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STANDARD DEVIATION OF VERTICAL WIND SHEAR IN THE LOWER ATMOSPHERE

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16. Abstract

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STANDARD DEVIATION OF VERTICAL WIND SHEAR IN THE LOWER ATMOSPHERE

SUMMARY

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I. INTRODUCTION

Statistical estimates of wind shear in the planetary boundary layer are important in the design of V/STOL aircraft (vertical/short take-off and landing aircraft). The STOL aircraft are sensitive to variations in lift forces resulting from vertical variations of the horizontal wind during take-off and particularly during landing operations. VTOL aircraft control systems must be designed to counteract extreme tipping moments resulting from vertical variations of the horizontal wind for specified acceptable values of risk of exceeding the design wind shear conditions. Similarly, wind shear information will also be necessary in the design of the forthcoming NASA Space Shuttle for both the launch and landing phases. During the launch phase the Shuttle will experience tipping moments; during the landing phase, it will experience fluctuating lift forces. The vertical two-point longitudinal velocity differences that will be experienced by these vehicles can be divided into two parts: the wind shear resulting from the mean flow, and the fluctuating or random part resulting from turbulence. The mean shear can be calculated with currently available models of the mean flow wind profile in the atmospheric boundary layer [1,2, and 3]. However, the fluctuating part must be specified statistically for engineering problems, and this requires knowledge about the statistical moments or the distribution function of the shear. A great deal of information can be gained by studying the second moment of wind shear. In addition, the square root of the second moment, the standard deviation, of a random variable is used as a standardizing parameter to render the random variable nondimensional. Accordingly, the analysis in this report is restricted to an examination of the second moment of vertical two-point longitudinal velocity differences, or, in other words, wind shear.

It is hypothesized that the standard deviation of the wind shear scaled with the surface friction velocity u_{*o} is a universal function of the nondimensional parameters $f\Delta z/u_{*o}$, $f\bar{z}/u_{*o}$, and u_{*o}/fL_o, where f is the Coriolis parameter, L_o is the surface Monin-Obukhov length, Δz is the vertical distance between the two points over which the shear is calculated, and $\bar{z}$ is the mean height of the mid-point of the shear interval Δz above natural grade.

From the eleven cases of turbulence in unstable air selected for analysis, it is concluded that σ/u_{*o} is indeed a function of the nondimensional parameters listed above. Empirical mathematical expressions are obtained to represent this function. It appears that σ/u_{*o} is proportional to $(f\Delta z/u_{*o})^{1/3}$, the coefficient of proportionality being a function of $f\bar{z}/u_{*o}$ and u_{*o}/fL_o. The results of the analysis are valid for $20 < -u_{*o}/fL_o < 2000$.

The author expresses his thanks to Mr. Archie Jackson and Mrs. Ella Mae McAllister of the Marshall Space Flight Center's Computation Laboratory. Mr. Jackson developed the program for the calculation of the two-point standard deviations. Mrs. McAllister developed the remaining programs which were used to calculate the non-dimensional quantities. In addition, thanks go to Mr. Julian Nelson and Mr. Douglas Mackiernan of the Aero-Astrodynamics Laboratory for their help in the preparation of this report and the analysis of the data.

II. THE DATA SOURCE

The data analyzed in this report consist of eleven sets of longitudinal turbulent velocity fluctuation time histories digitized at 0.2-second intervals with 18,000 data points per time history. The longitudinal velocity fluctuations were calculated with horizontal wind speed and direction data measured at the 18-, 30-, 60-, 90-, 120-, and 150-meter levels at the NASA meteorological tower site at the Kennedy Space Center with Climet (model Cl-14) wind sensors. The measurements were taken during the daytime in unstable air. The computation procedures used to calculate the longitudinal turbulent velocity fluctuations are discussed by Fichtl and McVehil in reference 4. Supporting temperature measurements were made at the 18- and 30-meter levels with Climet (model Cl-016) aspirated thermocouples. (Details concerning the instrumentation at the NASA tower site can be found in a report by Kaufman and Keene [5]; the surface roughness lengths associated with this site are discussed by Fichtl and McVehil in reference 4.)

III. THEORETICAL CONSIDERATIONS

The instantaneous longitudinal velocity $u(x,y,z,t)$ in a horizontally homogeneous atmospheric boundary layer can be represented as

$$u(x,y,z,t) = \bar{u}(z) + u'(x,y,z,t), \tag{1}$$

where $\bar{u}(z)$ is the temporal mean wind at height z, and $u'(x,y,z,t)$ is a longitudinal turbulent velocity fluctuation. Upon evaluating equation (1) at heights z_1 and z_2 ($z_2 > z_1$) for fixed values of x,y, and t, we may calculate the two-point velocity difference

$$\Delta u(z_1,z_2,t) = \Delta\bar{u}(z_1,z_2) + \Delta u'(z_1,z_2,t), \tag{2}$$

where

$$\Delta u(z_1,z_2,t) = u(z_2,t) - u(z_1,t) \tag{3}$$

$$\Delta\bar{u}(z_1,z_2) = \bar{u}(z_2) - \bar{u}(z_1) \tag{4}$$

$$\Delta u'(z_1,z_2,t) = u'(z_2,t) - u'(z_1,t). \tag{5}$$

The dependence of Δu and $\Delta u'$ on x and y is understood. For engineering purposes, the quantity $\Delta\bar{u}(z_1,z_2)$ can be treated as a known quantity that can be calculated with a power law wind profile like

$$\bar{u}(z) = \bar{u}(z_1)(z/z_1)^{\alpha}, \tag{6}$$

where α is a positive constant, or with the more elegant wind profile formulations for the barotropic and baroclinic Ekman planetary boundary layers by Blackadar [1]. For values of $z_2 \lesssim 30$ m, the mean wind profile law appropriate for a Monin layer can be used:

$$\bar{u}(z) = \frac{u_{*o}}{k_1}\left(\ln\frac{z}{z_o} - \psi(z/L_o)\right), \tag{7}$$

where u_{*o} and L_o denote the surface friction velocity and Monin-Obukhov stability length, z_o is the surface roughness length, k_1 is von Karman's constant with numerical value approximately equal to 0.4 and $\psi(z/L_o)$ is a universal function of z/L_o [3]. On the other hand, the quantity $\Delta u'(z,z_2,t)$ is a rapidly varying random function of time, and thus must be treated statistically.

