

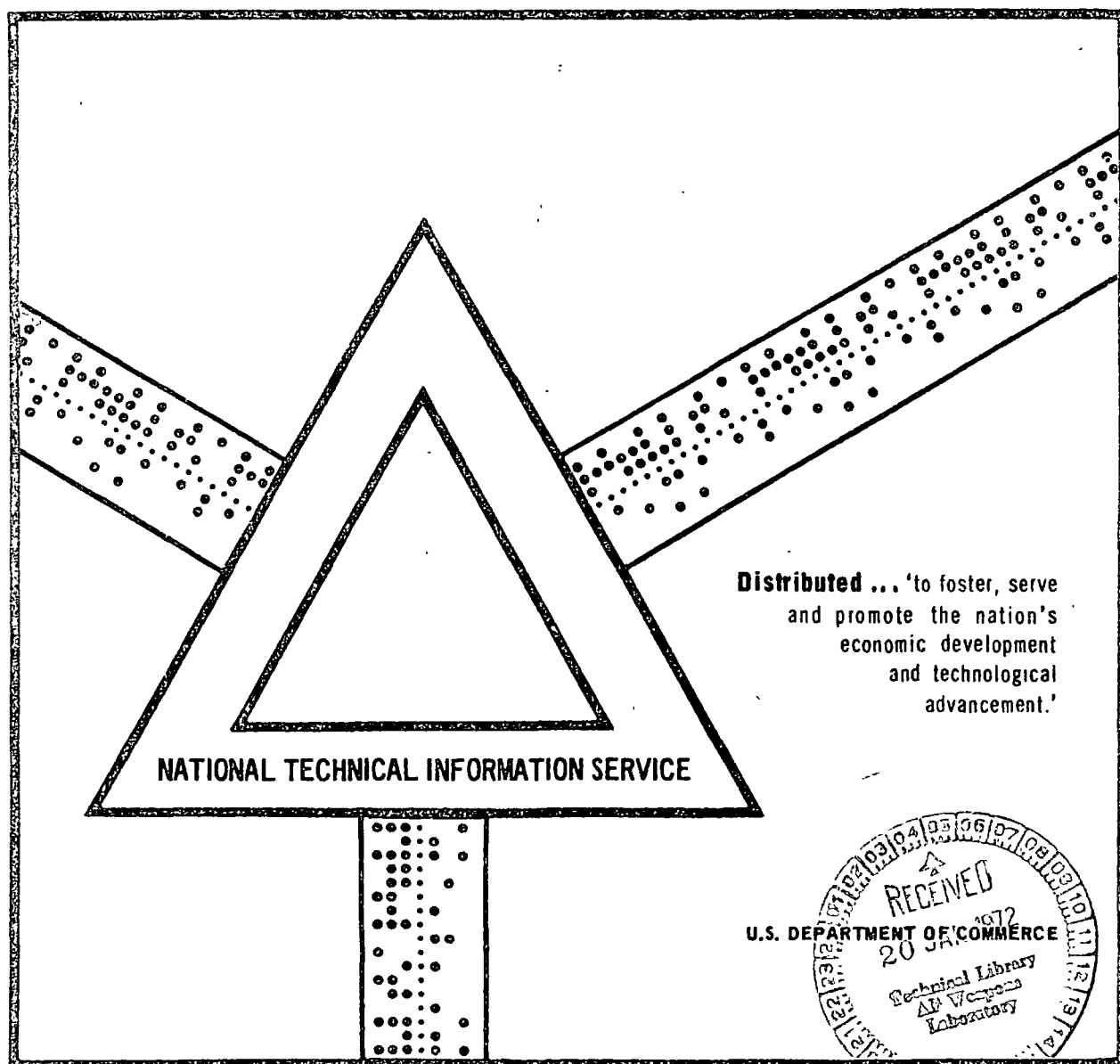
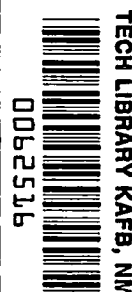
A NONSIMILAR SOLUTION FOR LAMINAR AND TURBULENCE
BOUNDARY LAYER FLOWS INCLUDING ENTROPY LAYER
AND TRANSVERSE CURVATURE

Ronald D. Grose, et al.

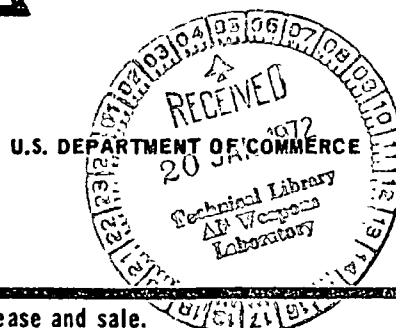
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FINAL REPORT

A NONSIMILAR SOLUTION FOR LAMINAR AND
TURBULENT BOUNDARY LAYER FLOWS
INCLUDING ENTROPY LAYERS AND
TRANSVERSE CURVATURE

by

Ronald D. Grose
Robert M. Kendall

June 1970

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ABSTRACT

A mathematical model of a nonsimilar laminar or turbulent boundary layer flow including entropy layer and transverse curvature effects is presented, and a method of solution is given. In the present analysis, a turbulent model is described which employs a mixing length model for eddy viscosity in the wall region with consideration of injection or suction effects. The wake region eddy viscosity is taken to be proportional to the free stream velocity and local velocity defect thickness and includes an intermittency factor. Transverse curvature effects are also incorporated into the present analysis. A modification of the Levy-Lees transformation is used to transform the equations of motion to the (ξ, η) coordinate plane, where the conservation equations are integrated across boundary layer strips. Derivatives in the normal direction are related to one another by Taylor series truncated to reflect a quadratic or cubic approximation, and stream-wise derivatives are expressed in finite difference form. The resultant set of equations is solved by general Newton-Raphson iteration.

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PRECEDING INFORMATION FILMED

SYMBOLS

A	parameter used in the solution of the mixing length equation (defined by Equation (124))
B	parameter used in the solution of the mixing length equation (defined by Equation (125))
c	constant introduced in the α_H constraint (Equation (35))
C	product of density and viscosity normalized by their reference values (defined by Equation (43))
$\bar{C}_p$	specific heat of the gas mixture
d_0, d_1, d_2	coefficients defined in finite-difference representation of streamwise derivatives (defined in Equations (110) and (111) for two- and three-point difference relations, respectively)
f	stream function (defined by Equation (36))
$\tilde{f}$	inviscid stream function (defined by equation (64))
h	static enthalpy of the gas
h_w	static enthalpy of the gas at the wall
h_g	enthalpy of gas which enters boundary layer
H_{T_i}	inviscid flow total enthalpy (relative to zeroth streamline reference total enthalpy)
h_g	inviscid flow static enthalpy difference commensurate with S_i (equation (75))
H_T	total enthalpy
ℓ	mixing length (defined by Equation (16))
$\tilde{\ell}$	dimensionless mixing length (defined by Equation (47))
L	parameter used in mixing length formulation (defined by Equation (121))
	molecular weight of the gas mixture
N	number of nodal points across the boundary layer selected for the purpose of the numerical solution procedure
p	dummy variable representing f' or H_T

SYMBOLS (continued)

P	pressure; also a parameter used in the mixing length formulation (defined by Equation (118))
Pr	frozen Prandtl number of the gas mixture (defined by Equation (60))
Pr _t	turbulent Prandtl number (defined by Equation (45))
P _{Tr}	local shock total pressure ratio
P _T '	total pressure as measured by a pitot probe
q _a	diffusional heat flux per unit area away from the surface
q _{cond}	heat conduction per unit area into the surface material
q _r	one-dimensional radiant heat flux (toward the surface), that is, the <u>net</u> rate per unit area at which radiant energy is transferred across a plane in the boundary layer parallel to the surface
r	local radius in the boundary layer in a meridian plane for an axisymmetric shape
r _o	local radius of body in a meridian plane for an axisymmetric shape
R	universal gas constant
Re	Reynolds number; subscripted with the length scale if other than s
R _{eff}	effective nose radius for Newtonian flow
s	distance along body from stagnation point or leading edge
S	entropy (relative to zeroth streamline reference entropy)
S _i	inviscid flow entropy (relative to zeroth streamline reference entropy)
t	parameter defined to simplify problems with transverse curvature; see Equation (42)
T	static temperature
u	velocity component parallel to body surface
u _τ	shear velocity, defined in Equation (24)
v	velocity component normal to body surface
XP ₂ ,...	truncated series obtained in Taylor series expansion of
	$\int_{i-1}^i f'p \, d\eta \text{ (defined by Equation (115))}$
y	distance from surface into the boundary layer, measured normal to the surface
y ⁺	dimensionless y-coordinate defined by Equation (24))
y _a ⁺	constant in the mixing length differential equation (see Equation (20))

SYMBOLS (continued)

Y	Cartesian vertical coordinate (see Figure 1)
α^*	flux normalizing parameter (defined by Equation (58))
α_H	normalizing parameter used in definition of $\bar{\eta}$ (see Equation (34)) defined implicitly by use of a constraint such as Equation (35)
β	streamwise pressure-gradient parameter (defined by Equation (44))
δ	y-dimension normalizing parameter (defined by Equation (46))
$\ell^{\Delta\ell-1}$	logarithmic distance between two streamwise positions denoted by the subscripts ℓ and $\ell-1$ (defined by Equation (112))
$\Delta f_i, \Delta f'_i, \dots$	corrections for $f_i, f'_i, \dots$, during Newton-Raphson iteration
δ^*	displacement thickness (defined by Equation 27))
δ_e^*	effective displacement thickness (defined by Equation (134))
$\hat{\delta}_i^*$	incompressible or velocity displacement thickness (defined by Equation (28))
$\delta\eta$	distance between two boundary layer nodal points (defined by Equation (103))
$\eta, \hat{\eta}$	transformed coordinate in a direction normal to the surface (defined by Equation (38)). Note: the hat is dropped from $\hat{\eta}$ throughout most of the report.
η^*	intermittency reference value of η (where intermittency effects vanish)
θ	angle between a surface normal and a normal to the body centerline
λ	thermal conductivity
μ	shear viscosity
ν	kinematic viscosity
$\xi, \hat{\xi}$	transformed streamwise coordinate (defined by Equation (38)). Note: the hat is dropped from $\hat{\xi}$ throughout most of the report.
ρ	density
$\rho_w v_w$	total mass flux per unit area into the boundary layer
$\rho \epsilon_H$	turbulent eddy conductivity (defined by Equation (7))
$\rho \epsilon_M$	turbulent eddy viscosity (defined by Equation (2))
$\tilde{\epsilon}_M$	dimensionless eddy viscosity (defined by Equation (49))
σ	Stefan-Boltzmann constant
σ_i	intermittency factor (defined by Equation (31))
τ	local shear stress

SYMBOLS (continued)

Subscripts

edge, e	pertains to boundary-layer edge.
i	inviscid flow, also index on normal coordinate
l	pertains to l^{th} streamwise position
m	pertains to m^{th} iteration during the Newton-Raphson iteration process
n	pertains to the n^{th} nodal points, corresponding to the outer edge of the boundary layer solution
sp	pertains to the stagnation point
T	stagnation condition
w	pertains to wall
l	reference condition, usually taken as zero streamline from inviscid solution (synonymous with boundary-layer edge in the absence of an entropy layer)
∞	freestream

Superscripts

κ	equal to unity for axisymmetric bodies and zero for two-dimensional bodies
*	signifies that quantity is normalized by α^* (e.g., $= j_k/\alpha^*$)
'	represents partial differentiation with respect to η or $\tilde{\eta}$ (usually $\hat{\eta}$ unless otherwise noted). Represents turbulent fluctuation in Section II.
$\tilde{\cdot}$	represents differentiation with respect to $\tilde{f}$

SECTION I

INTRODUCTION

A computational procedure is described which is suitable for obtaining accurate numerical solutions of the nonsimilar laminar and/or turbulent boundary layer with radiation absorption and emission, second order transverse curvature effects, and a variety of boundary edge conditions including intimate coupling with the inviscid flow energy and entropy gradients. A Fortran IV computer program has been developed in accordance with this analysis with the exception that radiation absorption and emission within the boundary layer is not permitted in the version of the program reported here. This computer program, designated BLIMP, for Boundary Layer Integral Matrix Procedure, is described in the literature. The analysis and computer program described herein are extensions of the previously developed BLIMP program for treating laminar and turbulent boundary layer flows with equilibrium and nonequilibrium chemical reactions described in references 1-2. The full development of the turbulent boundary layer transverse curvature model which has been incorporated is also described in the literature.

The computational procedure was developed while attempting to take advantage of the most attractive features of other boundary-layer procedures. In light of the general application of the procedure (including chemistry), certain specific requirements were appropriate. In particular, minimization of the number of "nodal points" required to obtain a solution was judged to be of prime importance as a consequence of the relatively large times associated with state calculations for a general chemical environment and, in the streamwise direction, because of the desire to couple the boundary layer procedure to a transient internal conduction or ablation solution.

For a given accuracy, the number of necessary "nodal points" in the surface normal direction is controlled primarily by the nature of the functions which relate the dependent variables (and their derivatives) to the independent variable. Thus the continuous functions typically used in integral relations approaches require fewer "nodal points" than the functions with discontinuous first derivatives implied by most finite difference approximations. In order to permit relatively flexible profiles, sets of connected quadratics and cubics were selected to represent enthalpy, velocity, and elemental concentrations. The first derivatives (and second derivatives with cubics) of these functions were made continuous at the connecting points. The advantages of such a "spline fit" are considered, for example, in reference 3.

If the general integral relations approach is followed, weighting functions must be selected. In the method presented, step weighting functions similar to those used by Pallone (reference 4) are used. That is, the conservation equations are integrated between nodal points (over strips) with a unity weighting function.

In the past when relatively large spacing in the streamwise direction has been desired, iterative procedures have generally been used to assure accuracy and stability. Some of these procedures have treated the solution in a manner resembling that used for a similar solution but with the addition of finite difference representations for the nonsimilar terms, a procedure which eliminates the necessity of special starting techniques. Using this basic approach, the specific treatment adopted in the current study follows most closely the matrix procedure used by Leigh (reference 5) wherein the iteration is a consequence of the solution of a set of linear and nonlinear algebraic relations. The general Newton-Raphson technique was used in the present procedure to solve these simultaneous equations. This technique results in linearized coupling between all relations required to characterize the boundary layer, and thus assures a more general, rapid and stable iterative convergence.

This document is one of a series of reports describing the analysis and solution procedure associated with the BLIMP program. The addition of an inviscid entropy and energy gradient capability and reduction to a homogeneous method provide the impetus for this report, therefore the entropy and energy layer topics will receive a disproportionate share of discussion. Much of the rest of the analysis and solution procedure is the same as is reported in the literature; the reader is referred to references 1-2 for more complete discussions.

This report concentrates on the fluid mechanical aspects of the problem and describes the basic numerical solutions procedure. The procedures employed for calculating the equilibrium state of a homogeneous gas are traditional and since they are conveniently treated as subroutines to the basic boundary layer computational procedure are not described here. Radiation absorption and emission enters directly into the conservation equations only as a net radiation flux term in the energy equation. The calculation of this term can be conveniently accomplished by a subroutine and is likewise not treated in this report.

Section II includes the entire mathematical modeling of the boundary layer flow including discussions of the general conservation equations, turbulent flow considerations, transverse curvature effects, coordinate transformations, and boundary conditions including the entropy/energy layer considerations. Section III outlines the integral matrix method for solving the simultaneous differential equations including the integral strip relations, the mixing length solution procedure, and the Newton-Raphson iteration technique. Section IV presents some of the results obtained with the program. Section V contains the summary, conclusions, and recommendations for further study.

SECTION II

MATHEMATICAL MODEL OF THE BOUNDARY LAYER

1. GENERAL CONSERVATION EQUATIONS

In the present analysis, the usual turbulent flow technique of breaking the velocity, and enthalpy fields into mean and fluctuating components, time averaging, and making appropriate order of magnitude approximations is used. Taking the results of these manipulations as a point of departure, the streamwise momentum equation becomes:

$$\rho u \frac{\partial u}{\partial s} + \rho v \frac{\partial u}{\partial y} = \frac{1}{r^\kappa} \frac{\partial}{\partial y} \left[\rho r^\kappa (v + \epsilon_M) \frac{\partial u}{\partial y} \right] - \frac{\partial P}{\partial s} \quad (1)$$

where s and y are the streamwise and normal coordinates, respectively, u and v are the velocity components in the s and y directions, respectively, r is the radius from the body centerline to the point of interest in a meridian plane for an axisymmetric shape, κ is zero for a flat plate and unity for a body of revolution, ρ is the density, ν is the viscosity, and P is the local static pressure. The eddy viscosity ϵ_M is defined in terms of the Reynolds' stresses of turbulent flow by

$$\rho \epsilon_M = - \frac{(\rho v)'u'}{\frac{\partial u}{\partial y}} \quad (2)$$

The transverse direction momentum equation reduces to zero when longitudinal curvature effects are ignored. Since transverse curvature is included in the present analysis, r must be treated as a function of y whereas in the typical boundary layer analysis, r is set equal to r_0 , the surface radius. The relationship between r , r_0 , and y is

$$r(s, y) = r_0(s) + y \cos \theta \quad (3)$$

Figure 1 helps orient the reader to the nomenclature being used.

In equation (1) and in the energy conservation equation to follow, turbulent transport terms are expressed in Boussinesq form, that is, in terms of an eddy

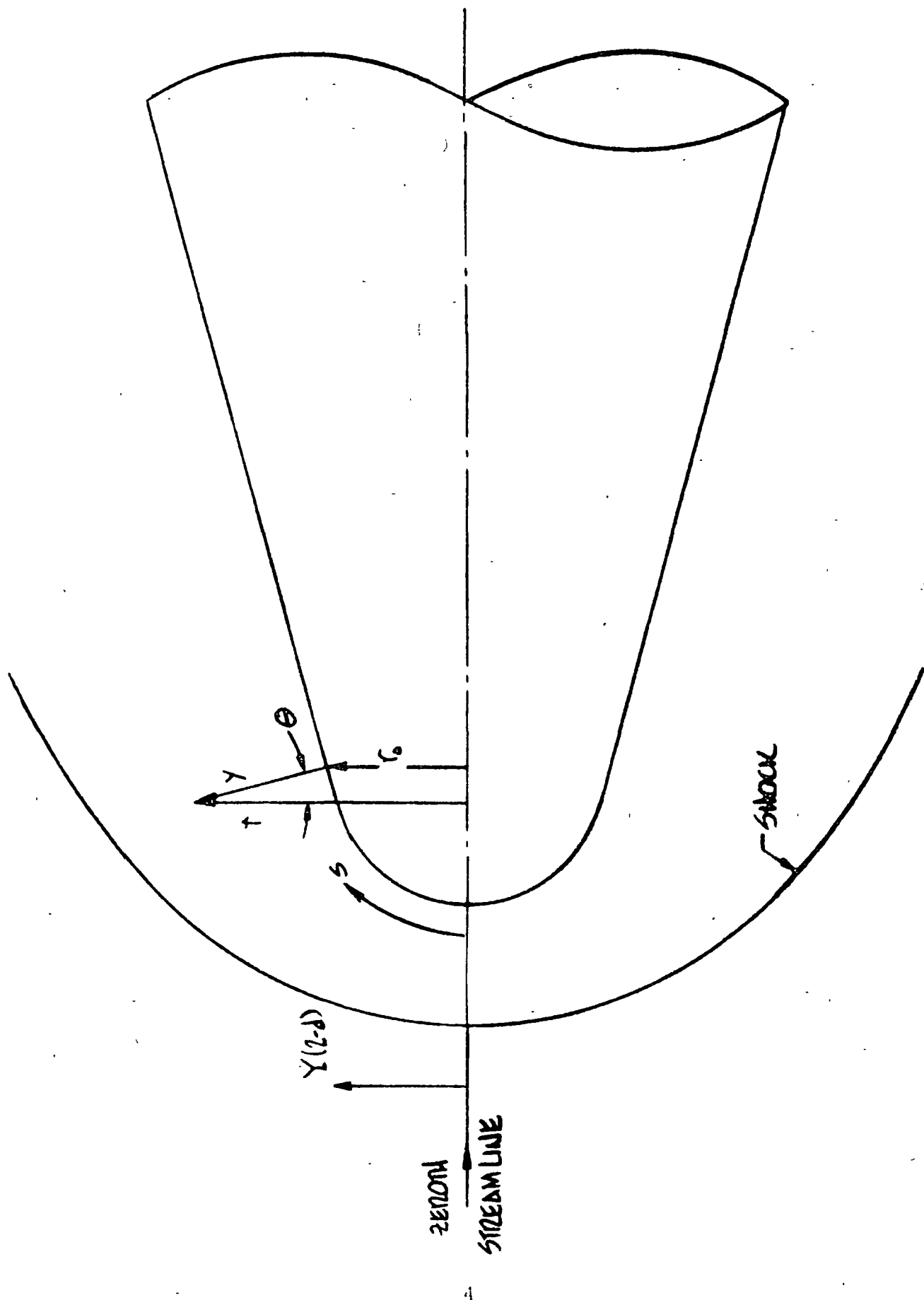


Figure 1. Definition of the Coordinate Nomenclature

viscosity and eddy conductivity. Hence all terms are time-averaged quantities and no need exists for using a superscript bar. In the order-of-magnitude arguments, terms of the following types have been eliminated: 1) triple correlations, 2) derivatives of turbulent correlations parallel to the wall, and 3) correlations involving turbulent components of molecular transport mechanisms.

The global continuity equation is:

$$\frac{\partial \rho u r^\kappa}{\partial s} + \frac{\partial \rho v r^\kappa}{\partial y} = 0 \quad (4)$$

and the energy equation for the homogeneous boundary layer is

$$\rho u \frac{\partial H_T}{\partial s} + \rho v \frac{\partial H_T}{\partial y} = \frac{1}{r^\kappa} \frac{\partial}{\partial y} \left[\rho r^\kappa (\epsilon_M + \nu) \frac{\partial u^2/2}{\partial y} + r^\kappa (\lambda + \rho \epsilon_H \bar{c}_p) \frac{\partial T}{\partial y} \right] \quad (5)$$

where H_T is the total enthalpy (static plus kinetic)

$$H_T = h + \frac{u^2}{2} \quad (6)$$

h is the static enthalpy, T is the temperature, $\bar{c}_p$ is the specific heat of the gaseous mixture, and λ is the thermal conductivity. The turbulent enthalpy transport coefficient is defined by

$$\rho \epsilon_H = - \frac{(\overline{\rho v})' h'}{\partial h / \partial y} \quad (7)$$

Diffusive energy flux through the boundary layer is given by

$$q_a = - \left\{ \rho (\epsilon_M + \nu) \frac{\partial \left(\frac{u^2}{2} \right)}{\partial y} + (\lambda + \rho \epsilon_H \bar{c}_p) \frac{\partial T}{\partial y} \right\} \quad (8)$$

such that the energy equation can be expressed as

$$\rho u \frac{\partial H_T}{\partial s} + \rho v \frac{\partial H_T}{\partial y} = - \frac{1}{r^\kappa} \frac{\partial}{\partial y} (r^\kappa q_a) \quad (9)$$

Equations (1), (4), and (9) comprise the boundary layer conservation equations. The equations are parabolic in nature, therefore requiring specifications of the dependent variables, their derivatives, (or a linear combination thereof) along the wall ($y = 0$); at the edge of the boundary layer; and at the initial body station. Typical sets of boundary conditions will be discussed later in this report. Also necessary in the mathematical formulation of the problem is the specification of the equation of state and a description of the eddy viscosity and conductivity. The turbulent flow model is reviewed in the next subsection.

2. TURBULENT FLOW CONSIDERATIONS

In the conservation equations developed above, the concepts of eddy viscosity, and eddy conductivity were used to express the correlations of fluctuating velocity, and enthalpy fields in terms of mean field quantities. This is only one of several possible techniques of closing the set of equations (assuming satisfactory expressions for the eddy parameters are available), and it does not provide any information regarding the evolution of the turbulent correlations as the flow progresses downstream. Admittedly, it would be more desirable to describe the turbulent fluctuations in a more complete manner such as with an entrainment relation or a turbulent kinetic energy relation (reference 6). However, these techniques are still in early stages of development even for incompressible single component flows, therefore a more proven approach was selected. The Boussinesq description of turbulent boundary layers has proved to be very useful, (particularly for complex reacting flows such as are dealt with in the general method) and is used exclusively in the analysis presented here.

There is a wide amount of latitude possible even within the eddy viscosity framework of turbulence, particularly in applying classical incompressible models to compressible flows. The following two subsections describe how the turbulence model described in references 2 and 7 was applied to the compressible flow problem.

a. Wall Region

Following the work of Clauser (reference 8) the boundary layer is divided into a law of the wall region and a wake region. The relatively thin wall region of the turbulent boundary layer is characterized by very steep gradients in the turbulent transport and mean field properties. Turbulent stress varies from zero at the wall to near its maximum value at the outer edge of the wall region. There is a vast amount of empirical evidence that these turbulent stresses and also the mean flow field properties can be described entirely in terms of the wall state, wall fluxes, thermodynamic and transport properties of the fluid, and the normal coordinate y . Since the streamwise coordinate does not enter the solution for this region, the problem becomes a one-dimensional initial value problem. Eliminating s derivatives from the continuity equation and neglecting variations in r due to the thinness of the layer results in

$$\frac{d(\rho v)}{dy} = 0 \quad (10)$$

or

$$\rho v = \rho_w v_w \quad (11)$$

where the subscript w refers to the wall value. Thus the wall injection rate, $\rho_w v_w$, which may be a function of s , determines the transverse mass flux through the entire wall region. Using the same technique for the momentum equation and substituting equation (11)

$$\rho_w v_w u = \rho(v + \epsilon_M) \frac{du}{dy} - \tau_w \quad (12)$$

where the wall shear, τ_w , is also typically a function of s . For flows over an impermeable wall with constant properties, this equation reduces to

$$\rho(v + \epsilon_M) \frac{du}{dy} = \tau_w \quad (13)$$

or

$$\tau = \tau_w \quad (14)$$

indicating that shear can be considered constant in the wall region. For flows with injection, it is seen that shear varies with the mass injection rate and local velocity, that is,

$$\tau = \tau_w + \rho_w v_w u \quad (15)$$

This one-dimensional description of the shear in the wall region is useful in formulating a mixing length model for eddy viscosity as described in the following paragraphs. It should be made clear however, that only the wall region turbulent shear stress is assumed to behave in a one-dimensional fashion. In the solution procedure, the complete two-dimensional equations of motion are solved over the entire boundary layer.

A complete investigation of the validity of the mixing length postulate for flows with injection has been reported in reference 9. The analysis used in this investigation is an extension of that work; therefore, the reader should refer to reference 9 for more details.

Because of the current lack of understanding of turbulent mechanisms, "theoretical" predictions of the variation of turbulence near the wall must rely on empirical input into relations based on some phenomenological dependence. The generality of the ultimate goals of this analysis and the desire to approximate the physical situation dictated certain prerequisites for the turbulent transport relations. These were:

- a) The relations must indicate a continuous variation of the turbulent transport properties from the wall to the fully turbulent region.

- b) The relations must be generally applicable to mass, momentum, and energy transport.
- c) The relations must be applicable to compressible or incompressible flows with real gas properties.
- d) The relations should be suitable for transpired and untranspired boundary layers without any, or a minimum, modification of form.

Two basic variations of the eddy viscosity hypothesis have been proposed in the past. The first type predicts the variation of turbulent viscosity from the wall to the fully turbulent region. Reichardt, Rannie, and Deissler, in references 10-12, have proposed such variations. The second type of hypothesis involves a variation of mixing length from the wall into the fully turbulent portion of the boundary layer. Rotta, von Kármán, and van Driest (references 13-15) have adopted this procedure. Data indicate that surface mass addition strongly affects the eddy viscosity profile, and it was found that the first type of hypothesis could not be simply modified to predict this variation. On the other hand, success of the mixing length theory in predicting profiles in the fully turbulent portion of the boundary layer with surface mass addition has been noted, for example, in references 16 and 17. It has generally been concluded that the slope of the linear relation between mixing length and distance from the wall is insensitive to surface mass addition. As a consequence of this apparent generality of the mixing length approach, it was adopted for the present studies.

The basic mixing length postulate can be expressed as

$$\overline{(\rho v)'u'} = \rho \ell^2 \left(\frac{du}{dy} \right)^2 = \rho \epsilon_M \frac{du}{dy} \quad (16)$$

where the mixing length, ℓ , is a combination of various correlations, but retains some relationship to the scale of turbulence. Prandtl proposed that this length will, in its simplest form, be related to the distance from a wall, at least in the region of development of turbulence. His proposition that

$$\frac{d\ell}{dy} = \text{constant}, K \quad (17)$$

has been tested under a variety of conditions and found to be quite adequate in the fully turbulent portion of the wall region.

As the wall is approached however, this simple relation is no longer appropriate, and, in fact, it can be shown theoretically that

$$\left. \begin{aligned} \lim_{y \rightarrow 0} \ell &= 0 \\ \lim_{y \rightarrow 0} \frac{d\ell}{dy} &= 0 \end{aligned} \right\} \quad (18)$$

This is a consequence of the Reichardt-Elrod criterion (see reference 9). Thus, two criteria are specified, namely Prandtl's hypothesis which is appropriate in the fully turbulent portion of the wall region and the Reichardt-Elrod wall criterion as expressed by equation (18).

Several means of expressing a relation covering the full range of y and including these limiting criteria have been used by other investigators. It is advantageous in considering extensions of mixing length theory to establish some physical logic for the selected relation. Unfortunately, the understanding of transition from the laminar to the turbulent portions of the layer has not reached a state permitting any quantitative specification. Therefore, the selected model can be based only on qualitative understanding of the process, dimensional considerations, and the above limiting criteria. These criteria are satisfied for incompressible flows by a simple implicit relation of the form

$$\frac{d\ell}{dy} \propto (Ky - \ell) \quad (19)$$

which implies that the rate of increase of the mixing length is proportional to the difference between the value postulated by Prandtl (Ky) and its actual value. This rate of increase is assumed to be augmented by the local shear and retarded by the local viscosity. Using these parameters to nondimensionalize the above relation yields

$$\frac{d\ell}{dy} = (Ky - \ell) \frac{\sqrt{\tau/\rho}}{y_a^+ v} \quad (20)$$

where y_a^+ is the constant of proportionality. The coefficients K and y_a^+ were shown in reference 9 to be invariant for a wide variety of flow conditions at values of 0.44 and 11.83, respectively.

