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Abstract

The accurate in situ measurement of the weak magnetic fields in interplanetary space and near the moon and planets by satellites, has often been limited more by the spacecraft generated magnetic field than by the zero level stability of the magnetometer or the quantization uncertainty of the telemetry data readout system. A new method is proposed for obtaining accurate results even in the presence of a large and variable spacecraft field. The method uses simultaneous data from two magnetometers whose sensors are placed at different positions along a moderately long boom. The analysis of the data yields a continuous measure of the spacecraft field and the unknown field in space. The accuracy is determined by the validity of assumptions concerning the spacecraft field and the zero level drifts of the sensors. It is assumed that the external field to be measured is spatially uniform on the distance scale of the sensors' separation distance. This method can be used on both spin stabilized and fixed attitude spacecraft. Specific application to the future NASA-JPL Mariner Venus Mercury mission in 1973 is presented, with an estimated accuracy of ±0.5 to ±1.0 gamma using a 6 meter boom.

Introduction

For many years, the accuracy of measurement of the weak magnetic fields in interplanetary space and near the moon and planets has been frequently limited more by the magnetic field of the spacecraft, on which the magnetometer instrumentation was placed, than by the intrinsic zero level stability of the magnetometer or the quantization uncertainty of the telemetry data readout system. Ness (1970) has recently reviewed past magnetometer experiments on spacecraft and their operating characteristics and performance. Several series of spacecraft, including Explorer, IMP, Pioneer, OGO, Electron and certain COSMOS satellites, have carried special booms on which the magnetometer sensors were remotely placed at distances ranging from 1 to 8 meters from the main body of the spacecraft in order to reduce the contribution of the spacecraft field to the measured values. When combined with sufficiently tight constraints on the mechanical and electrical design and fabrication of the spacecraft and its subsystems, maximum values of the spacecraft field have been achieved which are less than 1 gamma at the sensor position.

However, the development of such magnetically clean spacecraft has increased total program costs as well as restricted the use of certain devices and materials which contain magnetic or magnetizable material. In addition, special attention to the use of self-compensation methods for power distribution cabling and solar array generation were required. Additional expensive testing of the magnetic properties of such spacecraft in various operational modes is also a requirement to certify the cleanliness of the unit after assembly. Fortunately, the same principles used to reduce the general level of electromagnetic interference on spacecraft also depend upon such self-compensation methods, and good engineeri g practices thus contribute to the overall magnetics restraint effort.

In the NASA-JPL Mariner series of planetary flyby missions to Venus in 1962 and 1967 and Mars in 1965, neither design nor fabrication constraints were employed and in the absence of a boom the residual spacecraft field at the sensor position was on the order of 10-100 gamma and variable during each mission. Various procedures were used to estimate the magnitude of the spacecraft field (and intrinsically include any variability of the zero level of the magnetometer) from inflight magnetic field data. On Mariner 2, Coleman (1965) used preflight test estimates of the spacecraft magnetic field and inflight roll maneuvers to determine the magnitude of the spacecraft field. In addition, the assumption of symmetries in the distribution of data sets for the interplanetary magnetic field averaged over a solar rotation period of 27 days was used to correct for the variable zero levels of the sensors.

There is no a priori reason, however, why the observed variations were entirely associated with the magnetometer and in fact may have been associated with a variable spacecraft field. (Note that on all spacecraft, the effective zero level of a magnetometer refers to the combined effects of a variable spacecraft field and a variable zero level). The effective zero levels on Mariner 2 were adjusted according to the theoretical Archimedean spiral angle expected for a steady-state solar wind using the onboard measurements of solar wind velocity.

On Mariners 4 and 5 similar roll manuevers were used to calibrate the effective zero levels transverse to the roll axis. In the case of Mariner 4 the third axis value was determined by forcing agreement between the measured and theoretically predicted near earth magnetic field assuming that the difference was only due to the spacecraft field (Coleman et al., 1966). More recently Davis and Smith (1968) have applied another method to inflight data again assuming certain symmetries in the interplanetary magnetic field. Rather than utilize the steady-state average, and require consistency with the spiral geometry to determine the spacecraft fields, their second method assumes that fluctuations of the field are such that on average the magnitude of the field is relatively constant while the field direction shows the most variability. In particular, the method determines the effective zero level of the magnetometer by choosing that value which minimizes, over many ambient field discontinuities, the sum of the squares of the field magnitude change.

While it is known that some such magnetic field discontinuities observed are in good agreement with classical MHD theory regarding their joint plasma-field behavior, it is also known that not all discontinuities preserve field magnitude. Since there was no preliminary selection of only that special subset of discontinuities that satisfy the field magnitude preservation constraint, there is no a priori reason to assume that the final results should be entirely correct. At present, no quantitative discussion of the results comparing the second method with the earlier technique exists, and thus it cannot be assumed that the effective zero level determined should be accurate for all time intervals of data chosen.

