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ACCURACY OF ESTIMATING THE LOCATION
OF A LANDED SPACECRAFT ON MARS
FROM RANGE AND RANGE-RATE DATA

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16. Abstract

Standard deviations of parameters defining the location of a landed spacecraft on the surface of Mars are generated by assuming earth-based range and range-rate tracking of the landed spacecraft. These statistics are presented parametrically for several tracking schedules. The covariance matrix for the spacecraft location includes effects of data measurement errors, and of model uncertainties in the tracking station locations, in the ephemeris of Mars, and in the Martian rotational period and rotational axis orientation. Comparisons are presented between the accuracies resulting from analysis of range and range-rate data independently and in combination.

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ACCURACY OF ESTIMATING THE LOCATION OF A LANDED SPACECRAFT

ON MARS FROM RANGE AND RANGE-RATE DATA

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SUMMARY

This paper presents an analysis of the accuracy of earth-based range-rate and range data for determining the location of a spacecraft landed on Mars. Statistics on the landed spacecraft location are presented parametrically for several tracking schedules. These statistics are based on linearized equations of motion and include effects of data measurement errors and of uncertainties in the tracking station locations, in the ephemeris of Mars, and in the Martian rotational period and rotational axis orientation.

It is shown that substantially improved estimates of the Martian pole location, spacecraft longitude, and distance off the Martian axis of rotation can be obtained from analysis of range-rate data only. This result is shown to be affected little by consideration of current uncertainties in the Martian ephemeris or by the uncertainties anticipated by 1976. The single exception to a substantially improved estimate is the distance of the landed spacecraft from the Mars equatorial plane. Accuracy of estimating this parameter is shown to be strongly dependent upon the Martian ephemeris uncertainties. It is also shown that range data are the primary source of information on this parameter.

INTRODUCTION

The Viking 1975 missions are designed to land two spacecraft on the surface of Mars. Precise analysis of subsequent telemetry data will require accurate knowledge of the position of the landed spacecraft. The purpose of this paper is to present results of preliminary studies of the accuracy of determining the spacecraft location from earth-based range and range-rate measurements.

Estimates of the position of the spacecraft are assumed to be obtained by a conventional weighted least-squares analysis in which the tracking data are unbiased, the data noise is uncorrelated, and the weighting matrix is the inverse of the noise convariance matrix. Covariance matrices for parameters defining the spacecraft location are calculated for range and range-rate data independently and in combination. These covariance matrices include the effects of certain model errors which may not be refined except by extensive postflight analysis. (See ref. 1.)

SYMBOLS

A	matrix of partial derivatives relating y to x
C	matrix of partial derivatives relating y to p
cov( )	covariance of ( )
E	expected value operator
P,Q	angular displacements of Mars axis of rotation
p	vector of parameters describing observation model errors
r_{spin}	normal distance from Mars axis of rotation
x	vector of parameters to be estimated
$\hat{x}$	an estimate of x
y	vector observable
Z	distance from Mars equator parallel to axis of rotation
α	angle from spacecraft zenith
ϵ	random noise vector
λ	longitude measured in Mars equatorial plane
Λ	a covariance matrix

Subscripts:

$1,2,3,\hat{x},\epsilon,p$	denotes parameter to which a matrix corresponds

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Superscripts:

-1   denotes matrix inversion

T   denotes transpose of a matrix

PROCEDURES AND METHOD OF ANALYSIS

The results presented herein are based on two possible types of tracking data: earth-based range (light travel time), and earth-based range rate (Doppler frequency). Both range and range-rate data are assumed to be unbiased and to have a random noise of 15 meters and 1 mm/sec, respectively. (See ref. 1.) Periods of occultation being excluded, both data types could be available almost continuously. However, it is anticipated that more refined mission definition will limit this availability because of power and telemetry considerations, and antenna pointing capabilities. Such data limitations have been simulated by specifying a cone with its vertex at the landed spacecraft; tracking data are assumed to be available when the earth tracking station lies within this cone. (See fig. 1.) The following sections discuss the sources of model errors and the procedures utilized to obtain standard deviations associated with the parameters of interest.

Lander Location

Location of a landed spacecraft on the surface of Mars is described relative to the Martian equator and axis of rotation. The three necessary coordinates are chosen to be: Z (distance along the axis of rotation), r_{spin} (distance normal to the axis of rotation), and λ (longitude). (See fig. 1.) Since the tracking data are those obtained between the landed spacecraft and the earth tracking stations, analysis of these data requires referencing both the spacecraft and tracking station to a common coordinate system. A logical system is one referred to the earth equator and equinox since this system is the one in which planetary observational data are taken to describe the planetary ephemerides.

The first phase of the transformation from body-fixed axes in Mars to body-fixed axes in the earth requires knowledge of the direction of the axis of rotation of Mars. Uncertainty in the current knowledge of this direction is on the order of 1°. (See ref. 2.) Errors in orientation of the axis of rotation are treated herein as two rotations, P and Q, relative to an assumed direction. The rotation P is an angular displacement in declination and $-Q$ is an angular displacement in aerocentric right ascension. These parameters are illustrated in figure 2.

The second phase of the transformation is the translation from the center of mass of Mars to that of the earth. This translation is obtained by use of the ephemeris of Mars.

Current uncertainties in the ephemeris position of Mars are of the order of 200 to 400 kilometers, but are expected to be reduced to 5 to 10 kilometers by postflight analysis of the 1971 Mars orbiter missions. (See refs. 1 and 3.) This position uncertainty is shown to affect the range data primarily. For this analysis it is assumed that the ephemeris was generated by numerical integration to yield a continuous Martian orbit, and velocity uncertainties are estimated at 5×10^{-7} km/sec. This uncertainty is shown to affect the range-rate data primarily.

The final phase of the transformation relates tracking station locations to the earth coordinate system. These locations have been well defined and no significant improvement is anticipated.

A summary of standard deviations associated with these error sources is given in table I. Values in this table were obtained from reference 1. Included in this table are a priori spacecraft uncertainties which can result from Martian entry trajectory analysis.

TABLE I.- A PRIORI STANDARD DEVIATIONS

Parameter	Standard deviation
Lander longitude	1.7×10^{-2} radian
Lander distance off axis of rotation	40 km
Lander Z-component	100 km
Martian polar locations	1.7×10^{-2} radian
Martian rotational period	5×10^{-7} hr
Tracking station distance off axis of rotation	0.0015 km
Tracking station Z-component	0.025 km
Tracking station longitude	4.7×10^{-7} radian
Ephemeris position of Mars:	
Anticipated	5 km
Current	200 km
Ephemeris velocity of Mars	5×10^{-7} km/sec

Statistical Model

The statistical model is based on the assumption that the estimation utilizes a weighted least-squares process in which the data are unbiased, the data noise is uncorrelated, and the weighting matrix is the inverse of the noise covariance matrix. This estimator is characteristic of processes currently in use for orbit determination and parameter estimation. These procedures generally assume linearized equations of motion and in this report only statistics associated with these linearized equations are

analyzed. The following discussion develops the covariance matrix associated with this estimator for a single data type and the covariance matrix associated with the minimum variance combination of estimates from independent data types.

Let x be a vector of parameters to be estimated; y, the vector observable; p, a vector of parameters which includes observation model uncertainties; and ϵ, the vector representing the noise on the data. To generate statistics on the estimator $\hat{x}$ of x, it is assumed that p is a random vector with mean zero ($E(p) = 0$) and known covariance $cov(p) = \Lambda_p$. Further, it is assumed that $E(\epsilon) = 0$, $cov(\epsilon) = \Lambda_\epsilon$, and $E(p\epsilon^T) = 0$. The latter assumption asserts that model errors and measurement errors are uncorrelated.

The linearized equation relating the observations to the parameters is

$$dy = A\,dx + Cp + \epsilon \tag{1}$$

where the true value of x is obtained by adding dx to the initial estimate of x, and A and C are matrices of partial derivatives. The weighted least-squares estimate $d\hat{x}$ of dx is given by (ref. 4)

$$d\hat{x} = \left(A^T \Lambda_\epsilon^{-1} A\right)^{-1} A^T \Lambda_\epsilon^{-1}\,dy \tag{2}$$

Note that $E(d\hat{x}) = dx$; hence, $d\hat{x}$ is an unbiased estimate of dx. Under these assumptions, the covariance of $d\hat{x}$ is

$$\Lambda_{\hat{x}} = E\left[(dx - d\hat{x})(dx - d\hat{x})^T\right]$$

$$= \left(A^T \Lambda_\epsilon^{-1} A\right)^{-1} + \left(A^T \Lambda_\epsilon^{-1} A\right)^{-1} A^T \Lambda_\epsilon^{-1} C \Lambda_p C^T \Lambda_\epsilon^{-1} A \left(A^T \Lambda_\epsilon^{-1} A\right)^{-1} \tag{3}$$

Equation (3) applies to the estimation of x from a single data source.

Suppose that two vector estimates $\hat{x}_1$ and $\hat{x}_2$ of x have been obtained from two independent data sources and that their respective covariances are $\Lambda_{\hat{x}_1}$ and $\Lambda_{\hat{x}_2}$. It is then desirable to combine these two independent estimates to yield one "optimal" estimate $\hat{x}_3$ of x.

If optimal is defined to mean linear, unbiased, minimum variance, then $\hat{x}_3$ is given by (ref. 4)

$$\hat{x}_3 = \left(\Lambda_{\hat{x}_1}^{-1} + \Lambda_{\hat{x}_2}^{-1}\right)^{-1}\left(\Lambda_{\hat{x}_1}^{-1}\hat{x}_1 + \Lambda_{\hat{x}_2}^{-1}\hat{x}_2\right) \tag{4}$$

and

$$\Lambda_{\hat{x}_3} = \Lambda_{\hat{x}_1}\left(\Lambda_{\hat{x}_1} + \Lambda_{\hat{x}_2}\right)^{-1}\Lambda_{\hat{x}_2} \tag{5}$$

5

Equation (5) is the form used in this paper to combine estimates resulting from range, range-rate, and a priori analyses.

RESULTS AND DISCUSSION

An analysis of the accuracy of determining the location of a Martian lander (landed spacecraft) has been performed for an assumed lander areographic location; 112.43^O longitude, 24.71^O latitude, and 3393.4-kilometer radius. This location passes through the Earth-Mars line on July 23, 1976. Range and range-rate tracking data are assumed to be available from this date. This location is possible for Viking 1975 landings and the selection of July 23 as an analysis epoch agrees with a possible landing time for the first Viking spacecraft.

Range-rate and range data are assumed to be sampled at 6-minute intervals. Three earth-based tracking stations of the NASA deep space network were considered: Goldstone, California; Woomera, Australia; and Madrid, Spain. These stations are separated by approximately 120° in longitude and thus provide continuous viewing of Mars. The availability of data is controlled by specifying the cone of visibility described earlier. This cone is specified by assigning a value of the permissible angle from the lander zenith α at which data can be taken. Three values of zenith angle were considered: 5°, 45°, and 90°. These angles correspond approximately to 45 minutes, 6 hours, and 12 hours of tracking per day. For each of these zenith angle values, data representing 1, 2, 5, 10, and 15 days of tracking were assumed.

The solution set corresponding to each of these tracking schedules generally consisted of lander longitude, r_{spin}, Z, and the components of Martian pole location, P and Q. The exceptions are the cases for 5° zenith angle for 1- and 2-day tracking intervals during which insufficient data were available to obtain a solution because of linear correlations between the parameters. Results of this analysis are presented in figures 3 to 7 in the form of standard deviations for each parameter in the solution set. These standard deviations represent the combined results of data analysis and a priori statistics. (See table I.)

Figure 3 shows standard deviations of the solution parameters, for each zenith angle value, as a function of time for range-rate and range data separately. In each of these cases, no model errors were assumed. Thus, the curves in figure 3 represent the minimum uncertainty in determining the lander location that can be achieved once all other significant sources of data bias have been removed. Figure 3 can be summarized by stating that range-rate and range are roughly equivalent data types with regard to determining lander location, the one important exception being the determination of the Z-component. By referring to figure 3(c), it can be seen that extensive range-rate

tracking would be required to reduce the Z-component uncertainty to the 1-kilometer level. To reduce the uncertainty to this level, it appears that sufficient time must be allowed for the Earth-Mars geometry to change significantly. In contrast to the range-rate data, the range data is seen to be an inherently stronger data type for determination of the lander Z-component.

The results in figure 3 are idealistic by current standards in that no model errors are assumed. As stated earlier, values of tracking station locations, Martian ephemeris position and velocity, and the rotational period of Mars are uncertain enough to create data biases. By using the formulas given in the section on the statistical model, the effects of these error sources on the accuracy of estimating the solution set parameters were calculated. Figure 4 presents these results by assuming the a priori standard deviations listed in table I and by using the 5-kilometer ephemeris position uncertainty. Comparison of figure 4 and figure 3 shows the range-rate data analysis to be little affected by these error sources. However, considerable degradation in the range data analysis is apparent. Comparison of the two data types indicates range rate to be far superior to range for the estimation of all parameters, excluding the Z-component. By referring to figure 4(c) the range-rate estimate of Z, under the influence of the stated error sources, is shown to be degraded by the extension of tracking time. In contrast, range is still a relatively strong data type for determination of Z. Again, it appears that if sufficient time is permitted to elapse so that the Earth-Mars geometry is significantly changed, an uncertainty of about 10 kilometers in Z may be achieved through analysis of range data.

As stated earlier, the 5-kilometer ephemeris position uncertainty is not a current estimate, but rather a value anticipated by 1976. The results shown in figure 5 represent reevaluations of the cases summarized in figure 4, utilizing a more current ephemeris uncertainty of 200 kilometers. These results again indicate that the range-rate analysis (excluding the Z-component estimate) is not severely degraded. However, in this situation the range data are sufficiently biased to make it a poor data type in comparison with range rate. Even for the Z-component estimation shown in figure 5(c), range does not yield a good independent estimate. The possibility of improving these results by including the ephemeris position and velocity in the solution set was considered. However, even the case for the 90° zenith angle and 15 days tracking provided insufficient data to obtain this solution. For example, an analysis of range-rate data representing 15 days with approximately 12 hours of tracking per day yielded standard deviations on lander longitude, distance off the axis of rotation, and pole location of approximately 2×10^{-5} radian, 0.1 kilometer, and 4×10^{-5} radian, respectively. These standard deviations are substantially less than the respective a priori values of 1.7×10^{-2} radian, 40.0 kilometers, and 1.7×10^{-2} radian. The possibility of achieving better results through combining data types is then an obvious area of investigation. This combination has been accomplished by use of equation (5).

