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FINAL REPORT

A STUDY OF PROBLEMS IN AIRCRAFT NAVIGATION AND CONTROL

by

Kenneth R. Britting

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ABSTRACT

This report presents a summary of the research performed under NASA Grant NGR 22-009-229. A general description of the research is followed by a publications list of the reports published under this grant. Abstracts from these reports appear in the final section.

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1. Introduction and Summary

For the past four years the Measurement Systems Laboratory has been the recipient of NASA grant which has supported research on the performance of the various types of pure inertial and hybrid navigation systems. In addition, research on flight control theory and statistical estimation techniques has been performed via doctoral level thesis research.

Since the Measurement Systems Laboratory is primarily an academically oriented laboratory in the Department of Aeronautics and Astronautics at M.I.T., the thrust of the grant has been directed toward both direct and supervisory support of students pursuing advanced degrees in the Department. Thus many of the research topics have been aimed at establishing a sound basis for research in the area of aircraft navigation and control systems.

The research level has consisted of the full time efforts of a research staff member, the writer, plus the services of, on the average, three research assistants. Six masters and three doctoral theses have been written through this grant.

The following list indicates the scope of the research to date. Reports which document the research performed on these topics appear in the publications list in Section 2.

MASTER'S LEVEL RESEARCH

1. Satellite Inertial System
2. Doppler Inertial Systems
3. Stellar Inertial Systems
4. Strapdown Systems
5. Statistical Estimation Techniques
6. Hybrid Altimetry

DOCTORAL RESEARCH

1. Flight Control Systems
2. Statistical Estimation
3. Inertial Navigation Theory

STAFF RESEARCH

1. Inertial Navigation Error Propagation
2. Strapdown Alignment Techniques
3. Strapdown Navigation Equations
4. Preflight Test Procedures
5. Navaid Statistical Modelling

Particular research topics have, in general, been closely allied with the sponsor's current interests. Since NASA's original interest was in the area of navigation systems for the supersonic transport, roughly the first year's effort was spent in developing the performance equations for navigation systems in the S.S.T. flight environment. In addition a number of hybrid systems; satellite-inertial, doppler-inertial, and stellar-inertial; were investigated via thesis research. Because the sponsor's most recent interest was in the area of V/STOL navigation systems, current research has been in support of this effort.

The Master's level research has emphasized the performance of hybrid systems including the important problem of altimetry. The doctoral level research has, by its nature, been concerned with broader questions of a more theoretical nature. For example, in order to apply modern estimation techniques, it is necessary to have a prior knowledge of the system's noise statistics. This is a very serious constraint in a practical situation. In an effort to ease this constraint, Abramson developed an optimal procedure for estimating the state of a linear dynamical system when the statistics of the measurement and process noise are poorly known. This procedure was subsequently applied via master's level research by Strunce to the determination of optimal estimates for an inertial system which was aided by a ground based radar. This work was intended to be directly applicable to the VTOL flight tests which were to be conducted at Wallops by ERC and Langley personnel.

Doctoral level research in the area of flight control systems has been directed toward the development of a analytical design technique which is based on engineering specifications rather than on some purely mathematical optimization process. This work was extended to the design of systems which will exhibit a low sensitivity to changes in the system's open loop parameters, while at the same time obtaining desirable closed loop response characteristics. The problem of designing flight control systems for varying flight conditions was of particular interest.

In addition, doctoral level research was performed on general inertial navigation theory as it applies to the error propagation in inertial systems of various configuration. This research developed an error propagation model which is applicable to a broad class of inertial navigation system configurations.

Staff research has been used both to supply a foundation for the thesis research and to directly aid NASA in its experimental flight test program. This "mission oriented" research has proven to be mutually beneficial to both NASA and the Measurement Systems Laboratory; the laboratory being able to supply technical supervision and financial aid to graduate students while, at the same time, performing research which is, for the most part, directly applicable to the NASA research effort. For example, the research on strapdown alignment, strapdown navigation equation, and navaid statistical modeling was directly aimed at the flight testing of an aided strapdown navigation system in a VTOL.

The research reported on concerning the error propagation in space stabilized inertial navigation systems was extensively used for system calibration and in the analysis of performance data collected during the phase I-B portion of ERC's program of aided inertial flight test experiments.