From the available results on Mariner 2 (Coleman, 1966) it is clear that the effective zero levels of the magnetometer were time variable. Therefore any method which must utilize symmetries in data distributions, or characteristics of fluctuations, must employ intervals sufficiently long that statistical stationarity of the assumed parameters is justified. The fact that use of such an assumption then shows a variable effective zero level, between successive time intervals selected, is an indication that a better method would be one which continuously permits an estimation of the spacecraft field independent of the characteristics of the unknown field to be measured in space. Even if the other methods of periodically determining the effective zero levels were satisfactory, the short time available for measurements during planetary flyby (on many planetary missions) renders them useless to deal with changes which occur then.

It is the purpose of this paper to discuss a method for performing magnetic field measurements on spacecraft with associated fields, which utilizes in-flight data from two magnetometers simultaneously to provide a continuous estimate of the spacecraft field and the unknown ambient field in space. The method uses two similar magnetometers, located on a moderate length boom, separated from each other by approximately half the boom length. The accuracy of the method is limited by:

1) The zero level drift of the sensors and,
2) The validity of certain assumptions concerning the spacecraft magnetic field, which will be discussed in greater detail later.

Different assumptions may be applied concerning removal of the spacecraft field. The assumptions used depend upon the relative behavior of the spacecraft field and the ambient magnetic field. The simplist assumption is that of approximating the spacecraft field by a centered magnetic dipole whose magnitude and direction vary with time. If roll maneuvers of the spacecraft are possible <u>in-flight</u>, or if the spacecraft itself is intrinsically spin-stabilized, then error source (1) can be eliminated and the validity of (2) established for those components perpendicular to the roll (or spin) axis.

The magnetic field of the spacecraft itself is analyzed in Section 2 and the mathematical basis for the new method is discussed in Section 3. The use of a dual magnetometer system is outlined in Section 4 and an analysis of the effect of errors in the assumed characteristics of the spacecraft field and zero level errors are presented in Section 5. Section 6 discusses the use of this dual magnetometer system with application to the future Mariner Venus Mercury 1973 mission.

2. Magnetic Field of Spacecraft

Due to the presence of magnetized material and electrical currents, all spacecraft possess a magnetic field which may be large enough to adversely affect the measurements performed at the position of the magnetometer sensor. If the magnetic field of the spacecraft varies slowly enough so that electromagnetic induction effects are negligible, then the magnetic field of the spacecraft can be derived uniquely from a scalar potential ψ which satisfies Laplace's equation. In spherical coordinates r, θ and ϕ

$$\psi(r,\theta,\varphi;t)=a \sum_{n=1}^{\infty} \sum_{m=o}^{n} \{[A_{nm}(t)\sin m\varphi + B_{nm}(t)\cos m\varphi] \cdot P_n^m(\theta)\left(\frac{a}{r}\right)^{n+1}\} \qquad (2.1)$$

for $r \geq a$,

where a is the distance of the furthest source from the spacecraft center and thus is a length characteristic of the spacecraft size. Nominally one may assume it to be approximately equal to the mean radius of the spacecraft exclusive of appendages such as solar arrays or booms.

The exact location of the coordinate origin is not important although, as shall be discussed in Section 4, its coincidence with the magnetic center of the spacecraft is desirable from the viewpoint of reduced errors. Note that the time variations of the spacecraft field are reflected in the coefficients $A_{nm}(t)$ and $B_{nm}(t)$ and that electromagnetic radiation and induction effects are neglected, quite a reasonable assumption in the present context.

The magnetic field of the spacecraft is then derivable as the gradient of the potential as $\vec{B}_{sc} = -\nabla\psi$. It is advantageous to choose the coordinate origin such that the two magnetometers lie on the same radial line from the origin, hence at the same angular coordinates ($\theta_1 = \theta_2$ and $\varphi_1 = \varphi_2$), and to consider the sensors' detector axes as parallel. If they are not parallel, then a simple matrix transformation can relate the two sets of measurements as though they were obtained from such a geometry.

The spacecraft magnetic field components are then given by the following equations for $r \geq a$:

$$B_r(r,\theta,\varphi;t) = +\sum_{n=1}^{\infty} \sum_{m=0}^{n} (n+1) \left(\frac{a}{r}\right)^{n+2} [A_{nm}(t)\cos m\varphi + B_{nm}(t) \sin m\varphi] P_n^m(\theta) \quad (2.2)$$

$$B_\theta(r,\theta,\varphi;t) = -\sum_{n=1}^{\infty} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+2} [A_{nm}(t)\cos m\varphi + B_{nm}(t)\sin m\varphi] \frac{d}{d\theta} P_n^m(\theta) \quad (2.3)$$

$$B_\varphi(r,\theta,\varphi;t) = \frac{1}{\sin\theta} \sum_{n=1}^{\infty} \sum_{m=0}^{n} m \left(\frac{a}{r}\right)^{n+2} [A_{nm}(t)\sin m\varphi - B_{nm}(t)\cos m\varphi] P_n^m(\theta) \quad (2.4)$$

One observes that these equations are of the form

$$B_i(r,\theta,\varphi;t) = \sum_{n=1}^{\infty} \left(\frac{1}{r}\right)^{n+2} f_{n,i} (\theta,\varphi; t) \quad (2.5)$$

where i is the i^{th} component and $f_{n,i}$ is a function of θ, φ and t for the n^{th} multipole moment and the i^{th} component.