Figure 6 presents the results of combining the range-rate and range estimates given in figure 4. Because of the accuracy of determination from range-rate analysis alone, little improvement was gained except in the Z-component. Comparison of figures 4(c) and 5(c) shows substantial improvement in the estimate of Z owing to the inclusion of range data. The rapid reduction of this uncertainty to an essentially constant value as shown by the curves in figure 6(c) illustrates that this improvement can be achieved with the use of very little range data. This result implies a possible mode of spacecraft tracking which would consist primarily of range-rate tracking, supplemented by range data taken on a noninterference basis.

The results obtained by utilizing the 200-kilometer ephemeris position uncertainty are given in figure 7 and should be compared with those of figure 4. As before, little improvement beyond the range-rate only analysis is noted except in the Z-component. Comparison of figures 5(c) and 7(c) shows range data to be essential in obtaining any significant reduction in the uncertainty in the Z-component. However, the estimation accuracy is still rather poor although indications are that considerably longer tracking intervals would yield further improvement. This large uncertainty points out the need for Martian ephemeris improvement. Based on the impact of ephemeris errors on this analysis, primarily through their effect on range data, it appears that analysis of range data could yield significant reductions in these errors. However, such analysis will probably require data taken over extended periods of time in order to take advantage of the changing Earth-Mars geometry.

CONCLUDING REMARKS

This paper has presented an analysis of the accuracy of range-rate and range data for determining the location of a Mars lander. It has been shown that relatively accurate estimates of the Martian pole location, lander longitude, and distance off the Martian axis of rotation can be obtained from analysis of range-rate data only. For example, an analysis of range-rate data representing 15 days with approximately 12 hours of tracking per day yielded standard deviations on lander longitude, distance off the axis of rotation, and pole location of approximately 2×10^{-5} radian, 0.1 kilometer, and 4×10^{-5} radian, respectively. These standard deviations are substantially less than the respective a priori values of 1.7×10^{-2} radian, 40.0 kilometers, and 1.7×10^{-2} radian. This result was shown to be little affected by the consideration of current uncertainties in the Martian ephemeris or by the uncertainties anticipated for 1976. The single exception to an accurate estimate is the lander Z-component (distance of landed spacecraft from the Mars equatorial plane).

When the Martian ephemeris errors anticipated for 1976 were considered, it was shown that the range-rate data only yielded an uncertainty of approximately 60 kilometers

in the Z-component after 15 days tracking. This value was reduced to approximately 10 kilometers by the inclusion of a very limited amount of range data. This result implies a possible tracking scheme consisting predominantly of range-rate data, augmented by limited amounts of range data taken on a noninterference basis.

When the Martian ephemeris errors of the magnitude currently estimated are considered, 15 days of range-rate tracking yields essentially no reduction in the uncertainty in the lander Z-component. Combining 15 days of range and range-rate data reduces this uncertainty to approximately 60 kilometers. This result points to a definite need for Martian ephemeris improvement. Such improvement could possibly be achieved through analysis of range data representing extended periods of time, during which the Earth-Mars geometry would have varied significantly.

Langley Research Center,
    National Aeronautics and Space Administration,
        Hampton, Va., November 12, 1970.

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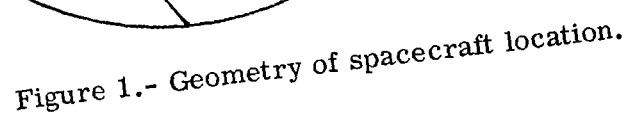
Figure 1.- Geometry of spacecraft location.
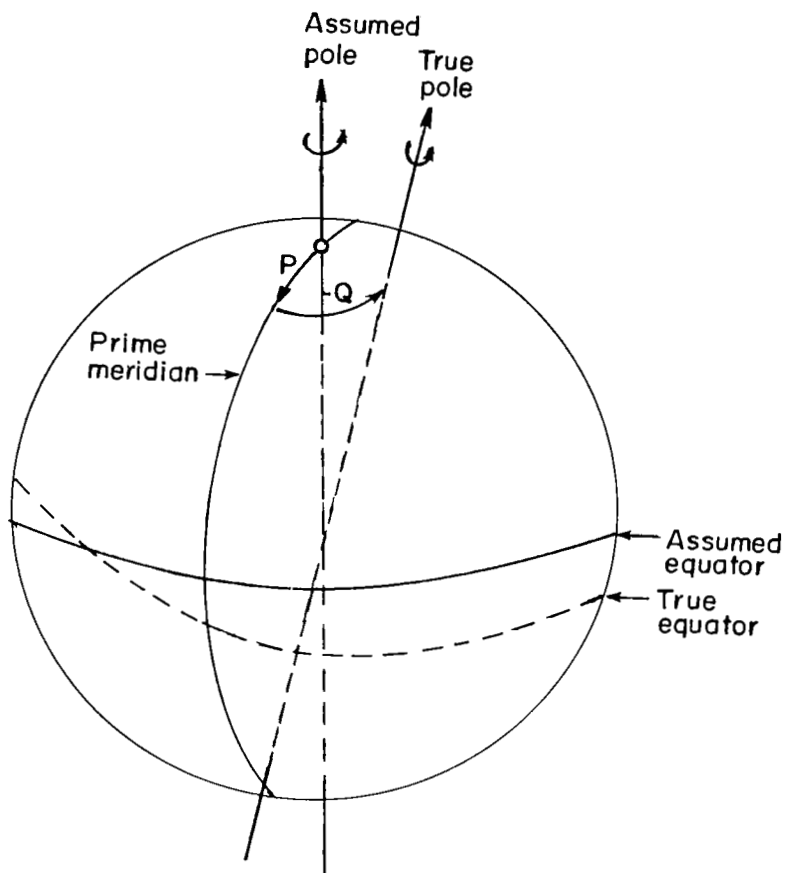
Figure 2.- Geometric relationship between the assumed
and true orientations of the Mars axis of rotation.

(a) λ.
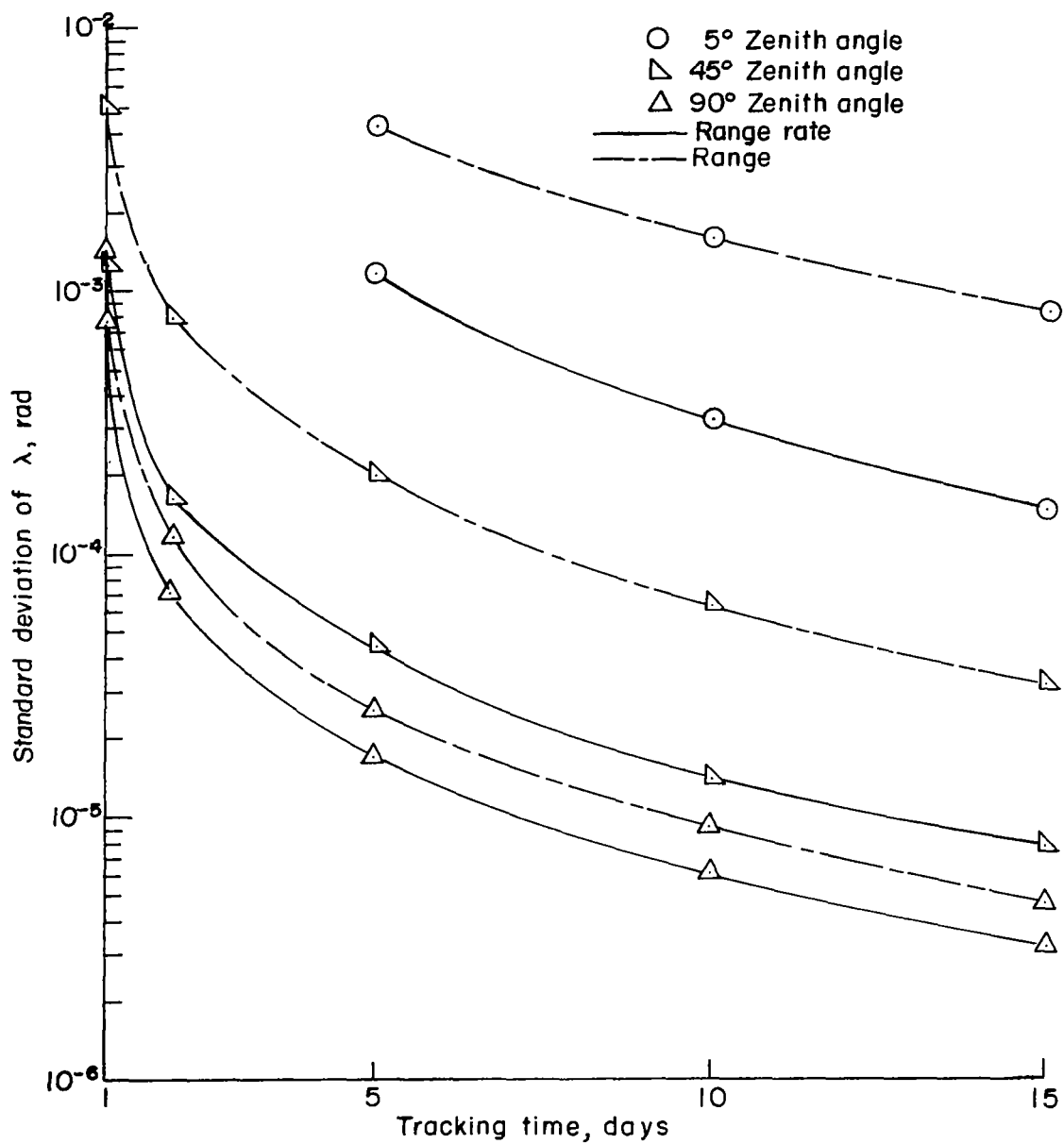
Figure 3.- Standard deviations of landed spacecraft location parameters from analysis of range and range-rate data independently. No model uncertainties are considered.
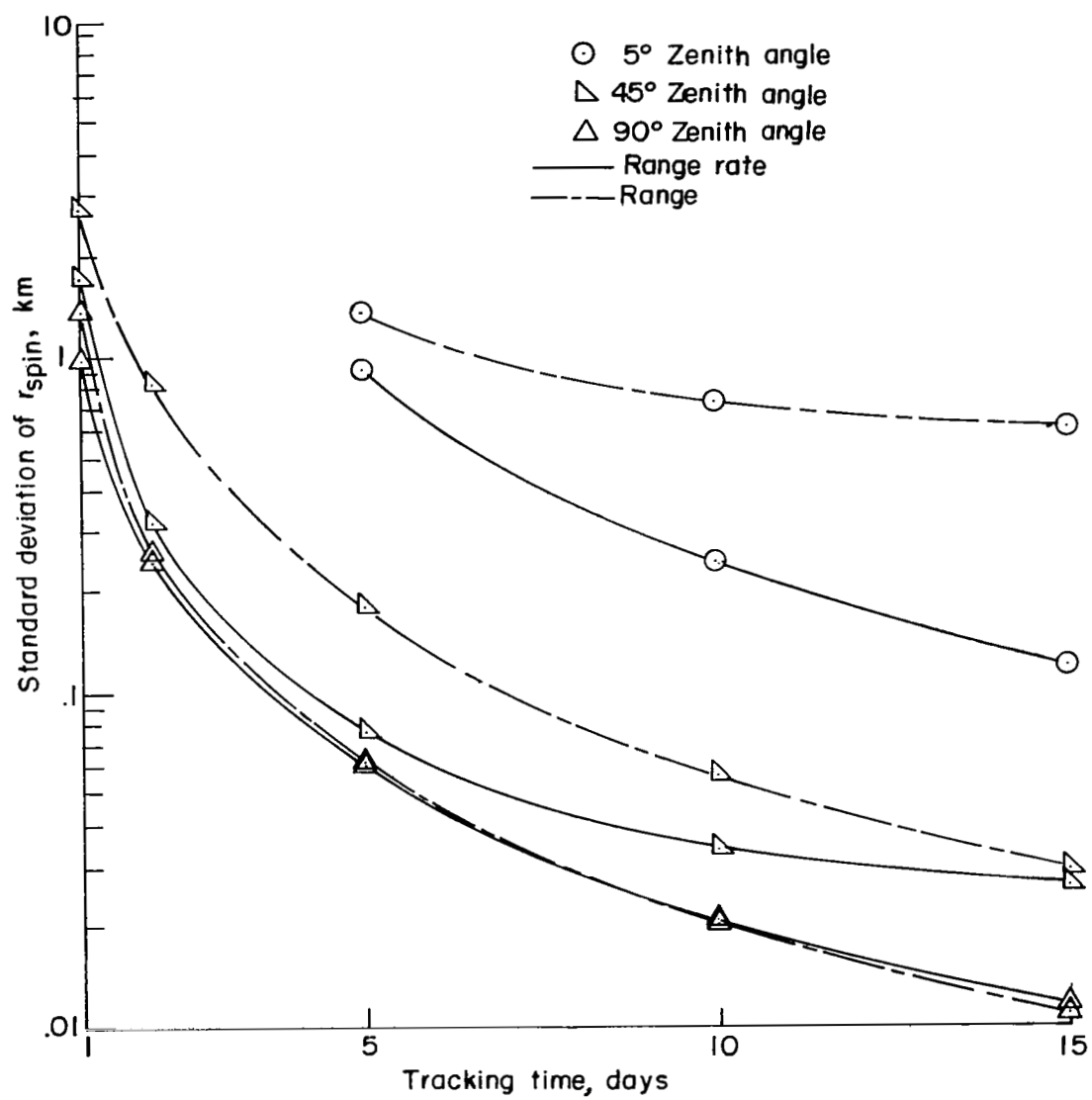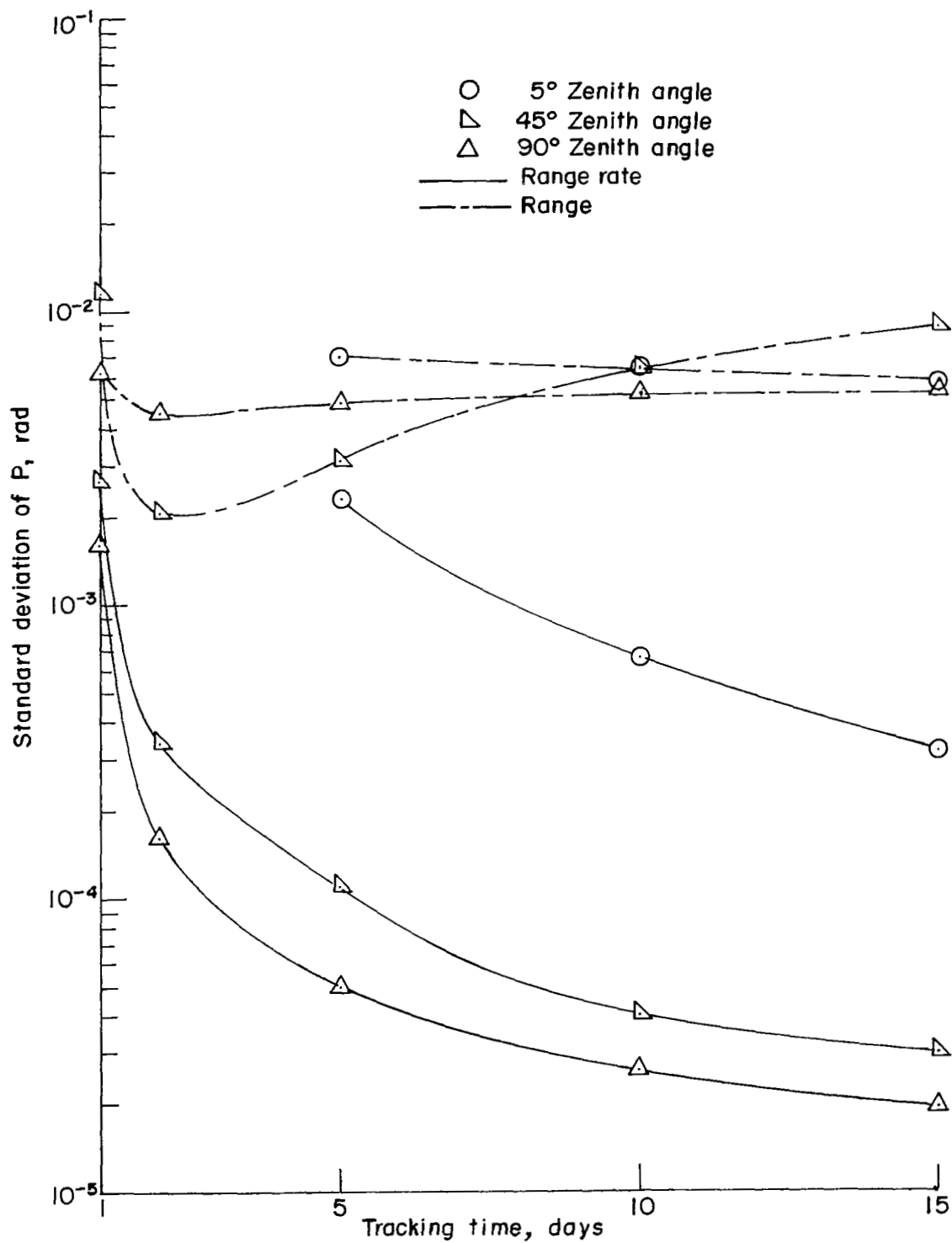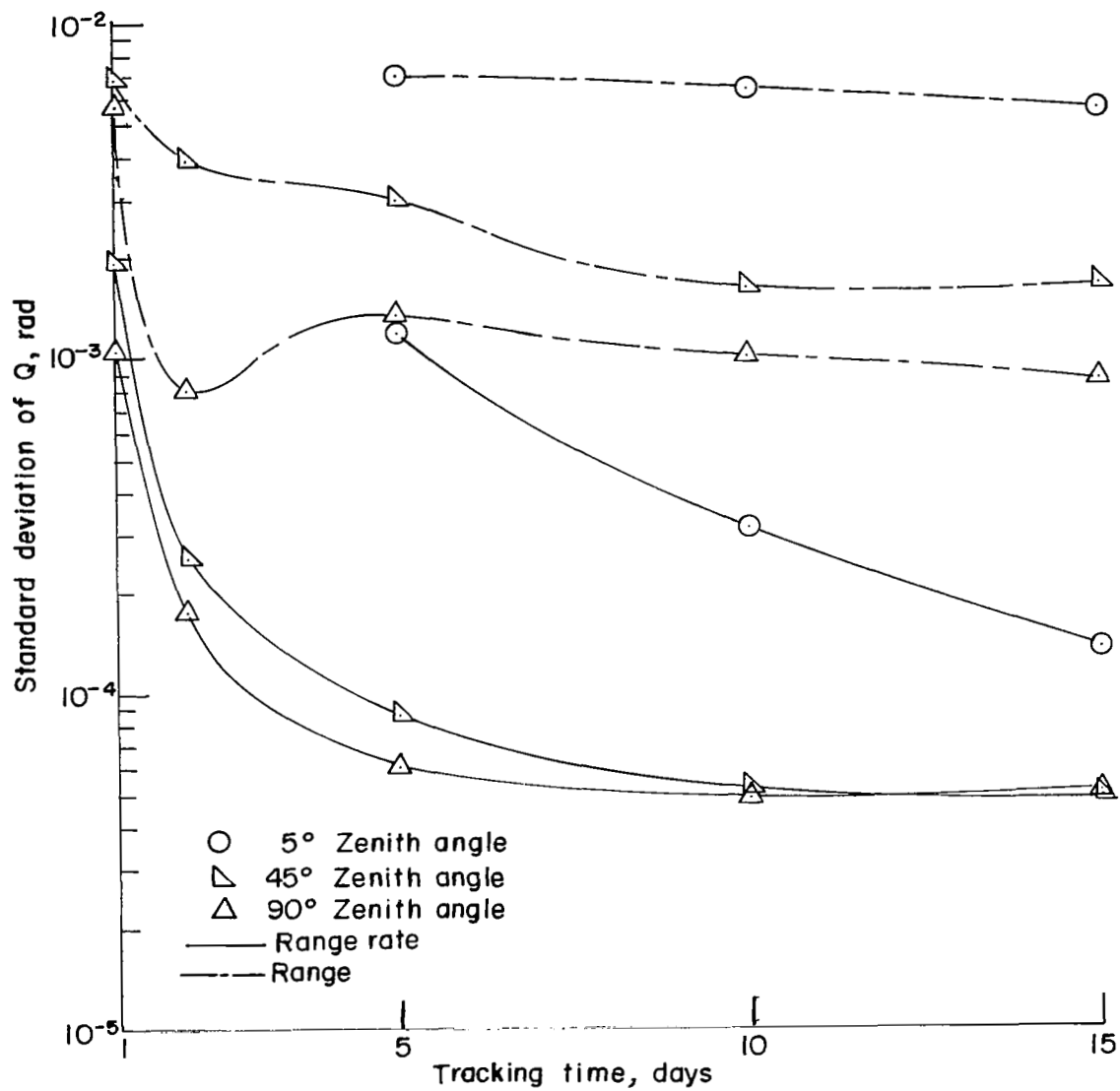
(b) r_{spin}.

