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ANALYSIS OF LATITUDE OBSERVATIONS FOR DETECTION OF CRUSTAL MOVEMENTS

by

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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and it is under the technical direction of Jerome D. Rosenberg, Project Manager, Geodetic Satellites Program, NASA Headquarters, Washington, D.C. The contract is administered by the Office of University Affairs, NASA, Washington, D.C. 20546.

An earlier version of this report was submitted to the Graduate School of The Ohio State University as partial fulfillment of the requirements for the Master of Science degree. It contained essentially the first three chapters of this report.

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CHAPTER I

INTRODUCTION

The phenomenon of variation of the astronomically determined latitude at a place is well known. Astronomic latitude is observed at the following five stations of the International Polar Motion Service (IPMS) frequently and in an uninterrupted program.

Station	Latitude	Longitude
Mizusawa, Japan	39°08'03".602	141°07'51"
Kitab, USSR	39 08 01 .850	66 52 51
Carloforte, Italy	39 08 08 .941	8 18 44
Gaithersburg, Maryland	39 08 13 .202	- 77 11 57
Ukiah, California	39 08 12 .096	-123 12 35

The primary role of these stations is to monitor the terrestrial position of the earth's instantaneous rotation axis. For this purpose the stations are located at nearly the same latitude and are well distributed in longitude so that the same stars can be observed and the derived coordinates of the pole can be freed of errors in star coordinates.

The data published by the IPMS (called the International Latitude Serviee prior to 1962) provides a source for investigations into all aspects of the phenomenon of variation of latitude.

The purpose of this thesis is to analyze the published data of a latitude station to find evidence, if any, of sudden short-term changes in latitude at a station, and to find to what extent these are compatible with the geophysical phenomenon of earthquakes. The investigation is confined to the latitude station Ukiah. Data for the years 1962-66 has been taken up for analysis in detecting sudden changes in latitude. This is reported in Chapter III. Chapter II gives a general appreciation of the problem and a historical background.

A few preliminary steps were also taken to analyze the published data for the years 1922-64, to study the secular variation in latitude and its compatibility with the geophysical phenomenon of continental drift. These have been described in Chapter IV.

CHAPTER II

GENERAL DISCUSSION

2.1 Latitude Variation

Astronomic latitude observation at a station at any epoch yields (on computation) the complement of the angular distance between the local vertical and the instantaneous rotation axis of the earth.

Apart from the systematic errors in the coordinates of the star pairs observed and the random errors of observations which are corrected for systematic errors, the variation in the values of the latitude at a station as reduced from daily observations could be ascribed mainly to

(a) the relative movement of the instantaneous rotation axis of the earth with respect to the earth's crust, termed as the polar motion

(b) local crustal displacement at the station position with respect to the earth's crust

(c) changes in the direction of the vertical at the station.

It is difficult to isolate the different factors. To date the variation of latitude has been explained partly by Chandler's theory according to which the motion of the pole is the resultant of two components. One is the counterclockwise revolution of the true pole around the principal moment of inertia axis as viewed from north with a period of about 1.2 years, and the other is a revolution in the same direction with an annual period [Chandler, 1891, p. 65; 1892, p. 97]. The IPMS determination of the pole positions also indicates a displacement of $0''.003$ to $0''.006$ per year in the direction of about 285° longitude [Mueller, 1969, p. 82]. We thus generally can contend that the variations have cyclic components and secular components. Considering local nonpolar variation of latitude, if physical phenomena like earthquakes cause a sudden crustal displacement at the station position this is likely to give rise to a sudden change (or jump) in latitude variation considered as a function of time. A physical phenomenon like continental drift is likely to cause a secular change in the values of latitude.

This investigation is confined to one station, Ukiah, and aims at finding evidence of such jumps if any and to see if these phenomena are compatible with the physical phenomena of earthquakes. The effect of such sudden changes on latitude variation considered as a function of time depends also on the duration of the sudden changes. Only short-period changes of the duration of about 30 days have been investigated.

The published data gives the values of latitude deduced from the individual star pairs observed on the specified calendar day concerned. The epochs of observations to individual star pairs are not available. It was therefore difficult to find the epoch to which the mean of the observations of every night refer. It has been assumed that the daily means of the observed latitude values refer to 0.0 hrs UT for the calendar day concerned. There are some days for which no data is available. Presumably no observations could be carried out on these nights due to meteorological conditions.

In the investigation for sudden changes the time argument in the functional relationship between latitude variation and time has been taken as the time interval in the number of mean solar days which have elapsed since 1962 January $0^{d}.00$ UT and data for the period from Besselian year 1962.00 to 1967.00 has been considered. For the investigation of secular changes the time interval is reckoned in terms of the number of mean solar days that have elapsed since 1922 January $0^{d}.00$ UT and data for the period Besselian year 1922.70 to 1964.70 has been considered.

The values of latitude have been coded by subtracting $39^{\circ}08'10''.0$ from the published values. The arithmetic mean of latitude values, deduced from the observations of the star pairs on a particular night, has been given the number of star pairs observed as the weight of the observation. This involved the assumption that there is no change in the value of latitude between observations on the same night.

Since the data of only one station has been taken up for analysis, the data is not freed of the errors in star coordinates. The errors in the declinations of the stars observed directly affect the latitude deduced (see Appendix B for method of reduction). The IPMS publishes declination corrections deduced from the observations at all the five stations themselves and are available for the years 1962 onward. These have not been applied for reasons explained in section 3.1, but their effect has been taken into account while arriving at the conclusions.

2.2 Historical Background

Several investigators have attempted in the recent past to analyze the cyclic and secular components of the latitude variation and also to discuss whether the existence of local variation in latitude which is not common to all the IPMS stations is possible or not. The annual component has been explained by the continuous redistribution of mass in meteorological and geophysical processes [Mueller, 1969, p. 80].

Mansinha and Smylie [1969, p. 4731; September 1968, p. 1127; December 1968, p. 7661] have hypothesized that earthquakes excite the Chandler wobble, the cyclic variation of 1.2 year period.

Markowitz [1967, p. 25] has analyzed concurrent latitude observations of the International Latitude Service for 66 years and has concluded that the mean pole has a secular motion which consists of a progressive component of about $0''.0035$/year (10 cm/yr) along the meridian 65° W and a librational component (oscillation) of 24-year period along the meridian 122° W (or 58° E).

From the analysis of the notable increases in the residual latitudes of the five stations of the International Latitude Service, Yumi and Wako [1967, p. 33] have derived local drifts of $-\ 0''.00156$/year for Mizusawa and of $+\ 0''.00105$/year for Ukiah.

On analysis of the data of the ILS stations for the period from 1933.0 to 1966.0, Okuda [1968, p. 231] has obtained results which show marked local variation of latitude with period of approximately 19 years with the same phase for Mizusawa and Gaithersburg and the opposite phase for Ukiah and Kitab.

On analysis of the residual latitudes at the IPMS stations during the years 1962 to 1967, Yumi, Ishii, and Sato [1968, p. 161] have obtained a local trend in latitude variation for each station, other than by the polar motion. They were found most likely to depend on terms which might rather be attributed to an assumed gradual deformation of the earth or a variation of geopotential surface than to the astronomical origins.

The whole aspect of local nonpolar variation of latitude is incompletely understood at present and requires further investigation.

CHAPTER III

EXPERIMENTATION FOR THE INVESTIGATION FOR BREAKS

3.1 Aim and Approach

The aim of the investigation was to identify existence of sudden short-period changes in the latitude variation taken as a function of time and to see if the occurrence of such sudden changes has a relation with the occurrrence of earthquakes. Short-period changes have been assumed as changes prevalent for a duration of about thirty days.

The daily means of observations plotted against the time argument showed frequent fluctuations of the order of 0".7 on an average. To smooth out the fluctuations and to get an idea of the latitude variation taken as a function of time, a plot of moving weighted means of ten consecutive values of daily means against time was obtained. This method is similar to the method employed in the statistical methods in economics [Brown, 1959, p. 12; Fletcher and Clarke, 1964, p. 91]. From this plot it was difficult to identify any breaks but some observations could be made regarding the nature of the function. In obtaining moving weighted means, weighted means of groups of ten consecutive values of daily means were taken in a sliding step of one day. Figs. 3.1a - 3.1d show these plots taken in four convenient overlapping sections covering the whole period 1962.00 to 1967.00. The four plots are on slightly different scales. The plots still showed fluctuations (vibrations), but judging from the lowest values reached periods of wavelengths of prominence (taken as periods between two consecutive trough points) were discernible as follows:

Minimum Value	Epoch Corresponding to Minimum Value	Period of Wave Length
1"528	Day 52.80	
1.556	450.80	Day 398.00
1.688	653.60	202.80
1.219	1080.90	427.30
1.008	1419.50	338.60

For identification of breaks simulation technique was adopted and the problem was approached as follows. Existing data for a period of 251 days reasonably expected to be free of sudden changes (breaks) was taken and a short-period break for 30 days was simulated by changing the data by a constant amount during the short period taken near the middle of the 251-day period. The effect of this simulation on the residuals of a polynomial fit was studied. This study was utilized to see if the data over the period 1962-66 showed any similar behavior of residuals against polynomial fits as shown in the case of the simulated break.

In its functional relationship latitude variation behaves as a dependent variable and time, a progressive independent argument. A polynomial approximates the true function over limited ranges of the variables involved [Draper and Smith, 1968, p. 2]. Since our purpose is to locate sudden changes by study of residuals and not to find the exact mathematical model fitting the data, polynomial fitting was adopted.

The daily observed values of latitude at a station as published by IPMS normally require to be corrected for the declination corrections of the stars observed (see Appendix B for details of how IPMS computes this correction). However, these declination corrections are based on the monthly mean of the values of latitude obtained from every star pair which involves the assumption that the variation of latitude during the course of one month is

Fig. 3.1a. Plot of Moving Weighted Means of Observed Values, Set I

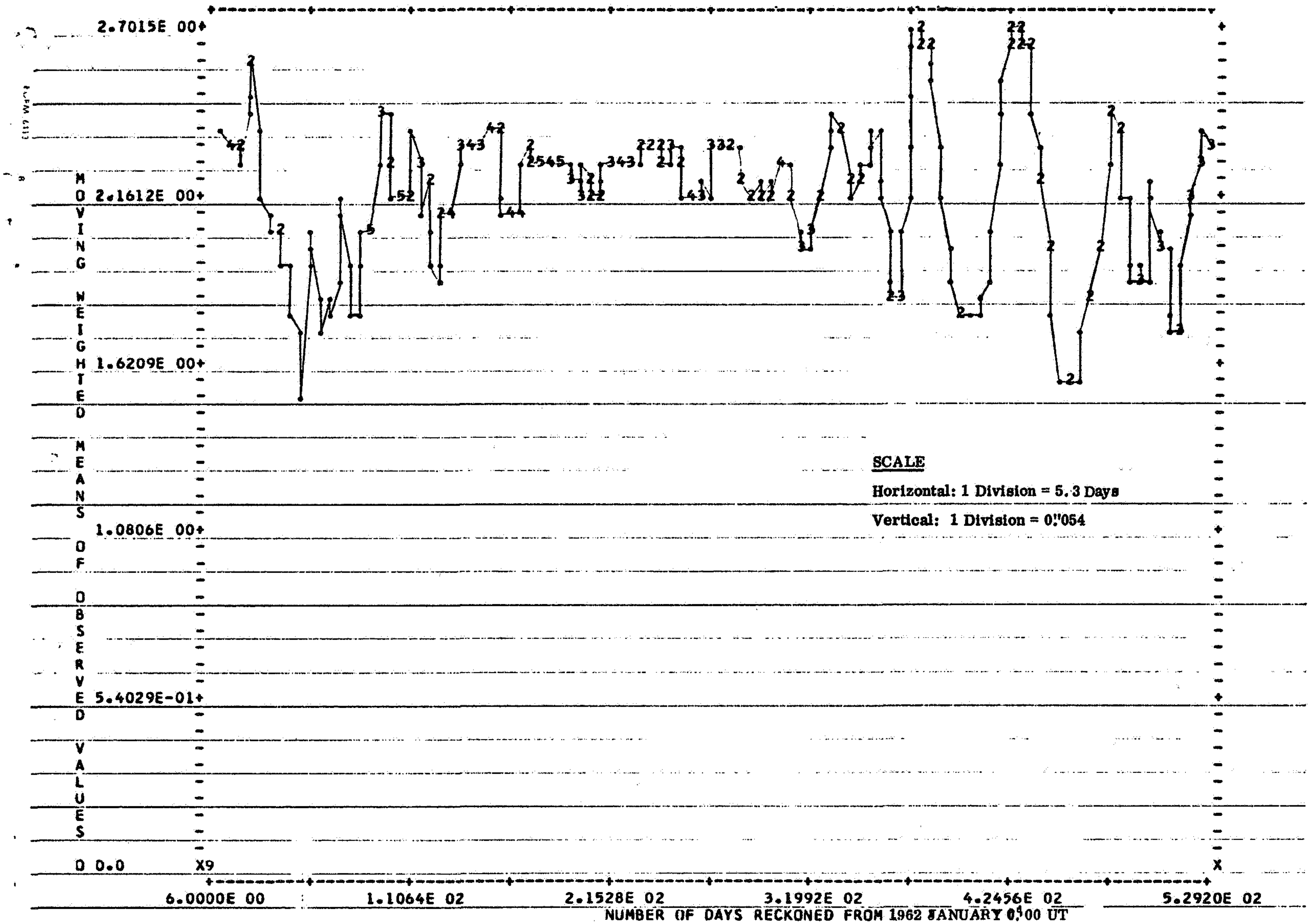


Fig. 3.1b. Plot of Moving Weighted Means of Observed Values, Set II

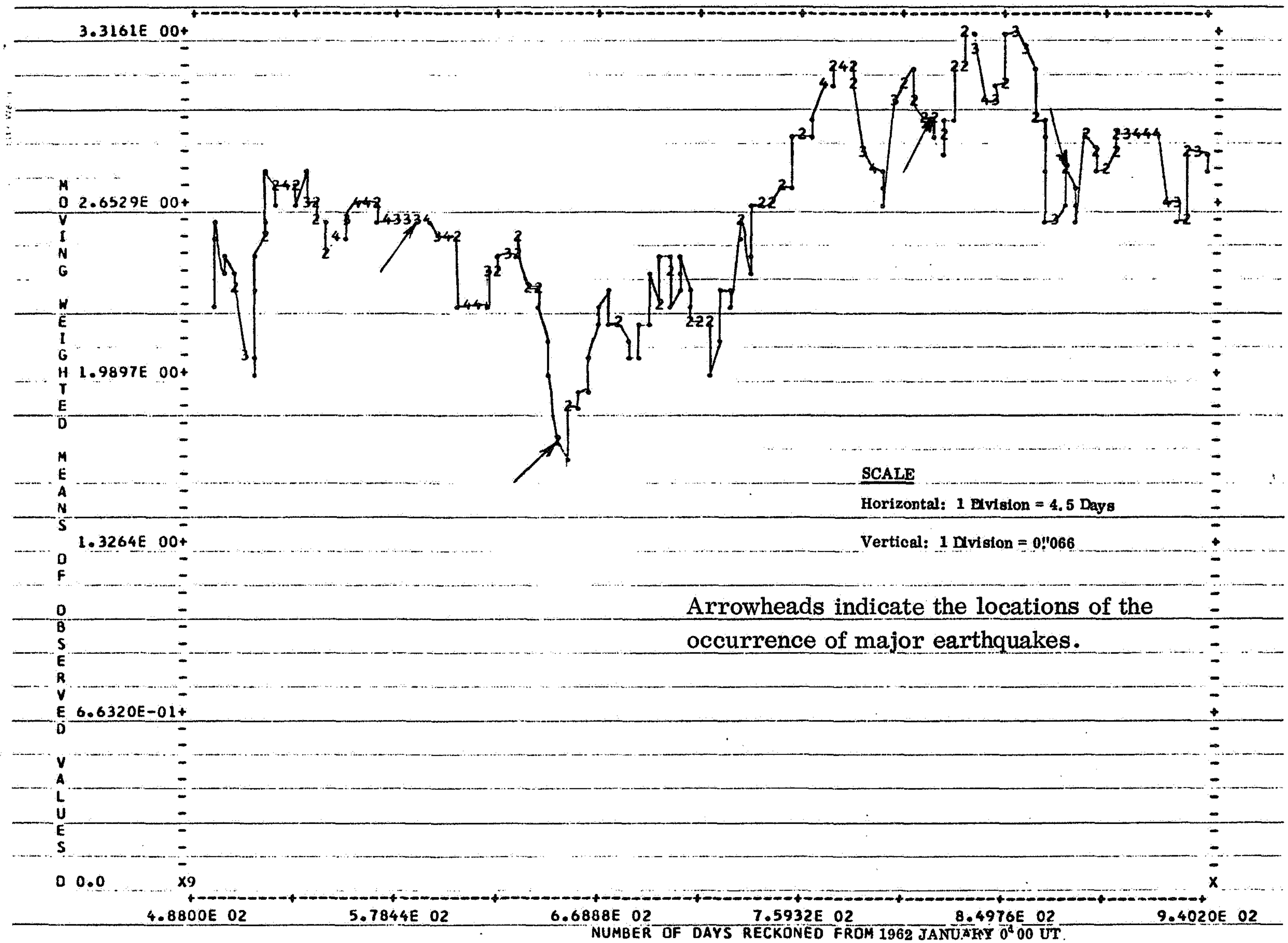


Fig. 3.1c. Plot of Moving Weighted Means of Observed Values, Set III

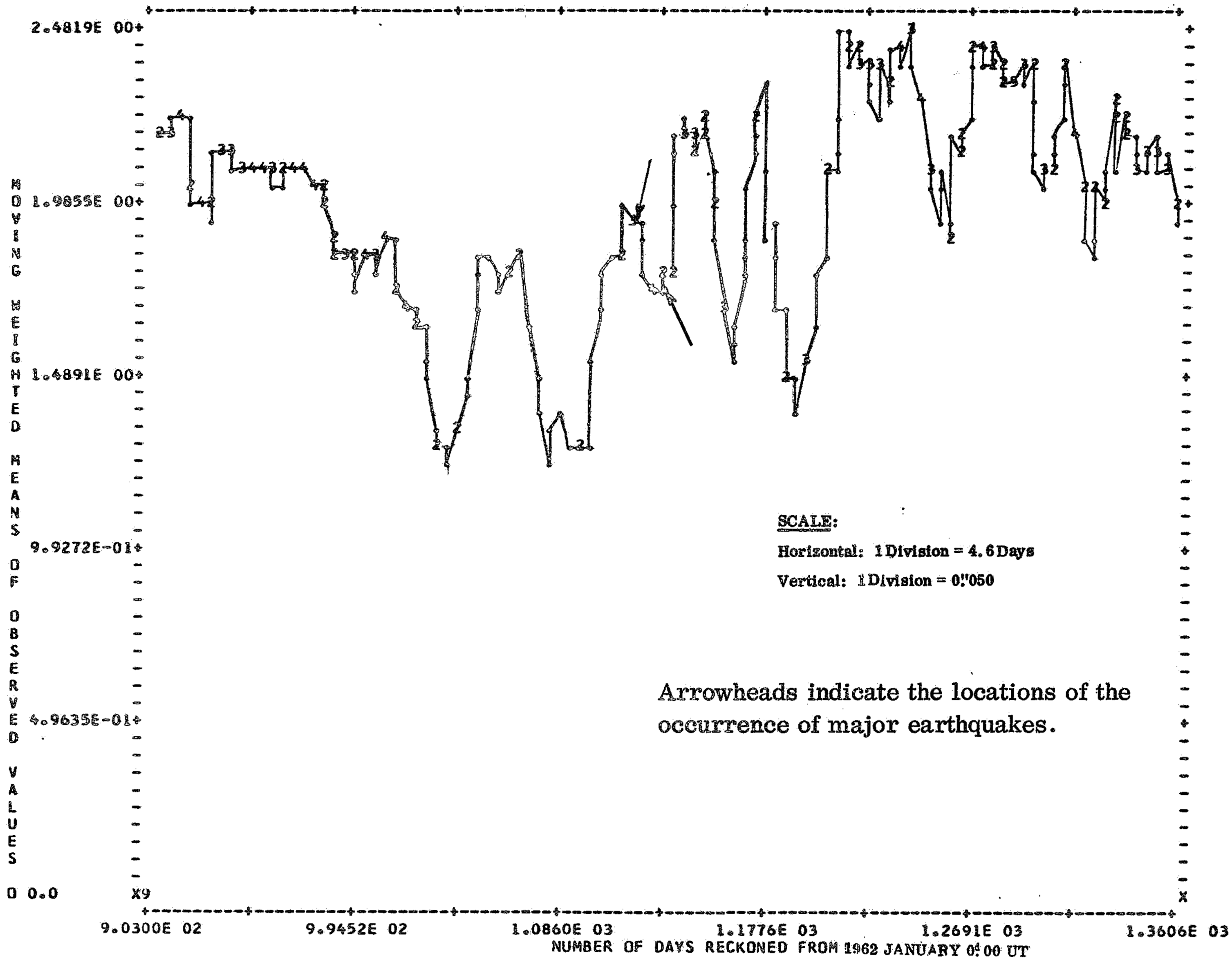
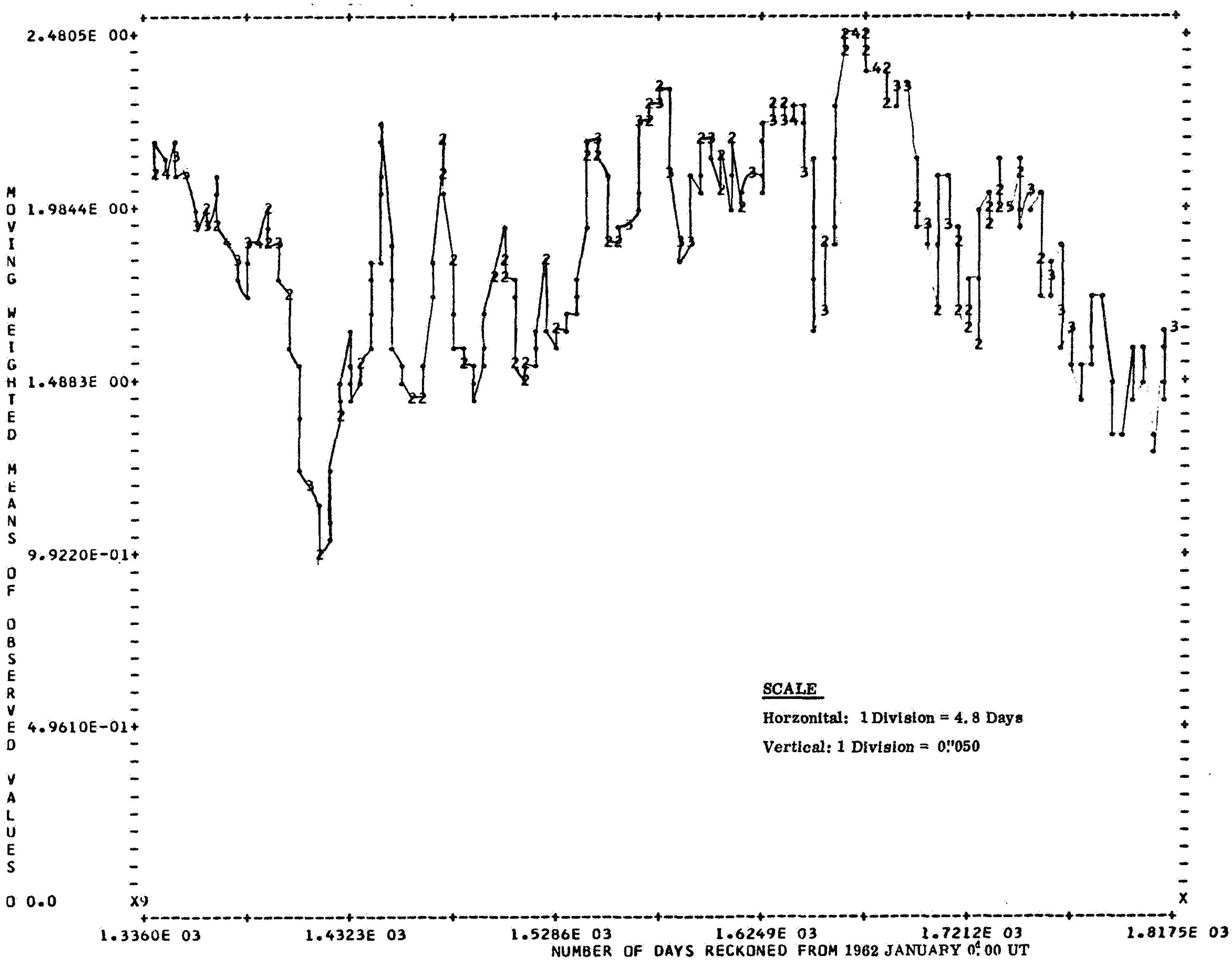


Fig. 3.1d. Plot of Moving Weighted Means of Observed Values, Set IV



linear. Our investigation aims at finding evidence of short-period sudden changes in latitude which would entail a nonlinear variation of latitude during a month. Secondly, for the investigation of the above type a systematic error like the declination correction is not likely to interfere with the investigation. Accordingly, the declination correction was not applied. However, since three groups of star pairs are observed every month in a rotating system in which only one new group is added every month and one deleted, in the program we had to make sure that the breaks that may be evidenced are not due to the differential declination errors applicable at every monthly changeover which is around the sixth day of every month.