The fields at each of the two sensors are given by

$$B_i(r_1,\theta_1,\varphi_1;t) = \sum_{n=1}^{\infty} \left(\frac{1}{r_1}\right)^{n+2} f_{n,i} (\theta_1,\varphi_1;t) \quad (2.6)$$

$$B_i(r_2,\theta_2,\varphi_2;t) = \sum_{n=1}^{\infty} \left(\frac{1}{r_2}\right)^{n+2} f_{n,i} (\theta_1,\varphi_1;t) \quad (2.7)$$

where $\theta_1 = \theta_2$ and $\varphi_1 = \varphi_2$. The spacecraft field at the outer magnetometer may be further reduced to

$$B_i(r_2,\theta_2,\varphi_2;t) = \sum_{n=1}^{\infty}\left(\frac{r_1}{r_2}\right)^{n+2}\left(\frac{1}{r_1}\right)^{n+2} f_{n,i}(\theta_1,\varphi_1;t) \tag{2.8}$$

With the introduction of a coupling matrix α_{ij}, this may be rewritten as:

$$B_i(r_2,\theta_2,\varphi_2;t) = \sum_{j=1}^{3}\alpha_{ij}\, B_j(r_1,\theta_1,\varphi_1;t) \tag{2.9}$$

$$\text{where}\quad \alpha_{ij} = \alpha_{ii}\delta_{ij} = \frac{\delta_{ij}\sum_{n=1}^{\infty}\left(\frac{r_1}{r_2}\right)^{n+2}\left(\frac{1}{r_1}\right)^{n+2} f_{n,i}(\theta_1,\varphi_1;t)}{\sum_{n=1}^{\infty}\left(\frac{1}{r_1}\right)^{n+2} f_{n,i}(\theta_1,\varphi_1;t)}$$

Thus $\underline{\underline{\alpha}}$ is a diagonal matrix in which the element α_{ii} is the coupling constant between the two magnetometers for the i^{th} component. In general, it is a quantity which indicates the effective multipole moment of the spacecraft, since it is a weighted average of the multipole moments.

For any pure multipole term, the three component values of α_{ii} are identical and equal to $(r_1/r_2)^{n+2}$. For a dipole field with inner magnetometer halfway between the spacecraft center and the outer magnetometer α_{ii} = 0.125. In general the three diagonal elements may not be equal since the term $(r_1/r_2)^{n+2}$ is weighted by different functions. However, in most cases, it is expected that they will be nearly equal. It is important to note that the α_{ii} values may vary with time as the spacecraft field changes, since this may affect the weighting factors; however, substantial variation is not expected. Both the component and temporal variations of $\underline{\underline{\alpha}}$ may be ascertained from inflight observations of spacecraft field changes and periodic roll maneuvers.

3. Mathematical Basis

Assume for any given instant of time that the magnetic field observed at each magnetometer is represented by

$$\vec{B}_{obs}(r_i) = \vec{B}_{am} + \vec{B}_{zo}(r_i) + \vec{B}_{sc}(r_i) \qquad (3.1)$$

where $\vec{B}_{am}$ is the ambient field to be measured,

$\vec{B}_{zo}(r_i)$ is the absolute error at position r_i of the field due to zero offset (including any possible quantization error),

$\vec{B}_{sc}(r_i)$ is the spacecraft field at position r_i,

and r_1, r_2 are the inner and outer positions of the magnetometers, respectively.

Now, under all circumstances, the magnetic field of the spacecraft at the two positions r_1 and r_2 can be related by a coupling matrix α_{ij} as

$$\vec{B}_{sc}(r_2)_i = \sum_{j=1}^{3} \alpha_{ij} [\vec{B}_{sc}(r_1)]_j \qquad (3.2)$$

In order to determine the ambient field, it is necessary to determine $\vec{B}_{sc}(r_i)$. We define the estimated $B_{sc}(r_i)$, which incorporates $\vec{B}_{zo}(r_i)$, from the observed fields in the following manner:

$$\vec{B}^{est}_{sc}(r_2) - \vec{B}^{est}_{sc}(r_1) = \vec{B}_{obs}(r_2) - \vec{B}_{obs}(r_1) \qquad (3.3)$$

Using (3.2) we obtain

$$\vec{B}^{est}_{sc}(r_1) = [\underline{\underline{1}} - \underline{\underline{\alpha}}]^{-1}[\vec{B}_{obs}(r_1) - \vec{B}_{obs}(r_2)] \qquad (3.4)$$

$$\vec{B}^{est}_{sc}(r_2) = \underline{\underline{\alpha}}\,[\underline{\underline{1}} - \underline{\underline{\alpha}}]^{-1}[\vec{B}_{obs}(r_1) - \vec{B}_{obs}(r_2)]$$

