entrainment, was observed to exist at start-up. Gas entrainment usually manifested itself by an abnormally large film thickness that decreased with time to an abnormally small film thickness. This can be qualitatively explained in the present test arrangement by recalling that the initial rotor motion is erratic and large in amplitude because of the absence of a fully developed lubricating film. Thus, the pumping action is non-uniform around the bearing at the groove entrances and intermittent film separation is likely to occur. If separation occurs at the groove entrance, the void will be filled by the ambient gas and, in some cases, the gas will be trapped in the grease film and transported forward by grease pumped in behind it. In a short time the grease pumping rate into the film clearance becomes zero but the temperature in the film continues to rise thus increasing the pressure of the trapped gas. For a time, the bearing operates with an abnormally large clearance, but when the gas eventually works its way out of the bearing area (possibly also displacing some of the grease from the groove entrance), the bearing is left in a starved condition.

In Figure 4-14, care was taken to avoid the anomalies described above.

Following break-away (at 98 seconds in Figure 4-14) there was a sharp reduction in friction moment as a partial fluid film began to develop, followed by a rapid increase in friction moment as the rotor speed increased and pumped cold, highly viscous grease into the clearance space. Subsequently, the friction moment reached a maximum and then began to fall off as the grease viscosity decreased due to the increased

shear rate and rising temperature. After about 144 seconds, with nozzle pressure unchanged from that at breakaway, the rotor speed reached equilibrium.

Figure 4-15 shows another feature influencing the start-up character, namely the affect of total axial clearance. These tests were performed with the motor driven rig shown in Figure 4-11. The bearing specifications are given at the beginning of the section. Note the inability of the motor to accelerate the rotor beyond 20,000 rpm and the onset of a vibration above 18,000 rpm. During acceleration, the maximum motor input power was measured at approximately 50 watts but for those cases where the rotor speed reached the maximum motor speed of 22,000 rpm, the power input was less than 15 watts. Coast down time for the run shown on Figure 4-15 with an axial clearance of 127 μ m was measured at 3.5 minutes. When operated with a lubricant with a lower dynamic viscosity, the coast down time was extended to about 15 minutes.

Based on making more than 100 start-up tests under various conditions, it is likely that a start-up curve similar to Figure 4-15 would be a simple but useful part of quality control for insuring proper rotor assembly and lubrication.

4.6 Rotor Operating Characteristics for Several Grease Lubricants

Figure 4-16 and 4-17 show changes in bearing characteristics caused by varying the lubricant constituents. The test was performed with mass loading on the grease development rig (Figure 4-9) and with bearing specifications given in Section 4.5. Table 4-1 is a list of comparative operating conditions at constant load.

An experiment, specifically designed to compare computed and measured spiral groove load-clearance data is currently being constructed, so further comments on comparison, other than those given in Sections 4.4 and 5.5, will be deferred until these measurements are taken.

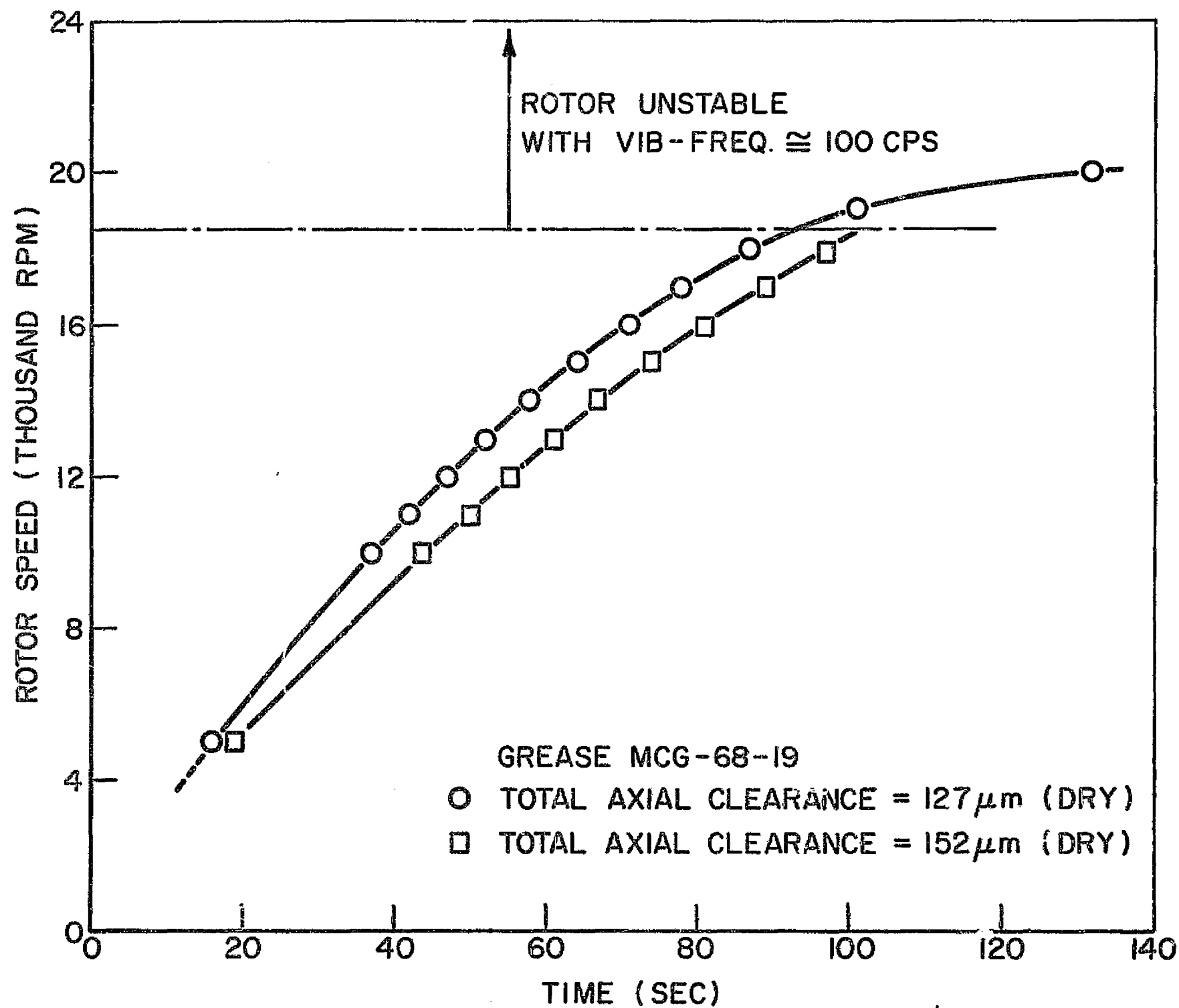


FIG. 4-15 INFLUENCE OF TOTAL AXIAL CLEARANCE ON ROTOR START-UP

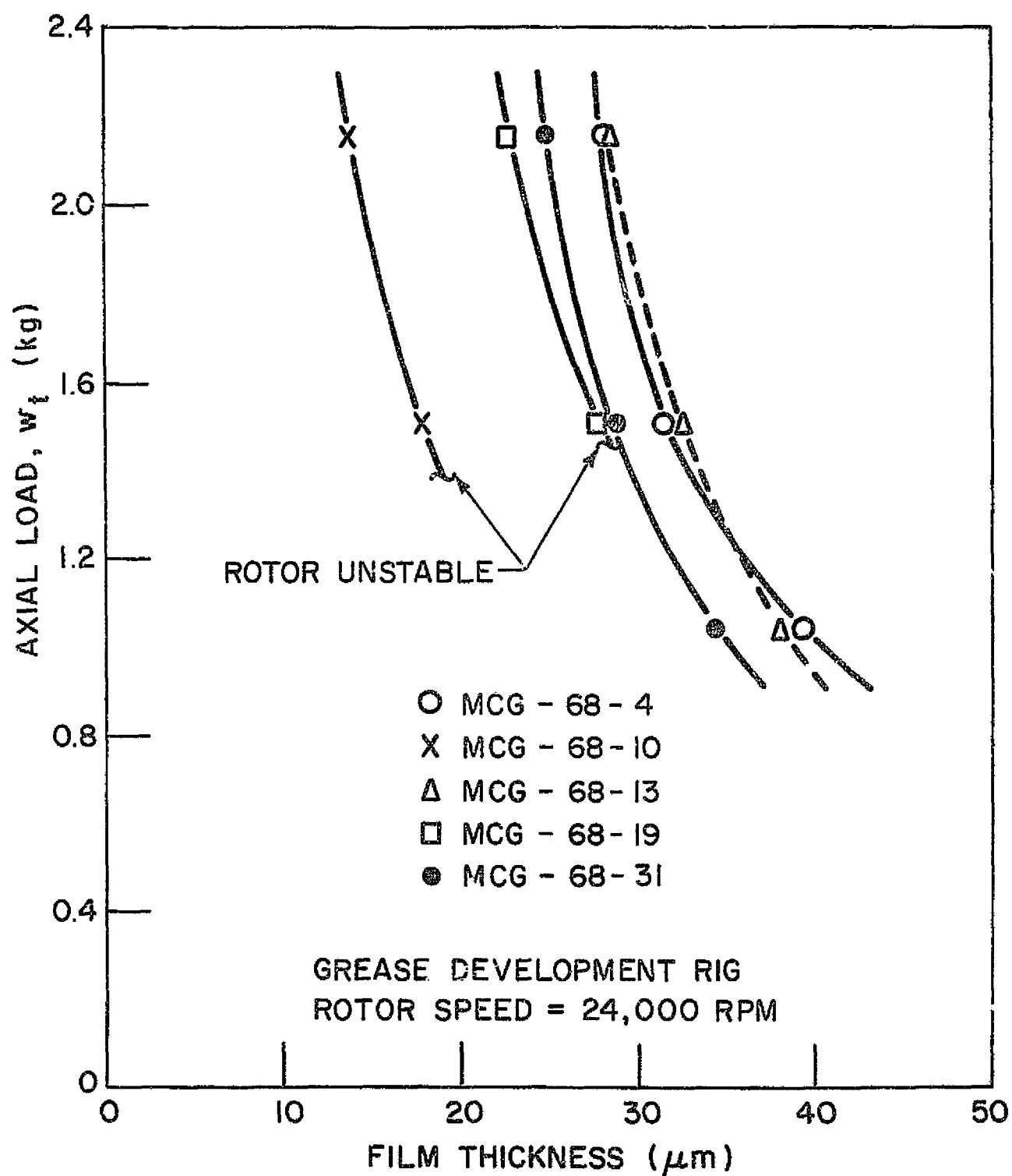


FIG. 4-16 MEASURED FILM THICKNESS VS. LOAD FOR FOUR DIFFERENT GREASE LUBRICANTS

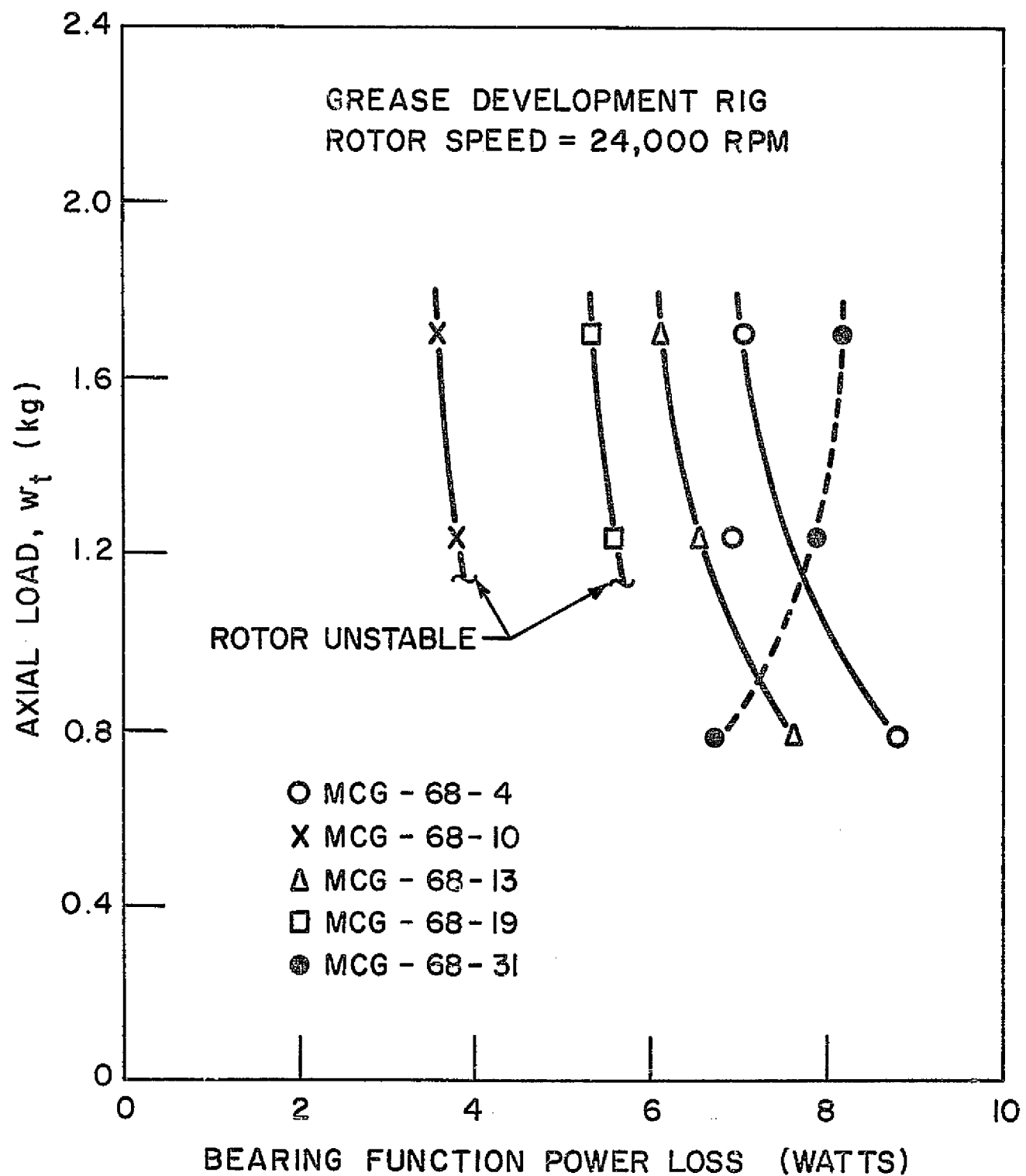


FIG. 4-17 MEASURED BEARING FRICTION POWER LOSS VS. LOAD
FOR FOUR DIFFERENT GREASE LUBRICANTS

Table 4-1
OPERATING CONDITIONS FOR CONSTANT LOAD⁽¹⁾ AND VARIOUS LUBRICANTS

<u>Grease Type</u>	<u>Film Thickness Microns</u>	<u>Lub.⁽²⁾ Temp. °C</u>	<u>Power Loss Watts</u>	<u>Compliance μm/N</u>	<u>Residual⁽³⁾ Film Thickness Microns</u>
MCG-68-13	28.2	27.2	5.9	0.524	3.0
MCG-68-4	27.7	29.4	6.8	0.303	7.6
MCG-68-31	24.9	28.3	8.2	0.406	11.7
MCG-68-19	23.6	27.8	5.2	0.633	1.5
MCG-68-10	13.7	26.7	3.4	0.467	1.0

Notes:

- 1) Bearing loaded by a mass of 1.96 Kg
- 2) Temperature read by Thermocouple in lubricant well (Figure 4-8). The bearing wall temperature could be about 15°C higher than the well (Ref. 3) and the maximum temperature in the film could be 15°C higher than the wall (Ref. 5).
- 3) 0.5 Kg dead-weight load with no shaft rotation for long period.

4.7 Influence of Load Cycling

Several tests were conducted to determine film thickness repeatability under load cycling. In these tests a weight was placed on the load platform shown in Figure 4-8 and sufficient time was allowed for the film to reach steady-state equilibrium. Next, a second weight was added. This weight was removed and added repeatedly and each time, several minutes were allowed to achieve an apparent steady-state film thickness. A typical result is plotted on Figure 4-18. Note there was a steady reduction in the film thickness for both loads. Later tests with the motor driven rotor, the bearings loaded only by the weight and residual unbalance of the rotor, also showed a steady reduction in film thickness. The reduction was apparently caused by a loss of lubricant

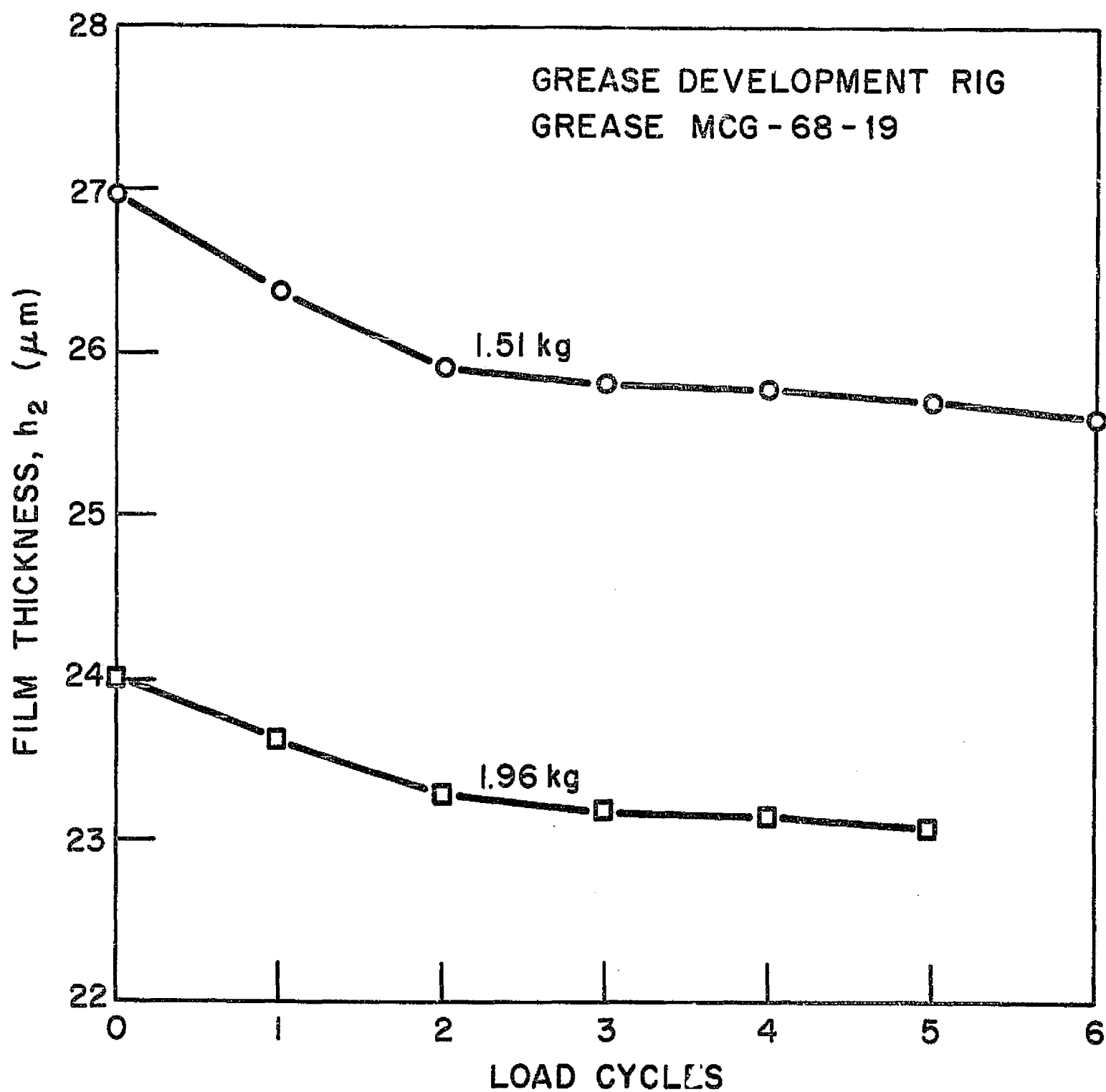


FIG. 4-18 CHANGE IN FILM THICKNESS AS LOAD IS CYCLED

via displacement from the pumping section rather than vaporization or as a consequence of entrained air. In order to confirm this hypothesis a bearing was built in which grease could be injected by a plunger from the lubricant well into the clearance space while the rotor was in operation. The plunger was butted against a set screw accessible from the outside of the bearing housing. Turning the set screw recharged the bearing with grease from the back end and insured a full film free of entrained gas. In a period of one to ten hours after greasing, the bearing film thickness was noticeably reduced and, in some cases, the rotor motion contained a distinct half frequency whirl component. Recharging always returned the bearing to its original operating condition after the newly injected lubricant reached thermal equilibrium. The tests implied that some design feature would be required to prevent the lubricant from migrating out of the pumping region of the cone. The mechanism of migration appeared to be a combination of the lubricant being squeezed out of the bearing pumping area due to rotor motion and centrifugal force on the lubricant attributable to the conical geometry. Centrifugal separation of the grease constituents (fluid from thickener) did not appear to account for short term lubricant loss since in some of the cases tested, the fluid and thickener were closely matched in density yet a relatively rapid film deterioration was observed. Problems associated with film-load repeatabilities lead to the redesign of the conical bearing, particularly in the area of the film-air-metal interface.

