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FINAL REPORT

NASA/ERC Contract No. NGR 23-054-003
March 15, 1969 to June 31, 1970

BIOSYSTEMS ENGINEERING RESEARCH

Volume IV. Augmentation of the Focus Control System of the Human Eye through Index of Refraction Variation of Liquid Crystals Materials.

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<u>VOLUME IV</u>

AUGMENTATION OF THE FOCUS CONTROL SYSTEM OF THE HUMAN EYE THROUGH INDEX OF REFRACTION VARIATION OF LIQUID CRYSTAL MATERIALS

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FOREWORD

This report was prepared under NASA Contract NGR 23-054-003, which provided institutional support for biosystems research conducted by the Oakland University School of Engineering, and is one of four volumes reporting research conducted during the period March 15, 1969 to June 31, 1970.

The four volumes of the series, together with their principal authors are listed below:

Volume I	A Dynamic Model of the Human Postural Control System (J.C. Hill).
Volume II	An Investigation of Two Hybrid Computer Identification Techniques for Use in Manual Control Research (G.A. Jackson).
Volume III	Pattern Recognition of Biological Photomicrographs Using Coherent Optical Techniques (R.E. Haskell).
Volume IV	Augmentation of the Focus Control System of the Human Eye through Index of Refraction Variation of Liquid Crystal Materials (R.H. Edgerton).

Professors J.C. Hill and J.E. Gibson, Dean of the School of Engineering, were principal investigators on the contract.

ABSTRACT

Research on the feasibility of developing a focus control device utilizing nematic liquid crytals is presented. Application to the development of a replacement for bifocal lenses is discussed. Experimental results obtained by applying a simple shear to nematic liquid crystals demonstrated that approximately a 5% change in index of refraction is realizable. A Theoretical model to predict changes in the index of refraction assuming the crystals behave as ellipsoids when in a shear field is developed. This model is used in the geometrical design of the shear element.

AUGMENTATION OF THE FOCUS CONTROL SYSTEM OF THE HUMAN EYE THROUGH INDEX OF REFRACTION VARIATION OF LIQUID CRYSTAL MATERIALS

Augmentation of the Focus Control System of the Human Eye through Index of Refraction Variation of Liquid Crystals Materials

by

Robert H. Edgerton

Introduction

The ability of people to perform everyday functions is often impaired by the biological limitations of ageing. The present pace of modern society requires the use of prosthetic devices. Perhaps the most common device is the ordinary spectacle. The use of spectacles is not limited to diseased or genetically deficient people. Their use is required in the present society due to the effects of ageing on the eyes. This report discusses the ageing effect on the focusing ability of the eye and attempts to produce a new prosthetic means for assisting in this function. Figure 1 shows the function of age on the ability of the lens to focus.[1] The scale is noted as accomodation in diopters. Accomodation represents the change in focus measured in diopters (or reciprocal focal length in meters). This ageing effects <u>everyone</u> and limits their ability to perform adequately such tasks as automobile driving, operating machinery while reading instructions, or continuing to maintain ability in sports such as baseball or tennis.

In operating transportation vehicles, whether of autos or airplanes, the human driver is (or should) continually focus on the outside distant objects, and also on the instruments at close range. The change of focus tasks involves factors other than total accomodation, however as for example, fatique can lead one to unconsciously avoid the observation usually of the near objects since the eye is relaxed when focused at infinity.

This report deals with an attempt to examine a means for augmenting the focus requirements of the eye by varying the index of refraction of an optical system. In particular liquid crystal materials have been examined experimentally and a theoretical model of their behavior described here.

Human Focus Control Requirements

The requirements for an augmented focus control system for spectacle wearers will be discussed in this section. This will provide rough estimates on the feasibility of utilizing liquid crystals materials for this application.

Classical lens optics relate the focal power of a lens to the index of refraction and radius of curvature of the lens element.

As:

$$\frac{1}{f} = \left(n-1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (1)$$

f is the focal length of the lens; n the index of refraction (relative to air in this case) and r_1 and r_2 are the radii of curvature of the lens surface. To provide control of the focal length or focus then requires either changing the radii of curvature or the index of refraction of the lens material. The latter method is proposed in this research since it would not be limited in time of response by mechanical motion of the lens itself, as would radius of curvature change techniques.

The relation of the index of refraction to the dielectric constant and the permeability in the media is known to be related through Maxwell's equations as:

$$n = \left(\frac{\varepsilon}{\varepsilon_o} \quad \frac{\mu}{\mu_o}\right)^{1/2} \tag{2}$$

If the media is nonmagnetic the permeability $\mu = \mu_o$, and $n = (k)^{1/2}$ where k is the dielectric constant of one media.

Liquid Crystals

Liquid crystals have been developed and experimented with for many years.(3) The applications have been limited up to the present with the notable exception of 1) temperature sensitive detectors(4,5,6) 2) a "dynamic scattering mode" display effect(7,8) and 3) aerodynamic shear field visualization.(9) The first are now well known sensors which exhibit color changes with temperature. The "dynamic scattering mode" materials are presently scheduled for production of displays in digital meters to replace "nixie" tubes.

Much research has also been directed toward the development of new liquid crystals materials. Notable are the developments of wide temperature range chloesteric materials, nematic (shear sensitive) materials which are responsive at room temperature,[10] and tunable diffraction materials.[11] The main thrust of this work is directed at applications of nematic materials to devices for index of refraction control.

Liquid crystals are classified as nematic, sematic or chloresteric, depending upon molecular structure in an undisturbed condition. Chloresteric liquid crystals are field intensity detectors in the same category as photographic films, and as such can be used for the mapping of fields of energy in which a film is used to absorb energy in the form of heat. Nematic liquid crystals, however, are vector field sensitive. Their physical changes are a function of the direction of the applied field. As an example, if an electric field is present it will tend to align molecules in the direction of the field. The detection of the change will then determine the <u>direction</u> of the field rather than its <u>intensity</u>. There is, of course, also an intensity effect since as the intensity increases; the alignment (which is opposed by Brownian method) also increases. Similar effects can be produced by a magnetic field. A vector shear field also can produce this effect in nematic liquid crystals. As a shear detector, it might be utilized to observe accoustic wave fields, or as a light modulation device with fast response characteristics. As an element in which the index of refraction can be changed rapidly; nematic crystals thus represent a possible means for focus control in zoom lens applications, and as replacements for biofocal spectacles. Preliminary data of of others, particularly Carr[2], show the dielectric constant (measured electrically) of thin films of liquid crystals can be altered by approximately 12% with a magnetic fields. If the optical index changes as well, this would predict a change in the index of refraction of

$$\frac{\eta_F}{\eta_o} = {\frac{K_F}{K_o}}^{1/2} = {\frac{1.12}{1}}^{1/2} = 1.07 \qquad (3)$$

or 7% would be possible with liquid crystals materials. <u>NOTE</u>: The subscript F refers to the property with a field applied and the subscript o refers to the property of the undisturbed material.

The requirements of a focus system are usually represented in diopter units. The diopter being defined as the reciprocal of the focal length in meters (i.e. $D \equiv \frac{1}{f}$). Using the equations (1) and (3), then the ratio of the focal power without the field to that with a field applied is approximately

$$\frac{D_o}{D_f} = 0.86.$$

If the focal power of a prosthetic lens were set at 20 diopters then

$$D_F = \frac{20}{.86} \quad 24$$

and a change of 24-20=4 diopters would be possible. From Fig. 1 the physiological accommodation as a function of age for people goes from 16 to 1 diopters. If a change of 4 is possible, this would extend the accommodation of a 68-year old to the equivalent of an average 36-year old. This demonstrates that the possibility exists that liquid crystal materials could be used as an augmentation device. The experimental section of this report demonstrates that a 5% change in optical index of refraction is possible with a shear field.

Important in an augmentation device discussed before is the response time. With shear induced effects 20 millisecond response times appear reasonably which would be over 20 times faster than human focus response. (400 milliseconds).

Before attempting to show the experimental results, a further explanation of liquid crystals their properties, and sensitivities will be developed. It is hoped this will make clear the role and advantages of liquid crystals in this application.

The influence of shear motion on the index of refraction and on the light scattering properties of nematic materials are described in the experimental section which follows. Shear motion as a mechanism was attempted for two specific reasons.

1) Shear motion effects need to be understood for predicting and controlling other effects since they all involve the motion of a fluid media.

2) Direct shear rather than indirect motion through a field effect was hypothesized as having faster response characteristics.

EXPERIMENTAL WORK ON SHEARED LIQUID CRYSTALS

Description of Experimental Apparatus

The two test cells shown in Fig. 2 and 3 were constructed for use in experimental study of the optical behavior of nematic liquid crystal materials in shear fields. The first cell (Fig. 2) shows tin oxide coated glass plate which is electrically heated to provide the temperature control to maintain the liquid crystal in the nematic meso-phase. On top of this is a pyrex glass plate which is attached to the core of a radio loudspeaker. The pyrex plate is driven in motion parallel to the bottom plate. A thin film of liquid crystal is placed between the plates with the film thickness maintained by mylar spacing strips. The second cell (Fig. 3) consists of two electrically heated glass plates in a sandwich configuration with the space between filled with liquid crystal. With the plates in a vertical position the quartz shear element is driven mechanically with a lead zirconate piezoelectric element in an oscillator circuit.

