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A REVIEW OF NASA MINITRACK DATA TIME-TAGGING

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CONTENTS

ILLUSTRATIONS

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ABSTRACT

The Minitrack tracking stations measure an analog of direction cosines in units called Minitrack counts. These counts are the scaled phase differences between corresponding points of the wave fronts arriving at the two antenna arrays comprising the given baseline. The process for generating Minitrack counts is examined and the time for proper time-tagging of the count value is identified.

SUMMARY

The purpose of this report is to interpret and document the data time-tagging associated with the Minitrack interferometer. Precise orbit determinations, such as those required in the current Mission and Trajectory Analysis Division, Goddard Space Flight Center (MTAD-GSFC) Earth physics and geodesy studies, need careful interpretation of the time associated with each tracking system measurement.

The intent of this report is to clearly define the value of time to be associated with each Minitrack measurement. This is shown to be the time that the counter "stop" command is generated, a time easily determined from knowledge of the start time and the value accumulated in the counter. The time-tagging procedure for the current GSFC preprocessor is described, and, it is pointed out in general, that for data which is not excessively noisy, it agrees with the description contained in this report.

Intuitively, one might expect that time-tagging is properly achieved at the mid-point of the counting interval. However, this report makes it clear that Minitrack data is properly time-tagged at the end of the counting interval, differing from the mid-point by as much as 5 milliseconds.

A REVIEW OF NASA MINITRACK DATA TIME-TAGGING

INTRODUCTION:

The Minitrack tracking stations measure an analog of direction cosines in units called Minitrack counts. These counts are the scaled phase differences between corresponding points of the wave fronts arriving at the two antenna arrays comprising the given baseline. Figure 1 shows one of these arrays and Figure 2 shows the antenna field arrangement. The system cannot distinguish between wave fronts separated by a time delay corresponding to integral wavelengths, hence, it measures only the decimal part of the true difference. The integer part is supplied by the Minitrack pre-processing computer program (Reference 3) which derives this number from the comparison of nearly simultaneous reading of the coarse, medium, and fine antenna arrays.

MINITRACK SYSTEM OPERATION

Definition of Minitrack Counts

The Minitrack tracking system is capable of measuring the decimal part of the phase difference between tracking signals arriving at opposite ends of an antenna base line. The system defines one Minitrack count as 10^{-3} wavelengths of the arriving signal.

The Minitrack count is also a measurement of time, each count being 10^{-5} seconds. This can be seen with the aid of the simplified block diagram shown in Figure 3.

Let us assume that a signal source is located at a very great distance from the receiving site, hence, we may consider that the radio frequency waves are collimated. The separation of the antenna arrays at the ends of the base line can be expressed as λ_L which is the number of wavelengths at the received frequency ω_1. This distance can also be expressed as $2\pi\lambda_L$ radians, which we shall call b. The signal arriving from the distant source makes an angle θ with the base line, and because the signal is collimated, θ is valid at either antenna.

At time t the signal arriving at the nearest antenna is $\sin(\omega_1 t + \phi)$, where ϕ is an arbitrary phase angle. (Since in theory the system does not measure amplitude, all signal amplitudes are considered unity.) The signal at the other