4.8 Variations in Design of Conical Bearing

Six, dimensional variations in the three basic designs shown on Figure 4-19 were studied experimentally. All were undertaken to improve the load-film thickness repeatability of the simple, dead-ended conical bearing which used only a centrifugal slinger to return displaced lubricant to the pumping groove entrance. The design variations were constructed to inhibit axial flow of the lubricant caused by a

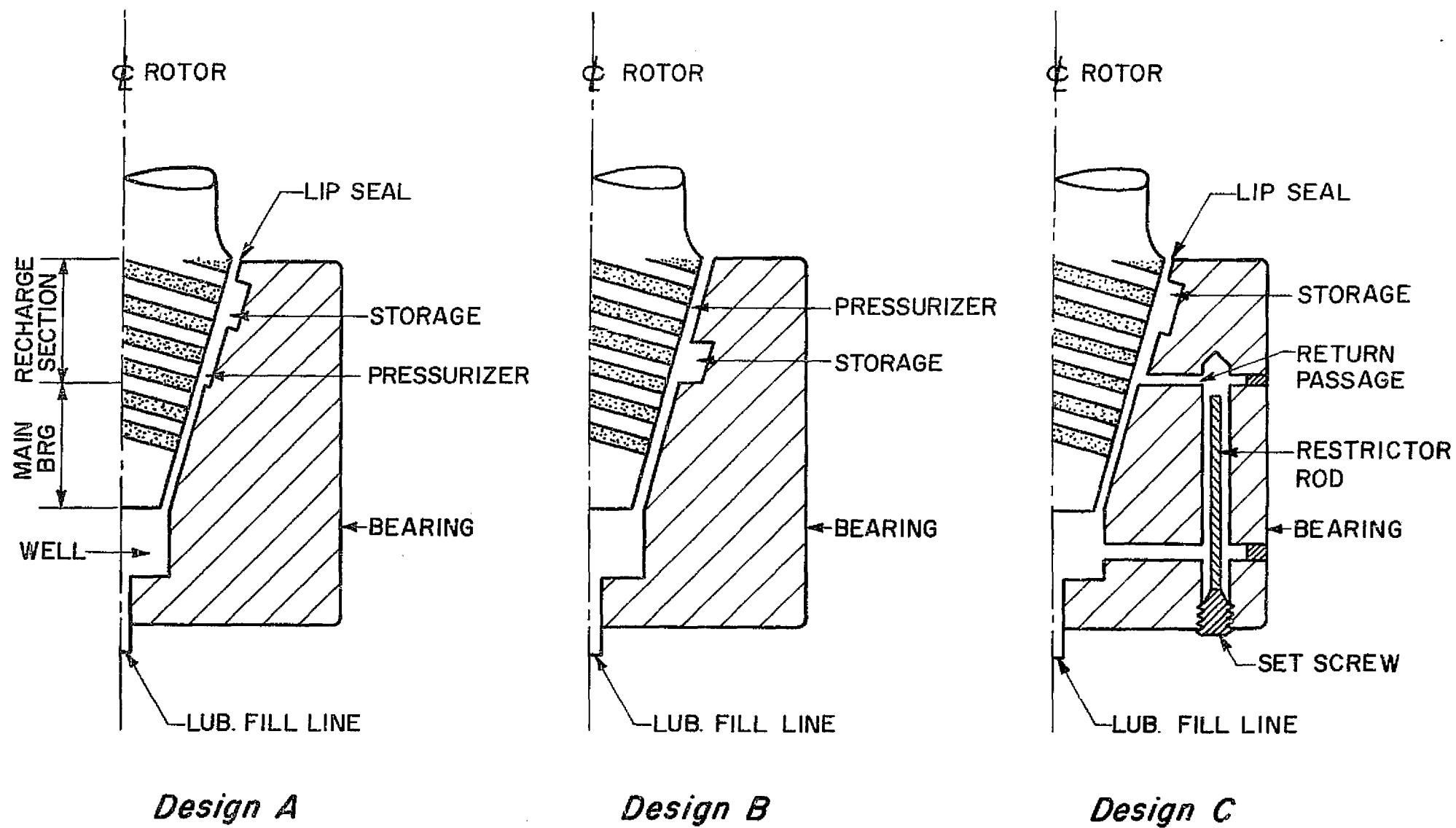


FIG. 4-19 VARIATIONS IN DESIGN OF CONICAL BEARING

redistribution of the clearance volume and to return or store lubricant in response to a change in the clearance volume.

Design A of Figure 4-19 had a lip seal and an annular cavity used both to store lubricant and to retard its vaporization. Below the storage volume was a section of bearing designed to operate with a clearance of about 1.5 times larger than the main bearing. The purpose of this section was to keep the entrance to the main bearing supplied continuously with pressurized lubricant.

In Design B the relative location of the storage and pressurizer were interchanged but the object was the same, namely, to supply the boundary of the main bearing continuously with pressurized lubricant.

Design C was constructed to counter two suspected deficiencies of the previous bearing models. One was lubricant constituent separation; the basis for the proposed solution being to circulate the lubricant in a manner that would mix the fluid with the thickener. As previously mentioned, the conical geometry is a natural centrifuge which causes the more dense constituents of the lubricant to travel along the surface of the cone in the direction of increasing radius. Circulation was achieved by using the high pressure available at the nose of the cone to force lubricant from the well back to the entrance region of the main bearing. The amount of grease circulated was governed by the diameter and length of the restrictor rods placed in the return circuits.

The second conceptual change afforded by Design C compared to previous designs was concerned with providing lubricant to fill the film cavities that tend to appear due to the relative displacement between the male and female cones. For example, under shock loading directed normal to the spin axis, the lubricant in the film clearance will be accelerated both axially and radially as the bearing surfaces come together. If the force from the lip seal cannot contain the lubricant, some may escape and be permanently lost to the bearing and

the lost lubricant must be replaced by material from a storage volume. On the maximum clearance side of the main bearing entrance, the film may separate and leave the pumping section without a full film across the clearance. Since grease lubricants are non-Newtonian at low shear rates, they will not readily flow into the separation but must be forced to do so. The second purpose of the circulating design was to alleviate this condition by discharging grease directly onto the grooved, rotating member.

Figure 4-20 shows start-up curves for a circulating bearing with a single return passage consisting in part of a 0.050" diameter rod in a 0.962" diameter passage.

Within the confines of the desired maximum power loss and an over-all axial bearing length of not more than twice the main bearing, none of the design modifications described above substantially changed the load-film thickness repeatability. It appears that the use of centrifugal force provided by the spinning rotor (as suggested in Ref. 6) offers the best solution for keeping the film clearance filled with lubricant. Current investigations are taking this direction.

Past experiments indicated (1) more interface understanding is required to produce a reliable design and (2), it is not necessary to treat the lubricant as non-Newtonian in the general clearance space for the usual steady-state or dynamic response calculations but it may be necessary to do so in order to establish the film boundary location and lubricant transient rheology. The latter is particularly important because there will always be a "fluid response time" regardless of the shape of the bearing. These problems, which are common to flat spiral groove thrust bearings and seals, are currently being studied experimentally and the results of these studies will be used as guide modifications to the analysis of Section 3.

The decision to terminate a rotor with cones should be made on the basis of a good understanding of the consequence of thermally induced



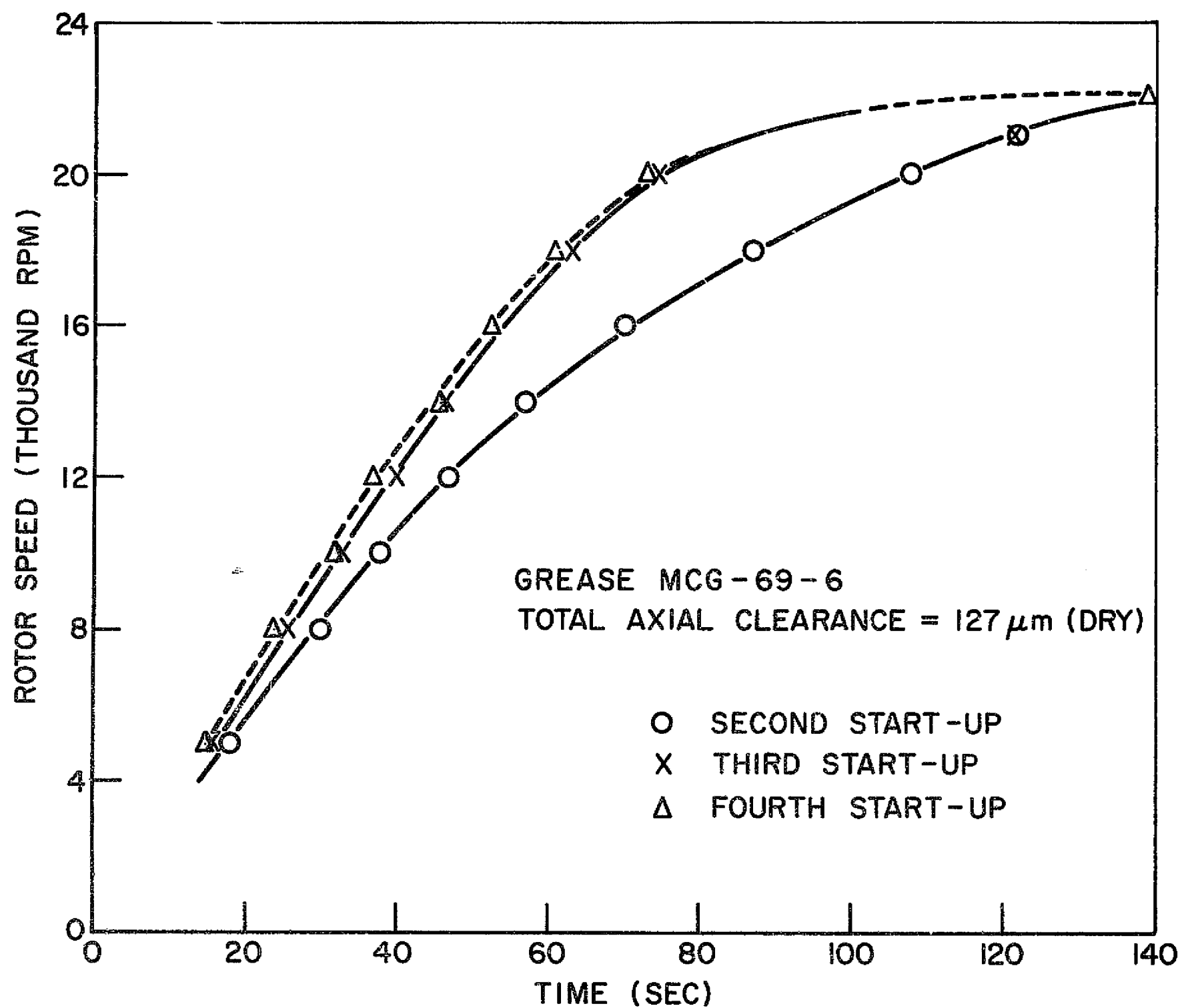


FIG. 4-20 ROTOR ACCELERATION FOR THREE CONSECUTIVE STARTS

dimensional changes. For instance, the gyro construction shown in Figure 4-10 presents two design problems associated with thermal effects. One is that the starting current is about three times the running current and the consequence is a film thickness transient where the clearance initially decreases to a minimum as the rotor becomes warm and subsequently increases as the frame temperature begins to rise. The second problem comes about because the motor end bearing runs warmer than the wheel and bearing. Thus, there is a difference in the size and cone angle match between the bearings at either end of the rotor. Although the latter problem is generally a typical design consideration for most rotor bearing systems, the thermally induced film transient presents the lubricant retention difficulties described previously. When the film clearance closes, the design must provide storage, and when the clearance opens the design must provide the pressure for lubricant injection. It should also be noted that similar design problems often exist for bearing configurations other than conical.

Film thickness transients, as here encountered, are often accommodated by flexible bearing mounts such as elastomeric supports which consist of a many-layered sandwich of metal and elastomer. Bearing dynamic supports influence the effective bearing stiffness and general rotor response and often introduce dynamical problems of a magnitude similar to the problem they try to circumvent. Thus, a passive bearing mount was chosen here to simplify the experimental effort, although when properly designed, a flexible support may ultimately provide reliability over a wider range of operating conditions than could be accomplished by a fixed support.

5. GREASE LUBRICANTS FOR SPIRAL GROOVE BEARINGS

5.1 Introduction

Significant differences in performance properties are often found between batches of greases that were intended to be identical. The purpose of this study was to formulate lubricants with consistent and predictable properties, specifically for high speed, low power loss spiral groove bearings. The character of the lubricant should be such that it has shear stability at high shear rates exhibits good start-stop performance over the required temperature range, have high load carrying capacity, provide good boundary lubrication, be self sealing and perform reliably for two or more years in a thin film without significant change in its physical or chemical properties.

While some of the information reported here has general lubrication interest, the principal motivation for the effort was the performance advantages offered by spiral groove bearings lubricated with grease (Ref. 1). Because of the high pressure obtained by pumping grease with spiral grooves, bearing designs tend to be small. The added attraction of reliable start-up due to an ever present boundary film make the quest for a suitable lubricant more inviting. Grease lubricant manufacturers seldom provide data relating shear stress, shear rate and temperature and since bearing performance cannot be computed without it, this data was measured for each class of lubricants studied. Because of the operating specifications, special greases were formulated and, in addition to apparent viscosity measurements, performance properties for 36 lubricants were compared in a typical high shear rate bearing application. Measurements taken consisted of relative breakaway torque, film thickness, power loss, temperature, compliance and coast down time. The data presented here is intended to expand on other potential grease formulations and test results already available in the literature (2,3,4,5).



5.2 Constituent and Formulation Properties

5.2.1 Base Fluids

Four classes of base fluids were selected for use in the grease formulations. The selection of the fluids was based on their ability to meet the majority of the requirements previously mentioned.

Three of the fluids were available in two to three viscosity grades; the mineral oil was available in one viscosity grade only. The fluid properties are given in Table 5-1.

5.2.2 Thickeners

The non-soap thickeners were selected for their ability to thicken the fluids and produce stable greases with low bleed characteristics. Fluorinated ethylenepropylene (FEP) was used as a thickener for all of the fluids. Benton clay was used as an auxiliary thickener with the FEP in the mineral oil and ester base greases to obtain the desired bleed and consistency characteristics. A triazine, 2,4-diamino-6-hydroxy-1,3,5-triazine, commonly known as "Ammeline" was used in producing greases from the perfluoro alkyl ether base fluids. The thickener properties are given in Table 5-2.

5.2.3 Formulations

Initially the greases were formulated to three consistencies each: 270, 300, and 330 ASTM unworked penetrations. It was desirable to study two of the basic perfluoro alkyl ether greases for shear stability under a wider consistency range, therefore they were each compounded to 350 and 400 ASTM unworked penetrations. Consistency measurements were

Table 5-1
FLUID PROPERTIES

<u>Fluid Type</u>	<u>Viscosity at 38°C Centistokes</u>	<u>Density at 25°C g/cc</u>
Mineral Oil	164	0.8774
Polyol Aliphatic Ester	5	0.9660
Polyol Aliphatic Ester	16	0.9840
Fluorosilicone	176	1.2610
Fluorosilicone	524	1.2610
Perfluoro Alkyl Ether	38	1.8700
Perfluoro Alkyl Ether	90	1.8900
Perfluoro Alkyl Ether	280	1.8900

Table 5-2
THICKENER PROPERTIES

<u>Thickener</u>	<u>Density g/cc</u>	<u>Max. Particle Size Microns</u>
Fluorinated ethylenepropylene (FEP)	2.182	0.13
Bentone clay	3.090	0.97
2,4-diamino-6-hydroxy-1,3,5-triazine (ammeline)	1.712	0.90

made after 60 and 10,000 double strokes on the penetrometer to provide assurance of shear stability. Careful selection of thickeners and fluids precluded the need for additives. Care was exercised in formulating the greases to allow studying the effects of base fluid type and viscosity, thickener type and grease consistency on performance in spiral groove bearings. Properties of the greases are given in Table 5-4.

5.2.4 Low Pressure Performance

In many application it is desirable to operate bearings in a partially evacuated environment. Although low pressure data has not been obtained on the greases described previously, data has been obtained on similar greases and is reported below.

- Greases very similar to MCG-68-3, 68-4, and 68-5 have performed for up to 6000 hours at 10^{-8} mm. Hg and 77°C.
- Greases similar to MCG-68-6 through 68-11 have performed for 800-2700 hours at 10^{-8} mm. Hg and 77-88°C.
- Greases like MCG-68-12 through 68-17 have performed for 48,000 hours at 10^{-8} mm. Hg and 77°C.
- Greases like MCG-68-21, 68-22, 68-23, 68-30, 68-31, and 68-32 have performed for up to 300 hours at 10^{-8} mm.Hg and 149°C.
- Greases similar to MCG-68-24, 68-25, 68-26, 68-33, 68-34 and 68-35 have performed for more than 7500 hours at 10^{-11} mm. Hg and 149°C.

The vapor pressure data for the base fluids used in the grease are given in Table 5-3.

It is important to note that long performance of a grease at low pressure and high temperature depends critically on the mechanical design to provide shielding from direct exposure to a low pressure ambient.

5.3 Apparent Dynamic Viscosity Measurements

5.3.1 Construction of Viscometer

Figure 5-1 shows the general arrangement of the viscometer used to determine the lubricant shear stress - shear rate - temperature relation.

