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POWER SPECTRAL ANALYSIS OF MAGNETIC FIELDS USING A TIME/DATA 100 SYSTEM

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I. Introduction

This report explains how a small digital computer, the Time/Data 100, that is specialized in the analysis of time-series data, is used to analyze magnetic field data.

First, the computer will be described with reference to its operation and some of the algorithms it uses. Second, the organization of the data is detailed. Third, the results of the analysis of the data are examined. And finally, the Time/Data 100's method is compared with the Blackman-Tukey method of the calculation of the power spectrum.

II. Introduction to the Time/Data 100 Computer and to its Algorithms.

The Time/Data 100 is a small digital computer that can process certain wired-in algorithms. These algorithms can be implemented by using the push-button controls on the console.

Input data can be analog or digital and put into one of two 1001 word memory sections, each word having 7 bits plus a sign bit. The output data can also be analog or digital and is taken from one of two 1001 word memory sections in which each word consists of 17 bits plus a sign bit. Nine track IBM-360 compatible tapes are used for digital input and output. The input tapes are organized into 1011 byte records where the first 10 bytes are 'header' data. The output tapes also have 1011 byte records also with the first 10 bytes as 'header' data.

The contents of one or two of the memory sections can be displayed on the cathode ray tube. All of the 1001 points can be shown or, with the use of the EXPANDED button, any 127 points of the memory can be displayed.

The value of any word can also be displayed on the binary lamps. For example, if the contents of the memory section in INPUT A is being displayed on the lower beam of the tube, then the WORD SELECT switch is set to DISPLAY LOWER and the appropriate word is set on the digit switch. The value of that word will then appear on the binary lamps. It can also be seen on the tube by pushing the MARKER button. This will brighten 127 points, the first point of which is the selected word. If the EXPANDED button is pushed, these 127 points will appear on the tube.

When any of the 18 bit output memories are displayed on the tube, the AMPLITUDE SCALING switch must be used because only 7 bits plus the sign bit of any word can be displayed. When the switch is set, the 7 bits shown on the tube are the bit number selected plus the next six higher valued bits. For example, if the switch is set on 3, then the bits displayed are 2^3 through 2^9 plus the sign bit.

There is also a plotter that will duplicate on paper any one of the memories that is displayed on the tube. Again, it can only display 7 bits plus the sign, so the AMPLITUDE SCALING switch must be used to select the dynamic range to be plotted.

Connected to this processor is the system controller. It controls the data flow and enables the operator to process data continuously. It also has the algorithm and memory select switches that the console has. It shows the 'header' data and has a switch for manual or automatic operation. When the computer is in manual, the operator must execute each step himself and also select the correct memory section that the algorithm needs. For automatic processing, the computer will automatically select the correct memory section and will input, process and output the data. The processing will continue automatically until the MASTER CLEAR button is pushed, the limits of the algorithm are met, or until the number of times the algorithm was processed is equal to the number in the REPEAT digit switch. This switch can be set from 00 to 99, 00 meaning indefinite operation.

There is also a SEARCH switch that allows the operator to find a record on the input tape that is greater than or equal to the record number set in the RECORD digit switches.

The three algorithms we are using are the Direct Fourier Transform, the Auto-Spectral Density and the Ensemble Average.

The Time/Data 100 uses this form of the Direct Fourier Transform for a discrete time series and finite time interval. The formula is:

$$S(Kf_o) = \sum_{n=-N}^{+N} f(nt_o) \cos 2\pi k f_o (nt_o)$$

$$Q(Kf_o) = -\left[\sum_{n=-N}^{+N} f(nt_o) \sin 2\pi K f_o (nt_o) \right]$$

where: $S(Kf_o)$ and $Q(Kf_o)$ are the real and imaginary components of the D.F.T. respectively.

$0 \le K \le 1000$

$2N + 1$ = number of data points

t_o = sampling rate

$f_o = f_N/1000$ f_N = Nyquist frequency = $1/2t_o$

The final form of the algorithm is the Rapid Fourier Transform that uses various time and frequency folds to shorten the calculations. The algorithm takes one input frame and outputs two frames of data containing the real and imaginary components of the D.F.T.

The Auto-Spectral Density algorithm uses the following formula:

$$P(Kf_o) = |S(Kf_o)|^2 + |Q(Kf_o)|^2$$

where $0 \le K \le 1000$.

The algorithm takes 7 bits plus the sign, specified by the AMPLITUDE SCALING switch, of each word of the real and imaginary components of the D.F.T. It squares and adds the two and puts it into one of the output memory sections.

The Ensemble Averaging Algorithm then adds together each successive output frame and puts it into an output memory section.

There is, however, one algorithm that uses all three. It is the Averaged Auto-Spectrum algorithm. It takes the D.F.T. of an input frame, scales the two output frames, takes the sum of the squares and adds each successive output frame to form the averaged auto-spectrum.

This algorithm can be run in a single cycle or continuous mode. When done continuously, it can accumulate at least 512 records and will stop automatically if there is an impending overflow in one of the records.

To test the algorithm, I made up a tape containing records of this function:

$$f(t) = 5\left[10\sin\left(\frac{2\pi t}{12}\right) + 5\sin\left(\frac{2\pi t}{6} + \frac{\pi}{6}\right) + 2.25\sin\left(\frac{2\pi t}{3} + \frac{\pi}{2}\right)\right]$$

Assuming the sampling rate is 1 second, then the corresponding Nyquist frequency is $f_N = \frac{1}{2\Delta t} = \frac{1}{2}$ Hz, $\Delta t = 1$ sec.
If the algorithm does work, then I should find three spikes at 0.083 Hz, 0.166 Hz, and 0.333 Hz corresponding to the three sine functions.