For compressible flows, the physical arguments must be changed somewhat. Rather than describing the scale of a turbulent eddy, it seems appropriate to describe the mass of the eddy with respect to the mass within the boundary layer. Thus, by analogy to equation (19), the rate of increase of the mass of an eddy will be taken to be proportional to the difference between the mass available between the wall and the point of interest (times an appropriate constant) and the mass of the eddy:

$$\frac{1}{\rho} \frac{d\rho\ell}{dy} = K \int_0^y \rho dy - \rho\ell \quad (21)$$

Nondimensionalizing as above,

$$\frac{1}{\rho} \frac{d\rho\ell}{dy} = \left(K \int_0^y \rho dy - \rho\ell \right) \frac{\sqrt{\tau/\rho}}{y_a^+ \mu} \quad (22)$$

The constants K and y_a^+ are left at their incompressible values of 0.44 and 11.83 for the time being. The integral-differential character of this mixing length equation indicates a difficult solution procedure in the physical coordinate plane. However, in the (η, ξ) coordinates introduced by the Levy-Lees transformation, the mixing length equation simplifies somewhat. This will be discussed further in Section II-3.

For the special case of constant properties and zero injection (constant shear), equation (22) can be integrated to yield

$$\ell = \frac{K\nu}{u_\tau} \left\{ y^+ - y_a^+ \left[1 - \exp \left(- \frac{y^+}{y_a^+} \right) \right] \right\} \quad (23)$$

where

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad (24)$$

$$y^+ = \frac{yu_\tau}{\nu}$$

It can be seen that the Reichardt-Elrod criteria is satisfied at the wall. For large y , Rotta's (reference 13) expression

$$\ell = \frac{K\nu}{u_\tau} (y^+ - y_a^+) \quad (25)$$

is obtained. This special case result for constant property zero injection flows is not used in the general analysis technique presented here.

b. Wake Region

The wake region of a turbulent boundary layer is so named because the flow in this region tends to have a wake-like character. In particular, in the outer 80 to 90 percent of the boundary layer the local turbulent eddies dominate the mixing processes within the flow, and the viscous effects

become second order. Gradients in the wake region are typically much smaller than those of the wall region. Since the pressure gradient and streamwise derivative terms are important in the wake region, the two-dimensional character of the turbulence must be considered in its entirety, as opposed to the approximations of the wall region.

A fortunate feature of the wake portion of the boundary layer is that eddy viscosity is nearly constant across this region, at least for equilibrium* incompressible flows. In particular, Clauser (reference 8) was able to relate the eddy viscosity to edge velocity and a length scale ℓ^*

$$\epsilon_M = 0.018 u_1 \delta^* \quad (26)$$

for a great quantity of experimental data taken in equilibrium flows.

The quantity δ^* in this relation is the displacement thickness

$$\delta^* = \int_0^{\infty} \left(1 - \frac{\rho u}{\rho_1 u_1} \right) dy \quad (27)$$

in which the densities cancel out for incompressible flows. For compressible flows, this length scale is inappropriate since under some conditions δ^* can be negative. Rather a velocity defect thickness

$$\delta_i^* = \int_0^{\infty} \left(1 - \frac{u}{u_1} \right) dy \quad (28)$$

is used such that the eddy viscosity in the wake portion of the flow is given by

$$\epsilon_M = 0.018 u_1 \delta_i^* \quad (29)$$

A satisfactory technique for choosing the ϵ_M expression at any particular body station is to use the wall region expression

$$\epsilon_M = \ell^2 \frac{du}{dy} \quad (30)$$

until ϵ_M exceeds the wake value, equation (29), at which point ϵ_M is held constant at the wake value for the remainder of the boundary layer thickness.

*Equilibrium as used here refers to a particular pressure gradient, $(\ell^*/r_w)(dP/dx)$, which results in self-similar velocity profiles (reference 8).

The foregoing turbulence calculation procedure must be modified slightly when entropy and energy gradients in the inviscid flow are included. Since the velocity gradient $\partial u/\partial y$ does not vanish for such situations, strong shear forces can erroneously remain in the momentum equation (for example near the boundary layer edge where in reality there should be little or no shear forces present). To correct this situation, the concept of turbulent intermittance is introduced in a very simple way such that the value of ϵ_M from equations (29) and (30) is modified. A simple factor, σ_i , is defined which linearly depends on the transformed normal coordinate. η to be defined later.

$$\begin{aligned} \sigma_i &= 1.0, \quad \eta < \eta^* \\ \sigma_i &= \frac{\eta_e - \eta}{(\eta_e - \eta^*)}, \quad \eta \geq \eta^* \end{aligned} \quad (31)$$

and ϵ_M is multiplied by σ_i before it is introduced to the conservation equations. The value of η^* appearing in equation (31) is arbitrarily chosen as $\bar{\eta}_c$ to be defined prior to equation (35).

c. Boundary Layer Transition

As can be seen from the form of the conservation equation, both the molecular and turbulent transport terms are considered simultaneously. This is necessary since an accurate description of the turbulent boundary layer requires that the time-averaged fluctuation terms disappear near the wall. Another reason for the inclusion of these terms is the description of laminar or transitional flows. From the form of equation (29), it can be seen that for very small δ_i^* the turbulent stresses will be small compared to the laminar ones. Without any constraints on the equations as stated above, kinematic and eddy viscosities are equal at a velocity displacement thickness Reynolds number of 56:

$$1 = \frac{\epsilon_M}{\nu} = \frac{0.018 u_1 \delta_i^*}{\nu} \quad (32)$$

$$Re_{\delta_i^*} \approx 56$$

This Reynolds number is much lower than typical transition values, therefore ϵ_M is artificially set to zero until some other criterion is satisfied. A Reynolds number on momentum thickness, Re_θ , is currently used to trigger transition. (Alternatly the streamwise station at which transition is desired to occur can be specified). Once the pre-specified transition value for Re_θ is exceeded, turbulent transport properties are immediately brought into the solution. Since the method provides for a nonsimilar solution, the influence of the upstream

laminar profile is felt for some distance downstream, thus simulating a transitional region.

3. COORDINATE TRANSFORMATIONS

The equations of motion for a boundary layer flow can be solved in the physical (s,y) plane by numerous techniques, however, it is generally advantageous to transform the problem to another coordinate system. The transformed coordinates offer the advantages of nondimensionalizing the solution, confining the solution to a narrower region, minimizing changes in the dependent variables, simplifying boundary conditions and occasionally result in the deletion of streamwise derivative terms. This latter possibility occurs only under very restrictive sets of boundary conditions. The coordinate transformation in the present analysis is a variation of the Levy-Lees transformation and is derived in the literature. The standard Levy-Lees transformation takes the form

$$\begin{aligned}\xi &= \int_0^s \rho_1 u_1 \mu_1 r_o^{2K} ds \\ \eta &= \frac{r_o^K u_1}{\sqrt{2\xi}} \int_0^y \rho dy\end{aligned}\tag{33}$$

The first alteration of this transformation is actually a mathematical convenience for carrying out the numerical solution. Introducing a stretching parameter α_H in the normal coordinate, a new coordinate system is defined by

$$\begin{aligned}\bar{\xi} &= \xi \\ \bar{\eta} &= \frac{\eta}{\alpha_H}\end{aligned}\tag{34}$$

The parameter α_H is taken as a function of $\bar{\xi}$ only and is determined implicitly during the solution. Its purpose is to stretch the η coordinate such that the boundary layer remains of constant thickness in the $\bar{\eta}$ coordinates.

Since a new variable $\alpha_H(\xi)$ is introduced an additional relation is required. This is conveniently supplied by constraining some arbitrary point near the boundary-layer edge, $\bar{\eta}_c$, to have a specified streamwise velocity, c , near (but something less than) the edge value:

$$f' \Big|_{\bar{\eta}_c} = c f' \Big|_{\bar{\eta}_{\text{edge}}}\tag{35}$$

where f is the transformed stream function defined as

$$f - f_w = \int_0^\eta \frac{u}{u_1} d\eta = \alpha_H \int_0^{\bar{\eta}} \frac{u}{u_1} d\bar{\eta} \quad (36)$$

and the prime denotes differentiation with respect to $\bar{\eta}$ so that

$$f' = \alpha_H \frac{u}{u_1} \quad (37)$$

Examples of the utility of the stretching parameter α_H are contained in reference 2.

The second change in the Levy-Lees transformation has to do with the transverse curvature effect. For very thin axisymmetric bodies, it is possible to have boundary layer thicknesses on the order of the body radius r_0 . In this instance, it is necessary to treat r as a function of y , thereby including its variation through the boundary layer. The coordinate transformations become

$$\hat{\xi} = \int_0^s \rho_1 u_1 \mu_1 r_0^{2K} ds \quad (38)$$

$$\hat{\eta} = \frac{u_1}{\alpha_H \sqrt{2\hat{\xi}}} \int_0^y \rho r^K dy$$

Utilization of the above coordinate transformation relations results in a new set of governing equations in the $(\hat{\xi}, \hat{\eta})$ coordinate plane which will be given below. The hat(^) notation will be dropped for the remainder of the text for simplicity, however ξ and η are given by equation (38). Primes will refer to derivatives with respect to η except when noted otherwise.

The global continuity equation is automatically satisfied by the definition of a transformed stream function $f(\xi, \eta)$, shown in equation (36), and redefined here in the final coordinate system:

$$f - f_w = \alpha_H \int_0^\eta \frac{u}{u_1} d\eta \quad (39)$$

$$f_w = -\frac{1}{\sqrt{2\xi}} \int_0^\xi \frac{\rho_w v_w}{\rho_1 u_1 \mu_1 r_0^K} d\xi \quad (40)$$

The other governing equations will be discussed separately.

Streamwise Momentum Equation

$$ff'' + \left[\frac{tC \left(1 + \frac{\epsilon_M}{\nu} \right)}{\alpha_H} f'' \right] + \beta \left(\alpha_H^2 \frac{\rho_1}{\rho} - f'^2 \right) = 2 \left(f' \frac{\partial f'}{\partial \ln \xi} - f'^2 \frac{\partial \ln \alpha_H}{\partial \ln \xi} - f'' \frac{\partial f}{\partial \ln \xi} \right) \quad (41)$$

In this equation, utilizing the technique of reference 18, the transverse curvature effect is included entirely in the coordinate transformation and in the definition of t :

$$t \equiv \left(\frac{r}{r_0} \right)^2 = 1 + \frac{2\alpha_H \sqrt{2\xi} \cos \theta}{u_1 r_0^2} \int_0^\eta \frac{1}{\rho} d\eta \quad (42)$$

where θ is the angle between the surface normal and a plane normal to the body centerline (see Figure 1). Other definitions of interest are:

$$C \equiv \frac{\rho \mu}{\rho_1 \mu_1} \quad (43)$$

$$\beta \equiv 2 \frac{\partial \ln u_1}{\partial \ln \xi} \quad (44)$$

For solutions without consideration of transverse curvature, t is set to 1.0 throughout the boundary layer. A rigorous development of the coordinate transformation with transverse curvature is given in the literature.

Turbulent Model Equations

The turbulent fluctuations are related to the mean field through the eddy models described in equations (2) and (7). Eddy viscosity is described by a wall law and wake law, while eddy conductivity is related to eddy viscosity by the turbulent Prandtl number:

$$Pr_t \equiv \frac{\epsilon_M}{\epsilon_H} \quad (45)$$

Defining

$$\delta \equiv \frac{\sqrt{2\xi}}{\rho_1 u_1 r_0} \quad (46)$$

$$\tilde{\ell} \equiv \frac{\rho \ell}{\rho_1 \delta} \quad (47)$$

$$Re_\delta \equiv \frac{\rho_1 u_1 \delta}{\mu_1} \quad (48)$$

$$\tilde{\epsilon}_M \equiv \frac{\rho^2 \epsilon}{\rho \mu} \quad (49)$$

The wall region eddy viscosity relation becomes

$$\tilde{\epsilon}_M = \frac{\rho (Re_\delta)}{\rho_1 \alpha_H^2} \tilde{\ell}^2 f'' \quad (\text{wall region}) \quad (50)$$

$$\tilde{\epsilon}_M = 0.018 \left(\frac{\rho}{\rho_1} \right)^2 Re_{\delta_1^*} \quad (\text{wake region}) \quad (51)$$

where

$$\delta_1^* = \delta \alpha_H \int_0^\infty \left(1 - \frac{f'}{\alpha_H} \right) \frac{\rho_1}{\rho} d\eta \quad (52)$$

Transverse curvature is not considered in determining the wake region length scale δ_1^* . The governing equation for mixing length, which must be solved for the entire boundary layer (although it is used only in the wall region), is

$$\frac{d\tilde{\ell}}{d\eta} = \frac{\alpha_H \rho_1 \delta \sqrt{\tau/\rho}}{y_a^+ \mu} \left(K \alpha_H \eta - \tilde{\ell} \right) \quad (53)$$

Since mixing length is used only in the wall region, it is valid to use the one-dimensional expression for shear stress, equation (15). In transformed coordinates, this becomes

$$\frac{\tau}{\rho} = \frac{u_1^2}{\alpha_H} \frac{\rho_1}{\rho} \left[\frac{C_w}{\alpha_H} \frac{f_w''}{Re_\delta} + \frac{\rho_w v_w}{\rho_1 u_1} f' \right] \quad (54)$$

Energy Equation

$$f H_T' + [t(-q_a^* + q_r^*)]' = 2 \left(f' \frac{\partial H_T}{\partial \ln \xi} - H_T' \frac{\partial f}{\partial \ln \xi} \right) \quad (55)$$

where q_a^* is the normalized diffusive energy flux away from the surface including turbulent fluxes and q_r^* is the normalized radiant energy flux toward the surface:

$$q_a^* = q_a / \alpha^* \quad (56)$$

$$q_r^* = q_r / \alpha^* \quad (57)$$

The flux normalizing parameter α^* is defined by

$$\alpha^* \equiv \frac{\rho_1 u_1 \mu_1 r_o^k}{\sqrt{2\xi}} = \frac{\mu_1}{\delta} \quad (58)$$

Diffusive Fluxes

The normalized diffusive energy flux is given by

$$q_a^* = -\frac{C}{\alpha_H} \left[\frac{f' f''}{\alpha_H^2} u_1^2 + \frac{\bar{C}_p}{Pr} T' \right] - \frac{\tilde{\epsilon}_M}{\alpha_H} \left[\frac{f' f''}{\alpha_H^2} u_1^2 + \frac{\bar{C}_p}{Pr_t} T' \right] \quad (59)$$

where Pr is the Prandtl number based on the mean specific heat

$$Pr \equiv \frac{\bar{C}_p \mu}{\lambda} \quad (60)$$

When certain groupings of parameters are constant so that the flow similarity assumption is valid the terms on the right-hand side of the conservation equations (equations (41) and (55)) vanish, in which case the conservation equations become ordinary differential equations. It should be emphasized that the equations as presented herein are equivalent to the corresponding boundary-layer equations presented in Section 2.1. That is, no similarity assumptions have been made in their development.

4. BOUNDARY CONDITIONS INCLUDING ENTROPY LAYER EFFECTS

The usual set of boundary conditions for the boundary layer flow problem consists of the specification of initial profiles for the dependent variables f' and H_T plus additional specifications of these quantities along the wall and at the edge of the boundary layer, and the specification of f_w along the wall. However, since the analytical techniques presented here are used to compute boundary layer properties for flows over ablating or transpired surfaces, these boundary conditions have been greatly generalized. Some of the numerous options resulting from this generalization are discussed below.

The boundary layer edge conditions typically are found from an isentropic expansion from known elemental gas composition and stagnation conditions. Thus,

given a set of stagnation conditions and a description of local static pressure along the surface of interest, thermodynamic techniques may be used to establish the entropy of the gaseous mixture which, when combined with the specified pressures, can be used to establish the complete equilibrium edge gas state at each body station. Edge boundary conditions then consist of

$$f'_{\text{edge}} = \frac{\alpha H'_{\text{edge}}}{u_1} \quad (61)$$

$$H_{T_{\text{edge}}} = H_{T_{\text{edge}}} \bigg|_{\text{actual}}$$

where the subscript "edge" refers to conditions specified at η_{edge} , chosen to be outside the boundary layer (see Section II-3). An additional constraint at the conventional boundary layer edge (which is necessary only when cubics are used) is the requirement of zero slope, i.e.,

$$f''_{\text{edge}} = 0$$

$$H'_{T_{\text{edge}}} = 0 \quad (62)$$

It is possible to specify edge entropy as well as pressure. Thermodynamic techniques are then used to establish the complete edge gas state for a non-isentropic expansion around the body of interest.

With the existence of energy or entropy gradients in the inviscid flow field, the establishment of the "edge" condition becomes arbitrary. The method described below couples the viscid and inviscid solutions through the stream function such that the edge condition "floats" depending upon the value of the stream function calculated at the edge η value. In this method the arbitrary reference condition (denoted by the subscript 1 in the preceding equations) is assigned to the wall stream line of the inviscid flow field (see Figure 1). In the boundary layer the corresponding stream line is represented by the zero stream function so that by assigning the same stream function value to the wall inviscid stream line the two flow fields can be coupled, since the changes in the inviscid flow field entropy and energy can be uniquely related to stream function. Thus, the relative inviscid flow field entropy, S_i , and total enthalpy H_{Ti} are defined by the functions:

$$S_i = S_i(\tilde{f})$$

(63)

and

$$H_{Ti} = H_{Ti}(\tilde{f})$$

where $\tilde{f}$ is the inviscid flow stream function which is given by:

$$\tilde{f} = \int_{y(f=0)}^y \rho u r^{\kappa} dy \quad (64)$$

and where the reference (zeroth stream function) of entropy and enthalpy are:

$$\begin{aligned} S_i(0) &= 0 \\ H_{Ti}(0) &= 0 \end{aligned} \quad (65)$$

Similarly the "edge" coupling can be shown in the functional form

$$\begin{aligned} S_{i_edge} &= S_{edge} = S_i(\tilde{f}_{edge}) \\ H_{Ti_edge} &= H_{T_edge} = H_{Ti}(\tilde{f}_{edge}) \end{aligned} \quad (66)$$

The exact functional form of Equations (66) are of course problem dependent -- depending on shock shape, strength, etc. in the inviscid flow field. The relationship between f and $\tilde{f}$ is obtained from the definition of the boundary layer stream function, f , given by equation (39) and the derivative of equation (38):

$$d\eta = \frac{u_1}{\alpha_H \sqrt{2\xi}} \rho u r^{\kappa} dy \quad (67)$$

From the definition of α^* (Equation 58)

$$\alpha^* = \rho_1 u_1 \mu_1 r_o^{\kappa} / (2\xi)^{1/2} \quad (68)$$

Equation (67) can be cast in the form:

$$d\eta = \frac{\alpha^* \rho u r^{\kappa} dy}{\rho_1 \mu_1 \alpha_H r_o^{\kappa}} \quad (69)$$

with $\delta = \frac{\mu_1}{\alpha^*}$ and recalling that α_H is a function of s alone, then (39) becomes:

$$f = \int_{y(f=0)}^y \frac{\rho u r^{\kappa}}{\rho_1 u_1 r_o^{\kappa}} \frac{dy}{\delta} = \frac{1}{\rho_1 u_1 r_o^{\kappa} \delta} \int_{y(f=0)}^y \rho u r^{\kappa} dy = \frac{\tilde{f}}{\rho_1 u_1 r_o^{\kappa} \delta} \quad (70)$$

Thus f and $\tilde{f}$ are related by the proportionality constant $u_1 \rho_1 r_o^{\kappa \delta} (= \sqrt{2\tilde{f}})$. The function forms can now be written (with the notation e for edge)

$$\begin{aligned} S_e(\tilde{f}) &\equiv S_i(\rho_1 u_1 r_o^{\kappa \delta} \tilde{f}_e) = S_i(\tilde{f}_e) \\ H_{T_e}(\tilde{f}) &\equiv H_{T_i}(\rho_1 u_1 r_o^{\kappa \delta} \tilde{f}_e) = H_{T_i}(\tilde{f}_e) \end{aligned} \quad (71)$$

The derivatives of these functions with respect to $\tilde{f}_e$ are then given by (with $\dagger$ now denoting differentiation with respect to $\tilde{f}$).

$$\begin{aligned} \frac{dS_i}{d\tilde{f}_e} &= \frac{dS_i}{d\tilde{f}} \frac{d\tilde{f}}{d\tilde{f}_e} = S_i^\dagger u_1 \rho_1 r_o^{\kappa \delta} \\ \frac{dH_{T_i}}{d\tilde{f}_e} &= H_{T_i}^\dagger u_1 \rho_1 r_o^{\kappa \delta} \end{aligned} \quad (72)$$

The edge velocity relative to the reference velocity, u_1 , is given by the change in energy expressed in the form

$$\begin{aligned} u_e^2 &= u_1^2 + 2(H_{T_i} - h_s) \\ \frac{f'_e}{\alpha_H} &\equiv \tilde{u}_e = \frac{u_e}{u_1} = \left(1 + \frac{2(H_{T_i} - h_s)}{u_1^2} \right)^{1/2} \end{aligned} \quad (73)$$

where h_s is the change in static enthalpy commensurate with the change in entropy. The relationships between h_s , velocity and total enthalpies can be displayed by writing the energy equation as:

$$\begin{aligned} u_e^2 - u_1^2 &= 2(H_{T_e} - h_e) - 2(H_{T_1} - h_1) \\ &= 2(H_{T_e} - H_{T_1} - h_e + h_1) \\ &= 2(H_{T_i} - h_s) \end{aligned} \quad (74)$$

so that it is seen that the change in static enthalpy is defined by

$$h_s \equiv h_e - h_1 \quad (75)$$

For a given solution iteration, the value of h_s can be obtained from the current value of h_e and S_e by a Taylor's series expansion around h_e (at constant pressure):

$$h_s = h_e - h_1 + \left. \frac{\partial h}{\partial S} \right|_p (S_i - S_e) + \frac{1}{2} \left. \frac{\partial^2 h}{\partial S^2} \right|_p (S_i - S_e)^2 + \dots \quad (76)$$

$$h_s = h_e - h_1 + T(S_i - S_e) (1 + (S_i - S_e)/2\bar{C}_p) + \dots$$

The reference static enthalpy, h_1 , is known from the reference expansion of the zeroth streamline. In the current version of the method only the terms in the expansion shown explicitly in equation (76) are used. The derivative of edge velocity in the normal direction due to entropy and energy gradient effects is given by

$$\tilde{u}'_e = \frac{\partial \tilde{u}_e}{\partial \eta} = \frac{\partial (u_e/u_1)}{\partial \eta} \quad (77)$$

which (by the use of equation (73)) can be cast in the form:

$$\tilde{u}_e \tilde{u}'_e = \frac{1}{2} \frac{\partial}{\partial \eta} \left(\frac{2(H_{Ti} - h_s)}{u_1^2} \right) = \frac{\partial}{\partial f_e} \left(\frac{(H_{Ti} - h_s)}{u_1^2} \right) \frac{df_e}{d\eta} \quad (78)$$

or:

$$\tilde{u}'_e = \alpha_H \frac{\partial}{\partial f_e} \left(\frac{(H_{Ti} - h_s)}{u_1^2} \right)$$

Because $h_s = h_s(S_i)$ and $S_i = S_i(f)$, it follows that $h_s = h_s(f)$ and also that $h_s = h_s(\rho_1 u_1 r_o^\kappa \delta f_e)$. Thus by use of equation (72) for $H_{Ti}^\dagger$ and by analogy to (72) for $h_s^\dagger$, $\tilde{u}'_e$ is given by:

$$\tilde{u}'_e = \frac{f''}{\alpha_H} = \frac{\alpha_H \rho_1 r_o^\kappa \delta (H_{Ti}^\dagger - h_s^\dagger)}{u_1} \quad (79)$$

In summary, when the inviscid flow field contains entropy and enthalpy gradients these effects may be conveniently treated within the present boundary layer method by appropriate modification of the "edge" boundary conditions. In particular equations (61) become

$$f'_{\text{edge}} = \alpha_H \tilde{u}_e \quad (80)$$

as given by Equation (73) and

$$H_{T_{edge}} = H_{T_1} + H_{T_i} \quad (80)$$

If it is desirable to prescribe gradients of these variables, cubic equations should be used at the edge and Equations (62) become

$$f''_{edge} = \alpha_H \tilde{u}'_e \quad (81)$$

as given by Equation (79) and the enthalpy gradient by

$$H'_{T_{edge}} = \frac{\partial H_{T_i}}{\partial \eta} = \frac{\partial H_{T_i}}{\partial \tilde{f}} \frac{\partial \tilde{f}}{\partial f_e} \frac{\partial f_e}{\partial \eta} \quad (82)$$

$$H'_{T_{edge}} = H_{T_i} \rho_l u_l r_o^k \tilde{u}_e \alpha_H$$

The energy and entropy gradient effects are implemented in the computer coding of the boundary layer equations apart from the matrix inversion solution through a table look up procedure. In effect, changes in entropy from the reference, S_i , and like changes in enthalpy, H_T , are input for discrete values of inviscid stream function, $\tilde{f}$. Local values of S_i and H_T and local derivatives S_i' and H_{T_i}' for a given boundary layer edge stream function value, f_e , are obtained from the tabular data by an overlapped cubic curve fitting procedure.

Initial profiles of f' and H_T are difficult to establish for the general problem, therefore calculations are often started with reasonable assumed profiles far upstream of the region of interest so that effects of erroneous assumptions will die out. Another possibility for initially laminar problems is to assume a similar solution as a starting profile. This assumption reduces the equations to ordinary differential equations at the starting point, which may be solved simultaneously for a set of profiles unique to the assumed edge and wall state. The similar solution is exact at a body stagnation point, therefore this approach is particularly valuable for blunt body problems.

The wall boundary conditions allow the widest selection of specifications. The simplest combination is the straightforward assignment of velocity, enthalpy, and stream function at the wall:

$$\begin{array}{ll} f'_w = 0 & \text{no slip} \\ f_w = f_w(\xi) & \text{specified } \rho_w v_w \\ H_{T_w} = h_w(\xi) & \text{specified enthalpy of gas at the wall} \end{array} \quad (83)$$

Wall temperature may be used to find wall enthalpy in the above formulation.

5. STAGNATION POINT CONSIDERATIONS

Equations (58), (44), and (40) for α^* , β , and f_w , respectively, are indeterminate at the stagnation point of a blunt body. Special forms for these equations valid at the stagnation point are shown in reference 1 to be given by

$$\alpha_{sp}^* = \left(\rho_1 \mu_1 \frac{du_1}{ds} / \beta \right)_{sp}^{1/2} \quad (84)$$

$$f_{w_{sp}} = - \left(\rho_w v_w / \alpha^* \right)_{sp} \quad (85)$$

where the Newtonian flow

$$\beta_{sp} = 1/(\kappa + 1) \quad (86)$$

and

$$\left. \frac{du_1}{ds} \right|_{sp} = (2P/\rho)_{sp}^{1/2} / R_{eff} \quad (87)$$

with R_{eff} an effective nose radius taking into account the shock shape. Alternatively, β_{sp} and $(du_1/ds)_{sp}$ can be computed from curve fits of the inviscid pressure distribution. The transverse curvature parameter t also requires some special treatment at a stagnation point. The troublesome term is $\cos \theta / r_o$ which is evaluated at a stagnation point by

$$\left(\frac{\cos \theta}{r_o} \right)_{sp} = \left[6 \frac{d^2 \left(1 - \frac{r_o}{s} \right)}{ds^2} \right]^{1/2} \quad (88)$$

In addition, to improve the accuracy of numerical integration procedures in the nose region, ξ and f_w can be computed by the following relations

$$\xi = \frac{1}{2(\kappa + 1)} \int_0^s s^{2\kappa+2} \left[\rho_1 \mu_1 \left(\frac{u_1}{s} \right) \left(\frac{r_o}{s} \right)^{2\kappa} \right] ds \quad (89)$$

$$f_w = - \frac{(2\xi)^{-1/2}}{(\kappa + 1)} \int_0^s s^{\kappa+1} \left[\rho_w v_w \left(\frac{r_o}{s} \right)^\kappa \right] ds \quad (90)$$

which take advantage of the fact that u_1/s and r_o/s vary more nearly linearly in the stagnation region than do u_1 and r_o . These equations are also discussed more thoroughly in reference 1.