Observation of optical effects is made with a spectra physics (Model 132) laser or a tungsten filament lamp. A photo detector element also shown (Fig. 2) is used for quantative evaluation.

Experimental Results

Four nematic liquid crystals listed below have been tested using the vibraitng shear apparatus of Fig. 2.

1. p-azoxyanisole (116°C)
2. APAPA (anisylidene para-aminophenylacetate) (83°C).
3. BPEOC (Butyl p (p-Ethoxyphenoxycarbonyl) Carbonate) (55°C).
4. Liquid Crystal Industries-proprietary room temperature nematic liquid crystals (22°C).

Note should be made at this point on the temperature required for the nematic state as this will be important in optical prostheses. A general comment concerning the optical properties of these mateirals is also included here.

The first material, p-azoxyanisole, is the most thoroughly studied liquid crystal material available.(3) It has a yellow green color in the nematic state (above 116°C). APAPA is a white cyrstal which in the nematic state is clear (above 83°C). BPEOC is also a white crystal which in the nematic state is clear but at a lower temperature (55°C). The proprietary liquid crystal is yellow-green at room temperature and quite opaque except in a very thin film. This material is much more viscous than the other materials and is thus easier to handle and contain. Difficulty was encountered in the use of all the above room temperature materials in their containment in an open vibration configuration. Their high capillarity and low viscosity cause them to "creep away" through spacing areas.

With the shear plate, shear rates from 0.02 sec^{-1} to 2000 sec^{-1} were produced and the optical behavior of these four materials observed. Index of refraction changes are observable in these materials at both high and low frequencies of shear. In the intermediate shear region between approximately 1 cps and 400 cps scattering of light by the rotational motion of the particles is present. This scattering, similar to that observed by others [7,8,10] with an electrical field, has been studied in this phase to help delineate the limits of index of refraction change and also to examine possible applications of the shear scattering effect. (For example, these might include pressure field detectors or optical demodulators). This effect is detrimental to the optical transparency of these materials and causes the material to appear opaque and cloudy. At the low frequencies index of refraction changes of approximately 5% are obtainable with small shear displacement of the glass plates. The directional changes have not as yet been put in quantative terms. It appears that at low frequencies this will require accurate positive displacement measurements. (Small displacements were not measurable with the present apparatus).

The orientation of molecules parallel to the plate surfaces (which are usually obtained by initially rubbing both surfaces with cloth or paper in one direction[12]) were produced in these experiments by mechanical shear motion of the plates for about 30 minutes followed by recrystallization. With the room temperature sensitive material the orientation was obtained by continuous shear for about 90 minutes without recrystallization. It is expected that longer time of shear with the other materials will also produce this alignment without recrystallization. This mechanical alignment without prior surface preparation can therefore be used to restore the alignment if the electrical alignment forces are disturbed.

Above 400 cps the scattering is appreciably decreased in all materials studied. The index of refraction variation with frequency could not be observed above this frequency. This could be due to several factors:

1). The amplitude of oscillation is severely reduced at these frequencies because of power limitations of the driving mechanism. (The scattering limits are in part a function of the shear and displacement as well as frequency).

2). The thickness of the film may restrict the movement of molecules at high frequencies. (This will be examined further with a direct displacement measurement). The index of refraction at the high frequencies also appears to be slightly higher than the aligned index which could be due to either rotational or vibratory crystal motion.

The scattering phenomena (which shows the same scattering as the dynamic scattering produced by an electric field) produced by the shear motion is shown in the photograph (Figure 4), as compared with the liquid crystal at rest and aligned (Figure 5). It was found that mechanical shear could be used to produce more rapid scattering (.4 millisec with BPEOC) than that reported for electric field scattering[7]. Figure 6 shows this rapid change from transparent to scattered light in terms of the intensity of transmitted laser light. The time for a change from scattered to transmitted light with a step change in frequency from 100 cps. to 1000 cps. (shown in Fig. 7) is approximately 20 milleseconds. This may be an important means for modulation of the intensity in light control devices.

All tests were performed with a film thickness of 25μ. Larger thickness should permit higher shear rate with less scattering effects. This is presently being attempted.

The shear strip element has so far not yielded any apparent index of refraction or scattering changes. The reasons are not clear and modifications noted below are to be attempted. A thicker and wider sandwich which contains the liquid crystal is to be tested. This will permit a longer light path for index of refraction observation and more energy transmission to the liquid crystal. One of the difficulties is the lack of a method of observation of the shear wave which is being produced by the element (in effect the liquid crystal effect is proposed to do this in this research).

The parameters examined to date are summarized in Table 1. Both BPEOC and APAPA appear promissing as possible materials for optical control with advantages over the other two materials examined. This is based on the considerations of temperature control, viscosity and capillarity as well as the data in Table 1. The numbers in this table are preliminary at this stage and further experimentation is needed to quantify the limits of the effects such as the influence of film thickness and amplitude of oscillation, This will require the construction of an accurate refractometer and a displacement measurement system.

In addition to this experimental work a theoretical model for use in predicting the behavior of these materials in a simple shear field has been attempted. This model is presented in the following section. The model together with the experimental evidence so far obtained are to be used to develop and improve the design of the index of refraction control system.

TABLE 1

Optical Properties of Nematic Materials in Mechanical Shear.

	Lower Scattering Limit in Shear	Upper Scattering Limit Frequency	Temp. reqd.	Index of Refrac. Change
BPEOC	500 sec^{-1}	400 cps	55°C	0.12
APAPA	250 sec^{-1}	300 cps	78°C	0.15
LCI	100 sec^{-1}	450 cps	22°C	0.13
p-Azoxanisole	200 sec^{-1}	475 cps	116°C	<0.1

Future experimental work required:

The immediate steps which appear necessary before proving the feasibility of these materials as focus control devices are outlined below.

1. To provide better quantative data, a measurement of the displacement of the shear plate is required. It is proposed to use a small reed strain gage element to determine this displacement and hence the shear at higher frequencies.
2. Index of refraction variation measurements in three principle directions need to be made. This can be done by varying the laser beam direction and recording the displacement as a function of angular position.
3. Construction of a special refractometer for better index of refraction measurements. Good quantative measurements will require considerable work and experience.
4. Construction of a rotating element shear device to produce continuous steady shear over longer times than available with a vibrating plate is also needed.
5. Testing of other and new room temperature nematic crystals as they become available (for example p-[N-(p-Methoxybenzylidene)amino]-n-butybenzene.[10]).
6. Further development of the theory of the shear effect on index of refraction change and its comparison with experiments including incorporation of models of

Helfrich[13], on the dynamic scattering effects.

Theoretical Analysis

The problem being attacked requires a means to rapidly change the index of refraction of materials. The orientation of long molecules of nematic liquid crystals by electrical, magnetic and active surfaces as well as shear possible means for effecting this change. Surface orientation can be achieved with nematic liquids by rubbing a glass surface with a cloth in one direction (the direction of orientation desired); then altered by one of the field effects noted. Alternatively starting from a random dispersion the fields can be used for alignment. The concern in the present research for using shear induced index of refraction changes rather than electrical or magnetic is that the possibility exists for partial or continuous control of the index of refraction. A theoretical understanding of the shear effect will help in the design of a means of control of molecular orientation.

The index of refraction variation in a shear flow is analyzed in this section by utilizing the relation between index of refraction and dielectric constant

$$n = \left(K \frac{\mu}{\mu_o} \right)^{1/2} \tag{4}$$

where n is the index of refraction, K the dielectric constant and μ/μ_o the relative magnetic permeability. In this analysis the relative permeability will be assumed fixed and the dielectric constant effect examined.

In order to analyze this index of refraction change possible in nematic materials, an analytical model of a liquid crystal as consisting long symmetrical ellipsoids approximating long molecules is assumed. The hydrodynamic effect of a simple shear field on these ellipsoids is examined here.

As a first approximation then, long molecules in a fluid will be assumed to be ellipsoids with different properties than the fluid in which they are contained. These ellipsoids will be assumed randomly dispersed in a media and free to rotate and translate under the influence of electrical or mechanical forces. The flow geometry of Figure (8) will be assumed with the properties of the fluid designated with subscript "s" and that of the ellipsoids with a subscript "p".

For a random dispersion of ellipsoids (a suspension at rest in the absence of a disturbance) the dielectric constant has been shown[14] to be approximated by the equations

$$K = K_s + \frac{\rho}{3}(K_p - K_s) \sum_{\alpha=a,b,c} \left[\frac{1 + x_\alpha}{x_\alpha + K_p/K_s} \right] \tag{5}$$

where

$$x_\alpha = \frac{2 - abc\, L_\alpha}{abc\, L_\alpha} \tag{6}$$

and

$$L_\alpha = \int_0^\infty \frac{d\lambda}{(a^2+\lambda)\sqrt{(a^2+\lambda)(b^2+\lambda)(c^2+\lambda)}} \tag{7}$$

where a, b, and c are the lengths of the ellipsoid axes.