3.2 Experimentation and Observations

A period of 251 days from Day 1350 to Day 1601 well between the dates of reported earthquakes was chosen. See Table 3.1 for the dates of major earthquakes reported during the period 1962 to 1967.

Table 3.1

Major Earthquakes in the Period 1962 - 1966

Date	Region	Magnitude	No. of Mean Solar Days from 1962.0
15 Aug 63	Peru, Bolivia	7.75	592
13 Oct 63	Kurile Islands	8.25	651
28 Mar 64	Southern Alaska	8.5	818
26 May 64	S. Sandwich Island	7.5 - 7.75	877
24 Jan 65	Ceram Sea	7.6	1120
4 Feb 65	Santa Cruz Island	7.75	1131
28 Dec 66	Off. N. Chile	7.75	1823

Information based on [Mansinha and Smylie, 1968]

Polynomial fittings up to degree 9 were obtained for this period to study the general behavior of the function by suitable programming on Omnitab/360. The printout shown in Table 3.2 was obtained for the 9th-degree polynomial.

Table 3.2

Set A - Polynomial Fit Degree 9

ANALYSIS OF VARIANCE

SOURCE		SUM OF SQUARES	D.F.	MEAN SQUARE	F
TOTAL		1.3588633E 04	192	7.0774124E 01	
TERM OF DEGREE	0	1.3483000E 04	1	1.3483000E 04	24379.29
RESIDUAL		1.0563281E 02	191	5.5305135E-01	
TERM OF DEGREE	1	2.7799957E 01	1	2.7799957E 01	67.86
RESIDUAL		7.7832855E 01	190	4.0964657E-01	
TERM OF DEGREE	2	1.2960892E 00	1	1.2960892E 00	3.20
RESIDUAL		7.6536758E 01	189	4.0495634E-01	
TERM OF DEGREE	3	2.2911158E 00	1	2.2911158E 00	5.80
RESIDUAL		7.4245636E 01	188	3.9492357E-01	
TERM OF DEGREE	4	4.5777643E-01	1	4.5777643E-01	1.16
RESIDUAL		7.3787857E 01	187	3.9458746E-01	
TERM OF DEGREE	5	2.9911846E-01	1	2.9911846E-01	0.76
RESIDUAL		7.3488739E 01	186	3.9510071E-01	
TERM OF DEGREE	6	3.7912786E-01	1	3.7912786E-01	0.96
RESIDUAL		7.3109604E 01	185	3.9518702E-01	
TERM OF DEGREE	7	3.1108916E-01	1	3.1108916E-01	0.79
RESIDUAL		7.2798508E 01	184	3.9564401E-01	
TERM OF DEGREE	8	2.8673804E-01	1	2.8673804E-01	0.72
RESIDUAL		7.2511765E 01	183	3.9623910E-01	
TERM OF DEGREE	9	2.6493776E-01	1	2.6493776E-01	0.67
RESIDUAL		7.2246826E 01	182	3.9696056E-01	
TOTAL REDUCTION		1.3516383E 04	10	1.3516382E 03	34049.68

For checking the significance of the 9th degree (10th parameters) fit at a level of significance of 5%, the tabular value for $F_{1, 182, 0.05} = 3.89$ [Bowker and Lieberman, 1960, p. 560] was compared against the computed value 0.67 in the printout, and this being less than the tabular value, the 10th parameter is insignificant.

The value of F in the printout was checked for degree 3 by computing the variance ratio

$$(n-p)\,\frac{R_{p-1} - R_p}{R_p} \qquad \text{[Hamilton, 1964, p. 169]}$$

where

$n-p$ = degree of freedom

n = number of observations

p = number of parameters

$p-1$ = degree of fit, under test

R_{p-1} = $V'PV$ = sum squares of residuals for $(p-1)$ parameter fit

R_p = sum squares of residuals for p parameter fit

For 3rd-degree fit (i.e., 4-parameter fit)

p = 4

n = 192

R_p = 74.245636

R_{p-1} = 76.536758

variance ratio = 5.80

which checks with the printout.

The tabular values for $F_{1, 183, 0.05}$ to $F_{1, 190, 0.05}$ are also about the same, i.e., 3.89. It can therefore be seen that 3rd degree (4th parameter) fit is significant. From the study of the printout it can be seen that the 3rd-degree term (4th parameter) is significant although the second-degree term (3rd parameter) is insignificant. A similar behavior could also be shown by the even-degree coefficients for polynomial fits for such a data.

In view of this it was decided that the criterion for selection of the degree of the polynomial should be the lowest degree polynomial showing significant variance ratio beyond which two consecutive higher-degree coefficients indicate no significant variance ratio (F-test). In this particular case, accordingly the third-degree polynomial was chosen for the fit.

The next consideration was the magnitude of the sudden change to be simulated and the manner of introducing it. The following printout obtained from the Omnitab program for the third-degree polynomial fit for the period of 251 days was studied.

Table 3.3

Set A - Polynomial Degree 3

TERM OF DEGREE	COEFFICIENT AND ITS	STANDARD DEVIATION
0	2.9223584E 02	1.1731436E 02
1	-5.8822793E-01	2.3931676E-01
2	3.9610080E-04	1.6251563E-04
3	-8.8578474E-08	3.6738520E-08
STANDARD DEVIATION	6.2779379E-01	

The standard deviation of the fit was 0".6278 for observation of unit weight. A χ^2 test was carried out to get an idea of the range within which variance σ^2 is likely to fall with a probability of 95%.

$$\text{(standard deviation)}^2 = 0.394$$

$$\text{degree of freedom} = 188$$

Formula:

$$P\left(\frac{\nu s^2}{\chi^2_{\nu,\,0.025}} < \sigma^2 < \frac{\nu s^2}{\chi^2_{\nu,\,0.975}}\right) = 0.95$$

where

ν = degree of freedom [Hamilton, 1964, p. 82]

s = standard deviation

From [Bowker and Lieberman, 1960, p. 563], $\chi^2_{188,\,0.025} = 227.52$ and $\chi^2_{188,\,0.975} = 152.84$. Thus

$$P\left(\frac{188 \times 0.394}{227.52} < \sigma^2 < \frac{188 \times 0.394}{152.84}\right) = 0.95$$

$$P(0.325 < \sigma^2 < 0.484) = 0.95$$

The probability is 95% that the variance of the fit lies between 0.325 and 0.484 The weight of a daily mean observation on an average is 10. Therefore, the corresponding standard deviation of the daily mean will lie between 0".18 and 0".22. Any break smaller than 0".22 is not likely to be detected. It was therefore decided to experiment with a simulated sudden change of 0".3. The direction of the sudden change could have different effects on the pattern of residuals. It was therefore decided to experiment both with a positive change of 0".3 and a negative change of 0".3.

Accordingly, in the test block (Days 1350 to 1601) the data for the period Day 1461 to Day 1491 was simulated by increasing the daily mean latitude by 0".3 in one set and decreasing the daily mean latitude by 0".3 in a second set. Thus there were three sets of data for study as follows:

Set A: Days 1350-1601, existing data

Set B: Days 1350-1460, existing data
Days 1461-1491, simulated data (existing values of daily mean latitude increased by 0".3)
Days 1492-1601, existing data

Set C: Days 1350-1460, existing data
Days 1461-1491, simulated data (existing values of daily mean latitude decreased by 0".3)
Days 1492-1601, existing data

Polynomial fits for sets B and C yielded the results shown in Tables 3.4 and 3.5. It was seen that the third-degree polynomial was maintained as the significant degree polynomial as per criterion decided upon.

For sets A, B, and C, plots of normalized residuals (square root of weights × residuals) against time were studied for polynomial fit of degree three. Similarly, plots of moving means of normalized residuals against time were also studied. For obtaining moving means of normalized residuals,

Table 3.4

Set B - Polynomial Fit Degree 5

ANALYSIS OF VARIANCE

SOURCE		SUM OF SQUARES	D.F.	MEAN SQUARE	F
TOTAL		1.4020664E 04	192	7.3024292E 01	
TERM OF DEGREE	0	1.3875902E 04	1	1.3875902E 04	18308.00
RESIDUAL		1.4476172E 02	191	7.5791472E-01	
TERM OF DEGREE	1	2.8889786E 01	1	2.8889786E 01	47.37
RESIDUAL		1.1587193E 02	190	6.0985225E-01	
TERM OF DEGREE	2	9.8146296E-01	1	9.8146296E-01	1.61
RESIDUAL		1.1489046E 02	189	6.0788602E-01	
TERM OF DEGREE	3	2.7647667E 00	1	2.7647667E 00	4.64
RESIDUAL		1.1212569E 02	188	5.9641320E-01	
TERM OF DEGREE	4	1.3319139E 00	1	1.3319139E 00	2.25
RESIDUAL		1.1079376E 02	187	5.9248000E-01	
TERM OF DEGREE	5	9.1847569E-01	1	9.1847569E-01	1.55
RESIDUAL		1.0987527E 02	186	5.9072727E-01	
TOTAL REDUCTION		1.3910785E 04	6	2.3184641E 03	39247.62

Table 3.5

Set C - Polynomial Fit Degree 5

ANALYSIS OF VARIANCE

SOURCE		SUM OF SQUARES	D.F.	MEAN SQUARE	F
TOTAL		1.3211313E 04	192	6.8808914E 01	
TERM OF DEGREE	0	1.3095750E 04	1	1.3095750E 04	21644.46
RESIDUAL		1.1556250E 02	191	6.0503924E-01	
TERM OF DEGREE	1	2.6731552E 01	1	2.6731552E 01	57.18
RESIDUAL		8.8830948E 01	190	4.6753126E-01	
TERM OF DEGREE	2	1.0677197E 01	1	1.0677197E 01	25.82
RESIDUAL		7.8153748E 01	189	4.1351187E-01	
TERM OF DEGREE	3	1.8619118E 00	1	1.8619118E 00	4.59
RESIDUAL		7.6291824E 01	188	4.0580755E-01	
TERM OF DEGREE	4	3.9641693E-02	1	3.9641693E-02	0.10
RESIDUAL		7.6252182E 01	187	4.0776563E-01	
TERM OF DEGREE	5	1.8350288E-02	1	1.8350288E-02	0.04
RESIDUAL		7.6233826E 01	186	4.0985924E-01	
TOTAL REDUCTION		1.3135078E 04	6	2.1891797E 03	53412.93

arithmetic mean of five consecutive values of normalized residuals was taken in a sliding step of one value.

Figs 3.2a,b,c show the plots of normalized residuals for sets A, B, and C. Figs. 3.3a,b,c show the plots of moving means of normalized residuals for sets A, B, and C. Printouts (below) were obtained for the 3rd-degree polynomial fits for sets B and C:

Table 3.6

Set B - Polynomial Fit Degree 3

TERM OF DEGREE	COEFFICIENT AND ITS	STANDARD DEVIATION
0	3.0261890E 02	1.4420032E 02
1	-6.2127107E-01	2.9416305E-01
2	4.2663910E-04	1.9976073E-04
3	-9.7304678E-08	4.5158203E-08
STANDARD DEVIATION	7.7167070E-01	

Table 3.7

Set C - Polynomial Fit Degree 3

TERM OF DEGREE	COEFFICIENT AND ITS	STANDARD DEVIATION
0	2.8185107E 02	1.1895644E 02
1	-5.5518144E-01	2.4266654E-01
2	3.6556041E-04	1.6479038E-04
3	-7.9851702E-08	3.7252757E-08
STANDARD DEVIATION	6.3658112E-01	

The following observations can be made:

Set A

(i) the standard deviation of observation of unit weight was 0".628,

(ii) as per values obtained in the Omnitab printouts and plots, residuals vary in value from + 0".594 to - 0".768, a range of 1".362 which is about 2.17 times the standard deviation.

(iii) the plot of normalized residuals indicates an approximately normal distribution and the maximum fluctuation over a short range of six

