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A CRITICAL EVALUATION OF ANALYTIC METHODS FOR PREDICTING LAMINAR BOUNDARY-LAYER, SHOCK-WAVE INTERACTION

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16. Abstract

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SYMBOLS

The symbols employed by the originators of the various theoretical methods have been employed in the discussion of these theories. While this may cause some confusion in the present report, it is felt that the use of these symbols will increase the usefulness of this paper as a cross reference to the original sources.

A $\dfrac{1 - M^2}{M^2}$

a velocity profile parameter employed in references 2 and 3

b enthalpy profile parameter employed in reference 3

C Chapman-Rubesin constant

$C_i(x)$ parameter of the shear profile employed in reference 4

C_f skin-friction coefficient, $\dfrac{\tau_w}{1/2(\rho_o u_o^2)}$

c acoustic velocity

D denominator of the right-hand side of moment equations in references 2 and 3

E_1 parameter of the enthalpy profile employed in reference 4

f dimensionless stream function

H total enthalpy

h static enthalpy or flow parameter employed in references 2 and 3

M Mach number

N_i numerator of the right-hand side of the moment equation in references 2 and 3

Pr Prandtl number

p pressure

q heat-transfer rate, Btu/ft^2 sec

Re unit Reynolds number based on boundary-layer-edge conditions at x_0

$\mathrm{Re}_{\delta_i^*}$ Reynolds number based on Re and δ_i^*, the transformed displacement thickness

S $\dfrac{H}{H_e} - 1$

T absolute temperature, °R

u streamwise component of velocity

v cross-stream component of velocity

x streamwise space variable

y cross-stream space variable

α_s $\dfrac{\partial u}{\partial \eta}$ on zero velocity streamline in separated region employed in references 4 and 5

$\bar{\beta}$ Stewartson transformation coefficient

β pressure gradient parameter

γ isentropic exponent

δ boundary-layer thickness

δ_i^* boundary-layer displacement thickness

δ_i^* transformed boundary-layer displacement thickness

η transformed y variable

μ viscosity

ρ density

ϕ flow turning angle across incident shock

ω exponent in temperature-viscosity relation

Subscripts

aw   adiabatic wall conditions (Pr = 1)

e   boundary-layer edge

f final pressure level immediately downstream of interaction

i shock impingement point

o beginning of interaction

s zero velocity streamline

w wall

∞ undisturbed free-stream conditions

δ evaluated at $\dfrac{u}{u_e} = 0.99$

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SUMMARY

The present paper is an evaluation of methods for predicting laminar boundary-layer characteristics in the presence of an impinging shock wave. Three analytic methods for describing the observed interaction phenomena are discussed, and results obtained with these methods are compared with selected experimental data. It is shown that the presently available theoretical methods are in essential agreement with each other at all flow conditions considered. The predictions agree with experimental data for weak shock waves at low Mach numbers, but not for strong shock waves at hypersonic speeds.

INTRODUCTION

The present paper reports on existing analytic methods for predicting laminar boundary-layer parameters in the presence of an oblique impinging shock wave with or without boundary-layer separation. A recent survey article by Brown and Stewartson (ref. 1) provides an introduction to the mathematical complexities of the problem as well as a historical introduction to the subject matter. In order to avoid repetition, and to keep the present paper to a reasonable size, some familiarity with the several theoretical methods on the part of the reader has been assumed.

The scope of this study is restricted to completely analytic methods in which both the boundary-layer parameters and the pressure distribution are computed for the entire interaction as part of the solution. As a result of this restriction, only three basic methods are considered. They are the method of Lees and Reeves (ref. 2) and its extension to nonadiabatic flows by Klineberg (ref. 3), the method of Nielsen, Lynes, and Goodwin (refs. 4 and 5), and the method of Reyhner and Flügge-Lotz (ref. 6). The methods differ in their mathematical structure but are virtually identical in their underlying physical assumptions. The methods are compared with each other and with carefully selected experimental data for Mach numbers from 2 to 9.7, wall temperature ratios (T_w/T_{aw}) from 0.2 to 1.0, and unit Reynolds numbers (based on undisturbed edge conditions) from 0.72×10^6 to 4.4×10^6 ft^{-1}.

DESCRIPTION OF THE ANALYTICAL METHODS

The analytic methods for predicting laminar boundary-layer, shock-wave interactions will be described in two stages. First, the physical model together with the governing equations will be

described, and second, the mathematical procedures employed in each of the methods will be discussed. Before embarking on these discussions, however, it is necessary to treat a mathematical point that has hindered analysis of the problem for the past 20 years.

The Separation Point Singularity

In 1948 Goldstein (ref. 7) performed an analysis which indicated that when the free-stream velocity distribution was specified, the boundary-layer equations contain a singularity at the separation point. Many analysts considered this to be the final word, despite the fact that in the same paper Goldstein remarked, "Another possibility is that a singularity will always occur except for certain special pressure variations in the neighborhood of separation, and that, experimentally, whatever we may do, the pressure variations near separation will always be such that no singularity will occur." This behavior of the pressure distribution (i.e., the local adjustment to avoid the singularity) is explicitly provided for in the methods considered here. Unfortunately, in earlier analyses Lees and Reeves and Nielsen, Lynes, and Goodwin employed integral approximation methods in their solutions so that their success in passing the separation point failed to convince some analysts. In 1966 Catherall and Mangler (ref. 8) carried out a careful numerical solution of the incompressible laminar boundary layer approaching separation wherein they relaxed the condition of the imposed pressure distribution in the vicinity of the separation point and instead imposed the condition that the displacement thickness δ^* be regular in the neighborhood of the separation point. This solution was found to pass smoothly through the separation point. With the publication of this last study it has been generally accepted that the boundary-layer equations need not contain a singularity at the separation point for all flow conditions.

Physical Model and Governing Equations

The description of the physical model and the governing equations is simplified by the fact that all the methods considered employ essentially the same model. The equations are those of the conservation of mass, momentum, and energy to the boundary-layer approximation, plus an equation of state and a so-called "free-interaction" relation that couples the local viscous and inviscid flows. This relation employs the Prandtl-Meyer equation to relate the local pressure gradient to the rate of change of the angular displacement of the inviscid flow. The term "free-interaction" is used since the pressure gradient is dependent only on local properties and is insensitive to the downstream cause of the perturbation. Implicit in this relation is the assumption that the viscous and inviscid flow interact only along a single coupling line. The equations can be written as

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{2a}$$

2

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\frac{\partial H}{\partial y}\right) - \frac{\partial}{\partial y}\left[\mu\left(\frac{1}{Pr}-1\right)u\frac{\partial u}{\partial y}\right] \tag{3}$$

$$p = p(\rho, T) \tag{4}$$

$$\frac{dp}{dx} = f\left(\frac{v_e}{u_e}\right) \quad \text{or} \quad f\left(\frac{d\delta^*}{dx}\right) \tag{5}$$

Each theoretical method discussed employs some form of the above equations. Initial conditions are prescribed at the assumed beginning of interaction x_o, and the interaction is initiated by one of the following procedures: The methods of Lees and Reeves, Klineberg, and Nielsen, Lynes, and Goodwin employ a small positive pulse in surface pressure, which causes an outward displacement of the local boundary-layer edge or the displacement thickness line;[1] in the method of Reyhner and Flügge-Lotz the initial portion of the calculation is carried out in a weak prescribed adverse pressure distribution. The outward displacement in turn increases the pressure through the free-interaction relation (eq. (5)), and the process amplifies in the streamwise direction until the shock impingement point, x_i, is reached (see sketch (a)). At this point, the coupling line and the entire inviscid flow are turned through an angle ϕ:[2]
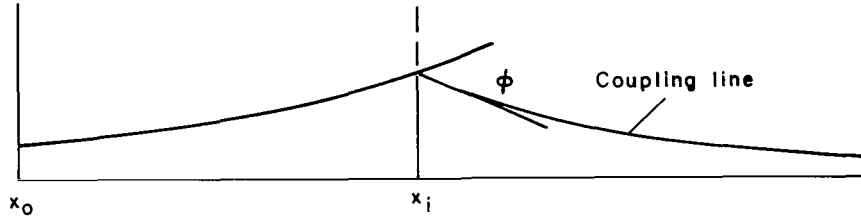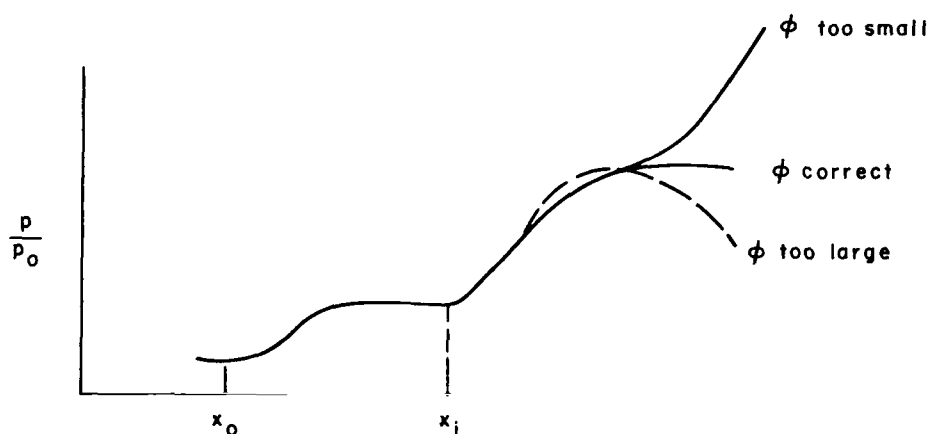

