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COMPUTER PROGRAM TO CALCULATE CONVECTIVE HEAT-TRANSFER COEFFICIENTS FROM HARMONIC TEMPERATURE OSCILLATIONS

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16. Abstract

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SUMMARY

Two FORTRAN IV version 13 programs are described which obtain the analytical solutions for convective heat-transfer coefficients related to either the phase lag or the amplitude ratio. The first, entitled BL, uses the ratio of the wall to driven hot fluid temperature to calculate the coefficients. The second, entitled JCL, uses the phase-lag angle between the sinusoidally driven fluid and wall temperature to calculate the heating fluid and coolant convective coefficients. Criteria for the choice of either of the two methods and their mathematical formulation are described in detail in reference 1.

INTRODUCTION

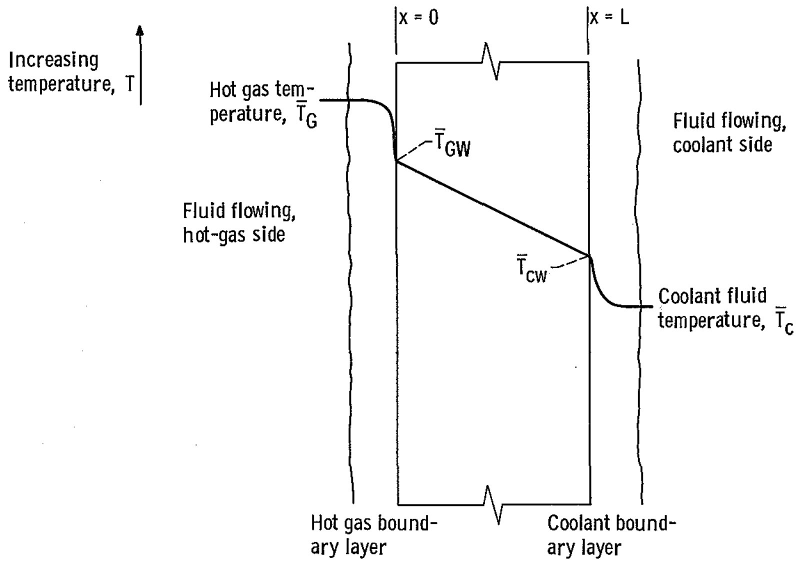
The analytical expressions to determine the convective heat-transfer coefficients for a wall separating two fluids, one having an oscillating temperature, were derived in reference 1 from a second-order partial differential equation. These equations lead to computational procedures that are iterative and, therefore, time consuming and tedious. To alleviate this difficulty, two computer programs, BL and JCL, were written to provide the means for obtaining the numerical solutions to either the amplitude ratio or the phase-lag angle method quickly and accurately.

The data published in reference 1 were obtained by the use of these programs, and their descriptions and listings, which are shown herein, should prove helpful to anyone desiring to investigate further.

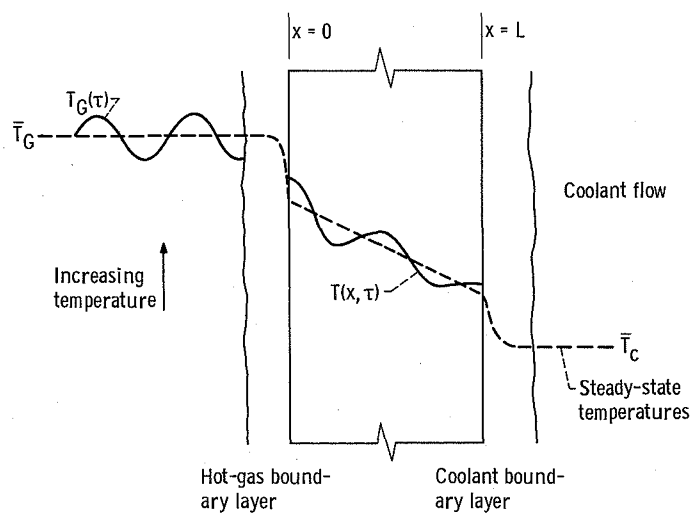
Specifically, the BL program calculates the convective heat-transfer coefficients related to the amplitude ratio for a plate separating two fluids, one of the fluids having an oscillating temperature. The JCL program, on the other hand, calculates the heat-transfer coefficients related to the phase lag.

STATEMENT OF THE PROBLEM

Figure 1(a) shows a wall that separates two moving fluids. The hot-gas-side fluid temperature T_G is greater than the coolant temperature T_c, causing heat to flow through the wall in the positive x direction. The problem is to determine the convective heat-transfer coefficients if either the hot-gas or coolant temperature is varied sinusoidally and if the temperature response of the wall is measured at only one point. Figure 1(b) shows what the wall temperature might look like at any given instant in time.



(a) Wall temperature distribution shown at time zero before start of hot-gas temperature oscillation.



(b) Transient temperature response of plate to sinusoidally forced fluid temperature.

Figure 1. - Basic heat-transfer model.

METHOD OF ATTACK

If the hot-gas temperature is made to vary sinusoidally, the wall temperature will respond sinusoidally but will lag the driven temperature by an angle φ. In addition, the amplitude of the wall temperature oscillation will be less than that of the driven temperature. Both the phase lag φ and the ratio of the amplitude of the wall temperature to the amplitude of the driven temperature $\theta/\Delta T$ are, among other things, functions of the convective heat-transfer coefficients h. Finding the relation between the convective heat-transfer coefficients and the phase lag φ, or the amplitude ratio $\theta_m/\Delta T$, is required in order to solve for the coefficients.

To find this relation, the temperature response of the wall to a sinusoidally driven fluid temperature is derived. From this solution and steady-state heat-transfer conditions (which relate the ratio of the convective heat-transfer coefficients to the ratio of the temperature drops between fluids and wall), the absolute values of the coefficients can be determined as functions of either phase lag or amplitude ratio. The quantities that must be known as a function of time are the hot-gas temperature T_G, the wall temperature at any point x $(T(x, \tau))$ and the coolant temperature T_c. From these quantities either the phase lag or the amplitude ratio can be determined as well as the ratio of the convective heat-transfer coefficients. With the ratio of the coefficients, the frequency of the temperature oscillation, the wall material properties, and either φ or $\theta_m/\Delta T_G$, the convective heat-transfer coefficients h_G and h_c can be calculated.

It is also possible to calculate the coefficients by oscillating the coolant temperature sinusoidally. This approach requires the measurement of the same quantities as when the hot fluid temperature is oscillated.

The controlling differential equations and boundary conditions are given next. The one-dimensional transient heat-conduction equation[1]

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau} \tag{1}$$

is solved for the temperature distribution in a wall that has convective heat transfer over its two surfaces (fig. 1). (Symbols are defined in the appendix.) The hot-gas side fluid temperature is driven sinusoidally and is given in equation form by

$$T_G = \Delta T_G e^{-i\omega\tau} + \overline{T}_G \tag{2}$$

[1]All equations are numbered to correspond with those in reference 1.

where $\overline{T}_G$ is given. It is not necessary to know the numerical value of ΔT_G if the phase lag is used for calculating the coefficients because the phase determination is independent of the absolute value of the temperature oscillation. If the amplitude ratio is used for calculating the coefficients, the ΔT_G must be computed from the change in chamber pressure. The boundary condition at the hot-gas side of the wall surface $x/L = 0$ equates the heat transferred by convection to that conducted away from the surface into the wall. This is given in equation form as

$$h_G\left[T_G - T(0, \tau)\right] = -K \frac{\partial T(0, \tau)}{\partial x} \tag{3}$$

The boundary condition at the coolant surface $(x/L = 1)$ equates the heat conducted to the surface with the heat transferred convectively to the coolant. In equation form this is given by

$$h_c\left[T(L, \tau) - T_c\right] = -\frac{K \partial T(L, \tau)}{\partial x} \tag{4}$$

The solution for the wall temperature as a function of time and location within the wall consists of the sum of the transient $\theta(x, \tau)$ and steady-state $T_s(x)$ solutions. The steady-state solution is found by setting $\partial T/\partial \tau$ equal to zero in equation (1) and assuming the wall temperature at $x/L = 0$ and 1 to be given as $\overline{T}_{GW}$ and $\overline{T}_{cw}$, respectively. This yields the usual linear temperature distribution with distance and can be written as

$$\overline{T}_s(x) = (\overline{T}_{cw} - \overline{T}_{GW}) \frac{x}{L} + \overline{T}_{GW} \tag{5}$$

The transient solution $\theta(x, \tau)$ is found by assuming the usual product solution: one a function of time only $F(\tau)$ and the other a function of the location within the wall measured from the heated side of the wall $X(x)$. Equations (1), (3), and (4) are modified to account for the change in variable from $T(x, \tau)$ to $\theta(x, \tau)$. The details of the solution of equation (1) using its pertinent boundary conditions (eqs. (2) to (4)) are shown in appendix B of reference 1.

The relation, derived from the steady-state (or mean) condition that requires the heat transferred convectively to the wall to equal the heat transferred convectively to the coolant, is expressed mathematically as

$$\frac{h_c}{h_G} = \frac{\overline{T}_G - \overline{T}(0)}{\overline{T}(L) - \overline{T}_c} \tag{6}$$

From the definition of ψ

$$h = \frac{K\eta}{\psi} \tag{7}$$

The ratio of convective heat-transfer coefficients in equation (6) is defined as R. Calculating the ratio R using equation (7) gives

$$R = \frac{\psi_G}{\psi_c} \tag{8}$$

where R is also given by

$$R = \frac{\overline{T}_G - \overline{T}(0)}{\overline{T}(L) - \overline{T}_c} \tag{9}$$

The following assumptions have been used to obtain the solution:

(1) The heat flows through the plate in the x direction only (one-dimensional heat conduction).

(2) The wall or plate properties (density, specific heat, and thermal conductivity) are constant.

(3) The convective heat-transfer coefficients on both sides of the wall are constant.

(4) The coolant temperature is constant.

The details of the analytical solutions are given in reference 1. However, the following equations are restated herein because they are used by the computer program to calculate the heat-transfer coefficients.

The equation involving amplitude ratio is

$$\begin{aligned}\frac{\theta_m}{\Delta T_G} = \frac{1}{a^2 + b^2}\Bigg(\bigg\{&-\left[(ca - db)\sin \eta L \frac{x}{L} - (da + cb)\cos \eta L \frac{x}{L}\right] e^{\eta L[2-(x/L)]} + \left[(fa - eb)\cos \eta L \frac{x}{L} - (ea + fb)\sin \eta L \frac{x}{L}\right] e^{\eta L(x/L)}\bigg\}^2 \\ &+ \bigg\{\left[(ea + fb)\cos \eta L \frac{x}{L} + (fa - eb)\sin \eta L \frac{x}{L}\right] e^{\eta L(x/L)} - \left[(ca - db)\cos \eta L \frac{x}{L} + (da + cb)\sin \eta L \frac{x}{L}\right] e^{\eta L[2-(x/L)]}\bigg\}^2\Bigg)^{1/2}\end{aligned} \tag{B37}$$

where

$$a = 1 - (\psi_G + \psi_c) + e^{2\eta L}\left[(\psi_G + \psi_c + 2\psi_G\psi_c)\sin 2\eta L - (1 + \psi_c + \psi_G)\cos 2\eta L\right] \tag{B28}$$

$$b = (\psi_G + \psi_c - 2\psi_G\psi_c) + e^{2\eta L}\left[(\psi_G + \psi_c + 2\psi_G\psi_c)\cos 2\eta L + (1 + \psi_c + \psi_G)\sin 2\eta L\right] \tag{B29}$$

$$c = (1 + \psi_c)\cos 2\eta L - \psi_c \sin 2\eta L \tag{B30}$$

$$d = \psi_c \cos 2\eta L + (1 + \psi_c)\sin 2\eta L \tag{B31}$$

$$e = 1 - \psi_c \tag{B32}$$

$$f = \psi_c \tag{B33}$$

The equation involving phase lag angle is

$$\psi_c^3 + \frac{\left[\underline{C} + R(\underline{C} + \underline{D})\right]}{R(F + G)}\psi_c^2 + \frac{2\underline{A} + R(\underline{A} + \underline{B})}{2R(F + G)}\psi_c + \frac{E}{2R(F + G)} = 0 \tag{B51}$$

where

$$\underline{A} \equiv -\sin(\varphi + \eta x) + e^{2\eta L}\cos(\varphi + \eta x - 2\eta L) + e^{2\eta(2L-x)}\sin(\varphi - \eta x) - e^{2\eta(L-x)}\cos(\varphi - \eta x + 2\eta L) \tag{B39}$$

$$\underline{B} \equiv \cos(\varphi + \eta x) - e^{2\eta L}\sin(\varphi + \eta x - 2\eta L) - e^{2\eta(2L-x)}\cos(\varphi - \eta x) + e^{2\eta(L-x)}\sin(\varphi - \eta x + 2\eta L) \tag{B40}$$

$$\underline{C} \equiv \sin(\varphi + \eta x) + e^{2\eta L}\sin(\varphi + \eta x - 2\eta L) + e^{2\eta(2L-x)}\sin(\varphi - \eta x) + e^{2\eta(L-x)}\sin(\varphi - \eta x + 2\eta L) \tag{B41}$$

$$D \equiv -\cos(\varphi + \eta x) + e^{2\eta L}\cos(\varphi + \eta x - 2\eta L) - e^{2\eta(2L-x)}\cos(\varphi - \eta x) + e^{2\eta(L-x)}\cos(\varphi - \eta x + 2\eta L) \tag{B42}$$

$$E \equiv \sin(\varphi + \eta x) - e^{2\eta L} \sin(\varphi + \eta x - 2\eta L) + e^{2\eta(2L-x)}\sin(\varphi - \eta x)$$
$$- e^{2\eta(L-x)}\sin(\varphi - \eta x + 2\eta L) \qquad (B43)$$

$$F \equiv \cos(\varphi + \eta x) + e^{2\eta L} \sin(\varphi + \eta x - 2\eta L) - e^{2\eta(2L-x)}\cos(\varphi - \eta x)$$
$$- e^{2\eta(L-x)}\sin(\varphi - \eta x + 2\eta L) \qquad (B44)$$

$$G \equiv -\sin(\varphi + \eta x) - e^{2\eta L} \cos(\varphi + \eta x - 2\eta L) + e^{2\eta(2L-x)}\sin(\varphi - \eta x)$$
$$+ e^{2\eta(L-x)}\cos(\varphi - \eta x + 2\eta L) \qquad (B45)$$