2. Publication List

1. Spicer, B. R., "Study of Aircraft Position Fixing Using the Navy Navigation Satellite System"; TE-18 M.S.; May 1967

2. Sakran, C. F.; "Hybrid Doppler Radar Inertial Navigation Techniques for the S. S. T."; TE-19; June 1967. (Navigation, Vol. 17, No. 1; 1970)

3. Ryan, T. J.; "An Optimum Stellar Monitored Strapdown Navigation System;" TE-22; M.S.; January 1968.

4. Britting, K. R.; "Analysis of Space Stabilized Inertial Navigation Systems"; RE-35; January 1968.

5. Britting, K. R., and Palsson, T.; "Self Alignment Techniques for Strapdown Inertial Navigation Systems with Aircraft Application:; RE-33; November 1968. (Journal of Aircraft, Vol. 7, No.4, 1970)

6. Abramson, P. D.; "Simultaneous Estimation of the State and Noise Statistics in Linear Dynamical Systems"; TE-25; Sc.D.; May 1968.

7. Killingsworth, W.; "Computation Frames for Strapdown Inertial Systems"; TE-28; M.S.; June 1968.

8. Ruth, J. C.; "Analysis of Error Propagation in Aircraft Inertial Navigation Systems"; RE-43; July 1968.

9. Redeiss, H.; "Design of Linear Flight Control Systems Using a Model Performance Index"; TE-26; Sc. D; December 1968. (AIAA Paper 69-885, 1969)

10. Britting, K. R.; "State Transition Matrix for Inertial Navigation Systems"; RE-53; March 1969.

11. Britting, K. R.; "Strapdown Navigation Equations for Geographic and Tangent Coordinate Frames"; RE-56; June 1969.

12. Johanson, H. K.; "Uncertainty Estimation Via Preflight Test Procedures for a Space Stabilized Inertial Navigation System"; RE-59; August 1969.

13. Strunce, R. R.; "Estimation of Measurement Noise Statistics in Aided Inertial Navigation Systems"; I. L. T-524; M. S.; September 1969.

14. Johansen, H. K.; "A Survey of General Coverage Navaids for V/Stol Aircraft - A VOR/DME Model"; RE-64; Oct. 1969.

15. Britting, K. R.; "Error Analysis of Strapdown and Local Level Inertial Systems which Compute in Geographic Coordinates"; RE-52; November 1969.

16. Jankowski, P. C.; "Hybrid Altimeter Using a Strapdown Inertial Navigation System"; TE-36; M.S.; January 1970.

17. Britting, K. R.; "Unified Error Analysis of Terrestrial Inertial Navigation Systems"; TE-42; Sc. D. Thesis; October 1970.

18. Marcus, F.J.;"Computational Comparison of Strapdown System Attitude Algorithms"; TE-43; M.S., February 1971.

19. Ryan, T.J.; "Alignment and Calibration of a Strapdown Inertial Measuring Unit"; RE-67; December 1970.

3. Report Abstracts

The following constitute the abstracts from the reports Listed in Section 2.

1. Spicer; "Study of Aircraft Position Fixing Using the Navy Navigational Satellite System"

The possibility of using the Navy Navigational Satellite System for position fixing of supersonic aircraft (speeds up to Mach 2. 7) is examined. The effects of errors in required input data on the computed position of the aircraft are determined for various pass angles of the satellite with respect to the aircraft. Results indicate this system could be a valuable aid to supersonic aircraft navigation.

2. Sakran; "Hybrid Doppler-Radar Inertial Navigation Techniques for the Supersonic Transport Aircraft"

Two particular hybrid navigation systems utilizing both inertial and doppler radar sensors are studied for use in high speed aircraft such as the supersonic transport. The systems postulated consist of a semigeometric inertial system receiving linear velocity corrections from a Janus Beam array doppler radar. System equations are linearized assuming constant aircraft velocity and quasistatic latitude. Response to constant component uncertainties of the two hybrid doppler-inertial systems is compared to that of a pure (undamped) inertial navigator. The predominant error sources are found to be either gyro drift or doppler radar bias propagating as a bounded oscillation in latitude and true heading, and unbounded in longitude.

3. Ryan; "An Optimum Stellar Monitored Strapdown Navigation System"

Two strapdown navigation systems are studied for use in the Supersonic Transport. The first system uses external altitude information to stabilize the gravitational computation, but otherwise operates in pure inertial mode. Linear error equations are obtained for the system. Root mean square position errors are computed for simply described random inputs of altimeter error, accelerometer bias, attitude computation errors, and gyro drift. The predominant error source is gyro drift as characterized statistically by a random walk process. A hybrid navigation system consisting of a body-mounted star tracker coupled to the first navigation system is also considered. The nature of this coupling is determined by the application of Kalman filtering techniques. For easy reference, a comparison of these two systems for combinations of various quality star trackers and gyros is presented in graphical form.