Sketch (a)

The angle ϕ is chosen in the methods of Lees and Reeves and of Klineberg such that an isentropic turn back to the original free-stream direction will provide the desired final pressure, and either x_o or x_i is employed as an iteration parameter. In the methods of Reyhner and Flügge-Lotz and of Nielsen, Lynes, and Goodwin, the angle ϕ itself is used as an iteration parameter. For the value of ϕ chosen, the calculation procedure employed upstream of shock impingement is resumed and carried on until some downstream conditions are satisfied or until it becomes obvious that they cannot be satisfied. The downstream compatibility condition in the methods of Reyhner and Flügge-Lotz and of Nielsen et al., is $dp/dx = d^2p/dx^2 = 0$, while in the methods of Lees and Reeves and of Klineberg, it is assumed that the solution passes through the Crocco-Lees point (see appendix) and approaches the flat-plate solution far downstream. The ϕ iteration process is

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[1] The sign of $d\delta^*/dp$ in laminar boundary layers is discussed in the appendix.

[2] By virtue of the rotation of the inviscid flow through the angle ϕ the coupling line inclination is a monotonically increasing function of x throughout the interaction as viewed by the inviscid flow.

illustrated in the computed pressure distributions of sketch (b). Depending on the direction of divergence of the solution, ϕ is either increased or decreased until the solution is bracketed. At this point the increment of ϕ, is successively halved until the desired convergence criteria are satisfied. As a result of imposing the downstream boundary condition on the system of parabolic equations, the physically elliptic problem is solved as a two-point boundary-value problem rather than the initial value problem of classical boundary-layer theory. The three parameters of the solution are x_0, the beginning of interaction, x_i, the shock impingement point, and ϕ, the flow turning angle (or shock strength). Once any two of these parameters are chosen, the third parameter is uniquely specified.

Sketch (b)

Mathematical Procedures

As noted above, the physical models employed by the various analytic methods are virtually indistinguishable. Consequently, differences in the predicted results of the several methods must be attributed to the different mathematical techniques employed in the solution of the governing equations. These differences arise primarily from the manner in which the system of equations (eqs. (1)-(5)) is reduced to a system that is amenable to computer solution.

The methods employed by Lees and Reeves, Klineberg, and Nielsen et al., fit into the broad category of moment methods. In these methods analytic or tabulated functions are assumed to describe the y dependence of the unknowns in terms of x-dependent parameters. These approximating functions are then substituted into the partial differential equations, and the resulting equations are multiplied by some weighting function (e.g., u^i, $i = 0, 1, \ldots n$) and integrated with respect to y. The result is a system of ordinary differential equations describing the variation of the x-dependent profile parameters. The accuracy of these methods then depends on the choice of the approximating functions (i.e., the assumed profiles) and on the weighting functions chosen (cf. ref. 9).

Method of Lees and Reeves— The Lees and Reeves' method employs the continuity, momentum, and first moment of momentum equations for flow over adiabatic walls. The first moment of momentum equation is obtained by choosing u as the weighting function. The

4

approximating function for the velocity profile is represented by the Falkner-Skan family of profiles for attached flow and by the Stewartson reverse-flow family of profiles (ref. 10) for separated flow. The dependence of these profiles on the pressure gradient parameter β is neglected in favor of a new parameter a defined as

$$a = \eta_\delta f_w'' \qquad \text{attached flow}$$

$$a = \frac{\eta_s}{\eta_\delta} \qquad \text{separated flow}$$

where

$$f_w'' = \frac{\partial u/u_e}{\partial \eta}\Bigg|_{\eta=0}$$

η is the cross-stream variable (transformed y) under the Stewartson transformation, and η_s is the value of this variable on the zero velocity streamline between the forward and reverse flow.[3] With these assumptions and definitions, the transformed continuity, momentum, and moment of momentum equations can be integrated with respect to η to yield the system of ordinary differential equations:

$$\frac{\delta_i^*}{M_e}\frac{dM_e}{dx} = \frac{\overline{\beta}C}{Re_{\delta_i^*}}\frac{M_\infty}{M_e}\frac{N_1(M_e, a, h)}{D(M_e, a)} \qquad (6)$$

$$\frac{d\delta_i^*}{dx} = \frac{\overline{\beta}C}{Re_{\delta_i^*}}\frac{M_\infty}{M_e}\frac{N_2(M_e, a, h)}{D(M_e, a)} \qquad (7)$$

$$\delta_i^*\frac{da}{dx} = \frac{\overline{\beta}C}{Re_{\delta_i^*}}\frac{M_\infty}{M_e}\frac{N_3(M_e, a, h)}{D(M_e, a)} \qquad (8)$$

where $\overline{\beta} = c_e p_e / c_\infty p_\infty$, c is the acoustic velocity, C is the Chapman-Rubesin constant, N_i and D are complicated functions of the arguments noted, and h is a parameter of both the viscous and inviscid flow. The streamwise integration is carried out in an iterative fashion until the downstream compatibility relation is satisfied. The integration is conceptually straightforward, but in its practical application is quite complicated. Readers interested in the iteration procedure are referred to references 2 and 3.

[3] A similar analysis was carried out by Bray, Gadd, and Woodger (ref. 11) wherein the β coupling was retained. It was found that the experimentally observed plateau pressure could not be sustained, apparently because only the Blasius and the Chapman profiles (ref. 12) are consistent with zero pressure gradient similarity flows.

Method of Klineberg– Klineberg's method (ref. 3) extends the method of reference 2 to the more general case of nonadiabatic flow with initial conditions characteristic of flow in a region of weak interaction. Holden (ref. 13) extends the method of Lees and Reeves to nonadiabatic flows but his method is not considered separately here since it is essentially contained within the method of Klineberg. The velocity and enthalpy profiles employed by Klineberg are those of Cohen and Reshotko (ref. 14). The addition of the enthalpy profile as an unknown in the analysis requires an additional equation (the energy equation) and a parameter b proportional to the enthalpy gradient at the wall. The system of equations becomes:

$$\frac{\delta_i^*}{M_e} \frac{dM_e}{dx} = \frac{\overline{\beta C}}{Re_{\delta_i^*}} \frac{M_\infty}{M_e} \frac{N_1(M_e,a,b,h)}{D(M_e,a,b)} \tag{9}$$

$$\frac{d\delta_i^*}{dx} = \frac{\overline{\beta C}}{Re_{\delta_i^*}} \frac{M_\infty}{M_e} \frac{N_2(M_e,a,b,h)}{D(M_e,a,b)} \tag{10}$$

$$\delta_i^* \frac{da}{dx} = \frac{\overline{\beta C}}{Re_{\delta_i^*}} \frac{M_\infty}{M_e} \frac{N_3(M_e,a,b,h)}{D(M_e,a,b)} \tag{11}$$

$$\delta_i^* \frac{db}{dx} = \frac{\overline{\beta C}}{Re_{\delta_i^*}} \frac{M_\infty}{M_e} \frac{N_4(M_e,a,b,h)}{D(M_e,a,b)} \tag{12}$$

Again streamwise integration is conceptually straightforward. In the case of flow over cooled walls, however, additional difficulties are encountered in the streamwise integration. These difficulties are associated with the behavior of so-called "supercritical" boundary layers. This concept and the related concept of a critical temperature ratio as proposed by Nielsen et al., are discussed in the appendix.