Stagnation point problems are encountered for entropy and energy layer problems as well. In this case f'_e and f''_e are also indeterminate since u_1 in the denominator of Equations (73) and (79) is zero at the stagnation point. It will be seen that the formulation for avoiding these indeterminate forms are more complicated -- for instance, axisymmetric and 2-dimensional shapes requiring separate treatment -- than is the case for α^* , β and f_w . In the following development, the factor $2(H_{Ti} - h_s)/u_1^2$ in Equation (73) will be operated on for illustrative purposes. Applying L'Hospital's rule to this term and performing the indicated differentiation as before yields

$$\begin{aligned} \left. \frac{2(H_{Ti} - h_s)}{u_1^2} \right|_{s \rightarrow 0} &= \left. \frac{2d(H_{Ti} - h_s)/ds}{du_1^2/ds} \right|_{s \rightarrow 0} = \left. \frac{d(H_{Ti} - h_s)/d\tilde{f}_e \cdot d\tilde{f}_e/ds}{u_1 du_1/ds} \right|_{s \rightarrow 0} \\ \left. \frac{2(H_{Ti} - h_s)}{u_1^2} \right|_{s \rightarrow 0} &= \frac{(H_{Ti}^+ - h_s^+) d(\rho_1 u_1 r_o^K \delta f_e)/ds}{u_1 du_1/ds} \end{aligned} \quad (91)$$

For plane blunt bodies $u_1 \sim s$, $r_o^K = 1.0$ and the derivatives of the remaining terms are zero at the stagnation point (symetry), thus

$$\frac{d(\rho_1 u_1 r_o^K \delta f_e)}{ds} = \rho_1 \delta f_e \frac{du_1}{ds} \quad (92)$$

Thus:

$$\left. \frac{2(H_{Ti} - h_s)}{u_1^2} \right|_{s \rightarrow 0} = \frac{(H_{Ti}^+ - h_s^+) \rho_1 \delta f_e}{u_1}$$

which is still indeterminate. However, a repeated differentiation yields:

$$\left. \frac{2(H_{Ti} - h_s)}{u_1^2} \right|_{s \rightarrow 0} = (H_{Ti}^{++} - h_s^{++}) (\rho_1 \delta f_e)^2 \quad (93)$$

and thus

$$f'_{e_{sp}} = \alpha_H \tilde{u}_e = \alpha_H \left(1 + (H_{Ti}^{++} - h_s^{++}) (\rho_1 \delta f_e)^2 \right)^{1/2} \quad [2-d] \quad (94)$$

A similar differentiation for $u_{e_{sp}}$ yields

$$\tilde{u}'_e \Big|_{s \rightarrow 0} = (H_{Ti}^{++} - h_s^{++}) (\rho_1 \delta)^2 f_e \alpha_H \quad (95)$$

and thus

$$f''_{e_{sp}} = f_e (\rho_1 \delta \alpha_H)^2 (H_{Ti}^{++} - h_s^{++}) \quad [2-d]$$

For axisymmetric bodies both u_1 and r_o vary with s so that

$$\frac{d(\rho_1 u_1 r_o \delta f_e)}{ds} \approx \rho_1 \delta f_e \left(r_o \frac{du_1}{ds} + u_1 \frac{dr_o}{ds} \right) \quad (96)$$

Now in the vicinity of the stagnation point $r_o \rightarrow s$ and $du_1/ds \rightarrow u_1/s$, so that

$$\begin{aligned} \frac{2(H_{Ti} - h_s)}{u_1^2} \Big|_{s \rightarrow 0} &= \frac{(H_{Ti}^+ - h_s^+) \rho_1 \delta f_e (r_o \frac{du_1}{ds} + u_1)}{u_1 \frac{du_1}{ds}} \\ &= \frac{(H_{Ti}^+ - h_s^+) \rho_1 \delta f_e}{\frac{du_1}{ds}} \left(\frac{s}{u_1} \frac{du_1}{ds} + 1 \right) \end{aligned} \quad (97)$$

$$\frac{2(H_{Ti} - h_s)}{u_1^2} \Big|_{s \rightarrow 0} = \frac{2(H_{Ti}^+ - h_s^+) \rho_1 \delta f_e}{\frac{du_1}{ds}}$$

Therefore

$$f'_{e_{sp}} = \alpha_H \left(1 + \frac{2(H_{Ti}^+ - h_s^+) \rho_1 \delta f_e}{\frac{du_1}{ds}} \right)^{1/2} \quad [\text{axisym}] \quad (98)$$

Similarly for $\tilde{u}'_e$

$$\tilde{u}'_e \Big|_{s \rightarrow 0} \approx \frac{\alpha_H \rho_1 \delta \left[(H_{Ti}^+ - h_s^+) \frac{dr_o}{ds} + r_o \frac{d(H_{Ti}^+ - h_s^+)}{ds} \right]}{\frac{du_1}{ds}} \quad (99)$$

which, neglecting the second term with second order derivatives, becomes

$$\tilde{u}'_e \Big|_{s \rightarrow 0} \approx \frac{\alpha_H \rho_1 \delta (H_{Ti}^+ - h_s^+)}{\frac{du_1}{ds}} \quad (100)$$

and finally

$$f''_{e_{sp}} \approx \frac{\alpha_H^2 \rho_1 \delta (H_{Ti}^+ - h_s^+)}{\frac{du_1}{ds}} \quad [\text{axisym}] \quad (101)$$

In the computer program, the second derivatives in these equations are approximated by simple two point differences of the cubic curve fit derivatives.

SECTION III

INTEGRAL MATRIX SOLUTION PROCEDURE

The solution of the transformed boundary layer equations presented in Section II uses an integral matrix method which has been developed specifically for the solution of the nonsimilar, coupled, chemically reacting boundary layer. A complete presentation of the integral matrix procedure was included in reference 1, where solution of laminar flow problems was discussed. In the present effort this technique has remained essentially unchanged, however new variables and equations have been added to describe the turbulent aspects of the flow and to include transverse curvature and the entropy and energy layer effects. The present discussion will therefore review only the highlights of the method, and the reader may refer to reference 1 for more details.

In the integral matrix procedure, the primary dependent variables and their derivatives with respect to η are related by Taylor series expansions such that these dependent variables are represented by connected quadratics or cubics (either option is available). That is, f' and H_T are expanded in Taylor series form and the series are truncated to reflect the proper polynomial representation. A nodal network is defined through the boundary layer and the Taylor series expansions are assumed valid between each set of nodes, with an additional requirement of continuous first and second derivatives (a spline fit). Primarily for convenience, the conservation equations are integrated across each "strip" (between nodal points) using a unity weighting function. The linear Taylor series expansions together with linear boundary conditions form a very sparse matrix which has to be inverted only once for a given problem. The nonlinear boundary layer equations and nonlinear boundary conditions are then linearized, the errors being driven to zero using Newton-Raphson iteration.

In Section III-1 the Taylor series expansions are presented, the integrated form of the momentum equation is discussed, and techniques for evaluating integral terms are demonstrated. In Section III-2 the special techniques applied to the mixing length differential equation are discussed, and in Section III-3 the actual simultaneous equation solution procedure is summarized.

1. INTEGRAL STRIP EQUATIONS WITH SPLINED INTERPOLATION FUNCTIONS

Consider the boundary layer in the region of a given streamwise station s as being divided into $N-1$ strips connecting N nodal points. These nodal

points are designated by η_i where $i = 1$ at the wall and N at the edge of the boundary layer. Consider a function $p(\eta)$ which with all its derivatives is continuous in the neighborhood of the point $\eta = \eta_i$. Then, for any value of η in this neighborhood, $p(\eta)$ may be expressed in a Taylor series expansion as

$$p_{i+1} = p_i + p_i' \delta\eta + p_i'' \frac{(\delta\eta)^2}{2} + p_i''' \frac{(\delta\eta)^3}{6} + p_i^{(4)} \frac{(\delta\eta)^4}{24} \dots \quad (102)$$

where

$$\delta\eta = \eta_{i+1} - \eta_i \quad (103)$$

Conventional finite difference schemes, in effect, typically truncate the Taylor series after the first term and use the resulting expression to relate p' to p , etc., that is

$$p_i' = \frac{p_{i+1} - p_i}{\delta\eta} \quad (104)$$

Round-off error is then of order $(\delta\eta)^2$ and many nodes must be chosen to bring this value down to acceptable limits. One can achieve a reduction in the number of nodes for a given accuracy by employing a quadratic or cubic relation representing the function p over the interval of interest. This can be achieved by truncating the Taylor series after the third or fourth term. The cubic approximation will be used for the remainder of this discussion. The p_i can be considered to be any of the f_i , f_i' , f_i'' , f_i''' , H_{T_i} , H_{T_i}' , or H_{T_i}'' . Since the highest derivatives of the dependent variables which appear in the boundary layer equations are f_i''' and H_{T_i}'' , it is reasonable to truncate the series at the next higher derivative and to consider that derivative as being constant between η_i and η_{i+1} , that is,

$$\begin{aligned} f_{i+1}^{(4)} &= \frac{f_{i+1}^{(3)} - f_i^{(3)}}{\delta\eta} \\ H_{T_{i+1}}^{(3)} &= \frac{H_{T_{i+1}}'' - H_{T_i}''}{\delta\eta} \end{aligned} \quad (105)$$

Thus, rather than using finite difference approximations similar to Equation (104) which are substituted directly into the governing differential equations, a set of linear relations between the dependent variables and their derivatives is obtained and is solved simultaneously with the governing differential equations. These linear relations are of the form

$$-f_{i+1} + f_i + f'_i + f''_i \frac{(\delta\eta)^2}{2} + f'''_i \frac{(\delta\eta)^3}{8} + f_{i+1}''' \frac{(\delta\eta)^3}{24} = 0 \quad (106)$$

$$-p_{i+1} + p_i + p'_i + p''_i \frac{(\delta\eta)^2}{3} + p_{i+1}'' + \frac{(\delta\eta)^2}{6} = 0 \quad (107)$$

$$-p'_{i+1} + p'_i + p''_i \frac{\delta\eta}{2} + p_{i+1}'' \frac{\delta\eta}{2} = 0 \quad (108)$$

where in equations (107) and (108) the p_i can represent f'_i , or H_{T1} .

Notice that f' has been taken to be a cubic over each strip, rather than the stream function, f , since it was desired to represent velocity ($u = u_1 f' / \alpha_H$) with the cubic. Equations (106) through (108) above, when written for each adjacent pair of nodes; give $5(N - 1)$ simultaneous algebraic equations for the $7N + 1$ unknowns, f_n , f'_n , f''_n , f'''_n , α_H , H_{Tn} , H'_{Tn} , H''_{Tn} at each streamwise station.* Additional relations must come from the governing differential equations and the boundary conditions. It is important to note that the f , f' , etc., are treated as individual variables related by algebraic equations. It is also important to note that the coefficients in Equations (106) through (108) are functions of $\delta\eta$ only; therefore, this portion of the resulting matrix need be inverted only once for a given problem.

The conservation equations (41) and (55) contain streamwise derivative or "nonsimilar" terms. In the present solution technique, two or three point finite difference formulas are considered sufficient to express these derivatives, since gradients in this direction are not severe. As in reference 1

$$s \left[\frac{d(\quad)}{d(\ln \xi)} \right]_l = d_0(\quad)_l + d_1(\quad)_{l-1} + d_2(\quad)_{l-2} \quad (109)$$

where $(\quad)_{l-1}$ refers to the previous streamwise station,

$$d_0 = \frac{2}{l^{\Delta_{l-1}}}, \quad d_1 = -\frac{2}{l^{\Delta_{l-1}}}, \quad d_2 = 0 \quad (110)$$

for two-point difference and

$$d_0 = 2 \frac{l^{\Delta_{l-1}} + l^{\Delta_{l-2}}}{l^{\Delta_{l-1}} l^{\Delta_{l-2}}}, \quad d_1 = -2 \frac{l^{\Delta_{l-2}}}{l^{\Delta_{l-1}} l^{\Delta_{l-2}}}, \quad d_2 = 2 \frac{l^{\Delta_{l-1}}}{l^{\Delta_{l-2}} l^{\Delta_{l-2}}} \quad (111)$$

for three-point difference where typically

$$l^{\Delta_{l-1}} = \ln \xi_l - \ln \xi_{l-1} = \ln(\xi_l / \xi_{l-1}) \quad (112)$$

*The mixing length is not included in this variables count since mixing length (as well as ϵ_M in the wake region) is treated as a state property.

The three-point difference relation is generally used unless a similar solution is desired (in which case $d_0 = d_1 = d_2 = 0$) or unless the point in question is the first point after either (1) a similar solution or (2) a discontinuity (e.g., where the body changes shape abruptly, or where mass injection is suddenly terminated).

The next step in the treatment of the conservation equations is their integration across the boundary layer "strips". The primary reason for this integration is to simplify the η -derivative terms in the energy and species conservation equations, since it is not convenient to express the complex q_a^* and j_k^* terms in derivative form. The solution can actually proceed very nicely without integrating across strips (see reference 7) without any noticeable change in speed, accuracy, or stability for simplified problems such as incompressible, nonreacting flows. The weighting function for integration between nodes in this integral method is unity. In the terminology of the general method of integral relations, where integrals are carried from 0 to ∞ in η (reference 19), a square wave weighting function is used which is unity across the strip in question and zero elsewhere. The equations are then integrated $N-1$ times with the square wave applied to each strip in succession. Using the momentum equation as an example, the integration from $i-1$ to i results in

$$\begin{aligned} & \int_{i-1}^i f f'' d\eta + \left[\frac{t(C + \tilde{\epsilon}_M)}{\alpha_H} f'' \right]_{i-1}^i + \beta \alpha_H^2 \int_{i-1}^i \frac{\rho_1}{\rho} d\eta - \beta \int_{i-1}^i f'^2 d\eta \\ &= \int_{i-1}^i f' (d_0 f' + d_1 f'_{\ell-1} + d_2 f'_{\ell-2}) d\eta - \int_{i-1}^i f'^2 [d_0 \ln \alpha_H \\ &+ d_1 (\ln \alpha_H)_{\ell-1} + d_2 (\ln \alpha_H)_{\ell-2}] d\eta - \int_{i-1}^i f'' (d_0 f + d_1 f_{\ell-1} + d_2 f_{\ell-2}) d\eta \quad (113) \end{aligned}$$

The Taylor series approximations introduced earlier can also be used to express the integral terms above. As demonstrated in reference 1, the term $\int_{i-1}^i f' p d\eta$ becomes

$$\int_{i-1}^i f' p d\eta = f'_i X P_1 + f'_{i-1} X P_2 + f'_{i-1} X P_3 + f'_{i-1} X P_4 \quad (114)$$

where

$$\begin{aligned}
 XP_1 &= \delta\eta \left(p_i - p_i' \frac{\delta\eta}{2} + p_i'' \frac{(\delta\eta)^2}{8} + p_{i-1}'' \frac{(\delta\eta)^2}{24} \right) \\
 XP_2 &= -(\delta\eta)^2 \left(\frac{p_i}{2} - p_i' \frac{\delta\eta}{3} + p_i'' \frac{11(\delta\eta)^2}{120} + p_{i-1}'' \frac{(\delta\eta)^2}{30} \right) \\
 XP_3 &= (\delta\eta)^3 \left(\frac{p_i}{8} - p_i' \frac{11(\delta\eta)}{120} + p_i'' \frac{11(\delta\eta)^2}{420} + p_{i-1}'' \frac{5(\delta\eta)^2}{504} \right) \\
 XP_4 &= (\delta\eta)^3 \left(\frac{p_i}{24} - p_i' \frac{\delta\eta}{30} + p_i'' \frac{5(\delta\eta)^2}{504} + p_{i-1}'' \frac{(\delta\eta)^2}{252} \right)
 \end{aligned} \tag{115}$$

This technique is used to rewrite each of the integral terms in equation (113) above of the form $\int_{i-1}^i f' p \, d\eta$. The remaining integral term in the momentum equation, $\int_{i-1}^i (\rho_1/\rho) d\eta$ is evaluated by approximating the functions as cubics over the strip and integrating directly. This yields

$$\int_{i-1}^i \frac{\rho_1}{\rho} d\eta = \left(\frac{\rho_1}{\rho_i} + \frac{\rho_1}{\rho_{i-1}} \right) \frac{\delta\eta}{2} + \left(\frac{\rho_1 \rho_i'}{\rho_i^2} - \frac{\rho_1 \rho_{i-1}'}{\rho_{i-1}^2} \right) \frac{(\delta\eta)^2}{12} \tag{116}$$

However, before discussing how the set of algebraic equations is solved, Section III-2 describes how the mixing length differential equation is solved.

2. SOLUTION OF THE MIXING LENGTH EQUATION

The mixing length equation is a first order linear differential equation whose solution can be written directly in general terms. The differential equation is

$$\frac{d\tilde{\ell}}{d\eta} = \frac{\alpha_H \rho_1 \delta \sqrt{\tau/\rho}}{y_{a\mu}^+} (K\alpha_H \eta - \tilde{\ell}) \tag{117}$$

Defining

$$P(\eta) \equiv \frac{\alpha_H \rho_1 \delta \sqrt{\tau/\rho}}{y_{a\mu}^+} \tag{118}$$

results in

$$\frac{d\tilde{\ell}}{d\eta} = (K\alpha_H \eta - \tilde{\ell}) P \tag{119}$$

The solution to this equation is

$$\tilde{\ell} = K\alpha_H \left[\eta - \frac{\int_e^\eta \int_0^{\eta'} P \, d\eta'' \, d\eta'}{\int_e^\eta P \, d\eta'} \right] \tag{120}$$

The remaining problem is to evaluate the integral terms. Defining

$$L(\eta) \equiv \frac{\int_0^{\eta_e} \int_0^{\eta'} P \, d\eta'' \, d\eta'}{\int_0^{\eta_e} P \, d\eta'} \quad (121)$$

yields

$$\tilde{\ell} = K\alpha_H (\eta - L) \quad (122)$$

A complete description of the technique used to evaluate $L(\eta)$ is available in the literature. In essence, $P(\eta)$ is assumed to vary linearly over the interval η_{i-1} to η_i , and the integrals are expressed in a more tractable form. The final expression is

$$L_i = BL_{i-1} + A \left\{ D_w\left(\frac{AP_i}{2}\right) - BD_w\left(\frac{AP_{i-1}}{2}\right) \right\} \quad (123)$$

where

$$A \equiv \left[\frac{2\Delta\eta_i}{P_i - P_{i-1}} \right]^{1/2} \quad (124)$$

$$B \equiv e^{-\Delta\eta_i \left[\frac{P_i - P_{i-1}}{2} \right]} \quad (125)$$

$$\Delta\eta_i \equiv \eta_i - \eta_{i-1} \quad (126)$$

$$D_w(\) = e^{-(\)^2} \int_0^{(\)} e^{+y^2} dy \quad (127)$$

The Dawson Integral, $D_w(\)$, can be evaluated from tables (reference 20) or by a series method. A series evaluation method is used in the present analysis. Thus, combining equations (122) and (123), an explicit recursion formula for mixing length at each node is obtained. This mixing length is a function of local shear, viscosity, and density through the variation of $P(\eta)$, and is re-evaluated at each node on each iteration during the course of a solution.

3. NEWTON-RAPHSON ITERATION FOR A SOLUTION

A complete description of the Newton-Raphson iteration procedure as applied to the laminar equations of motion was given in reference 1. Since the procedure is basically unchanged with the addition of turbulent flow, transverse curvature, entropy and energy layer equations, it will be reviewed only briefly here.

To illustrate the Newton-Raphson method, consider two simultaneous non-linear algebraic equations

$$F(x,y) = 0 \quad G(x,y) = 0 \quad (128)$$

the solution for which is given by $x = \bar{x}$, $y = \bar{y}$. Define x_m and y_m as the values of x and y for the m^{th} iteration. The desired solution $f(\bar{x}, \bar{y})$ can be expressed in a Taylor series expansion

$$\begin{aligned} 0 = F(\bar{x}, \bar{y}) &= F(x_m, y_m) + (\bar{x} - x_m) \frac{\partial F(x_m, y_m)}{\partial x} \\ &\quad + (\bar{y} - y_m) \frac{\partial F(x_m, y_m)}{\partial y} + \dots \\ 0 = G(\bar{x}, \bar{y}) &= G(x_m, y_m) + (\bar{x} - x_m) \frac{\partial G(x_m, y_m)}{\partial x} \\ &\quad + (\bar{y} - y_m) \frac{\partial G(x_m, y_m)}{\partial y} + \dots \end{aligned} \quad (129)$$

The Newton-Raphson method consists of replacing $(\bar{x}, \bar{y})$ by (x_{m+1}, y_{m+1}) on the right-hand side of these expressions and neglecting nonlinear terms in $x_{m+1} - x_m$ and $y_{m+1} - y_m$. This yields the set of simultaneous equations

$$\begin{aligned} \Delta x_m \frac{\partial F(x_m, y_m)}{\partial x} + \Delta y_m \frac{\partial F(x_m, y_m)}{\partial y} &= -F(x_m, y_m) \\ \Delta x_m \frac{\partial G(x_m, y_m)}{\partial x} + \Delta y_m \frac{\partial G(x_m, y_m)}{\partial y} &= -G(x_m, y_m) \end{aligned} \quad (130)$$

or in matrix form

$$\begin{bmatrix} \frac{\partial F(x_m, y_m)}{\partial x} & \frac{\partial F(x_m, y_m)}{\partial y} \\ \frac{\partial G(x_m, y_m)}{\partial x} & \frac{\partial G(x_m, y_m)}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x_m \\ \Delta y_m \end{bmatrix} = \begin{bmatrix} -F(x_m, y_m) \\ -G(x_m, y_m) \end{bmatrix} \quad (131)$$

where

$$\Delta x_m \equiv x_{m+1} - x_m \quad \Delta y_m \equiv y_{m+1} - y_m \quad (132)$$

The Δx_m and Δy_m are the corrections to be added to x_m and y_m , respectively, to yield the values of the dependent variables for the $m + 1^{\text{th}}$ iteration. Here $F(x_m, y_m)$ and $G(x_m, y_m)$ are the values of the original functions $F(x, y)$ and $G(x, y)$ evaluated for $x = x_m$ and $y = y_m$. As the corrections approach zero, the $F(x_m, y_m)$ and $G(x_m, y_m)$ approach zero. Hence, it is appropriate to look upon these as errors associated with the original equation (128). It is apparent that this procedure can be extended to an arbitrary number of functions and a corresponding number of primary variables.

For the purpose of the present analysis, it has been found most convenient to consider the primary variables as $f_i, f'_i, f''_i, f'''_i, H_{Ti}, H^1_{Ti}, H^2_{Ti}$ and α_H . This amounts to $N + 1$ unknowns where N is the number of nodes to be considered in the boundary layer. Recounting the number of equations, we have

	<u>Eqn. Numbers</u>	<u>No. of Equations</u>
Taylor series expansions	(106) - (108)	$5(N - 1)$
Boundary layer equations	(113) and equivalent energy equations	$2(N - 1)$
Boundary conditions	(61), (62), (83) or equivalent	7
α_H definition	(35)	<u>1</u>
Total		$7N + 1$

Other secondary variables such as ϵ, ρ, T , etc. are expressed in terms of those listed above. The corrections in these secondary variables are therefore found in terms of the corrections to the primary variables.

The use of the Newton-Raphson technique for the current set of equations requires the evaluation of the partial derivatives of each equation with respect

to each variable. The partial derivatives of the Taylor series equations and linear boundary conditions are exactly the same as in reference 1. Again, the reader is directed to reference 1 for the full exposition of these lengthy and tedious equations which will not be repeated here.

The technique of relating corrections on secondary variables such as C, ρ , T, PR, etc., to corrections in primary variables was fully explained in reference 1. The same techniques are used for the new corrections Δt and $\Delta \tilde{e}_M$ (and Δu_e for energy and entropy layers).

Once the correction coefficients (partial derivatives with respect to each primary variable) for each equation at each nodal point are found, they are arranged in matrix form for further manipulation. The order of the primary variables and the order of the equations is of some importance in the matrix formulation. It is most convenient to divide the variables into "linear" (symbol L) and "nonlinear" (symbol NL) sets, namely

$$\begin{bmatrix} AL & BL \\ \hline ANL & BNL \end{bmatrix} \begin{bmatrix} \Delta VL \\ \hline \Delta VNL \end{bmatrix} = - \begin{bmatrix} EL \\ \hline ENL \end{bmatrix} \quad (133)$$

where the linear equations are the Taylor series equations and some of the boundary conditions. The purpose of the partitioning is to allow operations on sections of the coefficient matrix which result in significant simplification of the overall inversion. In particular, since the coefficients of the linear equations are all constant or functions of the fixed nodal spacing, this portion of the matrix (the AL portion) can be diagonalized once and for all in any given problem. In essence, the corrections on the linear variables ΔVL are always expressed in terms of the nonlinear variable corrections ΔVNL . The choice of linear and nonlinear labels for the variables is somewhat arbitrary, but care must be taken that the AL matrix not be singular. It has been found convenient to arrange the variables into the linear and nonlinear groups as follows:

$\Delta VL_F(\Delta f_2, \Delta f_3, \dots, \Delta f_n, \Delta f_2'', \Delta f_3'', \dots, \Delta f_n'', \Delta f_1''', \Delta f_2''', \dots, \Delta f_n''')$; and $\Delta VL_H(\Delta H_{T_1}, \Delta H_{T_2}, \Delta H_{T_3}, \dots, \Delta H_{T_n}, \Delta H_{T_w}, \Delta H_{T_2}'', \dots, \Delta H_{T_n}'')$. The nonlinear variables are then arranged in the following order: $\Delta VNL_F(\Delta \alpha_H, \Delta f_w, \Delta f_w'', \Delta f_1', \Delta f_2', \dots, \Delta f_n')$; and $\Delta VNL_H(\Delta H_{T_w}, \Delta H_{T_w}', \Delta H_{T_2}, \dots, \Delta H_{T_{n-1}})$. The order of the linear equations in the present matrix procedure is:

<u>No. of Equations</u>	<u>Description of Equations</u>
3N-2	Linear boundary conditions and Taylor series for f, f', f'', f'''
2N	Linear boundary conditions and Taylor series for H_T, H_T', H_T''

The nonlinear equations are sequenced as follows:

<u>No. of Equations</u>	<u>Description of Equations</u>
4	Nonlinear boundary conditions and α_H constraint
$N - 1$	Momentum equation for each pair of nodes
N	Energy equation for each pair of nodes plus wall enthalpy equation

Special logic has been written for the matrix inversion, taking advantage of the regular sparseness of the matrix. Once the corrections for the linear and nonlinear variables are found, these corrections are added to the variables to form the new guesses. The magnitude of the errors for each equation are checked and the procedure advances to the next iteration if the absolute values of the errors exceed prescribed upper limits. If the errors are acceptable, iteration is completed for the current streamwise position ξ . Typically, three to six iterations are required to reach a satisfactory solution.

4. SOME RESULTS FOR BOUNDARY LAYERS WITH INVISCID ENTROPY GRADIENT

A total of five sample problems are presented here to demonstrate the use of the boundary layer theory and computational methods on problems with entropy gradients in the inviscid flow field adjacent to surfaces of interest. A limited comparison of the theoretical results with experimental data obtained in the Ames 3.5 ft. hypersonic wind tunnel is also presented. The problems are presented in order of increasing entropy layer effect. The free stream Mach number for all problems is 10.5 and air is the working fluid.

It is pertinent to point out here that a number of conventional boundary layer parameters traditionally calculated (and which are found in the results presented here) lose their physical meaning in flows with entropy gradients. For example, momentum thickness and displacement thickness (as traditionally defined) have no physical significance for flows with entropy and energy layer effects. A modified definition of displacement thickness which does have the usual physical definition is presented in the results to follow. This effective displacement thickness, δ_e^* , is given by:

$$\delta_e^* \equiv y_e - \frac{1}{r_0} \int_0^e \frac{df}{\rho_i u_i} \quad (134)$$

where the inviscid variation of ρ and u as a function of stream function, f , is included in the integration.

Reynolds number and the transfer coefficients are based on the reference streamline property values. This choice is arbitrary and one of convenience since free stream values (or some other arbitrary reference) could be used equally as well.

Only the results from a few selected output stations are presented here. The total number of stations for which calculations were performed ranged from 8 for the sharp bodies to about 20 for the blunt bodies. The stations selected for presentation here correspond to the instrumentation stations employed in the Ames hypersonic facility testing (references 22 and 23).

a. Flat Plate with Sharp Leading Edge - Laminar Flow

A schematic of the physical situation is shown in Figure 2. This figure shows a flat plate with a sharp leading edge (attached shock wave) at a 3° compression angle, the shock having a slightly curved shape because of boundary layer effects. In this and subsequent problems, the theoretical inviscid flow field data were obtained from computer solutions generated at Ames prior to the conduct of this study. The boundary layer displacement corrections made in these inviscid solutions were approximate and do not reflect the boundary layer properties calculated in this study as iterations of this kind were not performed between the inviscid and BLIMP solutions. The inviscid computer code used at Ames which produced the data used in this study is based on the method of Lomax (reference 21), and the method of Soremsen (reference 24).

In the computer output which follows special attention is drawn to the shock-entropy table in the first part of the output and the entropy and total pressure profile data supplied at each solution station.

Perusal of the output for the eight solution stations shows little effect of entropy layer. The velocity overshoot above the reference velocity is only $1/2$ of 1% at the last station considered. Part of the reason for the low magnitude of the entropy layer effect is due to the assumed shape of the entropy versus stream function curve (or alternately the shock shape). In the vicinity of the $\tilde{f} = 0$ streamline the entropy curve (or shock pressure ratio) is quite flat as shown in Figure 3, just as for a blunt nosed configuration. This is tantamount to the presumption of an infinitesimal shock standoff distance. Such a shape was required to satisfy compatibility with the inviscid solution. In any case, a finite amount of fluid must be entrained by the boundary layer before entropy effects occur.