The longitudinal time histories were differenced in the vertical to yield fifteen longitudinal velocity difference time histories for each case. Unbiased estimates of the second moment were calculated with the formula

$$\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(\Delta u_i')^2, \tag{8}$$

where σ is the standard deviation of the random longitudinal shear and n is the number of data points used in the calculation.

IV. DIMENSIONAL ANALYSIS

The Monin-Obukhov similarity hypothesis [3] predicts that the standard deviation of two-point longitudinal velocity differences, scaled with the surface friction velocity u_{*o}, are universal functions of z_1/L_o and z_2/L_o or $\bar{z}/L_o$ and $\Delta z/L_o$, where

$$\begin{aligned} \bar{z} &= \frac{z_1 + z_2}{2} \\ \Delta z &= z_2 - z_1 \end{aligned} \quad . \tag{9}$$

Thus, the Monin-Obukhov similarity hypothesis leads to the conclusion that

$$\sigma/u_{*o} = F(\bar{z}/L_o, \Delta z/L_o), \tag{10}$$

where F is a universal function of $\bar{z}/L_o$ and $\Delta z/L_o$. However, as one proceeds upward out of the Monin layer into the Ekman layer, it is reasonable to hypothesize the σ/u_{*o} is a function of $\bar{z}/L_o$, $\Delta z/L_o$ and other parameters that characterize the action of Coriolis forces and baroclinic effects. Blackadar and Tennekes [6] have shown that, in the horizontally homogeneous, neutral, barotropic Ekman layer, u_{*o}/f is the appropriate length scale, where f is the Coriolis parameter. This result was produced by analyzing the mean flow momentum conservation equations along with the turbulent energy equation. Thus, as a working hypothesis, one might assume that σ/u_{*o} is a function of the independent variables

$$\Delta z,\ \bar{z},\ L_o,\ u_{*o}/f. \tag{11}$$

According to Buckingham's theorem [7], the number of independent nondimensional quantities that can be constructed from this list of variables is three. Now there are eighteen possible ways to select this group of three independent nondimensional quantities such that $\bar{z}$ and Δz appear within each group. However, any one of the possible groups can be derived from the remaining seventeen groups. Accordingly, we are free to select any one group we like to perform a data analysis; thus, we hypothesize that

$$\frac{\sigma}{u_{*o}} = F\left(\frac{f\Delta z}{u_{*o}}, \frac{f\bar{z}}{u_{*o}}, \frac{u_{*o}}{fL_o}\right) . \tag{12}$$

This hypothesis includes the dominant effects in the Ekman layer: atmospheric stability, and the action of Coriolis forces in the horizontal mean flow momentum conservation equations. It does not include baroclinic effects resulting from a height-dependent horizontal pressure-gradient force. Nevertheless, the hypothesis given by equation (12) seems to be appropriate in view of the fact that the outstanding feature of the Ekman layer is the action of Coriolis forces and that inclusion of baroclinic effects will only lead to refinements. This hypothesis will be used to analyze the standard deviation of vertical two-point longitudinal velocity fluctuation differences. The procedures used in the calculation of u_{*o} and L_o are given in appendix A. The data used in these calculations and the results of the calculations are given in appendix B.

V. DATA ANALYSIS

To estimate the universal function F on the right-hand side of equation (12), the nondimensional standard deviation σ/u_{*o} was plotted as a function of $f\Delta z/u_{*o}$ for various categories of $f\bar{z}/u_{*o}$ and u_{*o}/fL_o. Each case is characterized by a single value of u_{*o}/fL_o; however, within each case, the quantity $f\bar{z}/u_{*o}$ can vary.

Figure 1 is a plot of σ/u_{*o} as a function of $f\Delta z/u_{*o}$ for the various categories of $f\bar{z}/u_{*o}$ and u_{*o}/fL_o. The data appear to show that, for each of the selected categories, σ/u_{*o} is proportional to $(f\Delta z/u_{*o})^{1/3}$. In addition, within each category of $f\bar{z}/u_{*o}$, the values of σ/u_{*o} tend to sort out according to the parameter u_{*o}/fL_o; i.e., for fixed values $f\bar{z}/u_{*o}$ and $f\Delta z/u_{*o}$, the quantity σ/u_{*o} tends to be a decreasing function of $-u_{*o}/fL_o$. There also appears to be a dependence of σ/u_{*o} on $f\bar{z}/u_{*o}$; i.e., for fixed values of $f\Delta z/u_{*o}$ and u_{*o}/fL_o, the quantity σ/u_{*o} is a decreasing function $f\bar{z}/u_{*o}$.

To see these effects better, it was assumed that σ/u_{*o} is proportional to $(f\Delta z/u_{*o})^{1/3}$ for all categories of the remaining independent variables, so that

$$\frac{\sigma}{u_{*o}} = G\left(\frac{f\bar{z}}{u_{*o}}, \frac{u_{*o}}{fL_o}\right)\left(\frac{f\Delta z/u_{*o}}{0.001}\right)^{1/3}, \qquad (13)$$

where G is a universal function of $f\bar{z}/u_{*o}$ and u_{*o}/fL_o. The function G was determined by fitting equation (13) to the data in figure 1 for each category of $f\bar{z}/u_{*o}$ and u_{*o}/fL_o. The results of these calculations are shown in figure 2. We see from this figure that G and thus σ/u_{*o} are clearly decreasing functions of $f\bar{z}/u_{*o}$ and increasing functions of u_{*o}/fL_o.