The solution of this cubic equation can be obtained using the classical approach. The positive, nonzero, root will yield the desired convective heat-transfer parameter ψ_c. From equation (8), then

$$\psi_G = R\psi_c \qquad (B52)$$

Also, from equations (6) and (9), the following relation must be satisfied:

$$R \equiv \frac{h_c}{h_G} = \frac{T_G - T(0)}{T(L) - T_c} \qquad (B49)$$

DESCRIPTION OF BL AND JCL

The computer program, BL, was written to calculate the heat-transfer coefficients h_c and h_G by use of values of the thermal conductivity K, the density of the wall material ρ, the specific heat of the wall material c, thickness of wall material L, the ratio of distance measured from the heated surface wall to the thickness of the wall x/L, the mean temperature of the hot gas $\overline{T}_G$, the wall temperature as a function of time at x = 0 $T(0, \tau)$, the mean temperature of the coolant $\overline{T}_c$, the frequency of temperature oscillation f, and the amplitude ratio $\theta/\Delta T_G$.

The coolant wall temperature T_{cw} is initially set to a positive perturbation value in subroutine BALAN for the coolant temperature $\overline{T}_c$. The ratio of the convective heat-transfer coefficients R, is then calculated, and with a linear relation of conductive heat transfer across the plate both heat-transfer coefficients h_c and h_G can be obtained. With these quantities, a calculated amplitude ratio is found and compared with the measured ratio. Then, by means of linear interpolation in subroutine BALAN, T_{cw} is altered until the difference between the two ratios is less than 10^{-4}.

It should be noted that the $\theta/\Delta T_G$ ratio as a function of the coolant wall temperature has a pole at $T(x, \tau)$ (fig. 2), which imposes a heavy restriction on the allowable error for the value of the amplitude ratio. If the error in the amplitude ratio exceeds 10 percent of the true value, the iteration will converge to a value for the coolant wall temperature in excess of the gas wall temperature.

The computer program JCL was written to calculate heat-transfer coefficients h_c and h_G by use of values of the thermal conductivity K, the density of wall material ρ, the specific heat of wall material c, thickness of wall material L, the ratio of distance measured from the heated surface of the wall to the thickness of the wall x/L, the mean temperature of the hot gas $\overline{T}_G$, the wall temperature as a function of distance (measured into wall from hot surface) $T(x, \tau)$, the mean temperature of the coolant $\overline{T}_c$, the frequency of temperature oscillation f, and the phase lag angle θ. The coolant wall temperature T_{cw} is initially set to equal the measured temperature of the wall $T(x, \tau)$. The ratio R, of the convective heat-transfer coefficients is calculated, and the coolant wall temperature value is corrected for the wall temperature drop due to conduction. Only the positive, nonzero root is used when solving the cubic equation.

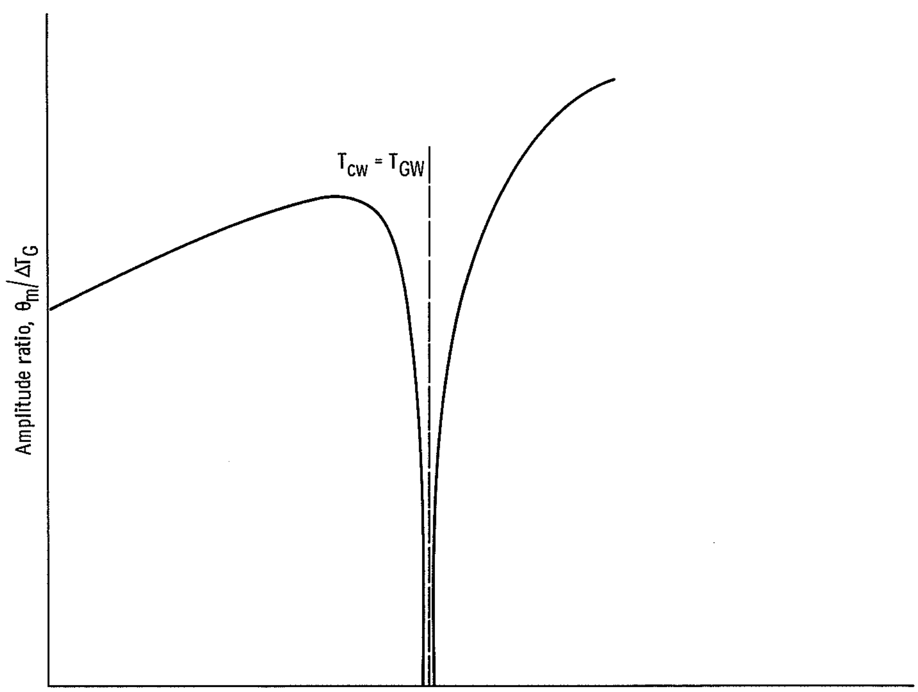


Figure 2. - Typical variation of amplitude ratio versus coolant wall temperature T_{CW} showing singularity at $T_{CW} = T_{GW}$.

These programs assume that the hot-gas-side temperature is measured. Therefore, the coolant wall temperature is calculated from the following equation:

$$T_{cw} = T(x, \tau) - \frac{(h_G)(L)\left[\overline{T}_G - T(x, \tau)\right]}{K}$$

If the wall temperature is measured at a point other than the hot-gas surface $x/L \approx 0$, the programs must be modified to account for the new location.

INPUT CARDS

Program BL

The BL program requires the following cards for input information:

Card 1: Three values with 3I2 format (table I, line 1) containing

IRDG	reading number
ISTA	station number
NR	number of readings to be processed

Card 2: Nine values with 10E8.4 format (table I, line 2) containing

K	conductivity
RHO	density of wall material
CC	specific heat of wall material
L	thickness of wall material
XL	ratio of distance measured from hot-gas side of wall and temperature measuring location to wall thickness
TG	mean temperature of hot gas
TWO	wall temperature, a function of time, at $x = 0$
TWL	wall temperature, a function of time, at $x = L$
TC	mean temperature of coolant

Card 3: Contains pairs of values (table I, line 3) under 10E8.4 format showing

WRAD(1)	frequency of temperature oscillation
OMDTG(1)	ratio of amplitude of wall temperature oscillation to amplitude of sinusoidally driven hot-gas temperature

.

.

.

WRAD(NR)

OMDTG(NR)

The program read in NR pairs of values, up to five pairs in card 3. If more than five pairs are desired, more cards may be added. The NR pairs will continue to be read under the same 10E8.4 format.

TABLE I. - SAMPLE INPUT FOR PROGRAM BL

LINE	STATEMENT NUMBER	CONT.	FORTRAN STATEMENT											IDENTIFICATION
	1 2 3 4 5	6	7 8 9 10 11 12	13 14 15 16 17 18	19 20 21 22 23 24	25 26 27 28 29 30	31 32 33 34 35 36	37 38 39 40 41 42	43 44 45 46 47 48	49 50 51 52 53 54	55 56 57 58 59 60	61 62 63 64 65 66	67 68 69 70 71 72	73 74 75 76 77 78 79 80
1	1 1	1												
2	.207	0	-3 .28	60+0 .	1170+0	.6000	-1 .16	67+0 .	9600+3	.6580	+3 .65	80+3 .	5706+3	
3	.197	0	+0 .15	38+0										

Program JCL

The JCL program requires the following cards for input information:

Card 1: Three values with 3I2 format (table II, line 1) containing

IRDG	reading number
ISTA	station number
NR	number of readings to be processed

Card 2: Eight values with 10E8.4 format (table II, line 2) containing

K	conductivity
RHO	density of wall material
CC	specific heat of wall material
L	thickness of wall material
XL	ratio of distance measured from hot-gas side of wall and temperature measuring location to wall thickness
TG	mean temperature of hot gas
TGW	wall temperature, a function of distance
TC	mean temperature of the coolant

Card 3: Contains pairs of values (table II, line 3) under 10E8.4 format showing

WRADA(1)	frequency of temperature oscillation
PHIRA(1)	phase lag angle between driving gas temperature and responding wall temperature

.

.

.

WRADA(NR)

PHIRA(NR)

The program reads in NR pairs of values up to five pairs in card 3. If more than five pairs are desired, more cards may be added. The NR pairs will continue to be read under the same 10E8.4 format.

TABLE II.. - SAMPLE INPUT FOR PROGRAM JCL

LINE	STATEMENT NUMBER	CONT.	FORTRAN STATEMENT											IDENTIFICATION
	1 2 3 4 5	6	7 8 9 10 11 12	13 14 15 16 17 18	19 20 21 22 23 24	25 26 27 28 29 30	31 32 33 34 35 36	37 38 39 40 41 42	43 44 45 46 47 48	49 50 51 52 53 54	55 56 57 58 59 60	61 62 63 64 65 66	67 68 69 70 71 72	73 74 75 76 77 78 79 80
1	1 1	1												
2	. 207	0	-3 .28	60+0 .	1170+0	.6000	-1 .16	67+0 .	9600+3	.6580	+3 .57	06+3		
3	.197	0	+04.39	65+1										

OUTPUT

The outputs of both BL and JCL programs display the descriptive title followed by a list of the input values with their program labels. The results of the calculations appear under the heading OUTPUT shown in the next sections.

Program BL

TCWA calculated coolant wall temperature

TEST1 heat flux to coolant

TEST2 heat flux from hot gas

TEST ratio of heat flux to coolant to heat flux from hot gas (ideally should be 1.0)

HCT time averaged convective heat-transfer coefficient on coolant side of wall

HGT time averaged convective heat-transfer coefficient on hot-gas side of wall

ERROR (1.0 - TEST) which is used by the program to test against the convergence criterion

R ratio of convective heat-transfer coefficients

THETA final calculated value of ratio of the amplitude of wall temperature to amplitude of sinusoidally driven hot-gas temperature

Program JCL

TCW	calculated coolant wall temperature
TEST1	heat flux to coolant
TEST2	heat flux from hot gas
TEST	ratio of heat flux to coolant to heat flux from hot gas (ideally should be 1.0)
HCT	time averaged convective heat-transfer coefficient on coolant side of wall
HGT	time averaged convective heat-transfer coefficient on hot gas side of wall
ERROR	(1.0 - TEST) which is used by the program to test against convergence criterion
R	ratio of convective heat-transfer coefficients
PSIC	convective heat-transfer parameter on coolant side of wall
PSIG	convective heat-transfer parameter on hot-gas side of wall
GR	check on the zero of eq. (B51) which should be equal to 0.0000

COMPUTER PROGRAMS

These programs are written in FORTRAN IV and are operational on the IBM 7094-2/7044 direct-coupled system of the Lewis Research Center. Program BL uses approximately $5187)_{10}$ storages and Program JCL uses approximately $4261)_{10}$ storages. Flow charts of the programs are shown in figures 3 and 4.

Machine running time for both programs is minimal, each using less than 1 minute per case.