These equations can be reduced to the dielectric constant for an aligned suspension by replacing $\delta/3$ by δ, and removing the summation sign. For example, for the dielectric constant of a suspension aligned with the a axis in the direction of the applied field,

$$K_a = K_s + (K_p - K_s)\rho \frac{1 - x_a}{x_a + K_p/K_s} \qquad (8)$$

We are concerned in this research with the dielectric constant of a suspension in which the ellipsoids are in motion produced by a field (in particular a simple mechanical shear field).

Similar equations with a replaced by b or c represent the dielectric constant of a solution with the axes b or c aligned with the field. These quantities, designated K_a, K_b, and K_c, give the principle dielectric constants of a suspension of uniform ellipsoids. The variation of the principle dielectric constants K_a and K_b if the dielectric constant of the particle (K_p) is assumed to be zero is shown in Figure 9 for a particle density of 0.5 as a function of the axis ratio r=a/b. To find the dielectric constant at any angle to the principle axes the dielectric constant is assumed to be a second order tensor which transforms according to the tensor relation

$$K_{ij}' = a_{ik}\, a_{j\ell}\, K_{k\ell} \qquad (9)$$

where the a_{ij} represent the cosines of the angles between the original axes and the new axes. In this case, the a_{ij} represent the cosines of the angles between the axes of the ellipsoid and the applied field.

Utilizing the tensor property of the dielectric constant, the dielectric constant in the direction of the x_3^1 axis is

$$K_{33}^1 = a_{31}a_{31}K_{11} + a_{32}a_{32}K_{22} + a_{33}a_{33}K_{33}$$

Setting $K_{22} = K_{33}$ (from symmetry) and using the geometrical relations

$$a_{31}^2 + a_{32}^2 + a_{33}^2 = 1 \qquad (10)$$

and

$$a_{31} = \cos\theta_3$$

then

$$a_{32}^2 + a_{33}^2 = 1 - \cos^2\theta_3$$

and

$$K_{33}^1 = K_a \cos^2\theta_3 + K_b (1 - \cos^2\theta_3) \qquad (11)$$

Similarly

$$K_{22}^1 = K_a \cos^2\theta_2 + K_b (1 - \cos^2\theta_2) \qquad (12)$$

and

$$K_{11}^1 = K_a \cos^2\theta_1 + K_b (1 - \cos^2\theta_1) \qquad (13)$$

These three equations define the dielectric constant of a suspension in which the electric field is in the direction of one of prime axes in terms of the principle dielectric constants and the cosine of the angle between the axis of symmetry and

and the direction of the field.

The dielectric constant is then obtained by proper averaging as a function of angle of the ellipsoids to the applied electric field. (See Appendix A).

The results of this analysis may be summarized by the following discussion and equations.

The dielectric constant of a sheared suspension in the three directions x_1', x_2', x_3' may be represented by the equations

$$\overline{K_{11}}' = K_b + (K_a - K_b)\,\overline{F}_1$$

$$\overline{K_{22}}' = K_b + (K_a - K_b)\,\overline{F}_2$$

$$\overline{K_{33}}' = K_b + (K_a - K_b)\,\overline{F}_3$$

The variation which can be produced by a shear field is summarized in Figure (10) where the shear factors $\overline{F}_1$, $\overline{F}_2$, and $\overline{F}_3$ are shown as a function of the ellipsoidal axis ratio r.

Since the factor $\overline{F}$ is 0.5 (Equation A5, Appendix A) when the suspension is randomly orientated (i.e., when the suspension is dispersed due to Brownian motion), the variation from this value with shear represents the magnitude of the effect. If we are concerned with long molecules (prolate ellipsoids) where $r \gg 1$ it is clear that slight variation with shear occurs in the direction perpendicular to shear planes ($\overline{F}_3$). The maximum variation occurs along the shear planes in the direction of the shear ($\overline{F}_1$). For control of properties by a shear field the maximum response can be obtained (for $r >> 1$) by motion in the x_1^1 direction and observation in the same direction. If the control is provided by a sandwich the plates should be moved parallel to each other and the observation made as shown in Figure (11,a). The other alternative is to vibrate a plane perpendicular to the sandwich and observe as shown in Figure (11,b). If we are concerned with a fluid with oblate particles (or discs with [$r << 1$]), then the response will depend upon the direction of observation. The effect is much greater for observation in the x_3 and x_2 directions than for prolate spheroids. If a choice of form of the particles is possible then oblate spheroids have a clear advantage from the control or detection point of view. For liquid crystals we do not have this choice, if the molecules are approximatable* by long prolate ellipsoids. In this case the best alternative at present is the system shown in Figure (12).

* NOTE: When these molecules act as "swarms" rather than individually then the oblate configuration may help in predicting their optical behavior also.

Discussion of Results and Conclusions

Index of refraction changes of approximately 5 percent have been produced by simple shear of nematic liquid crystals in the laboratory. This change in magnitude is not considered sufficient for the purposes outlined in this report although it is an "order of magnitude" larger than solid index of refraction changes possible. Further experimentation is indicated to provide optimization of the effects observed. This would include variation of the film thickness and the displacement or shear rate to include a non-symmetrical driving signal. (That is a signal whose rise time differs from its return time). Light scattering which is detrimental to the purpose of this work also appears to be a more difficult problem then expected. The solution to this problem appears to be operation with smaller displacements then originally planned. Displacements need now to be measured to substantiate this expectation.

The theoretical model presented has illustrate the importing of one design parameter, (the direction of oscillation) on the physical effect. It has failed however to take into account the experimental variation of index with shear rate or the effect of frequency on the index of refraction. These effects which also lead to the scattering observed must be examined by recourse to a changed model which includes the rotational inertia of the fluid particles and the interaction of individual particles in producing light scattering.

The introduction of an oscillatory shear and its effect on the fluid are first to be examined through the use of perturbation techniques. A shear of the form $G + ge^{i\omega t}$ is to be introduced and the resultant effects observed through separation of oscillatory and steady state components.

The calculation of the magnitude of the effect as predicted compared to the experimental results is not possible as yet since the dielectric constant or the equivilant ellipsoidal ratios of the liquid crystals are unknown. If the model is sufficient however the model might be used to calculate these factors from the experimental results. This would also assist in determining particle sizes for 1 other application such as display devices and field visualization techniques. Future theoretical analysis should also include consideration of the models of Helfrich[13] which attempt to account for the scattering effects in these materials.

Index of refraction variation has thus been observed and predicted by a theoretical model in a simple shear flow element using nematic liquid crystals as a media.

The use of these materials to control focus is possible but has not as yet been proven feasible. There are general considerations which need to be examined before the design of a useful focus control device.

1). Practical considerations of containment and temperature control as well as element configuration must be solved.

2). Scattering effects must be minimized by more careful experimentation to examine the scattering mechanisms and the limits of amplitude and frequency as well as materials.

3). Comparison with electrical field index changes (if present), must also be attempted to demonstrate the advantages of each technique.

4). The focus control mechanism of the eye is poorly understood. If an augmentation scheme is to be successful experimentally a device must be designed to provide an understanding of this control.

5). Experimental studies of response speeds for human focus are very incomplete. The construction of an optometer for measurements and perhaps the use of liquid crystal elements in this optometer could provide this information.

6). The detrimental effects of poor focus or slow focus ability on the part of people driving automobiles or aircraft for example is so far unknown. Studies on this aspect could prove valuable in providing measurements of human performance capabilities.

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APPENDIX A

CALCULATION OF THE DIELECTRIC CONSTANT OF A SUSPENSION OF SYMMETRIC ELLIPSOIDS IN A SIMPLE SHEAR FLOW.

Flow Analysis

In sheared suspensions at steady state in couette flows the motion of ellipsoids has been mathematically described by Jeffreys[1] and experimentally verified by Mason[2,3]. These descriptions give the motion in terms of the angles ϕ and λ shown in Figure (8) in the body of the report. If steady state is assumed, then in rotational diffusion the probability density functions for distribution in the angles ϕ and λ are inversely proportional to the angular velocity ω of the particles. Hence

$$p(\phi) = \frac{\text{const}}{\omega\phi}$$

$$p(\lambda) = \frac{\text{const}}{\omega\lambda}$$

Now given that the particle is orientated somewhere in the field

$$\int_0^{\pi} \int_0^{2\pi} p(\phi)p(\lambda)\,d\phi d\lambda = 1; \qquad \text{(A1)}$$

the conductivity in a given direction may be then found as

$$\bar{K}_\alpha = \int_0^\pi \int_0^{2\pi} p(\phi)p(\lambda)K_{\alpha\alpha}{}^1(\phi,\lambda)\,d\phi d\lambda \qquad (A2)$$

If the particles are randomly distributed then $p(\phi) = \frac{1}{2\pi}$ and $p(\lambda) = \frac{1}{\pi}$. From geometry the angle θ_3 can be written in terms of ϕ and λ since

$$\cos^2 \theta_3 = \frac{\tan^2\lambda}{\tan^2\phi} . \qquad (A3)$$

Then

$$\bar{K}_r = \int_o^\pi \int_o^{2\pi} K_b + (K_a - K_b)\cos^2\theta_3 \quad \frac{1}{2\pi} \quad \frac{1}{\pi}\,d\phi d\lambda \qquad (A4)$$

$$\bar{K}_r = \frac{K_a + K_b}{2} \qquad (A5)$$

Given the random uniform dispersion of ellipsoids the dielectric constant measured will be assumed to be $\bar{K}_r$ given by this equation. It should be noted that the dielectric constant of a suspension of axially symmetric ellipsoids which are uniformly dispersed at random orientations is the average of the dielectric constant when aligned. This is independent of whether they are oblate or prolate ellipsoids. This average quantity which is easily measured can be used for determining the shape of particles and for estimating

K_a and K_b and hence the relative dimensions of particles.