Method of Nielsen, Lynes, and Goodwin– In their method Nielsen, Lynes, and Goodwin employ a "Crocco like" coordinate system, and for attached flow approximate the shear profile as:

$$\frac{\partial \hat{u}}{\partial n} = \frac{(1 - \hat{u}) \sqrt{\hat{u} + C_4(x)}}{C_1(x) + C_2(x)\hat{u} + C_3(x)\hat{u}^2}$$

and the enthalpy profile as:

$$S = S_w(1 - \hat{u}) + E_1(x)(1 - \hat{u}) \left[\sqrt{C_4(x)} - \sqrt{\hat{u} + C_4(x)}\right]$$

where

$$\hat{u} = u/u_e.$$

6

In the separated flow regime the above equations are employed above the zero velocity streamline. Below this line the velocity is assumed to have the form:

$$\hat{u} = -\alpha_s \eta \left(1 - \frac{\eta}{\eta_s} \right)$$

and the enthalpy profile is taken as:

$$S = S_w + (S_s - S_w) \frac{\eta}{\eta_s}$$

Conditions are imposed so that the first and second derivatives of the velocity profile and the first derivative of the enthalpy profile are continuous across the $u = 0$ line. The equations employed are the first four moments of the momentum equation and the first moment of the total energy equation, giving a system of five ordinary differential equations in the five unknowns $C_i(x)$, $i = 1,4$, and $E_1(x)$. Streamwise integration is carried out by a standard fourth-order Adams-Moulton routine.

Method of Reyhner and Flügge-Lotz — The only method considered that applies a full finite difference technique to equations (1) – (5) is that of Reyhner and Flügge Lotz. They represent derivatives in the cross-stream direction by a finite difference approximation. The result is a system of equations that can be solved as ordinary differential equations in the streamwise direction. The y-dependent problem is solved iteratively at each x location and the streamwise integration is carried out by the Crank-Nicolson method. This procedure provides an exact numerical solution to equations (1)–(5) in the attached flow region, and with the additional assumption that $u(\partial u / \partial x) = u(\partial H / \partial x) = 0$ below the zero velocity streamline, it provides an exact numerical solution in the region of separated flow. This last assumption was made necessary by the appearance of nondamping eigenvalues in this flow region which results in an inherently unstable system of equations. The streamwise integration is subject to the conditions described in the preceding section. Initial velocity and temperature profiles may be input in tabular form or, alternatively, provision is made within the program to compute a flat plate initial profile. Wall temperature and/or mass transfer distributions may be input as functions of the streamwise variable. As a result, the Reyhner and Flügge-Lotz method is the most general method considered here.

COMPUTER PROGRAMS

In terms of generality, the method of Reyhner and Flügge-Lotz is clearly superior to the others because of the transport property options, which include non-unity Prandtl number, and both the Sutherland and power laws of viscosity, and the capability of employing tabulated initial profiles as well as allowing for nonisothermal walls with or without mass transfer. The transport properties employed by the remaining methods are $Pr = 1$ and the Chapman-Rubesin viscosity law. The boundary conditions imposed at the wall are $u_w = v_w = 0$ and $T_w/T_{aw} = \text{const}$. Initial conditions employed by these methods are either flat-plate similarity profiles as employed by Nielsen, Lynes, and Goodwin, and by Lees and Reeves or the hypersonic strong- or weak-interaction solutions (i.e., leading-edge viscous interaction solutions) employed by Klineberg. The transport properties and boundary condition options are summarized in table 1.

Each theoretical method discussed in the present study is embodied in a computer program. In each case the program used in the comparisons was provided by the respective authors. However, certain modifications were required to make the program compatible with the NASA-Ames DCS 7040–7094 computer. These modifications were made with reasonable care to prevent any loss of accuracy; however, the program for the method of Reyhner and Flugge-Lotz was written for the CDC 6600 employing 15 significant figures, and the 7040–7094 employs only 9 significant figures. Since double precision was not used in the program conversion, for some conditions the convergence criteria employed in this program had to be relaxed somewhat.

In terms of user convenience, the method of Nielsen, Lynes, and Goodwin is superior to the others considered. Only three input cards are required, and all setup and iteration procedures are carried out internally. Klineberg's is the most difficult to use for two reasons: First, the program was written as a research program and as such was never intended for "batch" calculations. The second reason is directly associated with the underlying theory. Since the approximating functions for velocity and enthalpy are tabulated functions of a and b, the functions appearing on the right-hand side of equations (9) through (12) must be generated externally and curve fitted. A separate subroutine must be written employing these curve fits for each value of wall temperature ratio, T_w/T_{aw}, considered. At the present time these subroutines are available only for $T_w/T_{aw} = 0.2$.

EXPERIMENTAL DATA

In choosing the experimental data for comparison with the analytic methods the following criteria were employed:

1. In addition to presenting the pressure distribution throughout the interaction region, the data source should provide supplementary information such as schlieren photographs, skin-friction distributions, or heat-transfer distributions.

2. The flow should be two-dimensional and laminar throughout the interaction. These criteria are necessarily qualitative in most cases. When the experimental data did not include downstream profiles or skin-friction distributions, the flow was considered laminar if a well-defined edge white line on the schlieren photograph could be traced throughout the interaction. A flow was considered to be two-dimensional if the predicted downstream mass flow profiles matched those obtained experimentally or, lacking measured downstream profiles, if the aspect ratio (i.e., model width divided by distance from the leading edge to shock impingement) was of the order of unity or greater. Lewis (ref. 15) has shown that, at $M_\infty = 6.06$, the flow over a flat-plate-ramp combination has substantial three-dimensional effects for aspect ratios less than 1.0.

COMPARISON OF EXPERIMENTAL AND ANALYTIC RESULTS

General

Before the results of the analytic methods are compared with experimental data, a few words are necessary to describe the manner in which the comparisons were made. As noted earlier, specifying any two of the three parameters x_O, x_i, and ϕ (or the shock strength) is sufficient to provide a unique solution to the analytic problem. Unfortunately, none of these parameters can be determined very precisely even when experimental data are available, so that a certain lack of uniqueness exists in the application of these analytic methods to the prediction of experimental data. In the present study each analytic method was used in an iterative fashion to obtain a reasonably good match to the pressure distribution over the entire interaction regime. For flow at high Mach numbers (i.e., $M_\infty \geq 7.4$) it was found that this procedure could not be followed except for very weak impinging shock waves. When the entire pressure distribution could not be matched, it was decided to match the pressure distribution upstream of shock impingement (i.e., $x_O \leq x \leq x_i$). When this procedure was followed, the final pressure ratio was generally underpredicted. It should also be pointed out that all predictions presented were obtained under the assumptions $Pr = 1.0$ and $\mu/\mu_O = T/T_O$.

The Data of Hakkinen, Greber, Trilling, and Abarbanel

In figures 1(a) through 1(c) the three analytic methods are compared with the data of Hakkinen, Greber, Trilling, and Abarbanel (ref. 16) at a Mach number of 2.0. An experimental measure of skin friction was obtained for these data employing the pitot probe as a Preston tube. It can be seen from these figures that the predicted pressure distributions are indistinguishable from each other and agree well with the experimental data. The predicted skin-friction distributions also agree closely with each other but only qualitatively with the data. It can be seen further in these figures that the analytic methods have a common tendency to overpredict the extent of separation relative to that observed experimentally.

The Data of Lewis

In figures 2(a) and 2(b) the three analytic methods are compared with the data of Lewis (ref. 15) at Mach numbers of 4 and 6.06. For these experiments, only surface pressure distributions were reported in reference 15. In figure 2 predicted skin friction is plotted for comparison among the theories even though experimental results are lacking. In figure 2(a) excellent agreement is found for surface pressures both among the theories and between theory and data. Some difficulty was experienced with the 7094 (single precision) version of the Reyhner and Flügge-Lotz program. In particular, convergence difficulties were encountered immediately downstream of shock impingement. The results shown for this program were obtained by Dr. Reyhner on the Boeing CDC 6600 using, what would be on the 7094, double precision calculations.

Figure 2(b) provides the only comparison, in the present study, of the method of Klineberg with other methods and with data, primarily because of the limitations on wall cooling ratio

implicit in Klineberg's method. Of particular interest is the discontinuity in both predicted surface pressure and skin friction in Klineberg's method which is brought about by the supercritical-subcritical jump mentioned earlier. The cause of the discrepancy in surface pressure distributions among the several theories in this case is difficult to pinpoint since slightly different values of x_O and x_i were used in each method in addition to the different profile descriptions, etc., described earlier. In any case, none of the theoretical methods depart from the data by more than 20 percent, which may be acceptable for many applications.