To give the experimental results mathematical expression, we assume that the curves in figure 2 are shape-invariant with respect to variation in u_{*o}/fL_o and that translation only with respect to the G-axis is required to produce a collapse of these curves. This means that there exists a function $H(u_{*o}/fL_o)$ such that

$$G\left(\frac{f\bar{z}}{u_{*o}}, \frac{u_{*o}}{fL_o}\right) = G\left(\frac{f\bar{z}}{u_{*o}}, -150\right) + H\left(\frac{u_{*o}}{fL_o}\right). \qquad (14)$$

In view of the experimental results in figure 2, these assumptions appear to be reasonable. To calculate the function $H(u_{*o}/fL_o)$, we subtract the experimental estimate of $G(f\bar{z}/u_{*o}, -150)$ from each curve in figure 2 and then average each of the resulting difference functions over the experimental range of variation $f\bar{z}/u_{*o}$. The experimental estimates of $H(u_{*o}/fL_o)$ for the various values of u_{*o}/fL_o are given in figure 3. The function

$$H(u_{*o}/fL_o) = -0.072 \ln\left(-\frac{u_{*o}/fL_o}{150}\right) \qquad (15)$$

appears to summarize these results reasonably well. To improve our estimate of $G(f\bar{z}/u_{*o}, -150)$, we subtract $H(u_{*o}/fL_o)$ as given by equation (15) from the experimental values of $G(fz/u_{*o}, u_{*o}/fL_o)$ given in figure 2 and average the resulting difference functions over the experimental range of variation of u_{*o}/fL_o for each category of $f\bar{z}/u_{*o}$. The revised estimate of the function $G(f\bar{z}/u_{*o}, -150)$ is depicted in figure 4, and the analytical expression

$$G(f\bar{z}/u_{*o}, -150) = 1 + 2.77 \left(\frac{f\bar{z}/u_{*o}}{0.001}\right)^{-5/2} \qquad (16)$$

summarizes these data. The results in figure 4 show that $G(f\bar{z}/u_{*o}, -150) \to 1$ as $f\bar{z}/u_{*o} \to \infty$. This means that σ/u_{*o} becomes asymptotically height-invariant as $f\bar{z}/u_{*o} \to \infty$. The function (16) contains this asymptotic behavior.

The combination of equations (13) through (15) yields

$$\frac{\sigma}{u_{*o}} = \left[1 + 2.77 \left(\frac{f\bar{z}/u_{*o}}{0.001}\right)^{-5/2} - 0.072 \ln\left(-\frac{u_{*o}/fL_o}{150}\right)\right]\left(\frac{f\Delta z/u_{*o}}{0.001}\right)^{1/3} \qquad (17)$$

According to equation (17) σ/u_{*o} becomes large as $f\bar{z}/u_{*o} \to 0$ and $f\Delta z/u_{*o} \to \infty$. This means that σ becomes large as the height $\bar{z}$ decreases and the interval Δz increases. In addition, σ/u_{*o} decreases as $-u_{*o}/fL_o$ increases. However, equation (17) predicts that $\sigma/u_{*o} < 0$ at sufficiently large values of $-u_{*o}/fL_o$. This is nonsense and is an apparent shortcoming of the empirical formula (17). Negative values of σ/u_{*o} are predicted with (17) when

$$f\bar{z}/u_{*o} > 0.001 \left[\frac{2.77}{0.072 \ln\left(-\frac{u_{*o}/fL_o}{150}\right) - 1}\right]^{2/5}, \qquad (18)$$

provided $-u_{*o}/fL_o > 4 \times 10^8$. However, values of $-u_{*o}/fL_o > 4 \times 10^8$ are very rare if we exclude the region of the earth on and about the equator. A physically realizable upper bound value on $-u_{*o}/fL_o$ in unstable air is probably on the order of 10,000 if we exclude the equatorial region of the earth. Thus, the shortcoming of predicting negative values of σ/u_{*o} with equation (17) is only apparent and would not arise in practical situations. The function $H(u_{*o}/fL_o)$ probably tends to approach a constant value as $-u_{*o}/fL_o$ becomes large and if this occurs in the interval $1{,}000 < -u_{*o}/fL_o < 10{,}000$, then the error in the estimate of σ/u_{*o} resulting from (17) is approximately 5 percent. As $-u_{*o}/fL_o \to 0$ from positive values of $-u_{*o}/fL_o$, $\sigma/u_{*o} \to \infty$. The function $H(u_{*o}/fL_o)$ should approach a finite value as $-u_{*o}/fL_o \to 0$ because of the finite nature of σ. Accordingly, $H(u_{*o}/fL_o)$ does not have logarithmic behavior as $-u_{*o}/fL_o \to 0$ as predicted by (15). The precise value of $-u_{*o}/fL_o$ where $H(u_{*o}/fL_o)$ departs from the behavior stated by (15) can only be determined with measurements of σ/u_{*o} associated with values of $-u_{*o}/fL_o < 20$. In view of these comments, it is recommended that (17) be used to obtain estimates of σ only for those situations in which u_{*o}/fL_o is contained within the experimental range of variation in this report, i.e., $20 \leqq -u_{*o}/fL_o \leqq 2{,}000$. Estimates of σ/u_{*o} can be obtained with equation (17) for $2{,}000 < -u_{*o}/fL_o < 10{,}000$ which probably have errors equal to 5 percent.