Figure 4 presents a comparison of the laminar boundary layer profile with experimental data taken from reference 22 at about the midway point down the plate. The good agreement between the experiment and theory must be viewed

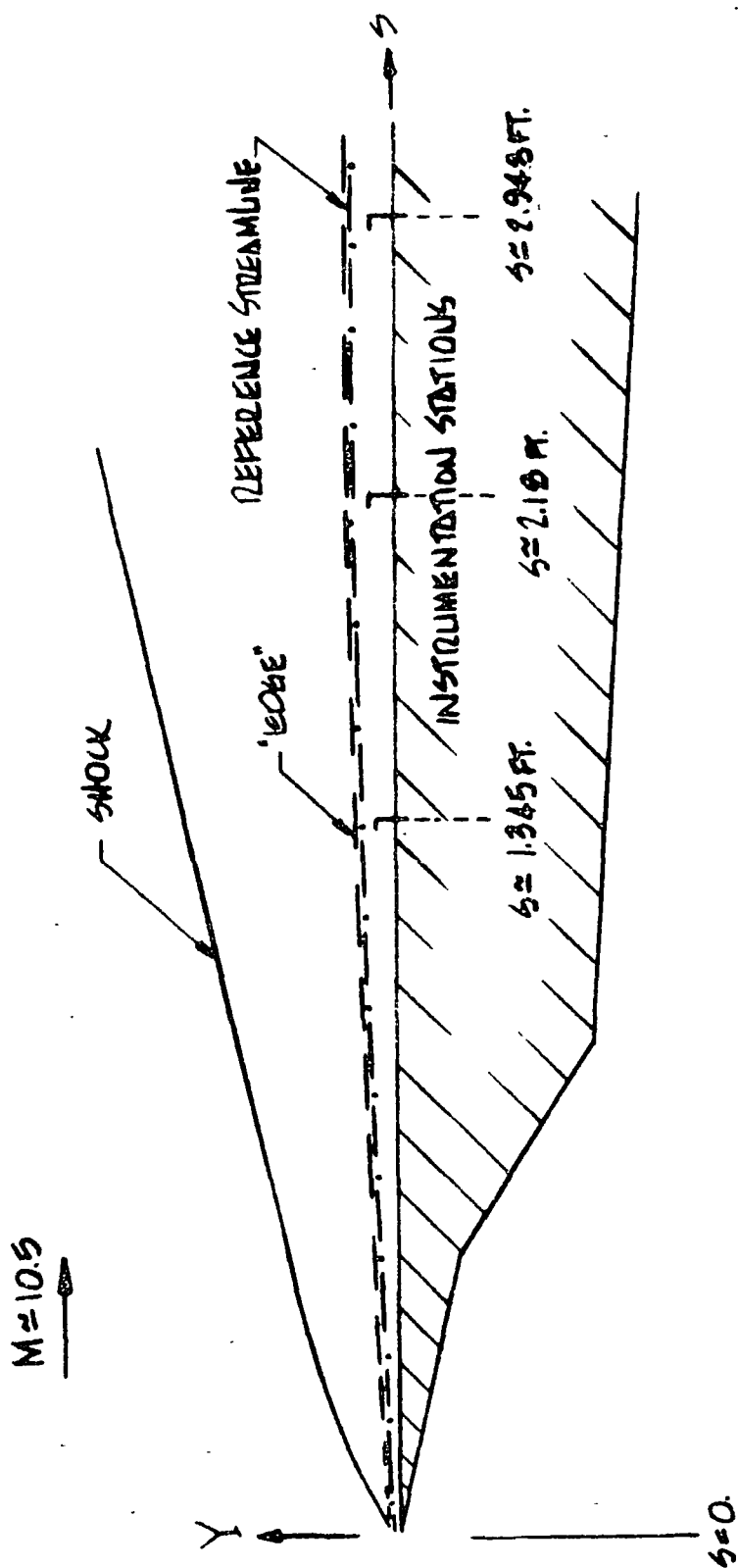


Figure 2. Schematic of the Configuration for Sample Problems 1 and 2

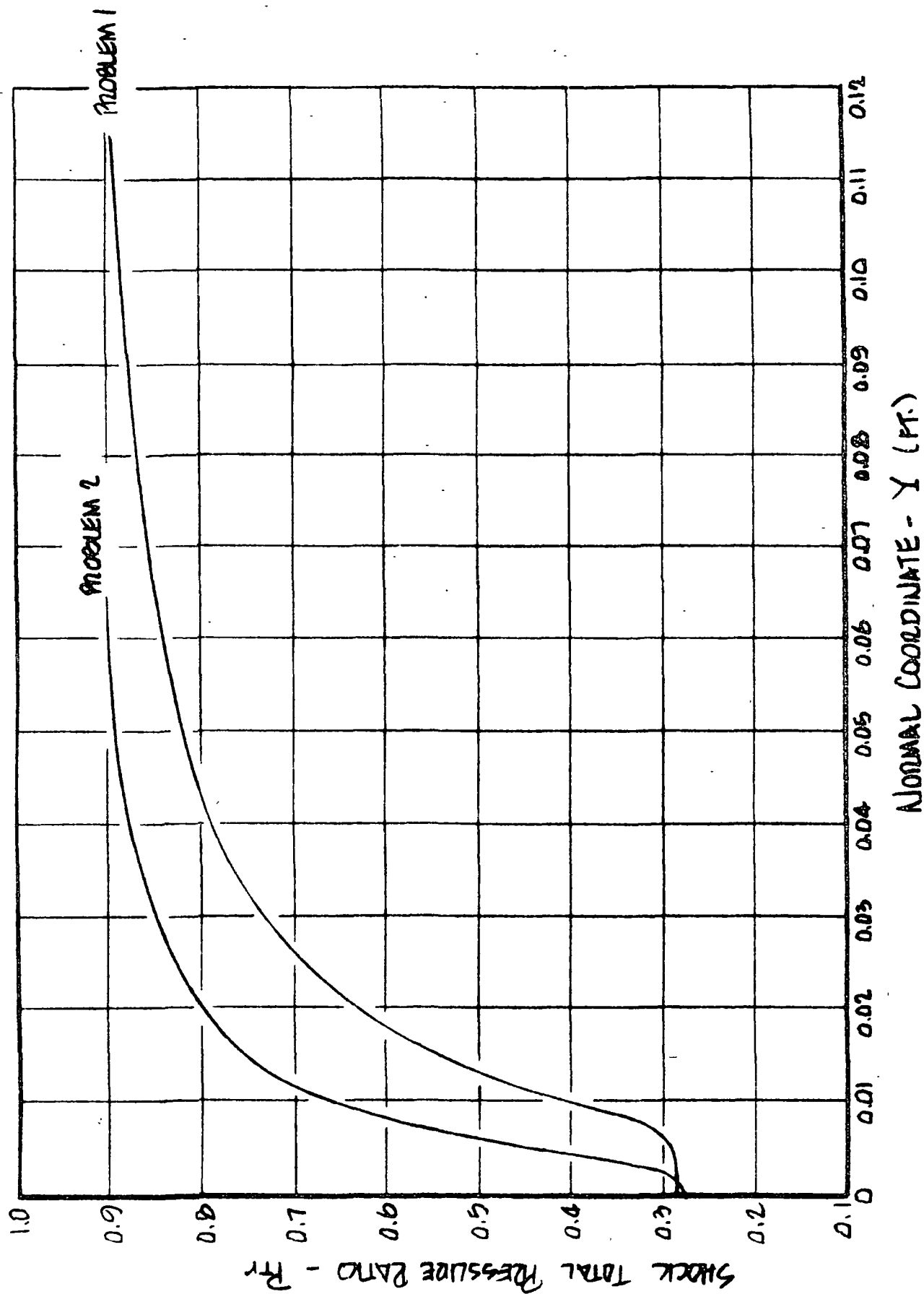


Figure 3. Presumed Shock Total Pressure Profile (Based on Inviscid Solution) for Problems 1 and 2

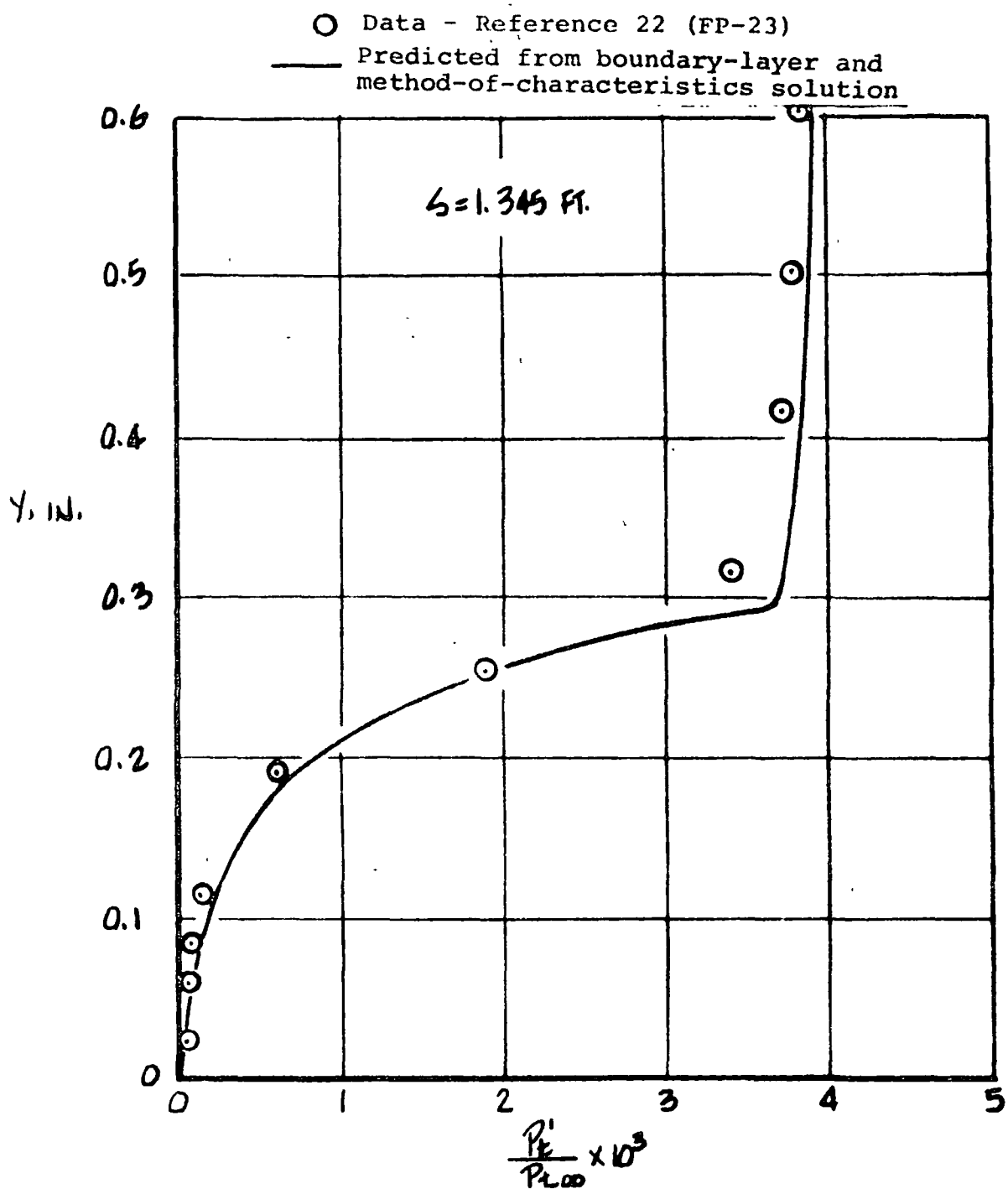


Figure 4. Comparison of Measured and Predicted Pitot Pressure at Probe Station a; Laminar Boundary Layer

as verification of the small entropy gradients in the experimental flow field.

Problem 1. Sample Output

Stations

- a. $s = 1.345$ ft.
- b. $s = 2.18$ ft.
- c. $s = 2.945$ ft.

Reference Stagnation Conditions

$$\begin{aligned}P_{T1} &= 12.5 \text{ atm.} \\H_{T1} &= 331.3 \text{ Btu/lb} \\T_{T1} &= 1851^\circ\text{R} \\S_{i1} &= - 0.4896 \text{ Btu/lb-}^\circ\text{R}\end{aligned}$$

b. A Flat Plate with Sharp Leading Edge - Turbulent Flow

The physical situation is that of the first problem with the exception of an increase in free stream total pressure from 12.5 to 35.13 atmospheres. Transition from laminar to turbulent flow occurs approximately at x/L of 0.5 ($s \approx 2$ ft.). About a factor of 2 increase in velocity overshoot (above reference) can be noted at the last station compared to case 1.

Figure 5 presents a comparison of the experimental and theoretical turbulent profiles near the end of the plate. The agreement displayed is surprisingly good considering the nature of the turbulent model employed and the fact that the empirical constants employed were obtained from operating regimes far removed from those of this example.

Problem 2. Sample Output

Stations

- a. $s = 1.345$ ft.
- b. $s = 2.18$ ft.
- c. $s = 2.945$ ft.

Reference Stagnation Conditions

$$\begin{aligned}P_{T1} &= 35.13 \text{ atm.} \\H_{T1} &= 354.3 \text{ Btu/lb} \\T_{T1} &= 1937^\circ\text{R} \\S_{i1} &= - 0.5483 \text{ Btu/lb-}^\circ\text{R}\end{aligned}$$

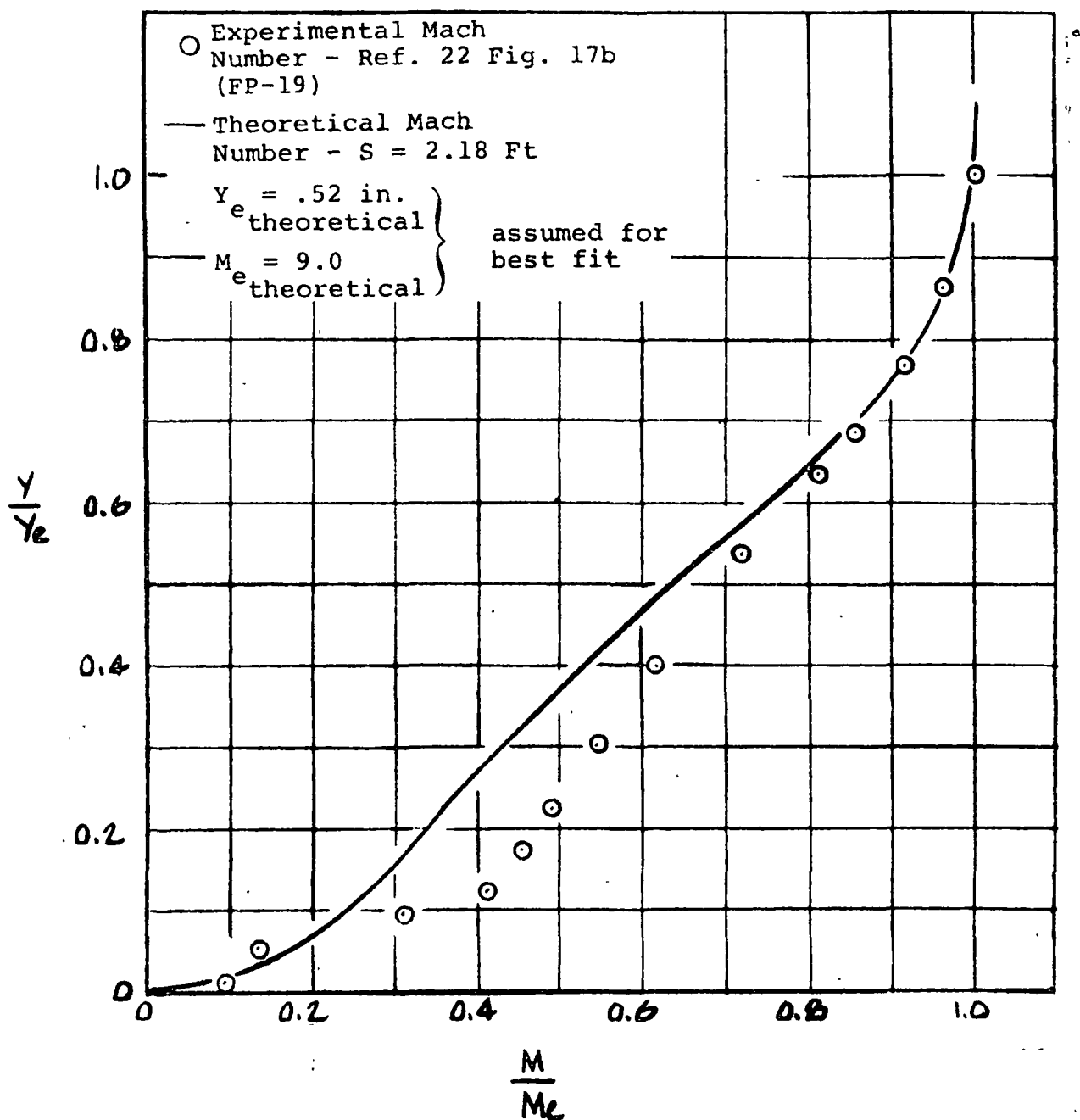


Figure 5. Comparison of Experimental and Predicted Mach Number Profiles; Turbulent Flow

ITERATED VALUES DAMP MAX,MIN MAX,ERRORS IN CONSERVATION EQS.

ITS TIME ALPH FPPM ERRUM MOMENTUM ENERGY
 1 .265 1.114 .5747 .4999 2.00 14 -7.8-04 10 -4.9-01
 2 .528 1.115 .5741.0000 8.04 14 -3.9-04 10 -2.5-01
 3 .787 1.115 .57431.0000 1.05 14 -2.1-06 11 1.3-03

ALPHA XI HOKAP PRESSURE REF. BETA FLUX NOR- HEAT FLUXES
 (LB /SEC**2) (FT) (ATM) (FT/SEC) MALIZING DIFFUSIONAL TOT ENTH RERAD
 1.11500 1.30505 1.00000 1.73103 4.60703 1.30003 1.78203 2.43301 2.23301 0.000 2.23301

WALL MASS FLUXES
 SHEAR INJECTED
 (LB/50 FT) (LB/50 SEC SU FT)
 9.76202 0.000

MOM TRANS HEAT TRANS
 COEFF, COEFF,
 (BASED ON CH) (BASED ON CH)
 6.81804 6.71204 0.000 0.000

MOMENTUM DISPLAC. EFFECTIVE ENTHALPY REYNOLDS
 THICKNESS, THICKNESS, BODY THICKNESS, NUMBER
 THETA DELSTAR DISPLAC. LAMUDA PER FOOT
 (FT) (FT) (FT) (REF.)
 9.06704 1.80402 1.80602 4.26604 4.83305

NODAL INFORMATION

DISTANCE, FROM WALL (FT)	ETA	F	FP (U/UK)	FPP	SHEAR (LB/FTSQ)	TOTAL ENTH- MALPY, G (BTU/LB)	UP (BTU/LB)	GPP (BTU/LB)	STATIC ENTHALPY (BTU/LB)	TEMP (DEG R)	ELECTRON COLL FREQ (1/SEC)
0.000	0.000	0.000	0.000	5.74301	9.76202	-1.37000	1.31702	1.26802	-1.37000	5.37002	5.82808
2.34904	2.67502	2.06004	1.54302	5.79001	9.75802	2.19700	1.35102	1.26702	2.09600	5.51402	5.75108
3.94904	4.45802	5.73404	2.57802	5.82001	9.75602	4.62600	1.37302	1.26302	4.36400	5.60802	5.70308
7.22804	8.02502	1.86402	4.66402	5.87601	9.75102	9.60400	1.41802	1.25302	8.68300	5.78802	5.61308
1.23303	1.33801	5.20403	7.82702	5.95101	9.74102	1.73701	1.48502	1.22402	1.47701	6.04102	5.49408
2.13003	2.22401	1.45602	1.31801	6.05001	9.71602	3.11001	1.59402	1.14102	2.37401	6.41202	5.33308
3.56303	3.56701	3.76202	2.13201	6.17101	9.64402	5.34501	1.74702	9.30201	3.41801	6.84202	5.16308
5.58403	5.35001	8.53802	3.22301	6.04701	9.52002	8.60901	1.91302	2.89401	4.20601	7.16602	5.04508
9.70303	8.91701	2.37801	5.28901	5.40201	8.54302	1.56202	2.01602	-1.09402	3.75701	6.98102	5.11108
1.81602	1.56000	6.96201	8.14701	3.05401	5.37802	2.66502	1.28502	-1.38402	-1.48001	4.80902	6.15808
2.02002	2.22900	1.29400	4.50501	1.00601	2.09602	3.21502	3.59101	-7.45001	-6.14101	2.84202	8.01008
2.44102	3.56700	2.61400	1.00100	-1.54104	-3.94605	3.31302	0.000	2.08101	-9.29701	1.49102	1.10609

DISTANCE FROM WALL (FT)	DENSITY, RHO (LB/50 FT)	VISCOSITY, MU (LB/SEC FT)	RHO*MU	SPECIFIC HEAT (BTU/LB R)	THERMAL COND (BTU /SEC FT R)	PRANDTL NUMBER	SCHMIDT NUMBER	MODIFIED MOLECULAR WEIGHT	RHOSQEPS /RHOR*MU	MACH NUMBER
0.000	1.28004	1.02805	6.66301	2.40101	3.52706	7.00001	7.00001	2.90001	0.000	0.000
2.34904	1.24704	1.04705	6.60701	2.40301	3.59506	7.00001	7.00001	2.90001	0.000	6.17902
3.94904	1.22604	1.05905	6.57101	2.40501	3.64006	7.00001	7.00001	2.90001	0.000	1.02401
7.22804	1.18804	1.08205	6.50501	2.40901	3.72406	7.00001	7.00001	2.90001	0.000	1.82401
1.23303	1.13804	1.11405	6.41701	2.41401	3.84206	7.00001	7.00001	2.90001	0.000	2.99801
2.13003	1.07204	1.16005	6.29601	2.42101	4.01306	7.00001	7.00001	2.90001	0.000	4.90301
3.56303	1.00504	1.21305	6.16601	2.43001	4.20906	7.00001	7.00001	2.90001	0.000	7.68501
5.58403	9.59405	1.25105	6.07501	2.43701	4.35606	7.00001	7.00001	2.90001	0.000	1.13500
9.70303	9.84705	1.22905	6.12601	2.44301	4.27306	7.00001	7.00001	2.90001	0.000	1.88800

Problem 1 - Sample Output

[illegible]

DISTANCE FROM WALL (FT)	STREAM FUNCTION (ACTUAL)	ENTROPY S (BTU/LB R)	ρ/ρ_{ref}	μ/μ_{ref}	γ/γ_{ref}
0.000	-0.000	-1.910-01	4.069-05	4.047-05	1.394+00
2.364-04	1.053-06	-1.866-01	4.060-05	4.040-05	1.394+00
3.949-04	2.930-06	-1.806-01	4.099-05	4.049-05	1.394+00
7.226-04	9.525-06	-1.729-01	4.104-05	4.144-05	1.397+00
1.233-03	2.659-05	-1.626-01	4.330-05	4.100-05	1.396+00
2.130-03	7.440-05	-1.442-01	4.792-05	4.702-05	1.395+00
3.564-03	1.922-04	-1.425-01	5.004-05	5.004-05	1.393+00
5.588-03	4.363-04	-1.212-01	9.055-05	9.042-05	1.391+00
9.703-03	1.215-03	-1.276-01	2.661-04	2.064-04	1.392+00
1.616-02	3.557-02	-2.174-01	3.193-03	6.895-04	1.402+00
2.020-02	6.612-03	-3.419-01	3.145-02	1.659-03	1.412+00
2.441-02	1.336-02	-4.925-01	3.062-01	3.679-03	1.418+00

ITERATED VALUES DAMP MAXIMUM MAXIMUM IN CONSERVATION EOS.

ITS	TIME	ALPHA	FPP#	ERROR	MOMENTUM	ENERGY	BETA	FLUX NOM- MALIZING	DIFFUSIONAL	TOT ENTH	REAR	QCOND
1	.265	1.110	.5746	4.000	3.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	.520	1.118	.5751	0.000	2.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	.787	1.119	.5751	0.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	1.046	1.119	.5751	0.000	2.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

ALPHA	XI	WOKAP	PRESSURE	REF.	BETA	FLUX NOM- MALIZING	DIFFUSIONAL	TOT ENTH	REAR	QCOND
1.119-00	2.057-05	1.000-00	1.000-00	1.000-00	1.000-00	1.000-00	1.000-00	1.000-00	1.000-00	1.000-00

WALL	SHEAR	MASS FLOUES	INJECTED	BLowing PARAMETERS	COEFF.	COEFF.	COEFF.	COEFF.	COEFF.	COEFF.
(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)
7.611-02	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

MOM TRANS	HEAT TRANS	COEFF.	COEFF.	COEFF.	COEFF.	COEFF.	COEFF.	COEFF.	COEFF.	COEFF.
(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)
5.314-04	5.228-04	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

MOMENTUM	DISPLAC.	EFFECTIVE	ENTHALPY	MEYNOLDS	THICKNESS	THICKNESS	THICKNESS	THICKNESS	THICKNESS	THICKNESS
(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)
1.151-03	2.300-02	2.312-02	1.185-03	4.776-05	0.000	0.000	0.000	0.000	0.000	0.000

MODAL INFORMATION	ETA	F	FP	FPP	SHEAR	TOTAL ENTH- MALPY	UP	GPP	STATIC	TEMP	ELECTRON
(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)	(FT)
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

DISTANCE	DENSITY	VISCOSITY	MU/MU	SPECIFIC	THERMAL	PRANDTL	MODIFIED	MOLECULAR	MU/MU	MACH
(FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)	(LB/50 FT)
0.000	1.250-04	1.028-05	0.648-01	2.401-01	3.527-06	7.000-01	7.000-01	2.900-01	0.000	0.000

1.25--02	4.030-05	1.220-05	6.116-01	2.411-01	7.000-01	7.000-01	2.400-01	0.000	1.899-00
2.00--02	1.409-04	9.470-06	6.500-01	2.140-01	7.000-01	7.000-01	2.400-01	0.000	3.520-00
2.50--02	2.430-04	6.543-04	6.220-01	2.340-01	7.000-01	7.000-01	2.400-01	0.000	5.370-00
3.110-02	4.425-04	4.104-05	1.024-00	2.311-01	7.000-01	7.000-01	2.400-01	0.000	7.945-00
DISTANCE	STREAM	ENTHALPY	PI/PTIME	PI/PILOT	GAMMA				
FROM WALL	FUNCTION	S							
(FT)	(ACTUAL)	(BTU/LB R)							
0.000	-0.000	-1.443-01	3.972-05	3.472-05	1.344-00				
3.031-04	1.335-06	-1.524-01	3.983-05	3.443-05	1.343-00				
5.097-04	3.714-06	-1.708-01	4.001-05	4.001-05	1.348-00				
9.330-04	1.207-05	-1.712-01	4.006-05	4.006-05	1.347-00				
1.542-03	3.370-05	-1.509-01	4.230-05	4.230-05	1.345-00				
2.750-03	4.431-05	-1.454-01	4.607-05	4.607-05	1.345-00				
4.000-03	2.437-04	-1.307-01	5.005-05	5.005-05	1.343-00				
7.213-03	5.529-04	-1.195-01	8.911-05	8.911-05	1.341-00				
1.252-02	1.539-03	-1.261-01	2.063-04	2.063-04	1.342-00				
2.042-02	4.502-03	-2.140-01	3.271-03	3.271-03	1.402-00				
2.594-02	8.463-03	-3.470-01	3.396-02	3.396-02	1.412-00				
3.114-02	1.067-02	-5.071-01	3.704-01	3.704-01	1.414-00				

Problem 1 - Sample Output (continued)

ITERATED VALUES DAMP MAXIMUM MAXIMUMS IN CALCULATION EUN.