The Data of Needham

In figures 3(a) through 3(c) the methods of Reyhner and Flügge-Lotz and of Nielsen, Lynes, and Goodwin are compared with the data of Needham (ref. 17). Measured heat transfer rates were reported for these data, and are shown instead of the skin-friction coefficient. The Mach number for these tests was 7.4. The only variable parameter in this series of data is the shock strength. At high Mach numbers discrepancies between theories and the data begin to appear in the pressure distribution, even for relatively weak shocks. In figure 3(a), for example, the predicted pressure distributions display a well-defined plateau, while no such behavior is noted in the data. This is consistent with the previously noted tendency to overpredict the extent of separation. A comparison of the measured and predicted heat transfer, however, indicates a more serious shortcoming in the analytic methods. Downstream of the shock-impingement point, very large errors in predicted heat transfer are apparent. In figures 3(b) and 3(c), for increasing shock strength, the errors in predicted heat transfer become even larger. Furthermore, it is found that one can no longer match the final downstream pressure level if the pressure distribution upstream of shock impingement is to be matched.

In figures 4(a) and 4(b) the methods of Nielsen, Lynes, and Goodwin, and Reyhner and Flügge-Lotz are compared with additional data of Needham obtained at a Mach number of 9.7. The difference between these two sets of data is shock strength. Comparisons between the theories and comparison of theory with data are qualitatively the same as those discussed for figures 3(a) through 3(c). Again, for a weak shock, the pressure distribution throughout the interaction is reasonably well matched, and the heat-transfer distribution is rather poorly predicted downstream of shock impingement. At somewhat higher shock strengths, imposing x_O and x_i inferred from the data results in a substantial underprediction of the final pressure ratio.

IMPLICATIONS OF PRESENT EVALUATION

When all of the foregoing comparisons of theory and data are considered, the following general observations can be made. First, all the theoretical predictions agree surprisingly well with each other, considering the differences in the mathematical procedures employed. Second, for very weak shock waves, all the methods provide an adequate representation of the observed pressure distribution but have a uniform tendency to overpredict the extent of separation and underpredict heat-transfer rates. Third, for strong shocks when x_O, the beginning of interaction, and x_i, the shock impingement point, are determined from the data, all of the methods substantially underpredict the final pressure ratio.

10

Whether these discrepancies result from a lack of two-dimensionality in the experimental flows, transition near reattachment, or some basic shortcoming in the underlying physical model cannot be unequivocally determined from the present study. While reasonably convincing arguments can be marshalled in favor of any of the above possibilities, it is the author's opinion that the inability of the methods to predict the experimental results is not wholly associated with shortcomings in the experimental data. As mentioned earlier, care was exercised to insure that all data considered were obtained in pure laminar interactions and on models of relatively large aspect ratio. While in the absence of side plates, a large but finite aspect ratio does not guarantee two-dimensionality, it is felt that the uniformity of the behavior of the experimental results over a wide range of parameters militates against random discrepancies in the data. In order to determine how the existing physical model should be modified to provide a better quantitative description of the details of the flow, it is useful to reconsider the validity of the underlying assumptions. The first assumption is that the boundary-layer equations are valid. Implicit in this assumption is the condition $\partial p/\partial y = 0$. This condition together with the second assumption that the viscous and inviscid flows interact only along some line at or near the boundary-layer edge imposes the physically unrealistic condition that the supersonic inviscid flow cannot respond to perturbations in the boundary layer except insofar as the perturbations affect the inclination of the local boundary-layer edge or the displacement thickness line. Only by carrying out an analysis wherein these assumptions have not been made can the validity of these assumptions be tested. Some efforts in this direction have been made by Rose (ref. 18). His success in predicting both the experimental pressure and heat-transfer distributions of Needham in the strong shock, high Mach number case is considered to be strong evidence that the cause of the failure of the analytic methods described here to predict these data lies within the assumptions cited rather than with the experimental results. The third and last assumption is that the so-called downstream compatibility conditions are meaningful and correct. Since the location of a downstream critical point is related to the specific assumptions made for velocity and enthalpy profiles and to the choice of the line along which the viscous and inviscid flows are coupled, the condition that the solution pass smoothly through this point, as in the methods of Lees and Reeves and of Klineberg, while consistent with the boundary-layer approximation, seems somewhat physically artificial (see refs. 19 and 20). The alternate condition employed by Reyhner and Flügge-Lotz and by Nielsen, Lynes, and Goodwin, is unfortunately equally artificial in that the simultaneous satisfaction of the conditions $dp/dx = d^2 p/dx^2 = 0$ is inconsistent with the free interaction model, and it is this inconsistency that prevents continued downstream integration in these latter methods. It is clear that some downstream boundary condition is required to provide a relation between upstream influence and shock strength if nonelliptic equations are to be employed, but as to which, if either, of the above downstream conditions is valid, remains to be demonstrated.

CONCLUSIONS

The methods considered for predicting laminar boundary-layer shock-wave interaction yield substantially the same results so that the choice of which method to use in a particular application must be dictated by considerations other than accuracy.

All the methods predict surface pressure distributions which are in excellent agreement with experimental data in the range $2 \leq M_\infty \leq 6$. At higher Mach numbers there is a tendency on the

part of all the methods to underpredict the final pressure level when *the beginning of interaction* and the shock impingement point are specified from the data.

There is a uniform tendency to underpredict both skin friction and *heat transfer at all Mach* numbers, and consequently, to overpredict the extent of separation. Poor predictions of these parameters are obtained at high Mach numbers.

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        Moffett Field, Calif., 94035, Aug. 19, 1970

APPENDIX

SUBCRITICAL AND SUPERCRITICAL BOUNDARY LAYERS AND THE

CRITICAL TEMPERATURE RATIO

As noted in the description of the analytic model employed in all the methods considered here, it is required that the boundary layer respond to an adverse pressure gradient by thickening. The resulting outward displacement of the inviscid flow causes an increase in pressure through the free-interaction relation (eq. (5)), and feeds back into the boundary-layer equations such that the whole process is self-supporting in the downstream direction. In 1955, Crocco (ref. 21) described this behavior in an extension of the Crocco-Lees mixing theory to the problem of shock-wave — boundary-layer interaction. From his analysis he deduced that the displacement of the boundary-layer edge under a positive pressure gradient $[(d\delta/dx)/(dp/dx) = d\delta/dp]$ can either increase or decrease depending on the details of the velocity and density profiles, the magnitude of the edge velocity and the choice of the boundary-layer edge δ. Boundary layers for which $d\delta/dp$ was positive were termed subcritical and boundary layers for which $d\delta/dp$ was negative were termed supercritical. The net result of this finding is that only subcritical boundary layers $(d\delta/dp > 0)$ are consistent with the free-interaction model employed by the methods considered here.

In applying their method to flows over a cooled wall, Nielsen, Lynes, and Goodwin (ref. 4) found that when the wall temperature ratio was reduced below some critical value, no free interaction could be induced by a pressure pulse $((d\delta/dp) \leq 0)$ or supercritical behavior. Fleeman (ref. 22) used this program to map the locus of the critical temperature ratio as a function of Mach number.

Lees and Reeves (ref. 2) discuss extensively the general theory of the application of moment methods to the prediction of shock-wave — boundary-layer interaction and discuss the concept in detail. Problems of supercritical-subcritical transition did not arise, however, in their treatment of adiabatic flows. When Klineberg (ref. 3) attempted to extend the Lees and Reeves procedure to flows with heat transfer he found that for cold walls, the initial profiles were supercritical. To circumvent this difficulty, Klineberg introduced a discontinuous jump from the supercritical to the subcritical state to permit free interaction when the boundary layer is initially supercritical. Many other investigators have carried out analyses with various degrees of approximation to determine when supercritical behavior is to be expected. One of the most interesting and complete of these analyses is by Weinbaum (ref. 23) who derived from the boundary-layer equations an expression of the form:

$$\frac{dp}{dx} = \frac{p\left(\dfrac{v_e}{u_e}\right) + \displaystyle\int_0^{\delta} \frac{h}{pu}\left[\frac{1}{u}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{1}{Pr^2 h}\frac{\partial}{\partial y}\left(\frac{\mu}{h}\frac{\partial h}{\partial y}\right) - (\gamma - 1)M_\infty^2 \frac{\mu}{h}\left(\frac{\partial u}{\partial y}\right)^2\right]dy}{\dfrac{1}{\gamma}\displaystyle\int_0^{\delta} A\,dy}$$

where

$$A = \frac{1 - M^2}{M^2}$$

It is clear from this expression that when $\int_0^\delta A \, dy = 0$, no finite value of dp/dx is consistent with the boundary-layer equations unless the numerator goes to zero at the same rate. This is equivalent to the condition used by Klineberg that N_i and D must simultaneously approach zero in order to pass through the downstream critical, or Crocco-Lees, point.