In a shear flow the particles are not randomly distributed in orientation and do not rotate at a constant angular velocity. The calculation of the dielectric constants then requires a knowledge of the distribution probabilities $p(\phi)$ and $p(\lambda)$. These probabilities can be calculated from Jeffreys analysis by using the relation between the angles or orientation and the orbit constant C for a particle:

$$\tan \theta_2 = \frac{C\, r_e}{(r_e^2 \cos^2 \phi + \sin^2 \phi)^{1/2}} \tag{A6}$$

where r_e is the axis ratio of the ellipsoid of revolution. The constant C of the orbit is determined by the angle at which the axis of the ellipsoid is oriented to the plane of shear; its distribution in a random dispersion would be uniform. In a shear situation there is no theoretical reason, in the limit of Stokes flow, that this will change.

The conductivity for a fixed orbit constant C may then be calculated as follows:

If the effect of Brownian motion on the particle distribution is neglected, then in the steady state

$$p\ (\phi) = \frac{\text{const}}{\omega\phi}$$

$$p\ (\theta_2) = \frac{\text{const}}{\omega_{\theta_2}}$$

The angular velocity of particles in Couette shear flow (derived theoretically by Jeffreys using Stokes flow approximations) may be expressed for the shear flow shown in Figure 1 in the body of the report as:

$$\omega_{\theta_2} = \frac{G}{4} \frac{\left(r_e^2 - 1\right)}{\left(r_e^2 + 1\right)} \sin 2\theta_2 \sin 2\phi \qquad \text{(A7a)}$$

$$\omega_{\phi} = \frac{G}{r_e^2 + 1} \left(r_e^2 \cos^2 \phi + \sin^2 \phi\right) \qquad \text{(A7b)}$$

for a symmetrical ellipsoid. Using the condition from Equation (A1) together with Equation (A7b) gives

$$p(\phi) = \frac{r_e}{2\pi(r_e^2 \cos^2 \phi + \sin^2 \phi)} \qquad \text{(A8)}$$

Let us first consider the dielectric constant in the X_2^1 direction (parallel to shear planes, perpendicular to the flow).

From Equation (8b)

$$K_{22}^1 = K_b + (K_a - K_b) \cos^2 \theta_2$$

this can be expressed as a function of the angle ϕ by using Equation (A6) expressed in the form:

$$\cos \theta_2 = \left(1 - \frac{\dfrac{C^2 \; r_e^2}{\left(r_e^2 - 1\right)}}{\dfrac{r_e^2 \; (1 + C^2)}{\left(r_e^2 - 1\right)} \sin^2 \phi}\right)^{1/2} \qquad \text{(A9)}$$

Then for fixed C the dielectric constant $\overline{K}_{22}{}^{1}$ can be expressed as

$$K_{22}{}^{1}\Big|_{C} = \int_{0}^{2\pi} p(\phi)\ K_{22}{}^{1}\ d\phi$$

Integrating using $p(\phi)$ from Equation (38) gives:

$$K_{22}{}^{1}\Big|_{C} = K_b + (K_a - K_b)\left(\frac{1}{(1 + C^2)\ [1 + (Cr_e)^2]}\right)^{1/2}$$

$$= K_b + (K_a - K_b)\ F_2. \qquad \text{(A10)}$$

The average dielectric constant for a suspension with orbits distributed with a probability distribution $p(C)$ can be found as

$$\overline{K_{22}{}^{1}} = \int_{0}^{\infty} p(C)\ K_{22}{}^{1}\Big|_{C}\ dC$$

$$= K_b + (K_a - K_b)\ \overline{F_2} \qquad \text{(A11)}$$

where

$$\overline{F_2} \quad \int_{0}^{\infty} p(C)\ F_2\ dC$$

Similarly the dielectric constant in the X_3^1 direction can be found from the relations

$$\tan \phi = r_e \tan \frac{2\pi t}{T} \qquad \text{(A12)}$$

and

$$\tan \lambda = Cr_e \sin \frac{2\pi t}{T} \qquad \text{(A13)}$$

(T is the period of revolution of a particle):

By solving for $\tan^2 \lambda$ in terms of $\tan^2 \phi$, $\tan^2 \lambda$ may be shown to be equal to

$$\tan^2\lambda = \frac{(Cr_e)^2 \tan^2 \phi}{(Cr_e)^2 + \tan^2\phi} \qquad \text{(A14)}$$

Now from geometry

$$\cos^2 \theta_3 = \frac{\tan^2\lambda}{\tan^2\phi} = \frac{C^2 r_e^2}{(Cr_e)^2 + \tan^2 \phi} \qquad \text{(A15)}$$

then

$$K_{33}^1 = K_b + (K_a - K_b) \cos^2 \theta_3$$

$$= K_b + (K_a - K_b) \frac{(C^2 r_e^2}{(Cr_e)^2 + \tan^2 \phi} \qquad \text{(A16)}$$

$$K_{33}^1 \Bigg|_C = K_b + (K_a - K_b) \int_0^{2\pi} p(\phi) \frac{C^2 r_e^2}{(Cr_e)^2 + \tan^2\phi} d\phi$$

(A17)

$$= K_b + (K_a - K_b) \frac{C}{1 + C} \qquad \text{(A18)}$$

and

$$K_{33}^1 = K_b + (K_a - K_b) \int_0^{\infty} p(C) \frac{C}{1 + C} dC \equiv K_b + (K_a - K_b)\overline{F_3}$$

(A19)

The dielectric constant in the X_1^1 direction is found using the relations

$$K_{11}^{1} = K_b + (K_a - K_b)\cos^2\theta_1.$$

From geometry

$$\cos\theta_1 = \cos\theta_2 \tan\lambda$$

Then using Equation (A9) for θ_2 and Equation (A14) for $\tan\lambda$

$$K_{11}^{1}\Big|_C = K_b + (K_a - K_b)\int_0^{2\pi} \frac{[(r_e^2-1)\cos^2\phi+1][(Cr_e)^2\sin^2\phi]\,p(\phi)\,d\phi}{[(Cr_e)^2+(r_e^2-1)\cos^2\phi+1]\,[(Cr_e)^2-1)\cos^2\phi+1]} \tag{A20}$$

Integrating with $p(\phi)$ from Equation (38) gives:

$$K_{11}^{1}\Big|_C = K_b + (K_a - K_b)\ \frac{2(Cr_e)^2\ r_e\qquad\qquad 3}{[2(Cr_e)^2+r_e^2+1]\ \ [Cr_e)^2+1][H-D]}\left[\frac{H+1}{(\overline{1-H}^2)^{1/2}} - \frac{D+1}{(1-D^2)^{1/2}}\right] \tag{A21}$$

$$K_{11}^{1}\Big|_C = K_b + (K_a - K_b)\,F_1$$

where

$$D \equiv \frac{(Cr_e)^2 - 1}{(Cr_e)^2 + 1} \qquad\qquad H \equiv \frac{r_e 2 - 1}{2(Cr_e)^2 + r_e^2 + 1}$$

then

$$\overline{K_{11}^{\ 1}} = \int_0^\infty p(C)\ K_{11}^{\ 1}\Big|_C\ dC$$

Since K_a and K_b are independent of C and $\int_0^\infty p(C) = 1$, this may be written as

$$\overline{K_{11}^{\ 1}} = K_b + (K_a - K_b)\ \overline{F_1}$$

where

$$\overline{F_1} = \int_0^\infty p(C)\ F_1\ dC \qquad \text{(A21a)}$$

The evaluation of the distribution p(C) of the orbit constant for a suspension is next required to determine the dielectric constant. The parameter C may range from 0 to ∞. C = 0 corresponds to the symmetric axis of the ellipsoid being perpendicular to the shear planes and C = ∞ corresponds to the symmetry axis of the ellipsoid rotating in the plane of the flow which corresponds to the ellipsoid having the angle $\theta_2 = 90°$.

No theoretical prediction of the distribution of C with flow rate has been made. Two hypotheses have been proposed, however.