Fig. 3.2a Plot of Normalized Residuals, Set A

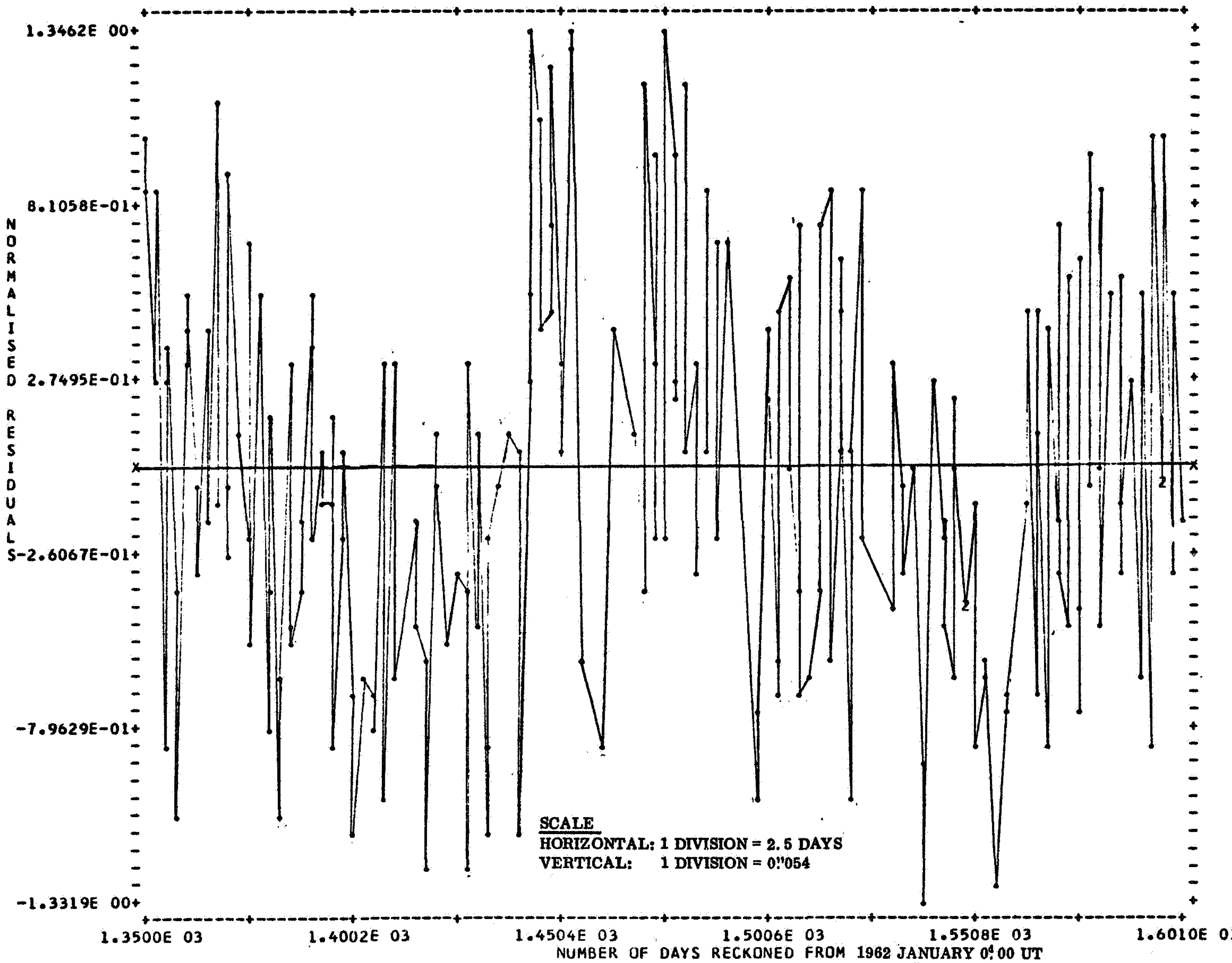


Fig. 3.2b. Plot of Normalized Residuals, Set B

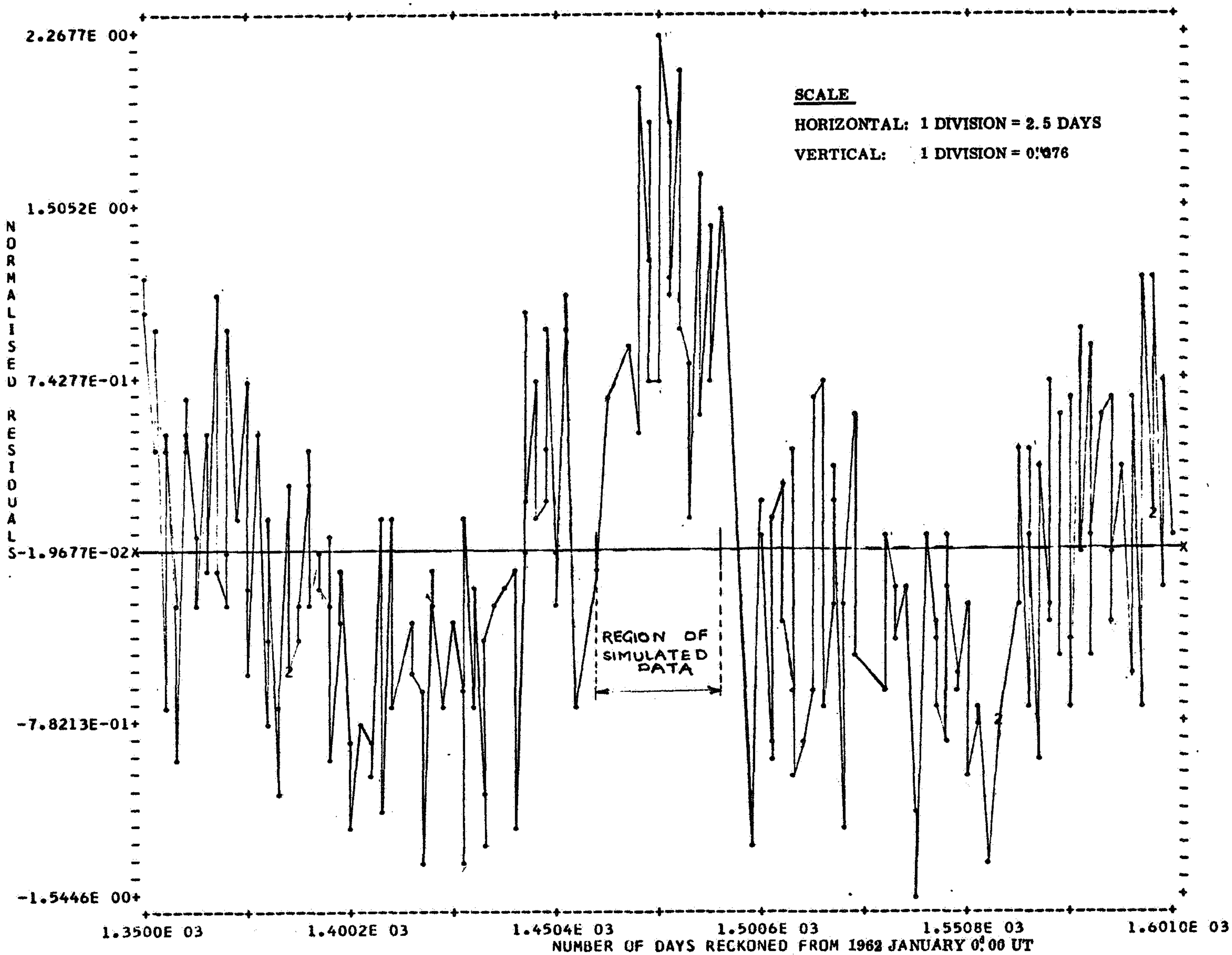


Fig. 3.2c. Plot of Normalized Residuals, Set C

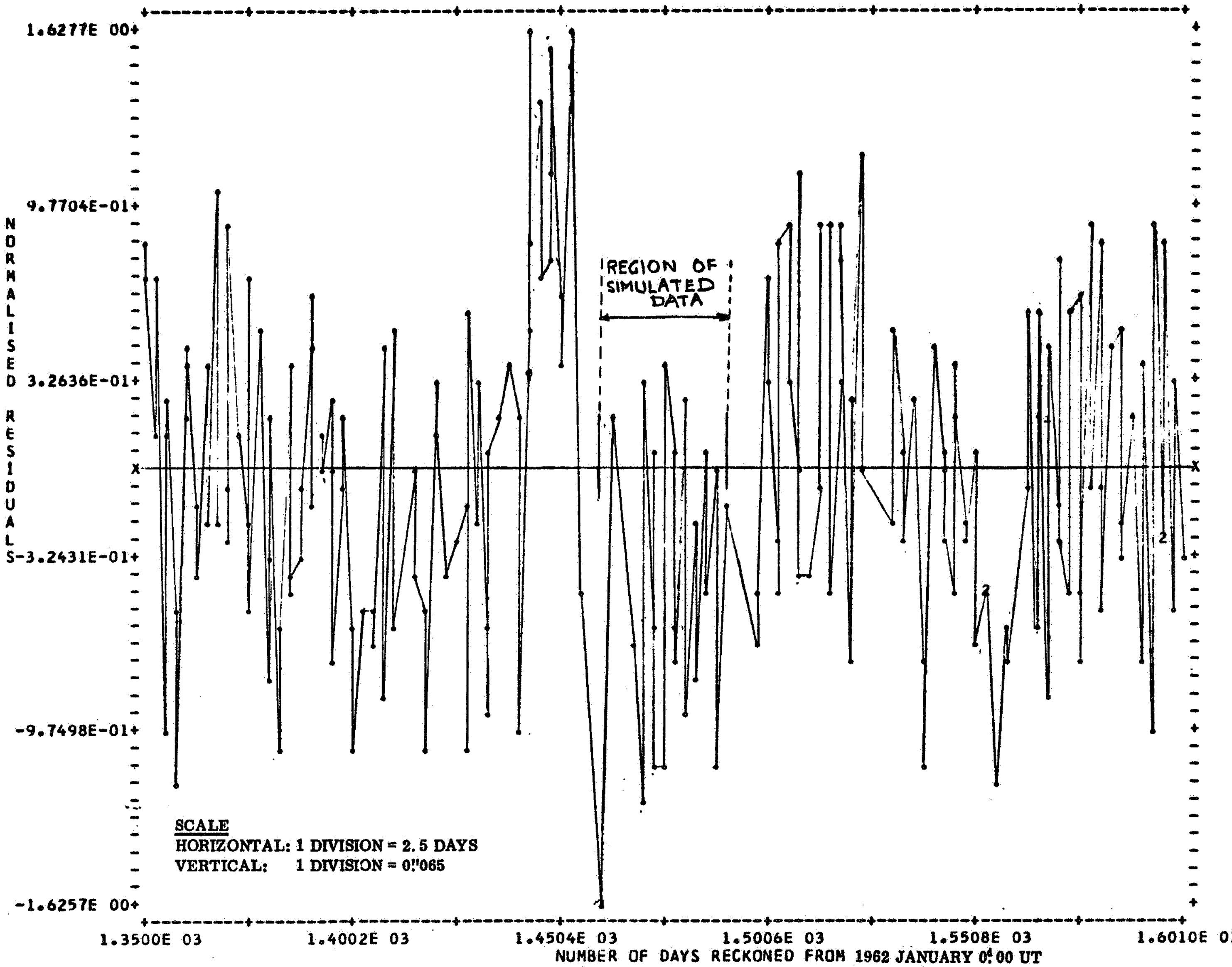


Fig. 3.3a. Plot of Moving Means of Normalized Residuals, Set A

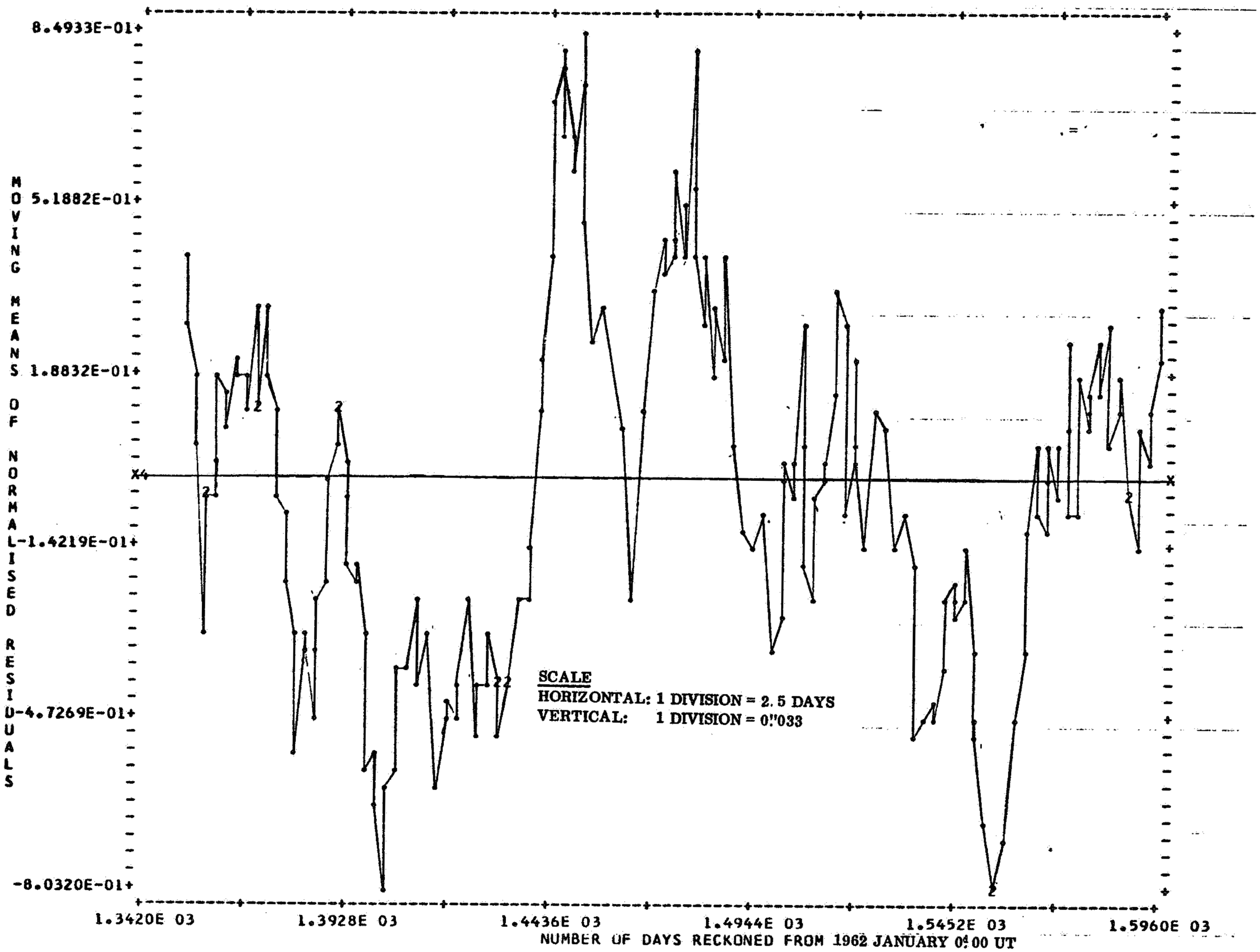


Fig. 3.3b. Plot of Moving Means of Normalized Residuals, Set B

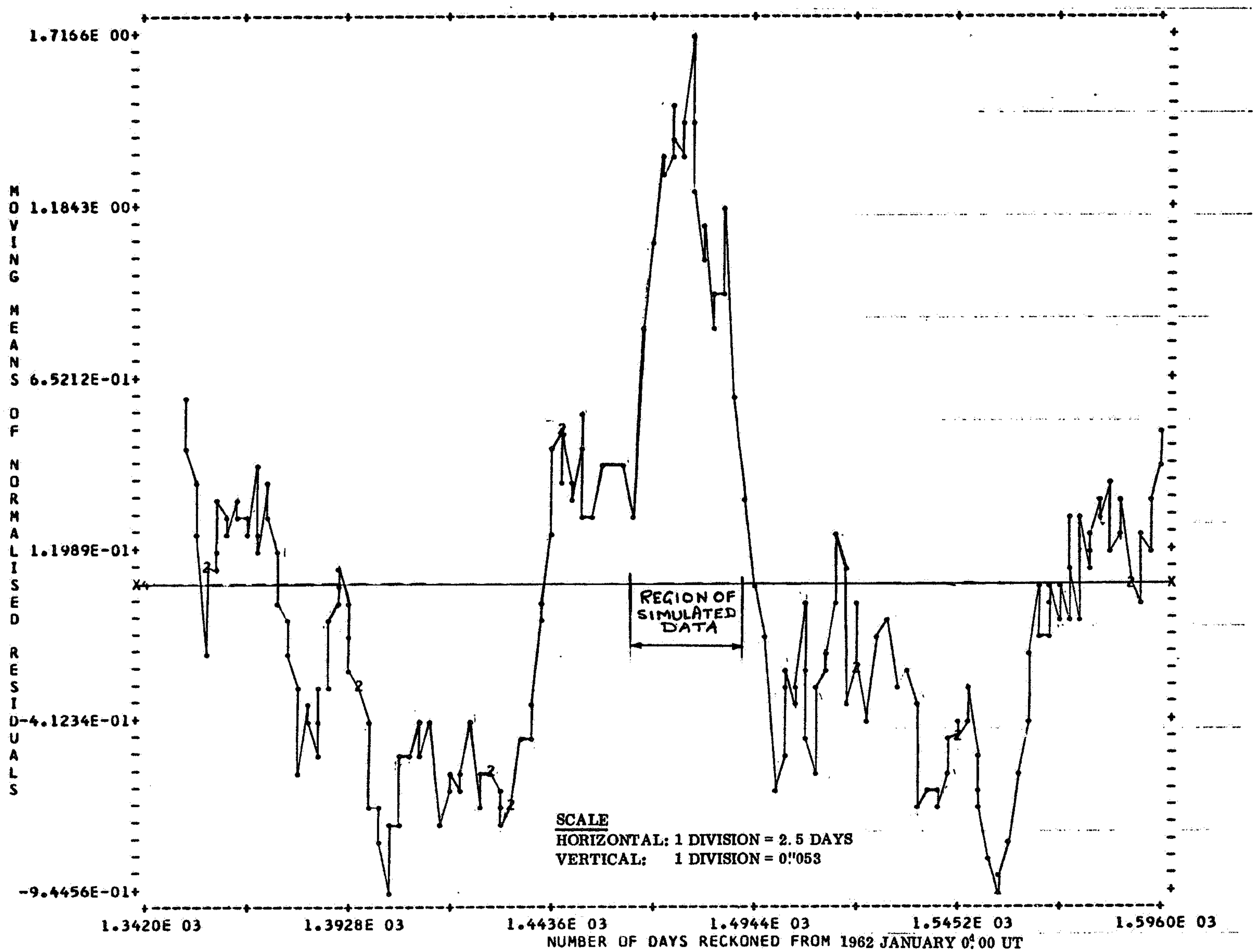
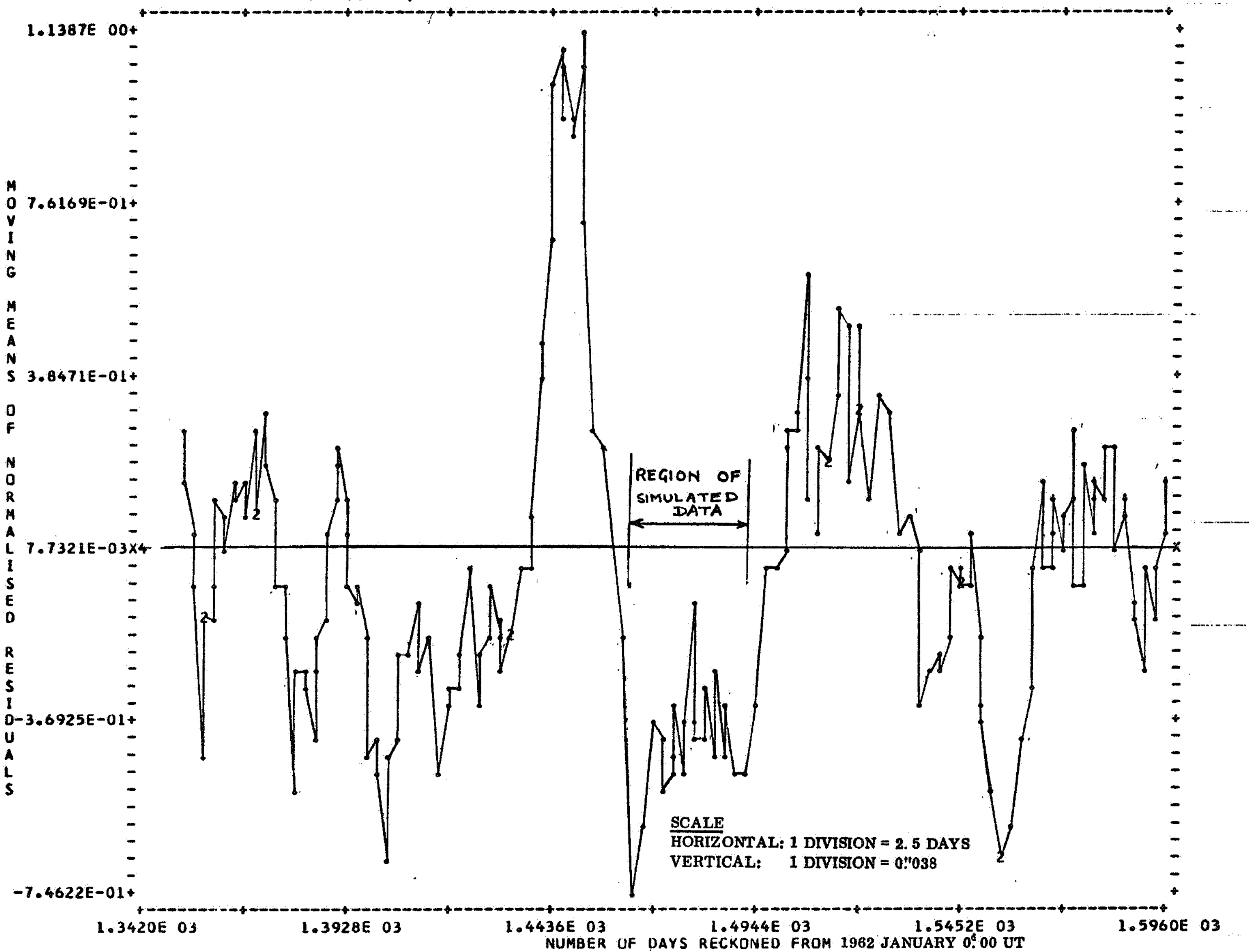


Fig. 3.3c. Plot of Moving Means of Normalized Residuals, Set C



consecutive values was + 2''.438 which is about 3.88 times the standard deviation. This was obtained as follows where maximum fluctuation was noticed by subtracting the value for an earlier day from the value for the later day.

minimum value:	Day 1441	value of normalized residual: − 1''.092
maximum value:	Day 1443	+ 1''.346
		fluctuation: + 2''.438

(iv) the moving means of normalized residuals fluctuate generally around the zero line between values + 0''.977 and − 0''.921. The maximum fluctuation over a range of 12 consecutive values was + 1''.269 which is 2.02 times the standard deviation. The fluctuation was obtained from values in the Omnitab printout as follows:

	Mean Value of Day	Mean Value of Normalized Residual
min. value in the range	1432.0	− 0''.501
max. value in the range	1445.0	+ 0.768
	fluctuation:	+ 1.269

At this stage it is useful to introduce a few notations. Let

r = the maximum fluctuation obtained as above in six consecutive values of normalized residuals

m = maximum fluctuation obtained as above in 12 consecutive values of moving means of normalized residuals

f = r/(standard deviation of the corresponding polyfit) = a measure for fluctuation of normalized residuals

g = m/(standard deviation of the corresponding polyfit) = a measure for fluctuation of moving means of normalized residuals

Set B

(i) the standard deviation of observation of unit weight was 0''.772,

(ii) the residuals vary in value from 0''.662 to − 0''.794, a range of 1''.456 which is 1.89 times the standard deviation,

(iii) normalized residuals fluctuate around the zero line except near the two ends of the period simulated for sudden change.

At the commencement of the simulated period (Day 1461), the value of r obtained was $+ 2''.752$, and the value of f obtained was 3.56. At the end of the simulated period (Day 1491), the value of r was $3''.007$, and the value of f was 3.90 as per details shown below:

Table 3.8

Set B - Values of r and f

* Region	Maximum Value of Normalized Residual		Minimum Value of Normalized Residual		Fluctu-ation r	Value of Stand. Dev.	Ratio f
	Day	Value	Day	Value			
Day 1461	1471	+2.033	1455	$-0''.719$	$+2''.752$	$0''.772$	3.56
Day 1491	1485	+1.682	1498	-1.325	-3.007	0.772	3.90

* Values in this column represent the means of the days for which the maximum and minimum values of residuals have been taken.

Let

u = ratio of f obtained in any case to 3.88 the value of f in case Set A
= a measure of comparison of the fluctuation of normalized residual observed in any arbitrary period with the similar fluctuation in the period assumed to be free of breaks

Thus, for Day 1461 the value of u is

$$3.56/3.88 = 0.92$$

and for Day 1491 the value of u is

$$3.90/3.88 = 1.01$$

(iv) the moving means of normalized residuals fluctuate around the zero line except near the two ends of the period simulated for sudden change. At the commencement of the simulation period (Day 1461) and the end of the simulation period, the values of m and g were obtained as follows:

Table 3.9

Set B - Values of m and g

Region	Maximum Value of Moving Means of Normalized Residual		Minimum Value of Moving Means of Normalized Residual		Fluctu- ation r	Value of Stand. Dev.	Ratio f
	Day	Value	Day	Value			
Day 1461	1475.0	1".501	1454.6	0".209	+1".292	0.772	1.67
Day 1491	1487.4	1.204	1500.0	-0.614	-1.818	0.772	2.35

(v) in the region of simulated change the values are positive.

Let

w = ratio of g obtained in any case to 2.02 the value of g in case of Set A
= a measure of comparison of fluctuation of the moving means of normalized residuals in any arbitrary period with the similar fluctuation in the period assumed to be free of breaks

Thus, for Day 1461 the value of w is

$$1.67/2.02 = 0.83$$

and for Day 1491 the value of w is

$$2.35/2.02 = 1.16$$

<u>Set C</u>

(i) the standard deviation of observation of unit weight was 0".637,

(ii) the residuals vary in value from 0".666 to - 0".741, a range of 1".407 which is about 2.21 times the standard deviation,

(iii) normalized residuals fluctuate around the zero line except near the two ends of the period simulated for sudden change.

Values of r, f, and u obtained for the commencement and end of the period simulated for break are shown in Table 3.10.

Table 3.10

Set C - Values of r, f, and u

Region	Maximum Value of Normalized Residual		Minimum Value of Normalized Residual		Fluctuation r	Val. of Stand. Dev.	Ratio f	Ratio u = f/3.88
	Day	Value	Day	Value				
Day 1461	1452	1".598	1461	- 1".626	- 3".224	0".637	5.06	1.30
Day 1491	1500	0.742	1488	- 1.113	+1.855	0 .637	2.91	0.75

(iv) the moving means of normalized residuals fluctuate around the zero line except near the two ends of the period simulated for sudden change.

Values of m, g, and w were obtained as follows for the commencement and end of the period simulated for break:

Table 3.11

Set C - Values of m, g, and w

Region	Maximum Value of Moving Means of Normalized Residual		Minimum Value of Moving Means of Normalized Residual		Fluctuation m	Val. of Stand. Dev.	Ratio g	Ratio w = g/2.02
	Day	Value	Day	Value				
Day 1461	1451.2	1".139	1463.4	- 0".746	- 1".885	0".637	2.96	1.47
Day 1491	1506.0	0.621	1490.0	- 0.480	+1.101	0.637	1.73	0.86

(v) in the period of simulated change the values of moving means are negative.

The observations of the experiment are given in tabular form in Table 3.12. From this table it can be seen that the simulated breaks do not show up clearly in every case. But it is not unreasonable to take the maximum values of

Table 3.12

Summary of Results - Simulation Program

Set	Stand. Dev. of Polyn. Fit	Region Day	r	f	u	m	g	w	Remarks
A	0"628	1439				+1"269	2.02	1.00	existing, unsimulated
		1442	+2"438	3.88	1.00				data
B	0.772	1461	+2.752	3.56	0.92	+1.292	1.67	0.83	start of simulation
		1491	-3.007	3.90	1.01	-1.818	2.35	1.16	end of simulation
C	0.637	1461	-3.224	5.06	1.30	-1.885	2.96	1.47	start of simulation
		1491	+1.885	2.91	0.75	+1.101	1.73	0.86	end of simulation

anomalous behavior due to the simulation of breaks as our criteria for identifying breaks similar to the ones introduced in the simulated data. Thus for detection of breaks the following approach was decided on:

(i) Regions where the moving means of normalized residuals remain positive or negative could be suspected.

(ii) For identification of a break of a nature used in the simulation, the following conditions must be satisfied: (a) the value of f must be at least 5.06 which corresponds to the value of u = 1.30, (b) the value of g must be at least 2.96 which corresponds to the value of w = 1.47.

To verify if the above criteria of f = 5.06 and g = 2.96, for identification of breaks, are statistically realistic, it was sought to obtain confidence intervals for μ_f and μ_g, the expected values of f and g respectively, in Set A, the set of data assumed to be free of breaks. If the values set for the criteria fall within the confidence interval of μ_f and μ_g respectively, then the criteria are not statistically realistic.

To obtain a confidence interval for μ_f with 95% probability, sample values of f were obtained in Set A by calculating values of f for successive groups of six consecutive values of normalized residuals of daily observed quantities. Table 3.13 shows the values of f thus obtained.

Table 3.13

Sample Values of f in Set A

Region Day*	Maximum Value of Normalized Residual		Minimum Value of Normalized Residual		Fluctu-ation r	Ratio f = r/stand. dev. **
	Day	Value	Day	Value		
1352	1350	0''999	1354	-0''851	-1.850	2.95
1358.5	1359	0.532	1358	-1.075	1.607	2.56
1365	1367	1.109	1363	-0.304	1.413	2.25
1370	1371	0.916	1369	-0.255	1.171	1.87
1379.5	1377	0.518	1382	-1.078	-1.596	2.54
1383.5	1384	0.353	1383	-0.661	1.014	1.61
1390.5	1390	0.537	1391	-0.190	-0.727	1.16
1397.5	1395	0.147	1400	-1.140	-1.287	2.05
1407.5	1407	0.303	1408	-1.017	-1.320	2.10
1419.5	1420	0.122	1419	-1.214	1.336	2.13
1428.5	1429	0.337	1428	-1.203	1.540	2.45
1432	1431	0.094	1433	-1.134	-1.228	1.96
1442	1443	1.346	1441	-1.092	2.438	3.88
1449.5	1449	1.260	1450	0.314	-0.946	1.51
1457.5	1454	1.324	1461	-0.858	-2.182	3.48
1470.5	1471	1.183	1470	-0.367	1.550	2.47
1475.5	1476	1.340	1475	-0.194	1.534	2.44
1485	1487	0.719	1483	-0.319	1.038	1.65
1494.5	1491	0.690	1498	-0.990	-1.680	2.68
1504	1505	0.580	1503	-0.680	1.260	2.01
1512	1515	0.840	1509	-0.705	1.545	2.46
1519	1518	0.638	1520	-1.001	-1.639	2.61
1526	1522	0.891	1530	-0.442	-1.333	2.12
1540	1541	0.257	1539	-1.332	1.589	2.53
1545.5	1546	0.220	1545	-0.639	0.859	1.37
1553.0	1550	-0.090	1556	-1.290	-1.200	1.91
1565.5	1566	0.498	1565	-0.668	1.166	1.86
1569.5	1571	0.775	1568	-0.866	1.641	2.61
1577	1578	0.963	1576	-0.741	1.704	2.71
1580.5	1580	0.872	1581	-0.502	-1.374	2.19
1593.5	1594	1.044	1593	-0.849	1.893	3.02
1597.5	1597	1.012	1598	-0.332	-1.344	2.14

*Values in this column represent the mean of the days on which the maximum and the minimum value of the residuals have been taken.

**Standard deviation = 0''628.

A statistical analysis of the sample values of f was obtained and the Omnitab printout is shown in Table 3.14.

Table 3.14

Set A - Statistical Analysis of Sample Values of f

Statistical Analysis of Column 4 N = 32

Frequency Distribution (1-6) 2 3 4 7 5 7 2 0 1 1

```
MEASURES OF LOCATION (2-2)

     UNWEIGHTED MEAN              =  2.2898464E 00
     WEIGHTED MEAN                =  2.2898464E 00
     MEDIAN                       =  2.2192497E 00
     MID-RANGE                    =  2.5211992E 00
     5 PCT UNWTD TRIMMED MEAN     =  2.2744246E 00
     5 PCT WTD TRIMMED MEAN       =  2.2744246E 00

MEASURES OF DISPERSION (2-6)

     STANDARD DEVIATION           =  5.7460988E-01
     S.D. OF MEAN                 =  1.0157764E-01
     RANGE                        =  2.7244005E 00
     MEAN DEVIATION               =  4.4054985E-01
     VARIANCE                     =  3.3017653E-01
     COEFFICIENT OF VARIATION     =  2.5093811E 01
```

A 2-sided 95% confidence interval for mean is 2.0827E 00 to 2.4970E 00 (2-2).

A 2-sided 95% confidence interval for standard deviation is 4.5609E -01 to 7.5546E -01 (2-7).

From the above it can be seen that

$$P\,(2.08 < \mu_f < 2.50) = .95$$

The criterion for break is f = 5.06 which is greater than 2.50, the higher bound of μ_f. Therefore, the criterion is not statistically unrealistic.

To obtain a confidence interval for μ_g with 95% probability, sample values of g were obtained in Set A by finding values of g for successive groups of 12 conseuctive values of moving means of normalized residuals. Since moving means represent the weighted means of five consecutive values of normalized residuals, four consecutive values of moving means were left out of consideration between two successive groups. This ensured that the sample values of g obtained have no correlation which otherwise exists between two consecutive values of moving means. Table 3.15 shows the values of g thus obtained.

Table 3.15

Set A - Sample Values of g

Region Day*	Maximum Value of Moving Means of Normalized Residual: Day	Maximum Value of Moving Means of Normalized Residual: Value	Minimum Value of Moving Means of Normalized Residual: Day	Minimum Value of Moving Means of Normalized Residual: Value	Fluctuation m	Ratio g = m/stand. dev. **
1354.0	1352.0	0.″431	1356.0	-0.″322	-0.″753	1.20
1366.1	1369.0	0.325	1363.2	0.080	0.245	0.39
1386.5	1392.0	0.036	1381.0	-0.539	0.575	0.92
1397.5	1393.0	-0.042	1402.0	-0.798	-0.757	1.21
1419.5	1423.4	-0.227	1415.6	-0.620	0.393	0.63
1438.5	1445.0	0.768	1432.0	-0.501	1.269	2.02
1457.2	1451.0	0.849	1463.4	-0.247	1.096	1.75
1487.1	1479.0	0.546	1495.2	-0.158	-0.704	1.12
1512.6	1515.8	0.342	1509.4	-0.255	0.597	0.95
1530.6	1525.4	0.114	1535.8	-0.513	-0.627	1.00
1559.7	1565.0	0.066	1554.4	-0.803	0.869	1.38
1577.5	1580.0	0.254	1575.0	-0.084	0.338	0.54
1593.1	1596.0	0.320	1590.2	-0.152	0.472	0.75

*Values in this column represent the mean of the days on which the maximum and the minimum value of the residuals have been taken.

**Standard deviation = 0.″628.

A statistical analysis of the sample values of g was obtained and the Omnitab printout is shown in Table 3.16. From this table it can be seen that

$$P(0.78 < \mu_g < 1.35) = .95$$

The criterion for the break is g = 2.96 which is greater than 1.35, the higher bound of μ_g. Therefore, the criterion is not statistically unrealistic.

It is of course recognized that this is only one of the possible methods for location of breaks.

Table 3.16

Set A - Statistical Analysis of Sample Values of g

Statistical Analysis of Column 5 N = 13

Frequency Distribution (1-6) 2 1 1 3 3 0 1 0 1 1

MEASURES OF LOCATION (2-2)

UNWEIGHTED MEAN	=	1.0655222E 00
WEIGHTED MEAN	=	1.0655222E 00
MEDIAN	=	9.9899995E-01
MID-RANGE	=	1.2057991E 00
5 PCT UNWTD TRIMMED MEAN	=	1.0655222E 00
5 PCT WTD TRIMMED MEAN	=	1.0655222E 00

MEASURES OF DISPERSION (2-6)

STANDARD DEVIATION	=	4.6431804E-01
S.D. OF MEAN	=	1.2877864E-01
RANGE	=	1.6323986E 00
MEAN DEVIATION	=	3.5170120E-01
VARIANCE	=	2.1559125E-01
COEFFICIENT OF VARIATION	=	4.3576553E 01

A 2-sided 95% confidence interval for mean is 7.8494E -01 to 1.3461E 00 (2-2).