First, I used the D.F.T. algorithm to determine the correct amplitude scaling so that the power spectral density algorithm will use the correct values. An amplitude scaling of 7 was sufficient for the power spectral density algorithm to take the highest value of the D.F.T. of the input function. Second, I ran the machine on automatic and accumulated 45 records. Third, I plotted the resulting averaged power spectrum and found out that there were three spikes at the expected frequencies.

III. The data and its preparation for analysis

The data we are using is from the OGO-E rubidium magnetometer experiment that measures the magnitude of the intensity of the magnetic field. The sampling rate is one point per 144 milliseconds.

In order to do the power spectral analysis we are assuming that the data satisfies the ERGODIC HYPOTHESIS. First, it is a stationary, random process or the ensemble averages of the data don't vary with time. Second, each record is statistically equivalent to every other record. And, third, the ensemble averages of many records can be replaced by one typical time average.

The data was sampled at three different rates:

1. Every point or one point per 144 milliseconds. The corresponding Nyquist frequency is 3.47 Hz.

2. Every seventh point or one point per 1.008 seconds. The Nyquist frequency is 0.496 Hz.

3. Every fourteenth point or one point per 2.016 seconds. The Nyquist frequency is 0.248 Hz.

I wrote a program to read the data tapes, sample them at these intervals and build 1001 point records for input to the Time/Data 100. Each record is 1011 bytes long where the first 10 bytes of header information contain the time of the first data point in each record, the record number, the identification number and the scale factor. The average of each record was subtracted to get rid of the DC component.

The output tape containing all of the data sampled at the three rates was then used as input to the Time/Data 100.

To select the correct amplitude scaling for the power spectrum, the D.F.T. of each record had to be checked so that the most significant bits were used. Without doing this first, some significant bits, especially in the lower frequencies, would be lost and erroneous results would be obtained. After the amplitude scaling setting and been determined, the spectral averaging was done.

IV. Time/Data 100 Results

Our examples are taken from October 26, 1968 from $6^{H}0^{M}15^{S}$ to $14^{H}8^{M}0^{S}$. The first graph is a sample of the first record of the data and the second graph is its corresponding power spectral density. The curve is very rough but it can be smoothed by accumulating many records. The third graph is the accumulated power spectrum of the above time interval sampled at 144 milliseconds. The resulting curve is much smoother than the power spectrum of one record. The fourth and fifth graphs show the time interval sampled at 1.008 seconds and 2.016 seconds respectively. These are not as smooth as the third graph, because of fewer accumulated records, but they still show the same general shape of the spectrum of the third graph.

V. Blackman-Tukey Method

We are also comparing the Time/Data method of computing the averaged power spectrum with the Blackman-Tukey method. They express the power spectral density in terms of the autocovariance of the time-series.

$$P(f) = \int_{-\infty}^{+\infty} C(\tau)\, e^{-i 2\pi f \tau}\, d\tau$$

where:

$$C(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} f(t)\, f(t+\tau)\, dt$$

We are using the following algorithm for our data: Given one record of the time-series f_t, $t = 0,1,\ldots,1000$; the total number of observations are 1001.

1. Prewhiten the data.

$$\tilde{f}_t = f_t - 0.6 f_{t-1} \qquad 1 \le t \le 1000$$

2. Calculate the autocovariance and subtract the mean.

$$C_r = \frac{1}{1000-r} \sum_{t=1}^{1000-r} \tilde{f}_t \tilde{f}_{t+r} - \left(\frac{1}{1000} \sum_{t=1}^{1000} \tilde{f}_t \right)^2$$

where $r = 0,1,\ldots,100$ are the numbers of lags.

3. Calculate the finite cosine series transform.

$$V_r = \left[C_0 + 2 \sum_{q=1}^{99} C_q \cos \frac{q r \pi}{1000} + C_{100} \cos r\pi \right]$$

4. Use of the "Hanning" lag window is equivalent to the convolving of the computed power according to:

$$U_0 = \frac{1}{2}(V_0 + V_1)$$

$$U_r = \frac{1}{4} V_{r-1} + \frac{1}{2} V_r + \frac{1}{4} V_{r+1} \qquad 1 \leq r \leq 99$$