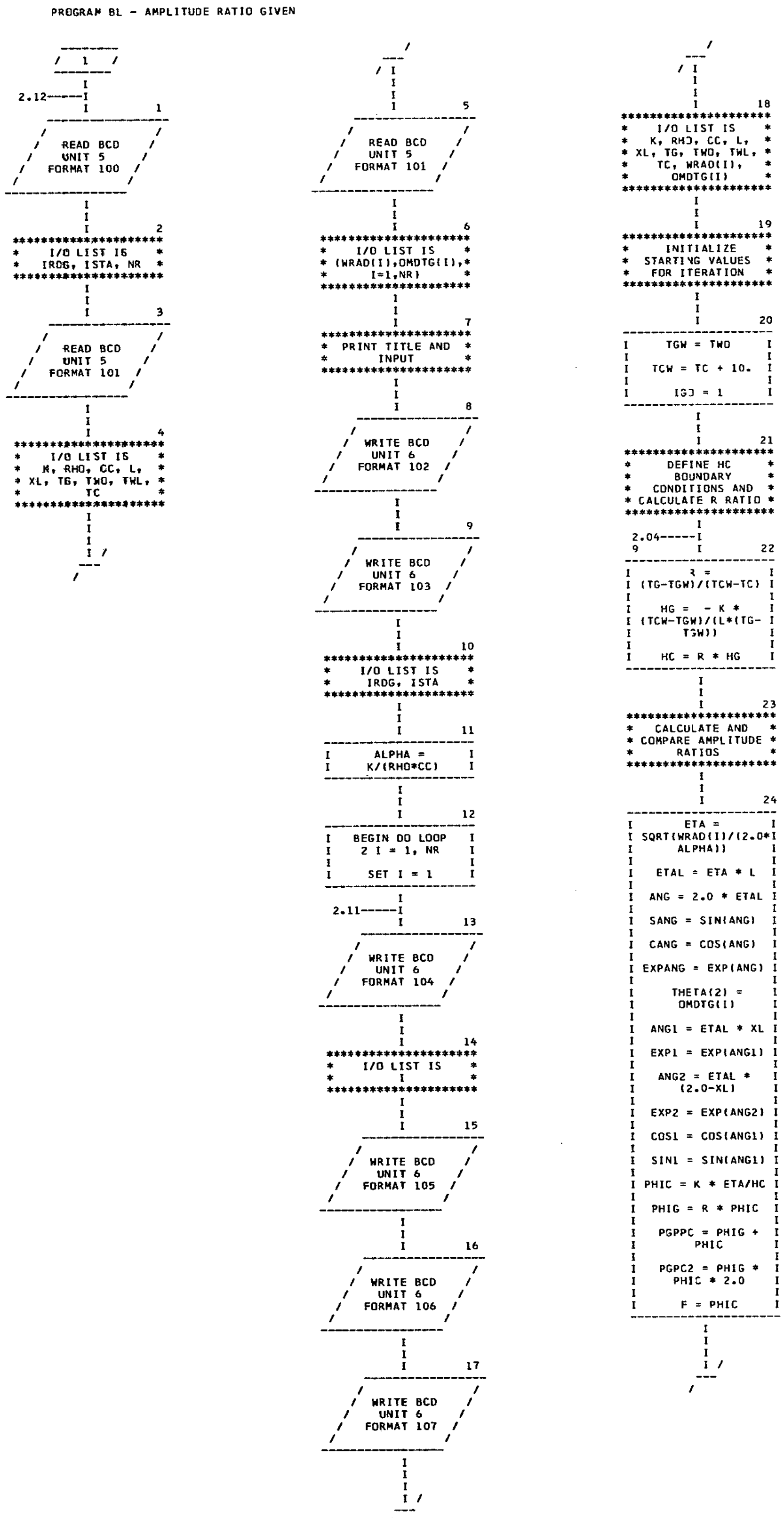


Figure 3. - Flow chart for program BL.

PROGRAM BL - AMPLITUDE RATIO GIVEN

```
           /
        ---
       / I
         I
         I
         I          1
 --------------------
 I    E = 1.0 - F   I
 I                  I
 I PCP1 = PHIC + 1.0 I
 I                  I
 I   D = F * CANG + I
 I    PCP1 * SANG   I
 I                  I
 I C = PGP1 * CANG - I
 I    PHIC * SANG   I
 I                  I
 I B = (PGPPC-PGPC2) I
 I    + EXPANG *    I
 I  ((PGPPC+PGPC2)* I
 I CANG+(PGPPC+1.0)* I
 I       SANG)      I
 I                  I
 I A = 1.0 - PGPPC + I
 I     EXPANG *     I
 I  ((PGPPC+PGPC2)* I
 I SANG-(1.0+PGPPC)* I
 I       CANG)      I
 I                  I
 I  DEL1 = C * A - D I
 I        * B       I
 I                  I
 I  DEL2 = D * A + C I
 I        * B       I
 I                  I
 I  DEL3 = F * A - E I
 I        * B       I
 I                  I
 I  DEL4 = E * A + F I
 I        * B       I
 I                  I
 I       NUM1 =     I
 I  (DEL1*SIN1-DEL2* I
 I    COS1) * EXP2  I
 --------------------
         I
         I
         I          2
 --------------------
 I       NUM2 =     I
 I  (DEL3*COS1-DEL4* I
 I    SIN1) * EXP1  I
 I                  I
 I NUM = NUM2 - NUM1 I
 I                  I
 I       DEN =      I
 I  (DEL4*COS1+DEL3* I
 I   SIN1) * EXP1 - I
 I  (DEL1*COS1+DEL2* I
 I    SIN1) * EXP2  I
 I                  I
 I   A2PB2 = A**2 + I
 I       B**2       I
 I                  I
 I       CON =      I
 I SQRT(NUM**2+DEN** I
 I        2)        I
 I                  I
 I      AMPDTG =    I
 I     CON/A2PB2    I
 I                  I
 I  DIFF = AMPDTG - I
 I      OMDTG(I)    I
 --------------------
         I
         I
         I          3
 --------------------
 I  I     BALAN     I
 I 3 I              I
 I . I(TCW,IGO,DIFF) I
 I 0 I              I
 I 1 I              I
 --------------------
         I
         I
         I
         *          4
       *   *
     *       *
   *           *
TRUE*            *
---*   IGO .LT. 10  *
I  *             *
I    *         *
I      *     *
I ****   *  *
-* 1 *     *
 * 22 *    IFALSE
  ****     I
   9       I
           I
           I
           I
           I /
          ---
         /
```

```
           /
        ---
       / I
         I
         I
         I          5
 --------------------
 I      IJK = 1     I
 I                  I
 I  TCWA(IJK) = TCW I
 I                  I
 I      HCT = HC    I
 I                  I
 I      HGT = HG    I
 I                  I
 I THETA(4) = AMPDTG I
 I                  I
 I   TEST1 = HCT *  I
 I      (TCW-TC)    I
 I                  I
 I   TEST2 = HGT *  I
 I      (TG-TGW)    I
 I                  I
 I      TEST =      I
 I    TEST1/TEST2   I
 I                  I
 I ERROR = 1. - TEST I
 --------------------
         I
         I
         I          6
*********************
*   PRINT OUTPUT    *
*********************
         I
         I
         I          7
      ----------------
     /              /
    /  WRITE BCD   /
   /    UNIT 6    /
  /   FORMAT 109 /
 /              /
 ----------------
         I
         I
         I          8
      ----------------
     /              /
    /  WRITE BCD   /
   /    UNIT 6    /
  /   FORMAT 110 /
 /              /
 ----------------
         I
         I
         I          9
      ----------------
     /              /
    /  WRITE BCD   /
   /    UNIT 6    /
  /   FORMAT 111 /
 /              /
 ----------------
         I
         I
         I          10
*********************
*    I/O LIST IS    *
* TCWA(IJK), TEST1, *
* TEST2, TEST, HCT, *
*   HGT, ERROR, R,  *
*      THETA(4)     *
*********************
         I
         I
         I
   2     *          11
       *   *
     *       *
   *           *
LO/EQ*           *
---*   ADD 1 TO I,  *
I  * COMPARE TO NR *
I    *         *
I      *     *
I ****   *  *
-* 1 *     *
 * 13 *    I HIGH
  ****     I
           I
           I
           I
           I          12
*********************
*   END OF DO LOOP  *
*    VARIABLE I     *
*********************
         I
         I
         I
        ...
       .   .
       . 1.01.
       .   .
        ...
         1
```

```
FORMAT STATEMENTS

   --------
  /       /
  --------
     I
     I
     I
     ... 13
    .   .
    . EXIT.
    .   .
     ...
```

Figure 3. - Continued.

```
PROGRAM BL - AMPLITUDE RATIO GIVEN

    SUBROUTINE BALAN


  PARAMETERS  (X,K,Y)

             --------
            / BALAN /
            --------
                I
    2.03-------I
                I          1
  ***********************
  * SUBROUTINE BALAN    *
  ***********************
                I
                I
                I          2
     ******************
     *COMPUTED GO TO *
     *  FOR K         *
     *  1   AT   3.03*
     *  2   AT   3.05*
     ******************
                I
                I
                I
               ...
             .     .
             .     .
             .     .
               ...

   BRANCH ACCORDING TO THE
   VALUE OF VARIABLE K



             ---------
            /   1   /
            --------
                I
    3.02-------I
                I          3
  ----------------------
  I        X1 = X      I
  I                    I
  I        Y1 = Y      I
  I                    I
  I    X = X + X/10.   I
  I                    I
  I        K = 2       I
  ----------------------
                I
                I
                I
               ...  4
             .     .
             . EXIT.
             .     .
               ...
```

```
                     ---------
                    /   2   /
                    --------
                        I
            3.02-------I
                        I          5
          ----------------------
          I        X2 = X      I
          I                    I
          I        Y2 = Y      I
          ----------------------
                        I
                        I
                        I
                        *          6
                      *   *
                    *       *
                  *           *
        FALSE*                 *
I-----------*    ABS(Y) .LT.     *
I            *     1.E-4        *
I              *               *
I                *           *
I                  *       *
I                    *   *
I                      *
I                      I  TRUE
I                      I
I                      I
I                      I
I                      I
I                      I          7
I         ----------------------
I         I       K = 15       I
I         ----------------------
V                      I
-----------------------I
                       I          8
          ----------------------
          I        X =         I
          I (X2-X1)/(Y2-Y1) *  I
          I     (-Y1) + X1     I
          I                    I
          I       X1 = X2      I
          I                    I
          I       Y1 = Y2      I
          ----------------------
                       I
                       I
                       I
                      ...  9
                    .     .
                    . EXIT.
                    .     .
                      ...
```

Figure 3. - Concluded.