ITS	TIME	ALPH	FRPM	ESPM	MOMENTUM	ENERGY
1	264	1.126	2.733	4.000	1.4E-03	1.1E-01
2	531	1.126	2.733	4.000	1.4E-03	1.1E-01
3	794	1.126	2.733	4.000	1.4E-03	1.1E-01

ALPHA	AL	HOKAP	PRESSURE	REF.	VELOCITY	BETA	FLUX MOR-	HEAT FLUXES	CLONO
(LB/50 FT)	(FT)	(ATM)	(ATM)	(FT/SEC)	(FT/SEC)		PARAME-	TOT ENTH	
1.126E-00	2.733E-05	1.000E-00	1.000E-00	4.000E-03	1.500E-03	1.100E-03	1.483E-01	1.483E-01	1.483E-01

WALL
SHEAR
(LB/50 FT)
6.499E-02

MASS FLUXES
INJECTED
(LB/SEC SQ FT)
0.000

MOM TRANS HEAT TRANS
COEFF. COEFF.
RHO*U*CF/2 RHO*U*CF/2
4.537E-04 4.537E-04

BLOWING PARAMETERS
(BASED ON CM)
0.000

MOMENTUM DISPLAC. EFFECTIVE ENTHALPY REYNOLDS
THICKNESS, THICKNESS, BODY THICKNESS, NUMBER
THETA DELTA DISPLAC. LAMBDA PER FOOT
(FT) (FT) (FT) (REF.)
1.341E-03 2.679E-02 2.703E-02 1.366E-03 4.734E-03

NODAL INFORMATION

DISTANCE, FROM WALL (FT)	ETA	F	FP	FPP	SHEAR (LB/FT SQ)	TOTAL ENTH- (BTU/LB)	GP (BTU/LB)	GMP (BTU/LB)	STATIC ENTHALPY (BTU/LB)	TEMP (DEG R)	ELECTRON COLL FREQ (1/SEC)
0.000	0.000	0.000	0.000	5.74E-01	6.499E-02	-1.370E-00	1.139E-02	1.277E-02	-1.370E-00	5.370E-02	5.588E-08
3.581E-04	2.733E-02	2.112E-04	1.565E-02	5.812E-01	6.497E-02	2.242E-00	1.354E-02	1.275E-02	2.137E-00	5.516E-02	5.513E-08
6.020E-04	4.505E-02	5.677E-04	2.615E-02	5.842E-01	6.495E-02	4.702E-00	1.377E-02	1.271E-02	4.411E-00	5.611E-02	5.467E-08
1.102E-03	8.109E-02	1.911E-03	4.730E-02	5.869E-01	6.491E-02	9.747E-00	1.423E-02	1.261E-02	8.795E-00	5.793E-02	5.360E-08
1.881E-03	1.352E-01	5.334E-03	7.939E-02	5.973E-01	6.488E-02	1.762E-01	1.491E-02	1.230E-02	1.494E-01	6.048E-02	5.265E-08
3.249E-03	2.253E-01	1.493E-02	1.337E-01	6.071E-01	6.465E-02	3.125E-01	1.602E-02	1.143E-02	2.397E-01	6.421E-02	5.110E-08
5.436E-03	3.604E-01	3.856E-02	2.162E-01	6.150E-01	6.414E-02	5.424E-01	1.756E-02	9.233E-01	3.440E-01	6.851E-02	4.947E-08
8.521E-03	5.400E-01	8.749E-02	3.267E-01	6.108E-01	6.279E-02	8.739E-01	1.922E-02	2.611E-01	4.210E-01	7.167E-02	4.837E-08
1.479E-02	9.010E-01	2.435E-01	5.354E-01	5.473E-01	5.682E-02	1.584E-02	2.010E-02	-1.129E-02	3.678E-01	6.949E-02	4.912E-08
2.451E-02	1.577E-00	7.114E-01	8.213E-01	2.990E-01	3.516E-02	2.689E-02	1.253E-02	-1.369E-02	1.730E-01	4.705E-02	5.970E-08
3.047E-02	2.253E-00	1.319E-00	9.547E-01	4.576E-02	1.343E-02	3.223E-02	3.284E-01	-6.763E-01	-6.437E-01	2.716E-02	7.857E-08
3.643E-02	3.664E-00	2.658E-00	1.005E-00	3.560E-03	6.303E-04	3.313E-02	0.000	1.903E-01	-9.715E-01	1.311E-02	1.131E-09

DISTANCE FROM WALL (FT)	DENSITY, RHO (LB/50 FT)	VISCOSITY, MU (LB/SEC FT)	RHO*U	SPECIFIC HEAT, C (BTU/LB R)	THERMAL COND (BTU/SEC FT R)	PRANDTL NUMBER	SCHMIDT NUMBER	MODIFIED MOLECULAR WEIGHT	RHOSPEPS /RHOMUR	MACH NUMBER
0.000	1.226E-04	1.028E-05	6.637E-01	2.401E-01	3.527E-06	7.000E-01	7.000E-01	2.900E-01	0.000	0.000
3.581E-04	1.192E-04	1.047E-05	6.240E-01	2.404E-01	3.598E-06	7.000E-01	7.000E-01	2.900E-01	0.000	6.269E-02
6.020E-04	1.175E-04	1.060E-05	6.544E-01	2.405E-01	3.641E-06	7.000E-01	7.000E-01	2.900E-01	0.000	1.039E-01
1.102E-03	1.138E-04	1.083E-05	6.478E-01	2.409E-01	3.727E-06	7.000E-01	7.000E-01	2.900E-01	0.000	1.850E-01
1.881E-03	1.098E-04	1.115E-05	6.389E-01	2.414E-01	3.845E-06	7.000E-01	7.000E-01	2.900E-01	0.000	3.041E-01
3.249E-03	1.027E-04	1.161E-05	6.268E-01	2.422E-01	4.018E-06	7.000E-01	7.000E-01	2.900E-01	0.000	4.971E-01
5.436E-03	9.622E-05	1.214E-05	6.139E-01	2.430E-01	4.214E-06	7.000E-01	7.000E-01	2.900E-01	0.000	7.792E-01
8.521E-03	9.197E-05	1.252E-05	6.051E-01	2.437E-01	4.356E-06	7.000E-01	7.000E-01	2.900E-01	0.000	1.152E-00
1.479E-02	9.488E-05	1.325E-05	6.111E-01	2.432E-01	4.258E-06	7.000E-01	7.000E-01	2.900E-01	0.000	1.916E-00

Problem 1 - Sample Output (continued)

2.451-02 1.401-04 9.394-06 0.924-01 2.367-01 3.203-05 7.000-01 2.900-01 0.000 3.559-00
 3.047-02 2.427-04 6.470-06 0.254-01 2.337-01 2.170-06 7.000-01 2.900-01 0.000 5.426-00
 3.643-02 3.027-04 3.943-06 1.042-00 2.319-01 1.307-06 7.000-01 2.900-01 0.000 8.199-00

DISTANCE FROM WALL (FT)	STREAM FUNCTION (ACTUAL)	ENTROPY S (BTU/LB R)	PT/PT,INF	PI(PILOT) PI,INF	GAMMA
0.000	-0.000	-1.881-01	3.901-05	3.901-05	1.399-00
3.581-04	1.561-06	-1.816-01	3.912-05	3.912-05	1.399-00
6.020-04	4.345-06	-1.776-01	3.931-05	3.931-05	1.398-00
1.102-03	1.413-05	-1.699-01	3.946-05	3.946-05	1.397-00
1.881-03	3.943-05	-1.595-01	4.159-05	4.159-05	1.396-00
3.249-03	1.103-04	-1.450-01	4.617-05	4.617-05	1.395-00
5.426-03	2.851-04	-1.293-01	5.817-05	5.817-05	1.393-00
8.521-03	6.468-04	-1.143-01	8.861-05	8.861-05	1.391-00
1.979-02	1.800-03	-1.258-01	2.686-04	2.078-04	1.392-00
2.451-02	5.259-03	-2.196-01	3.377-03	6.871-04	1.403-00
3.047-02	9.753-03	-3.497-01	3.547-02	1.647-03	1.412-00
3.643-02	1.965-02	-5.195-01	4.541-01	4.043-03	1.419-00

Problem 1 - Sample Output (concluded)

ITERATED VALUES DAMP MAX. LIN. MAX. ERRORS IN CONSERVATION FNS.

ITS	TIME	ALPH	FPW	ERROR	MOMENTUM	ERROR
1	.315	1.150	.5660	.4999	1.000	1.000
2	.625	1.151	.56691	.0000	1.000	1.000
3	.933	1.151	.56691	.0000	2.000	2.000

ALPHA	KL	MORAP	PRESSURE	REF.	BETA	FLUX NORM	HEAT FLUXES	UCUND
1.15100	3.37405	1.00000	4.50703	4.72403	5.72504	PARAMETER	(BTU/SEC SQ FT)	3.689-01
								0.000

WALL	SHEAR	INJECTED	U	0.000
(LB/50 FT)	(LB/SEC SQ FT)			
1.643-01				

MOM TRANS-HEAT TRANS	COEFF.	COEFF.	CH
RMOR=URALF/2	PHUR=URACH		
1.120-03	1.107-03		0.000

MOMENTUM	DISPLAC.	EFFECTIVE	ENTHALPY	REYNOLDS
THICKNESS	THICKNESS	BODY	THICKNESS	NUMBER
THETA	DELTA	DISPLAC.	LAMDA	PER FOOT
(FT)	(FT)	(FT)	(REF.)	(REF.)
5.271-04	1.046-02	1.114-02	5.641-04	1.236-05

MODAL INFORMATION

DISTANCE	ETA	F	FP	FPW	SHEAR	TOTAL ENTHALPY	UP	GPP	STATIC	TEMP	ELECTRON
FROM WALL	(FT)	(=U/UM)	(=U/UM)	(BTU/LB R)	(LB/FTSQ)	(BTU/LB)	(BTU/LH)	(BTU/LB)	(BTU/LB)	(DEG R)	(1/SEC)
0.000	0.000	0.000	0.000	5.664-01	1.645-01	2.954-00	1.378-02	1.314-02	2.954-00	5.550-02	1.512-09
1.491-04	2.762-02	2.168-04	1.572-02	5.718-01	1.645-01	6.809-00	1.414-02	1.312-02	6.809-00	5.708-02	1.491-09
2.507-04	4.603-02	6.035-04	2.628-02	5.750-01	1.644-01	9.434-00	1.438-02	1.309-02	9.434-00	5.806-02	1.478-09
4.592-04	8.266-02	1.463-03	4.756-02	5.804-01	1.644-01	1.482-01	1.486-02	1.298-02	1.481-01	8.001-02	1.454-09
7.844-04	1.381-01	5.480-03	7.987-02	5.888-01	1.643-01	2.323-01	1.558-02	1.268-02	2.038-01	6.273-02	1.422-09
1.350-03	2.302-01	1.534-02	1.346-01	5.993-01	1.640-01	3.611-01	1.675-02	1.178-02	3.004-01	6.671-02	1.379-09
2.272-03	3.683-01	3.967-02	2.179-01	6.041-01	1.628-01	6.236-01	1.837-02	9.487-01	4.119-01	7.130-02	1.334-09
3.565-03	5.524-01	9.009-02	3.246-01	6.048-01	1.596-01	9.779-01	2.012-02	2.472-01	4.918-01	7.466-02	1.304-09
6.191-03	9.205-01	2.510-01	5.406-01	5.411-01	1.443-01	1.736-02	2.103-02	-1.205-02	4.334-01	7.218-02	1.326-09
1.023-02	1.611-00	7.332-01	8.271-01	2.884-01	8.777-02	2.400-02	1.669-02	-1.384-02	-1.488-01	4.806-02	1.625-09
1.266-02	2.302-00	1.357-00	9.574-01	8.860-02	3.226-02	3.444-02	3.093-01	-3.040-01	-6.404-01	2.731-02	2.156-09
1.507-02	3.683-00	2.732-00	1.010-00	-1.266-02	-5.689-03	3.582-02	-1.103-01	1.585-01	-9.639-01	1.344-02	3.072-09
1.767-02	5.754-00	4.817-00	1.003-00	9.777-04	4.558-04	3.543-02	0.000	-5.199-00	-9.832-01	1.261-02	3.172-09

DISTANCE	DENSITY	VISCOSITY	RHO MU	SPECIFIC	THERMAL	PRANDTL	MODIFIED	MOLECULAR	KHUSOEPS	MACH
FROM WALL	(LB/50 FT)	(LB/SEC FT)	(LB/SEC FT)	(BTU/LB R)	CUND (BTU/SEC FT)	NUMBER	SCHMIDT	WEIGHT	/MHORUM	NUMBER
0.000	3.248-04	1.052-05	6.664-01	2.404-01	3.612-06	7.000-01	7.000-01	2.400-01	0.000	0.000
1.491-04	3.178-04	1.072-05	6.606-01	2.407-01	3.586-06	7.000-01	7.000-01	2.400-01	0.000	6.352-02
2.507-04	3.123-04	1.085-05	6.569-01	2.409-01	3.733-06	7.000-01	7.000-01	2.400-01	0.000	1.053-01
4.592-04	3.022-04	1.109-05	6.503-01	2.413-01	3.824-06	7.000-01	7.000-01	2.400-01	0.000	1.874-01
7.844-04	2.891-04	1.143-05	6.408-01	2.414-01	3.944-06	7.000-01	7.000-01	2.400-01	0.000	3.080-01
1.350-03	2.710-04	1.192-05	6.283-01	2.427-01	4.132-06	7.000-01	7.000-01	2.400-01	0.000	5.035-01
2.272-03	2.544-04	1.247-05	6.151-01	2.436-01	4.339-06	7.000-01	7.000-01	2.400-01	0.000	7.893-01
3.565-03	2.425-04	1.287-05	6.061-01	2.443-01	4.440-06	7.000-01	7.000-01	2.400-01	0.000	1.167-00

6.131-03	6.512-04	1.258-05	6.127-01	6.438-01	4.374-05	7.000-01	7.000-01	2.000-01	0.000	1.946-00
1.023-02	3.776-04	9.536-06	6.479-01	2.349-01	3.255-05	7.000-01	7.000-01	2.000-01	0.000	3.635-00
1.266-02	6.642-04	6.493-06	6.362-01	2.348-01	2.178-05	7.000-01	7.000-01	2.000-01	0.000	5.502-00
1.507-02	1.348-03	4.011-06	1.643-00	2.328-01	1.323-05	7.000-01	7.000-01	2.000-01	0.000	6.341-00
1.767-02	1.438-03	3.840-06	1.071-00	2.318-01	1.272-05	7.000-01	7.000-01	2.000-01	0.000	6.592-00
DISTANCE STREAM ENTALPHY PT/PTINF PTINF GAMMA										
FROM WALL	FUNCTION	S	PT/PTINF	PTINF						
(FT)	(ACTUAL)	(BTU/LB M)								
0.000	-0.000	-2.495-01	3.680-05	3.680-05	1.349-00					
1.491-04	1.781-06	-2.428-01	3.670-05	3.670-05	1.349-00					
2.507-04	4.457-05	-2.385-01	3.704-05	3.704-05	1.347-00					
4.592-04	1.612-05	-2.307-01	3.771-05	3.771-05	1.345-00					
7.844-04	4.502-05	-2.200-01	3.429-05	3.429-05	1.345-00					
1.356-03	1.260-04	-2.051-01	4.372-05	4.372-05	1.343-00					
2.272-03	3.259-04	-1.889-01	5.539-05	5.539-05	1.341-00					
3.555-03	7.400-04	-1.777-01	8.523-05	8.445-05	1.340-00					
6.191-03	2.062-03	-1.859-01	2.657-04	1.978-04	1.341-00					
1.023-02	6.023-03	-2.840-01	3.563-03	6.785-04	1.402-00					
1.266-02	1.115-02	-4.178-01	3.510-02	1.646-03	1.412-00					
1.507-02	2.244-02	-5.430-01	4.640-01	4.013-03	1.419-00					
1.767-02	3.957-02	-5.979-01	5.837-01	4.290-03	1.420-00					

Problem 2 - Sample Output (continued)

TIME .10000+01 SECONDS - - - STREAMWISE DIMENSION .21800+01FEET - - - 27 APR 70 17:55:46
 ITERATED VALUES DAMP MAX.LIN MAX.ERRORS IN CONSERVATION EQS.
 TIME ALPH FPM LAMP MOMENTUM LAMPGY
 1 .315 1.152 .5671.4999 2.-04 15 -2.3-03 10 -2.0-01
 2 .627 1.152 .56761.0000 4.-05 15 -2.6-03 10 -1.0-01
 3 .934 1.152 .56761.0000 4.-07 2 1.2-06 12 3.5-04

Problem 2 - Sample Output (continued)

PRIOR LAMINAR SOLUTION AFTER TRANSITION. TURBULENCE WILL BE INCLUDED AND SOLUTION CONTINUED

[illegible]

Problem 2 - Sample Output (continued)

ITERATED VALUES DAMP MAX. LIN MAX. MOMENTS IN CONSERVATION EQU.

ITS	TIME	ALPHA	FPPW	EMOM	MOMENTUM	ENERGY
1	.379	2.180	2.2802	.3041	4.000	12
2	.739	2.397	2.4028	.5035	1.004	15
3	1.105	2.637	2.5434	.9595	1.004	15
4	1.468	2.665	2.6131	1.0000	2.004	15
5	1.830	2.667	2.6107	1.0000	1.005	7
6	2.192	2.667	2.6107	1.0000	8.007	5
7	2.553	2.667	2.6107	1.0000	5.007	5

ALPHA	AI	POKAP	PRESSURE	REF.	BETA	FLUX NUM-	HEAT FLUXES
	(LB	(FT)	(ATM)	VELOCITY		MALIZING DIFFUSIONAL	TOT ENTH
	/SEC)*2			(FT/SEC)		PARAMETER	(BTU/SEC SQ FT)
2.66700	1.24205	1.00000	4.49703	4.72503	4.22404	1.99603	1.07400
							0.000

WALL	SHEAR	MASS FLUXES
(LB/SQ FT)	(LB/SEC SQ FT)	
5.10000	0.000	

MOM TRANS HEAT TRANS

COEFF. COEFF.

RHOREF/F2 RHOREF/CH

3.47903 3.07103

MOMENTUM DISPLAC. EFFECTIVE ENTHALPY VISCOSITY

THICKNESS, THICKNESS, BODY THICKNESS, NUMBER	THETA DELTATAM	DISPLAC.	LAMDA	PER FOOT
(FT)	(FT)	(FT)	(FT)	(REF.)
1.45603	1.91002	2.49402	1.96703	1.22606

NODAL INFORMATION

DISTANCE, DISTANCE, ETA	F	FP	FPP	SHEAR	TOTAL ENTH-	UP	GPP	STATIC	TEMP	ELECTRON
FROM WALL	(FT)	(=U/UR)		(LB/FTSQ)	HALYU	(BTU/LB)	(BTU/LB)	(BTU/LB)	(DEG R)	(1/SEC)
0.000	0.000	0.000	2.61900	5.10701	2.95000	5.65602	2.03103	2.95400	3.55002	1.48900
5.64904	0.40102	1.64901	2.66000	5.10801	4.35101	6.96602	8.00902	3.07801	6.70202	1.35500
9.84904	1.06701	2.74201	2.27600	5.10801	7.25601	6.63102	2.59503	3.90301	7.04102	1.32800
1.84003	1.92001	4.56602	1.33400	5.10201	1.19702	4.41702	-1.57103	3.79301	6.99502	1.37700
3.07803	1.09401	2.56101	6.62901	5.10501	1.63602	2.40602	-5.58402	2.55101	6.48502	1.46200
4.93503	5.33601	2.40401	3.04801	5.11901	2.02602	1.21402	-1.73902	7.84500	3.75502	1.55800
7.39303	6.53401	4.64901	7.34601	5.14901	2.31902	6.57401	-6.00201	-4.61200	3.08802	1.55800
1.03002	1.29000	7.90601	7.67401	5.17901	2.54502	4.01301	-1.24601	-2.18701	4.51302	1.65100
1.53102	2.13400	1.49200	6.52401	5.05501	2.84202	2.94601	-4.55700	-3.96801	3.76302	1.80800
2.27202	3.33400	4.92100	4.60201	4.21501	3.19402	1.45101	-4.01600	-6.21101	2.81202	2.04200
2.84902	5.33400	4.43300	4.60201	4.22001	3.16702	7.12000	-1.50200	-7.43301	2.24102	2.31700
3.74002	8.53400	7.56800	6.56901	1.36301	3.51902	2.31300	-1.24300	-8.90601	1.66002	2.72300
4.65902	1.33301	1.23901	1.01100	8.24805	3.54302	0.000	3.14601	-1.01202	1.13802	3.28800

DISTANCE	DENSITY	VISCOSITY	RHOREF	SPECIFIC	HEAT	COND	MOLECULAR	PHUSOREPS	MACH
FROM WALL	(LB/CU FT)	(LB/SEC FT)	(MU)	(BTU/LB R)	(BTU/LB R)	(BTU/SEC FT R)	(BTU/LB)	(BTU/LB)	NUMBER
0.000	3.21704	1.05205	6.65501	2.40000	3.61206	7.00001	2.90001	0.000	0.000
5.64904	2.66404	1.19605	6.16701	2.42701	4.14606	7.00001	2.90001	2.87502	6.30801
9.84904	2.53604	1.23705	6.16701	2.44401	4.30000	7.00001	2.90001	1.46601	9.99501
1.84003	2.55204	1.23105	6.17901	2.44301	4.27906	7.00001	2.90001	6.46901	1.56600

3.078-03	2.754-04	1.169-05	6.331-01	2.423-01	4.047-06	7.000-01	7.000-01	2.900+01	1.994+00	2.110+00
4.935-03	3.103-04	1.078-05	6.578-01	2.408-01	3.709-06	7.000-01	7.000-01	2.900+01	4.979+00	2.655+00
7.393-03	3.523-04	9.887-06	6.851-01	2.395-01	3.382-06	7.000-01	7.000-01	2.900+01	1.050+01	3.145+00
1.030-02	3.956-04	9.138-06	7.110-01	2.383-01	3.111-06	7.000-01	7.000-01	2.900+01	1.881+01	3.570+00
1.531-02	4.745-04	8.076-06	7.535-01	2.348-01	2.732-06	7.000-01	7.000-01	2.900+01	2.715+01	4.226+00
2.272-02	6.349-04	6.625-06	8.272-01	2.349-01	2.223-06	7.000-01	7.000-01	2.900+01	4.861+01	5.247+00
2.849-02	7.794-04	5.763-06	8.832-01	2.339-01	1.926-06	7.000-01	7.000-01	2.900+01	7.325+01	8.086+00
3.740-02	1.076-03	4.626-06	9.792-01	2.326-01	1.538-06	7.000-01	7.000-01	2.900+01	6.979+01	7.397+00
4.658-02	1.569-03	3.580-06	1.105+00	2.316-01	1.185-06	7.000-01	7.000-01	2.900+01	0.000	9.072+00

DISTANCE FROM WALL (FT)	STREAM FUNCTION (ACTUAL)	ENTROPY S (BTU/LB R)	PT/PTINF	PT(PITOT) PTNF	GAMMA
0.030	-0.000	-2.484-01	3.623-05	3.623-05	1.399+00
5.549-04	6.490-05	-2.029-01	4.731-05	4.731-05	1.393+00
9.849-04	1.794-04	-1.909-01	6.837-05	6.837-05	1.392+00
1.845-03	5.471-04	-1.925-01	1.463-04	1.327-04	1.392+00
3.078-03	1.316-03	-2.109-01	3.388-04	2.264-04	1.394+00
4.935-03	2.894-03	-2.397-01	8.020-04	3.520-04	1.398+00
7.393-03	5.595-03	-2.702-01	1.705-03	4.946-04	1.401+00
1.030-02	9.515-03	-2.979-01	3.177-03	6.417-04	1.403+00
1.531-02	1.796-02	-3.411-01	7.800-03	9.155-04	1.407+00
2.272-02	3.516-02	-4.098-01	2.857-02	1.448-03	1.412+00
2.849-02	5.335-02	-4.578-01	6.609-02	2.010-03	1.414+00
3.740-02	9.108-02	-5.330-01	2.224-01	3.067-03	1.417+00
4.658-02	1.492-01	-6.207-01	8.139-01	4.741-03	1.420+00

Problem 2 - Sample Output (conc.)

c. A Curved Plate with Blunt Leading Edge - Laminar Flow

A schematic of the physical situation for this sample problem is shown in Figure 6. The principal difference is the fact that the blunt leading edge causes a strong detached shock to form and the curvature of the shock introduces large entropy gradients into the flow behind the shock wave. Figure 7 shows the total pressure ratio across the shock as a function of the vertical coordinate. Comparison with Figure 3 shows the significant increase in total pressure loss and the flat region at $y = 0$ which is characteristic of detached bow shocks. Note also that the boundary layer must achieve a significant height before strong entropy gradients are encountered because of this flat region.

Problem 3. Sample Output

Stations

- a. $s = 0.95$ ft.
- b. $s = 1.086$ ft.
- c. $s = 2.21$ ft.
- d. $s = 3.273$ ft.

Reference Stagnation Conditions

$$\begin{aligned}P_{T1} &= 0.2818 \text{ atm.} \\u_{T1} &= 365.1 \text{ Btu/lb} \\T_{T1} &= 1977^\circ\text{R} \\S_{11} &= - 0.2122 \text{ Btu/lb-}^\circ\text{R}\end{aligned}$$

d. A Curved Plate with a Blunt Leading Edge - Turbulent Flow

This sample problem is the same as the foregoing in all respects with the exception that turbulence is introduced by the use of the generators shown in the Figure 6 schematic (in the computer code, the transition station was specified).

The inclusion of turbulence again causes the influence of the entropy gradient to be greater than for the laminar case.

$M_{\infty} \approx 10.5$

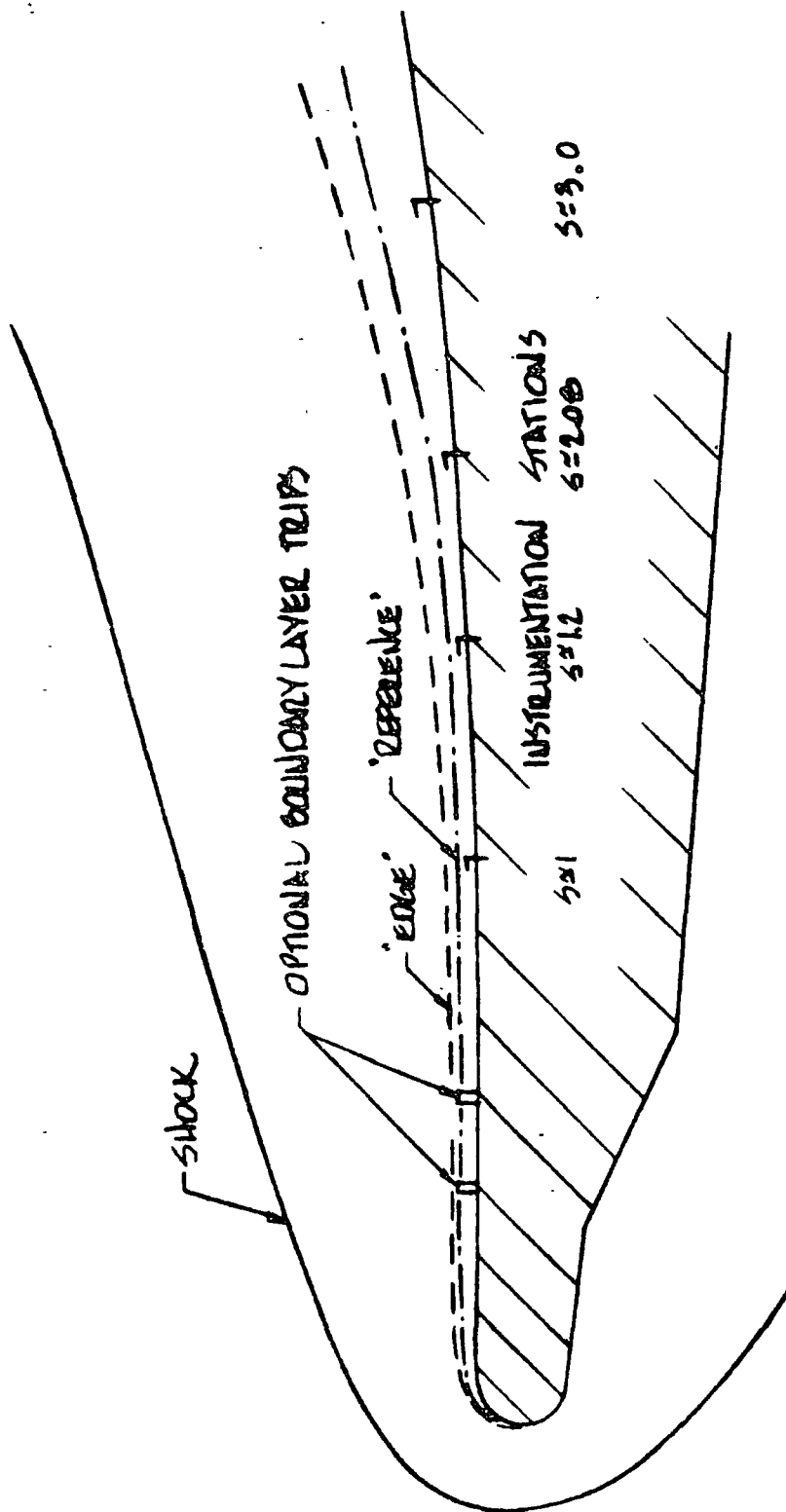


Figure 6. Schematic of the Configuration for Sample Problems 3, 4 and 5

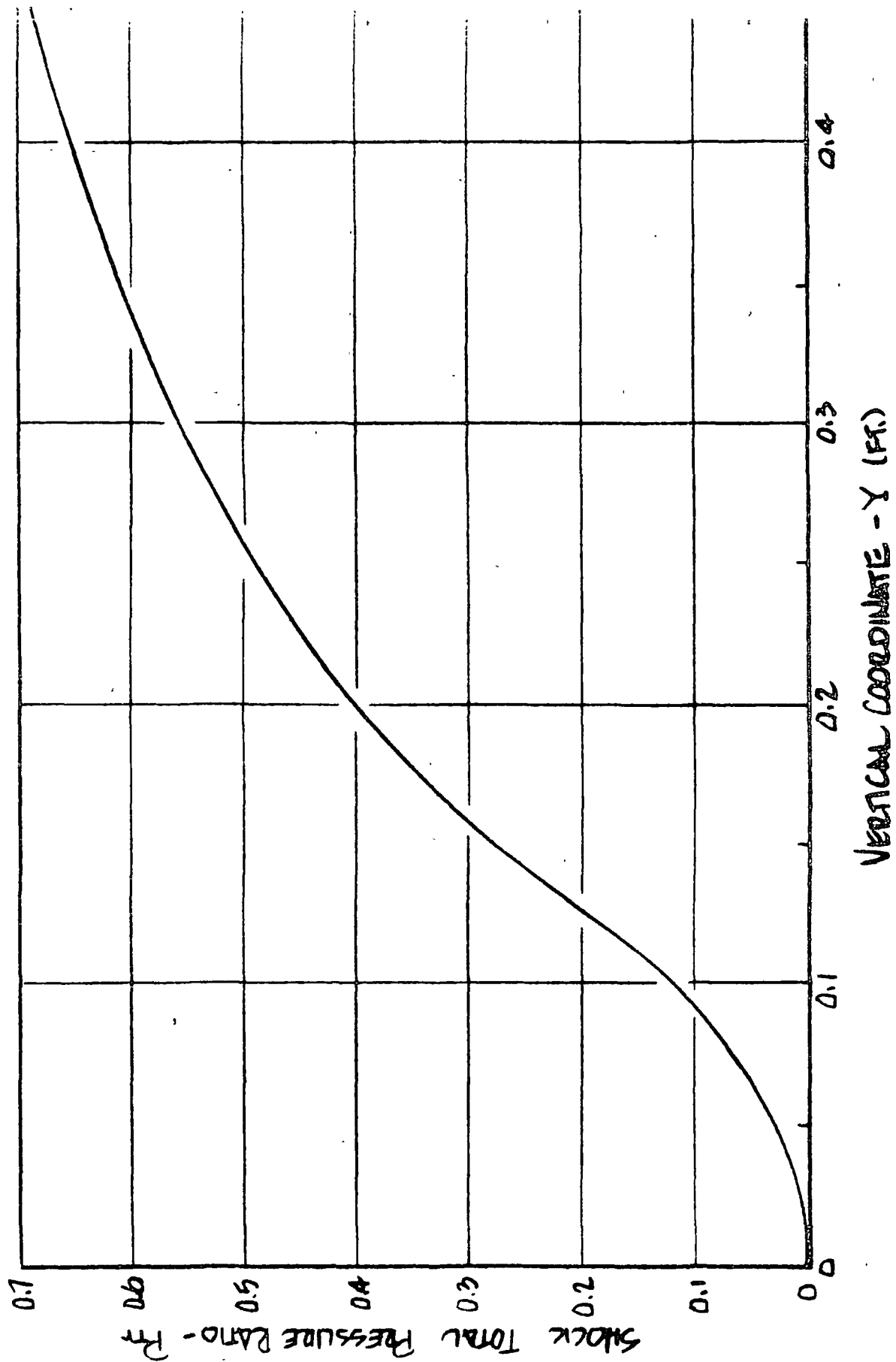


Figure 7. Shock Total Pressure Profile for Problems 3 and 4

TIME .1000+01 SECONDS - - - STREAMWISE DIMENSION .95000-00FEET - - - 30 APR 70 09:40:10

ITERATED VALUES DAMP MAX-LIN MAX-ERRORS IN CONSERVATION EQS.

