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A PLAN FOR COMPUTER-ASSISTED OPTIMIZATION OF STRUCTURES

by

R. J. Melosh

Prepared under Contract No. NAS5-11779
by
Philco-Ford Corporation WDL
Palo Alto, California

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
Goddard Spaceflight Center
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TABLE OF CONTENTS

TABLE OF CONTENTS (Cont)

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF TABLES (Cont)

Section 1
INTRODUCTION

The 1960's was a fruitful decade of research in the implementation of computers in structural optimization. Studies were addressed usually to minimizing structural weight by selecting element sizes from a continuous spectrum. Trusses or frames with relatively few elements, few degrees of freedom, and few constraints were optimized.

This report describes a plan for automating structural design based on an extension of this research. The plan identifies the theoretical basis, associated software components, data management, and the sequence calculations for improving a given structural design.

Structural Optimization

Structural optimization consists of modifying a given structure to improve some measure of the design. Modifications may consist of changes to any of the structural parameters: geometry, topology, material composition or boundary conditions. The measure of design has usually been simple, weight for example, but it could be complex, such as dollar cost, structural volume, cost ineffectiveness, or a weighted measure. Ideally, the objective of optimization is to find a design which minimizes (or maximizes) that measure.

The optimization problem can be concisely stated as a problem in the calculus of variations as follows:

Find the components of a vector $\overrightarrow{X}$, X_v, such that $C(X)$ is minimized and

$$f_k(\overrightarrow{X}) \le 0 \qquad k = 1, 2, 3 \ldots K \tag{1-1}$$

where each X_v is one of the V design variables and $C(X)$ is the design measure. This will be referred to as "cost."

Equation (1-1) presents design inequality constraints. They include such behavioral constraints as strength-of-materials requirements and such variable constraints as element size limits. Equality behavioral constraints (equilibrium and compatibility conditions) and equality variable constraints (prescribed variables) are satisfied explicitly and, therefore, are omitted from Equation (1-1).

Generally, optimization equations are nonlinear and nonlinear at such a high order (due to coupling and large K) that a solution can be developed only by an iterative process. Moreover, since an infinite number of solutions can exist and it is costly to locate all solutions, the analyst usually must be content with only one or two solutions.

Computer-Assisted Optimization Plan

The optimization plan described in this report is based on state-of-the-art design technology, computer software, and computer hardware. It addresses itself to the design of multicomponent, multidegree-of-freedom, linear, finite-element models of structures having static loadings.

Automatic optimization entails the following three subplans:

1.	Input/Output:	Communicating data between the designer and the computer.
2.	Design-Analysis:	Predicting the structural behavior of candidate designs.
3.	Redesign:	Reassigning design variables to reduce system cost.

The optimization program plan was developed to meet two objectives: low implementation risk and high optimization efficiency. Low-risk methods are those which have been proven in practice. Moderate-risk methods have not been proven in practice and possess aspects in question which may involve only minor programming revisions. High-risk methods may involve major reprogramming as technology advances. Optimization efficiency is measured by the number of useful calculations per data word processed. Meeting the first objective requires providing program development and execution flexibility; meeting the second, planning efficient file transfer, minimizing file search, and maximizing parallel data processing.

Program-development risk is associated with the evolution of the designer/computer interface relations and the continuing advancement of optimization technology. Data processing inefficiency is prompted by the nonlinearity of the design problem and the large quantity of data to be processed. Thus, the plan involves some measure of risk and some inefficiency compared with an analysis program plan.

Report Organization

This report proceeds from the general to the particular. Section 2 reviews the state-of-the-art of optimization technology, and Section 3 provides an overview of the implementation plan. Section 4 describes the Input/Output Plan; Section 5 defines and justifies the Design/Analysis Plan; and Section 6, the Redesign Plan. Section 7 validates the plan by reviewing the reformulation of several problems under the plan. Section 8 summarizes the major features of the plan.

Acknowledgements

Acknowledgements for assistance in preparing this report are due many persons. Dave Kelley of Philco-Ford prepared material for Section 4. Don Smith of Philco-Ford was responsible for Section 7. Many of the ideas for this plan were made by authors listed in the references and many others who have contributed to advancing the state-of-the-art of optimization. Special acknowledgement is addressed to the engineers of the Flight Dynamics Laboratory at Wright-Patterson Air Force Base and in the NASA family of centers -- Goddard, Langley, Marshall and JPL, who, recognizing the value of computer-assisted optimization, performed optimization research and development work and campaigned for continued government sponsorship of development work in the field.

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Section 2
STATE-OF-THE-ART OF STRUCTURAL OPTIMIZATION

This section reviews the state-of-the-art with respect to the development of an optimization implementation plan. It identifies the state for each of the subplans listed in Section 1. The purpose of this review is to provide visibility for the selection of methods incorporated in the subplans. The state-of-the-art is such that developing an Input/Output Plan will involve risk in graphics; an Analysis Plan, low risk; and a Redesign Plan, Moderate risk.

Input/Output Plan

Development of the Input/Output Plan involves little risk for the passive plan: management of card input and printed taped and plotted output. Various approaches are available in existing programs, have been tried, and can be evaluated. A proven passive subplan can be selected and modified as necessary. Development of a subplan for interactive use of computer graphics hardware with optimization, however, involves some risk. Active use of graphics in conjunction with automatic redesigning has been demonstrated only in a research environment. Since problems of interest require simulation of three-dimensional structures with thousands of degrees of freedom, the active interface subplan described here must be constructed of rational but largely unproven concepts.

Passive input and output subplan. - There are at least four nonproprietary design programs: two created by Bell Aerosystems personnel,[1,2] one by MIT faculty and students[3,4] and one by Philco-Ford employees[5,6]. Salient features of these programs are summarized in Table I. These data show that small-problem optimization capabilities are available for the principal integrity constraints with a variety of finite element models. All the codes are based on the displacement method of analysis.

TABLE I

CHARACTERISTICS OF AVAILABLE OPTIMIZATION CODES

Item \ Code	Bell Aerosystems Linear Merit	Bell Aerosystems Nonlinear Merit	M.I.T. STRUDL	Philco/Ford SAFER
Design Measure	Weight	Weight	Weight and Cost[1]	Modularized Cost[1]
Design Variables	Continuous: Element size	Continuous: Element size Joint site	Quasi-continuous: Element size Proportioning	Continuous or Quasi-Continuous: Element size, Proportions, Material
Con-straints	Stress Deflection Gages	Stress Deflection Gages	Stress Gages	Stress Deflection Gages
Structural Model	Rod, shear panels, triangular mem-brane, quad. plate, tube beam.	Rod, shear panels, triangular, mem-brane, quad. plate.	Rod, beam plate	Rod, triangular shear panel, mem-brane, and plate, general line element.
Design Processes	Fully-stressed, optimum vector	Fully-stressed, optimum vector	Fully stressed	Fully-stressed
Problem Size[2]	$D = 170$ $E \leq 200$	$D = 450$ $E \leq 200$	Undefined	$D \simeq 500$ $E \leq 300$
Context	None	None	Civil Engineer problem spectrum	Fail-safety, vulnerability evalua-tion
Refer-ences	(1), (2)	(1), (2)	(3), (4)	(5), (6)

[1]Both weight and cost are evaluated but neither influences the search.

[2]Based on 32k word core. D = degrees of freedom, E = number of elements.

Input and output of these optimization programs include the following additional features over conventional analysis programs such as NASTRAN, (7, 8, 9):

1. Input: The design measure must be identified and its parameters defined; the design search algorithm and its control values must be specified; failure criterion and parameters must be prescribed, and constraints on selection of design variables must be particularized. (Data defining initial guesses of all design variables, except joint locations, may be omitted.)

2. Output: System cost, structural integrity, progress of the design iterations, and the current value of design variables must be characterized.

In general, the input/output data for each of the optimization programs is the same. Each group of developers, however, has based his implementation on a different plan. Bell Aerosystems' plan is founded on vectorized, MIT's on verbalized, and Philco-Ford's on interrelated tabular data.

Vectorized data: With vectorized input, each like input item is grouped in a vector. For example, in the Bell Aerosystems process, one group of input consists of a vector of element numbers identifying all elements whose size is not permitted to be changed by redesign. This approach results in a requirement for 19 groups of data in Bell Aerosystems' programs. Bell's plan also provides for reading redundant summary information as a gross check on the quantity of input describing the problem.

Output is also based on vectorized data. Two-phase printing is provided: one phase includes all but search subroutines, the second phase reports optimization progress. In each phase, two printout levels are available: one for normal operation and a second for debugging.

Verbalized input: With verbalized input, as in STRUDL, the user describes the problem in a "problem oriented language." This is, more or less, the language of the structural engineer with definitions made more precise. To describe a load with an X component of 1200 and Y of 900 at joints A1 and A2, for example, the input would be

JOINTS 'A1,' 'A2,' LOAD FORCE X 1200. Y 900

The STRUDL program is part of a bigger system developed for solving a wide variety of small civil engineering problems. As such, the Input/Output Plan is based on low-input volumes, nonreport-form output, and user/computer interaction at execution time. Input is free-field but can be considered grouped into eight tables. Calculation and print control information is verbalized.

Interrelated tabular input: In the interrelated tabular input, as in Philco-Ford's SAFER, data is introduced in cross-referenced form. For example, the cross section for a given element is designated by a number; a cross section with this number is defined in the candidate element table. Some cross-referencing is used in the other optimization codes but the SAFER plan results in many cross-references. Input is organized into eight tables.

Output for SAFER consists of report-form tables of problem description data and results, a running commentary on calculation progress, and special features of the problem solution. Four levels of printout are provided: minimum, intermediate, detailed, and debug. At each level, printout of the previous level is augmented. For example, the intermediate level printout for Baseline Analysis adds data defining joint displacement to the stress printout.

Active graphics subplan. - Following some pilot studies on use of computer graphics with structural analysis by Sutherland [10] in 1963, Lockheed developed a production capability for analyzing airframe structures. This capability is currently limited to 200 joints and two-dimensional structures. Plans have been formulated, however, to extend this capability to 6000 to 8000 joints and three-dimensional structures[11].

Details of the existing hardware and software are provided in several published articles[11, 12, 13]. Salient features of the approach are as follows:

1. The software system was developed under the concept of a dedicated computer with an active interface. The computer is an IBM System/360 Model 50. Three 2250 Model 3 display units are tied in and four 2311 disk storage drives, two tape drives and a card reader are provided as peripheral input/output devices.
2. The hardware is time-shared. The feasibility of doing background calculations when three users are on line concurrently was demonstrated with a research version of the present code and occurs on a production basis now.

3. Software is organized in three segments under an overlay monitor and is almost entirely in FORTRAN IV. The first segment provides graphic display and interchange for reviewing and modifying the computer model of the structure to be analyzed. The data may be entered in card form or through the graphics link. The second segment directs the analyses. The third segment provides graphic displays of solution results.
4. Plots, data, and text are displayed on the scope. Plots include views of the structure's original geometry, deformed shape, and internal load distribution. Any input can be displayed and primary element loads and stresses can be depicted. Problem input can be changed by graphics entries. Text includes error comments, educed from the computer's review of input, and prompting messages to assist the operator in making the proper selections with the light pen.
5. The graphics system is supplemented by the chain printer. Upon completion of his evaluation, the engineer can require that part or all of his complete file of data be printed off line.

This system has been well received by Lockheed engineers. Therefore, the developers are anticipating checkout of the more comprehensive system previously described. They believe that it is important to allow the user to create his own graphics programs in the new system and to create a common data base accessible for a variety of uses besides structural analysis and design. According to them, development of this new system will be a "prodigious task."

Analysis Plan

The subplan for optimization must provide for three analysis tasks:

1. Baseline Analysis - prediction of behavior of a structure using only geometry, material, and boundary condition data.
2. Influence Analysis - prediction of changes in behavior induced by changes in design.
3. Reanalysis - prediction of response of the redesigned structure using response data from previous analysis.

The last two analysis tasks will be grouped under the term "Design-Analysis." In general, analysis approaches to accomplish each of these tasks have been well researched.

Baseline analysis subplan. - The state-of-the-art of Baseline Analysis is represented by the NASTRAN Code. This code is based on the finite-element concept and the direct stiffness approach. It is intended to represent the fruits of over 14 years of research in the field of numerical analysis of structures using digital computers. The program provides the intelligence to direct Baseline Analysis of structures of any geometry and any of the conventional structural materials for any holonomic boundary conditions. A subplan for Baseline Analysis is implicit in the extensive documentation of the program. An alternate subplan will not be presented here.

Influence analysis subplan. - The general characteristics of some methods of Influence Analysis are cataloged in Table II. Based on the data of Sack et al.[14] and Sobieszczanski[15], any of these methods would be expected to be much more efficient than obtaining influences by finite difference evaluations using Baseline Analyses.

The series expansion method[16] is the only iterative method presented. The iterative process is guaranteed to yield exact response, barring manipulation error, if the modulii of $K^{-1}k_i$ are all less than 1, where K is the total stiffness matrix and k_i the change in the stiffness matrix. The first term of the expansion has been successfully used by Von Hoerner[17] in designing reflectors so they will deform under gravity loads with minimum loss of reflector gain.

The other four methods listed in Table II are direct methods. The matrix modification method is credited to Sherman and Morrison[18] and recommended by Sack et al. In this approach, each changed row (or column) of the stiffness matrix is treated, one at a time, until the flexibility matrix has been updated to reflect all changes. The direct derivative evaluation method proposed by Fox[19] is incorporated in the Bell Aerosystems codes[1]. It multiples the flexibility matrix by the vectors of change of the stiffness matrix and the existing deflections to obtain exact evaluations of response (deflection) derivatives with respect to element changes. Reference (20) describes a process which performs operations with a set of self-equilibrating vectors associated with structural elements to reflect the stiffness changes and evaluate responses for any magnitude change. This method is used in the SAFER[5] code for fail-safe and vulnerability analyses. Closely related to this method is the parallel element method recently reviewed by Sobieszczanski[15]. He compares the number of simple arithmetic calculations for this method with those required for Baseline Analysis and concludes that the parallel element method may reduce calculations by factors of 100 or more.

TABLE II

INFLUENCE ANALYSIS METHODS

Item \ Method	Series Expansion	Matrix Modification	Direct Differentiation	Self-Loading Vectors	Parallel Element Concept
Basic Approach	Expand solution in power series of original stiffness and change in total stiffness	For each row change, update influence matrix by subtracting change	Evaluate differential of load-deflection equations directly	Superimpose responses of self-equilibrating loads associated with the change	Finds changes in forces so system with design change will satisfy compatibility and equilibrium
Calculations	Evaluate successive terms of the expansion of $\delta = (I + K^{-1} k_i)^{-1} \delta^1$	Update all flexibility coefficients for each change of row element in stiffness matrix by a matrix formed by matrix multiplications	Multiply inverse by product of change and original solution $\frac{\partial \delta}{\partial A_i} = -K^{-1} k_i \delta$	Obtain response of baseline structure to self-loads and superimpose to simulate any change desired	Updates force redundants influence matrix for each change of interest, sequentially.
Advantages	• Yields derivative in first step • Can be self-correcting for some round-off	• All changes to a run treated as a module • Provides changes to entire inverse	• Obtains exact values of all derivatives concurrently • Involves little data processing	• Minimizes calculations • Incorporates kinematic instability check	• Few calculations required • Each change treated sequentially
Disadvantages	• Involves multiple passes of stiffness matrix • Only converges for small changes of stiffnesses	• Requires inverse matrix to be available • Involves multiple passes of matrices	• Only finds derivatives, not response • Requires flexibility matrix, as defined.	• Involves solution of subset simultaneous equations • Requires rigid modes in element models	• Force flexibility (or modified stiffnesses) must be generated and processed as well as stiffness matrix • Data processing non-optimum
References	(16)	Sherman and Morrison, (18)	(19)	(20)	(15)

Table II lists the principal advantages and disadvantages as well as the general features of each method. All these methods develop corrections to the current solution by performing matrix operations on available stiffness or flexibility matrices. The differences between the methods involve differences in data processing and calculation efficiency.

Reanalysis subplan. - Most optimum design studies perform Reanalysis by Baseline Analysis. However, the principle that Reanalysis may be less accurate than Baseline Analysis has been espoused by a number of authors. Moreover, since changes to the design will become small as optimization continues, response of a given design often may be changed little from that of the previous design. Thus, candidate methods for Reanalysis might include those which are approximate because of analysis approximations and those which are approximate because an iterative solution process is truncated.

Before reviewing the state-of-the-art of these methods, it is useful to identify the decisions that must be made in selecting an approximate reanalysis process. The decisions can be grouped into those which particularize the analysis approach and those which identify the approach for solving the resulting structural equations. The first set of decisions have an impact on the analysis accuracy and interpretation of analysis results; the second, on manipulation error and the efficiency of the solution process.

Analysis decisions: In general, the analyst chooses to represent the behavior by a set of functions,

$$\alpha_k = \sum_{i=1,2}^{N} c_i \, \phi_i \, (x, y, z) \quad k = 1, 2, \ldots K \tag{2-1}$$

where

α_k represents displacements or stresses over the points of three-dimensional space. Each point is located by its coordinates x_j, y_j, z_j.

ϕ_i are a set of functions,

c_i are arbitrary constants which are either generalized displacement or generalized stress coordinates,

N is the number of generalized coordinates, i   is a dummy subscript, and

K   in classical elasticity has a maximum value of three if the α_k are displacements and a maximum value of six if the α_k are stress components.

Assuming the set of ϕ_i is mathematically complete and satisfies certain continuity conditions (see Reference 21) with appropriate choices of the c_i, Equation (2-1) can represent the solution of the elasticity equations as accurately as desired, as N approaches infinity.

In approximate analyses, N is finite. Then, the choice of the analysis method affects the values that will be assigned to the c_i. As pointed out by Crandall[22], selection of the method constitutes choosing a weighting function for analysis error. The solution process then evaluates the c_i to minimize the weighted error.

Table III provides a decision ladder which groups mathematical modeling decisions for the analysis approach. The top three rungs of the ladder involve selection of interpolating functions; the lower two, selection of error criteria. Decisions at all levels, however, affect analysis efficiency.

Decisions at the highest rung fix the analyst's goal by identifying the equations whose solution is being sought. Either a differential (D. E.) or integral equation (I. E.) approach may be taken. The differential equations will be the equilibrium, constitutive, and compatibility equations. The I. E. approach involves finding the solution of these equations by minimizing an integral (variational approach) or solution of a Fredholm integral equation. Since it is always possible to transform from the differential equation form to the integral, and conversely, the analyst can always choose either formulation for his analysis.

The selection of formulation identifies specifications for the α functions. The D. E. approach requires functions which can be differentiated and will provide good estimates of the variation of the differentials over the structure. The I. E. approach requires functions which are integrable and whose integrals are good estimates of the corresponding exact integral of structure behavior.

TABLE III

ANALYSIS DECISION LADDER

Formulation:	Differential Equations	Integral Equation	Hybrid Equations	Mixed Equations
Behavior Model:	Stress or Force	Strain or Displacements	Hybrid Functions	Mixed Functions
Operators:	Intersecting	Disjoint	Both	
Articulation:	Subdegree	Least Degree	Extra Degree	Mixed
Total Error Criterion:	Uniform Weighting	Galerkin Weighting	Positive Weighting	Mixed Weighting

NOTE: In defining an analysis method, a selection is made among entries at each ladder level.

The analyst can also choose to use any of a spectrum of hybrid approaches. In these approaches, functions are chosen which can be both differentiated and integrated. The approach can be to choose functions which would make zero particular terms of the integral and find the c_i to satisfy differential equations. Alternately, the approach could be to minimize the integral subject to differential equation conditions on the functions. These hybrid approaches are not popular, though they offer a great deal of analysis flexibility.

Though not usually done, the analyst could choose to mix the two approaches. He could use the differential equation approach for part of the structure and the integral for another part, and hybrid over a third part.

The second decision level limits the type and form of the behavior functions. The most important of these decisions is the choice of the α_k. These may be stress components, strain components, or hybrid functions of both stress and strain components. The selection can also be spatially mixed over the structure.

This decision establishes the form of the equations and additional conditions on the ϕ_i. For example, if the differential equation approach is taken and the α_k are stresses, the equations take the Beltrami-Mitchell form. The functions must be differentiable through the second derivatives. If the corresponding integral equation approach is taken, the functions must have integrable second order derivatives, satisfy the differential equations in the regions of definition, and satisfy the homogeneous conditions at the boundaries. If the differential equation approach is taken and the α_k are deflection components, the equations take the Navier form.(21)

If both stress and displacement functions are included (hybrid), the differential equations of elasticity in unreduced form are to be solved. Alternately, non-extremum variational principles (such as Reissner Energy) define the equations of interest.

The behavior model is further particularized by the decision to attack the equations in microscopic or macroscopic form. In microscopic form, stress or strain (or displacement) variables are retained. To write the equations in macroscopic form, they are integrated over some of the dimensions of the structure. Stress variables are replaced by force resultants and strains with displacements.

The third decision level involves the selection of difference and integral operators. The difference operators will transform the differential equations into difference equations. The integral operators will replace the integration with summation. These operators will form a collection from which operators will be picked for given systems.

At this level, an important decision is whether the collection will contain disjoint, intersecting, or both types of operators. Each disjoint operator can be uniquely identified with a particular region of a structure. The region can be delineated by fictitious cuts. Inclusion of only these operators limits the analysis method to finite element operators. The collection must include an operator model for every element topology and material model that may arise. Intersecting operators, on the other hand, are defined among mesh points. They need not be based on functions which are uniquely defined over a region nor be associated with fictitious cuts. A complete set of these operators requires subsets of operators for the boundaries of the structure and for the interior. The finite difference method uses intersecting operators.

The operators may also be classified by characteristics of the functions upon which they are based. This, in turn, can distinguish between analysis methods. Use of only harmonic functions is a hallmark of the Treffetz and Rafalson methods(21). Finite difference methods are based on first order estimates of the derivatives based on their definitions, which, using Taylor's series, is comparable to a polynomial basis. The complementary energy method restricts operators to those based on functions which satisfy the stress equations of equilibrium everywhere in the interior and match surface tractions across boundaries. The potential energy approach requires functions which satisfy displacement continuity (not compatibility) everywhere. Except for the finite difference method, specialization of the basis in these methods insures a solution bound. If the analyst will forego bounding, practically any piecewise continuous ϕ functions can be used.

The fourth level of decisions establishes the difference equations which model the structure. Consider that numerical modeling identifies mesh points on the system and the requirement to produce specific data at these points, say displacements. Then, in the differential equation approach, the fourth level of decisions defines at what points, across what lines, or over what regions the difference equations will be expressed. In the integral equation approach, this level of decisions determines where and how many fictitious cuts will interlace the mesh points. These cuts delineate the integration boundaries. Minimizing the sum of the parts of the integral with respect to the generalized coordinates produces the difference equations.

One important decision is what degree operators shall be used. The method is called subdegree if the number of degrees of freedom in the analysis is less than that specified by the idealization. In this case, interpolation must again be used on the numerical results to obtain response evaluations at points specified by the analyst. In least-degree analysis, generalized coordinates are only associated with idealization points. In extra-degree, the operators are based on other coordinates than those specified. In the finite element approach, these elements have been referred to as super elements.[7] Of course, the analyst may choose to vary the degree of operator over the structure, using a mixed approach.

Another articulation decision determines the order of the operators to be used. These may be least-order or refined. Least order operators are based on functions which imply the simplest elastic behavior. The rod, beam, and Turner[23] triangular membrane finite elements are least-order. Refined operators involve higher order behavior states. The six-joint triangular membrane model of Argyris[24] illustrates this type of operator.

The final analysis decisions concern the basis for minimizing analysis errors. The errors may be considered components of an error vector.

The analyst must decide how many error components to use. He can define as many as there are generalized coordinates (determined set) or more (over determined set). The analyst can also choose to evaluate the c_i to minimize any norm measure of this error vector. If there are an equal number of error components (difference equations) and c_i, the system of equations is determinate and the c_i can be evaluated so all components of the vector vanish. If there are more error components than c_i, the c_i can be found so the sum of the squares of the error is minimum (Euclidean vector norm), the maximum error is minimum (min-max norm), or the sum of the absolute value of the error is minimum.

A more important error decision defines how the error shall be weighted over the system. All methods evaluate error by measuring an inadequacy of the assumed behavior in satisfying the equations of elasticity, but they differ on how these errors shall be weighted in combining them into a single error criterion. Uniform weighting may be used (finite difference and Biezeno-Koch methods); weighting the error by the behavior states, Equation (2-1), may be used (Galerkin's method). Other positive weightings can be used, or these weightings can be mixed over the structure.

Solution decisions: Figure 1 shows the decision tree for particularizing the solution approach. There are two major decisions: choosing between solution approach and selecting the solution algorithm.

The solution approach can involve the direct minimization of an integral which measures solution error or energy, or the solution of the set of equations associated with stationary values of the error. In analyzing linear systems, the stationary value of the error is an absolute minimum as long as the error kernel is of quadratic form, which is usually the case.

Selection of the solution algorithm is the consequence of choices between relaxation and gradient iterative methods or triangularization and orthogonalization direct methods. In relaxation methods, components of the solution vector are reduced, more or less independently, endeavoring to minimize the error with respect to the components. In gradient methods, on the other hand, a relationship between the components is established (by gradient calculations, for example) and all components changed simultaneously. In triangularization methods, successive row or column (or row and column) operations are performed to transform the matrix of coefficients to triangular form. In orthogonalization methods, on the other hand, a transformation is sought which diagonalizes the matrix of coefficients.

Figure 1 shows that selection of function minimization limits the selection of algorithms to an iterative method. The same iterative methods available for the analyst choosing minimization are also available if the Euler equation approach is chosen. Furthermore, it is noted that like analysis approaches, Figure 1 implies a spectrum of approaches lying between the extremes of minimization and Euler equations and a spectrum of processes lying between pure iteration and completely noniterative algorithms.

Analysis state-of-the-art: Several researchers have reported studies of the effect of analysis decisions on solution accuracy. Reference 25 examines each issue in turn, using a membrane problem for illustrative purposes. It concludes that, to minimize calculations/accuracy, a combination of integral formulation, intersecting operators, low-degree articulation, and non-uniform error weighting is best.

Pian[26, 27] reports studies of the effect on solution accuracy of the choice of assumed behavior functions using energy weighting. He considers the effect of choosing functions satisfying the compatibility or equilibrium across element boundaries or in the interior of the finite element and concludes that basing analysis on assumed stresses can provide more accurate stress predictions than when using assumed displacement states. In terms of accuracy, however, results form a spectrum ranging from good assumed stress functions and displacement functions to bad assumed stress functions and displacement functions. Hybrid models, which use functions which satisfy compatibility across element boundaries and equilibrium in the interior, or conversely, are regarded as more likely to provide accurate results than either compatible or equilibrium models simply because the latter models are associated with extremum solutions.

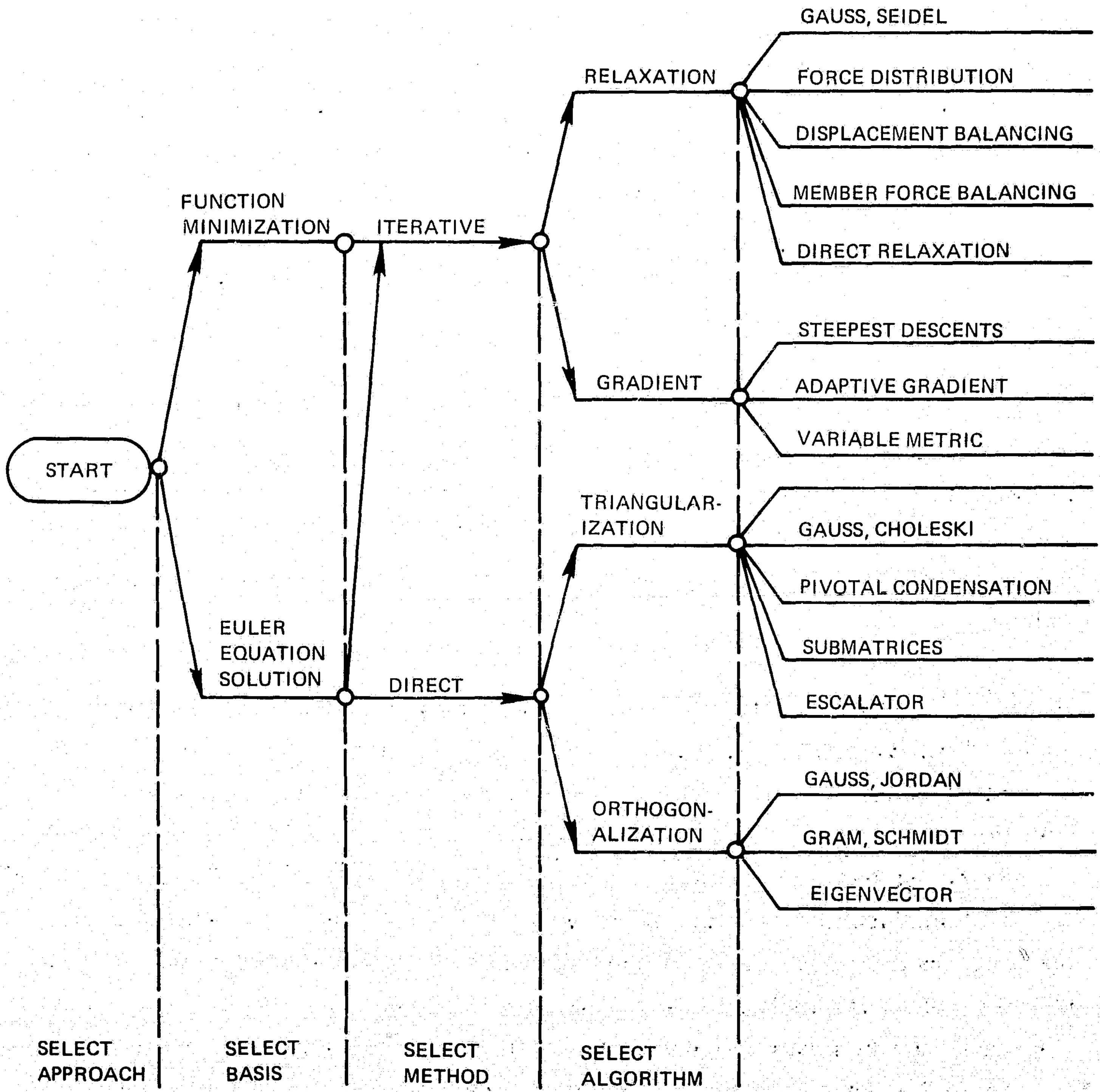


Figure 1. Solution Decision Tree

Reference 28 cites responses for a truss using two alternate sets of assumed vectors: one a set of displacement vectors implying satisfaction of compatibility everywhere and the other a set of stress vectors implying satisfaction of equilibrium everywhere. The stress and displacements are comparable in the sense that they are associated with solutions of the same truss geometry. It is concluded that the solution improvement per vector for the assumed displacement approach is much less than for the stress approach. Solution improvement is measured by the change in strain energy.

Leissa et.al. [29] report a study of the use of various weighting functions for a plate based on assumed displacement functions. They rate nine methods on 11 characteristics. An extract of the comments and ratings for the five best in their opinion, is given in Table IV. Among the methods, they favor point matching and boundary-point least squares and examine these methods more carefully in subsequent reports [30, 31]. It is noteworthy that though each method was studied from a theoretical point of view, and used to solve problems, the 11 characteristics did not include conclusions with respect to relative accuracy of the methods.

Solution state-of-the-art: Of the solution approaches, two are popular. One involves function minimization using gradient iterative methods. This approach has been described by Bogner et al. [32] Mallett and Schmit, [33] and Fox and Stanton [34]. Orientation of these authors is toward use of the approach for nonlinear analyses. The more popular approach involves attack on the Euler equations by direct triangularization of the coefficient matrix in the load-deflection equations. Gauss and Choleski algorithms have both been widely accepted. Techniques for efficient data handling in this approach have recently been described in Reference 35 and by Whetstone, [36] and by Jensen and Parks [37]. Reference 38 provides a review of problems involved in maintaining solution accuracy in solving the simultaneous equations of structural analysis.

Table V provides an extract of these reports. It provides for comparing the efficiency, flexibility, accuracy, and automation of these two approaches. The important distinctions between them concern efficiency and flexibility. The Euler equation approach involves fewer calculations but more data handling than the minimization

TABLE IV

FEATURES OF SOME WEIGHTING METHODS

Item \ Method	Boundary Point Matching	Boundary Point Least Squares	Interior Collocation	Interior Least Squares	Ritz
Applicability when Differential Equation is Complicated	POOR: Requires functions which satisfy differential equations exactly in interior	POOR: Requires functions which satisfy differential equations exactly in interior	GOOD: Straight-forward	GOOD: Straight-forward	GOOD: Straight-forward
Ability to Represent Loading Singularities	GOOD: Singularity functions can be introduced	GOOD: Singularity functions can be introduced	FAIR: Difficult to find singularity functions which satisfy boundary conditions	FAIR: Difficult to find singularity functions satisfying boundary conditions	FAIR: Difficult to find singularity functions satisfying boundary conditions
Ability to Obtain Higher Derivatives (Stresses)	FAIR: Has the advantage of singularity function incorporation	FAIR: Has the advantage of singularity function incorporation	POOR	POOR	POOR
Capability of Assessing Solution Accuracy	GOOD: Measures error by edge load values so it is easily interpreted	GOOD: Measures error by edge load values so easily interpreted	POOR: Difficult to interpret residuals	POOR: Difficult to interpret residuals	FAIR: Residuals hard to interpret in some problems
Ease of Understanding the Method and Use	GOOD: Error measure and math easily grasped	GOOD: Error measure and math easily grasped	GOOD: Error measure and math easily grasped	GOOD: Error measure and math easily grasped	FAIR: Requires more mathematical facility
Suitability for Digital Computer Programming	GOOD: Given the assumed functions, Euler equation coefficients are easily coded	GOOD: Given the assumed functions, Euler equation coefficients are easily coded	GOOD: Given the assumed functions, Euler equation coefficients are easily coded	GOOD: Given the assumed functions, Euler equation coefficients are easily coded	FAIR: Requires more integrations than other methods

TABLE V

FEATURES OF EXISTING SOLUTION IMPLEMENTATIONS

Item \ Approach	Minimization-Iteration	Euler Equation-Direct
EFFICIENCY		
• No. of Calculations	• More: Regenerates coefficients, each iteration, uses structural partitioning, scales variable coordinates	• Less: No regeneration, or structural partitioning or scaling but needs coefficient identification and indexing
• Volume of Data	• Less: Develops answers with 27E items of saved data and small program where E is the number of elements.	• More: Uses 27E + 2Nw storage locations and moderate size program where N the number of equations, w is the wavefront.*
• Calculation Complexity	• Worse: Slowed by double-precision arithmetic.	• Better: Can usually use single-precision arithmetic.
• Critical Factors	• Slow convergence due to bad scaling or tough problem.	• Extensive data handling when wavefront* so large that spilling of core occurs.
ACCURACY		
• Error Magnitudes	• Accuracy demonstrated. No extensive study of errors made though need for scaling is established.[34]	• Accuracy demonstrated relation between accuracy and precision has been established.
• Error Sources	• A principal source of error and devices to minimize error are known but error measures and interpretation not well founded.	• Principal sources, measures, minimization devices, and interpretation of errors have been documented.
FLEXIBILITY		
• Adaptability	• Adaptible to any structural analysis approach.	• Adaptable to any structural analysis approach.
• Capability	• Software limits the number of finite elements that can be in core with the solution vector.	• Software limits the number of degrees of freedom in solution vector fitting in core.
• Use with Design	• Bad: Will not admit efficient sensitivity analysis.	• Good: Well-suited to sensitivity analysis.
• Extendability	• Good: Proven for use with geometric nonlinearities, material nonlinearities, and nonlinear transient response predictions.	• Fair: Proven for use with geometric and material nonlinearities but cumbersome for nonlinear transient response analysis.
AUTOMATION		
• Development Cost	• Low: Simple data management, since little data and few types.	• Low: Though data management complex and many items to handle, details have been worked out and documented.
• Development Time	• Low: Simple program logic should facilitate development.	• Moderate: More complex program logic will slow development.

*Wavefront is defined as the maximum number of degrees of freedom referenced when a row of the matrix is last treated during decomposition.[35]

approach. The Euler approach incurs relatively efficient Influence Analysis. On the other hand, the proven facility of direct minimization in nonlinear analysis makes it the current choice for nonlinear transient analyses.

Redesign Plan

Ten years of research in optimization of structures by mathematical programming methods has demonstrated the suitability of a number of methods for this purpose. Few comparisons have been reported, however, and new methods are evolving. Selection of a redesign method, thus, involves some risk.

Table VI classifies existing optimization methods. As observed by Berke[39], approaches include minimization and optimality avenues. In the minimization approach, the objective of Redesign is to reduce the design measure in each step. Usually, this is achieved using derivatives of the design measure with respect to the design variables. The approach is general; any design measure with measurable derivatives can be considered. In the optimality approach, on the other hand, algorithms seek a design which satisfies the optimality equations. These are the Euler equations of the variational statement of the problem. Methods using this approach need not use gradients of the design measure, but may assign each variable to satisfy the optimality requirements with respect to that variable. This develops an optimum design by a relaxation process. A special algorithm may be required for each design measure used. There is no assurance that the relaxation process used for one design measure will yield a converging sequence of designs for another.

There are three types of minimization methods: trial-and-error, explicit, and implicit. Trial-and-error methods pick values of the design variables at random, rejecting designs which violate constraints. Some use a learning technique to focus choice of variables on those associated with acceptable designs of low design measure as designing proceeds. These methods usually require analyzing many designs to locate a good one. Since they are appropriate principally when analysis incurs relatively few calculations compared with Redesign, they will be excluded from further discussions here.

TABLE VI

OPTIMIZATION APPROACHES AND METHODS

Minimization Approach (gradient, direct, general)	Optimality Approach (non-gradient, relaxation, special)
TRIAL AND ERROR	**SUBSTITUTE OPTIMALITY**
• Steepest Descent	• Fully Stressed
• Gunshot	• Uniform Strain-Energy-Density
• Random Walk	• Simultaneous Buckling
• Monte Carlo	• Limit Design
• Learning Trials	**IMPLICIT METHODS**
EXPLICIT METHODS	• Newton-Rhapson
• Gradient Projection	
• Feasible-Usable	
• Optimum Vector	
• Linear Programming	
• Allocation	
IMPLICIT METHODS	
• Steepest Descents	
• Adaptive Gradient	
• Variable Metric	
• Conjugate Gradients	

Explicit methods involve direct solution of the "auxiliary problem" for each design cycle. This problem consists of determining the direction of travel to take in the design space such that the design measure will be rapidly reduced while no constraints will be violated. The auxiliary problem usually involves minimizing a first-order model of the design measure subject to the set of constraint equations which are active for the current design. "Active constraints" are those constraints which are violated by small changes to the design variables.

