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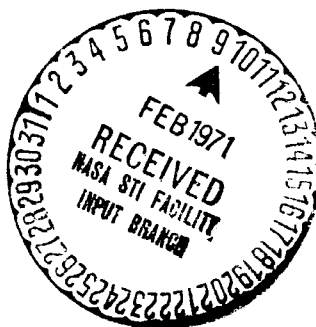
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THE GERT SIMULATION PROGRAMS:
GERTS III, GERTS IIIQ, GERTS IIIC, AND GERTS IIIR

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ABSTRACT

This report describes the procedures for using a digital computer program for simulating GERT networks. New and advanced GERT concepts are introduced.

The simulation program can accommodate GERT networks which have logical operations associated with the input side of a node and branching operations associated with the output side of a node. Logical operations associated with a node are defined in terms of the number of realizations of activities incident to the node that must occur before the node can be realized. A similar quantity is required for realizing the node after its first realization. The branching operation associated with a node is either DETERMINISTIC (all branches are taken that emanate from the node) or PROBABILISTIC (a selection of one of the branches emanating from the node is taken when the node is realized).

Branches of a GERT network are described in terms of a probability that the branch is realized; a time to perform the activity represented by the branch; a count designation and an activity number. The time associated with a branch can be a random variable. The count designator identifies a count set for which a counter is indexed every time the branch is realized. The activity number identifies nodes that are affected by the realization of the branch. Through activity numbers, a network can be modified during the simulation of the network.

GERT networks having the above characteristics are simulated by a program labeled GERTS III. GERTS III is a fundamental package and as such provides the foundation for building advanced network simulation programs. In this report GERTS III has been extended in three directions. First, a queue node capability was added resulting in the GERTS IIIQ program. Then cost information was added to obtain GERTS IIIIC. The third extension involved the

inclusion of resource requirements for each activity and limited resources to perform the project. The simulation package to study resource allocation has been labeled GERTS IIIIR.

Examples of the use of GERTS III, GERTS IIIIQ, GERTS IIIIC, and GERTS IIIIR are presented in the report. The GERTS III programs are written in FORTRAN IV. The program has been exercised on the IBM 360/65 system. GERT networks with up to 1,000 nodes can be analyzed.

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THE GERT SIMULATION PROGRAMS:

GERTS III; GERTS IIIQ; GERTS IIIC; and GERTS IIIR

INTRODUCTION

The GERT simulation program is a general purpose program for simulating networks. The program is written in FORTRAN IV. The input to the program is a description of the network in terms of its nodes and branches along with control information for setting up the simulation conditions. Applications of earlier GERT simulation programs [7,8,9] resulted in the need for new network concepts and additional capability. This need has been satisfied with the completion of the GERT Simulation Program III hereafter referred to as GERTS III.

The following list describes the features in GERTS III.

1. Branches that are characterized by:
 - a. A probability of being included in the network;
 - b. A time required to complete the activity represented by the branch. The time is specified by defining a parameter set number and a distribution type;
 - c. A counter type to identify the branch as belonging to a particular group of branches; and
 - d. An activity number.
2. Nodes that are characterized by:
 - a. The number of releases required to realize the node for the first time;
 - b. The number of releases required to realize the node after the first time;
 - c. The removal of events that are scheduled to release the node;
 - d. The method for scheduling the activities emanating from the node (DETERMINISTIC or PROBABILISTIC); and
 - e. The statistical quantities to be estimated for the node.

3. Modification of the network based on the occurrence of end of activity events during the simulation of the network.
4. A method for tracing a set of simulation runs.
5. Automatic printout of the description of the network and the final results.

During the research leading to GERTS III, the following concepts were explored:

- 1) nodes that provided a storage or queue capability - a Q-node;
- 2) costs associated with the performance of activities; and
- 3) activities that required resources.

It was found that GERTS III could be modified to allow the simulation of networks that involved these concepts and implementation proceeded on a limited scale. It was felt that separate programs should be maintained for these new concepts but that each should contain the basic GERT simulation program, GERTS III. The results of the exploratory research are: 1) GERTS IIIQ, a GERT network simulation program that includes Q-nodes; 2) GERTS IIIC, a GERT network simulation program that collects cost statistics; and 3) GERTS IIIR, a GERT network simulation program that involves resource allocation decisions.

The main purpose of this report is to describe the procedure for using GERTS III, GERTS IIIQ, GERTS IIIC, and GERTS IIIR. Since many new concepts associated with GERT have been developed it is necessary to describe these before proceeding with examples illustrating the use of the new programs.

OVERALL PROGRAM OPERATION

The GERTS III program performs a simulation of a network by advancing time from event to event. In simulation parlance this is termed a next event simulation. The events associated with a simulation of a GERT network are:

(1) Start of the simulation; (2) End of an activity; and (3) Completion of a simulation run of the network. Since GERTS III is a FORTRAN IV program the operating procedure is the standard FORTRAN operating procedure. Many concepts of GERTS III were adopted from GASP IIA [10].

The start event causes all source nodes to be realized and schedules the activities emanating from the source nodes according to the output type of the source node. The output type for all nodes is either DETERMINISTIC or PROBABILISTIC. In the former case, all activities emanating from the node are scheduled and in the latter case, only one of the activities emanating from the node is scheduled. By scheduling an activity is meant that an event "end of activity" is caused to occur at some future point in time. The simulation proceeds from event to event until the conditions which indicate that the simulation of the network is completed are obtained. The above process is then repeated for a specified number of simulations of the network.

As part of the input data, the number of releases required to realize a node is specified. Each time an end of activity event occurs, the number of releases for the end node of that activity is decreased by one. When the number of releases remaining is zero, the node is realized. At this time the number of releases is set equal to the number of releases required to realize the node after the first time, and the activities emanating from the node are scheduled. Again, the number of activities scheduled depends on the output type for the node.

For each activity scheduled, an end of activity event is put in a file containing all events in chronological order. The end of activity events are removed from the event file one at a time and at each removal instant, a test is performed to determine if a node is realized. If a node is not

realized, the next event is removed from the event file. If a node is realized, activities from that node are scheduled and the simulation is continued. The simulation ends when a prescribed number of sink nodes have been realized. As part of the input data, the number of source nodes, sink nodes and nodes on which statistics are collected as well as their node numbers and the number of nodes required to realize the network are defined.

The above process describes one simulation of a network. The program is written to allow multiple simulations to be performed. The number of simulation runs to be performed is part of the input data. The GERT simulation program automatically initializes the pertinent variables in order that consecutive simulations of the same network can be performed and, if desired, permits simulations of different networks to be performed consecutively.

GERTS III NETWORK CHARACTERISTICS

GERT networks consist of nodes and directed branches. First consider the characteristics that describes a node. The number of releases associated with a node specifies the number of times activities incident to the node must be realized before the node can be realized. When the number of releases is 1, the input side of the node can be thought of as an OR operation. If the number of releases equals the number of activities incident to the node, the node can be thought of as an AND operator. However, it is permissible to specify the number of releases to be less than or greater than the number of activities incident to the node. For example, the number of releases can be 2 whereas the number of activities incident to the node could be 3. This would represent the case where if 2 of the 3 activities were realized, the node is realized. Alternatively, the number of releases can be 2 and the number of activities incident to the node could be 1. This would represent the case where the activity must be realized twice before the node

is realized.

Figure 1 illustrates the node symbolism for GERTS III.

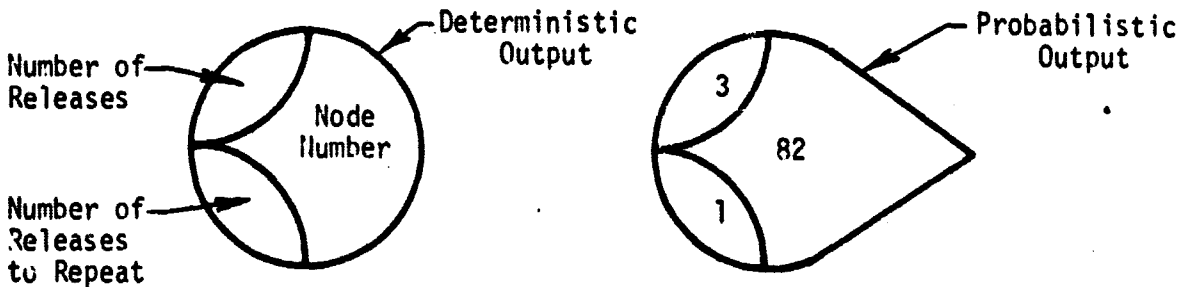


Figure 1. Node Symbolism for GERTS III

In Figure 1 it is seen that the semicircle, \smile , on the output side of a node is used to represent a DETERMINISTIC output, and a lazy V, \succ , for a PROBABILISTIC output. Nodes are also characterized by their function in the network. A GERT analyst can specify a node as:

1. A Source Node;
2. A Sink Node;
3. A Statistics Node; or
4. A Mark Node.

Activities emanating from a source node are started at time zero. A sink node is a node that indicates that the network may be realized when it is realized. (NOTE: a sink node may have successor activities.) A statistics node is one on which statistics are maintained. All sink nodes are automatically made statistics nodes. A mark node establishes a reference time and permits the calculation of the time it takes to go between two nodes of the network.

For statistics nodes, GERTS III obtains statistical estimates associated with the time a node is realized. Five types of time statistics are possible:

- F. The time of first realization of a node;
- A. The time of all realizations of a node;
- E. The time between realizations of a node;
- I. The time interval required to go between two nodes in the network; and
- D. The time delay from first activity completion on the node until the node is realized.

The nodes on which statistics are to be collected and the type of statistics desired are part of the description given to a node by the input to GERTS

III. They are not part of the graphical representation.

The branches of GERT networks represent activities and/or information transfers. The term activity will be used to identify both. Activities emanate from a start node and are incident to an end node. Associated with activities are a probability that the activity will be realized given its start node is realized and a time to perform the activity given the activity is realized. For GERTS III the time variable is specified by a parameter set number and a distribution type. The following nine distribution types are available:

1. Constant;
2. Normal;
3. Uniform;
4. Erlang;
5. Lognormal;
6. Poisson;
7. Beta;
8. Gamma; and
9. Beta fitted to three parameters as in PERT.

The parameter set number along with the distribution type completely describe the time variable associated with an activity. Each distribution type specifies the arrangement of the parameters in a parameter set. With GERTS III, two additional characteristics can be associated with an activity.

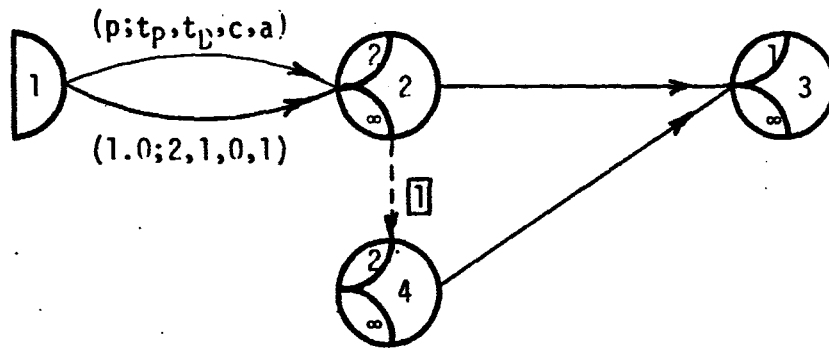
These are a counter type and an activity number.

The counter type number specifies the counter to be increased by 1 every time the activity is realized. The number of counter types permitted is limited to 4*. Any number of activities may be associated with a counter type.** Statistics are automatically kept on the counter types. At the end of all simulation runs, the average and standard deviation of the number of times a counter type was realized prior to the realization of each node for which statistics are collected is determined and printed. In addition, the minimum and maximum number of times activities having the specified counter type were realized during a simulation is printed. Since the number of counts is always referenced to the realization of a node, the number of counts occurring prior to the realization of a node may be different in different simulation runs due to the sequence in which the nodes are realized.

Activity numbers are given to activities to permit network modifications based on the realization of the activity. Specification of an activity number does not automatically indicate that the network will be modified. However, only activities with activity numbers can cause the network to be modified. Network modification involves the replacing of a node by another node on the output side only.*** Thus when a node is realized, the activities to be started depend on the modifications that have taken place. For example if node 8 replaces node 5 then when node 5 is realized the activities emanating from node 8 are scheduled to start. A node may be changed many times before it is actually realized.

-
- * Changes in the dimensions of two arrays can be made to increase this value.
 - ** Activities incident to nodes on which delay statistics are collected or to Q-nodes may not have counter types associated with them.
 - *** The program can be modified to change the input side of a node also, [9, p. 57]. This involves decisions on the part of the user as to the number of releases remaining on the input side. Ref. [6] contains an example in which the input side of a node was modified.

The activity number causing the network modification along with all the nodes to be replaced, and the nodes to be inserted, are specified by the user. The method for incorporating network modifications is described later in the program operating procedure section. Figure 2 illustrates the branch and node modification notation that will be used throughout this report. Modifications will be shown by a dashed branch with the activity number attached in a square. The modification in Figure 2 is read "the output of node 2 is replaced by node 4 when activity 1 is realized".



LEGEND

p = probability of realization
 t_p = parameter set for time
 t_D = distribution type
 c = counter type
 a = activity number


 Node A is replaced by node B when activity with activity number 1 is realized.

Figure 2. Illustration of Branch Descriptors and Network Modification Symbolism.

As an illustration of these new characteristics, consider the network of Figure 3.

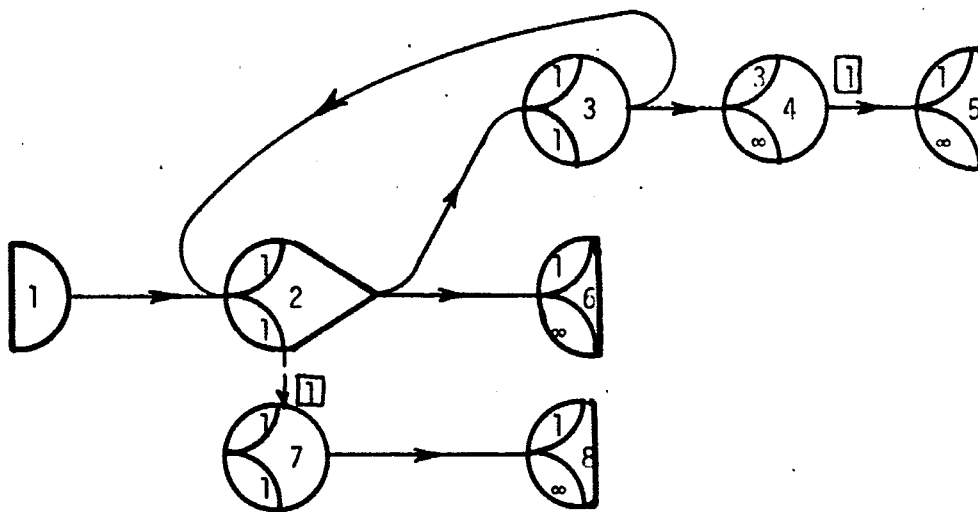


Figure 3. Network Containing Information Branches in Addition to Activities.

This network represents the changing of the network structure when the self-loop about node 2 is taken three times and is accomplished in the following manner. The output of node 3 is DETERMINISTIC so that every time node 3 is realized both branches emanating from node 3 are taken. The branch from node 3 to node 4 is used to count the number of times node 3 is realized. Node 4 is only realized when the branch incident to it is realized three times (of course, this corresponds to three traversals of the self-loop). When node 4 is realized, the activity labeled activity number 1 (a 1 on the network in this case) causes node 2 to be replaced by node 7, and the objective of changing the network is achieved.* Of significant importance in the above network is the incorporation on the network of branches representing activities and branches representing

* Care is required here to ensure that the branch from node 4 to node 5 is realized prior to the realization of the branch from node 3 to node 2. If a zero time is associated with both branches, normal operation would have the branch from node 3 to node 2 realized first since it was scheduled first. By assigning a small negative time (-.000001) to branch from node 4 to node 5, the desired ordering can be obtained.

information transfers. The inclusion of different types of branches within a GERT network expands the network modeling capability within the GERT framework.

Input to GERTS III and Limitations

The input requirements for GERTS III consist of at most 7 different types of data cards. These seven cards describe the network and the control information for performing the simulation. A general description of each card is provided below. In Appendix A, a complete description for each field of each Data Card type is presented.

<u>Data Card Type</u>	<u>General Description</u>
1	Identification Information, number of times simulation is to be performed and an initial random number seed (1 card).
2	General node, counter and network modification data (1 card).
3	Description of each node (1 card for each node).
4	Parameters of time variables associated with activities (1 card for each parameter set).
5	Description of each activity (1 card for each activity).
6	Network modifications desired (1 card for each activity that modifies network. If none, no Data Card Type 6 is required).
7	Run numbers to be traced (1 card only if tracing is requested by using a negative project number).

The dimensions of the GERTS III program have been set to allow for a maximum of 999 nodes, 999 activities, 4 counter types, collections of statistics on $100/(\text{number of counter types} + 1)$ nodes, and 300 parameter sets.

Examples of GERTS III

In previous reports examples were given that illustrated the use of the GERT Simulation Program to model to:

1. The modification of a project based on elapsed time [9];
2. The modification of a network based on the realization of the first of two activities [9];
3. The starting of a phase of a project based on progress to date [9];

4. Multiple modifications of a node during one realization of a network [9];
5. The Planning R & D Projects [3];
6. An advertising promotion in studying consumer brand choice [4]; and
7. A manufacturing process [11].

During the past year, the GERTS program has been used to analyze segments of a University [1], a product development problem for a large computer manufacturer, the R & D program for a weapons system and maintenance and checkout operations.

In this report, two examples are presented that demonstrate the new concepts included in GERTS III. The examples are:

1. Illustration of the features of GERTS III; and
2. Analysis and sequencing of space experiments.

Example 1. Illustration of the Features of GERTS III.