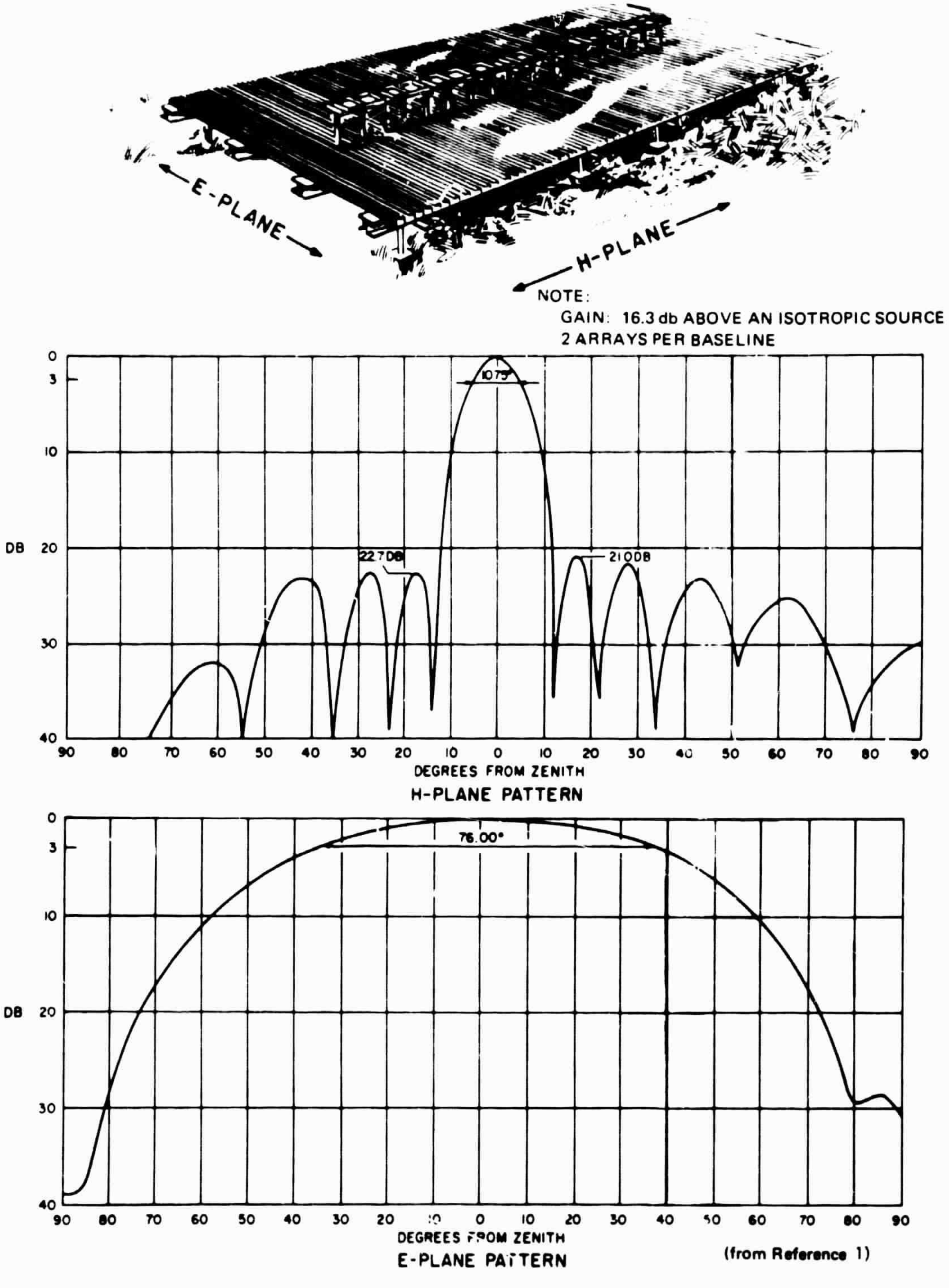

Figure 1. Typical E- and H-Plane Patterns, 136 MHz Minitrack Antenna, Serial No. 001

(from Reference 2)