Program BL

```
$IEFTC BL

C
C
C***************************************************************************
C     CONVECTIVE HEAT-TRANSFER COEFFICIENTS FROM HARMONIC TEMPERATURE
C     OSCILLATIONS  -  AMPLITUDE RATIC GIVEN
C***************************************************************************
C
C
C          IRDG=READING NUMBER
C          ISTA=STATION NUMBER
C            NR=NUMBER OF READINGS TO BE COMPUTED
C             K=THERMAL CONDUCTIVITY OF WALL MATERIAL
C           RHO=DENSITY OF WALL MATERIAL
C            CC=SPECIFIC HEAT OF WALL MATERIAL
```

```
C           L=THICKNESS OF WALL MATERIAL
C          XL=RATIO OF DISTANCE MEASURED FROM HOT GAS SIDE OF THE WALL
C                 TEMPERATURE MEASURING LOCATION TO WALL THICKNESS
C          TG=MEAN TEMPERATURE OF HOT GAS
C         TWO=WALL TEMPERATURE, FUNCTION OF TIME, X=0.0
C         TWL=WALL TEMPERATURE, FUNCTION OF TIME, X=L
C          TC=MEAN TEMPERATURE OF THE COOLANT
C        WRAD=FREQUENCY OF TEMPERATURE OSCILLATIONS
C       OMDTG=RATIO OF THE AMPLITUDE OF THE WALL TEMPERATURE TO THE
C                 AMPLITUDE OF THE SINUSOIDALLY DRIVEN HOT GAS TEMPERATURE
C       ALPHA=THERMAL DIFFUSIVITY
C         ETA=FREQUENCY AND WALL PROPERTY PARAMETER
C      HCA(1)=LOWER LIMIT OF COOLANT SIDE CONVECTIVE HEAT-TRANSFER
C                 COEFFICIENT
C      HCA(3)=UPPER LIMIT OF COOLANT SIDE CONVECTIVE HEAT-TRANSFER
C                 COEFFICIENT
C         HCT=TIME AVERAGED CONVECTIVE HEAT-TRANSFER COEFFICIENT ON
C                 COOLANT SIDE OF THE WALL
C         HGT=TIME AVERAGED CONVECTIVE HEAT-TRANSFER COEFFICIENT ON HOT
C                 GAS SIDE OF THE WALL
C         IJK=NUMBER OF ITERATIONS
C           R=RATIO OF CONVECTIVE HEAT-TRANSFER COEFFICIENTS
C        TCWA=CALCULATED COOLANT WALL TEMPERATURE
C       TEST1=HEAT FLUX TO THE COOLANT
C       TEST2=HEAT FLUX FROM THE HOT GAS
C
C
C***********************************************************************
      DIMENSION TCWA(300),WRAD(300),OMDTG(300),HCA(4),THETA(4)
      EQUIVALENCE (PSIC,PHIC),(PSIG,PHIG)
      REAL K,L,NUM1,NUM2,NUM,NEWERR
C***********************************************************************
C     READ INPUT
C***********************************************************************
    1 READ(5,100) IRDG,ISTA,NR
      READ(5,101) K,RHO,CC,L,XL,TG,TWO,TWL,TC
      READ(5,101) (WRAD(I),OMDTG(I),I=1,NR)
C***********************************************************************
C     PRINT TITLE AND INPUT
C***********************************************************************
      WRITE (6,102)
      WRITE(6,103) IRDG,ISTA
      ALPHA=K/(RHO*CC)
      DO 2 I=1,NR
      WRITE(6,104) I
      WRITE(6,105)
      WRITE(6,106)
      WRITE(6,107) K,RHO,CC,L,XL,TG,TWO,TWL,TC,WRAD(I),OMDTG(I)
C***********************************************************************
C     INITIALIZE STARTING VALUES FOR ITERATION
C***********************************************************************
      TGW=TWO
      TCW=TC+10.
      IGO=1
C***********************************************************************
C     DEFINE HC BOUNDARY CONDITIONS AND CALCULATE R RATIO
C***********************************************************************
    9 R=(TG-TGW)/(TCW-TC)
      HG=-K*(TCW-TGW)/(L*(TG-TGW))
      HC=R*HG
```

```
C***********************************************************************
C     CALCULATE AND COMPARE AMPLITUDE RATIOS
C***********************************************************************
      ETA=SQRT(WRAD(I)/(2.0*ALPHA))
      ETAL=ETA*L
      ANG=2.0*ETAL
      SANG=SIN(ANG)
      CANG=COS(ANG)
      EXPANG=EXP(ANG)
      THETA(2)=CMDTG(I)
      ANG1=ETAL*XL
      EXP1=EXP(ANG1)
      ANG2=ETAL*(2.0-XL)
      EXP2=EXP(ANG2)
      COS1=COS(ANG1)
      SIN1=SIN(ANG1)
      PHIC=K*ETA/HC
      PHIG=R*PHIC
      PGPPC=PHIG+PHIC
      PGPC2=PHIG*PHIC*2.0
      F=PHIC
      E=1.0-F
      PCP1=PHIC+1.0
      D=F*CANG+PCP1*SANG
      C=PCP1*CANG-PHIC*SANG
      B=(PGPPC-PGPC2)+EXPANG*((PGPPC+PGPC2)*CANG+(PGPPC+1.0)*SANG)
      A=1.0-PGPPC+EXPANG*((PGPPC+PGPC2)*SANG-(1.0+PGPPC)*CANG)
      DEL1=C*A-D*B
      DEL2=D*A+C*B
      DEL3=F*A-E*B
      DEL4=E*A+F*B
      NUM1=(DEL1*SIN1-DEL2*COS1)*EXP2
      NUM2=(DEL3*COS1-DEL4*SIN1)*EXP1
      NUM=NUM2-NUM1
      DEN=(DEL4*COS1+DEL3*SIN1)*EXP1-(DEL1*COS1+DEL2*SIN1)*EXP2
      A2PB2=A**2+B**2
      CON=SQRT(NUM**2+DEN**2)
      AMPDTG=CON/A2PB2
      DIFF=AMPDTG-OMDTG(I)
      CALL BALAN(TCW,IGO,DIFF)
      IF(IGO.LT.10) GO TO 9
      IJK=1
      TCWA(IJK)=TCW
      HCT=HC
      HGT=HG
      THETA(4)=AMPDTG
      TEST1=HCT*(TCW-TC)
      TEST2=HGT*(TG-TGW)
      TEST=TEST1/TEST2
      ERROR=1.-TEST
C***********************************************************************
C     PRINT OUTPUT
C***********************************************************************
      WRITE(6,109)
      WRITE(6,110)
      WRITE(6,111) TCWA(IJK),TEST1,TEST2,TEST,HCT,HGT,ERROR,R,THETA(4)
    2 CONTINUE
      GO TO 1
C***********************************************************************
```

```
C      FORMAT STATEMENTS
C*****************************************************************************
  100 FORMAT(3I2)
  101 FORMAT(10E8.4)
  102 FORMAT(120H1CONVECTIVE HEAT-TRANSFER COEFFICIENTS FROM HARMONIC TE
     1MPERATURE OSCILLATIONS -  AMPLITUDE RATIO GIVEN                    )
  103 FORMAT(18H0READING NUMBER = ,I2,10X,18H STATION NUMBER = , I2)
  104 FORMAT(4H0RUN,I2)
  105 FORMAT(6H0INPUT)
  106 FORMAT(1H0,4X,1HK,8X,3HRHO,8X,2HCC,8X,1HL,9X,2HXL,8X,2HTG,7X,3HTWO
     1,7X,3HTWL,7X,2HTC,7X,4HWRAD,6X,5HOMDTG)
  107 FORMAT(1H0,F8.6,4F10.4,4F10.1,2F10.4)
  108 FORMAT(55H0ITERATIONS EXCEED 50                                    )
  109 FORMAT(7H0OUTPUT)
  110 FORMAT(1H0,3X,4HTCWA,5X,5HTEST1,5X,5HTEST2,6X,4HTEST,6X,3HHCT,7X,3
     1HHGT,6X,5HERROR,7X,1HR,7X,5HTHETA)
  111 FORMAT(1H0,F8.1,3F10.4,2F10.6,3F10.4)
  112 FORMAT(55H0SOLUTION IS DIVERGING                                   )
      END


$IBFTC BALAN

      SUBROUTINE BALAN(X,K,Y)
      GO TO (1,2),K
    1 X1=X
      Y1=Y
      X=X+X/10.
      K=2
      RETURN
    2 X2=X
      Y2=Y
      IF(ABS(Y).LT.1.E-4)K=15
      X=(X2-X1)/(Y2-Y1)*(-Y1)+X1
      X1=X2
      Y1=Y2
      RETURN
      END
```

Program BL Output

CONVECTIVE HEAT-TRANSFER COEFFICIENTS FROM HARMONIC TEMPERATURE OSCILLATIONS - AMPLITUDE RATIO GIVEN

READING NUMBER = 1 STATION NUMBER = 1

RUN 1

INPUT

K	RHO	CC	L	XL	TG	TWO	TWL	TC	WRAD	OMDTG
0.000207	0.2860	0.1170	0.0600	0.1667	960.0	658.0	658.0	570.6	1.2390	0.1538

OUTPUT

TCWA	TEST1	TEST2	TEST	HCT	HGT	ERROR	R	THETA
619.0	0.1344	0.1344	0.9998	0.002775	0.000445	0.0002	6.2350	0.1538

01 UNITC5, EOF. REC= 00000 FIL= 00002

PROGRAM JCL - PHASE LAG ANGLE GIVEN

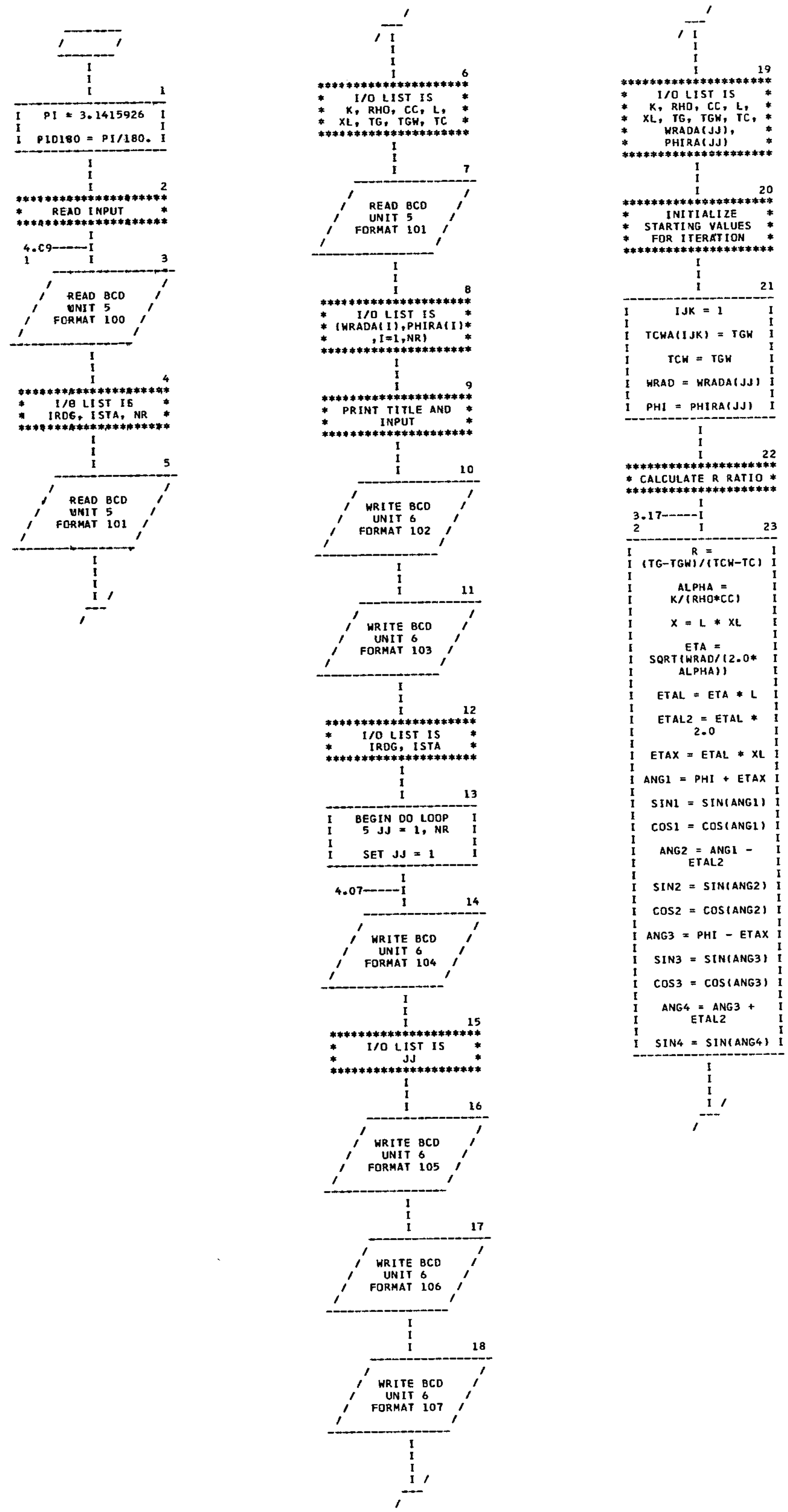


Figure 4. - Flow chart for program JCL.