A 2-sided 95% confidence interval for standard deviation is 3.2482E -01 to 7.4186E -01 (2-7).

The next step was to take up the existing data for the whole period 1962 to 1966 to see if there is any evidence of a similar behavior pattern of residuals as noticed in the simulated data. Initially a polynomial fit for the whole data over the period 1962 to 1966 was attempted. It was noticed that at degree six of the polynomial the normal matrix began to get unstable. At degree seven the discrepancy between the normal matrix and the matrix obtained by reinverting the inverse of the normal matrix was as high as 6%. The results obtained are given in Table 3.17. At this stage it was realized that it was not necessary

Table 3.17

Polynomial Fits for the Whole Data 1962.00 to 1967.01

Degree of Polynomial	Sum Squares of Residuals ($V'PV$)	Degree of Freedom	Variance Ratio
0	116822	1289	
1	802.3	1288	186253
2	802	1287	0.15
3	788	1286	23.3
4	787	1285	2.08
5	782.8	1284	7.25
* 6	782.4	1283	0.69
** 7	746	1282	62.55

*Inversion of normal matrix disagrees by 0.04%.
**Inversion of normal matrix disagrees by 6%.

to seek a polynomial fit for the whole data since our aim was only to identify sudden changes in latitude. So it was possible to consider the whole data in several sections. This would also obviate the need for a high degree polynomial. In dividing the whole data into several sections, two considerations came to light:

(i) It was possible to divide the whole data into sections of 250 days with overlaps to correspond with the exact period taken in the simulation program. This would open the possibility that for each section we could go only up to the third-degree polynomial as in the case of the simulation program to study the pattern of residuals. But this involved the assumption that a 250-day period in any region required only a third-degree polynomial fit as per our criterion and assuming there are no sudden changes of latitude in that region.

(ii) It was possible to divide the whole data into convenient overlapping sections and to consider for each section a degree of polynomial fit as required by our statistical criterion. This method gave rise to the possibility that some sudden changes in latitude may be absorbed in a higher degree polynomial as could be demanded by our criterion.

It was finally decided to adopt method (ii) for the following reasons:

(1) Plots of moving means of observed values indicate that the wave-lenghts in the mean functional curve are not of uniform period indicating that equal periods of time in different regions may normally require a different degree of polynomial for significant fit.

(2) In the simulation program the sets A, B, and C had required the same degree of polynomial for significant fit, and the normalized residuals had shown some behavior pattern where the data had been simulated. It was therefore reasonable to assume that in any other period also the significant degree of the polynomial fit would remain the same whether there are sudden changes in the values of latitude or not (considering short-period sudden changes of the magnitude of about $0\overset{''}{.}3$).

Accordingly, the whole data was divided into four sets as follows:

Set I	period up to Day 542
Set II	period from Day 488 to Day 948
Set III	period from Day 903 to Day 1379
Set IV	period from Day 1336 to Day 1831

Polynomial fits to the individual sets yielded the significant degree polynomial as degree six for Set I, degree five for Set II, degree three for Set III, and degree five for Set IV. These are shown in the following tables.

Table 3.18

Set I - Degree of Polynomial 8

ANALYSIS OF VARIANCE

SOURCE		SUM OF SQUARES	D.F.	MEAN SQUARE	F
TOTAL		2.5976082E 04	341	7.6176193E 01	
TERM OF DEGREE	0	2.5848109E 04	1	2.5848109E 04	68673.69
RESIDUAL		1.2797266E 02	340	3.7639016E-01	
TERM OF DEGREE	1	5.4943171E 00	1	5.4943171E 0C	15.21
RESIDUAL		1.2247833E 02	339	3.6129296E-01	
TERM OF DEGREE	2	1.3324718E 01	1	1.3324718E 01	41.26
RESIDUAL		1.0915361E 02	338	3.2293963E-01	
TERM OF DEGREE	3	5.3160400E 01	1	5.3160400E 01	319.95
RESIDUAL		5.5993210E 01	337	1.6615194E-01	
TERM OF DEGREE	4	2.0103622E 01	1	2.0103622E 01	188.21
RESIDUAL		3.5889587E 01	336	1.0681421E-01	
TERM OF DEGREE	5	1.4970903E 00	1	1.4970903E 00	14.58
RESIDUAL		3.4392487E 01	335	1.0266411E-01	
TERM OF DEGREE	6	3.7395515E 00	1	3.7395515E 00	40.75
RESIDUAL		3.0652924E 01	334	9.1775179E-02	
TERM OF DEGREE	7	2.7441329E-01	1	2.7441329E-01	3.01
RESIDUAL		3.0378510E 01	333	9.1226697E-02	
TERM OF DEGREE	8	1.2770265E-01	1	1.2770265E-01	1.40
RESIDUAL		3.0250793E 01	332	9.1116846E-02	
TOTAL REDUCTION		2.5945828E 04	9	2.8828696E 03	316392.58

Table 3.19

Set II - Degree of Polynomial 7

ANALYSIS OF VARIANCE				
SOURCE	SUM OF SQUARES	D.F.	MEAN SQUARE	F
TOTAL	2.4902336E 04	319	7.8063736E 01	
TERM OF DEGREE 0	2.4675648E 04	1	2.4675648E 04	34615.30
RESIDUAL	2.2668750E 02	318	7.1285373E-01	
TERM OF DEGREE 1	7.0990570E 01	1	7.0990570E 01	144.54
RESIDUAL	1.5569693E 02	317	4.9115747E-01	
TERM OF DEGREE 2	9.9542755E-01	1	9.9542755E-01	2.03
RESIDUAL	1.5470149E 02	316	4.8956168E-01	
TERM OF DEGREE 3	1.2157257E 02	1	1.2157257E 02	1155.95
RESIDUAL	3.3128922E 01	315	1.0517114E-01	
TERM OF DEGREE 4	6.0576215E-02	1	6.0576215E-02	0.58
RESIDUAL	3.3068344E 01	314	1.0531318E-01	
TERM OF DEGREE 5	7.0320539E 00	1	7.0320539E 00	84.54
RESIDUAL	2.6036285E 01	313	8.3182991E-02	
TERM OF DEGREE 6	8.0343902E-02	1	8.0343902E-02	0.97
RESIDUAL	2.5955933E 01	312	8.3192050E-02	
TERM OF DEGREE 7	7.5539410E-02	1	7.5539410E-02	0.91
RESIDUAL	2.5880386E 01	311	8.3216667E-02	
TOTAL REDUCTION	2.4876453E 04	8	3.1095566E 03	373669.92

Table 3.20

Set III - Degree of Polynomial 5

ANALYSIS OF VARIANCE

SOURCE		SUM OF SQUARES	D.F.	MEAN SQUARE	F
TOTAL		2.9672152E 04	366	8.1071442E 01	
TERM OF DEGREE	0	2.9382961E 04	1	2.9382961E 04	37085.41
RESIDUAL		2.8919141E 02	365	7.9230517E-01	
TERM OF DEGREE	1	6.0267548E 01	1	6.0267548E 01	95.83
RESIDUAL		2.2892386E 02	364	6.2891167E-01	
TERM OF DEGREE	2	1.7373337E 01	1	1.7373337E 01	29.81
RESIDUAL		2.1155052E 02	363	5.8278376E-01	
TERM OF DEGREE	3	1.2472591E 02	1	1.2472591E 02	520.02
RESIDUAL		8.6824615E 01	362	2.3984694E-01	
TERM OF DEGREE	4	7.6548874E-01	1	7.6548874E-01	3.21
RESIDUAL		8.6059113E 01	361	2.3839086E-01	
TERM OF DEGREE	5	6.1918920E-01	1	6.1918920E-01	2.61
RESIDUAL		8.5439911E 01	360	2.3733306E-01	
TOTAL REDUCTION		2.9586711E 04	6	4.9311172E 03	207772.03

Table 3.21

Set IV - Degree of Polynomial 7

ANALYSIS OF VARIANCE

SOURCE		SUM OF SQUARES	D.F.	MEAN SQUARE	F
TOTAL		2.9846188E 04	393	7.5944489E 01	
TERM OF DEGREE	0	2.9636586E 04	1	2.9636586E 04	55426.79
RESIDUAL		2.0960156E 02	392	5.3469783E-01	
TERM OF DEGREE	1	1.6850266E 01	1	1.6850266E 01	34.18
RESIDUAL		1.9275130E 02	391	4.9297005E-01	
TERM OF DEGREE	2	2.8232910E 01	1	2.8232910E 01	66.93
RESIDUAL		1.6451839E 02	390	4.2184198E-01	
TERM OF DEGREE	3	1.1711095E 01	1	1.1711095E 01	29.81
RESIDUAL		1.5280728E 02	389	3.9282072E-01	
TERM OF DEGREE	4	9.6225233E 00	1	9.6225233E 00	26.07
RESIDUAL		1.4318475E 02	388	3.6903286E-01	
TERM OF DEGREE	5	3.1669455E 00	1	3.1669455E 00	8.75
RESIDUAL		1.4001781E 02	387	3.6180311E-01	
TERM OF DEGREE	6	1.1412325E 00	1	1.1412325E 00	3.17
RESIDUAL		1.3887657E 02	386	3.5978383E-01	
TERM OF DEGREE	7	7.1156657E-01	1	7.1156657E-01	1.98
RESIDUAL		1.3816499E 02	385	3.5887009E-01	
TOTAL REDUCTION		2.9708020E 04	8	3.7135024E 03	103477.62

The following printouts were obtained for the polynomial fits of degrees 6, 5, 3, 5 for Sets I, II, III, and IV respectively.

Table 3.22

Set I - Degree of Polynomial 6

TERM OF DEGREE	COEFFICIENT	AND ITS STANDARD DEVIATION
0	2.3993416E 00	3.4776065E-02
1	-5.6684278E-03	1.7647229E-03
2	8.7609864E-05	2.7364702E-05
3	-7.9155899E-07	1.8413681E-07
4	3.2380960E-09	6.0449268E-10
5	-5.7693607E-12	9.5213247E-13
6	3.6868024E-15	5.7555381E-16
STANDARD DEVIATION	3.0188781E-01	

Table 3.23

Set II - Degree of Polynomial 5

TERM OF DEGREE	COEFFICIENT	AND ITS STANDARD DEVIATION
0	-2.2392415E 02	2.7281876E 01
1	1.7157221E 00	1.9694555E-01
2	-5.0819814E-03	5.6239730E-04
3	7.3425699E-06	7.9423273E-07
4	-5.1790821E-09	5.5486771E-10
5	1.4288345E-12	1.5346697E-13
STANDARD DEVIATION	2.8482234E-01	

Table 3.24

Set III - Degree of Polynomial 3

TERM OF DEGREE	COEFFICIENT	AND ITS STANDARD DEVIATION
0	9.9227707E 01	4.4785089E 00
1	-2.6406282E-01	1.1923648E-02
2	2.3662907E-04	1.0494189E-05
3	-6.9843793E-08	3.0547800E-09
STANDARD DEVIATION	4.8846179E-01	

Table 3.25

Set IV - Degree of Polynomial 5

TERM OF DEGREE	COEFFICIENT AND ITS	STANDARD DEVIATION
0	6.4658545E 02	1.1618214E 02
1	-1.4719839E 00	3.0210173E-01
2	1.1452679E-03	3.1180331E-04
3	-2.7857413E-07	1.6850134E-07
4	-4.8112944E-11	5.2101198E-11
5	2.2446736E-14	7.4864634E-15
STANDARD DEVIATION	5.9353179E-01	

Plots of normalized residuals and moving means of normalized residuals for Sets I to IV were then studied for evidence of breaks (sudden changes). For convenience the plot for normalized residuals for each set was obtained in two sections. Figs. 3.4 a - 3.4 c show the plots of normalized residuals and moving means of normalized residuals for Set I. Figs. 3.5 - 3.7 show similar plots for Sets II, III, and IV respectively. At regions where sudden changes were suspected, the values of r, f, m, g, u, and w were calculated as shown in Tables 3.26 and 3.27.

Fig. 3.4a. Plot of Normalized Residuals, Set I

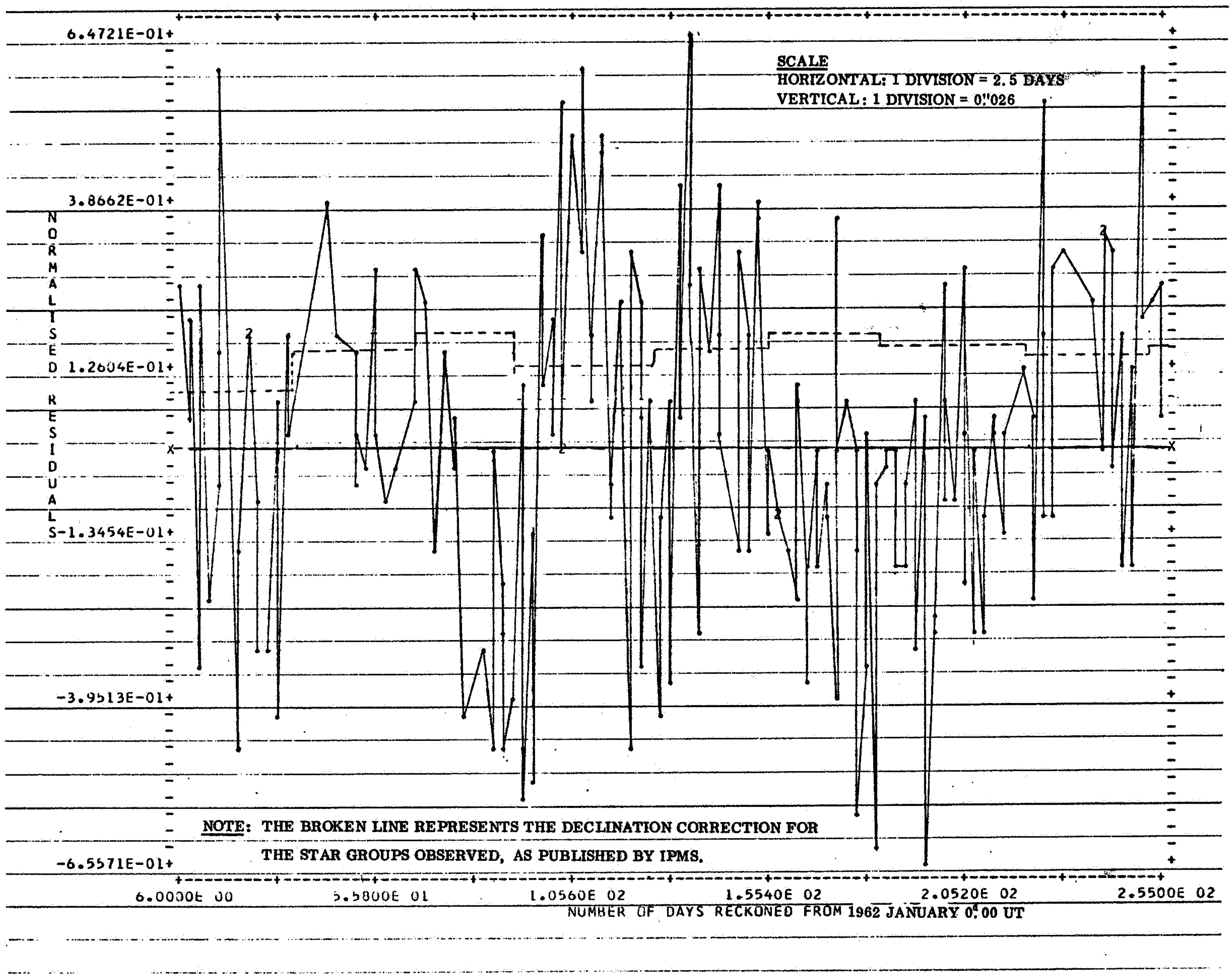


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Fig. 3.4b. Plot of Normalized Residuals, Set I