Implicit methods use penalty functions to transform the constrained minimization problem to an unconstrained one. For example, Fiacco and McCormick[40] express the problem described by Equation (1-1) as follows for the case when the starting design does not violate constraints.

Find the components of a vector $\vec{X}$, X_V, such that $C^*(\vec{X}, r)$ is minimized where,

$$C^*(\vec{X}, r) = C(\vec{X}) - \gamma \sum_{k=1,2\ldots}^{K} \frac{1}{f_k(\vec{X})} \tag{2-2}$$

with

$C^*(\ )$ the modified cost function,
$\gamma \geq 0$ is a constant, and
X_V are such that $f_k(\vec{X}) \leq 0$.

The first term on the right of Equation (2-2) is the design measure and the second, the inequality constraint repulsion term. In minimizing C* the penalty encumbers the measure unless the constraints of Equation (1-1) are all satisfied. By performing a sequence of minimizations, with progressively smaller values of γ, the solution to the problem defined by Equation (1-1) is approached. Thus the auxiliary problem is solved implicitly in the minimization process.

Generally minimization methods have been limited to problems where design variables are continuous. A common difficulty with continuous variables involves selecting a basis for measuring derivatives yielding efficient search.

Optimality methods can be divided into substitute and implicit subclasses. Substitute methods choose optimality conditions independently of a design measure.

Of these, the fully stressed process is the most venerable. Implicit optimality methods are concerned with finding a solution to the Euler equations associated with penalty function formulations of the design problem. No researchers have yet reported on the characteristics of this approach. Explicit optimality methods are omitted in the classification because they are appropriate for only a limited class of structural optimization problems.

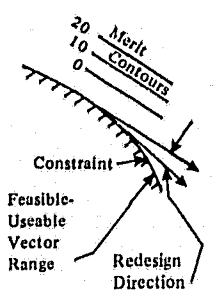
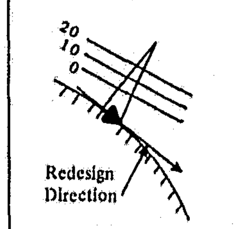
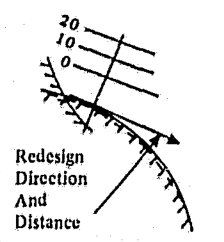
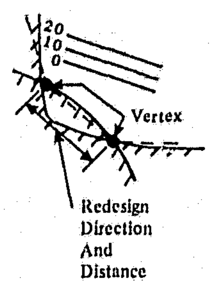
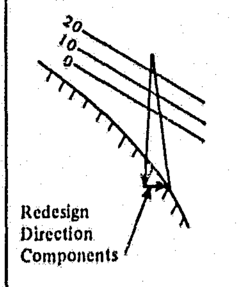
Schmit[41] recently presented a thorough review of minimization methods. A review of his paper and those of Kowalik[42] and Pope[43] yield the evaluations furnished in Table VII. This table cites features of the four proven explicit methods of structural optimization. The feasible-usable directions approach, described by Zoutendijk[44], and Rosen's gradient projection method[45] find the distance of travel in the design space in each redesign cycle by a curve fitting the design measure using trial step sizes. The optimum vector approach of Gellatly[1] and the linear programming approach [43] find both the direction and distance of travel from a completely linearized auxiliary problem solution.

Each method has the special difficulties noted. All, however, share two shortcomings: (1) they involve numerical instability when some of the constraint equations are linearly or nearly linearly dependent, as represented by their gradients, and (2) arbitrariness is introduced in linearizing the constraints when they are nonlinear. Constraint gradients are usually based on finite difference measures so the methods become more effective when constraints are linear.

Table VIII identifies features of four implicit methods. The steepest-descent method, attributed to Cauchy[46], is the forerunner of almost all nonlinear search methods. It is unpopular because it often entails many iterations to locate a relative minimum. The adaptive gradient method, ascribed to Rosenbrock[47], is similar but avoids direct calculation of gradients and is burdened by calculations to normalize a set of N variables, where N is the number of design parameters. Fletcher and Powell[48] describe the variable metric method, and Fletcher and Reeves[49] present the method to conjugate gradients for the nonlinear problem. Both of these latter methods accelerate iteration convergence such that if the cost function is quadratic, the solution is obtained in at most N iterations.

TABLE VII

FEATURES OF EXPLICIT MINIMIZATION METHODS

Item \ Method	Feasible-Usable Dir.[44]	Gradient Projection[45]	Optimum Vector[1],[2]	Linear Programming[43]	Allocation[28]
Basis for Search	DIRECTION Vector between tangent to constraints and isomer π contour DISTANCE Golden section[87] or curve fit to minimize	DIRECTION Vector projection of gradient of merit in tangent space DISTANCE Golden section or curve fit to minimize	DIRECTION AND DISTANCE Choose linear combination of gradient of merit function and active constraints which minimizes the merit function in linear model.	DIRECTION AND DISTANCE Move to adjacent vertex which lies in direction of lower design measure in linear model	DIRECTION AND DISTANCE Reassign each design variable (each element) to minimize the design measure with respect to that variable thus defining direction components
Schematic					
Calculations*	1. Solve order auxiliary minimization problem for direction.* 2. Evaluate design measure and acceptability for several distances.	1. Update projection matrix (if new active constraints) and multiply by merit gradient found. 2. Evaluate design for trial distances.	Solve linear homogeneous simultaneous equations.*	Solve for constraint gradients and take steps as part of linear programming solution algorithm after linearizing order.	Compare each variable independently against the most effective variable in reducing the measure and meeting constraints.
Advantages	• All designs are feasible • Treats non-linear merit directly	• Avoids solving simultaneous equations. • Treats non-linear merit directly	• Test of design derivatives equal to zero is intrinsic in solution process • Avoids trial and error	• Very few to define redesign. • Avoids trial and error	• Few calculations and design cycles required • Little storage space required
Disadvantages	• Zigzagging from successive steps (concave space) • Small boundary steps may occur.	• Updating projection matrix costly in non-linear case • Infeasible designs may occur	• Equations singularity may occur due to optimality or dependent constraints • Design measure must be linearized.	• Space may not be convex and algorithm fails • Design measure must be linearized	• Unproven for general design measure • May be limited to size and material selection variables.

*Order of problem is the number of active constraints. This is the number of design variables which are prescribed by at least one of the constraint equations.

TABLE VIII

FEATURES OF IMPLICIT MINIMIZATION METHODS

Method[2] / Item	Steepest Descents(46)	Adaptive Gradient(47)	Variable Metric(48)	Conjugate Gradients(49)
Search Basis	Move in direction with greatest rate of design improvement.	Deduce a gradient estimate from two previous steps and move in this and orthogonal directions.	Move based on measure gradient and continually improving estimates of space second derivatives.	Move based on current and previous gradients such that if design measure is quadratic current move will be orthogonal to previous ones.
Calculations[1]	Find components of the gradient.	Construct an orthonormal subspace basis based on changes made to variables in last cycle for space of dimension N.	Find gradient vector and use to improve second derivatives using these and trials to define next design.	Find gradient vector and modify using vector of previous travel direction using trials to define distance.
Advantages	• Simple coding and few calcs. per redesign. • Very reliable	• No gradients calculated explicitly. • Each component individually treated.	• Quadratic convergence • Accumulates curvature knowledge for few calculations and explicitly.	• Quadratic convergence • Requires little storage space for repetitively used data.
Disadvantages	• Extremely slow (many cycles required) • Only trial and error control on travel distance.	• Many calculations per redesign cycle. • Many steps in a cycle with small improvement.	• Requires storing an N x N matrix (triangular) • Coordinates must be rescaled in each redesign step.	• Coordinates must be rescaled in each redesign step. • Does not take advantage of accumulated curvature data to speed convergence.

[1]N is the number of design variables

[2]Numbers in parentheses refer to publication references.

The implicit methods have been used successfully in solving problems where equality constraints are missing[50,51]. However, additional study is necessary to evaluate how γ should be varied in the sequence of minimizations to avoid excessive design cycles with small improvements. When equality constraints exist, the implicit methods are encumbered by the intrinsic numerical singularity of the penalty function as $f_k(\vec{X})$ approaches zero.

Table IX lists some of the features of the substitute optimality methods. Each method is addressed to developing a design where material mechanical capabilities are exploited.

The fully stressed optimality condition is founded in the work of Maxwell[52], Michell[53], and Cilley[54]. Its expedient extension to multiple loading has been described by Young and Christiansen[55], Gellatly et al.[56], Venkayya et al.[57], and others. Reference 58 identifies the characteristics of the sequence of designs generated by the process and exhibits results for a successful extrapolation process for the method. This method involves fewer calculations to perform redesign than any of the other methods.

Prager[59] and Taylor[60] have shown that uniform-strain-energy-density designs are designs of minimum weight under a variety of conditions including designs with stiffness or deflection limitations. If the same material is used throughout the structure and fully-stressed designs are of minimum weight, then uniform-strain-energy-density designs will be of minimum weight for they will be the same as the fully stressed designs. Venkayya et al.[57] report that these designs can be developed as rapidly as fully-stressed designs. The process used for their development is only slightly different than that used for developing fully-stressed designs. Thus, with minor additions, a single computer code can be used for both.

The simultaneous buckling criterion forms the basis for extensive studies by Gerard[61] and Shanley[62]. They use the criterion to determine "optimum" stiffener spacing in semimonocoque systems. Their approach and calculations have been the basis for proportioning aircraft and missile critical buckling structure for the past decade. Little work has been done to implement the philosophy in the

TABLE IX

FEATURES OF SUBSTITUTE OPTIMALITY METHODS

Item \ Method	Fully Stressed Design	Uniform Strain-Energy-Density	Simultaneous Buckling	Limit Design
Basis for Search	Resize each element so it is at maximum stress at one point for at least one loading.	Resize each element so it is at maximum strain-energy-density for at least one loading.	Resize all elements so all buckling modes occur at same load level.	Size all elements so they are all fully-yielded at the same load level.
Calculations	For each element, assume generalized forces do not change and find its size so maximum stress will be attained. Of sizes for an element, pick largest.	For each element assume generalized forces do not change and find it size so energy density is at maximum. Of sizes for an element, pick largest.	Equate non-linear expressions relating buckling load and geometry and solve for geometry.	Solve linear programming problem associated with the failure modes.
Advantages	• Few calculations (few per cycle, few cycles) • Modular data processing - discrete sizes easily handled.	• Few calculations (few per cycle, few cycles) • Modular data processing - discrete sizes easily handled.	• Few equations since failure modes are macroscopic. • Nonlinear equations are explicitly stated.	• Convergence to absolute minimum is guaranteed. • Only linear equations are involved so few design cycles.
Disadvantages	• Convergence not guaranteed. • Not appropriate for some deflection critical designs.	• Convergence not guaranteed. • Not appropriate for some stress critical designs.	• Not demonstrated for large complex systems. • Stress distribution must be established *ab initio*.	• Incompatible with deflection limitation. • Failure mechanisms must be isolated *ab initio*.

computer environment, however. In addition, simultaneous buckling designs are often not minimum weight designs—the usual aircraft design objective.

Limit designs are absolute minimum weight designs when the failure criterion is the existence of yield stress everywhere. Fundamental work in analysis and design based on the full-yield concept has produced many papers. Among the most important works are those of Von Mises[63], Nadai[64], Hodge and Prager[65], and Hill[66]. Charnes and Greenberg[67] show that limit designs can be developed using the linear programming algorithm. Unfortunately, the limit design criterion is inconsistent with the goal of optimizing a design for multiple loading conditions.

The principal disadvantage of all designs based on substitute optimality conditions is that often they are not optimum, even with respect to weight. The simplicity of the methods and the fact that they incur few calculations has made them popular despite this known deficiency.

Criterion Design Capability

The referenced papers imply the existence of many computer programs for optimizing structural design. From these papers, a composite state-of-the-art computer program can be synthesized. Its features will represent proven capabilities; i.e., capabilities validated by more than one researcher. Where these features represent analysis or design approach decisions, the choice made by the majority will be cited. The existing capability is as follows:

Design Variables: element sizes, selected from a continuous or discrete spectrum. Size selection includes cross section proportions and scale factor.

Constraints: failure modes for allowable stress, element buckling, and kinematic instability; lower bounds on design variables (minimum gages). (A number of researchers report designing to meet dynamic requirements, but this has generally been restricted to meeting frequency requirements.)

Design Measure: weight. (A few papers describe use of other design measures, but weight is the most popular.)

Loadings: non-circulatory loads which are specified ab initio.

Structure: systems composed of line and surface finite-elements.

Analysis Approach: Baseline Analysis, based on potential energy weighting, for influence and reanalysis, employing the Euler equation approach and triangular decomposition.

Design Approach: no majority choice; many have been tried.

Problem Size: 200 design variables, 450 structural degrees of freedom, 10 loadings, and a single design measure. This is a computer code which performs optimization by retaining most of the needed data in core.

Computer Hardware: 32K with a minimum of a 36-bit, floating-point number representation.

As opposed to this existing capability, this study addresses itself to defining a plan for optimizing more complex structures. This entails state-of-the-art advances in problem scope and size. To particularize the problems of interest, the following criterion capability is identified:

Design Variables: element sizes, element material, joint positions, and element joint kinematic boundary conditions.

Constraints: failure modes for allowable stress, element bucking, kinematic instability, and excessive deflection; upper and lower bounds on design variables.

Design Measure: a general modular cost function. This will include weight as a special case but also provides for optimizing with respect to cost or cost effectiveness using simple cost models. The ability to combine this measure with user-introduced measures will be planned.

Loadings: noncirculatory loads which are specified ab initio.

Structure: systems composed of line, surface, and solid finite-elements.

Analysis Approach: a special influence analysis method and a reanalysis method chosen for efficiency.

Design Approach: alternate methods of low risk and high optimization efficiency.

Problem Size: up to 2000 design variables, 6000 structural degrees of freedom, and 40 loadings.

Computer Hardware: 64 to 128K words with a minimum of a 48-bit floating point number representation.

The criterion capability will shape the characteristics of the computer program which implements the plan. For example, small problems will not run as efficiently with the code as with a less general program. Hardware characteristics imply that single-precision arithmetic will be sufficient.

To provide a basis for storage allocation and data management decisions, timing data are assumed for the hardware. These data are given in Table X.

TABLE X

COMPUTER TIMING DATA

Operation	Units of Time	Basis
Arithmetic	8	Includes logic time
Store a word in core	1	Normalization basis
Store a word on tape	$40 + \frac{6500}{w*}$	Includes start-stop and access time
Store a word in addressable bulk storage	$26 + \frac{47000}{w*}$	Based on average access time of Fastram

* = number of words

These data are based on existing computer hardware. The availability of parallel arithmetic hardware would lower arithmetic time. This consideration has been disregarded. Improvement in computer bulk storage access and transmission time can be anticipated, so numbers have been rounded down. The data imply that for maximum efficiency, records longer than 3000 words should be put on bulk storage rather than tape if peripheral units are used. If search can be avoided, all records less than 3000 words should be put on tape units.

Section 3
GENERAL DESCRIPTION OF THE OPTIMIZATION PROCESS

This section describes the prominent features of the optimization process. It presents the flow chart for the operations; relates the operations to the components of the implementation plan; provides a list of subroutines required in the program; and identifies the operations in which they are evoked. It also identifies that part of the plan which provides more detailed description of the operations and, in passing, presents the flow charting conventions used in this report.

The Process

Data processing to perform optimization is grouped in four operations: Initialization, Baseline Analysis, Design, and Active Review. Initialization performs reading, checking, and interpreting problem definition data. Baseline Analysis evaluates the "exact" behavior of a given design. Design includes Design-Analysis and Redesign steps. The Active Review Operation provides scope displays of the current design and its behavior.

The relations between operations during calculations are illustrated in Figure 2. The L shaped part includes that logic which resides in the Primary Computer. The logic in the upper right-hand area resides in the Graphics Computer. The decision logic between these parts is at the interface and could reside in either the Primary or Graphics Computer.

Sequence of operations.- The optimization calculations are initiated on the primary computer by executing Operation I. Operation II then predicts response of the initial design. Operation III follows and then is executed a multiple number of times. Each time, a single "design cycle" is completed. This design is restricted in the sense that usually an optimum design is produced assuming that, using a reduced degree-of-freedom model, the behavior of the structure is accurately represented. The Baseline Analysis Operation then is reentered to develop an improved Design-Analysis basis. Each pass through Baseline Analysis

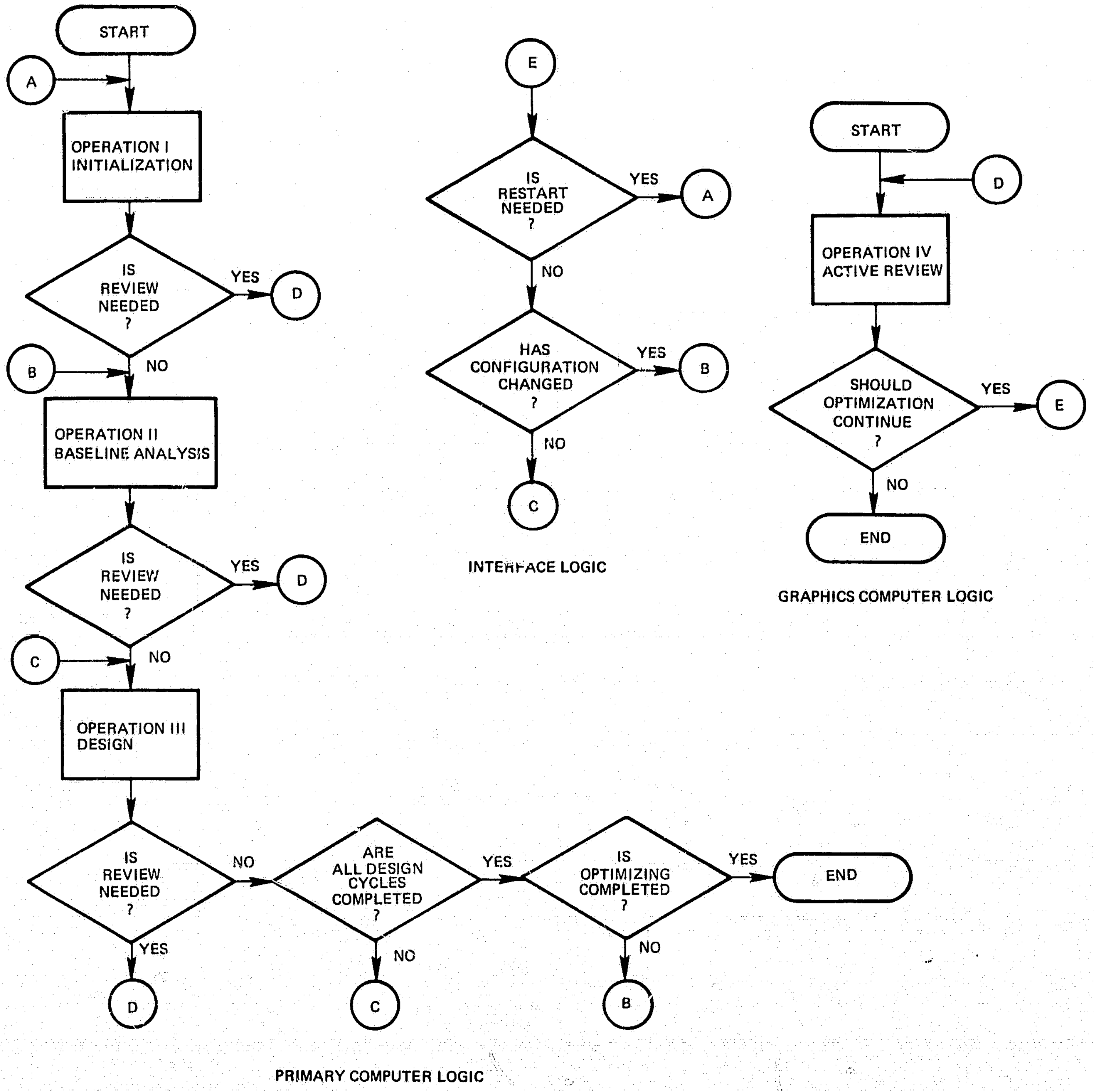


Figure 2 Master Flowchart of Operations

defines an optimization cycle. As the number of optimization cycles is increased, the design will approach a relative optimum for the structure of interest. In some problems one cycle will suffice. The optimized design will be restricted only by the fidelity of the analysis based on the mesh refinement and element representation chosen for the original structure.

The Active Review Operation will be executed at the analyst's option. It will be implemented on a separate arithmetic unit and in conjunction with the calculations on the Primary Computer. After each operation, the designer will be given an opportunity to review data produced by the operation using a scope, printed, or punched display of data. Logic at the interface will provide for the continuation of Primary Computer processing at the appropriate operation and, if required, under delayed restart conditions. Because two arithmetic units are anticipated, the Primary Computer calculations can also proceed, at the user's discretion, while the review of results continues.

Operations and implementation plan.- The relation between these operations and the components of the implementation plan are summarized in the Table XI below. The first two columns identify the primary function of each operation. Both Operations I and IV are primarily Input/Output operations. The last two columns show which operation has the software implementing each of the subplans.

A summary of the functions to be performed in developing the optimum design is provided by the data in Tables XII - XV. Each table identifies a group of functions, the operations within which the functions are performed, and the particular subplan containing subroutine specifications for implementing the calculations. Subplans are noted by I/O, Input/Output; DA, Design-Analysis; and RD, Redesign. To clarify requirements, only those functions required for Operations I, III, and IV are included. Notations indicate when these functions are also required in Operation II.

NASTRAN Interface

To avoid duplication, it will be assumed that the optimization program will complement the NASA Structural Analysis system, NASTRAN. To facilitate this, program

TABLE XI

RELATION BETWEEN OPERATIONS AND SUBPLANS

Operation	Primary Functions	Subplan	Operations Affected
I	Input/Output	Input/Output	I, II, III, IV
II	Baseline Analysis	Baseline Analysis	III
III	Design-Analysis, Redesign	Redesign	III
IV	Input/Output		

TABLE XII

READ AND WRITE FUNCTIONS

Function	Operations	Subplans*
Read, echo, and check design control data	I	I/O
Read, echo, and check incremental load data	I	I/O
Read, echo, and check incremental material data	I	I/O
Read, echo, and check candidate sections data	I	I/O
Read, echo, and check incremental element data	I	I/O
Read, echo, and check incremental joint data	I	I/O
Read, echo, and check joint relocation data	I	I/O
Read, echo, and check allowable displacements	I	I/O
Read, check and compile cost function subroutine	I	I/O
Read, check and compile displacement constraint subroutine	I	I/O
Read, check, and compile material constants subroutine	I	I/O
Read, check, and compile cross section properties subroutine	I	I/O
Read a block of data from peripheral storage and unpack	II, III	I/O
Print special redesign summary design data and comments	III	I/O
Print general redesign summary design data and comments	III	I/O
Print response data critical for design	III	I/O
Print response data critical in each loading	III	I/O
Print deformations	III	I/O
Print critical states of stress	III	I/O
Print generalized forces	II, III	I/O
Print debug data	I, II, III	I/O
Write a block of data onto peripheral storage after packing	I, II, III	I/O
Write new design data into problem descriptive data	III	I/O
Read and write spilled data	III	I/O

*I/O = Input/Output
RD = Redesign
D-A = Design-Analysis

TABLE XIII

SORT AND LOCATE FUNCTIONS

Function	Operation	Subplans*
Sort and store design variable data	I, II	I/O, RD
Sort, compact, and store element data	I	I/O
Sort and store deflection limits data	I	I/O
Sort and store row listed load vectors	I	I/O
Sort data into tabular form	IV	I/O
Locate material constants and interpolate for a particular material	III	D-A
Locate an entry in the candidates section table of a given type	III	D-A, RD
Locate a matrix in peripheral storage	III	D-A
Locate class data for a particular element	III	D-A
Locate gang data for a joint	III	D-A
Locate deflection limit data for a particular loading	III	D-A
Locate load defining parameters for a particular loading	III	D-A
Locate and select from element displacement data	IV	I/O
Locate and select from element state-of-stress data	IV	I/O
Locate and select from joint coordinates	IV	I/O
Locate and select from joint displacements	IV	I/O

*I/O = Input/Output
RD = Redesign
D-A = Design-Analysis

TABLE XIV

CALCULATION FUNCTIONS

Function	Operations	Subplans*
Develop element direction cosines	III	D-A
Develop element stiffness matrices	II, III	D-A
Add two matrices	II, III	D-A
Form the union of two nonconforming arrays	II, III	D-A
Multiply two matrices	II, III	D-A
Add the transpose of a matrix to a matrix	II, III	D-A
Scalar multiply a matrix and add another	II, III	D-A
Multiply two sparse matrices	II, III	D-A
Generate element state of stress coefficients	III	D-A
Transform stresses into principal stresses	III	D-A
Evaluate the Von Mises yield criterion	III	D-A
Evaluate the stress interaction formulae	III	D-A
Scale size design variables	III	RD
Replace design with realizable hardware	III	RD
Extrapolate design variables by rational polynomials	III	RD
Decompose a square, symmetry positive matrix in sparse form	II, III	RD
Perform forward substitution	II, III	D-A
Perform backward substitution	II, III	D-A
Evaluate built-in cost function	III	RD
Calculate derivatives of built-in cost function	III	RD
Calculate candidate element performance measure	III	RD
Form initial element selections	I	I/O
Form list of synthesis candidates	I	I/O
Calculate design penalty	III	RD
Determine minimum by Davidson's Method	III	RD
Scale variables	III	RD
Unscale variables	III	RD
Extract eigenvalues (Givens[1] - Householder)	III	D-A
Calculate eigenvectors	III	D-A
Generate element Q matrices	III	D-A
Calculate strain energy	III	D-A
Generate displacements by integration	III	D-A
Evaluate element interior displacement	III	D-A
Calculate buckling allowable	III	D-A
Form element group influence vector	III	D-A
Form joint gang influence vector	III	D-A
Define critical stress region	III	D-A
Resize for element buckling integrity	III	RD
Calculate Gradient	III	RD
Correct curvatures	III	RD
Form subspace stiffness matrix	III	D-A
Form subspace flexibility matrix	III	D-A

*I/O = Input/Output
RD = Redesign
D-A = Design-Analysis

TABLE XV
CONTROL FUNCTIONS

Function	Operations	Subplans*
Test input data magnitudes	I	I/O
Test input data format	I	I/O
Test input data inconsistency	I	I/O
Test input data completeness	I	I/O
Test and define design-analysis strategy	III	D-A
Determine subspace construction data processing mode	III	D-A
Control graphics file data accumulation	III	D-A, RD
Test numerical singularity	III	D-A
Test kinematic stability	III	D-A
Test for local rigidities	III	D-A
Test all loads completed	III	D-A, RD
Test all elements completed	III	D-A, RD
Control buckling integrity check	III	RD
Control fracture integrity test	III	RD
Control endo-element deflection integrity check	III	RD
Control interjoint deflection integrity check	III	RD
Test accuracy of stress prediction	III	RD
Control branching upon return from graphics	III, IV	RD

*I/O = Input/Output
RD = Redesign
D-A = Design-Analysis

specification and flowcharting, where given, will generally comply with NASTRAN documentation standards. However, a hexagonal box in a flow chart will designate a function which is defined in more detail on another chart.

For many of these functions, implementing computer code is already available. In particular, most of the read and write functions and the sort and locate functions are required in structural analysis and are represented in the NASTRAN code.

It is assumed that the NASTRAN executive performs all supervisory functions in accordance with NASTRAN documentation. These functions include:

1. Establishing and controlling the sequence of module executions.

2. Establishing, protecting, and communicating arguments for each module.

3. Allocating system files to all data blocks. (A data block is a set of data, matrix or table, occupying a file.)

4. Providing full restart capability.

Since the code implementing these functions is suitable for optimization as well as analysis, an alternate plan and its description would be redundant. Thus, subplan descriptions will concentrate on analysis aspects of optimization.

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Section 4

INPUT/OUTPUT PLAN

This section describes the Input/Output Plan used in the optimization process. It includes a description of special input data, printed and taped output data, and data used for interactive graphic displays. Generally, the description omits data for and from Baseline Analysis since these depend on the parent program. The last paragraphs indicate special considerations in integrating the plan with a NASTRAN parent and review salient features of the Input/Output Plan.

General Description

Optimization requires data in addition to that needed for Baseline Analysis. Information is formatted into interrelated tables, and the additional data is organized independently and treated as incremental input.

The incremental input is divided into nine types. Each type is designated as an input data set. The name and purpose of each input set is summarized in Table XVI. The data of these sets completely define the optimization process control, cost function, and behavioral and design variable constraints.

Output is displayed in print and on CRTs and is stored on tape and in graphics files. Printed output in report form is planned to provide a complete record of (1) the problem and its solution, and (2) messages of progress and difficulties of solution. Taped output is used to accommodate optimization restart. This output will be produced under control of the optimization software in Operations I and III. (See Figure 2.) Some of these data will be controlled by the parent program under Operation II.

The Graphics File will provide data for accessing at the Active Graphics Interface. In addition to data describing the current design configuration, this file includes information on the behavior of the design structure. A second file includes

TABLE XVI

INPUT DATA SETS

Number	Data Name	Purpose
1	Design Control	Define logic options, quantify overall problem parameters.
2	Design Loadings	Identify loading to be considered in designing.
3	Candidate Materials	Define admissible structural materials and their capabilities.
4	Candidate Sections	Define admissible element sizes and their material composition.
5	Undesigned Elements	Identify elements for redesign.
6	Undesigned Joints	Identify joint relocation variables.
7	Gang Constraints	Define acceptable joint relocation range.
8	Displacement Limits	Define acceptable range for deformation.
9	End File Card	Indicate end of incremental input.

information to facilitate monitoring and direct solution progress. The analyst/computer interface can be activated only at the end of Operations I, II, and III.

The Active Graphics Option gives the designer the opportunity to reduce optimization costs by contributing decisions on a supervisory level; interfacing is admitted only at planned points. The Graphics File emphasizes summary information.

Input Data

Table XVII defines all the input data sets. Those not required for a particular problem may be omitted entirely. However, since an optimization problem must be adequately defined, as a minimum, sets (1) through (5) must be included. If unreferenced data (data not called during the design process) is included, it will be printed with other input and occupy storage space.

Magnitude checks.- All these data are checked with respect to magnitude, consistency and completeness. Magnitude checks are performed only when the data is first read. These checks ensure that reasonable numbers are selected. For example, "The maximum allowable tensile stress is required to lie between zero and 200,000 psi.". Because these checks are dependent on dimensional units, the checking subroutine contains conversion factors to accommodate a variety of dimension standards. These constants are the only problem-dependent constants in the computer code.

Consistency checks.- Consistency and completeness checks are performed both when input is first read and in subsequent operations. Consistency checks preclude conflicting problem-descriptive data. For example, the existence of two distinct sets of coordinates for a particular joint is inadmissible. Checking for an active structural kinematic instability is an example of a consistency check during calculations. Failure to satisfy either check will cause processing to abort.

Completeness checks.- Completeness checks ensure that an optimization problem is completely defined. A completeness test will ensure that a failure criterion is identified for each kind of material in the structure. Requiring that the cost function, candidate cross-section geometry, and optimization control information are included are other examples of input completeness tests. Determining if a joint exists for each joint referenced in the topology is a completeness check that may be made during calculation, depending on data management for the parent program at input time.

Generic identifiers.- Incremental input is reduced by using generic identifiers. The generic identifier is entered as a zero on a blank. In general, the program interprets generic identifiers (zero or blank) as generic whenever its interpretation as "zero" is meaningless. It is interpreted to mean that the corresponding data is appropriate for all of the possible identifications. For example, if a joint for a load is identified by a blank, the specified load will be applied to all joints of the structure. If the load component identifier is generic, the given load is applied to all components of the joint identified. The load vector, joint numbers, displacement components, cross-section form restriction, and admissible material identifiers may all be indicated by a generic.

Input Data Sets

The following paragraphs describe the input in each data set in more detail. A review of these paragraphs will provide the reader with an understanding of the options in the Optimization Program Plan. Format details are defined for clarity of exposition and will be modified to make them compatible with the parent program and Baseline Analysis input.

In the tables that summarize the input data, each entry is designated by a symbol to condense data. Symbols imply completeness tests and are interpreted as given in Table XVII.

Also, as a convention, when several items in a list are enclosed in brackets, the analyst must choose one of the items for the input entry.

TABLE XVII

INPUT DATA SET KEY

Symbol	Meaning
.A.	The character A must be punched.
"A"	Any alphanumeric character except a blank is involved.
A	A floating decimal value is expected.
$\underline{A}$	The value may be omitted.
I_r	Ignored data. Read but disregarded.
I_N	Ignored. Not read, printed, or used.

Set 1: Design control data.- Table XVIII summarizes the input for these data. Generally, the information is that required to direct the optimization process and define overall problem parameters. Cards in this set are of two types: quantifying and titling. The quantifying cards include the data in Table XVIII. Any number of titling cards may follow the quantifying cards. Titling cards are printed with all output as job identification information. This titling is optional with the designer.

The first ten items in the data set control program branching. The Specialized Redesign Option is selected for efficient optimization when element size is the only design variable, many elements must be designed, and primary interest is in structures of high-performance efficiency. This option permits either a fully-stressed or least-cost design objective. General Redesign is selected when there are few design variables and the cost function is well-known. For optimization efficiency, General Redesign is addressed toward optimization of relatively few design variables. In General Redesign, a number of methods could be distinguished though the only one described in this plan uses direct minimization.

Items 3, 4, and 5 provide for cognizance of FORTRAN statement definitions of the cost function, displacement limits, and material characteristics. Thus, the User's cost function may supplant the built-in function and displacement limits and materials characterized in ways other than those in this program plan. Item 6 permits using Baseline Analysis for Design-Analysis as discussed in Section 5.

TABLE XVIII

SET 1: DESIGN CONTROL DATA

Item	Format	Symbol	Interpretation
1	A1	.S. .G.	Specialized Redesign required General Redesign required.
2	A1	.0. .L.	Fully stressed design if item 1 is .S.; direct minimization otherwise Least cost design if item 1 is .G.; I_g otherwise.
3	A1	.P. .F.	Program modular cost function to be used Cost function to be introduced as FORTRAN subroutine
4	A1	.P. .I. .T.	Displacement limits to be only those in program Displacement limits to include those in program and those defined by a user-introduced FORTRAN subroutine: incremental form User FORTRAN subroutine: total form
5	A1	.P. .F.	Programmed material models only Some materials defined by user-introduced FORTRAN subroutine
6	A1	1	Skip Design-Analysis subspacing
7	E2.0	$\underline{P}$	Printout level
8	E2.0	$\underline{G_1}$	Active Graphics level
9	E2.0	$\underline{G_2}$	Graphics cutoff level
10	E2.0	$\underline{T}$	Tape output option
11	E6.0	$\underline{N_1}$	Number of optimization cycles permitted
12	E6.0	$\underline{C_1}$	Optimization nondimensional cutoff criterion
13	E6.0	$\underline{N_2}$	Number of design cycles permitted.
14	E6.0	$\underline{C_2}$	Design nondimensional cutoff criterion.

TABLE XVIII (Continued)

Item	Format	Symbol	Interpretation
15	A1	.$\underline{L}$.	Unit of Length L = C or blank: centimeters L = M: meters L = F: feet L = I: inches
16	A1	.$\underline{F}$.	Unit of Force F = G or blank: grams F = K: kilograms F = P: pounds F = T: tons
17	A1	.$\underline{T_1}$.	Unit of Time T_1 = S or blank: seconds T_1 = M: minutes T_1 = H: hours
18	A1	.$\underline{T_2}$.	Temperature Scale T_2 = C or blank: Centigrade T_2 = K: Kelvin T_2 = F: Farenheit T_2 = R: Rankine
19	E6.0	C_1	Cost parameters for program or FORTRAN cost function
20	E6.0	C_2	

Items 7 through 10 control output options. Printed output and data selected for the Active Graphics File are managed using the leveling concept. Under this concept, higher numerical levels include all data from lower levels and additional items. The graphics cutoff level is a nondimensional number which defines an output filter passband. The tape option provides for restart by saving data defining current values of design variables.

Items 11 through 14 control the optimization cycling. The nondimensional cutoff criteria define the value of the convergence measure at which iterations will be stopped. Both these items and the number of design cycles are default variables. (If undefined, built-in program controls will be active.)

The basic analysis units are specified by Items 15 through 18. In default, the CGS system is assumed. In any event, the user must introduce all his input using these basic units. These units will be implied in all calculations and used in defining the dimensions of output data. Data introduced into the parent program for the Baseline Analysis must be in the same units as used for incremental data.

The last two entries of Set 1 are the values of two of the cost-function parameters. The built-in cost function takes the form,

$$C = \sum_{e=1,2\ldots}^{E} L_{e} \left\{ \alpha (c_{e} + \bar{c}_{e} A_{e}) + \beta A_{e} \rho_{e} \right\} \tag{4-1}$$

where

C	is the system cost,
E	is the number of finite elements,
α , β	are optimization cost parameters (entries 18 and 19 in Input Set 1),
c_{e}	is the cost per unit length of the candidate selected for element e (introduced with the candidate cross section: Input Set 4),
$\bar{c}_{e}$	is the cost per unit volume of material (introduced with the candidate materials Input Set 3),

L_e is the length of element e if a line element or a surface dimension, otherwise

A_e is the cross sectional area of element e such that $L_e A_e$ is the volume, and

ρ_e is the density of the material selected for element e (from Input Set 3).

Set 1, because of its nature, contains a number of cards. Each may have a different format. This characteristic is not shared with the other data sets where a number of cards of the same form are involved.

Set 2: Design loadings data.- Table XIX cites the data required in this set. This set identifies which of the Baseline Analysis load vectors are to be considered in the optimization and how. Item 4 indicates only strength integrity checks. The requirement for displacement checks is implicit in the inclusion of displacement limits. (Set 8.)