$$U_{100} = \frac{1}{2} V_{99} + \frac{1}{2} V_{100}$$

5. Correct for prewhitening and correction for the mean

$$\left(\frac{1}{1.36 - 1.20 \cos \pi/300}\right) U_0$$

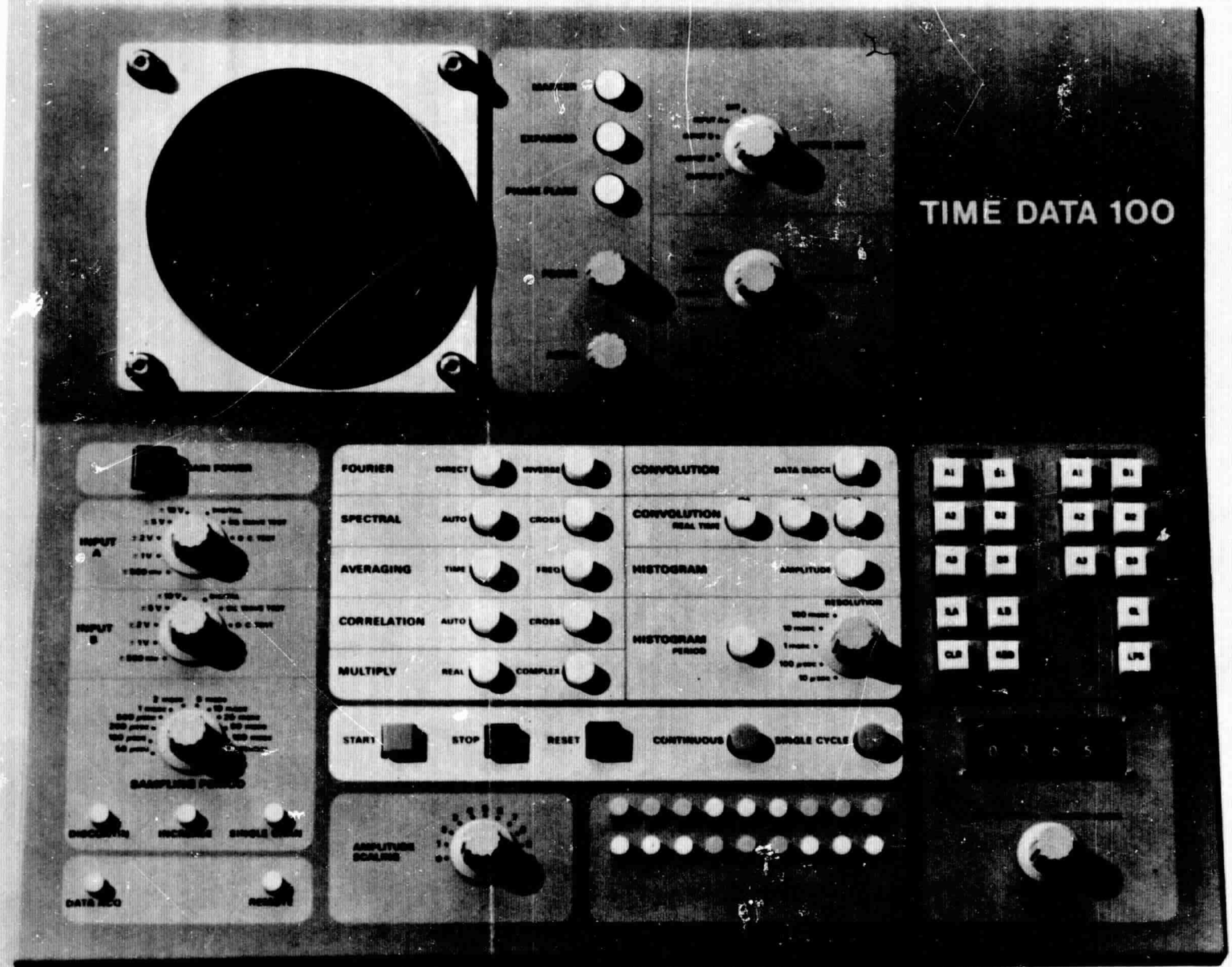
$$\left(\frac{1}{1.36 - 1.20 \cos r\pi/100}\right) U_r \qquad 1 \leq r \leq 99$$

$$\left(\frac{1}{1.36 - 1.20 \cos[(1 - 1/600)2\pi]}\right) U_{100}$$

VI. Conclusion

We are presently debugging the program that prepares the data for the Blackman-Tukey method of power spectral analysis and will eventually be using both methods to analyze the data.

Since the Auto-Spectral Averaging algorithm of the Time/Data 100 can process one record in one second, we can accumulate many records in a very short time. We can also selectively accumulate records and can compare their results in just a few minutes. So, the speed of computation and the immediate return of results are important factors in using the Time/Data 100 for power spectral analysis. Therefore, with these two factors in mind and from the results we have already obtained, we feel confident that the Time/Data 100 computer can be successfully used to analyze our data.


TIME DATA 100
FOURIER
DIRECT
INVERSE
SPECTRAL
AUTO
CROSS
AVERAGING
TIME
FREQ
CORRELATION
AUTO
CROSS
MULTIPLY
REAL
COMPLEX
CONVOLUTION
DATA BLOCK
CONVOLUTION
REAL TIME
HISTOGRAM
AMPLITUDE
RESOLUTION
HISTOGRAM
PERIOD
INPUT
A
INPUT
B
START
STOP
RESET
CONTINUOUS
SINGLE CYCLE
AMPLITUDE
SCALING