The zeros of the integral $\int_0^\delta A \, dy$ are interpreted as the streamwise locations where the boundary layer passes from supercritical to subcritical or vice versa. Since the integrand $(1 - M^2)/M^2$ is a strong function of the velocity and temperature profiles, it is not surprising that methods employing different approximate representations of the velocity and temperature profiles provide conflicting testimony regarding the condition of a given flow (i.e., whether it be subcritical or supercritical). To avoid the effects of the assumed velocity and temperature profile forms, the method of Reyhner and Flügge-Lotz was used in conjunction with the numerical integration of the Cohen and Reshotko Mach number profiles to deduce the effects of perturbations on zero pressure gradient flows for several Mach numbers, $2 \leq M_\infty \leq 10$, and wall temperature ratios, $0.03 \leq T_w/T_{aw} \leq 1.0$. It was found that while initially zero pressure gradient boundary layers frequently exhibit supercritical characteristics for high cooling rates even at moderate Mach numbers, they can undergo a smooth supercritical-subcritical transition as a result of a short streamwise exposure to a mild adverse pressure gradient. This transition is possible because the flow near the wall, having very little momentum, reacts rapidly to small adverse pressure gradients and yields a large positive contribution to the integral. The resulting condition is such that computing methods employing a small pulse in surface pressure to induce the interaction will not generate adverse pressure gradients unless specific allowance is provided for a supercritical-subcritical jump. The method of Reyhner and Flügge-Lotz, which induces the interaction by prescribing some initial region of small adverse pressure gradient, effectively bypasses the problem. With regard to the other analytic methods considered, only the method of Nielsen, Lynes, and Goodwin is limited to initially subcritical flows. In the present application, however, this does not constitute a serious shortcoming since the condition occurs only for $T_w/T_{aw} \leq 0.2$ at $M = 10$, and at even lower wall temperature ratios for Mach numbers less than 10.

14

REFERENCES

1. Brown, S. N.; and Stewartson, K.: Laminar Separation. Vol. 1 of Annual Reviews Fluid Mechanics, edited by W. R. Sears and M. Van Dyke, 1969, Annual Reviews, Inc., Palo Alto, California, pp. 45-72.

2. Lees, L.; and Reeves, B. L.: Supersonic Separated and Reattaching Laminar Flows: I. General Theory and Application to Adiabatic Boundary-Layer/Shock-Wave Interactions. AIAA J., vol. 2, no. 11, Nov. 1964, pp. 1907-1920.

3. Klineberg, J. M.: Theory of Laminar Viscous-Inviscid Interactions in Supersonic Flow. Ph.D. Thesis, Calif. Inst. Tech., June 1968.

4. Nielsen, J. N.; Lynes, L. L.; and Goodwin, F. K.: Calculation of Laminar Separation With Free Interaction by the Method of Integral Relations (AFFDL TR 65−107). Part I: Two-Dimensional Supersonic Adiabatic Flow, June 1965, Part II: Two-Dimensional Supersonic Nonadiabatic Flow and Axisymmetric Supersonic Adiabatic and Nonadiabatic Flows. Oct. 1965.

5. Goodwin, F. K.; Nielsen, J. N.; and Lynes, L. L.: Calculation of Laminar Boundary Layer − Shock-Wave Interaction by the Method of Integral Relations. Nielsen Engineering and Research, Inc., NEAR Rep. TR 2, July 25, 1967.

6. Reyhner, T. A.; and Flügge-Lotz, I.: The Interaction of a Shock Wave With a Laminar Boundary Layer. Internal Tech. Rep. 163, Div. Engin. Mech., Stanford Univ., Nov. 1966, published in abbreviated form in Int. J. Non-Linear Mech., vol. 3, no. 2, June 1968, pp. 173-199.

7. Goldstein, S.: On Laminar Boundary-Layer Flow Near a Position of Separation. Quart. J. Mech. and Appl. Math., vol. 1, 1948, pp. 43-69.

8. Catherall, D.; and Mangler, K. W.: The Integration of the Two-Dimensional Laminar Boundary-Layer Equations Past the Point of Vanishing Skin Friction. J. Fluid Mech., Sept. 1966, vol. 26, pt. 1, pp. 163-182.

9. Murphy, John D.; and Rose, William C.: Application of the Method of Integral Relations to the Calculation of Incompressible Turbulent Boundary Layer. Proc. Computation of Turbulent Boundary Layers, 1968, AFOSR−IFP Stanford Symposium, vol. 1, edited by S. J. Kline, G. Sovran, M. V. Morkovin, and D. J. Cockrell, Aug. 1968, Stanford Univ., Calif., pp. 54-75.

10. Stewartson, K.: Further Solutions of the Falker-Skan Equation. Proc. Cambridge Phil. Soc., vol. 50, pt. 3, 1954, pp. 445-465.

11. Bray, K. N. C.; Gadd, G. E.; and Woodger, M.: Some Calculations by the Crocco-Lees and Other Methods of Interactions Between Shock Waves and Laminar Boundary Layers, Including the Effects of Heat Transfer and Suction. Aeronautical Research Council ARC 21,834, April 5, 1960.

12. Chapman, D. R.: Laminar Mixing of a Compressible Fluid. NACA TR 958, 1950.

13. Holden, M. S.: An Analytical Study of Separated Flows Induced by Shock Wave Boundary Layer Interaction. NASA CR 600, 1966.

14. Cohen, Clarence B.; and Reshotko, Eli: Similar Solutions for the Compressible Laminar Boundary Layer With Heat Transfer and Pressure Gradient. NACA Rep. 1293, 1956.

15. Lewis, John E.: Experimental Investigation of Supersonic Laminar, Two-Dimensional Boundary Layer Separation in a Compression Corner With and Without Cooling. Ph.D. Thesis, Calif. Inst. Tech., 1967.

16. Hakkinen, R. J.; Greber, I.; Trilling, L.; and Abarbanel, S. S.: The Interaction of an Oblique Shock Wave With a Laminar Boundary Layer. NASA MEMO 2–18–59W, 1959.

17. Needham, D. A.: Laminar Separation in Hypersonic Flow. Ph.D. Thesis, Univ. of London, 1965.

18. Rose, W. C.: A Method for Analyzing the Interaction of an Oblique Shock Wave and a Boundary Layer. NASA SP 228, 1970, pp. 541-567.

19. Shamroth, S. J.: On Integral Methods for Predicting Shear Layer Behavior. ASME paper 69–WA/APM–11, 1969.

20. Shamroth, S. J.; and McDonald, H.: A New Solution of the Turbulent Near-Wake Recompression Problem. AIAA Paper 70-228, 1970.

21. Crocco, L.: Considerations on the Shock–Boundary Layer Interaction. Proceedings of the Conference on High-Speed Aeronautics, edited by A. Ferri, N. J. Hoff, and P. A. Libby, Polytechnic Inst. of Brooklyn, Jan. 20–22, 1955, pp. 75–112.

22. Fleeman, Eugene L.: Incipient Separation for Highly-Cooled Walls in a Hypersonic Free Stream. FDCC TM 66–13, Air Force Flight Dyn. Lab., Dec. 1966.

23. Weinbaum, Sheldon: Near Wake Uniqueness and a Re-examination of the Throat Concept in Laminar Mixing Theory. AIAA Paper 67–65, AIAA, Jan. 1967.