The correlation coefficient of the longitudinal wind fluctuations at z_1 and z_2 is

$$R(u'(z_1), u'(z_2)) = \frac{\sigma_u^2(z_1) + \sigma_u^2(z_2) - \sigma^2(z_1, z_2)}{2\sigma_u(z_1)\ \sigma_u(z_2)}\ , \tag{19}$$

where $\sigma_u(z)$ is the standard deviation of the longitudinal wind fluctuations at height z. Fichtl and McVehil [4] find that for the unstable boundary layer

$$\frac{\sigma_u}{u_{*o}} = 1.9(z/18)^{-0.14}. \tag{20}$$

This result was obtained by effectively averaging together measurements of σ_u/u_{*o} obtained during various degrees of instability in the first 150 m of the atmospheric boundary layer at Cape Kennedy, Florida. However, the recent results of Panofsky and Mirabella [9] appear to show that σ_u/u_{*o} is relatively insensitive to changes in stability and thus u_{*o}/fL_o. Now, (20) shows that σ_u/u_{*o} is a slowly varying function of z so that we will assume $\sigma_u(z_1) \simeq \sigma_u(z_2)$ and set $\sigma_u/u_{*o} = 1.5$. This value of σ/u_{*o} occurs at $z \simeq 80$ m. Thus, we may write (19) as

$$R(u'(z_1), u'(z_2)) \simeq 1 - \frac{(\sigma/u_{*o})^2}{4.5}\ . \tag{21}$$

We may conclude from (17) and (21) that in the unstable boundary layer the interlevel correlation coefficient of the longitudinal velocity fluctuations increases as the instability of the boundary layer and $f\bar{z}/u_{*o}$ increase and as $f\Delta z/u_{*o}$ decreases.

VI. CONCLUDING COMMENTS

1. We have experimentally analyzed the standard deviation of vertical two-point longitudinal velocity fluctuation differences, and have found that σ/u_{*o} is indeed a function of $f\bar{z}/u_{*o}$, $f\Delta z/u_{*o}$, and u_{*o}/fL_o. In particular, σ/u_{*o} is proportional to $(f\Delta z/u_{*o})^{1/3}$. Although the analysis did not include baroclinic effects, we believe them to be of second order. The fact that σ^2 is proportional to $(\Delta z)^{2/3}$ is, in all likelihood, related to the fact that the structure function of longitudinal velocity fluctuations at a given height is proportional to $(\Delta x)^{2/3}$ for sufficiently small values of the horizontal space lag Δx when an inertial subrange is present. The precise nature of this relationship is not obvious; however, it will be the subject of future investigation by the author.

2. This study is the first part of a four-part study to define the statistical properties of fluctuating wind shears in the atmospheric boundary layer. The second part of the study will be concerned with determining the dependence of the third and fourth moments, μ_3 and μ_4, on $f\Delta z/u_{*o}$, $f\bar{z}/u_{*o}$, and u_{*o}/fL_o in unstable air. The third part will be concerned with determining a class of distribution functions which contain four parameters that will adequately describe the observed wind shear distribution functions in unstable air. Preliminary results appear to show that the Pearson system of distribution functions can be used to represent the data. The four parameters which will appear in the distribution function will be completely specified upon specifying the first four moments of the wind shear. The data in this report will be used in parts two and three of the total study. The fourth part will be concerned with extending the results of parts one, two, and three to (1) the neutral boundary layer with turbulence data for relatively high wind speed conditions ($\bar{u}(18m) \geq 15$ m sec^{-1}, say) and (2) stable boundary layers associated with nighttime conditions. Accordingly, the overall goal is to develop models of σ, μ_3 and μ_4 such that these parameters can be calculated for given values of the height parameters $\bar{z}$ and Δz once the external parameters L_o, u_{*o}, and f are specified. Substitution of σ, μ_3, μ_4 and the mean wind shear (see section III) into the distribution function of wind shear will facilitate the calculation of wind shear for any percentile level of occurrence for a given state of the boundary layer. Furthermore, the model will permit climatological studies of wind shear with rather straightforward statistical procedures for specified locations upon specifying

the climatological statistics of L_o and u_{*o} (joint distribution function). The distribution function of these external parameters could be extracted from standard meteorological observations.