PROGRAM JCL - PHASE LAG ANGLE GIVEN

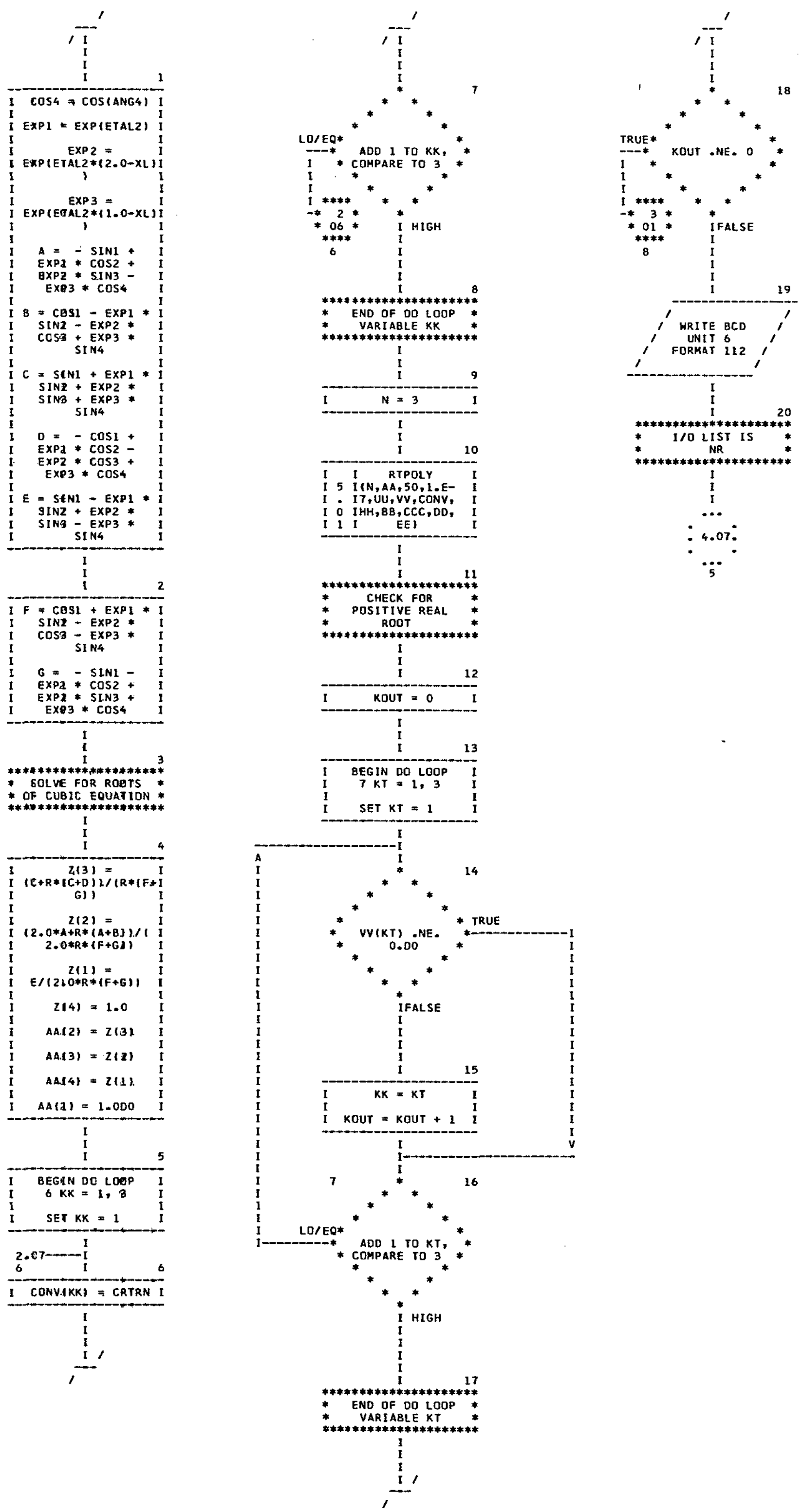


Figure 4. - Continued.

PROGRAM JCL - PHASE LAG ANGLE GIVEN

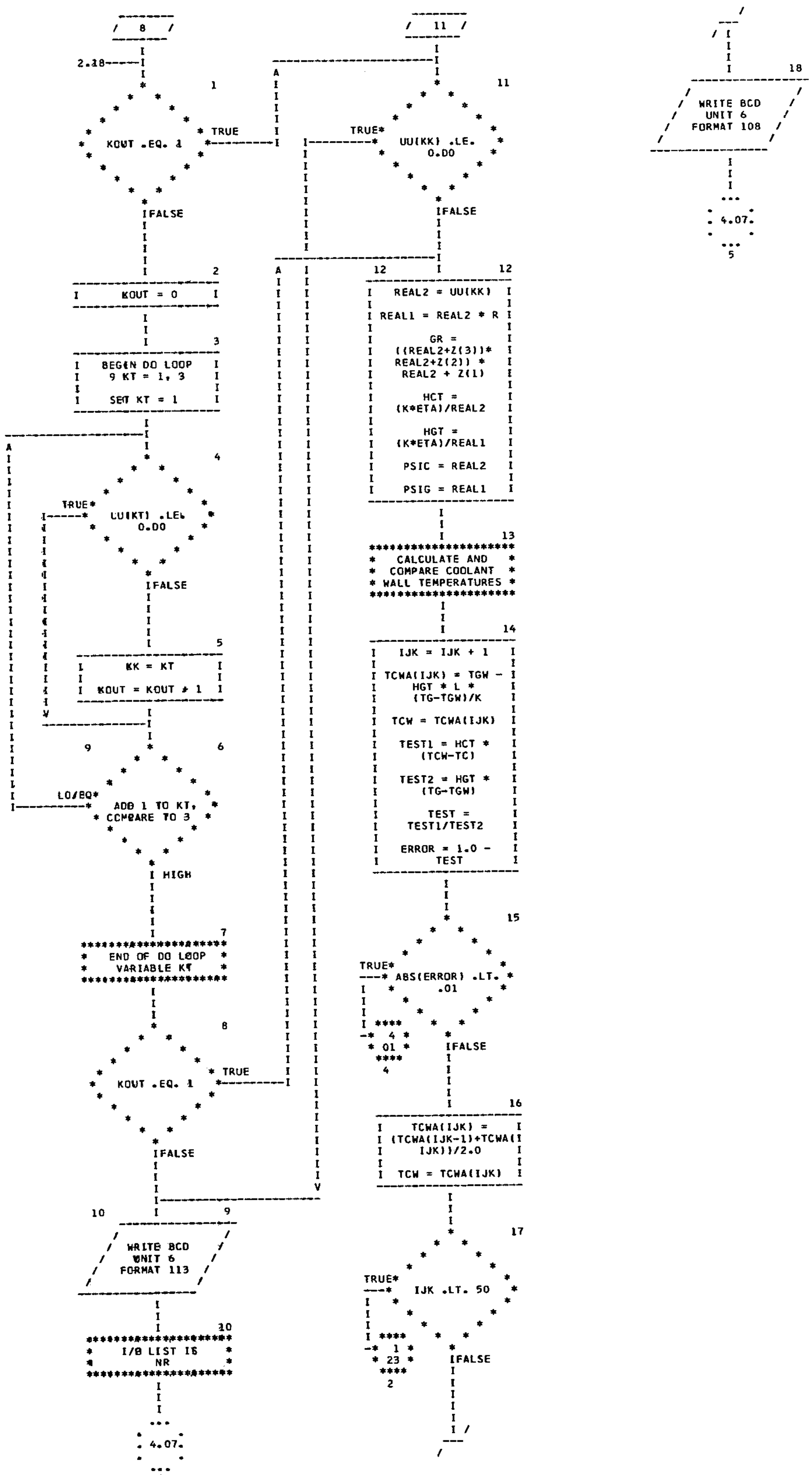


Figure 4. - Continued.

PROGRAM JCL - PHASE LAG ANGLE GIVEN

```
              --------
             /   4   /
             --------
                 I
      3.15-------I
                 I         1
       ----------------------
       I         R =        I
       I (TG-TGM)/(TCW-TC)  I
       ----------------------
                 I
                 I
                 I         2
       **********************
       *   PRINT OUTPUT     *
       **********************
                 I
                 I
                 I         3
            -----------------
           /               /
          /  WRITE BCD    /
         /    UNIT 6     /
        /   FORMAT 109  /
       /               /
       -----------------
                 I
                 I
                 I         4
            -----------------
           /               /
          /  WRITE BCD    /
         /    UNIT 6     /
        /   FORMAT 110  /
       /               /
       -----------------
                 I
                 I
                 I         5
            -----------------
           /               /
          /  WRITE BCD    /
         /    UNIT 6     /
        /   FORMAT 111  /
       /               /
       -----------------
                 I
                 I
                 I         6
       **********************
       *    I/O LIST IS     *
       * TCWA(IJK), TEST1,  *
       * TEST2, TEST, HCT,  *
       *   HGT, ERROR, R,   *
       *   PSIC, PSIG, GR   *
       **********************
                 I
       2.20---*--I
                 I
       5         *         7
               *   *
             *       *
           *           *
    LO/EQ*               *
    ---*   ADD 1 TO JJ,   *
    I    * COMPARE TO NR *
    I      *           *
    I        *       *
    I ****     *   *
    -* I *       *
     * 14 *      I HIGH
      ****       I
                 I
                 I
                 I
                 I         8
       **********************
       *   END OF DO LOOP   *
       *    VARIABLE JJ     *
       **********************
                 I
                 I
                 I         9
            -----------------
           /               /
          /  WRITE BCD    /
         /    UNIT 6     /
        /   FORMAT 114  /
       /               /
       -----------------
                 I
                 I
                 I
                ...
              .     .
              . 1.03.
              .     .
                ...
                 I
```

FORMAT STATEMENTS

```
     --------
    /       /
    --------
        I
        I
        I
       ... 10
      .     .
      . EXIT.
      .     .
       ...
```

Figure 4. - Continued.

PRØGRAM JCL - PHASE LAG ANGLE GIVEN

SUBROUTINE RTPOLY

```
                              ---------
                             / RTPOLY/
                              ---------
                                  I
                         2.10-----I
                                  I           1
                       ********************** 
                       * SUBROUTINE RTPOLY *
                       **********************
                                  I
                                  I
                                  I           2
                       ----------------------
                       I     NSAVE = N      I
                       I                    I
                       I     NN = N + 1     I
                       ----------------------
                                  I
                                  I
                                  I           3
                       ----------------------
                       I   BEGIN DO LOOP    I
                       I    1 I = 1, NN     I
                       I                    I
                       I     SET I = 1      I
                       ----------------------
                                  I
            ----------------------I
            A            1        I           4
            I          ----------------------
            I          I    H(I) = A(I)     I
            I          ----------------------
            I                     I
            I                     I
            I                     I
            I                     *           5
            I                   *   *
            I                 *       *
            I               *           *
            I     LO/EQ*                  *
            I-----------*  ADD 1 TO I,      *
                         * COMPARE TO NN *
                           *           *
                             *       *
                               *   *
                                 *
                                 I HIGH
                                 I
                                 I
                                 I
                                 I
                                 I            6
                       **********************
                       *   END OF DO LOOP   *
                       *     VARIABLE I     *
                       **********************
                                 I
                                 I
                                 I            7
                       ----------------------
                       I      XK = F        I
                       I                    I
                       I      LT = 1        I
                       ----------------------
                                 I
                                 I
                                 I            8
                       **********************
                       *    EXTRACT ZERO    *
                       *       ROOTS        *
                       **********************
                                 I
                                 I
                                 I
                                 I /
                                ---
                               /
```

Figure 4. - Continued.

PROGRAM JCL - PHASE LAG ANGLE GIVEN

SUBROUTINE RTPOLY

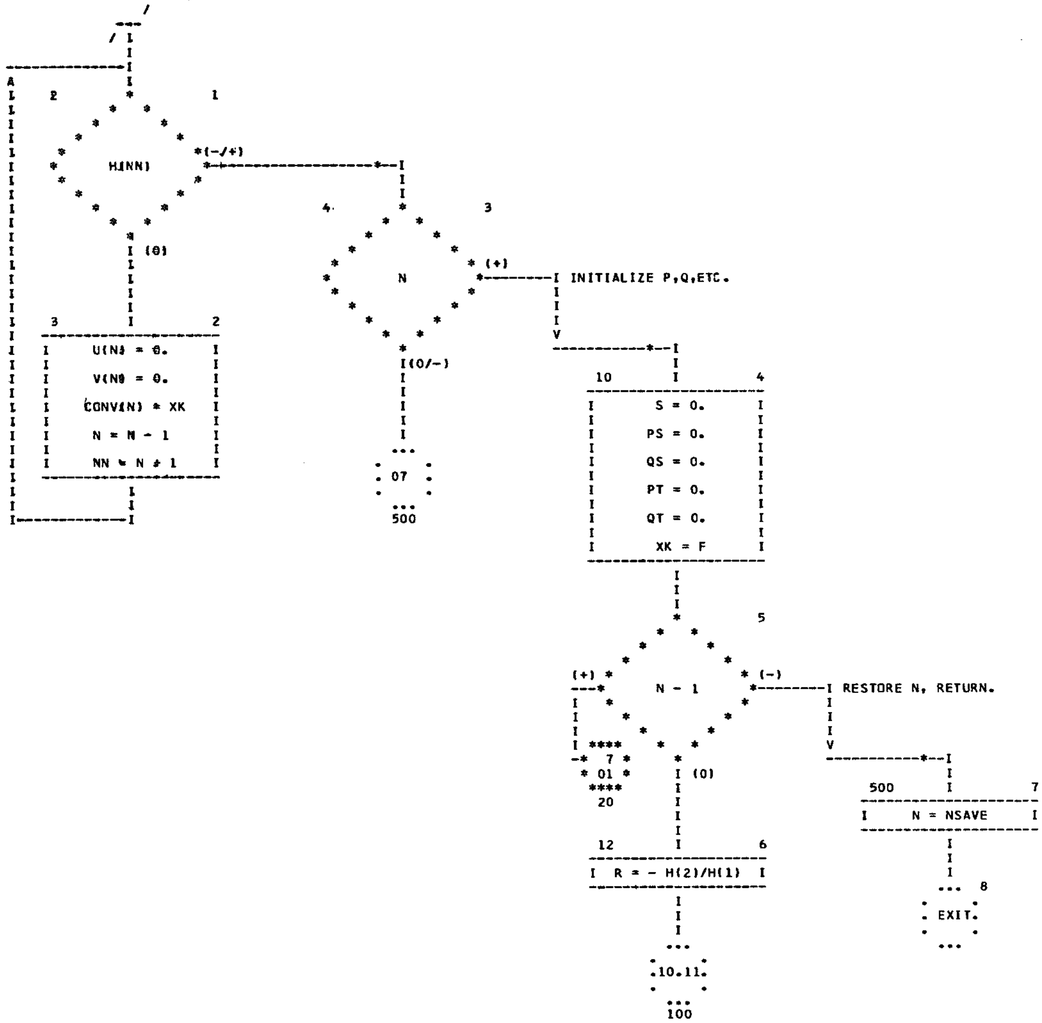


Figure 4. - Continued.