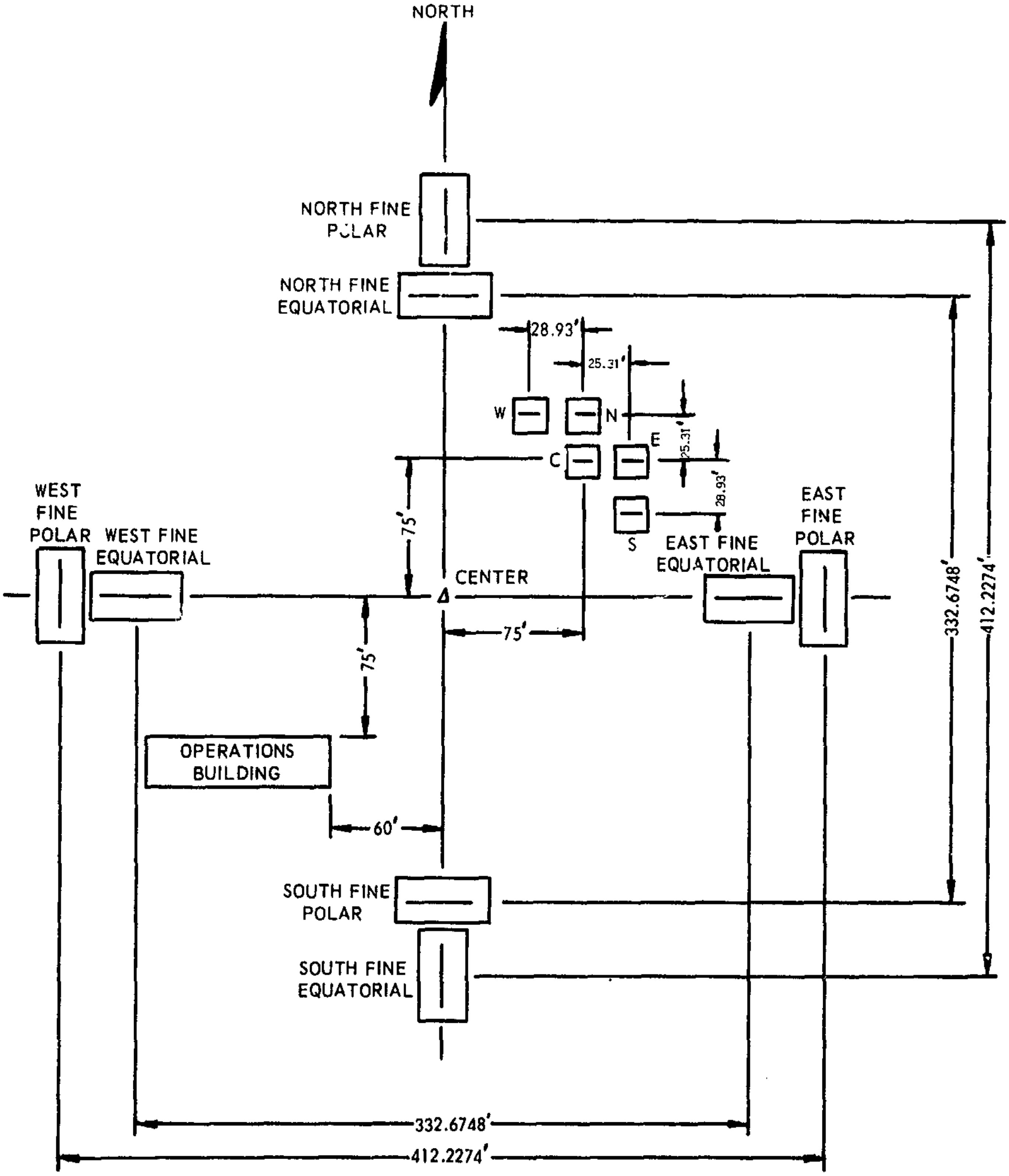

Figure 2. Antenna Field, Relative Location and Function of Antennas

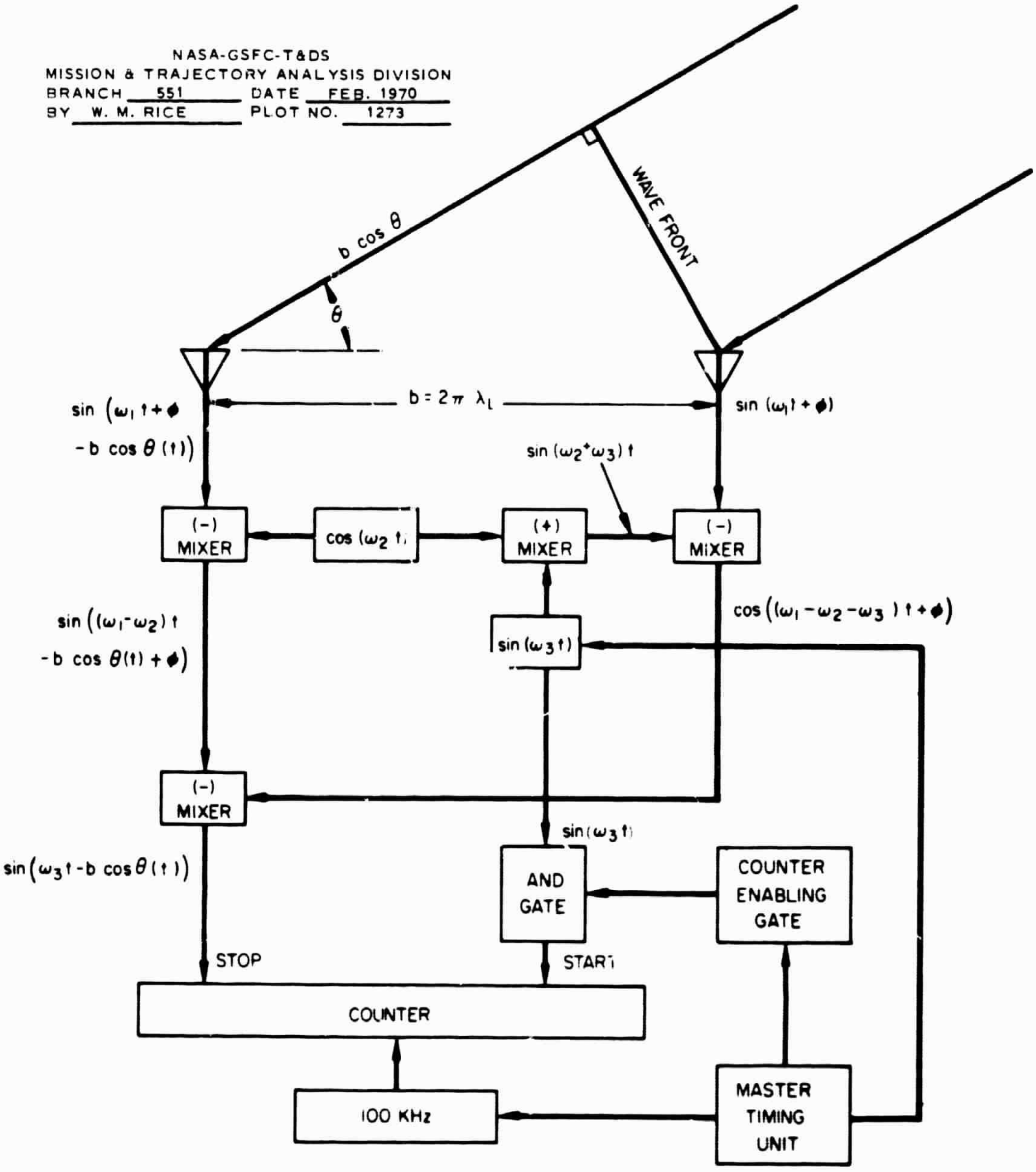

Figure 3. Minitrack Simplified Block Diagram

antenna is sin $(\omega_1 t + \phi - b \cos\theta(t))$ where $\theta(t)$ indicates that θ is a function of time. The antennas are connected to the receiving equipment by equal length cables. Phase shift introduced by these cables, being equal on both paths, may be ignored.