Now the ambient field is estimated from either magnetometer from

$$\vec{B}_{am}^{est}(r_i) = \vec{B}_{obs}(r_i) - \vec{B}_{sc}^{est}(r_i) \tag{3.5}$$

Substituting (3.4) into (3.5) yields, for <u>either</u> magnetometer, the same result,

$$\vec{B}_{am}^{est} = [\underline{\underline{1}} - \underline{\underline{\alpha}}]^{-1}[\vec{B}_{obs}(r_2) - \underline{\underline{\alpha}}\,\vec{B}_{obs}(r_1)] \tag{3.6}$$

From (3.4) we see that if the difference between $\vec{B}_{obs}(r_1)$ and $\vec{B}_{obs}(r_2)$ is constant, there is no spacecraft field variation. If this quantity shows variations, then the spacecraft field or zero levels are varying.

It was shown in Section 2 that the matrix α_{ij} may be represented by a scalar times the unit matrix for any simple magnetic multipole spacecraft field. For a more complex spacecraft field distribution (i.e., an arbitrary superposition of multipoles) α_{ij} will in general be a diagonal matrix with each of the diagonal elements reflecting the influence of the effective multipole moment for that component.

For diagonal matrices, the inverse is particularly simple; the inverse of the matrix $\underline{\underline{\alpha}}$ whose diagonal elements are α_{ii} is the matrix $[\underline{\underline{\alpha}}]^{-1}$ whose diagonal elements are $\frac{1}{\alpha_{ii}}$. This allows equations 3.4 and 3.6 to be reduced simply to the following equations governing the ith field components.

$$B_{sc}^{est}(r_1)_i = \frac{1}{1-\alpha_{ii}}\,[\vec{B}_{obs}(r_1) - \vec{B}_{obs}(r_2)]_i \tag{3.7}$$

$$B_{sc}^{est}(r_2)_i = \frac{\alpha_{ii}}{1-\alpha_{ii}}\,[\vec{B}_{obs}(r_1) - \vec{B}_{obs}(r_2)]_i$$

$$B_{am}^{est} = \left[\frac{B_{obs}(r_2)_i - \alpha_{ii}B_{obs}(r_1)_i}{1 - \alpha_{ii}}\right] \tag{3.8}$$

4. Use of Two Magnetometers.

A dual magnetometer system is especially valuable on those missions which involve a single pass of the spacecraft past an interesting object, or region of space, simultaneous with significant operational mode changes of the spacecraft with possible associated changes in the spacecraft field.

It does not seem prudent to require nor possible to achieve an accurate magnetic map of the spacecraft magnetic field prelaunch in every conceivable operational mode, as demanded by a single magnetometer experiment. Furthermore, a nearly continuous series of mode changes during the encounter phase along with a complex series of variations in the ambient field make it mandatory that some type of coincidence technique be employed on the spacecraft to uniquely identify variations of scientific interest. The two magnetometer system will permit estimation, with high confidence, of an observed event as being either a spacecraft (or instrument) associated perturbation or an ambient field phenomenon.

4.1 Unique Identification of Events

The observation of a significant event may appear as an abrupt change in field magnitude or direction, a sinusoidal wave phenomenon with associated field component variations, or any general combination of these time changes. A dual magnetometer system may be employed to distinguish between the two types of magnetic field events as follows. Figure 1 shows the ratio of the temporal changes of the two magnetometer observations as a function of the ratio of the two magnetometer positions. If the observed field variation is due to a spacecraft (or instrument) perturbation, then the outer magnetometer will measure a variation in each of its field components that is substantially less than that of the inner magnetometer. This is true for a spacecraft centered dipole field approximation and a radial distance of the inner magnetometer r_1 less than 0.8 times the outer magnetometer distance. For the ratio of $r_1/r_2 = 0.5$ the ratio of the observed field changes is 0.125 for the dipole approximation.

If the observed field variation is due to an ambient field change, however, then the ratio of the two magnetometer variations will be unity in each component. Thus it becomes a straightforward task to dissociate spacecraft perturbations from real events by taking the ratio of the changes in the two magnetometers' observations. Once one identifies an event as being spacecraft associated, its removal is straightforward and the ambient field data results in a form limited by errors in the coupling coefficients α_{ii} and by the zero level uncertainty (see Section 5). Dual magnetometer identifications, it should be noted, might also be useful to other experiments should they have trouble distinguishing their "events" from spacecraft related effects as monitored by the spacecraft magnetic field.