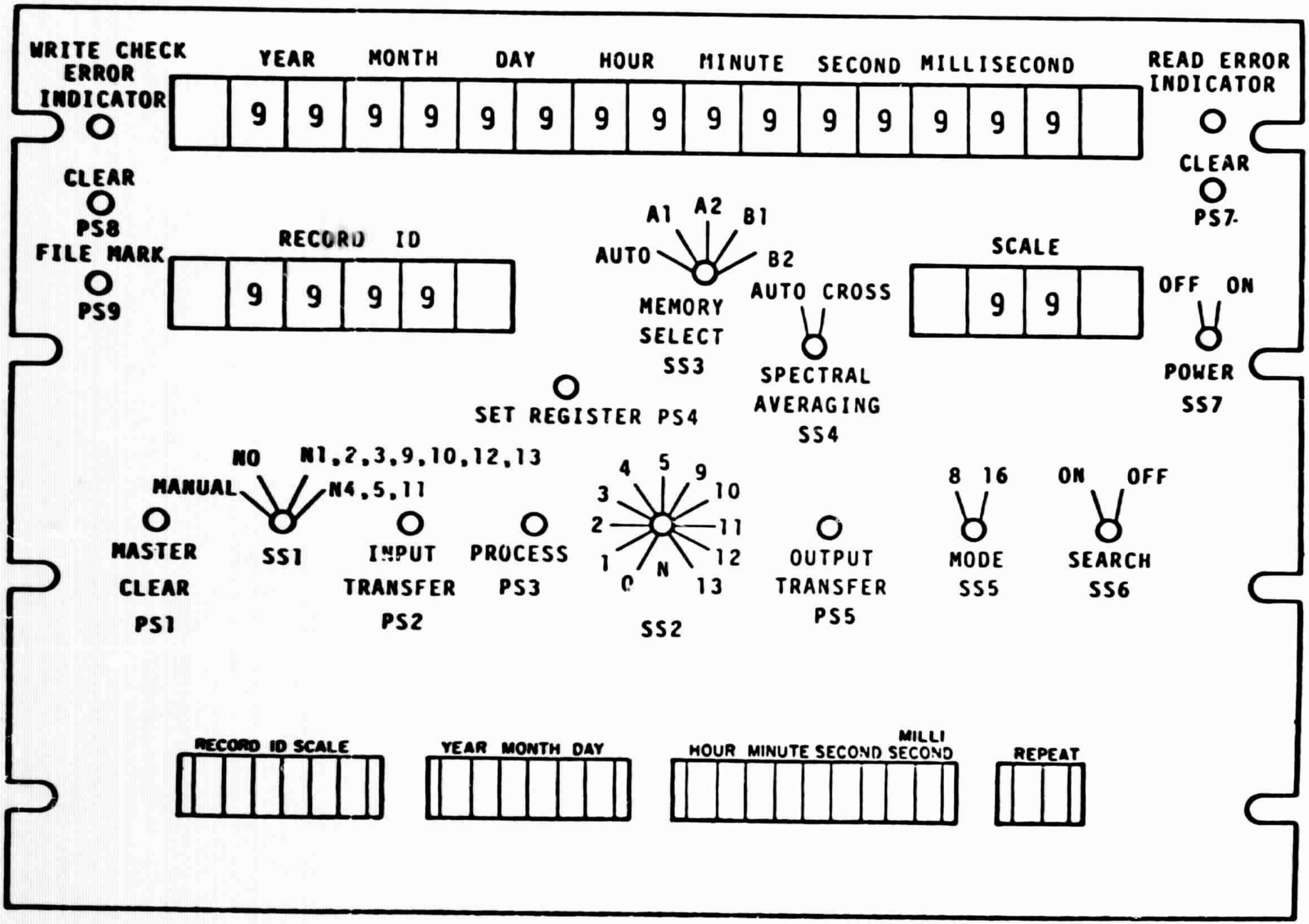