ITS	TIME	ALPH	FPV	ERROR	MOMENTUM	ENERGY
1	.266	1.310	.5139	4.999	2.03 13	4.0-02 11 -1.4+01
2	.533	1.327	.48581	.0000	1.03 13	2.0-02 11 -6.9+00
3	.796	1.328	.48541	.0000	4.05 12	4.1-04 11 1.1-01
4	1.055	1.328	.48541	.0000	3.06 2	1.8-05 11 5.1-04

ALPHA	XL	ROKAP	PRESSURE	REF.	BETA	FLUX NOR-	HEAT FLUXES	QCOND
	(LB	(FT)	(ATM)	VELOCITY		PARAMETER	TOT ENTH	
	/SEC)	**2		(FT/SEC)			(BTU/SEC SQ FT)	
1.328+00	2.542-05	1.000+00	6.256-03	4.075+03	2.698-02	2.469-03	4.253-01	4.253-01

WALL	MASS FLUXES
SHEAR	INJECTED
(LB/SO FT)	(LB/SEC SQ FT)
1.621-01	0.000

NOM TRANS	HEAT TRANS	BLOWING PARAMETERS
COEFF.	COEFF.	(BASED ON CM)
RHOREURCF/2 RHOREUR*CH		
1.293-03	1.174-03	0.000

MOMENTUM	DISPLAC.	EFFECTIVE ENTHALPY	REYNOLDS
THICKNESS	THICKNESS	BODY THICKNESS	NUMBER
TMETA	DELSTAR	DISPLAC.	LAMBDA
(FT)	(FT)	(FT)	(REF.)
2.167-03	6.259-03	6.432-03	2.855-03
			1.141+05

NODAL INFORMATION

DISTANCE	ETA	F	FP	FPP	SHEAR	TOTAL ENTH-	GP	GPP	STATIC	TEMP	ELECTRON
FROM WALL	FROM WALL	(FT)	(=U/UR)	(U/UR)	(LB/FTSQ)	HALPY/G	(BTU/LB)	(BTU/LB)	(BTU/LB)	(DEG R)	COLL FREQ
(FT)										(1/SEC)	
0.000	0.000	0.000	0.000	4.854-01	1.621-01	2.954+00	1.118+02	7.577+01	2.954+00	5.550+02	2.071+09
1.275-04	3.187-02	2.471-04	1.552-02	4.848-01	1.619-01	6.550+00	1.142+02	7.553+01	6.476+00	5.696+02	2.045+09
2.143-04	5.312-02	6.875-04	2.593-02	4.909-01	1.618-01	8.997+00	1.158+02	7.516+01	8.778+00	5.792+02	2.028+09
3.923-04	9.562-02	2.234-03	4.688-02	4.950-01	1.615-01	1.399+01	1.190+02	7.436+01	1.327+01	5.978+02	1.996+09
6.695-04	1.594-01	6.232-03	7.861-02	5.004-01	1.610-01	2.172+01	1.237+02	7.248+01	1.971+01	6.245+02	1.953+09
1.157-03	2.656-01	1.742-02	1.321-01	5.072-01	1.599-01	3.527+01	1.314+02	6.770+01	2.960+01	6.653+02	1.892+09
1.942-03	4.250-01	4.694-02	2.134-01	5.122-01	1.576-01	5.707+01	1.422+02	5.643+01	4.227+01	7.174+02	1.822+09
3.068-03	6.375-01	1.018-01	3.217-01	5.075-01	1.527-01	8.850+01	1.542+02	2.330+01	5.492+01	7.692+02	1.759+09
5.480-03	1.062+00	2.828-01	5.268-01	4.576-01	1.352-01	1.562+02	1.641+02	-5.391+01	6.598+01	8.143+02	1.710+09
9.998-03	1.859+00	8.276-01	8.151-01	2.661-01	8.022-02	2.698+02	1.211+02	-9.089+01	5.382+01	7.647+02	1.765+09
1.415-02	2.656+00	1.543+00	9.575-01	9.136-02	2.826-02	3.375+02	4.871+01	-5.704+01	3.942+01	7.057+02	1.837+09
2.201-02	4.250+00	3.127+00	1.008+00	5.393+03	1.683+03	3.651+02	0.000	-4.079+00	3.481+01	6.868+02	1.862+09

DISTANCE	DENSITY	VISCOSITY	RHO*MU	SPECIFIC	THERMAL	PRANDTL	MODIFIED	MOLECULAR	RHUSQ*EPS	MACH
FROM WALL	RHO	MU	/RHO*MU*	HEAT	COND	NUMBER	SCHMIDT	WEIGHT	/RHOR*EUR	NUMBER
(FT)	(LB/CU FT)	LB/SEC FT	C	(BTU/LB R)	(BTU		NUMBER			
0.000	4.476-04	1.052-05	1.081+00	2.440+01	3.612+06	7.000-01	7.000-01	2.900+01	0.000	0.000
1.275-04	4.361-04	1.071-05	1.072+00	2.407-01	3.681+06	7.000-01	7.000-01	2.900+01	0.000	5.360-02
2.143-04	4.289-04	1.083-05	1.066+00	2.409-01	3.726+06	7.000-01	7.000-01	2.900+01	0.000	6.881-02
3.923-04	4.155-04	1.106-05	1.056+00	2.413-01	3.813+06	7.000-01	7.000-01	2.900+01	0.000	1.581-01
6.695-04	3.978-04	1.140-05	1.041+00	2.418-01	3.937+06	7.000-01	7.000-01	2.900+01	0.000	2.595-01
1.157-03	1.734-04	1.190-05	1.020+00	2.426-01	4.124+06	7.000-01	7.000-01	2.900+01	0.000	4.228-01
1.942-03	3.463-04	1.252-05	9.958-01	2.477-01	4.360+06	7.000-01	7.000-01	2.900+01	0.000	6.580-01
3.068-03	3.230-04	1.313-05	9.738-01	2.447-01	4.591+06	7.000-01	7.000-01	2.900+01	0.000	9.589-01

5.480-03	3.051-04	1.365-05	9.562-01	2.456-01	4.789-06	7.000-01	7.000-01	2.900-01	0.000	1.527-00
9.998-03	3.248-04	1.308-05	9.757-01	2.446-01	4.571-06	7.000-01	7.000-01	2.900-01	0.000	2.437-00
1.415-02	3.520-04	1.238-05	1.001-00	2.419-01	4.307-06	7.000-01	7.000-01	2.900-01	0.000	2.977-00
2.201-02	3.617-04	1.216-05	1.010-00	2.411-01	4.221-06	7.000-01	7.000-01	2.900-01	0.000	3.176-00
DISTANCE	STREAM	ENTROPY	PT/PTINF	PT(PITOT)	GAMMA					
FROM WALL	FUNCTION	S		PTINF						
(FT)	(ACTUAL)	(BTU/LB R)								
0.000	-0.000	-2.711-01	5.111-05	5.111-05	1.399-00					
1.275-04	1.762-06	-2.648-01	5.121-05	5.121-05	1.398-00					
2.143-04	4.902-06	-2.608-01	5.139-05	5.139-05	1.397-00					
3.923-04	1.593-05	-2.532-01	5.201-05	5.201-05	1.397-00					
6.695-04	4.444-05	-2.426-01	5.355-05	5.355-05	1.395-00					
1.157-03	1.242-04	-2.273-01	5.777-05	5.777-05	1.393-00					
1.942-03	3.205-04	-2.089-01	6.824-05	6.824-05	1.391-00					
3.068-03	7.259-04	-1.919-01	9.198-05	9.198-05	1.389-00					
5.480-03	2.017-03	-1.780-01	1.946-04	1.791-04	1.387-00					
9.998-03	5.901-03	-1.934-01	8.055-04	4.198-04	1.389-00					
1.415-02	1.100-02	-2.130-01	1.890-03	6.241-04	1.392-00					
2.201-02	2.230-02	-2.196-01	2.565-03	7.153-04	1.393-00					

Problem 3 - Sample Output (continued)

TIME .1000+01 SECONDS - - - STREAMWISE DIMENSION .10860+01FEET - - - 30 APR 70 09:40:11

ITERATED VALUES DAMP MAX-LIN MAX-ERRORS IN CONSERVATION EQS.

ITS	TIME	ALPH	FPW	ERROR	MOMENTUM	ENERGY
1	.266	1.329	.4874	.4999	4-.04 13	9.5-03 11 -1.2+01
2	.524	1.331	.48951	.0000	2-.04 13	4.7-03 11 -5.8+00
3	.786	1.331	.48951	.0000	4-.07 2	3.3-05 11 2.3-02

ALPHA	XI	ROKAP	PRESSURE	REF.	BETA	FLUX NOR-	HEAT FLUXES	OCONO
(LB/50 FT)	(LB/SEC)*2	(FT)	(ATM)	VELOCITY	(FT/SEC)	MALIZING DIFFUSIONAL TOT ENTH	PARAMETER	PERAD
(LB/50 FT)	(LB/SEC SQ FT)					(BTU/SEC SQ FT)		
1.331+00	2.779-05	1.000+00	6.101-03	4.042+03	4.353-02	2.308-03	3.998-01	0.000
1.531-01							3.998-01	0.000

WALL
SHEAR
(LB/50 FT)
1.531-01

MASS FLUXES
INJECTED
(LB/SEC SQ FT)
0.000

MOM TRANS HEAT TRANS
COEFF, COEFF,
RHOR*UR*CF/2 RHOR*UR*CH
1.218-03 1.104-03

BLOWING PARAMETERS
(BASED ON CH)
0.000

MOMENTUM DISPLAC. EFFECTIVE ENTHALPY REYNOLDS
THICKNESS, THICKNESS, BODY THICKNESS, NUMBER
THETA DELSTAR DISPLAC. LAMBDA PER FOOT
(FT) (FT) (FT) (REF.)
2.302-03 6.754-03 6.958-03 3.016-03 1.128+05

MODAL INFORMATION

DISTANCE, FROM WALL (FT)	ETA	F	FP	FPF	SHEAR (LB/FTSQ)	TOTAL ENTH- (BTU/LB)	GP	GPP	STATIC ENTHALPY (BTU/LB)	TEMP (DEG R)	ELECTRON COLL FREQ (1/SEC)
0.000	0.000	0.000	0.000	4.895-01	1.531-01	2.954+00	1.124+02	7.703+01	2.954+00	5.550+02	2.020+09
1.367-04	3.194-02	2.501-04	1.568-02	4.925-01	1.528-01	6.584+00	1.149+02	7.668+01	6.503+00	5.698+02	1.994+09
2.299-04	5.323-02	6.958-04	2.619-02	4.945-01	1.525-01	9.047+00	1.165+02	7.617+01	8.823+00	5.794+02	1.977+09
4.207-04	9.581-02	2.260-03	4.732-02	4.941-01	1.521-01	1.408+01	1.198+02	7.512+01	1.335+01	5.982+02	1.946+09
7.182-04	1.597-01	6.302-03	7.928-02	5.027-01	1.514-01	2.188+01	1.246+02	7.281+01	1.983+01	6.250+02	1.904+09
1.242-03	2.661-01	1.760-02	1.331-01	5.043-01	1.500-01	3.556+01	1.323+02	6.743+01	2.977+01	6.650+02	1.844+09
2.084-03	4.258-01	1.535-02	2.145-01	5.118-01	1.474-01	5.754+01	1.431+02	5.562+01	4.252+01	7.185+02	1.775+09
3.294-03	6.387-01	1.026-01	3.229-01	5.040-01	1.425-01	8.927+01	1.549+02	2.228+01	5.524+01	7.706+02	1.714+09
5.884-03	1.065+00	2.844-01	5.278-01	4.542-01	1.261-01	1.573+02	1.644+02	-5.473+01	6.637+01	8.159+02	1.666+09
1.073-02	1.863+00	8.310-01	8.159-01	2.648-01	1.201-02	2.711+02	1.207+02	-9.136+01	5.343+01	7.648+02	1.721+09
1.517-02	2.661+00	1.549+00	9.582-01	9.048-02	2.629-02	3.383+02	4.777+01	-5.667+01	3.871+01	7.028+02	1.795+09
2.354-02	4.258+00	3.137+00	1.009+00	5.620+03	1.647-03	3.651+02	0.000	-3.154+00	3.311+01	6.798+02	1.825+09

DISTANCE FROM WALL (FT)	DENSITY, RHO (LB/CU FT)	VISCOSITY, MU (LB/SEC FT)	RHO*MU (LB/SEC FT)	SPECIFIC HEAT (BTU/LB R)	THERMAL COND (BTU/SEC FT R)	PRANDTL NUMBER	MODIFIED SCHMIDT NUMBER	MOLECULAR WEIGHT	RHOSQEPS /RHOR*MR	MACH NUMBER
0.000	4.365-04	1.052-05	1.079+00	2.404-01	3.612-06	7.000-01	7.000-01	2.900+01	0.000	0.000
1.367-04	4.252-04	1.071-05	1.070+00	2.407-01	3.682-06	7.000-01	7.000-01	2.900+01	0.000	5.424-02
2.299-04	4.181-04	1.083-05	1.064+00	2.409-01	3.727-06	7.000-01	7.000-01	2.900+01	0.000	4.984-02
4.207-04	4.050-04	1.107-05	1.053+00	2.413-01	3.815-06	7.000-01	7.000-01	2.900+01	0.000	1.598-01
7.182-04	3.876-04	1.140-05	1.038+00	2.416-01	3.939-06	7.000-01	7.000-01	2.900+01	0.000	2.621-01
1.242-03	3.637-04	1.191-05	1.017+00	2.426-01	4.127-06	7.000-01	7.000-01	2.900+01	0.000	4.265-01
2.084-03	3.372-04	1.254-05	9.931-01	2.441-01	4.364-06	7.000-01	7.000-01	2.900+01	0.000	6.624-01
3.294-03	3.144-04	1.315-05	9.711-01	2.447-01	4.596-06	7.000-01	7.000-01	2.900+01	0.000	9.635-01
5.884-03	2.969-04	1.367-05	9.535-01	2.450-01	4.796-06	7.000-01	7.000-01	2.900+01	0.000	1.531+00

1.365.00
2.327.00
2.971.00
3.234.00

0.000
0.000
0.000
0.000

2.900+01
2.900+01
2.900+01
2.900+01

7.000-01
7.000-01
7.000-01
7.000-01

7.000-01
7.000-01
7.000-01
7.000-01

5.038-06
4.792-06
4.360-06
4.146-06

2.448-01
2.456-01
2.417-01
2.437-01

9.580-01
9.583-01
9.981-01
1.020+00

1.429-05
1.366-05
1.253-05
1.196-05

2.929-04
3.131-04
3.555-04
3.807-04

8.117-03
1.506-02
2.124-02
3.240-02

DISTANCE FROM WALL (FT) STREAM FUNCTION (ACTUAL) ENTROPY S (BTU/LB R) PT/PTINF PT(PILOT) GAMMA

0.000	-0.000	-2.729-01	5.249-05	5.249-05	1.399+00
1.824-04	2.039-06	-2.665-01	5.256-05	5.256-05	1.398+00
3.068-04	5.684-06	-2.623-01	5.268-05	5.268-05	1.397+00
5.618-04	1.855-05	-2.543-01	5.308-05	5.308-05	1.396+00
9.599-04	5.208-05	-2.431-01	5.410-05	5.410-05	1.395+00
1.664-03	1.472-04	-2.262-01	5.695-05	5.695-05	1.393+00
2.806-03	3.864-04	-2.050-01	6.423-05	6.423-05	1.390+00
4.470-03	8.960-04	-1.838-01	8.117-05	8.117-05	1.387+00
8.117-03	2.601-03	-1.632-01	1.583-04	1.529-04	1.384+00
1.506-02	8.084-03	-1.796-01	6.951-04	3.943-04	1.387+00
2.124-02	1.557-02	-2.107-01	1.924-03	6.403-04	1.391+00
3.240-02	3.227-02	-2.274-01	2.875-03	7.627-04	1.393+00

Problem 3 - Sample Output (continued)

TIME .10000+01 SECONDS - - - STREAMWISE DIMENSION .32730+01FEET - - - 30 APR 70 09:40:15

ITERATED VALUES DAMP MAX.LIN MAX.ERRORS IN CONSERVATION EQS.

ITS	TIME	ALPHA	FPPA	ERROR	MOMENTUM	ENERGY
1	.266	1.519	.2839	.4999	1.7-01 10	-7.0+00
2	.533	1.619	.2074	1.0000	7.0-03 13	9.6-02 10 -3.5+00
3	.792	1.643	.1913	1.0000	1.0-03 13	1.9-02 11 1.2+00
4	1.052	1.647	.1904	1.0000	3.0-04 13	1.9-03 11 6.8-02
5	1.313	1.647	.1904	1.0000	6.0-05 2	2.4-04 11 7.3-03
6	1.575	1.648	.1904	1.0000	1.0-05 2	4.6-05 11 5.5-04

ALPHA	XI	ROKAP	PRESSURE	REF.	BETA	FLUX NOR-	HEAT FLUXES	QCNO
	(LB	(FT)	(ATM)	VELOCITY		PARAMETER	TOT ENTH	
	/SEC)*2			(FT/SEC)			(BTU/SEC SQ FT)	
1.648+00	6.874+05	1.000+00	9.243+03	3.909+03	-2.093+01	2.071+03	3.055+01	0.000

WALL	SHEAR	MASS FLUXES
(LB/SQ FT)	INJECTED	
5.364+02	(LB/SEC SQ FT)	
	0.000	

MON TRANS	HEAT TRANS	BLOWING PARAMETERS
COEFF.	COEFF.	(BASED ON CM)
RHO*UR*CF/2	RHO*UR*CH	
9.415-04	8.436-04	0.000

MOMENTUM	DISPLAC.	EFFECTIVE ENTHALPY	REYNOLDS
THICKNESS, THICKNESS,	BODY	THICKNESS,	NUMBER
THEIA	DELSTAR	DISPLAC.	LAMBDA
(FT)	(FT)	(FT)	(REF.)
2.891+03	1.063+02	1.177+02	3.400+03
			1.358+05

MODAL INFORMATION	ETA
DISTANCE,	FROM WALL
(FT)	
0.000	0.000
1.818+04	3.954+02
3.058+04	6.590+02
5.602+04	1.186+01
9.581+04	1.977+01
1.664+03	3.295+01
2.823+03	5.272+01
4.539+03	7.908+01
8.439+03	1.318+00
1.620+02	2.307+00
2.300+02	3.295+00
3.482+02	5.272+00

TEMP	STATIC	GPP	GP	ENTH-	SHEAR	FPP	FP	FPP	ELECTRON
(DEG R)	ENTHALPY	(BTU/LB)	(BTU/LB)	HALPY.G	(LB/FTSQ)	(BTU/LB)	(=U/UR)		COLL FREQ
5.550+02	(BTU/LB)	(BTU/LB)	(BTU/LB)	(BTU/LB)	(LB/FTSQ)	(BTU/LB)			(1/SEC)
5.702+02	2.954+00	2.723+01	9.223+01	2.954+00	5.364+02	1.904+01	0.000	1.904+01	3.060+09
5.803+02	6.604+00	2.758+01	9.331+01	6.622+00	5.512+02	1.974+01	7.668+03	1.974+01	3.019+09
6.005+02	9.040+00	2.807+01	9.404+01	9.091+00	5.613+02	2.021+01	1.293+02	2.021+01	2.943+09
2.871+09	1.391+01	2.904+01	9.404+01	1.409+01	5.820+02	2.119+01	2.385+02	2.119+01	2.942+09
2.766+09	2.121+01	3.093+01	9.552+01	2.173+01	6.140+02	2.271+01	4.121+02	2.271+01	2.807+02
2.629+09	3.327+01	3.466+01	1.019+02	3.469+01	6.693+02	2.536+01	7.288+02	2.536+01	6.804+02
2.489+09	5.078+01	3.925+01	1.087+02	5.571+01	7.543+02	2.952+01	1.271+01	2.952+01	7.523+02
2.332+09	7.200+01	3.824+01	1.190+02	8.573+01	8.627+02	3.495+01	2.121+01	3.495+01	8.388+02
2.378+09	1.010+02	2.536+01	1.392+02	1.538+02	1.005+01	4.247+01	4.162+01	4.247+01	9.559+02
2.593+09	9.195+01	-8.589+01	1.141+02	2.790+02	7.610+02	7.831+01	7.831+01	3.176+01	9.196+02
7.053+02	5.588+01	-3.580+01	2.923+01	3.499+02	2.134+02	8.425+02	1.732+00	9.617+01	7.731+02
7.715+09	3.932+01	6.224+00	0.000	3.651+02	4.211+03	1.614+02	1.033+00	1.614+02	2.715+09

DISTANCE	DENSITY,	VISCOSITY,	RHO*MU	SPECIFIC	THERMAL	PRANDTL	MODIFIED	MOLECULAR	RHOS*EPS	MACH
FROM WALL	RHO	LB/SEC FT	LB/SEC FT	HEAT	COND	NUMBER	SCHMIDT	WEIGHT	/RHO*MU	NUMBER
(FT)	(LB/CU FT)			(BTU/LB R)	(BTU/SEC FT R)		NUMBER			
0.000	6.613+04	1.052+05	1.120+00	2.404+01	3.612+06	7.000+01	7.000+01	2.900+01	0.000	0.000
1.818+04	6.437+04	1.071+05	1.110+00	2.407+01	3.684+06	7.000+01	7.000+01	2.900+01	0.000	2.564+02
3.058+04	6.325+04	1.084+05	1.104+00	2.409+01	3.731+06	7.000+01	7.000+01	2.900+01	0.000	4.287+02
5.602+04	6.112+04	1.110+05	1.092+00	2.413+01	3.826+06	7.000+01	7.000+01	2.900+01	0.000	7.773+02
9.581+04	5.819+04	1.147+05	1.075+00	2.414+01	3.965+06	7.000+01	7.000+01	2.900+01	0.000	1.311+01
1.664+03	5.394+04	1.208+05	1.049+00	2.429+01	4.193+06	7.000+01	7.000+01	2.900+01	0.000	2.235+01

2.823-03	4.879-04	1.293-05	1.016-00	2.444-01	4.515-06	7.000-01	7.000-01	2.900-01	0.000	3.711-01
4.539-03	4.376-04	1.393-05	9.810-01	2.441-01	4.847-06	7.000-01	7.000-01	2.900-01	0.000	5.872-01
8.439-03	3.840-04	1.522-05	9.408-01	2.544-01	5.403-06	7.000-01	7.000-01	2.900-01	0.000	1.081-00
1.620-02	3.991-04	1.483-05	9.525-01	2.477-01	5.247-06	7.000-01	7.000-01	2.900-01	0.000	2.073-00
2.300-02	4.747-04	1.318-05	1.007-00	2.448-01	4.608-06	7.000-01	7.000-01	2.900-01	0.000	2.828-00
3.482-02	5.204-04	1.238-05	1.037-00	2.434-01	4.305-06	7.000-01	7.000-01	2.900-01	0.000	3.113-00

DISTANCE FROM WALL	STREAM FUNCTION	ENTROPY S	PT/PTINF	PT(P1101)	GAMMA
(FT)	(ACTUAL)	(BTU/LB R)			
0.000	-0.000	-2.978-01	7.551-05	7.551-05	1.399-00
1.818-04	1.767-06	-2.913-01	7.535-05	7.555-05	1.398-00
3.058-04	4.948-06	-2.871-01	7.501-05	7.561-05	1.397-00
5.602-04	1.629-05	-2.788-01	7.584-05	7.584-05	1.396-00
9.581-04	4.636-05	-2.670-01	7.643-05	7.643-05	1.395-00
1.664-03	1.341-04	-2.486-01	7.818-05	7.818-05	1.393-00
2.823-03	3.643-04	-2.241-01	8.300-05	8.300-05	1.390-00
4.539-03	8.849-04	-1.974-01	9.517-05	9.517-05	1.386-00
8.439-03	2.807-03	-1.651-01	1.565-04	1.564-04	1.381-00
1.620-02	9.859-03	-1.747-01	6.601-04	4.549-04	1.382-00
2.300-02	2.031-02	-2.174-01	2.219-03	8.335-04	1.389-00
3.482-02	4.393-02	-2.398-01	3.449-03	1.014-03	1.392-00

Problem 3 - Sample Output (concluded)

Problem 4. Sample Output

Stations

- a. $s = 0.95$ ft.
- b. $s = 1.086$ ft.
- c. $s = 2.21$ ft.
- d. $s = 3.273$ ft.

Reference Stagnation Conditions

$$\begin{aligned}P_{T1} &= 0.2818 \text{ atm.} \\H_{T1} &= 341.8 \text{ Btu/lb} \\T_{T1} &= 1890^\circ\text{R} \\S_{i1} &= - 0.2242 \text{ Btu/lb-}^\circ\text{R}\end{aligned}$$

e. A Curved Plate with a Relatively Sharp Edge (Detached Shock)

In this problem, the configuration is different than in problems 3 and 4 because of a factor of 3 reduction in nose radius. This causes the shock curvature and hence entropy gradient to be closer to the body than in the previous cases. As would be expected, then, entropy layer effects are more pronounced.

Figure 8 presents a demonstration of the coupling between the viscous and inviscid flow fields. Velocity and density of both flow fields are presented as a function of the normal coordinate. Note that both the absolute values of velocity and density as well as the gradients of these quantities are matched at the boundary layer "edge" (i.e., the outermost boundary layer solution station).

Problem 5. Sample Output

Stations

- a. $s = 0.945$ ft.
- b. $s = 1.07$ ft.
- c. $s = 2.19$ ft.
- d. $s = 3.26$ ft.

Reference Stagnation Conditions

$$\begin{aligned}P_{T1} &= 0.2818 \text{ atm.} \\H_{T1} &= 365.1 \text{ Btu/lb} \\T_{T1} &= 1977^\circ\text{R} \\S_{i1} &= - 0.2122 \text{ Btu/lb-}^\circ\text{R}\end{aligned}$$

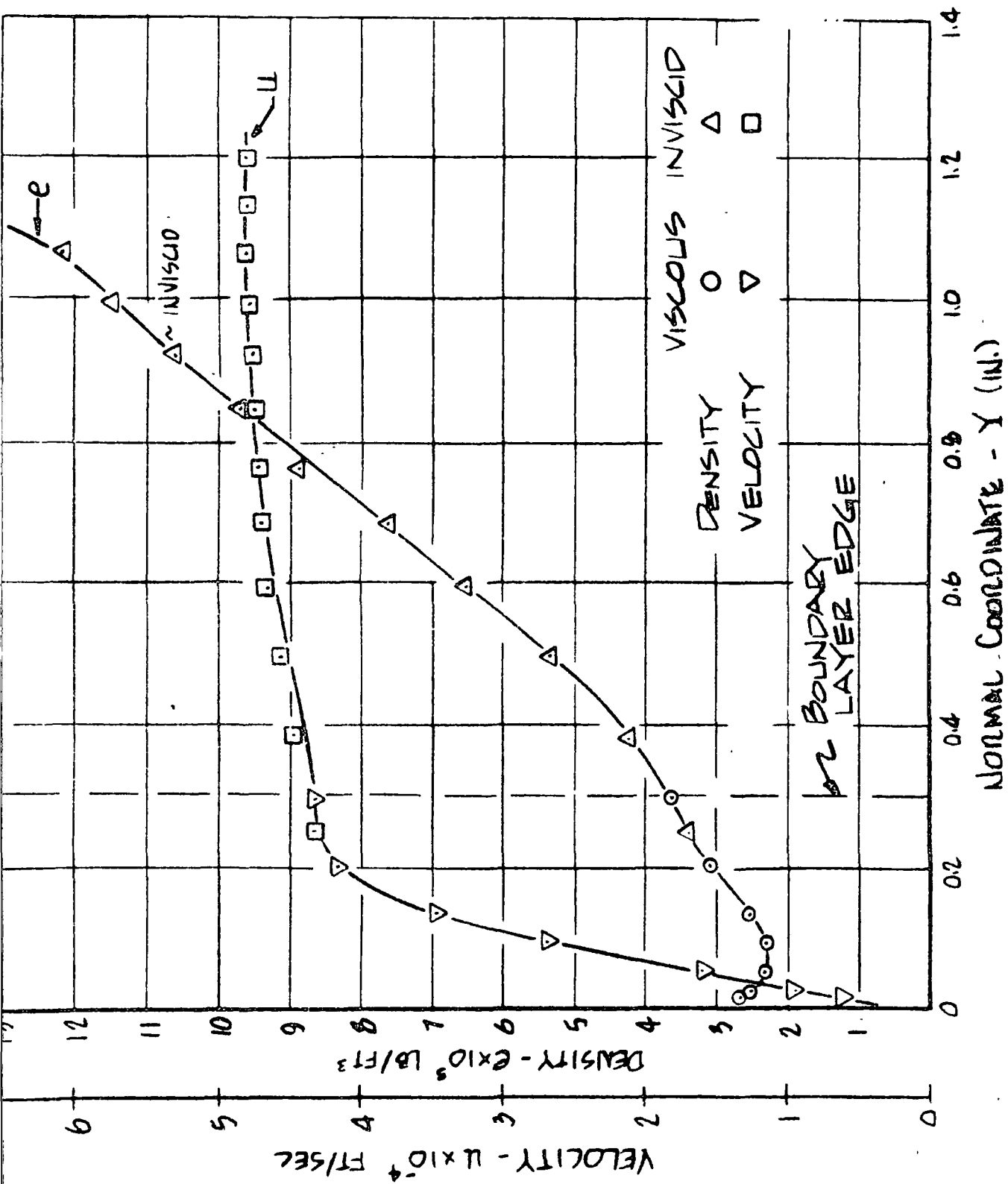


Figure 8. Demonstration of Flow Field Coupling

TIME .10000*01 SECONDS - - - STREAMWISE DIMENSION .95000-00FEET - - - 01 MAY 70 09:28:35

ITERATED VALUES DAMP MAX-LIN MAX-ERRORS IN CONSERVATION EQS.