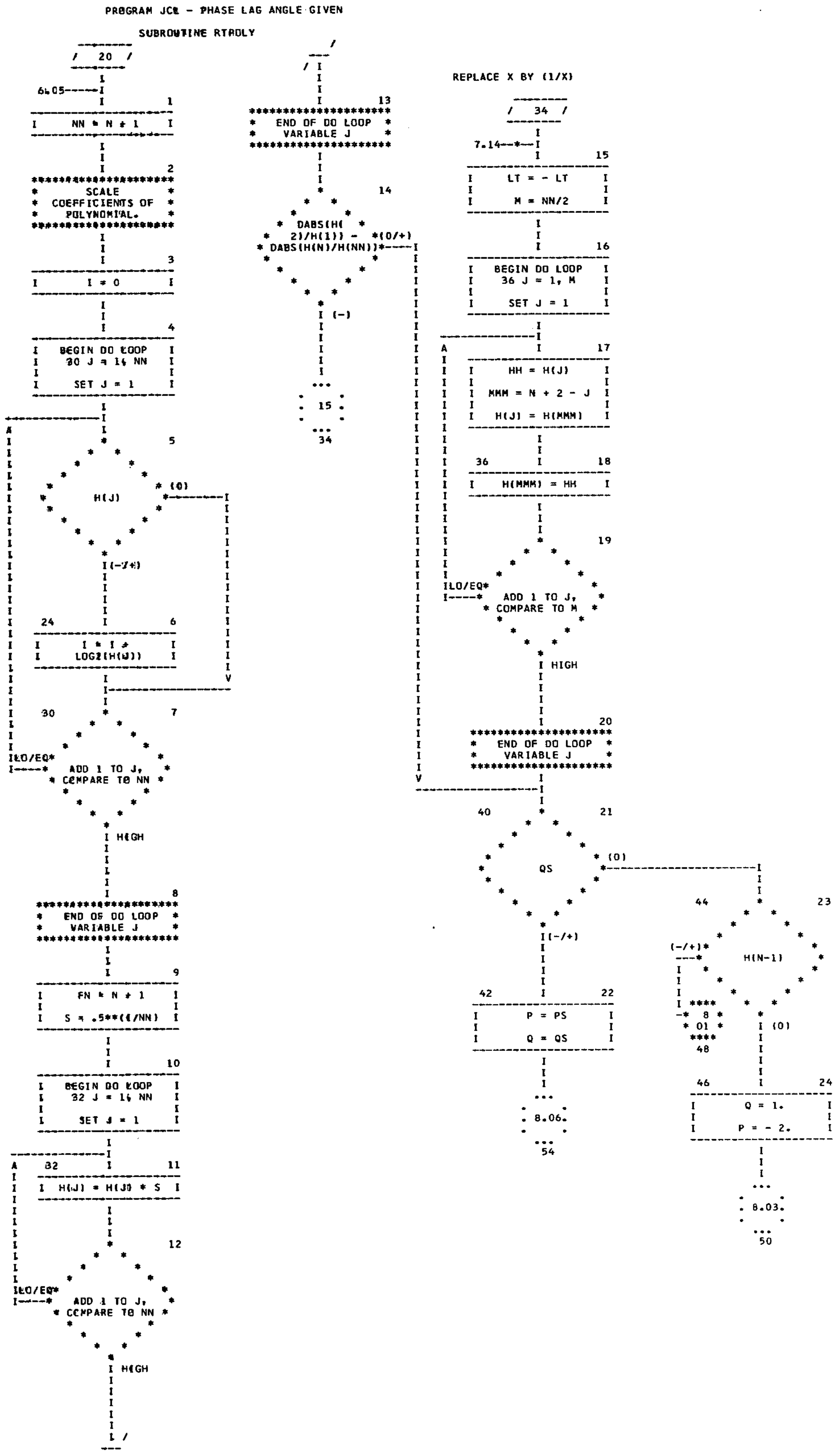


Figure 4. - Continued.

PROGRAM JCL - PHASE LAG ANGLE GIVEN

SUBROUTINE RTPOLY

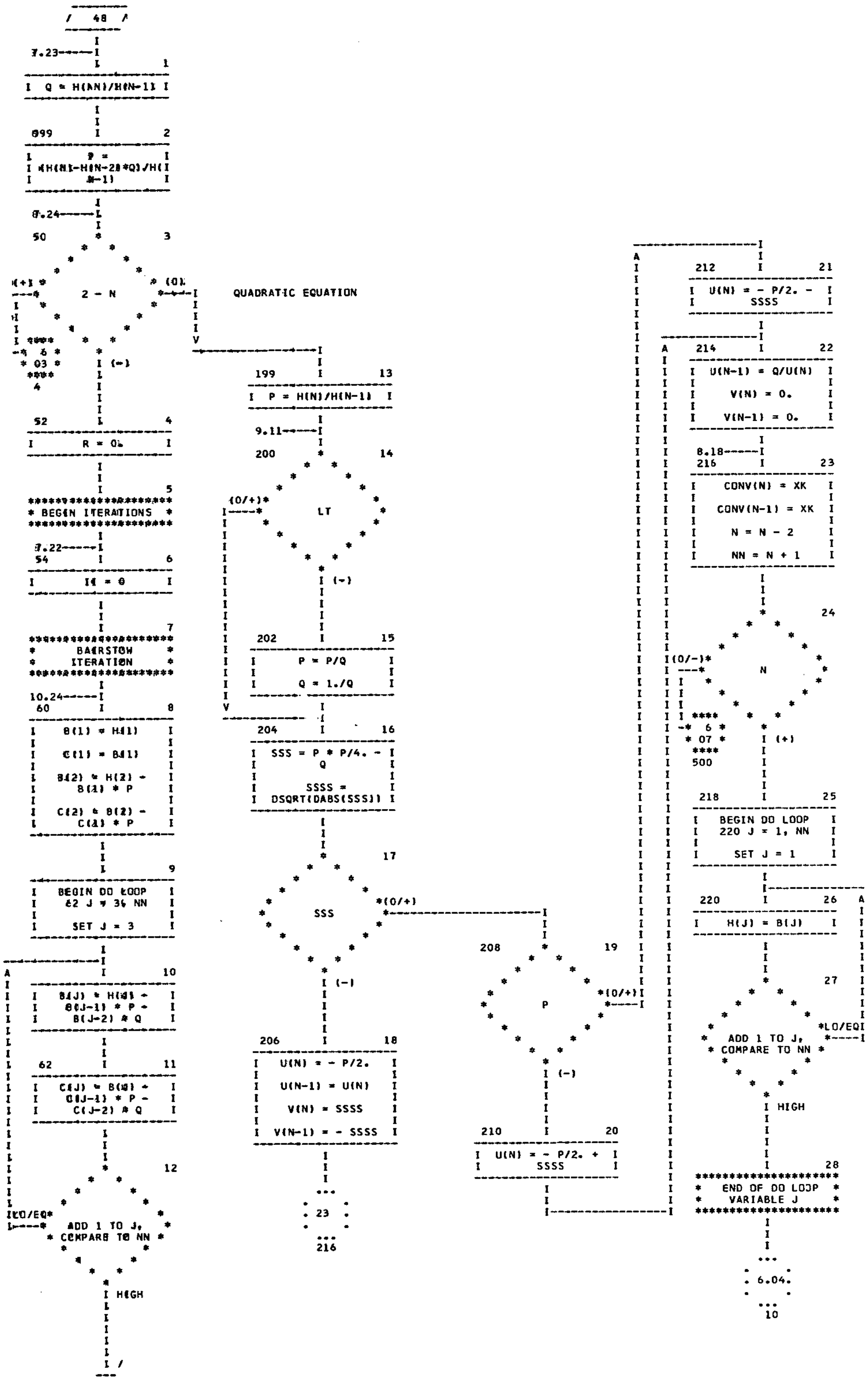


Figure 4. - Continued.

PROGRAM JCL - PHASE LAG ANGLE GIVEN

SUBROUTINE RTPOLY

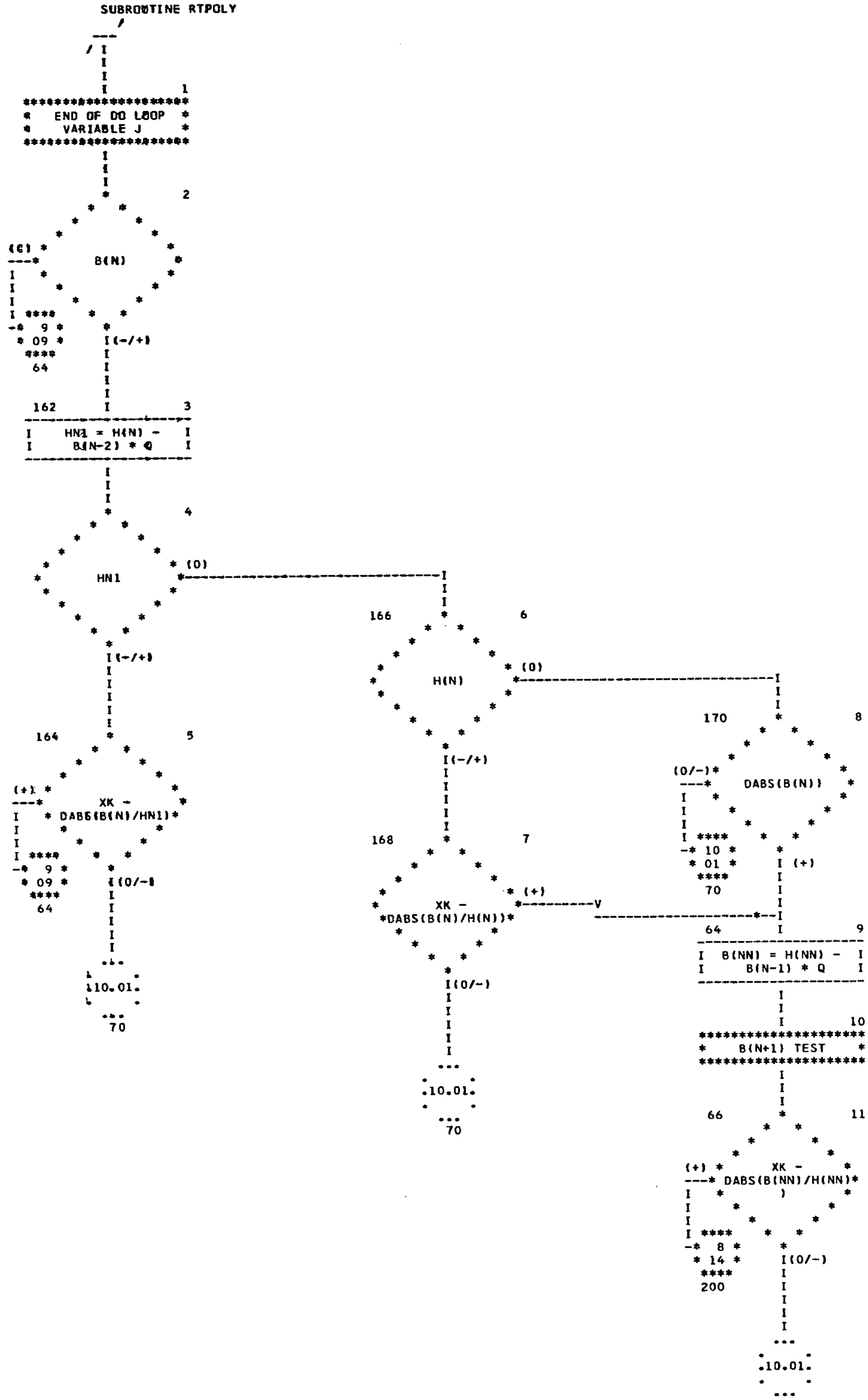


Figure 4. - Continued.