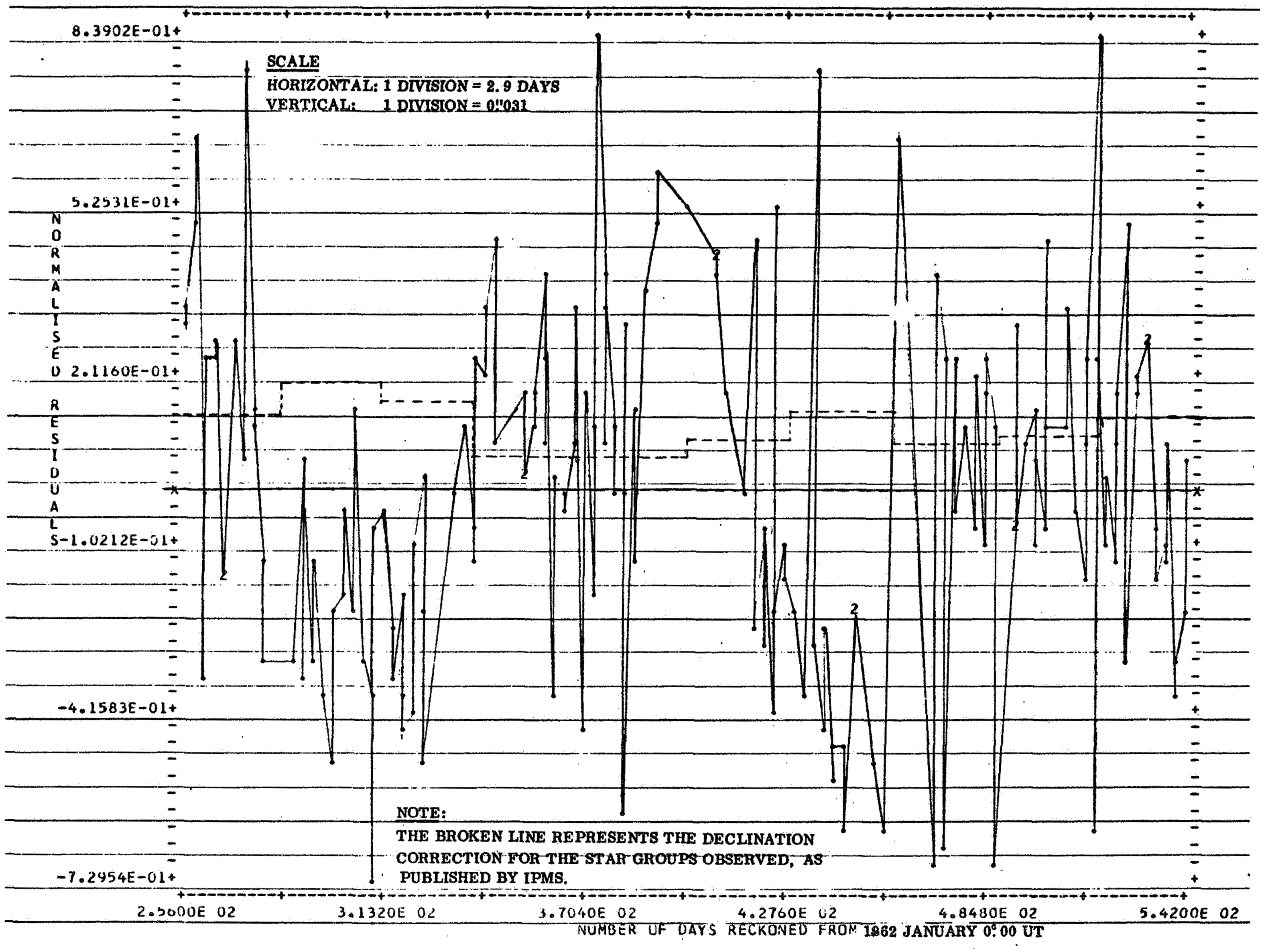


Fig. 3.4e.— Plot of Moving Means of Normalized Residuals, Set I

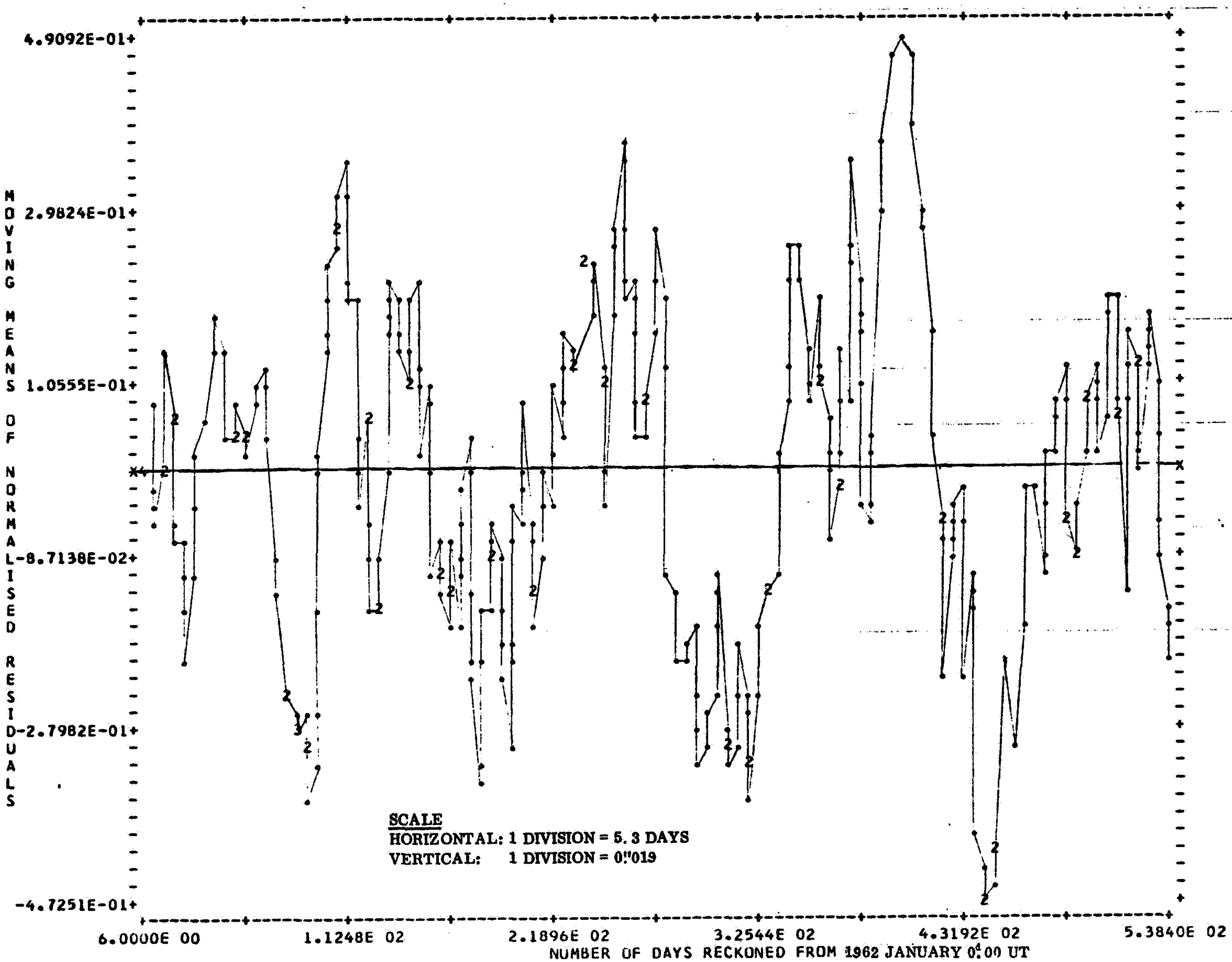


Fig. 3.5a. Plot of Normalized Residuals, Set II

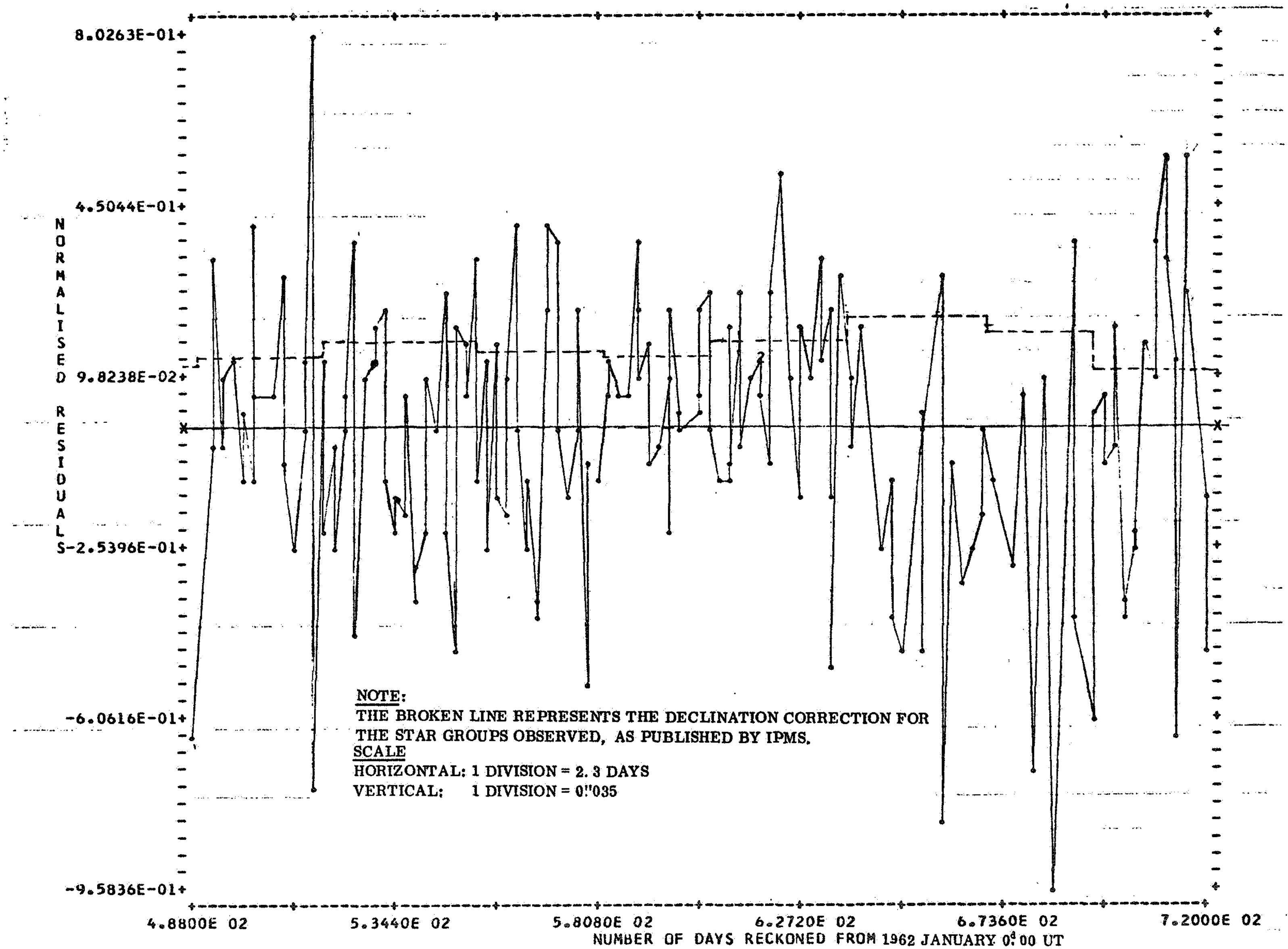


Fig. 3.5b. Plot of Normalized Residuals, Set II

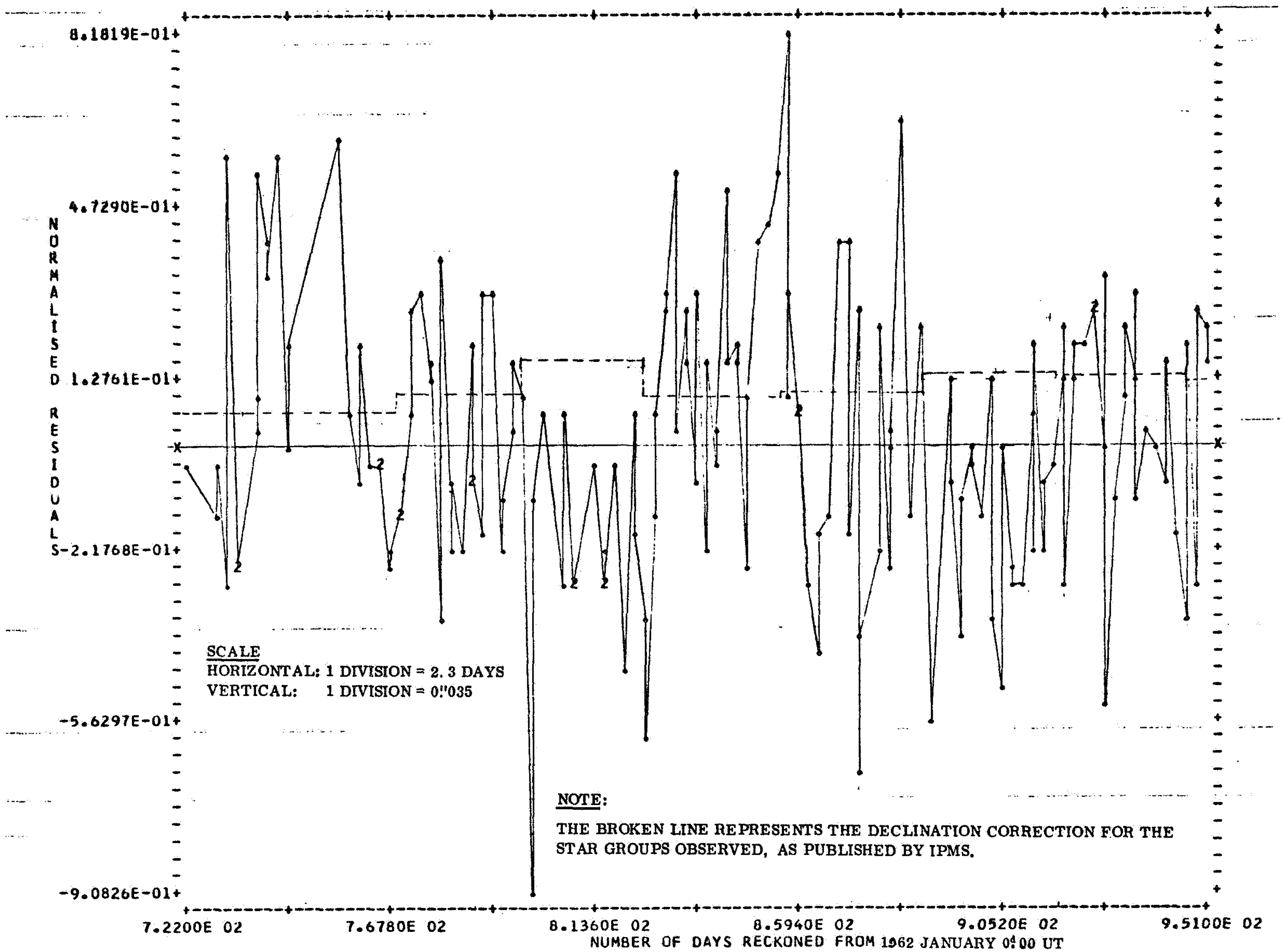


Fig. 3.5 c. Plot of Moving Means of Normalized Residuals, Set II

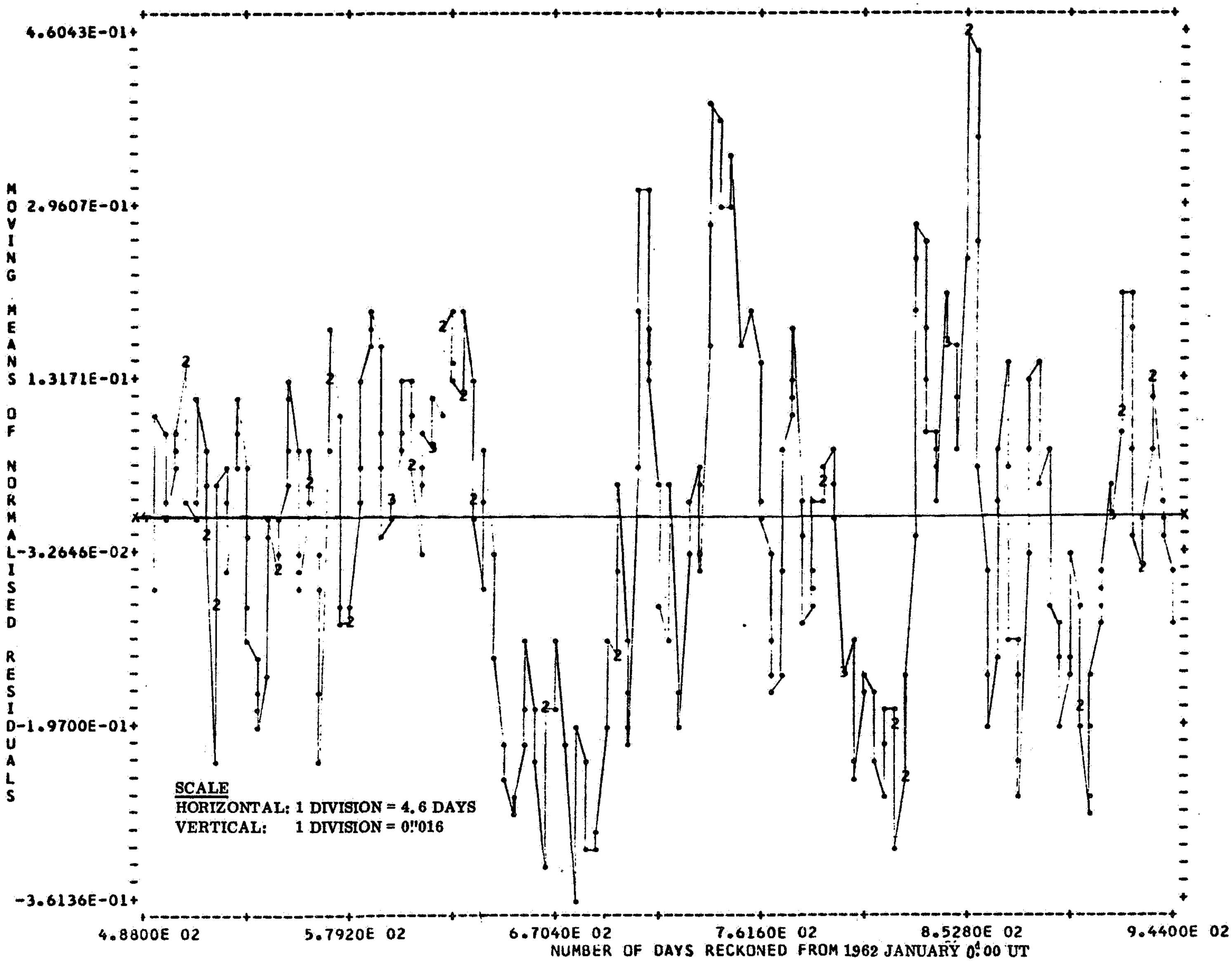


Fig. 3.6a. Plot of Normalized Residuals, Set III

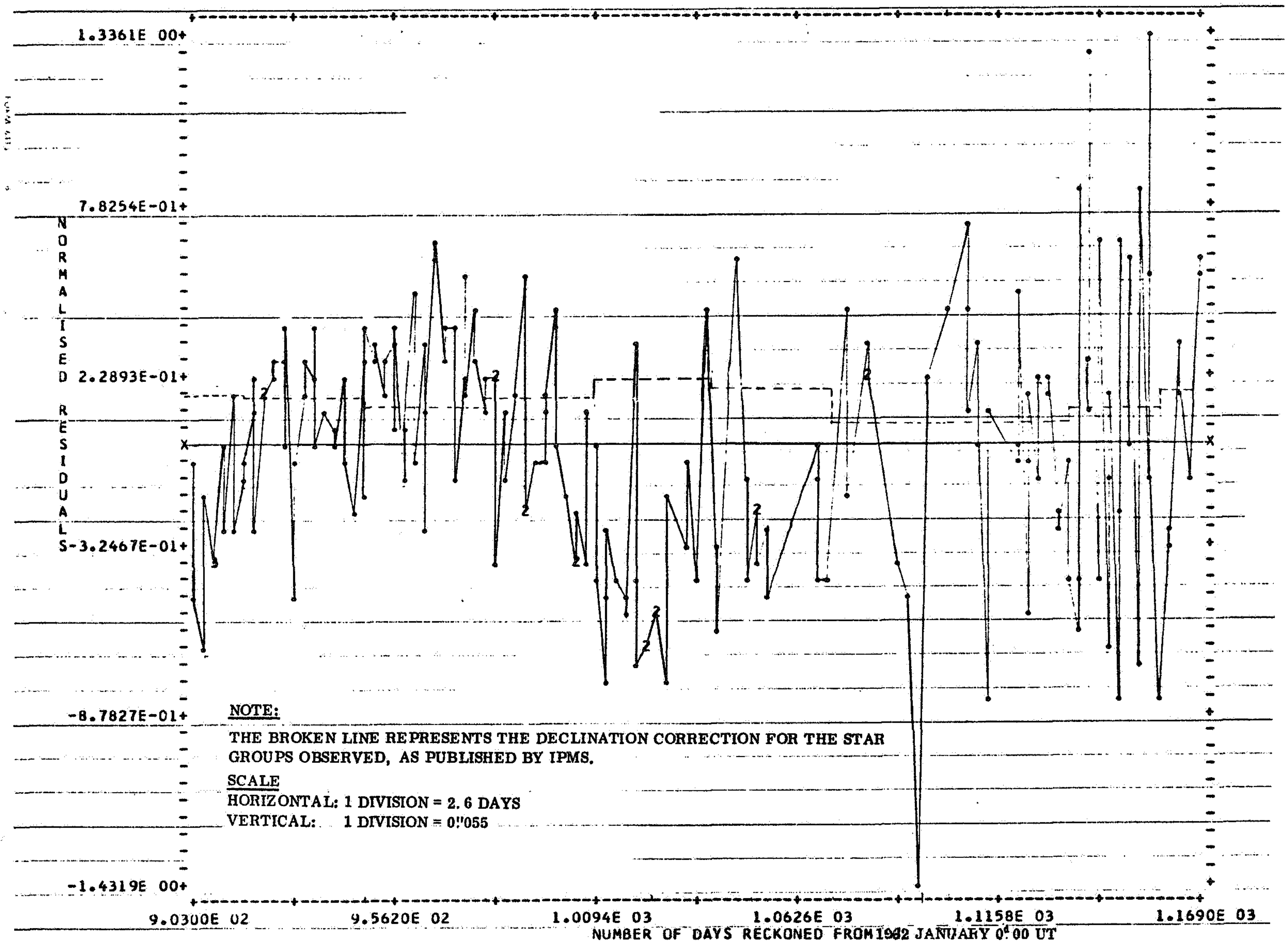


Fig. 3.6b. Plot of Normalized Residuals, Set III

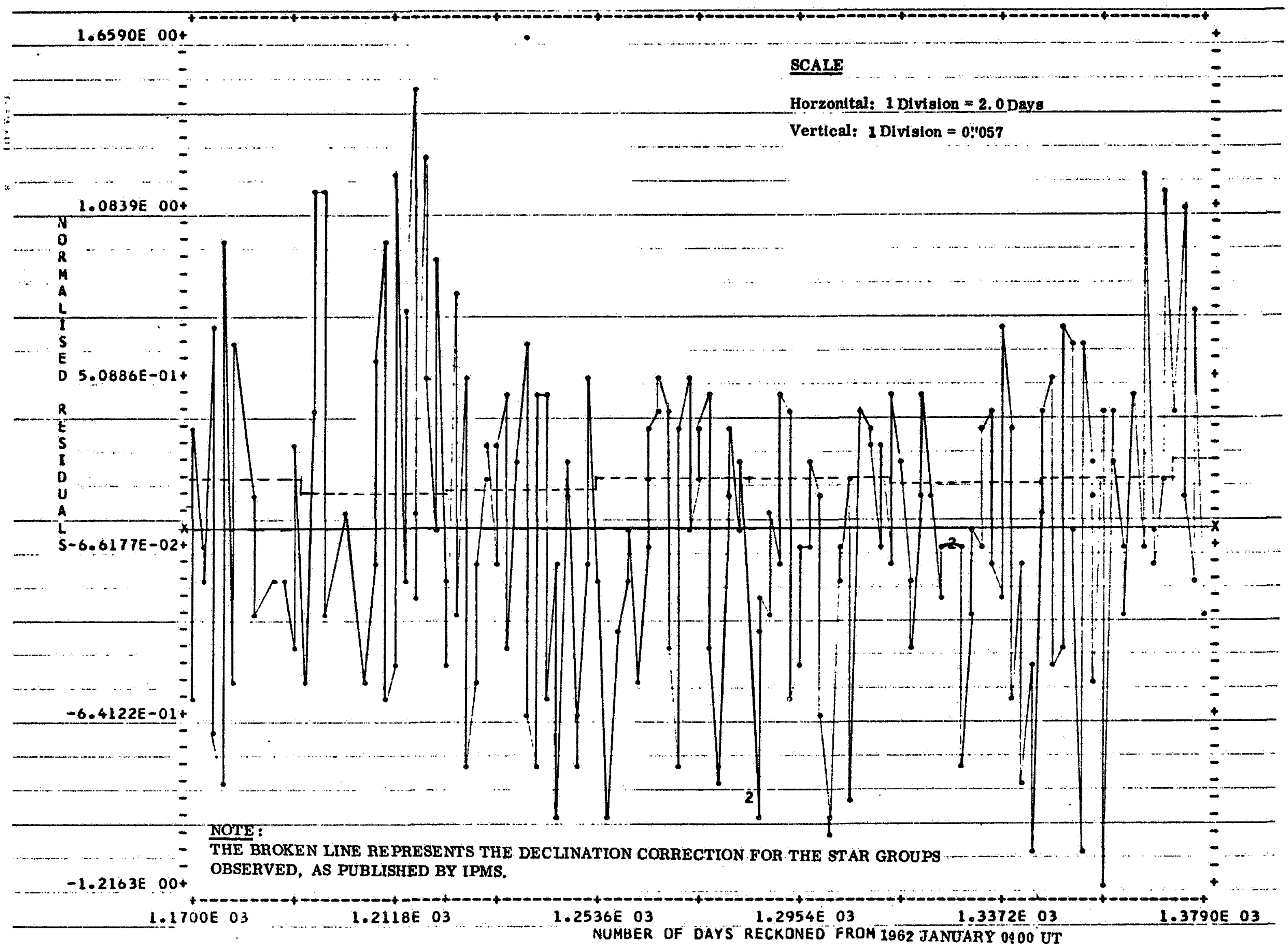


Fig. 3.6c. Plot of Moving Means of Normalized Residuals, Set III

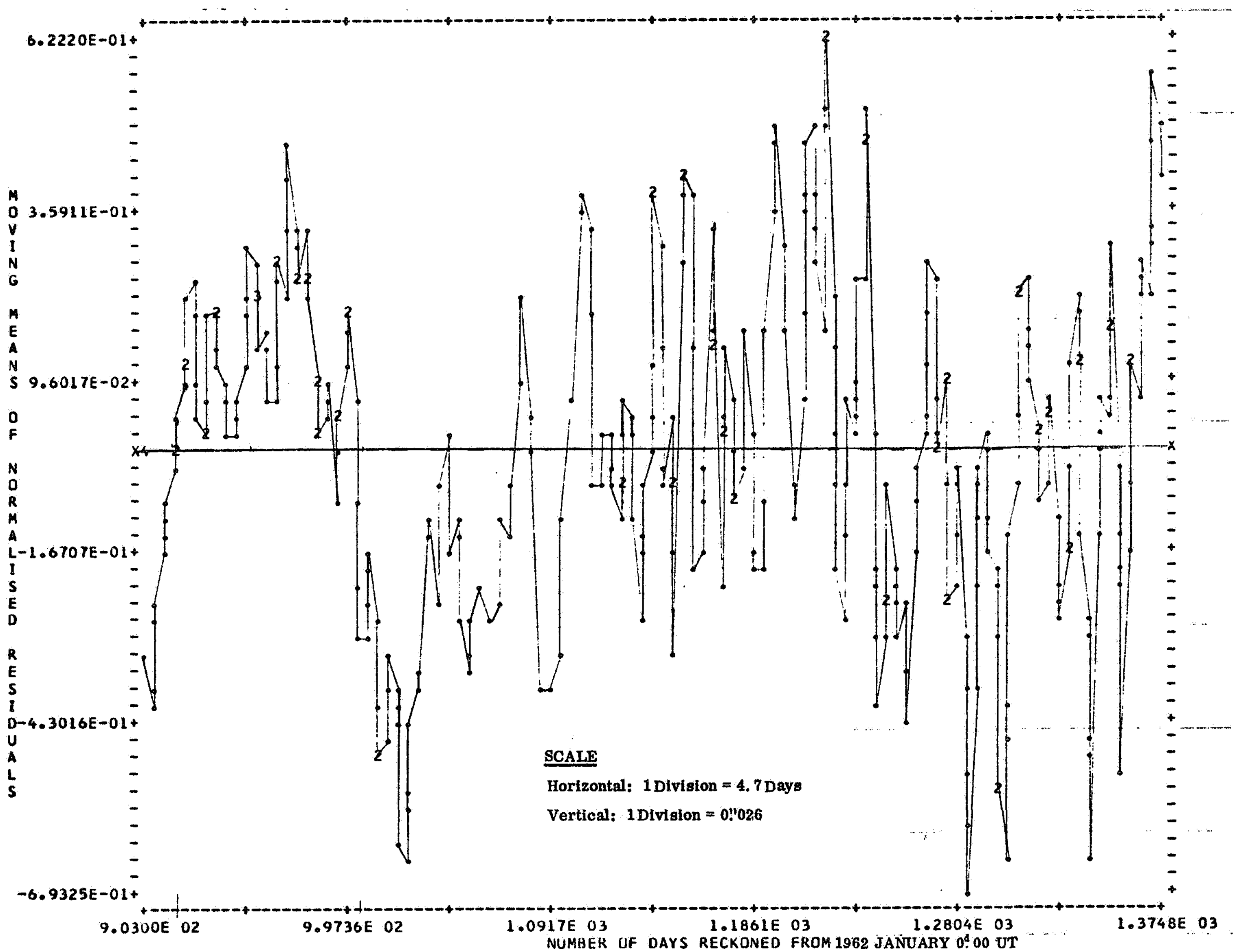


Fig. 3.7a. Plot of Normalized Residuals, Set IV

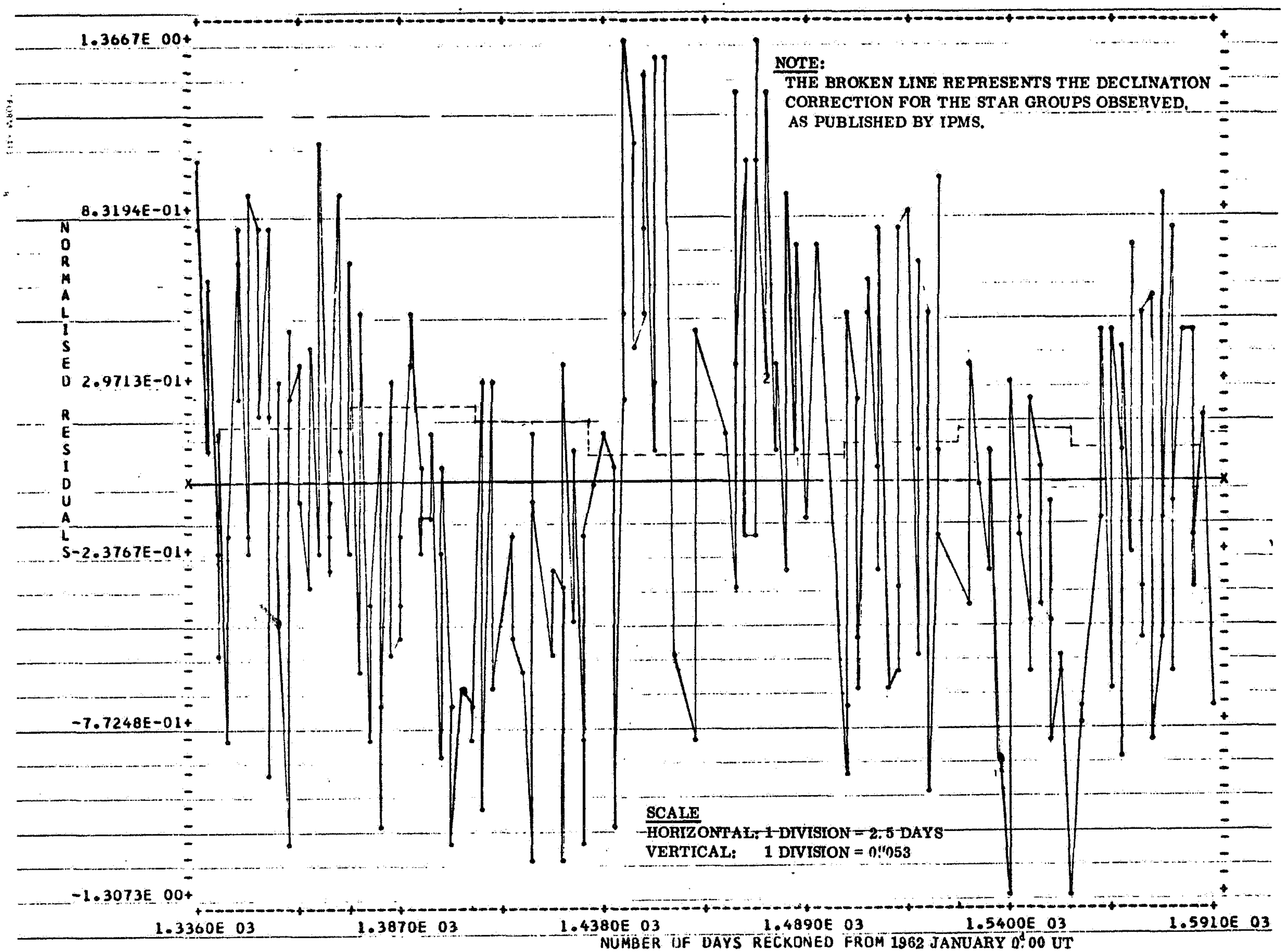


Fig. 3.7b. Plot of Normalized Residuals, Set IV

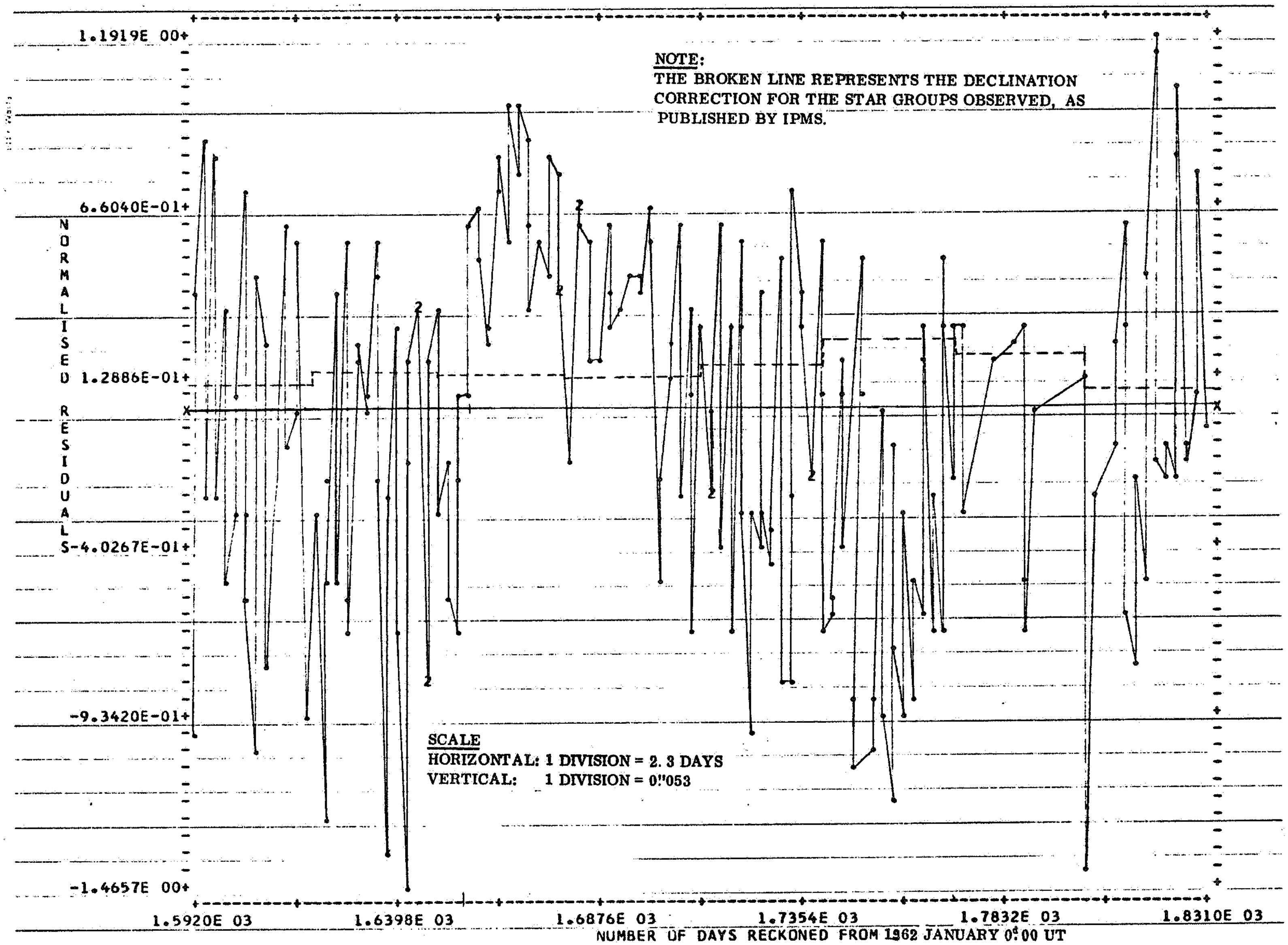


Fig. 3.7c. Plot of Moving Means of Normalized Residuals, Set IV

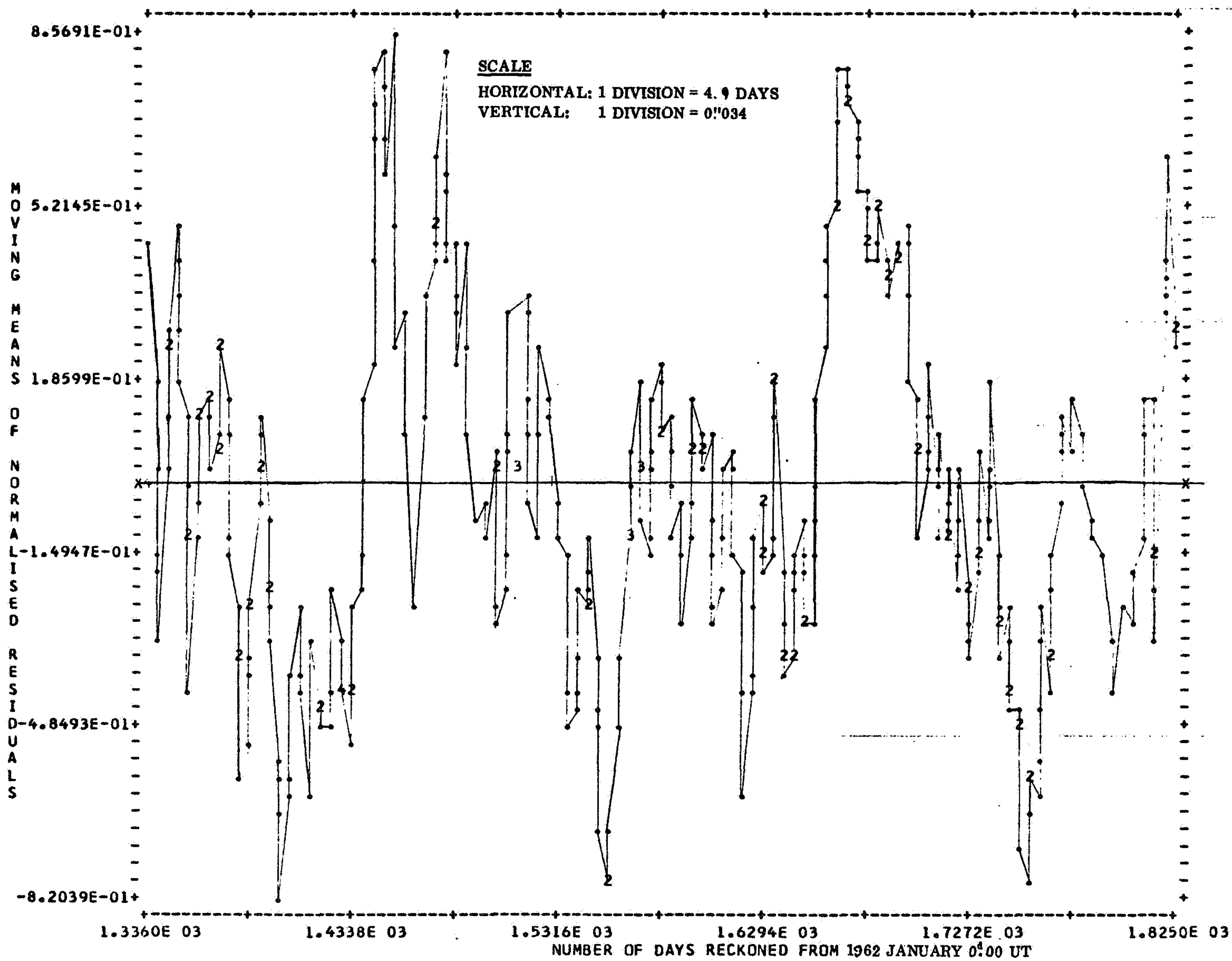


Table 3.26

Values of r, f, and u at Regions Suspected of Breaks

Set	Region Day*	Maximum Value of Normalized Residual		Minimum Value of Normalized Residual		r	Stand. Dev. of Fit	f	u
		Day	Value	Day	Value				
I	438	435	0''773	442	-0''526	-1''299	0''302	4.30	1.11
II	516	515	0.803	516	-0.731	-1.534	0.285	5.38	1.39
	687	689	0.384	685	-0.958	1.342		4.71	1.21
	828	831	0.526	825	-0.604	1.130		3.96	1.02
	859	856	0.818	863	-0.421	-1.239		4.35	1.12
	886	883	0.638	890	-0.560	-1.198		4.20	1.08
III	1100	1107	0.738	1094	-1.432	2.170	0.488	4.45	1.15
	1154	1155	1.336	1152	-0.695	2.031		4.16	1.07
IV	1442	1443	1.350	1441	-1.090	2.440	0.594	4.10	1.06

Table 3.27

Values of m, g, w at Regions Suspected of Breaks

Set	Region Day*	Maximum Value of Moving Means of Normalized Residual		Minimum Value of Moving Means of Normalized Residual		m	Stand. Dev. of Fit	g	w
		Day	Value	Day	Value				
I	410	398.8	0''491	421.8	-0''230	-0''721	0''302	2.39	1.18
II	706	708.6	0.311	703.0	-0.211	0.522	0.285	1.83	0.91
	857	854.0	0.460	861.8	-0.198	-0.658		2.31	1.14
IV	1439	1445.0	0.773	1432.0	-0.505	1.278	0.594	2.15	1.06

*Values in this column represent the mean of the days on which the maximum and minimum values of the residuals have been taken.

The values in the above tables have been taken from the Omnitab printouts.

Against our criteria for evidence of a short-period break of a magnitude used in the simulation program, there is only one region (Day 516) where the value of f is 5.38 against our criterion of 5.06. However, Day 516 lies in the overlap portion between Sets I and II. The value given in Table 3.26 pertains to the polynomial fit for Set II. In the polynomial fit for Set I, the value of f near Day 516 is only 3.19.

Thus we can conclude that the data does not contain a break of the magnitude and duration introduced in the simulation program. It is also noticed from the above Tables 3.26 and 3.27 that around Days 1439, 1441, the value of f is 4.10 and the value of g is 2.15. In our simulation program we have assumed that the period of 251 days from Day 1351 to 1601 is free of breaks. Therefore, the values of $f = 4.10$ and $g = 2.15$ cannot represent any anomalous behavior.

This prompted an attempt to locate and tabulate only those regions where the values of f and g are equal to or larger than 4.2 and 2.2 respectively. Table 3.28 shows such regions.

Table 3.28

Regions Where $f \geq 4.20$ and/or $g \geq 2.2$

Region/Day	Corresponding Date	f	u	g	w
410	Feb 14, 1963			2.39	1.18
438	Mar 14, 1963	4.30	1.11		
516	May 31, 1963	5.38	1.39		
687	Nov 18, 1963	4.71	1.21		
857	May 6, 1964			2.31	1.14
859	May 8, 1964	4.35	1.12		
886	Jun 4, 1964	4.20	1.08		
1100	Jan 4, 1965	4.45	1.15		

Of the regions tabulated above, no break can be suspected around Day 516 for reasons already explained. Comparison of the remaining regions in the table with the days of the reported earthquakes gives an interesting observation.

Days 857 and 859 in the above table are about 40 days beyond Day 818 of the reported earthquake. Day 886 in the above table is about 10 days beyond Day 877 of the reported earthquake. Day 1100 in the above table is 20 days before Day 1120 and 31 days before Day 1131, which are the days of the reported earthquakes. Day 687 in the above table is 36 days beyond Day 651 of the reported earthquake. Such a correspondence between the days of anomalous behavior of residuals and the days of the earthquakes is not likely to be a mere coincidence. However, it is not possible to draw any definite conclusions on the basis of the above observations as to whether earthquakes cause local crustal displacements, which give rise to such anomalous patterns.

Out of the regions listed in Table 3.3, Days 857, 859, 886 and 1100 fall close to the 6th of the month which is generally the day there is a change in the star groups observed and consequentially a differential declination error in the observed value of the latitude. With the help of the published values of the declination corrections, it was investigated whether the large values of f and g could be due to the differential declination error involved. It was found that the effect of the differential declination error was negligible. See Appendix E for these calculations.

CHAPTER IV

EXPERIMENTS WITH SECULAR VARIATIONS

4.1 General Discussion

The aim of the investigation was to isolate the secular element in the latitude variation and to see to what extent the secular changes are compatible with the recent theory of continental drift.