4.2 Determination of Coupling Coefficients α_{ii}.

The simplest method of determining α_{ii} is to assume that the spacecraft field is represented by a single multipole term, and with a knowledge of the relative position of the two sensors calculate $\alpha_{ii} = (r_1/r_2)^{n+2}$. The most plausible estimate is that the spacecraft field is dipolar, i.e., n = 1. The second method, and the one that is to be used to check the validity of the first, is to estimate α_{ii} from in-flight data, obtained during roll maneuvers. Assuming that the spacecraft field is stationary at the two sensors during roll maneuvers and that the ambient field does not vary, then by definition

$$\alpha_{ii}^{est} = \frac{[\vec{B}_{obs}(r_1) - \vec{B}_{am}]_i}{[\vec{B}_{obs}(r_2) - \vec{B}_{am}]_i} \qquad (4.1)$$

During the roll (or spin) maneuvers, an accurate determination of the ambient field components transverse to the roll (or spin) axis is possible, independent of spacecraft field and zero level drift (Ness 1970). Thus α_{ii} is obtained with an error depending on the zero level errors. While this might appear to offer an opportunity for significant amplification of errors in the final determination of $\vec{B}_{am}$, this is not true, as will be shown in Section 5.0.

The third method for determining α_{ii} in-flight is to assume that if the <u>difference</u> in the observed field does change significantly at a given moment, all of the variation in the magnetometers is due to a spacecraft field change. The validity of this assumption relates to the probability of simultaneous variations occurring in the spacecraft magnetic field, the zero level of the instrument, the ambient magnetic field and the relative size of any such changes.

In the most general case, a change in the spacecraft field during a short time interval Δt can also be accompanied by a change in the coupling coefficient α_{ii} for each magnetometer axis. From equation 3.2

$$[\vec{B}_{sc}(r_2,t)]_i = \alpha_{ii}(t)\ [\vec{B}_{sc}(r_1,t)]_i \qquad (4.2)$$

and at time t + Δt a similar expression:

$$[\vec{B}_{sc}(r_2, t+\Delta t)]_i = \alpha_{ii}(t+\Delta t) \quad [\vec{B}_{sc}(r_1, t+\Delta t)]_i \tag{4.3}$$

Using estimates of the spacecraft field and (3.5), or by definition, (4.3) can be put into the form

$$\alpha_{ii}^{est}(t+\Delta t) = \frac{[\vec{B}_{obs}(r_2, t+\Delta t)]_i - [\vec{B}_{am}^{est}(t+\Delta t)]_i}{[\vec{B}_{obs}(r_1, t+\Delta t]_i - [\vec{B}_{am}^{est}(t+\Delta t)]_i} \tag{4.4}$$

If it can be assumed that there is no change in the ambient field during the period Δt, this may be written

$$\alpha_{ii}^{est}(t+\Delta t) = \frac{[\vec{B}_{obs}(r_2, t+\Delta t) - \vec{B}_{am}^{est}(t)]_i}{[\vec{B}_{obs}(r_1, t+\Delta t) - \vec{B}_{am}^{est}(t)]_i} \tag{4.5}$$

Thus an estimate of the new value of α_{ii} is determined from the magnetometer observations after the spacecraft field change and the estimated ambient field before the change.

4.3 Spacecraft Field

In the event that the spacecraft field changes are small relative to the ambient field changes, and preflight magnetic maps of the spacecraft do not exist, then it is necessary to depend upon the centered-dipole approximation of the spacecraft field for the most reliable estimate of α_{ii}. In this approximation, all the non-dipole moments of the spacecraft are neglected and it is assumed that $\alpha_{ii} = (r_1/r_2)^3$.

The exact nature of the higher order moments in the spacecraft field representation is not known satisfactorily for real spacecraft and each one is expected to have a different relative magnitude and orientation from spacecraft to spacecraft. For conservative estimates, where the maximum error is required, it seems reasonable to assume that the higher order moments are of magnitude equal to the dipole moment. Under the condition that all moments add so as to maximize their field contribution at the sensor position then the magnetic dipole moment determined from the observed fields will be in error by a factor determined from

$$\vec{M}_{sc}[1 + \frac{a}{r} + (\frac{a}{r})^2 + \cdots] = \vec{M}_{sc} \sum_{n=0}^{\infty} (\frac{a}{r})^n \quad (4.6)$$

This can be summed to yield:

$$\vec{M}_{apparent} = \vec{M}_{sc} \cdot [\frac{1}{1 - a/r}] \quad (4.7)$$

The error will be directly reflected in the spacecraft field predicted at the two positions in the same ratio as equation 4.7.

Depending upon the ratio of the two distances, r_1/r_2, an error in the choice of the origin may or may not be significantly reflected into the predicted spacecraft field at the two locations. That the error depends upon the change in the ratio, and not on the change in the individual values of r_1, is beneficial since both r_1 and r_2 will change in the same sense as the origin is changed (due to the sensors being positioned along the same radial line). The change in r_1 is very sensitive to the direction in which the origin is shifted, by an amount d; it is maximum parallel to the boom axis and a minimum when transverse to the boom axis.

A number of factors enter into the actual selection of specific values of the ratio r_1/r_2. They depend upon the spacecraft geometry or specific value of a, the boom length ($\cong r_2$), and the estimated value of d for the spacecraft under consideration. There is also the interaction with the spacecraft structure and the impact on its dynamics for a boom in both the stowed and erected configuration as well as the more obvious matter of weight for the boom and the boom cables.