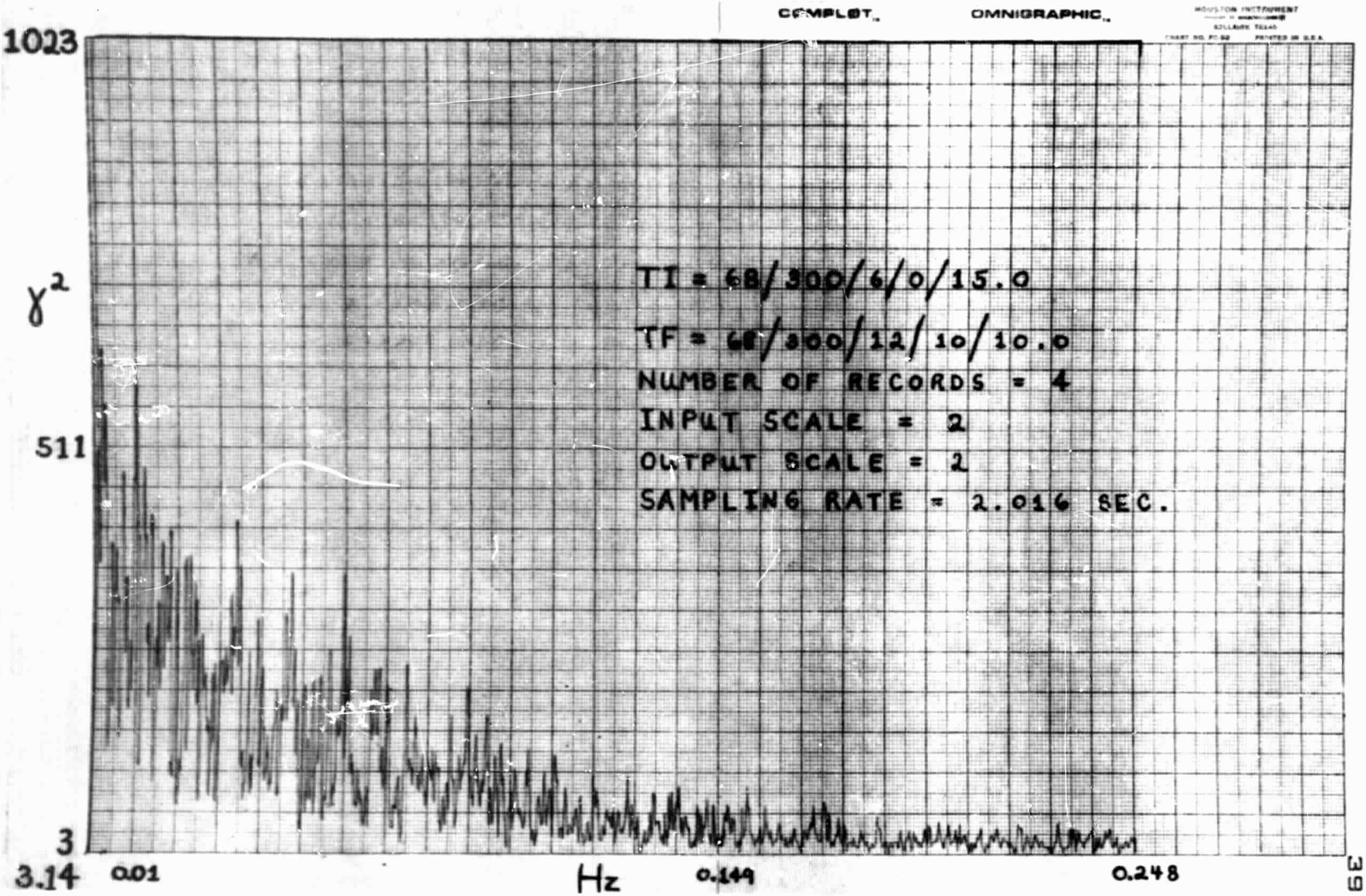
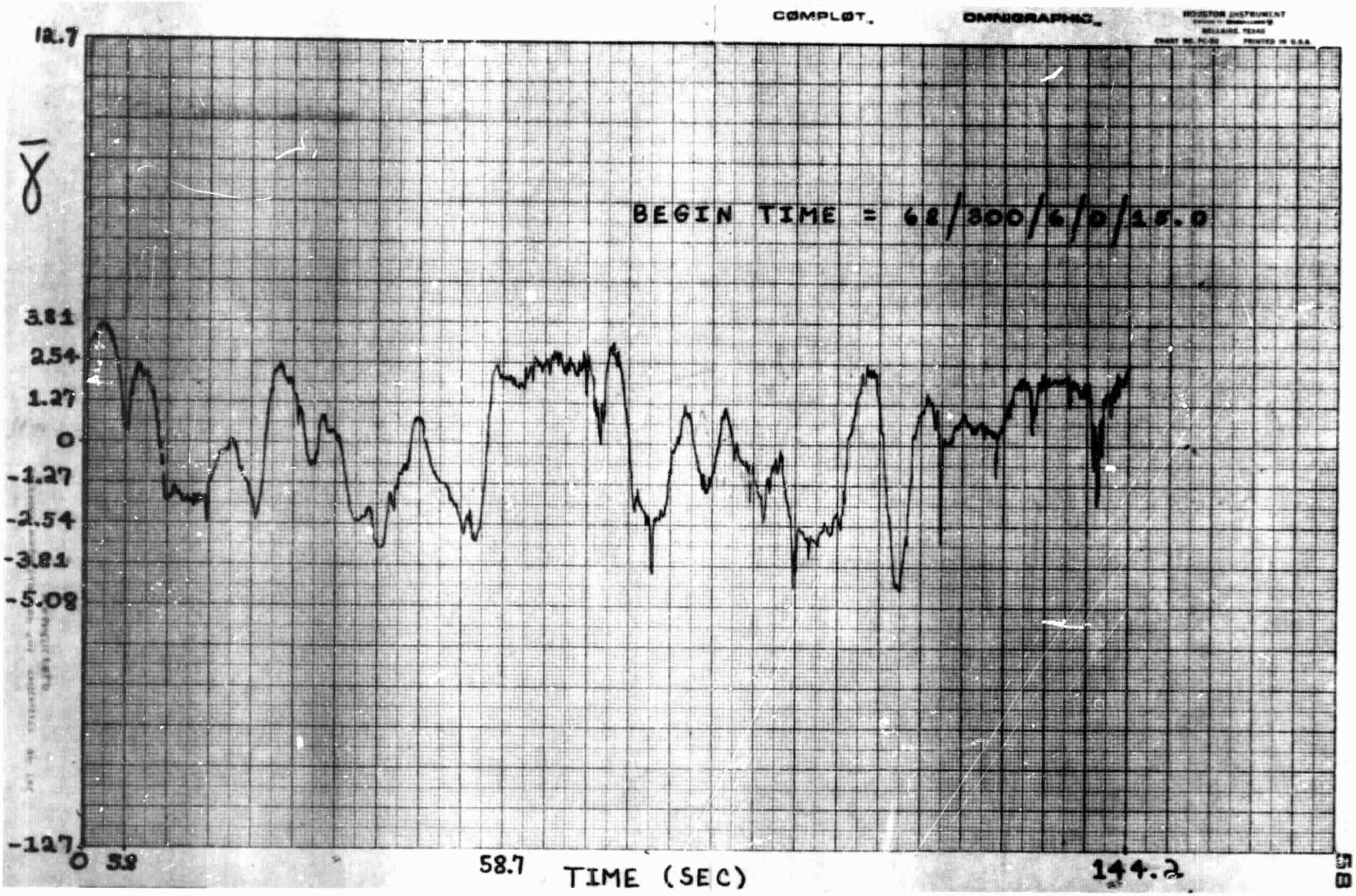