As already mentioned the latitude variation at a place considered as a function of time consists mainly of a periodic portion and an aperiodic portion apart from sudden changes considered in Chapter III. It has been observed that over a period of about six years the periodic motion of the true pole with respect to the earth's crust averages out [Mueller, 1969, p. 81]. Since this observation is directly deduced from latitude observations, it is reasonable to assume that the periodic portion of the latitude variation averages out over a period of about six years. Therefore a regression line fitted over well-selected observations in blocks of six-year periods should give a reasonable estimate of the direction and magnitude of the aperiodic (or secular) portion of the latitude variation. Markowitz [1967, p. 25] has published corrected values for latitude variation at each of the five latitude stations at six-year intervals from 1903.0 to 1963.0. Not all intervening values have been given. A straight line fit has been given to the values published by Markowitz and the slope of the line obtained has been taken as the secular variation of latitude at each station, with the uncertainties as obtained in the fitting of the regression line.

The secular variation obtained as above was then sought to be compared with the magnitude and direction of the continental drift as hypothesized by Le Pichon [1968, p. 3661]. According to Le Pichon large blocks of the earth's surface undergo displacements and the only modifications of the blocks occur along some or all of their boundaries, that is, the crests of the mid-ocean ridges where crustal material may be added, and their associated transform faults and the active trenches and regions of active folding or thrusting where crustal material may be lost or shortened. Then the relative displacement of any block with respect to another is a rotation on the spherical surface of the earth. Le Pichon [1968, p. 3674] explains the determination of the movements between blocks as follows:

> If we assume that the earth is spherical and the the length of its radius does not change with time, we can then proceed to the complete determination of the movements of the major crustal blocks relative to each other. This, of course, presupposes the determination of the boundaries of the blocks, other than ridge crests, i.e., lines of compression or shear between blocks. It is further necessary to assume that all blocks and consequently all ridge crests and other boundaries, may migrate over the surface of the earth. To make the problem entirely determinate, we divide the earth's surface into six rigid blocks which stay undeformed except at their boundaries where surface may be added or destroyed. These simplifications will lead to a mathematical solution which can be considered a first-approximation solution to the actual problem of the earth's surface displacements.

Le Pichon has divided the earth model into six large rigid blocks and has obtained the parameters of the rotations.

Fig. 4.1 shows a map showing the surface of the globe divided into the six rigid blocks. Fig. 4.2 shows the data available on sea floor spreading and the location of the ridges, trenches, and centers of rotation.

Le Pichon has calculated the centers of rotation based on the rates of sea floor spreading and assuming relative movements of two plates (considered two at a time) about the axes of the ridges. Certain anomolous situations are obtained. For example, if there is an opening about the mid-Atlantic ridge and the American continent is moving relatively away from Africa and there is also a similar relative movement between the Pacific and the American continent away from each other about the ridges west of the continent, it is difficult to decide the relative movement of the North American continent with respect to the rotation axis of the earth. There are no active trenches corre-

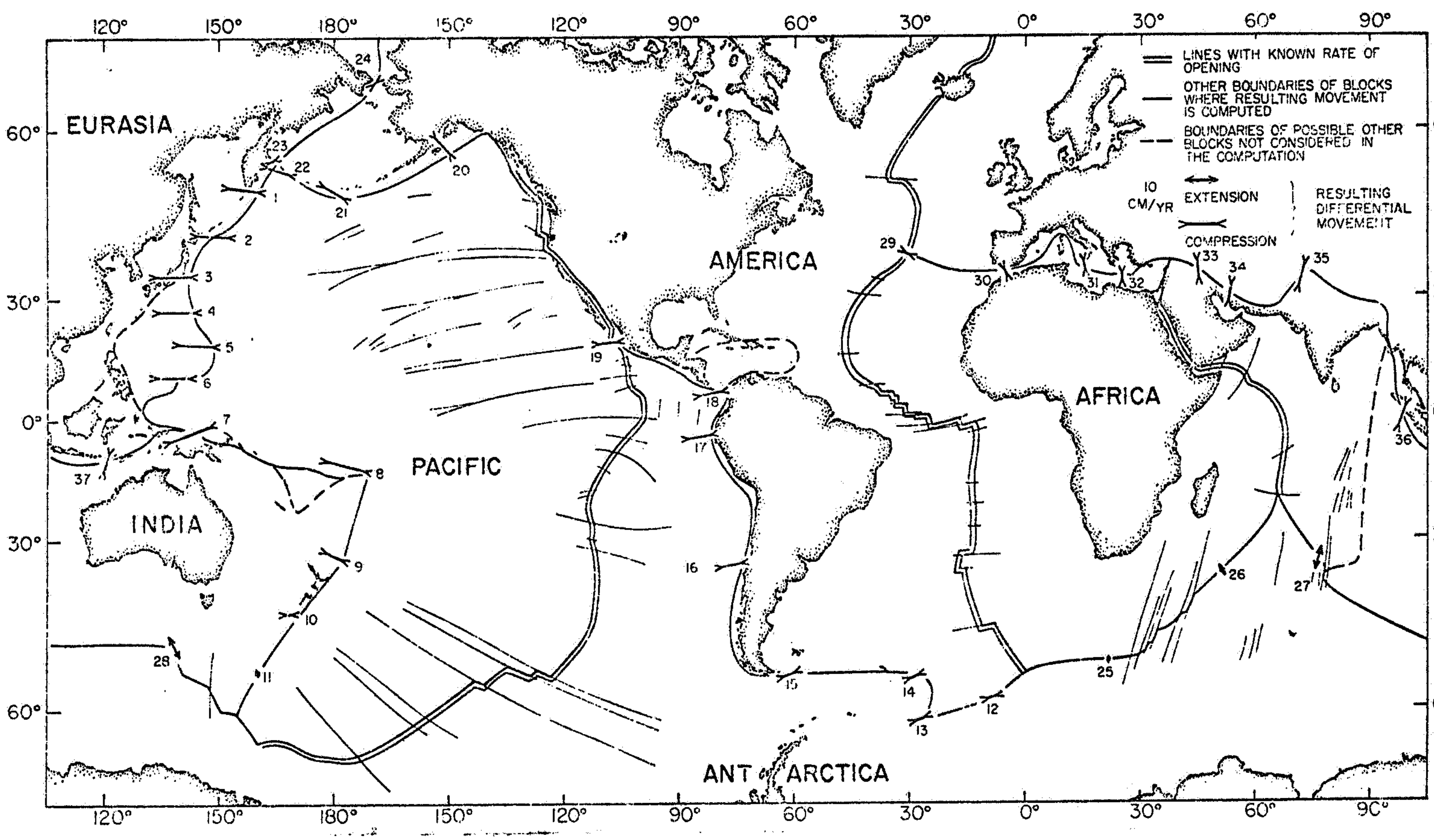
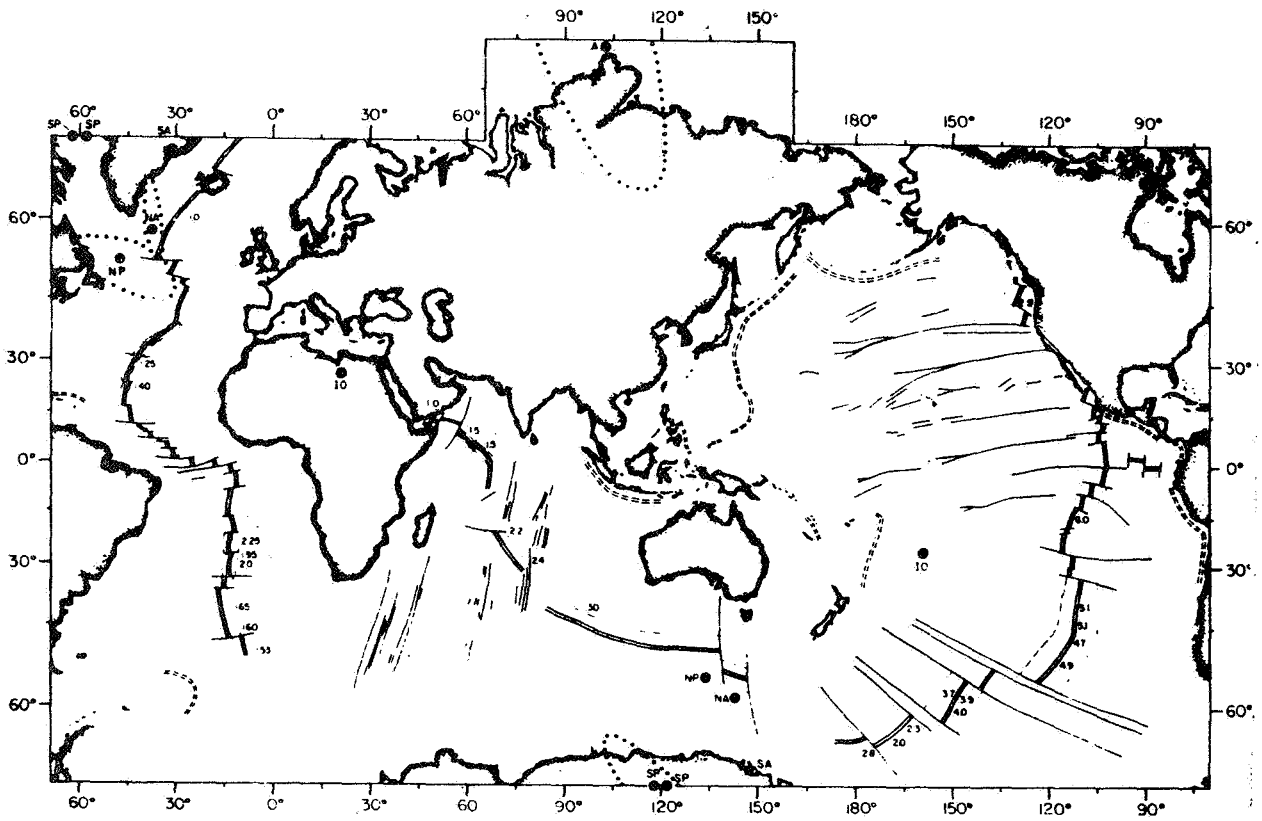


Fig. 4.1 Location of the boundaries of the six blocks



Available data on sea-floor spreading. The axes of the actively spreading mid-ocean ridges are shown by a double line; the fracture zones by a single line; the active trenches by a double dashed line. The spreading rates are given in centimeters per year. The locations of the centers of rotation are shown by X; NA stands for North Atlantic; SA for South Atlantic ; NP for North Pacific; SP for South Pacific; IO for Indian Ocean; A for Arctic.

Fig. 4.2 Sea floor spreading

sponding to every active ridge to explain such movements.

The problem of finding out the drift at any point with respect to the rotation axis of the earth from geophysical data for comparing this with the secular variation of latitude can be approached in one of the following ways.

(i) The probable direction of the continental drift at a station can be worked out separately based on the relative movements of each pair of plates on which the station is located and assuming in each case that the ridge is fixed. The magnitude of the drift can be taken as the actual sea floor spreading rate near the station location and the center of rotation can be taken from LePichon's calculation. Comparison of these with the secular latitude variation obtained will indicate which of the pairs of the plate motions correspond to the secular variation of latitude.

If this process is carried out for all the latitude stations it should be possible to arrive at a consistent interpretation of the relative movement of plates in terms of movements with respect to an earth-fixed coordinate system and to clarify in case of which plate motion there is doubt about the ridge being stationary provided of course that our assumption that a part of the secular variation in latitude is due to the continental drift is correct.

(ii) It can be assumed that at any station the velocity vectors due to the relative movements of the pairs of plates concerned (assuming in each case that the ridge is stationary) act at the point simultaneously and from the parameters of rotation given by LePichon the resultant magnitude and direction of the drift can be arrived at.