From a consideration of the use of a dual magnetometer system it is desirable that the spacecraft field nominally be about 5-10 times larger at the remote sensor which implies that r_1/r_2 be approximately 0.45-0.65. The quality of a dipole approximation is expected to increase as the ratios a/r_1 and d/r_1 are reduced.

5.0 Error Analyses

There are two possible sources of error in the two magnetometer system:

a) Coupling coefficient errors and

b) Zero level errors

5.1 Coupling Coefficient Errors

If the improper values of α_{ii} are used, they directly affect the estimated ambient field. The field component error resulting from an error $\Delta\alpha_{ii}$ in α_{ii} is given by differentiating 3.8 to obtain for the i^{th} component

$$\text{Error}_i = \frac{\Delta\alpha_{ii}}{[1 - \alpha_{ii}]^2} [\vec{B}_{obs}(r_2) - \vec{B}_{obs}(r_1)]_i \tag{5.1}$$

Substituting from 3.1 and 3.2, the error is found to be

$$\begin{aligned}\text{Error}_i = & \left[\frac{1}{\alpha_{ii}-1}\right] \left[\frac{\Delta\alpha_{ii}}{\alpha_{ii}}\right] [\vec{B}_{sc}(r_2)]_i \\ & + \frac{\Delta\alpha_{ii}}{(1-\alpha_{ii})^2} [\vec{B}_{zo}(r_2) - \vec{B}_{zo}(r_1)]\end{aligned} \tag{5.2}$$

Here the relative importance of the spacecraft field and the zero level errors are seen in the context of errors in the coupling coefficient.

The maximum error occurs when $\vec{B}_{zo}(r_1) = -\vec{B}_{zo}(r_2)$ and yields

$$\begin{aligned}\text{Error}_i = & \left[\frac{1}{\alpha_{ii}-1} \frac{\Delta\alpha_{ii}}{\alpha_{ii}}\right] [\vec{B}_{sc}(r_2)]_i \\ & + \left[\frac{2\Delta\alpha_{ii}}{(1-\alpha_{ii})^2}\right] [\vec{B}_{zo}(r_2)]_i\end{aligned} \tag{5.3}$$

The terms involving $\Delta\alpha_{ii}$ represent amplification factors. They are tabulated in Table I, assuming that $\Delta\alpha_{ii} = r^3 - r^4$ where $r = (r_1/r_2)$. This corresponds to assuming a dipole representation for the spacecraft field when it is in fact quadrupole (or vice versa). It is seen that the amplification factors are less than unity for $r \geq 0.55$, which means in fact that the errors are less than they would be with the use of a single magnetometer. Thus there will always be an improvement in the accuracy of the measurements by use of the two magnetometer method.

5.2 Zero Level Errors.

In addition to coupling coefficient errors, the magnetometers themselves may introduce errors due to zero level drifts. The effect of errors in the zero levels of the magnetometers will lead to errors in the estimated spacecraft field and thence to errors in the estimated ambient field. These errors can be derived by substituting from equations (3.1) and (3.2) into (3.8) to obtain for the i^{th} component

$$[\vec{B}_{am}^{est}]_i = [\vec{B}_{am}]_i + \frac{1}{(1-\alpha_{ii})} [\vec{B}_{zo}(r_2)]_i - [\frac{\alpha_{ii}}{1-\alpha_{ii}}] [\vec{B}_{zo}(r_1)]_i \qquad (5.4)$$

Here it is seen that the error in the estimated ambient field is weighted less heavily, by α_{ii}, for the inner magnetometer at position r_1 than the outer one at r_2. The maximum error occurs under worst case conditions, when

$$\vec{B}_{zo}(r_1) = -\vec{B}_{zo}(r_2) \qquad (5.5)$$

The error is then given by

$$\text{Error}_i = + [\frac{1+\alpha_{ii}}{1-\alpha_{ii}}] [B_{zo}(r_2)]_i \qquad (5.6)$$

The magnitude of the coefficient $\frac{1+\alpha_{11}}{1-\alpha_{11}}$ is of interest for it is a measure of the amplification of the zero level error. The variation of this error term as a function of r is also given in Table I for nominal values of r_1/r_2. It is seen that the amplification is always greater than unity, although by less than a factor of 2, for $0.45 \le r \le 0.65$. However, it is felt that this is an acceptable situation since the spacecraft field, which is expected to be the principal error source, is correspondingly reduced.

It should be noted that if a "flipper" mechanism is included at the sensor to physically invert the sensors while in-flight, then the zero level can be determined accurately. This eliminates all errors associated with zero level drift.

Error levels of the sensors are often stable within ±0.50 gamma from preflight to post-launch operation. Such stability has been demonstrated inflight in the Explorer 33, 34, 35 and 41 and the Pioneer 6, 7 and 8 spacecraft. Over a period of 40 weeks, the average two week drift on Explorer 33 was 0.16 gamma, while the total drift varied between ±0.5 gamma over this interval. Thus even with an amplification factor >1, relatively frequent updating of zero levels will insure that $|\vec{B}_{zo}|$ is always small and hence by equation 5.6 that zero level errors are always within acceptable limits.