4. Britting; "Analysis of Space Stabilized Inertial Navigation Systems"

This report is a tutorial exposition of the class of inertial navigation systems which instrument an inertially nonrotating coordinate frame. A detailed discussion of the possible types of system mechanizations is followed by a discussion and error analysis of self-contained alignment schemes. Stabilization of the vertical error is investigated.

Perturbation methods are used to derive linear error equations which apply to a high performance vehicle such as the Supersonic Transport. The error sources treated consist of:

1. Altimeter uncertainty
2. Deflection of the vertical
3. Accelerometer uncertainty and scale factor error
4. Gyro drift
5. Initial misalignment error
6. Initial condition errors

The error equations are solved for the case of constant gyro drift and altimeter uncertainty.

5. Britting and Palsson; "Self Alignment Techniques for Strapdown Inertial Navigation Systems with Aircraft Application"

One of the more critical problem areas in the application of strapdown inertial techniques to the navigation of commercial aircraft is that of initial alignment. A two stage self-alignment scheme which appears promising in this regard is explored. The first or "coarse" alignment stage utilizes the measurement of the gravity and earth rotation vectors to directly compute the transformation matrix relating the body frame to a reference frame. A linearized error analysis is presented. The second "fine" alignment stage corrects the initial estimate of the transformation by feeding back a computed angular velocity command to the transformation computer. This correction signal is computed by using estimates of the error angles between a known reference frame and the corresponding computed frame. Kalman filtering techniques are used to inplement this technique and an error analysis is presented.

6. Abramson; "Simultaneous Estimation of the State and Noise Statistics in Linear Dynamical Systems"

An optimal procedure for estimating the state of a linear dynamical system when the statistics of the measurement and process noise are poorly known is developed. The criterion of maximum likelihood is used to obtain a optimal estimate of the state and noise statistics. These estimates are shown to be asymptotically unbiased, efficient, and unique, with the estimation error normally distributed with a known covariance. The resulting equations for the estimates cannot be solved recursively, but an iterative procedure for their solution is presented. Several approximate solutions are presented which reduce the necessary computations in finding the estimates. Some of the approximate solutions allow a real time estimation of the state and noise statistics.

Closely related to the estimation problem is the subject of hypothesis testing. Several criteria are developed for testing hypotheses concerning the values of the noise statistics that are used in the computation of the appropriate filter gains in a linear Kalman type state estimator. If the observed measurements are not consistent with the assumptions about the noise statistics, then estimation of the noise statistics should be undertaken using either optimal or suboptimal procedures.

Numerical results of a digital computer simulation of the optimal and suboptimal solutions of the estimation problem are presented for a simple but realistic example.

7. Killingsworth; "Computation Frames for Strapdown Inertial Systems"

Three configurations of a strapdown navigation system are analyzed to determine the effect of mechanizing different computation frames. For systems computing in the navigation frame and in an inertial frame, a linear analysis is developed. These linear theories are verified as accurate analytical descriptions of the systems by a computer solution of the system differential equations by numerical methods. Gyro drift and torquing uncertainty are found to be the predominant error sources. In the system computing in the geographic frame, these two error sources result in bounded latitude error but unbounded longitude error. For inertial frame computation these two error sources result in unbounded errors for both longitude and latitude.

8. Ruth; "Analysis of Error Propagation in Aircraft Inertial Navigation Systems"

An analysis is presented of the dominant sources of error in three different aircraft inertial navigation systems. The three systems studied, all of which are undamped, are:

(1) A local-level free azimuth system

(2) A space-stabilized system

(3) A strap-down system

Linearized mathematical error models describe each system, while the use of a digital computer is necessary to perform the required simulations.

Various flight paths and maneuvers are simulated, and the position errors are analyzed on the basis of the sources contributing to the error. It is assumed that the sources of the errors are contained in the instruments mounted in each system. The error models representing the gyros and accelerometers for each system are described in the paper. The gyros are assumed to be of the single-degree-of-freedom floated, integrating type; and the accelerometers are assumed to be of the pulsed, integrating, floated, pendulum type. The effects of gravitational attraction, inertial acceleration, linear velocity, angular velocity, and angular acceleration on the position error of the systems are presented and analyzed. The factor of elapsed time, short duration missions versus long duration missions, is discussed. The dominant sources of error for the various systems are identified and analyzed.

For the specific systems studied in this paper, conclusions are drawn pertaining to the value gained by flight testing of these systems. It is shown that the effects of the dynamics of the flight path on the systems' performance do not produce the dominant terms, forcing the position errors. In fact, adequate knowledge of the systems' performance may be gained by laboratory and van-road testing of the system. It is shown that the dominant error sources are primarily affected by gravitational attraction and elapsed time; therefore, the additional costs of flight testing systems of these types for performance parameters appear to be unjustified.