Figure 4 shows the network to be analyzed in Example 1. The source node for the network is node 2 and the sink node is node 12. From node 2 three activities emanate which are performed simultaneously. These activities cause nodes 3, 4, and 5 to be realized. The activities emanating from nodes 3, 4, and 5 are all incident to node 6. The number of releases required to realize node 6 is three, therefore node 6 will only be realized when all three activities incident to node 6 are realized. For this example, we desire to obtain statistics on the time delay between the completion of the first activity incident to node 6 and the time node 6 is realized. To obtain these statistics, node 6 is defined as a statistics node with delay statistics (code D) desired.

It is also desired to collect statistics on the time required to go from node 7 to node 11. To accomplish this node 7 is defined as a mark node (node type 4) and node 11 is defined as a statistics node with the statistics

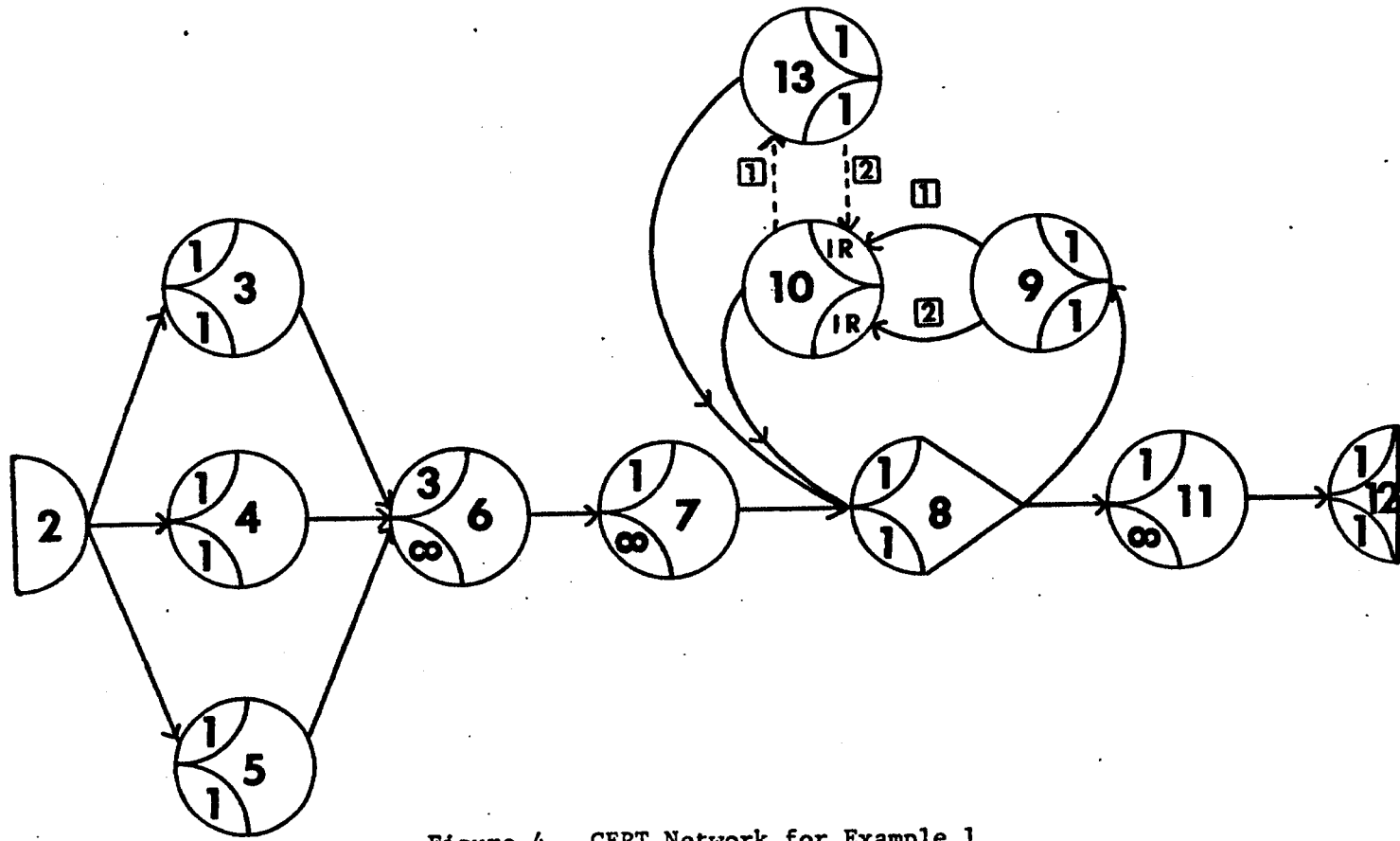


Figure 4. GERT Network for Example 1

calculated on an interval basis (code I). The statistical quantities collected at node 11 will be the interval of time from the realization of node 7 to the realization of node 11. If alternative paths existed between node 7 and 11, the interval node would collect statistics on the time required to traverse the separate paths. This will be further illustrated in Example 3.

Node 8 has a probabilistic output side so that either activity emanating from node 8 can be taken. For those situations where the feedback path is taken, it is desired to determine the time required to traverse the feedback path. This is accomplished by making node 8 a statistics node with statistics collected on time between realization of node 8 (code B). Nodes 9 and 10 were also defined as statistics nodes. For node 9 statistics are collected on its first realizations (code F) and for node 10 statistics are collected on all realizations (code A). Thus, this example includes all the types of statistical calculations that can be included within the GERTS III program. Since node 12 is a sink node, statistics will automatically be collected on it. For this example, statistics on the first realization of node 12 were specified. If the type of statistics desired is not specified by the input information, the GERTS III program assumes that statistics on first realization are desired, i.e., the default condition is first realization.

Also included in this example is the network modification feature and the stopping of an activity in progress. If the top activity (activity 1) from node 9 to node 10 is realized first then node 10 is replaced by node 13*. If the bottom activity (activity number 2) is realized first then node 10 remains in the network. If node 10 had been replaced by node 13 then

* For nodes on feedback paths, the input and output sides are reversed. However, the number of realizations to cause the node to be realized for the first time is still indicated by the top number.

node 10 is reinstated into the network. The network modification is implemented by assigning activity numbers to the branches between node 9 and node 10. When either of these activities are completed, the network modification is implemented and the other activity is stopped since node 10 has an "R" assigned to it. The removal of scheduled activities incident to a node applies to all realizations of the node.

A listing of the input cards for this example is shown in Figure 5. The description of the network that is printed by the GERTS III program is shown in Figures 6 and 7. Figure 8 presents a trace of a simulation run for this network. We will use this trace to describe the operating procedure of the GERTS III program in simulating the network shown in Figure 14.

The simulation begins by scheduling end of activity completion events from each source node. For Example 1, the source node is node 2 and end of activity events are scheduled for the activities from node 2 to node 3, node 2 to node 4, and node 2 to node 5. To obtain the time for each of these events, samples are drawn from: 1) a normal distribution using parameter set 1, 2) the Erlang distribution using parameter set 2, and 3) the uniform distribution using parameter set 3.

The trace of the simulation starts with the first end of activity event. This is seen to be the activity that is incident to node 5 and the event occurs at time 1.88. Since node 5 had its number of releases equal to 1, node 5 is realized and the activity from node 5 to node 6 can be initiated. An end of activity event for this activity is then scheduled by the program. At time 4.31, the activity on node 4 is completed as shown by the second line in the trace of Figure 8. At time 7.88 the activity from node 5 to node 6 was completed and we have the first activity incident to node 6 being completed. (To determine from the trace that this was the activity from node 5

ALL FEATURES -1 5201970 500 11 40 1267										1	EX 1 10
13	1	1	1	6	1	1				2	EX 1 20
2	1		0							3	EX 1 30
3		1	10								EX 1 40
4		1	10								EX 1 50
5		1	10								EX 1 60
6	3	3	D		1	1	D				EX 1 70
7	4	1	10								EX 1 80
8	3	1	1P		1	1	B				EX 1 90
9	3	1	10		27		5 F				EX 1 100
10	3	1	1DR		30		2 A				EX 1 110
11	3	1	D		8		2 I				EX 1 120
12	2	1	D		35		2 A				EX 1 130
13		1	10								EX 1 140
0											EX 1 150
10			0		100			1		4	EX 1 160
2			0		100			2			EX 1 170
3			0		5			5			EX 1 180
4			0		100			1			EX 1 190
5			0		100						EX 1 200
6			0		100						EX 1 210
.6425			-5.0		5.0			1.492			EX 1 220
8			0		100			6			EX 1 230
3			2		5						EX 1 240
0											EX 1 250
1.358			0.0		100.0			0.218		EX 1 260	
1	2	3	1	2						5	EX 1 270
1	2	4	2	4							EX 1 280
1	2	5	3	3							EX 1 290
1	3	6	4	4							EX 1 300
1	4	6	5	6							EX 1 310
1	5	6	6	1							EX 1 320
1	6	7	11	5							EX 1 330
1	7	8	8	1							EX 1 340
6	8	9	9	9							EX 1 350
4	8	11	7	8							EX 1 360
1	9	10	3	7		1					EX 1 370
1	9	10	9	3		2					EX 1 380
1	10	8	10	1							EX 1 390
1	11	12	1	2						EX 1 400	
1	13	8	1	1	1					EX 1 410	
0										EX 1 420	
1	10	13	0							6	EX 1 430
2	13	10	0								EX 1 440
0											EX 1 450
1	5									7	EX 1 460

Figure 5. Input Data for Example 1

GERT SIMULATION PROJECT -1 BY ALL FEATURES
 DATE 5/ 20/ 1970

****NETWORK DESCRIPTION****

NUDE CHARACTERISTICS

HIGHEST NODE NUMBER IS 13
 NUMBER OF SOURCE NODES IS 1
 NUMBER OF SINK NODES IS 1
 NUMBER OF NODES TO REALIZE THE NETWORK IS 1
 STATISTICS COLLECTED ON 6 NODES
 NUMBER OF PARAMETER SETS IS 11
 INITIAL RANDOM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRFD AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	D		
4	1	1	D		
5	1	1	D		
6	3	9999	D		D
7	-1	-1	D		
8	1	1	P		B
9	1	1	D		F
10	1	1	D	R	A
11	1	9999	D		I
12	1	9999	D		A
13	1	1	D		

SOURCE NODE NUMBERS
 2

SINK NODE NUMBERS
 12

STATISTICS COLLECTED ALSO ON NODES
 6 11 10 9 8

Figure 6. Echo Check for Example 1

****ACTIVITY PARAMETERS****

PARAMETER NUMBER	PARAMETERS			
	1	2	3	4
1	10.0000	0.0	100.0000	0.1000
2	2.0000	0.0	100.0000	2.0000
3	3.0000	0.0	5.0000	0.5000
4	4.0000	0.0	100.0000	1.0000
5	5.0000	0.0	100.0000	0.0
6	6.0000	0.0	100.0000	0.0
7	0.6425	-5.0000	5.0000	1.4920
8	8.0000	0.0	100.0000	0.6000
9	3.0000	2.0000	5.0000	0.0
10	0.0	0.0	0.0	0.0
11	1.3580	0.0	100.0000	0.2180

****ACTIVITY DESCRIPTION****

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	2	0	0	1.0000
2	4	2	4	0	0	1.0000
2	5	3	3	0	0	1.0000
3	6	4	4	-1000	0	1.0000
4	6	5	6	-1000	0	1.0000
5	6	6	1	-1000	0	1.0000
6	7	11	5	0	0	1.0000
7	8	8	1	0	0	1.0000
8	9	9	9	0	0	0.6000
8	11	7	8	0	0	0.4000
9	10	3	7	0	1	1.0000
9	10	9	3	0	2	1.0000
10	8	10	1	0	0	1.0000
11	12	1	2	0	0	1.0000
13	8	1	1	1	0	1.0000

****NETWORK MODIFICATIONS****

ACTIVITY NODE FILE NODE FILE NODE FILE NODE FILE NODE FILE NODE FILE NODE FILE NODE FILE NODE

1	10	13
2	13	10

Figure 7. Further Echo Check for Example 1

to node 6, the attributes of the activity are examined). Since node 6 is a delay node, the program records the time 7.88 as the time of first completion of an activity that is incident to node 6. At time 8.31, the activity from node 4 to node 6 is completed. At time 10.16 the activity incident to node 3 is completed and node 3 is realized. The activity from node 3 to node 6 is then scheduled and is completed at time 12.09. This is the third activity that is realized incident to node 6 hence node 6 is realized. The time from the first activity completion on node 6 to the time that node 6 is realized is the delay time. This value is 4.21 ($12.09 - 7.88$) and is one sample of the delay time associated with node 6. Next the activity from node 6 to node 7 is scheduled. This activity is completed at time 13.78.

Node 7 is a mark node and the time 13.73 is identified with the path of activities following node 7. The activity emanating from node 7 is then completed at time 21.73 and node 8 is realized. This value is recorded for node 8 as the first time node 8 is realized since the time between realizations of node 8 is desired. From the trace, we see that node 9 is realized next at time 25.56. This indicates that the branching operation took the branch from node 8 to node 9 for this simulation of the network. Statistics are collected on node 9 which was realized at time 25.56. The two activities emanating from node 9 are then scheduled. Activity 1, the upper branch is completed first at time 28.79. This causes node 10 to be realized and the value of 28.79 is recorded as a time of realization of node 10. Since node 10 removes all activities scheduled to be completed that are incident to node 10, activity 2 is halted. Since activity 1 has been completed, node 10 is replaced by node 13 according to the prescribed network modification.

Branching from node 13 is now done. This is indicated by the trace by the attributes associated with the end of activity event on node 3

being those from node 13 to node 8. The time between realization of node 8 is collected and the current time used as the last time node 8 was realized. Again the branching process selects the activity from node 8 to node 9 and the loop around node 8 is traversed again. On this second traversal of the loop activity 1 again was completed before activity 2, and the branch from node 13 to node 8 is included in the network. On the third traversal of the loop, activity 2 was completed before activity 1 and the branch from node 10 to node 8 which involved no time delay is included in the network. Finally at time 107.56, node 8 is realized and the branching process directs that the activity from node 8 to node 11 be completed. Node 11 is realized at time 112.56. Since node 11 is an interval node, a value is calculated which represents the time to go from node 7 to node 11. In this case it is 98.78 (112.56 - 13.78). The activity from node 11 to node 12 is scheduled and completed at 122.65. At this time node 12 is realized. Since node 12 is the sink node of the network and since it only takes one realization of the sink node to realize the network, the network is realized. The value of 122.65 is then recorded as 1 sample of the time to realize node 12 or equivalently the time to realize the network. This completes one simulation run of the network.

Several comments on the statistics collected on nodes 8, 9, and 10 are in order. For run 1 node 8 was realized seven times, therefore, six values were calculated for the time between realizations of node 8. For node 9, statistics are collected on the time of first realization therefore only the value 25.56 is recorded as the appropriate sample on run 1 for node 9. For node 10, all realization times are collected since node 10 is an ALL node. Thus the values 28.79, 43.98, 59.74, 65.39, 81.89, and

97.56 are sample values regarding the realization of node 10.

In this example, all 9 distribution types were utilized to obtain samples for the time required to perform an activity. In Figure 8, a trace of 4 additional simulation runs is presented to indicate both the variability of the time required to perform an activity and the variability involved in the network structure due to the branching process and the network modification procedures.

The final GERTS summary report for Example 1 is presented in Figure 9 for 500 simulations of the network. The statistics presented for node 12 represents the values associated with the completion time of the network. From Figure 9, it is seen that node 12 has a probability of one of being realized as expected. The average time to realize node 12 was approximately 53.55 time units with a standard deviation of approximately 23.43 time units. In one simulation the network was realized in less than 30 time units and in another simulation it required over 148 time units to realize the network. Since node 12 is only realized once in each simulation there is no difference between statistics based on first realization and all realizations. In this simulation the branch from node 13 to node 8 was designated with a counter type 1. Statistics are automatically collected on the number of times that branch was taken prior to the realization of the node on which the statistics are collected. For node 12 it is seen that the average number of times the branch from node 13 to node 8 was taken prior to the realization of the network was .894. In some cases, the branch was never taken and in at least one simulation the branch was taken 7 times before the network was realized.

GERT SIMULATION PROJECT -1 BY ALL FEATURES
 DATE 5/20/1970

****FINAL RESULTS FOR 500 SIMULATIONS****

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
12	1.0000	53.5455	23.4290	500.	29.5463	148.3445	A
12	1	0.8940	1.2923	500.	0.0	7.0000	
6	1.0000	14.2856	3.8987	500.	4.2100	35.3449	D
6	1	0.0	0.0	500.	0.0	0.0	
11	1.0000	27.8881	23.1954	500.	8.0286	125.6182	I
11	1	0.8940	1.2923	500.	0.0	7.0000	
10	0.5660	46.5107	21.7802	697.	23.3519	132.5558	A
10	1	0.8838	1.2373	697.	0.0	6.0000	
9	0.5660	26.7693	3.7985	283.	21.7731	46.9250	F
9	1	0.0	0.0	283.	0.0	0.0	
8	1.0000	17.0576	7.1713	1197.	4.1900	44.8769	B
8	1	0.8881	1.2601	1197.	0.0	7.0000	

****HISTOGRAMS****

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
			1	2	3	4	5	6	7	8	9	10	
12	35.00	2.00	90	46	41	29	14	14	17	22	14	36	12
			9	14	7	8	12	4	12	11	9	3	5
			5	5	6	4	4	5	2	4	4	32	
6	1.00	1.00	0	0	0	0	1	0	0	0	0	5	84
			70	74	64	35	27	35	30	27	8	12	5
			6	2	5	2	2	0	3	0	0	3	
11	8.00	2.00	0	97	52	73	7	12	13	2	15	22	25
			32	8	11	6	5	3	13	9	13	6	5
			1	2	7	4	2	5	10	3	3	34	
10	30.00	2.00	183	45	52	23	20	10	29	41	27	27	27
			10	15	12	14	23	10	7	11	7	4	8
			7	11	7	6	4	2	3	4	5	43	
9	27.00	0.50	181	12	6	8	10	6	9	8	16	4	3
			2	5	1	0	2	2	1	0	0	0	0
			1	2	0	0	0	0	1	1	0	2	
8	1.00	1.00	0	0	0	0	24	124	82	20	0	0	0
			0	0	4	63	174	159	45	2	56	80	73
			71	41	31	33	29	33	12	12	6	23	

Figure 9. GERTS III Summary Report for Example 1

Statistics on node 6 show that the time between the first completion of an activity on node 6 and the time node 6 is realized required almost 7 time units. For the count statistics listed under node 6, it is seen that the branch from node 13 to node 8 was never taken prior to the realization of node 6. This is as expected since that branch follows node 6. Other items of interest from the final GERTS summary report will now be described. The probability associated with nodes 9 and 10 represent the probability that either of these nodes were realized in any simulation run. It is seen that branching around the loop from node 8 occurred in 56.6 percent of the runs. Even though statistics for node 10 are collected for all realizations, the probability of realizing node 10 on a simulation run is the probability of every realizing node 10, in that simulation run. If it is desired to obtain the average number of times node 10 was realized, this can be calculated from the number of observations divided by the number of simulation runs (697 divided by 500 for this example). The average time of realizing node 10 in a simulation run is the sum of all realization times of node 10 divided by the number of times node 10 is realized. This statistic is not an ordinary one for network models since it combines the time of first realization, second realization, and so on. Care must be taken when using these values.