In this approach, analysis at one station can prove the validity or otherwise of the following assumptions, taken together:

(a) relative movements take place according to the hypothesis of LePichon,

(b) the relative plate movements take place, in each case, with the ridge stationary,

(c) continental drifts account for part of the secular variation of latitude at a station, as derived from the data of the latitude stations,

(d) there are no trenches between the ridge and the station location along the direction of rotation.

According to this approach also, from analysis at all the latitude stations we should be able to achieve the same aim as in approach (i) above, by assuming that the velocity vector at a station as derived from the latitude variation is a linear combination of the components of the relative velocity vectors of all the pairs of plates concerned in the direction of the meridian. Analysis of all the stations would help evaluate the coefficients in the linear combinations.

4.2 Aim and Approach of Analysis

Of the five I P M S stations, Ukiah and Gaithersburg are situated on the America-block while Carloforte, Kitab and Mizusawa are located on the Eurasia-block. The values of the coordinates of the pole of rotation for America-Eurasia-blocks, and the relative angular velocity between the blocks, as obtained by Le Pichon, have been utilized in the analysis, in respect of continental drift.

I P M S determination of the pole positions, also indicates a displacement of $0''.003$ to $0''.006$ per year in the direction of the 285° meridian [Mueller, 1969, p. 82]. This information has been utilized in the adjustment procedures, in respect of the actual secular motion of the pole.

It was aimed to subject this information to an adjustment procedure, to find out, to what extent, the astronomical information is compatible with the geophysical information.

Following notations have been used:

Subscripts $_{1}$, $_{2}$, $_{3}$, $_{4}$, $_{5}$, stand for I P M S stations Ukiah, Gaithersburg, Carloforte, Kitab and Mizusawa respectively.

Subscripts $_{\mathrm{I}}$, $_{\mathrm{II}}$ stand for America-block and Eurasia-block respectively.

s_j Secular variation of latitude at the I P M S station

"$_J$" as obtained from the values published by Markowitz (a positive sign indicating an increase in latitude).

φ_o Latitude of the pole of rotation between blocks $_I$ and $_{II}$.

λ_o Longitude of the pole of rotation between blocks $_I$ and $_{II}$.

ω Angular velocity of the block, around the pole of rotation (φ_o, λ_o) (a positive sign indicating counterclockwise rotation).

α East longitude, indicating the direction of the actual secular polar motion.

a Rate of the actual secular polar motion.

r $= \omega_I - \omega_{II}$ or the relative angular velocity between blocks $_I$ and $_{II}$ (a positive value of r indicates that the blocks are getting closer to each other causing compression. A negative value of r will indicate extension or spreading, at the boundary between the two blocks).

λ_J Longitude of the I PMS station "$_J$".

φ_J Latitude of the I PMS "$_J$".

L_a Array of adjusted values of observed quantities.

X_a Array of adjusted values of parameters.

m_o^2 Variance of unit weight.

In a spherical earth model, consider two geocentric right-handed orthogonal coordinate systems U,V,W, and X_o, Y_o, Z_o, defined as follows:

U axis - in the equatorial plane, in the direction of the Greenwich Mean Astro-meridian

V axis - in the equatorial plane perpendicular to U in an easterly direction

W axis - through the Conventional International Origin (C.I.O.).

X_o axis - in the line of intersection between the meridian plane through the pole of block rotation (φ_o,λ_o) and the plane secondary to this pole of rotation (positive south of the equator)

Y_o axis - in the plane secondary to the pole of block rotation (φ_o,λ_o) and perpendicular to X_o, in an easterly direction

[It can be seen that Y_o axis is also in the line of intersection between the equatorial plane and the plane secondary to the pole of block rotation]

Z_o axis - through the pole of block rotation

Angular velocity ω_I of block $_I$ can be completely described as a vector in the X_o, Y_o, Z_o system, by

$$\vec{\omega}_I = 0(\vec{i}_o) + 0(\vec{j}_o) + \omega_I(\vec{k}_o)$$

where $\vec{i}_o$, $\vec{j}_o$, $\vec{k}_o$ are the unit vectors in directions X_o, Y_o, Z_o.

This can be transformed to the U,V,W system giving

$$\begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix}_{\omega_I} = R_3(-\lambda_o)\, R_2(-(90-\varphi_o)) \begin{bmatrix} 0 \\ 0 \\ \omega_I \end{bmatrix}$$

$$= \begin{bmatrix} \cos\lambda_o & -\sin\lambda_o & 0 \\ \sin\lambda_o & \cos\lambda_o & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(90^\circ-\varphi_o) & 0 & \sin(90^\circ-\varphi_o) \\ 0 & 1 & 0 \\ -\sin(90^\circ-\varphi_o) & 0 & \cos(90^\circ-\varphi_o) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \omega_I \end{bmatrix}$$

$$= \begin{bmatrix} \omega_I \cos\varphi_o \cos\lambda_o \\ \omega_I \cos\varphi_o \sin\lambda_o \\ \omega_I \sin\varphi_o \end{bmatrix}$$

The elements of this column matrix represent the components of the angular velocitites of the crust about U,V,W axes, due to the angular velocity ω_I of block $_I$, about the pole (φ_o, λ_o), and affects all points on block $_I$.

Now consider U,V,W system along with X_p, Y_p, Z_p geocentric Cartesian coordinate system described below

X_p axis - in the equatorial plane of the earth in the direction of longitude α

Y_p axis - in the equatorial plane of the earth perpendicular to X_p, and in an easterly direction

Z_p axis - through the C. I. O.

A polar secular motion of magnitude, a, in the direction α, can be represented as a negative crustal rotation about Y_p axis as far as effect on latitude observation is concerned, and affects all points on both blocks. This rotation can be represented as

$$\vec{a} = 0(\vec{i}_p) - a(\vec{j}_p) + 0(\vec{k}_p)$$

where $\vec{i}_p$, $\vec{j}_p$, $\vec{k}_p$ are the unit vectors in the directions X_p, Y_p, Z_p.

This transformed to the U,V,W system gives

$$\begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix}_p = R_3(-\alpha) \begin{bmatrix} 0 \\ -a \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -a \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} a \quad \sin\alpha \\ -a \quad \cos\alpha \\ 0 \end{bmatrix}$$

The elements of this column matrix represent components of the angular velocities of the crust about U,V,W axes due to secular motion of the pole, a, in the direction α, and is applicable to all points on both the blocks.

Therefore the combined effect of ω_I and, a, at any point on block $_I$, will be given by

$$\begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix}_I = \begin{bmatrix} \omega_I \cos\varphi_o \cos\lambda_o + a \sin\alpha \\ \omega_I \cos\varphi_o \sin\lambda_o - a \sin\alpha \\ \omega_I \sin\varphi_o \end{bmatrix}$$

Now consider the effect of ω_I and, a, at a particular IPMS station on block $_I$, say station 1 (Ukiah), with the help of a geocentric Cartesian coordinate system G_1, E_1, N_1 defined as follows:

G_1 axis - through the location of the IPMS station 1, in the approximate direction of the local vertical

E_1 axis - in the line of intersection between the equatorial plane and the plane secondary to station 1 as a pole (positive north from the secondary plane)

N_1 axis - in the line of intersection of the meridian plane of station 1 and the secondary plane (positive north of the equator).

The angular velocities of the crust about the U, V, W, axes due to the effects of ω_1 and, a, can be transformed as under to the G_1, E_1, N_1 system at station 1.

$$\begin{bmatrix} \dot{G}_1 \\ \dot{E}_1 \\ \dot{N}_1 \end{bmatrix} = R_2(-\varphi_1)\ R_3(\lambda_1) \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix}_1$$

$$= \begin{bmatrix} \cos\varphi_1 & 0 & +\sin\varphi_1 \\ 0 & 1 & 0 \\ -\sin\varphi_1 & 0 & \cos\varphi_1 \end{bmatrix} \begin{bmatrix} \cos\lambda_1 & \sin\lambda_1 & 0 \\ -\sin\lambda_1 & \cos\lambda_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{U} \\ \dot{V} \\ \dot{W} \end{bmatrix}_1$$

$$= \begin{bmatrix} \omega_1 (\cos\varphi_0 \cos\lambda_0 \cos\varphi_1 \cos\lambda_1 + \cos\varphi_0 \sin\lambda_0 \cos\varphi_1 \sin\lambda_1 + \sin\varphi_0 \sin\varphi_1) \\ + a(\sin\alpha\cos\lambda_1 \cos\varphi_1 - \cos\alpha\sin\lambda_1 \cos\varphi_1) \\ \\ \omega_1 (\cos\varphi_0 \sin\lambda_0 \cos\lambda_1 - \cos\varphi_0 \cos\lambda_0 \sin\lambda_1) \\ - a(\sin\alpha\sin\lambda_1 + \cos\alpha\cos\lambda_1) \\ \\ -\omega_1 (\cos\varphi_0 \sin\lambda_0 \sin\varphi_1 \sin\lambda_1 + \cos\varphi_0 \cos\lambda_0 \sin\varphi_1 \cos\lambda_1 \\ - \sin\varphi_0 \cos\varphi_1) - a(\sin\alpha \cos\lambda_1 \sin\varphi_1 - \cos\alpha \sin\lambda_1 \sin\varphi_1) \end{bmatrix}$$

The elements of this column vector represent the components of the angular velocities of the crust about G_1, E_1, N_1 axes at station 1, due to the effects of ω_1 and, a.

The secular variation of latitude at station 1, s_1, can be represented as a negative angular velocity of the crust about E_1 axis, giving

$$\vec{s}_1 = 0(\vec{i}_1) - s_1 (\vec{j}_1) + 0(\vec{k}_1)$$

where $\vec{i}_1$, $\vec{j}_1$, $\vec{k}_1$ are unit vectors in the G_1, E_1, N_1 directions.

$$\therefore \quad \begin{bmatrix} \dot{G}_1 \\ \dot{E}_1 \\ \dot{N}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -s_1 \\ 0 \end{bmatrix}$$

Therefore, the mathematical model for the secular variation of latitude at station 1 can be formed as under

$$-s_1 = \omega_{\mathrm{I}} \cos\varphi_o (\sin\lambda_o \cos\lambda_1 - \cos\lambda_o \sin\lambda_1) - a(\sin\alpha \sin\lambda_1 + \cos\alpha \cos\lambda_1)$$

similar expressions for stations 2, 3, 4, and 5 will be,

$$-s_2 = \omega_{\mathrm{I}} \cos\varphi_o (\sin\lambda_o \cos\lambda_2 - \cos\lambda_o \sin\lambda_2) - a(\sin\alpha \sin\lambda_2 + \cos\alpha \cos\lambda_2)$$

$$-s_3 = \omega_{\mathrm{II}} \cos\varphi_o (\sin\lambda_o \cos\lambda_3 - \cos\lambda_o \sin\lambda_3) - a(\sin\alpha \sin\lambda_3 + \cos\alpha \cos\lambda_3)$$

$$-s_4 = \omega_{\mathrm{II}} \cos\varphi_o (\sin\lambda_o \cos\lambda_4 - \cos\lambda_o \sin\lambda_4) - a(\sin\alpha \sin\lambda_4 + \cos\alpha \cos\lambda_4)$$

$$-s_5 = \omega_{\mathrm{II}} \cos\varphi_o (\sin\lambda_o \cos\lambda_5 - \cos\lambda_o \sin\lambda_5) - a(\sin\alpha \sin\lambda_5 + \cos\alpha \cos\lambda_5)$$

These five equations formed the basis for adjustments.

The adjustment was handled separately in four ways.

In method (a), s_1, s_2, s_3, s_4, s_5 were taken as observed quantities, ω_{I}, ω_{II}, and, a, were taken as unknown parameters. The observed quantities were assumed to have equal weights. φ_o, λ_o, and α were given fixed values as known to us. The five equations of the form $L_a = F(X_a)$, with three unknowns yield a least square solution.

In method (b), s_1, s_2, s_3, s_4, s_5, φ_o, λ_o, and α were taken as observed quantities with known variances. ω_{I}, ω_{II}, and, a, were taken as unknowns and a least square solution was obtained in a system $F(L_a, X_a) = 0$.

In method (c), a special case of method (b), ω_{I} and ω_{II} were taken as zero and, a, as the only unknown.

In method (d), another special case of method (b), a, was taken as zero and ω_{I} and ω_{II} as the only unknowns.

4.3 Data and Calculations

For obtaining values of s_1, s_2, s_3, s_4, s_5, a regression line was fitted to the values of observed latitude for the stations published by Markowitz [1967, p. 25].

These, with their uncertainties as obtained from the straight line fit, are given below in units of 10^{-3} arc sec/year

$$s_1 = 2.60 \pm 0.72$$
$$s_2 = 3.06 \pm 0.41$$
$$s_3 = 0.35 \pm 0.42$$
$$s_4 = -2.14 \pm 0.42$$
$$s_5 = -3.13 \pm 0.24$$

The values for the coordinates of the pole of block rotations, was taken from Le Pichon [1968, p. 3665]. The computation of these coordinates is based on the spreading rates away from ridge lines which are generally in the North South direction. Hence the uncertainty in latitude is assumed to be larger than in longitude. The uncertainty in the position of the pole, is $9\overset{\circ}{.}1$ as given by Le Pichon. The following values, with their uncertainties were used in the calculations:

$$\varphi_o = 78^\circ \text{ N} \pm 8\overset{\circ}{.}8$$
$$\lambda_o = 102^\circ \text{ E} \pm 2\overset{\circ}{.}2$$

The value for α was taken as 285° [Mueller, 1969, p. 82]. Taking into consideration, the plots of pole positions, the uncertainty in α was taken as $\pm 10^\circ$.

The known coordinates of the IPMS latitude stations Ukiah, Gaithersburg, Carloforte, Kitab, and Mizusawa were used for the values φ_j and λ_j.

Method (a):

The observation equations reduce to the form of

$$AX + L = V$$

where $A = \partial L/\partial X$

$X = X_a - X_o$, the array of corrections to unknowns

X_o = Array of approximate values of unknowns

$L = L_o - L_b$

$L_o = F(X_o)$

L_b = Array of observed values of observable quantities

V = Array of residuals

The weight matrix of observed quantities, P, was taken as an identity.

The normal matrix $N = A'A$

$$X = -N^{-1} A'L$$

D. F. = 2, as there are five observed quantities and three unknowns, variance of unit weight $m_o^2 = V'V$

Variance covariance matrix of parameters $\Sigma_x = m_o^2$

The A matrix takes the form of

$$\begin{array}{ccc} \omega_I & \omega_{II} & a \end{array}$$

$$\begin{bmatrix} \cos\varphi_o(\sin\lambda_o\cos\lambda_1-\cos\lambda_o\sin\lambda_1) & 0 & -(\sin\alpha\sin\lambda_1+\cos\alpha\cos\lambda_1) \\ \cos\varphi_o(\sin\lambda_o\cos\lambda_2-\cos\lambda_o\sin\lambda_2) & 0 & -(\sin\alpha\sin\lambda_2+\cos\alpha\cos\lambda_2) \\ 0 & \cos\varphi_o(\sin\lambda_o\cos\lambda_3-\cos\lambda_o\sin\lambda_3) & -(\sin\alpha\sin\lambda_3+\cos\alpha\cos\lambda_3) \\ 0 & \cos\varphi_o(\sin\lambda_o\cos\lambda_4-\cos\lambda_o\sin\lambda_4) & -(\sin\alpha\sin\lambda_4+\cos\alpha\cos\lambda_4) \\ 0 & \cos\varphi_o(\sin\lambda_o\cos\lambda_5-\cos\lambda_o\sin\lambda_5) & -(\sin\alpha\sin\lambda_5+\cos\alpha\cos\lambda_5) \end{bmatrix}$$

X_o was taken as zero.

Method (b):

The general mathematical model is

$$F(L_a, X_a) = 0$$

The observation equations take the form of

$$BV + AX + W = 0$$

where $B = \partial F/\partial L \ (L_b, X_o)$

$A = \partial F/\partial X \ (L_b, X_o)$

$W = F(L_b, X_o)$

$$P = \text{weight matrix} = \begin{bmatrix} \frac{1}{m_{s_1}^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_{s_2}^2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{m_{s_3}^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{m_{s_4}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{m_{s_5}^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{m_{\varphi_o}^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{m_{\lambda_o}^2} & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{m_{\alpha}^2} \end{bmatrix}$$

where m_j = uncertainty in the observed quantity j

$M = BP^{-1}B'$

$R = A'M^{-1}A$

$X = -R^{-1} A'M^{-1}W$

$K_L = -M^{-1}(AX + W)$

$V = P^{-1}B'K_L$

$m_o^2 = \dfrac{V'PV}{D.F.}$ D.F.= 2 in this case

The B matrix takes the following form:

$$
\begin{array}{ccccccccc}
s_1 & s_2 & s_3 & s_4 & s_5 & \varphi_\circ & \lambda_\circ & \alpha \\
\end{array}
$$

$$
\begin{bmatrix}
+1 & 0 & 0 & 0 & 0 & \omega_{I}\sin\varphi_\circ(\cos\lambda_\circ\sin\lambda_1-\sin\lambda_\circ\cos\lambda_1) & \omega_{I}\cos\varphi_\circ(\sin\lambda_\circ\sin\lambda_1+\cos\lambda_\circ\cos\lambda_1) & a(\sin\alpha\cos\lambda_1-\cos\alpha\sin\lambda_1) \\
0 & +1 & 0 & 0 & 0 & \omega_{I}\sin\varphi_\circ(\cos\lambda_\circ\sin\lambda_2-\sin\lambda_\circ\cos\lambda_2) & \omega_{I}\cos\varphi_\circ(\sin\lambda_\circ\sin\lambda_2+\cos\lambda_\circ\cos\lambda_2) & a(\sin\alpha\cos\lambda_2-\cos\alpha\sin\lambda_2) \\
0 & 0 & +1 & 0 & 0 & \omega_{II}\sin\varphi_\circ(\cos\lambda_\circ\sin\lambda_3-\sin\lambda_\circ\cos\lambda_3) & \omega_{II}\cos\varphi_\circ(\sin\lambda_\circ\sin\lambda_3+\cos\lambda_\circ\cos\lambda_3) & a(\sin\alpha\cos\lambda_3-\cos\alpha\sin\lambda_3) \\
0 & 0 & 0 & +1 & 0 & \omega_{II}\sin\varphi_\circ(\cos\lambda_\circ\sin\lambda_4-\sin\lambda_\circ\cos\lambda_4) & \omega_{II}\cos\varphi_\circ(\sin\lambda_\circ\sin\lambda_4+\cos\lambda_\circ\cos\lambda_4) & a(\sin\alpha\cos\lambda_4-\cos\alpha\sin\lambda_4) \\
0 & 0 & 0 & 0 & +1 & \omega_{II}\sin\varphi_\circ(\cos\lambda_\circ\sin\lambda_5-\sin\lambda_\circ\cos\lambda_5) & \omega_{II}\cos\varphi_\circ(\sin\lambda_\circ\sin\lambda_5+\cos\lambda_\circ\cos\lambda_5) & a(\sin\alpha\cos\lambda_5-\cos\alpha\sin\lambda_5)
\end{bmatrix}
$$

The A matrix takes the following form:

$$
\begin{array}{ccc}
\omega_{I} & \omega_{II} & a \\
\end{array}
$$

$$
\begin{bmatrix}
\cos\varphi_\circ(\sin\lambda_\circ\cos\lambda_1-\cos\lambda_\circ\sin\lambda_1) & 0 & -(\sin\alpha\sin\lambda_1+\cos\alpha\cos\lambda_1) \\
\cos\varphi_\circ(\sin\lambda_\circ\cos\lambda_2-\cos\lambda_\circ\sin\lambda_2) & 0 & -(\sin\alpha\sin\lambda_2+\cos\alpha\cos\lambda_2) \\
0 & \cos\varphi_\circ(\sin\lambda_\circ\cos\lambda_3-\cos\lambda_\circ\sin\lambda_3) & -(\sin\alpha\sin\lambda_3+\cos\alpha\cos\lambda_3) \\
0 & \cos\varphi_\circ(\sin\lambda_\circ\cos\lambda_4-\cos\lambda_\circ\sin\lambda_4) & -(\sin\alpha\sin\lambda_4+\cos\alpha\cos\lambda_4) \\
0 & \cos\varphi_\circ(\sin\lambda_\circ\cos\lambda_5-\cos\lambda_\circ\sin\lambda_5) & -(\sin\alpha\sin\lambda_5+\cos\alpha\cos\lambda_5)
\end{bmatrix}
$$

The following values were taken for X_o

$$X_o = \begin{bmatrix} \omega_{Io} \\ \omega_{IIo} \\ a_o \end{bmatrix} = \begin{bmatrix} 0.1 \times 10^{-5} \\ 0.1 \times 10^{-5} \\ 0.1 \times 10^{-4} \end{bmatrix} \quad \text{seconds of arc per year.}$$

Methods (c) and (d) were just variations of method (b) assigning specific values to the unknowns.

Table 4.1 gives the results obtained.

Table 4.1 RESULTS OF ANALYSIS

Method	Details of the Adjustment Procedure				Results			
	Unknowns	Observed Quantities	Constraints	Degrees of Freedom	Values of Unknowns 10^{-3} sec/yr	Adjusted Values of Observed Quantities 10^{-3} sec/yr	degrees	$V'PV$
a	$\omega_{I}, \omega_{II}, a$	s_1, s_2, s_3, s_4, s_5	$\varphi_o = 78°N$ $\lambda_o = 102°E$ with zero variance	2	$\omega_{I} = 3.3 \pm 2.8$ $\omega_{II} = -1.5 \pm 1.4$ $a = 3.3 \pm 0.2$	$s_1 = 2.6$ $s_2 = 2.9$ $s_3 = 0.0$ $s_4 = -1.9$ $s_5 = -3.5$		0.00
b	$\omega_{I}, \omega_{II}, a$	s_1, s_2, s_3, s_4, s_5 $\varphi_o, \lambda_o, \alpha$		2	$\omega_{I} = 2.8 \pm 5.5$ $\omega_{II} = -2.1 \pm 1.5$ $a = 3.3 \pm 0.3$	$s_1 = 2.6$ $s_2 = 3.3$ $s_3 = 0.8$ $s_4 = -2.3$ $s_5 = -2.9$	$\varphi_o = 78°N$* $\lambda_o = 102°E$* $\alpha = 285°E$*	2.43
c	a	s_1, s_2, s_3, s_4, s_5	$\omega_{I} = 0$ $\omega_{II} = 0$ with zero variance	4	$a = 3.5 \pm 0.2$	$s_1 = 2.3$ $s_2 = 3.5$ $s_3 = 0.4$ $s_4 = -2.7$ $s_5 = -2.8$	$\alpha = 285.°0$	4.98
d	ω_{I}, ω_{II}	s_1, s_2, s_3, s_4, s_5 φ_o, λ_o	$a = 0$ with zero variance	3	$\omega_{I} = -16.4 \pm 39.0$ $\omega_{II} = -9.7 \pm 10.2$	$s_1 = 2.4$ $s_2 = -0.04$ $s_3 = 2.0$ $s_4 = 1.2$ $s_5 = -1.3$	$\varphi_o = 78°N$* $\lambda_o = 102.°0$	194.01

*Residuals negligibly small.

CHAPTER V

CONCLUSIONS

5.1 Short Period Variations

As observed in Chapter III, on the basis of the results obtained, the IPMS data analyzed shows no incidence of sudden changes in the values of latitude of a magnitude introduced in the simulation program. However, some anomalous behavior in the residuals of the polynomial fits to the data were observed in periods which have a very rough correspondence with the dates of earthquakes, but no definite conclusions can be drawn to correlate them.

It has been demonstrated that by the approach indicated in the thesis, it is possible to identify short-period breaks of a predetermined magnitude if the observations are sensitive to short-period breaks of that magnitude.

The main difficulties in the detection of crustal movements from the analysis of latitude data are the very small crustal movements and the large noise level in the latitude observations. As seen in our experiment, the existing data is sensitive to short-period breaks of $0\overset{''}{.}3$. But breaks of such a large magnitude are improbable in physical reality.

For detection of breaks of much smaller magnitude, the observations must be more refined and the star coordinates must be more accurately known. In this respect, satellite methods open a new possiblity in detection of crustal movements.

The experiment carried out in this work should be repeated by introducing smaller amounts of breaks in the simulation program. In order to ascertain the smallest amount of break to which the existing data is sensitive, similar analysis of existing latitude observations at other IPMS stations should also be carried out to strengthen the findings presented in this thesis.

5.2 Secular Variations

In a study of this nature positive conclusions cannot be drawn becuase of the following reasons:

(a) The astronomically observed quantities are very limited in number.

(b) The geophysical data regarding the coordinates of the pole of block rotation and the relative angular velocity of the blocks, have large uncertainties. Le Pichon [1970, p. 2793] points out clearly that the different computations of relative movements, indicated by him in the earlier paper [1968, p. 3676] and used in this investigation, cannot be taken as precise estimates of the actual movements, due to the inadequacy of the data and the simplified model used by him.