Set 3: Candidate materials data.- Table XX summarizes items for the input set. These data reference Baseline Analysis input for material properties (elastic constants, density.) Thus, baseline data must include coefficients for candidate materials as well as those selected for the initial design. The array of yield and ultimate strength criteria cited is intended to include those most frequently used. Others can easily be added, as necessary. Yield and ultimate strength parameters particularize the strength criteria. These data are also supplied as arguments to the user's FORTRAN subroutine for material characterization.

Set 4: Candidate sections data.- Table XXI lists entries for this input set. This set defines all admissible sizes and construction materials.

The size name (number) is used as the basis for ordering the candidates in terms of increasing structural performance capabilities. Such an ordering is impossible for multiple degree-of-freedom elements without knowledge of the way in which the element deforms. Thus, ordering is performed in accordance with the analyst's judgment. Should the analyst input size numbers that increase while corresponding performance decreases, the Redesign process will be slowed. Moreover, the

TABLE XIX

SET 2: DESIGN LOADING DATA

Item	Format	Symbol	Interpretation
1	A1	"A"	Loading name. (To correspond with name in Baseline Analysis.)
2	A1	.C.	Loading criticality C = Y or blank: yield C = U: ultimate
3	E4.0	S	Loading safety factor
4	A1	.I.	Applicable integrity checks I = A or blank: all checks I = F: fracture only I = B: buckling only I = N: neither fracture nor buckling

TABLE XX

SET 3: CANDIDATE MATERIALS DATA

Item	Format	Symbol	Interpretation
1	A1	"A"	Name of the material (To correspond with name in Baseline Analysis.)
2	E6.0	<u>B</u>	Material cost per unit volume (c_e of Eq. (4-1)).
3	A1	.C.	Yield criterion C = V or blank: Von Mises C = T: Tresca C = P: maximum principal stress
4	A1	.D.	Ultimate strength criterion D = P: maximum principal stress D = B: brittle failure criterion D = V: Von Mises D = T: Tresca
5,6,7	3E6.0	$E_{i=1,2,3}$	Yield criterion parameters
8,9,10	3E6.0	$F_{i=1,2,3}$	Ultimate strength criterion parameters

TABLE XXI

SET 4: CANDIDATE SECTIONS DATA

Item	Format	Symbol	Interpretation
1	A1	.L. .S. .T.	Line element Surface element Three dimensional solid element
2	A1	.A. .B. .C. .D. .I. .J. .O. .R. .W. .Z. .H. .S.	Single angle cross section Box section Channel section Double angle section I beam section J section Circular cross section Rectangular cross section Wide flange section Z section Homogeneous solid surface Sandwich shell surface
3	A1	"A"	Material for this candidate (may be generic)
4	A6	"B"	Size name (or number)
5	E60	C	Cost per unit length for this section (c_e of Eq. 4-1))
6-13	7E60	$N_{i=1,2..7}$	Dimensions describing the geometry of the cross section.

discrete sizes chosen in Redesign will tend to increase sizes in the final design since the design search normally will not consider all possible discrete sizings in the candidate list.

The last seven entries in Table XXI particularize the cross section. Figure 3 illustrates the interpretation of these data for a Tee cross section. These data are sufficient for development of stiffness and stress models and generation of dead weight and thermal load vectors for the element.

Set 5: Undesigned elements.- Table XXII lists incremental element data to identify the elements to be designed and the constraints on the variables . If any elements in the Baseline Analysis are omitted from the Set 5 incremental data, it is implied that the elements can not be changed with respect to size or material.

The first six entries identify the element and its redesign restrictions. The sixth entry is used to require that a number of elements be redesigned to the same size and material. All elements with the same group name must be the same.

The element connection reference permits defining the assembly of the structure's pieces. Joint locations define points in space. The connection reference data defines which points on the cross section coincide with these joint locations. The Tee section of Figure 3, for example, can be attached to a joint at its neutral axis, reference point A, or at its upper face, reference B. Similarly, reference A defines neutral axis attachment for any cross section and B and C alternate attachment joints. A blank entry for all elements is, by default, an election of A references. Thus, if these entries are all blank, no joint eccentricities other than those explicitly defined, will be considered.

Items 9 and 10 permit designation of the element as a subspace synthesis candidate. This allows the analyst to use his knowledge of structural behavior in the analysis of the structure as discussed in Section 5.

Note that elements may be used to determine optimum joint boundary conditions by selecting spring elements. These elements can connect joints of the elements

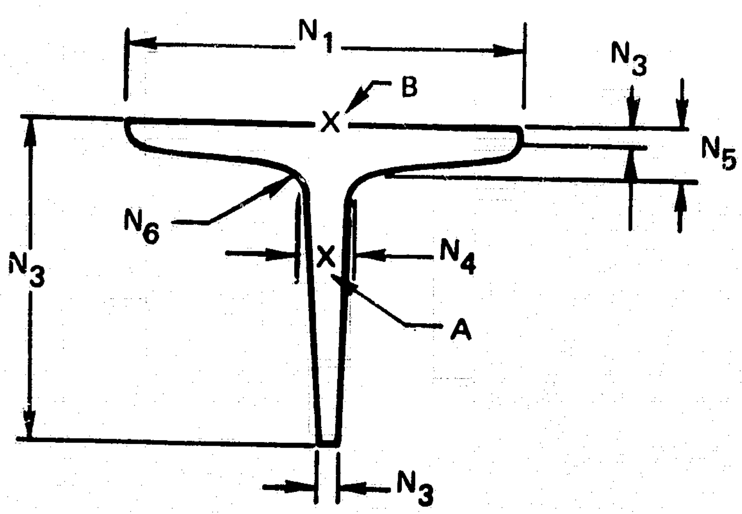


Figure 3. Tee Section Dimension Data

TABLE XXII

SET 5: UNDESIGNED ELEMENTS

Item	Format	Symbol	Interpretation
1	A1	.L. .S. .T.	Line element Surface element Three dimensional element
2	A1	"A"	Name of acceptable materials for redesign Blank = generic: any of given materials A→Z = one choice this material only 0→9 = exclusion:exclude this material
3	A6	"A"	Element name (number) identification
4	A1	.N. .Y. . .	Element cannot change size Element may be any size
5	A1	"X"	Element restricted to be of section type X (e.g. if X = U, only tubes are admissible as sections)
6	A1	"Y" . .	Element group name Not associated with an element group
7	A1	.A. .B. .C. . .	Element connected at reference point A Element connected at reference point B Element connected at reference point C Element connected at reference point A
8	A1	. . A	Element may not be eliminated Element is a candidate for removal
9	A1	"A" . .	A proposed subspace synthesis candidate Not suggested as a candidate
10	A1	. . .D. .C.	All modes of behavior are candidates Only direct force behavior (e.g. membrane) Only couple behavior (e.g. plate)

or an element joint to a fixed joint. By optimizing the sizing prescribed for these elements in Design, element connection fixity and structural support constraints can be optimized.

Set 6: Undesigned joints.- Table XXIII lists the two entries required in the input set. These data relate joints to joint gangs. A joint may belong to only one gang. If a joint does not have a gang assignment, its coordinates may not be changed in redesign.

Set 7: Gang relocation data.- Table XXIV summarizes the data defining the nature and limits of the gang relocation variables. The first three items of this data need be entered only once for each gang. Items 9 through 18 may require up to eight cards to define each of the possible eight octants of joint relocation.

Items 3 through 8 define the orientation of the joint relocation axes. If these data are omitted, the coordinate axes will be assumed to coincide with the displacement axes. If more than one displacement axis is involved for joints in the gang, the input is inconsistent.

Items 9 through 12 provide data for interpreting the relocation limits given as items 13 through 18. An inclusive octant is one in which relocation is possible within the given limits. An exclusive octant is one from which relocation is excluded within the given limits. Items 10 through 12 provide a convention to abbreviate the definition of the quadrant for which the limits are given. If these data are not plus and minus signs (e.g., left blank), the first quadrant is implied.

Set 8: Displacement limits data.- Table XXV describes the items for each card entry in this input set. There are two types of displacement limits: endo-element and system. Endo-element limits restrict deformations in the interior of an element relative to the bounding joints. These are requirements such as the need to limit lateral deflections of a beam to less than 1/360 of the span. System displacement limits require the design to preclude the relative motion of any two joints. Since an immobile joint can be included in the optimization, system-relative displacement limits can be converted to absolute limits by selecting an immobile joint as a member of the joint pair specified in items 1 and 2.

TABLE XXIII

SET 6: UNDERSIGNED JOINTS DATA

Item	Format	Symbol	Interpretation
1	A6	"A"	Joint name (number)
2	A6	"B"	Joint gang name

TABLE XXIV
SET 7: GANG RELOCATION DATA

Item	Format	Symbol	Interpretation
1	A6	"B"	Joint gang name
2	A6	"C"	Coordinate system referenced
3-8	6E6.0	$\underline{N}_i$	Direction cosines of the relocation triad with respect to the reference axis
9	A1	.I. .E.	Octant defines inclusive limits Octant defines exclusive limits
10	A1	.+. .-.	Pertinent octant is on plus x coordinate side Pertinent octant is on minus x coordinate side
11	A1	.+. .-.	Pertinent octant is on plus y coordinate side Pertinent octant is on minus y coordinate side
12	A1	.+. .-.	Pertinent octant is on plus z coordinate side Pertinent octant is on minus z coordinate side
13	E6.0	X_1	Upper x limit on joint relocation
14	E6.0	X_2	Lower x limit on joint relocation
15	E6.0	Y_1	Upper y limit on joint relocation
16	E6.0	Y_2	Lower y limit on joint relocation
17	E6.0	Z_1	Upper z limit on joint relocation
18	E6.0	Z_2	Lower z limit on joint relocation

TABLE XXV

SET 8: DISPLACEMENT LIMITS DATA

Item	Format	Symbol	Interpretation
1	A6	"A"	Element name (number) or first joint number (name)
2	A6	"A"	Second joint number (name)
3	A6	"B"	Loading name (may be a generic)
4	E6.0	C	Maximum relative x displacement
5	E6.0	D	Maximum relative y displacement
6	E6.0	E	Maximum relative z displacement
7	E6.0	F	Maximum relative x rotation
8	E6.0	G	Maximum relative y rotation
9	E6.0	H	Maximum relative z rotation

*Limits are absolute value conditions relative to the element local axes when elements are named. They are absolute value conditions relative to displacement axes of the first joint when joints are designated.

Set 9: End file card.- This card is a single card designating the end of the card input file.

FORTRAN subroutine input.- Three FORTRAN subroutines may be introduced and will have access to the same data required by the built-in subprograms as well as any data the user wants them to contain. In essence, the user subroutines substitute for the system subroutines when required. Thus, since the cost function is a single function, the user's FORTRAN IV Subroutine "replaces" the built-in subroutine. The User's Displacement Limits Subroutine is called either instead of or in addition to the built-in function, as specified in Input Set 1. The User's Materials Subroutine is used whenever a material is considered whose elastic constants are not included in the baseline data.

The functions which the subroutines perform impose special requirements on the subroutines. The Cost Routine must supply system cost and accurate evaluations of the change in cost for indicated design variable changes. The Displacement Limits Program must check the loading identification and only return measures of acceptability if the appropriate loading is indicated. The Materials Subroutine must indicate an incompleteness check if it is called upon to particularize material characteristics for materials other than those for which it is coded.

For efficiency, the Displacement Limits Subroutine may take one of two forms. In "incremental" form, this subroutine accepts the deflection data for one joint at a time for all loadings. After an entry is made for every joint, it returns the factor by which the structure must be scaled to satisfy the deflection limit. In "total" form, the set of all deflections for all loadings are given and the subroutine retains acceptability data. Since the total form requires extra data transfers, it will prolong calculations beyond those required for the incremental form.

Offline Output

Offline output consists of printed, plotted and taped data produced in Operations I, II and III and printed and plotted output from Operation IV. These outputs consist of abbreviated reports and tabular and graphical problem information.

Abbreviated reports.- These consist of capsule and diagnostic reports. Capsule reports provide summary information on problem characteristics and calculation progress. Diagnostics report unusual problem features and faults. Diagnostics are distinguished by the fact that the message is followed by an alphanumeric code enclosed in parentheses. No code appears with capsule reports. Both types of reports are short printed messages. Capsule reports may involve one to three lines of output. Diagnostics rarely exceed one line.

Table XXVI lists some typical capsule reports. In actual output, underlined blanks would be replaced by numbers. These reports are included in the normal output stream. Capsule reports vary depending on the optimization options selected.

Table XXVII lists some typical diagnostic reports. These reports are triggered by tests made to check input and calculation accuracy. These reports usually are printed at the beginning or end of other printed output.

The diagnostic report code serves to identify the report and define its source in the program. The first letter indicates whether the fault induces calculation abort (A) or continuation (C). Other characters define the program subroutine and the number of the message in the subroutine.

Tabular information.- The designer will have the option of selecting logic in the parent and optimization coding which will produce printed output of calculation results. The printout option will be based on the "level" approach. With this approach, a given level includes printout of all data required at lower numerical levels plus that designated for the level called. A level may be defined independently for each of the operations.

Data available at each level is cited in Table XXVIII. The zero level for all operations requires printing all abbreviated reports. In addition, two renditions of problem input are produced: one in the form of an echo of card data, the second as a complete and labeled tabular form description of the problem. This description includes a citation of structural initial geometry; material composition; boundary conditions and optimization cost function; candidate sizes; candidate materials; relocation variables and limits; and displacement and strength allowables.

TABLE XXVI

SAMPLE CAPSULE REPORTS

Capsule Report
NO ERRORS WERE FOUND IN INPUT.
THE STRUCTURE INVOLVES _____ ELEMENTS, _____ JOINTS, _____ MATERIALS AND _____ LOADINGS.
THERE ARE _____ ELEMENT SIZE VARIABLES AND _____ JOINT RELOCATION VARIABLES.
THE BASELINE ANALYSIS WAS COMPLETED SUCCESSFULLY.
THE SUBSPACE BASIS CONSISTS OF _____ DEGREES OF FREEDOM.
AFTER DESIGN CYCLE _____: COST = _____, CONVERGENCE MEASURE = _____.
AFTER OPTIMIZATION CYCLE _____: COST = _____, CONVERGENCE MEASURE = _____.
COST OF FINAL DESIGN = _____.
THE FINAL DESIGN INVOLVES _____ ELEMENTS WHICH ARE RECOMMENDED FOR REMOVAL.
THE FINAL DESIGN INVOLVES _____ JOINT GANGS WHICH ARE AT RELOCATION LIMITS.
TAPED OUTPUT DEFINES THE IDEALIZED OPTIMUM DESIGN.
TAPED OUTPUT DEFINES THE RELIABLE DESIGN.
JOINT ECCENTRICITY IS IGNORED.
BUCKLING INTEGRITY IS DISREGARDED.
FORTRAN STATEMENTS DEFINE COST FUNCTION.
FORTRAN STATEMENTS DEFINED DISPLACEMENT LIMITS.
NO DEFLECTION LIMITS ARE SPECIFIED.

TABLE XXVII

SAMPLE DIAGNOSTIC REPORTS

Diagnostic Report	Code
INPUT DATA IS INCOMPLETE. COORDINATES FOR JOINT _____ ARE REQUIRED.	(AF3.12)
ELEMENT _____ HAS IMPROPER COORDINATES. AS STATED, ELEMENT HAS NO VOLUME.	(AG1.01)
ORIENTATION OF ELEMENT _____ IS UNDEFINED. IT HAS BEEN ASSUMED TO BE IMMATERIAL.	(CB0.12)
IMPOSED LOAD AT JOINT _____ IS IGNORED. NO SUCH DEGREE OF FREEDOM EXISTS.	(CZ0.02)
FOR THREE SUCCESSIVE CYCLES, ELEMENT _____ REQUIRED A SIZE GREATER THAN AVAILABLE.	(AK1.05)
JOINT _____ IS GIVEN TWO CONFLICTING SETS OF COORDINATES.	(AS27.03)
MATERIAL _____ HAS A DENSITY WHICH IS NOT BETWEEN 0. AND 1.0, AS REQUIRED.	(AS23.01)
IMPOSED DEFLECTION FOR LOADING _____, JOINT _____ IS MEANINGLESS. THIS DEGREE OF FREEDOM IS NONEXISTENT.	(CB4.10)
THE CROSS SECTION TYPE _____, FOR ELEMENT _____ HAS NO PROPERTIES SUBROUTINE AVAILABLE.	(AA01.01)
NO LOADINGS HAVE BEEN FOUND FOR BASELINE ANALYSIS.	(AB01.04)
THE STRUCTURE IS PASSIVELY UNSTABLE IN AT LEAST DEGREE OF FREEDOM _____.	(CK02.01)
THE STRUCTURE IS ACTIVELY UNSTABLE IN DEGREE OF FREEDOM _____.	(AW03.01)
A FULLY STRESSED DESIGN DOES NOT EXIST FOR THIS SYSTEM.	(AQ02.11)
THE DESIGN SEQUENCE FAILS TO CONVERGE IN THE _____ CYCLES PERMITTED.	(AB01.03)

TABLE XXVIII

PRINTED OUTPUT LEVELS

Level	Operation I Initialization	Operation II Baseline Analysis	Operation III Design
0	Input and abbreviated reports	Abbreviated reports	Abbreviated reports
1	Input and abbreviated reports	Add response deformations	All optimization problem and current design variables
2	Input and abbreviated reports	Add response generalized forces	Add design criteria for each element
3	Input and abbreviated reports	Add response state of stress	Add design criteria for each loading
4	Input and abbreviated reports	Add response state of stress	Add deformation response of design
5	Input and abbreviated reports	Add response state of stress	Add generalized forces of response
6	Input and abbreviated reports	Add response state of stress	Add peak stress and endo-element deflection data
7	Input and abbreviated reports	Add response state of stress	Add peak stress and endo-element deflection data
8	Input and abbreviated reports	Add response state of stress	Add peak stress and endo-element deflection data
9	Add debug dumps	Add debug dumps	Add debug dumps

Problem debug printout may be selected for each Operation. This printout provides extensively labeled printouts of problem details to facilitate determining the reason for calculation faults when program capsule reports are inadequate.

The most extensive printout is reserved for the Design Operation. Here data on both structural response and critical design conditions is produced. Critical design data may include only the conditions which result in each element size selection and joint position. If desired, this may be augmented to identify the criticality of each loading. Then, the critical design conditions for each element and loading are printed.

All printout is fully labeled. Dimensional units of all tabulated quantities is specified and will comply with the user's selection. Any unusual abbreviations are defined by footnotes, and the user's attention to special conditions, such as excessive stresses or deflections, is attracted by asterisks or special footnoted symbols.

Graphical output.- In addition to data for plotting produced by the parent program in Operations I and II, the optimization program generates information for offline plots. Plots include graphical renditions of critical design conditions for each element. These plots are rendered as the geometry of the structure with symbols indicating critical element conditions.

Taped output.- Output of each design cycle will be taped to permit continuation of designing after any design cycle. These data will define the current value of the design variables and control data needed for restart. No cards will be punched for restart.

Online Graphics Display

The "level" approach also is used for controlling online Graphic Display File contents. Table XXIX indicates the additional output for each level of online graphics for each operation. Levels provide for obtaining all offline output through the Active Graphics File, except debug output.

TABLE XXIX

ON-LINE GRAPHIC DISPLAY LEVELS

Level	Operation I Initialization	Operation II Baseline Analysis	Operation III Design
0	No on-line graphics	No on-line graphics	No on-line graphics
1	All abbreviated reports	All abbreviated reports	All abbreviated reports
2	Add off-line plots	Add off-line plots	Add off-line plots
3	Add complete optimization problem description	Add off-line plots	Add current design description data
4	Add complete optimization problem description	Add off-line plots	Add deflection response data
5	Add complete optimization problem description	Add off-line plots	Add specified force response data
6	Add complete optimization problem description	Add off-line plots	Add peak stress data

The online Graphics Displays are controlled under Operation IV. This operation is performed using a Graphics Computer which communicates with the Primary Computer through a shared peripheral storage device such as a disk. The Primary Computer controls entering of basic problem data into the display file and directs the optimization process. The Graphics Computer's primary role is to search this file, retrieve required information, and organize these data into scope displays and special offline printouts for the user.

Calculations on the Primary Computer are delayed for designer action if a non-zero online graphic display level is optioned. This is effected by requesting designer direction at the end of the calculations of an operation. At this time, the designer can redefine problem input and select the subsequent operation to be performed. If the designer does not respond, optimization continues in accordance with card input control data of Input Set 1. Thus, if the designer defaults control through the active interface, optimization continues.

The Graphics Computer software enables the designer to display any portion of the data in the Graphics File. He can communicate his desires directly (using a mouse, light pen, or similar device). Prompting messages define display options and suggest needed control data. Any data organized for display, including tabular and graphical display, can also be rendered offline in appropriate form using graphics software.

With online graphic displays, the policy is to emphasize clarity and simplicity of information in display. Multiple displays provide the full spectrum of data. For example, in presenting the design conditions for the elements, two colors are used in the primary display of the data. The first color designates elements designed by endo-element criteria. The second designates elements designed by system requirements. In the secondary display, for endo-element critical elements only a multiple-color display is used. For example, elements which are marginal for stress might be in red; those sensitive to buckling instability, yellow; those designed by deflection limits, blue; and those of minimum size, green. A tertiary display would distinguish stress intensities in five colors, one for each range of intensities. Green might designate those of the lowest stress intensity. Any overstressed elements could be colored "unfortunate."

Input Set 1 includes control information to reduce the amount of uninformative data transferred into the Graphics File. This involves the graphics cutoff parameter. Specifying this permits the user to eliminate filing information on stresses, buckling loads, and endo-element and system deflection unless it falls in the user-defined range. This number is non-dimensional and defines the lower cutoff value for response data. For example, if the number is .20, all stresses and displacements less than 0.2 of allowables are omitted in the Graphics File.

The NASTRAN Input/Output Interface

In attaining the simulation capabilities implied by this Input/Output Plan, the program developer will be confronted with a number of interface problems. Some of these are listed in Table XXX when the parent program is NASTRAN. These interface problems involve making optimization and analyses capabilities consistent, compatible and complementary.

Most of the interfacing can be handled by requiring that the optimization programming adapt to NASTRAN as it exists. However, some changes are desirable in the initialization operation to provide checking of input on a consistent basis. Restart should also be simplified to provide for the "normal" restart encountered in optimization. Substantial NASTRAN changes may be required to provide Graphics File storage capabilities in Baseline Analysis.

A significant technical advantage of the optimization code is its capability to distinguish strength and deflection inadequacies of a given structural design. Baseline Analysis changes are recommended to make the capabilities available to the analyst as well as the designer.

Adapting to NASTRAN also seems inappropriate in treating joint eccentricities. Present NASTRAN capabilities are too limited in flexibility of use to serve as a basis for optimization. Thus, these capabilities should be reworked.

Plan Justification

Efficiency and flexibility justify major decisions of the Input/Output Plan. Efficiency dictates the form of input and offline output. Graphics File policies are primarily based on flexibility considerations.

TABLE XXX

SPECIAL NASTRAN INTERFACE PROBLEMS

Number	Operation	Problem
1	I	Initial element size supplied by preface, if omitted.
2	I	Data form made NASTRAN consistent, i.e., card formats, card identification.
3	I	Data input made NASTRAN compatible - redundancies removed (output controls, end file card), identification of joints, elements, and loading made to conform.
4	I	Redundant data must be acceptable - cripple unreferenced data checks.
5	I	Magnitude checks of primary input are needed.
6	I, II	Graphics file output must be provided, as required.
7	II	Dimensional units should be printed with output and consistency checked in matrix operations.
8	I, II	Coordinate system capabilities of NASTRAN need be made applicable to incremental input.
9	II	The redesign output must be organized into Baseline Analysis input file.
10	I, II	Joint eccentricity treatment must be made applicable to all element types and consistent with automatic treatment.
11	II	"Stress" analysis of NASTRAN must be made consistent with Design-Analysis or deleted.
12	I, II	Diagnostics should be complementary and consistent.
13	I, II	Restart should be simplified to provide for optimization continuation.

The integrated tabular form for input and output is relatively efficient as well as being compatible with NASTRAN data organization. Because input data is identified by card position and form, rather than requiring independent identification as does verbalized input, punched input is reduced. Use of integrated tabular form also reduces input redundancy. The same materials and cross sections often are used throughout the structure to reduce fabrication costs. By tending to minimize input, these decisions will reduce input preparation time and errors.

The leveled nature of output simplifies the user decisions and permits flexible control of offline and graphics output. This permits the analyst to tailor output volume to his personal requirements. The more proficient analyst will obtain less output than the average user. The range of output includes labeled debug dumps to permit rapid diagnosis of special problem difficulties.

Limiting data in the Graphics File to status information on the current design and response summaries improves optimization efficiency. The intent thereby is to use the designer's activity on the scope to provide overall optimization surveillance and policy decisions - tasks for which the computer is ill-suited. On the other hand, the Primary Computer will be restricted to detailed design and optimization search decisions involving thousands of variables - a task for which it is well-suited. Thus, the computer/designer will be an optimization team each doing the task for which he is proficient.

Salient Input/Output Features

Table XXXI summarizes the principal features of the Input/Output Plan. All these features can be implemented with little programming risk. The subplan for graphics, however, involves some risk due to the continuing evolution of graphics hardware and development of understanding of how the user and computer should work together in structural optimization.

TABLE XXXI

FEATURES OF THE INPUT/OUTPUT PLAN

Number	Feature
1	Integrated tabular input
2	Incremental input
3	Magnitude, consistency and completeness checks of input
4	FORTRAN statement cost function
5	FORTRAN statement displacement limits
6	Off-line output leveling including labelled debug data
7	Graphic file storage leveling for user surveillance
8	Diagnostic reports grouped and coded
9	Capsule progress and summary reports
10	"Built-in" modular cost function
11	Detailed cross section input
12	Yield and fracture strength criteria
13	Multiple element attachment references
14	Gang relocation limits by prism space control
15	Relative and implicit-absolute displacement limits

Section 5
DESIGN-ANALYSIS PLAN

This section describes and demonstrates the necessity for a special subplan to predict the behavior of the redesign structure. It provides a general description of the Influence and Reanalysis Process, its mathematical basis and sequential steps. It describes the process's economy, efficiency, accuracy and compatibility and concludes with a summary of features of the Design-Analysis Plan.

General Description

Design-Analysis is executed under Operation III of the master flowchart shown in Figure 2. A flowchart showing the relationship of Design-Analysis tasks to other tasks in Operation III is given in Figure 4. This operation includes all tasks for performing a design cycle: selection of the subspace basis; evaluating subspace, stiffness and flexibility matrices; analyzing the subspace mathematical model; converting the solution from the subspace to the element and joint response data; and redesigning the structure. Design-Analysis tasks accomplish Influence and Reanalysis and are described in this section. Interfaces of Design-Analysis and Redesign are indicated in this section (noted as hexagonal boxes in flowcharts), but details of these tasks appear in Section 6.

With Design-Analysis response evaluation can be performed using fewer degrees-of-freedom than in Baseline Analysis. Since the vector basis chosen for Design-Analysis is a subspace of that used in Baseline Analysis, this method is characterized as a "subspace" method.

The approach is to develop an analysis process which permits use of the pure strategies of Complementary Energy and Potential Energy Analysis or a mixed strategy. Selection of the appropriate strategy is made automatically so that estimates of deflection and internal force involve minimum error.

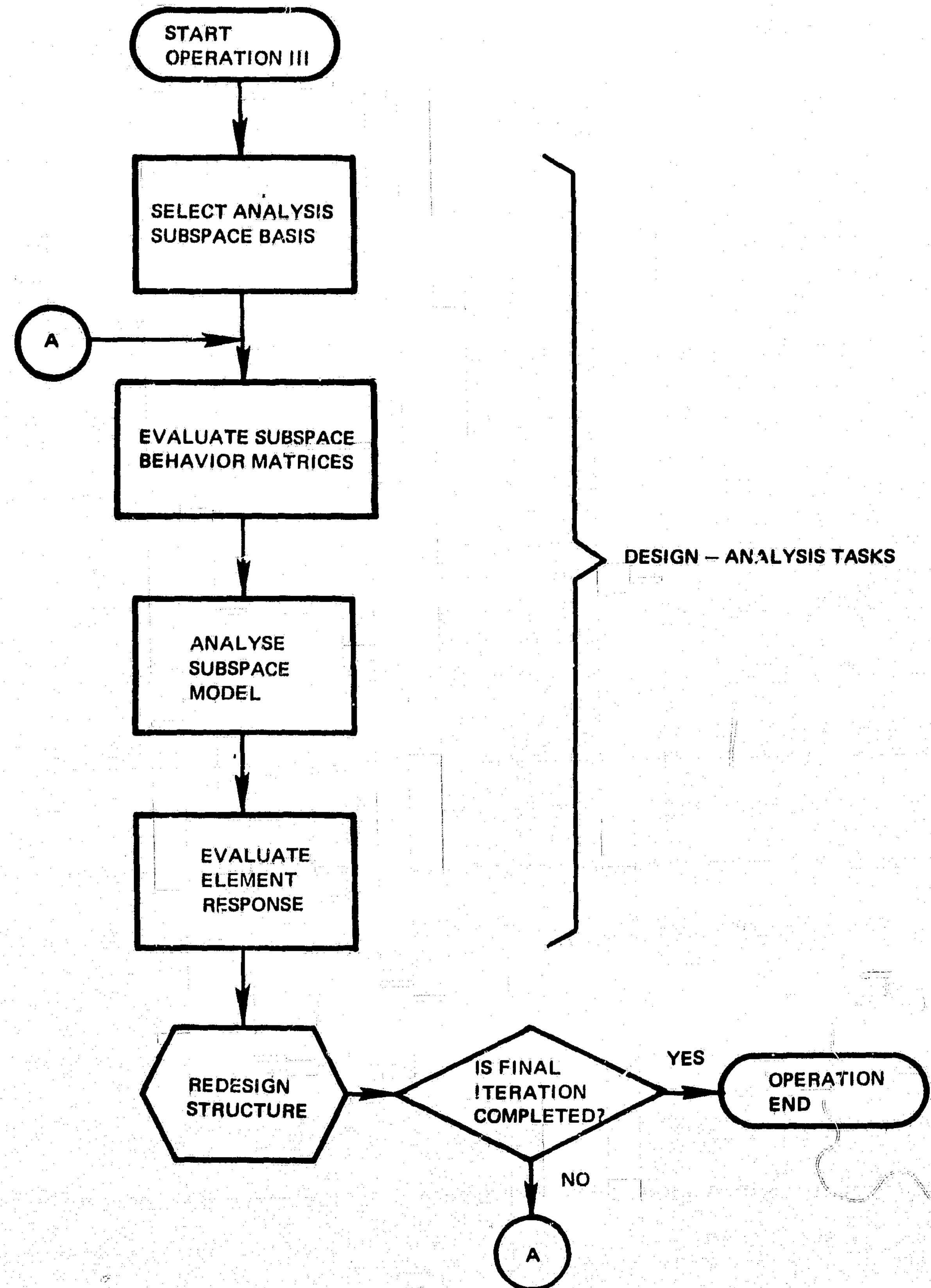


Figure 4. Operation III Flowchart

The accuracy of the subspace Design-Analysis predictions depends upon a good selection for the subspace basis. To ensure that a good choice is made the analyst may propose some synthesis candidates by designating critical elements of the structure. Then, at the option of the analyst, the computer identifies additional candidates until a full complement is designated. (The full complement consists of as many vectors as can be handled with in-core data processing.) This procedure provides for a spectrum of candidates ranging from all nominations by the user to all by computer-program direction.

The computer, using eigenvalue analysis, then is directed to the subset of force vectors which will comprise the working subspace basis for Complementary Energy Analyses. Because the eigenvalue analysis demands less core-storage space than other Design-Analysis tasks, this set will be usually much smaller than the set of synthesis candidate vectors. Displacement vectors, consisting of the displacements in the original structure for the selected force vectors, are used for the Potential Energy Analyses. The close relationship between the subspace work and displacement vectors permits performing both Complementary and Potential Energy Analyses at little more data processing cost than either one alone.

Mathematical Basis

The equations of this section are complicated by the need to include many similar variables. Thus, the notation has been selected to simplify interpretation of variables using subscripts and punctuation to distinguish components.

Table XXXII includes the major notation conventions. The meaning of each symbol will be defined when introduced. This table should accelerate acquaintance with the notation.

TABLE XXXII

DESIGN-ANALYSIS NOTATION CONVENTION

Convention Selected	Example	
	Symbol	Meaning
Forces are identified by majuscules. Miniscules of the same letter identify associated displacements.	F, f	Self-equilibrating loading, displacement.
	P, p	Real loading, displacement.
	S, s	Internal loading, displacement.
In general, Greek letters pertain to subspace-generalized coordinates and Latin to joint-generalized coordinates.	Ψ ψ	Generalized loadings, displacements in the subspace bases.
	P, p	Generalized loadings, displacements for joints in the structure.
Dummy indexes are often miniscules of the index upper limit.	b, B	Dummy base-vector index, number of base vectors.
	e, E	Dummy element index, number of elements.
	r, R	Dummy subspace vector index, number of subspace vectors used for response productions.
Subscripts without parentheses denote vectors.	S_i	The i^{th} vector of internal forces
Subscripts in parentheses denote element partition vectors.	$S_{i(e)}$	The partition of S_i coefficients associated with element e.
Circled subscripts indicate particular coefficients.	C_{ij}	The coefficient of matrix C in row i and column j .
Standard matrix notation is used:		
Braces $\{\ \}$ mean a column matrix.	$\{\Gamma\}$	The column vector of Γ 's
Open brackets $\lfloor\ \rfloor$ mean a row matrix	$\lfloor\Psi\rfloor\{\phi\}$	The dot product of the Ψ row vector and ϕ column vector
Closed brackets $[\ ]$ mean a rectangular matrix (usually square).	$[K]^{-1}$	The inverse of the K matrix.
	$[T]^T$	The transpose of the T matrix.

Complementary Energy reanalysis basis.- Assume the element joint forces can be expressed as,

$$\left|S\right| = \left|S_o\right| + \sum_{b-1,2\ldots}^{B} \Psi_{(b)} \left|S_b\right| \tag{5-1}$$

where

S	is a vector of internal forces containing partitions of joint forces for every element of the structure,
S_o	is the vector of internal forces corresponding to a given external load, P_o , on the original configuration,
$\Psi_{(b)}$	is an, as yet, undefined scalar,
S_b	is a self-equilibrating synthesis candidate base vector of internal loads, and
B	is the number of synthesis candidate base vectors.

Regardless of the values chosen for the Ψ_b , S is an admissible set of internal forces. S satisfies the stress boundary conditions and macroscopic equilibrium equations at every joint.

Weighting the error in satisfying integral conditions of compatibility in the Complementary Energy sense, the Ψ_b can be evaluated by solving,

$$\left[C\right]\left|\Psi\right| = \left|\psi\right| \tag{5-2}$$

with

$$C_{(ji)} = \sum_{e=1,2\ldots}^{E} \left|S_{i(e)}\right|\left[A_{(e)}\right]\left|S_{j(e)}\right| ,$$

$$\psi_{(j)} = - \sum_{e=1,2\ldots}^{E} \left|S_{o(e)}\right|\left[A_{(e)}\right]\left|S_{j(e)}\right| ,$$

and $i = 1,2,\ldots B; \; j = 1,2,\ldots B$

where

C	is a symmetric square generalized flexibility matrix of order BxB with coefficients $C_{(ij)}$,
ψ	is a generalized displacement vector formed from known coefficients,
E	is the total number of elements in the structure,
$S_{j(e)}$	denotes the partition of the S_j vectors containing joint forces of element e and similarly $S_{o(e)}$, and
$A_{(e)}$	is the influence matrix for element e alone.

NOTE: Here, and in the sequel, subscripts in parentheses denote particular coefficients; without parentheses, they denote particular vectors. (Equation (5-2) is given in scalar form for a truss as Equation (2-12) in Reference 24.)

Now, suppose the solution of Equation (5-2) is expanded in eigenvalues and vectors of the C space. Then Ψ can be expressed by

$$\{\Psi\} = \sum_{r=1,2,\ldots}^{\bar{R}} \frac{\lfloor V_r \rfloor \{\psi\}}{\lambda_r} \{V_r\} \tag{5-3}$$

where

V_r	is the r^{th} orthonormal eigenvector of C and λ_r the associated eigenvalue, and
$\bar{R} \leq B$	by the number of eigenvectors associated with zero eigenvalues. $\bar{R}$ is the dimension of the reduced space.

(Zero eigenvalue vectors are discarded because they identify dependent force vectors.)

A rational basis for selecting R vectors from the reduced set of base vectors, $\bar{R}$, consists of electing those associated with the lower, non-zero, eigenvalues of the C matrix. The R base vectors will be called working vectors. In accordance with Equation (5-3), these can be expected to induce the larger contributions to Ψ , and hence, by Equation (5-1) provide more accurate response prediction per vector.

With this choice of working vectors, the accuracy of joint force predictions depends on the choice of candidate S_b's ; the value of R and $\bar{R}$ and the force redundancy of the system, Y . The relationship between these variables and solution accuracy is summarized in the table below. This summary reflects that exact solutions are nearly always possible but not always guaranteed.

TABLE XXXIII

EXACT SOLUTION POTENTIALITIES

Parameter Condition	Solution Accuracy
$R < Y$ (or $R < \bar{R}$)	Exact solution possible, if best S_b vectors selected and $B \geq 1$.
$R = Y$	Solution exact; guaranteed regardless of choice of S_b vectors
$R > Y$	Impossible condition. (Computer error)

Therefore, a key to efficiency using this analysis basis is the selection and generation of candidate S_bs. Consider an element of the structure. Then all distinct prestress states for this element are included in

$$\left\{F_{(e)}\right\} = \left[T_{(e)}\right]\left[k_{(e)}\right]\left[Q_{(e)}\right] \qquad (5\text{-}4)$$

where

$\left\{F_{(e)}\right\}$ is a set of "self-straining" joint forces for element e ,

$\left[k_{(e)}\right]$ is the stiffness matrix for element e in the local coordinate system,

$\left[T\right]$ transforms joint forces from the local to the global coordinate* system and

$\left[Q_{(e)}\right]$ is the "qualifying matrix" for element e .