Table 5-3
VAPOR PRESSURE DATA

Temperature °C	Vapor Pressure mm Hg	Temperature °C	Vapor Pressure mm Hg
a. Mineral Oil, 164 cs		e. Fluorosilicone, 524 cs	
176.7	0.4	285.3	10.0
204.4	1.0	305.0	20.0
232.2	2.2	319.7	32.0
260.0	4.4	325.5	41.0
287.8	9.4	329.0	54.0
315.6	18.0	331.7	75.0
b. Polyol Aliphatic Ester, 5 cs		f. Perfluoroalkyl Ether, 38 cs	
148.9	2.0	224.8	10.0
176.7	4.4	247.0	20.0
204.4	8.7	260.0	30.0
232.2	15.2	268.2	40.0
260.0	25.5	283.2	60.0
287.8	52.0	288.8	70.0
315.6	110.0	299.5	100.0
c. Polyol Aliphatic Ester, 16 cs		g. Perfluoroalkyl Ether, 90 cs	
121.1	0.9	215.7	40.0
148.9	2.0	254.1	50.0
176.7	4.3	276.2	60.0
204.4	7.6	299.4	80.0
232.2	12.8	313.5	100.0
260.0	21.0	324.3	120.0
287.8	33.0	332.8	140.0
315.6	57.0	343.4	170.0
343.3	120.0	351.0	200.0
d. Fluorosilicone, 176 cs		355.5	220.0
		h. Perfluoroalkyl Ether, 280 cs	
198.0	10.0	314.6	10.0
222.2	18.0	333.7	20.0
250.2	34.0	342.0	30.0
261.8	43.0	350.1	40.0
272.8	54.0	356.9	50.0
287.3	70.0	365.8	70.0
297.2	90.0	372.6	100.0
301.0	110.0	374.2	120.0

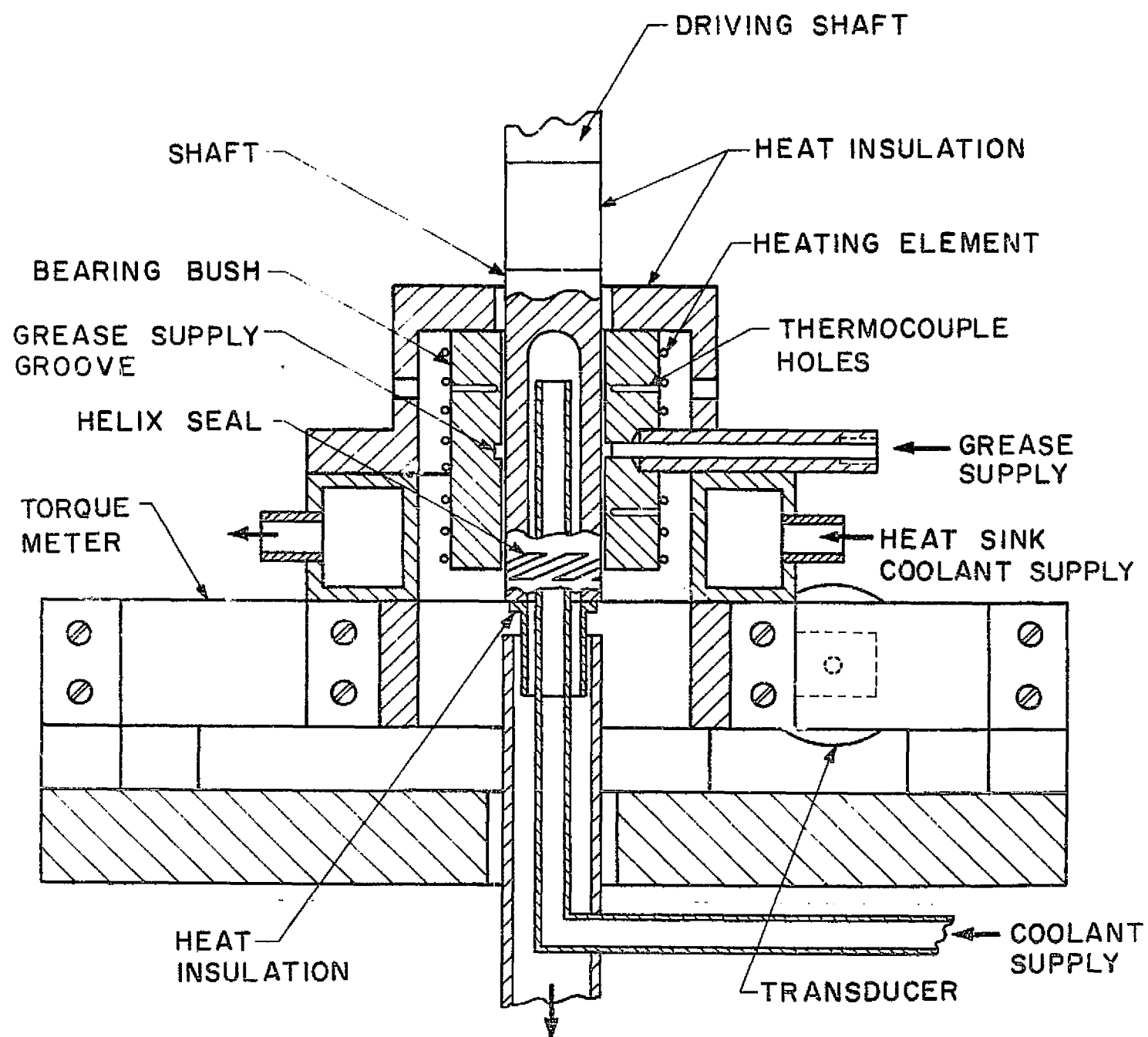


FIG. 5-1 LAYOUT OF GREASE VISCOMETER

Table 5-4
GREASE PROPERTIES

Code No.	Fluid	Fluid (1) Viscosity, CS	Thickener	Penetration, Unworked	Penetration, 60 Strokes	Penetration, 10,000 Strk	Grease Density, g/cc	Thermal (1) Conductivity J/(m sec °C)	Specific (1) Heat J/(g °C)
MCG 68-3	Mineral Oil	164	FEP & Clay	271	292	301	1.120	0.05 ⁽²⁾	(3)
MCG 68-4	Mineral Oil	164	FEP & Clay	304	317	325	1.110	"	"
MCG 68-5	Mineral Oil	164	FEP & Clay	332	339	348	1.101	"	"
MCG 68-6	Polyol Aliphatic Ester	5	FEP & Clay	264	278	348	1.088	0.05 ⁽²⁾	"
MCG 68-7	Polyol Aliphatic Ester	5	FEP & Clay	305	345	400	1.073	"	"
MCG 68-8	Polyol Aliphatic Ester	5	FEP & Clay	335	374	400	1.052	"	"
MCG 68-9	Polyol Aliphatic Ester	16	FEP & Clay	273	276	283	1.140	"	"
MCG 68-10	Polyol Aliphatic Ester	16	FEP & Clay	294	320	340	1.111	"	"
MCG 68-11	Polyol Aliphatic Ester	16	FEP & Clay	321	324	342	1.106	"	"
MCG 68-12	Fluorosilicone	176	FEP	269	297	297	1.461	0.044	0.451
MCG 68-13	Fluorosilicone	176	FEP	300	311	311	1.444	"	"
MCG 68-14	Fluorosilicone	176	FEP	326	338	338	1.427	"	"
MCG 68-15	Fluorosilicone	524	FEP	268	296	292	1.437	"	"
MCG 68-16	Fluorosilicone	524	FEP	307	333	339	1.410	"	"
MCG 68-17	Fluorosilicone	524	FEP	334	363	371	1.384	"	"
MCG 68-18	Perfluoro Alkyl Ether	38	FEP	272	297	288	1.966	0.026	0.313
MCG 68-19	Perfluoro Alkyl Ether	38	FEP	301	324	318	1.960	"	"
MCG 69-11 (Similar to MCG 68- 19)	Perfluoro Alkyl Ether	38	FEP	300	331	339	1.963	"	"
MCG 69-20	Perfluoro Alkyl Ether	38	FEP	326	343	344	1.944	"	"

F-C2028

Notes: (1) at 38°C
(2) approximate
(3) not available

Table 5-4
GREASE PROPERTIES (Con't)

Code No.	Fluid	Fluid (1)		Penetration, Unworked	Penetration, 60 Strokes	Penetration, 10,000 Strk	Grease Density, g/cc	Thermal (1)	Specific (1)
		Viscosity, CS	Thickener					Conductivity J/(m sec °C)	Heat J/(g °C)
MCG 69-12	Perfluoro Alkyl Ether	38	FEP	347	366	363	1.953	0.026	0.313
MCG 69-13	Perfluoro Alkyl Ether	38	FEP	412	438	442	1.942	"	"
MCG 68-27	Perfluoro Alkyl Ether	38	Ammeline	264	268	254	1.806	"	"
MCG 68-28	Perfluoro Alkyl Ether	38	Ammeline	300	317	308	1.820	"	"
MCG 69-14 (Similar to MCG 69- 28)	Perfluoro Alkyl Ether	38	Ammeline	295	306	309	1.863	"	"
MCG 68-29	Perfluoro Alkyl Ether	38	Ammeline	331	341	332	1.828	"	"
MCG 69-15	Perfluoro Alkyl Ether	38	Ammeline	344	348	362	1.873	"	"
MCG 69-16	Perfluoro Alkyl Ether	38	Ammeline	393	380	403	1.884	"	"
MCG 68-21	Perfluoro Alkyl Ether	90	FEP	275	300	314	1.972	0.028	0.302
MCG 68-22	Perfluoro Alkyl Ether	90	FEP	298	325	302	1.964	"	"
MCG 69-17 (Similar to MCG 68- 22)	Perfluoro Alkyl Ether	90	FEP	296	333	332	1.958	"	"
MCG 68-23	Perfluoro Alkyl Ether	90	FEP	332	355	361	1.955	"	"
MCG 69-18	Perfluoro Alkyl Ether	90	FEP	345	375	371	1.945	"	"
MCG 69-19	Perfluoro Alkyl Ether	90	FEP	394	408	420	1.937	"	"
MCG 68-30	Perfluoro Alkyl Ether	90	Ammeline	275	310	298	1.842	"	"
MCG 68-31	Perfluoro Alkyl Ether	90	Ammeline	300	315	309	1.844	"	"

Notes: (1) at 38°C
(2) approximate
(3) not available

F-C2028

Table 5-4
GREASE PROPERTIES (Con't)

Code No.	Fluid	Fluid (1) Viscosity, CS	Thickener	Penetration, Unworked	Penetration, 60 Strokes	Penetration, 10,000 Strk	Grease Density, g/cc	Thermal (1) Conductivity J/(m sec °C)	Specific (1) Heat J/(g °C)
MCG 69-20 (Similar to MCG 68- 31)	Perfluoro Alkyl Ether	90	"Ammeline"	299	329	338	1.869	"	"
MCG 68-32	Perfluoro Alkyl Ether	90	Ammeline	334	343	334	1.848	"	"
MCG 69-21	Perfluoro Alkyl Ether	90	Ammeline	350	363	391	1.877	"	"
MCG 69-22	Perfluoro Alkyl Ether	90	Ammeline	394	421	439	1.883	"	"
MCG 68-24	Perfluoro Alkyl Ether	280	FEP	273	293	286	1.969	0.029	0.291
MCG 68-25	Perfluoro Alkyl Ether	280	FEP	299	331	330	1.958	"	"
MCG 68-26	Perfluoro Alkyl Ether	280	FEP	337	367	369	1.947	"	"
MCG 68-33	Perfluoro Alkyl Ether	280	Ammeline	264	385	288	1.846	"	"
MCG 68-34	Perfluoro Alkyl Ether	280	Ammeline	293	307	303	1.847	"	"
MCG 68-35	Perfluoro Alkyl Ether	280	Ammeline	335	348	365	1.860	"	"

Notes: (1) at 38°C
(2) approximate
(3) not available

The instrument is a Couette type viscometer consisting essentially of a motor driven hollow shaft and a bearing bush made of identical materials. The same ratio of inner and outer shaft and bush diameters are incorporated to minimize gap distortion. A variable speed motor is employed and its membrane coupling was insulated thermally from the shaft while the bearing bush is shrouded with a heating element and instrumented with thermocouples and a torque meter. The torque meter consists of three springs isolated from thermal distortion by a heat sink and an inductive displacement pick-up.

The shear stress, τ , was determined by measuring the moment, M , exerted on the bearing bush as a result of the viscous forces in the lubricating film when the shaft was rotated at a fixed speed. Thus

$$\tau = K M$$

The shear rate is given by

$$\Gamma = V/\Delta r$$

and the apparent viscosity is defined as

$$\eta = \tau/\Gamma$$

5.3.2 Test Procedure

With the temperature held constant to within $\pm 1^\circ\text{C}$ the grease sample was first sheared at a rate of $\Gamma = 7.2 \times 10^4 \text{ sec}^{-1}$ until no change in the torque motor read out could be detected. The moment was then measured for shear rates of 7.2, 5.4, 3.6, $1.8 \times 10^4 \text{ sec}^{-1}$ holding the temperature to within 0.15°C of the desired value. The temperature range investigated was from 25 to 99.4°C and the measuring accuracy of τ and Γ was estimated at ± 2 percent.

5.3.3 Test Results

Not all of the grease specimens were tested for their dynamic viscosity but a sampling was taken which essentially indicated how all of the greases would probably perform. The sampling included each of the four fluids thickened with FEP (or FEP and clay) to the same unworked penetration of about 300. One of the fluids, perfluoro alkyl ether was thickened to an unworked penetration of 300 with Ammeline for a direct comparison with the other principal thickener.

Figures 5-2 through 5-6 give the measured results for shear stress vs. shear rate at one or more temperatures. Note that at the shear rates shown, the shear stress at constant temperature is essentially proportional to the shear rate for all tests and therefore the lubricants can be treated as Newtonian for computational purposes. Figure 5-7 shows a plot of apparent viscosity vs. temperature for greases MCG-68-4, 10, 13, 19 and 31 at a shear rate of $7.2 \times 10^4 \text{ sec}^{-1}$.

5.4 Lubricant Performance in Spiral Groove Bearing

The test rig shown schematically in Figure 5-8 was used to mutually compare 35 greases and one oil specimen under the same test conditions. The test rotor was 5 cm in diameter, with a mass of 0.63 kg and was driven at 24,000 rpm. The test bearing geometry is given in Figure 5-9.

Test results which include breakaway torque, film thickness, power loss, lubricant temperature, residual film thickness for boundary lubrication, bearing compliance and coast-down time are given by Table 5-5.

Table 5-5
LUBRICANT PERFORMANCE IN SPIRAL GROOVE BEARING

Col. 1 Code No. MCG-68	2 Fluid and Thickener	3 (1) Breakaway Torque N m x 10 ³	4 (2) Film Thickness, Microns load, kg					5 (3) Power Loss, Watts load, kg					6 (4) Lub. Vell Temp., °C load, kg					7 (5) h microns	8 (6) Compliance m/N x 10 ⁶	9 (7) Count down See.
			1.05	1.51	1.96	1.51	1.05	0.79	1.24	1.70	1.24	0.79	1.05	1.51	1.96	1.51	1.05	↑/↓	6	
3	Mineral Oil; PEP & Clay	1.754	30.73	25.65	21.08	24.84	30.23	6.0	7.2	9.3	6.5	4.9	25.5	26.1	26.7	27.2	27.2	0.13	1.04/0.87	42.
4	Mineral Oil; PEP & Clay	1.039	39.12	32.51	27.69	32.00	38.86	8.8	6.9	7.1	6.5	5.8	29.4	29.4	29.4	30.0	30.0	7.62	1.09/1.08	47.
5	Mineral Oil; PEP & Clay	2.744	43.43	32.00	26.92	30.99	54.14	6.9	7.0	7.4	6.5	6.0	28.3	29.4	31.1	31.1	31.1	6.60	1.15/0.92	45.
6	Polyol Aliphatic Ester; PEP & Clay	1.764	*	15.49	11.43	13.46	*	*	2.8	2.8	2.3	*	*	25.0	26.1	26.1	*	1.78	0.92/0.46	117.
7	Polyol Aliphatic Ester; PEP & Clay	1.970	*	12.45	8.12	10.92	*	*	3.1	2.9	2.3	*	*	26.1	26.7	26.7	*	0.51	0.98/0.63	108.
8	Polyol Aliphatic Ester; PEP & Clay	2.538	*	11.43	6.60	9.14	*	*	3.1	2.9	2.5	*	*	26.1	26.7	26.7	*	0.25	0.81/0.52	96.
9	Polyol Aliphatic Ester; PEP & Clay	0.608	*	21.08	17.53	19.81	*	*	9.2	11.2	7.2	*	*	26.7	27.2	27.2	*	2.03	0.92/0.52	74.
10	Polyol Aliphatic Ester; PEP & Clay	0.764	*	17.78	13.72	16.00	*	*	3.8	3.6	3.1	*	*	26.1	26.7	26.7	*	1.08	0.92/0.52	105.
11	Polyol Aliphatic Ester; PEP & Clay	0.510	*	17.78	13.72	16.00	*	*	4.6	4.2	3.4	*	*	26.1	26.7	26.7	*	0.76	0.92/0.63	42.
12	Fluorosilicone; PEP	0.539	38.86	32.77	28.96	34.77	37.59	*	7.2	7.4	6.6	*	23.9	26.1	26.7	26.7	26.7	3.05	0.86/0.86	30.
13	Fluorosilicone; PEP	1.352	38.10	32.26	27.19	32.26	*	7.6	6.5	6.1	5.7	*	25.0	26.1	27.2	27.2	28.3	3.05	0.92/0.92	—
14	Fluorosilicone; PEP	1.088	36.83	30.99	26.42	30.23	36.07	6.0	7.0	7.6	6.1	5.9	26.7	27.8	28.3	28.3	28.3	0.76	1.03/0.87	53.
15	Fluorosilicone; PEP	0.774	*	43.69	38.35	43.43	*	*	9.1	9.4	8.7	*	*	28.9	30.0	30.0	*	2.03	1.21/1.15	45.
16	Fluorosilicone; PEP	0.608	50.55	42.16	37.59	37.59	49.28	9.2	8.6	9.0	8.5	7.6	27.8	28.9	29.4	30.5	29.4	0.25	1.14/1.14	38.
17	Fluorosilicone; PEP	0.402	48.77	40.64	35.56	40.64	47.75	8.9	9.3	8.8	9.0	8.3	28.9	29.4	30.5	30.5	30.5	0.76	1.15/1.15	39.
18	Perfluoro Alkyl Ether; PEP	0.461	*	29.97	24.98	28.96	*	*	6.1	6.6	6.1	*	*	26.7	28.3	28.3	*	1.52	1.26/1.04	68.
19	Perfluoro Alkyl Ether; PEP	0.304	*	27.94	23.62	26.92	*	*	5.6	5.3	5.3	*	*	26.7	27.7	27.7	*	1.52	0.87/0.81	76.
20	Perfluoro Alkyl Ether; PEP	1.166	*	27.12	22.61	26.07	*	*	4.8	5.2	4.9	*	*	26.1	26.7	26.7	*	3.05	1.04/0.92	65.
27	Perfluoro Alkyl Ether; Amelene	1.313	*	21.59	20.83	20.32	*	*	7.4	7.2	5.6	*	*	27.7	27.7	27.7	*	13.21	0.17/-0.11	67.
28	Perfluoro Alkyl Ether; Amelene	2.332	*	23.88	21.34	23.37	*	*	9.2	7.6	4.7	*	*	27.7	27.7	27.7	*	12.45	0.57/0.46	55.
29	Perfluoro Alkyl Ether; Amelene	3.253	*	22.10	18.29	21.08	*	*	4.8	5.5	4.6	*	*	28.9	28.9	28.9	*	9.14	0.87/0.63	57.
21	Perfluoro Alkyl Ether; PEP	0.608	*	35.56	30.48	34.80	*	*	7.1	7.6	6.6	*	*	28.3	28.9	28.9	*	1.02	1.15/0.98	52.
22	Perfluoro Alkyl Ether; PEP	1.117	*	37.08	31.50	36.07	*	*	7.1	6.7	6.4	*	*	27.2	27.7	27.7	*	3.30	1.26/1.04	52.
23	Perfluoro Alkyl Ether; PEP	1.470	38.86	32.00	26.92	31.50	37.59	6.0	6.5	6.5	5.5	5.1	27.2	27.7	27.7	27.7	27.7	0.25	1.15/1.04	55.
30	Perfluoro Alkyl Ether; Amelene	0.559	*	32.51	27.12	31.24	*	*	7.9	7.9	7.2	*	*	28.3	29.4	29.4	*	13.46	1.21/0.92	45.
31	Perfluoro Alkyl Ether; Amelene	1.009	34.29	28.70	24.89	28.70	34.54	6.7	7.9	8.1	6.4	5.7	27.7	27.7	28.3	27.7	27.7	11.68	0.87/0.87	48.
32	Perfluoro Alkyl Ether; Amelene	1.519	*	31.24	25.15	28.96	*	*	6.4	6.1	5.7	*	*	27.2	28.3	28.3	*	12.70	1.38/0.87	48.
24	Perfluoro Alkyl Ether; PEP	1.803	50.04	41.40	37.59	42.16	50.29	8.1	9.8	12.4	13.7	9.9	27.2	27.2	28.3	28.3	28.3	1.78	0.87/1.04	11.
25	Perfluoro Alkyl Ether; PEP	2.176	48.77	41.40	34.80	40.13	47.24	8.5	11.2	14.0	11.2	8.9	28.3	29.4	30.0	30.0	30.0	1.02	1.44/1.21	28.
26	Perfluoro Alkyl Ether; PEP	1.970	48.01	40.13	34.80	40.13	47.24	7.6	8.1	8.2	7.7	7.0	28.3	29.4	30.5	30.5	30.5	0.25	1.21/1.21	13.
33	Perfluoro Alkyl Ether; Amelene	2.695	*	38.86	32.77	38.10	*	*	10.7	12.2	10.7	*	*	28.3	29.4	29.4	*	10.16	1.38/1.21	36.
34	Perfluoro Alkyl Ether; Amelene	2.803	49.28	40.64	34.80	39.37	*	8.9	11.2	12.2	9.7	*	30.0	30.0	30.5	30.5	*	13.97	1.33/1.04	20.
35	Perfluoro Alkyl Ether; Amelene	3.077	44.70	38.10	33.53	38.10	44.70	8.0	8.0	8.5	7.2	6.9	25.0	25.5	26.7	27.2	27.2	10.67	1.04/1.04	18.
100	Lithium Base Automotive Lub.	0.656	*	25.65	21.08	24.98	*	*	6.6	6.1	4.8	*	*	25.5	25.5	25.5	*	3.05	1.04/0.75	57.
101	100 w/MoS ₂	0.510	*	32.26	26.67	31.24	*	*	7.8	7.1	6.5	*	*	27.2	27.2	27.2	*	3.05	1.27/1.04	51.
102	SAE 90 Oil	1.313	*	24.98	19.81	23.37	*	*	4.7	4.7	3.8	*	*	28.8	25.5	25.5	*	0.00	1.04/0.79	—

Notes:

* Test rig vibration prevented reading.