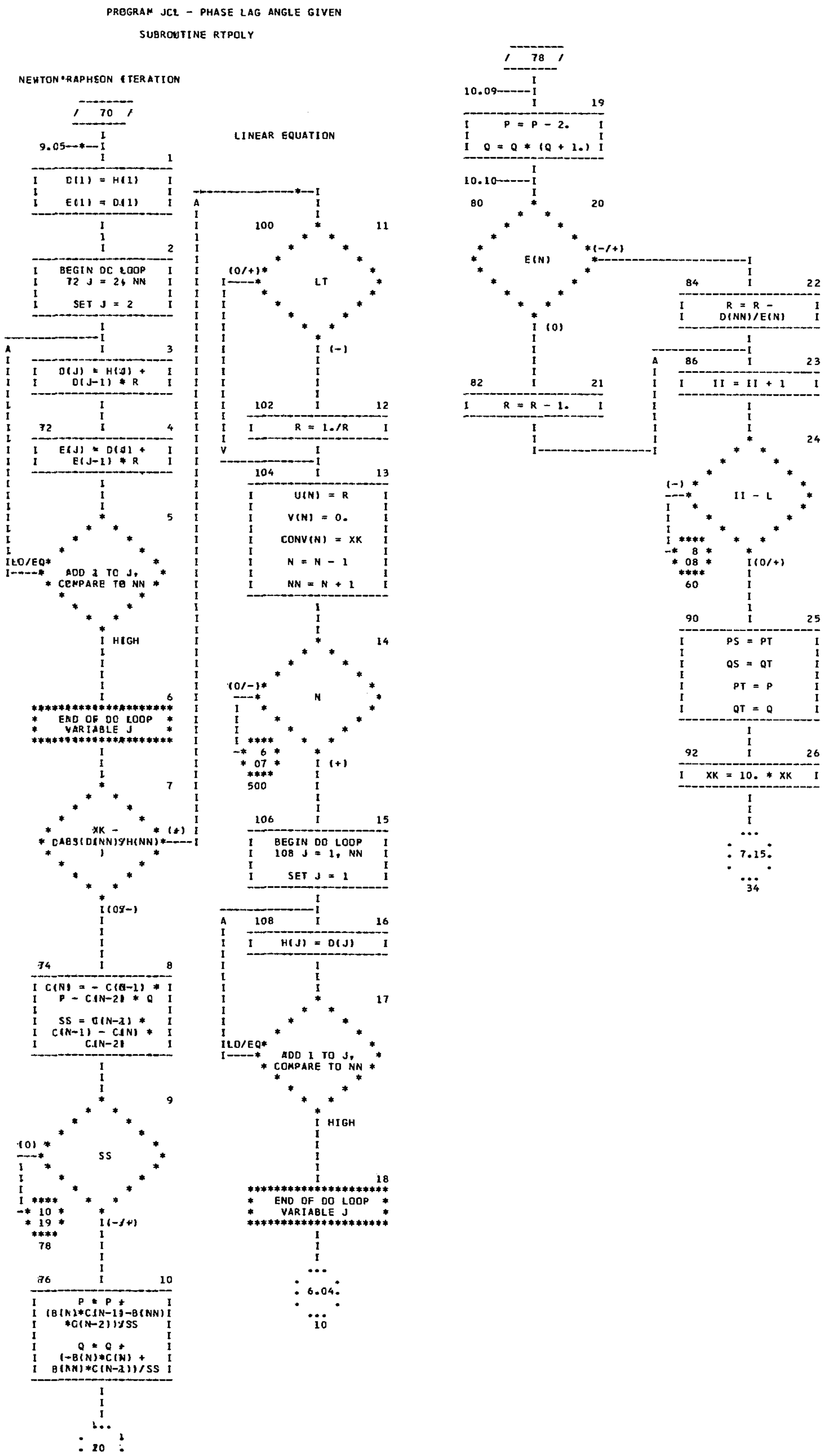


Figure 4. - Concluded.

Program JCL

```
$IEFTC JCL

C
C
C*************************************************************************
C     CONVECTIVE HEAT-TRANSFER COEFFICIENTS FROM HARMONIC TEMPERATURE
C     OSCILLATIONS  -  PHASE LAG ANGLE GIVEN
C*************************************************************************
C
C
C        IRDG=READING NUMBER
C        ISTA=STATION NUMBER
C          NR=NUMBER OF DATA POINTS TO BE COMPUTED
C           K=THERMAL CONDUCTIVITY OF WALL MATERIAL
C         RHO=DENSITY OF WALL MATERIAL
C          CC=SPECIFIC HEAT OF WALL MATERIAL
C           L=THICKNESS OF WALL MATERIAL
C          XL=RATIO OF DISTANCE MEASURED FROM HOT GAS SIDE OF THE WALL
C               TEMPERATURE MEASURING LOCATION TO WALL THICKNESS
C          TG=MEAN TEMPERATURE OF HOT GAS
C         TGW=WALL TEMPERATURE,FUNCTION OF DISTANCE
C          TC=MEAN TEMPERATURE OF THE COOLANT
C       WRADA=FREQUENCY OF TEMPERATURE OSCILLATIONS
C       PHIRA=PHASE LAG ANGLE BETWEEN THE DRIVING GAS TEMPERATURE AND THE
C               RESPONDING WALL TEMPERATURE
C       ALPHA=THERMAL DIFFUSIVITY
C         ETA=FREQUENCY AND WALL PROPERTY PARAMETER
C          GR=CUBIC EQUATION CHECK OF ROOTS, SHOULD APPROXIMATE ZERO
C         HCT=TIME AVERAGED CONVECTIVE HEAT-TRANSFER COEFFICIENT ON
C               COOLANT SIDE OF THE WALL
C         HGT=TIME AVERAGED CONVECTIVE HEAT-TRANSFER COEFFICIENT ON HOT
C               GAS SIDE OF THE WALL
C         IJK=NUMBER OF ITERATIONS
C           R=RATIO OF CONVECTIVE HEAT-TRANSFER COEFFICIENTS
C        PSIG=CONVECTIVE HEAT-TRANSFER PARAMETER ON THE HOT GAS SIDE OF
C               THE WALL
C        PSIC=CONVECTIVE HEAT-TRANSFER PARAMETER ON THE COOLANT SIDE OF
C               THE WALL
C         TCW=CALCULATED COOLANT WALL TEMPERATURE
C       TEST1=HEAT FLUX TO THE COOLANT
C       TEST2=HEAT FLUX FROM THE HOT GAS
C        CONV=CONVERGENCE CRITERION USED FOR EACH ROOT
C
C
C*************************************************************************
      DIMENSION WRADA(100),PHIRA(100),TCWA(100),Z(4),CONV(4)
      DOUBLE PRECISION HH(4),BB(4),CCC(4),DD(4),EE(4),AA(4),UU(4),VV(4)
      REAL L,K
      DATA CRTRN/1.E-5/
      PI=3.1415926
      PID180=PI/180.
C*************************************************************************
C     READ INPUT
C*************************************************************************
    1 READ(5,100) IRDG,ISTA,NR
      READ(5,101) K,RHO,CC,L,XL,TG,TGW,TC
```

```
      READ(5,101) (WRADA(I),PHIRA(I),I=1,NR)
C*********************************************************************
C     PRINT TITLE AND INPUT
C*********************************************************************
      WRITE (6,102)
      WRITE(6,103) IRDG,ISTA
      DO 5 JJ=1,NR
      WRITE(6,104) JJ
      WRITE(6,105)
      WRITE(6,106)
      WRITE(6,107) K,RHO,CC,L,XL,TG,TGW,TC,WRADA(JJ),PHIRA(JJ)
C*********************************************************************
C     INITIALIZE STARTING VALUES FOR ITERATION
C*********************************************************************
      IJK=1
      TCWA(IJK)=TGW
      TCW=TGW
      WRAD=WRADA(JJ)
      PHI=PHIRA(JJ)
C*********************************************************************
C     CALCULATE R RATIO
C*********************************************************************
    2 R=(TG-TGW)/(TCW-TC)
      ALPHA=K/(RHO*CC)
      X=L*XL
      ETA=SQRT(WRAD/(2.0*ALPHA))
      ETAL=ETA*L
      ETAL2=ETAL*2.0
      ETAX=ETAL*XL
      ANG1=PHI+ETAX
      SIN1=SIN(ANG1)
      COS1=COS(ANG1)
      ANG2=ANG1-ETAL2
      SIN2=SIN(ANG2)
      COS2=COS(ANG2)
      ANG3=PHI-ETAX
      SIN3=SIN(ANG3)
      COS3=COS(ANG3)
      ANG4=ANG3+ETAL2
      SIN4=SIN(ANG4)
      COS4=COS(ANG4)
      EXP1=EXP(ETAL2)
      EXP2=EXP(ETAL2*(2.0-XL))
      EXP3=EXP(ETAL2*(1.0-XL))
      A=-SIN1+EXP1*COS2+EXP2*SIN3-EXP3*COS4
      B=COS1-EXP1*SIN2-EXP2*COS3+EXP3*SIN4
      C=SIN1+EXP1*SIN2+EXP2*SIN3+EXP3*SIN4
      D=-COS1+EXP1*COS2-EXP2*COS3+EXP3*COS4
      E=SIN1-EXP1*SIN2+EXP2*SIN3-EXP3*SIN4
      F=COS1+EXP1*SIN2-EXP2*COS3-EXP3*SIN4
      G=-SIN1-EXP1*COS2+EXP2*SIN3+EXP3*COS4
C*********************************************************************
C     SOLVE FOR ROOTS OF CUBIC EQUATION
C*********************************************************************
      Z(3)=(C+R*(C+D))/(R*(F+G))
      Z(2)=(2.0*A+R*(A+B))/(2.0*R*(F+G))
      Z(1)=E/(2.0*R*(F+G))
      Z(4)=1.0
      AA(2)=Z(3)
```

```
      AA(3)=Z(2)
      AA(4)=Z(1)
      AA(1)=1.0D0
      DO 6 KK=1,3
    6 CONV(KK)=CRTRN
      N=3
      CALL RTPOLY(N,AA,50,1.E-7,UU,VV,CONV,HH,BB,CCC,DD,EE)
C***********************************************************************
C     CHECK FOR POSITIVE REAL ROOT
C***********************************************************************
      KOUT=0
      DO 7 KT=1,3
      IF(VV(KT).NE.0.D0) GO TO 7
      KK=KT
      KOUT=KOUT+1
    7 CONTINUE
      IF(KOUT.NE.0) GO TO 8
      WRITE(6,112) NR
      GO TO 5
    8 IF(KOUT.EQ.1) GO TO 11
      KOUT=0
      DO 9 KT=1,3
      IF(UU(KT).LE.0.D0) GO TO 9
      KK=KT
      KOUT=KOUT+1
    9 CONTINUE
      IF(KOUT.EQ.1) GO TO 12
   10 WRITE(6,113) NR
      GO TO 5
   11 IF(UU(KK).LE.0.D0) GO TO 10
   12 REAL2=UU(KK)
      REAL1=REAL2*R
      GR=((REAL2+Z(3))*REAL2+Z(2))*REAL2+Z(1)
      HCT=(K*ETA)/REAL2
      HGT=(K*ETA)/REAL1
      PSIC=REAL2
      PSIG=REAL1
C***********************************************************************
C     CALCULATE AND COMPARE COOLANT WALL TEMPERATURES
C***********************************************************************
      IJK=IJK+1
      TCWA(IJK)=TGW-HGT*L*(TG-TGW)/K
      TCW=TCWA(IJK)
      TEST1=HCT*(TCW-TC)
      TEST2=HGT*(TG-TGW)
      TEST=TEST1/TEST2
      ERROR=1.0-TEST
      IF(ABS(ERROR).LT..01) GO TO 4
      TCWA(IJK)=(TCWA(IJK-1)+TCWA(IJK))/2.0
      TCW=TCWA(IJK)
      IF(IJK.LT.50) GO TO 2
      WRITE(6,108)
      GO TO 5
    4 R=(TG-TGW)/(TCW-TC)
C***********************************************************************
C     PRINT OUTPUT
C***********************************************************************
      WRITE(6,109)
      WRITE(6,110)
```

```
      WRITE(6,111) TCWA(IJK),TEST1,TEST2,TEST,HCT,HGT,ERROR,R,PSIC,PSIG,
     1GR
    5 CONTINUE
      WRITE(6,114)
      GO TO 1
C***********************************************************************
C     FORMAT STATEMENTS
C***********************************************************************
  100 FORMAT(3I2)
  101 FORMAT(10E8.4)
  102 FORMAT(120H1CONVECTIVE HEAT-TRANSFER COEFFICIENTS FROM HARMONIC TE
     1MPERATURE OSCILLATIONS -   PHASE LAG ANGLE GIVEN                  )
  103 FORMAT(18H0READING NUMBER = ,I2,10X,18H STATION NUMBER = , I2)
  104 FORMAT(4H0RUN,I2)
  105 FORMAT(6H0INPUT)
  106 FORMAT(1H0,4X,1HK,8X,3HRHO,8X,2HCC,8X,1HL,9X,2HXL,8X,2HTG,7X,3HTGW
     1,8X,2HTC,6X,5HWRADA,5X,5HPHIRA)
 107 FORMAT(1H0,F8.6,4F10.4,3F10.1,2F10.4)
 108 FORMAT(55H0ITERATIONS EXCEED 50                                   )
 109 FORMAT(7H0OUTPUT)
 110 FORMAT(1H0,4X,3HTCW,5X,5HTEST1,5X,5HTEST2,6X,4HTEST,6X,3HHCT,7X,3H
    1HGT,6X,5HERROR,7X,1HR,8X,4HPSIC,6X,4HPSIG,7X,2HGR)
 111 FORMAT(1H0,F8.1,3F10.4,2F10.6,5F10.4)
 112 FORMAT(40H0ERROR IN SUBR RTPOLY FOR DATA POINT NO. I5)
 113 FORMAT(63H0A UNIQUE POSITIVE REAL ROOT CANNOT BE FOUND FOR DATA PO
    1INT NO. I5)
 114 FORMAT(1H1)
     END

$IBFTC POLRT

      SUBROUTINE RTPOLY(N,A,L,F,U,V,CONV,H,B,C,D,E)                     IB400010
C     N          ORDER OF INPUT POLYNOMIAL                         FIXEDIB400020
C     A          COEFFICIENTS OF INPUT POLYNOMIAL IN DESCENDIN ORDER  D.P.IB400030
C     L          MAXIMUM NUMBER OF ITERATION FOR EACH ROOT         FIXEDIB400040
C     F          DESIRED TOLERANCE (REMAINDER / CONSTANT TERM)      S.P.IB400050
C     U          REAL PART OF ROOT                                  D.P.IB400060
C     V          IMAGINARY PART OF ROOT                             D.P.IB400070
C     CONV       CONVERGENCE CRITERION USED FOR EACH ROOT           S.P.IB400080
C                IF AFTER L ITERATIONS CONVERGENCE WITHIN F FAILS,  S.P.IB400090
C                THE TOLERANCE IS RELAXED BY 10.                    S.P.IB400100
C     H          WORKING STORAGE                                    D.P.IB400110
C     B          WORKING STORAGE                                    D.P.IB400120
C     C          WORKING STORAGE                                    D.P.IB400130
C     D          WORKING STORAGE                                    D.P.IB400140
C     E          WORKING STORAGE                                    D.P.IB400150
      DIMENSION A(40),U(40),V(40),CONV(40),H(40),B(40),C(40),D(40),E(40)IB400160
      DOUBLE PRECISION  A,U,V,H,B,C,D,E                                 IB400170
      DOUBLE PRECISION  P,Q,R,S,PS,QS,PT,QT,HH,HN1,XK,SSS,SSSS          IB400180
      NSAVE = N                                                         IB400190
      NN = N+1                                                          IB400200
      DO 1 I = 1,NN                                                     IB400210
    1 H(I) = A(I)                                                       IB400220
      XK = F                                                            IB400230
      LT = 1                                                            IB400240
C     EXTRACT ZERO ROOTS                                                IB400250
    2 IF(H(NN)) 4,3,4                                                   IB400260
    3 U(N) = 0.                                                         IB400270
      V(N) = 0.                                                         IB400280
      CONV(N) = XK                                                      IB400290
      N = N-1                                                           IB400300
```

```
      NN = N+1
      GO TO 2
    4 IF(N) 500,500,10
C     INITIALIZE P,Q,ETC.
   10 S = 0.
      PS = 0.
      QS = 0.
      PT = 0.
      QT = 0.
      XK = F
      IF (N-1) 500,12,20
   12 R = -H(2)/H(1)
      GO TO 100
   20 NN = N+1
C     SCALE COEFFICIENTS OF POLYNOMIAL.
      I=0
      DO 30 J = 1,NN
      IF(H(J)) 24,30,24
 24   I=I+LOG2(H(J))
   30 CONTINUE
      FN = N + 1
      S=.5**(I/NN)
      DO 32 J = 1,NN
  32  H(J)=H(J)*S
      IF(DABS(H(2)/H(1)) - DABS(H(N)/H(NN)))  34,40,40
C     REPLACE X BY (1/X)
   34 LT = - LT
      M = NN/2
      DO 36 J = 1, M
      HH = H(J)
      MMM =N+2-J
      H(J) = H(MMM)
   36 H(MMM) = HH
   40 IF(QS) 42,44,42
   42 P = PS
      Q = QS
      GO TO 54
   44 IF(H(N-1)) 48,46,48
   46 Q = 1.
      P = -2.
      GO TO 50
   48 Q = H(NN)/H(N-1)
  999 P = (H(N)-H(N-2)*Q)/H(N-1)
   50 IF(2-N) 52,199,4
   52 R = 0.
C     BEGIN ITERATIONS
   54 II = 0
C     BAIRSTOW ITERATION
   60 B(1) = H(1)
      C(1) = B(1)
      B(2) =  H(2) - B(1)*P
      C(2) = B(2) - C(1)*P
      DO 62 J = 3,NN
      B(J) = H(J) - B(J-1)*P - B(J-2)*Q
   62 C(J) = B(J) - C(J-1)*P - C(J-2)*Q
      IF(B(N)) 162,64,162
  162 HN1 = H(N) - B(N-2) *Q
      IF(HN1) 164,166,164
  164 IF(XK - DABS(B(N)/HN1)) 70,70,64
  166 IF(H(N)) 168,170,168
  168 IF(XK - DABS(B(N)/H(N))) 70,70,64
  170 IF(DABS(B(N)))70,70,64
   64 B(NN) = H(NN) - B(N-1)*Q
C     B(N+1) TEST
```

```
   66 IF(XK - DABS(B(NN)/H(NN))) 70,70,200
C      NEWTON-RAPHSON ITERATION
   70 D(1) = H(1)
      E(1) = D(1)
      DO 72 J = 2, NN
      D(J) =  H(J) + D(J-1)*R
   72 E(J) = D(J) + E(J-1)*R
      IF(XK - DABS(D(NN)/H(NN))) 74,74,100
   74 C(N) = -C(N-1)*P - C(N-2)*Q
      SS = C(N-1)*C(N-1) - C(N)*C(N-2)
      IF(SS) 76,78,76
   76 P = P  + (B(N)*C(N-1)-B(NN)*C(N-2))/SS
      Q = Q  + (-B(N)*C(N) + B(NN)*C(N-1))/SS
      GO TO 80
   78 P = P - 2.
      Q = Q*(Q + 1.)
   80 IF(E(N)) 84,82,84
   82 R = R - 1.
      GO TO 86
   84 R = R - D(NN)/E(N)
   86 II = II + 1
      IF( II - L ) 60,90,90
   90 PS = PT
      QS = QT
      PT = P
      QT = Q
   92 XK = 10.*XK
      GO TO 34
C      LINEAR EQUATION
  100 IF(LT) 102,104,104
  102 R = 1./R
  104 U(N) = R
      V(N)     =  0.
      CONV(N) = XK
      N = N-1
      NN = N + 1
      IF(N) 500,500,106
  106 DO 108 J = 1, NN
  108 H(J) = D(J)
      GO TO 10
C      QUADRATIC EQUATION
  199 P = H(N)/H(N-1)
  200 IF(LT) 202,204,204
  202 P = P/Q
      Q = 1./Q
  204 SSS = P*P/4. - Q
      SSSS = DSQRT(DABS(SSS))
      IF(SSS) 206,208,208
  206 U(N) = - P/2.
      U(N-1) = U(N)
      V(N) = SSSS
      V(N-1) = - SSSS
      GO TO 216
  208 IF(P) 210,212,212
  210 U(N) = - P/2. + SSSS
      GO TO 214
  212 U(N) = - P/2. - SSSS
  214 U(N-1) = Q/U(N)
      V(N) = 0.
      V(N-1) = 0.
  216 CONV(N) = XK
      CONV(N-1) = XK
      N = N-2
      NN = N +1
      IF(N) 500,500,218
  218 DO 220 J = 1,NN
```

```
  220 H(J) = B(J)                                                       IB401610
      GO TO 10                                                          IB401620
C     RESTORE N, RETURN.                                                IB401630
  500 N = NSAVE                                                         IB401640
      RETURN                                                            IB401650
       END                                                              IB401660
```