It is realized that certain assumptions made in the development of this topic may influence the results. However, the assumptions and their possible ramifications on the systems performance are discussed.

9. Rediess; "A New Model Performance Index Engineering Design of Control Systems"

The theory and application of a new performance index, the Model PI, that brings engineering design specifications into the analytical design process is developed. A parameter optimization design procedure is established that starts with practical engineering specifications and uses the Model PI as a synthesis tool to obtain a satisfactory design. Although the techniques apply to linear, time invariant, deterministic control systems in general, the thesis is developed in the context of flight control systems in order to emphasize the relationship of realistic design requirements to the synthesis process. The Model PI represents a new criterion for approximating one dynamical system by another, based on a novel geometrical representation of linear autonomous systems. It is shown to be an affective performance index in designing practical systems and is shown to be substantially more efficient to use than a comparable model-referenced integral error performance index. A general digital computer program for control system design using the Model PI is developed. Its usefulness is demonstrated by three practical flight control system design examples.

Some interesting developments in linear optimal control resulting from the Model PI theory are presented. The Model PI is shown to provide a means of interpreting the state vector weighting matrix in terms of a model which the optimal system will approach in a limiting case. An interestingly simple solution of the linear optimal control synthesis procedure using one root square locus is presented.

10. Britting; "State Transition Matrix for Inertial Navigation Systems"

This report formulates and solves in state space notation the error equation for inertial navigation systems. The system is assumed to be moving at a constant celestial longitude rate. The state transition matrix is explicitly derived both for long-term and short-term operation. Examples are included to demonstrate the ease with which the state transition matrix can be used for error analysis.

11. Britting; "Strapdown Navigation Equations for Geographic and Tangent Coordinate Frames"

Six coordinate frames relevant to the operation of a radar aided strapdown inertial navigation system are defined and the relationships between these frames are established. Analytic expressions for the specific force are derived for the cases of computation in the local geographic frame and in the tangent coordinate frame. An algorithm for the solution of the direction cosine matrix is indicated. Approximate analytic relations are derived which relate the change in latitude and longitude to the radar coordinates.

12. Johanson; "Uncertainty Estimation Via Preflight Test Procedures for a Space Stabilized Inertial Navigation System"

A space stabilized inertial navigation system is used in VTOL aircraft where flight times of a maximum of one and a half hours duration is expected. This report deals with the usefulness of a preflight test run on a stationary base lasting less than 20 minutes in estimating the I.N.S. uncertainties. The use of data from the alignment phase is also discussed. A simple estimation procedure not including any optimal filtering techniques is assumed. The error propagation for a maximum one and a half hour run is derived when compensation signals with and without a succeeding realignment of the platform is applied.

13. Strunce; "Estimation of Measurement Noise Statistics in Aided Inertial Navigation Systems"

A study was made of an iterative procedure for determining an optimal estimate of measurement noise statistics. Random noise was incorporated in the position fixes used to update a Kalman-Filtered inertial navigation system. A second system was updated with position fixes obtained from a ground based radar where random noise was introduced in range, elevation angle, and azimuth angle. The variance of the measurement noises was estimated by a procedure based on the criterion of maximum likelihood.

14. Johansen; "A Survey of General Coverage Navaids for V/STOL Aircraft -A VOR/DME Error Model"

A promising navigation concept for the V/STOL aircraft is to make a hybrid system comprising a low cost inertial navigation system which is updated by a radio navigation aid. This report gives a short description of suitable en route and terminal radio navaids which are available or may be available during the next decade. A statistical model for the VOR/DME errors is derived together with other information required by a Kalman filter approach to estimate the hybrid navigation system errors.

15. Britting; "Error Analysis of Strapdown and Local Level Inertial Systems which Compute in Geographic Coordinates"

This report is a tutorial exposition of two broad classes of strapdown and local level inertial navigation systems which perform their navigational computations in the local geographic coordinate frame. The strapdown chapter includes discussions of the direction cosine update procedure, alignment techniques and instrument redundancy. An analysis of error sources peculiar to the strapdown mechanization is followed by a perturbation type error analysis which shows that the basic error equations are identical to those which describe the behavior of the local level platform system. The error analysis of the local level system is followed by a rather complete set of analytic and computer solutions for both the stationary and moving navigation cases. The effect of the Foucault mode on the validity of the analytic solutions is discussed. The results of the error analysis are applicable to both navigation system mechanizations.