1. Jeffreys assumed that the ellipsoids would eventually rotate in such a way as to minimize the energy dissipation. This theory predicts that

   $p(C) = 1$ for $C = 0$

   $p(C) = 0$ for $C > 0$

All the ellipsoids would be alligned with their symmetrical axes perpendicular to the shear planes if $a/b < 1$ or parallel to the shear planes if $a/b > 1$. The conductivity with time across shear planes would approach K_a with $\frac{a}{b} < 1$ or K_b with $\frac{a}{b} > 1$ using this assumption.

2. Eisenshitz[4] assumed that p(C) remained the same whether at rest or in motion, so that every orbital constant C was equally probable.

Since it has been demonstrated that Jeffreys' hypotheses is not correct, we will assume a uniform distribution of the orbital constant(2,3) (the Eisenschitz assumption). In this case the probability that a particle axis is between $C = 0$ and C is

$$P(C) = \frac{2}{\pi} \int_0^{\pi/2} [1 - \cos\theta_2(\phi)]\, d\phi \qquad \text{(A22)}$$

Substituting Equation (A7) into Equation (A22) gives

$$P(C) = 1 - \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \left(\frac{1 - \frac{r_e^2 - 1}{r_e^2} \sin^2\phi}{1 + C^2 - \frac{r_e^2 - 1}{r_e^2} \sin^2\phi} \right)^{1/2} d\phi. \qquad \text{(A23)}$$

Since the probability density distribution $p(C) \equiv \frac{dP(C)}{dC}$ is needed, differentiation gives

$$P(C) = \frac{2}{\pi}\,\frac{C}{(1 + C^2)^{3/2}} \int_0^{\frac{\pi}{2}} \frac{\left(1 - \dfrac{r_e^{\,2} - 1}{r_e^{\,2}}\,\sin^2\phi\right)^{1/2}}{\left(1 - \dfrac{r_e^{\,2} - 1}{r_e^{\,2}(1 + C^2)}\,\sin^2\phi\right)^{3/2}}\, d\phi. \tag{A24}$$

This equation is of the form of an elliptic integral if the coefficients of the $\sin^2\phi$ terms range between 0 and 1. For $r_e < 1$ this is not true and a change in form must therefore be made. First the numerator is expanded as:

$$P(C) = \frac{2}{\pi}\,\frac{C}{(1 + C^2)^{3/2}} \int_0^{\frac{\pi}{2}} \frac{\left(1 - \sin^2\phi + \dfrac{\sin^2\phi}{r_e^{\,2}}\right)^{1/2}}{\left(1 - \dfrac{r_e^{\,2} - 1}{r_e^{\,2}\,(1 + C^2)}\,\sin^2\phi\right)^{3/2}}\, d\phi$$

Changing the variable by letting $\phi = \frac{\pi}{2} - \psi$ gives:

$$P(C) = \frac{2}{\pi}\,\frac{C\,r_e^{\,2}}{[(Cr_e)^2 + 1]^{3/2}} \int_0^{\frac{\pi}{2}} \frac{[1 - (1 - r_e^{\,2})\,\sin^2\psi]^{1/2}}{\left(1 - \dfrac{1 - r_e^{\,2}}{1 + (Cr_e)^2}\,\sin^2\psi\right)}\, d\psi \tag{A25}$$

This is a complete elliptic integral of the second kind.

$$P(C) = \frac{2}{\pi} \frac{Cr_e^2}{[(Cr_e)^2 + 1]^{3/2}} \left[\frac{1}{\left(1 - \frac{1 - r_e^2}{1 + (Cr_e)^2}\right)^{1/2}} E\left[\left[\frac{\pi}{2}, C\left(\frac{1 - r_e^2}{1 + C^2}\right)^{1/2}\right]\right] \right]$$

$$= \frac{2}{\pi} \frac{2\, Cr_e}{[(Cr_e)^2 + 1]\ (1 + C^2)^{1/2}} \left[E\left[\frac{\pi}{2}, C\left(\frac{1 - r_e^2}{1 + C^2}\right)^{1/2}\right]\right]$$

where E [ ] is the complete elliptical integral of the second kind. The other case of interest, (i.e., $r_e > 1$) is found directly from Equation (A26) as:

$$P(C) = \frac{2}{\pi} \frac{Cr_e}{(1 + C^2)\ [(Cr_e)^2 + 1]^{1/2}} \quad E\left[\frac{\pi}{2}, \left[\frac{C^2\ (r_e^2 - 1)}{(Cr_e)^2 + 1}\right]^{1/2}\right] \tag{A27}$$

There are two cases where these integrals can be expressed as simple closed solutions.

1. The first is the case where $r_e = 1$. This corresponds to a sphere (hydrodynamically). For this case

$$P(C) = \frac{C}{(1 + C^2)^{3/2}} \tag{A28}$$

2. For the second case, $r_e = \infty$ (an infinitely long rod). Then

$$P(C) = \frac{2}{\pi(1 + C^2)} \tag{A29}$$

The factors $\overline{F}_i$ are then calculated using the probability distributions with Equations (A11), (A19) and (A21a). The results are shown plotted in Figure 10 in the body of the report.

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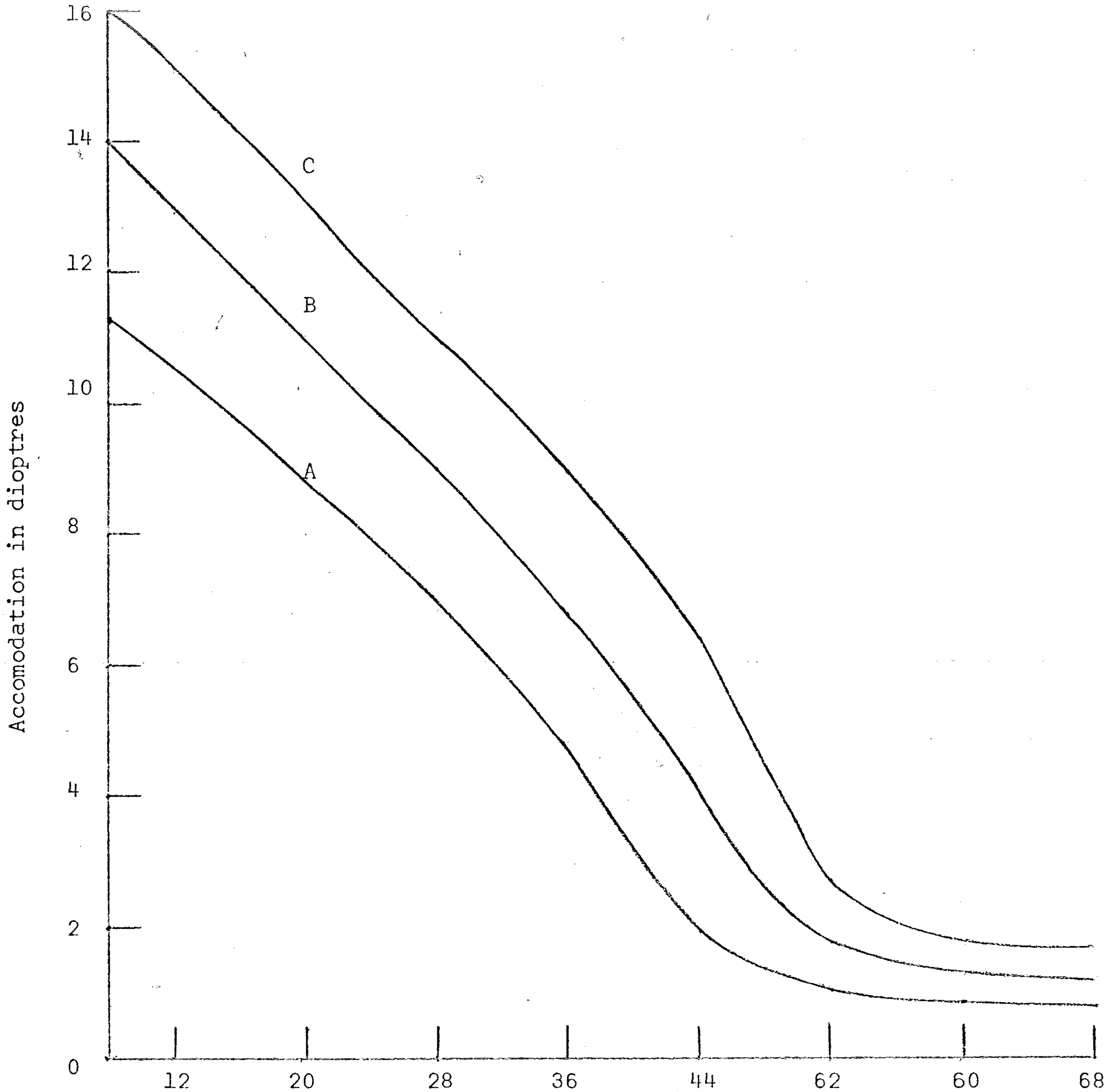

Figure 1. Accomodation of the human eye as a function of age (1).
Curves A and C are physiological limits and B is the normal.