16

TABLE 1.– ASSUMPTIONS EMBODIED IN ANALYTIC METHODS

METHODS CONSIDERED	TRANSPORT PROPERTIES	WALL CONDITIONS
MOMENT METHODS LEES AND REEVES KLINEBERG NIELSEN, LYNES, AND GOODWIN	$Pr = 1, \ \dfrac{\mu}{\mu_0} = C\,\dfrac{T}{T_0}$	$\dfrac{T_w}{T_{aw}} = 1.0$ $\dfrac{T_w}{T_{aw}} = 0.2$ $\dfrac{T_w}{T_{aw}} = $ ARBI-TRARY* $\left.\begin{array}{c} \\ \\ \\ \\ \\ \\ \\ \end{array}\right\}$ $\dfrac{\partial H_w}{\partial x} = 0$ $V_w = 0$
FINITE DIFFERENCE REYHNER AND FLÜGGE-LOTZ	$Pr = $ CONST $\dfrac{\mu}{\mu_0}\left\{\begin{array}{l} = C\,\dfrac{T}{T_0} \\[2mm] = \left(\dfrac{T}{T_0}\right)^{\omega} \\[2mm] = \text{SUTHERLAND} \end{array}\right.$	$\dfrac{\partial H_w}{\partial x} = $ ARBITRARY $V_w(X) = $ ARBITRARY $\dfrac{T_w}{T_{aw}} = $ ARBITRARY

*THERE IS AN EFFECTIVE LOWER BOUND ON $\dfrac{T_w}{T_{aw}}$ FOR WHICH PHYSICALLY MEANINGFUL SOLUTIONS CAN BE OBTAINED. (SEE APPENDIX)
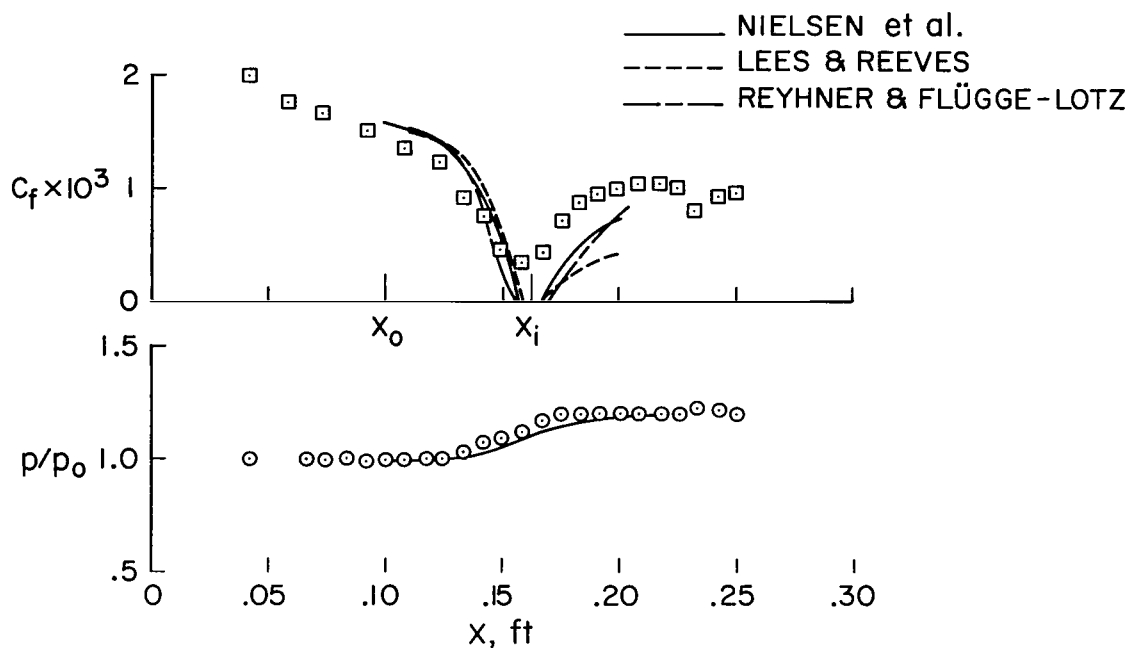
(a) $M_\infty = 2.0$; $T_w/T_{aw} = 1.0$, $Re = 1.78 \times 10^6$ ft^{-1}, $p_f/p_0 = 1.20$
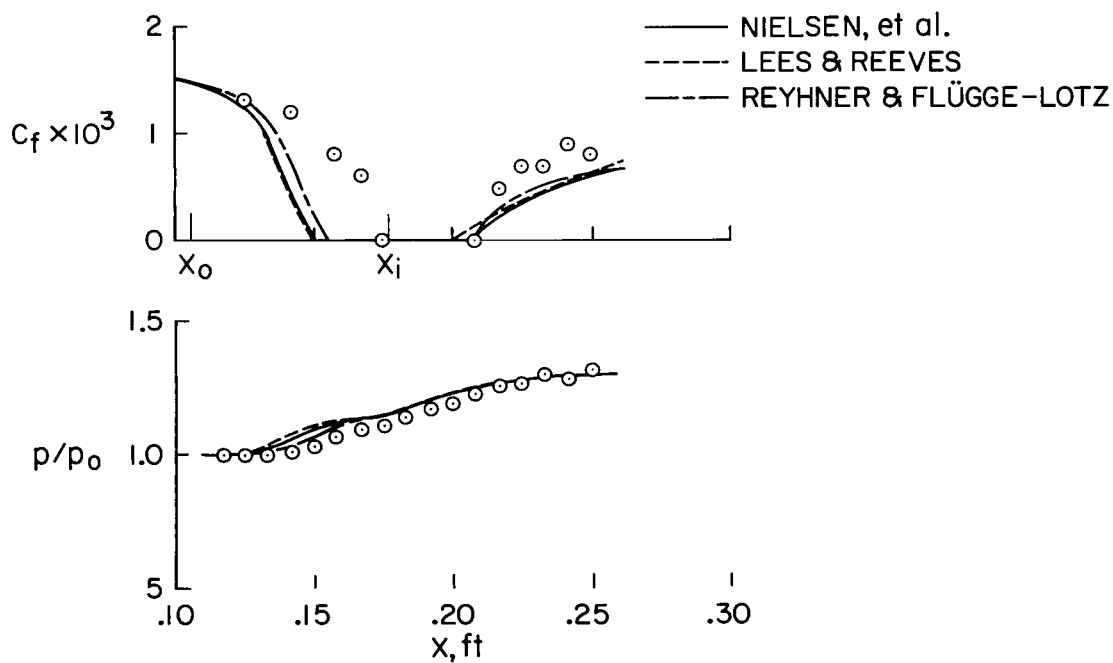
(b) $M_\infty = 2.0$; $T_w/T_{aw} = 1.0$, $Re = 1.92 \times 10^6$ ft^{-1}, $p_f/p_0 = 1.32$

Figure 1.— Comparison of analytic results with the data of Hakkinen et al. (ref. 16).