Figure 3.- Continued.
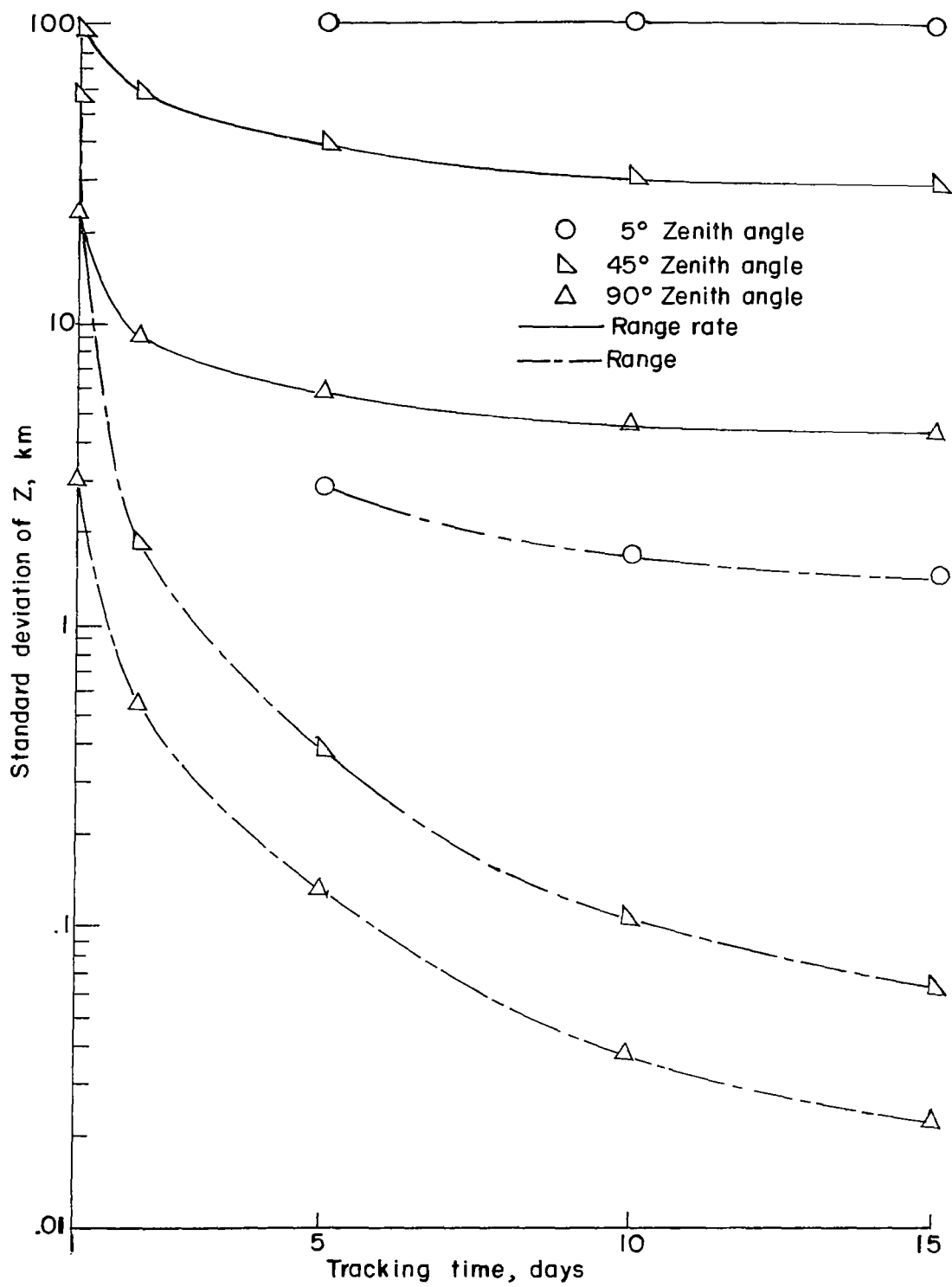
(c) Z.