Histograms for each of the statistics nodes are also presented in Figure 9. Consider the histogram for node 12 where the lower limit of the second cell is 35 and the cell width of each cell is 2. From the data presented, is seen that in 90 of the 500 simulation runs the realization time for node 12 and hence the network was less than 35 time units. In 46 other simulation runs the time to complete the network was between 35 and 37 time units. Other values can be read directly from Figure 9. This example demonstrates that a great deal of data can be obtained from GERTS III.

Example 2. Analysis and Sequencing of Space Experiments*

The performance of experiments in space by a spacecraft crew are almost always severely constrained by time. Many experiments are usually proposed by the scientific community and of those proposed a subset must be chosen for a given space mission. The sequencing of these experiments can be an important factor in determining the number of experiments that can be completed.

A GERT network of the sequence of experiments will be developed that permits the assessment of the time required to perform the experiments. In addition, information regarding the number of experiments that can be completed in a specified period of time will be determined. By modifying the sequence of experiments (which involves modifying the GERT network) an analysis can be performed on proposals for different sequencing procedures.

It will be assumed that there are three possible outcomes from the performance of an experiment: 1) successful completion; 2) failure; and 3) inconclusive results. If an experiment is successfully completed the next experiment in the sequence is performed. If a failure occurs, the experiment is scrubbed and the next experiment is then performed. If the results of an experiment are inconclusive, the experiment is repeated n times or until a success or failure occurs. The experiment is scrubbed if it is tried n times and the results are still inconclusive.

The GERT network for a three experiment program is shown in Figure 10. Node 2 is the start node and initiates a transfer to node 3. Node 3 represents the decision point for the first experiment. If the first experiment is successful, the activity from node 3 to node 4 is traversed. If the first experiment fails, then the activity from node 3 to node 19 is taken. The

* This example was developed by Mr. J. Ignizio in a seminar on GERT based on Mr. Ignizio's experience [5].

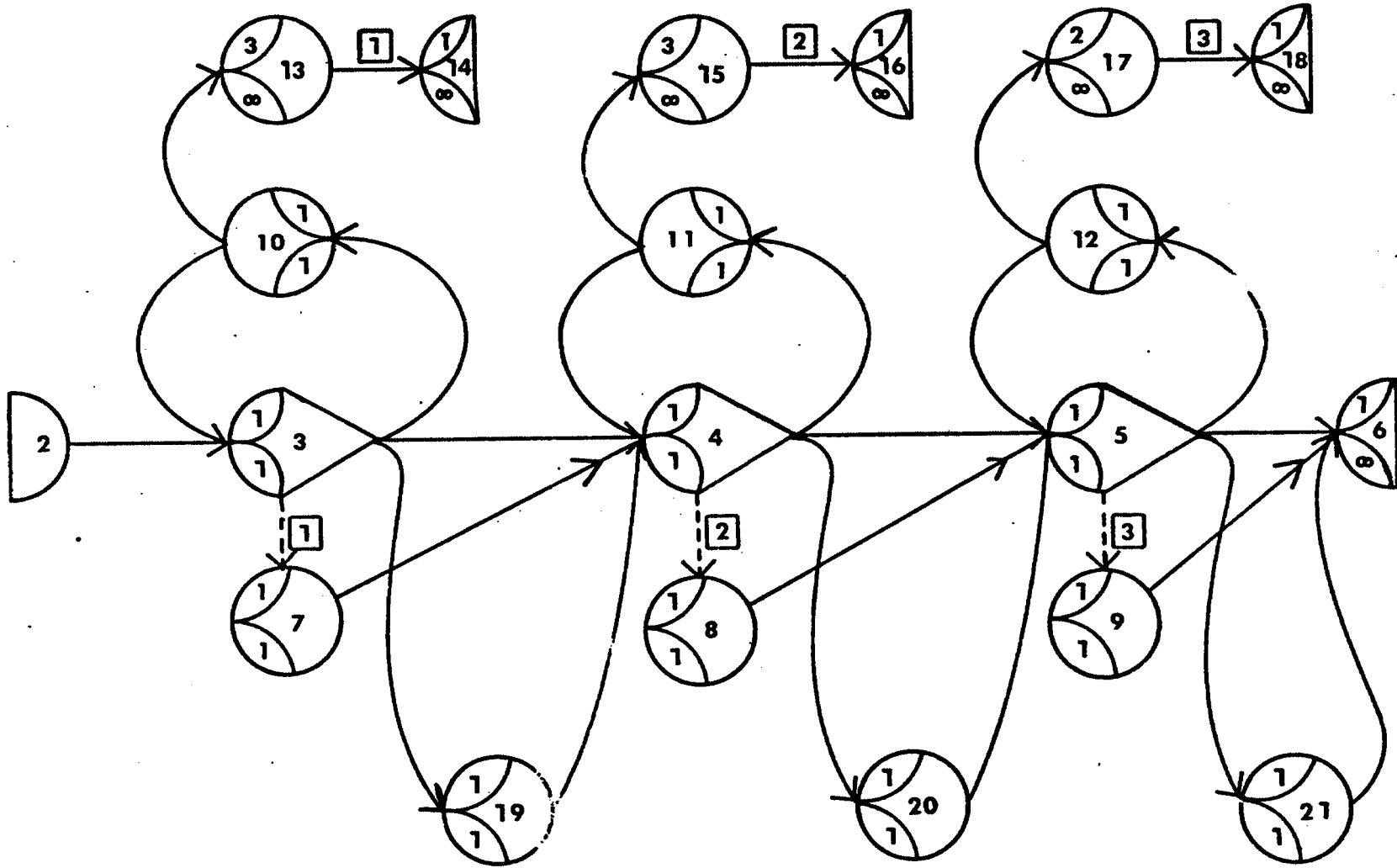


Figure 10. GERT Network for the Analysis and Sequencing of Space Experiments

second experiment is started by transferring from node 19 to node 4. If the results of the first experiment are inconclusive, the activity from node 3 to node 10 is taken. The output of node 10 is DETERMINISTIC; and both the first experiment is performed again and a signal to node 13 is sent to indicate that the first experiment has been performed once. Thus, for each experiment we will either transfer to node 4 or reach node 13. When node 13 is realized three times, the activity from node 13 to node 14 is traversed. This activity is labeled as activity 1 and causes the network to be modified by replacing node 3 with node 7. After this occurs when node 3 is realized, node 7 is in the network and a transfer to node 4 is caused. A similar discussion holds for experiments 2 and 3. From the network it is seen that nodes 19, 20, and 21 represent the failure of experiments 1, 2, and 3, respectively. Nodes 14, 16 and 18 represent the outcomes that inconclusive results were obtained after the maximum number of experiments could be performed for experiments 1, 2, and 3, respectively.

Table 1 presents the data for the experimental characteristics to be analyzed. In Figure 11 the input for Example 2 is presented. The GERTS III echo check for the description of the network is presented in Figures 12 and 13. Figure 14 presents a summary report describing the results from the GERTS III simulation of the sequence proposed for the space experiment program. From the output it is seen that experiment 1 failed 15.75% of the time and had inconclusive results 3.5% of the time. Therefore, it was successful 81% of the time. The time to complete experiment 1 is the time to reach node 4 of the network. Figure 14 shows that on the average it took over 46 time units to reach node 4 with a standard deviation of over 18 time units. In some instances it took as little as 24.5 time units and in others it took over 114 time units to complete the experiment. The number of times that experiment 1 was completed within given time intervals is presented in

SPACE EXPS 2 5201970 400 4 40 1267							1	EX 2 10
21	1	1	1	1	1		2	EX 2 20
2	1	0	D				3	EX 2 30
3	3	1	1P	5	1	A		EX 2 40
4	3	1	1P	25	3	A		EX 2 50
5	3	1	1P	37	3	A		EX 2 60
6	2	1	D	37	3	A		EX 2 70
7		1	10					EX 2 80
8		1	10					EX 2 90
9		1	10					EX 2 100
10		1	10					EX 2 110
11		1	10					EX 2 120
12		1	10					EX 2 130
13		3	D					EX 2 140
14	3	1	D	1	1	F		EX 2 150
15		3	D					EX 2 160
16	3	1	D	65	1	F		EX 2 170
17		2	D					EX 2 180
18	3	1	D	50	2	F		EX 2 190
19	3	1	10	1	1	A		EX 2 200
20	3	1	10	27	2	A		EX 2 210
21	3	1	10	41	3	A		EX 2 220
0								EX 2 230
10			5	20		2	4	EX 2 240
20			15	25		1		EX 2 250
15			10	30		3		EX 2 260
0								EX 2 270
1	2	3	1	2			5	EX 2 280
6	3	4	2	2				EX 2 290
3	3	10	4	1				EX 2 300
1	3	19	4	1				EX 2 310
5	4	5	3	2				EX 2 320
4	4	11	4	1				EX 2 330
1	4	20	4	1				EX 2 340
7	5	6	4	1				EX 2 350
2	5	12	4	1				EX 2 360
1	5	21	4	1				EX 2 370
1	7	4	2	2				EX 2 380
1	8	5	3	2				EX 2 390
1	9	6	4	1				EX 2 400
1	10	3	1	2				EX 2 410
1	10	13	4	1				EX 2 420
1	11	4	2	2				EX 2 430
1	11	15	4	1				EX 2 440
1	12	5	3	2				EX 2 450
1	12	17	4	1				EX 2 460
1	13	14	4	1	1			EX 2 470
1	15	16	4	1	2			EX 2 480
1	17	18	4	1	3		EX 2 490	
1	19	4	2	2			EX 2 500	
1	20	5	3	2			EX 2 510	
1	21	6	4	1			EX 2 520	
0							EX 2 530	
1	3	7					6	EX 2 540
2	4	8						EX 2 550
3	5	9						EX 2 560
0								EX 2 570

Figure 11. Input Data for Example 2

****NETWORK DESCRIPTION****

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 21
 NUMBER OF SOURCE NODES IS 1
 NUMBER OF SINK NODES IS 1
 NUMBER OF NODES TO REALIZE THE NETWORK IS 1
 STATISTICS COLLECTED ON 10 NODES
 NUMBER OF PARAMETER SETS IS 4
 INITIAL RANDOM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		A
4	1	1	P		A
5	1	1	P		A
6	1	9999	D		A
7	1	1	D		
8	1	1	D		
9	1	1	D		
10	1	1	D		
11	1	1	D		
12	1	1	D		
13	3	9999	D		
14	1	9999	D		F
15	3	9999	D		
16	1	9999	D		F
17	2	9999	D		
18	1	9999	D		F
19	1	1	D		A
20	1	1	D		A
21	1	1	D		A

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
6

STATISTICS COLLECTED ALSO ON NODES
21 20 19 18 16 14 5 4 3

Figure 12. Echo Check for Example 2

****ACTIVITY PARAMETERS****

PARAMETER NUMBER	PARAMETERS			
	1	2	3	4
1	10.0000	5.0000	20.0000	2.0000
2	20.0000	15.0000	25.0000	1.0000
3	15.0000	10.0000	30.0000	3.0000
4	0.0	0.0	0.0	0.0

****ACTIVITY DESCRIPTION****

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	2	0	0	1.0000
3	4	2	2	0	0	0.6000
3	10	4	1	0	0	0.3000
3	19	4	1	0	0	0.1000
4	5	3	2	0	0	0.5000
4	11	4	1	0	0	0.4000
4	20	4	1	0	0	0.1000
5	6	4	1	0	0	0.7000
5	12	4	1	0	0	0.2000
5	21	4	1	0	0	0.1000
7	4	2	2	0	0	1.0000
8	5	3	2	0	0	1.0000
9	6	4	1	0	0	1.0000
10	3	1	2	0	0	1.0000
10	13	4	1	0	0	1.0000
11	4	2	2	0	0	1.0000
11	15	4	1	0	0	1.0000
12	5	3	2	0	0	1.0000
12	17	4	1	0	0	1.0000
13	14	4	1	0	1	1.0000
15	16	4	1	0	2	1.0000
17	18	4	1	0	3	1.0000
19	4	2	2	0	0	1.0000
20	5	3	2	0	0	1.0000
21	6	4	1	0	0	1.0000

****NETWORK MODIFICATIONS****

ACTIVITY	NODE	FILE	NODE	FILE	NODE	FILE	NODE	FILE	NODE	FILE	NODE	FILE	NODE
1	3	7											
2	4	8											
3	5	9											

Figure 13. Further Echo Check for Example 2

GERT SIMULATION PROJECT 2 BY SPACE EXPS
DATE 5/ 20/ 1970

****FINAL RESULTS FOR 400 SIMULATIONS****

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
6	1.0000	66.1562	21.8904	400.	36.7050	137.6164	A
21	0.1300	68.2321	21.8631	52.	37.9771	118.8453	A
20	0.1525	44.6018	16.2177	61.	25.6203	96.1071	A
19	0.1575	13.3597	6.6827	63.	5.5631	32.2204	A
18	0.0350	77.9196	22.1003	14.	52.1680	119.6703	F
16	0.0675	73.4611	7.2378	27.	66.5126	93.7269	F
14	0.0350	29.9889	3.0599	14.	25.3156	36.5681	F
5	1.0000	55.8379	21.5760	493.	36.7050	137.6164	A
4	1.0000	46.1740	16.7907	670.	24.5228	114.1765	A
3	1.0000	14.1291	7.5545	576.	5.0000	43.2401	A

****HISTOGRAMS****

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
			2	10	32	44	30	18	17	11	28	25	21
6	37.00	3.00	17	18	12	11	14	19	13	7	4	5	2
			6	5	4	8	1	5	3	3	0	5	2
21	41.00	3.00	4	5	1	5	2	2	0	5	2	0	3
			5	2	1	1	0	4	2	1	3	0	1
			0	1	0	0	2	0	0	0	0	0	0
20	27.00	2.00	4	5	7	4	5	1	2	2	3	3	2
			2	2	4	2	1	0	0	3	3	0	
			2	0	2	0	0	0	1	0	0	1	
19	1.00	1.00	0	0	0	0	0	2	3	4	7	8	10
			9	2	0	1	2	0	0	0	5	0	0
			2	3	0	0	0	2	0	1	0	2	
18	50.00	2.00	0	0	1	0	1	1	2	0	2	0	1
			0	0	0	0	0	0	0	1	0	1	1
			0	0	0	0	1	0	0	0	0	2	
16	65.00	1.00	0	0	4	1	1	2	6	2	4	1	0
			0	0	0	1	0	1	1	0	0	1	0
			0	0	0	0	0	0	1	1	0	0	
14	1.00	1.00	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	1	0	1	7	0	1	4	
5	37.00	3.00	2	12	39	49	41	24	21	15	36	35	27
			22	22	13	13	17	20	13	9	5	5	3
			10	5	6	9	1	6	4	4	0	5	
4	25.00	3.00	2	59	135	73	17	31	28	30	44	55	35
			8	20	15	15	14	23	10	4	8	8	4
			10	10	2	1	6	1	0	0	2	0	
3	5.00	1.00	0	11	20	41	59	74	81	55	37	13	11
			8	10	8	11	20	13	19	15	10	10	2
			1	4	10	3	2	3	5	2	3	15	

Figure 14. GERTS III Summary Report for Example 2

the histogram for node 4. Similar statistical quantities are available for the other nodes of the network.

Another interesting feature that could be incorporated into the network model for the sequencing of experiments is the changing of the sequence depending on the results of some of the experiments. This would be accomplished through the network modification procedures of the GERTS III program.

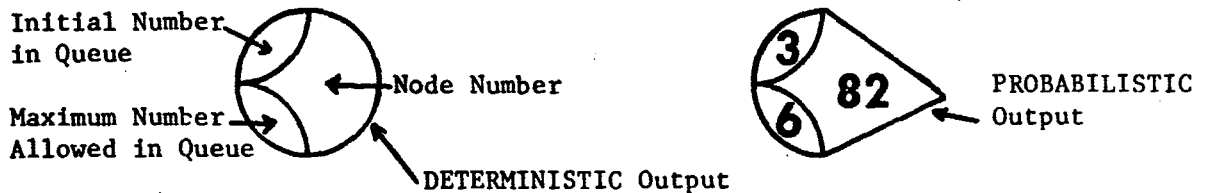
Table 1. Experiment Characteristics

Experiment	Probability of Success	Probability of Failure	Probability of Inconclusive Results	Allowable Numbers of Repeats
1	0.6	0.1	0.3	3
2	0.5	0.1	0.4	3
3	0.7	0.1	0.2	2

Experiment	Mean Time	Minimum Time	Maximum Time	Standard Deviation
1	10.0	5.0	20.0	2.0
2	20.0	15.0	25.0	1.0
3	15.0	10.0	30.0	3.0

GERTS IIIQ

In modeling some systems there is a need for a node which provides a storage capability. To meet this need, a Q-node has been developed and added to the basic GERTS III program. The simulation program which includes the capability of modeling systems with this Q-node has been labeled GERTS IIIQ. The Q-node is the only new feature included in GERTS IIIQ. The graphical representation of a Q-node is shown below.