Equation (5-4) develop self-equilibrating joint loadings as long as the element stiffness matrix implies that any rigid body motions of the element induces no

*In writing the load-deflection equations for the completed structure, displacements at each joint are referenced to particular coordinate area. The set of all these joint coordinate axes comprises the global coordinate system.

elastic work. This requirement is met by most stiffness matrixes given in the literature and all those included in the first release of NASTRAN (level 12).

The qualifying matrix is a rectangular Boolean operator. It elects independent self-equilibrating vectors from among those composing the stiffness matrix. Illustrations of this matrix for the line and a flat triangular shell element are given in Reference 5. For any given topology, the Q matrix is invariant with changes in assumed behavior states. It depends only on the generalized displacement coordinates used as long as only rigid body modes result in zero elastic work.

As a consequence of applying the $F_{(e)}$ loads to the structure, a set of internal joint forces, $S_{(e)}$, are generated. Then, subtracting the original $F_{(e)}$, a set of self-equilibrating loads implying zero external forces are developed. This defines a vector of Equation (5-1) by

$$\left\{S_{(e)}\right\} = \left\{\bar{S}_{(e)}\right\} - \left\{F_{(e)}\right\} \tag{5-5}$$

where $\bar{S}_{(e)}$ is the vector of internal joint forces induced by $F_{(e)}$ when treated as externally applied joint loads.

Because $F_{(e)}$ is a sparsely populated vector, it is convenient to calculate the $C_{(ji)}$ and $\psi_{(j)}$ using Equation (5-5) to describe $\left\{S_{(e)}\right\}$. Substituting it in Equation (5-2) gives,

$$C_{(ji)} = \sum_{e=1,2\ldots}^{E} \Big(\left\lfloor \bar{S}_{j(e)} \right\rfloor \left[A_{(e)}\right] \left\{\bar{S}_{i(e)}\right\} - \left\lfloor \bar{S}_{j(e)} \right\rfloor \left[A_{(e)}\right] \left\{F_{i(e)}\right\} - \left\lfloor F_{j(e)} \right\rfloor \left[A_{(e)}\right] \left\{\bar{S}_{i(e)}\right\} + \left\lfloor F_{j(e)} \right\rfloor \left[A_{(e)}\right] \left\{F_{i(e)}\right\} \Big) \tag{5-6}$$

$$\psi_{\text{(j)}} = - \sum_{e=1,2\ldots}^{E} \lfloor S_{o(e)} \rfloor \left[A_{(e)} \right] \left\{ \bar{S}_{j(e)} - F_{j(e)} \right\} \tag{5-7}$$

where $i = 1,2\ldots R; \; j = 1,2,\ldots R$

Note that the i and j limits in Equations (5-6) and (5-7) are R, reflecting limitation of base vectors to the working vectors.

To simplify Equation (5-6), observe that the first term on the right-hand side is the external work of the $\left\{ F_{j(e)} \right\}$ loads,

$$\sum_{e=1,2\ldots}^{E} \lfloor \bar{S}_{j(e)} \rfloor \left[A_{(e)} \right] \left\{ \bar{S}_{i(e)} \right\} = \lfloor F_j \rfloor \left\{ f_{i(e)} \right\} \tag{5-8}$$

where $\left\{ f_{i(e)} \right\}$ are the joint deformations induced by $F_{i(e)}$. Note also that the second term on the right of Equation (5-6) can be expressed as

$$\sum_{e=1,2\ldots}^{E} \lfloor \bar{S}_{j(e)} \rfloor \left[A_{(e)} \right] \left\{ F_{i(e)} \right\} =$$

$$\lfloor f_{j(e)} \rfloor \left[T_{(e)} \right] \left[k_{(e)} \right] \left[T^{T}_{(e)} \right] \left[Q_{(e)} \right] \left[Q^{T}_{(e)} k_{(e)} Q_{(e)} \right]^{-1} Q^{T}_{(e)} T k_{(e)} T_{(e)} \tag{5-9}$$

since

$$\left[A_e \right] = \left[Q_{(e)} \right] \left(\left[Q^{T}_{(e)} \right] \left[k_{(e)} \right] \left[Q_{(e)} \right] \right)^{-1} \left[Q_{(e)} \right]^{T} \tag{5-10}$$

$$\left\lfloor \bar{S}_{i(e)} \right\rfloor = \lfloor f_{i(e)} \rfloor \left(\left[T_{(e)} \right] \left[k_{(e)} \right] \left[T_{(e)} \right]^{T} \right) . \tag{5-11}$$

Equation (5-10) relates the element stiffness and flexibility matrices. Equation (5-11) relates joint forces and deformations.

Furthermore, the matrix of coefficients from the third term on the right of Equation (5-6) is the transpose of that of the second. Thus, the third term is easily formed by evaluation of Equation (5-9). The fourth term vanishes unless $i = j$.

Equations (5-6) through (5-11) provide for calculating $C_{(ji)}$ and $\psi_{(j)}$ with few calculations and data. Development of the (e) contribution of coefficients requires only the vector of displacements for a particular loading (such as $U_{j(e)}$ or $U_{o(e)}$) and element stiffness and qualifying matrices for all other elements selected as self-straining candidates.

The $\psi_{(j)} = 0$ for all j, for the original configuration. The calculation of ψ_j can be simplified when few elements are changed by referencing flexibilities to their original values. Thus,

$$\psi_{(j)} = - \sum_{e=1,2\ldots}^{E} \left|S_{o(e)}\right| \left[\Delta A_{(e)}\right] \left|\bar{S}_{j(e)} - F_{j(e)}\right| \tag{5-12}$$

with

$$\left[\Delta A_{(e)}\right] = \left[A_{(e)}\right] - \left[A_{o(e)}\right]$$

$\Delta A_{(e)}$ is the change in flexibility of element e and $A_{o(e)}$ the flexibility of element e in the original structure.

These equations lead to values of joint forces throughout the structure. If compatibility is satisfied, the corresponding displacements are found from the element load - deflection equations,

$$\left|\bar{S}_{i(e)}\right| = \left[T_{(e)}\right] k_{(e)} \left]\left[T_{(e)}\right]^T \left|f_{i(e)}\right| \tag{5-13}$$

If a sufficient number or more displacements are known for the element than there are rigid body modes in $k_{(e)}$, the equations in (5-13), (with row numbers corresponding to unknown displacement column numbers), are solved directly for the unknown displacements. If there are less displacements, two or more element load-deflection relations may need to be solved simultaneously to evaluate displacements.

This process of finding displacements does not lead to exact displacements when compatibility is violated. This violation can be detected by checking how well the calculated deflections satisfy the unused element load-deflection equations.

Complementary influence analysis basis.- The influence of design changes on behavior, in the subspace basis, depends on the change to ψ_j. Differentiating Equation (5-2) and evaluating the differential at the original design point,

$$\left\{\frac{d\Psi}{dX_{v}}\right\} = \left[C^{-1}\right]\left\{\frac{\partial \psi}{\partial X_{v}}\right\} \tag{5-14}$$

where X_{v} is the design variable being changed, and $\frac{\partial \psi}{\partial X_v}$ is the derivative of the design variable X_v and is the limiting value of x_j in Equation (5-12) as X_v approaches zero.

Equation (5-14) is the force method equivalent of the method of Fox[15] for evaluating response derivatives. Using the derivative, Equation (5-1) yields internal force changes and Equation (5-13) may evaluate displacement changes.

Potential energy reanalysis basis.- If the internal forces and the element load-deflection relations imply a violation of displacement continuity, determining a unique value of displacements requires additional assumptions. It is convenient, then to let

$$\{p\} = \{p_o\} + \sum_{p=1,2,\ldots}^{B} \phi_b \{p_b\} \tag{5-15}$$

where

p_o is a vector of displacements for the real loads, P_o ,

ϕ_b is an, as yet, undefined scalar, and

p_b is a base vector of displacements which imply satisfaction of continuity requirements for the structure.

Regardless of the values of $\phi_{(b)}$, p is an admissible displacement state in the Potential Energy sense.

Weighting the error in satisfying integrals of the equilibrium equations in the Potential Energy sense, the error is zero when the $\phi_{(b)}$ are found from

$$\left[K\right]\left\{\phi\right\} = \left\{\phi\right\} \tag{5-16}$$

where

$$K_{(ji)} = \sum_{e=1,2...}^{E} \lfloor p_j \rfloor \left[K_{(e)}\right] \left\{p_i\right\} \tag{5-17}$$

$$\phi_{(j)} = \lfloor p_j \rfloor \left\{ \left\{\phi_o\right\} - \left[K\right]\left\{p_o\right\} \right\} \tag{5-18}$$

and $i = 1,2,\ldots B$; $j = 1,2,\ldots B$ with $K_{(e)}$ the element stiffness matrix and K the total generalized stiffness matrix.

It is noted that if no element stiffnesses change, $\phi_{(b)} = 0$ and $\phi_{(b)} = 0$ for all b.

Selection of the best subset of a given set of B base vectors as the R working vectors could again be made from a spectral analysis of the characteristic matrix. In this case, however, vectors associated with low eigenvalues are the more representative. Zero eigenvalue vectors cannot be discarded.

Equations (5-15) through (5-18) yield values of joint displacements throughout the structure. If equilibrium is satisfied exactly, element forces are given from the load-deflection relations,

$$\left\{S_{(e)}\right\} = \left[K_{(e)}\right]\left\{p_{(e)}\right\} \tag{5-19}$$

For most structures, however, use of Equation (5-19) will only provide approximations to the true forces because K_e is only approximate.

Potential influence analysis basis.- The influence of design changes on behavior, in the subspace basis, depend on the change to ϕ_b. Differentiating Equation (5-16) and evaluating differentials at the original design point, related to p_o and P_o,

$$\left\{\frac{d\phi}{dX_v}\right\} = \left[-K^{-1}\right]\left[p_j\right]\left[dK\right]\left\{p_o\right\} \tag{5-20}$$

where dK designates the change in K induced by a change of the design variable, X_v.

Solution in the total space.- With the Ψ or ψ known, there are two ways to evaluate the displacements associated with the solution. In the first way, an "effective" loading is developed which, if imposed on the baseline configuration, will produce the solution displacements. In the second way, the response is found by superimposing subspace vectors according to Equations (5-1) and (5-15).

In both cases, the solution in the subspace basis must be transformed to one in the joint coordinate basis. The implication of Equation (5-3) is that external forces in the subspace are related to element joint forces by

$$\left\{P\right\} = \left[F_b\right]\left[V_r\right]\left\{\Gamma\right\} \quad r = 1,2,\ldots R; \; b = 1,2,\ldots B \tag{5-21}$$

where P is the r^{th} vector of external joint loads and V_r the matrix of eigenvectors, Γ_r is ψ_r for the Complementary approach and ϕ_r for the Potential. The corresponding self-equilibrating internal forces are given by

$$\left|S\right| = \left|\bar{S}_b - F_b\right|\left|V_r\right|\left|\Gamma\right| \quad r = 1,2\ldots R;\ b = 1,2\ldots B \tag{5-22}$$

The associated transformed displacements are

$$\left|p_r\right| = \left|f_b\right|\left|V_r\right| r = 1,2,\ldots R;\ b = 1,2\ldots B \tag{5-23}$$

where p_r is the r^{th} vector of joint displacements and f_b is the b^{th} candidate synthesis vector. This synthesis vector is associated with the F_b load vector.

In the first way, the P_r vectors of Equation (5-21) are used to form effective loadings. An effective loading is then given by

$$P_f = P_o + \sum_{r=1,2\ldots}^{R} \Gamma_r P_r \tag{5-24}$$

where P_f is the effective loading on the baseline structure.

Applying the effective loading to the baseline structure produces the displacements p_f and, by Equation (5-19), a set of internal forces. For the Complementary approach these forces must be corrected by the self-equilibrating forces; ie,

$$\left|S\right| = \left|S_f\right| - \sum_{r=1,2\ldots}^{R} \Psi_{r} \left|P_r\right| \tag{5-25}$$

where S_f are the internal forces associated with p_f.

In the second way, the Complementary Energy internal force solution is given by

$$\left|S\right| = \left|S_o\right| + \sum_{r=1,2\ldots}^{R} \Psi_{r} \left|S_r\right| \tag{5-26}$$

Equation (5-26) in the subspace representation of Equation (5-1). The subspace model of Equation (5-15) for the Potential Energy solution is

$$\left|p\right| = \left|p_o\right| + \sum_{r=1,2...}^{R} \phi_{r} \left|p_r\right| \tag{5-27}$$

Steps of Design-Analysis

As shown in Figure 4, Design-Analysis includes three tasks:

1. Selection of the analysis base vectors

2. Development of influence data defining response changes as a function of changes to the design variables;

3. Reanalysis of the configuration after redesign.

Calculations supporting these tasks are grouped into those for selecting the subspace basis, developing the subspace generalized flexibility and stiffness matrices, analyzing the subspace model, and evaluating finite element stresses and deformations. The equations in these calculations are Equations (5-1) through (5-19). Additional equations are cited in detailing the steps of each calculation in the paragraphs that follow.

Selection of subspace analysis basis.- Figure 5 is a chart of the sequence of evaluations in selecting the base vectors for Design-Analysis. Details of the steps are as follows:

1. The decision to use or not use subspacing may be made by the user or, through default, by computer logic. When based on computer logic, the decision will be predicated on a comparison of the amount of data processing with and without subspacing. Subspacing is usually advantageous whenever the number of

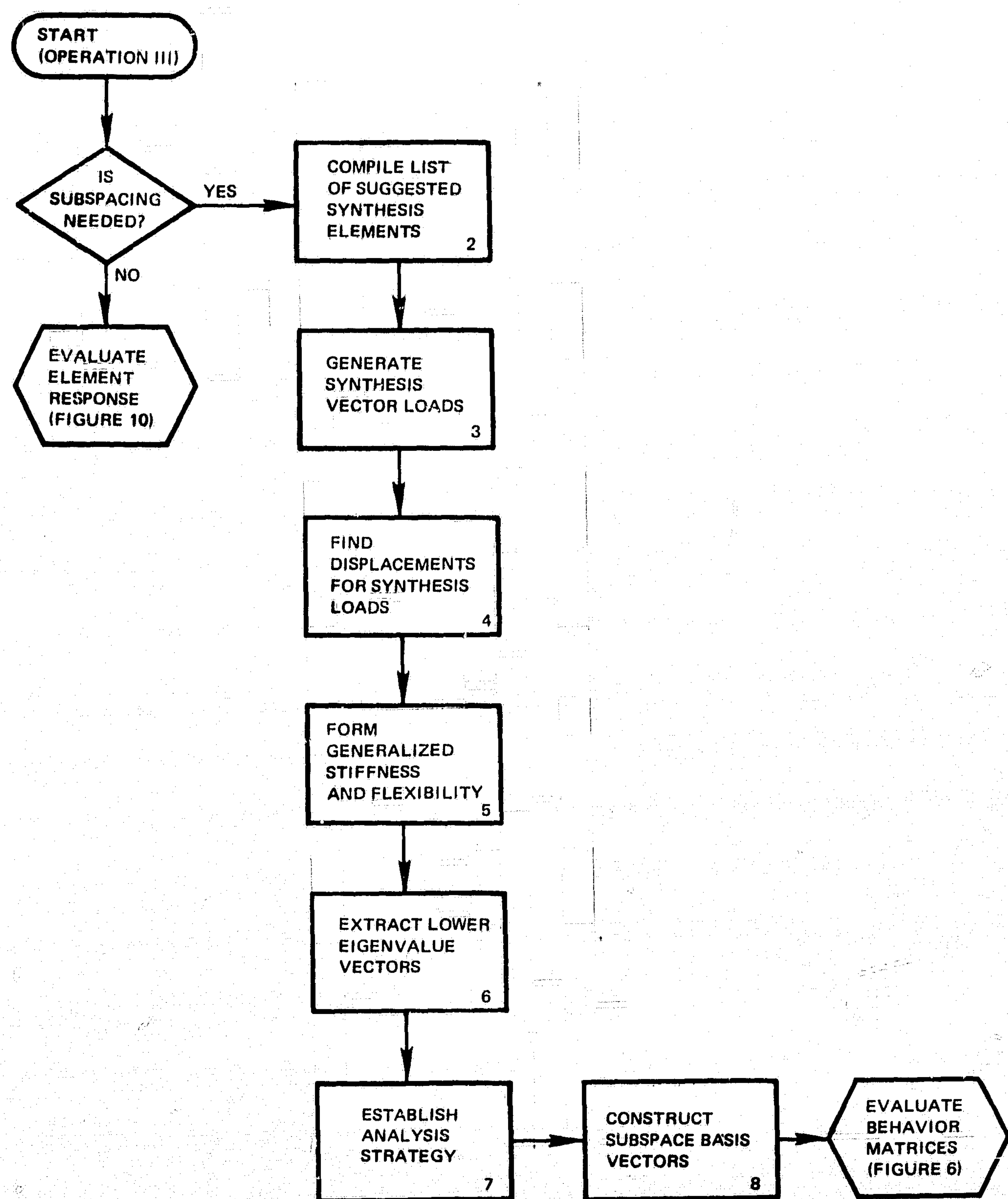


Figure 5. Steps in Developing the Subspace Basis

degrees-of-freedom in the subspace is significantly less than in the original analysis.

2. A list of synthesis structural elements is read or automatically generated. This list identifies the subset of elements and the associated B self-equilibrating loadings that will be used as synthesizing subspace base vectors.

   When the user selects the elements, he has the opportunity to improve the subspace basis using his knowledge of the structural behavior under the loads of interest. This knowledge will lead to improving Design-Analysis efficiency by identifying the more important element response modes, by suggesting the region where largest design changes are expected, and by limiting the number of working vectors, R, used. Near discontinuities in pressurized shells, both element bending and membrane modes must be included. Away from discontinuities only membrane modes are important. If only elements near the discontinuities are to be designed (all other components are to remain unchanged during Redesign), only vectors associated with these elements are needed for the subspace basis. If the structure is determinate, no self-equilibrating vectors are required.

   The computer's selection of synthesis elements will tend to be poorer than that of the engineer. Without knowledge of the total problem, it will choose more or less uniformly distributed elements and use the lower modes of element responses.

3. Values for the candidate synthesis element self-equilibrating loads are calculated in accordance with Equation (5-4). The qualifying matrix is modified, in accordance with the user data accepted in Step 2.

For efficiency, the total number of candidate synthesis vectors should be limited so the associated generalized influence matrix is contained in high-speed storage (core). This matrix will be symmetric and fully-populated. Therefore, the number of synthesis base vectors, $\bar{R}$, should be

$$\bar{R} \leq -1 + \sqrt{.5 + 2W_A} \approx \sqrt{2W_A} \qquad (5\text{-}28)$$

where W_A is the number of core storage words allocated for the matrix. For a 32k core, $\bar{R}$ will have a value of about 200 assuming 20,000 cells are allocated to the matrix.

The synthesis loading vectors have zero coefficients except in degrees of freedom corresponding to the loaded element joints. Thus, the non-zero coefficients for the total set of B vectors can easily be accommodated in core. For example, with 200 vectors for a three dimensional solid model by the simple eight-jointed prism element[68], a maximum of 9,600 locations would be needed, regardless of how many degrees of freedom existed in the total structure problem definition (Baseline Analysis).

4. The equation solver of the parent program is used in finding deflections caused by the synthesis loads. Because deflections of the original configuration are needed, efficiency of this calculation is enhanced by saving the decomposition developed when the response of the system to the real loads has been found. This decomposition then is reused in finding synthesis element load responses.

For efficiency, the column vectors obtained by this calculation should be row-listed. Because the stiffness matrix usually is sparse, the forward and back substitution can be performed by exploiting the bandedness. Using the wavefront concept,[31]* only as many rows

*When the i^{th} diagonal of the decomposition matrix is evaluated, the joints in the wavefront are those represented by non-zero coefficients in row i .

need be in core at any time as are contained in the wavefront for the row being treated. With this consideration, many column vectors can be developed in a single forward and backward pass of the decomposed stiffness matrix.

5. The generalized stiffness matrix contains the coefficients defined by Equation (5-17). The generalized flexibility coefficients are given by Equation (5-6) and evaluated by subtracting twice the values given by Equation (5-9) and adding the last term of Equation (5-6) to the terms of Equation (5-8).

It is convenient to develop the generalized stiffness coefficients during the forward substitution of the synthesis vectors. The load-deflection equations of the baseline structure take the form

$$\left[K_G\right]\left\{p\right\} = \left\{P\right\} \tag{5-29}$$

where

K_G is the stiffness matrix in global coordinates,

p is the vector of joint displacements, and

P is the vector of external loads.

The relation between the stiffness and its decomposition is

$$\left[K_G\right] = \left[L\right]\left[D\right]\left[L\right]^T \tag{5-30}$$

where

$\left[L\right]$ is a lower triangular matrix with those on the diagonal,
$\left[D\right]$ is a diagonal matrix, and
$\left[L\right]^T$ is the transpose of L .

Substituting Equation (5-30) in (5-29) and performing forward substitution gives

$$\{y\} = [L]^{-1}\{P\} = [D][L]^{T}\{p\} \tag{5-31}$$

where y is the vector resulting from forward substitution. But, the strain energy can be expressed by

$$U = \frac{1}{2}\{p\}[L][D][L]^{T}\{p\} = \frac{1}{2}\{y\}[D]^{-1}\{y\} \tag{5-32}$$

where $[D]^{-1}$ is the inverse of the D matrix. Then, using Equation (5-32), the stiffness coefficients, as given by Equation (5-17), can be calculated conveniently during forward substitution without requiring an additional pass of the solueion vectors. At the completion of forward substitution, the generalized stiffness matrix is saved for use in Step 7.

The terms to be added to form the generalized flexibility matrix are formed and added during the backward substitution. This requires accessibility to the F_i. Note that the flexibility and stiffness matrices are not inverses of each other.

6. The lower eigenvalue vectors of the generalized flexibility matrix can be extracted by the process available in the parent program. Zero eigenvalues can be discarded since they are associated with null vectors. This is true because vectors are chosen from the complementary viewpoint.

7. The subspace analysis strategy, if not specified by the user, can be determined from spectral analyses of the generalized flexibility and stiffness matrices. The factors defining the strategy are the number of non-zero eigenvalues in the flexibility and stiffness matrices on the subspace basis, compared with R, the dimension of the working subspace.

Let λ_K and λ_C be the number of non-zero eigenvalues of the generalized stiffness and generalized flexibility matrices respectively. Then, the distinct strategies are selected as follows:

a. Evaluate both stresses and displacements from the Potential Energy solution as long as $\lambda_K \leq R$.

b. Use the Complementary Energy solution for internal forces and the Potential Energy for displacements when $\lambda_K > R$ and $\lambda_C > R$.

c. Use the Complementary Energy solution for both internal force and joint displacement predictions when $\lambda_K > R$ and $\lambda_C \leq R$.

Selection of the strategy, then requires an eigenvalue analysis of the generalized stiffness matrix in addition to that performed for the generalized flexibility in Step 6. In this case, however, the eigenvectors are of no interest.

For most multidegree-of-freedom systems (>1000), the dual approach, b , above, will occur because $R < Y$.

8. The working displacement vectors, p_r , can be constructed by either using an effective loading or direct superposition. In the first way, Equations (5-21) are evaluated. The resulting loads then are used with the decomposed stiffness matrix of the baseline configuration to obtain required subspace vectors. In the second way, the subspace vectors are constructed by Equation (5-23) and Equation (5-27). These displacement vectors will be used for either the Complementary or Potential Energy Analysis.

The selection of the way to construct working subspace vectors is determined automatically in order to minimize data processing. The relative efficiencies depend on data parameters. Data volumes are summarized in Table XXXIV. The multiplication method generally involves

TABLE XXXIV

DATA FOR SUBSPACE BASIS FORMATION[a]

MULTIPLICATION METHOD

Item	Volume	No. of Transfers	Total Volume
F_b , b=1, 2...B	$N \cdot B$	$(w-B \cdot R)/N$	$(W_D \cdot B-B^2R)$[b]
V_r , r=1, 2...R	$B \cdot R$	1	$B \cdot R$
S_r , r=1, 2...R	$N \cdot R$	1	$N \cdot R$

EFFECTIVE LOADS METHOD

Item	Volume	No. of Transfers	Total Volume
F_b , b=1, 2...B	$40 \cdot B$[c]	1	40B
V_r , r=1, 2...R	$B \cdot R$	1	$B \cdot R$
K matrix	$N \cdot w$	$2(W_D-40B-B \cdot R)/w$	$2N(W_D-40B-B \cdot R)$[d]
forward solution	$N \cdot R$	2	$2N \cdot R$
P_r , r=1, 2...N	$N \cdot R$	1	$N \cdot R$

[a] W_D = number of words of core allocated to data; w = wavefront of stiffness matrix; N = the number of load-deflection equations in the joint coordinate system.

[b] Assuming the V_r vectors are retained in core during the multiplication.

[c] Including coefficient codes and assuming four joint-shell finite elements.

[d] Assuming $F_{i(e)}$ and V_r are retained in core during substitutions.

full matrices while the effective-loads method involves sparse matrices. Comparison of the formulas leads to the conclusion that the multiplication method is preferred when

$$W_D \cdot B - B^2 R < 2N(W_D - 40B - B \cdot R) + 40 \cdot B \qquad (5\text{-}33)$$

where

W_D is the number of words of core allocated to data and

N is the number of degrees of freedom in the joint coordinate basis.

Evaluation of subspace behavior matrices.- Whether Influence, Reanalysis or both are to be performed, coefficients representing the system in the working basis must be developed when the subspacing basis is elected. Since both the flexibility and stiffness matrices are of small order, R, they can be developed conveniently and concurrently as shown in Figure 6. The calculation involves sequentially developing contributions to the behavior matrix coefficients for each of the finite elements in turn. Details of the steps are as follows:

1. Initialization consists of acquiring previously determined data needed for calculation. This includes the F_b and V_r sets of vectors, the p_r vectors, and the data defining the finite elements of the analysis. F_b and V_r can be moved into core. The p_r vectors are available in row listed form in auxiliary storage. Element data may be brought into core or read one element at a time.

2. If all necessary data is in core, including the p_r rows, the contributions to the new flexibility and stiffness matrices are calculated in accordance with Equation (5-6) and (5-17) or Equations (5-6), (5-8), (5-9) and (5-17). These coefficients are accumulated in two upper triangular matrices - one for the flexibility and one for the stiffness.

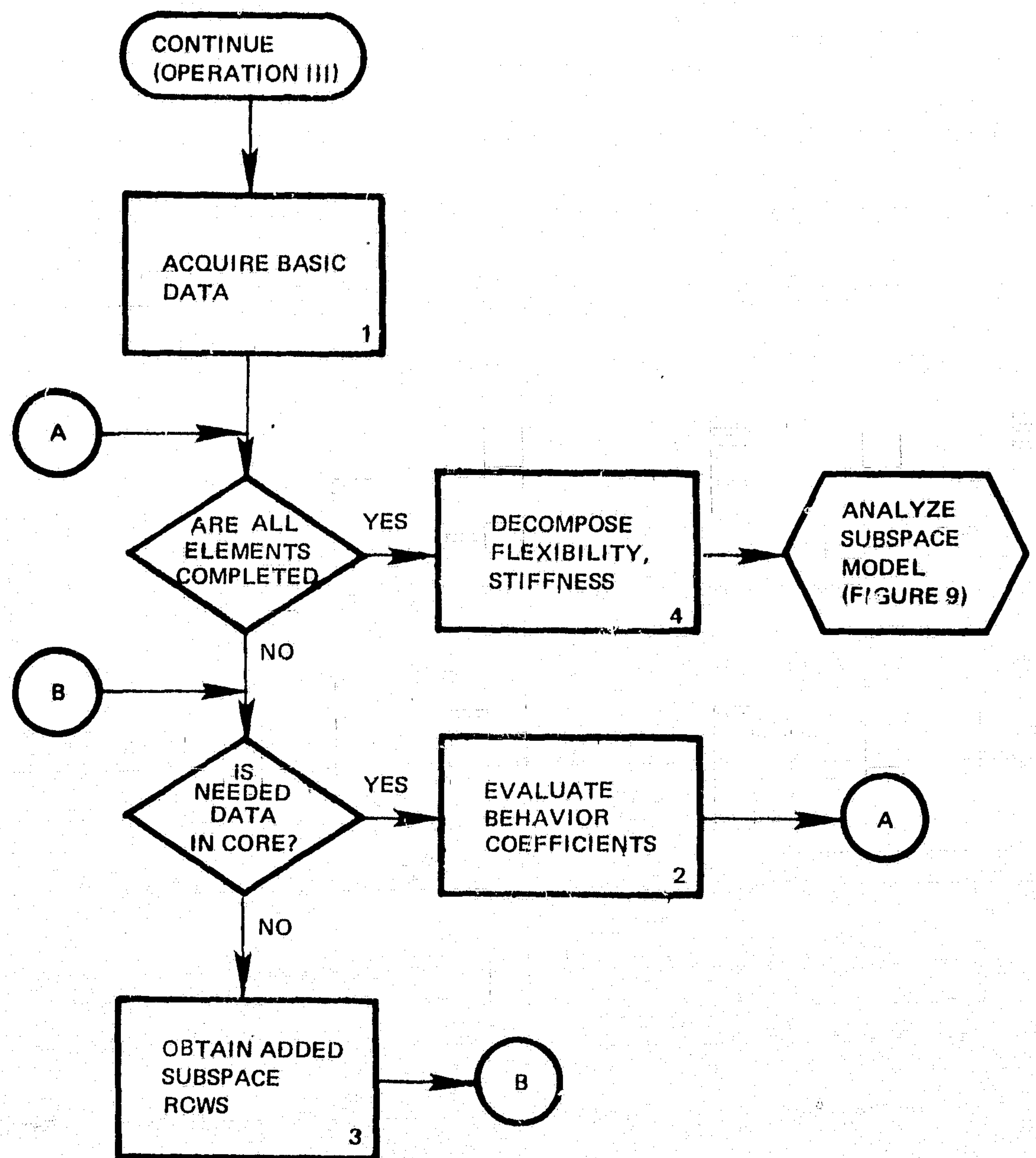


Figure 6. Evaluation of Behavior Matrices

3. If the core-contained rows of the working basis vectors, p_r, do not include enough joints so the next element contribution to the stiffness and flexibility matrices can be calculated, additional data must be acquired. The logic in this step must provide for reading in additional rows if space is available, save rows to be written over if they will be referenced later, and calling previously spilled rows when necessary.

   This type of data management strives for efficiency by exploiting the relation between the element numbering sequence and the joint numbering order. Suppose joints are numbered to minimize the wavefront of the stiffness matrix. Then there exists an element numbering which will result in requiring a minimum number of (sequential) joint displacements to be stored as the energy of each element is calculated in turn. This number will be called the element wavefront number.

   To fix ideas, consider the membrane shown in Figure 7. Joints are numbered across the short dimension (topologically). Assuming only one equation per joint, this numbering results in a stiffness matrix with a maximum wavefront of seven. If elements are numbered as indicated, the maximum element wavefront is also seven. When the energy of element 2 is calculated, for example, displacements for joint 3, 5, 7, 9, 11, 13, and 15 are contained in the element wavefront. All joint displacements are included in the element wavefront which exists between the lowest joint number for the element and the highest. If joints are carefully numbered to minimize joint wavefront, element wavefront will be equal or greater than joint. Figure 8 shows a simple truss illustrating this conclusion.

4. Decomposition of the flexibility and stiffness matrices, can be taken to be of the form of Equation (5-30). Usually, the subspace matrices will be fully populated so use of a subroutine which exploits sparseness is a strategy of dubious merit.

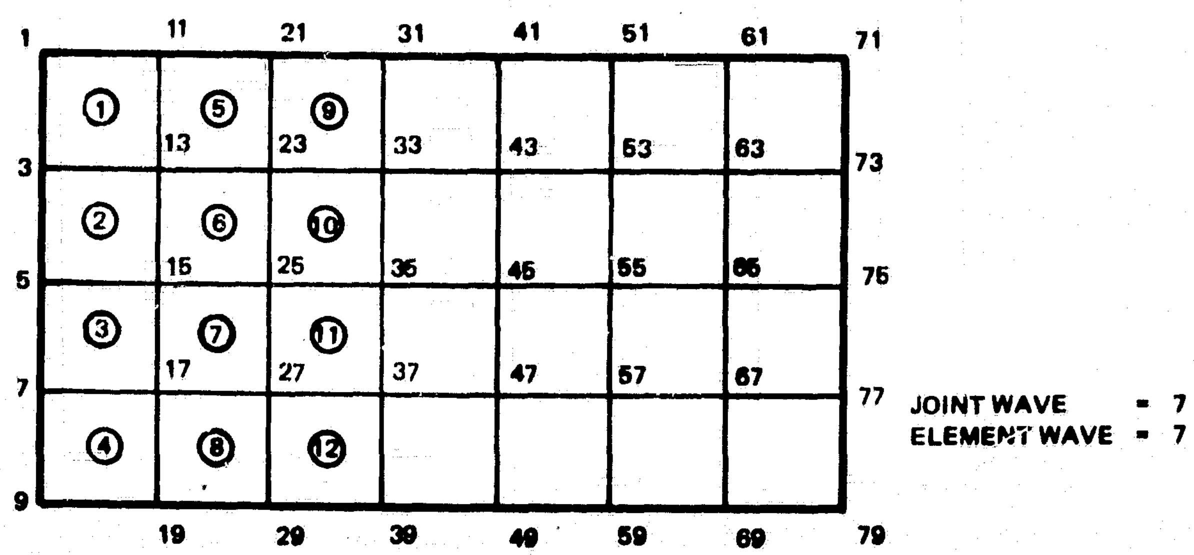


Figure 7. Simple Membrane Problem

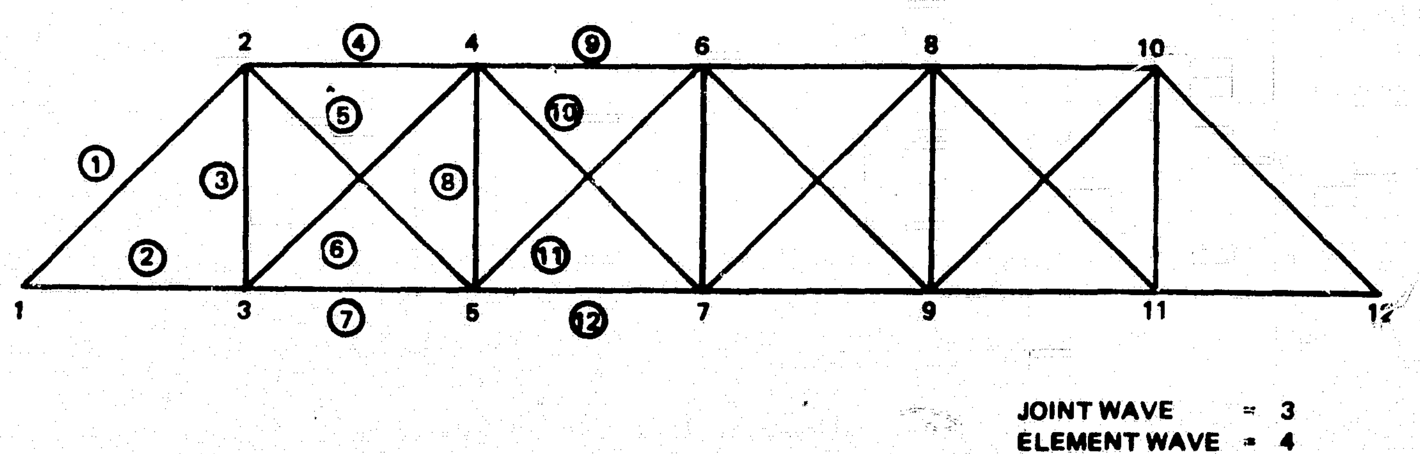


Figure 8. Simple Truss Problem

Analysis of subspace model.- For both Influence and Reanalysis the subspace mathematical model must be manipulated to obtain predictions of structural behavior. These manipulations include developing loading vectors in subspace coordinates and solving the load-deflection equations.

Figure 9 is a chart showing the relation of logic and tasks in the subspace analysis. The modular basis is again the finite element. Each of these is treated successively in the logic loop for a given loading. The analysis is completed when all loadings have been treated.

The logic presented in Figure 9 provides for simultaneously generating subspace equation coefficients and transforming solutions to the joint coordinate basis. Consider a number of sets of loading vectors. For the first set, the transformation coefficients are passed through to develop the coefficients of ψ and ϕ. The last pass through the logic, the transformation coefficients are used to evaluate the joint responses from the coefficients of Ψ and ϕ. In all other passes, joint responses for the previous pass and ψ and ϕ coefficients for the current pass are evaluated.

Details of the analysis steps are as follows:

1. Basic data needed for the analysis consists of the V_r, F_b, P_o, p_o and p_r vectors and the element data. The V_r and F_b data are of small volume and can be retained in core. The P and p vectors can be read in row-listed form in two separate arrays. The element data can be read one element at a time, as required. Rows of the row listed p_r vectors can be read as required for element analysis, exploiting the element wavefront concept, as described in Step 3 of the Evaluation of Subspace Behavior matrices.

2. If insufficient subspace rows are available for a given element, additional rows are read. This step requires logic to provide for spill and recovery of subspace rows of p_r when the element wavefront is exceeded as well as to treat the usual case where only the next set of rows for a joint are needed.