16. Jankowski; "Hybrid Altimeter Using a Strapdown Inertial Navigation System"

Several methods of combining the output of a strapdown inertial navigation system with an external measurement of altitude are investigated. This combined information is used in the computation of the gravity field vector of the earth. The computation uses the two sources of information in a manner which minimizes the error in the indicated position due to the component errors in the altimeters and the gyros. Two sub-optimal hybrid altimeter configurations are considered. The first altimeter mixes radar altitude, which contains no dynamic lag, with vertical acceleration; the second combines barometric altitude, which contains dynamic lag, with vertical acceleration. The steady state mean squared error of either hybrid system is not reduced by the inclusion of vertical acceleration data. The results of linear optimal filtering theory for discrete systems are reviewed. The linear optimal filtering theory is applied to the problem of hybrid altimetry. It is shown that a combination of a radio altimeter, barometric altimeter and vertical acceleration reduces the mean squared error in the estimate of altitude to a great extent. It is also shown that this accuracy is only very lightly sensible to the error angle in tracking the vertical.

17. Britting; "Unified Error Analysis of Terrestrial Inertial Navigation Systems"

The error equations for a broad class of terrestrial inertial navigation systems are developed using linear perturbation techniques. A system of matrix notation is developed which unambiguously describes the interaction of the physical system with the navigation computer without the artifice of associating a coordinate frame with the computer. The developed analytical techniques are validated by comparing results obtained using the linear perturbation methods with a closed-form analytic solution of the large angle system differential equations for a restrictive case. The error equations for space stabilized and local level mechanizations are developed and the effects of externally supplied altitude information are studied. The analysis shows that these two rather diverse mechanizations have identical characteristic matrices, differing only in their forcing functions. If the error state vector consists of the system's attitude and position errors, the error equations are said to be in the canonical form. The analysis techniques are then brought to bear on a general terrestrial navigation system configuration for which both the mechanized and computational reference frames are arbitrary. It is found that if a nonlinear estimator is used to combine the external and inertially derived altitude information, the error equations can again be written in the canonical form. The unified theory is applied to obtain the error equations for space stabilized, local level, free azimuth, rotating azimuth, and strapdown configurations.

18. Marcus; "Computational Comparison of Strapdown System Attitude Algorithms"

The accuracies of two strapdown system attitude algorithms are compared by digital computer simulation of the algorithms' response to specified angular input rates. For constant rate cases, both the direction cosine matrix (D.C.M.) technique and the Euler parameter technique gave rise to linearly growing drift angle errors. The errors incurred by the Euler parameter technique were significantly less than those incurred by the direction cosine matrix scheme.

For the cases where the simulated body motions were purely sinusoidal, the drift errors incurred by both techniques were bounded sinusoids having the same periods as the applied angular inputs. The magnitudes of the bounds were again less for those routines using Euler parameters rather than D.C.M. schemes. The magnitudes of the bounds for this case were also found to be proportional to the input angular rate raised to the power of the order of the numerical integration scheme employed.

19. Ryan; "Alignment and Calibration of a Strapdown Inertial Measuring Unit"

Alignment refers to a self-contained process of calculating the direction cosine transformation that relates the axes in the strapdown inertial measuring unit (IMU) to the local geographic coordinates. Three alignment methods are analyzed.

The first technique, open loop in nature, uses the accelerometer and gyro measurements to evaluate the direction cosine matrix. The outputs of the inertial instruments are averaged for a period of time so as to reduce the effect of base motion disturbances and instrument noise. The inherent accuracy of this method is rather low, chiefly because even a small amount of angular rotation completely swamps the earth rate signal. Application of this technique is limited to very benign environments such as might exist in a laboratory.

The second method investigates a closed loop strapdown gyrocompass, the design of which is based on classical automatic control considerations. The results show that gyrocompassing of a strapdown IMU is completely analogous to fixed base gyrocompassing of a conventional gimballed IMU. The "leveling" accuracy of this system is limited by the horizontal projections of specific force uncertainty, and the azimuth accuracy is bounded by the east copponent of gyro drift.

The final scheme employs optimal filtering techniques to specify a closed loop gyrocompass and to estimate instrument uncertainties. The accuracy of this system is limited by the "unobservable state variables", the east component of gyro drift, and the north and east components of accelerometer bias. The effect is entirely similar to that which exists in the deterministic case. The statistical filter is able to deduce the north and vertical components of gyro drift and the vertical accelerometer bias. Alignment time for this scheme is faster than the setting times for a conventional gyrocompass design.