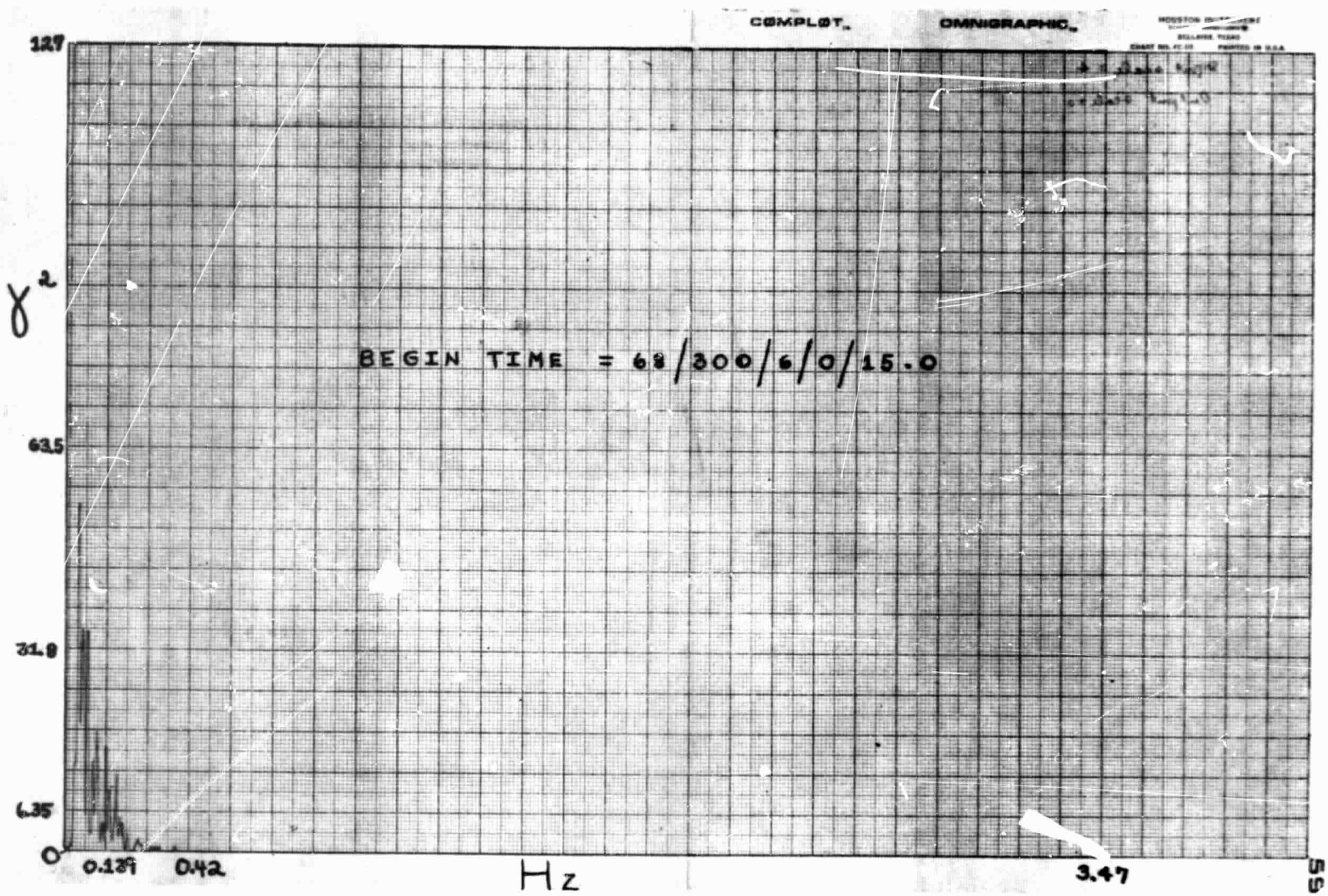