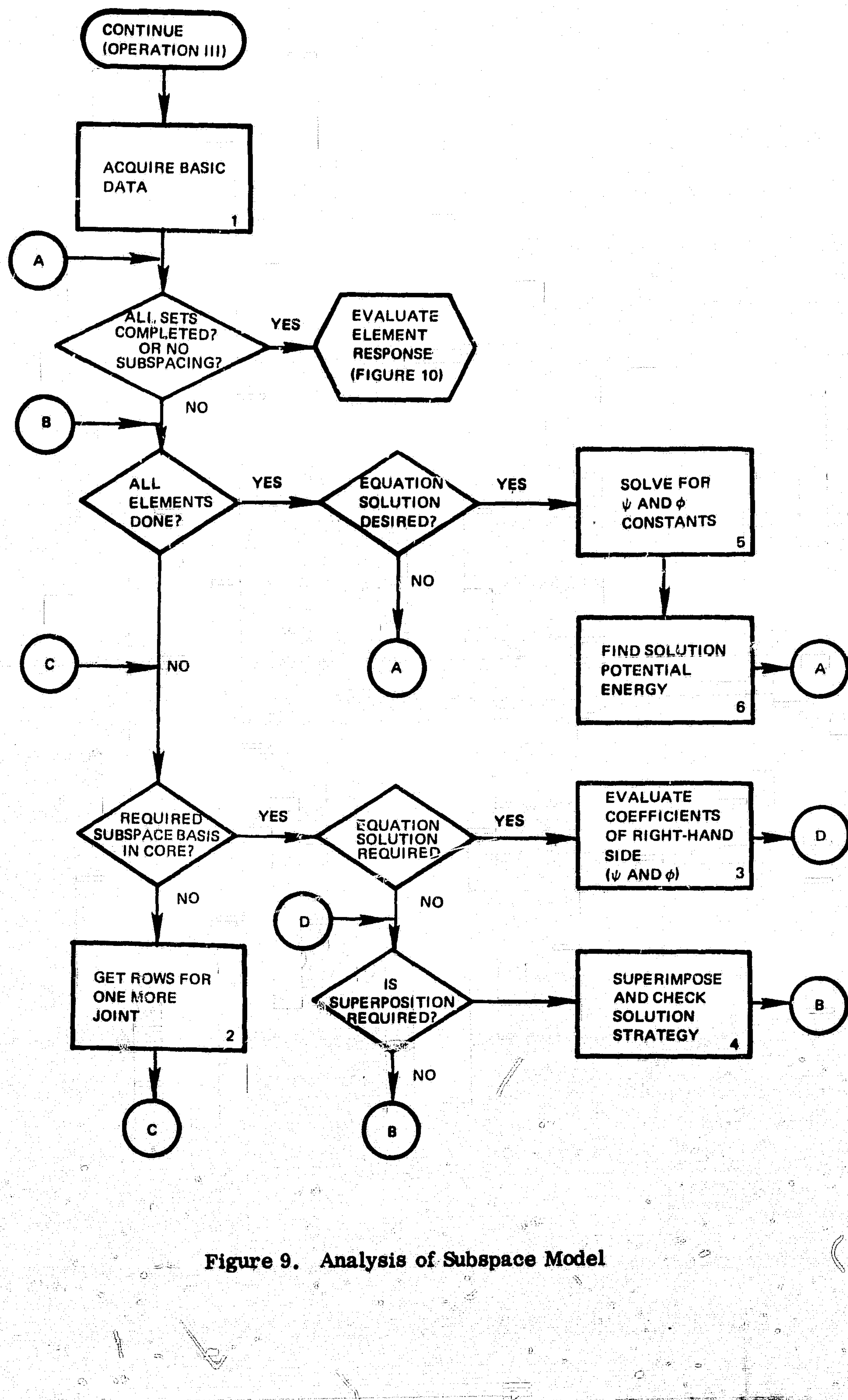


Figure 9. Analysis of Subspace Model

3. Evaluation of the coefficients for the right-hand side of the load deflection equations, Equations (5-2) and (5-16), is performed in accordance with Equations (5-12) and/or (5-18) or Equation (5-14) and/or (5-20) depending on the analysis objective and strategy. The first pair of equations is used if the objective is Reanalysis and the second pair for Influence Analysis. If a pure strategy for a Complementary Energy formulation is involved, Equation (5-12) or (5-14) only are required. If the pure Potential Energy strategy is chosen, only Equations (5-18) or (5-20) are evoked. With the mixed strategy all four equation sets are implemented.

In Reanalysis, all loading cases are treated together. Thus, there are L columns of right-hand side vectors developed where L is the number of real loads. A single pass of the element data and the p_r vectors are sufficient to obtain all needed coefficients.

In Influence Analysis, all the loading cases also are treated together. The number of design variable changes in a trip through the logic is fixed by the available storage space and the number of changes still not evaluated.

Equations (5-14) and (5-20) are adequate for determining the influence of any of the design variables of interest. The design variables of the criterion problem fall into two classes: Those associated with element changes and those associated with joint changes. For Influence Analysis purposes, however, both classes of variables can be encompassed by a capability which predicts the effect on responses of changes to groups of elements. The element size change then is a special case where only one element is in the group. For a material change all elements which must have a common material comprise the group. For a change of joint location, all elements connected to the joint lie in the group. The joint restraint variables can be treated as springs attached to the joints. Assignment of sizes for these spring elements then determines the optimum joint displacement boundary conditions.

4. If required, responses referenced to joint variables, joint internal forces, and displacement components, can be determined concurrently with development of the ψ and ϕ coefficients. If many design variables or real loadings are involved, this option can reduce the number of times the p_r vectors will be transferred by nearly a factor of two.

The joint referenced responses are developed by evaluating Equation (5-26) and/or (5-27) depending on the analysis strategy. This evaluation requires knowing the Ψ_r and ϕ_r from a previous pass of the subspace analysis logic of Figure 9. Since both formation of the $\psi_{(r)}$ and $\phi_{(r)}$ constants and the superposition can be performed using the row listed form of the p_r, they can be efficiently performed together as previously noted.

It is also advisable to check the validity of the analysis strategy concurrently with superposition. Definition of the analysis strategy as described for Step 7 of Figure 5 will be correct for almost all structures. However, it is possible for the analyst to select synthesis vectors for which Step 7 yields an incorrect strategy.

A necessary and sufficient check of the validity of the strategy of pure Potential Energy Analysis is to ensure that the sum of the internal loads at each joint be in equilibrium with the applied loads at the joint. Calculations for this type of check would require $N \cdot L$ storage locations - as many as required for the solutions. This would mean the check would involve as much data management as the superposition. Since the probability that the strategy selected is invalid is low, this costly a check is incommensurate with the objective of analysis efficiency. An alternate check is advisable.

The alternate necessary check of the validity of the strategy selected is to ensure that the external work represented in the subspace analysis match the external work of the corresponding real structure response. Thus, the real structure work is calculated during the superposition. This is conveniently developed by accumulating the energy contribution of each element. Each element stiffness matrix is multiplied by the superimposed displacements to produce joint forces, in accordance with Equation (5-19). These forces in turn multiply the joint displacements to calculate the element contribution to the external work; i.e.,

$$W_{\text{(j)}(e)} = \lfloor S_{j(e)} \rfloor \{p_j\} \tag{5-34}$$

where

$W_{\text{(j)}(e)}$ is the contribution to the external work due to the joint forces of element e for loading j; and

p_j are the displacements of the real structure under loading j

The total external work is given by

$$W_{\text{(j)}} = \sum_{e=1,2\ldots}^{E} W_{j(e)} \tag{5-35}$$

with $W_{\text{(j)}}$ the total work for loading j .

A necessary and sufficient check of the validity of a Complementary Energy strategy is that the sum of the elongations, from any joint to any other, is invariant with the joint path taken. Again, implementation of this check results in checking costs that are incommensurate with the efficiency objective.

The alternate necessary check selected is to ensure that no joint incompatibilities are revealed for a sample of the solutions associated with the real loads. Since this calculation is most efficient if all displacement components for a particular vector are in core, the size of the sample is based on the number of degrees of freedom, N , and the available storage space.

The check can be performed by a numerical model of the Williot-Mohr diagram.[69] Element end loads are calculated by Equation (5-26). Relative element displacements are found by an equation of the form of Equation (5-13). In piecing together the deformed structure, rigid body motions of the elements are admissible. If these motions and rigid body motions of the total structure are not sufficient to obtain a unique evaluation of the location of every joint, the Complementary Energy cannot be used for predicting deflections.

5. Solving the subspace generalized load-deflection equations for the Ψ and ϕ (Equations (5-8) and (5-10)), is achieved as it is in Baseline Analysis. The decomposition, of the form of Equation (5-30) is used in the forward substitution to produce the y vectors of Equation (5-31). Then, with p playing the role of Ψ or ϕ, the unknowns are found by diagonal division and back substitution; i.e.,

$$\{p\} = [L]^{T-1}[D]^{-1}\{y\} \qquad (5\text{-}36)$$

where the -1 power denotes matrix inversion. In practice, of course, L^T is not inverted explicitly - rather the set of Equations (5-31) are solved simultaneously for p.

If, during Redesign, the size of some finite elements are set to zero or a relative infinite number, special difficulties may be encountered in solving the load-deflection equations. The determinate of the behavior matrix may become zero or infinite.

Table XXXV summarizes the effect of setting sizes to the extremes for the Complementary and Potential Analysis. The infinity of the Complementary Energy matrix is avoided by neglecting energy calculations for elements with zero size. Then, for both zero and infinite size, the flexibility matrix may be singular. The infinity of the Potential Energy Analysis will manifest itself on the computer as a matrix singularity because the computer number set is not closed.

Because all other stiffnesses are zero compared with those of the infinite size element, and the stiffness matrix of any isolated element is singular because rigid body modes are included, the singularity is intrinsic.

TABLE XXXV

EXTREME SIZE EFFECTS ON BEHAVIOR MATRIX

Energy Method	Element Size	Matrix Determinate	Analysis Action
Complementary	Zero	Infinity	Test kinematic stability
	Infinity	Zero	Test kinematic stability
Potential	Zero	Zero	Test kinematic stability
	Infinity	Zero	Correct for infinite stiffness

It is necessary to distinguish the last case in Table XXXV from the rest to take appropriate action when this singularity arises. Furthermore, a problem-dependent numerical singularity can also be induced by loss of accuracy in the decomposition.[35] Action for this singularity involves analysis abortion so it must be distinguished from the kinematic and infinite stiffness types of singularity indicated in Table XXXV.

In both Complementary and Potential Energy cases, the singularity is assumed to be of the extreme size type (zero or infinity) if it is not of the numerical singularity type. Thus, when the singularity is encountered (during diagonal division), the numerical singularity test is applied. The matrix is numerically non-singular as long as $e_s < 1$ where

$$e_s = b^{1-\phi}/\text{min.}\,(d_{jj}/k_{jj}) \quad j = 1,2,\ldots N \tag{5-37}$$

when

e_s is the relative singularity error,

b is the computer number base (usually 2.0),

p is the number of places (bits) in the mantissa of the floating point number,

d_{jj} is the j^{th} diagonal of the D matrix,

N is the total number of degrees of freedom,

k_{jj} is the j^{th} diagonal of the generalized behavior matrix (either flexibility or stiffness), and

min (...) denotes the minimum of all included candidates.

For the Complementary Energy Analysis, if singularity is not due to manipulation error, the kinematic stability test is applied.[5] This test determines if for every zero d_{jj} there is a corresponding zero for y_j for every loading. If so, analysis proceeds. If not, the structure is actively kinematically unstable and the analysis is aborted.

In the Potential Energy Analysis, the same kinematic stability test is applied if the infinite stiffness case is not involved. Relative infinite stiffnesses are identified by examining diagonals of the D matrix. Infinite stiffness exist if

$$d_{jj}/d_{ii} < b^{-p} \quad j > i , \quad j \neq i \tag{5-38}$$

where j and i are dummy indexes which may have values of 1, 2, 3...N . If infinite stiffnesses exist, they can be identified and their values reduced so the analysis can proceed with the available computer precision. Note that the problem of relatively infinite stiffness is not peculiar to subspace analysis. The device proposed here to eliminate the problem in Design-Analysis could be used in Baseline Analysis.

6. The Potential Energy of the solution is evaluated directly from the subspace Potential Energy. Thus, the strain energy can be expressed as,

$$U_{(j)\,P.E.} = \left|\phi_j\right|^T \left|\phi_j\right| \qquad (5\text{-}39)$$

where

$U_{P.E.}$ is the strain energy of the Potential Energy solution.

This value is used in checking analysis strategy by comparing it with the results of Equation (5-35). If $U_{j.P.E.} = W_j$, then the Potential Energy solution is exact. Note that $\left|U_{P.E.}\right| \neq \left|U_{C.E.}\right|$ where $U_{C.E.}$ is the Complementary Energy strain energy, for most structures.

Evaluating element response.- To obtain data for Redesign, stresses in the finite elements and joint deflections must be calculated. These calculations include evaluating element stress, stiffness, and deformation integrity. In addition, these calculations include checks of system deflection integrity.

Figure 10 is a chart showing the logic relation between tasks in evaluating response. The modular basis is again the finite element. Each of these is treated successively for a set of subspace solutions. A set consists of a manageable collection of loading case calculations. Subroutines implementing these calculations interface directly with Redesign routines (indicated by hexagonal boxes) in compiling response data. The evaluation is completed when all loads, real and influence, have been treated. The way in which the last three tasks are performed depends on the Redesign approach selected. These tasks also include saving data for printing and graphical display.

Details of the evaluation steps are as follows:

1. Basic data to be acquired depends on what has been produced by the subspace analysis steps. There are two possibilities. Subspace analysis may produce deflections, which have been transformed to the joint coordinate system, for all loadings - real and influence. This occurs when the number of loadings is less than the number of design variables. Alternately, the subspace analysis produces only the Ψ and ϕ coefficients for all the loadings. In the first option, the solution vectors must be accessed. In the second, the

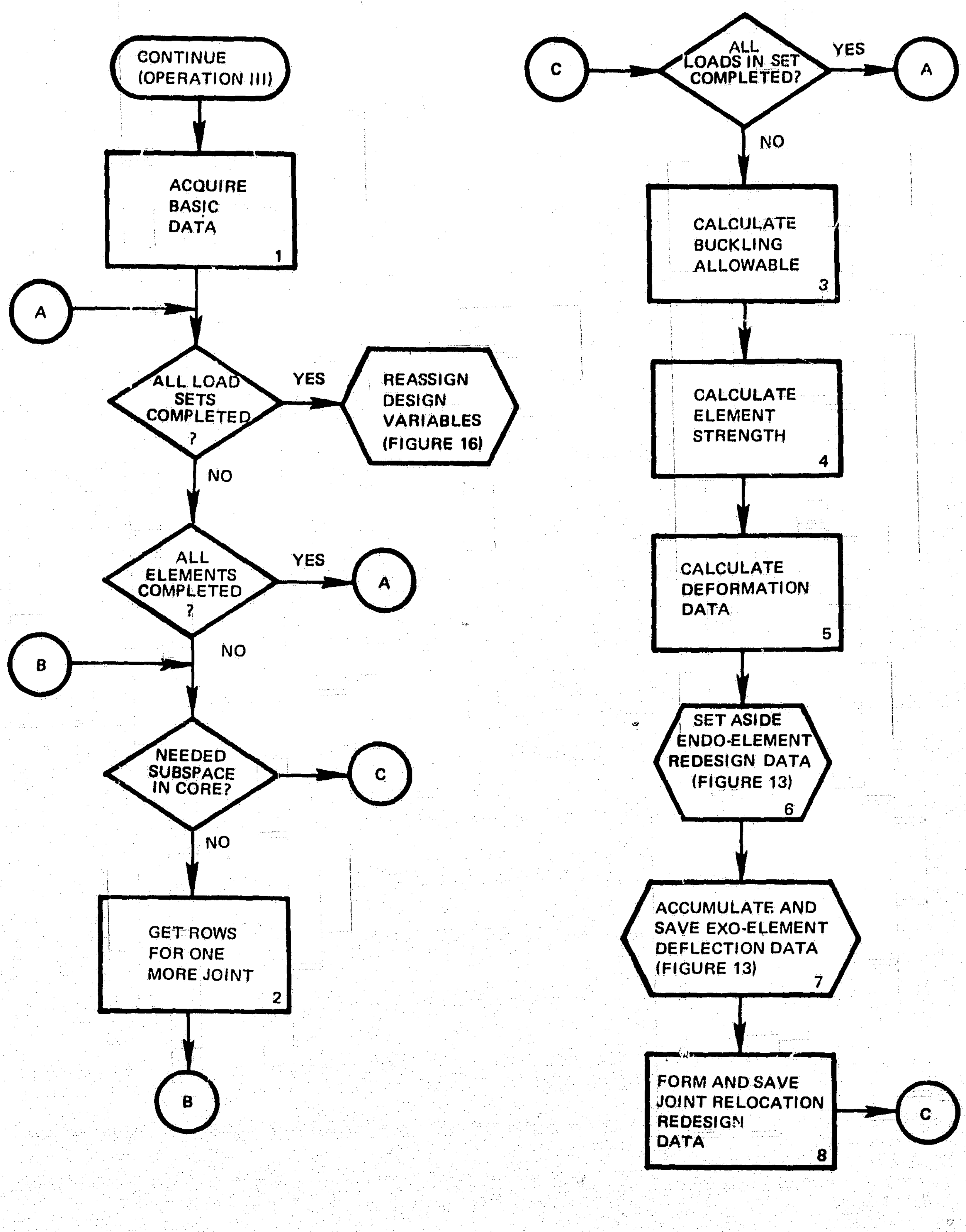


Figure 10. Evaluation of Element Response

F_i , V_r , p_r , ψ_r and ϕ_r coefficients are required. Efficient management of these data for response evaluation is similar to that described for subspace analysis.

2. Additional subspace rows are read as required using the element wavefront approach. If superposition is involved, these rows involve subspace vectors. If superposition has been performed in the subspace analysis, these rows involve solution vectors in joint displacement and/or internal force components.

3. Evaluation of buckling integrity of an element involves several subtasks. Element joint forces must be evaluated, if not available. Buckling admissibility must be checked. The appropriate buckling formula must be selected and the allowable stress calculated.

   Depending on analysis strategy, joint forces are evaluated by the Complementary Analysis, Equation (5-16), or the Potential, Equation (5-19). To simplify subsequent calculations, joint forces are transformed to a local coordinate system imbedded in the finite element.

   Buckling is considered to be admissible if the element is not a three-dimensional solid and if only compressive forces are applied. The second criteria implies that line elements which have no bending moments and surface elements which have no edge moments are buckling candidates.

   Selection of a buckling formula depends on the element type, material, and cross-section dimensions. For example, a line element of steel usually involves the Johnson formula while aluminum evokes the straight line formula.[71] Either of these formulas depends on the member length to radius of gyration of the cross section, the material compressive yield stress, and Young's modulus. For the surface element, formulas for simple equivalent surfaces such as the rectangle will be used. In both cases, the formula will define the maximum compressive load on the element.

4. Evaluating element relative strength requires selecting and adapting an appropriate failure criterion, surveying the element to locate the failure critical region and quantifying the tendency for failure. The evaluation is performed using the best estimate of joint displacements and forces in accordance with the Design-Analysis strategy.

   The selection of the failure criterion is based on the user's designation of criterion for the element material. As a minimum, Hencky-Von Mises, maximum strain-energy, and the ANC5 criteria should be available for ductile materials, and the maximum normal stress for brittle.[74]

   Based on the allowable stress values given with the incremental material input data and the allowable beam-column and crippling stresses, the failure value is determined from the failure criterion. For example, the Hencky-Von Mises theory failure criterion is

$$(\sigma_1 + \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \sigma_0^2 \tag{5-40}$$

   where

   σ_1, σ_2 and σ_3 are principle stresses and

   σ_0 is the failure value.

   The failure value is found by Equation (5-40). The allowable principle stresses are determined using Mohr's circle relations from the allowable uniaxial tension, compression, and shear data. Ultimate or yield allowable stresses are used depending on the nature of the relevant loading. Then, where beam-column effects occur, σ_0^2 is modified[71] to reflect nonlinearity. Similar relations can be constructed for each of the failure criteria.

The critical failure region is located by identifying the position where strain-energy density is maximum. Using the assumed displacement function (or stress function) and the elastic constants, strain energy is expressed as a function of coordinates of points on the element and the critical region identified.

To illustrate, consider a beam element. The energy is given by

$$U(x) = \frac{2EI}{a} \left[B\right]\left[B\right]^T \tag{5-41}$$

where

$$B = \left[\theta_1 \omega_1 \; \theta_2 \omega_2\right]\left[\frac{3x}{a} -2, \; \frac{6x}{a^2} - \frac{3}{a}, \; \frac{3x}{a} -1, \; -\frac{6x}{a^2} + \frac{3}{a}\right]^T$$

with

(x), the strain energy at point x on the beam,

E, Young's modulus

I, the bending moment of inertia

a, the length of the span,

$\theta_1, \omega_1, \theta_2, \omega_2,$ the rotation and lateral displacement at end 1 and 2 and

x the coordinate along the beam axis with the origin at end 1.

Then, differentiating Equation (5-41) with respect to x and setting the differential to zero locates the position of maximum (or minimum) energy. This will indicate that the failure critical section occurs at one of the beam ends.

The tendency for failure is quantified by examining the failure critical region and comparing evaluations of the failure criterion with the failure value. For the line and surface elements, several points on the critical cross section are examined to find that which maximizes the evaluation of the failure criterion. For three-dimensional solids, only the failure critical point requires examination. In either event, the failure value is multiplied by the factor γ before comparing with the failure criterion. Here

$$\gamma = \frac{1}{f_s} \sqrt{\frac{\tilde{W}}{U}} \tag{5-42}$$

where

- U is the strain energy implied by the displacement function and joint displacements and
- $\tilde{W}$ is the external work associated with the best estimate of joint forces and displacements in accord with the analysis strategy, and
- f_s is the factor of safety associated with the loading condition: yield or ultimate.

The factor γ accounts approximately for cross section geometry details.

Admissible strength failure modes described here include yield or fracture and slip for compression, tension, and shear conditions under static loads. Though inclusion of impact and repeated loading-stress failure criteria are not currently within the optimization state-of-the-art, they would fit easily into this part of the analysis plan.

5. The strategy for calculating joint displacements requires either using an equation of the form of Equation (5-13) and summing elongations or superimposing in accordance with Equation (5-27). Development of joint displacements, in either case, is accomplished on the element modular basis without special data management problems.

Requirements pertinent to only one element (endo-element requirements) are checked using the displacement function for the element. The maximum rotation or displacement relative to the joints is calculated easily and compared with the allowable. The ratio of actual to maximum relative displacement provides a non-dimensional measure of deflection integrity.

0

When system deflections are limited, information for the pertinent joints must be accumulated in core or spilled to auxiliary storage and recalled later. Both Reanalysis and Influence Analysis data can be handled concurrently. (With a large number of system relative deflection limits, the repetitive transfer-of-deflection data may make treatment-of-deflection limits unduly costly. To reduce these penalties it may be desirable to develop a special set of subroutines to develop deflection Redesign data independently of element strength and deflection integrity analyses.)

6. Endo-element redesign data includes information to ensure that the element will not buckle, will not fracture, and deformations in the interior will not be excessive relative to the joint displacements for each and every loading. Basic data on the endo-element integrity is developed in Steps 3, 4, and 5 above. Since the details of this step depend on the Redesign subsystem selected, calculations performed and data saved are described in Section 6.

7. Joint deflection data is pertinent to Redesign so the structure does not exceed system relative deflection limits. Basic data for system deflection integrity is developed in Step 5, above. Details depend on the Redesign subplan and are described in Section 6.

8. Joint relocation data is response information needed to improve the design by changing joint positions. These data are developed in Steps 3 and 5 and modified and saved in accordance with the Redesign subplan selected. Details of this step are described in Section 6 for each subplan.

Justification of Design-Analysis Plan

The Design-Analysis plan is a compromise among competing objectives. The overriding consideration in decision making is economy; i.e., providing a plan for an optimization program that will iteratively improve designs at low computer cost. This consideration dictates the subspacing decision and the solution decision. It determines the data management process. Design-Analysis efficiency (accuracy/calculation) plays a secondary role. It is most influential in selection of the dual analysis approach but also impacts on the choice of Influence method and solution approach. The need for accuracy (meaningful digits in calculation results) influences selection of the dual approach details and the choice of a non-iterative solution approach. The desire for low program development cost has required that the plan be able to use finite element generation routines of the parent program (NASTRAN) and be compatible with its Baseline Analysis.

The following paragraphs justify each of the major Design-Analysis decisions in terms of these competing objectives. Though each is considered in turn, the decisions are interactive.

Subspace approach. The decision to setup Design-Analysis on the basis of a reduced degree-of-freedom model is a salient feature of the plan. This decision is prompted by economy. The computer costs associated with developing the subspace basis are much less than the cost reductions for design analyses in the subspace basis. Consequently, with subspacing, the cost of analysis is reduced by an order of magnitude over that of the original basis.

Tables XXXVI through XL list data for comparing design analysis using joint generalized coordinates directly and using the subspace approach. These tables cite the number of words transferred between core and auxiliary storage for each of the Design-Analysis tasks. These data pertain to the maximum criterion problem of Section 2. They make tabulated numbers interpretable in terms of problem parameters.

The criterion problem characteristics are defined in prime numbers as follows:

$$\begin{aligned} E &= 2003 & \qquad N &= 6007 \\ B &= 199 & \qquad R &= 53 \\ L &= 41 & \qquad w &= 401 \end{aligned} \tag{5-46}$$

where w is the stiffness matrix wavefront and the 2003 elements are to be sized by the optimization process.

TABLE XXXVI

BASELINE REANALYSIS DATA TRANSFER VOLUMES (a)

Task	Words Moved (b)	Record Length	Record Penalty	Effective Volume
1. Form and tape coded stiffness matrix	4.8^{6}	2406	1.01	4.84^{6}
2. Read, decompose, and tape decomposition	9.6^{6}	2406	1.01	9.70^{6}
3. Read decomposition and forward substitute taping solutions	4.8^{6} 4.92^{5}	2406 246	1.01 1.58	4.84^{6} 7.55^{5}
4. Read decomposition and back substitute taping solutions	4.8^{6}x3 (c) 4.92^{5}	2406 246	1.01 1.58	1.45^{7} 7.55^{5}
5. Read solutions and find element responses and tape (d)	2.46^{5} 1.0^{6}	246 2000	1.58 1.05	3.88^{5} 1.30^{6}
TOTALS	3.6^{7}			3.7^{7}

NOTES:

(a) Numerical exponents imply a base of 10.0; e.g., $2^{4} = 2x10^{4}$

(b) Loadings treated in one column partition.

(c) Including penalty factor of 3 for backspacing and rewriting decomposition for subsequent forward read.

(d) Saving 50 words/element for 1/4 of elements.

TABLE XXXVII

BASELINE INFLUENCE ANALYSIS DATA TRANSFER VOLUMES[a]

Task	Words Moved	Record Length	Record Penalty	Effective Volume
1. Read decomposition and forward substitute taping solutions.	4.8^{6} 6.0^{5}	2406 600	1.01 1.24	4.84^{6} 7.45^{5}
2. Read decomposition and back substitute taping solutions.	4.8^{6} 1.2^{6}	2406 600	1.01 1.24	4.84^{6} 1.49^{6}
3. Read solutions and find element responses and tape redesign data	6.0^{5} 5.0^{5} [b]	600 1000	1.24 1.13	7.45^{5} ---- [c]
SUBTOTAL	1.25^{7}			1.27^{7}
4. Repeat for 2003x41 cases in groups of 100	x 820			x 820
TOTALS	1.0^{10}			1.1^{10}

NOTES:

[a] Numerical exponents imply A base of ten; e.g., $2^{4} = 2\text{x}10^{4}$

[b] Saving 10 words per element per loading for 1/4 of elements

[c] No penalty since read-write concurrent so maximum case governs.

TABLE XXXVIII

DATA TRANSFER VOLUMES IN DEVELOPING SUBSPACE MODEL (a)

Task	Words Moved (b)	Record Length	Record Penalty	Effective Volume
1. Read decomposition and forward substitute taping solutions.	4.8^{6} x 2 6.0^{5} x 2	2406 600	1.01 1.24	9.70^{6} 1.49^{6}
2. Read decomposition and back substitute taping solutions.	4.8^{6} x 2 1.2^{6} x 2	2406 600	1.01 1.24	9.70^{6} 2.98^{6}
3. Extract lower R eigenvectors and disc	1.0^{4}	10000	0.74	7.40^{3}
4. Read joint displacement vectors and tape subspace basis vectors	1.2^{6} 3.0^{5}	600 300	1.24 1.58	1.49^{6}
5. Read subspace vectors, form and disc behavior matrixes	3.0^{5} 5.0^{3}	300 5000	1.58 0.86	4.74^{5} 4.30^{3}
TOTALS	2.5^{7}			2.6^{7}

NOTES:

(a) Numerical exponents imply a base of 10, e.g., $2^{4} = 2\text{x}10^{4}$

(b) Treating the 199 vectors in column partitions of 100.

(c) Exploiting simultaneous read-write capabilities so only maximum.

TABLE XXXIX

DATA TRANSFER VOLUMES FOR SUBSPACE REANALYSIS[a]

Task[b]	Words Moved[c]	Record Length	Record Penalty	Effective Volume
1. Read subspace vectors and find ψ_r and ϕ_r	3.0^{5}	300	1.58	4.74^{5}
2. Read behavior matrices, solve for ψ_r and ϕ_r	5.0^{3}	5000	0.86	4.30^{3}
3. Read subspace vectors and find element responses and tape[d]	3.0^{5} 1.0^{6}	300 2000	1.58 1.05	4.74^{5} 1.30^{6}
TOTALS	1.6^{6}			2.2^{6}

NOTES:

(a) Numerical exponents imply a base of 10.0; e.g., $2^4 = 2x10^4$.

(b) Responses obtained by superposition, since best in this case.

(c) All 41 loadings treated in a single pass.

(d) Saving 50 words for 1/4 of elements/load.

TABLE XL

DATA TRANSFER VOLUMES FOR SUBSPACE DESIGN-ANALYSIS (a)

Task (b)	Words Moved (c)	Record Length	Record Penalty	Effective Volume
1. Read subspace vectors to form ψ_r and ϕ_r and find and tape element responses (d)	3.0^5 x 411 1.0^6 x 410	300 2000	1.58 1.05	1.95^8 4.30^8
2. Read behavior matrices	5.0^3	5000	0.86	4.30^3
TOTALS	5.3^8			6.2^8

NOTES:

(a) Numerical exponents imply a base of 10.0; e.g., $2^4 = 2 \times 10^4$.

(b) Using superposition to obtain joint responses.

(c) Treating 200 loadings at a time and concurrently evaluating b_j and c_j for one set while superimposing for the previous set.

(d) Saving 10 words/element/load for 1/4 of the elements.

The tables cite effective data volumes as an index of computer time. The effective volume is defined as:

$$V_f = V_a R_f \tag{5-47}$$

where

V_f if the effective number of words transferred,

V_a is the actual number of words transferred, and

R_f is a factor to reflect the effect of the record size on transfer time.

R_f is calculated using the timing data in Table X, Section 2, and normalizing $R_f = 1.0$ for 3000 words of data.

In determining data effective volumes, record sizes have been selected to provide as large a size as possible consistent with core utilization. The effectiveness of this planning is reflected in the small difference in effective and relative volumes in all totals. This small difference also implies that the effective size of records is about 3000 words. Data management implied by the volumes is consistent with the Design-Analysis plan described in this section.

A summary of the total effective volumes from these tables is in Table XLI.

TABLE XLI

SUMMARY OF DESIGN ANALYSIS DATA TRANSFER VOLUMES

Approach	Basis Selection	Reanalysis	Influence	Design Analysis
Baseline	0	3.7^7	1.1^{10}	1.1^{10}
Subspace	2.6^7	2.2^6	6.2^8	6.2^8

Analysis of these data leads to the following conclusions:

a. Depending on the number of optimization and design cycles, the subspace approach can reduce data transfer volume by a factor of between one and 18 as compared with Baseline Analysis. The high factor occurs when Influence Analysis is avoided and no optimizations are involved. The low factor occurs when many iterations of optimization and few design cycles are involved.

b. Influence Analysis involves about 300-times more words in data transfer than Reanalysis whichever Design-Analysis basis is used. This fact is due to the factor of 2003 more loadings for the criterion problem. The reduction of the factor to 300 reflects economies in handling the larger data volumes.

c. The amount of data transferred in performing a single reanalysis in the subspace basis is less than for the Baseline Analysis despite data transfers in developing the subspace basis.

The improvement in economy of the subspace approach is attributed to the smaller number of equations in the subspace basis. The fact that matrices and vectors in the subspace are fully populated reduces identification information from about half the data transferred in Baseline Analysis to a small fraction in subspace analysis, thereby further improving analysis efficiency.

Analysis decisions. The Design-Analysis plan is based on an integral equation formulation, use of a dual behavior model including, separately, internal forces and displacements, disjoint operators, and Galerkin error weighting. These decisions represent a modest compromise of efficiency for adaptability.

The choice of an integral formulation is based on efficiency. Differential formulations yield solution results which are more sensitive to joint location and generally less accurate, for a given number of comparable degrees of freedom, than integral formulations.[25]

The dual approach, involving both Complementary and Potential Energy formulations provides a Design-Analysis method choice of relatively high efficiency, economy and accuracy. The dual approach yields uniquely defined internal forces and displacements - something not guaranteed with either approach alone.

The dual approach is efficient because whenever the subspace basis includes all force redundants or all kinematic redundants, the exact solution is obtained with fewer calculations and data transfer than in the Baseline Analysis. Since a closely related basis is used for both the Complementary and Potential Energy approaches, little extra data processing is involved in taking the dual approach as opposed to ab initio selection of one or the other. When the exact solution is involved, a single optimization cycle is sufficient. This efficiency is improved by reducing the data processing per design cycle without the penalty of successive optimization cycles.

The selection of basis vectors is biased toward the complementary energy to further enhance efficiency. Experience has shown that analysis accuracy for the Complementary and Potential Energy approaches is a function of the number of degrees of freedom in the subspace compared with the total number of redundants.[28] Table XLII cites the number of force and displacement redundants for two structures analyzed by both methods illustrating the fact that in practical analyses the number of force redundants is about half the number of kinematic redundants.[75] This implies that Complementary Energy Analysis may be intrinsically more efficient for practical structures analyses and justifies the biased selection of subspace vectors.

There is at least one class of structures for which this bias must result in highly efficient analyses. This is the class of structures for which St. Venant's principle is valid. This principle can be stated as follows:

> If a self-equilibrating load is applied to a region of a structure, it will cause negligible changes in stress at locations far removed from the region of load application. Distances are measured in terms of the greatest dimension of the region of load application.

If this principle is applicable, the internal force distribution in the structure will be little changed by changing element local geometry. In this case, a Complementary Energy Analysis will yield efficient estimates of system behavior.

TABLE XLII

REDUNDANTS IN PRACTICAL ANALYSES

Structure	Analysis Method	No. of Equations	No. of Redundants
Swept Wing	Force Displacement	390 360	101 354
Unswept Box	Force Displacement	390 300	161 294

Since the subspace basis is picked by choosing self-equilibrating vectors associated with finite elements of the structure, additional analysis efficiency can be achieved in late design cycles. If the synthesis vectors are developed from self-equilibrating loads for the elements being changed in the design, the Design-Analysis will be exact.

The use of the dual Complementary and Potential Energy strategy is also more economical than the use of a non-extremum variational principle encompassing use of both displacement and internal force variables such as the Reissner principle. If the exact solution is to be obtained by the Reissner approach, the number of base vectors must generally be at least equal to the sum of the number of force and kinematic redundants. Thus, the non-extremum principles can be expected to incur higher Design-Analysis cost than the dual approach.

The dual approach also admits developing solutions with known error bounds and with smaller maximum error. Since both extremum principles are available, minor additions to the plan would provide solution bounds for internal force and deflection at a joint.[(76),(77),(78),(79)] Interpolation then could be used to produce answers with smaller maximum error. Thus, the dual approach can be more accurate than alternate approaches.

The dual approach is also compatible with a variety of finite element bases. It develops rigorous estimates of internal forces and joint displacements regardless of the basis of the finite elements used. It requires only the behavior matrix for each finite element. As long as these matrices imply satisfaction of macroscopic equilibrium and joint deformation compatibility for the baseline analysis, the dual analysis need not develop different element models. Thus, if the analyst has used Complementary Energy finite elements in his Baseline Analysis, he can continue to use them in Design-Analysis. This compatibility exists also for mixed and potential energy models.

The decision to use disjoint operators and Galerkin weighting in optimization is also implicit in NASTRAN finite element analysis. It avoids development costs for new finite element models especially for optimization. Both the decisions, though not associated with maximum analysis efficiency,[25] are widely accepted and their limitations known.

When necessary, related detail Design-Analysis decisions were made to be consistent with the major decision described above. Otherwise, they were made for efficiency. The decision to locate the failure criteria region on an energy basis is consistent with the energy approaches. The decision to evaluate influences directly (though avoiding matrix inversion) is illustrative of a decision made for efficiency.

Solution decisions.- Efficiency and accuracy justify the solution decisions. The Euler equation approach, direct solution, and the modified Gauss decomposition algorithm are prescribed. This combination has already been accepted by most engineers. The efficiency of direct solution is brought out in a recent study of iterative methods.[5] This study shows that even if only one digit improvement in accuracy is required for a guess supplied to an iterative approach, the direct method is more efficient. The accuracy advantage of modified Gauss decomposition over Choleski has been established.[35] Error checks for interpreting accuracy of results also have been validated.[70]

Special Advantages of the Design-Analysis Plan

Special advantages of a computer program implementing the plan are its flexibility in operation, extendability to other failure criteria, and an increased understanding of a structure's behavior. These advantages are a consequence of the choice of the dual energy approach and the data management method.

Flexibility is associated with the ability of the implementing computer code to accommodate any number of equations and any number of elements. The first accommodation is available because no matter how many equations occur in the Baseline Analysis, the number of subspace equations, R, can be limited to a fixed value. This value can be chosen as a function of the available core storage space to provide easy adaptation among computer hardware. The second accommodation exists because logic is planned on a modular basis with the finite element being the module. Treatment of each element in turn makes computer time a function of the number of elements but introduces few limitations (in the computer logic) on the number of elements.

Other failure criteria of interest could include specified minimum resonant frequency, integrity under transient response, and safety against dynamic instability. The Design-Analysis basis selected is a microcosm of the Baseline Analysis approach. The analysis reflects, to some extent, the effect of any design variable change on structural response. This is also true if the process is extended to predict resonance, transient response, and stability of the system. Moreover, because it is a microcosm, the extension need involve no new or untried approximations. Thus the approach is extendible to other failure criteria simply by adding Design-Analysis subroutines.

Use of the dual approach can yield additional information of intrinsic characteristics of the structure. The eigenvector analysis identifies the principal "static behavior modes" of the structure. It can define the nature and number of redundant force and displacement systems and can measure the quality of St. Venant's principle as a function of structure and loading. When these data are interpreted by the engineer, he may not only understand better the behavior of his structure but also suggest major configuration changes which will permit the computer to evolve much better designs.

Salient Design-Analysis Features

Table XLIII summarizes the principal technical features of the Design-Analysis plan. Despite the fact that most of these features are unique to this plan, the plan can be implemented with little risk. With few exceptions, structural research has been performed insuring the successful implementation and use of each of the features. The exceptions are associated with features which, if unsuccessful, would result in only minor coding changes in the computer program. The last column in the table indicates the relative risk in each feature.