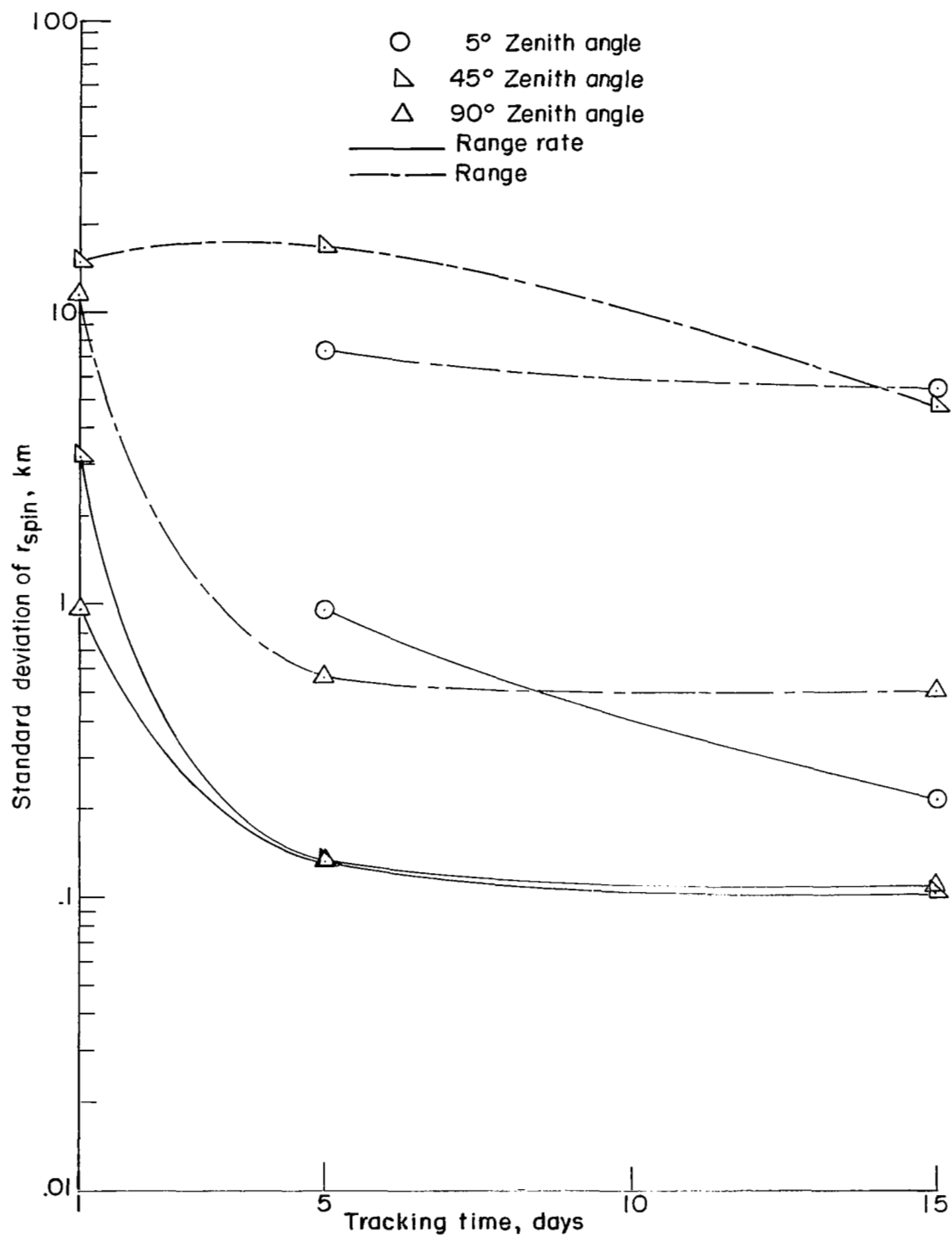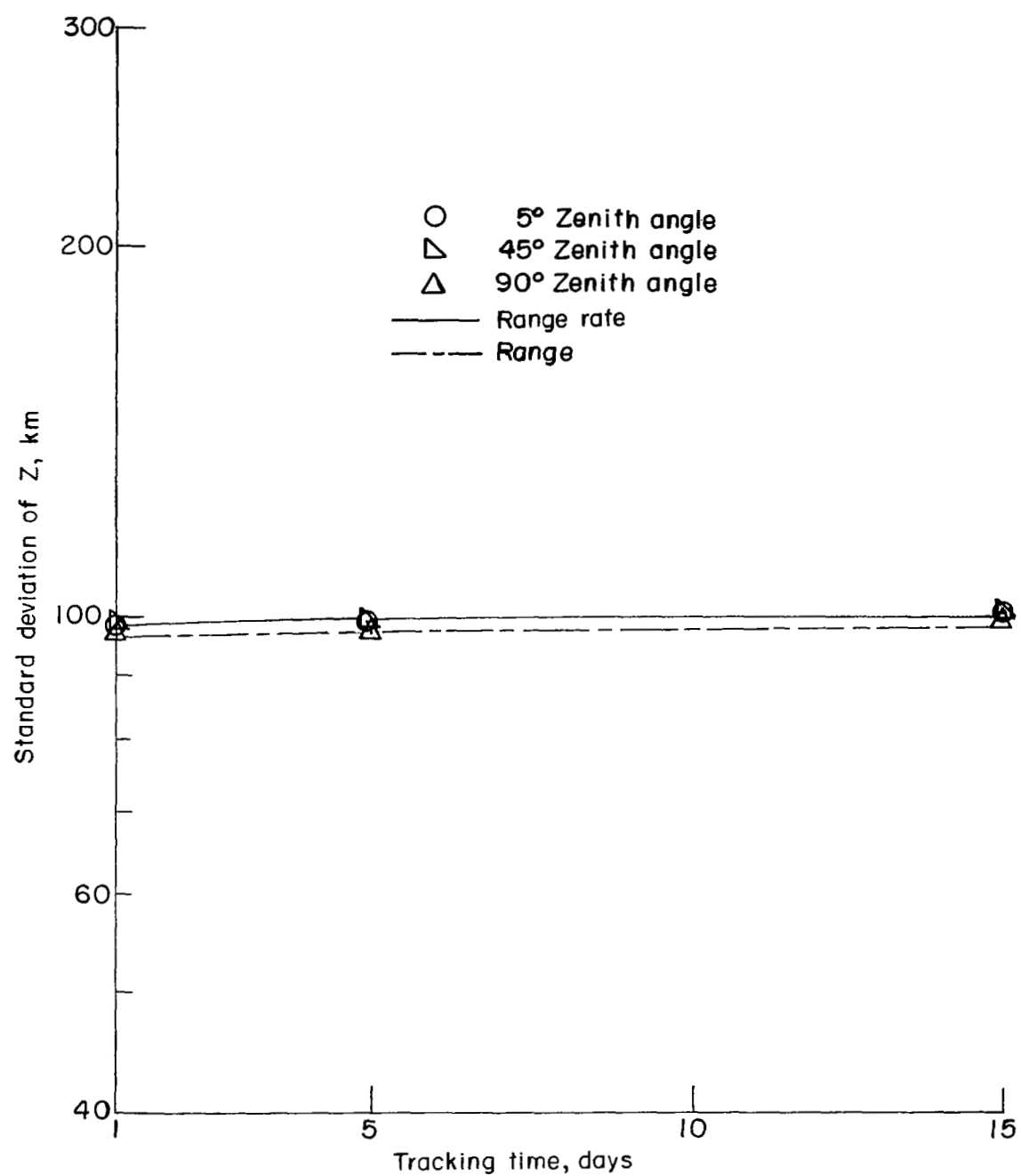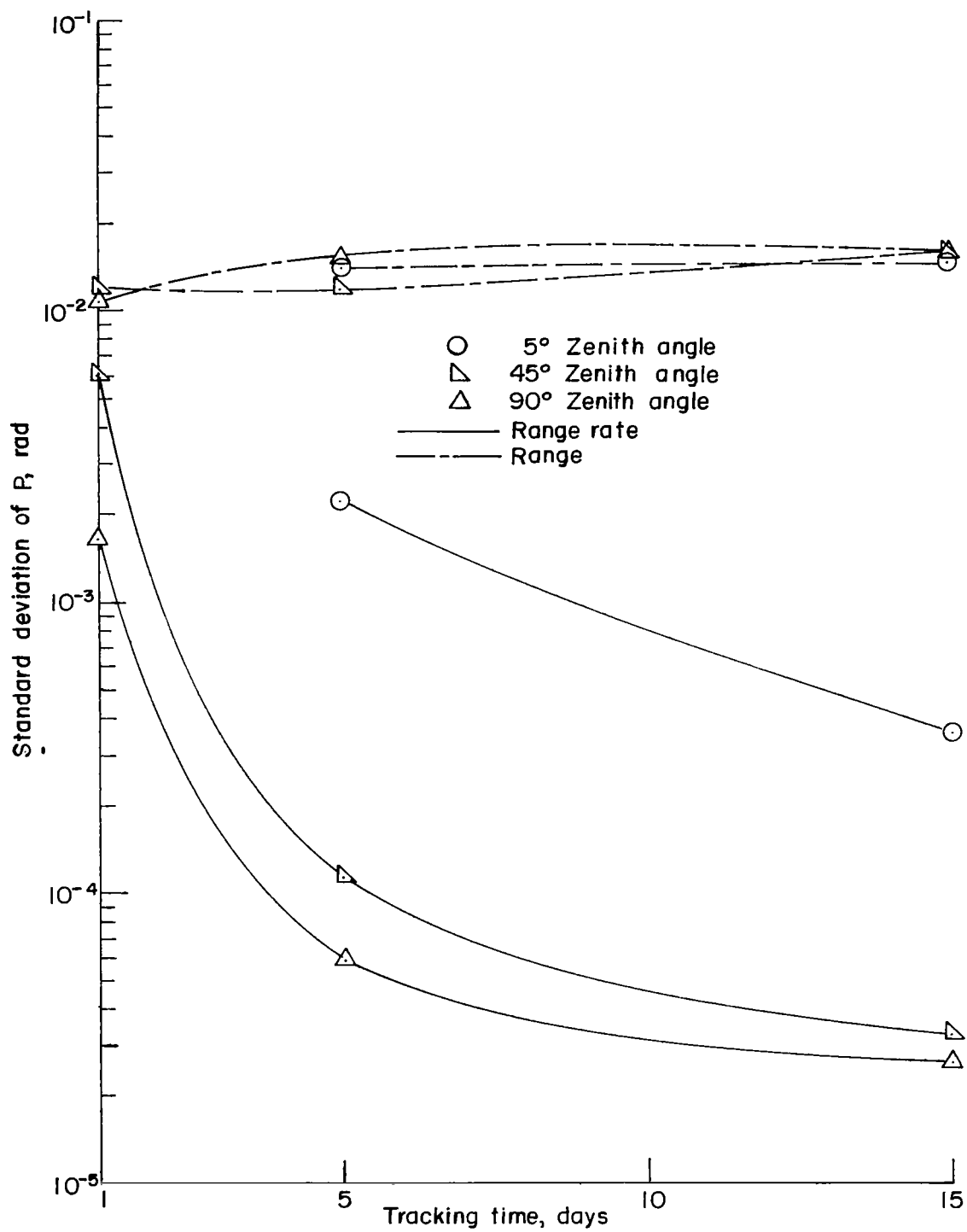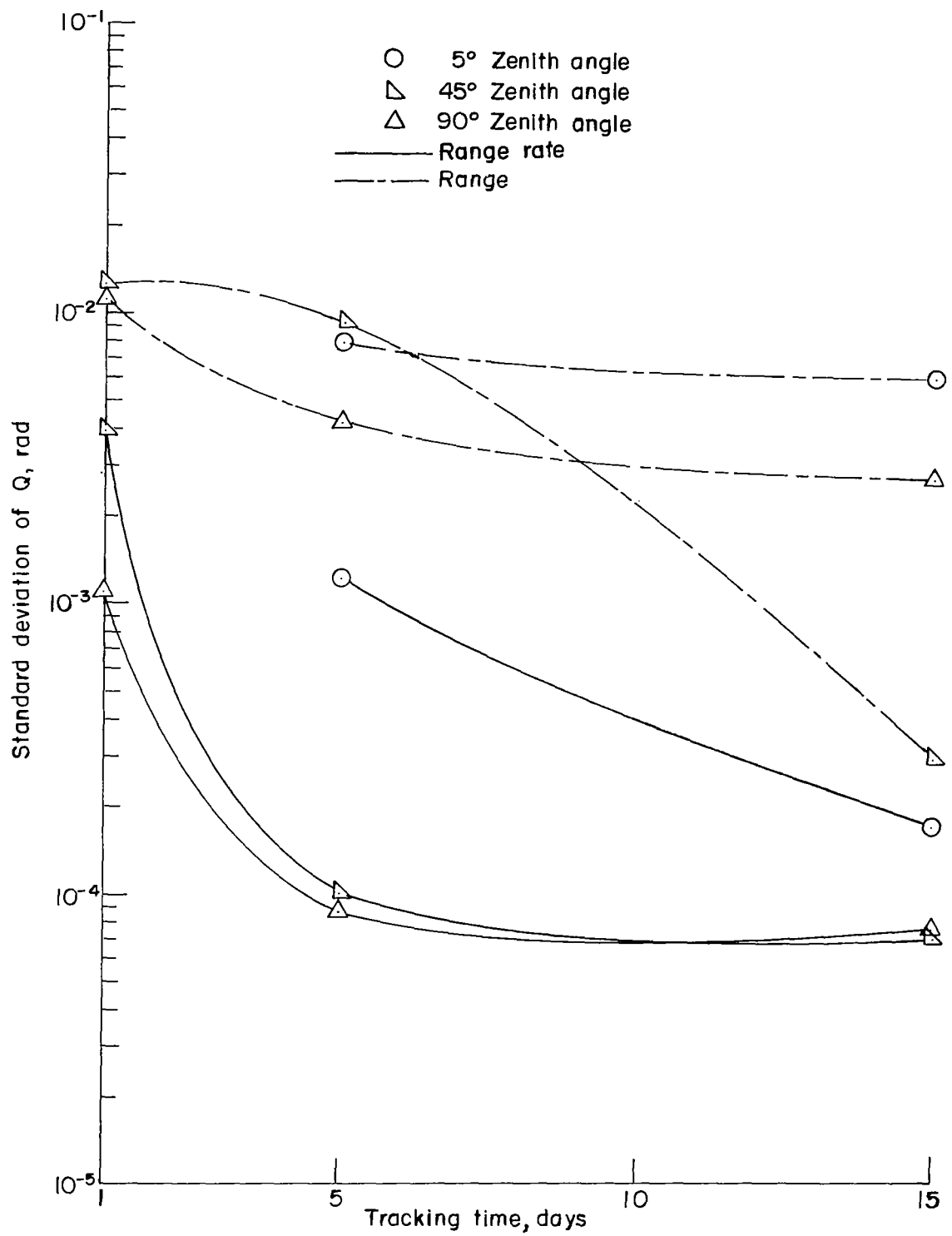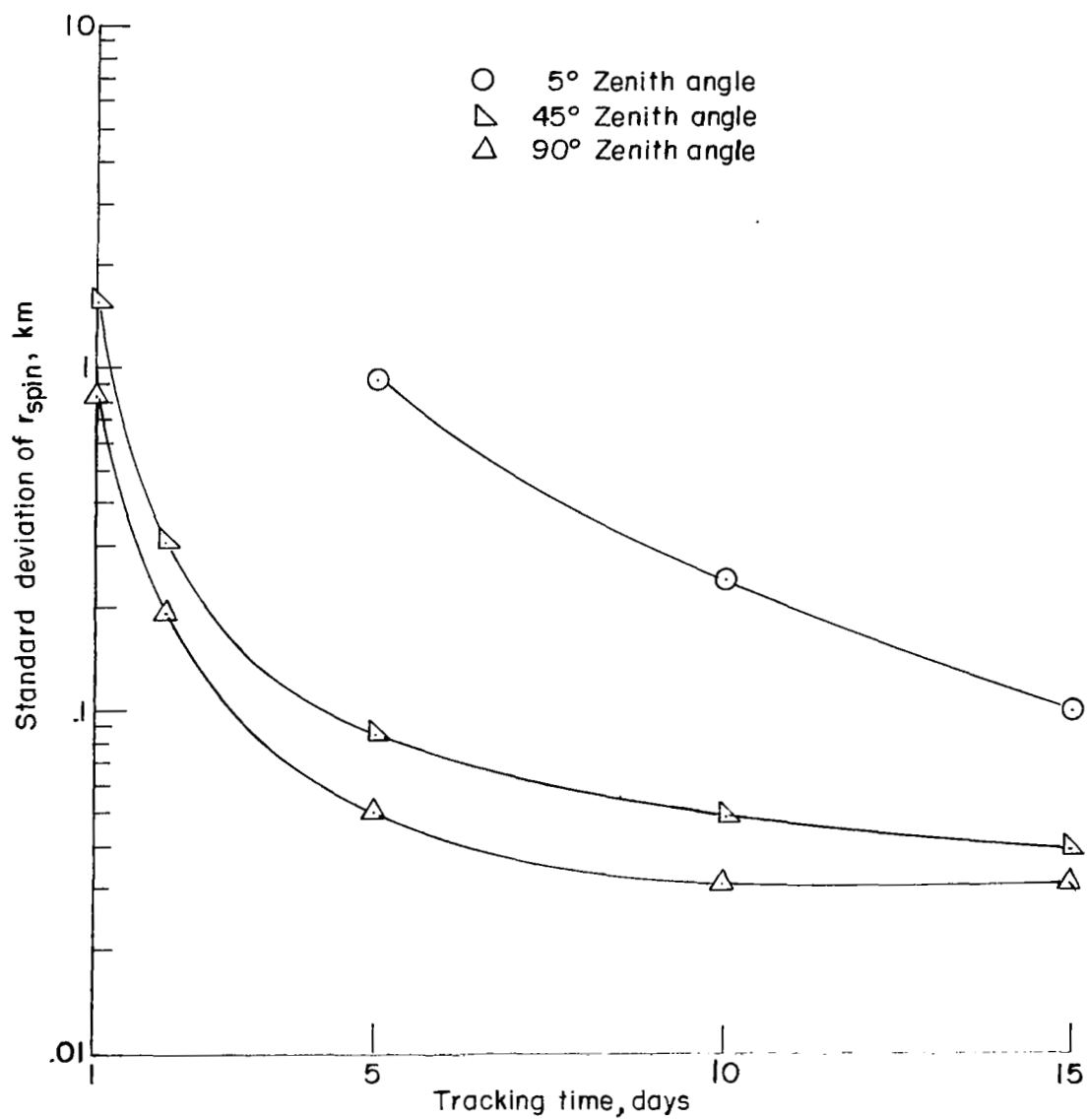Figure 3.- Continued.
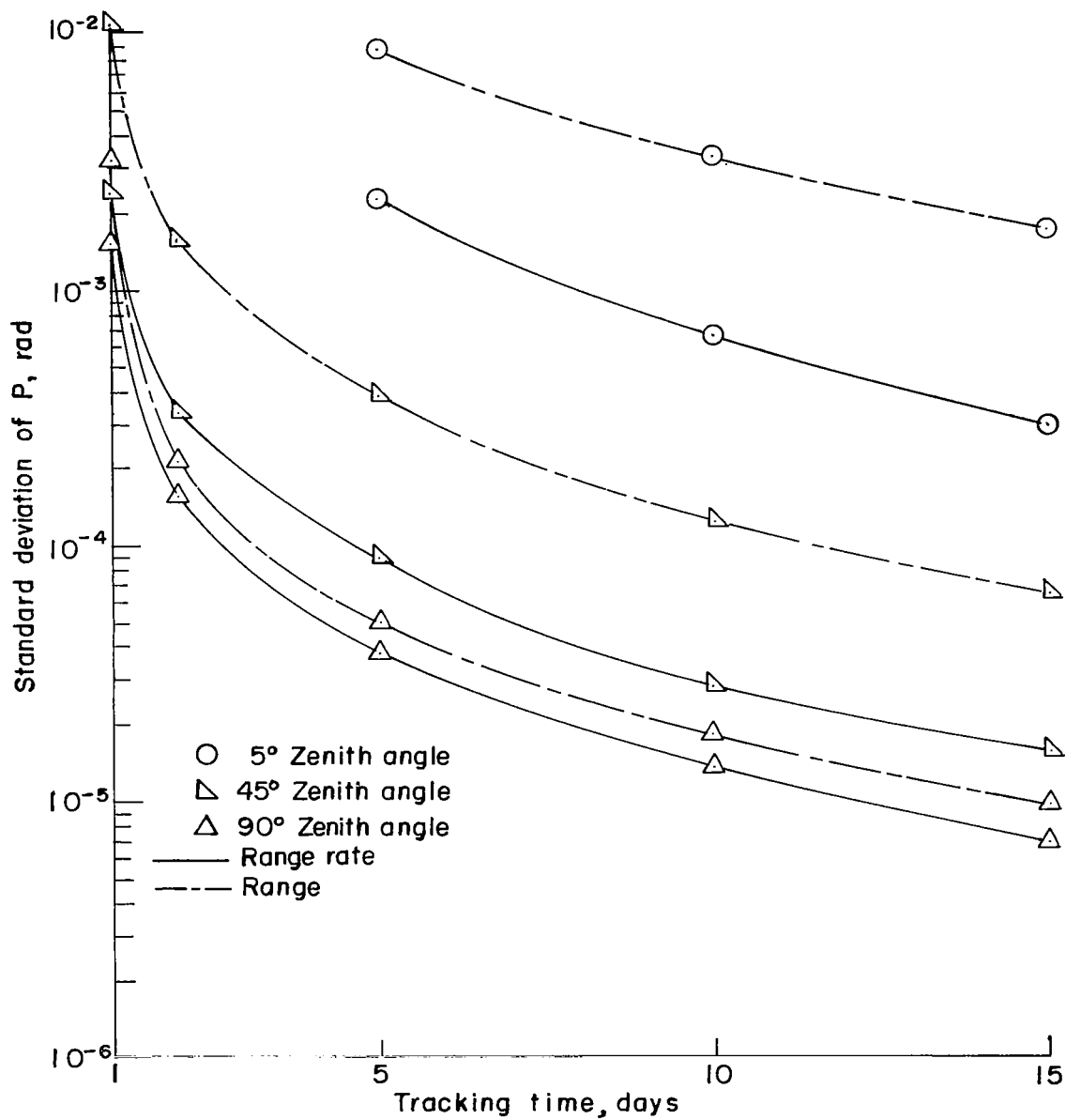
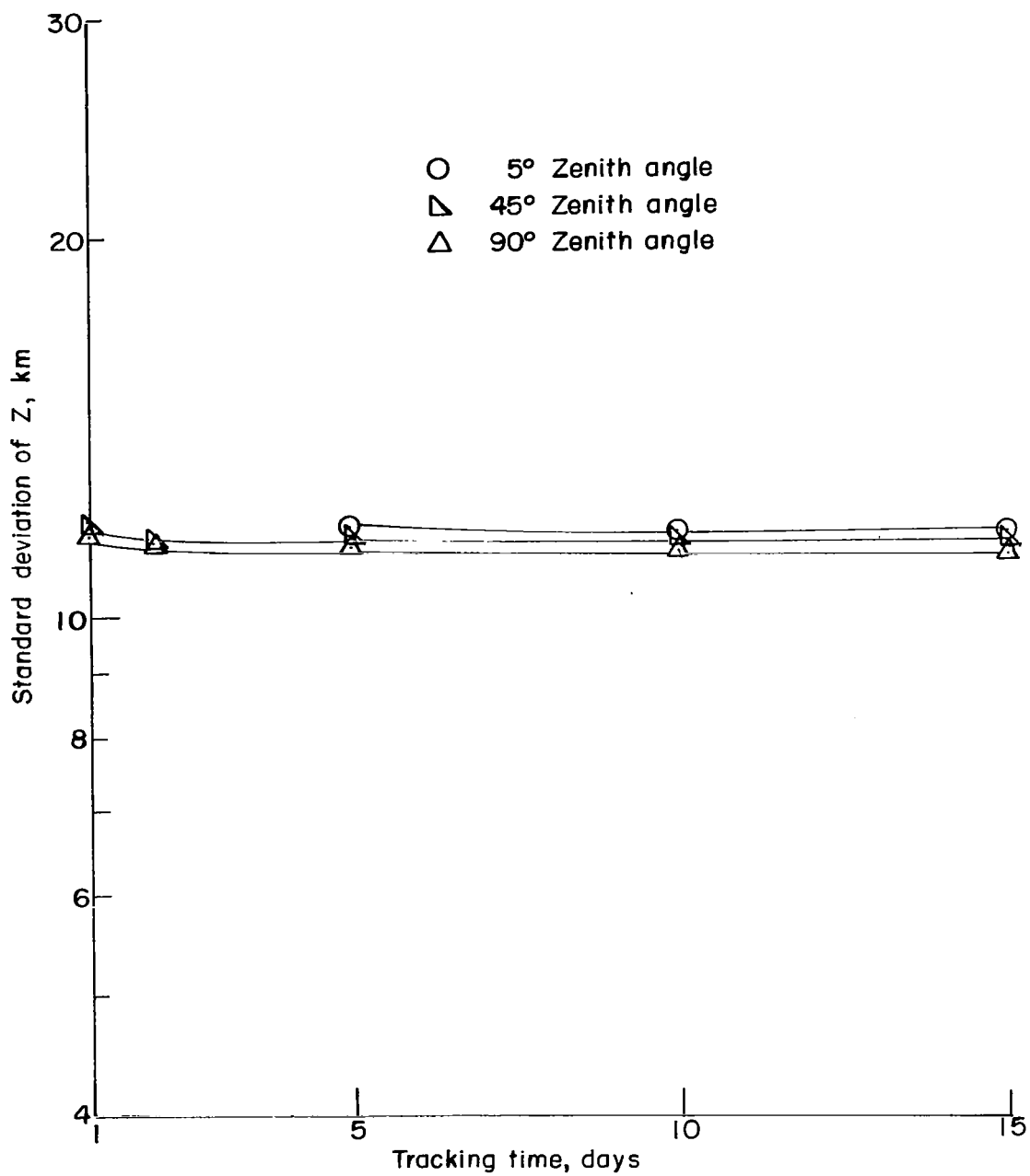(d) P.