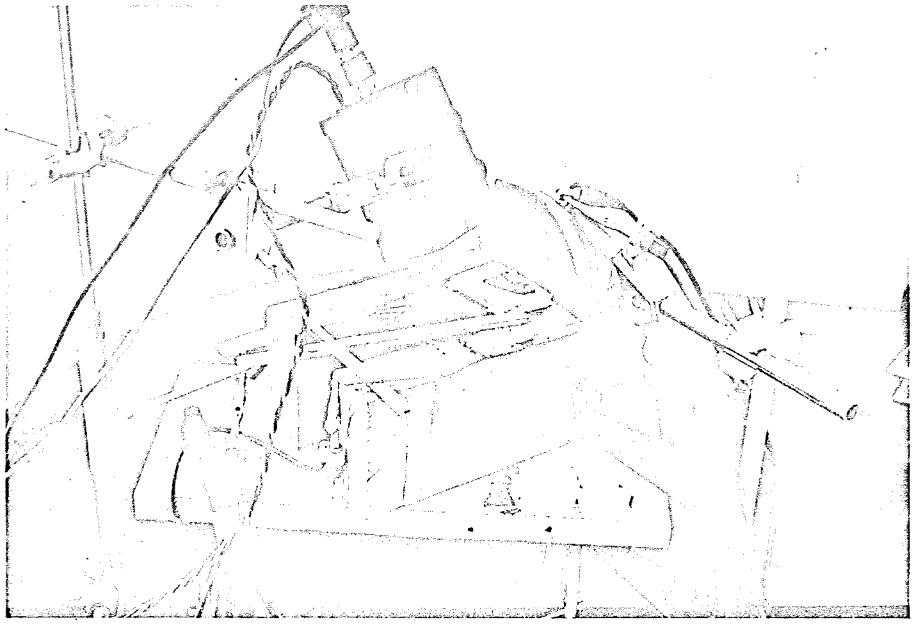

Fig. 2: Photo of experimental test apparatus for determination of index of refraction change in liquid crystals with linear shear.

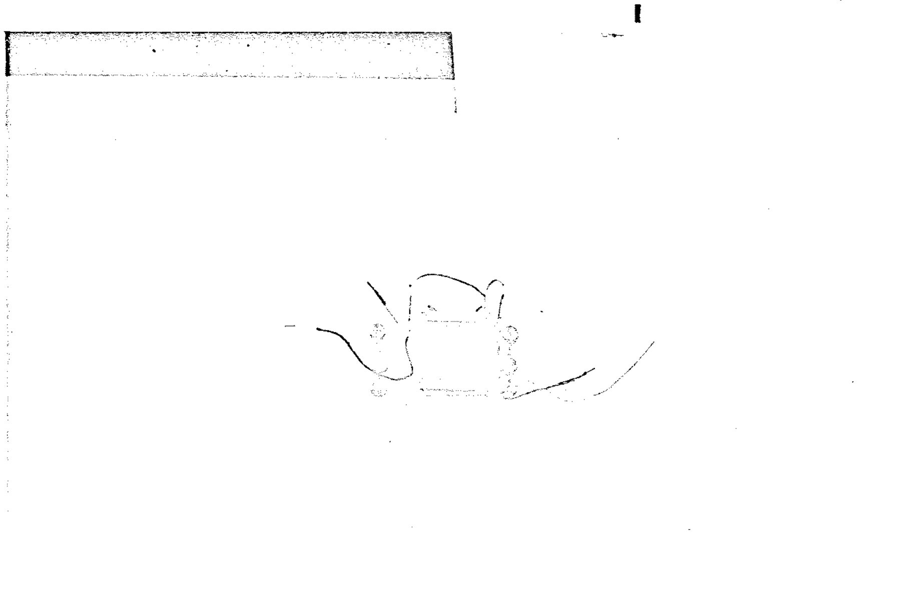

Fig. 3: Photo of experimental piezo electric shear wave generation cell.

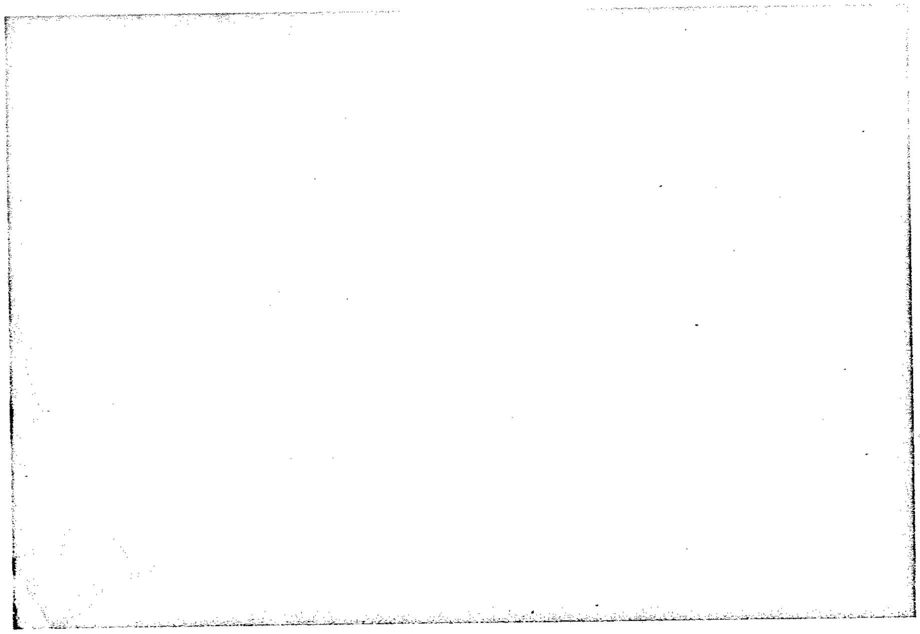

Fig. 4: Photo of scattering with shear of Butyl p. (p-Ethoxyphenoxy carbonyl) carbonate (25μ layer between glass plates).

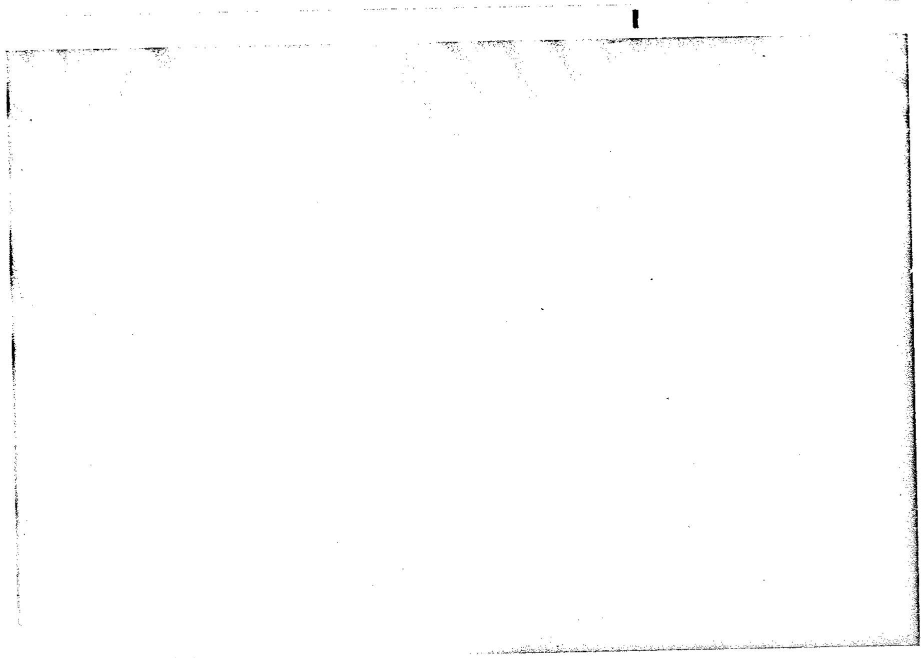

Fig. 5: Photo of optical transparency of Butyl p (p-Ethoxyphonoxy carbonyl) carbonate with molecules aligned. Same configuration as Figure 9.

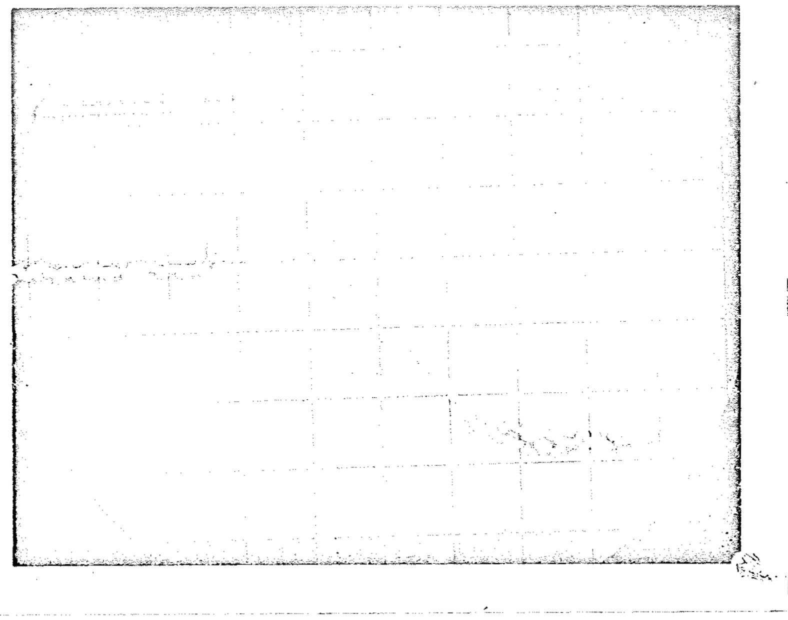

Fig. 6: Response of BPEOC to step in shear rate. At the left is optically transparent response with material at rest. At the right is optically opaque scattered light response. Horizontal scale is 0.2 milliseconds/cm.