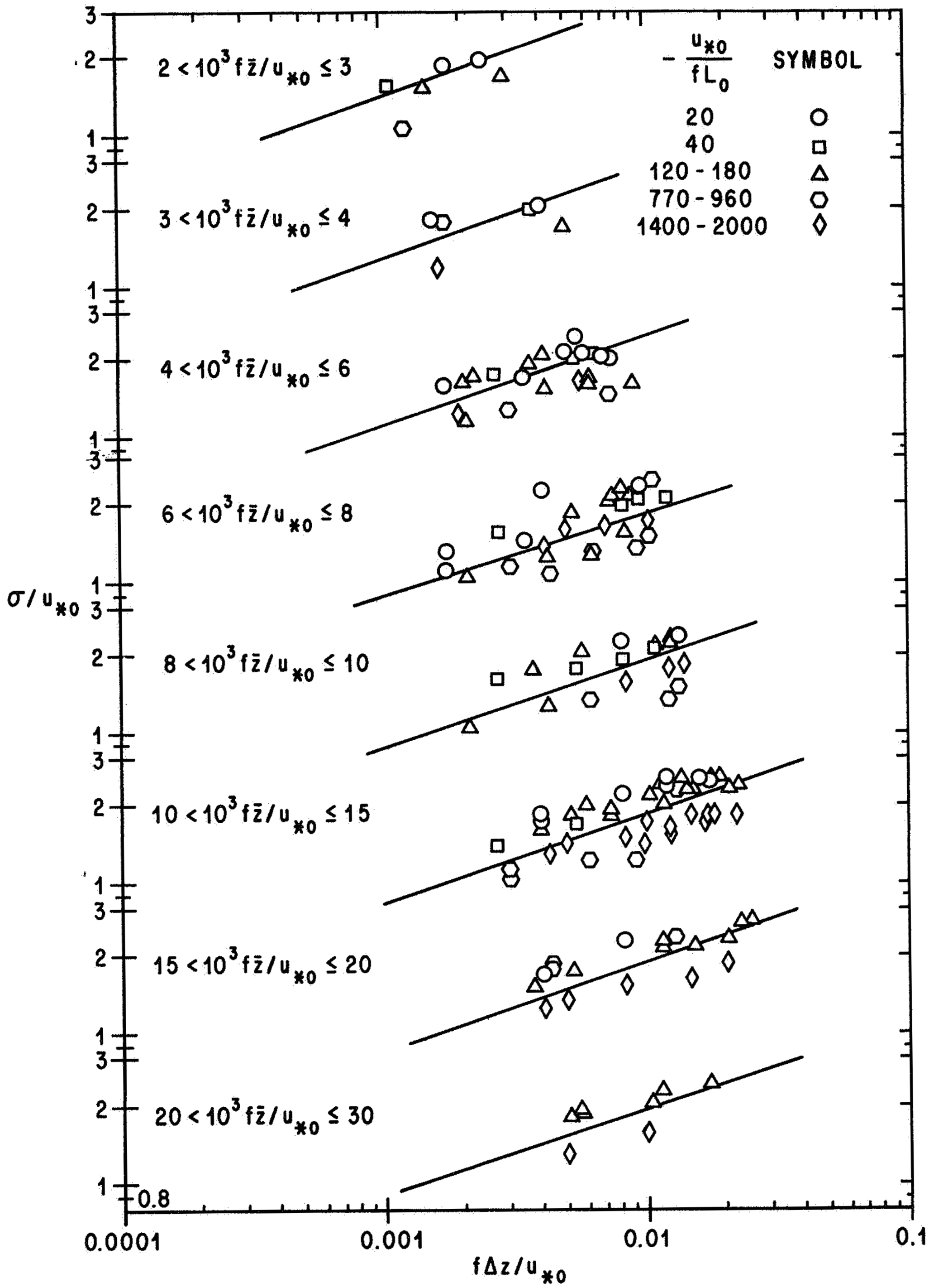

Figure 1. Experimental values of σ/u_{*o} for various categories of $f\bar{z}/u_{*o}$ and $-u_{*o}/fL_o$