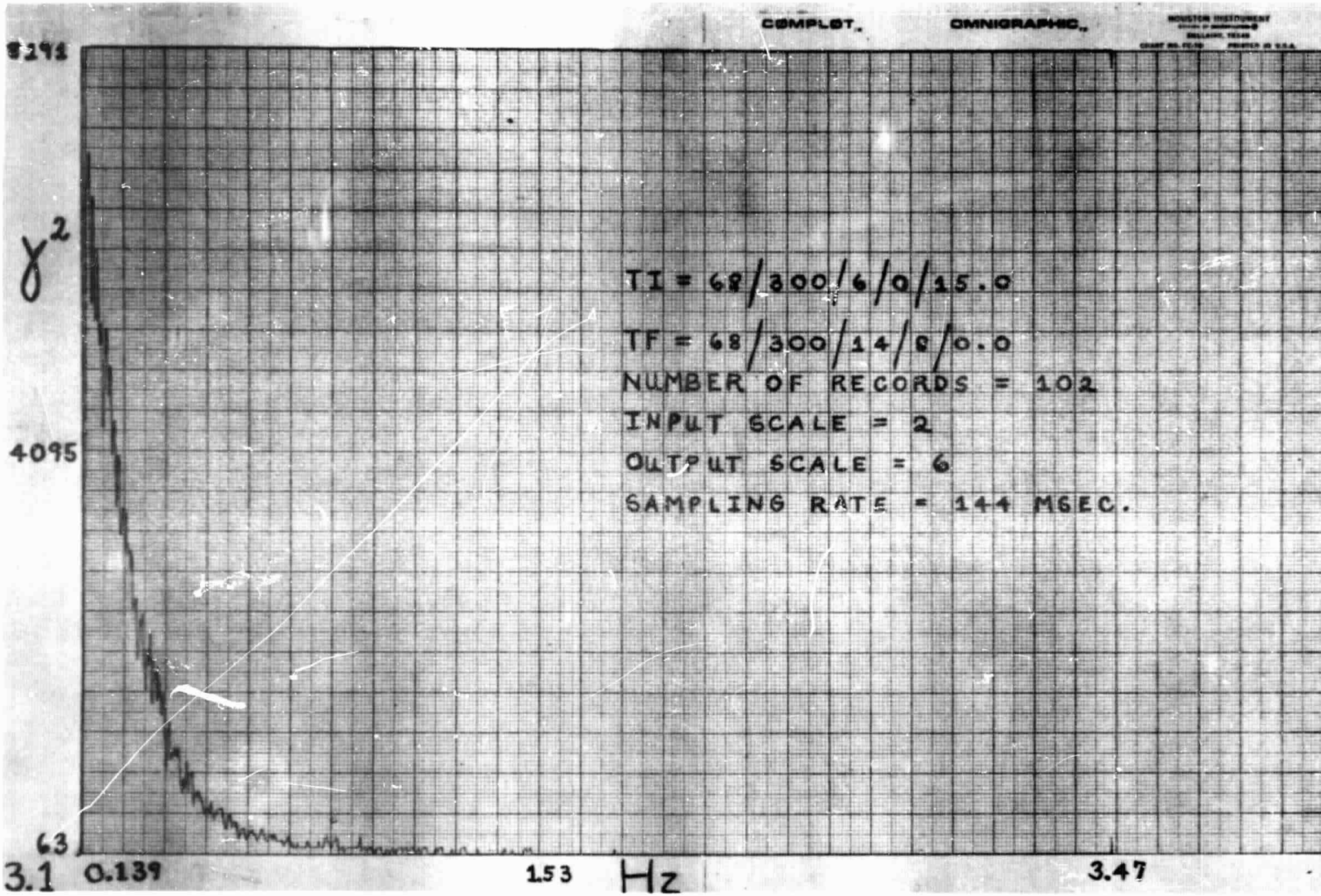
Figure 3. Interface controller front panel