19
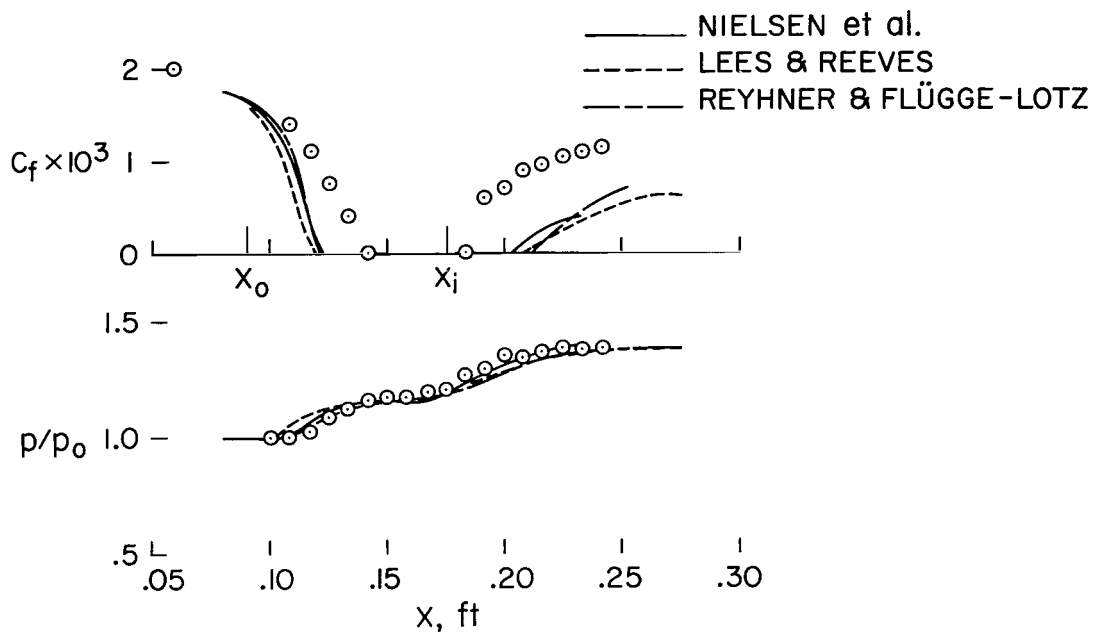
(c) $M_\infty = 2.0$; $T_w/T_{aw} = 1.0$, $Re = 1.78 \times 10^6$ ft^{-1}, $p_f/p_0 = 1.40$
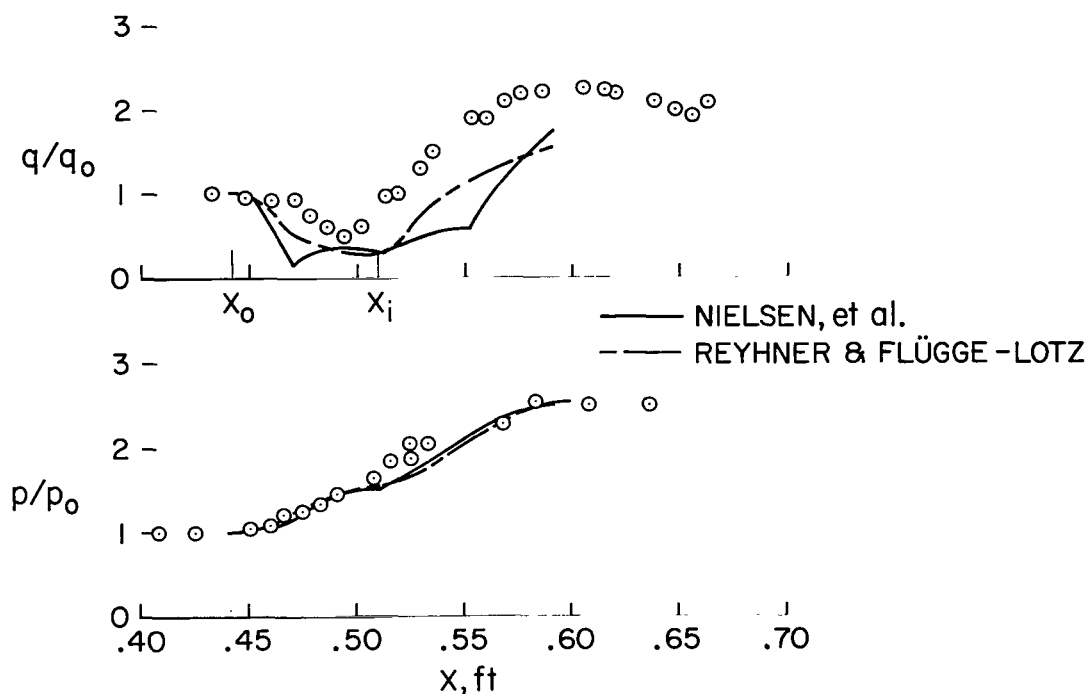
Figure 1.— Concluded.
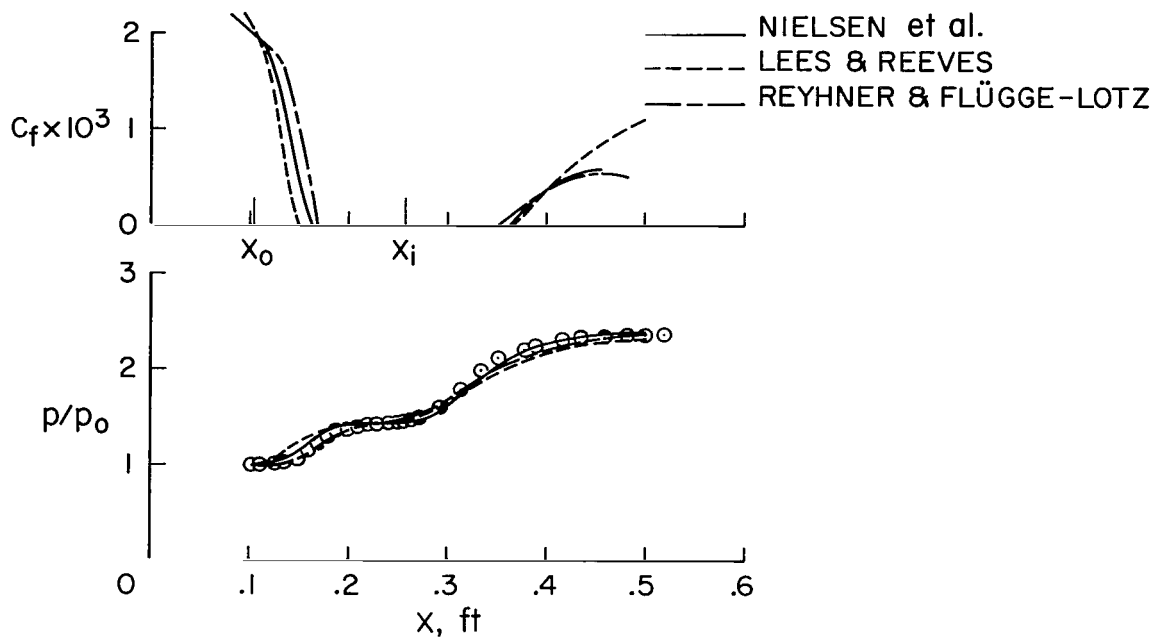
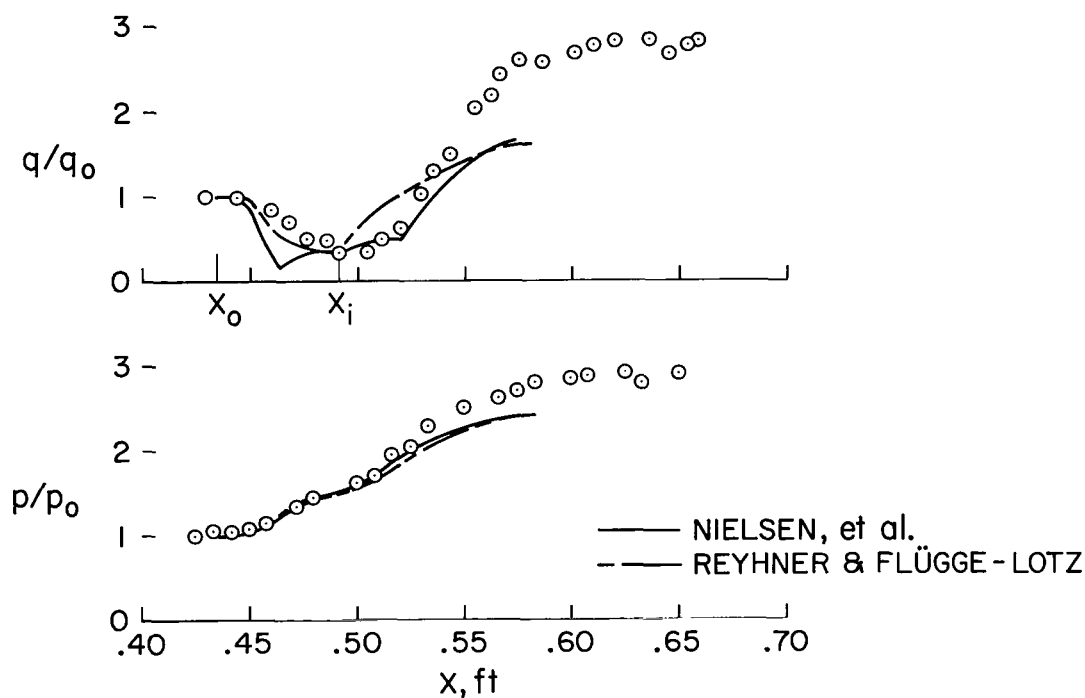(a)$M_\infty = 4$; $T_w/T_{aw} = 1$, $Re = 1.08 \times 10^6$ ft^{-1}, $p_f/p_0 = 2.35$
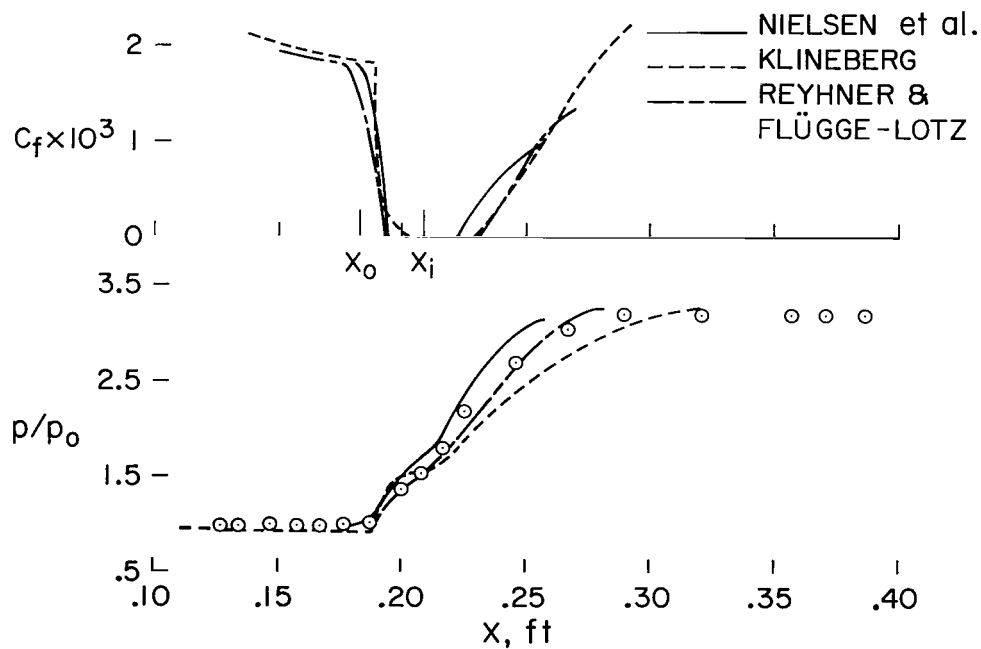
(b) $M_\infty = 6.06$; $T_w/T_{aw} = 0.2$, $Re = 0.72 \times 10^6$ ft^{-1}, $p_f/p_0 = 3.20$

Figure 2.— Comparison of analytic results with the data of Lewis (ref. 15).