ITS	TIME	ALPHA	PPW	DAMP	MAX-LIN ERROR	MOMENTUM	ENERGY
1	.369	2.225	1.2800	.2154	1.0-02	14	1.6-00 11 3.3-02
2	.739	2.447	1.3201	.3020	2.0-02	14	1.2-00 11 2.7-02
3	1.100	2.692	1.3716	.4644	2.0-02	14	8.3-01 11 2.0-02
4	1.466	2.961	1.4458	.8475	6.0-03	14	4.4-01 11 1.1-02
5	1.829	3.095	1.50971.0000		2.0-02	7	2.4-01 4 -2.6-01
6	2.189	3.172	1.53261.0000		1.0-02	7	6.8-02 4 -6.5-00
7	2.556	3.225	1.54331.0000		4.0-03	6	2.5-02 5 2.4-00
8	2.920	3.264	1.55011.0000		2.0-03	6	1.6-02 5 1.4-00
9	3.281	3.294	1.55501.0000		6.0-04	6	1.1-02 5 1.0-00
10	3.644	3.318	1.55871.0000		2.0-04	6	8.5-03 5 7.7-01
11	4.003	3.336	1.56161.0000		4.0-05	6	6.6-03 5 5.9-01
12	4.366	3.351	1.56391.0000		2.0-04	6	5.2-03 5 4.6-01
13	4.731	3.363	1.56571.0000		2.0-04	8	4.2-03 5 3.7-01
14	5.095	3.373	1.56711.0000		2.0-04	8	3.4-03 5 2.9-01
15	5.458	3.380	1.56831.0000		2.0-04	8	2.7-03 5 2.3-01
16	5.821	3.387	1.56921.0000		2.0-04	8	2.2-03 5 1.9-01
17	6.181	3.392	1.57001.0000		2.0-04	8	1.8-03 5 1.5-01
18	6.546	3.396	1.57061.0000		1.0-04	8	1.4-03 5 1.2-01
19	6.908	3.399	1.57111.0000		1.0-04	8	1.2-03 5 1.0-01
20	7.268	3.402	1.57151.0000		1.0-04	8	9.5-04 5 8.1-02
21	7.633	3.404	1.57181.0000		8.0-05	8	7.7-04 5 6.5-02
22	7.995	3.406	1.57211.0000		7.0-05	8	6.3-04 5 5.3-02
23	8.360	3.407	1.57231.0000		6.0-05	8	5.1-04 5 4.3-02
24	8.722	3.409	1.57251.0000		5.0-05	8	4.1-04 5 3.5-02
25	9.081	3.410	1.57261.0000		4.0-05	8	3.4-04 5 2.9-02
26	9.442	3.410	1.57281.0000		3.0-05	8	2.7-04 5 2.3-02
27	9.807	3.411	1.57281.0000		3.0-05	8	2.3-04 5 1.9-02
28	10.165	3.412	1.57291.0000		2.0-05	8	1.8-04 5 1.5-02
29	10.527	3.412	1.57301.0000		2.0-05	8	1.5-04 5 1.2-02

ALPHA	XI	ROKAP	PRESSURE	REF.	BETA	FLUX NOR-	HEAT FLUXES	QCOND
3.412*00	2.523*05	1.000*00	6.256-03	3.941*03	2.690-02	2.455-03	1.209*00 1.209*00 0.000	1.209*00

WALL SHEAR (LB/SQ FT) 5.032-01
 MASS FLUXES INJECTED (LB/SEC SQ FT) 0.000

MOM TRANS HEAT TRANS BLOWING PARAMETERS (BASED ON CM)
 COEFF. COEFF. RMOR*UR*CF/2 RMOR*UR*CH
 4.108-03 J.568-03

MOMENTUM, DISPLAC. EFFECTIVE ENTHALPY REYNOLDS THICKNESS, THICKNESS, BODY THICKNESS, NUMBER THETA DELSTAR DISPLAC. LAMBDA PER FOOT (FT) (FT) (FT) (REF.)
 -J.558-04 -1.270-02 3.888-03 5.731-03 1.211*05

MODAL INFORMATION
 DISTANCE, ETA
 FROM WALL (FT)

STATIC ENTHALPY (BTU/LB)
 TEMP (DEG R)
 ELECTRON COLL FREQ (1/SEC)

Problem 4 - Sample Output

ITERATED VALUES DAMP MAX-LIN MAX-ERRORS IN CONSERVATION EQS.

ITS	TIME	ALPH	FPP	DAMP	MAX-LIN	MAX-ERRORS	IN CONSERVATION	EQS.
1	.370	3.536	1.5818	.4999	5.0-03	14	1.0-00	11 1.5-02
2	.737	3.777	1.59471	.0000	1.0-02	14	5.0-01	11 8.2-01
3	1.102	3.968	1.61531	.0000	3.0-02	6	-1.1-01	11 2.6-01
4	1.465	4.094	1.63041	.0000	1.0-02	6	-8.1-02	12 1.7-01
5	1.828	4.171	1.64091	.0000	2.0-03	6	-4.9-02	11 6.3-00
6	2.189	4.217	1.64701	.0000	8.0-04	6	-2.8-02	11 2.1-00
7	2.552	4.245	1.65061	.0000	1.0-03	6	-1.6-02	3 -1.1-00
8	2.915	4.263	1.65271	.0000	1.0-03	5	-4.6-03	2 -6.4-01
9	3.278	4.275	1.65401	.0000	8.0-04	5	-6.1-03	2 -4.1-01
10	3.641	4.283	1.65491	.0000	6.0-04	5	-4.0-03	2 -2.6-01
11	4.005	4.288	1.65551	.0000	4.0-04	5	-2.6-03	2 -1.7-01
12	4.371	4.292	1.65581	.0000	3.0-04	5	-1.8-03	2 -1.2-01
13	4.734	4.294	1.65611	.0000	2.0-04	5	-1.2-03	2 -7.8-02
14	5.097	4.296	1.65631	.0000	1.0-04	5	-8.0-04	2 -5.3-02
15	5.459	4.297	1.65641	.0000	9.0-05	5	-5.5-04	2 -3.6-02
16	5.821	4.298	1.65651	.0000	6.0-05	5	-3.7-04	2 -2.5-02
17	6.180	4.298	1.65651	.0000	4.0-05	5	-2.5-04	2 -1.7-02
18	6.546	4.299	1.65651	.0000	3.0-05	5	-1.7-04	2 -1.1-02
19	6.908	4.299	1.65661	.0000	2.0-05	5	-1.2-04	2 -7.8-03

ALPHA	XI	HOKAP	PRESSURE	REF.	BETA	FLUX NOR-	HEAT FLUXES	COND
	(LB	(FT)	(ATM)	VELOCITY		MALIZING DIFFUSIONAL TOT ENTH	REARAD	
	/SEC)**2			(FT/SEC)		PARAMETER	(BTU/SEC SQ FT)	
4.299+00	2.757-05	1.000+00	6.101-03	3.948+03	4.342-02	2.299-03	1.190+00	1.190+00

WALL	SWEAK	INJECTED	MASS FLUXES
(LB/SO FT)	(LB/SEC SQ FT)		
4.962-01	0.000		

NOM TRANS	HEAT TRANS	BLOWING PARAMETERS
COEFF.	COEFF.	(BASED ON CH)
RHOREUR*CF/2 RHOREUR*CH		
4.043-03	3.511-03	0.000

MOMENTUM	DISPLAC.	EFFECTIVE	ENTHALPY	REYNOLDS
THICKNESS	THICKNESS	BODY	THICKNESS	NUMER
THETA	DELTA	DISPLAC.	LAMBDA	PER FOOT
(FT)	(FT)	(FT)	(FT)	(REF.)
-2.840-03	-2.592-02	1.219-03	6.544-03	1.197+05

NODAL INFORMATION	ETA	F	FP	FPP	SWEAK	TOTAL ENTH-	GP	GPP	STATIC	TEMP	ELECTRON
DISTANCE	FROM WALL		(=U/UH)		(LB/FTSQ)	HALPY,G	(BTU/LB)	(BTU/LB)	ENTHALPY	(DEG R)	COLL FREQ
(FT)									(BTU/LB)		(1/SEC)
0.000	0.000	0.000	0.000	1.657+00	4.962-01	2.954+00	3.412+02	5.867+02	2.954+00	5.550+02	2.020+09
4.963-04	1.032-01	8.459-03	1.721-01	1.680+00	4.948-01	4.128+01	4.018+02	-3.808+02	3.206+01	6.754+02	1.831+09
8.711-04	1.720-01	2.449-02	2.795-01	1.441+00	4.936-01	6.802+01	3.756+02	-9.087+02	4.370+01	7.233+02	1.769+09
1.666-03	3.095-01	7.473-02	4.376-01	8.575-01	4.908-01	1.111+02	2.505+02	-5.143+02	5.148+01	7.552+02	1.732+09
2.874-03	5.159-01	1.603-01	5.713-01	4.384-01	4.874-01	1.514+02	1.403+02	-1.931+02	4.980+01	7.483+02	1.740+09
4.834-03	4.598-01	3.963-01	6.834-01	2.194-01	4.831-01	1.882+02	7.388+01	-6.186+01	4.284+01	7.198+02	1.774+09
7.648-03	1.376+00	7.747-01	7.673-01	1.121-01	4.784-01	2.181+02	4.197+01	-2.204+01	3.487+01	6.870+02	1.816+09
1.124-02	2.064+00	1.325+00	8.248-01	6.683-02	4.742-01	2.418+02	2.681+01	-2.922+00	2.795+01	6.585+02	1.855+09
1.884-02	3.439+00	2.524+00	9.105-01	5.147-02	4.185-01	2.759+02	2.279+01	-4.655+00	1.786+01	6.168+02	1.916+09
2.975-02	6.019+00	5.013+00	1.007+00	2.277-02	2.232-01	3.192+02	1.078+01	-2.949+00	3.697+00	5.581+02	2.014+09

Problem 4 - Sample Output (continued)

4.054-02	8.598-00	7.670-00	1.047-00	8.018-03	4.152-02	3.312-02	3.176-00	-8.118-01	-4.353-00	5.246-02	2.018-00
6.048-02	1.376-01	1.316-01	1.076-00	2.138-03	1.511-02	3.427-02	-1.053-00	4.494-01	-1.755-01	4.694-02	2.196-00
8.621-02	2.150-01	2.159-01	1.103-00	3.030-03	1.017-03	3.418-02	0.000	-1.800-01	-3.664-01	3.892-02	2.412-09
DISTANCE FROM WALL (FT)											
0.000	4.365-04	1.052-05	1.042-00	2.404-01	3.612-06	7.000-01	7.000-01	2.900-01	0.000	0.000	
4.963-04	3.587-04	1.202-05	9.170-01	2.428-01	4.170-05	7.000-01	7.000-01	2.900-01	4.693-02	5.350-01	
8.711-04	3.349-04	1.259-05	9.754-01	2.438-01	4.386-06	7.000-01	7.000-01	2.900-01	2.384-01	6.400-01	
1.666-03	3.208-04	1.297-05	9.620-01	2.444-01	4.528-06	7.000-01	7.000-01	2.900-01	1.066-00	1.288-00	
2.874-03	3.238-04	1.289-05	9.648-01	2.443-01	4.497-06	7.000-01	7.000-01	2.900-01	2.971-00	1.684-00	
4.834-03	3.366-04	1.255-05	9.769-01	2.437-01	4.370-06	7.000-01	7.000-01	2.900-01	7.066-00	2.059-00	
7.648-03	3.526-04	1.216-05	9.916-01	2.431-01	4.222-06	7.000-01	7.000-01	2.900-01	1.413-01	2.365-00	
1.124-02	3.679-04	1.181-05	1.005-00	2.435-01	4.093-06	7.000-01	7.000-01	2.900-01	2.415-01	2.608-00	
1.864-02	3.928-04	1.130-05	1.026-00	2.417-01	3.901-06	7.000-01	7.000-01	2.900-01	2.757-01	2.958-00	
2.975-02	4.341-04	1.056-05	1.060-00	2.405-01	3.627-06	7.000-01	7.000-01	2.900-01	3.368-01	3.436-00	
4.058-02	4.618-04	1.012-05	1.081-00	2.398-01	3.468-06	7.000-01	7.000-01	2.900-01	3.812-01	3.685-00	
6.048-02	5.161-04	9.386-06	1.120-00	2.387-01	3.201-06	7.000-01	7.000-01	2.900-01	2.380-01	3.997-00	
8.621-02	6.226-04	8.262-06	1.189-00	2.371-01	2.798-06	7.000-01	7.000-01	2.900-01	0.000	4.493-00	

DISTANCE FROM WALL (FT)	STREAM FUNCTION (ACTUAL)	ENTROPY S (BTU/LB R)	PT/PTINF	PT(PILOT) PTINF	GAMMA
0.000	-0.000	-2.693-01	4.984-05	4.984-05	1.393-00
4.963-04	6.579-05	-2.219-01	6.051-05	6.051-05	1.393-00
8.711-04	1.818-04	-2.052-01	7.893-05	7.893-05	1.391-00
1.666-03	5.549-04	-1.947-01	1.354-04	1.354-04	1.389-00
2.874-03	1.339-03	-1.970-01	2.418-04	2.418-04	1.390-00
4.834-03	2.958-03	-2.064-01	4.301-04	2.973-04	1.391-00
7.648-03	5.753-03	-2.178-01	6.994-04	3.865-04	1.393-00
1.124-02	9.843-03	-2.281-01	1.029-03	4.678-04	1.394-00
1.864-02	1.875-02	-2.439-01	1.778-03	6.018-04	1.396-00
2.975-02	3.723-02	-2.680-01	3.650-03	8.192-04	1.398-00
4.058-02	5.696-02	-2.829-01	5.231-03	9.494-04	1.400-00
6.048-02	9.774-02	-3.095-01	8.045-03	1.127-03	1.403-00
8.621-02	1.603-01	-3.541-01	1.532-02	1.445-03	1.406-00

Problem 4 - Sample Output (continued)

TIME .10000*01 SECONDS - - - STREAMWISE DIMENSION .22100*01FEET - - - 01 MAY 70 09120159

ITERATED VALUES DAMP MAX-LIN MAX-ENMOM IN CONSERVATION EDS.

ITS	TIME	ALPH	FPP#	EMOR	MOMENTUM	E-ENGR
1	.375	6.006	1.8943	.4099	1.002	15 -3.2*00 10 1.5*02
2	.743	6.261	1.95691	.0000	3.003	15 -1.7*00 10 7.9*01
3	1.105	6.376	1.99181	.0000	6.003	5 -4.1*01 2 -2.1*01
4	1.470	6.429	2.00311	.0000	3.003	5 -1.5*01 2 -7.1*00
5	1.835	6.453	2.00761	.0000	2.003	5 -0.0*02 2 -2.9*00
6	2.197	6.464	2.00951	.0000	7.004	5 -2.6*02 2 -1.2*00
7	2.563	6.468	2.01041	.0000	2.004	5 -1.1*02 2 -5.4*01
8	2.930	6.470	2.01071	.0000	9.005	5 -3.0*03 2 -2.3*01
9	3.292	6.471	2.01091	.0000	4.005	5 -2.2*03 2 -1.0*01
10	3.655	6.472	2.01101	.0000	2.005	5 -9.4*04 2 -4.5*02
11	4.022	6.472	2.01101	.0000	8.006	5 -4.1*04 2 -1.9*02
12	4.385	6.472	2.01101	.0000	3.006	15 -2.4*04 12 2.7*02
13	4.752	6.472	2.01101	.0000	1.006	5 -0.0*05 2 -3.8*03

ALPHA	XI	HOKAP	PRESSURE	REF.	BETA	FLUX NOR-	HEAT FLUXES	QCONO
	(LB	(FT)	(ATM)	VELOCITY		MALIZING DIFFUSIONAL TOT ENTH	PARAMETER	
6.472*00	4.632*05	1.000*00	6.425*03	3.913*03	-1.087*01	1.852*03	1.250*00	0.000
								1.250*00

WALL
SHEAR
(LB/50 FT)
4.857*01

MASS FLUXES
INJECTED
(LB/SEC 50 FT)
0.000

WOM TRANS HEAT TRANS
COEFF, COEFF,
RHOR=URCF/2 RHOR=URCH

3.973*03 3.688*03 0.000

MOMENTUM, DISPLAC, EFFECTIVE ENTHALPY REYNOLDS

THICKNESS, THICKNESS, BODY THICKNESS, NUMREH
THETA DELSTAR DISPLAC, LAMBDA PER FOOT
(FT) (FT) (FT) (REF.)
-1.670*02 -9.334*02 2.072*03 9.790*03 1.225*05

NODAL INFORMATION
DISTANCE, ETA
FROM WALL
(FT)

	F	FP	FPP	SHEAR	TOTAL ENTH-	GP	GPP	STATIC	TEMP	ELECTRON
		(=U/UK)		(LB/FTSQ)	HALPY.G	(BTU/LB)	(BTU/LB)	ENTHALPY	(DEG R)	COLL FREQ
0.000	0.000	0.000	2.011*00	4.857*01	2.954*00	4.427*02	1.584*01	2.954*00	5.550*02	11/SEC1
9.858*04	1.553*01	2.273*02	2.829*01	4.889*01	7.192*01	4.452*02	-1.416*03	4.719*01	7.376*02	1.845*09
1.741*03	2.589*01	5.966*02	4.194*01	4.905*01	1.104*02	2.985*02	-7.227*02	5.608*01	7.760*02	1.801*09
3.285*03	4.660*01	1.642*01	5.715*01	4.613*01	4.920*01	1.568*02	-2.322*02	5.586*01	7.731*02	1.802*09
5.551*03	7.766*01	3.603*01	6.788*01	4.918*01	1.918*02	7.673*01	-7.012*01	4.948*01	7.470*02	1.834*09
9.162*03	1.294*00	7.372*01	1.149*01	4.875*01	2.221*02	4.042*01	-2.229*01	4.023*01	7.091*02	1.882*09
1.824*02	2.071*00	1.363*00	8.367*01	4.766*01	2.468*02	2.311*01	-6.510*00	3.055*01	6.693*02	1.937*09
2.074*02	3.107*00	2.260*00	8.923*01	4.562*01	2.672*02	1.637*01	-2.267*00	2.130*01	6.311*02	1.995*09
3.573*02	5.178*00	4.192*00	9.693*01	3.059*02	2.963*02	1.167*01	-1.846*00	6.024*00	5.678*02	2.103*09
5.196*02	9.061*00	8.143*00	1.054*00	2.392*01	3.277*02	4.505*00	-8.279*01	-1.576*01	4.769*02	2.295*09
6.854*02	1.294*01	1.232*01	1.092*00	6.072*03	3.389*02	1.290*00	-2.003*01	-2.936*01	4.198*02	2.446*09
9.862*02	2.071*01	2.095*01	1.127*00	2.965*03	3.429*02	-2.856*01	4.237*02	-0.944*01	3.351*02	2.738*09
1.295*01	3.236*01	3.423*01	1.149*00	1.617*03	3.418*02	0.000	3.224*03	-6.627*01	2.635*02	3.087*09

DISTANCE DENSITY, VISCOSITY, RHOMU SPECIFIC THERMAL PRANDTL MODIFIED MOLECULAR RHOMSQEPS HACH

FROM WALL (FT)	MMO (LB/CU FT)	MU LB/SEC FT	MMO/SEC C	HEAT (BTU/LB R)	COND (BTU /SEC FT R)	NUMBER	SCHMIDT NUMBER	WEIGHT	MMO/SEC	NUMBER
0.000	4.597-04	1.052-05	1.067-00	2.404-01	3.612-06	7.000-01	7.000-01	2.900-01	0.000	0.000
9.858-04	3.459-04	1.276-05	9.738-01	2.421-01	4.450-06	7.000-01	7.000-01	2.900-01	3.494-01	8.390-01
1.741-03	3.296-04	1.319-05	9.589-01	2.428-01	4.612-06	7.000-01	7.000-01	2.900-01	1.196-00	1.215-00
3.285-03	3.300-04	1.318-05	9.593-01	2.428-01	4.608-06	7.000-01	7.000-01	2.900-01	3.730-00	1.656-00
5.551-03	3.416-04	1.287-05	9.699-01	2.443-01	4.492-06	7.000-01	7.000-01	2.900-01	8.588-00	2.001-00
9.162-03	3.598-04	1.242-05	9.862-01	2.475-01	4.322-06	7.000-01	7.000-01	2.900-01	1.774-01	2.320-00
1.429-02	3.812-04	1.195-05	1.005-00	2.477-01	4.142-06	7.000-01	7.000-01	2.900-01	3.211-01	2.602-00
2.074-02	4.043-04	1.148-05	1.024-00	2.419-01	3.967-06	7.000-01	7.000-01	2.900-01	4.494-01	2.856-00
3.263-02	4.444-04	1.058-05	1.054-00	2.407-01	3.872-06	7.000-01	7.000-01	2.900-01	5.552-01	3.268-00
5.196-02	5.350-04	9.487-06	1.120-00	2.387-01	3.237-06	7.000-01	7.000-01	2.900-01	7.869-01	3.872-00
6.859-02	6.077-04	8.694-06	1.166-00	2.377-01	2.954-06	7.000-01	7.000-01	2.900-01	1.015-02	4.270-00
9.662-02	7.614-04	7.463-06	1.258-00	2.340-01	2.516-06	7.000-01	7.000-01	2.900-01	7.971-01	4.926-00
1.295-01	9.682-04	6.338-06	1.354-00	2.326-01	2.124-06	7.000-01	7.000-01	2.900-01	0.000	5.658-00

DISTANCE FROM WALL (FT)	STREAM FUNCTION	ENTHALPY S (BTU/LB R)	PT/PTINF	PT(PITOT)	GAMMA PTINF
0.000	-0.000	-2.729-01	5.249-05	5.249-05	1.399-00
9.858-04	2.188-04	-2.040-01	8.303-05	8.303-05	1.390-00
1.741-03	5.742-04	-1.922-01	1.293-04	1.242-04	1.389-00
3.285-03	1.580-03	-1.925-01	2.423-04	2.414-04	1.369-00
5.551-03	3.468-03	-2.009-01	4.129-04	2.947-04	1.340-00
9.162-03	7.095-03	-2.136-01	6.859-04	3.923-04	1.392-00
1.429-02	1.311-02	-2.277-01	1.074-03	4.904-04	1.393-00
2.074-02	2.175-02	-2.419-01	1.598-03	5.903-04	1.395-00
3.263-02	4.035-02	-2.674-01	2.993-03	7.768-04	1.398-00
5.196-02	7.837-02	-3.092-01	7.129-03	1.109-03	1.402-00
6.859-02	1.186-01	-3.398-01	1.213-02	1.365-03	1.405-00
9.662-02	2.016-01	-3.930-01	2.728-02	1.853-03	1.409-00
1.295-01	3.294-01	-4.493-01	6.172-02	2.494-03	1.413-00

Problem 4 - Sample Output (continued)

ITERATED VALUES DAMP MAX-LIN MAX-ERRORS IN CONSERVATION EOS.

ITS	TIME	ALPH	FPP	ERROR	MOMENTUM	ENERGY
1	.374	6.626	2.1334	.4999	1.03	15 -7.6+00 10 1.6+02
2	.746	6.855	2.29211+0000	6.04	15 -3.8+00 10 7.8+01	
3	1.109	6.946	2.33461+0000	4.03	5 -6.8-01 2 -3.1+01	
4	1.477	6.977	2.35021+0000	2.03	5 -1.7-01 2 -7.6+00	
5	1.839	6.986	2.35311+0000	5.04	5 -4.7-02 2 -2.1+00	
6	2.206	6.989	2.35391+0000	2.04	5 -1.4-02 2 -6.3-01	
7	2.570	6.990	2.35421+0000	5.05	5 -4.4-03 2 -2.0-01	
8	2.931	6.990	2.35431+0000	2.05	5 -1.4-03 2 -6.2-02	
9	3.299	6.991	2.35431+0000	5.06	5 -4.4-04 12 2.9-02	
10	3.660	6.991	2.35431+0000	2.06	5 -1.4-04 12 -1.8-02	

ALPHA	XI	HOKAP	PRESSURE	PEF.	BETA	FLUX NON-MALIZING DIFFUSIONAL PARAMETER	HEAT FLUXES TOT ENTH	QCOND
6.991+00	6.821-05	1.000+00	9.243-03	3.618+03	-2.088-01	2.063-03	1.716+00	1.716+00

WALL
SHEAR
(LB/SQ FT)
6.351-01

MASS FLUXES
INJECTED
(LB/SEC SQ FT)
0.000

MOM TRANS HEAT TRANS
COEFF. COEFF.
(BASED ON CH)
RMORUR*CF/2 RHORUR*CH
5.353-03 5.065-03

MOMENTUM DISPLAC. EFFECTIVE ENTHALPY REYNOLDS
THICKNESS, THICKNESS, BODY THICKNESS, NUMBER
THETA DELSTAR DISPLAC. LAMBDA PER FOOT
(FT) (FT) (FT) (REF.)
-2.703-02 -1.219-01 -2.148-03 1.040-02 1.461+05

MODAL INFORMATION

DISTANCE FROM WALL (FT)	ETA	F	FP	FPP	SHEAR (LB/FTSQ)	TOTAL ENTHALPY (BTU/LB)	GP	GPP	STATIC ENTHALPY (BTU/LB)	TEMP (DEG R)	ELECTRON CULL FREQ (1/SEC)
0.000	0.000	0.000	0.000	2.354+00	6.351-01	2.454+00	5.285+02	-5.413+02	2.954+00	5.550+02	3.060+09
9.513-04	1.678-01	2.938-02	3.278-01	1.554+00	6.422-01	8.400+01	4.377+02	-1.569+03	5.271+01	7.602+02	2.615+09
1.679-03	2.796-01	7.433-02	4.632-01	8.662-01	6.462-01	1.231+02	2.622+02	-6.116+02	6.068+01	7.928+02	2.561+09
3.162-03	5.033-01	1.957-01	6.037-01	3.943-01	6.520-01	1.665+02	1.253+02	-1.794+02	6.038+01	7.915+02	2.563+09
5.348-03	8.389-01	4.166-01	7.022-01	1.944-01	6.570-01	1.984+02	6.514+01	-5.409+01	5.492+01	7.692+02	2.600+09
8.850-03	1.398+00	8.351-01	7.859-01	1.071-01	6.594-01	2.264+02	3.489+01	-1.769+01	4.662+01	7.353+02	2.659+09
1.385-02	2.237+00	1.525+00	8.538-01	5.872-02	6.554-01	2.495+02	2.005+01	-4.880+00	3.728+01	8.964+02	2.731+09
2.018-02	3.355+00	2.514+00	9.106-01	4.245-02	6.400-01	2.688+02	1.461+01	-1.458+00	2.749+01	6.566+02	2.814+09
3.171-02	5.592+00	4.648+00	9.926-01	3.050-02	5.757-01	2.965+02	1.015+01	-1.458+00	9.726+00	5.831+02	2.986+09
5.001-02	9.787+00	9.031+00	1.046+00	1.413-02	4.071-01	3.263+02	4.031+00	-6.281+01	-1.718+01	4.710+02	3.322+09
6.513-02	1.398+01	1.369+01	1.130+00	6.933-03	2.775-01	3.377+02	1.403+00	-1.727-01	-3.428+01	3.991+02	3.609+09
8.950-02	2.237+01	2.337+01	1.172+00	3.102-03	1.073-01	3.434+02	-4.374+02	-4.558-02	-5.679+01	3.039+02	4.136+09
1.178-01	3.495+01	3.826+01	1.190+00	1.518-03	5.380-04	3.418+02	0.000	5.253-02	-7.032+01	2.462+02	4.595+09

DISTANCE FROM WALL (FT)	DENSITY RHO (LB/CU FT)	VISCOSITY MU (LB/SEC FT)	WMO*WU	SPECIFIC HEAT C (BTU/LB R)	THERMAL COND (BTU/SEC FT R)	PRANDTL NUMBER	MODIFIED SCHMIDT NUMBER	MOLECULAR WEIGHT	WMO*WU	MACH NUMBER
0.000	6.613-04	1.052-05	1.102+00	2.404-01	3.612-06	7.000-01	7.000-01	2.730+01	3.000	0.000

Problem 4 - Sample Output (continued)

9.513-04	4.828-04	1.303-05	7.985-01	2.455-01	50-06	7.000-01	7.000-01	2.900-01	5.918-01	7.301-01
1.679-03	4.630-04	1.340-05	7.831-01	2.452-01	4.945-06	7.000-01	7.000-01	2.900-01	2.064-00	1.287-00
3.162-03	4.637-04	1.339-05	7.837-01	2.452-01	4.949-06	7.000-01	7.000-01	2.900-01	5.840-00	1.679-00
5.346-03	4.772-04	1.313-05	7.928-01	2.447-01	4.591-05	7.000-01	7.000-01	2.900-01	1.268-01	1.981-00
8.850-03	4.992-04	1.273-05	1.007-00	2.444-01	4.439-06	7.000-01	7.000-01	2.900-01	2.513-01	2.266-00
1.385-02	5.267-04	1.228-05	1.025-00	2.473-01	4.267-06	7.000-01	7.000-01	2.900-01	4.458-01	2.527-00
2.016-02	5.590-04	1.179-05	1.044-00	2.475-01	4.084-06	7.000-01	7.000-01	2.900-01	5.996-01	2.775-00
3.171-02	6.294-04	1.088-05	1.085-00	2.470-01	3.745-06	7.000-01	7.000-01	2.900-01	7.602-01	3.206-00
5.001-02	7.793-04	9.407-06	1.162-00	2.387-01	3.208-06	7.000-01	7.000-01	2.900-01	1.165-02	3.896-00
6.513-02	9.196-04	8.405-06	1.225-00	2.373-01	2.849-06	7.000-01	7.000-01	2.900-01	1.623-02	4.399-00
8.950-02	1.208-03	6.983-06	1.336-00	2.354-01	2.348-06	7.000-01	7.000-01	2.900-01	1.400-02	5.221-00
1.178-01	1.491-03	6.052-06	1.429-00	2.342-01	2.025-06	7.000-01	7.000-01	2.900-01	0.000	5.890-00
DISTANCE FROM WALL (FT)	STREAM FUNCTION (ACTUAL)	ENTROPY S (BTU/LB R)	PT/PTINF	PT(PITOT) PTINF	GAMMA					
0.000	-0.000	-2.978-01	7.552-05	7.552-05	1.399-00					
9.513-04	3.432-04	-2.216-01	1.316-04	1.316-04	1.389-00					
1.679-03	8.683-04	-2.113-01	2.048-04	2.048-04	1.388-00					
3.162-03	2.285-03	-2.117-01	3.608-04	3.114-04	1.388-00					
5.346-03	4.866-03	-2.187-01	5.754-04	4.184-04	1.389-00					
8.850-03	9.754-03	-2.297-01	9.056-04	5.394-04	1.390-00					
1.385-02	1.782-02	-2.428-01	1.372-03	8.640-04	1.392-00					
2.016-02	2.936-02	-2.572-01	2.028-03	8.016-04	1.394-00					
3.171-02	5.429-02	-2.859-01	3.928-03	1.075-03	1.397-00					
5.001-02	1.055-01	-3.371-01	1.059-02	1.616-03	1.402-00					
6.513-02	1.599-01	-3.765-01	2.060-02	2.062-03	1.406-00					
8.950-02	2.730-01	-4.409-01	5.512-02	3.020-03	1.411-00					
1.178-01	4.468-01	-4.903-01	1.120-01	3.896-03	1.413-00					

Problem 4 - Sample Output (concluded)

TIME .10000*01 SECONDS - - - SIMLAW-WISE DIMENSION .94500-00FLEET - - -

ITERATED VALUES DAMP MAX-LIN MAX-ERRORS IN CONSERVATION EOS.