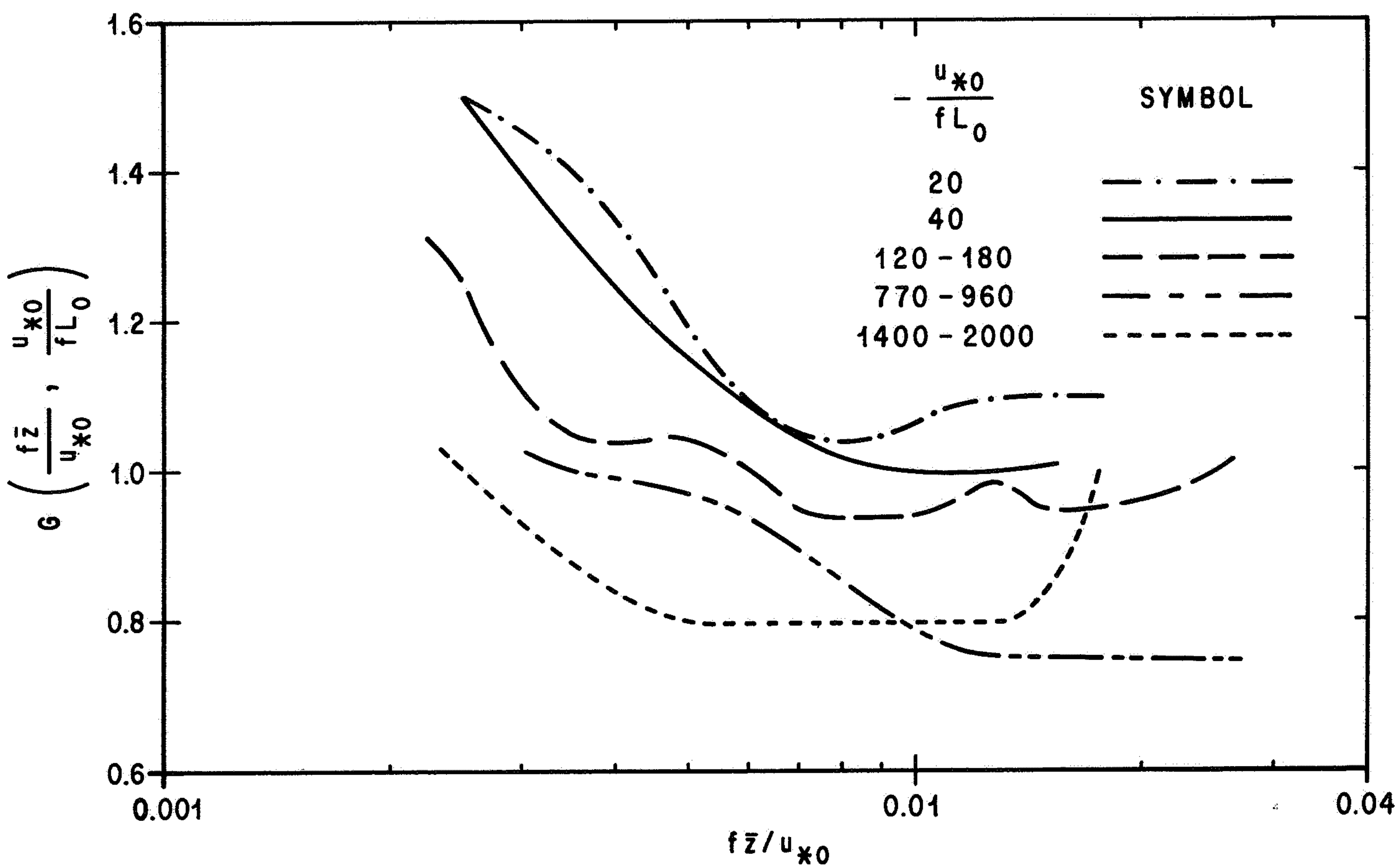

Figure 2. Estimate of the function $G(f\bar{z}/u_{*o}, u_{*o}/fL_o)$

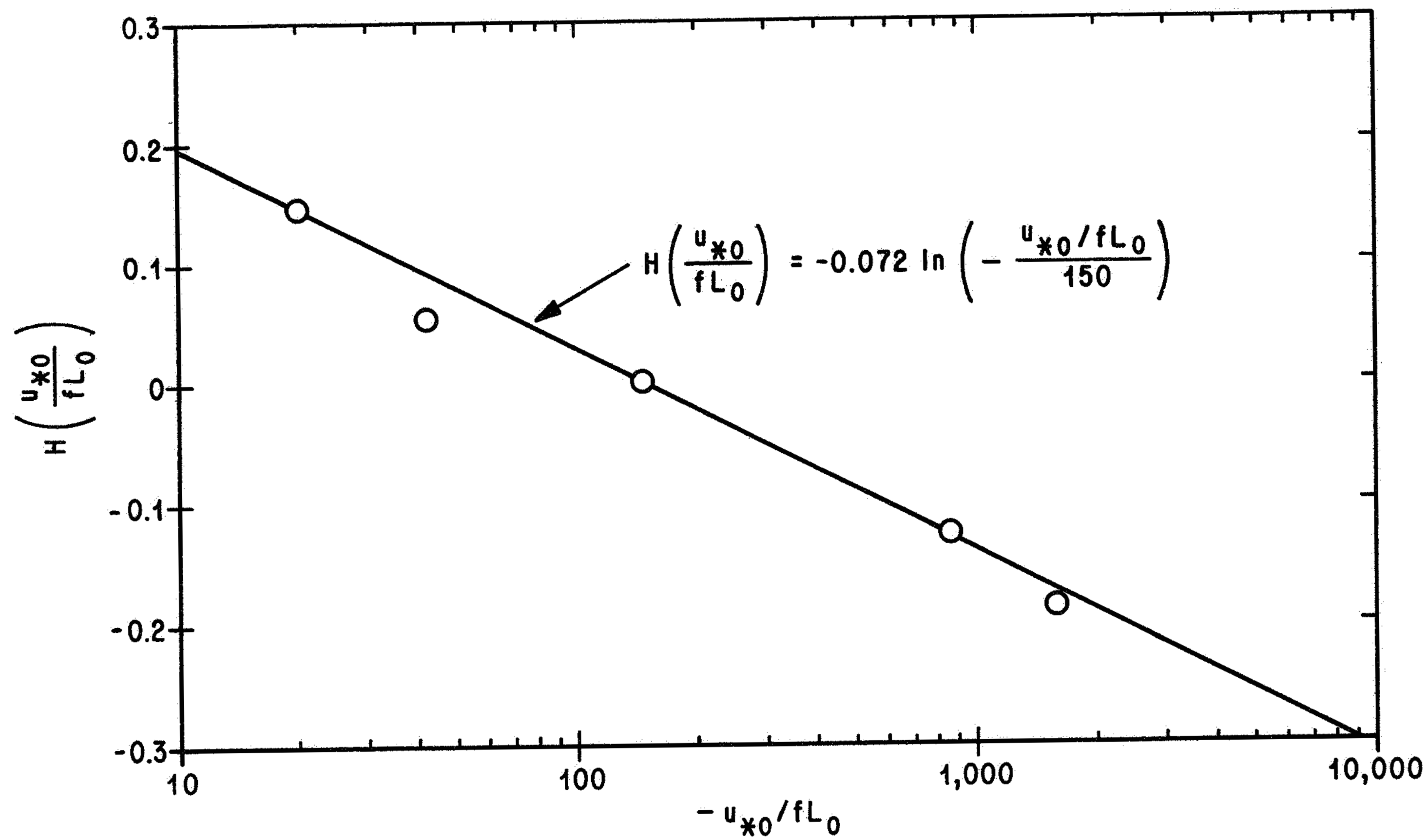

Figure 3. Estimate of the function $H(u_{*o}/fL_o)$

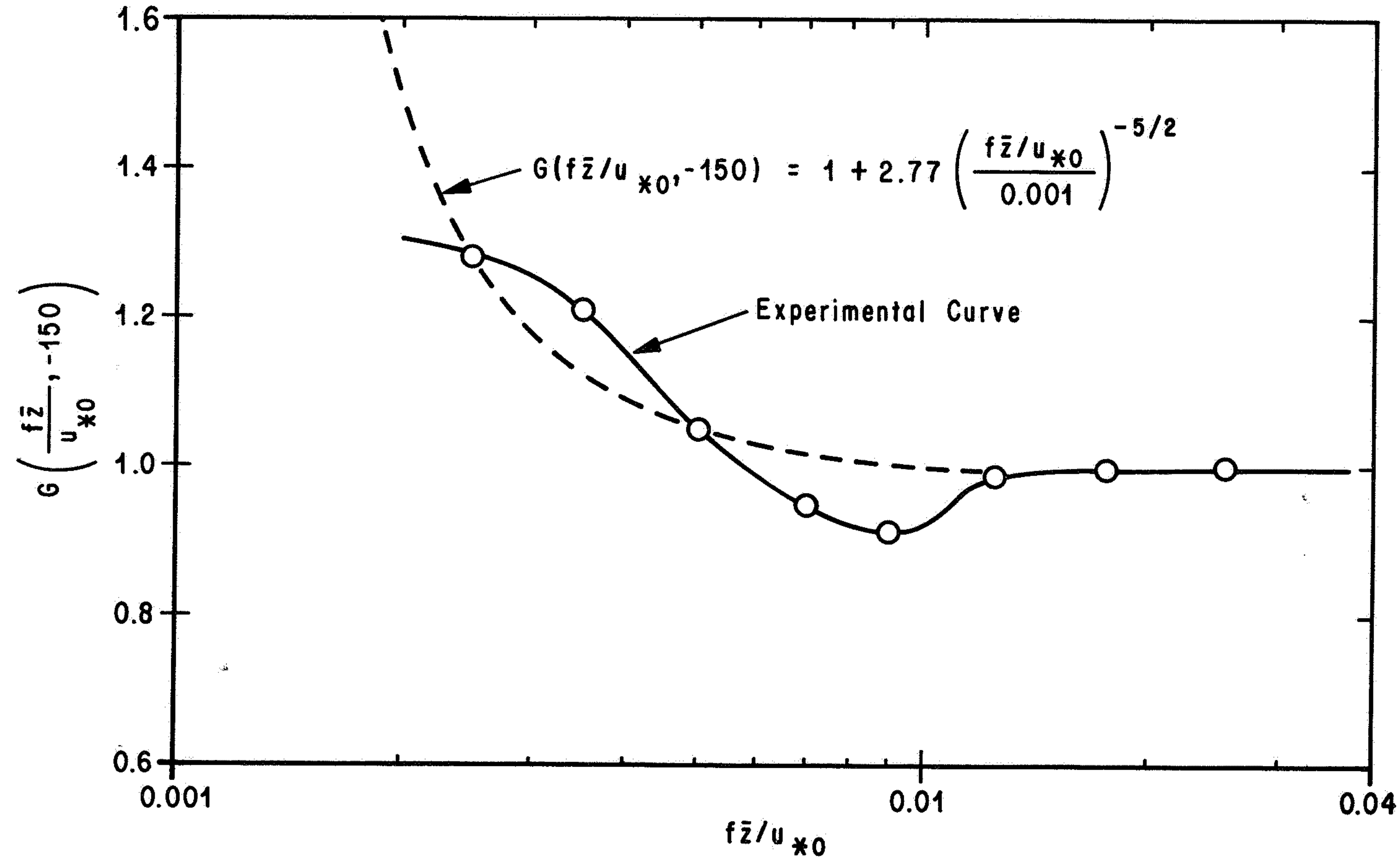

Figure 4. Estimate of the function $G(f\bar{z}/u_{*o}, -150)$

APPENDIX A

Calculations of the Scaling Parameters u_{*o} and L_o

In the unstable Monin layer, the dimensionless mean flow shear is a universal function of z/L_o, so that

$$\frac{k_1 z}{u_{*o}} \frac{\partial \bar{u}}{\partial z} = \phi_u(z/L_o), \tag{A-1}$$

where $\bar{u}(z)$ is the mean wind speed at height z, k_1 is von Karman's constant with numerical value approximately equal 0.4, u_{*o} is the surface friction velocity, and $\phi_u(z/L_o)$ is a universal function of z/L_o. The quantity L_o is the surface Monin-Obukhov stability length, namely,

$$L_o = - \frac{u_{*o}^3 \, C_p \bar{\rho} \, \bar{T}}{k_1 g \, H_o} . \tag{A-2}$$

In this equation H_o is the surface heat flux, $\bar{\rho}$ and $\bar{T}$ denote the mean flow density and Kelvin temperature, g is the acceleration of gravity, and C_p is the specific heat at constant pressure. The dimensionless shear ϕ_u is related to the flux Richardson number through the experimentally derived relationship

$$\phi_u = (1 - 18Ri)^{1/4}, \tag{A-3}$$

which is given in reference 3. The flux Richardson number is defined as

$$Ri = \frac{\frac{g}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z}}{(\partial \bar{u}/\partial z)^2} , \tag{A-4}$$

where $\bar{\theta}$ is the mean potential temperature at height z. The flux Richardson number is a function of z/L_o. We shall invoke the Businger hypothesis [8] to relate Ri to z/L_o, so that

$$Ri = z/L_o. \tag{A-5}$$

Upon combining equations (A-1), (A-3), and (A-5) and integrating the resulting relationship, we find that