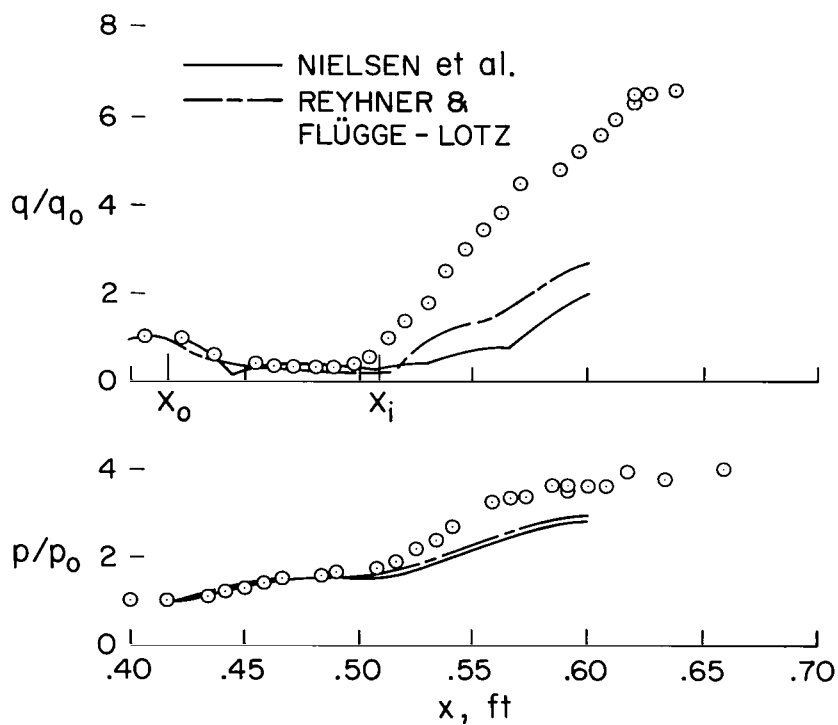(a) $M_\infty = 7.4$; $T_w/T_{aw} = 0.264$, $Re = 4.4 \times 10^6$ ft^{-1}, $p_f/p_0 = 2.50$

(b) $M_\infty = 7.4$; $T_w/T_{aw} = 0.264$, $Re = 4.4 \times 10^6$ ft^{-1}, $p_f/p_0 = 2.90$

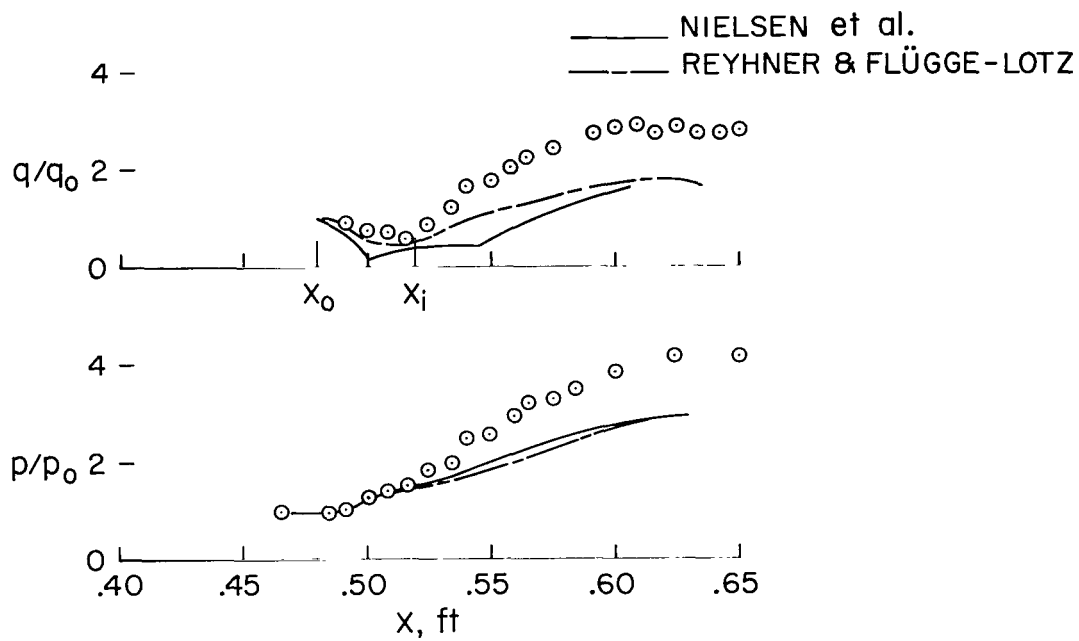Figure 3.— Comparison of analytic results with the data of Needham (ref. 17).
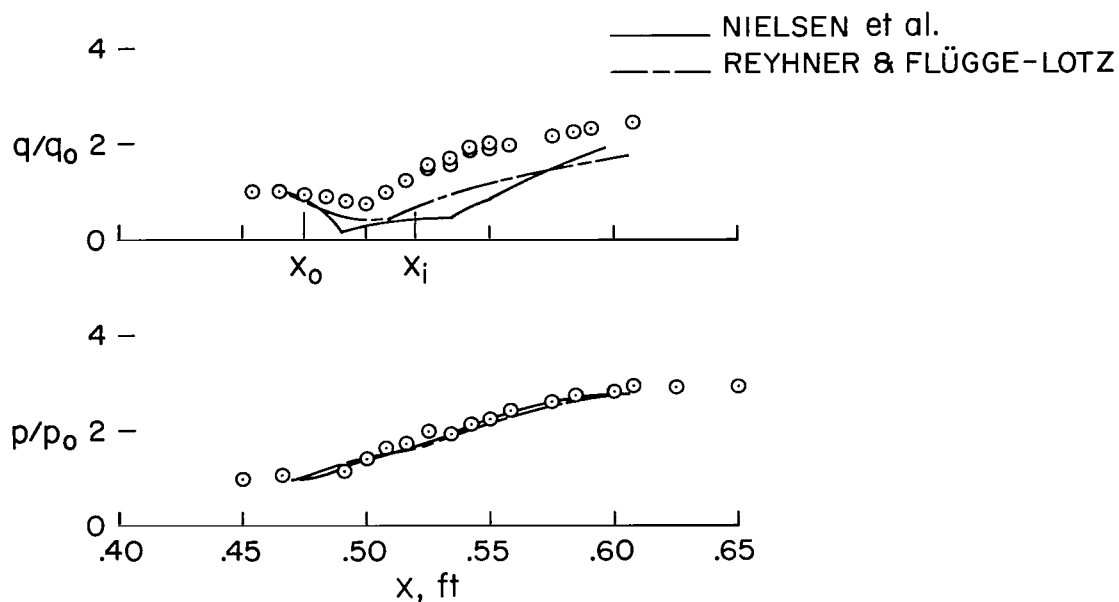
(a) $M_\infty = 9.7$; $T_w/T_{aw} = 0.223$, Re $= 1.91 \times 10^6$ ft^{-1}, $p_f/p_0 = 2.90$

(b) $M_\infty = 7.4$; $T_w/T_{aw} = 0.264$, Re $= 4.4 \times 10^6$ ft^{-1}, $p_f/p_0 = 3.80$

Figure 4.— Comparison of analytic results with the data of Needham (ref. 17).

(c) $M_{\infty} = 9.7$; $T_w/T_{aw} = 0.223$, $Re = 1.91 \times 10^6$ ft^{-1}, $p_f/p_0 = 4.0$

Figure 4.— Concluded.

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