The local oscillator provides two signals, one differing from the other by a precise frequency ω_3. The basic local oscillator signal is cos $(\omega_2 t)$. This signal is supplied to two mixers, one to heterodyne the RF signal to an intermediate frequency and the second to generate a second local oscillator signal. In the second mixer cos $(\omega_2 t)$ heterodynes with sin $(\omega_3 t)$ and the sum frequency provides sin $(\omega_2 + \omega_3) t$ to be used to translate the second RF signal to an intermediate frequency. The two intermediate frequencies are

$$\sin\left[(\omega_1 - \omega_2)\, t - b \cos\theta(t) + \phi\right],$$

and

$$\cos\left[(\omega_1 - \omega_2 - \omega_3)\, t + \phi\right].$$

These frequencies are mixed to produce $\sin\left[\omega_3 t - b \cos\theta(t)\right]$.

If we compare the RF signals,

$$\sin(\omega_1 t - b \cos\theta(t) + \phi)$$

and

$$\sin(\omega_1 t + \phi)$$

with the output of the last mixer and the ω_3 source,

$$\sin\left[\omega_3 t - b\cos\theta(t)\right] \text{ and } \sin(\omega_3 t),$$

we note that the phase difference has been preserved. Measurement of phase differences can be made quite readily using the latter pair of signals because ω_3 may be selected for optimization of this measurement.

The method used to measure phase is to count the cycles of a 100 KHz clock between corresponding phase positions of sin $(\omega_3 t)$ and $\sin[\omega_3 t - b\cos\theta(t)]$. The frequency of ω_3 is 100 Hz so that in the corresponding period of 0.01 second there are 1000 cycles of the clock. This, in effect, divides the RF wavelength into 1000 parts so that we can achieve a measurement resolution of 10^{-3} wavelengths.

The counter is also a measure of time, each count being 10^{-5} seconds. Hence, knowing the counter start time and the count time duration, which is readily available from the value of the count, the counter stop time can be determined by summing these time values.

The 100 Hz and 100 KHz signals are derived from a master timing unit which, in turn, is synchronized with a time standard such as WWV. The counter is started at intervals of 0.2 seconds, coinciding with the required triggering level of sin $(\omega_3 t)$.

An alternate way of looking at the expression

$$\sin(\omega_3 t - b\cos\theta(t))$$

can be realized by transforming the term $b\cos\theta(t)$.

It had been stated that $b = 2\pi\lambda_L$, where λ_L is the number of wavelengths of the received frequency separating the two antennas. The wavelength of frequency f_1, $(\omega_1 = 2\pi f_1)$, is c/f_1. Hence the number of wavelengths can also be expressed as $f_1 d/c$, where d is the metric length of the antenna base line.

We can write the equations

$$b = 2\pi\lambda_L ,$$

$$\lambda_L = \frac{f_1 d}{c} ,$$

and

$$b\cos\theta(t) = 2\pi f_1 \frac{d}{c}\cos\theta(t) ,$$

but $d/c \cos\theta(t) = \tau(t)$, which is the time required for the wave front to reach the second antenna after arriving at the first antenna. Therefore,

$$\omega_1 = 2\pi f_1 ,$$

$$b \cos\theta(t) = \omega_1 \tau(t) ,$$

and

$$\sin\left[\omega_3 t - b\cos\theta(t)\right] = \sin\left[\omega_3 t - \omega_1 \tau(t)\right] .$$

It can be seen that $\omega_1 \tau(t)$ is the phase shift introduced by frequency ω_1 propagating the extra distance necessary to arrive at the second antenna. The term $\tau(t)$ is a slowly varying function of time whose value can be considered a constant for the interval $\Delta t = \tau$.

Figure 3 also shows, in a simplified diagramatic way, the various operations needed to obtain a count measurement. The many needed amplifiers, shaping circuits, filters, and other circuits in the hardware implementation have been omitted for clarity.

Count Generation Analysis

Certain assumptions are made in the following analysis:

1. Source motion for time interval $\tau \leq b/c$ is considered negligible.

2. Antenna cable length differences are assumed to be zero.

3. Circuit pass bands are assumed to be broad compared to the signal rate of change.

4. Signal amplitude variations and noise are neglected.

5. Antenna field irregularities are ignored.

These assumptions have the following effects on the analysis:

1. Propagation time: The Minitrack system measures the line of sight angle* to a signal source. It cannot measure range nor can the preprocessor determine range. Hence, we are not concerned with the separation between the Minitrack site and the RF source.