(1) Gears sheared for several minutes; breakaway torque applies to top bearing under dead weight load of 0.5 kg.

(2) Readings taken after breakaway without re-greasing and consecutively for increasing then decreasing load; speed held constant at 24,000 rpm; load and film thickness taken from lower bearing of test rig.

(3) Power loss computed from torque reading on upper bearing.

(4) See Figure 4-8 for thermocouple location.

(5) Residual film thickness after standing for long period with 1.2 kg load on bearing.

(6) Compliance measured on first load cycle; ↑ = compliance for increasing load; ↓ = compliance for decreasing load.

(7) Count down time from 24,000 rpm to zero with weight of torque shaft and load platform (0.5 kg total) on top bearing.

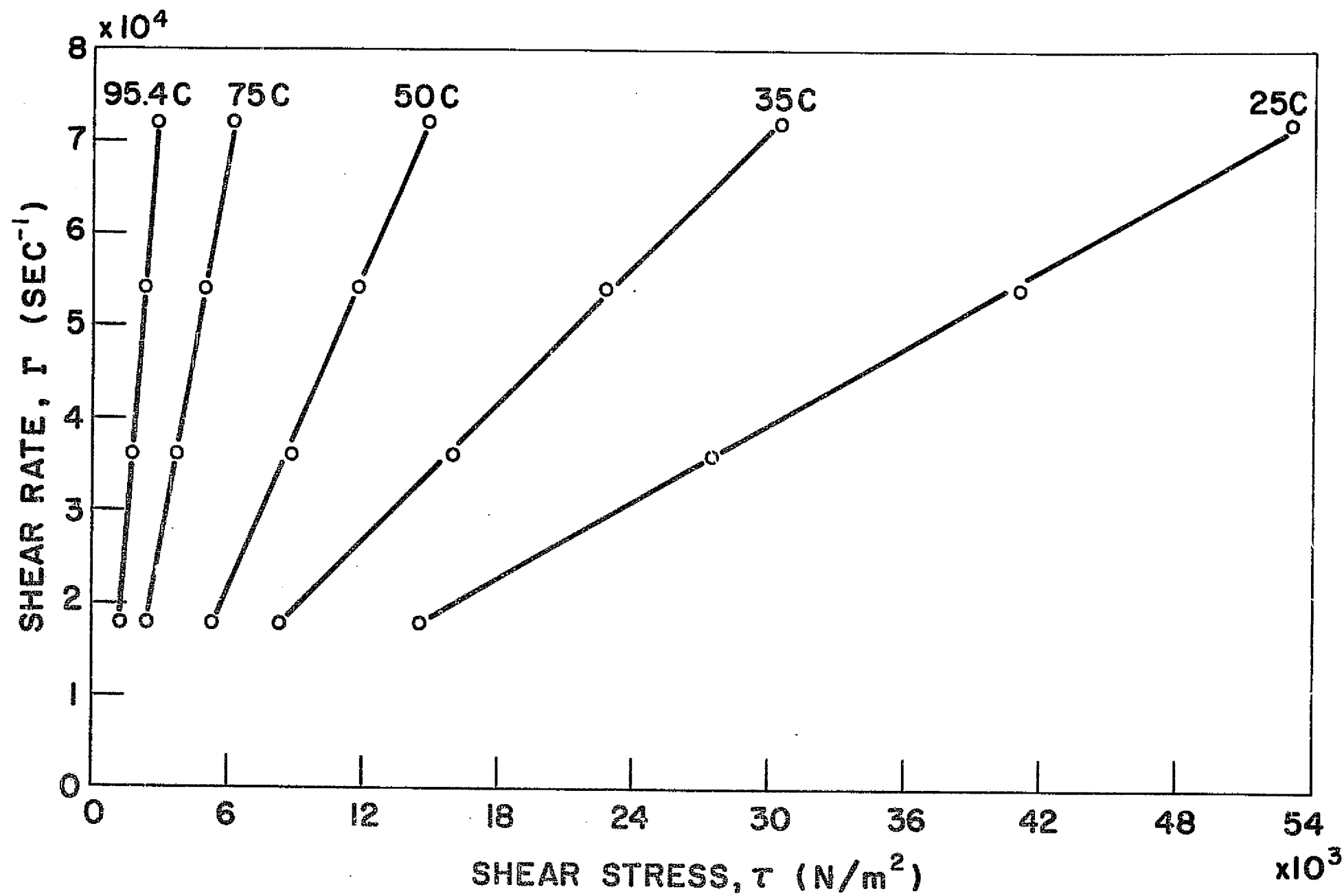


FIG. 5-2 SHEAR STRESS VS. SHEAR RATE FOR MCG-68-4

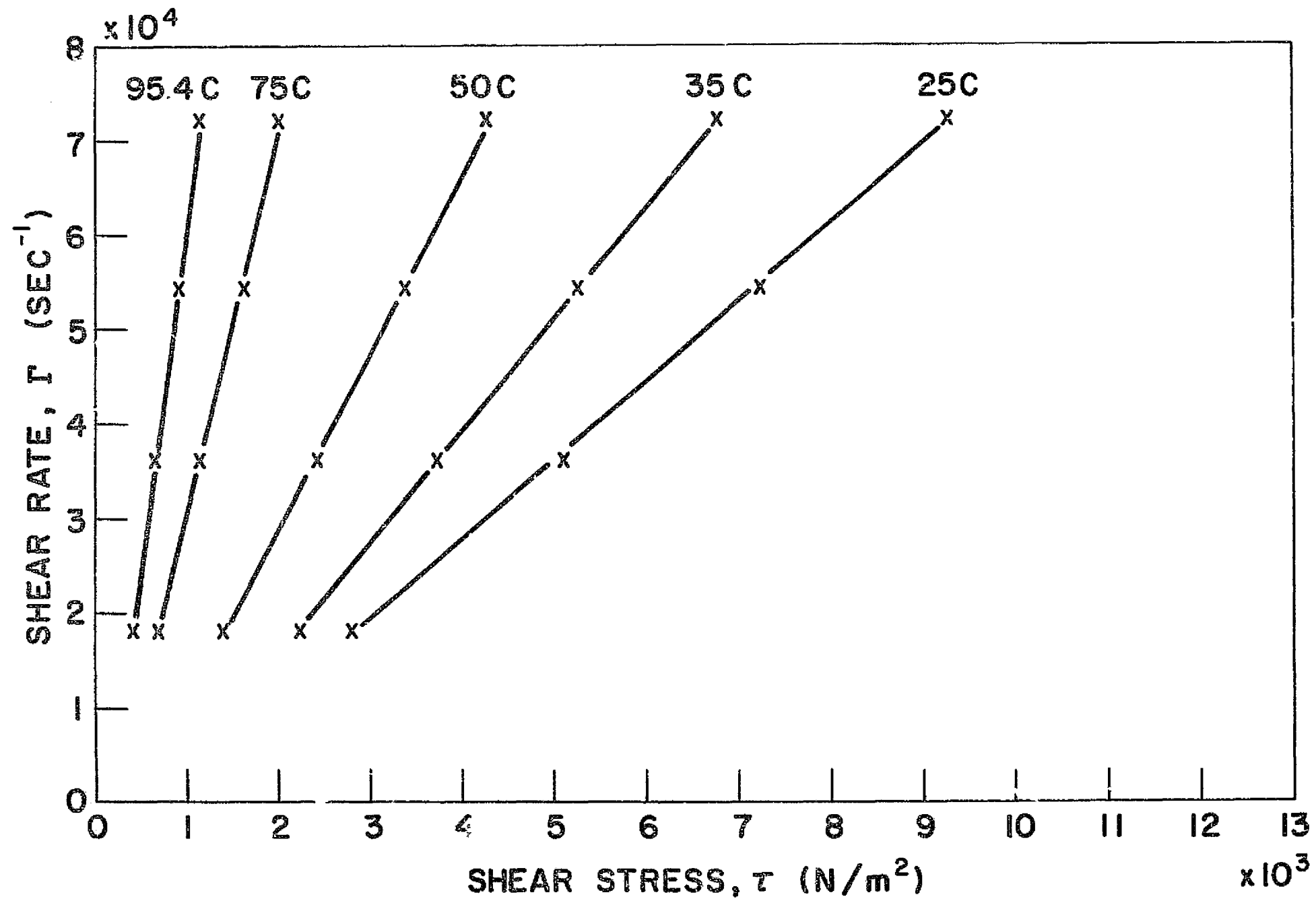


FIG. 5-3 SHEAR STRESS VS. SHEAR RATE FOR MCG-68-10

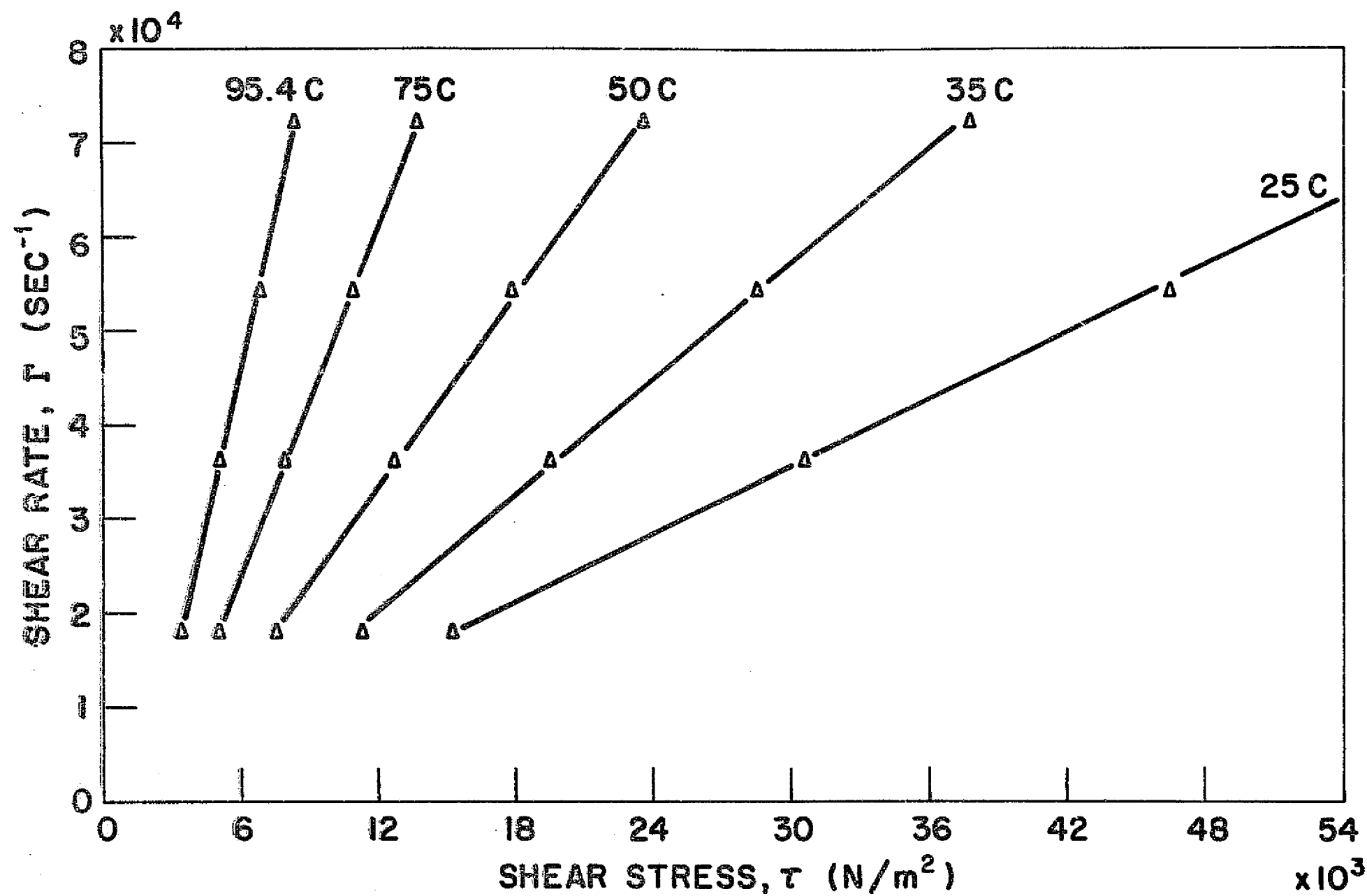


FIG. 5-4 SHEAR STRESS VS. SHEAR RATE FOR MCG-68-13

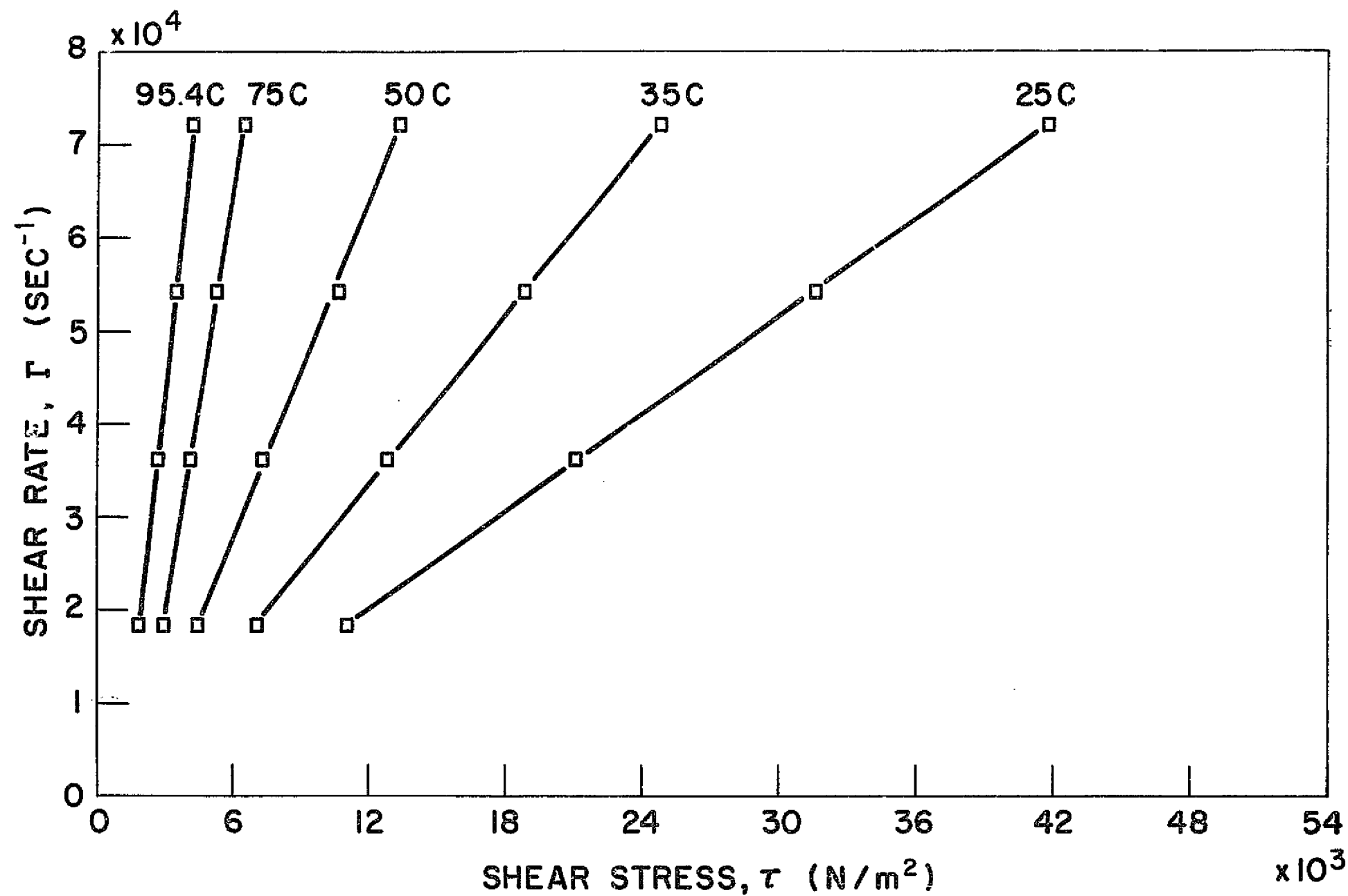


FIG. 5-5 SHEAR STRESS VS. SHEAR RATE FOR MCG-68-19

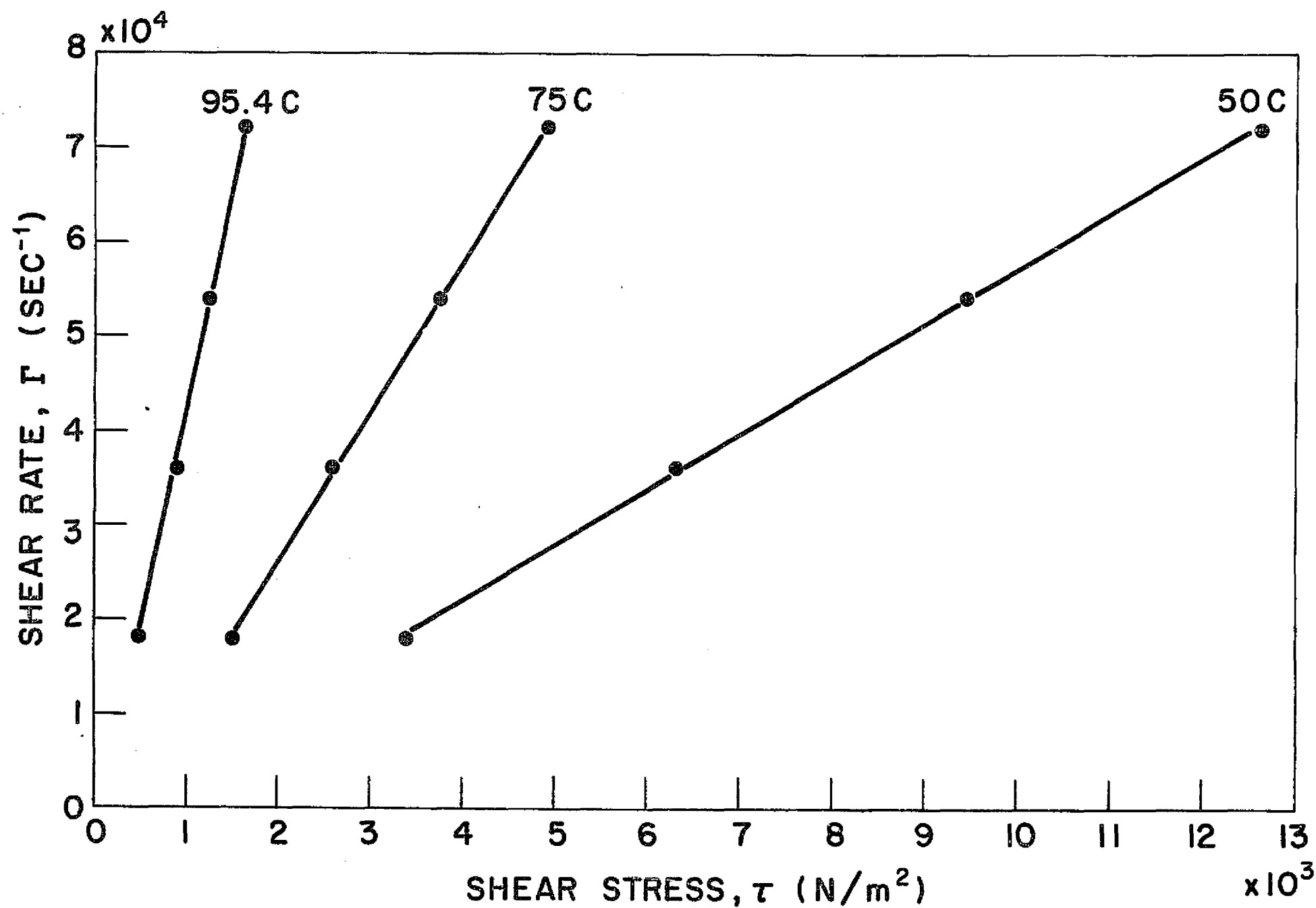


FIG. 5-6 SHEAR STRESS VS. SHEAR RATE FOR MCG-68-31

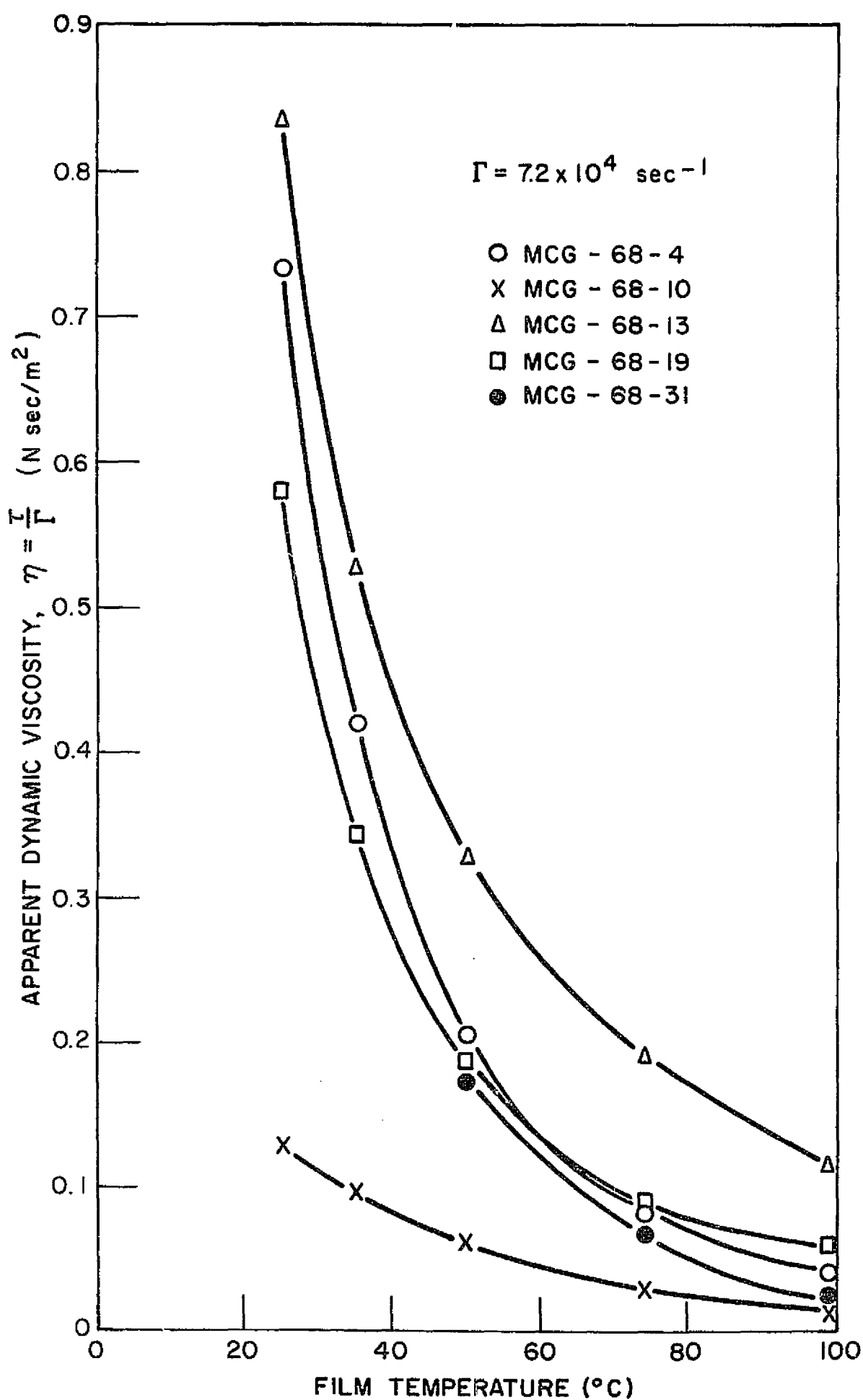


FIG. 5-7 APPARENT DYNAMIC VISCOSITY VS. TEMPERATURE
AT HIGH SHEAR RATE

5.5 The Effect of Grease Properties on Film Temperature and Load Capacity

Figure 5-8 shows a typical load-temperature relation taken at three bearing locations: two in the wall of the ungrooved stationary member and the third in the lubricant well. The test was one of many made on the rig shown in Figure 4-3 and it was often found that when moderately loaded, the wall temperature was about 50°C. Reference 7 gives an approximate expression for the increase in film temperature relative to the wall temperature* as

$$\Delta T \approx 0.09 \frac{\eta \Gamma^2 h_2^2}{\lambda} \quad 5.1$$

If the bearing shown in Figure 5-20 is assumed to operate with the viscosity of the base oil

$$\eta \approx 0.1 \text{ N sec/m}^2 \text{ (viscosity at 50°C)}$$

$$\Gamma \approx 9 \times 10^4 \text{ sec}^{-1} \text{ (avg. shear rate)}$$

$$h_2 \approx 38 \text{ microns (avg. land clearance)}$$

$$\lambda \approx 0.05 \text{ J/(m sec °C) (Thermal conductivity)}$$

then

$$\Delta T = 1^\circ\text{C}$$

Thus, for the above application, viscosity based on the bearing wall temperature differs little from that based on the maximum film temperature. On the other hand, some of the lubricants listed in Table 5-4, as well as many other grease lubricants, can have a significant film temperature rise relative to the wall. In the above example, if the lubricant MCG-68-10 is substituted and the wall temperature, shear rate and film thickness remain unchanged, the temperature rise relative to the wall would increase by a factor of six. For the latter case, the assumption of an isoviscous lubricant is questionable and shows the potential

*Both bearing walls are assumed to have a constant temperature.



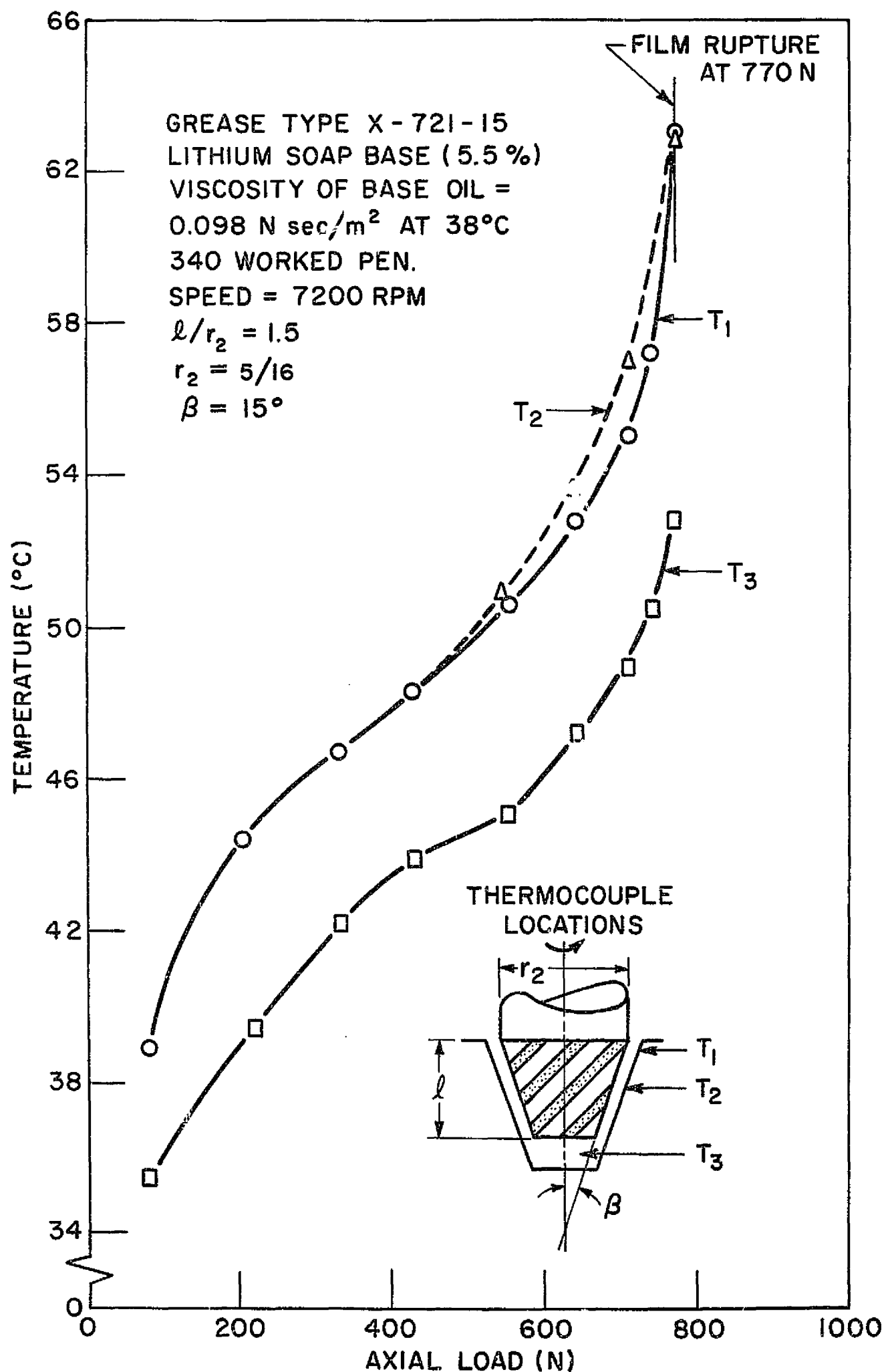


FIG. 5-8 TEMPERATURE VS. LOAD FOR A GREASE LUBRICATED SPIRAL GROOVE BEARING

of a substantial difference. Full account of the energy equation to accommodate temperature variations in the film represents a formidable task for the general case. Since the majority of available bearing design data is based on the isoviscous assumption, an expedient method for sizing a bearing is to base the calculations on an "effective viscosity". The temperature used in determining the effective viscosity should be based on the heat transfer particular to the bearing mounting design and consideration should be given to the possibility of a temperature rise in the film itself (e.g., Eq. [3]).

5.6 Selection of Optimum Lubricant for Spiral Groove Bearing

Table 5-6 lists the four fluids together with their thickeners for each fluid viscosity grade. The film thickness to power loss ratio h_2/P was computed for each grease listed in Table 5-4. Each combination of fluid, fluid viscosity and thickener was mixed in several consistencies and only the consistency, designated by "unworked penetration" resulting in the largest h_2/H is given.

The first column of Table 5-6 ranks each grease in terms of h_2/H . Several observations can be made based on the data of Table 5-6:

- Eight of the eleven greases listed are thickened with FEP.
- Six of the eleven greases contain the fluid perfluoro alkyl ether.
- There is an optimum fluid viscosity and penetration rating for the combination of perfluoro alkyl ether and FEP (MCG-68-22) which results in the largest h_2/H of all greases tested.
- There is a 40 percent difference in h_2/H between the grease ranked first and the grease ranked eleventh; greases ranked one through four differ by 10 percent.