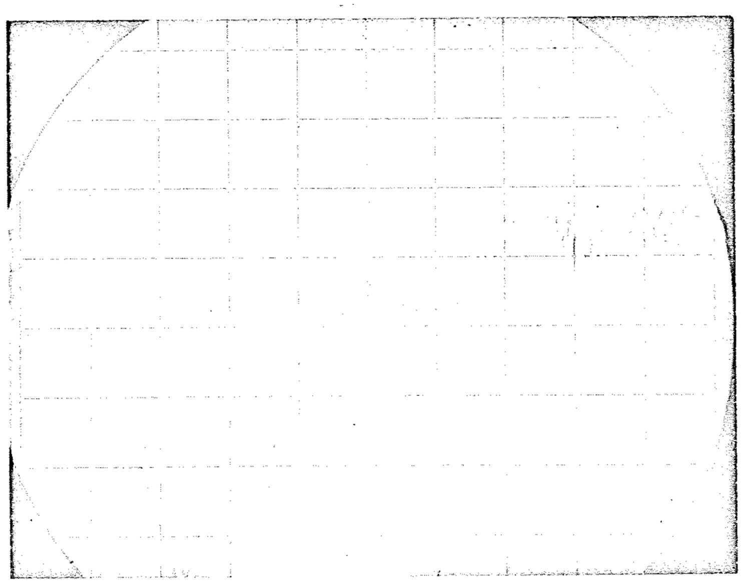

Fig. 7: Response of BPEOC to a step change in shear frequency from 100 Hz to 1000 Hz. Optical transparency changes from opaque on the μft to transparent on the right. Horizontal scale is 10 millisec/cm.

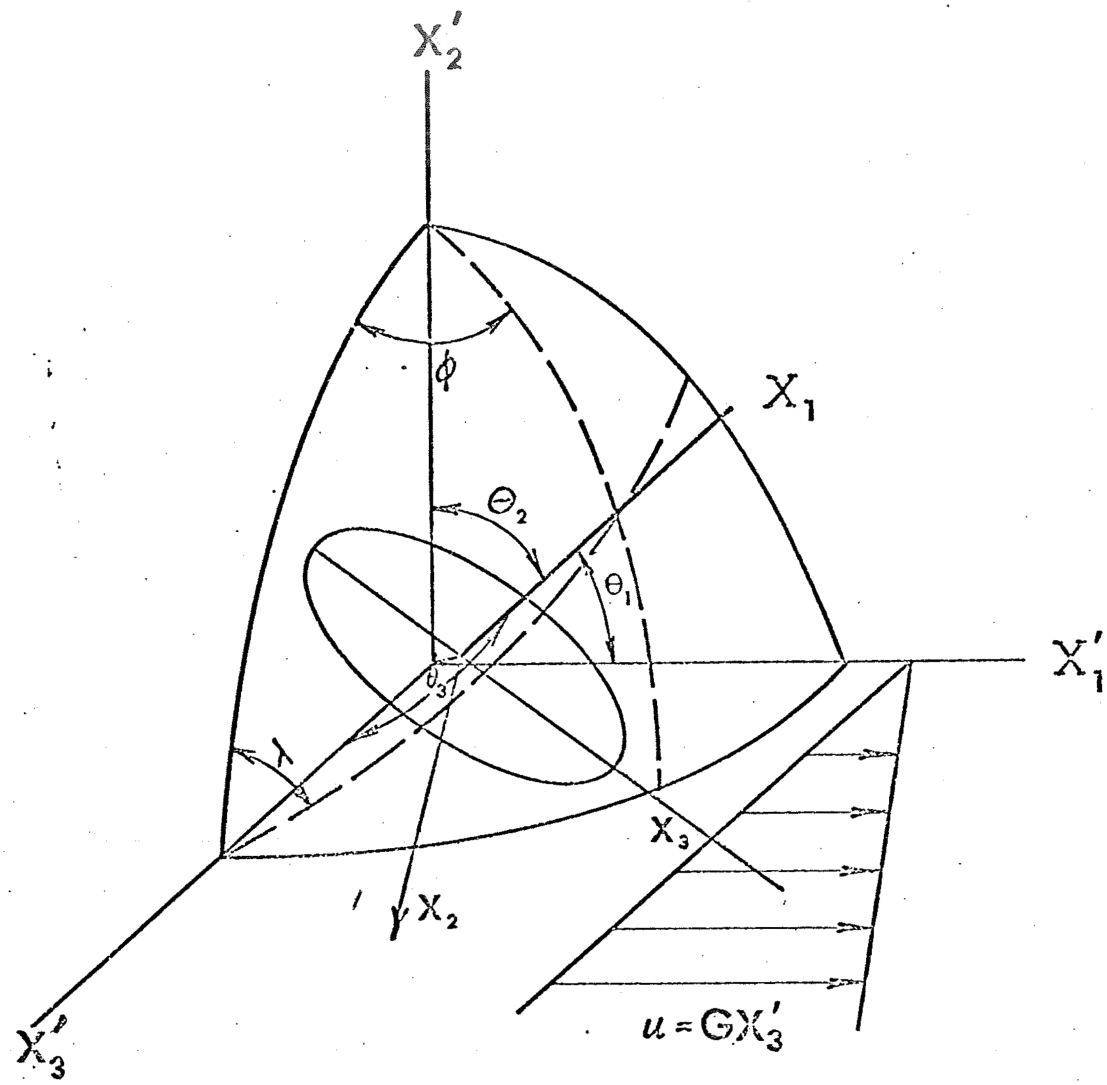

Fig. 8 Coordinate Geometry for an Ellipsoid of Revolution in a Shear Flow

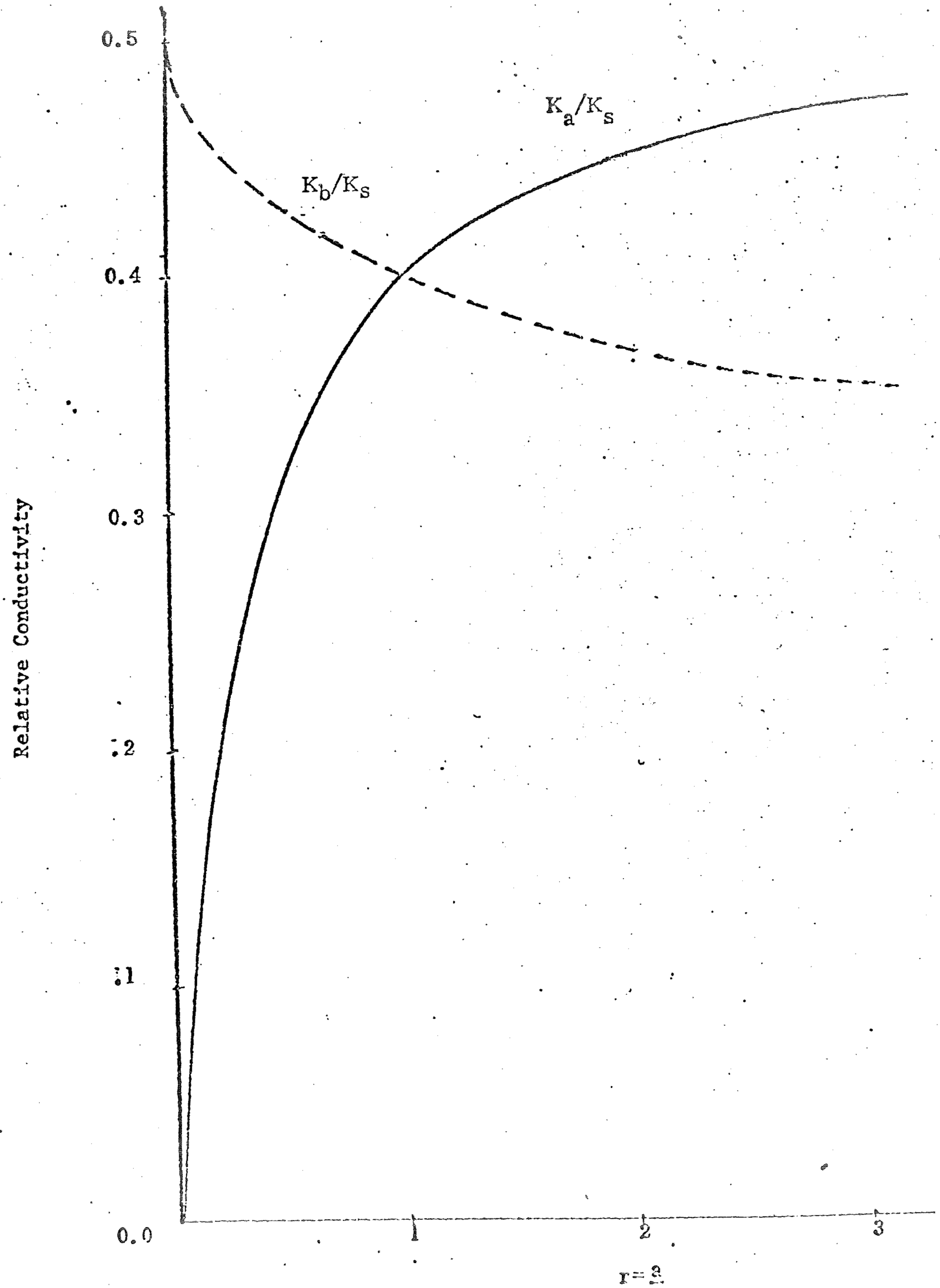

Fig. 9 Theoretical variation of the principal dielectric constant of symmetrical ellipsoids in suspension as a function of the axis ratio r = a/b. The volume density shown is ζ = 0.5.