Figure 3.- Continued.
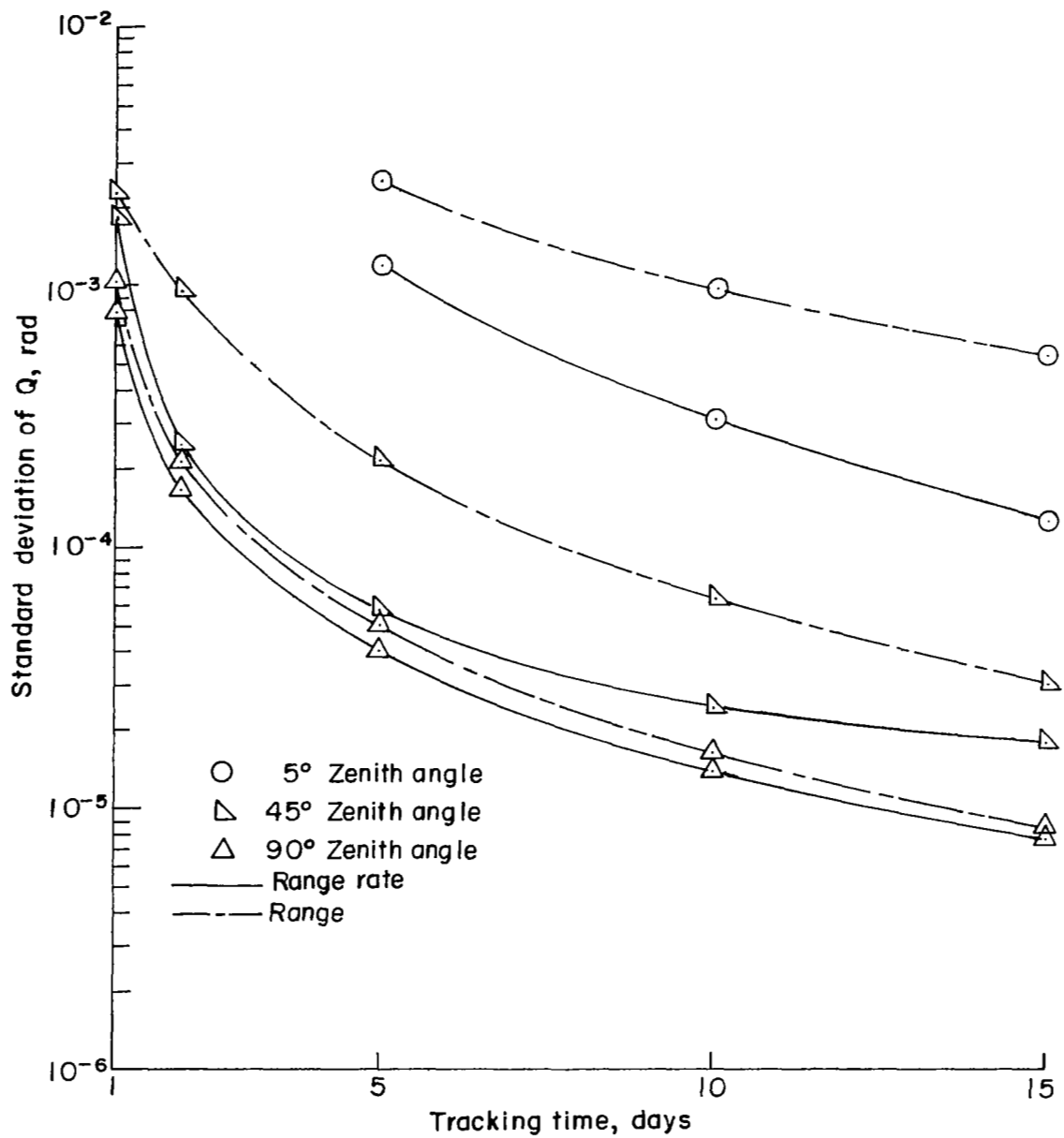
(e) Q.

Figure 3.- Concluded.