- Since there is little difference in the h_2/H ratio for the upper half of the rated greases, other quantiles such as absolute film thickness, power loss or cost should be considered in selecting among this group.

Table 5-6

RATING OF GREASES IN TERMS OF FILM THICKNESS TO POWER LOSS RATIO

Perfluoro Alkyl EtherFEP

<u>Rating</u>	<u>Code MCG-68-</u>	<u>h_2/H $\mu\text{m/watt}$</u>	<u>Viscosity cs</u>	<u>Unworked Penetration</u>
3	19	4.46	38	301
1	22	4.70	90	298
4	26	4.24	280	334

Ammeline

11	29	3.33	38	331
6	32	4.12	90	334
8	35	3.94	280	335

FluorosiliconeFEP

2	13	4.62	176	300
5	16	4.18	524	307

Polyol Aliphatic EsterFEP & Clay

7	6	4.08	5	264
10	10	3.81	16	273

Mineral OilFEP & Clay

9	4	3.90	164	304
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At the bottom of Table 5-4, test results for two soap base greases and an oil are given for comparison. The automotive grease 100 containing a lithium soap and petroleum fluid, had an h_2/H of 3.46. Grease number 101 was essentially the same as grease 100 except it contained the additive MoS_2 , and for this grease, $h_2/H = 3.76$. The final lubricant was SAE 90 oil which had an h_2/H of 4.21 but lacked the self-sealing quality provided by the greases. Thus, greases rated one through four in Table 5-6, showed a higher h_2/H than any of the lubricants tested for comparison.

Preliminary results for greases designated MCG-69-11 to 22 indicate the viscosity-temperature relation can be influenced by consistency variations approximately to the degree that MCG-68-13 differs from MCG-68-4 as shown in Figure 5-9. Also, the value of h_2/H can be considerably reduced for mixtures with off-optimum consistencies.

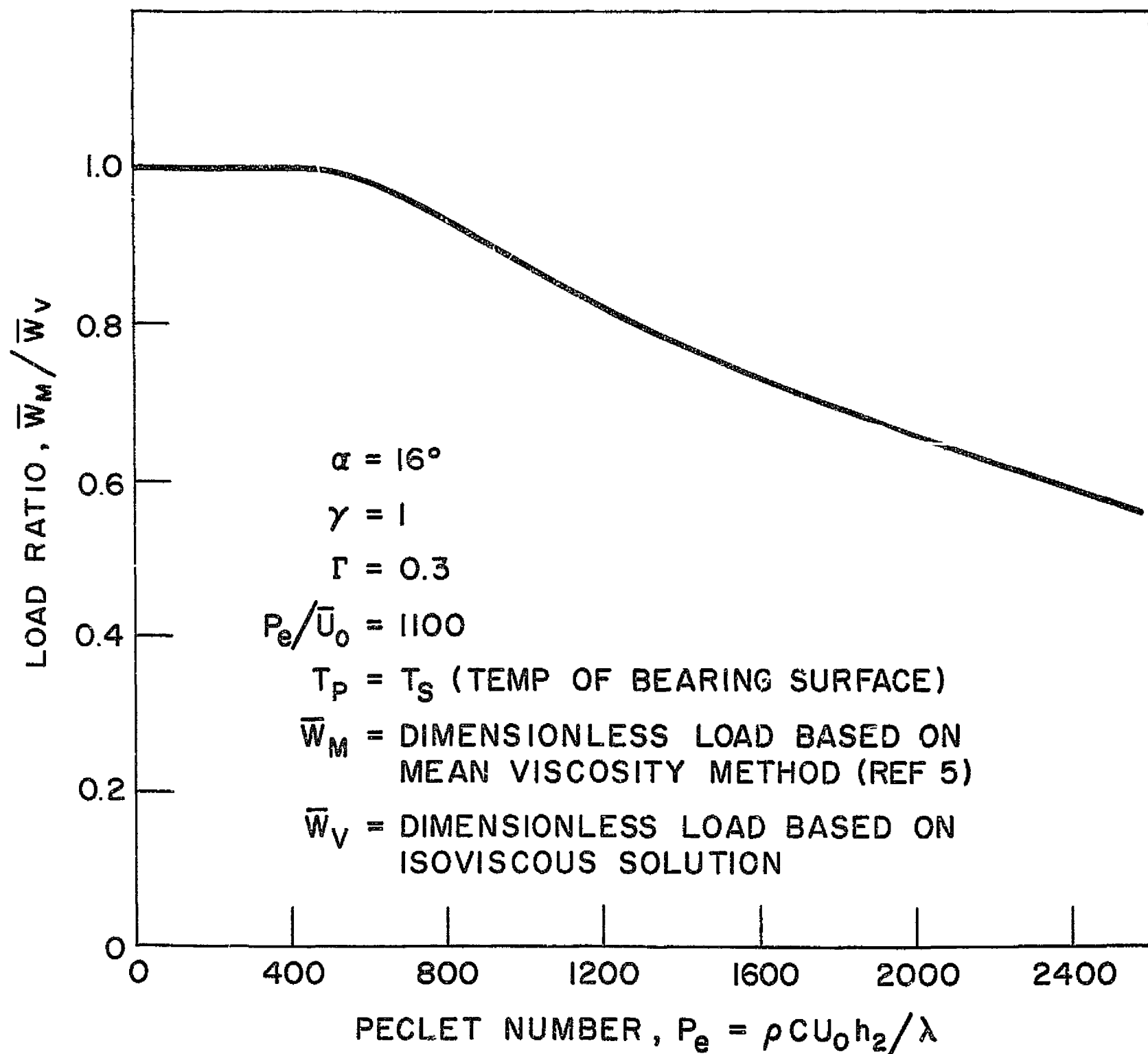
For small high speed applications where power loss and the cost of holding manufacturing tolerances are important, the polyol aliphatic ester grease MCG-68-6 thickened with FEP and Clay offers a good all around compromise.

For application where load capacity takes precedence over power loss, the perfluoro alkyl ether greases thickened MCG-68-22 with FEP are more suitable from a film thickness standpoint but for this class of grease, it is advisable to consider the low thermal conductivity in sizing the bearing.

5.7 Summary of Grease Development Testing

Approximately 50 greases were formulated specifically for small high speed spiral groove bearings. For each combination of constituents there was an optimum in terms of film thickness to power loss ratio.



Fig. 5-9. Peclet Number vs. $\bar{W}_M / \bar{W}_V$

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An unworked penetration rating of about 300 appears to be the best consistency for the application. A penetration rating of 250 or below was too stiff; 400 and above was too fluid.

The greases tested for apparent viscosity vs. temperature showed good shear stability and had a Newtonian character for shear rates above $2 \times 10^4 \text{ sec}^{-1}$. By virtue of their chemical nature, the grease physical properties can be expected to be repeatable from batch to batch.

Although neither the fluid nor the thickener is a particularly good lubricant, the combination of 90 cs perfluoro alkyl ether thickened with FEP to a consistency of 300 ASTM unworked penetration was outstanding among the lubricants tested in a high speed spiral groove bearing application. This grease demonstrated a good combination of shear stability, load capacity, film thickness, power loss and boundary lubrication.

6. EXPERIMENTAL VALIDATION OF THE FINITE ELEMENT COMPUTER PROGRAM

A special test set-up was constructed to validate the finite element computer program. In order to duplicate the calculation procedure, the grooved journal was fixed and the bearing rotated with its axis in a known orientation relative to the journal. The film pressures and temperatures corresponding to the pre-set clearance distribution were determined experimentally. The groove dimensions and clearance were selected to provide the limiting case for a conical bearing with a 15 degree groove angle and a land to groove depth ratio of 0.25.

6.1 Description of the Test Arrangement

The test arrangement is shown schematically on Figure 6-1 and photographically in Figure 6-2. The journal dimensions are shown on Figure 6-3 and the pressure tap and thermocouple junction locations are shown on Figure 6-4.

Pressure were sensed via 0.062 inch ID tubing finished flush with the journal surface. About a one inch length of grease was maintained in the journal end of each tube while the remainder of the tube was filled with oil. Pressures were read with calibrated gages whose Bourdon tubes were also filled with oil. Film temperatures were sensed by thermocouples which were potted with 0.5 mm thick epoxy in an insert. The inserts were pressed into the journal and ground flush to expose the junction (about 0.8 mm diameter).