The analysis does assume that, during the time required for a wave front to move from the nearest to the farthest antenna (an interval less than 0.5 microseconds), the change in angle to the source is too small to be significant. In other words, the two signals used to generate the counter stop command must travel unequal distances. The time difference required for this distance difference is too small to be measured by the system.

2. Cable length inequality: The effect of the RF line length inequality causes a phase shift of one signal with respect to the other prior to the generation of the stop-counter signal. This phase shift carries over directly to the stop-counter signal. In actual station operation, compensation for this effect is achieved by careful calibration of station equipment.

3. Circuit pass bands: In the dynamics of tracking a source, the electronic circuits are required to follow or track the signal while introducing a minimum of error. Phase integrity must be maintained throughout all circuits. As the source motion produces doppler variations, for example, it is assumed that all circuits are ideal and do not introduce tracking errors.

4. Signal amplitude variations and noise: Any electronic circuit variation that could possibly cause a phase variation in the actual equipment has been ignored. Such problems, if they do exist, are not a part of this analysis. The effects of noise of any kind are ignored for the same reason.

5. Antenna field irregularities: Irregularities in the antenna field could have the effect of causing the source to appear to be angularly displaced from its true direction. This effect could appear as improperly time-tagged data. Hence, for this analysis, the antenna fields are assumed to be perfect.

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*The angle that Minitrack measures is a function of the signal wavelength implicit in the preprocessor and the refraction corrections employed in obtaining the station constants from the calibration aircraft overflights. This problem is discussed in Reference 4.

Referring to Figure 3, the counter start condition can be established as

$$\sin(\omega_3 t_1) = 0 \ ,$$

and the counter stop condition as

$$\sin\left[\omega_3 t_2 - b\cos\theta(t_2)\right] = 0 \ ,$$

both valid for the positive slope of the function.

The arguments for the sine functions are, then,

$$\omega_3 t_1 = 2\,n\pi \ ,$$

and

$$\omega_3 t_2 - b\cos\theta\,(t_2) = 2\,m\pi \ ,$$

where n and m are integers, t_1 is the counter start time, and t_2 is the counter stop time. Let Δt = counting interval time, and

$$t_2 = t_1 + \Delta t \ .$$

next,

$$\omega_3(t_1 + \Delta t) - b\cos\theta(t_2) = 2\,m\pi \ ,$$

and

$$\omega_3 t_1 + \omega_3 \Delta T - b\cos\theta\,(t_2) = 2\,m\pi \ ,$$

but

$$\omega_3 t_1 = 2\,n\pi \ ,$$

$$2n\pi + \omega_3 \Delta t - b\cos\theta\,(t_2) = 2\,m\pi \ ,$$

and

$$\cos\theta(t_2) = \frac{2\pi(n - m) + \omega_3 \Delta t}{b} \ .$$

Because $b = 2\pi\lambda_L$ and $\omega_3 = 2\pi f_3$,

$$\cos \theta(t_2) = \frac{(n - m) + f_3 \Delta t}{\lambda_L} . \tag{1}$$

In Equation 1 the quantity $(n - m)$ is an integer. Its value is closely related to the path length difference $b \cos \theta$. However, n is evaluated at time t_1, and m at the later time t_2. The value $t_2 - t_1 = \Delta t$ may, at times, approach one period of f_3, which must be taken into account if the significance of $n - m$ is to be understood.

Let us rewrite Equation 1 so that

$$n - m = \lambda_L \cos \theta (t_2) - f_3 \Delta t ,$$

and

$$n - m = \lambda_L \cos \theta (t_2) - \frac{\Delta t}{T_3} ,$$

where the period of f_3 is $T_3 = 1/f_3$. We see that (n-m) is the path length difference at time t_2 reduced by a factor that is the ratio of the count accumulation time to the period of frequency f_3. Generally, $\Delta t / T_3$ will not exceed 1, but there are remote circumstances which will permit this value to exceed unity. For example, if $\lambda_L \cos \theta (t_2)$ is exactly an integer, then the counter will receive a start command simultaneously with a stop command. Depending upon the characteristics of the counter, one or the other commands will be dominant, possibly the start command. If the source doesn't move, the next stop command will be simultaneous with a start command and the counting process will continue until a stop command is displaced from the start command. Fortunately the sources for which Minitrack was designed are moving and successive simultaneous occurrence of start and stop commands are not possible, but it is within the realm of possibility for a single simultaneous occurrence to exist. If the next stop command should occur after the start command, then $\Delta t / T_3$ will be greater than unity. Furthermore, if $\theta (t)$ is near $\pi/2$, $(n - m)$ will be a negative number. This is demonstrated in the next section called Numerical Example.