(a) λ.
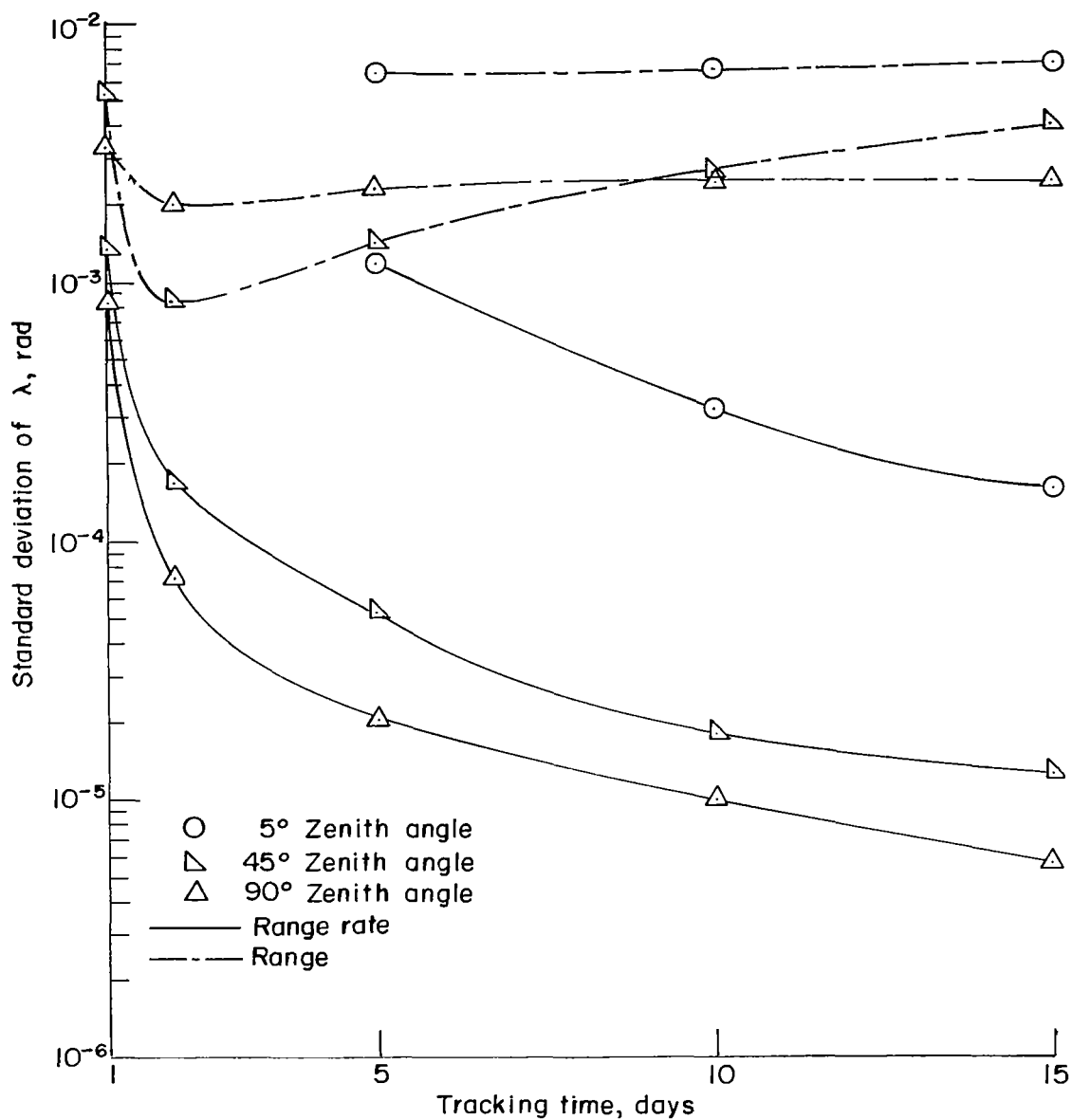
Figure 4.- Standard deviations of landed spacecraft location parameters from analysis of range and range-rate data independently. Model uncertainties are considered, the Mars ephemeris position uncertainty assumed to be 5 kilometers.
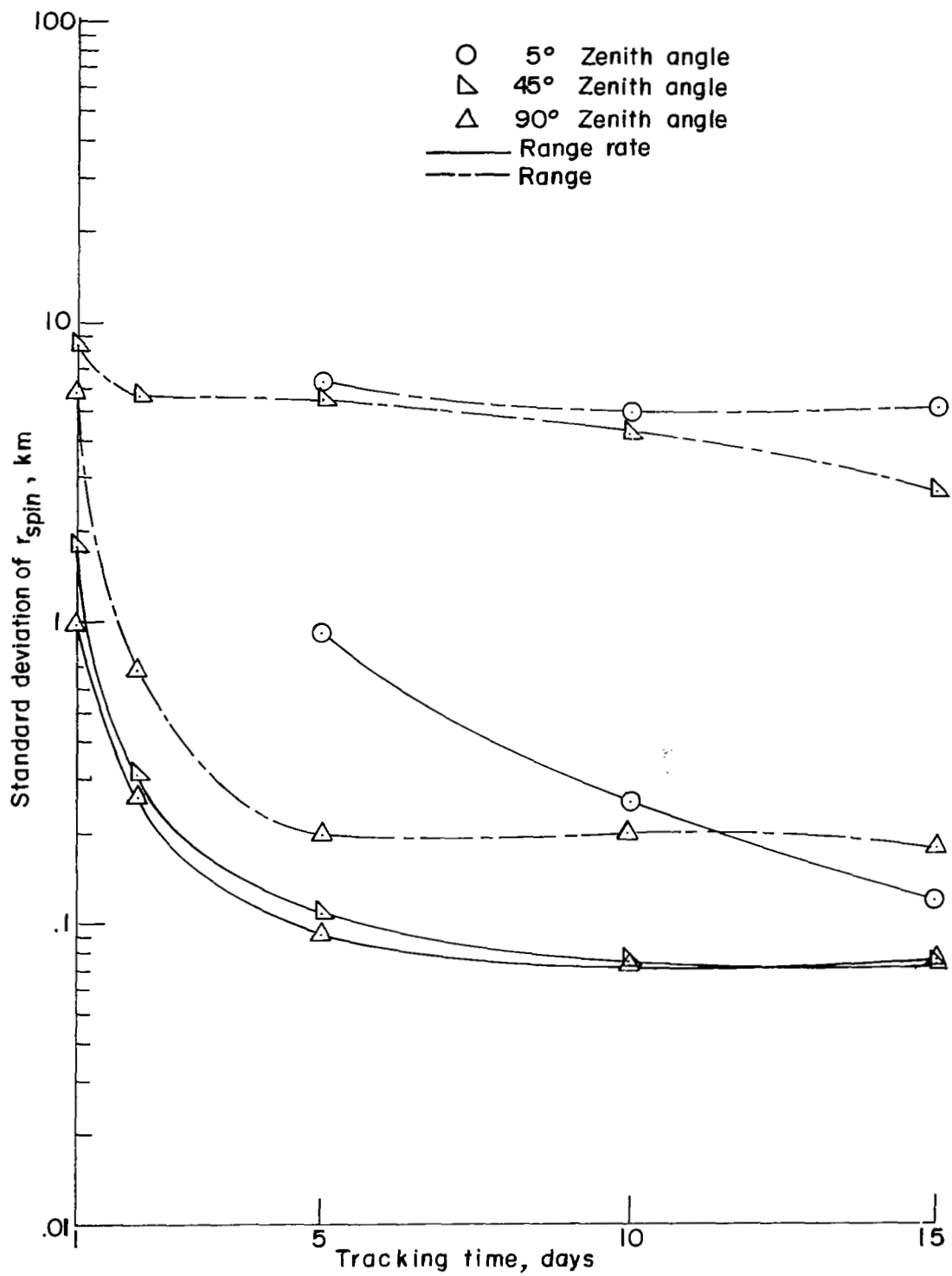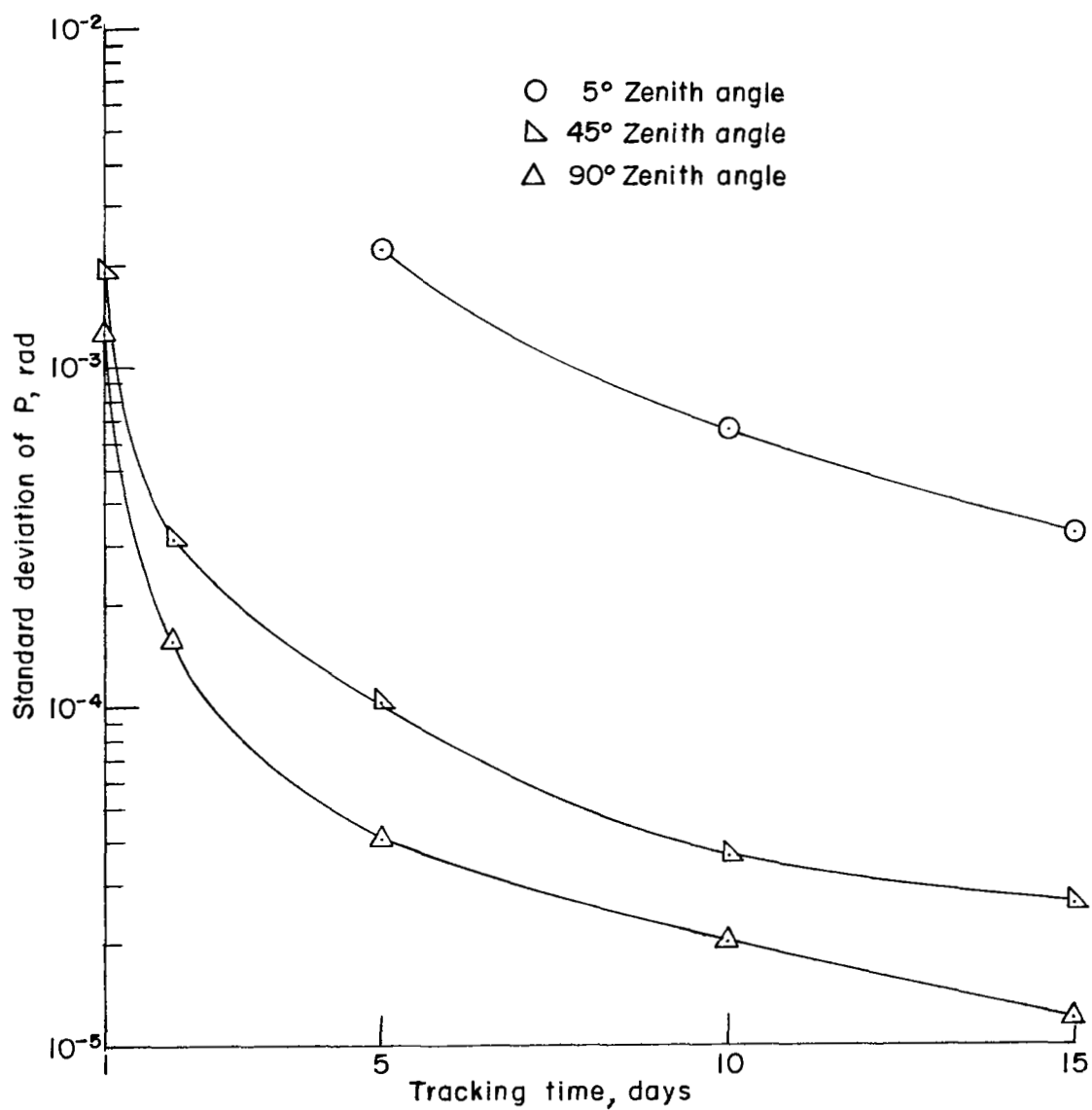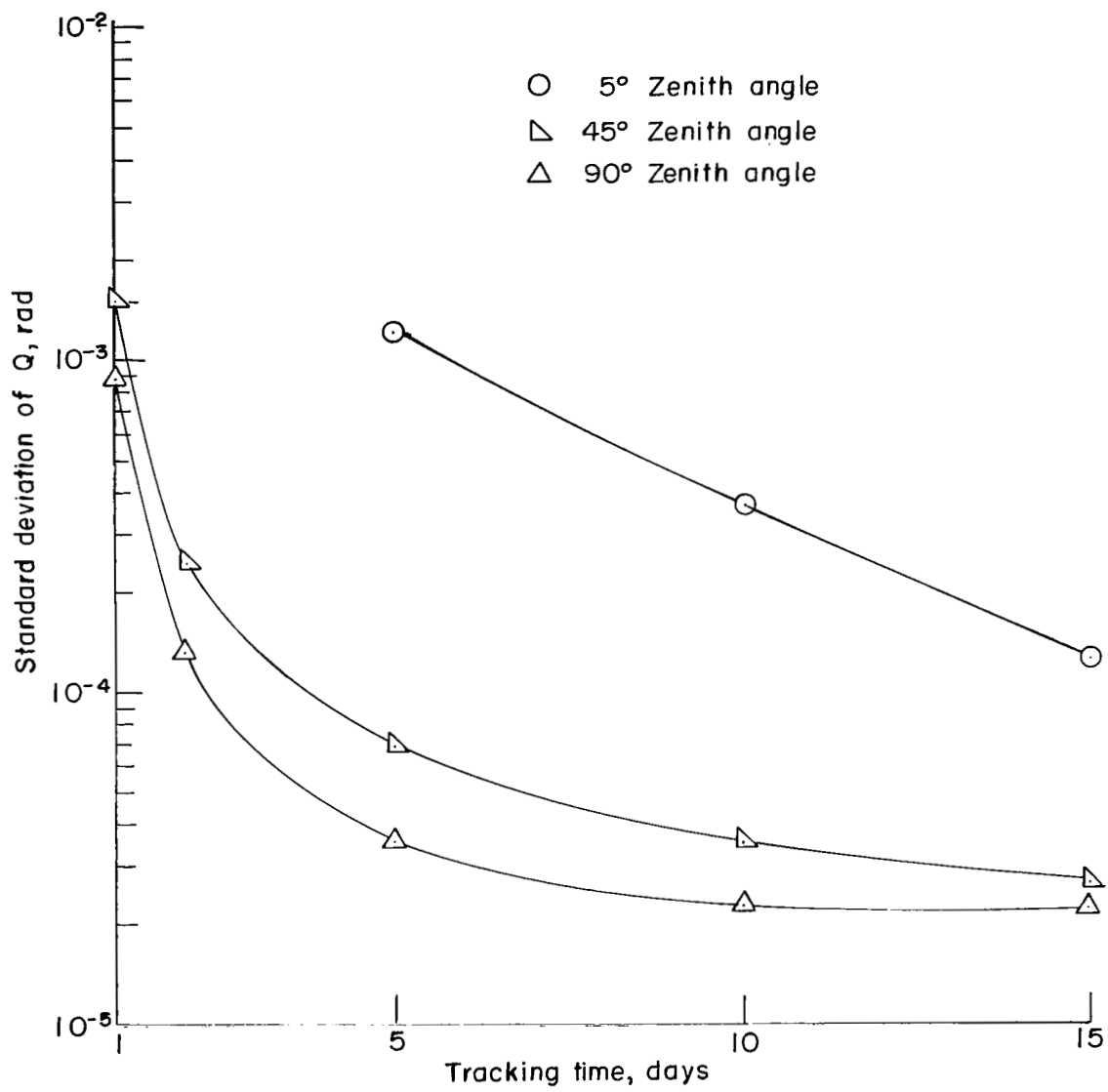
(b) r_{spin}.

Figure 4.- Continued.
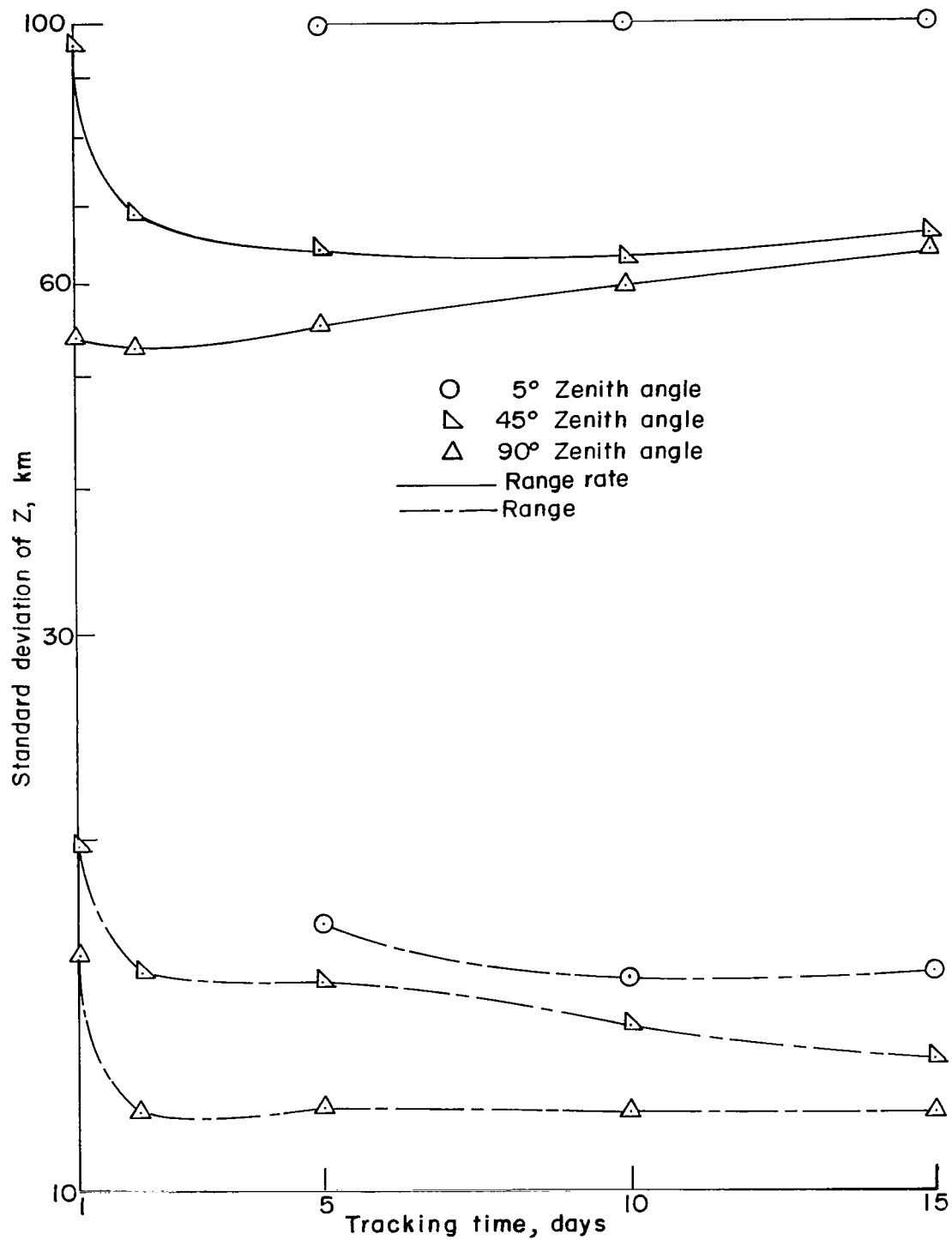
(c) Z.