$$\bar{u}(z) = \frac{u_{*o}}{k_1}\left\{\ln\frac{z}{z_o} - \psi\left(\frac{z}{L_o}, \frac{z_o}{L_o}\right)\right\}, \qquad \text{(A-6)}$$

where

$$\psi\left(\frac{z}{L_o}, \frac{z_o}{L_o}\right) = \int_{-z_o/L_o}^{-z/L_o} \frac{1 - (1 + 18\xi)^{-1/4}}{\xi}\, d\xi. \qquad \text{(A-7)}$$

We have used the condition that $\bar{u}(z_o) = 0$ in the derivation of (A-6), where z_o is the surface roughness length. Equation (A-7) can be evaluated numerically for any value of z/L_o. The lower bound of this integral may be set equal to zero because the contribution to ψ from the region $0 < -z/L_o < -z_o/L_o$ is negligibly small.

Equations (A-4) through (A-7) can be used to calculate the scaling velocity u_{*o} and L_o. The procedure for calculating these quantities is as follows: (1) calculate the gradient Richardson number, equation (A-4), with the mean flow wind and potential temperature profile data in the Monin layer, (2) calculate L_o with equation (A-5), and finally (3) calculate u_{*o} with (A-6) and (A-7).

The 18- and 30-meter temperature and wind data were used to estimate the Richardson number. The distributions of $\bar{u}(z)$ and $\bar{\theta}(z)$ between the 18- and 30-meter levels were assumed to be logarithmic profiles. The expressions for $\bar{u}(z)$ and $\bar{\theta}(z)$ were differentiated with respect to z and evaluated at the geometric height z = 23 meters to yield estimates of $\partial\bar{u}/\partial z$ and $\partial\bar{\theta}/\partial z$ for the calculation of Ri. This Richardson number was used to calculate L_o in step (2) above. The 18-meter level wind speed and $\psi(18\text{m}/L_o)$ were used to calculate u_{*o} in step (3).

The surface roughness lengths z_o that were associated with the NASA 150-meter meteorological tower site and that were used in the calculation of u_{*o} are given in reference 4.