Program JCL Output

CONVECTIVE HEAT-TRANSFER COEFFICIENTS FROM HARMONIC TEMPERATURE OSCILLATIONS - PHASE LAG ANGLE GIVEN

READING NUMBER = 1 STATION NUMBER = 1

RUN 1

INPUT

K	RHO	CC	L	XL	TG	TGW	TC	WRADA	PHIRA
0.000207	0.2860	0.1170	0.0600	0.1667	960.0	658.0	570.6	1.2380	0.7673

OUTPUT

TCW	TEST1	TEST2	TEST	HCT	HGT	ERROR	R	PSIC	PSIG	GR
624.6	0.1146	0.1152	0.9949	0.002121	0.000381	0.0051	5.5903	0.9764	5.4305	-0.0000

Lewis Research Center,

National Aeronautics and Space Administration,

Cleveland, Ohio, July 13, 1970,

129-04.

APPENDIX - SYMBOLS

Machine code	Engineering symbol	
ALPHA	α	thermal diffusivity, $K/\rho C$, in.^{2}/sec; m^2/sec
CC	c	specific heat of wall material, Btu/(lbm)($^{\circ}$R); J/(kg)(K)
ERROR	---	1.0 - (TEST1)/(TEST2)
ETA	η	frequency and wall property parameter, 1/in.; 1/m
GR	---	cubic equation check of roots, equals zero for correct solution
HCA(1)	---	lower limits of the coolant-side convective heat-transfer coefficient
HCA(3)	---	upper limits of the coolant-side convective heat-transfer coefficient
HCT	h_c	time averaged convective heat-transfer coefficient on the coolant side of the wall, Btu/(in.^{2})(sec)($^{\circ}$R); W/(m^2)(K)
HGT	h_G	time averaged convective heat-transfer coefficient on hot-gas side of wall, Btu/(in.^{2})(sec)($^{\circ}$R); W/(m^2)(K)
IJK	---	number of iterations
IRDG	---	reading number
ISTA	---	station number
K	K	thermal conductivity of wall material, Btu/(in.)(sec)($^{\circ}$R); J/(m)(sec)(K)
L	L	thickness of wall material, in.; m
NR	---	number of readings to be computed
OMDTG	$\theta/\Delta T_G$	ratio of amplitude of wall temperature to amplitude of sinusoidally driving hot-gas temperature
PHIRA	φ	phase-lag angle between driving gas temperature and responding wall temperature, rad
R	R	ratio of convective heat-transfer coefficients, h_c/h_G
REAL1	ψ_G	convective heat-transfer parameter on hot-gas side of wall (JCL program), $K\eta/h_G$

Machine code	Engineering symbol	
REAL2	ψ_c	convective heat-transfer parameter on coolant side of the wall, $K\eta/h_c$
RHO	ρ	density of wall material, lbm/in.^{3}; g/m^3
XL	x/L	ratio of distance measured from the hot-gas side of wall to wall temperature measuring location to the wall thickness
TC	$\overline{T}_c$	mean temperature of the coolant, $^{\circ}$R; K
TCW	T_{cw}	calculated coolant wall temperature (JCL program), $^{\circ}$R; K
TCWA	T_{cw}	calculated coolant wall temperature (BL program), $^{\circ}$R; K
TEST1	q_c	heat flux to coolant, Btu/(in.^{2})(sec); W/m^2
TEST2	q_G	heat flux from hot gas, Btu/(in.^{2})(sec); W/m^2
TG	$\overline{T}_G$	mean temperature of hot gas, $^{\circ}$R; K
TGW	$T(x, \tau)$	wall temperature, function of distance (measured into wall from hot surface) and time (JCL program), $^{\circ}$R; K
TWL	$T(L, \tau)$	wall temperature, function of time at $x = L$ (BL program), $^{\circ}$R; K
TWO	$T(O, \tau)$	wall temperature, function of time at $x = 0$ (BL program), $^{\circ}$R; K
WRAD	f	frequency of temperature oscillations (BL program), rad/sec
WRADA	f	frequency of temperature oscillations (JCL program), rad/sec

REFERENCE

1. Huff, Ronald G.: Convective Heat-Transfer Coefficients from a Solution of the Conduction Equation for a Wall Separating Two Fluids, One Having an Oscillating Temperature. NASA TN D-5520, 1969.

NASA-Langley, 1970 —— 8 E-5785

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