6. Applications to Mariner-Venus-Mercury 1973

Specific application to the future MVM-73 spacecraft shall be made in the following paragraphs. Although the final design of the boom system is not complete, approximately realizable distances for r_1 shall be used. In preliminary studies of the spacecraft magnetic field based upon past experience with similar Mariner spacecraft, it has been estimated that the maximum spacecraft field shall be 12.5 gammas at a distance of 12 feet (NASA Proposal Briefing Material). Assuming this to be a measure of the dipole moment of the spacecraft yields a value of 3.05×10^3 Gauss-cm^3 for the case where the maximum field is specified to be the radial component. However, in order to be conservative, it will be assumed that the dipole moment is twice this value or 6.1×10^3 Gauss-cm^3, which means that the maximum field was the azimuthal component.

The spacecraft main structure is an eight-sided truncated cone of approximately 50 cm height and diameter 150 cm. This implies a characteristic scale length, a, of approximately 50 cm, the mean radius of the structure (independent of the large separated solar arrays). It is proposed to place the magnetometer sensors on a boom such that they are approximately 300 and 600 cm from the spacecraft structure. Until the exact structural configuration is known, this suggests using the values of 350 and 650 cm., respectively for r_1 and r_2. This yields a ratio of r_1/r_2 of 0.54 and a ratio of spacecraft fields at the two positions of 6.5 to one.

The two solar arrays are assumed to lie transverse to the boom axis with their geometrical centers approximately 200 cm from the spacecraft geometrical center. Thus the origin shifts considered to be maximum will be 200 cm transverse and ±50 cm parallel to the boom axis. Note that the ratios a/r_2 and d/r_2 are less than 0.2 under all circumstances. Table II gives the computed results for the errors to be expected under these conditions for various combinations of origin offset (all values in gammas). It is seen that under almost all conditions the maximum error at the outboard sensor position, r_2, is less than 0.5 gamma. Only when the equivalent dipole is displaced by 50 cm towards the sensors does the error exceed the nominally desired limit of 0.5 gamma.

Thus it appears certain that the two magnetometer method will work successfully on MVM-73 without special procedures to clean up the spacecraft magnetically. However, it remains for inflight data to determine this hoped for result.

7. Conclusions

A two magnetometer system allows the separation of the observed magnetic field at the two sensors into an estimated spacecraft magnetic field and the ambient magnetic field. The dual system, using coincident techniques with simultaneous observations, can uniquely identify transient events as being either associated with the spacecraft (or magnetometer instrumentation) or an ambient magnetic field change. The mathematical basis for the method is founded upon the existence of a coupling matrix between magnetic field observations at the two sensors. Either a theoretical assumption of the multipole representation of the spacecraft magnetic field or inflight experimentally determined values of the coupling coefficients will allow the spacecraft magnetic field to be removed. There is amplification of the zero level uncertainty in the two magnetometers, under certain conditions. The overall absolute accuracy of the method is expected to be on the order of $\pm 1/2$ to ± 1 γ for a typical spacecraft mission, such as Mariner Venus Mercury 1973. It is anticipated that the dual magnetometer method has sufficient generality to be adopted for other spacecraft missions, especially those that probe the solar system at heliocentric distances greater than 1 AU.

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Discussions of the HEOS-1 magnetometer system designed by Dr. P. Hedgecock of Imperial College are appreciated. This system used a triaxial sensor and a mono-axial sensor with its axis parallel to the spin axis for providing an in-flight check on the constancy of the difference field parallel to the spin axis to verify the magnetic cleanliness of the spacecraft.

TABLE I

Assumed Field / Real Field r	Coupling Coefficient Errors: S/C Field Quad. / Dipole	Coupling Coefficient Errors: S/C Field Dipole / Quad.	Coupling Coefficient Errors: Zero Level Quad. / Dipole	Coupling Coefficient Errors: Zero Level Dipole / Quad.	Zero Level Errors: Quad.	Zero Level Errors: Dipole
0.45	-0.605	1.274	0.120	-0.112	-1.08	-1.20
0.50	-0.571	1.066	0.162	-0.150	-1.13	-1.29
0.55	-0.539	0.900	0.214	-0.198	-1.20	-1.40
0.60	-0.510	0.765	0.280	-0.262	-1.30	-1.55
0.65	-0.482	0.655	0.364	-0.346	-1.43	-1.76
0.70	-0.456	0.563	0.476	-0.468	-1.63	-2.04

Amplification Factors under worst case

for $\alpha_{ii} = r^3$ (Dipole) and $\alpha_{ii} = r^4$ (Quadrupole). See text

TABLE II

Offset Distance=	d = 0	d = -50 (Parallel)	d = +50 (Parallel)	d = 100 (Transverse)	d = 200 (Transverse)	Position
Nominal Field of SC	14.2	22.6	9.5	12.7	9.3	r_1
	2.2	2.8	1.8	2.1	1.9	r_2
Difference Field	12.0	19.8	7.7	10.6	7.4	
Predicted Field of SC	14.2	23.8	9.1	12.6	8.8	r_1
	2.2	4.0	1.4	2.0	1.4	r_2
Origin Errors	0	+1.2	-0.4	-0.1	-0.5	r_1
	0	+1.2	-0.4	-0.1	-0.5	r_2
Higher Moment Errors (±)	1.5	2.4	0.9	1.3	0.9	r_1
	0.2	0.4	0.1	0.2	0.1	r_2