When an activity is completed that is incident to a Q-node, two things can occur: 1) the activity following the Q-node can be initiated; or 2) the number in the queue can be increased by 1. The activity emanating from a Q-node represents a service activity. It is assumed in GERTS IIIQ that the service activity can only serve one item at a time. (The word item will be used to represent the concept of a customer, a transaction, etc.). If the service activity is currently serving an item then the arriving item is put in the queue by increasing the number in the queue by one. The position of the arriving item in the queue can be done either on a first-in-first-out (FIFO) or last-in-first-out (LIFO) basis. The GERTS IIIQ program also permits the specification of a maximum number of items in a queue. When this maximum value is obtained and a new item arrives, the item can either balk from the system or balk to a node as specified by the analyst. If the item balks from the system, no new nodes are realized and the path representing the item is completed. If the item balks to a node, the node is realized

immediately and the activities emanating from the node are scheduled. From the above it can be seen that the concept of number of releases is not appropriate for a Q-node.

For each Q-node used in the GERTS IIIQ program, the following information is required:

1. Initial number of items in the queue;
2. The maximum number of items allowed in the queue;
3. The node an item balks to if it cannot join the queue; and
4. Whether the items joining the queue are to be ranked according to the FIFO or LIFO priority procedure.

The above characteristics are associated with a node and as such would be described on Data Card Type 3 of the input cards (see Appendix A for format).

If the initial number in the queue is greater than zero, it is assumed that the server is busy. If it is desired to have 0 in the queue but the server busy then an activity from a source node to the Q-node will accomplish this result. No description of the activities incident to a Q-node nor activities emanating from a Q-node are required. The GERTS IIIQ program automatically identifies these activities when the activities for the network are inserted into the computer.

Restrictions Associated with Q-Nodes

The major restriction with regard to Q-nodes is that only one service activity can be associated with a Q-node. However, a probabilistic output from a Q-node is allowed where each of the activities emanating from the Q-node can be considered to represent the same server. The other restrictions associated with a Q-node are due to the desire to save core storage space and to alleviate the data input problem. These restrictions are:

1. A service activity cannot be an input to a Q-node;
2. Activities which are either incident to a Q-node or represent a service activity cannot have counter types associated with them; and

3. If the maximum number allowed in the queue is not specified, a large value is assigned. If no queue is really desired a -1 must be inserted for the maximum number allowed in the queue.

Statistics are collected on each Q-node. The number of Q-nodes plus the number of statistics nodes must be less than or equal to 100. To increase this number, the dimensions of selected arrays must be changed.

Statistics Collected

The GERTS IIIQ program is written to simulate the same network a given number of times. For each simulation of the network, values are collected on the average number in each queue as opposed to the number in a queue. As an automatic output of the GERTS IIIQ program, statistical estimates are given for the average number in each queue, the average busy time or utilization of each server, and the average rate of balkers per unit of time for each Q-node.

Examples of GERTS IIIQ

Three examples presented in this section of the use of the GERTS IIIQ program are:

1. A single channel queueing system;
2. A conveyor system; and
3. A network of queues.

A model of a registration system for a university which involves Q-nodes is given in Ref. [1].

Example 3: A Simple Queueing Situation

The GERT network representing a single server, single queue situation is presented in Figure 15. Node number 2 is the source node and is used to activate node 3 which represents the generation of arrivals to the system.

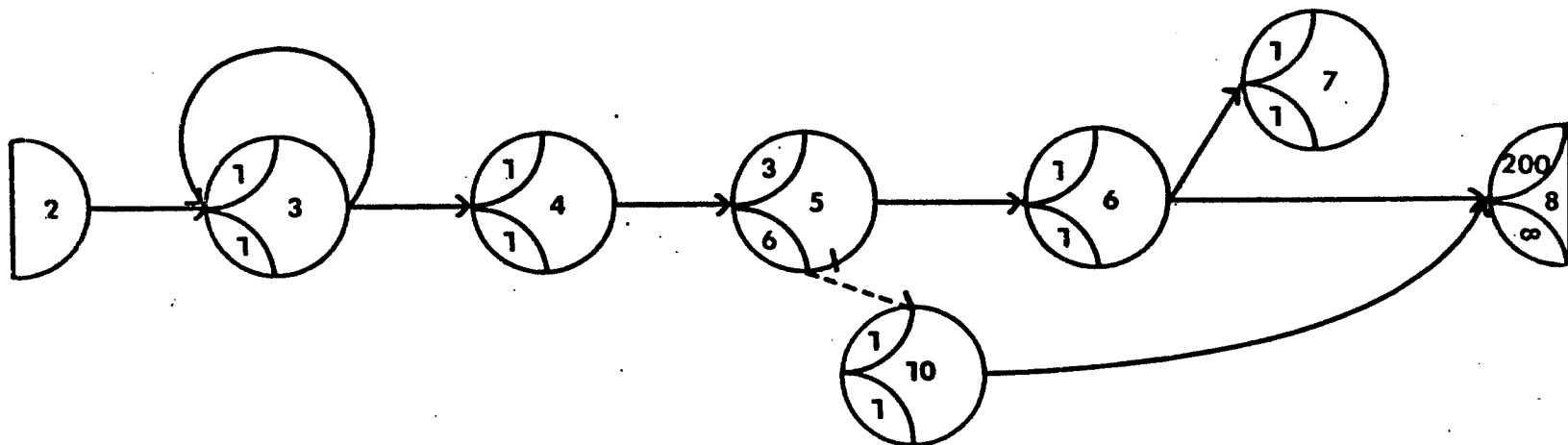


Figure 15. GERT Network for a Simple Queueing Situation, Example 3

The output side of node 3 is DETERMINISTIC, hence, the branch from node 3 to node 4 and the branch from node 3 back to node 3 are realized everytime node 3 is realized. The branch from node 3 to node 3 represents the time between the arrival of items to the system. Since we desire to establish the time of arrival of items to the system, node 3 is made a mark node. Node 4 is inserted into the system in order to collect statistics on the times between arrival of items. This node will provide a check on the interarrival time distribution represented by the branch from node 3 to node 3.

The output of node 4 is then the input to the Q-node, node 5. It is assumed that there are 3 items in the queue initially and that the queue has a capacity of 6. The branch from node 5 to node 6 represents a service activity. If items are in the queue at time 0 then it is assumed that the service activity is on-going. The time required for the item to traverse this branch is the service time. Node 6 is used to collect statistics on the time an item spends in a system. Node 6 is identified as a statistics node with statistics collected on a time interval. Thus the time it takes for the item that was marked at node 3 until the time the item reached node 6 is a value that is collected and associated with node 6. The output side of node 6 is DETERMINISTIC and the branches from node 6 to node 7 and node 6 to node 8 are both realized. Node 7 is a statistics node and collects samples on the time between realizations of node 6. This represents the time between departures of items from the system. Node 8 is a sink node and is used to specify the number of items required to realize the network. In this example, the number of releases for node 8 is set at 200. This corresponds to the number of items to be included in one simulation of the network.

Figure 16 presents the input data for the network. Figures 17 and 18 illustrate the GERTS IIIQ echo check of the network. Note that the echo

SIMPLE QUEUE 3 5201970 50 3 25 1267										1	EX 3 10
10	1	1	1	4						2	EX 3 20
2	1									3	EX 3 30
3	4										EX 3 40
4	3	1	1	1	3	B					EX 3 50
5	5	3	6	1	2		10				EX 3 60
6	3	1	1	1	3	I					EX 3 70
7	3	1	1	1	1	B					EX 3 80
8	2200		1000		50	A					EX 3 90
10	1	1									EX 3 100
0											EX 3 110
0					75			1			4
60		0			75			1		EX 3 130	
5		0								EX 3 140	
1		2	3	1	1					5	EX 3 150
1		3	3	2	4						EX 3 160
1		3	4	1	1						EX 3 170
1		4	5	1	1						EX 3 180
1		5	6	3	4						EX 3 190
1		6	7	1	1						EX 3 200
1		6	8	1	1						EX 3 210
1	10	8	1	1							EX 3 220
0										EX 3 230	

Figure 16. Input Data for Example 3

check indicates a count type of -1 for the branch from node 4 to node 5. A -1 is associated with the variable representing a count type for all activities incident to Q-nodes. For branch 5-6, the count type variable is given the negative of the source node number. Note that these values are not inserted as input information but are automatically set in the GERTS IIIQ program. The storage savings resulting from using the same variables for two different purposes is significant. In Figure 17, the code -1 is given for the number of releases for node 3. This identifies node 3 as a mark node. In the section on queue nodes in Figure 17, node 5 is identified as a Q-node with the queue priority being first-in-first-out (FIFO). The 10 in the node for balkers column indicates that items balk to node 10 if the queue is filled when they arrive. The capacity of the queue is printed under the column heading "maximum number allowed".

The final results for 50 simulations of the simple queueing network are given in Figure 19. These results are given for the input information presented in Figure 16. The interarrival time distribution was exponential and the service times were also exponentially distributed (Erlang with $k = 1$). From the values obtained for node 4 it is seen that the interarrival time distribution is close to the theoretical distribution. The time spent in the system by a customer is collected at node 6. The average value is 15.93. The standard deviation, minimum, maximum and a histogram for the time spent in the system is also shown in Figure 19. For exponential interarrival times with an unlimited queue it is well known that the departure process is also exponential with the same parameter as for the arrival process. With a limited queue this is not the case and one would expect slightly higher average times between departures. This is confirmed by the statistics collected on node 7. The statistics collected on node 8 represent the time required to process 200 items including those items that balked.

GERT SIMULATION PROJECT 3 BY SIMPLE QUEUE
 DATE 5/ 20/ 1970

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 10
 NUMBER OF SOURCE NODES IS 1
 NUMBER OF SINK NODES IS 1
 NUMBER OF NODES TO REALIZE THE NETWORK IS 1
 STATISTICS COLLECTED ON 4 NODES
 NUMBER OF PARAMETER SETS IS 3
 INITIAL RANDOM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	-1	-1	D		
4	1	1	D		B
6	1	1	D		I
7	1	1	D		B
8	200	9999	D		A
10	1	1	D		

QUEUE NODES

NODE	INITIAL # IN QUEUE	MAXIMUM # ALLOWED	OUTPUT TYPE	NODE FOR BALKERS	PRIORITY SCHEME
5	3	6	D	10	FIFO

SOURCE NODE NUMBERS
2

SINK NODE NUMBERS
8

STATISTICS COLLECTED ALSO ON NODES
7 6 4

Figure 17. Echo Check for Example 3

****ACTIVITY PARAMETERS****

PARAMETER NUMBER	PARAMETERS			
	1	2	3	4
1	0.0	0.0	0.0	0.0
2	6.0000	0.0	75.0000	1.0000
3	5.0000	0.0	75.0000	1.0000

****ACTIVITY DESCRIPTION****

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	3	2	4	0	0	1.0000
3	4	1	1	0	0	1.0000
4	5	1	1	-1	0	1.0000
5	6	3	4	-5	0	1.0000
6	7	1	1	0	0	1.0000
6	8	1	1	0	0	1.0000
10	8	1	1	0	0	1.0000

Figure 18. Further Echo Check for Example 3

GERT SIMULATION PROJECT 3 BY SIMPLE QUEUE
DATE 5/ 20/ 1970

****FINAL RESULTS FOR 50 SIMULATIONS****

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
8	1.0000	1195.4866	77.5555	50.	1043.5808	1412.6975	A
7	1.0000	6.3787	6.3492	9355.	0.0	62.3682	B
6	1.0000	15.9314	24.0739	9355.	0.0012	284.9275	I
4	1.0000	5.9883	5.9670	9943.	0.0	56.4771	B

****QUEUE NODES****

NODE	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	
5	1.8260	0.4464	50.	0.8555	2.7119	AVERAGE NUMBER IN THE QUEUE
5	0.7870	0.0588	50.	0.5975	0.9061	AVERAGE BUSY TIME OF PROCESSOR
5	0.0111	0.0061	50.	0.0	0.0263	AVERAGE BALKERS PER UNIT TIME

****HISTOGRAMS****

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
			0	1	3	11	14	9	8	2	1	1	0
8	1000.00	50.00	0	1	3	11	14	9	8	2	1	1	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0
7	1.00	1.00	1368	1134	1040	812	685	605	558	452	401	342	289
			259	226	160	137	122	110	93	91	72	76	43
			38	40	32	18	16	13	20	20	15	68	
6	1.00	3.00	553	1838	1593	1237	944	659	472	364	225	184	141
			123	118	83	77	62	63	55	36	41	46	33
			26	39	32	24	14	11	20	13	20	209	
4	1.00	3.00	1544	3317	1932	1252	790	429	279	156	96	55	28
			25	16	15	5	1	0	2	0	1	0	0
			0	0	0	0	0	0	0	0	0	0	0
5	1.00	0.20	2	1	4	9	7	13	5	1	5	3	0
			0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0

Figure 19. GERT IIIQ Summary Report for Example 3

Statistics collected on the Q-node indicate that the average of the average number in the queue was 1.826. A histogram for this variable is shown under node 5 in the histogram section of the final report. It is seen that the standard deviation of the average number in the queue is 0.4464. The high and low values observed for the average number in the queue are also recorded in Figure 19. The average of the fraction of the time the server or processor was busy was 0.787.

Figure 20 shows the network for a queueing situation in which the queue is empty initially and the server is idle. Figure 21 illustrates the network in which the queue is empty and the server is busy.

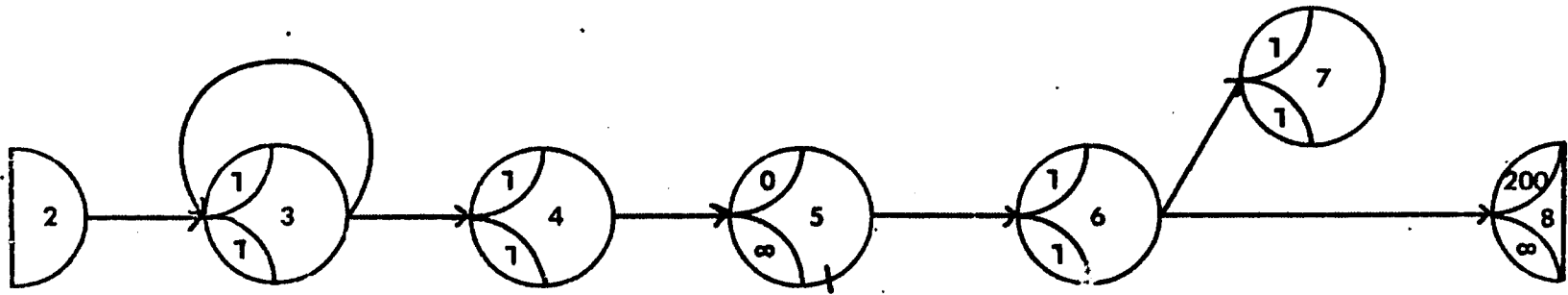


Figure 20. GERT Network for Simple Queueing Situation with the Server Initially Idle

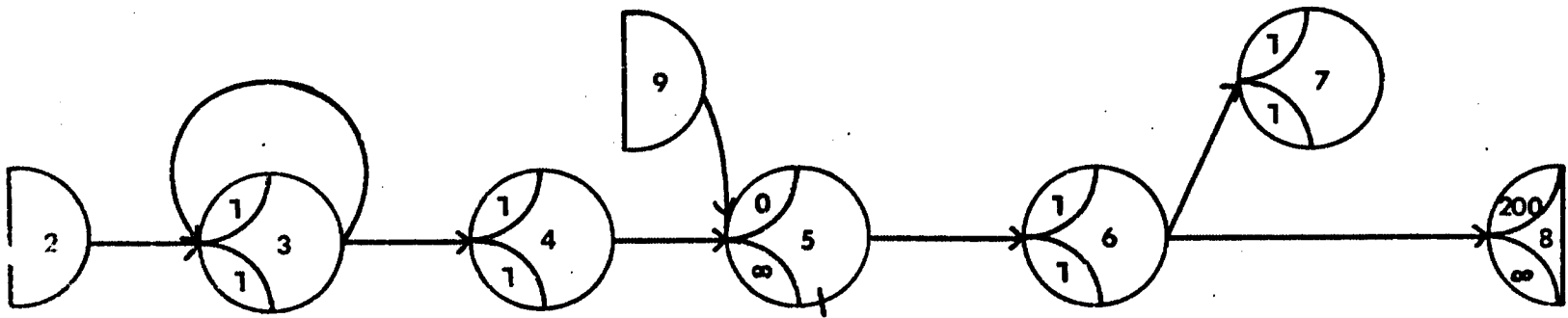


Figure 21. GERT Network for Simple Queueing Situation with the Server Initially Busy and No line in the Queue

Example 4: A Conveyor System

The network representing a continuous belt conveyor system with five servers is shown in Figure 22. The queue for the first server is represented by node 3. This server is not permitted to have any items in his queue. The second server is represented by node 4 and he is allowed one item in his queue. When an item arrives to the first server and the server is busy, it balks to node 13 and the activity from node 13 to node 4 represents the delay incurred in traveling from the first server to the second server. Nodes 5, 6 and 7 represent the queues of the third, fourth, and fifth servers respectively. The service activities are represented by the branches from nodes 3 to 9, 4 to 9, 5 to 9, 6 to 9, and 7 to 9. When an item balks from the queue of the fifth server, it is routed around the conveyor to the first server. This is represented by the activities from node 17 to node 8 and from node 8 to node 3.