Figure 4.- Continued.

(d) P.

Figure 4.- Continued.

(e) Q.

Figure 4.- Concluded.

(a) λ.
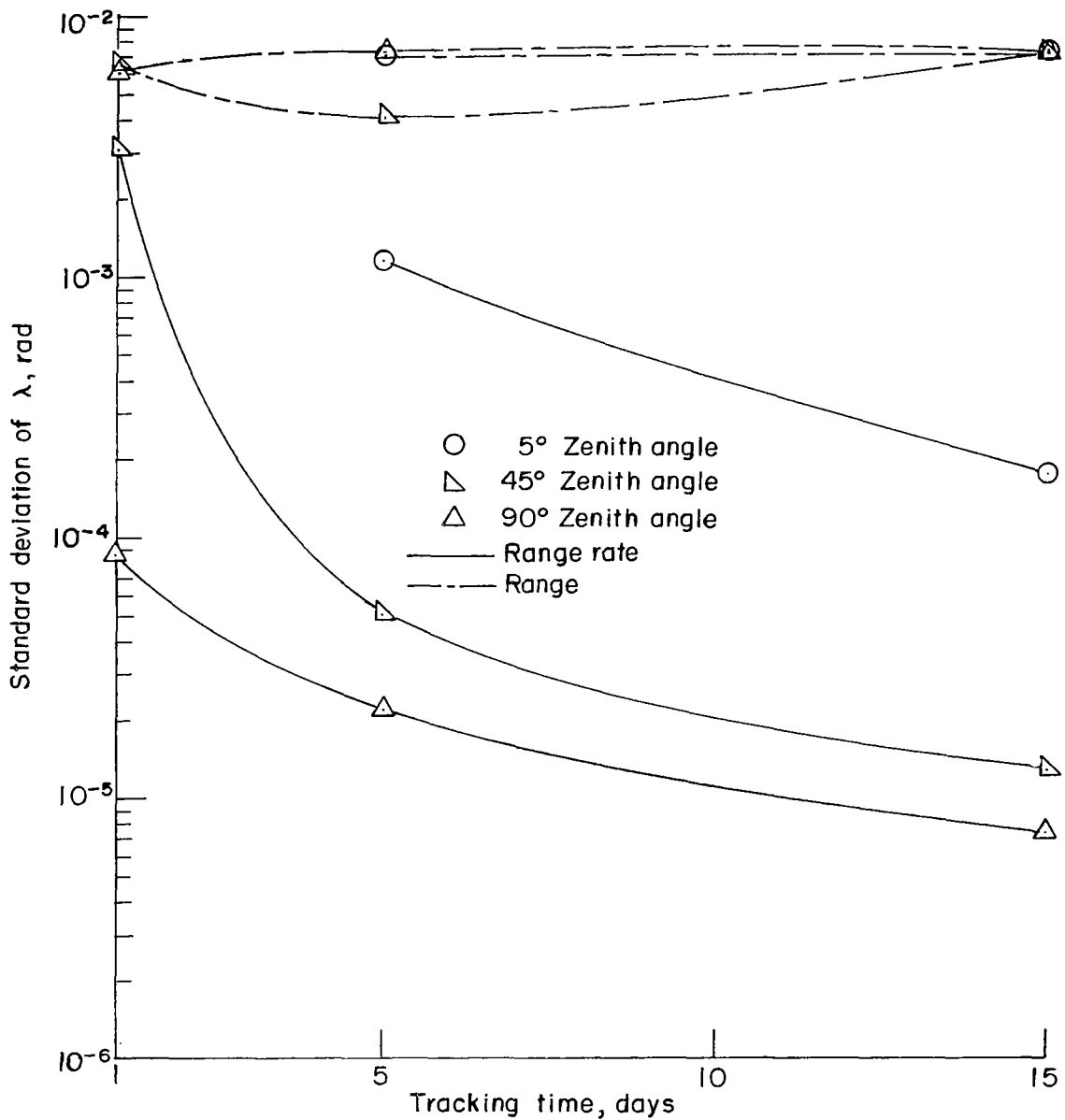
Figure 5.- Standard deviations of landed spacecraft location parameters from analysis of range and range-rate data independently. Model uncertainties are considered, the Mars ephemeris position uncertainty assumed to be 200 kilometers.

(b) r_{spin}.

Figure 5.- Continued.

(c) Z.
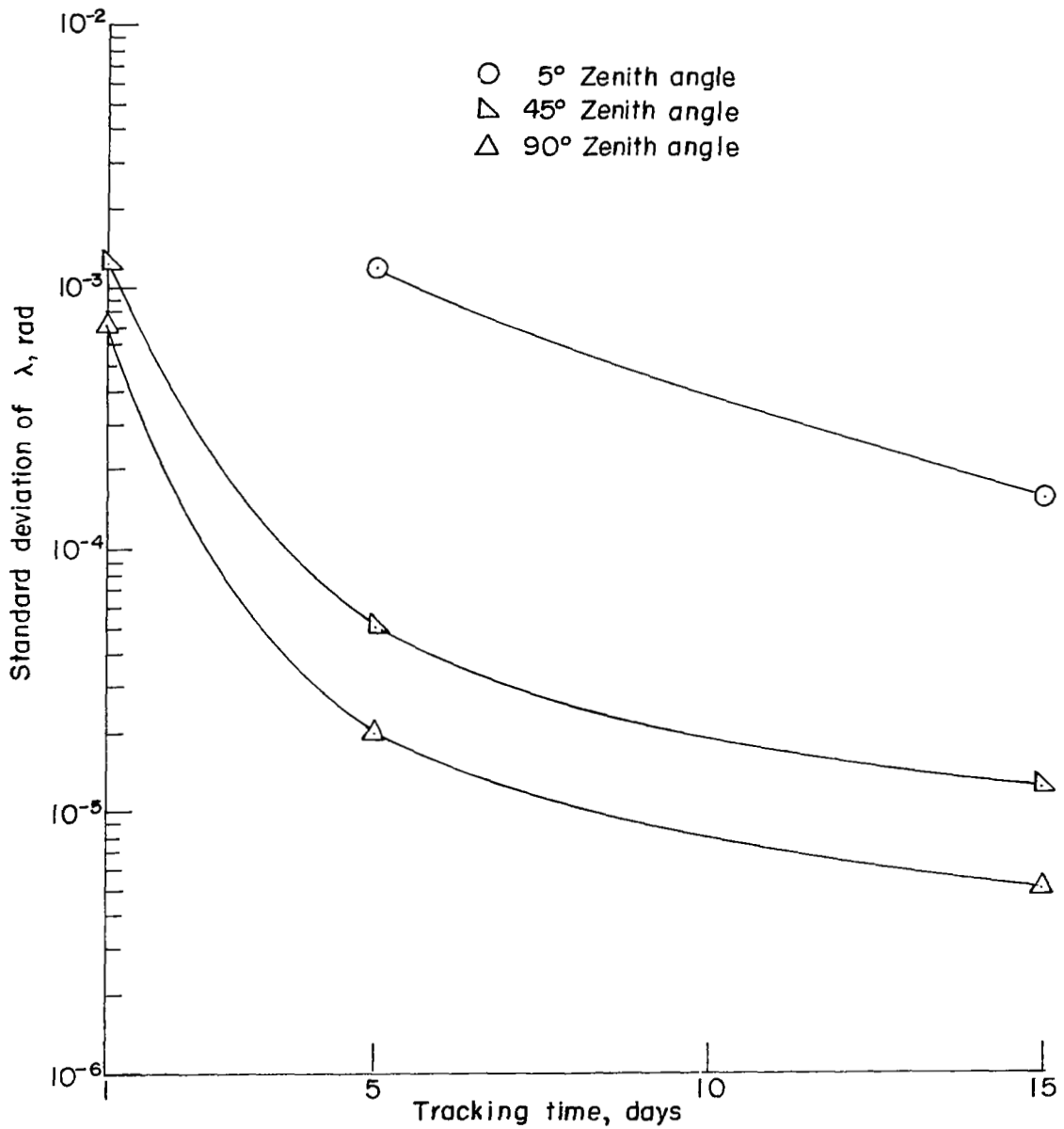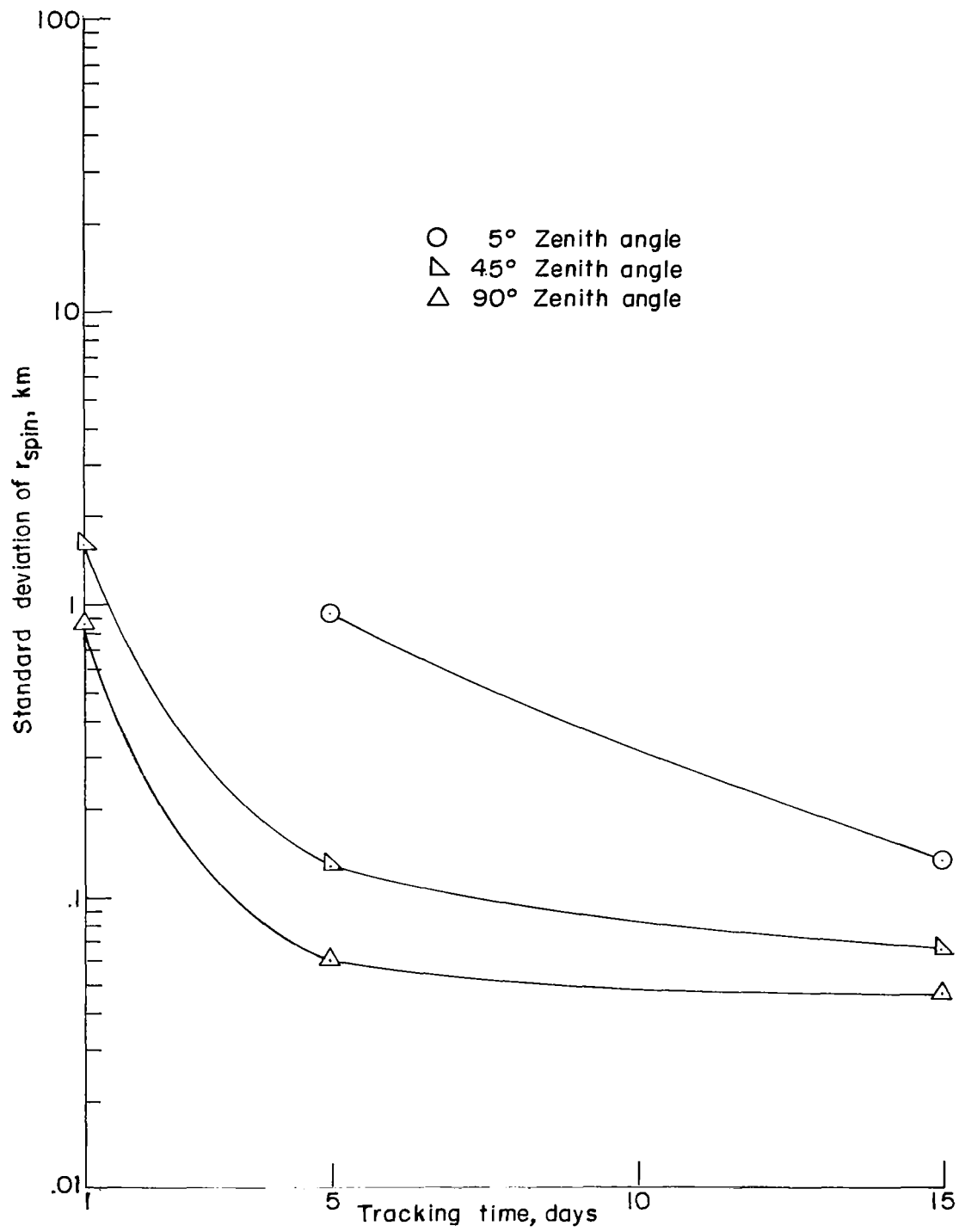
Figure 5.- Continued.