TABLE XLIII

TECHNICAL FEATURES OF THE DESIGN ANALYSIS PLAN

No.	Feature	Risk
1.	Subspace analysis replacing N equations with R	Low
2.	Dual Complementary and Potential Energy approach	Low
3.	Analysis strategy leading to exact solutions when possible.	Low
4.	Candidate synthesis vectors based on user or automatically selected finite elements and modes	Low
5.	Subspace synthesis vectors by eigenvalue election	Low
6.	Influence analysis using exact derivative evaluation	Low
7.	Direct solution for reanalysis and influence analysis	Low
8.	Direct treatment of zero and infinite stiffnesses	Moderate
9.	Automatic checking of buckling integrity when buckling can occur	Low
10.	Energy survey of element to locate fracture critical region	Moderate
11.	Fracture failure criteria which depend on loading, safety factor, material, and element geometry	Low
12.	Non-dimensional buckling, fracture, and deformation failure measures.	Low

Section 6
REDESIGN PLAN

This section describes and justifies two subplans for Redesign. It first describes common features of the Redesign approaches. It provides a subplan specialized for a multiple design variable capability (≤ 2000) and a subplan for optimization with few design variables (<200). For each subplan it defines the mathematical basis, logical connections among tasks, and redesign steps. It justifies the Redesign plans by economy, generality, and state-of-the-art limitations. It concludes with a citation of plan features and their implementation risk.

Common Subplan Considerations

The function of Redesign is to reassign design variables to improve the design measure. Redesign is performed under Operation III of the master flowchart shown in Figure 2. The general flowchart of Operation III is shown in Figure 4. Redesign logic interfaces directly with design analysis in the evaluation of element response as shown in Figure 10.

The redesign subplans work with the same design measure, constraints, and variables. They differ in their interpretation of the design measure and the mathematical basis, data, and steps used in redesign.

The design measure.- Each Redesign process is planned so virtually any design measure can be carried to its extreme. Thus, the user can define the measure in the form of FORTRAN statements. To ensure that changes will be prescribed during Redesign, the design measure must be a function of the values of the design variables. Since the Redesign processes require derivatives of the measure as a function of the design variables, the design measure evaluation must produce meaningful derivatives in the neighborhood of discontinuities. The FORTRAN program may produce derivatives directly or they will be developed by differencing. If the differencing option is elected by the user, the FORTRAN program must provide an evaluation of accuracy of the cost values so the controlling program can stop calculations if accuracy is inadequate.

The design measure defined by Equation (4-1) will be "built-in." Use of this function will eliminate the need for FORTRAN statements and provide a variety of design objectives as special cases. If, for example, $\alpha = 0$ and $\beta = 1$, minimizing the C measure minimizes weight of the finite elements. If $\alpha = 1$ and $\beta = 0$ and c_e and $\bar{c}_e$ are element dependent unit dollar costs, minimizing dollar cost is the objective. If, in this case β represents system dollar cost penalties related to weight, cost effectiveness is the design goal.[73] Though, in this case, the design objective is to minimize C, and C is a modular function, neither of these limitations is imposed on the FORTRAN defined design measure.

<u>Design variables</u>.- The criterion design capability requires treating four design variables: element size, element material selection, joint boundary conditions, and joint locations. In accordance with the input specifications of Section 4, the first two variables are discrete and the last two continuous.

For Redesign purposes there are only two quasi-continuous variables. The first variable is element selection. Each candidate for a given finite element is considered to have a particular size and be composed of a particular material. This interpretation eliminates independent treatment of material selection as a design variable. The element selection also encompasses the joint boundary condition variable if the user simulates it with a finite element whose selection is a variable. Thus, the candidate selected for a clock spring at a joint determines the desirable joint rotation fixity.

The second variable is joint location. This is naturally a continuous function of the position coordinates of the joint.

Only the element selection variable requires special consideration to interpret it as a continuous variable. Element selection is simulated by the following process:

1. Assume that the stiffness of each element is given by:

$$k_{(e)} = X_{(e)} k'_{(e)} \qquad e = 1, 2, 3 \ldots E \tag{6-1}$$

where

k_e is the linearly factored stiffness matrix for element e , this stiffness is used for every Reanalysis,

k'_e is the reference stiffness matrix for element e based on a particular candidate,

$X_{(e)}$ is the element current effective utilization factor. It serves as the design variable for element size for candidate e . To simplify the discussion, it will be assumed that all E elements must be designed.

2. Redesign by finding a new value for each of the $X_{(e)}$ assuming $C_{(e)}$ is an invariant for a particular candidate.

To define the relation between design cost and size, assume that the design measure is such that

$$\frac{\partial C}{\partial X_{(e)}} = c_{(e)} \qquad (6\text{-}2$$

3. Replace $k'_{(e)}$ with a candidate which has an equal or better structural capability and an equal or better design measure.

For this replacement, structural capability is measured by energy participation. The acceptability of a given candidate as a replacement is assayed as follows:

a. Using the current estimates of behavior, find the maximum energy density and total energy for the replacement candidate.

b. Determine the value of $X_{(e)}$ for the candidate such that the total energy for the candidate, $k_{(e)}$, equals that of the element being replaced. If $X_e > 1$, the candidate is rejected as a replacement.

c. Scale the maximum energy density to reflect the choice of $X_{ⓔ}$ in Step a. If the energy density is greater than the allowable strength value, reject the candidate.

From those which are acceptable replacements, that one which has the lowest value of $c_{ⓔ}$ is selected.

To fix ideas, suppose the candidates are sequenced so the $c_{ⓔ}$ increases monotonically with the candidate number. (This can be done once and for all at problem initiation.) Then the relation between the candidate sequence number and the element design measure is represented, for a typical set of data, by the graph of Figure 11. Equations (6-1) and (6-2) imply the continuous lines connecting the origin and the candidate characteristics.

Suppose the energy measures have been evaluated for a particular element and $X_{ⓔ}$ has been assigned by the redesign process. Then, the characteristics of the element to be replaced are indicated in Figure 12. Here, the horizontal line is the locus of the required stiffness $k_{ⓔ}$. Every line intersected by the horizontal is associated with a candidate with comparable or better stiffness. As shown, the point of intersection defines the candidate's $X_{ⓔ}$. Of those elements which are intersected, those with a higher maximum energy density than allowable, based on material allowables, must be disregarded. Of the remainder, that which has the lowest cost, when $X_{ⓔ} = 1.0$, is used for the replacement. This candidate is circled in Figure 12.

In considering replacement elements, selection of candidates is biased by the assumption that as the design measure increases, performance increases. Thus, if an element of higher performance is required, few candidates of lower measure are examined. Candidates of increasingly higher measure are examined until an acceptable one is found. If lower performance is acceptable, candidates of lower design measure are examined first. This reduces search for replacement candidates to nearly a one-sided search.

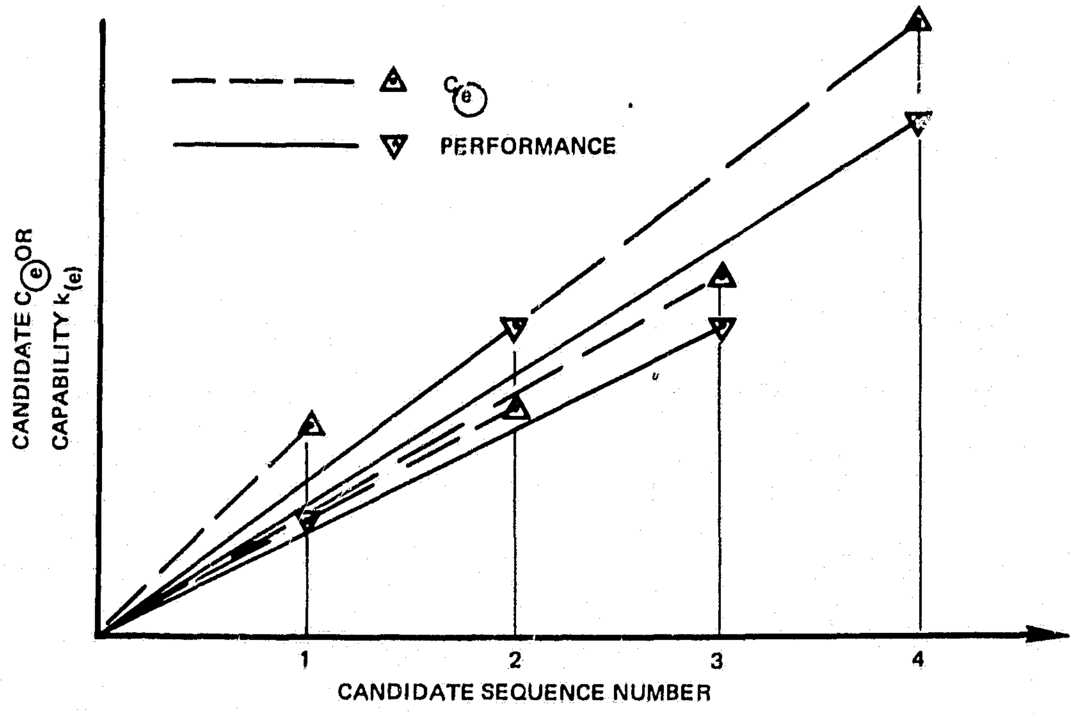


Figure 11. Candidate Performance and Design Measure Relationship

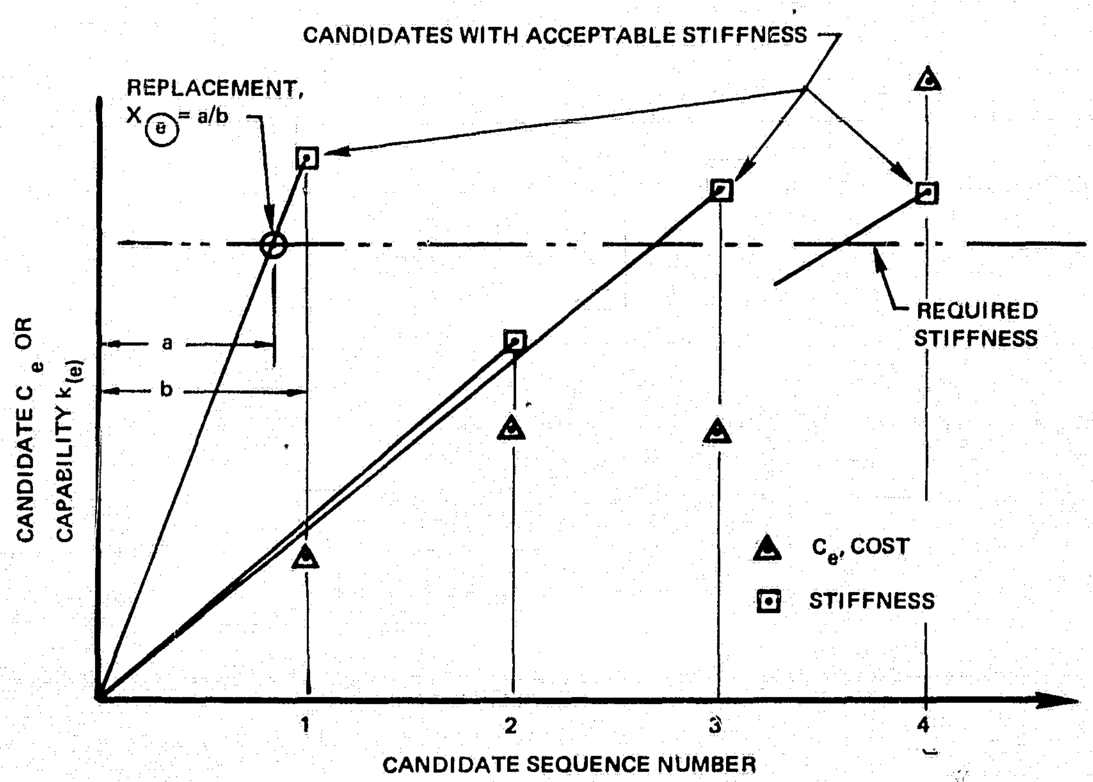


Figure 12. Selection of Equivalent Replacement

Design constraints.- Design constraints will include both behavioral and design variable requirements. Behavioral constraints are circumscribed by the Design-Analysis plan. It provides for insuring fracture integrity, kinematic stability, element buckling integrity and explicit or FORTRAN statement displacement limits. Design variable constraints include limits on joint relocation variables and element selection.

In each subplan, joint variable limitations are treated by assigning the appropriate limiting value of the variable whenever the assignment would otherwise exceed the limit. Thus, if Redesign recommends that the joint be relocated three inches from its initial position and only two inches are admissible, two will be assigned.

In each subplan, element selection limitations are treated indirectly. The selection variable is treated as a quasi-continuous variable during the Redesign and Design-Analysis processes. Assignments are allowed to exceed maximum and minimum candidates. When designing is terminated, either an ideal design (using the quasi-continuous variable value) or a realizable design (using the current reference candidate for each element) is produced at the user's discretion.

In general, each Redesign subplan will exclude the possibility of automatic changes of the configuration class even though the user's limit or control values do not make this inadmissible. The user can always introduce configuration changes by redefining the problem using the interactive communication link.

A change of configuration class is a change that results in a change in the number or form (not coefficient magnitudes) of the equations involved in the optimization. Of particular interest are the changes of configuration class induced by elements vanishing and joints coalescing. When an element vanishes, its equilibrium and compatibility equations must be eliminated. When two joints coalesce, equations must be eliminated because the number of independent degrees of freedom is reduced.

The vanishing of an element will be permitted if its associated equations thereby become degenerate. In the usual case, replacing an element with a null candidate makes either the equilibrium or compatibility equations null, depending on whether the force or displacement analysis method is used. In the degenerate case, both sets of equations are null. The degeneracy case will be included because it is not uncommon. For example, Venkayya[80] encountered it in fout out of 13 structural optimization problems.

Joints will not be permitted to be relocated so two are in juxtaposition. As two joints approach each other, the stiffness of an element connecting them approaches infinity. Based on this fact a simple test can ensure that each joint is distinct. Thus selection of unacceptable joint coalescing will be precluded.

Both element selection and joint location design variables will provide for variable equivalencing. Elements which must use the same candidate will be designated by an element group. All joints with the same relocation data will be denoted by the same joint gang name. All elements of a group and all joints of a gang will be assigned the same value of the design variable during Design-Analysis and when Redesign is completed in each design cycle.

Element grouping involves simply a reduction in the number of elements for which candidates can be selected independently. In developing influence data in Design-Analysis, all elements in the group are considered to be changed simultaneously. In redesigning, assignment of the group variable value is made with respect to the response and design measure implications of each and every element in the group.

Joint ganging reduces the number of independent joint location variables. Depending on relocation constraints, this variable has one, two, or three components: one if relocation is restricted to a vector magnitude; two if the relocated joint is restricted to be in a prescribed plane; and three if the joint may be relocated anywhere in a specified volume. Like the element group variable, during Design-Analysis required influence data are developed by perturbing all joints in the gang simultaneously. Redesign is executed considering the implications on response and the cost of changes to the ganged variable.

Redesign Subplans

The paragraphs that follow describe two Redesign subplans. The "Specialized Redesign Subplan" will lead to a computer program for rapid design of structures with many elements to be designed and few displacement limits. The "General Redesign Subplan" will lead to a program for optimizing structures with relatively few design variables and many displacement limits. In accordance with the Input/Output Plan, selection of the subplan and its options are at the analyst's disposal subject to limitations of data processing capabilities for each subplan.

Specialized Redesign subplan.- This subplan provides for optimizing element selection to satisfy all endo-element constraints and a few (less than about 40) system relative displacement limitations. Redesign uses the fully-stressed method modified to take cognizance of the design measure effects and accommodate displacement limits. Data processing for Redesign to meet endo-element constraints is integrated with the Design-Analysis process.

Mathematical basis: Redesign to satisfy endo-element constraints consists of selecting the appropriate candidate for each element of the structure independent of every other element. An element's "size" is selected so the element makes the smallest contribution to cost, subject to the limitation that none of the endo-element constraints are violated. Violation of constraints is assayed assuming the internal joint forces for the element will be unchanged by Redesign.

Changes of sizes to produce a structure which will comply with system relative displacement limits can be achieved by scaling all elements of the system by a factor; i.e., let

$$X^{SD}_{(e)} = fX'_{(e)} \qquad e = 1, 2, \ldots E \qquad (6\text{-}3)$$

where

$X^{SD}_{(e)}$ is the $X_{(e)}$ required to satisfy system deflection limitations,

$X'_{(e)}$ is the element effective utilization factor chosen for endo-element design, and

f is the largest ratio of analyzed deflection to its corresponding allowable maximum, among all deflection limits for all loadings.

This scaling provides a basis for Redesign when complex displacement constraints are introduced as FORTRAN statements. For example, suppose the FORTRAN statements indicate a violation of constraints. Then the FORTRAN statements are reused to determine a factor by which displacements can be scaled to be acceptable. Then the user subroutine indirectly defines the f of Equation (6-3). Since scaling provides an efficient basis for developing acceptable designs, the scaling option is also available even when no FORTRAN displacement constraints are introduced.

Changes of sizes to comply with displacement limits can also be selected using virtual work to determine the influence of size changes on deflection response. This promising approach has recently been described by Berke.[39]

Assume the displacement constraints are prescribed in the form

$$\lfloor \overline{P}_t \rfloor \{ p_\ell \} = W_x \le W^* \qquad (6\text{-}4)$$

where

- W_x is the "external vertical work" and W^* is the limiting value chosen by the designer either by discrete limits or by his FORTRAN subprogram,
- $\overline{P}_t$ is an influence loading vector, t, whose components are selected by the designer,
- p_ℓ is the displacement vector due to the particular P_ℓ real load.

Since the summation in Equation (6-4) evaluates work, multiplying components of P_t and p_ℓ must be at the same joint and coincident directions. Since there are no restrictions on $\overline{p}_\ell$, the choice,

$$\overline{P}_{(i)} = 1, \ \overline{P}_{(j)} = 0, \ j = 1, 2, \ldots N \text{ but } j \neq i \qquad (6\text{-}5)$$

where N is the total number of joint displacement degrees of freedom, can be made. This reduces (6-4) to the requirement that a particular displacement component, $p_{(i)}$, be limited.

The internal work corresponding to Equation (6-4) is

$$W_I = \sum_{e=1,\,2,\ldots}^{E} \frac{1}{X_{(e)}} \lfloor S_{\ell(e)} \rfloor \left[A_{(e)} \right] \left\lfloor \bar{S}_{t(e)} \right\rfloor \qquad (6\text{-}6)$$

where

W_I is the internal work

$\bar{S}$ are internal forces associated with the $\bar{P}$ loads and

S are internal forces associated with P loads.

$X_{(e)}$ appears in Equation (6-6) to reflect the utilization of the element e,

The Redesign problem consists of reassigning the $X^i_{(e)}$ $e = 1, 2, \ldots E$ as $X^{i+1}_{(e)}$, the sizing design variables for the $i + 1^{st}$ design cycle. Each $X^{i+1}_{(e)}$ must be equal to or greater than that required to meet all endo-element requirements. The increases in the $X_{(e)}$ are distributed to satisfy Equation (6-6) and minimize cost.

For Redesign, two types of $X_{(e)}$ enter into Equation (6-6): those which are prescribed by endo-element requirements, and those which are free to be optimized. Thus, Equation (6-6) can be expressed as

$$W_I = \sum_{e=1,2,\ldots}^{\bar{e}} \frac{1}{X_{(e)}} \lfloor S_{\ell(e)} \rfloor \left[A_{(e)} \right] \left\lfloor \bar{S}_{t(e)} \right\rfloor + \sum_{\bar{e}+1,\ldots}^{E} \frac{1}{X_{(e)}} \lfloor S_{\ell(e)} \rfloor \left[A_{(e)} \right] \left\lfloor \bar{S}_{t(e)} \right\rfloor$$

or

$$W_I = \sum_{e=1,2\ldots}^{\bar{e}} \frac{1}{X_{(e)}} \lfloor S_{\ell(e)} \rfloor \left[A_{(e)} \right] \left\lfloor \bar{S}_{t(e)} \right\rfloor + W^E \qquad (6\text{-}7)$$

with

$$W^E = \sum_{\bar{e}+1,}^{E} \frac{1}{X_{(e)}} \lfloor S_{\ell(e)} \rfloor \lfloor A_{(e)} \rfloor \lfloor \bar{S}_{t(e)} \rfloor$$

where the first summation includes those $\bar{e}$ elements for which $X_{(e)}$ is free to be optimized, the second summation includes elements for which $X_{(e)}$ is prescribed, and W^E is the value of the second summation; the internal energy for element size variables prescribed.

Then, minimizing the cost with respect to $X_{(e)}$ subject to the constraint of Equation (6-4), assuming a design measure monotonically increasing with $X_{(e)}$, $e \leq \bar{e}$, gives

$$X_{(e)}^{i+1} = \frac{1}{(W^* - W^E)} \sqrt{\frac{\lfloor S_{\ell(e)} \rfloor \lfloor A_{(e)} \rfloor \lfloor \bar{S}_{i(e)} \rfloor}{c_{(e)}}} \sum_{e=1,2\ldots}^{\bar{e}} \sqrt{c_{(e)} \lfloor S_{\ell(e)} \rfloor \lfloor A_{(e)} \rfloor \lfloor \bar{S}_{i(e)} \rfloor} \qquad (6\text{-}8)$$

where the design measure derivative $c_{(e)}$ is evaluated at the reference design point in accordance with the assumptions on the variation of cost with $X_{(e)}$.

Use of Equations (6-7) and (6-8) for Redesign implies an iterative procedure to establish prescribed variables. Thus, the $X_{(e)}^{i}$ and Equation (6-8) lead to assignments for each of the $X_{(e)}^{i+1}$ for each loading, $\ell = 1, 2, \ldots L$, and each displacement limit, $t = 1, 2, \ldots T$. The $X_{(e)}^{i+1}$ for a particular element is taken to be the harmonic mean of the L times T values determined.

Redesign process: Figures 13 and 14 show the connections between tasks of the Specialized Redesign process. Figure 13 includes those components which are an integral part of the evaluation of element response - the Design-Analysis objective charted in Figure 9. Figure 14 shows components which are activated upon completion of all the tasks cited in Figure 9.

The three functions to be directed by integrated logic include Redesign for element failure, Redesign to meet system deflection limits, and joint relocation. The last option is excluded from the Specialized Redesign. The logical connection among tasks for the other two functions is shown in Figure 13. Steps for these tasks are as follows:

1. Redesign for buckling integrity is achieved by defining the maximum scale factor for the element to preclude buckling for any of the real loads. Assuming joint forces are invariant with Redesign, this is found as

$$X^B_{ⓔ} = \max.\left(\frac{f_{c(\ell)}}{F_c}\right) \quad \ell = 1, 2, 3 \ldots L \tag{6-9}$$

where

$X^B_{ⓔ}$ is the value of $X_{ⓔ}$ to preclude buckling,

$f_{c(\ell)}$ is element compressive load under loading ℓ ;

F_c is the allowable compressive load, and

max ( ) designates the maximum value of the argument.

This step requires finding $(f_{c\ell}/F_c)$ and saving it if it is greater than the previous maximum for the element.

The load used in evaluating Equation (6-9) is formed directly from the element joint forces, taking cognizance of topology. Thus, for a line element it is simply the end load. For surface elements, the buckling load can be expressed as a function of the loads in two orthogonal directions, the panel proportions and the material properties. Thus, a survey can be performed with various orientations of coordinate axes to locate the largest value of $X^B_{ⓔ}$ for a particular element and loading.

2. Redesign for fracture integrity involves finding the scale factor so the element is strength-critical for one or more loadings.

This factor is evaluated, if the Hencky-Von Mises failure criterion is elected, by

$$X^F_{ⓔ} = \max.\left[\left(\frac{\overline{\sigma}_o}{\sigma_o}\right)_\ell + \frac{f_{c(\ell)}}{F_c}\right]_{\ell=1,\ 2,\ \ldots L} \qquad (6\text{-}10)$$

where $X^F_{ⓔ}$ is the value of $X_{ⓔ}$ for element e to preclude fracture due to overstress. Equation (6-10) is an interaction formula. The first term represents the tendency for strength failure and the second represents the tendency for beam-column overstress. The second term may be neglected if $f_{c(\ell)}$ is tensile. As described in Section 5, $\overline{\sigma}_o$ is the value of the failure criterion for the region of the element for which $\overline{\sigma}_o/\sigma_o$ is maximum. This step requires evaluating $(\overline{\sigma}_o/\sigma_o)$ and saving if it is greater than the previous maximum.

3. Redesign for endo-element deformation integrity also involves finding a maximum in accordance with

$$X^D_{ⓔ} = \max.\left(\frac{\overline{s}_{\ell ⓔ}}{s_{aⓔ}}\right) \quad \ell = 1,\ 2,\ \ldots L \qquad (6\text{-}11)$$

where

$X^D_{ⓔ}$ is the value of X_e to preclude excessive deflection in the interior of element e relative to displacements of its boundary joints,

$\overline{s}_{\ell ⓔ}$ is the calculated relative deflection in the interior of element (e) under loading ℓ, and

$s_{aⓔ}$ is the allowable relative displacement in element e under loading ℓ.

As described in Section 5, $\overline{s}_{\ell ⓔ}$ is determined for the point in the element using a suitable displacement interpolation function for the topology of interest. This step requires finding $(\overline{s}_{\ell ⓔ}/s_{aⓔ})$ for a loading and saving it if it is greater than the previous maximum.

4. Saving element Redesign data requires storing all data pertinent to redesign of the element and information for the active review file. Redesign data includes the largest of $X^B_{(e)}$, $X^F_{(e)}$, and $X^D_{(e)}$ for the element under scrutiny. In addition, the loading case specifying this design is identified. These data also comprise the minimum Graphic File output. If the higher level graphics review is required, $X^B_{(e)}$, $X^F_{(e)}$, and $X^D_{(e)}$ are stored for each loading along with appropriate identification. As a Redesign option, these data are scanned to find a single scale factor to use under the scaled Redesign option.

5. If scaling of the structure is the technique of accounting for system relative displacement limits, the scale factor, f, of Equation (6-3) is selected. Scaling is used if FORTRAN statements define deflection limitations or the user elects scaling directly.

   Data processing can take one of two forms depending on the form of the deflection criterion and independence of selection of the Redesign Method. In incremental form, the deflection data is treated one joint at a time. In total form, all deflections must be treated together. In the first form, the determination of f can be done one joint at a time for all loads, and no special data management is needed. In total form, the displacements at each joint for each loading must be saved while element responses are generated. If less than LxN locations are available in core for this purpose, spill is incurred.

   The total form is evoked only when a FORTRAN specification of system deflection limits is used and is in total form. When FORTRAN statements are not used or the FORTRAN deflection limit subroutine is in incremental form, the first form can be used.

6. Deflection Redesign data is accumulated for subsequent Redesign if the work equations, Equation (6-8), are to be used as the basis for Redesign. During the evaluation of element response, the terms

$$\left\lfloor S_{\ell(e)} \right\rfloor \left[A_{(e)} \right] \left\{ \bar{S}_{i(e)} \right\} \tag{6-11}$$

are saved for each loading and deflection criterion along with appropriate identification. Each record contains all the terms of the form (6-11) for an element.

7. When all loadings have been treated, deflection Redesign information collected in Step 6 is saved for user displays or Redesign. If scaling is elected, no data is saved on auxiliary storage.

Redesign tasks to select design variables consistent with system performance requirements are concerned with system deflection limits. Logical connections among tasks are shown in Figure 14. These tasks yield new values for element selection consistent with available candidates. Tasks provide for extrapolating a sequence of designs.

Details of the steps are as follows:

1. Internal work Redesign, if elected by the user, is performed in accordance with Equations (6-7) and (6-8).

   Data developed by the Design-Analysis interface tasks is read into core storage. Equations (6-7) and (6-8) then are evaluated iteratively until all the $X_{(e)}$ are reassigned. For the modular built-in cost function, the $c_{(e)}$ $e = 1, 2, \ldots E$ can be evaluated once and for all. For the FORTRAN statement design measure, their evaluation is incorporated in the iteration process and defined for the harmonic mean of each $X_{(e)}$ for all load cases.

2. Replacement of elements with candidates is a task required in each optimization process. The basis for the replacement is described by the discussion centered around Equations (6-1) and (6-2).

3. Scaling elements to meet deflection limits is performed in accordance with Equation (6-3). The X'_e are left in core from Step 4 of the Design-Analysis interface tasks.

4. The criterion for stopping the designing provides two bases for termination. In one, the user limits the number of cycles by passive input or by using the graphics interface to modify the passive input or terminate. In the other, cutoff is controlled by quantifying the quality of the design.

One of the two design quality measures are used, depending on the form of the design measure, C . The two measures are

$$D_{Q1} = \frac{(X_X - 1)}{(X_X - X_N)} \leq C_1 \qquad (6\text{-}12)$$

$$D_{Q2} = \frac{C^{i+1} - C^i}{C^i} \leq C_1 \qquad (6\text{-}13)$$

$$\text{with } C^{i+1} \leq C^i \leq C^{i-1}$$

where

D_Q is the design quality,

X_X is the maximum X'_e for e = 1, 2...E, before element replacement.

X_N is the minimum X'_e for e = 1, 2...E, before element replacement,

C^i is the design cost for the i^{th} design cycle design, and

C_1 is a constant less than 1.0. selected by the designer.

The first quality measure, D_{Q1} is used if the cost measure is zero or independent of the value of the design variables. It measures how close the design is to being a feasible design. This measure is applied directly to the design when the work expressions are used for deflection Redesign. It is applied to the endo-element design when scaling is used to treat system deflection requirements. If there are no deflection requirements, the process becomes a fully stressed Redesign process,

The second quality measure, D_{Q2} , is used when the cost measure varies with design variables. It ensures that designing is ended when little progress is made in improving the design in an iteration.

5. Replacing the ideal design with a realizable one is performed simply by replacing each X_{e} by 1.0. The difference between the cost of the design before the X_{e} are reset and the cost with all $X_{e} = 1.0$ provides a measure of the penalty associated with the resolution of the candidate element table.

6. Extrapolation of design variables is admissible if the active constraints do not change for three successive cycles and the size of each element is varying monotonically. To determine admissibility, the size and element design condition (active constraint) must be kept for each element for three design cycles.

7. If admissible, the element selection is extrapolated by a rational polynomial fit of the form,

$$X_{e}^{i+1} = (a_0 + a_1 i)/(a_2 + i) \tag{6-14}$$

where

X_{e}^{i+1} is the desired scalar for element e ,

a_0, a_1, a_2 are curve fitting constants, and

i is the design number counting as zero the first design with repetitive active constraints

Solving Equation (6-14) for a_1 , the value of X_{e} when i is infinite, by curve fitting existing data gives

$$a_1 = X^1_{e} + \frac{2\left(X^0_{e} - X^1_{e}\right)\left(X^2_{e} - X^1_{e}\right)}{\left(X^0_{e} - 2X^1_{e} + X^2_{e}\right)} \qquad (6\text{-}15)$$

Thus, extrapolation requires referencing three successive designs for an element to a common candidate, using the replacement criteria, and evaluating Equation (6-15) for each element using the data. If Equation (6-15) yields a negative value for a_1 , the smallest acceptable value is used for the relevant element.

General Redesign subplan.- This subplan provides for optimizing element selection or joint location to satisfy endo- and system integrity constraints for any cost function. It is addressed to problems where relatively few independent design variables must be assigned (less than about 200, for the 64k core implementation). Redesign uses an implicit direct minimization. Though data for Redesign is developed by Design-Analysis, most of the calculations are independent of it.

Mathematical basis: The modified cost function to be minimized is defined by,

$$C^*(\vec{X}_v) = C(X_q, \rho \vec{X}_{e}) + \sum_{e=1,2\ldots}^{E} \left| C(X_q, X_{e}\gamma_{e}) - C(X_q, X_{e}) \right| \qquad (6\text{-}16)$$

$$v = 1, 2, \ldots V$$

$$q = 1, 2, \ldots V-E.$$

where

C^* is the modified cost function,

X_q are relocation design variables, with

$$\rho = 1 \quad \text{if } G \geq 1$$

$$\rho = \pm G \quad \text{if } G < 1 \tag{6-17}$$

$$G = \max\left(\left|\frac{W^*}{W^E}\right|_a\right) \quad a = 1, 2, \ldots A$$

A is the number of system displacement constraints, and

$X_{ⓔ}$ are the element "size" design variables, with

$$\gamma_{ⓔ} = 1 \quad \text{if } H \leq 1$$

$$\gamma_{ⓔ} = \pm H \quad \text{if } H > 1 \tag{6-18}$$

$$H = \max\left(X^B_{ⓔ} X^F_{ⓔ} X^D_{ⓔ}\right) \text{ over all loadings}$$

The upper set of signs for G and H are used when C* is to be minimized and the lower, when maximized.

Equation (6-16) defines the cost function as modified by an exterior penalty function. The function is adaptive in the sense that all parameters are defined by the current design and its capabilities. Equations (6-17) define the system constraint penalties and Equations (6-18), endo-element. These penalties transform the design objective to minimizing the cost of that acceptable design which can be scaled from the current design. Scaling implied is a direct stiffness scaling of every element to attain system deflection integrity and an individual scaling of each element independently to attain endo-element integrity.

In Redesign, new values of the design variables are assumed to be expressed by

$$X_v^{i+1} = X_v^i + \Omega Y_v^i \tag{6-19}$$

where

X_v^{i+1} is the value of X_v assigned for the $i+1^{st}$ Redesign cycle,

Ω is a scalar chosen to extremize C*, and

Y_v^i is a vector component related to the corresponding component of the gradient vector.

Choosing the Fletcher-Powell method[48] for defining the Y_v^i requires that

$$Y_v^i = \mp B^i \cdot \vec{\nabla} C^* \qquad (6\text{-}20)$$

where

Y_v^i is the v^{th} component for the i^{th} Redesign cycle,

B^i is the i^{th} value of a matrix which for B^i, i=1, may be taken as any positive definite matrix (including the identity) and

$\vec{\nabla} C^*$ is the gradient of the modified cost function

Ω is then chosen to be the Ω which extremizes C^* by substituting Equation (6-18) in (6-15). (Cubic interpolation has been found to be satisfactory for this process.)

Then B^{i+1} is generated by

$$B^{i+1} = B^i + \frac{\Omega \left\{Y_v^i\right\} \left\lfloor Y_v^i \right\rfloor}{\left\lfloor Y^i \right\rfloor \left\{Z^i\right\}} - \frac{H^i \left\{Z^i\right\} \left\lfloor Z^i \right\rfloor H^i}{\left\lfloor Z^i \right\rfloor H^i \left\{Z^i\right\}} \qquad (6\text{-}21)$$

where

$$Z_v^i = \vec{\nabla} C^{*\,i+1} - \vec{\nabla} C^{*\,i} \text{ and}$$

∇ is the gradient operator.

The B matrix of Equation (6-19) effects extrapolation in successive Redesign cycles. Information is accumulated in B in accordance with Equation (6-21) so that if the variable space is quadratic, B coefficients approach second derivatives of C* with respect to design variables,[48] i.e.,

$$B_{ij} \rightarrow \frac{\partial C^{*2}}{\partial X_i \partial X_j} \tag{6-22}$$

as the number of cycles approaches V , the number of design variables.

Redesign process: Tasks to perform Redesign by direct minimization fall into three functions. The first consists of collecting data describing the behavior of the current design. The second requires determining the direction of travel in the design space. The third engenders reanalysis cycles to determine the distance of travel.

Figures 13, 14, and 15 show the logical connections between tasks to perform the three functions. Figure 14 charts the relations among functional parts of redesign within Operation III. Figure 15 shows tasks corresponding to those of Figure 13 of the Specialized Redesign Approach. Thus, the direct minimization involves subroutines that parallel those of the Specialized Redesign and are elected by the user's choice of General Redesign.

Figure 14 shows how the parts of Design-Analysis and Redesign interface for direct minimization. The Redesign functions are detailed in Figure 16. The Design-Analysis functions are charted in Figures 6, 9, and 10. As shown in Figure 13, four of these functions are executed iteratively to establish the value of the scalar, Ω . This iteration defines the "direct minimization loop" lying within the design and optimization loops. When Ω is established, another design or optimization cycle can be initiated.

Collection of data for Redesign involves the steps shown in Figure 13. Two types of data are required for Redesign: data leading to evaluations of the change in cost of the structure due to a change in each design variable and data to determine the cost of a given design. The first type of information is required to establish the direction of travel; the second to evaluate trials to assay the distance of travel.