The branch from node 2 to node 2 represents the interarrival time and, hence, the interarrival process of items to the system. Node 2 is also a mark node. Node 9 represents the departure point for items from the system. Node 9 is a statistics node with interval statistics being calculated. Thus the results obtained regarding the statistics for node 9 describe the time in the system for each item.

The input required for this network is given in Figure 23. Note that the maximum number of items allowed for node 3 has been set at -1. This is required to specify that no one can be in the queue of server one. (A blank would indicate that an infinite queue would be possible). The GERTS IIIQ echo check is given in Figures 24 and 25. The results from 50 simulations with each simulation containing 300 items is shown in Figure 26.

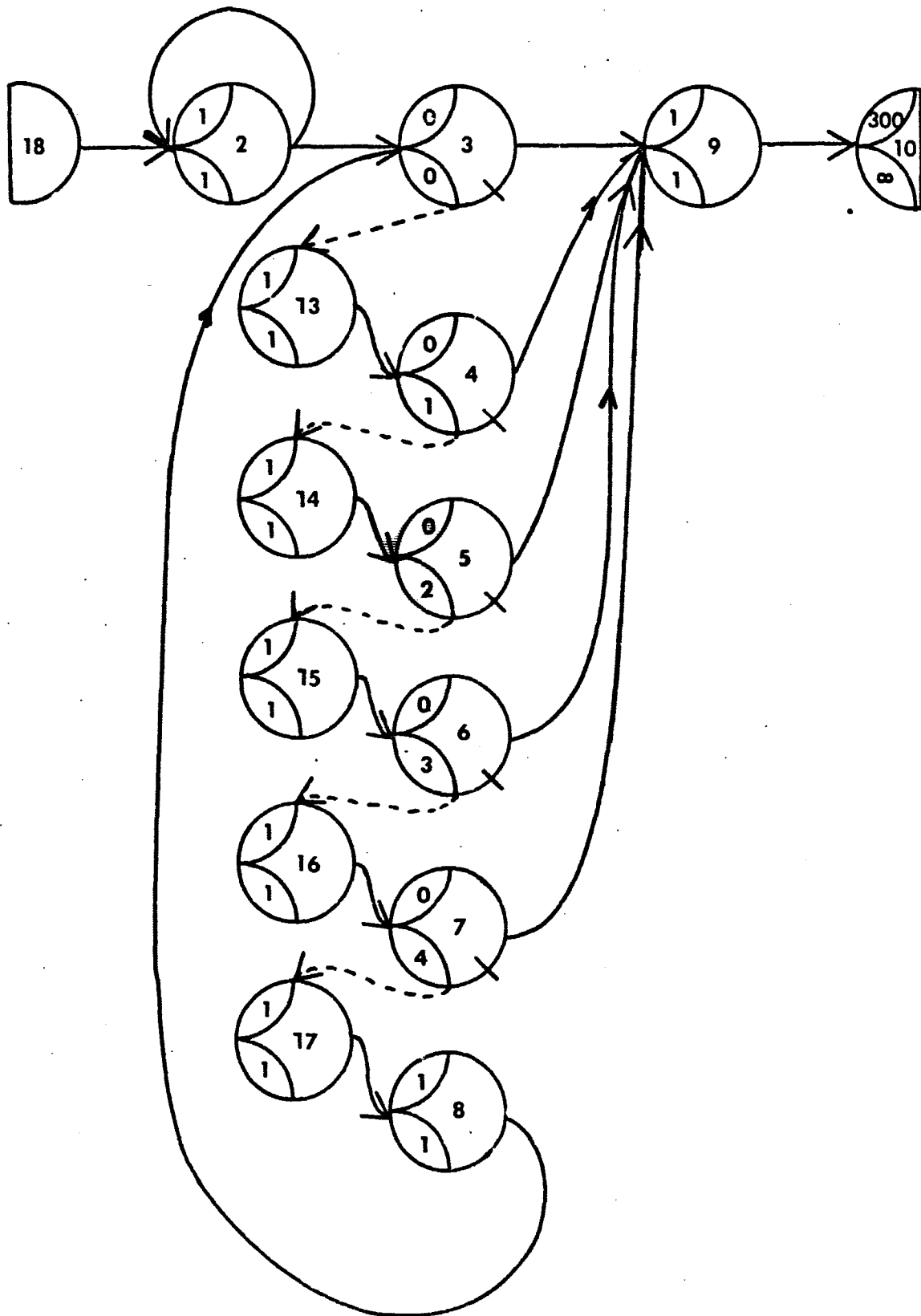


Figure 22. GERT Network for a Five Server Conveyor System, Example 4

CONVEYOR						4 5201970	50	4 200	1267		
18	1	1	1	2	1	0					
2	4	0	10								
3	5	0	-10	0.0	1.0			13			
4	5	0	10	.4	.01			14			
5	5	0	20	.3	.03			15			
6	5	0	30	0.0	.05			16			
7	5	0	40	0.0	.03			17			
8			10								
9	3	1	10	1.0	0.25	I					
10	2300			30.0	1.0	A					
13	1	1									
14	1	1									
15	1	1									
16	1	1									
17	1	1									
18	1	0	10								
0											
0.0											
0.32333	0.0				100.0			1.0			
1.0	0.0				100.0			1.0			
5.0											
1.0		2	2	2	4						
1.0		2	3	1	1						
1.0		3	9	3	4						
1.0		4	9	3	4						
1.0		5	9	3	4						
1.0		6	9	3	4						
1.0		7	9	3	4						
1.0		8	3	4	1	1					
1.0		9	10	1	1						
1.0		13	4	3	1						
1.0		14	5	3	1						
1.0		15	6	3	1						
1.0		16	7	3	1						
1.0		17	8	3	1						
1.0		18	2	1	1						
0											

Figure 23. Input Data for Example 4

GERT SIMULATION PROJECT 4 BY CONVEYOR
 DATE 5/ 20/ 1970

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 18
 NUMBER OF SOURCE NODES IS 1
 NUMBER OF SINK NODES IS 1
 NUMBER OF NODES TO REALIZE THE NETWORK IS 1
 STATISTICS COLLECTED ON 2 NODES
 NUMBER OF PARAMETER SETS IS 4
 INITIAL RANDOM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	-1	-1	D		
8	1	1	D		
9	1	1	D		I
10	300	9999	D		A
13	1	1	D		
14	1	1	D		
15	1	1	D		
16	1	1	D		
17	1	1	D		
18	0	1	D		

QUEUE NODES

NODE	INITIAL # IN QUEUE	MAXIMUM # ALLOWED	OUTPUT TYPE	NODE FOR BALKERS	PRIORITY SCHEME
3	0	0	D	13	FIFO
4	0	1	D	14	FIFO
5	0	2	D	15	FIFO
6	0	3	D	16	FIFO
7	0	4	D	17	FIFO

SOURCE NODE NUMBERS
 18

SINK NODE NUMBERS
 10

STATISTICS COLLECTED ALSO ON NODES
 9

Figure 24. Echo Check for Example 4

****ACTIVITY PARAMETERS****

PARAMETER NUMBER	PARAMETERS			
	1	2	3	4
1	0.0	0.0	0.0	0.0
2	0.3333	0.0	100.0000	1.0000
3	1.0000	0.0	100.0000	1.0000
4	5.0000	0.0	0.0	0.0

****ACTIVITY DESCRIPTION****

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	2	2	4	0	0	1.0000
2	3	1	1	-1	0	1.0000
3	9	3	4	-3	0	1.0000
4	9	3	4	-4	0	1.0000
5	9	3	4	-5	0	1.0000
6	9	3	4	-6	0	1.0000
7	9	3	4	-7	0	1.0000
8	3	4	1	-1	0	1.0000
9	10	1	1	0	0	1.0000
13	4	3	1	-1	0	1.0000
14	5	3	1	-1	0	1.0000
15	6	3	1	-1	0	1.0000
16	7	3	1	-1	0	1.0000
17	8	3	1	0	0	1.0000
18	2	1	1	0	0	1.0000

Figure 25. Further Echo Check for Example 4

****FINAL RESULTS FOR 50 SIMULATIONS****

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
10	1.0000	103.1165	5.4792	50.	91.2783	117.4670	A
10	1	0.0	0.0	50.	0.0	0.0	
9	1.0000	3.2255	2.4347	15000.	0.0000	28.1721	I
9	1	0.0	0.0	15000.	0.0	0.0	

****QUEUE NODES****

NODE	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	
3	0.0	0.0	50.	0.0	0.0	AVERAGE NUMBER IN THE QUEUE
3	0.7473	0.0296	50.	0.6818	0.8178	AVERAGE BUSY TIME OF PROCESSOR
3	2.2669	0.1633	50.	1.8814	2.6341	AVERAGE BALKERS PER UNIT TIME
4	0.5801	0.0501	50.	0.4792	0.6972	AVERAGE NUMBER IN THE QUEUE
4	0.8430	0.0374	50.	0.7570	0.9249	AVERAGE BUSY TIME OF PROCESSOR
4	1.4013	0.1679	50.	1.0812	1.8544	AVERAGE BALKERS PER UNIT TIME
5	0.8984	0.1377	50.	0.5545	1.2069	AVERAGE NUMBER IN THE QUEUE
5	0.7436	0.0689	50.	0.5521	0.8755	AVERAGE BUSY TIME OF PROCESSOR
5	0.6266	0.1516	50.	0.3661	1.0115	AVERAGE BALKERS PER UNIT TIME
6	0.6415	0.2487	50.	0.2146	1.2762	AVERAGE NUMBER IN THE QUEUE
6	0.4614	0.1102	50.	0.2786	0.7230	AVERAGE BUSY TIME OF PROCESSOR
6	0.1418	0.0779	50.	0.0177	0.3840	AVERAGE BALKERS PER UNIT TIME
7	0.1552	0.1227	50.	0.0	0.5686	AVERAGE NUMBER IN THE QUEUE
7	0.1368	0.0714	50.	0.0	0.3145	AVERAGE BUSY TIME OF PROCESSOR
7	0.0079	0.0143	50.	0.0	0.0623	AVERAGE BALKERS PER UNIT TIME

****HISTOGRAMS****

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES											
			0	1	2	3	4	5	6	7	8	9	10	
10	30.00	1.00	0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	50	0
9	1.00	0.25	2433	731	639	606	558	836	717	659	620	791	686	
			669	494	553	501	429	390	324	281	251	226	182	
			171	178	126	101	92	60	68	71	58	519		
3	0.0	1.00	0	50	0	0	0	0	0	0	0	0	0	
			0	0	0	0	0	0	0	0	0	0	0	
			0	0	0	0	0	0	0	0	0	0	0	
4	0.40	0.01	0	0	0	0	0	0	0	0	1	0	1	
			3	1	2	2	3	4	4	5	3	6	4	
			2	0	2	2	1	0	2	1	1	0		
5	0.30	0.03	0	0	0	0	0	0	0	0	0	1	1	
			0	0	1	2	3	2	2	4	3	6	4	
			6	3	4	2	1	3	0	0	1	1		
6	0.0	0.05	0	0	0	0	0	1	2	2	2	2	8	
			5	5	1	3	2	6	3	1	0	2	0	
			2	1	1	0	1	0	0	0	0	0	0	
7	0.0	0.03	0	6	6	3	7	8	1	6	7	4	3	
			1	0	0	0	1	1	0	0	1	0	0	
			0	0	0	0	0	0	0	0	0	0	0	

Figure 26. GERTS IIIO Summary Report for Example 4

Example 5: A Network of Queues

A model of a network of queues is shown in Figure 27. This example is included to illustrate how queueing networks can be built up from the fundamental symbols and concepts included in the GERTS IIIQ program. It also offers another example of the input and output information obtained from the GERTS IIIQ program. This example is simply illustrative, and no discussion will be given regarding the network nor the output.

The input for this network, the echo check, and the final results are given in Figures 28 through 31.

The only novel feature included in this network of queues is the representation of a server by two activities. Q-node 5 is seen to have a PROBABILISTIC output side. Each activity emanating from node 5 represents the service activity. The service activity can direct items to different locations in the network and the time characteristics of the service activity can be different. Thus, the branch from node 5 to node 6 and the branch from node 5 to node 8 represent the same service activity but allow the modeling of different routings for items and different distributions for the service times for items.

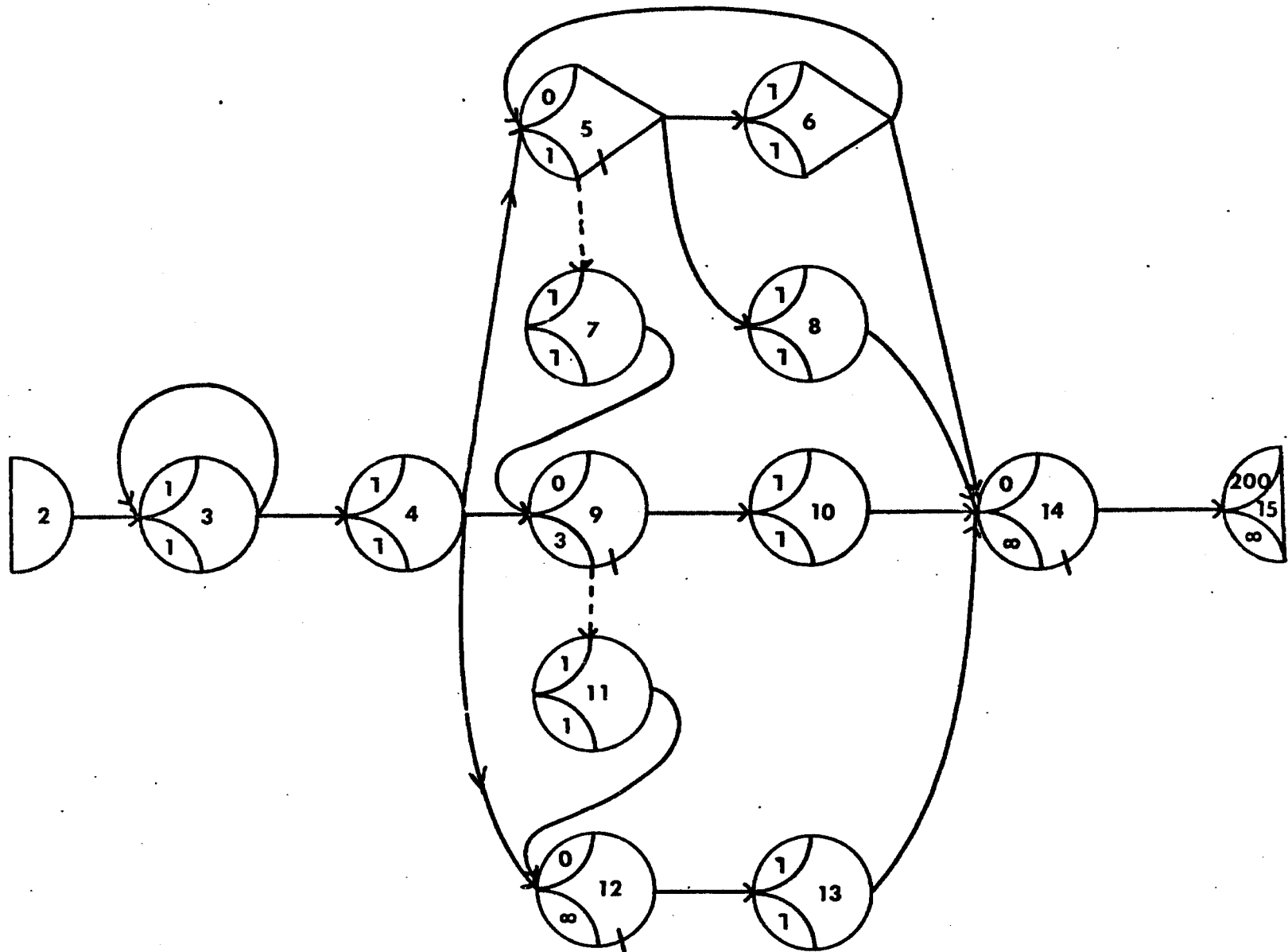


Figure 27. GERT Network for Representing a Network of Queues, Example 5

0 - NETWORK	5 5201970	50	5 100	1267		EX 5 10
15 1 1 1 1					1	EX 5 20
2 1 1 0					2	EX 5 30
3 1 10						EX 5 40
4 1 10						EX 5 50
5 5 1P 3 1 0 7						EX 5 60
6 1 1P						EX 5 70
7 1 10						EX 5 80
8 1 10					3	EX 5 90
9 5 30 0 01 1 11						EX 5 100
10 1 10						EX 5 110
11 1 10						EX 5 120
12 5 D 01 02 0						EX 5 130
13 1 10						EX 5 140
14 5 D 3 3 0						EX 5 150
15 2200 D 60 1 A						EX 5 160
0						EX 5 170
0						EX 5 180
1 0 100 1					4	EX 5 190
33 0 100 1						EX 5 200
30 0 100 1						EX 5 210
1 0 100 25						EX 5 220
1 2 3 1 1						EX 5 230
1 3 3 2 4						EX 5 240
1 3 4 1 1						EX 5 250
1 4 5 5 2						EX 5 260
1 4 9 5 2						EX 5 270
1 4 12 5 2						EX 5 280
5 5 6 3 4						EX 5 290
5 5 8 4 4						EX 5 300
25 6 5 5 2						EX 5 310
75 6 14 3 4						EX 5 320
1 7 9 1 1					5	EX 5 330
1 8 14 3 4						EX 5 340
1 9 10 4 4						EX 5 350
1 10 14 3 4						EX 5 360
1 11 12 1 1						EX 5 370
1 12 13 3 4						EX 5 380
1 13 14 3 4						EX 5 390
1 14 15 3 4						EX 5 400
0						EX 5 410