We can see from Equation 1 that the cosine of $\theta (t)$ at time t_2 can be calculated from the counting time interval Δt. The value of Δt is, quite simply, the accumulated count multiplied by the period of the 100 KHz signal. Hence, the conclusion is that the Minitrack count should be time-tagged at the occurrence of the counter stop command.

An examination of Reference 3 discloses that the preprocessor used by GSFC comes close to time-tagging data in this way. In fact, the difference may be less than the computational errors due to round-off and loss of significance. The GSFC preprocessor compresses each data line (frame) containing 5 data points to a single point. The five data points are assumed to be equally spaced. A parabola is fitted to these points in a least squares sense and an estimate is made for the smoothed midpoint value. The time associated with this value is the counter-stop time of the mid-point reading in that frame. For various reasons, one of which is for ease of resolving ambiguities, the value of the count at 0.4 seconds after the frame start time is to be obtained. The smoothed value obtained from the parabolic fit is located at a time of 0.4 seconds after the frame start time plus the counter accumulation time. Therefore, an interpolation must be made to obtain a count value at 0.4 seconds after the frame start. The GSFC preprocessor now proceeds to estimate this value. A new assumption is made regarding a frame of data. It is assumed that the 5 readings in any given frame can be fitted with a straight line passing through the smoothed value previously derived. The slope to be used is the same as that for a line connecting the frame's first and last counts, having a time base of 0.8 seconds. The compressed frame point is then just the smoothed value, minus the product of the slope multiplied by the counter accumulation time. This completes the preprocessor's time-tagging of the data.

Each line of tracking data (frame) is time-tagged in the same way for both east-west and north-south data. For arcs of 0.8 second duration, the difference between parabolic fitting and linear fitting should be quite small. However, as the data becomes noisy, both the linear slope and the mid-point count-accumulation time may become adversely affected; both are used in the final estimate of the smoothed count.

NUMERICAL EXAMPLE

Let us consider a two-dimensional example in which a radiating source moves in a circle using a Minitrack site as a center (Figure 4). (Although this is physically unrealizable it provides ease of calculation and demonstrates the desired results.) The plane of motion is defined by using y as the local vertical, and x as any direction normal to the vertical.

The sources position is then defined as

$$r^2 = y^2 + x^2 \ .$$

The tracking site is located at coordinates (0.0). Let us assume that the location of the source from the y axis can be expressed as

$$x = r \sin (10^{-3}\, t). \tag{2}$$

Then, at $t = 0$ the source is at zenith. From Figure 4, the angle θ is shown to be the elevation angle, whose direction cosine is x/r. Thus,

$$\cos \theta = \frac{x}{r} \quad ,$$

and from (2)

$$\sin (10^{-3}\, t) = \frac{x}{r} \quad ,$$

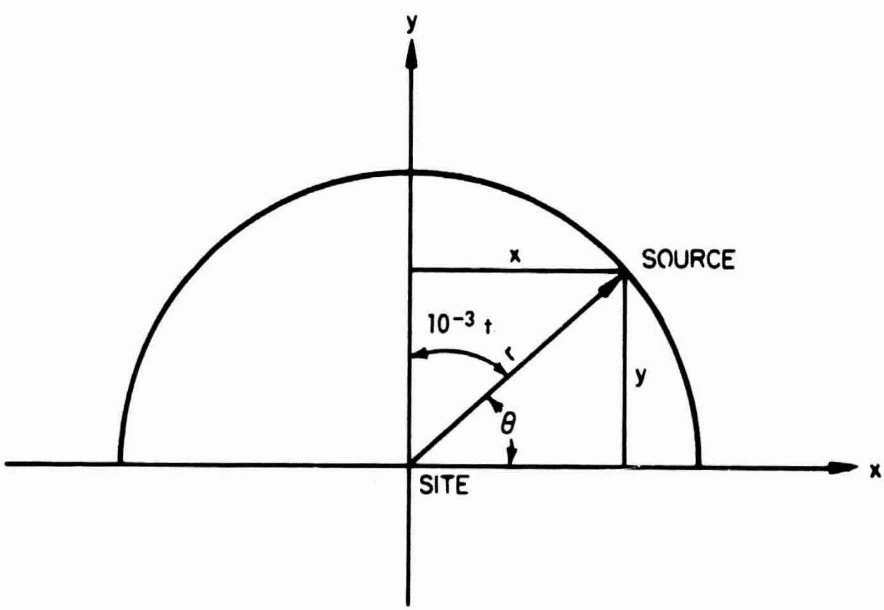

Figure 4.