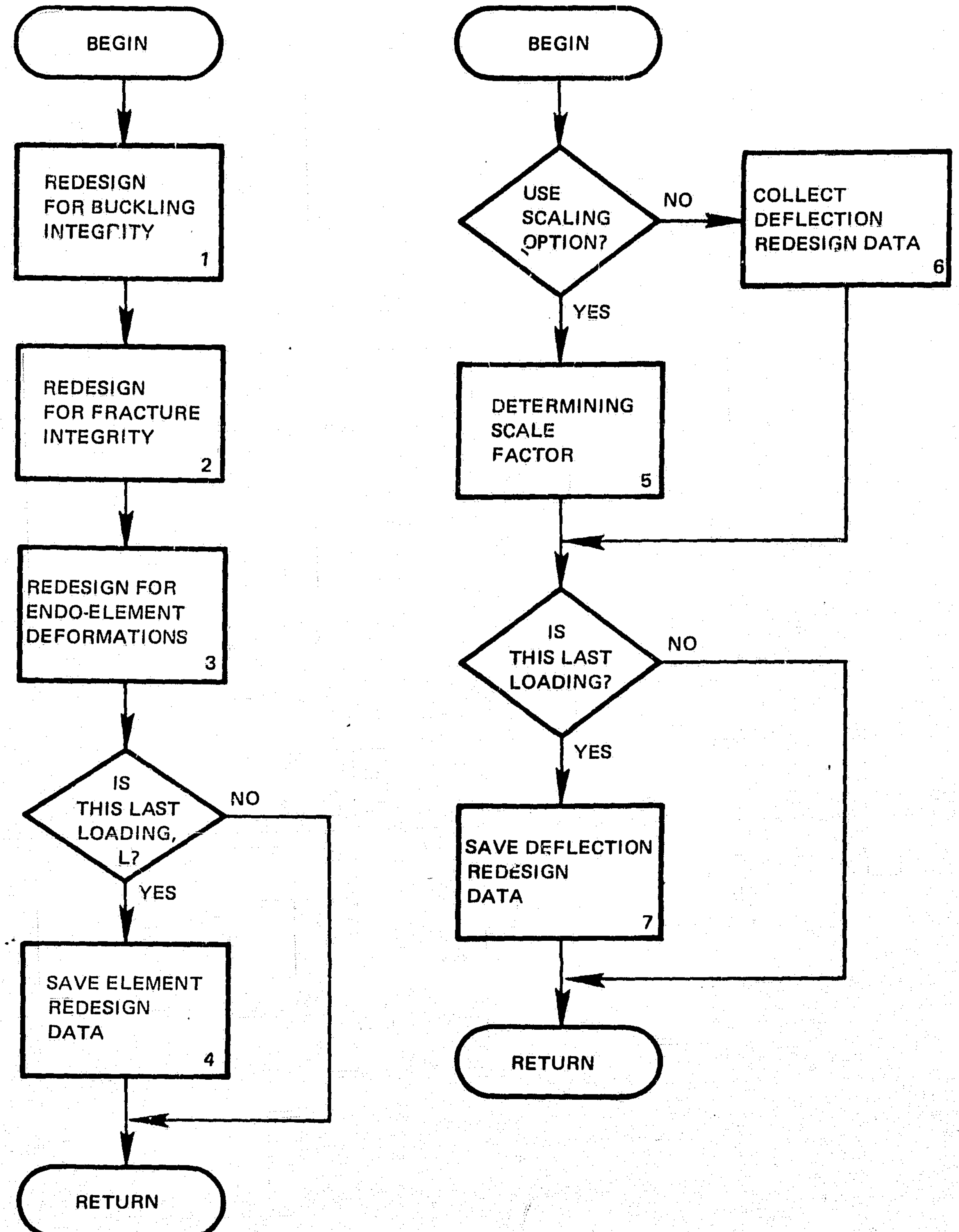


Figure 13. Redesign Design-Analysis Interface

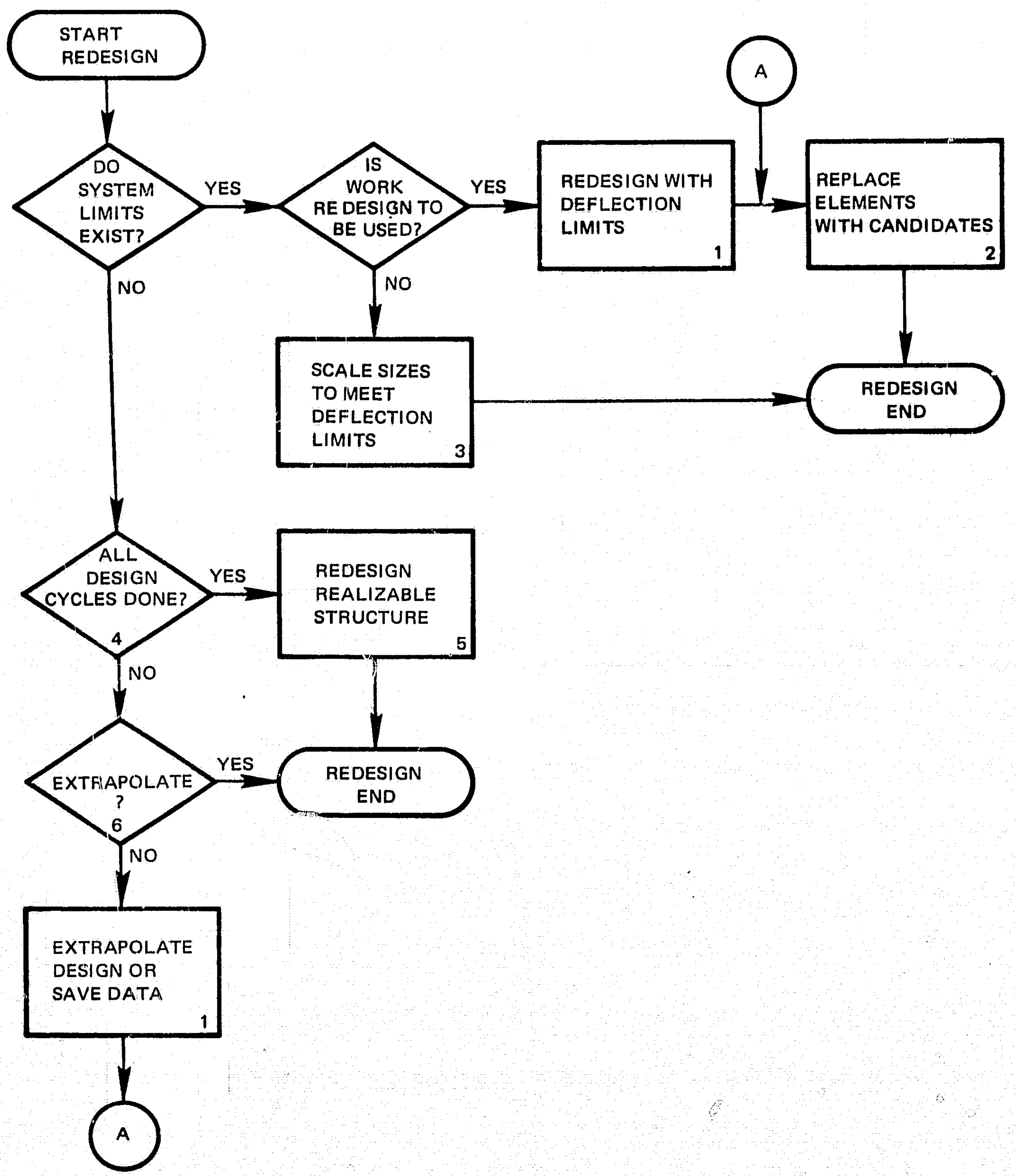


Figure 14. Inclusion of System Requirements in Specialized Design

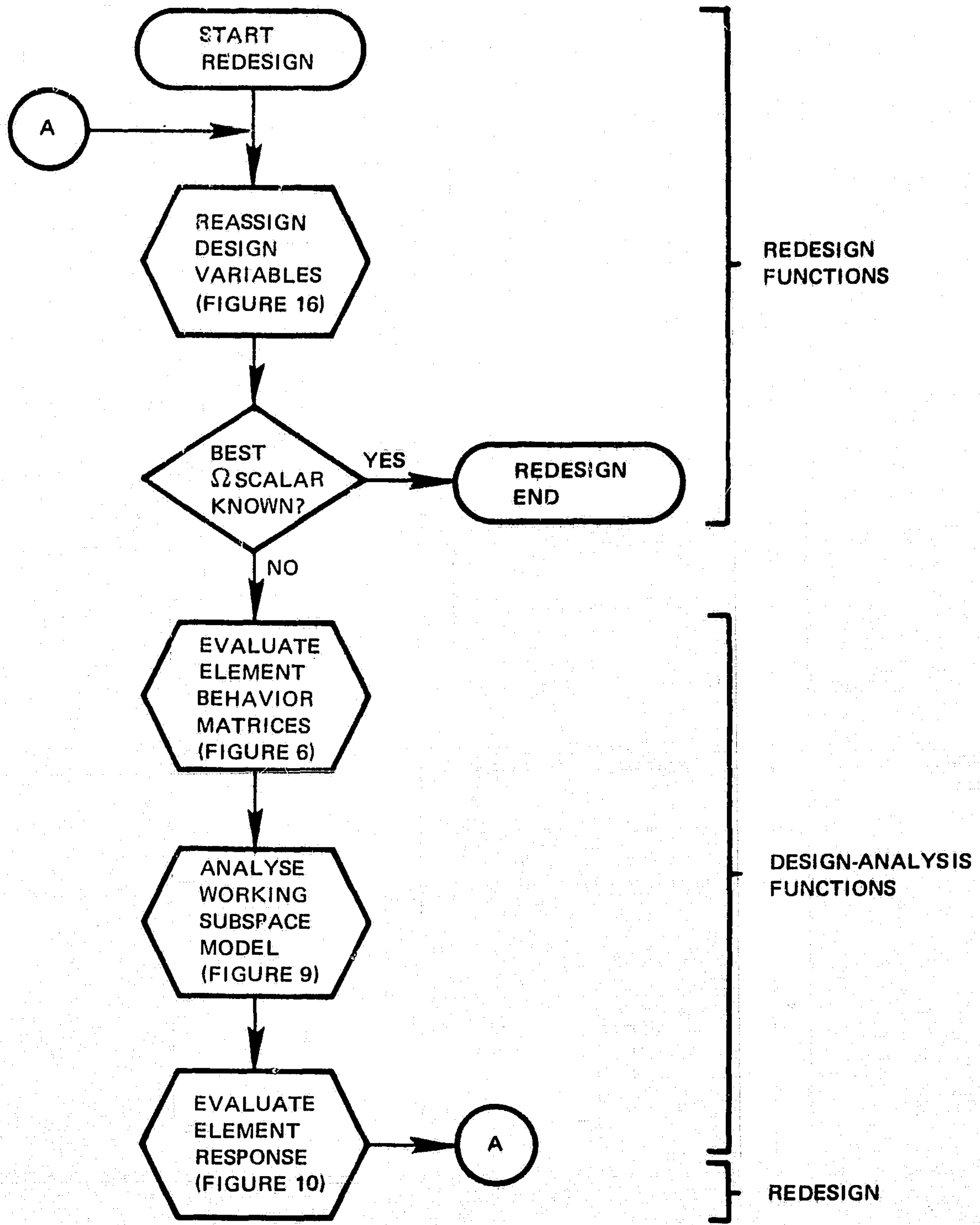


Figure 15. Direct Minimization Functional Relations

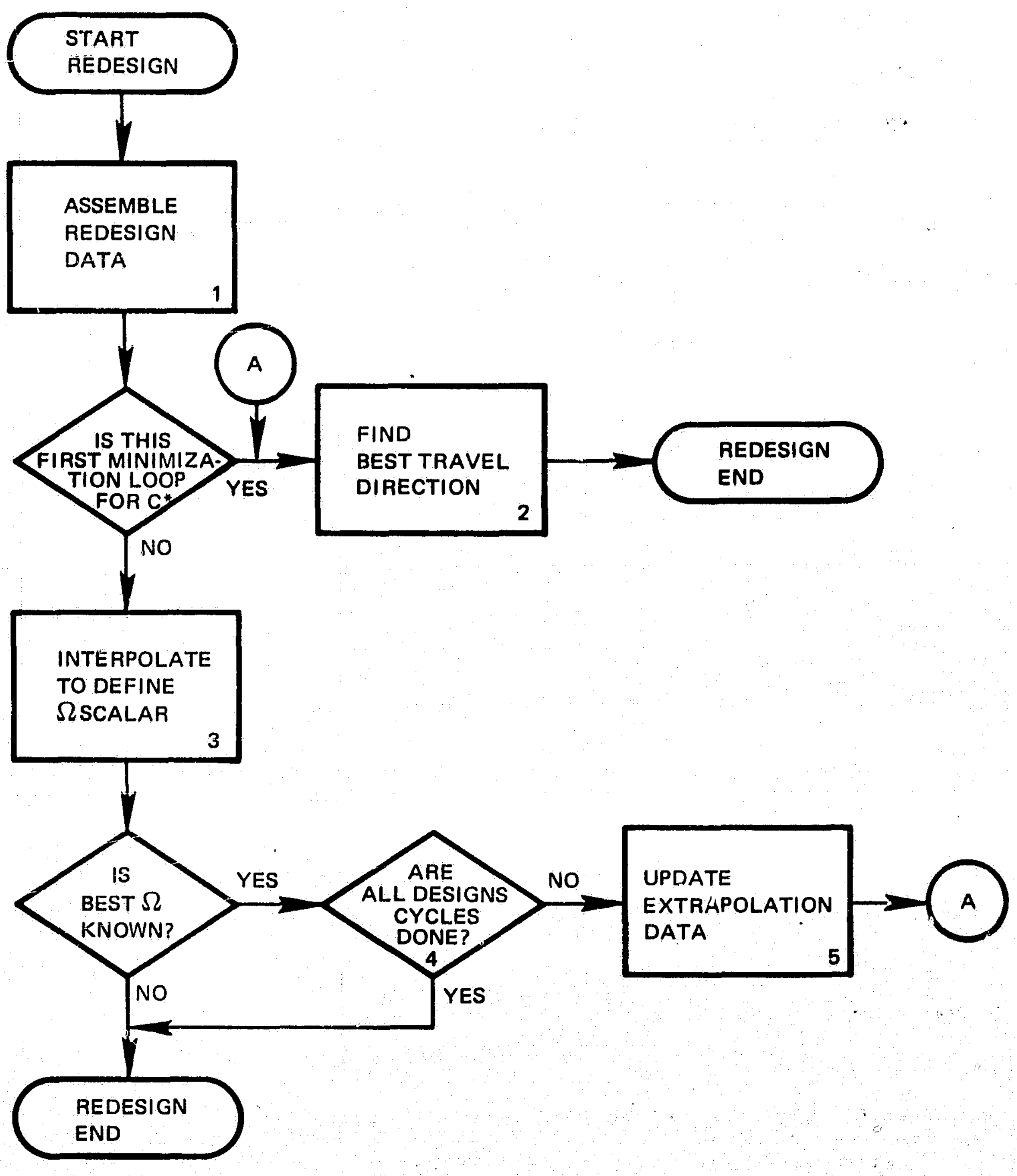


Figure 16. Reassignment of Design Variables by Direct Minimization

The changes to accommodate direct minimization of Redesign involve 4, 5, 6 and 7 of the steps described for the Specialized Design process. Vectors treated in evaluation of element behavior include both real loads and influence loads for each design variable. Thus, besides saving data describing response to the real loads, influence data must be saved.

In accordance with Equations (6-20) and (6-21) finding the direction of travel requires evaluating components of the gradient vector. These are given by the chain rule as,

$$\frac{dC^*}{dX_{(e)}} = \frac{\partial C}{\partial X_{(e)}} + \frac{\partial C}{\partial \rho}\frac{d\rho}{dX_{(e)}} + \frac{\partial \overline{C}}{\partial X_{(e)}} + \frac{\partial \overline{C}}{\partial \gamma_{(e)}}\frac{d\gamma_{(e)}}{dX_{(e)}} \tag{6-23}$$

where $\overline{C} = C(X_q, X_{(e)}\ \gamma_{(e)}) - C(X_q, X_{(e)})$

with $q = 1, 2, \ldots V-E$

To evaluate the second and fourth terms on the right-hand side of Equation (6-23), data describing the change response for each variable $X_{(e)}$ must be compiled. The change of system deformations leads to $d\rho/dX_{(e)}$. The change of element stresses and deflection leads to $d\gamma_{(e)}/dX_{(e)}$.

Not all possible derivatives need to be evaluated. The real loads define the critical design conditions and only the effect of changes on these conditions are needed. For example, suppose Element 4 is over-stressed only under Loading 6, and element deformation limits never exceeded. Then, when the change of stress in Element 4 due to a change of sizing of Element 4 is considered, only the change for Loading 6 needs evaluation.

The set of data required when changes to element e are examined include:

a. $\frac{\partial \sigma j}{\partial X_{(e)}}$ for the loading in which element j defines γ_j under the real loadings

b. $\dfrac{\partial p_j}{\partial X_{e}}$ for the loading in which the deformation p in element j defines γ_j under the real loadings

c. $\dfrac{\partial p_o}{\partial X_{e}}$ for the loading and deflection limit which define ρ under the real loadings.

Data described by a, b, and c above is produced only in the first evaluation of element response in a given direct minimization iteration. In additional iterations required for determining Ω , only the adequacy of the design with respect to the real loadings is needed.

Figure 16 shows the steps which complete the tasks for direct minimization of Redesign. Tasks 1, 2, and 3 control the minimization loop. Task 4 introduces extrapolation in the design cycles.

Details of the steps are as follows:

1. Assembling design data consists of bringing into core data defining the ρ and γ_{e} $e = 1, 2 \ldots E$ of the current design and the derivatives defined by a, b, and c above. These data will usually have to be read from auxiliary storage since they cannot be contained in core with other data required in evaluating element behavior.

2. The best direction of travel is found in accordance with Equation (6-20). This must be evaluated once for each design cycle. If no design cycles have been performed previously, B^1 is taken to be the identity matrix. If other cycles have occurred, B^i of the previous cycle is read from auxiliary storage.

   In evaluating the direction components, and, in fact, for all steps in the Fletcher Powell process, it is desirable to perform analysis in a transformed coordinate basis to reduce manipulation errors and to increase the rate of convergence. Using Stanton's experience,[81] the unknowns are replaced by

$$x_{e} = \sqrt{|h_{ee}|}\; X_{e} \qquad (6\text{-}24)$$

where

x_{e}		replace X_{e} in Equations (6-20) through (6-22),
$\lvert h^k_{ee} \rvert$		is the absolute value of the i^{th} diagonal of B^k
k	=	1, 2, 4, 8 ...

Thus, evaluation of the gradient components is done in the x_{e} basis and transformed to X_{e} by the inverse relation to Equation (6-24). As noted after Equation (6-24) scaling is changed every time the number of cycles doubles so that B approaches the values of the curvatures of the C* space.

3. Davidon's method[82] will provide for interpolating among results for various Ω to find the value of Ω which extremizes cost. This method sequentially prescribes successive trial values of Ω until C* stops decreasing (or increasing) and then uses cubic interpolation to locate the Ω associated with the minimum (or maximum) of C*

Davidon's method involves three phases of calculation as follows:

a. Estimate the Ω associated with the minimum of C*
   This estimate is based on a quadratic. Assume

$$C^* = a_0\Omega^2 + a_1\,\Omega + a_2 \qquad (6\text{-}25)$$

where a_i are constants. Then the point at which C* is extremized is

$$\Omega_x = -a_1/2a_0 \qquad (6\text{-}26)$$

where Ω_x is the value of Ω which extremizes C^* and the extreme is given by

$$C_x^* = -\frac{a_1^2}{4a_0} + a_2 \,. \tag{6-27}$$

C_x^* is the extreme value of C^*.

Knowing the cost of the starting design $C^*(\Omega = 0)$, the rate of change of the cost with respect to Ω, $\partial C^*/\partial \Omega$, and an estimate of C^*, the value of a_0 and a_1 can be found, and Ω_x estimated by

$$\tilde{\Omega}_x = 2\frac{\left(\tilde{C}_x^* - C_o^*\right)}{(\partial C^*/\partial X_v)\,\frac{\partial C^*}{\partial \Omega}} \tag{6-28}$$

where

$\tilde{\Omega}_x$ is the estimate of the extreme value of Ω.

$\tilde{C}_x^*$ is the estimate of the extreme value of C^*, and

$\frac{\partial C^*}{\partial \Omega}$ is the value of the derivative of C^* evaluated at the starting design point.

Then an estimate of the value of Ω_x is taken to be

$$\Omega_x = \begin{cases} \tilde{\Omega}_x \text{ if } \left|\tilde{\Omega}_x\right| < \left|\frac{\partial C^*}{\partial \Omega}\right| \\ \pm\left|\frac{\partial C^*}{\partial \Omega}\right| \text{ (using the sign of } (\tilde{\Omega}_x)\text{) otherwise} \end{cases} \tag{6-29}$$

b. Successive trials for Ω are chosen for Ω_x, $2\,\Omega_x$, $4\,\Omega_x$, $8\,\Omega_x$, ... until the associated C^* ceases to become more extreme. Search is then narrowed to the two values of Ω between which Ω_x is known to lie.

c. The true value of Ω_x is then located by passing a cubic through the four known data: $C^*(\Omega_1)$, $C^*(\Omega_2)$, $\left.\frac{\partial C^*}{\partial \Omega}\right|_{\Omega_1}$ and $\left.\frac{\partial C^*}{\partial \Omega}\right|_{\Omega_2}$ where Ω_1 and Ω_2 are the values bounding the region of the maximum C^*. The optimum Ω is then given by

$$\Omega_{OPT} = \Omega_2 - (\Omega_2 - \Omega_1)\left[\frac{\left.\frac{dC^*}{d\Omega}\right|_{\Omega_2} + \Sigma_2 - \Sigma_1}{\left.\frac{dC^*}{d\Omega}\right|_{\Omega_2} - \left.\frac{dC^*}{d\Omega}\right|_{\Omega_1} + 2\Sigma_2}\right] \tag{6-30}$$

where

$$\Sigma_1 = \left.\frac{dC^*}{d\Omega}\right|_{\Omega_1} + \left.\frac{dC^*}{d\Omega}\right|_{\Omega_2} + 3\frac{C^*(\Omega_1) - C^*(\Omega_2)}{\Omega_2 - \Omega_1} \tag{6-31}$$

$$\Sigma_2 = \left(\Sigma_1^2 - \left.\frac{dC^*}{d\Omega}\right|_{\Omega_1} \cdot \left.\frac{dC^*}{d\Omega}\right|_{\Omega_2}\right)^{1/2} \tag{6-32}$$

with Ω_{OPT} the value of Ω which extremizes C^*. With Ω_{OPT} known, accuracy can be improved by reapplying Equations (6-30) through (6-32) for either the (Ω_1, Ω_{OPT}) pair of points or the (Ω_{OPT}, Ω_2) pair. Note that $dC^*/d\Omega$ can be evaluated directly by adding an influence loading since, in accord with Equation (6-23) its evaluation requires finding the response derivatives.

4. Design cycles are terminated when the user-specified limit on cycles is attained or the design quality of the redesign loop is adequate. The design quality measure is

$$D_Q = \frac{\|\nabla C^*\|}{\|\nabla C^*\|_{max}} \leq C_1 \tag{6-33}$$

where

$\|\nabla C^*\|$ is the Euclidean length of the gradient of C^*, evaluated for the current design point,

$\|\nabla C^*\|_x$ is the maximum (or minimum) value of $\|\nabla C^*\|_x$ attained in the set of design cycles, excluding the first design cycle and C_1 is a constant ≤ 1, specified by the user.

5. Extrapolation data updating is performed in compliance with Equation (6-21). This requires the previous B^i and the values of Y^i and Ω known to optimize the design in the current design cycle.

Justification of the Redesign Plan

The Redesign Plan represents a collection of concepts tested in previous studies. New ideas are advanced to reduce known problems and provide for optimizing complex structures efficiently. The principal decision criterion is economy. This results in selecting two Redesign subplans. It biases formulation decisions and affects detailed decisions in the two subplans. Efficiency considerations influence all decisions but were of primary import in defining details of the Specialized Redesign. The advanced stage of development of the Direct Minimization approach for structural optimization recommends it for the General Redesign method. The next paragraphs justify the major decisions in these terms.

The major formulative decisions involve treatment of the discrete element-material variable as if it is a continuous variable and use of variable equivalencing. Both these decisions are dictated by the desire to optimize at relatively low cost.

Research in discrete variable optimization (integer programming) so far only has produced evidence that it is relatively costly. The best known methods are based on implicit enumeration. These processes enumerate the candidate designs and exclude large numbers of these from consideration by establishing a hierarchy. The exclusion techniques imply that if all cases are tried, those excluded must be non-optimum. Thus, examination of all designs is implicit.

The number of design cycles using implicit enumeration is very much greater than the number of cycles with continuous variables. Gue, Liggett, and Cain[83] cite experience for problems with linear constraints showing 1,829 cycles with 50 integer variables. Though this is a small fraction of the total number of designs (2^{50}), it is very large compared with 50 cycles indicated by Fletcher Powell's[34] experience for direct minimization with continuous variables of a comparable problem. Moreover, the number of design cycles for implicit enumeration increases rapidly with the number of zero-one variables. For example, over 15,000 cycles were required for a problem with 111 zero-one variables. In the comparable direct minimization, the number of cycles is proportional to the number of design variables.

When nonlinear constraints are considered, discrete variable optimization becomes even more costly. A nonlinear implicit ennumeration process is described by Lawler and Bell[24]. Their limited experience indicates it takes much longer: a factor of seven for samples chosen. Fletcher and Powell show a series of problems where nonlinear constraints result in an increase in the number of design cycles by only a factor of three or four over the linear.

Using quasi-continuous variables not only results in a major reduction in search time, but results in little loss in the quality of the optimum design: In one problem Balintfy[85] indicates a factor of 15 reduction in search time with a difference of less than five percent of the approximate optimum with that found by implicit enumeration. Reinschmidt[86] shows the same type of results for a simple truss minimized with respect to weight. Indications are that the curvature of the design space is usually small near the optimum. Thus one of many feasible designs can be found which will be nearly optimum.

Equivalencing of variables improves economy by reducing Design-Analysis data processing and reducing the number of design cycles. It reduces Design-Analysis processing time because it reduces the number of distinct influence vectors. It reduces the number of design cycles because these are proportional to the number of design variables. Then the Specialized Redesign is reduced by a factor proportional to the number of design variables, since relatively few influence vectors are involved for the optimization with respect to displacements. The General Redesign is reduced by a factor proportional to the number of design variables squared.

General Redesign is reduced in proportion to the square of the number of variables.

Considerations of economy, flexibility and state-of-the-art limits prompt including two Redesign processes. Research in substitute optimality methods has shown that their economy over other methods cannot be ignored. Because of limitations of the specialized Redesign, a general method must also be provided.

Though the Specialized Redesign does not necessarily search for an optimum design[58] its basis is readily accepted by engineers and its economy documented. It reduces the number of influence vectors drastically over those required for other methods. As many authors have reported,[55,56,57,58] it usually requires less than 20 design cycles to develop an optimum regardless of the number of design variables. (This, despite the fact that the maximum number of design cycles may equal the number of variables.)

The state-of-the-art limits the Specialized Redesign implicitly to an optimum structural efficiency objective.

Details for treating joint location as a design variable have not been worked out or validated. The significance of designs involving finite elements with multiple coupled elastic modes (shell and solid elements) has not been established. The General Redesign method provides the user with an optimization process which overcomes these objections.

The assumptions upon which the Specialized Redesign is based have been validated by previous studies. The key assumption is that internal forces will not change due to Redesign. This is used in redesigning for endo-element integrity constraints and is implied in the Redesign equations for system deflection integrity. This assumption is validated by the experienced persistent identification of a given active constraint with a particular element during successive design cycles. The cutoff and extrapolation basis is founded in convergence characteristics of the process. The basis has been validated for multiple design variable systems.[5]

Decisions in the General Redesign method are made to provide a validated Redesign process of high efficiency. The form of the penalty function does not restrict designs to feasible designs. It references design merit to a feasible design and seeks the optimum of that design. As such, every design cycle produces a reasonable reference design. By replacing the staged design process (wherein the optimization is performed in stages by varying a penalty scalar as discussed in Section 2) with a realizable penalty, singularities associated with constraint satisfaction are avoided and the penalty is adaptive. By permitting infeasible designs the sequence of designs is expected to be more regular and extrapolation more effective.

The Fletcher-Powell search process is especially suited for the structural optimization environment. Here evaluation of the performance of a given design incurs the biggest part of the data processing in a design cycle. These are the conditions under which Fletcher-Powell search has proven most effective.[87] It has the advantage that it may yield information on the curvatures of the design space in the neighborhood of the optimum. Both Fletcher-Powell search and Davidon interpolation have been validated for the structural optimization problem.[51]

Special Advantages of the Redesign Plan

Special advantages of a computer program implementing the redesign plan will be its economy, its easy extension in scope and its flexibility in applications. These advantages are a consequence of provision of parallel Redesign options and the search procedures selected.

Economy is associated with providing the Specialized Redesign process, including extrapolation and cutoff features, and integrating Design-Analysis and Redesign data processing. The Specialized Redesign provides a low cost optimization tool for the problem of widest interest: selection of many element sizes and material under multiple loading. Both the Specialized and General methods include extrapolation features, (Equations 6-15) and (6-21)), to eliminate inordinate calculations when in the neighborhood of the optimum. Redesign is integrated with element behavior evaluation from Design-Analysis to avoid duplication of calculations and data transfer in reanalysis and Redesign.

The Redesign Plan can be extended easily to provide an optimization tool of wider application. The penalty function can be extended to deal with parameter inequality constraints, or time or space dependent constraints.[88] Improvements in non-linear search methods can replace the Fletcher-Powell-Davidon search in the General Redesign method on a modular basis. If necessary, an implicit optimality method could be incorporated in the plan.

Features of the plan provide for considerable flexibility in optimization. The inclusion of the FORTRAN-defined design measure and deflection integrity constraints permits the user to select parochial design measures. If, for example, he chooses potential energy as the design measure he can optimize a given structural idealization. Complicated displacement integrity criteria, such as aeroelastic divergence, can be treated. For many problems both the Specialized Redesign and the General Redesign will be capable of optimizing the structure. Thus, the designer can use the Specialized Redesign for broad preliminary optimizations, and General for final optimization when the design measure has become well-defined.

Salient Redesign Features

Table XLIV summarizes the principal technical features of the Redesign Plan Many of the features which are unique to this plan involve moderate risk. Risk is associated primarily with the effect of these features on the rate and success of convergence of design cycles. In some cases significant computer program changes would be needed if the feature as planned causes difficulty, but in general changes to eliminate the feature or make it workable would be minor.

All features judged to involve low risk have been examined by at least one investigator. Special problems with the feature have been worked out and described in the references except for joint ganging. Like element grouping, however, this involves no technical problems, only data processing planned to accommodate it.

TABLE XLIV

TECHNICAL FEATURES OF THE REDESIGN PLAN

No.	Feature	Risk
1.	Use of input FORTRAN statement or a built-in modular design measure	Low
2.	Simulation of discrete candidate selection as a quasi-continuous variable	Moderate
3.	Use of input FORTRAN statement displacement constraints.	Low
4.	Inclusion of joint relocation as a design variable	Moderate
5.	Grouping of elements and ganging joints	Low
6.	Redesigning for deflection by the work method	Moderate
7.	Cutoff and extrapolation of the Specialized Redesign	Low
8.	Use of adaptive penalty function	Moderate
9.	Use of Fletcher-Powell search and extrapolation	Low
10.	Use of Davidon interpolation	Low
11.	Cutoff of Fletcher-Powell process	Moderate
12.	Selection of the Direct Minimization Approach as opposed to Optimality	Low

The items of highest risk involve features 2 and 4. Simulation of discrete variables as continuous has been demonstrated for selecting among discrete element sizes for trusses. This is not true for the general finite element nor for the element-material variable concept. Development work is required to ensure that the technique proposed results in satisfactory convergence trends and cutoff.

Little work has been done in treating joint location as a design variable. Though no special difficulties are anticipated for trusses and frames, continuum finite elements raise a problem. The optimization must distinguish between joint changes which modify discretization and those which result in real improvements to structural design.

Redesign using the work method was recently suggested by Berke and requires development. In preliminary studies; Gellatly has experienced periodicity and divergence in sequential designs using the method. There is no doubt that these shortcomings will be overcome. In some problems, Gellatly has experienced an order of magnitude reduction in the number of design cycles over those required with a minimization search method. It is expected that these benefits will be extended with elimination of convergence difficulties.

Risk is associated with the adaptive penalty function only because it is untried. There is no reason to believe it cannot be successfully used or the penalty modified to make it usable.

The cutoff criterion, Equation (6-29), for Fletcher Powell may result in stopping designing while considerable improvement in design is still possible. It is the experience of Fletcher and Powell[48] that at least as many designs should be produced as there are distinct variables. Since each design cycle will be relatively costly, it is judged that their conservative view should be abandoned for structural optimization.

The direct minimization approach provides an optimization process of proven validity for structures. Nevertheless, few study results of optimality approaches have appeared in the literature. If these should prove more efficient or more general, it may be desirable to incorporate them in the program and discard the direct minimization approach. If so, considerable program change would be needed. This change would occur at the Design-Analysis and Redesign interface (Figure 9) and in replacement of the logic outlined by Figure 15.

Section 7
VALIDATION PLAN

The purpose of this section is to validate the proposed plan for various types of engineering problems encountered in both the aerospace and the commercial engineering fields. A set of problems is selected to test the versatility and efficiency of the proposed plan. These problems are considered in order to determine if the plan will yield a computer code capable of solving them and describe, generally, how the solution would be effected.

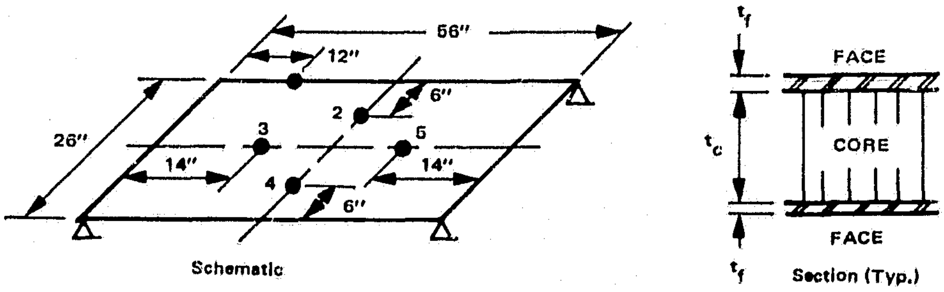
Validation Problems

The five problems discussed in this section are intended to be typical engineering problems. No attempt is made to consider all possible classes of problems. Optimization with respect to either cost or structural weight is emphasized in the belief that these two criteria commonly are of the greatest interest. Other optimization criteria could be used, however, as dictated by individual requirements of specific design problems. Because cost functions can vary so widely from company to company, no attempt is made to attach a dollar value to cost functions considered in these validation problems. Geometric configuration of the problems and assumed materials and materials properties have been selected on the basis of general reasonableness in accordance with each problem considered.

Discussion of each problem is presented in two tables: one table defines the problem and the second gives the problem as reformulated for computer analysis under the plan. The problem definitions are developed based on the criterion optimization capabilities described in Section 2. The reformulation is made to make the problems suitable for the plan as defined in Sections 3, 4, 5 and 6.

Design of a composite sandwich panel. - Table XLV describes a spacecraft equipment mounting panel for which the crosssectional geometry and material parameters are to be specified to yield a structure of minimum weight. Solution of this problem is deemed within the state-of-the-art.

TABLE XLV
STATEMENT: COMPOSITE SANDWICH PANEL DESIGN PROBLEM



Problem Precis

1. Design objective: minimum structural weight

2. Design variables: t_f, t_c, and orientation of material fibers.

Prescribed Data

1. Geometry: as shown

2. Materials:

   a. Face Sheets - $E_{11} = 15 \times 10^6$ psi, $E_{22} = 10 \times 10^6$ psi, $G = 4 \times 10^6$ psi, Density - .054#/in^3, Allowable = 20,000 psi in Tension, 10,000 psi in Shear.

   b. Core - $E_{11} = E_{22} = 0$, G = 32,000 psi, 16,000 orthogonal directions, Density = 3.1#/ft^3

3. Boundary Conditions:

   Force - 10# weights at points 1, 2, 3, 4, and 5; uniformly distributed weight of 0.1#/in^2 over panel; panel weight to be neglected. All weights under 15 g's acceleration normal to panel plane.

   Displacement - Laterally Supported at the four panel corners.

Design Constraints:

1. Behavioral: Von Mises failure criteria for face sheets, maximum principal shear for core.

2. Variable: Uniformly thick face skin of gage greater than .05 core of minimum thickness zero.

Table XLVI defines the reformulation. Different orthotropic material orientations are accommodated by providing candidate materials with different particular orientations. Different thicknesses for core and face sheets are admitted by treating these components as independent sets of finite elements. Only maximum and minimum gages are specified to reduce input. These are sufficient to developed the ideal design.

Element groupings ensure that a single-fiber orientation is chosen independently for the top face, core, and bottom face.

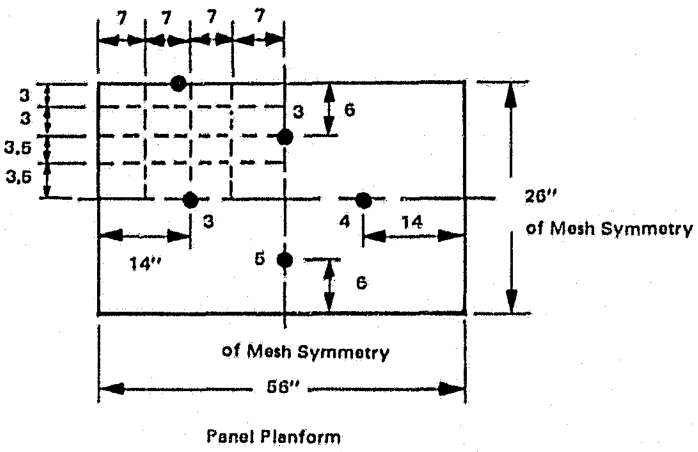
This problem involves no special treatment. Thus, minimum printout and no active graphics are chosen. Design could be performed by General Redesign rather than by the Specialized Redesign for this problem. Thus, the designer might wish to optimize both ways and compare results, though only Specialized Redesign is indicated in Table XLVI.

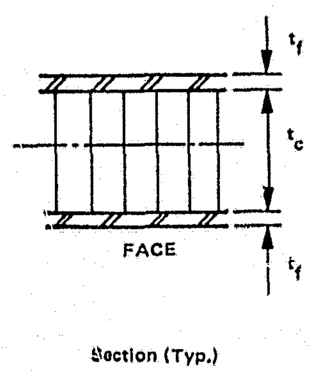
Design of a satellite dispenser structure. - Table XLVII describes a structural design problem involving a truss system whose function is to hold and release six space satellites into synchronous orbits. A detailed description of the geometry and design requirements for this structure is given by Young and Christiansen. [55] The system is required to be the minimum weight structure which meets specified minimum resonant frequency requirements and does not fracture under two acceleration conditions.

This design problem involves two special considerations. The first is the requirement to design to a given minimum frequency. The second is to accommodate the arbitrariness of the direction of the lateral rigid body acceleration. Both considerations can be treated under the optimization plan, though the problem is nominally beyond the scope of the criterion problem.

The frequency requirement is encompassed by extra calculations in the baseline analysis. These develop the D'Alembert forces and deflection limits by the following steps:

TABLE XLVI
REFORMULATION: COMPOSITE SANDWICH PANEL DESIGN PROBLEM





Design Control Data

1. Design measure: built-in modular cost function (Eq (4-1)) with $\alpha = 0$, $\beta = 1$, $c_i = 1$.
2. Alternate element to be used as subspace candidates with only membrane stiffness for the face sheets and shear for the core.
3. Specialized Redesign with least cost option, idealized design.
4. Minimum printout level.
5. No graphics interface.