Figure 28. Input Data for Example 5

GERT SIMULATION PROJECT 5 BY Q - NETWORK
 DATE 5/ 20/ 1970

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 15
 NUMBER OF SOURCE NODES IS 1
 NUMBER OF SINK NODES IS 1
 NUMBER OF NODES TO REALIZE THE NETWORK IS 1
 STATISTICS COLLECTED ON 1 NODES
 NUMBER OF PARAMETER SETS IS 5
 INITIAL RANDCM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	D		
4	1	1	D		
6	1	1	P		
7	1	1	D		
8	1	1	D		
10	1	1	D		
11	1	1	D		
13	1	1	D		
15	200	9999	D		A

QUEUE NODES

NODE	INITIAL # IN QUEUE	MAXIMUM # ALLOWED	OUTPUT TYPE	NODE FOR BALKERS	PRIORITY SCHEME
5	0	1	D	7	FIFO
9	0	3	D	11	LIFO
12	0	9999	D	0	FIFO
14	0	9999	P	0	FIFO

SOURCE NODE NUMBERS
 2

SINK NODE NUMBERS
 15

Figure 29. Echo Check for Example 5

****ACTIVITY PARAMETERS****

PARAMETER NUMBER	PARAMETERS			
	1	2	3	4
1	0.0	0.0	0.0	0.0
2	1.0000	0.0	100.0000	1.0000
3	0.3300	0.0	100.0000	1.0000
4	0.3000	0.0	100.0000	1.0000
5	1.0000	0.0	100.0000	0.2500

****ACTIVITY DESCRIPTION****

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY
2	3	1	1	0	0	1.0000
3	3	2	4	0	0	1.0000
3	4	1	1	0	0	1.0000
4	5	5	2	-1	0	1.0000
4	9	5	2	-1	0	1.0000
4	12	5	2	-1	0	1.0000
5	6	3	4	-5	0	0.5000
5	8	4	4	-5	0	0.5000
6	14	3	4	-1	0	0.7500
6	5	5	2	-1	0	0.2500
7	9	1	1	-1	0	1.0000
8	14	3	4	-1	0	1.0000
9	10	4	4	-9	0	1.0000
10	14	3	4	-1	0	1.0000
11	12	1	1	-1	0	1.0000
12	13	3	4	-12	0	1.0000
13	14	3	4	-1	0	1.0000
14	15	3	4	-14	0	1.0000

Figure 30. Further Echo Check for Example 5

GERT SIMULATION PROJECT 5 BY Q - NETWORK
DATE 5/ 20/ 1979

FINAL RESULTS FOR 50 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
15	1.0000	75.9332	6.1999	50.	64.0588	87.9079	A

QUEUE NODES

NODE	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	
5	0.0857	0.0341	50.	0.0332	0.1907	AVERAGE NUMBER IN THE QUEUE
5	0.3239	0.0532	50.	0.2298	0.4992	AVERAGE BUSY TIME OF PROCESSOR
5	0.0869	0.0448	50.	0.0119	0.1953	AVERAGE BALKERS PER UNIT TIME
9	0.1713	0.0865	50.	0.0265	0.3757	AVERAGE NUMBER IN THE QUEUE
9	0.3231	0.0526	50.	0.2138	0.4461	AVERAGE BUSY TIME OF PROCESSOR
9	0.0269	0.0353	50.	0.0	0.1704	AVERAGE BALKERS PER UNIT TIME
12	0.1954	0.1179	50.	0.0494	0.7191	AVERAGE NUMBER IN THE QUEUE
12	0.3269	0.0519	50.	0.2242	0.4668	AVERAGE BUSY TIME OF PROCESSOR
12	0.0	0.0	50.	0.0	0.0	AVERAGE BALKERS PER UNIT TIME
14	13.3648	10.0472	50.	2.8222	56.4267	AVERAGE NUMBER IN THE QUEUE
14	0.8884	0.0744	50.	0.6877	0.9826	AVERAGE BUSY TIME OF PROCESSOR
14	0.0	0.0	50.	0.0	0.0	AVERAGE BALKERS PER UNIT TIME

HISTOGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
			0	1	2	3	4	5	6	7	8	9	
15	60.00	1.00	0	0	0	0	0	2	0	2	1	1	
			3	4	1	3	4	1	4	3	2	3	
			0	1	2	1	3	1	2	0	0	0	
5	0.03	0.01	0	2	2	6	8	8	7	4	4	3	
			1	1	1	0	0	1	1	0	0	0	
			0	0	0	0	0	0	0	0	0	0	
9	0.0	0.01	0	0	0	1	0	0	1	2	6	0	
			4	0	2	2	4	2	2	3	2	1	
			1	3	2	1	1	1	1	0	0	5	
12	0.01	0.02	0	0	1	1	6	2	5	5	4	6	
			4	2	1	3	1	1	2	0	0	1	
			0	1	0	0	0	0	0	0	0	1	
14	3.00	3.00	1	11	5	10	7	5	4	1	3	0	
			0	0	1	0	0	0	0	0	1	0	0
			0	0	0	0	0	0	0	0	0	0	0

Figure 31. GERTS IIIQ Summary Report for Example 5

2

THE GERTS IIIC PROGRAM

The GERTS III program was modified to permit the calculation of total cost expended until a node of the network is realized. The cost information for each activity is a start-up cost and a cost per unit time. These two cost parameters are appended to the parameters associated with a branch of the network. Total cost expended until a node is realized is calculated for each node of the network which is designated as a statistics node. The total cost expended includes the start-up cost for all activities started prior to the realization of the node and the variable cost expended to the time the node is realized.

The changes in input information for GERTS IIIC are on Data Card Type 5 on which the attributes for an activity are defined. The start-up cost and the variable cost per unit time are added (see Appendix A for format). No other data cards have been changed. For GERTS IIIC, statistical information is collected on both the time and cost values. This necessitates an extra row for each statistics node on the standard GERTS summary report. If GERTS IIIC is used to solve a problem in which no cost information is involved, an extra line will be printed on the GERTS summary report.

The GERTS IIIC program requires additional storage since the cost parameters for each activity must be stored. In addition, a slight increase in computation time can be expected since each time a statistics node is realized, the total cost expended to that time must be updated to include the cost of the on going activities. An example of the GERTS IIIC program is given in the next section.

Example 6: Maintenance Procedures for Multiple Completion Oil Wells

To illustrate the GERTS IIIC program the time and cost involved in fixing a leak in a multiple completion oil well will be analyzed [2]. A

schematic diagram of an oil well is shown in Figure 32. It is assumed that three different compositions of fluid are being produced through three separate tubing strings from three different geological formations or zones. When any zone of the well has an anomalous change in production or a sudden increase in the percent water, a leak is suspected. A leak can be caused by a flaw in the casing, tubing, or in the packers that separate the zones. When a leak is suspected, production from the well must be shut off in compliance with government regulations.

The repair of the well consists of the following steps:

- 1) Pull all three strings of tubing and the three packers out of the well. If a packer becomes lodged, it has to be drilled out. This requires much longer than the simple pulling job. The probability of drilling each packer is 0.20; the time is normally distributed with a mean of 24 hours and a standard deviation of 3 hours.
- 2) After all of the tubing and packers are removed, tools are lowered into the well to close off different sections of the well between producing zones and test for leaks. Each section has the same probability of having a leak, 0.20. It is not uncommon to find leaks in different sections of the well.
- 3) If a leak is located, cement is forced into the leak under high pressure. The well sits without work while the cement hardens. After hardening, the cement remaining in the well bore is drilled out and the repair job is tested. The total time for this procedure is normally distributed with a mean of 48 hours and a standard deviation of 8 hours. If the test indicates that the leak has not been corrected, the cementing procedure is repeated until successful. The probability of a successful cement job is 0.50

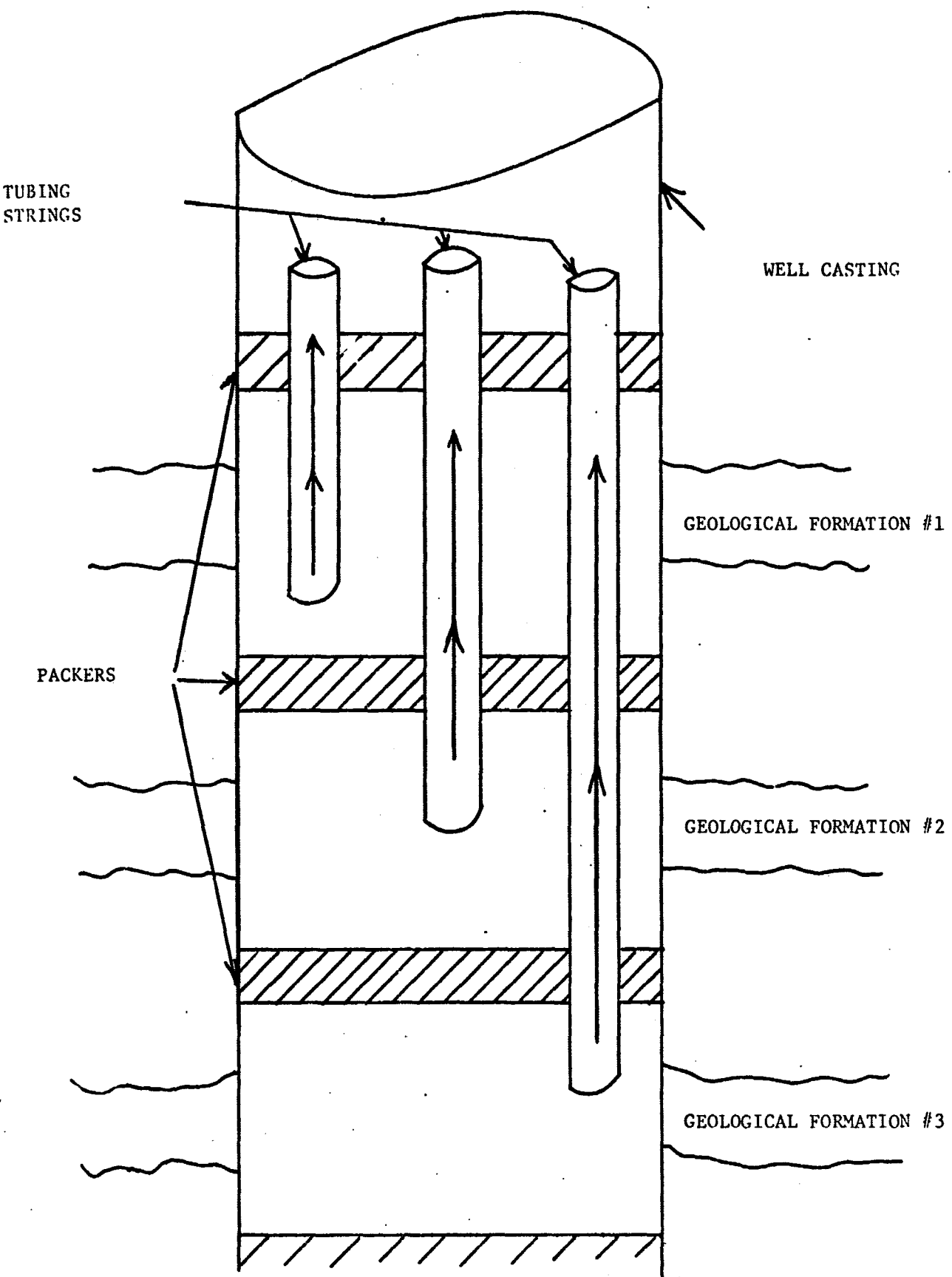


Figure 32. Schematic Diagram of a Multiple Completion Oil Well

is independent of the number of previous cement jobs. After a leak is repaired, Step 2 is repeated for the next section of casing until each section of casing is checked and all leaks are repaired.

- 4) The tubing is then run back into the well. After each string of tubing is in place it is pressure tested for leaks. A leak could occur, even with a new string of tubing, through a faulty connection or difficulty setting the packer. The probability of a leak in each tubing string is 0.10. If a leak is found, the problem has to be corrected before more tubing is run.
- 5) When all of the tubing is in place, the equipment is installed on the head of the well and it is placed in production.

The GERT network representing the repair of the well is shown in Figure 33. Nodes 2 through 6 indicate the removal of the strings and packers from the well. When node 5 is realized three times, all three packers have been removed from the well. The two activities between nodes 3 and 4 represent the actual removal of the tubing and packers. One activity represents the direct removal of the tubing and packer whereas the other activity assumes that the packer becomes lodged and has to be drilled out. Nodes 7 through 12 represent the cementing operation. A successful cementing operation is represented by the activity from node 10 to node 9 whereas an unsuccessful cement job is represented by the feedback branch from node 10 to node 8. When all four zones are successfully sealed, node 11 is realized which causes a transfer to node 12 and a modification of the network by replacing node 7 by node 13. Node 13 represents the initiation of the replacement of the tubes and packers into the well. Nodes 13 through 18 represent the rebuilding of the well by inserting the casing, tubing and packers. The activity from node 19 to node 20 represents the final well head installation.

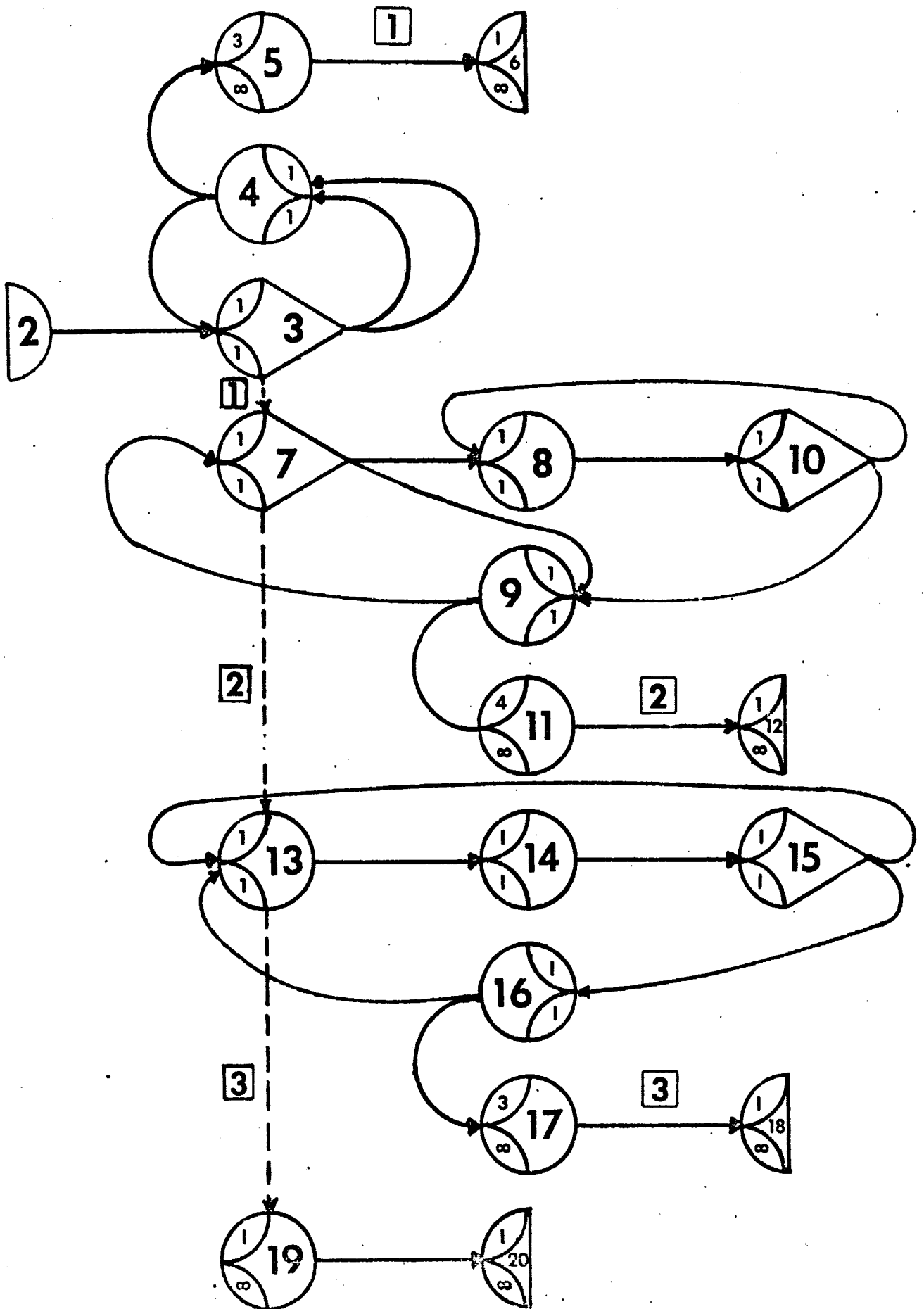


Figure 33. GERT Network of Maintenance Procedure for Multiple Oil Well, Example 6

Hypothetical time and cost data have been assigned to the various activities of the network. The cost include both equipment and labor cost. A listing of the input data to the GERTS IIIC program is given in Figure 34. The description of the network obtained from the output of the GERT IIIC program is presented in Figures 35 and 36. The final results for the simulation of the maintenance of the repair of an oil well are given in Figure 37. All cost results are given in thousands of dollars.