(c) The pole of block rotation being very close to the rotation axis of the earth, the effect of block rotations on latitude observations, is very small.

However the following observations can be made.

(i) Because of very large residuals and variances in case (d), it is improbable that the secular variation in latitude is caused only by block rotations.

(ii) It is possible that the secular variation in latitude is caused by secular motion of the pole only, or as a combined effect of the secular motion of the pole and the block rotations, as shown in cases (a), (b), and (c).

In cases (a) and (b) ω_{I} has a positive sign indicating that the American plate moves in an easterly direction; ω_{II} has a negative sign indicating that the Eurasian plate moves in a westerly direction. This may appear to give rise to a compression where an extension (spreading) is expected, as per geophysical information. However, noting the order of the uncertainties in ω_{I} and ω_{II} and the low degree of freedom, the values of relative angular velocity, computed by Le Pichon falls within the bounds of 95% confidence interval for μ_r, the expected value of r. For example, taking the case (b),

$\omega_{\text{I}} = 0''.00280 \pm 0''.00546$ per year

$\omega_{\text{II}} = 0\ 00211 \pm 0\ 00152$ per year

Degree of freedom = 2

student statistic $t_{0.025,2} = 4.303$

$r = \omega_{\text{I}} - \omega_{\text{II}}$

$m_r^2 = m^2_{\omega_{\text{I}}} + m^2_{\omega_{\text{II}}} - 2m_{\omega_{\text{I}}\omega_{\text{II}}} = 0.0000,333,850.$

where $m_{\omega_{\text{I}}\omega_{\text{II}}}$ is the covariance between ω_{I} and ω_{II}.

$r = 0''.00491$ (compression)

$P(r - t_{0.025,2}\ (m_r) < \mu_r < r + t_{0.025,2}\ (m_r)) = 95\%$

This works to

$P(+0.00297 > \mu_r > -0.0199) = 95\%$

The value for r given by Le Pichon is 2.8×10^{-7} degrees/year [1968, p. 3665] which works to -0.001 (extension). This value thus falls within the bounds of the 95% confidence interval for μ_r as obtained from astronomical data.

The value of, a, is consistent with the order of the values already known and the variance is quite small.

Thus both continental drift and secular motion of the pole fit into the general picture of secular variation of latitude and there is no appearent contradiction.

(iii) In the final transformation matrix, the component angular velocity around N axis is a simple function of the secular variation in longitude due to the effects of ω_{I}, ω_{II}, and a.

Thus the expression

$$-\omega_{\text{I}}\,(\cos\varphi_0 \sin\lambda_0 \sin\varphi_1 \sin\lambda_1 + \cos\varphi_0 \cos\lambda_0 \sin\varphi_1 \cos\lambda_1 - \sin\varphi_0 \cos\varphi_1)$$
$$- a\,(\sin\alpha \cos\lambda_1 \sin\varphi_1 - \cos\alpha \sin\lambda_1 \sin\varphi_1)$$

is a linear function of the secular variation in longitude, at Ukiah, due to the effects of ω_{I}, ω_{II}, and, a.

Longitude observations at the latitude stations can help separate the effects of ω_{I}, ω_{II}, and, a, more reliably.

APPENDIX A

PROGRAM FOR OBTAINING DAILY MEAN LATITUDE

360/Watfor Compiler

```
C     PROGRAM TO COMPUTE MEAN OBS LAT AT UKIAH      1963
      PRINT  1000
 1000 FORMAT(' DAT1963        MEANLAT  UK       STARPAIR  SCATTER ')
      DIMENSION OBLAT(18)
 3000 READ,JDATE,KPAIR,(OBLAT(I),I=1,KPAIR)
      JDATE=JDATE+365
      SUM=0.0
      DO 2000  I=1,KPAIR
      SUM=SUM+OBLAT(I)
 2000 CONTINUE
      AVELAT=SUM/KPAIR
      OBMAX=0.0
      DO 6000 I=1,KPAIR
      IF(OBLAT(I).LE.OBMAX) GO TO 6000
      OBMAX=OBLAT(I)
 6000 CONTINUE
      OBMIN=10.0
      DO 7000 I=1,KPAIR
      IF(OBLAT(I).GE.OBMIN) GO TO 7000
      OBMIN=OBLAT(I)
 7000 CONTINUE
      SCATR=OBMAX-OBMIN
      PRINT,JDATE,AVELAT,KPAIR,SCATR
      WRITE(7,1200)JDATE,AVELAT,KPAIR
 1200 FORMAT(I5,E16.7,I3)
      GO TO 3000
      STOP
```

APPENDIX B

METHOD OF REDUCTION USED BY IPMS

1. Star List and Observing Program

The 5 ILS stations carry out observations of latitude with the ordinary visual zenith telescopes. The star list consists of 144 stars forming 72 pairs in total and consisting of 12 groups each containing 6 pairs and occupying 2 hours of right ascension. The stars are listed according to Boss's General Catalogue. In this list the centennial variations and one-half of the secular variations of the centennial variations are given as CV and SV respectively and proper motions are given for one year.

For the right ascension, hundred times of the annual variation and fifty times of the secular variation in the GC are given as CV and SV for 1950.0. CV and SV in declination are calculated by the following formulas:

$$(CV)_\delta = \frac{d\delta}{dT} = n_o \cos\alpha_o + 100\,\mu'$$

$$(SV)_\delta = \frac{1}{2}\frac{d^2\delta}{dT^2} = \frac{1}{2}\left(\frac{dn}{dT}\right)_o \cos\alpha_o - \frac{1}{2}\sin 1''(m_o n_o \sin\alpha_o + 200\, n_o\, \mu \sin\alpha_o$$
$$+ 5000\,\mu^2 \sin 2\,\delta_o + n_o^2 \sin^2\alpha_o \tan\delta_o)$$

where α_o and δ_o are the right ascension and declination for 1950.0 given in the GC and m_o, n_o, $(dn/dT)_o$ are the centennial values for 1950.0 calculated by the following formulas based on Newcob's precession:

$$m = 4608''.506 + 2''.7945\,T + 0''.00012\,T^2$$
$$n = 2004''.685 - 0''.8533\,T - 0''.00037\,T^2$$
$$dn/dT = 0''.8533 - 0''.00074\,T$$

in which T is measured in tropical centuries from 1900.0. Numerical values of m, n, and dn/dT for 1950.0 are

$$m_o = 4609''.903$$
$$n_o = 2004''.258$$
$$dn/dT = -0''.8533$$

Using the values in the list, the mean places of stars at any other epoch t are calculated by the following formulas:

$$\alpha_t = \alpha_0 + (CV)_\alpha \cdot t + (SV)_\alpha \circ t^2 + (\text{3rd term})_\alpha \cdot t^3$$

$$\delta_t = \delta_0 + (CV)_\delta \cdot t + (SV)_\delta \cdot t^2 + (\text{3rd term})_\delta \cdot t^3$$

where t is measured in tropical centuries from 1950.0.

Duration of observing days for all combinations of any three successive groups which are distributed symmetrically with respect to midnight is one month. During a typical year (1962) the groups observed were as follows:

Date	Groups	Date	Groups
Jan 6 - Feb 5	IV,V,VI	Jul 7 - Aug 5	X,XI,XII
Feb 6 - Mar 6	V,VI,VII	Aug 6 - Sept 5	XI,XII,I
Mar 7 - Apr 6	VI,VII,VIII	Sept 6 - Oct 5	XII,I,II
Apr 7 - May 6	VII,VIII,IX	Oct 6 - Nov 5	I,II,III
May 7 - Jun 6	VIII,IX,X	Nov 6 - Dec 5	II,III,IV
Jun 7 - Jul 6	IX,X,XI	Dec 6 - Jan 5	III,IV,V

2. Atmospheric Conditions

Atmospheric conditions are usually observed every hour during the latitude observation. The following values are obtained and individual hourly values of T_{ex}, T_{tel}, and B are used to calculate the refraction and the temperature correction for the instrumental constants in the course of reduction.

${}_nT_{ex}$, ${}_sT_{ex}$	=	exterior (outdoor) temperature in degrees centigrade, north and south sides respectively
T_{ex}	=	mean of the above two quantities
ΔT_{ex}	=	hourly change of T_{ex}
${}_nT_{ex} - {}_sT_{ex}$	=	difference between exterior temperatures, north minus south
${}_nT_{in}$, ${}_sT_{in}$	=	interior (indoor) temperature, north and south sides respectively
T_{in}	=	mean of the above two quantities
ΔT_{in}	=	hourly change of T_{in}
${}_nT_{in} - {}_sT_{in}$	=	difference between interior temperatures, north minus south
T_{tel}	=	temperature of the telescope
ΔT_{tel}	=	hourly change of T_{tel}
$T_{ex} - T_{in}$	=	difference between exterior and interior temperatures
$T_{ex} - T_{tel}$	=	difference between interior and telescope temperatures

B	=	barometer reading in mmHg, corrected for both temperature and gravity of Mizusawa and Ukiah, but only for temperature of Kitab, Carloforte and Gaithersburg
ΔB	=	hourly change of B
SE	=	seeing of star image, scale 1-4 (best-worst)
ST	=	steadiness of star image, scale 1-4 (best-worst)
WV	=	wind velocity near the telescope in meters per second
WD	=	wind direction in angle reckoned clockwise from north
NS-comp	=	mean wind component velocity along NS-direction during the observation
EW-comp	=	mean wind component velocity along EW-direction during the observation
humidity	=	relative humidity during the observation

3. Method of Reduction

(a) General Formula.

Individual values of observed latitude are calculated by the formula

$$\varphi = \frac{1}{2}(\delta_s + \delta_n) + \frac{1}{2}(Z_s - Z_n)$$

where δ and Z are the apparent declination and zenith distance of the star. Suffixes s and n denote the southern and northern stars in a given pair.

(b) Apparent Declination of the Star.

Apparent declinations of the stars were calculated to one-thousandth of a second of arc by the following formula in which the correction for small quantity of second-order terms was considered.

$$\begin{aligned}\delta_r = {} & \delta_t + \tau\mu' + Aa' + Bb' + Cc' + Dd' \\ & + A^2 \sin 1'' \times \frac{1}{2}\left(-\frac{m}{n}\sin\alpha_t - \sin^2\alpha_t \tan\delta_t\right) \\ & + AB \sin 1'' \times \left(-\frac{m}{n}\cos\alpha_t - \frac{1}{2}\sin 2\alpha_t \tan\delta_t\right) \\ & + B^2 \sin 1'' \times \left(-\frac{1}{2}\cos^2\alpha_t \tan\delta_t\right) \\ & + AC \sin 1'' \times \left(-\frac{1}{2}\sin 2\alpha_t \sec\delta_t - \frac{m}{n}\cos\alpha_t \sin\delta_t - \tan\epsilon_t \cos\alpha_t \sin\delta_t\right) \\ & + \ldots\end{aligned}$$

$$+ AD \sin 1'' \times \left(- \sin^2 \alpha_t \sec \delta_t - \frac{m}{n} \sin\alpha_t \sin\delta_t + \cos\delta_t\right)$$

$$+ BC \sin 1'' \times (- \cos^2\alpha_t \sec\delta_t + \tan\epsilon_t \sin\alpha_t \sin\delta_t + \cos\delta_t)$$

$$+ BD \sin 1'' \times \left(-\frac{1}{2} \sin 2\alpha_t \sec\delta_t\right)$$

$$+ C^2 \sin 1'' \times \left(-\frac{1}{2} \cos^2\alpha_t \tan\delta_t - \tan\epsilon_t \sin\alpha_t \cos 2\delta_t - \frac{1}{2}\tan^2\epsilon_t \sin 2\delta_t + \frac{1}{2}\sin^2\alpha_t \sin 2\delta_t\right)$$

$$+ CD \sin 1'' \times \left(-\frac{1}{2} \sin 2\alpha_t \tan\delta_t - \frac{1}{2} \sin 2\alpha_t \sin 2\delta_t + \tan\epsilon_t \cos\alpha_t \cos 2\delta_t\right)$$

$$+ D^2 \sin 1'' \times \frac{1}{2} (\cos^2\alpha_t \sin 2\delta_t - \sin^2\alpha_t \tan\delta_t)$$

where

α_t, δ_t are the mean place at the nearest beginning of the Besselian year t which is measured in tropical centuries from 1950.0

τ is the epoch of observation measured in tropical years from the nearest beginning of the Besselian year t and should not exceed half a year

μ' is the proper motion in declination

a', b', c', d' are the star constants in right ascension at the nearest beginning of the Besselian year t

ϵ_t is the mean obliquity of the ecliptic calculated by

$$\epsilon_t = 23^\circ 26' 44''.84 - 46''.850\, t - 0''.0032\, t^2 + 0''.00181\, t^3$$

m, n are calculated for the required beginning of the Besselian year

Among the Besselian Day Numbers, C and D were calculated by the following formulas instead of using the values given in the Astronomical Ephemeris.

$$C = +1189''.80 \{-\dot{Y}_\odot - \dot{y} + 0.0000553\} + 0''.0092\, t$$

$$D = -1189''.80 \{-\dot{X}_\odot - \dot{x} + 0.0002815\} + 0''.0028\, t$$

where

$\dot{X}_\odot$, $\dot{Y}_\odot$ are the components of the sun's velocity referred to the true equator and equinox of the date

$\dot{x}, \dot{y}$ are those of the center of gravity of the solar system

(c) Reduction of Zenith Distance Difference.

Zenith distance difference of star pair is calculated by the formula

$$\frac{1}{2}(Z_s - Z_n) = \frac{1}{2}(M + \beta T_{tel}\{R_E + (IS)_E + (WI)_E - R_W - (IS)_W - (WI)_W\}$$

$$+\frac{1}{2}\text{ (level correction)}$$

$$+\frac{1}{2}\text{ (curvature correction)}$$

$$+\frac{1}{2}\text{ (differential refraction)}$$

where

M is the micrometer constant, the value of one revolution of the micrometer in seconds of arc

β is the temperature coefficient of the micrometer constant

T_{tel} is the temperature of the telescope

R is the mean of bisection values in the unit of revolution of the micrometer

(IS) is the correction for the progressive inequality of the micrometer screw

(WI) is the correction for the inclination of the moving wire

subscripts E and W denote the position of the telescope, east or west

Curvature correction is calculated for each star by the formula

$$C = \frac{15^2}{2}\sin 1'' F^2 \tan\delta_t$$

where

F is the equatorial distance of the measuring point from the meridian in seconds of time, that is, $\pm 20^s$ and $\pm 6\ 2/3^s$ are adopted for all the stations in common

Correction for differential refraction is calculated for each pair by the formula

$$\text{ref} = 60''\!.154 \times \frac{1}{1 + 0.00367\, T_{ex}} \cdot \frac{B}{760} \cdot \sec^2 Z\, \Delta Z \sin 1''$$

where

$60''\!.154$ is the constant of refraction

T_{ex} is the outdoor temperature in degrees centigrade at the time of observation

B	is the atmospheric pressure in mmHg at the time of observation, calibrated and reduced to 0° C
Z	is the mean zenith distance of the pair which is, in practice, replaced by one-half of the declination difference at the beginning of the year
ΔZ	is the difference of observed zenith distances measured in seconds of arc

Corrections for the inclination of the moving wire (WI) are applied to each mean value of micrometer readings as follows only when they were broken:

Bisection	(WI)	Bisection	(WI)
1,2,3,	$-\frac{1}{3} I_4$	1,-,3,-	$-\frac{1}{2}(I_2 + I_4)$
1,2,-,4	$-\frac{1}{3} I_3$	-,2,3,-	$-\frac{1}{2}(I_1 + I_4)$
1,-,3,4	$-\frac{1}{3} I_2$	1,-,-,4	$-\frac{1}{2}(I_2 + I_3)$
-,2,3,4	$-\frac{1}{3} I_1$	-,2,-,4	$-\frac{1}{2}(I_1 + I_3)$
1,2,-,-	$-\frac{1}{2}(I_3 + I_4)$	-,-,3,4	$-\frac{1}{2}(I_1 + I_2)$

In the above, I_1, I_2, I_3, I_4 are the reductions of each bisection value to the reduced path of the star due to inclination of themoving wire. Each of I_i is obtained from the bisection values of latitude observations made during the successive three months by the following formulas and was used for the middle month of the three.

$$I_i = \frac{1}{n}\left[\frac{1}{4}\sum_{j=1}^{n}(R_{1j} + R_{2j} + R_{3j} + R_{4j}) - \sum_{j=1}^{n}(R_i)_j \mp \sum_{j=1}^{n}(C_{1j} - C_{2j})\right]$$

upper sign: $i = 1,4$ of tel E

$i = 2,3$ of tel W

lower sign: $i = 2,3$ of tel E

$i = 1,4$ of tel W

$$C_{1j} - C_{2j} = \frac{15}{2}\,\frac{\sin 1''}{M}\,(F_1^2 - F_2^2)\tan\delta_j$$

$$F_1 = F_4 = 20^s$$

$$F_2 = F_3 = 6\ 2/3^s$$

where

i	=	position of bisection, $1 \sim 4$
j	=	serial number of star observed, $1 \sim n$
n	=	total number of observations
$R_{1j} \sim R_{4j}$	=	respective bisection values of star j
$(R_i)_j$	=	bisection value of star j at the position i
C_{1j}	=	curvature correction in the unit of micrometer turn of star j at the position $i = 1$ and is equal to C_{4j}
C_{2j}	=	curvature correction in the unit of micrometer turn of star j at the position $i = 2$ and is equal to C_{3j}
M	=	micrometer constant

4. Declination Corrections

In its annual reports the IPMS computes and publishes values of declination corrections which it applies to the individual values of latitude for obtaining the monthly mean latitudes. These declination corrections are obtained as follows [Yumi, 1964, p. 11]. Monthly pair means are obtained for every pair at every station on the basis of the individual values of the latitude deduced from the same star pair over a period of one month. See section 1 of this appendix for details of observing program and method of reduction. On the basis of the monthly means for the different star pairs in a group (a group consisting of six star pairs) the monthly group mean latitude for each group at the corresponding mean epoch is obtained separately for each of the five stations. Twelve common epochs are fixed in a year and the group mean latitude at the common epoch is obtained by linear interpolation or extrapolation between two successive mean latitudes of the same group.

Latitude variation $\Delta\varphi$ at the station of longitude λ is expressed by

$$\Delta\varphi = x \cos\lambda + y \sin\lambda + z$$

where x and y are the coordinates of the instantaneous pole, and z is the nonpolar variation, common to each of the five stations;

$$\Delta\varphi = \varphi - \Phi$$

where Φ is the adopted latitude value for the station, and φ is the group mean latitude at the common epoch.

The values of x, y, and z are obtained for each group by a least square solution knowing the $\Delta\varphi$ for the same group at the common epoch for each of the five stations.

The value of z in the above solution is considered as the "group correction," to which "reduction to group mean" is added to give the declination correction for each star pair. The "reduction to group mean" for the ith star pair is taken as the difference between the monthly group mean latitude of the group in which the ith pair is included at the mean epoch t_o and the monthly mean latitude of the ith pair at the mean epoch t_o.

APPENDIX C

OMNITAB PROGRAM FOR OBTAINING POLYNOMIAL FIT AND PLOTS FOR NORMALIZED RESIDUALS

```
DIMENSION THE WORKSHEET TO HAVE 415 ROWS AND 12 COLUMNS
READ 2 3 4
POLYFIT 3 4 2 3    7      11
TITLEX                          NUMBER OF DAYS RECKONED FROM 31 JAN 1961
TITLEY                     RESIDUALS
PLOT COL 11 VS COL 2
SUB  11  3  10
TITLEX                          NUMBER OF DAYS RECKONED FROM 31 JAN 1961
TITLEY                   ADJUSTED VALUES OF LATITUDE
PLOT COL 10 VS COL 2
RAISE 4  0.5 AND STORE IN COL 8
MULTIPLY COL 8 BY COL 11 AND STORE IN COL 12
TITLEX                          NUMBER OF DAYS RECKONED FROM 31 JAN 1961
TITLEY                 NORMALISED RESIDUALS
PLOT COL 12 VS COL 2
PRINT COLS 2, 10, 11, 12
PUNCH 2  12
POLYFIT 3 4 2 4    7      11
PLOT COL 11 VS COL 2
SUB  11  3  10
PLOT COL 10 VS COL 2
RAISE 4  0.5 AND STORE IN COL 8
MULTIPLY COL 8 BY COL 11 AND STORE IN COL 12
PLOT COL 12 VS COL 2
PRINT COLS 2, 10, 11, 12
POLYFIT 3 4 2 5    7      11
PLOT COL 11 VS COL 2
SUB  11  3  10
PLOT COL 10 VS COL 2
RAISE 4  0.5 AND STORE IN COL 8
MULTIPLY COL 8 BY COL 11 AND STORE IN COL 12
PLOT COL 12 VS COL 2
PRINT COLS 2, 10, 11, 12
STOP
```

APPENDIX D

OMNITAB PROGRAM TO OBTAIN MOVING MEANS OF NORMALIZED RESIDUALS

```
   DIMENSION THE WORKSHEET TO HAVE 190 ROWS AND 20 COLUMNS
   DIMENSION THE WORKSHEET TO HAVE 190 ROWS AND 20 COLUMNS
   READ 2 3
1.0/SUM 2 FROM ROW  1  TO ROW 5   AND STORE IN COL 6
1.2/DIVIDE COL 6  BY 5.0  ND STORE IN COL  7
1.3/ DEFINE ROW 1 OF COL 7  INTO ROW 1 OF COL 8
1.4/SUM 3  FROM ROW  1  TO ROW 5    AND STORE IN COL 9
1.8/DIVIDE 9 BY 5.0  ND STORE IN 11
1.9/ DEFINE ROW 1 OF COL 11 INTO ROW 1 OF COL 12
3.0/INCREMENT INSTRUCTION 1.0 BY ( 0, 1 , 1 ,0 )
3.1/INCREMENT INSTRUCTION 1.3 BY(1,0,1,0)
3.2/INCREMENT INSTRUCTION 1.4 BY ( 0, 1 , 1 ,0 )
3.4/INCREMENT INSTRUCTION 1.9 BY(1,0,1,0)
EXECUTE INSTRUCTION 1.0 THRU 3.4, 186 TIMES
DEFINE 1342.0 IN ROW 187 OF COL 8
DEFINE 1342.0 IN ROW 188 OF COL 8
DEFINE 1342.0 IN ROW 189 OF COL 8
DEFINE 1342.0 IN ROW 190 OF COL 8
PRINT 12  8
TITLEX                       NUMBER OF DAYS RECKONED FROM 31 DEC 1961
TITLEY              MOVING MEANS OF NORMALISED RESIDUALS
PLOT 12  8
STOP
```

APPENDIX E

CALCULATIONS FOR THE EFFECT OF DECLINATION CORRECTIONS

Days 857, 859

These days correspond to May 6, 1964, and May 8, 1964. From April 7 to May 6 groups VII, VIII, and IX were observed and the declination correction to each daily mean observation = $0''.110$ [Yumi, 1966, p. 84]. From May 7 to June 6 groups VIII, IX, and X were observed and the corresponding declination correction to each daily mean observation = $+ 0''.124$. Thus the likely break due to the differential declination error is of the order of $+ 0''.014$ which corresponds to an amount of $(0''.014 \times \sqrt{18}) = 0''.059$ of fluctuation of normalized residuals which is negligible compared to the values of r ($1''.198$) considered in the breaks above.

Day 886

This day corresponds to June 4, 1964. From May 7 to June 6 groups VIII, IX, and X were observed and the corresponding declination correction is $+ 0''.124$. From June 7 to July 6, groups IX, X, and XI were observed and the declination correction to each daily mean observation (assuming that all the 18 stars are observed every day which is generally the case) is $+ 0''.152$. Thus the likely break in the region of Day 886 due to differential declination error is of the order of $0''.028$ which corresponds to an amount of $(0''.028 \times \sqrt{18}) =$ $0''.118$ of fluctuation of normalized residual which is negligible compared to the value of r ($1''.198$) considered in the breaks above.

Day 1100

This day corresponds to January 4, 1965. From December 6, 1964, to January 5, 1965, groups III, IV, and V were observed. The corresponding declination correction to each daily mean observation = $+ 0''.072$ [Yumi, 1966,

p. 84]. From January 6, 1965, to February 5, 1965, groups IV, V, and VI were observed. The corresponding declination correction to each daily mean observation = + 0$''$.069 [Yumi, 1967, p. 88]. Thus the likely break in the region of Day 1100 due to the differential declination error is of the order of + 0$''$.003 which corresponds to the fluctuation of $(0.003 \times \sqrt{18}) = 0''.013$ as a normalized residual which is negligible compared to the value of r (1$''$.198) considered in the breaks.

Therefore our earlier observations regarding the breaks are not significantly affected by the likely breaks due to the differential declination error.

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