APPENDIX A

Calculations of the Scaling Parameters u_{*o} and L_o

In the unstable Monin layer, the dimensionless mean flow shear is a universal function of z/L_o, so that

$$\frac{k_1 z}{u_{*o}} \frac{\partial \bar{u}}{\partial z} = \phi_u(z/L_o), \tag{A-1}$$

where $\bar{u}(z)$ is the mean wind speed at height z, k_1 is von Karman's constant with numerical value approximately equal 0.4, u_{*o} is the surface friction velocity, and $\phi_u(z/L_o)$ is a universal function of z/L_o. The quantity L_o is the surface Monin-Obukhov stability length, namely,

$$L_o = - \frac{u_{*o}^3 \, C_p \bar{\rho} \, \bar{T}}{k_1 g \, H_o} . \tag{A-2}$$

In this equation H_o is the surface heat flux, $\bar{\rho}$ and $\bar{T}$ denote the mean flow density and Kelvin temperature, g is the acceleration of gravity, and C_p is the specific heat at constant pressure. The dimensionless shear ϕ_u is related to the flux Richardson number through the experimentally derived relationship

$$\phi_u = (1 - 18Ri)^{1/4}, \tag{A-3}$$

which is given in reference 3. The flux Richardson number is defined as

$$Ri = \frac{\frac{g}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial z}}{(\partial \bar{u}/\partial z)^2} , \tag{A-4}$$

where $\bar{\theta}$ is the mean potential temperature at height z. The flux Richardson number is a function of z/L_o. We shall invoke the Businger hypothesis [8] to relate Ri to z/L_o, so that

$$Ri = z/L_o. \tag{A-5}$$

Upon combining equations (A-1), (A-3), and (A-5) and integrating the resulting relationship, we find that

$$\bar{u}(z) = \frac{u_{*o}}{k_1}\left\{\ln\frac{z}{z_o} - \psi\left(\frac{z}{L_o}, \frac{z_o}{L_o}\right)\right\}, \tag{A-6}$$

where

$$\psi\left(\frac{z}{L_o}, \frac{z_o}{L_o}\right) = \int_{-z_o/L_o}^{-z/L_o} \frac{1 - (1 + 18\xi)^{-1/4}}{\xi}\, d\xi. \tag{A-7}$$

We have used the condition that $\bar{u}(z_o) = 0$ in the derivation of (A-6), where z_o is the surface roughness length. Equation (A-7) can be evaluated numerically for any value of z/L_o. The lower bound of this integral may be set equal to zero because the contribution to ψ from the region $0 < -z/L_o < -z_o/L_o$ is negligibly small.

Equations (A-4) through (A-7) can be used to calculate the scaling velocity u_{*o} and L_o. The procedure for calculating these quantities is as follows: (1) calculate the gradient Richardson number, equation (A-4), with the mean flow wind and potential temperature profile data in the Monin layer, (2) calculate L_o with equation (A-5), and finally (3) calculate u_{*o} with (A-6) and (A-7).

The 18- and 30-meter temperature and wind data were used to estimate the Richardson number. The distributions of $\bar{u}(z)$ and $\bar{\theta}(z)$ between the 18- and 30-meter levels were assumed to be logarithmic profiles. The expressions for $\bar{u}(z)$ and $\bar{\theta}(z)$ were differentiated with respect to z and evaluated at the geometric height z = 23 meters to yield estimates of $\partial\bar{u}/\partial z$ and $\partial\bar{\theta}/\partial z$ for the calculation of Ri. This Richardson number was used to calculate L_o in step (2) above. The 18-meter level wind speed and $\psi(18\text{m}/L_o)$ were used to calculate u_{*o} in step (3).

The surface roughness lengths z_o that were associated with the NASA 150-meter meteorological tower site and that were used in the calculation of u_{*o} are given in reference 4.

APPENDIX B

Wind Speed and Temperature Profile Data and Other Parameters

The wind profile and temperature data that were used in this report are given in Tables B-1 and B-2. The 18-meter level wind direction is tabulated here because the surface roughness length at the NASA 150-meter tower facility is a function of wind direction. The values of Ri(23 m), L_o, and u_{*o} that were calculated with these data are given in Table B-3.

TABLE B-1

Table of Temperature Profile Data*

(Temperature in °F)

Case No.	Date	Time (EST)	T(3m)	$(\Delta T)_1$	$(\Delta T)_2$
299	1/23/68	1315-1400	76.0	-1.41	-1.73
305	1/26/68	1130-1230	52.4	-2.19	-2.66
310	2/8/68	915-1015	40.5	- .32	-1.82
319	2/26/68	1030-1130	59.0	-1.28	-2.63
355	3/20/68	1330-1430	75.0	-1.82	-2.21
365	3/28/68	905-932	73.5	-1.44	-2.42
366	3/28/68	1145-1245	74.5	-2.63	-3.13
406	4/7/68	1507-1537	79.0	-2.02	-2.57
445	5/7/68	1400-1415	78.2	-2.29	-2.54
551	6/29/68	947-1027	79.8	- .93	-1.90
554	6/30/68	933-952	83.1	-1.40	-1.91

*$(\Delta T)_1$ = T(18m) - T(3m).
$(\Delta T)_2$ = T(30) - T(3m).

TABLE B-2

Table of Wind Speed and Direction Data

Case No.	18 m Wind Direction	$\bar{u}$(18 m) (m sec^{-1})	$\bar{u}$(30 m) (m sec^{-1})
299	219°	5.95	7.55
305	338°	8.50	9.31
310	298°	9.98	11.34
319	320°	4.81	5.13
355	83°	3.87	4.13
365	95°	4.98	5.25
366	87°	6.12	6.48
406	71°	4.36	4.73
445	93°	9.62	10.34
551	34°	2.92	3.05
554	86°	4.01	4.12

TABLE B-3

Table of Boundary Layer Parameters

Case No.	Ri(23 m)	L_o(m)	u_{*o}(m sec^{-1})
299	-0.065	-357	0.52
305	-0.090	-259	0.76
310	-0.032	-716	1.20
319	-2.448	- 9	0.48
355	-0.557	- 42	0.36
365	-2.234	- 10	0.67
366	-0.504	- 46	0.56
406	-0.546	- 43	0.40
445	-0.234	- 99	0.98
551	-8.977	- 3	0.41
554	-5.100	- 5	0.49

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APPROVAL

STANDARD DEVIATION OF VERTICAL WIND SHEAR IN THE LOWER ATMOSPHERE

by George H. Fichtl

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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