OIL WELL										1	EX 6 10	
20	1	1	1	3	3	1					2	EX 6 20
2	1	0	D									EX 6 30
3	1	1P										EX 6 40
4	1	1D										EX 6 50
5	3	D										EX 6 60
6	1	1										EX 6 70
7	3	1	1P	80	12	F	1000					EX 6 80
8	1	1D										EX 6 90
9	1	1D										EX 6 100
10	1	1P										EX 6 110
11	4	D									3	EX 6 120
12	1	D										EX 6 130
13	3	1	1	80	12	F	2000					EX 6 140
14	1	1										EX 6 150
15	1	1P										EX 6 160
16	1	1										EX 6 170
17	3											EX 6 180
18	1	D										EX 6 190
19	1											EX 6 200
20	2	1		120.0	12.0	F	10000					EX 6 210
J												EX 6 220
48.0	0.0	0.0	0.0	0.0	0.0							EX 6 230
23.0	11.0	35.0	3.0									EX 6 240
5.0	0.0	0.0	0.0									EX 6 250
0.0	0.0	0.0	0.0									EX 6 260
7.0	0.0	0.0	0.0									EX 6 270
47.0	23.0	71.0	8.0									EX 6 280
2.0	0.0	0.0	0.0									EX 6 290
24.0	0.0	0.0	0.0									EX 6 300
1.0	0.0	0.0	0.0									EX 6 310
14.0	0.0	0.0	0.0									EX 6 320
1.0	2	3	1	1								EX 6 330
.8	3	4	3	1			0					EX 6 340
0.2	3	4	2	2			0					EX 6 350
1.0	4	3	9	1			0					EX 6 360
1.0	4	5	4	1			0					EX 6 370
1.0	5	6	4	1	0	1	0					EX 6 380
0.8	7	9	9	1			0					EX 6 390
0.2	7	8	5	1			0					EX 6 400
1.0	8	10	6	2	1	0	200					EX 6 410
1.0	9	7	9	1			0					EX 6 420
1.0	9	11	4	1			0					EX 6 430
0.5	10	8	9	1	2		0					EX 6 440
0.5	10	9	4	1			0					EX 6 450
1.0	11	12	4	1	0	2	0					EX 6 460
1.0	13	14	3	1			100					EX 6 470
1.0	14	15	7	1			0					EX 6 480
0.9	15	16	4	1			0					EX 6 490
0.1	15	13	10	1	3	0	0					EX 6 500
1.0	16	13	9	1			0					EX 6 510
1.0	16	17	4	1			0					EX 6 520
1.0	17	18	4	1	0	3	0					EX 6 530
1.0	19	20	8	1			0					EX 6 540
0												EX 6 550
1	3	7										EX 6 560
2	7	13										EX 6 570
3	13	19										EX 6 580
0												EX 6 590

Figure 34. Input Data for Example 6

GERT SIMULATION PROJECT 6 BY OIL WELL
 DATE 5/ 20/ 1970

NETWORK DESCRIPTION

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 20
 NUMBER OF SOURCE NODES IS 1
 NUMBER OF SINK NODES IS 1
 NUMBER OF NODES TO REALIZE THE NETWORK IS 1
 STATISTICS COLLECTED ON 3 NODES
 NUMBER OF PARAMETER SETS IS 10
 INITIAL RANDOM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	0	9999	D		
3	1	1	P		
4	1	1	D		
5	3	9999	D		
6	1	1	D		
7	1	1	P		F
8	1	1	D		
9	1	1	D		
10	1	1	P		
11	4	9999	D		
12	1	9999	D		
13	1	1	D		F
14	1	1	D		
15	1	1	P		
16	1	1	D		
17	3	9999	D		
18	1	9999	D		
19	1	9999	D		
20	1	9999	D		F

SOURCE NODE NUMBERS

2

SINK NODE NUMBERS

20

STATISTICS COLLECTED ALSO ON NODES

13 7

Figure 35. Echo Check for Example 6

****ACTIVITY PARAMETERS****

PARAMETER NUMBER	PARAMETERS			
	1	2	3	4
1	48.0000	0.0	0.0	0.0
2	23.0000	11.0000	35.0000	3.0000
3	5.0000	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	7.0000	0.0	0.0	0.0
6	47.0000	23.0000	71.0000	8.0000
7	2.0000	0.0	0.0	0.0
8	24.0000	0.0	0.0	0.0
9	1.0000	0.0	0.0	0.0
10	14.0000	0.0	0.0	0.0

****ACTIVITY DESCRIPTION****

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY	SETUP COST	VARIABLE COST
2	3	1	1	0	0	1.0000	0.0	0.0
3	4	3	1	0	0	0.8000	0.0	125.00
3	4	2	2	0	0	0.2000	0.0	125.00
4	3	9	1	0	0	1.0000	0.0	125.00
4	5	4	1	0	0	1.0000	0.0	0.0
5	6	4	1	0	1	1.0000	0.0	0.0
7	9	9	1	0	0	0.8000	0.0	0.0
7	8	5	1	0	0	0.2000	0.0	90.00
8	10	6	2	1	0	1.0000	200.00	90.00
9	7	9	1	0	0	1.0000	0.0	90.00
9	11	4	1	0	0	1.0000	0.0	0.0
10	8	9	1	2	0	0.5000	0.0	90.00
10	9	4	1	0	0	0.5000	0.0	90.00
11	12	4	1	0	2	1.0000	0.0	0.0
13	14	3	1	0	0	1.0000	100.00	150.00
14	15	7	1	0	0	1.0000	0.0	150.00
15	16	4	1	0	0	0.9000	0.0	150.00
15	13	10	1	3	0	0.1000	0.0	150.00
16	13	9	1	0	0	1.0000	0.0	150.00
16	17	4	1	0	0	1.0000	0.0	0.0
17	18	4	1	0	3	1.0000	0.0	0.0
19	20	8	1	0	0	1.0000	0.0	90.00

****NETWORK MODIFICATIONS****

ACTIVITY NODE FILE NODE FILE NODE FILE NODE FILE NODE FILE NODE FILE NODE FILE

1	3	7
2	7	13
3	13	19

Figure 36. Further Echo Check for Example 6

GERT SIMULATION PROJECT 6 BY OIL WELL
DATE 5/ 20/ 1970

FINAL RESULTS FOR 200 SIMULATIONS

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
20	1.0000	212.1116	94.2245	200.	122.0000	550.8337	F
COST (000)		17.9767	9.1826	200.	8.6700	49.1551	
20	1	1.4350	1.8282	200.	0.0	9.0000	
20	2	0.7050	1.3811	200.	0.0	8.0000	
20	3	0.3600	0.6422	200.	0.0	4.0000	
13	1.0000	165.5267	92.6895	200.	82.0000	510.8337	F
COST (000)		12.1930	8.8654	200.	3.9100	44.3951	
13	1	1.4350	1.8282	200.	0.0	9.0000	
13	2	0.7050	1.3811	200.	0.0	8.0000	
13	3	0.0750	0.2641	200.	0.0	1.0000	
7	1.0000	95.4845	52.5304	200.	68.0000	439.5588	F
COST (000)		5.2549	5.1346	200.	2.3400	38.1400	
7	1	0.3550	1.0268	200.	0.0	8.0000	
7	2	0.1800	0.7814	200.	0.0	7.0000	
7	3	0.0	0.0	200.	0.0	0.0	

HISTDGRAMS

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES										
			0	35	34	4	12	14	9	8	8	4	11
20	120.00	12.00	7	4	11	8	2	4	4	1	2	0	1
			2	1	0	2	2	2	2	1	1	4	
COST	10000.00	1000.00	35	9	27	3	17	6	5	8	9	11	6
			7	4	5	11	4	3	1	3	3	2	3
			0	1	0	2	2	0	2	0	1	10	
13	80.00	12.00	0	47	30	7	6	14	9	7	6	7	10
			7	6	7	10	3	0	3	1	3	1	2
			0	2	1	1	1	3	1	0	1	4	
COST	2000.00	1000.00	0	0	47	0	13	18	5	17	5	7	4
			8	6	11	6	4	7	9	4	2	2	2
			2	2	2	2	1	0	2	0	7	10	
7	80.00	12.00	95	50	12	12	6	7	2	1	3	0	2
			2	1	1	0	0	1	0	1	0	2	0
			0	0	1	0	0	0	0	0	1	0	
COST	1000.00	1000.00	0	0	95	3	42	11	13	11	4	2	3
			2	2	0	2	1	1	1	1	0	0	1
			0	0	1	1	0	1	0	0	1	1	

Figure 37. GERTS IIIC Summary Report for Example 6

THE GERTS IIIR PROGRAM

GERT networks considered previously do not allow for the situation in which there are limited resources available to perform a project. When this is the case, the activities of the project compete for the resources. To introduce resources into the GERTS program, the resource requirements for each activity are added to the description of each activity. The maximum number of each type of resource that is available must also be specified. The introduction of resource requirements and resource limitations introduces a scheduling problem into the GERT framework.

With resource limitations, it is no longer possible to schedule an activity whose start node has been realized since resource requirements may not be available. Also when an end of activity event occurs, resources that were used on that activity are now available for use on other activities and these activities may be considered for scheduling. Thus, when end of activity events occur, a selection is to be made from among those activities that can be started (the activities which have all their predecessor activities completed). This selection should be made on the basis of the performance measures associated with the completion of the project.

There has been a large amount of research performed on resource allocation problems for projects described in terms of PERT networks. No general conclusions about a method for scheduling for such projects has been developed although several good rules have been proposed [12]. It has been found that the performance associated with a rule is dependent on the structure of the network. Since the GERT network structure is much more complex than the PERT network structure it is expected that the scheduling task will be more difficult. The purpose of the GERTS IIIR program is to provide a vehicle with which research can be performed in the development of scheduling rules

for GERT networks. The GERTS IIIR program is considered as a research vehicle and is in a preliminary form. It is being reported on at this time to present the resource allocation problem and to allow for discussion on the general requirements for a vehicle to perform research in this area.

GERTS IIIR Program Philosophy

The GERTS IIIR program requires that the user write two programs. These are Subroutine SCHDL and Function CALAT. SCHDL is written to define the scheduling rules to be used during the simulation of a GERT network. CALAT is used for calculating an attribute value on which activities can be ranked. GERTS IIIR attempts to perform all the bookkeeping operations required for the simulation and requires of the user only a small amount of programming. When GERTS IIIR calls subroutine SCHDL, it expects a selection of an activity from among those activities that can be scheduled. An activity can be scheduled if all its predecessors have been completed and sufficient resources are available. All activities that can be scheduled, are stored as entries in file NOQ. The first entry in file NOQ is stored in column MFE(NOQ). To obtain the successor of an entry, the function KSUCC is provided. The call to this function is shown below to obtain the successor of the first entry in file NOQ:

```
KCOL = KSUCC(MFE(NOQ), NSET, QSET)
```

In general, the successor to an entry can be obtained by using the FUNCTION KSUCC. When the column number of an entry's successor is 7777, there are no more entries in the file.

The attributes associated with an activity can be obtained by use of the subroutine TRNSF. If it is desired to obtain the attributes of the activity stored in column KCOL, the following statement is used:

```
CALL TRNSF(KCOL, NSET, QSET)
```

The attributes associated with the activity in column KCOL are:

- ATRI(1) = the time required to perform the activity
- ATRI(2) = the time the activity was marked
- ATRI(3) = specified by user in Function CALAT
- JTRI(1) = the end node for the activity
- JTRI(2) = the parameter set number for the activity
- JTRI(3) = the distribution type for the time required to perform the activity
- JTRI(4) = the counter type
- JTRI(5) = the activity number
- JTRI(6) = the number of resources of type one required to perform the activity
- JTRI(7) = the number of resources of type two required to perform the activity
- JTRI(8) = the number of resources of type three required to perform the activity

The subprograms KSUCC and TRNSF are provided to facilitate the programming of Subroutine SCHDL by the user in which he programs a scheduling rule to select among the available activities.

For the current GERTS IIIR program, only three resources per activity are permitted. The total number of resources available for resource type I is stored in the array NRESA(I). The number of resources currently in use is stored in the array NREST(I). File 1 contains those activities that are currently in progress. The attribute description for entries in file 1 is the same as the one given above except that ATRI(1) is the time that the activity will be completed. Information concerning the on-going activities can be obtained by using KSUCC and TRNSF. Those activities whose predecessors have all been completed but for which there are not sufficient resources available are stored in file (NOQ - 1). Whenever the user schedules an activity, files NOQ and NOQ - 1 are updated and if file NOQ is not empty, subroutine SCHDL is called. This process is continued until file NOQ is empty or Subroutine SCHDL sets KCOL = 0 indicating no activity should be

scheduled. Whenever an activity is completed, the resources that were used for that activity, are made available and the file NOQ is updated. Again subroutine SCHDL would be called to determine if the user desires to schedule an activity.

The entries in files NOQ and (NOQ - 1) are ranked on an attribute as specified by the user as part of the input data. The ranking is either low-value-first (LVF) or high-value-first (HVF). The attribute specified can be any one of the ATRIB or JTRIB values listed above.

The additional input requirements for GERTS IIIR are:

1. On Data Card Type 2, the following additional variables have been added:
 - a) The number of resource types involved in the project. If this value is 0, the program will operate as if it were GERTS III and no additional input is required (the largest value permitted without altering dimensions is 3).
 - b) The attribute number on which ranking is to be performed in file NOQ and NOQ - 1. If this value is 1, 2 or 3 ranking is done on the ATRIB value with the corresponding subscript. If the value is between 101 and 108, ranking is done on the JTRIB value with a subscript of the input value minus 100.
 - c) The priority ranking code. If the input value is 1, ranking is on low-value-first. If it is 2, ranking is on high-value-first.
 - d) The number of resources available for each resource type.
2. The resource requirements for each activity must be defined on Data Card Type 5.

Appendix A describes the formats for these new inputs to the program. Examples of the use of GERTS IIIR are given in the next section.

Example 7: Resource Allocation for a Maintenance Problem [10]

The network to be used for illustrating GERTS IIIR is presented in Figure 38. In Table 2 each activity is described in terms of its start node and end node, average duration, and resource requirements. The FORTRAN programs that