and

$$\cos \theta = \sin\left(10^{-3} t\right) . \tag{3}$$

As previously discussed,

$$\cos \theta(t) = \frac{n - m + f_3 \Delta t}{\lambda_L} . \tag{1}$$

Therefore, let

$$f_3 = 100 \text{ Hz}$$

and

$$\lambda_L = 50 \text{ wavelengths} .$$

Then,

$$\sin\left(10^{-3} t_2\right) = \frac{n - m + 100 \Delta t}{50} ,$$

or

$$\sin\left(10^{-3} t_2\right) = 2\left(10^{-2}(n - m) + \Delta t\right) . \tag{4}$$

next, since

$$t_2 = t_1 + \Delta t .$$

Thus,

$$\sin\left[10^{-3}(t_1 + \Delta t)\right] = 2(10^{-2}(n - m) + \Delta t)\ .$$

If the count is started at $t_1 = 0$, the source being at zenith, we can solve for the counter elapsed time Δt as follows:

$$\sin(10^{-3}\,\Delta t) = 2(10^{-2}(n - m) + \Delta t)\ ,$$

$$\sin(10^{-3}\Delta t) \approx 10^{-3}\,\Delta t\ ,$$

$$10^{-3}\,\Delta t = 2 \times 10^{-2}\,(n - m) + 2\Delta t\ ,$$

and

$$\Delta t = \frac{-2 \times 10^{-2}\,(n - m)}{(2 - 10^{-3})}\ . \qquad (5)$$

At zenith,

$$n - m = 0\ ,$$

hence,

$$\Delta t = 0.$$

Perhaps the counter cannot be started and stopped simultaneously. Let us assume that $\Delta t \neq 0$. This implies that $n - m$ must have a value other than zero. Let

$$n - m = -1 \qquad \text{(see count generation analysis)}\ .$$

Then,

$$\Delta t = \frac{-2 \times 10^{-2}(-1)}{(2 - 10^{-3})} = 10^{-2}\,(1 + 5 \times 10^{-4} + (5 \times 10^{-4})^2 + (5 \times 10^{-4})^3 + \ldots ,$$

or

$$\Delta t = 1.000500250125 \times 10^{-2} ,$$

which corresponds to approximately 0.5 minitrack counts greater than one full cycle of 100 Hz. Using actual hardware, the counter, responding to integral counts, will overflow and have a value of zero; but for this example, we must assume that the counter can measure time to a finer resolution than the 10^{-5} seconds of the Minitrack system. To verify that the direction cosine measured by the system is the correct value, let us evaluate Equations 3 and 4.

From Equation 3 we find that

$$\cos\theta = \sin(10^{-3}\Delta t)_{\Delta t = .010005} = 1.00050025 \times 10^{-5} .$$

From Equation 4 we find that

$$\cos\theta = 2(10^{-2}(n - m) + \Delta t) ,$$

or

$$\cos\theta = 2(-10^{-2} + 1.000500250125 \times 10^{-2}) ,$$

which becomes

$$= 1.00050025 \times 10^{-5} .$$

If, in Equation 5, we let

$$n - m = +1 ,$$

then Δt becomes a negative value; however, this will never occur because of the Minitrack equipment configuration. As one more example let $t_1 = 100$; then $t_2 = t_1 + \Delta t$. Thus,

$$\sin\left[10^{-3}(t_1 + \Delta t)\right] = 2(10^{-2}(n - m) + \Delta t) ,$$

expanding, we get

$$\sin 10^{-3} t_1 \cos 10^{-3} \Delta t + \cos 10^{-3} t_1 \sin 10^{-3} \Delta t = 2(10^{-2}(n-m) + \Delta t) .$$

Now, because

$$\cos 10^{-3} \Delta t \approx 1.0 ,$$

and

$$\sin 10^{-3} \Delta t \approx 10^{-3} \Delta t ,$$

we find that

$$\sin 10^{-1} + 10^{-3} \Delta t \cos 10^{-1} = 2(10^{-2}(n - m) + \Delta t) ,$$

and

$$\Delta t = \frac{\sin 10^{-1} - 2 \times 10^{-2}(n - m)}{2 - 10^{-3} \cos 10^{-1}} ,$$

using

$$\left(2 - 10^{-3} \cos 10^{-1}\right)^{-1} \approx 0.5 \left(1 + \frac{10^{-3}}{2} \cos 10^{-1}\right) .$$

we get

$$\Delta t \approx 10^{-2}\left[4.9916708 - (n-m)\right](1 + .00049750208\,26)\ .$$

Since

$$0 \leq \Delta t \leq 10^{-2}\ ,$$

then

$$n - m = 4\ ,$$

and

$$\Delta t = .0099216416\ .$$

This would be a total of 992 Minitrack counts.

As a check, evaluating Equations 3 and 4 with these values of $(n-m)$ and Δt we find that

$$\cos(\theta) = \sin\left[10^{-3}(100.0099216416)\right]\ .$$

or

$$\cos(\theta) = .0998433\ .$$

Also,

$$\cos(\theta) = 2(10^{-2}(n-m) - \Delta t)\ .$$

$$= 8 \times 10^{-2} - .0198433\ .$$

or

$$\cos(\theta) = .0998433 .$$

which verifies that the Minitrack counts are applicable at time $t = t_2$.