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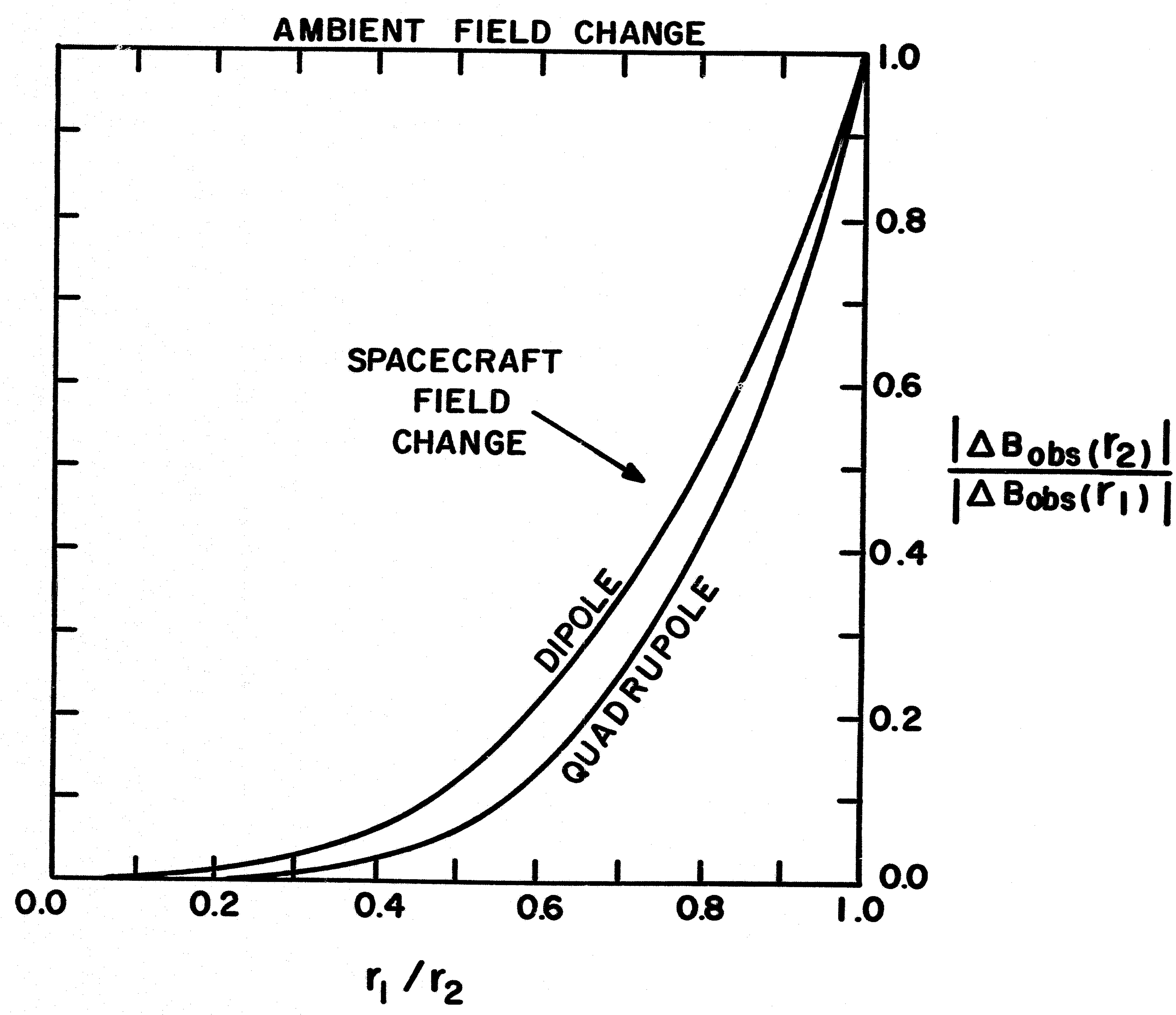
Figure Caption

1. Ratio of the changes in the observations of the two magnetometers as a function of the ratio of their distances from the spacecraft center. As can be seen, a substantial difference occurs between the magnetometer observations for an ambient field change relative to a spacecraft field change. The two curves shown, labeled spacecraft field change, represent the expected variation in the idealized cases of a centered dipole or a centered quadrupole. Inflight observations and ground testing will allow the spacecraft field to be more accurately determined.


AMBIENT FIELD CHANGE
SPACECRAFT FIELD CHANGE
DIPOLE
QUADRUPOLE
0.0
0.2
0.4
0.6
0.8
1.0
r_1 / r_2
$\frac{|\Delta B_{obs}(r_2)|}{|\Delta B_{obs}(r_1)|}$