ITS	TIME	ALPH	FRPM	ERR	MOM	ENERGY
1	.318	.953	.4999	3.03	15	-1.3-02 12 -1.3-01
2	.634	.966	.48151.0000	1.03	15	-6.5-03 12 -5.6-00
3	.944	.968	.48151.0000	5.05	2	3.6-04 12 4.8-01
4	1.252	.968	.48151.0000	7.05	2	1.3-04 12 1.3-02

ALPHA	XL	ROKAP	PRESSURE	REF.	BETA	FLUX NOR-	HEAT FLUXES	QCOND
	(LB	(FT)	(ATM)	(FT/SEC)		MALIZING DIFFUSIONAL TOT ENTH	(BTU/SEC SQ FT)	
9.680-01	1.767-05	1.000+00	4.573-03	4.123+03	1.967-02	2.280-03	3.863-01	3.862-01

WALL	SHEAR	MASS FLUXES
(LB/50 FT)	(LB/SEC SQ FT)	
1.479-01	0.000	

NOM TRANS	HEAT TRANS	BLOWING PARAMETERS
COEFF.	COEFF.	(BASED ON CH)
1.154-03	1.067-03	0.000

MOMENTUM DISPLAC.	EFFECTIVE ENTHALPY	REYNOLDS
THICKNESS	THICKNESS	NUMER
DELTA	LAMBDA	PER FOOT
(FT)	(FT)	(REF.)
1.841-03	5.432-03	7.294-03

MODAL INFORMATION	ETA	F	FP	FPP	SHEAR	TOTAL ENTH-	GP	GPP	STATIC	TEMP	ELECTRON
DISTANCE FROM WALL	(FT)		(=U/UK)		(LB/FTSQ)	(BTU/LB)	(BTU/LB)	(BTU/LB)	(BTU/LB)	(DEG R)	(1/SEC)
0.000	0.000	0.000	0.000	4.815-01	1.479-01	2.954+00	1.128-02	7.765+01	2.954+00	5.550+02	1.521+09
1.029-04	2.323-02	1.302-04	1.122-02	4.815-01	1.478-01	5.595+00	1.146-02	7.750+01	5.552+00	5.658+02	1.506+09
1.726-04	3.872-02	3.620-04	1.673-02	4.858-01	1.477-01	7.380+00	1.158-02	7.727+01	7.260+00	5.724+02	1.497+09
3.146-04	6.970-02	1.176-03	3.382-02	4.890-01	1.476-01	1.100+01	1.182-02	7.680+01	1.061+01	5.868+02	1.479+09
5.339-04	1.162-01	3.277-03	5.665-02	4.934-01	1.473-01	1.658+01	1.218-02	7.575+01	1.549+01	6.070+02	1.454+09
9.153-04	1.936-01	9.150-03	9.511-02	4.998-01	1.468-01	2.624+01	1.276-02	7.321+01	2.316+01	6.388+02	1.418+09
1.522-03	3.098-01	2.359-02	1.536-01	5.048-01	1.458-01	4.156+01	1.361-02	6.745+01	3.354+01	6.816+02	1.372+09
2.387-03	4.646-01	5.347-02	2.324-01	5.108-01	1.438-01	6.345+01	1.466-02	4.977+01	4.571+01	7.291+02	1.327+09
4.255-03	7.744-01	1.498-01	3.887-01	4.943-01	1.367-01	1.112+02	1.620+02	-6.738+00	5.976+01	7.898+02	1.275+09
7.934-03	1.355+00	4.541-01	6.500-01	4.015-01	1.100-01	2.042+02	1.581+02	-7.963+01	6.079+01	7.932+02	1.272+09
1.142-02	1.936+00	8.910-01	6.400-01	2.577-01	7.134-02	2.626+02	1.118-02	-8.297+01	4.311+01	7.209+02	1.334+09
1.734-02	3.098+00	1.989+00	1.010+00	4.041-02	1.221-02	3.565+02	1.546+01	-1.854+01	1.013+01	5.848+02	1.482+09
2.487-02	4.840+00	3.789+00	1.050+00	2.092-02	6.624-03	3.651+02	0.000	7.950+01	-9.097+00	5.048+02	1.595+09

DISTANCE FROM WALL	DENSITY	VISCOSITY	MU	RHO*U	SPECIFIC	THERMAL	PRANDTL	MODIFIED	RHOS*EPS	MACH
(FT)	(LB/50 FT)	LB/SEC FT	C	(BTU/LB R)	HEAT	COND (BTU /SEC FT R)	NUMBER	SCHMIDT	/RMOR*EUR	NUMBER
0.000	3.286-04	1.052-05	1.051+00	2.404-01	3.612-06	7.000-01	7.000-01	2.900+01	0.000	0.000
1.029-04	3.224-04	1.066-05	1.045+00	2.406-01	3.663-06	7.000-01	7.000-01	2.900+01	0.000	3.971-02
1.726-04	3.164-04	1.075-05	1.041+00	2.408-01	3.697-06	7.000-01	7.000-01	2.900+01	0.000	6.584-02
3.146-04	3.108-04	1.092-05	1.033+00	2.411-01	3.762-06	7.000-01	7.000-01	2.900+01	0.000	1.176-01
5.339-04	3.005-04	1.118-05	1.022+00	2.415-01	3.856-06	7.000-01	7.000-01	2.900+01	0.000	1.937-01
9.153-04	2.856-04	1.157-05	1.005+00	2.421-01	4.002-06	7.000-01	7.000-01	2.900+01	0.000	3.172-01
1.522-03	2.676-04	1.209-05	9.844-01	2.421-01	4.198-06	7.000-01	7.000-01	2.900+01	0.000	4.963-01

Problem 5 - Sample Output

DISTANCE FROM WALL	STREAM FUNCTION (FT)	ENTROPY S (BTU/LB R)	P(T/PINF	P(T/PILOT)	GAMMA				
2.387-03	2.502-04	1.266-05	9.634-01	2.437-01	4.412-06	7.000-01	7.000-01	2.900-01	0.000
4.255-03	2.309-04	1.337-05	9.391-01	2.441-01	4.681-06	7.000-01	7.000-01	2.900-01	0.000
7.534-03	2.249-04	1.341-05	9.378-01	2.442-01	4.697-06	7.000-01	7.000-01	2.900-01	0.000
1.142-02	2.510-04	1.256-05	9.669-01	2.447-01	4.375-06	7.000-01	7.000-01	2.900-01	0.000
1.734-02	3.119-04	1.099-05	1.034-00	2.410-01	3.752-06	7.000-01	7.000-01	2.900-01	0.000
2.487-02	3.614-04	9.860-06	1.084-00	2.364-01	3.372-06	7.000-01	7.000-01	2.900-01	0.000
0.000	-0.000	-2.499-01	3.753-05	3.753-05	1.399-00	1.398-00	1.398-00	1.398-00	1.398-00
1.029-04	7.738-07	-2.453-01	3.757-05	3.757-05	1.398-00	1.398-00	1.398-00	1.398-00	1.398-00
1.726-04	2.152-06	-2.423-01	3.764-05	3.764-05	1.398-00	1.398-00	1.398-00	1.398-00	1.398-00
3.146-04	6.489-04	-2.365-01	3.789-05	3.789-05	1.397-00	1.397-00	1.397-00	1.397-00	1.397-00
5.334-04	1.948-05	-2.283-01	3.852-05	3.852-05	1.396-00	1.396-00	1.396-00	1.396-00	1.396-00
9.153-04	5.439-05	-2.160-01	4.023-05	4.023-05	1.395-00	1.395-00	1.395-00	1.395-00	1.395-00
1.522-03	1.402-04	-2.003-01	4.437-05	4.437-05	1.393-00	1.393-00	1.393-00	1.393-00	1.393-00
2.387-03	3.178-04	-1.839-01	5.322-05	5.322-05	1.391-00	1.391-00	1.391-00	1.391-00	1.391-00
4.255-03	8.902-04	-1.643-01	6.704-05	6.704-05	1.388-00	1.388-00	1.388-00	1.388-00	1.388-00
7.534-03	2.699-03	-1.632-01	2.727-04	2.727-04	1.384-00	1.384-00	1.384-00	1.384-00	1.384-00
1.142-02	5.296-03	-1.866-01	8.177-04	8.177-04	1.391-00	1.391-00	1.391-00	1.391-00	1.391-00
1.734-02	1.183-02	-2.373-01	3.118-03	3.118-03	1.397-00	1.397-00	1.397-00	1.397-00	1.397-00
2.487-02	2.252-02	-2.727-01	5.560-03	5.560-03	1.401-00	1.401-00	1.401-00	1.401-00	1.401-00

Problem 5 - Sample Output (continued)

2.557-03	2.464-04	1.269-05	9.614-01	2.434-01	4.421-06	7.000-01	7.000-01	2.900-01	0.000	7.247-01
4.552-03	2.272-04	1.340-05	9.357-01	2.452-01	4.695-06	7.000-01	7.000-01	2.900-01	0.000	1.168-00
4.511-03	2.266-04	1.343-05	9.360-01	2.452-01	4.705-06	7.000-01	7.000-01	2.900-01	0.000	1.956-00
1.224-02	2.505-04	1.254-05	9.666-01	2.437-01	4.367-06	7.000-01	7.000-01	2.900-01	0.000	2.657-00
1.453-02	3.120-04	1.081-05	1.037-00	2.409-01	3.718-06	7.000-01	7.000-01	2.900-01	0.000	3.553-00
2.441-02	3.642-04	9.653-06	1.043-00	2.391-01	3.297-06	7.000-01	7.000-01	2.900-01	0.000	4.011-00
DISTANCE FROM WALL (FT)	STREAM FUNCTION (ACTUAL)	ENTHALPY S (BTU/LB H)	P1/PTINF	PT(P1TOT) PTINF	GAMMA					
0.000	-0.000	-2.491-01	3.707-05	3.707-05	1.399-00					
1.101-04	4.133-07	-2.444-01	3.711-05	3.711-05	1.398-00					
1.846-04	2.242-06	-2.413-01	3.718-05	3.718-05	1.398-00					
3.465-04	7.348-06	-2.355-01	3.742-05	3.742-05	1.397-00					
5.712-04	4.048-05	-2.273-01	3.804-05	3.804-05	1.396-00					
9.745-04	5.122-05	-2.148-01	3.971-05	3.971-05	1.395-00					
1.630-03	1.476-04	-1.989-01	4.378-05	4.378-05	1.393-00					
2.557-03	3.349-04	-1.823-01	5.248-05	5.248-05	1.391-00					
4.552-04	9.347-04	-1.625-01	8.546-05	8.546-05	1.388-00					
8.511-03	2.857-03	-1.618-01	2.718-04	2.718-04	1.388-00					
1.224-02	5.613-03	-1.463-01	8.246-04	8.246-04	1.391-00					
1.453-02	1.244-02	-2.395-01	3.234-03	3.234-03	1.398-00					
2.441-02	2.349-02	-2.793-01	6.133-03	6.133-03	1.402-00					

Problem 5 - Sample Output (continued)

TIME -1.0000+01 SECONDS - - - STREAMWISE DIMENSION -21900+01FEET - - -

ITERATED VALUES OMP MAX.LIN MAX.ERRORS IN CONSERVATION EQS.

ITS	TIME	ALPH	FPW	OMP	MAX.LIN	MAX.ERRORS	IN CONSERVATION	EQS.
1	.369	1.116	1.3307	.4999	3.-03	15	1.8+00	12 6.4+02
2	.735	1.161	1.45591	.0000	3.-03	15	7.0-01	12 3.3+02
3	1.098	1.173	1.48521	.0000	4.-03	8	-4.3-02	10 2.0-01
4	1.459	1.174	1.44311	.0000	4.-04	8	-8.5-03	5 -2.6+00
5	1.819	1.175	1.49441	.0000	1.-04	8	-1.5-03	5 -4.5-01
6	2.178	1.175	1.49461	.0000	2.-05	8	-2.6-04	5 -7.8-02
7	2.543	1.175	1.49461	.0000	3.-06	8	-4.4-05	5 -1.3-02

ALPHA	XI	ROKAP	PRESSURE	REF.	BETA	FLUX MOR-	HEAT FLUXES
	(LB	(FT)	(ATM)	VELOCITY		MAKING DIFFUSIONAL TOT ENTH	RERAD
	/SEC)**2			(FT/SEC)		PARAMETER	QCOND
1.175+00	3.525-05	1.000+00	5.642-03	4.043+03	-1.801-01	1.934-03	1.173+00 0.000
							1.173+00

WALL	MASS FLUXES
SHEAR	INJECTED
(LB/SO FT)	(LB/SEC SU FT)
3.912-01	0.000

MOM TRANS	HEAT TRANS
COEFF.	LOEFF.
RHOR*UR*CF/2	RHOR*UR*CH
3.098-03	3.239-03
	0.000

MOMENTUM	DISPLAC.	EFFECTIVE	ENTHALPY	REYNOLDS
THICKNESS	THICKNESS	BOU	THICKNESS	NUMREK
THETA	DELSTAR	DISPLAC.	LAMBDA	PER FOOT
(FT)	(FT)	(FT)	(FT)	(REF.)
1.981-03	1.769-03	9.776-03	4.353-03	1.093+05

MODAL INFORMATION	ETA	F	FP	FPP	SHEAR	TOTAL ENTH-	GP	GPP	STATIC	TEMP	ELECTRON
DISTANCE,						HALPY-G			ENTHALPY	(DEG R)	COLL FREQ
FROM WALL						(BTU/LB)	(BTU/LB)	(BTU/LB)	(BTU/LB)		(17/SEC)
(FT)						(LB/FTSQ)					
0.000	0.000	0.000	0.000	1.495+00	3.912-01	2.954+00	3.962+02	8.119+02	2.954+00	5.550+02	1.855+09
1.489-04	2.819-02	5.995-04	4.272-02	1.536+00	3.922-01	1.445+01	4.191+02	7.666+02	1.384+01	6.002+02	1.812+09
2.546-04	4.699-02	1.675-03	7.180-02	1.559+00	3.930-01	2.246+01	4.335+02	5.659+02	2.076+01	6.288+02	1.771+09
4.799-04	8.458-02	5.479-03	1.307-01	1.574+00	3.947-01	3.915+01	4.547+02	-1.093+02	3.352+01	6.815+02	1.701+09
8.486-04	1.410-01	1.529-02	2.165-01	1.470+00	3.978-01	6.482+01	4.486+02	-9.704+02	4.917+01	7.451+02	1.626+09
1.521-03	2.349-01	4.157-02	3.368-01	1.092+00	4.050-01	1.025+02	3.574+02	-9.008+02	6.508+01	8.107+02	1.559+09
2.594-03	3.759-01	9.849-02	4.607-01	6.648+01	4.206-01	1.439+02	2.304+02	-4.621+02	7.396+01	8.468+02	1.526+09
6.959-03	5.639-01	1.953-01	5.607-01	3.995+01	4.470-01	1.791+02	1.435+02	-1.750+02	7.543+01	8.528+02	1.521+09
1.216-02	1.645+00	9.518-01	7.985-01	1.418+01	5.016-01	2.207+02	7.778+01	-3.236+01	7.047+01	8.326+02	1.539+09
1.698-02	2.348+00	1.547+00	4.879-01	1.118+01	5.571+01	3.022+02	5.497+01	-1.596+01	5.727+01	7.788+02	1.591+09
2.537-02	3.759+00	2.896+00	1.016+00	6.938+02	5.130+01	3.022+02	4.372+01	-1.449+01	4.237+01	7.179+02	1.657+09
3.433-02	5.873+00	5.166+00	1.110+00	7.068+03	2.472-01	3.495+02	2.330+01	-2.308+01	9.499+00	5.822+02	1.840+09
						3.651+02	0.000	1.043+00	-4.091+01	3.712+02	2.305+09

DISTANCE FROM WALL	DENSITY, RHO (LB/CU FT)	VISCOSITY, MU LB/SEC FT	KHO*MU /RHO*KMUK, C	SPECIFIC HEAT (BTU/LB R)	THERMAL COND (BTU /SEC FT R)	PRANDTL NUMBER	MODIFIED SCHMIDT NUMBER	MOLECULAR WEIGHT	RHOSQ*EPS /RHO*HUR	MACH NUMBER
0.000	4.073-04	1.052-05	1.072+00	2.404-01	3.612-06	7.000-01	7.000-01	2.900+01	0.000	0.000
1.489-04	3.766-04	1.109-05	1.045+00	2.413-01	3.824-06	7.000-01	7.000-01	2.900+01	4.048-04	1.448-01
2.546-04	3.595-04	1.145-05	1.030+00	2.419+01	3.957-06	7.000-01	7.000-01	2.900+01	2.692-03	2.378-01
4.799-04	3.317-04	1.209-05	1.004+00	2.430-01	4.197-06	7.000-01	7.000-01	2.900+01	2.337-02	4.161-01

Problem 5 - Sample Output (continued)

8.445-04	3.031-04	1.284-05	9.752-01	2.442-01	4.445-04	7.000-01	7.000-01	2.900-01	1.332-01	6.597-01
1.521-03	2.756-04	1.361-05	9.475-01	2.445-01	4.774-06	7.000-01	7.000-01	2.900-01	5.697-01	9.855-01
2.544-03	2.655-04	1.402-05	9.363-01	2.443-01	4.932-06	7.000-01	7.000-01	2.900-01	1.655-00	1.319-00
4.053-03	2.751-04	1.405-05	9.342-01	2.443-01	4.945-06	7.000-01	7.000-01	2.900-01	3.648-00	1.600-00
7.755-03	2.715-04	1.386-05	9.414-01	2.440-01	4.870-06	7.000-01	7.000-01	2.900-01	8.901-00	1.949-00
1.210-02	2.902-04	1.324-05	9.618-01	2.445-01	4.633-06	7.000-01	7.000-01	2.900-01	1.513-01	2.382-00
1.655-02	3.143-04	1.253-05	9.572-01	2.447-01	4.361-06	7.000-01	7.000-01	2.900-01	1.781-01	2.756-00
2.537-02	3.853-04	1.067-05	1.056-00	2.410-01	3.743-06	7.000-01	7.000-01	2.900-01	1.354-01	3.493-00
3.737-02	6.090-04	8.000-06	1.219-00	2.347-01	2.705-06	7.000-01	7.000-01	2.900-01	0.000	4.764-00
DISTANCE FROM WALL	SINEM FUNCTION	ENTROPY S	PT/PTINF	PT(PILOT) PTINF	GAMMA					
(FT)	(ACTUAL)	(BTU/LB M)								
0.000	-0.000	-2.646-01	4.651-05	4.641-05	1.399-00					
1.485-04	5.033-06	-2.457-01	4.719-05	4.719-05	1.396-00					
2.545-04	1.406-05	-2.345-01	4.837-05	4.837-05	1.395-00					
4.799-04	4.600-05	-2.150-01	5.236-05	5.236-05	1.393-00					
8.460-04	1.254-04	-1.930-01	6.217-05	6.217-05	1.390-00					
1.521-03	3.491-04	-1.726-01	8.622-05	8.622-05	1.387-00					
2.594-03	8.270-04	-1.619-01	1.317-04	1.285-04	1.385-00					
4.055-03	1.639-03	-1.601-01	1.972-04	1.763-04	1.385-00					
6.759-03	3.608-03	-1.660-01	3.373-04	2.503-04	1.386-00					
1.210-02	7.492-03	-1.824-01	6.720-04	3.655-04	1.385-00					
1.696-02	1.249-02	-2.023-01	1.217-03	4.873-04	1.391-00					
2.537-02	2.431-02	-2.531-01	3.721-03	7.928-04	1.397-00					
3.433-02	4.338-02	-3.605-01	2.008-02	1.534-03	1.407-00					

Problem 5 - Sample Output (continued)

ITERATED VALUES DAMP MAX.LIN MAX.ERRORS IN CONSERVATION EQS.

ITS	TIME	ALPH	FPPW	ERROR	MOMENTUM	ENERGY
1	.372	1.242	1.5237	.1835	1.02 15	1.300 12
2	.737	1.421	1.6048	.3595	5.03 15	1.000 12
3	1.094	1.564	1.8084	.9300	2.02 15	6.301 12
4	1.465	1.580	1.95741	.0000	2.02 8	-2.001 5
5	1.826	1.591	1.98001	.0000	2.03 7	-4.602 4
6	2.187	1.594	1.98661	.0000	1.03 7	-1.001 4
7	2.548	1.595	1.98861	.0000	5.04 7	-3.303 4
8	2.908	1.596	1.98931	.0000	2.04 7	-1.103 4
9	3.275	1.596	1.98951	.0000	6.05 7	-3.604 4
10	3.634	1.596	1.98961	.0000	2.05 7	-1.204 4

ALPHA	KI	ROKAP	PHRESSURE	REF.	BETA	FLUX NUR- MALIZING DIFFUSIONAL PARAMETER	HEAT FLUXES TOT ENTH	RERAD	OCONO
1.59600	5.92705	1.00000	1.20602	3.81203	-2.83101	2.77203	2.25600	2.25600	2.25500

WALL
SHEAR
(LB/50 FT)
7.49001

MASS FLUXES
INJECTED
(LB/SEC SQ FT)
0.000

MOM TRANS HEAT TRANS
COEFF. COEFF.
(BASED ON CH)

RHO*UR*CF/2 RHO*UR*CH
6.32203 6.22803

0.000

MOMENTUM DISPLAC. EFFECTIVE ENTHALPY REYNOLDS
THICKNESS, THICKNESS, BODY THICKNESS, NUMBER
THETA DELSTAR DISPLAC. LAMDA PER FOOT
(FT) (FT) (FT) (REF.)
2.56604 -7.16303 8.75403 5.24603 1.52605

MODAL INFORMATION

DISTANCE, FROM WALL (FT)	ETA	F	FP (=U/UN)	FPP	SHEAR (LB/FTSQ)	TOTAL ENTH- MALPY.G (BTU/LB)	GP (BTU/LB)	GPP (BTU/LB)	STATIC ENTHALPY (BTU/LB)	TEMP (DEG R)	ELECTION COLL FREQ (1/SEC)
0.000	0.000	0.000	0.000	1.99003	7.49001	2.95400	4.96902	1.22703	2.95400	5.55002	3.94309
1.35004	3.83002	1.48003	7.77802	2.07200	7.51501	2.28901	5.43902	8.62002	2.11301	6.30402	3.74709
2.35004	6.38402	4.14303	1.30801	2.04300	7.53201	3.70501	5.65402	-3.65702	3.20801	6.75502	3.62009
4.52504	1.14901	1.34602	2.32301	1.89100	7.56801	6.54501	5.46702	1.69803	4.97901	7.48202	3.43909
8.10904	1.91501	3.62702	3.56201	1.34300	7.62301	1.02302	4.16702	-1.32703	6.55301	8.12502	3.30909
1.44903	3.19201	9.10802	4.89701	7.44901	7.71101	1.44702	2.47202	-5.73202	7.51401	8.51602	3.22409
2.42803	5.10701	1.96501	5.99801	4.00101	7.83201	1.81602	1.37402	-2.18602	7.71901	8.59902	3.20809
3.73203	7.66101	3.60901	6.80501	2.32401	7.96701	2.09502	8.16201	-7.51501	7.51601	8.51702	3.22404
6.28903	1.27700	7.33901	7.70901	1.21301	8.13501	2.41402	4.32401	-1.41901	6.90001	8.26602	3.27209
1.08802	2.23400	1.52200	8.68801	8.32302	8.09101	2.76302	2.96501	-6.15800	5.73101	7.79002	3.37104
1.51802	3.19200	2.39000	9.41201	6.80002	7.54001	3.01902	2.37501	-2.81000	4.48501	7.28002	3.48104
2.28102	5.10700	4.31200	1.06400	5.97102	4.92401	3.42202	1.83701	-8.94800	1.40601	6.01102	3.83704
3.12102	7.98000	7.55800	1.17700	1.86602	7.86803	3.65102	0.000	-3.84200	-3.65201	3.89702	4.76604

DISTANCE FROM WALL (FT)	DENSITY, RHO (LB/CU FT)	VISCOSITY, MU (LB/SEC FT)	WISCONSIN, RHO*MU (LB/SEC FT)	SPECIFIC HEAT (BTU/LB R)	THERMAL COND (BTU /SEC FT R)	PRANDTL NUMBER	MODIFIED SCHMIDT NUMBER	MOLECULAR WEIGHT	RHOSO*EPS /RHO*EUR	MACH NUMBER
0.000	6.63004	1.05205	1.14600	2.40401	3.61206	7.00001	7.00001	2.90001	0.000	0.000

Problem 5 - Sample Output (continued)

1.355-04	7.548-04	1.147-05	1.101-00	2.441-01	3.964-06	7.000-01	7.000-01	2.900-01	3.918-03	2.414-01
2.350-04	7.090-04	1.202-05	1.077-00	2.428-01	4.170-06	7.000-01	7.000-01	2.900-01	2.459-02	3.926-01
4.525-04	6.401-04	1.289-05	1.042-00	2.443-01	4.497-06	7.000-01	7.000-01	2.900-01	1.768-01	6.631-01
8.104-04	5.894-04	1.363-05	1.015-00	2.456-01	4.782-06	7.000-01	7.000-01	2.900-01	7.139-01	9.767-01
1.440-03	5.624-04	1.407-05	9.997-01	2.464-01	4.952-06	7.000-01	7.000-01	2.900-01	2.136-00	1.312-00
2.425-03	5.570-04	1.417-05	9.966-01	2.465-01	4.989-06	7.000-01	7.000-01	2.900-01	4.966-00	1.600-00
3.732-03	5.624-04	1.407-05	9.997-01	2.464-01	4.953-06	7.000-01	7.000-01	2.900-01	9.439-00	1.824-00
6.283-03	5.794-04	1.379-05	1.009-00	2.459-01	4.843-06	7.000-01	7.000-01	2.900-01	1.941-01	2.096-00
1.083-02	6.148-04	1.324-05	1.027-00	2.449-01	4.634-06	7.000-01	7.000-01	2.900-01	2.858-01	2.432-00
1.518-02	6.574-04	1.265-05	1.051-00	2.449-01	4.407-06	7.000-01	7.000-01	2.900-01	3.272-01	2.723-00
2.281-02	7.968-04	1.110-05	1.118-00	2.413-01	3.828-06	7.000-01	7.000-01	2.900-01	2.400-01	3.379-00
3.121-02	1.229-03	8.269-06	1.284-00	2.371-01	2.801-06	7.000-01	7.000-01	2.900-01	0.000	4.626-00
				PT(P/T)NF	PT(P/T)OT	GAMMA				
				PI/PTNF	PTNF					
				ENTROPY S	(BTU/LB R)					
				STREAM FUNCTION	(ACTUAL)					
				FROM WALL	(FT)					
				0.000	-0.000	9.854-05	9.854-05	1.399-00		
				1.355-04	1.611-05	1.026-04	1.026-04	1.395-00		
				2.350-04	4.510-05	1.045-04	1.045-04	1.393-00		
				4.525-04	1.465-04	1.321-04	1.321-04	1.390-00		
				8.104-04	9.949-04	1.808-04	1.808-04	1.387-00		
				1.440-03	9.916-04	2.763-04	2.763-04	1.385-00		
				2.425-03	2.139-03	4.172-04	3.732-04	1.385-00		
				3.732-03	3.929-03	5.871-04	4.698-04	1.385-00		
				6.283-03	7.990-03	9.014-04	8.067-04	1.386-00		
				1.083-02	1.657-02	1.541-03	8.054-04	1.388-00		
				1.518-02	2.602-02	2.446-03	1.007-03	1.391-00		
				2.281-02	4.695-02	2.968-01	6.667-03	1.396-00		
				3.121-02	8.228-02	-4.004-01	3.595-02	1.406-00		

Problem 5 - Sample Output (concluded)

SECTION IV

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

The BLIMP boundary layer code has been successfully extended to include turbulent flows with energy and entropy gradients and flows with transverse curvature. The basic analytical technique offers the advantages of completely general multicomponent chemistry, preservation of nonsimilar terms in the equations of motion, and intimate coupling to the surface material behavior.

There are several areas where the code and the analytical model which it includes can be improved or extended. One of these is in the general area of modeling turbulence in the equations of motion. The model that is currently used is perhaps the best available for general hypersonic flows with arbitrary injection; however, an extensive investigation into the generality of the model in all flow situations is called for. Experimental data comparisons at high Mach numbers and possible model changes to match these data would be most valuable.

A change in the program which would make it more accurate and convenient to use on some problems involves the α_H coordinate stretching parameter. On very long bodies, the boundary layer thickness varies over two or three orders of magnitude. The α_H parameter forces the boundary layer thickness to remain constant in the solution plane, thereby eliminating the need for a large number of nodes which are unused near the stagnation point or leading edge. For turbulent flows, the laminar sublayer grows at a much slower rate than the total boundary layer. As the normal coordinate is stretched in the solution plane, the nodes nearest the wall eventually can be pulled outside the sublayer altogether, resulting in poor convergence of the numerical technique and inaccurate solutions. A different stretching technique scaled to the sublayer as well as the boundary layer edge must be invented to avoid this problem.

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