The time-tagging can be shown in another way by considering a plot of the phase angles of the start signal, $\sin(\omega_3 t)$, and the stop signal, $\sin[\omega_3 t - b \cos \theta(t)]$. Figure 5 shows these phase angles, modulo 2π versus time. (The curvature in the trace of $\omega_3 t - b \cos \theta(t)$ has been exaggerated to make the time-tagging problem more apparent.) If the time at generation of the start signal is defined as $\omega_3 t = 0$, then the time at generation of the stop signal may be defined as $\omega_3 t - b \cos \theta(t) = 0$, both modulo 2π. Since ω_3 is derived from a precision source, phase angle $\omega_3 t$ is known at any time. If the 100 kHz counter is started at t_1 and stopped at t_2, it has measured time interval $t_2 - t_1$, which is the base of right triangle t_1, t_2, P. Side Pt_2 is the phase difference we seek at time t_2, which is just $\omega_3 (t_2 - t_1)$.

The Minitrack count that is accumulated between t_2 and t_1 is the scaled measure of this time interval in which each count is ten microseconds. The Minitrack count is also the scaled phase angle of ω_3 at time t_2 (1000 counts equals 2π radians). Hence, knowing the counter start time t_1 and the accumulated count, the phase difference at time t_2 can be determined. Thus the Minitrack counts are properly time-tagged at occurrence of the stop pulse.

CONCLUSIONS

This analysis is a practical treatment of that part of the Minitrack system which generates the Minitrack counts. The analysis shows that the time-tagging of the counts should coincide with the occurrence of the counter stop signal. The current GSFC preprocessor produces a data time-tag value that is close to being correct: in fact, the errors are smaller than the current system resolution.

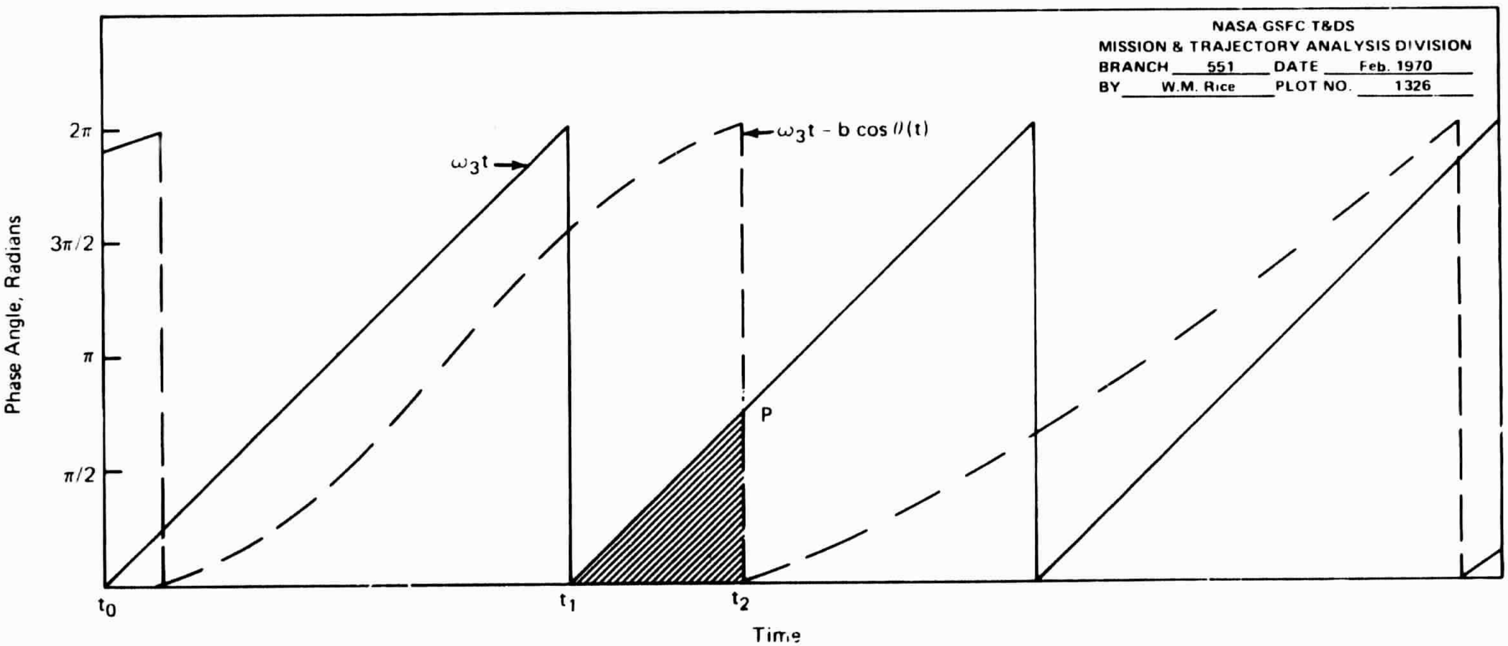

Figure 5. Phase Angle of Counter Start and Stop Signals Versus Time

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