Prescribed Data

1. Geometry: as defined by 9 x 9 mesh shown. No guesses for t_f, t_c, and no material orientation selected.
2. Materials: starting selection not prescribed.
3. Boundary conditions:

   Force - $150^{\#}$ weights at points 1, 2, 3, 4, and 5; uniformly distributed weight of $0.1^{\#}/in^2$ over upper face skin.

   Displacement - zero displacements at four panel corners.

Design Constraints

1. Behavioral: Maximum principal stress failure criterion for face sheet materials, maximum sheer for core.
2. Variable:

   a. Materials - a material defined, and its properties specified, for orientation angles of 0, 10, 20, 30, 40, 50, 60, 70, 80, and 90 degrees: one set for the skins and a second for the core.

TABLE XLVI (Continued)

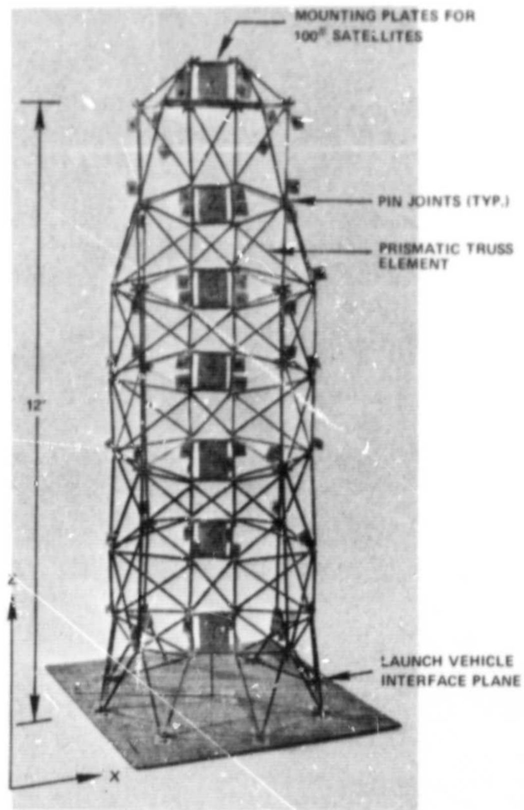
b. Element candidates - skin and core candidates of appropriate materials

- skin gages limits of 0.05 and 2.0 in.*
- core thickness limits of 0.1^{-4} and 10 in.*

c. Equivalencing - three element groups assigned: one for the upper panel face, one for the core, and one for the lower face.

*Limits are entered by citing the minimum and maximum candidates. No element is represented by a size less than the smallest size candidate during designing.

TABLE XLVII
STATEMENT: SATELLITE DISPENSER DESIGN PROBLEM



Problem Precis

1. Design objective: minimum structural weight.

2. Design variables: cross sectional area of each truss member.

Prescribed Data

1. Joint locations (as detailed in Reference 55.)

2. Material: all elements made of 2024T4 aluminum.

3. Boundary conditions:

   Force: 0°g acceleration in the z direction, 2g acceleration in any direction normal to z with dead load of truss and 100 pound satellites excited.

   Displacement: All joints on the interface plane are prevented from displacing.

Design Constraints

1. Behavioral: Buckling and overstress (Von Mises criteria) precluded treating (8g, 2g) load as ultimate, minimum frequency of 10 Hz admissible.

2. Variable: All elements to be prismatic tubes with a minimum gage of .05" and minimum O.D. of 0.5" and a maximum O.D. of 6".

1. Calculate the first resonant frequency, ω, and mode shape $\hat{x}$ for the given design. (To anticipate this calculation and accelerate the design process, an initial guess of element sizes is specified rather than permitting the program to make an arbitrary selection.)

2. Form the D'Alembert force vector y. This is given by $1/C_1$ [M][x] where M is the mass matrix and C_1 is an arbitrary constant chosen to be sufficiently large so the D'Alembert forces will be much smaller than the loads due to rigid body accelerations.

3. Calculate the value of the deflection limit by finding deflections under the D'Alembert forces. This limit is selected for the degree of freedom of maximum magnitude in the first mode. The limit is developed as $\omega_\phi{}^2/C_1 \; y_{max}$ where ω_ϕ is the lowest desired resonant frequency in radians per unit of time. The deflection is required to lie between 0 and this limiting value.

These steps transform the dynamic criteria into static under the assumption that element changes in Redesign have little effect in changing the mode shape. The factor C_1 is introduced to ensure that the stresses associated with the D'Alembert forces will not influence candidate selection by inducing overstress or buckling failure for the D'Alembert forces.

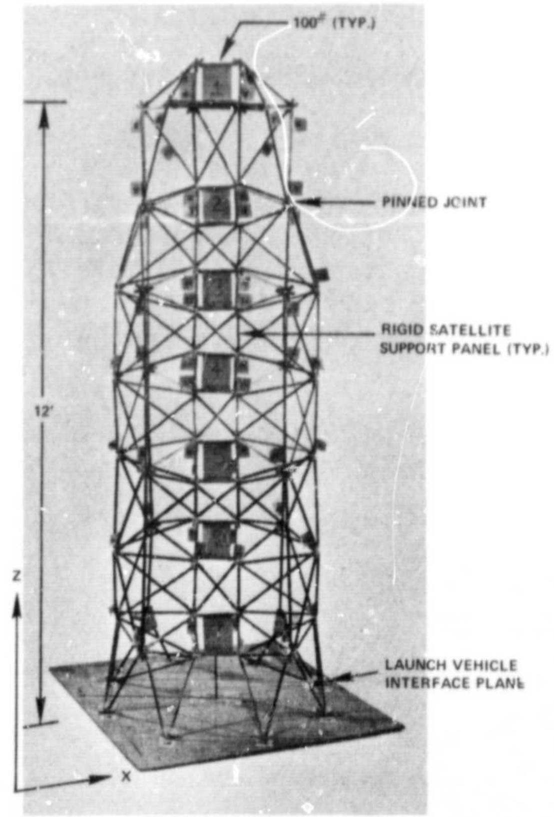
Since during design iterations the D'Alembert forces are not changed, the designer may wish to monitor the deformations due to the D'Alembert forces. Then he can enforce calculation of new D'Alembert forces when dramatic mode shape changes occur by stepping up to the optimization loop after any particular design cycle. Alternately, he could limit design cycles to two or one and approach the frequency design process successfully used by Young and Christiansen.

Arbitrariness of the lateral acceleration direction is treated by specifying a multiple number of loading conditions. Each relates to a particular orientation of the acceleration vector over a 90° sector.

Table XLVIII recapitulates the dispenser design problem as reformulated for the optimization program. It implies extension of Baseline Analysis to define the frequency requirement in static form. It includes the multiple loadings to accommodate four orientations of the lateral acceleration vector. Because this structure is so simple, the analyst can designate all the redundant bar elements for working subspace candidates and Design-Analysis will be exact. Because the design variables are only the areas of the truss elements, the minimum and maximum sizes are sufficient to define cross section characteristics for the idealized design desired.

Treatment of the displacement constraints in implicit in the Design-Analysis. The requirement that the satellite support panels remain planar is introduced only in the Baseline Analysis. (In NASTRAN it takes the form of a multipoint constraint.) This requirement is implicit in the subspace vectors and need not be considered explicitly in Reanalysis.

TABLE XLVIII
REFORMULATION: SATELLITE DISPENSER DESIGN PROBLEM



Design Control Data

1. Design measure: built-in modular cost function (Eq (4-1)) with $\alpha = 0$, $\beta = 1$.

2. Redundant bar elements to be used for subspace candidates.

3. Specialized Redesign with least cost option, buckling activated, idealized design.

4. Minimum printout level.

5. Graphics interface at minimum level.

Prescribed Data

1. Geometry: as detailed in Ref. 55 all real joints modelled. Non zero guesses given for element sizes.[a]

2. Materials: 2024-T4

3. Boundary Conditions:

   Force -
   a. $100^{\#}$ weights on the six support panels, dead weight under 8g axial 2g lateral with four loading vectors required for a lateral acceleration oriented at 0, 30, 60, and 90° with respect to the x axis.

   b. D'Alembert forces from the lowest resonant mode.

   Displacement -
   a. All four points of each support panel remain in a plane.

   b. All points on interface plane cannot displace.

TABLE XLVIII (Continued)

Design Constraints

1. Behavior:

   a. Fracture Criterion: Von Mises yield parameters specified for 2024-T4

   b. Displacement limits for only the D'Alembert loading found in the Baseline Analysis for the degree of freedom with maximum deflection [a].

2. Variable: elements include two limiting candidates - one with minimum limits (gage of 0.05 and O.D of .5), and one at maximum (solid bar with 6" O.D).

(a) A good initial guess of sizes is useful in reducing the number of design cycles. In each cycle, the sizes establish the dead weight in the Baseline Analysis and this weight is used in the design iterations. Thus, a good guess is important in establishing the resonant mode shape and the status loading.

Though design for the frequency requirement is satisfactory, the plan does not accommodate the arbitrary lateral acceleration in an efficient way. If the truss element stresses are known for two orthogonal directions of the lateral acceleration vector, it can be expressed for any direction as a linear combination of these results. Then the stress for the worst orientation of the vector for a particular truss element is given by

$$\sigma_{max_{(e)}} = \sqrt{\sigma_1^2 + \sigma_2^2}$$

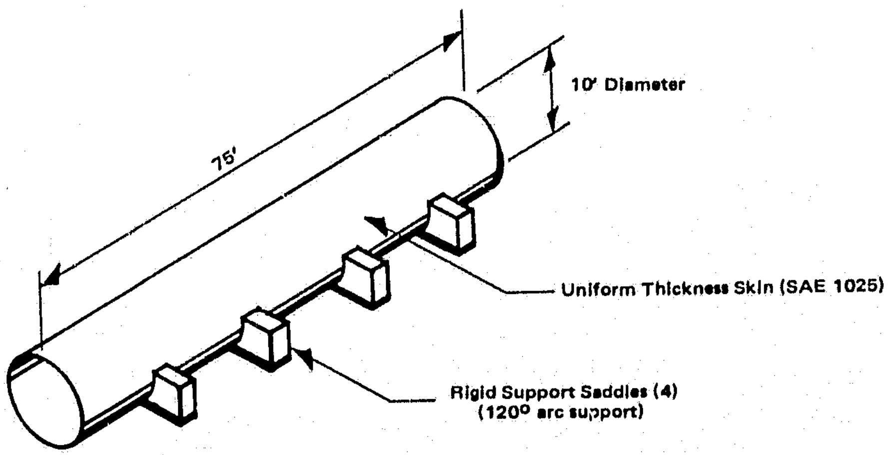
where σ_1 and σ_2 are the stress in the element due to acceleration in the orthogonal directions. Thus two static loadings would be sufficient to insure fracture and buckling integrity, regardless of the direction of lateral acceleration, if the failure analysis software were enlarged to encompass this possibility.

Design of a storage vessel.- Table XLIX defines an optimization requiring determining optimum skin thickness and support locations for a cylindrical storage vessel. Solution of this problem is currently beyond the state of the art of existing optimization software.

Table L defines how this problem could be reformulated for solution by a computer program implementing this plan. Symmetry of geometry and boundary conditions admits consideration of only half of the system for the design. Optimization of skin gage is intrinsic with the plan. The support optimization is addressed by seeking the position of the four support sections indicated by A, B, C, and D in the table. Joints at a support are ganged and each gang restricted to axial relocation to simulate the support relocation variable. All cylinder elements are in one group to insure uniform gage design.

This problem highlights an intrinsic problem in the state-of-the-art, which the plan does not resolve. The problem is that relocating the support planes is represented in the mathematical model by changes in the mesh geometry. These changes, while simulating relocation of the supports, also change the discretization error in the solution. Without a process to discriminate between the effect of changes to discretization errors and real changes in geometry, the success of

TABLE XLIX
STATEMENT: TANK SUPPORT DESIGN PROBLEM



Problem Precis

1. Design objective: minimum structural weight.
2. Design variables: skin gage, support positions.

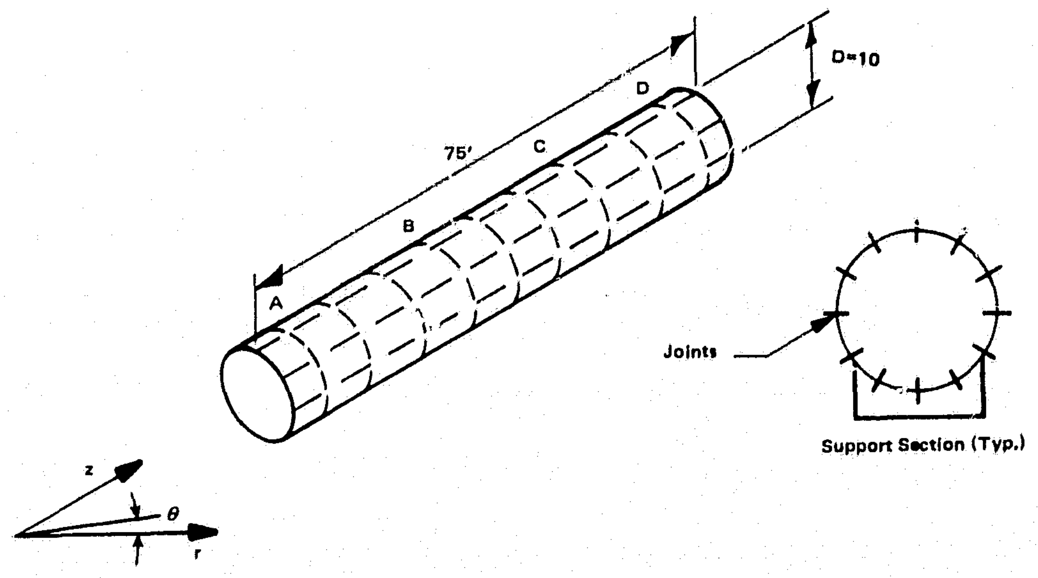
Prescribed Data

1. Geometry: as shown.
2. Material: SAE 1025 steel
3. Boundary conditions:
   a. Force - dead weight of the vessel plus $45^{\#}/ft^3$ for the fluid in the filled vessel. (No overpressure)
   b. Displacement: Simply - supported over the 120° arc of the saddle at four saddle positions.

Design Constraints

1. Behavioral:
   a. Fracture is defined by the maximum principal stress. No point on the skin may fracture under the loading with a safety factor of 2.0.
   b. Vertical deflections must be less than 0.15 in. everywhere under the loading.
2. Variable: Skin must be uniformly thick of standard gage greater than .10".

TABLE L
REFORMULATION: TANK SUPPORT DESIGN PROBLEM



<u>Design Control Data</u>

1. Design measure: built-in modular cost function (Equation (2-1), with $\alpha = 0$, $\beta = 1$.

2. Alternate elements to be used for subspace vector candidates with membrane elasticity only for panel far removed from the supports.

3. Direct minimization redesign option, realizable design required.

4. Minimum printout level.

5. Active graphics with computer delay.

<u>Prescribed Data</u>

1. Geometry: as defined by 6 joints around the periphery (using symmetry) and at least six joints along the length. Guessed gage of 0.25 in.

2. Material: elastic constants and stress allowables for 1025 Steel.

3. Boundary Conditions

   a. Force - Lumped loads at the joints calculated assuming the tank is rigid; uniformly distributed loads for the dead weight of the vessel, based on the assumed gage, geometry, and material density.

   b. Displacement - pinned to permit rotation about the tangent to the cylinder at the five joints of each support section.

TABLE L (Continued)

<u>Design Constraints</u>

1. Behavioral:

   a. Material allowables for maximum principal stress fracture, safety factor of 2.0 for loading given.

   b. Deflection constraint defined by FORTRAN statements which scan all deflections for the maximum vertical and compare with 0.15 in.

2. Variable:

   a. Element candidates include only standard gages; none smaller than 0.10 in.

   b. Equivalencing - Four joint gangs assigned - one for each of the sections A, B, C, and D. The gang relocation data for each of these gangs will limit relocation to points along the z axis such that the topological relation between the gangs cannot change. For example, the gang for section A must stay to the left of that of section B and cannot pass off the cylinder.

      - All elements in the same group.

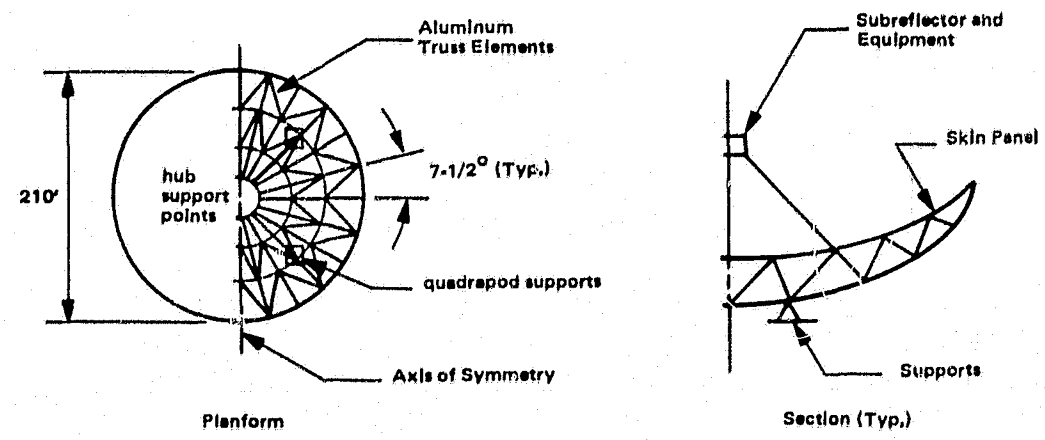
the optimization may be meaningless for the real system. Thus, though this problem is nominally within the scope of the plan, the success of optimization will hinge on the designer's guidance through the active interface.

Design of a Steerable Antenna Reflector. - Table LI defines an optimization which requires selecting element sizes for a reflector to minimize the R. M. S. of deviations of deformations from a best fit paraboloid. This table gives a representation of the problem, omitting non-critical details. Details of the geometry of a typical reflector are given by Weaver and Kane.[89] Details of a successful design process for this problem are given by Von Hoerner.[17]

To reduce optimization costs, it is performed in two phases. During the first phase, the material is redistributed to reduce the R. M. S. error without changing the structural weight. During the second phase, the structure is scaled to insure structural integrity. This approach is possible because R. M. S. error is dependent only on the distribution of weight, not on the magnitude. Thus the material distribution is developed considering only dead weight acting with the reflector in the zenith and horizon positions. The scale factor is found considering structural integrity under the wind loads. The wind is not considered in the design. The designer, communicating through the active interface, provides for problem redefinition for the second phase when the first phase optimization is complete.

Tables LII and LIII cite the problem formulations for the two phases of optimization. In Phase I (Table LII) the design objective requires special treatment. The user provides his cost function in the form of FORTRAN statements. These statements calculate the R. M. S. error as a function of the deformations due to the first two loading conditions: the zenith and horizon dead loads. The R. M. S. is calculated under the assumption of a constant weight by scaling deformations assuming they vary linearly with weight. This scaling requires knowledge of the current design weight (developed by the FORTRAN program) and the weight budget (a constant in the FORTRAN code). Programming of the cost function is simplified by forcing the Baseline Analysis to produce deformations in a parabolic coordinate system. Note that the choice of the displacement coordinate system would persist in the subspace analysis if it is used in baseline.

TABLE LI
STATEMENT: ANTENNA REFLECTOR DESIGN PROBLEM



Problem Precis

1. Design objective: deformations to deviate from a best fit paraboloid with a minimum R.M.S. for a given weight.

2. Design Variables: cross sectional area of each truss element.

Prescribed Data

1. Geometry: as shown schematically and further detailed in Reference 89.

2. Material: 6061-T6 aluminum.

3. Boundary conditions:

   Force - dead weight of the truss, reflector skin panels, subreflector and equipment for the zenith and horizon positions; three wind loadings.

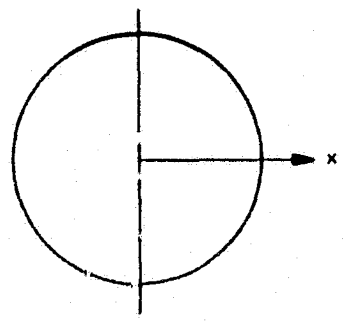
   Displacement - zero displacements at four hub support joints.

Design Constraints

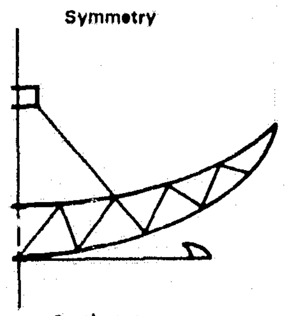
1. Behavioral: element buckling is precluded. Fracture is defined by beam-column and principal stress interaction.

2. Variable: All rib surface elements to be of the same size. All rib bracing to be of the same size. All cross bracing to be of the same size. No cross section is to be less than 0.5 square inches and no gage less than 0.10". All elements are to be double angle sections.

TABLE LII
REFORMULATION: PHASE I-REFLECTOR DESIGN PROBLEM



Planform



x Section

Design Control Data

1. Design measure: introduced as FORTRAN statements.

2. No subspacing permitted.

3. General Redesign with buckling disregarded, idealized design required.

4. Minimum printout level.

5. Graphics interface at highest level.

Prescribed Data

1. Geometry: as shown and detailed in Reference 89. All real joints modelled.

2. Material: 6061-T6.

3. Boundary conditions:

   Force - panel, subreflector, feed equipment, and structure dead weights under z and x axis gravity (two loadings).

   Displacement - zero displacements at four hub support joints.

Design Constraints

1. Behavior: None. Fracture and displacement criterion disregarded by omitting their specification.

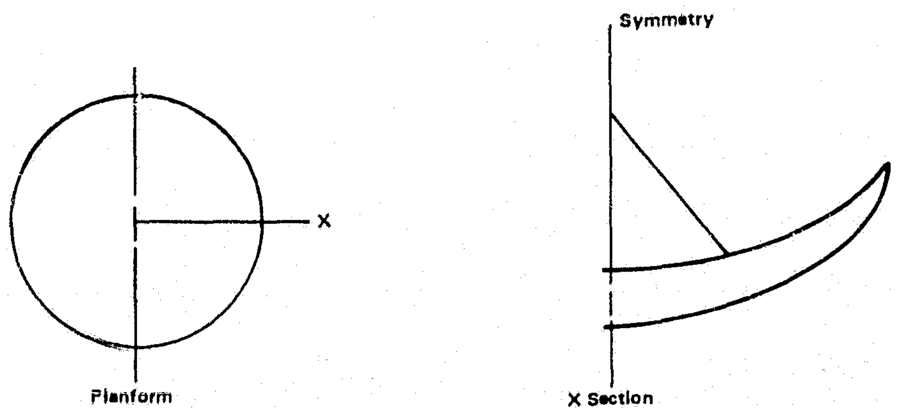
2. Variable:

   a. Material - elastic constants given for 6061-T6 aluminum.

   b. Element candidates - one cross section given of very large area.

   c. Equivalencing - three element groups assigned: one for rib surface elements, one for rib bracing, and one for other bracing.

TABLE LIII
REFORMULATION: PHASE II-REFLECTOR DESIGN PROBLEM



Design Control Data

1. Design measure: full stressing (No parameters given for built-in function)
2. No subspacing permitted. (Arbitrary choice of user)
3. Specialized Redesign involving scaling only option, realizable design.
4. Minimum printout level.
5. No active graphics.

Prescribed Data

1. Geometry: as shown and detailed in Reference 89. All real joints modelled.
2. Material: 6061-T6.
3. Boundary conditions:

   Force - panel, subreflector, feed equipment and structure dead weights under x and z axis gravity (two loadings).
   - wind loadings (three loadings)

   Displacement - zero displacements at four hub support joints.

Design Constraints

1. Behavior: Buckling precluded, fracture with interaction allowables under principle stress.
2. Variable:
   a. Material - 6061-T6 aluminum data.
   b. Element candidates - double-angle sections with gage ≥ .10in and area ≥0.5 in^2.
   c. Equivalencing - three element groups assigned: one for rib surface elements, one for rib bracing, and one for all other bracing.

For Phase I, the designer has chosen to avoid design iterations using subspace analysis and performs iterations only in the total optimization loop to avoid approximate Design-Analysis. He also decides to monitor deformation contours in active graphics to better understand the structural behavior and participate in problem reformulation for Phase II. Since only relative element sizes are required of Phase I, a simplified set of candidates are defined to reduce calculations in redesign. To retain the detailed description of the design for Phase II, the designer specifies that he wants the idealized design produced in Phase I optimization.

Phase II of optimization is entered after the Phase I optimization cycles are completed. This is determined either by design convergence criteria or the designer, since he elects to be active in this design problem.

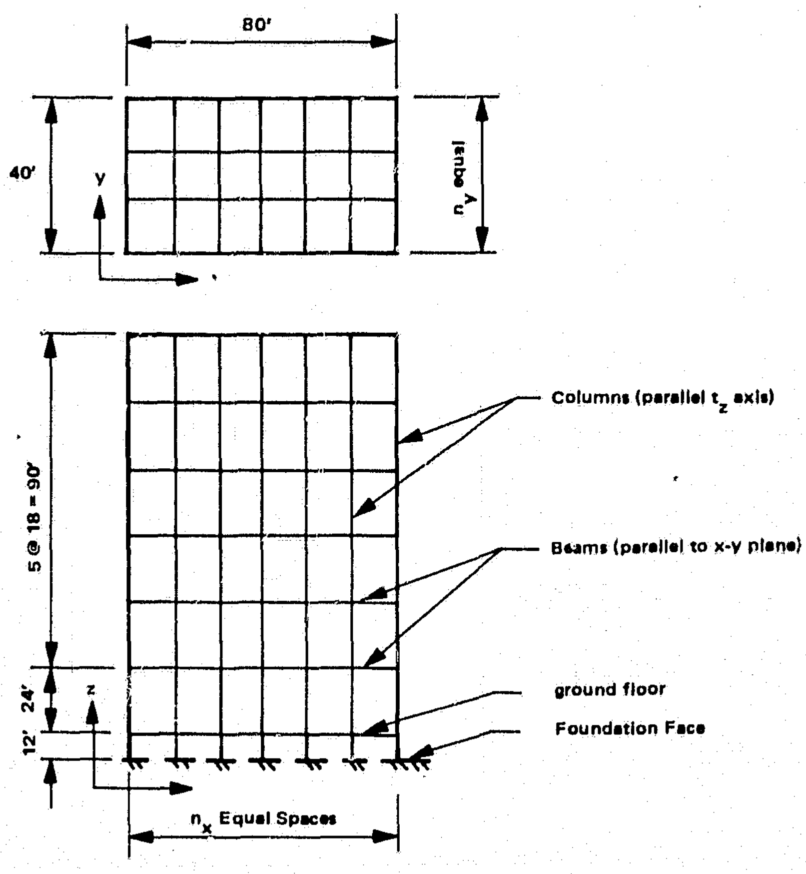
Table LIII cites the formulation for the Phase II calculations. A single design cycle is permitted, the designer intervenes, and the final sizes are selected. During the single cycle, under the scaling only option, the scale factor is determined, design scaling performed and elements selected from candidates. Another cycle could be used to produce the R. M. S. of the final design.

The plan provides for efficient treatment of this design problem. The flexibility to accommodate the complex design objective is embodied in the FORTRAN statement mode of cost function specification and displacement constraints and the ability to interface actively with the designer during optimization.

Design of an office building frame considering earthquake loads.- Table LIV describes a problem involving design of an office building for maximum cost effectiveness (minimum cost per usable volume). Various aspects of this problem have been discussed by Khan et al.[90] and Hill[91]. In the form defined here, the problem is beyond the scope of existing optimization software.

Table LIV summarizes the principal features of the design problem. The usable volume is a function of the total volume in a prism delineated by four column lines, a ceiling and a floor. The building cost is expressed as a function of costs per pound of steel costs and per joint connection. Both the costs are a function of the number of columns.

TABLE LIV
STATEMENT: BUILDING FRAME DESIGN PROBLEM



Schematic of Building

Problem Precis

1. Design objective: the number of column lines, equally spaced, and element sizes such that the building will be of minimum cost per unit of usable volume.

2. Design variables: cross sections of column and beam elements of the main frame, N_x and N_y.

Prescribed Data

1. Geometry: elevation of each floor, dimensions of outside envelope as shown above.

2. Material: ASTM-A7 Steel.

3. Boundary conditions:

   Force -
   a. Combined wind (in worst direction), dead load, live load, and snow load with a safety factor of 1.0 for a uniformly distributed live load of 150 psf.

   b. Same as a. but with checkerboard live loading.

   c. Dead load, live load and snow load with a safety factor of 1.5 for uniformly distributed live load of 150 psf.

TABLE LIV (Continued)

d. Same as c. but with checkerboard live loading.

e. Distributed earthquake accelerations (in worst direction) with a safety factor of 1.0 and with magnitude based on first resonant frequency of building.

Displacement - each column clamped at the foundation face.

Design Constraints

1. Behavioral:

   a. element buckling and overstress is precluded. Fracture is defined by principal stress considering int raction of buckling for columns.

   b. Maximum endo-element deflections are limited to 1/360 of span for beam elements for loadings a through d.

   c. Under earthquake loads accelerations displacements limited by building code.

2. Variable: All sections AISC standard WF and H. The following components must have a common size:

   a. All interior columns between any pair of floors.

   b. All floor beams except roof and ground floor.

   c. All roof beams.

   d. All ground floor beams.

This problem has two special features. First, the design variable, the number of column lines, is intrinsically a discrete variable. Second, the earthquake loading magnitudes, loading f and g, are a function of the resonant frequency of the structure. Thus, the loading is design dependent; a condition beyond the scope of the criterion problem and the current state-of-the-art.

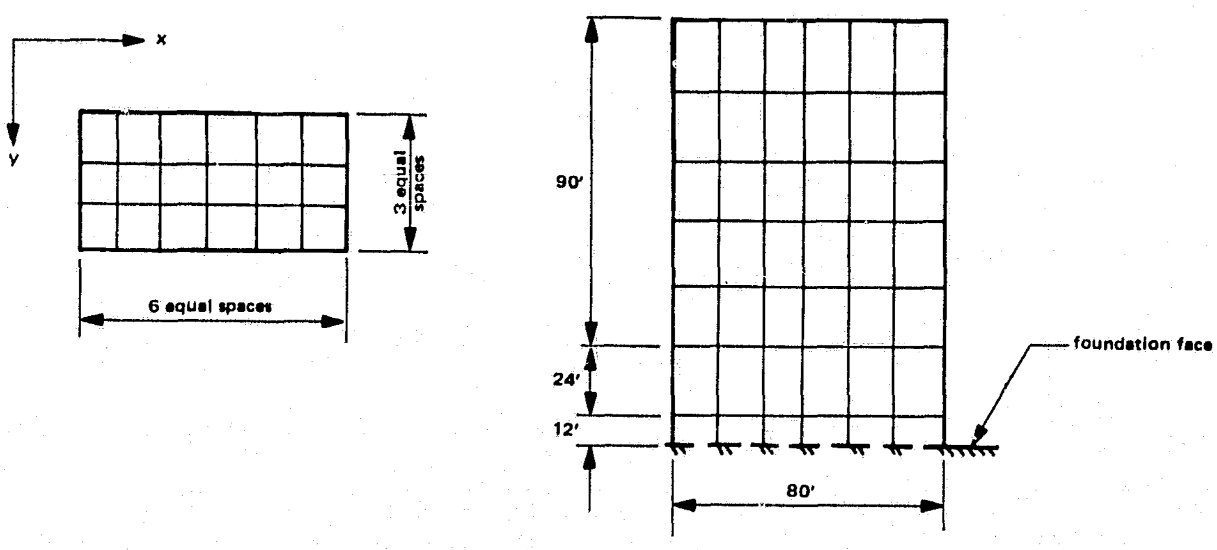
The discrete variable is accommodated by performing a set of optimizations for various column spacings and picking the best of these for the final design. Since the number of columns in the x and y directions is limited by minimum spacing requirements, from nine to 12 optimizations will probably be sufficient thus making the multiple optimization approach economical.

Table LV defines the design problem formulation for one of these optimizations. More subspace element candidates are selected from the lower floor levels because design of the elements is expected to be more interactive. Active Graphics with the delay option is prescribed to permit the designer to adjust the earthquake loading magnitude to correspond with the current building design. This is achieved by including the resonant frequency extraction and load superposition calculations in the Baseline Analysis.

One of several alternative formulations of the force boundary conditions are given in Table LV. Instead of scaling loads, the allowables could have been adjusted to accommodate the 1.0 and 1.5 safety factor loads. Alternately, one set of loadings could be specified as working and one as ultimate and safety factors introduced directly.

This formulation is expected to lead to an efficient optimization process. The problem is within the scope of the program plan. However, the multiplicity of the loading conditions recommends an addition to the plan. To reduce the number of calculations, the capability to superimpose stress conditions should be added to the Design-Analysis subplan. In this problem, this would reduce the number of loadings from eight to six. The six would be:

TABLE LV
REFORMULATION: BUILDING FRAME DESIGN PROBLEM



Building Plan

Building Elevation

Design Control Data

1. Design measure: introduced as FORTRAN statements.
2. Subspace element candidates selected from every floor level with most at lower levels.
3. General Redesign.
4. Minimum printout level.
5. Graphics with delay.

Prescribed Data

1. Geometry: as shown. All joints modelled.
2. Material: ASTM-A7 steel.
3. Boundary conditions:

   Force -
   a. Combined x direction wind, dead load, 150 psf uniform live load scaled by 3/4.
   b. Combined y direction wind, dead load, 150 psf uniform live load scaled by 3/4.
   c. Same as a but with checkerboard live load distribution.
   d. Same as b but with checkerboard live load distribution.
   e. Dead load, 150 psf uniform live load and snow load.
   f. Same as e with checkerboard live load distribution.
   g. x acceleration of masses based on 1.5 cps first mode and scaled by 3/4.

TABLE LV (Continued)

h. y acceleration of masses based on 1.0 cps first mode and scaled by 3/4.

Displacement - each column fixed in displacements and rotations at foundation face.

Design Constraints

1. Behavior:

   a. Fracture and buckling precluded based on safety factor of 1.0 for all loads. Fracture defined by principal stress with allowables modified using interactive equation.

   b. Beam endo-element relative deflections limited to less than ±1/360 of span for loadings a through f.

   c. System deflections under loadings g and h (earthquake).

2. Variable: limited per building code.

   a. All candidates AISC standard WF and H sections.

   b. Equivalencing - ten element groups defined as follows:

      1) All interior columns between each pair of floors (7 groups).

      2) All floor beams except roof and ground floor (1 group).

      3) All roof beams (1 group).

      4) All ground floor beams (1 group).

1) Uniform live load plus dead load.
2) Checkerboard live load plus dead load.
3) Wind acting in the x direction.
4) Wind acting in the y direction.
5) x earthquake accelerations
6) y earthquake accelerations

Since data processing for Design Analysis works on all real loadings together, addition of this component to the subplan will yield a more efficient optimization process.

Adequacy of the Plan

These design problems suggest the versatility and efficiency of an optmization program which implements this plan. The most common structural design problem is to select element size and material for a structure with given loads and geometry. As exemplified by the sandwich panel problem, this design is efficiently handled both with respect to input required and optimization calculations. The ability to integrate user-supplied FORTRAN statements which specify the cost function admits treatment of complicated objective functions like that of the reflector and building design problems. The Active Graphics provision allows application of the optimization to problems such as the tank support dispenser, and building design problems which are nominally beyond the scope of the criterion problem. Control of the number of design and optimization cycles and the subspace option provides for use of either the exact or subspace basis to be used for Design-Analysis. The Specialized Design capability provides directly for evaluation of structural integrity margins, thus permitting the designer to perform optimization directly, if he chooses, or to use the design subroutines only to identify integrity critical regions of this structure.

Treating joint relocation as a design variable requires further study to provide discrimination between real changes and changes that arise due to changes in idealization. This discrimination is required only for the Redesign process since the Design-Analysis plan will faithfully produce the required derivatives.

In addition, it appears desirable to modify the plan to include a load superposition capability in Design-Analysis. Admissibility of a FORTRAN definition of superposition would accommodate parametric load definition - a desirable feature.

Section 8
KEY ASPECTS OF THE OPTIMIZATION PLAN

Reveiwing the material presented in Sections 2 through 7 leads to the following conclusions relative to key aspects of the plan:

1. Formulation of a plan for an optimization program at this time (1970) involves risk. The state-of-the-art of Design-Analysis is well advanced and only minor risk is involved in constructing this subplan. An Input/Output subplan involves some risk because of limits on work with the graphics interface. Implementing the Redesign subplan described here involves risk due to lack of crystallization of optimization technology.

2. The plan contemplates two types of iterations. An inner (design) loop develops a sequence of designs in what is primarily an in-core operation using approximate analyses. The outer (optimization) loop permits increasing analysis accuracy and accounts for configuration changes.

   The designer can interact with the process at the end of either a design loop or an optimization loop to stop calculations, inspect the design and its performance, or redefine the design problem.

3. The Input/Output plan describes those data, beyond that needed for a Baseline Analysis required for optimization. An integrated tabular input plan is chosen. Levels of passive and active input/output are prescribed. The concept of consistency, completeness and magnitude checks is espoused.

4. The plan describes the basis and processing for Design-Analysis using a subspace basis and incorporating a dual approach. The dual approach provides response predictions based on complementary and/or potential energy analysis. It evokes a non-iterative reanalysis method and exact evaluation of derivatives on the subspace basis for influence analysis.

5. Redesign includes alternate subplans: Specialized Redesign for efficient optimization when element selection is the design variable and high structural efficiency the primary goal, and General Redesign for optimization with respect to any goal when only a few design variables are involved. In both plans discrete variables are simulated quasi-continuous and equivalencing of design variables is included.

6. Reformulation of validation problems for finite element simulation suggests the implementing computer code will have a scope beyond those stated as plan objectives. On the other hand, problems involving joint relocation evoke difficulties not resolved by state-of-the-art methods.

This report has presented a plan for augmenting a general purpose structural analysis program with a capability for structural optimization. An evaluation of the plan based on typical optimization problems inidcates the main risk in developing coding is due to lack of crystalization of optimization search technology. The plan minimizes this risk by including alternative optimization searches. Thus, implementing software should be compatible with a variety of optimization search methods.

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