6.2 Test and Test Results

6.2.1 Degasing Grease

To avoid entraining air in the grease when charging of the bearing with lubricant, the following procedure was used. The grease

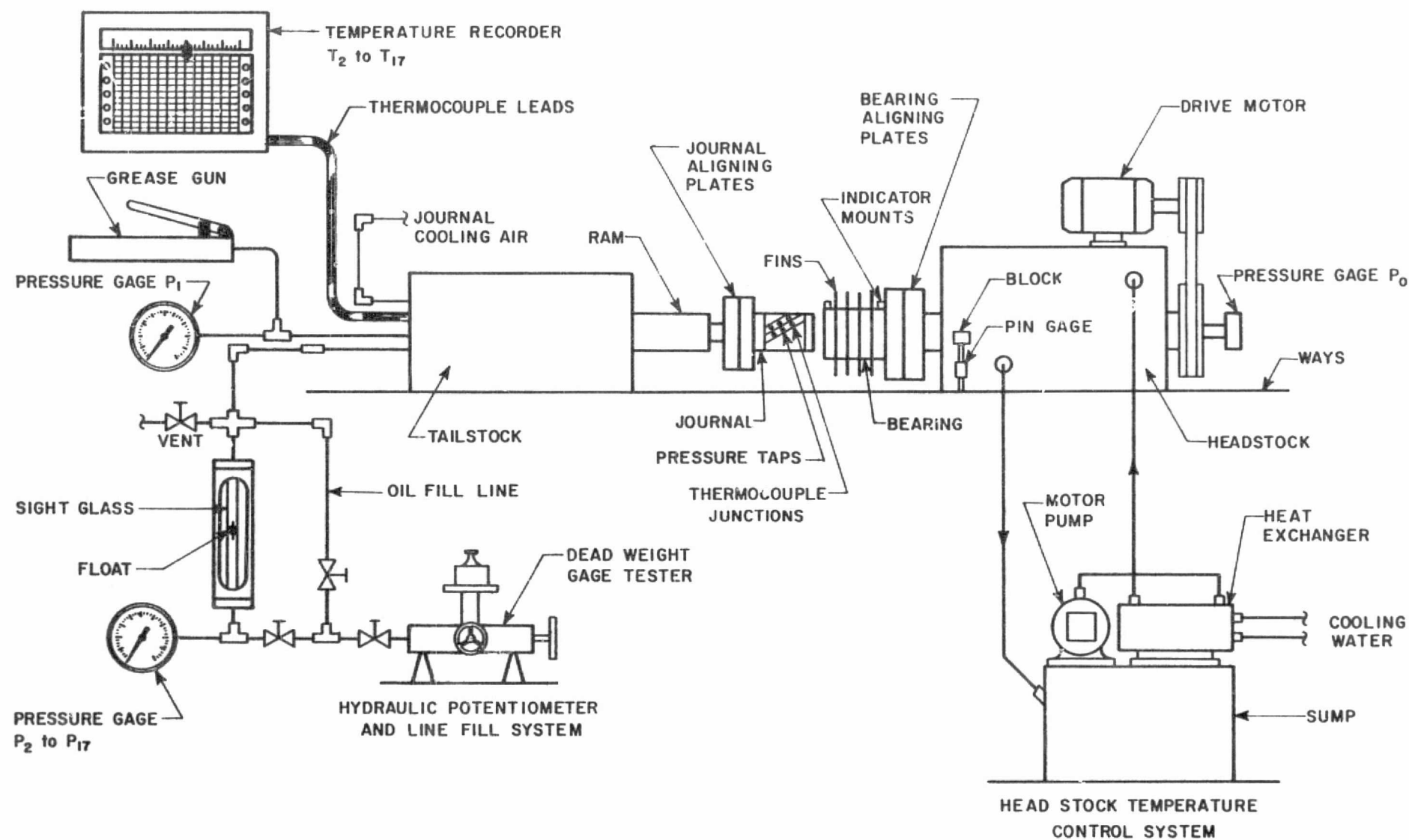
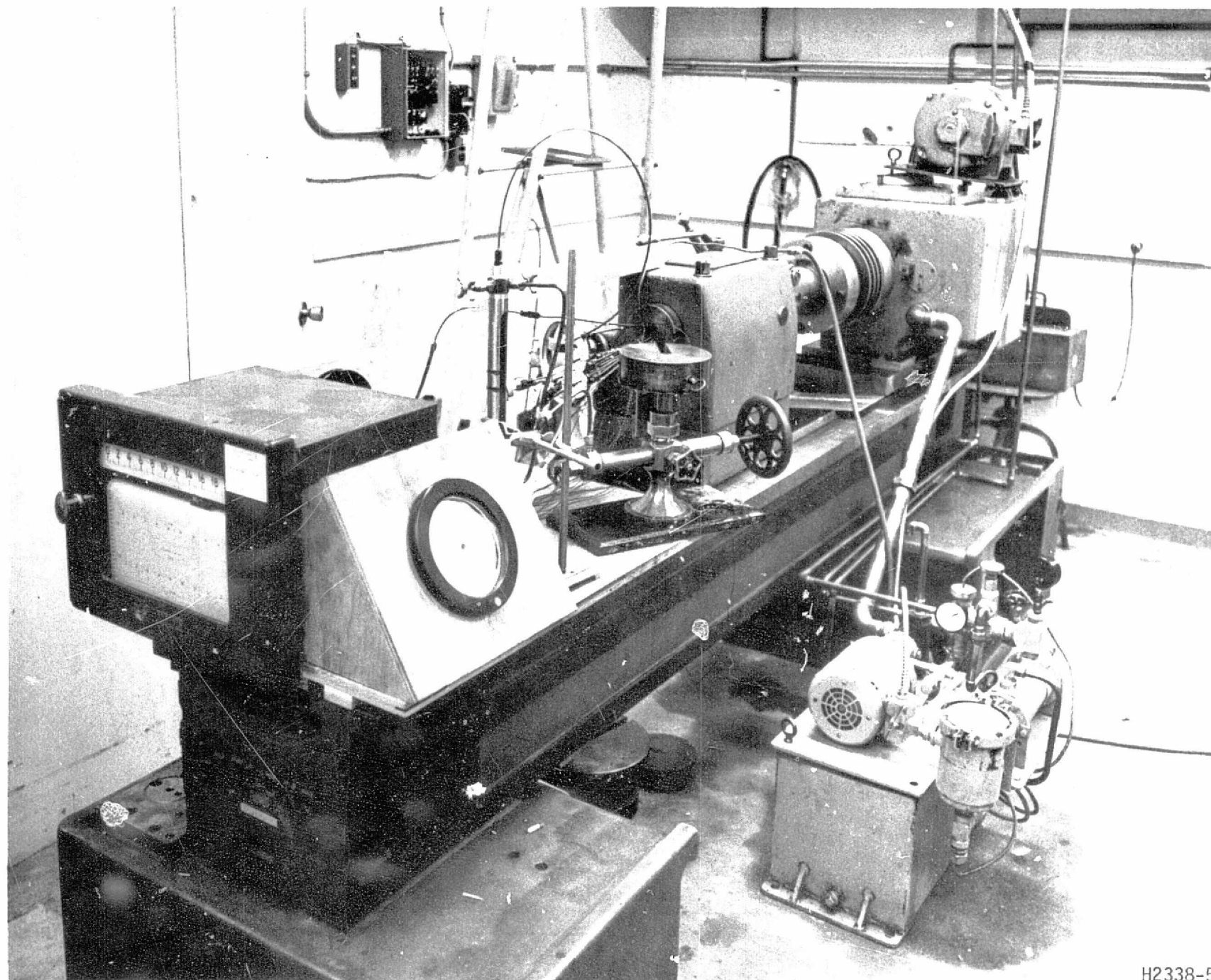


FIG. 6-1 SCHEMATIC OF TEST ARRANGEMENT FOR VALIDATION OF FINITE COMPUTER PROGRAM



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Figure 6-2. Photograph of Test Arrangement for Validation of Finite Element Computer Program

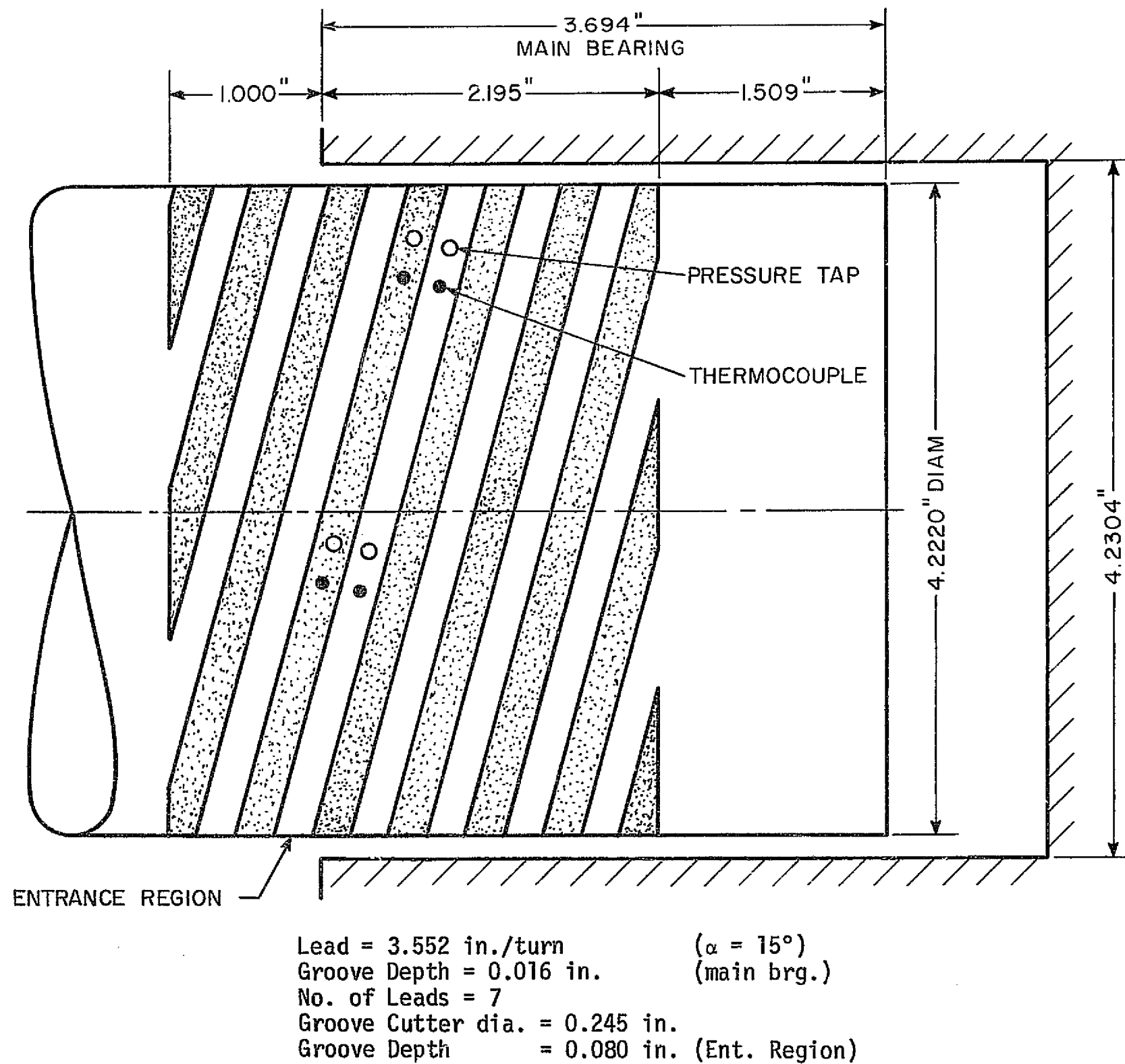
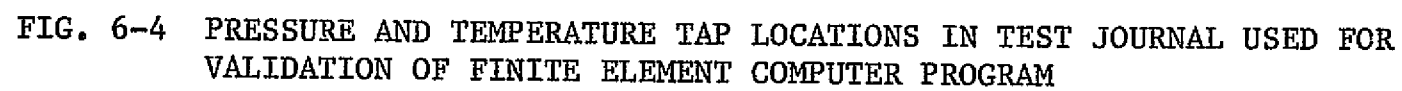


FIG. 6-3 TEST JOURNAL FOR VALIDATION OF FINITE ELEMENT COMPUTER PROGRAM



was passed through a funnel with a nozzle diameter of 0.6 mm into a mold which was housed in a bell jar at 1 mm Hg and 80°F. The mold diameter was 0.2 mm larger than the bore of the bearing and about 3 mm longer than the space between the front of the dead-ended journal and the bottom of the bearing (called the bearing cavity). The mold was placed in liquid nitrogen and the grease frozen solid. The frozen grease slug was pressed into the bearing while a vacuum was being drawn through a port at the base of the bearing. The bearing clearance was subsequently filled by thermal expansion of the melting grease*.

6.2.2 Rate of Pressure Build-up

With the bearing clearance charged with grease and with zero (or slightly positive) gage pressure at P_1 , the pressure rise in the film after start-up was found to be directly related to the grease temperature. The average pressure rise was about 100 psi per hour. If the pressure at P_1 was raised to 20 psig or above by preloading**, the pressure build-up was almost instantaneous. Thus, in all the test conducted in this set of experiments, an increase in film pressure above ambient had to be induced either by thermal expansion of the grease trapped in the bearing cavity, or by preloading.

6.2.3 Rate of Pressure Decay

In all experimental tests the pressures were found to decay with time. For instance, using grease MCG-68-10 and running at 1180 rpm†, P_{13} (see Fig. 6-4) decayed, non-linearly from a preloaded value of 41 psig

*For applications requiring through degasing it appears feasible to power the grease while frozen and subject the power to a 10^{-6} mm Hg ambient without damage due to liquid vapor loss.

**Moving the tailstock towards the headstock before startup causing an increase in the cavity pressure.

†Corresponds to a shear rate of $6.5 \times 10^4 \text{ sec}^{-1}$.

to the computed steady-state value of 14.2 psig in 168 minutes. At 206 minutes after start P_{13} read -1 psig. From the data collected, the rate of decay appeared to be effected by the journal eccentricity. (Note: as the headstock housing temperature increased the journal eccentricity changed. After several trial runs, the journal could be initially off-set to run at thermal equilibrium with an eccentricity ratio of about 0.1.)

Preloading while running at an eccentricity ratio of about .25 P_{13} was changed from the -1 psig mentioned above to +26 psig. P_{13} decayed from this point at a linear rate of 0.5 psi per minute for 50 minutes at which time there was an abrupt change in decay rate to 0.065 psi per minute. At the end of the test run P_{13} read -2 psig. The inference that the grease-air interface was moving into the bearing given by a plot of P_{13} vs. time was confirmed by other test, other gage readings and the "wiping" lines visible on the journal when extracted from the bearing. Thus, to compare the experimental reading with theory it was necessary to inject grease in order to re-establish the interface at the beginning of the bearing.

6.2.4 Grease Circulation In the Bearing Clearance

Tracer dye was injected into the bearing clearance through several of the pressure tap ports while the bearing was running. These tests indicate the initial flow of lubricant was from the cavity towards the open, or ambient, end of the bearing and that the probable cause was thermal expansion of the grease in the bearing cavity.

Dye was also placed in the 0.4 mm deep grooves. The dye flow pattern indicated a circulation in the groove cross-section from the upper trailing edge of the groove, down the groove side-wall, across the bottom of the groove, up the trailing edge wall and across to the starting point. The dye transferred circumferentially from groove to groove and also axially if there was any change in the bearing land clearance due to

thermal effects. In summary, these tests indicated that steady-state would not be established for the purpose of analytical validation until the thermally sensitive lubricant flow stabilized. With the present test set-up, this required at least 30 minutes at 1180 rpm.

6.2.5 Grease Voids in Grooves

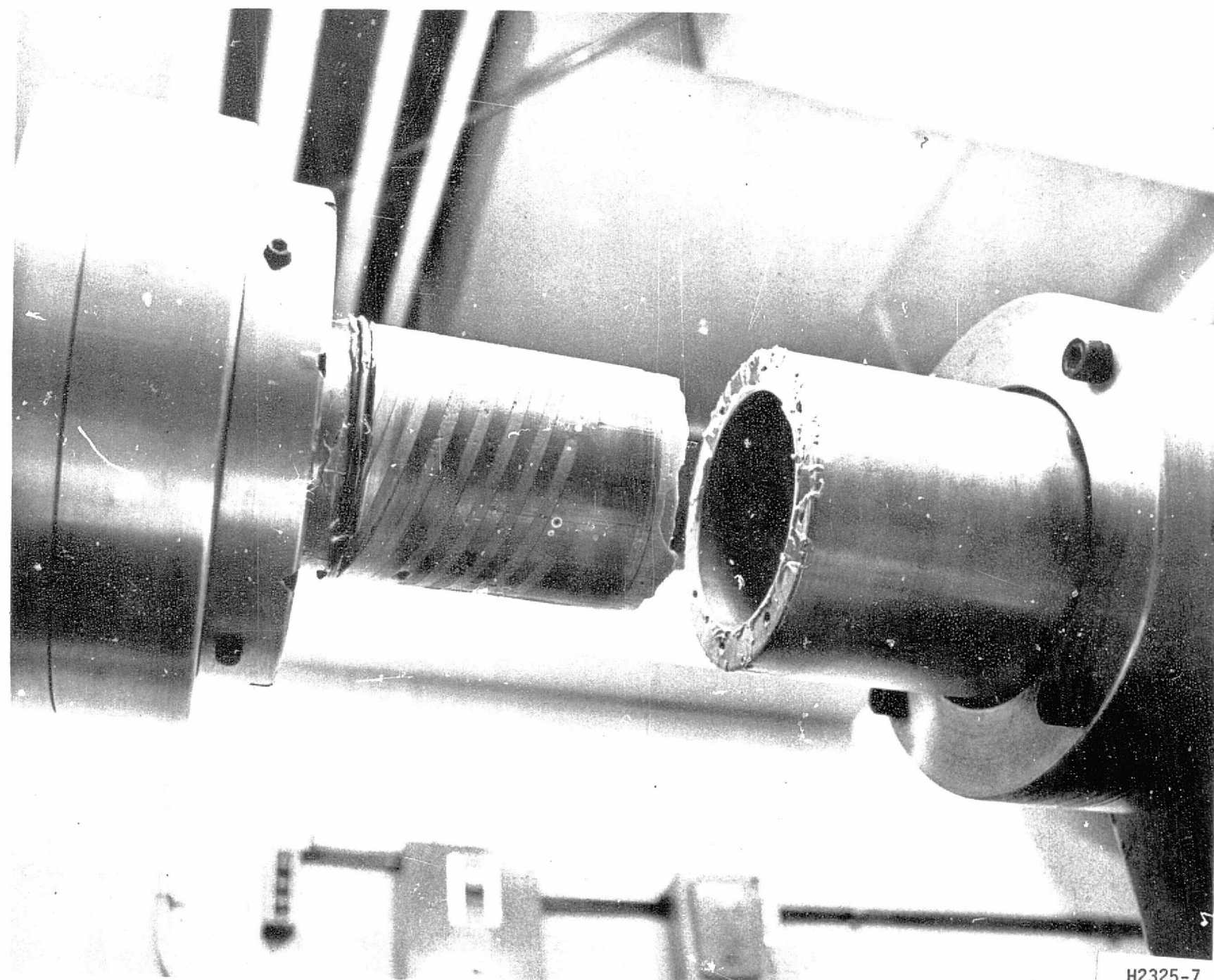
Upon disassembly after running for 5 to 15 minutes, the grooves were found to be packed with grease. Disassembly after running for more than 2 hours showed the grooves contained voids such as typically shown on Figure 6-5. The voids are believed to be associated with air entering from the ambient end of the bearing. Air appears to enter the clearance space during the thermal transient with the rate of entry increasing with increasing eccentricity. In order to validate the steady-state analysis it was necessary to force grease from the bearing cavity into the clearance with the tailstock ram and to take pressure time averages.

6.2.6 Experimental and Theoretical Comparison

Figure 6-6 shows the theoretical pressure profiles along the centerline of a land and a groove for a uniform radial clearance. Superimposed are the experimental readings. During the test run, the maximum separation between the axis of the journal and the axis of the bearing was 0.0127 mm (0.0005") with the minimum clearance located at $\bar{S} = 0.34$ on the groove pressure profile of Figure 6-6. This degree of eccentricity did not appear to have a significant influence on the pressure profile.

The deviation between measured and computed pressure near $\bar{S} = 0$ can be accounted for completely if the clearance was 15% smaller at $\bar{S} = 0$ than at $\bar{S} = 0.6$ (a difference of 0.0006"). In the latter case the theoretical $\bar{P}$ at $\bar{S} = 0$ would be about 9.5%.

*Corresponds to an end pressure of 150 psi or an axial force on the test journal of about 2000 lb.