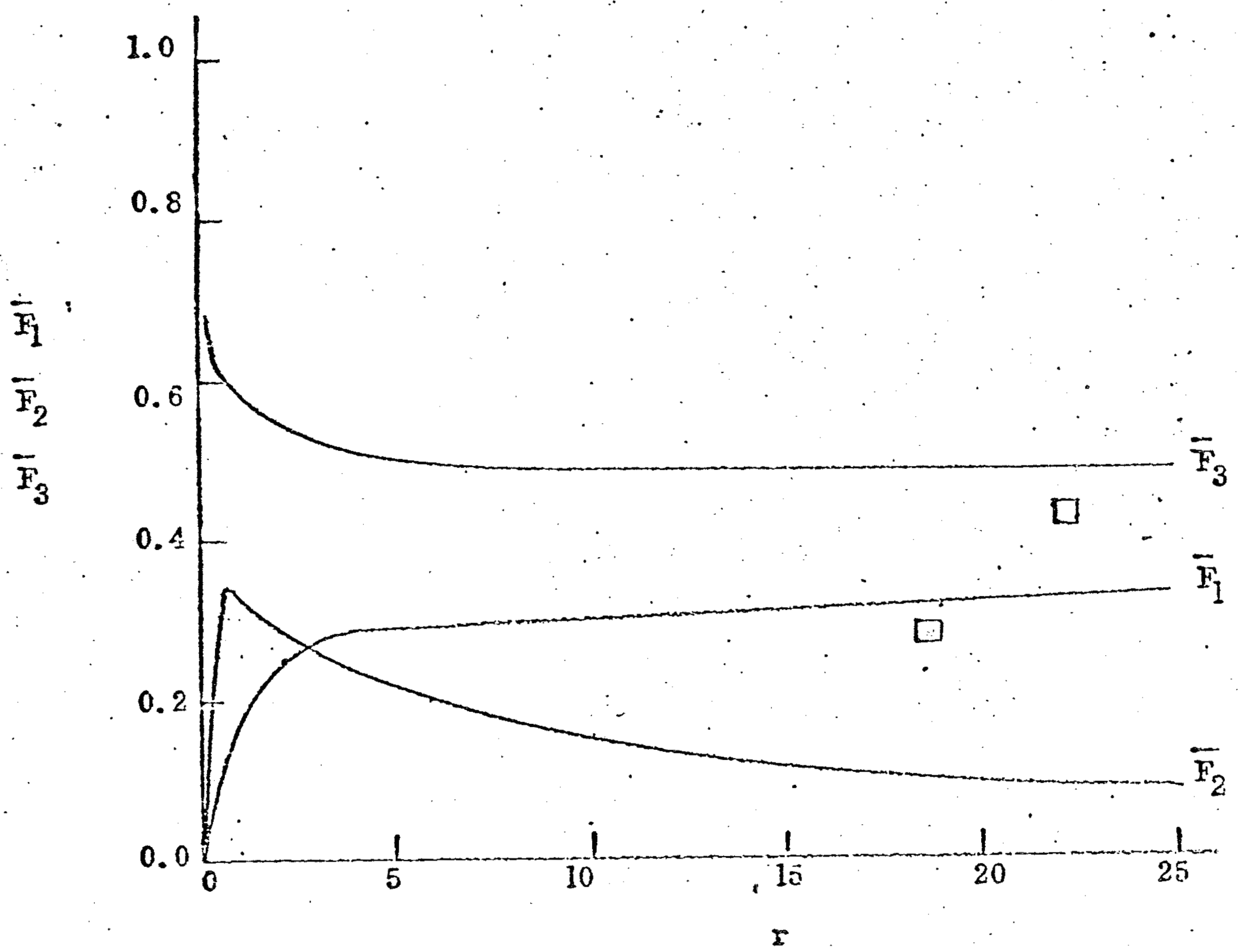

Fig. 10 Computed theoretical conductivity factors for conductivity as a function of the ellipsoid axis ratio r = a/b.

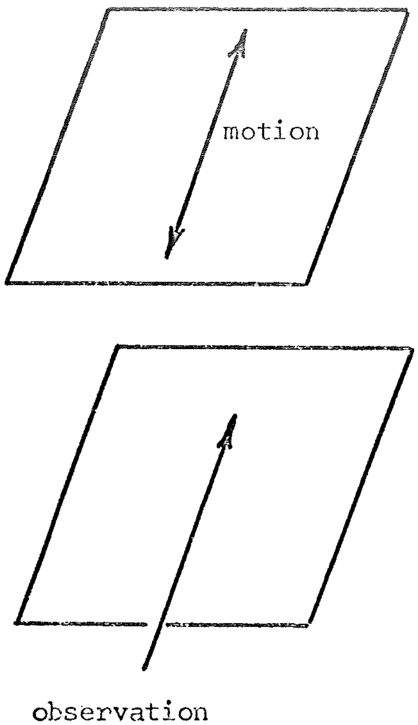

Fig. (11a)

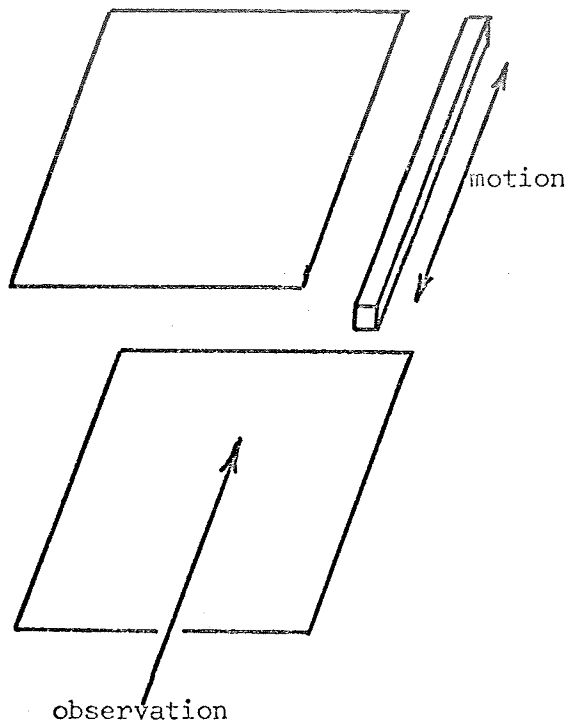

Fig. (11b)

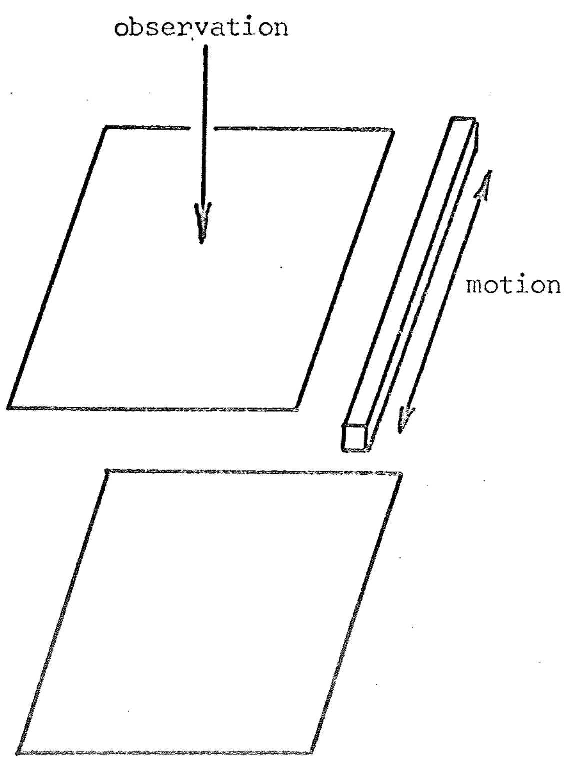

Fig. 12