COMPLOT
OMNIGRAPHIC
1023
γ²
511
3
3.14
0.01
Hz
0.149
0.248
TI = 68/300/6/0/15.0
TF = 68/300/12/10/10.0
NUMBER OF RECORDS = 4
INPUT SCALE = 2
OUTPUT SCALE = 2
SAMPLING RATE = 2.016 SEC.

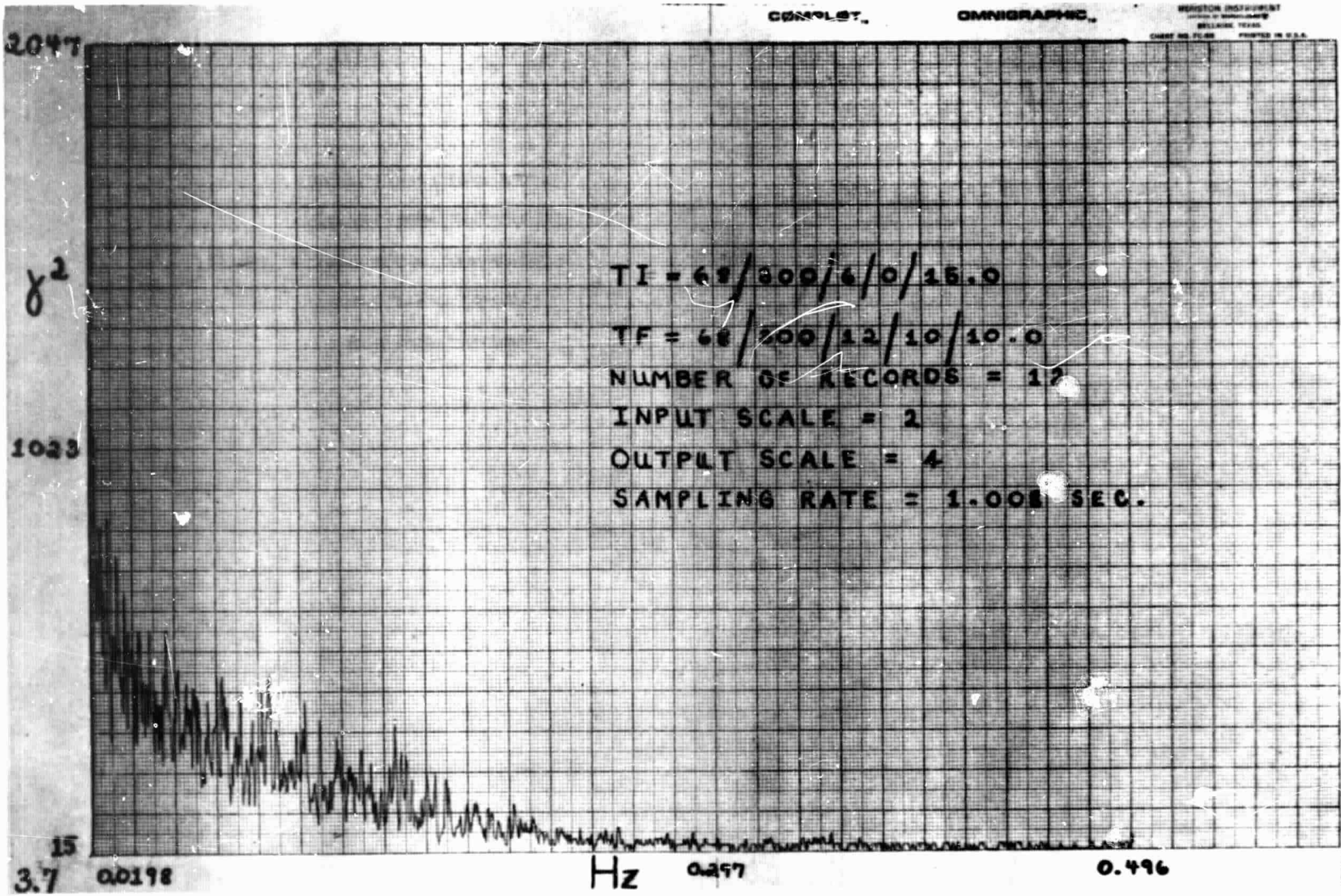


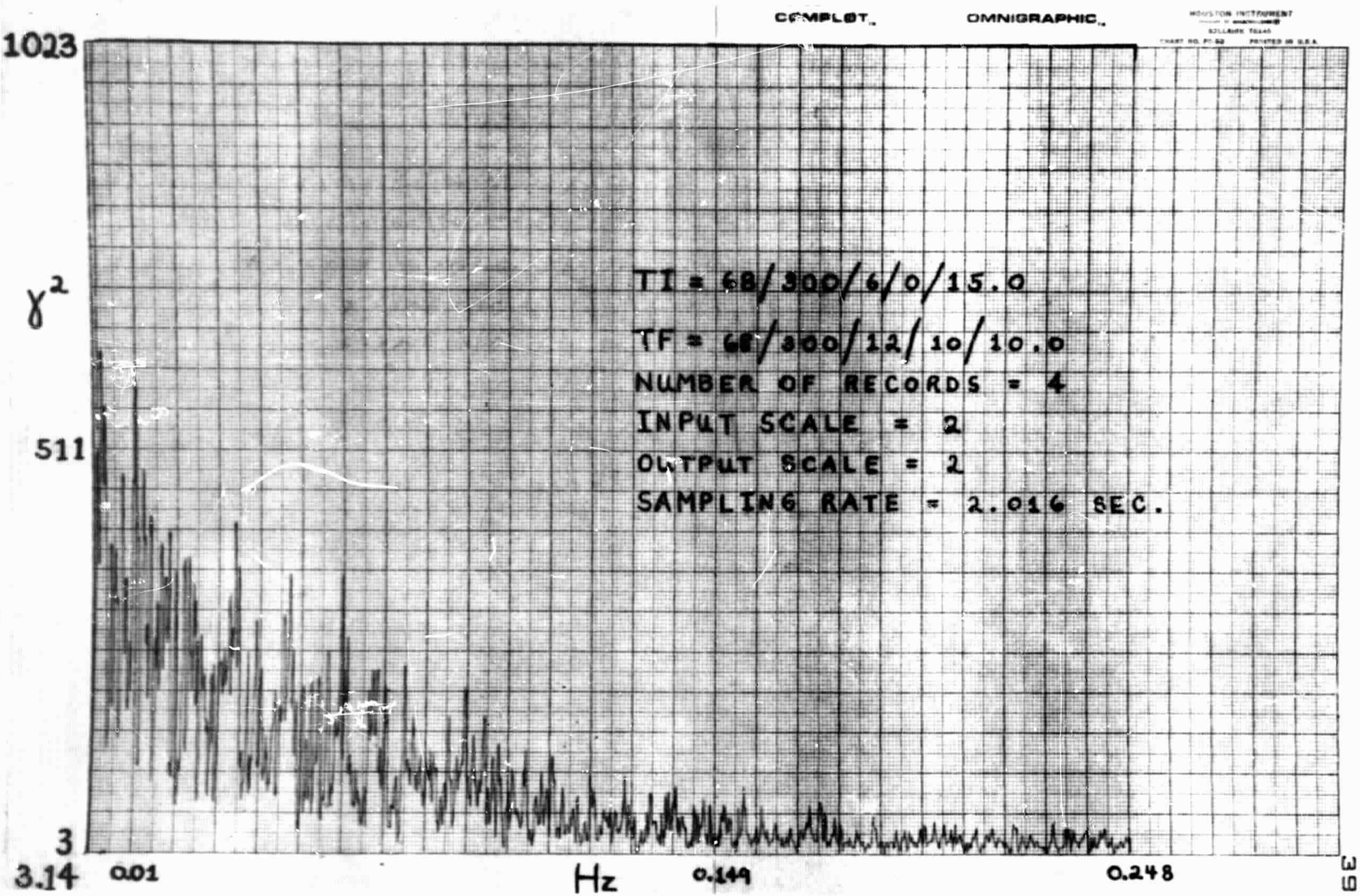
CØMPLØT.
OMNIGRAPHIC
12.7
$\bar{\gamma}$
BEGIN TIME = 68/300/6/0/15.0
3.81
2.54
1.27
0
-1.27
-2.54
-3.81
-5.08
-12.7
0
5.8
58.7
TIME (SEC)
144.2
58


COMPLOT
OMNIGRAPHIC
127
γ^2
63.5
31.8
6.35
0
BEGIN TIME = 68/300/6/0/15.0
0.139
0.42
Hz
3.47
5.5



COMPLOT
OMNIGRAPHIC
HOUSTON INSTRUMENT
8191
γ^2
4095
63
3.1
0.139
1.53
Hz
3.47
TI = 68/300/6/0/15.0
TF = 68/300/14/8/0.0
NUMBER OF RECORDS = 102
INPUT SCALE = 2
OUTPUT SCALE = 6
SAMPLING RATE = 144 MSEC.


COMPLOT
OMNIGRAPHIC
2047
γ^2
1023
15
3.7
0.0198
Hz
0.297
0.496
TI = 68/300/6/0/15.0
TF = 68/300/12/10/10.0
NUMBER OF RECORDS = 12
INPUT SCALE = 2
OUTPUT SCALE = 4
SAMPLING RATE = 1.008 SEC.



COMPLOT
OMNIGRAPHIC
HOUSTON INSTRUMENT
1023
511
3
3.14
γ^2
0.01
Hz
0.149
0.248
TI = 68/300/6/0/15.0
TF = 68/300/12/10/10.0
NUMBER OF RECORDS = 4
INPUT SCALE = 2
OUTPUT SCALE = 2
SAMPLING RATE = 2.016 SEC.

69