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Figure 6-5. Typical Condition of Grease in Grooves After Three Hours of Operation at 1180 rpm

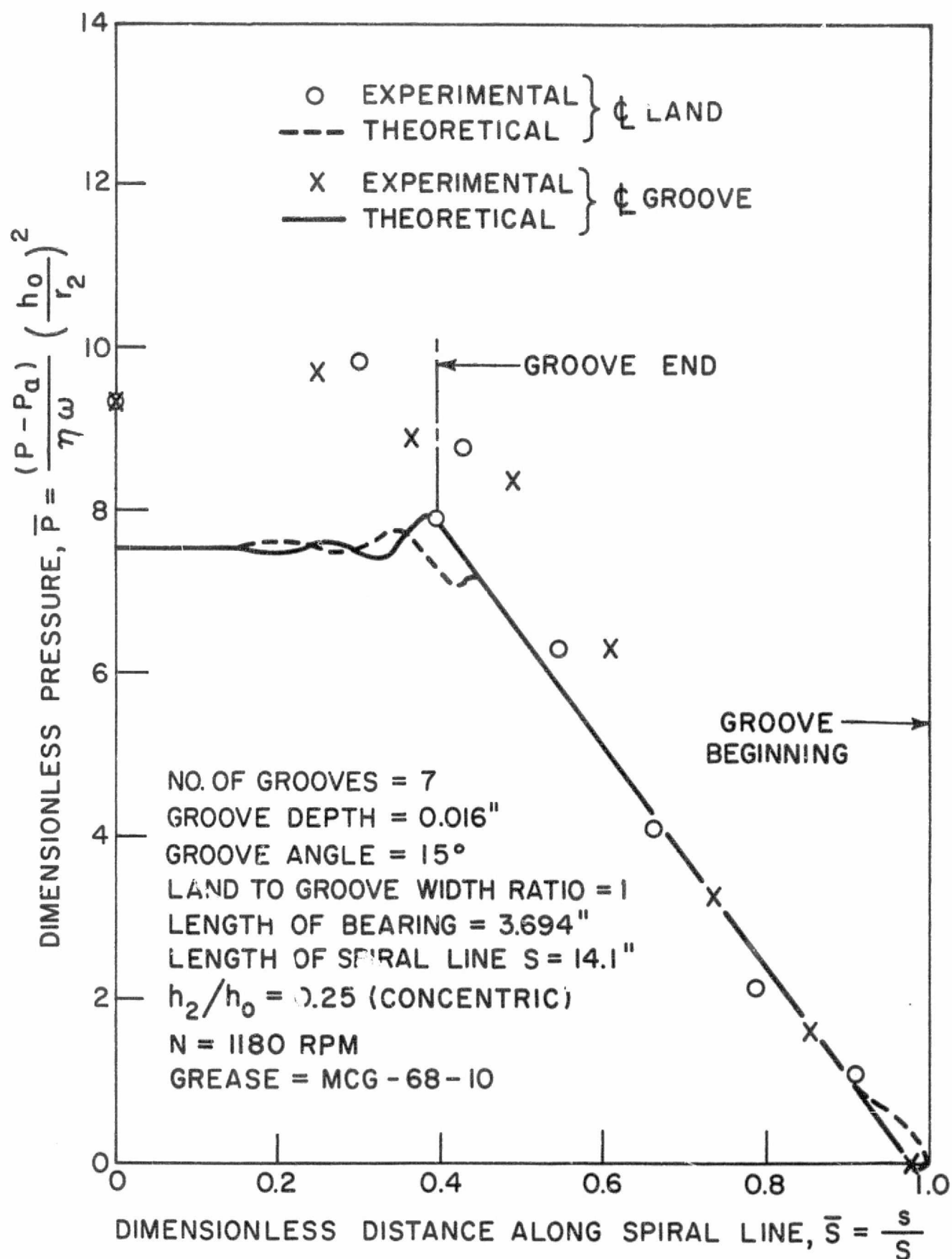


FIG. 6-6 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL PRESSURE PROFILE

Figure 6-7 shows the temperature profile corresponding to Figure 6-6. The profile was typical for all test runs in that the groove temperatures were lower than the land temperatures with the maximum difference occurring at the ambient end of the bearing and, after a sufficient period of time, the temperature was maximum at the end of the grooves.

Test runs were also made at 810 rpm and 1575 rpm. The general conclusion was:

If a steady-state geometry can be reached with the clearance space fully packed with grease, and if the film temperature is known and if the relation between lubricant temperature and viscosity is also known, then the theoretical analysis will accurately predict the bearing pressure distribution.

6.3 Implication of Validation Tests on Bearing Design

It appears very likely, the small dead-end conical bearing tested and described in Section 4 of this report acted in a manner similar to that observed with the larger cylindrical bearing described in this section. That is, after a certain period of time, the bearings ceased to operate on a full fluid film and, in some cases, operated for relatively long periods of time with boundary lubrication only. Nevertheless, the load capacity and other advantages offered by a grease lubricated spiral groove bearing do exist and are achievable. The basic problem is one of design; the bearing must be run in a flooded condition. Lubricant at a positive pressure must be available to all points on the circumference of the bearing film entrance. The interface must be located far enough from the bearing entrance so that ambient gas that migrates into the lubricant will not reach the film entrance within the design life of the bearing.



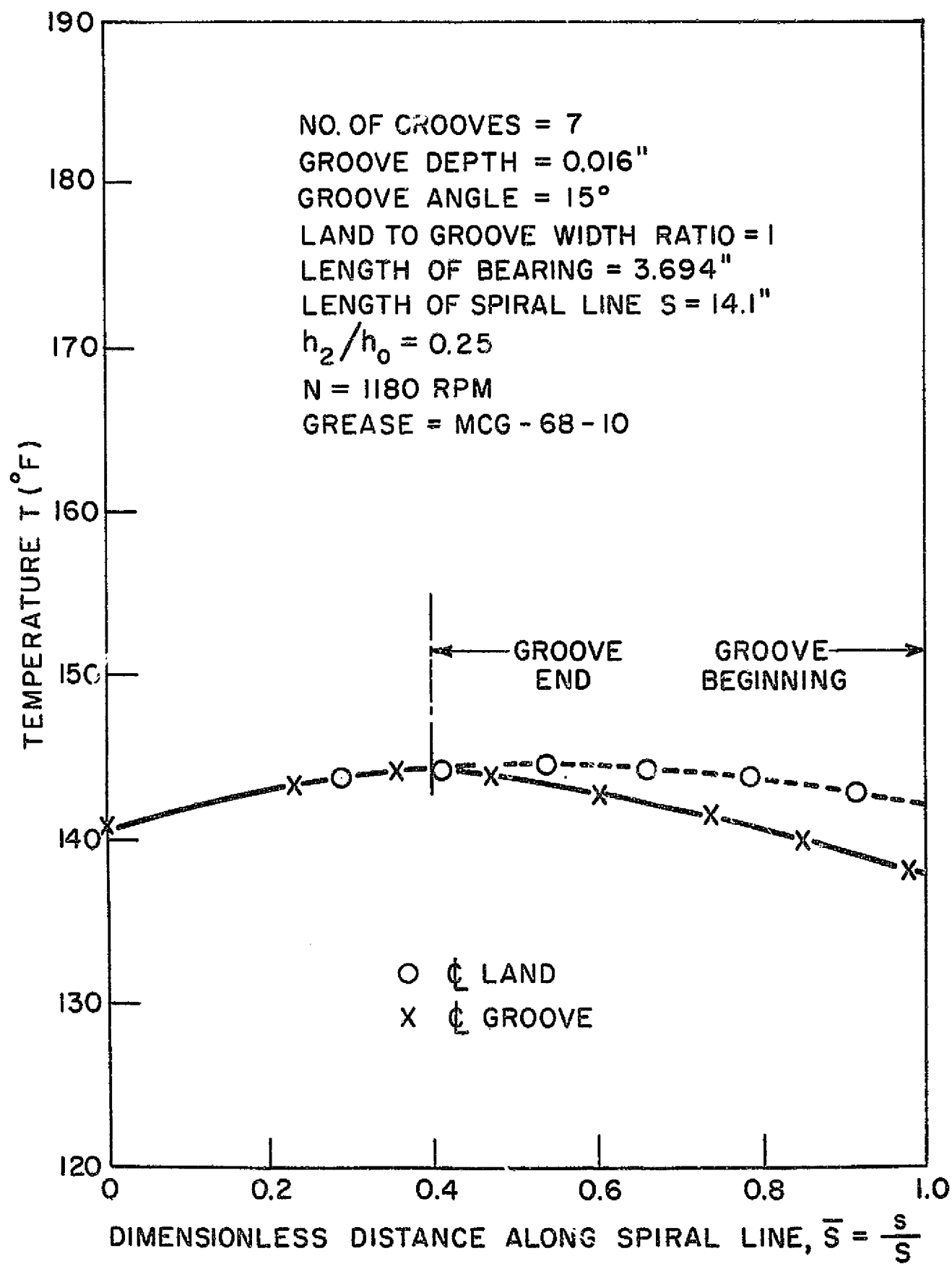


FIG. 6-7 TEMPERATURE PROFILE IN TEST BEARING

7. CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

• Based on the analyses of Ref.10 and this report, fitting a self-acting bearing surface with spiral grooves provides the greatest zero eccentricity stiffness and film thickness to power loss ratio of all surface geometries investigated.

• Compared to a *gas* lubricated bearing, a grease lubricated bearing has the following general advantages and disadvantages:

Advantages

- 1) higher load capacity to power loss ratio.
- 2) better boundary lubrication.
- 3) better high speed run and stop-start characteristics.
- 4) greater tolerance to dirt.
- 5) greater safety margin for overload.
- 6) lower rotor windage losses due to ability of bearing to operate in low ambient pressure.
- 7) cheaper to make because of less stringent manufacturing tolerances accrued from greater operating film thickness
- 8) less stringent surface finish and material dimensional stability.
- 9) greater film damping.
- 10) bearing surface porosity or accidental scratches produce less of a change in performance characteristics.
- 11) component handling and clean room assembly conditions less critical.

Disadvantages

- 1) sensitivity to temperature changes.
- 2) possibility of loss of lubricant or vaporization of fluid in lubricant.
- 3) possibility of cavitation.

- 4) possibility of alteration in lubricant properties due to chemical change.
- 5) change in rotor angular momentum due to lubricant mass shift.
- 6) possibility of gas entrainment.
- 7) analytical performance predictions less accurate due to non-Newtonian lubricant and possibility of significant temperature rise in film.

• Compared to an *oil* lubricated bearing, a grease lubricated bearing has the following general advantages and disadvantages:

Advantages

- 1) self-sealing.
- 2) higher film thickness to power loss ratio.
- 3) better lubricant property repeatability (for the grease formulated in this study).

Disadvantages

- 1) possible separation of grease constituent.
- 2) lubricant more expensive and less available

• Compared to *ball bearings*, a grease lubricated bearing has the following general advantages and disadvantages.

Advantages

- 1) longer life at higher rotor speeds.
- 2) less noise and vibration.

Disadvantages

- 1) not "off-the-shelf" item (currently) and must be designed rather than selected from a manufacturers catalog.
- 2) less over-load safety margin.

• Many of the unsolved problems associated with grease lubricated spiral groove bearings, such as interface location and the effects on performance of gas entrainment and cavitation, are also common to other types of liquid lubricated bearings.



- Grease lubricated spiral groove bearings show excellent potential for applications such as the spin axis of a momentum wheel and particularly, for devices that run continuously.
- If the bearing entrance region is directly exposed to the ambient, the quantity of fluid in the clearance space will decrease and the load capacity will drop to a fraction of that predicted by the steady state analysis.

7.2 Recommendations

Analysis

Further analytical studies should be done in the following areas:

- dynamics of grease lubricated spiral groove bearings,
 - 1) obtain film stiffness and damping coefficients using existing finite element analysis.
 - 2) adapt existing rotor response and stability analyses to incorporate film coefficients from item 1) above.
- thermal effects
 - 1) determine conditions under which the isoviscous assumption is invaled (Note: significant errors in load capacity calculations are possible if the isoviscous assumption is invalid).
 - 2) modify the existing analysis to include the energy equation.
- cavitation
 - 1) incorporate an appropriate treatment of cavitation into the existing analysis (Note: data in this report is limited to a radial eccentricity ratio of 0.5 because of the existence of significantly large subambient film pressures for eccentricity ratios greater than 0.5).
 - 2) determine the influence of cavitation on radial load and stiffness
- lubricant-gas-metal interface
 - 1) determine the shape, location and influence on performance of the film boundary at the groove entrance (Note: the assumption that the film boundary lies in a plane perpendicular to the bearing axis at the film entrance has been observed experimentally to be untrue).

Experimentation

- Develop and test a spiral groove bearing entrance assembly that will consume a minimum of power and result in zero net flow out of the clearance space during start-up, cold restart*, and normal operation.
- Develop and test a buffer seal to prevent evaporation of a lubricant from a spiral groove gyro spin axis bearing into a cavity that is below the vapor pressure of the lubricant.
- Analyze and design a control moment gyro in the 1000-5000 ft-lb-sec range that is integrated with the test stand, electronics, motor development and mission requirement of the Astrionics Laboratory. The design should incorporate a buffer seal and a hydrodynamic spiral groove grease lubricated bearings with the entrance assembly cited above.
- Construct and test the control moment gyro. Building the momentum wheel as a competitor of the existing ball bearing units will provide the best possible evaluation of the technology developed thus far for the application of grease lubricated spiral groove bearings to a spin axis.

*Cold restart can be a particularly severe operating condition if the grease and bearing members contract upon cooling at different rates and leave voids that are not filled because of the grease yield strength.

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APPENDIX A
COMPARISON BETWEEN THE APPROXIMATE
AND EXACT ANALYSES

A

The analytical method based on an assumed pressure profile and constant clearance is referred to as the approximate analysis. The method based on the finite element approach is referred to as the exact analysis.

Figure A-1 shows the pressure profiles computed by the approximate method with and without the end corrections described in Ref. 1. Also shown is a profile computed by the exact method. The profile applies to a line which follows the leading edge of a groove. Not shown on Figure A-1 is the pressure increase that exists as the groove is traversed circumferentially to its trailing edge. Here, the value of $\bar{P}$ at $\bar{S} = 0$ from the exact analysis reaches a maximum of 19.85.

The approximate solution with end corrections was used as one of the check cases for verification of the finite element computer program. Figure A-1 shows that the mutual agreement is good.

Using the corrected approximate solution, the computed axial load coefficient was conservative and differed by 0.4% from the exact value of

$$\frac{w_t h_2^2}{\eta \omega r_2^4} = 0.3674$$

The moment coefficient was also conservative and differed by 1% from the exact value of

$$\frac{m h_o}{\eta \omega r_2^4} = 3.47$$

An additional check was made using data for a dead ended conical bearing from Ref. 1. Here, approximate results without end corrections are given as

$$\frac{w_t h_2^2}{\eta \omega r_2^4} = 1.4 \text{ (or 1.23 if the suggested simplifications to the approx. equations are not made)}$$

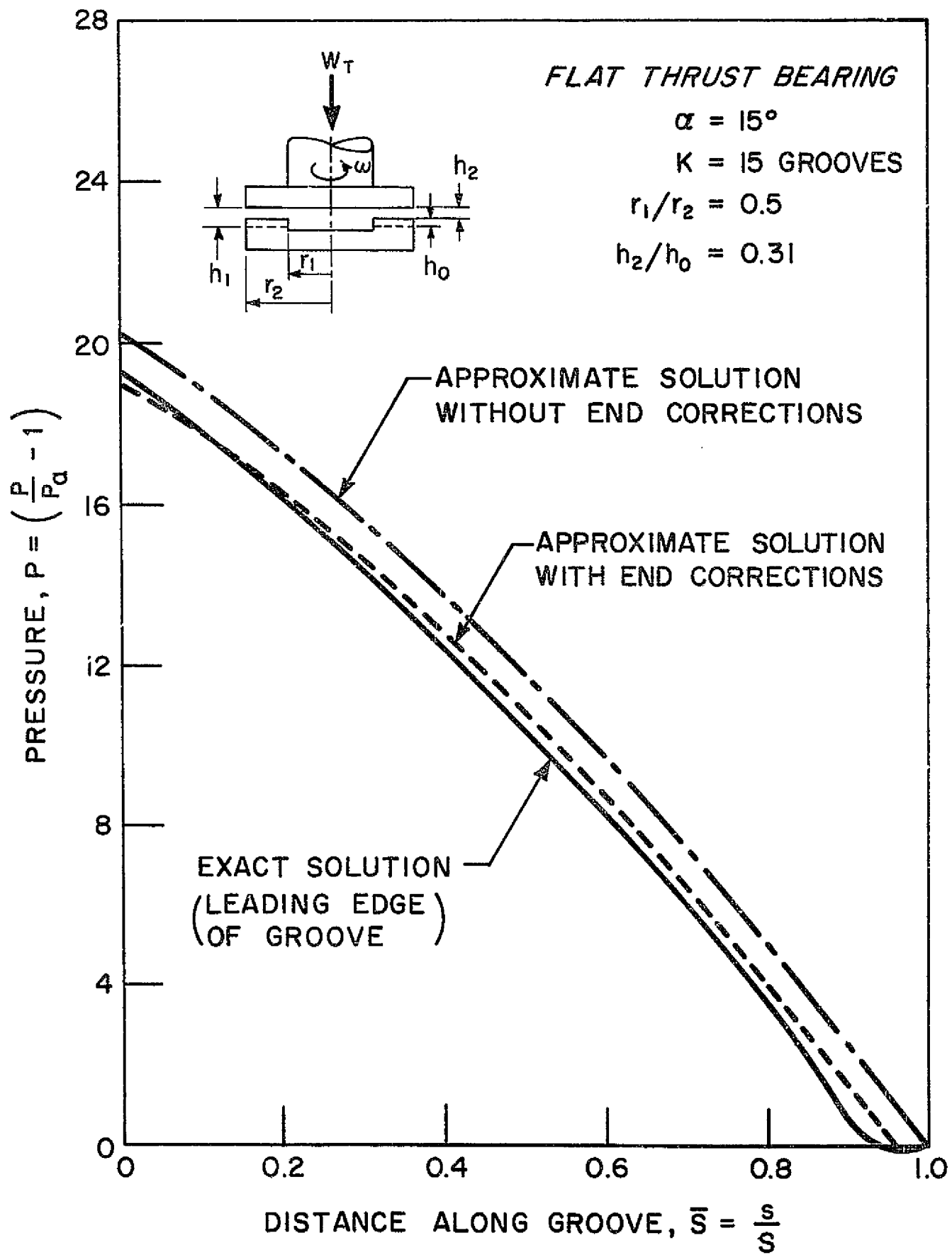


FIG. A-1 PRESSURE DISTRIBUTION ALONG SPIRAL GROOVE

where

$$\tan t = 1/6 \text{ (half cone angle)}$$

$$\alpha = 15^\circ$$

$$\delta = 0.4$$

$$\sigma = 1$$

For 7 lands and grooves, the exact solution was

$$\frac{w_t h_2^2}{\eta \omega r_2^4} = 1.29$$

In computing the friction moment coefficient for the cone, the approximate result (without end corrections) was 17% higher than was computed by the exact method.

Several checks were made to determine if the approximate analysis could be used to obtain information on radial load capacity and stiffness for small eccentricities. This violation of the basic assumption of constant clearance led to significant errors and incorrect trends.

The conclusions of the comparisons between the approximate and exact solutions were:

- for constant clearance, the approximate and exact analysis were mutually confirmative.
- for constant clearance, the approximate analysis gives good results for load capacity and fair results for the friction moment unless end corrections are applied.
- the approximate analysis can be used only for the case of constant clearance.