(d) P.
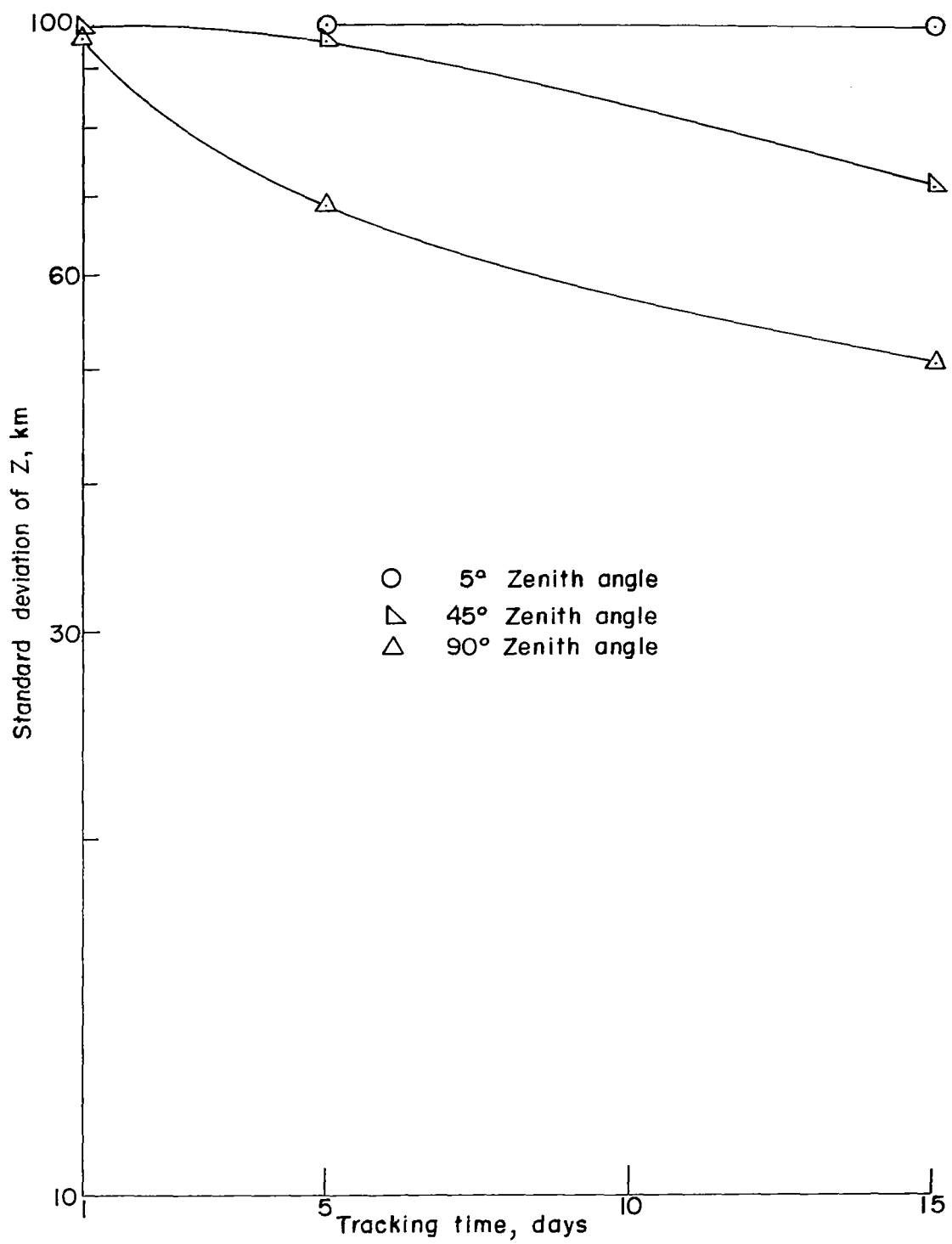
Figure 5.- Continued.

(e) Q.

Figure 5.- Concluded.

(a) λ.
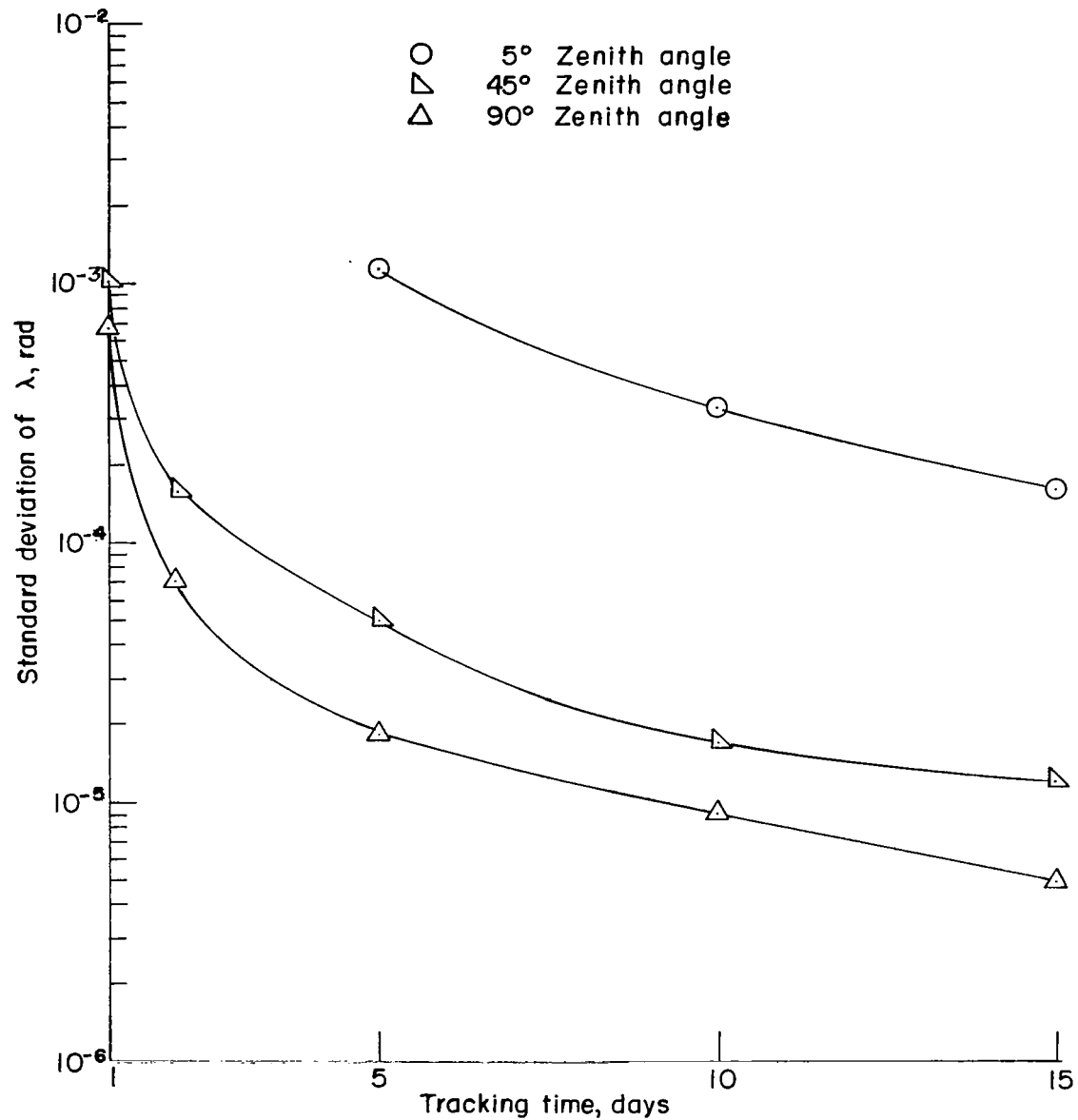
Figure 6.- Standard deviations of landed spacecraft location parameters based on minimum variance combination of independent range and range-rate estimates. Model uncertainties are considered, the Mars ephemeris position uncertainty assumed to be 5 kilometers.

(b) r_{spin}.

Figure 6.- Continued.

(c) Z.

Figure 6.- Continued.

(d) P.

Figure 6.- Continued.

(e) Q.

Figure 6.- Concluded.

(a) λ.

Figure 7.- Standard deviations of landed spacecraft location parameters based on minimum variance combination of independent range and range-rate estimates. Model uncertainties are considered, the Mars ephemeris position uncertainties assumed to be 200 kilometers.
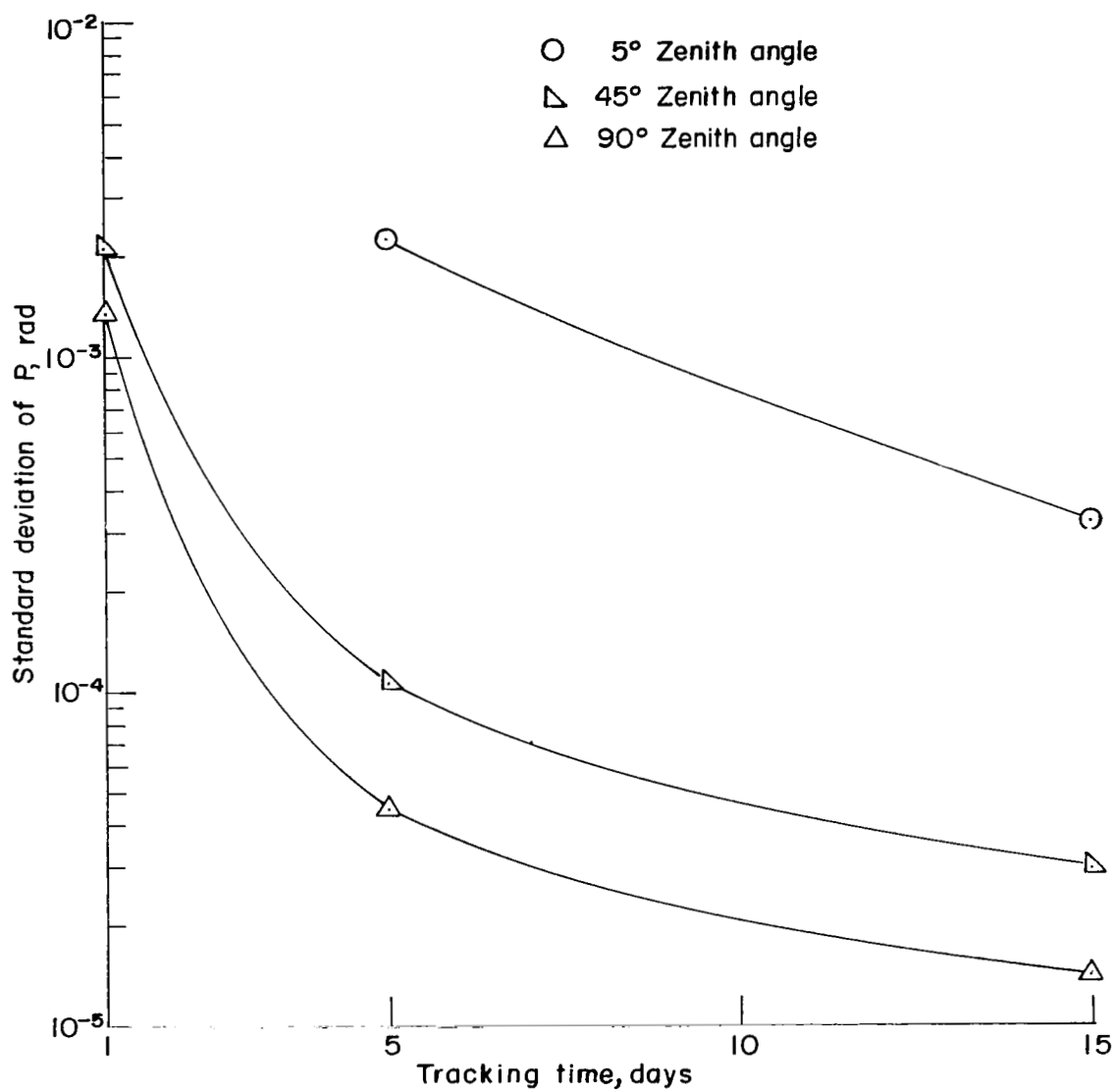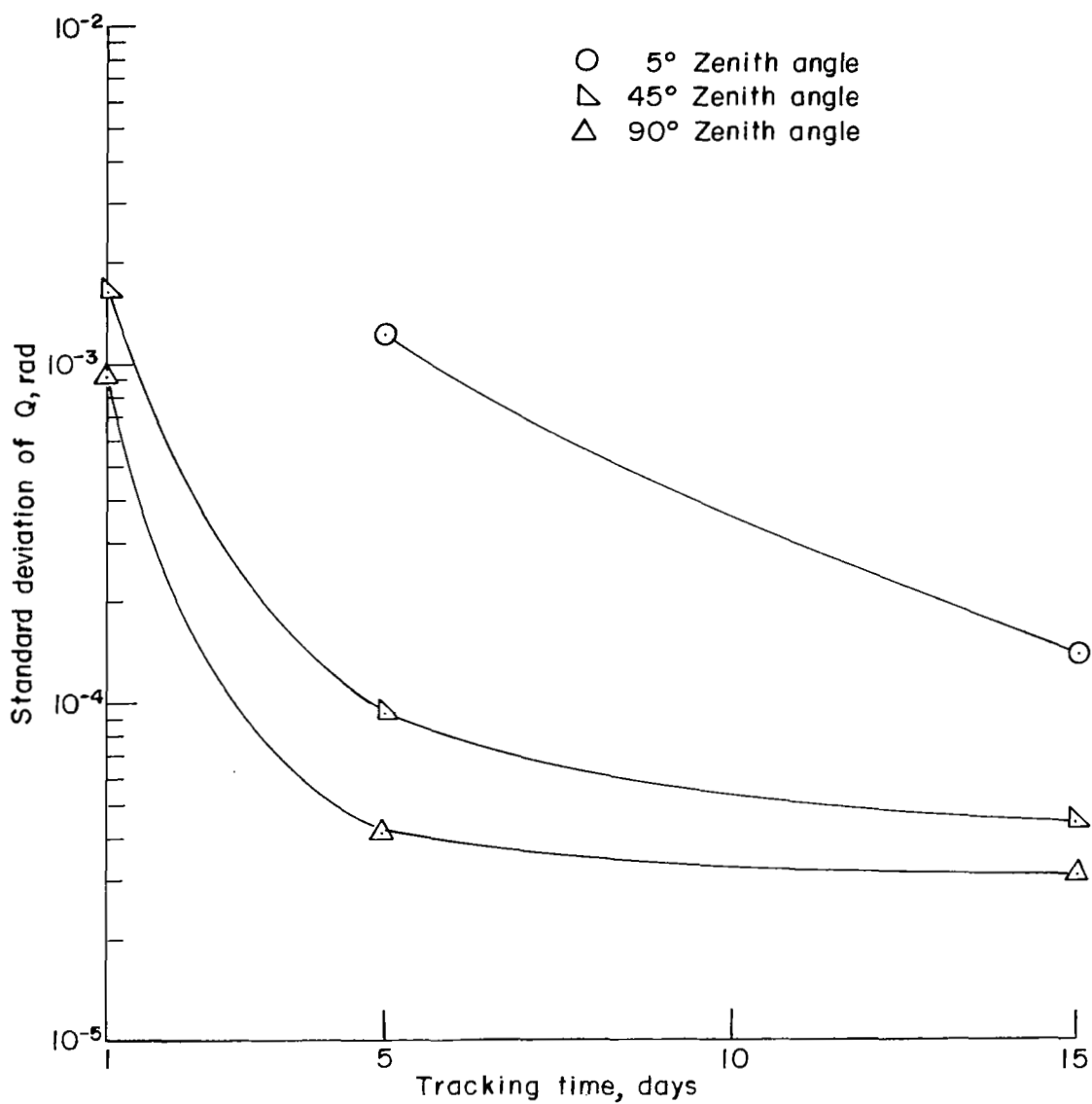
(b) r_{spin}.

Figure 7.- Continued.

(c) Z.

Figure 7.- Continued.

(d) P.

Figure 7.- Continued.

(e) Q.

Figure 7.- Concluded.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

— NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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