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APPENDIX B
APPLICATION OF THE FINITE ELEMENT METHOD
TO CONICAL SPIRAL GROOVE BEARINGS

B



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Governing Equations

For a conical bearing, the incompressible Reynold's equation can be written in terms of the curvilinear coordinates shown in Figure B-1 as (Ref. 4)

$$\begin{aligned} & \frac{1}{(\xi_3 \sin t)^2} \frac{\partial}{\partial \theta} \left(\frac{h^3}{6\eta} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial \xi_3} \left(\frac{h^3}{6\eta} \frac{\partial p}{\partial \xi_3} \right) + \frac{h^3}{6\eta} \frac{\partial p}{\partial \xi_3} \left(\frac{1}{\xi_3} \right) \\ & = 2\dot{h} + \frac{1}{\xi_3 \sin t} \frac{\partial}{\partial \theta} (h v_\theta) + \frac{\partial}{\partial \xi_3} (h v_{\xi_3}) + \frac{h}{\xi_3} v_{\xi_3} \end{aligned} \quad (B.1)$$

where t is the half cone angle. Let

$$\phi = \theta \sin t$$

The Reynold's equation becomes

$$\begin{aligned} & \frac{1}{\xi_3^2} \frac{\partial}{\partial \phi} \left(\frac{h^3}{6\eta} \frac{\partial p}{\partial \phi} \right) + \frac{\partial}{\partial \xi_3} \left(\frac{h^3}{6\eta} \frac{\partial p}{\partial \xi_3} \right) + \frac{h^3}{6\eta} \frac{\partial p}{\partial \xi_3} \left(\frac{1}{\xi_3} \right) \\ & = 2\dot{h} + \frac{1}{\xi_3} \frac{\partial}{\partial \phi} (h v_\theta) + \frac{\partial}{\partial \xi_3} (h v_{\xi_3}) + \frac{h v_{\xi_3}}{\xi_3} \end{aligned} \quad (B.2)$$

From Ref. 2, the Euler's equation of the variational problem is found to be

$$\nabla \cdot \left(\frac{h^3}{6\eta} \nabla p \right) = \nabla \cdot h \vec{u} + 2\dot{h} \quad (B.3)$$

It is noted that if Eq. (B.3) is written in plane polar coordinates (ρ, ψ) , it becomes identical with Eq. (B.2). Thus, by identifying the independent variables ρ, ψ with ξ_3, ϕ and the velocity components v_ρ, v_ψ with v_{ξ_3}, v_θ the variational principle given in Ref. 2 can be applied directly

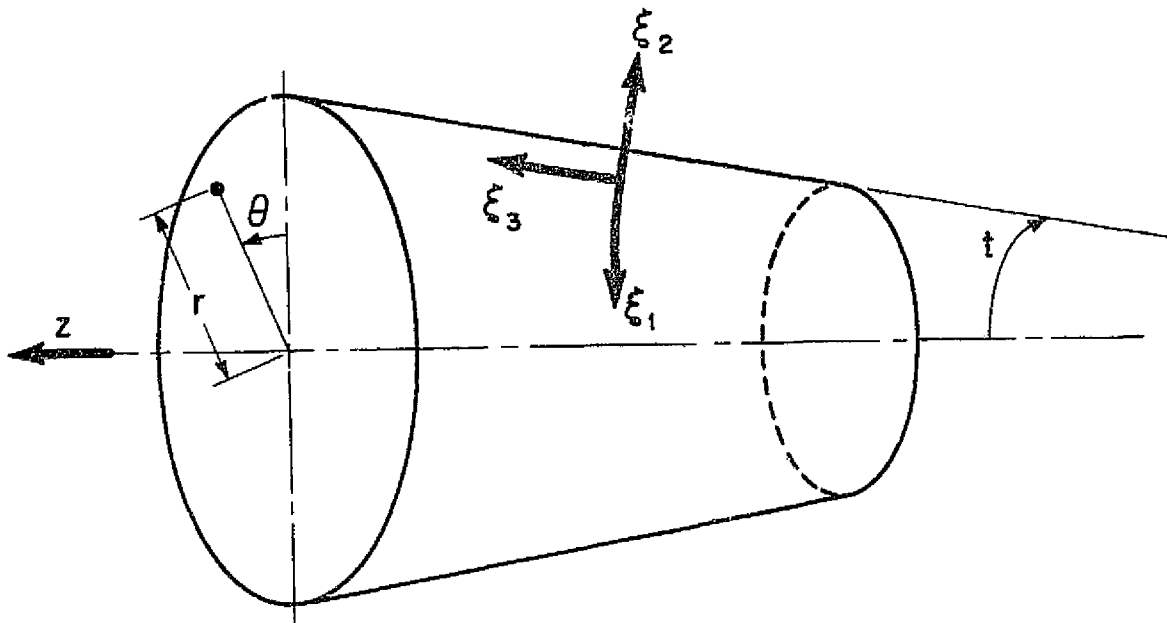


FIG. B-1 COORDINATE REFERENCE FRAME

to the problems of conical bearings. Since the conical surface is developable onto a plane, the domain of interest can be regarded as a plane region occupied by the developed conical surface (Fig. B.2).

The equations of transformation are

$$\begin{aligned}\rho &= \xi_3 = r/\sin t \\ \psi &= \phi = \theta \sin t\end{aligned}\tag{B.4}$$

The equation for a spiral groove on the surface of the cone is transformed to

$$\frac{\rho}{\rho_0} = e^{(\psi - \psi_0) \tan \alpha}\tag{B.5}$$

for the plane region. Here, α is the groove angle and (ρ_0, ψ_0) are the coordinates at which the spiral begins. The length of the spiral is given by

$$\begin{aligned}S &= \int_{\psi_0}^{\psi} \left\{ (d\rho)^2 + \rho^2 (d\psi)^2 \right\}^{1/2} \\ &= \frac{1}{\sin \alpha} [\rho - \rho_0]\end{aligned}\tag{B.6}$$

The expression for the film thickness over the land area when the rotor is eccentrically located in the bearing is found to be

$$h_\ell = h_2 - e \cos t \cos \theta\tag{B.7}$$

where h_2 is the concentric land clearance, e the eccentricity. Note that the above equation is derived with the assumption that the axes of the rotor and the bearing are parallel to each other. Let h_0 denote the groove depth. The film thickness over the groove is given by

$$h_g = h_\ell + h_0\tag{B.8}$$

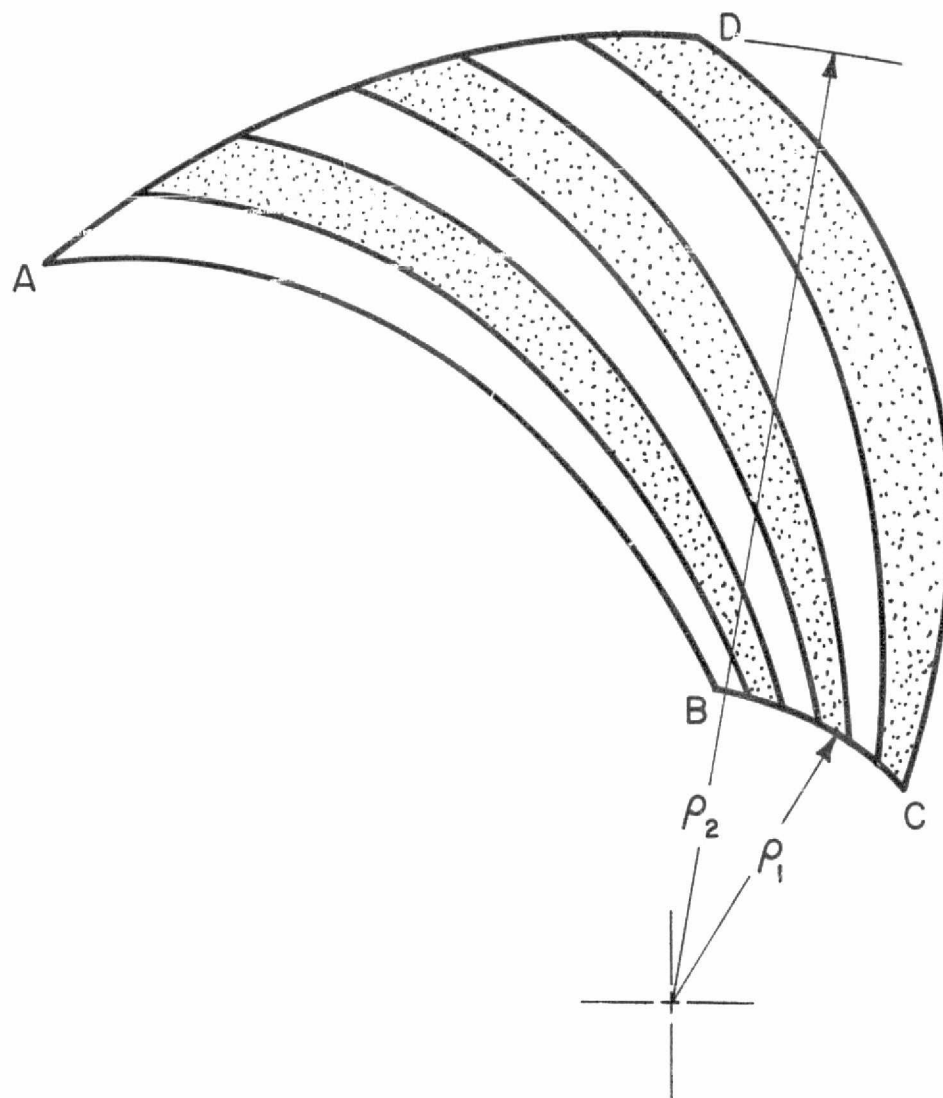


FIG. B-2 DEVELOPED CONICAL SURFACE

Defining the dimensionless quantities as follows

$$H = h/h_o \quad \vec{U} = \vec{u}/r_2 \omega$$

$$T = \omega t \quad \bar{\rho} = \rho/r_2$$

$$P = \frac{p h_o^2}{\eta \omega r_2^2}$$

$$\bar{q} = \frac{q}{r_2 \omega h_o}$$

Eqs. (B.3), (B.7) and (B.8) can be written in dimensionless form as

$$\nabla \cdot \frac{H^3}{6} \nabla P = \nabla \cdot H \vec{U} + 2 \frac{dH}{dT} \quad (B.9)$$

$$H_\ell = H_2 - \epsilon \cos \theta$$

$$H_g = 1 + H_\ell \quad (B.10)$$

where

$$\epsilon = \frac{e \cos t}{h_o} \quad (B.11)$$

The generating functional (Eq. 3.8) now becomes

$$\begin{aligned} \Phi\{P\} = \frac{r_2^4 \omega^2}{h_o} \left\{ \int_R \left[\frac{H^3}{12} \nabla P \cdot \nabla P - H \vec{U} \cdot \nabla P + 2P\dot{H} \right] dA \right. \\ \left. + 2 \int_{C_2} \bar{q} P ds \right\} \quad (B.12) \end{aligned}$$

Note that the rotational speed of the rotor, ω , and the viscosity of the lubricant, η , do not appear explicitly in Eq. (B.9). Thus, the resulting nondimensional pressure obtained for a particular film geometry is valid for all values of these parameters.

Finite Element Equations

In the following, the formulation of a system of linear algebraic equations using the finite element approach will be illustrated. These equations are derived based on composite quadrilateral elements (Fig. B-3). For the sake of convenience a cartesian coordinate system will be used. This is allowed because Eq. (B.9) and (B.12) are invariant with respect to coordinate transformation. It should be noted, however, that the velocity component u_x and u_y are given by

$$\begin{aligned}u_x &= v_\rho \cos \psi - v_\psi \sin \psi \\u_y &= v_\rho \sin \psi + v_\psi \cos \psi\end{aligned}$$

Within each triangle of the composite element, the pressure distribution is assumed to be of the form

$$p(x,y) = a_1 + a_2x + a_3y \quad (B.13)$$

Evaluating Eq. (B.13) at the nodes of triangle No. 1 yields

$$\begin{aligned}p_1 &= a_1 + a_2x_1 + a_3y_1 \\p_2 &= a_1 + a_2x_2 + a_3y_2 \\p_5 &= a_1 + a_2x_5 + a_3y_5\end{aligned} \quad (B.14)$$

where p_1 , p_2 and p_5 are the unknown nodal coordinates. Coefficients a_1 , a_2 and a_3 can be expressed in terms of the nodal pressures and nodal coordinates by solving Eq. (B.14). Substituting the resulting expressions for the coefficients into (B.13), the pressure distribution within triangle No. 1 is found to be

$$p(x,y) = \{t\}^T \begin{Bmatrix} p_1 \\ p_2 \\ p_5 \end{Bmatrix} \quad (B.15)$$

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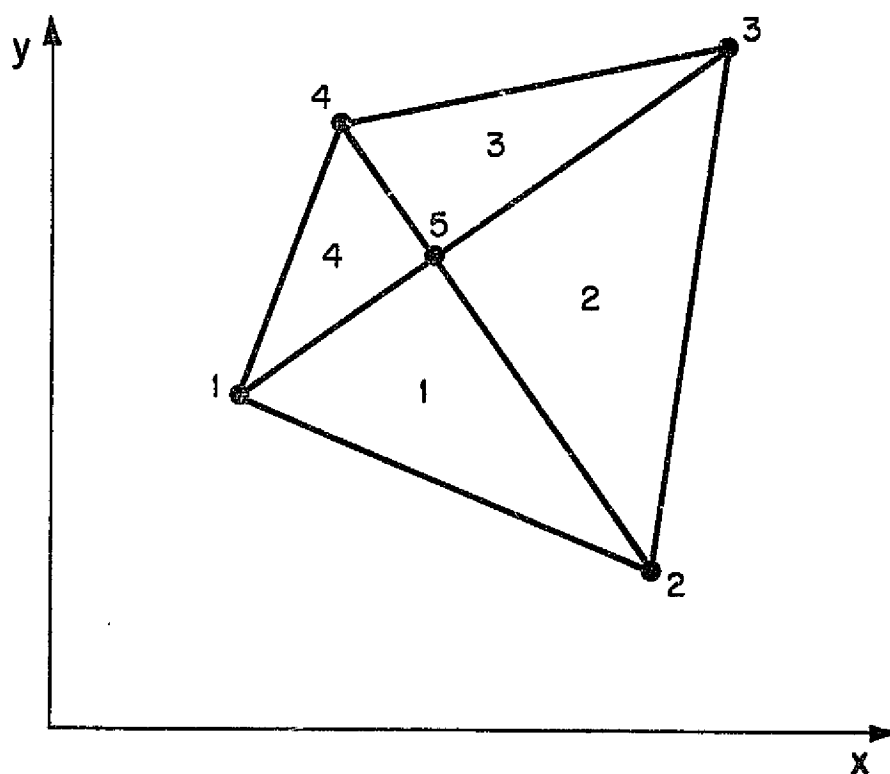


FIG. B-3 QUADRILATERAL ELEMENT

where

$$\{t\} = \begin{Bmatrix} t_1 \\ t_2 \\ t_3 \end{Bmatrix} = \frac{1}{D} \begin{Bmatrix} [(x_2 y_5 - x_5 y_2) + (y_2 - y_5)x + (x_5 - x_2)y] \\ [(x_5 y_1 - x_1 y_5) + (y_5 - y_1)x + (x_1 - x_5)y] \\ [(x_1 y_2 - x_2 y_1) + (y_1 - y_2)x + (x_2 - x_1)y] \end{Bmatrix} \quad (B.16)$$

$$D = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} \quad (B.17)$$

The first derivative of the pressure distribution within the triangular element is given by

$$\begin{Bmatrix} p_{,x} \\ p_{,y} \end{Bmatrix} = \begin{bmatrix} \frac{\partial t_1}{\partial x} & \frac{\partial t_2}{\partial x} & \frac{\partial t_3}{\partial x} \\ \frac{\partial t_1}{\partial y} & \frac{\partial t_2}{\partial y} & \frac{\partial t_3}{\partial y} \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \\ p_3 \end{Bmatrix} \quad (B.18)$$

In a similar manner, expressions for the pressure distributions and their derivatives in other triangles can be derived.

Let

$$\{p\} = \begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{Bmatrix} \quad (B.19)$$

Expressions of pressure and its derivatives for each triangle can be written as

$$p^m(x,y) = \{T^m\}^T \{p\} \quad (B.20)$$

$$\begin{Bmatrix} p^m_x \\ p^m_y \end{Bmatrix} = [R^m] \{p\} \quad (B.21)$$

where the superscript "m" denotes element number. The vector $\{T^m\}$ and matrix $[R^m]$ contain zero at places corresponding to nodes that do not belong to the element. Thus for element 1

$$\{T^1\} = \begin{Bmatrix} t_1 \\ t_2 \\ 0 \\ 0 \\ t_3 \end{Bmatrix} \quad (B.22)$$

and

$$[R^1] = \begin{bmatrix} \frac{\partial t_1}{\partial x} & \frac{\partial t_2}{\partial x} & 0 & 0 & \frac{\partial t_3}{\partial x} \\ \frac{\partial t_1}{\partial y} & \frac{\partial t_2}{\partial y} & 0 & 0 & \frac{\partial t_3}{\partial y} \end{bmatrix} \quad (B.23)$$

Substituting (B.20) and (B.21) into the generating functional, we have for the quadrilateral

$$\begin{aligned} \Phi\{p\} = & \sum_{m=1}^M \left\{ \int_A \left(\{p\}^T [R^m]^T \begin{bmatrix} h_m^3/12\eta & 0 \\ 0 & h_m^3/12\eta \end{bmatrix} [R^m] \right. \right. \\ & - h_m \langle U_x^m, U_y^m \rangle [R^m] + 2 h_m \{T^m\}^T \Big) dA \\ & \left. + 2 \int_{C_2} q_m \{T^m\}^T ds \right\} \{p\} \end{aligned}$$

where M is the total number of triangular elements.

Defining

$$[K] = \sum_{m=1}^M \int_A \left\{ [R^m]^T \begin{bmatrix} h_m^3/12\eta & 0 \\ 0 & h_m^3/12\eta \end{bmatrix} [R^m] \right\} dA \quad (B.24)$$

$$\langle V \rangle = \sum_{m=1}^M \int_A \left\{ h_m \langle U_x^m, U_y^m \rangle [R^m] \right\} dA \quad (B.25)$$

$$\langle H \rangle = \sum_{m=1}^M 2 \int_A (h_m \{T^m\}^T) dA \quad (B.26)$$

$$\langle Q \rangle = \sum_{m=1}^M 2 \int_{C_2} q_m \{T^m\}^T ds \quad (B.27)$$

where A is the area of a triangular element; C_2 the portion of the boundary where flow is prescribed. Thus

$$\phi\{p\} = \left[\{p\}^T [K] - \langle V \rangle + \langle H \rangle + \langle Q \rangle \right] \{p\}$$

The first variation of the functional with respect to independent variations in the nodal pressure, p_i yields

$$\delta\phi\{p\} = \sum_{i=1}^5 \frac{\partial\phi}{\partial p_i} \delta p_i = 0$$

Since δp_i is an independent variation

$$\frac{\partial\phi}{\partial p_i} = 2 [K] \{p\} - \langle V \rangle^T + \langle H \rangle^T + \langle Q \rangle^T = 0$$

$$2[K] \{p\} = \langle V \rangle^T - \langle H \rangle^T - \langle Q \rangle^T \quad (B.28)$$



Eq. (B.28) is a system of linear algebraic equations for the unknown nodal pressures of a composite quadrilateral. The same procedure can be used to derive the system of equations for an assemblage of quadrilateral elements. In this case, the column vector $\{p\}$ is expanded to include all nodal pressures. Thus,

$$\{p\} = \begin{Bmatrix} p_1 \\ p_2 \\ \cdot \\ \cdot \\ p_N \end{Bmatrix}$$

where N is the total number of nodes in the assemblage. Similarly, $\{T^m\}$ and $[R^m]$ are also expanded.

With these definitions Eq. (B.28) becomes valid for an assemblage of quadrilaterals. It should be noted that in Eq. (B.24) through (B.27) the summation is performed over the total number of triangles.

Boundary Conditions

Referring to Figure B-2, the boundary conditions are:

- cyclic conditions along sides AB and CD
- $P = P_a$ along AD
- $\left. \begin{array}{l} P = \text{constant} \\ \text{net flow} = 0 \end{array} \right\}$ along BC

The first two conditions can be prescribed in the manner outlined in [2]. The third condition, however, requires special attention since it is not one of the natural boundary conditions associated with the variational principle.

Since side BC is the only boundary where flow is prescribed, the generating functional (Eq. B.12) can be written as



$$\phi^v\{P\} = \frac{\phi}{r_2^4} \frac{h}{\omega^2} = \int_R \left[\frac{H^3}{12} \nabla P \cdot \nabla P - H \vec{U} \cdot \nabla P + 2\dot{H}P \right] dA$$

$$+ 2P' \int_{BC} \bar{q} \, ds$$

where P' is the unknown constant pressure along BC. It follows that the flow boundary condition is satisfied if the last term in the above equation is set to zero.

The remaining condition of constant pressure can be satisfied by assigning the same node number to all nodes on BC since there is a one-to-one correspondence between the nodes number and the nodal pressure.