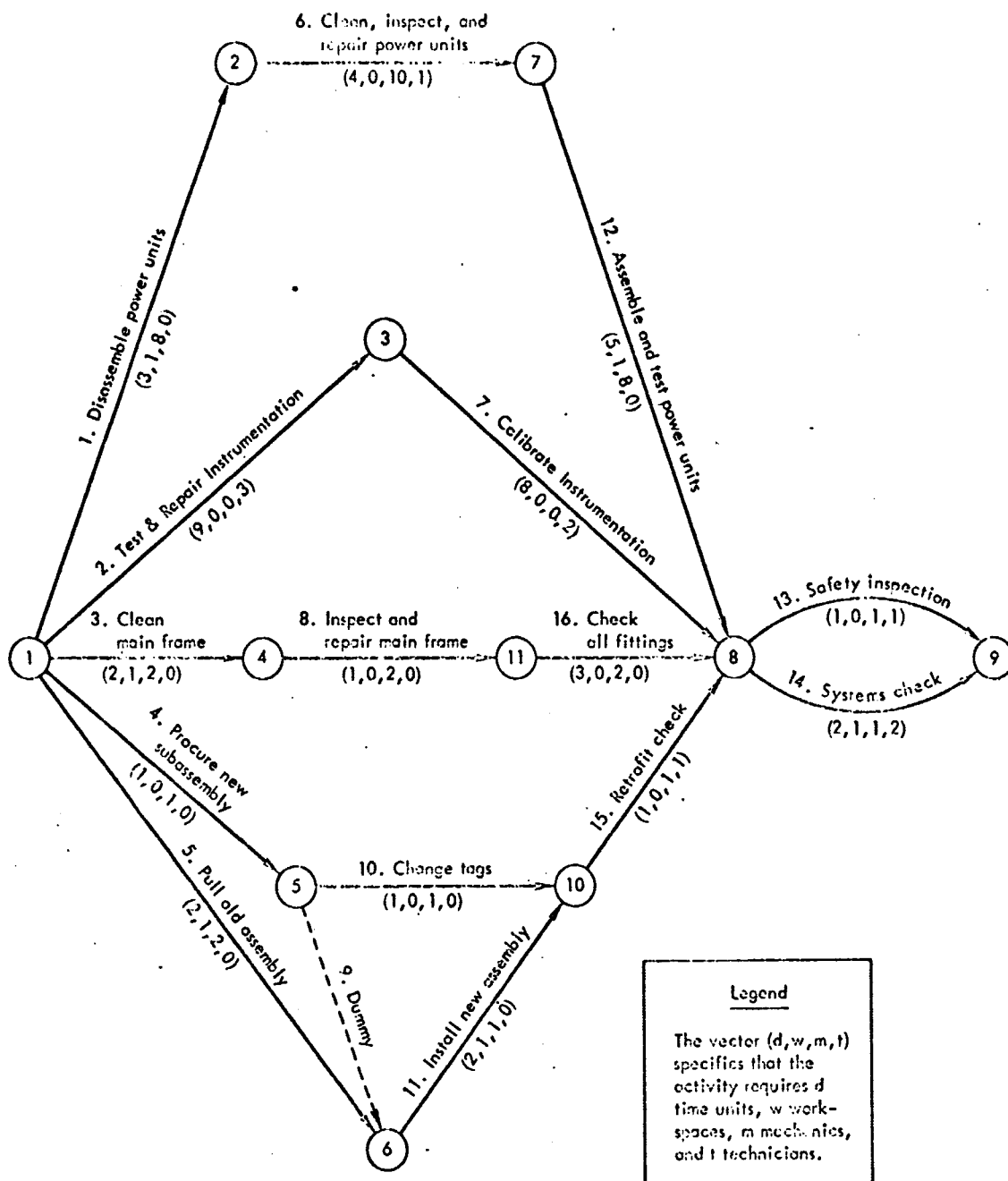


Figure 38. GERT Network for a Maintenance Problem, Example 7

must be added to the GERTS IIIR program are shown in Figure 39. The main program is standard and will be identical for all uses of GERTS IIIR. In Subroutine SCHDL, the scheduling rule specifies that the first activity in file NOQ should be given priority. The entries in file NOQ are stored in order of ATRIB(3) with high values first. These values are established by

```

DIMENSION NSET(10000),QSET(3000)                                GRTS 10
COMMON ID,IM,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NPRNT,NCRDR,NNM,IMN,NPD,GRTS 20
1NDQ,NPRMS,NRUN,NRUNS,ISEED,TNOW,MXX,IMM,NYR,SEED,           GRTS 20
2MAXQS,MAXNS,ATRI(3),JTRIB(8),NAME(6),NPROJ,MCN,NDAY,JCELS(100,32)GRTS 40
COMMON MFE(999),MLE(999),NQ(999),PARAM(300,4),SUMA(500,5)   GRTS 50
COMMON NSINK(100),NSKST,NSRC,XLOW(100),NREL(999),NREL(999),NN, GRTS 60
1NSKS,NSKSR,NSORC(20),NTYPE(999),WIDTH(100),TOTIM,NCTS,KOUNT(100), GRTS 70
2NPO(999),NFTBU(999),NABA(999),NREL2(999),NSNR(100),JSINK(100) GRTS 80
COMMON NDT,NDTL,NDTU,NSKD,NSTND,XSTUS(100)                   GRTS 90
COMMON NRESA(3),NREST(3),ARESU(3),TLRC(3),NRESC,KRNL,KPRTY   GRTS 100
C                                                                GRTS 110
C****SET EQUIPMENT NUMBERS FOR CARD READER(NCRDR) AND PRINTER (NPRNT). GRTS 120
C                                                                GRTS 130
NCRDR=5                                                        GRTS 140
NPRNT=6                                                        GRTS 150
IMN=999                                                        GRTS 160
7 CALL GASP(NSET,QSET)                                        GRTS 170
GO TO 7                                                        GRTS 180
END                                                            GRTS 190

SUBROUTINE SCHDL (KCOLL,NSET,QSET)                               SCDL 10
DIMENSION NSET(1),QSET(1)                                     SCDL 20
COMMON ID,IM,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NPRNT,NCRDR,NNM,IMN,NPD,SCDL 30
1NDQ,NPRMS,NRUN,NRUNS,ISEED,TNOW,MXX,IMM,NYR,SEED,           SCDL 40
2MAXQS,MAXNS,ATRI(3),JTRIB(8),NAME(6),NPROJ,MCN,NDAY,JCELS(100,32)SCDL 50
COMMON MFE(999),MLE(999),NQ(999),PARAM(300,4),SUMA(500,5)   SCDL 60
COMMON NSINK(100),NSKST,NSRC,XLOW(100),NREL(999),NREL(999),NN, SCDL 70
1NSKS,NSKSR,NSORC(20),NTYPE(999),WIDTH(100),TOTIM,NCTS,KOUNT(100), SCDL 80
2NPO(999),NFTBU(999),NABA(999),NREL2(999),NSNR(100),JSINK(100) SCDL 90
COMMON NDT,NDTL,NDTU,NSKD,NSTND,XSTUS(100)                   SCDL 100
COMMON NRESA(3),NREST(3),ARESU(3),TLRC(3),NRESC,KRNL,KPRTY   SCDL 110
KCOLL = MFE(NDQ)                                             SCDL 120
RETURN                                                         SCDL 130
END                                                            SCDL 140

FUNCTION CALAT(NSET,QSET)                                       CLAT 10
DIMENSION NSET(1),QSET(1)                                     CLAT 20
COMMON ID,IM,MFA,MSTOP,MX,MXC,NCLCT,NHIST,NPRNT,NCRDR,NNM,IMN,NPD,CLAT 30
1NDQ,NPRMS,NRUN,NRUNS,ISEED,TNOW,MXX,IMM,NYR,SEED,           CLAT 40
2MAXQS,MAXNS,ATRI(3),JTRIB(8),NAME(6),NPROJ,MCN,NDAY,JCELS(100,32)CLAT 50
COMMON MFE(999),MLE(999),NQ(999),PARAM(300,4),SUMA(500,5)   CLAT 60
COMMON NSINK(100),NSKST,NSRC,XLOW(100),NREL(999),NREL(999),NN, CLAT 70
1NSKS,NSKSR,NSORC(20),NTYPE(999),WIDTH(100),TOTIM,NCTS,KOUNT(100), CLAT 80
2NPO(999),NFTBU(999),NABA(999),NREL2(999),NSNR(100),JSINK(100) CLAT 90
COMMON NDT,NDTL,NDTU,NSKD,NSTND,XSTUS(100)                   CLAT 100
COMMON NRESA(3),NREST(3),ARESU(3),TLRC(3),NRESC,KRNL,KPRTY   CLAT 110
JOT=0                                                         CLAT 120
DO 20 J=1,NRESC                                             CLAT 130
20 JOT=JOT+JTRIB(J+5)                                       CLAT 140
CALAT=ATRI(1) * FLOAT(JOT)                                  CLAT 150
RETURN                                                         CLAT 160
END                                                            CLAT 170

```

Figure 39 FORTRAN Listing of the Main Program, Subroutine SCHDL, and Function CALAT

inserting the values of 3 and 2 on data card type 2 in fields 9 and 10. This can be seen in Figure 40. The value assigned to ATRIB(3) is calculated in the Function CALAT. In the function, CALAT is defined as the processing time multiplied by the resource requirements for resource types 1, 2, and 3, that is, the resource-hours required for the first two resources performing the activity. The GERTS IIIR program automatically sets ATRIB(3) equal to the value established for CALAT.

The echo check of the network for this sample is shown in Figures 41 and 42. The final results for allocating resources in accordance with Subroutine SCHDL and Function CALAT are shown in Figure 43.

Activity Number	Start Node	End Node	Duration	Workspace	Mechanics	Technicians
1	1	2	3	1	8	0
2	1	3	9	0	0	3
3	1	4	2	1	2	0
4	1	5	1	0	1	0
5	1	6	2	1	2	0
6	2	7	4	0	10	1
7	3	8	8	0	0	2
8	4	11	1	0	2	0
9	5	6	0	0	0	0 (Dummy)
10	5	10	1	0	1	0
11	6	10	2	1	1	0
12	7	8	5	1	8	0
13	8	9	1	0	1	1
14	8	9	2	1	1	2
15	10	8	1	0	1	1
16	11	8	3	0	2	0

Table 2. Resource Requirements for a Network of Activities Describing a Maintenance Problem

GERT SIMULATION PROJECT 7 BY RES ALLOC
 DATE 5/ 19/ 1970

****NETWORK DESCRIPTION****

NODE CHARACTERISTICS

HIGHEST NODE NUMBER IS 12
 NUMBER OF SOURCE NODES IS 1
 NUMBER OF SINK NODES IS 1
 NUMBER OF NODES TO REALIZE THE NETWORK IS 1
 STATISTICS COLLECTED ON 1 NODES
 NUMBER OF PARAMETER SETS IS 16
 INITIAL RANDOM NUMBER IS 1267 0.0

NODE	NUMBER RELEASES	NUMBER OF RELEASES FOR REPEAT	OUTPUT TYPE	REMOVAL DESIRED AT REALIZATION	STATISTICS BASED ON REALIZATIONS
2	1	9999	D		
3	1	9999	D		
4	1	9999	D		
5	1	9999	D		
6	2	9999	D		
7	1	9999	D		
8	4	9999	D		
9	2	9999	D		A
10	2	9999	D		
11	1	9999	D		
12	0	9999	D		

SOURCE NODE NUMBERS
 12

SINK NODE NUMBERS.
 9

Figure 41 Echo Check for Example 7

ACTIVITY PARAMETERS

PARAMETER NUMBER	PARAMETERS			
	1	2	3	4
1	3.0000	0.0	100.0000	0.3000
2	9.0000	0.0	100.0000	0.9000
3	2.0000	0.0	100.0000	0.2000
4	1.0000	0.0	100.0000	0.1000
5	2.0000	0.0	100.0000	0.2000
6	4.0000	0.0	100.0000	0.4000
7	8.0000	0.0	100.0000	0.8000
8	1.0000	0.0	100.0000	0.1000
9	0.0	0.0	100.0000	0.0
10	1.0000	0.0	100.0000	0.1000
11	2.0000	0.0	100.0000	0.2000
12	5.0000	0.0	100.0000	0.5000
13	1.0000	0.0	100.0000	0.1000
14	2.0000	0.0	100.0000	0.2000
15	1.0000	0.0	100.0000	0.1000
16	3.0000	0.0	100.0000	0.3000

ACTIVITY DESCRIPTION

START NODE	END NODE	PARAMETER NUMBER	DISTRIBUTION TYPE	COUNT TYPE	ACTIVITY NUMBER	PROBABILITY	FIRST RESOURCE	SECOND RESOURCE	THIRD RESOURCE
2	7	6	2	0	0	1.0000	0	10	1
3	8	7	2	0	0	1.0000	0	0	7
4	11	8	2	0	0	1.0000	0	2	3
5	6	9	2	0	0	1.0000	0	0	0
5	10	10	2	0	0	1.0000	0	1	0
6	10	11	2	0	0	1.0000	1	1	0
7	8	12	2	0	0	1.0000	1	8	0
8	9	13	2	0	0	1.0000	0	1	1
8	9	14	2	0	0	1.0000	1	1	2
10	8	15	2	0	0	1.0000	0	1	1
11	8	16	2	0	0	1.0000	0	2	0
12	2	1	2	0	0	1.0000	1	8	0
12	3	2	2	0	0	1.0000	0	0	3
12	4	3	2	0	0	1.0000	1	2	0
12	5	4	2	0	0	1.0000	0	1	0
12	6	5	2	0	0	1.0000	1	2	0
RESOURCE AVAILABILITY							1	11	4

RANKING FOR FILE NOQ IS BASED ON ATTRIBUTE 3 WITH HVF

Figure 42 Further Echo Check for Example 7

From Figure 43 it is seen that the average time to complete the network using the rule described above is approximately 21.76 time units with a standard deviation of approximately 0.83 time units. The minimum and maximum values along with the histogram for the time to complete the project are also shown. For GERTS IIIR, a new section has been added to the standard GERTS summary report to give the final results for resource utilization. Resource utilization is defined as the resource-hours used divided by the project completion time. For each resource type, the average, standard deviation, number of observations, minimum and maximum values observed are displayed. For resource type 1, the work space, the average utilization was approximately 74 percent. For mechanics for which there were 11 available, the average utilization was only 5.87 and for technicians for which there were 4 available, the average utilization was 2.41. Other estimates given on the GERTS IIIR summary report are for the standard deviation, the minimum, and maximum values of the resource utilization.

A second scheduling rule was tested which involved scheduling jobs by the shortest processing time but deviating from this rule if completion time of an activity exceeded 19 time units. To implement this rule, file NOQ is ranked on attribute 1, the processing time for the activity, with low values first (code = 1). Subroutine SCHDL is then coded to test each activity in file NOQ beginning with the first activity to see if the current time plus the processing time for the activity is greater than 19. When this occurs, the activity is selected for scheduling. The coding for this version of subroutine SCHDL is given below.

```

KCOLL = MFE(NOQ)
10 CALL TRNSF (KCOLL,NSET,QSET)
IF((TNOW + ATRIB(1)).GE.19.0) RETURN
KCOLL = KSUCC(KCOLL,NSET, QSET)
IF(KCOLL.LT.7777) GO TO 10
KCOLL = MFE(NOQ)
END

```

GERT SIMULATION PROJECT 7 BY RES ALLOC
DATE 5/ 19/ 1970

****FINAL RESULTS FOR 100 SIMULATIONS****

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	21.5651	0.8196	100.	19.6154	23.2011	A

****HISTOGRAMS****

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES											
			0	0	0	0	4	4	19	18	22	21	8	
9	18.00	0.50	0	0	0	0	4	4	19	18	22	21	8	
			4	0	0	0	0	0	0	0	0	0	0	
			0	0	0	0	0	0	0	0	0	0	0	

FINAL RESULTS FOR RESOURCE UTILIZATION

RESOURCE	AVERAGE	STD.DEV.	# OF OBS.	MIN.	MAX.
1	0.7406	0.0256	100.0000	0.6769	0.8055
2	5.9544	0.1945	100.0000	5.4666	6.3852
3	2.4573	0.1597	100.0000	2.1194	2.7923

Figure 43 GERTS IIIR Summary Report for Example 7 Using Scheduling Rule 1

GERT SIMULATION PROJECT 77 BY RES ALLOC
DATE 5/ 19/ 1970

****FINAL RESULTS FOR 100 SIMULATIONS****

NODE	PROB./COUNT	MEAN	STD.DEV.	# OF OBS.	MIN.	MAX.	NODE TYPE
9	1.0000	20.4557	0.8931	100.	18.4188	22.5189	A

****HISTOGRAMS****

NODE	LOWER LIMIT	CELL WIDTH	FREQUENCIES											
			0	1	3	13	14	20	21	16	7	4	1	
9	18.00	0.50	0	1	3	13	14	20	21	16	7	4	1	
			0	0	0	0	0	0	0	0	0	0	0	
			0	0	0	0	0	0	0	0	0	0	0	

FINAL RESULTS FOR RESOURCE UTILIZATION

RESOURCE	AVERAGE	STD.DEV.	# OF OBS.	MIN.	MAX.
1	0.7931	0.0235	100.0000	0.7057	0.8220
2	6.2734	0.2217	100.0000	5.6967	6.8220
3	2.5707	0.1725	100.0000	2.0412	2.9520

Figure 44 GERT IIIR Summary Report for Example 7 Using Scheduling Rule 2

For this scheduling rule, ATRIB(3) is not used and any Function CALAT can be used. For this reason, a standard Function CALAT is provided with the GERTS IIIR package that sets ATRIB(3) equal to the resource-hour requirements for an activity as was done for the scheduling rule discussed previously. The final results for this second scheduling rule are presented in Figure 44.

SUMMARY

The GERTS III program is a flexible and efficient tool for analyzing GERT networks. During the development of the GERTS III program, the modeling concepts and symbols included within GERT networks were expanded and amplified. These concepts and symbols are the basis for the information requirements of the GERTS III programs.

In this report, the simulation program was described in brief and the methods for calculating statistical quantities of interest were presented. The main emphasis of this report has been on illustrating the method for using GERTS III and its extensions. Seven example problems were described.

The basic GERTS III package was extended to include queue nodes, resource requirements for activities, and cost information. The need and desirability of including these concepts as a fundamental part of the GERTS package has not been determined. With the advent of the additional complexity as represented by the extensions, the modeling capability of GERT networks is increased. This increase in modeling capability is obtained at a cost in terms of computer efficiency and in communication difficulty. Further research is required to evaluate the need for the extensions and their worth as a systems modeling tool.

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APPENDIX A

INPUT DATA WORKSHEET AND DESCRIPTION

FOR

GERTS III, GERTS IIIQ, GERTS IIIC, and GERTS IIIR.

Type	FORMAT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	6A2, I4, 2I2, 4I4, I8, F10.4																								
2	12I3																								
3	4I3, 2A1, 2F6.2, A1																								
	2I3																								
	2F10.4																								
4	4F10.4																								
5	F8.3, 6I3																								
	2F9.2																								
	3I3																								
6	25I3																								
7	2I3																								

(For IIR)

(For IIC)

(For IIC)

(For IIC)

(For IIR)

DESCRIPTION OF DATA INPUT FOR GERTS III, IIIQ, IIIC, and IIIR.

DATA CARD 1

Field 1 The analyst's name (6A2,1)

Field 2 The project number (I4,1) (If negative, data card 7 is required to indicate the runs to be traced).

Field 3 The month number (I2,1)

Field 4 The day number (I2,1)

Field 5 The year (I4,1)

Field 6 The number of times the network is to be simulated (I4,1)

Field 7 The number of activities with different time characteristics (I4,1)

Field 8 The number of branches in the network plus an estimate of the maximum number of activities which can occur simultaneously (I4,1)

Field 9 An integer random number seed (I8,1)

Field 10 A floating point random number seed (F10.4,1)

DATA CARD 2

Field 1 The largest node number of the network including all possible modifications to the network (I3,1): The smallest node number permitted is 2.

Field 2 Number of source nodes (I3,1).

Field 3 Number of sink nodes (I3,1).

Field 4 Number of sink nodes that must be realized before the network is realized (I3,1).

Field 5 Number of nodes which statistics are to be collected on, including all sink nodes (I3,1).

Field 6 Number of types of counts (I3,1).

Field 7 A 1 if network modifications exist; a 0 otherwise (I3,1).

For GERTS IIIR

- Field 8 Number of different resource types (I3,1).
- Field 9 The attribute on which ranking is to be done for files NOQ and (NOQ-1). Add 100 to attribute number if a JTRIB value is to be ranked on.
- Field 10 The priority system to be used for files NOQ and (NOQ-1). A 1 indicates low-value first. A 2 indicates high-values first.
- Field 11 Number of available resources of Type 1 (I3,1).
- Field 12 Number of available resources of Type 2 (I3,1).
- Field 13 Number of available resources of Type 3 (I3,1).

DATA CARD 3

- Field 1 The node number (descriptor) associated with the node characteristics given on this card (I3,1).
- Field 2 Special characteristic of the node. Codes for special characteristics are:
 1. Source node
 2. Sink node
 3. Node on which statistics are collected
 4. A mark node
If Field 2 is left blank, no special characteristic is associated with the node (I3,1).
- Field 3 The number of releases required to realize the node for the first time (I3,1).
- Field 4 The number of releases required to realize the node after the first realization (I3,1).
- Field 5 Output characteristic of the node. Codes for input are: P for PROBABILISTIC; and D for DETERMINISTIC (A1,1).
- Field 6 If events that have been scheduled to end on this node are to be removed (cancelled) when this node is realized, an "R" should be put in this field. If removal is not desired, leave blank (A1,1).

Fields 7, 8 and 9 are used only if

The node is a sink node or a statistics node (code 2 or 3 in field 2).

- Field 7 The lower limit of the second cell for the histogram to be obtained for this node. The first cell of the histogram will contain the number of times the node was realized in a time less than the value given in this field (F6.2,1).
- Field 8 The width of each cell of the histogram. Each histogram contains 32 cells. The last cell will contain the number of times the node was realized in a time greater than or equal to the lower limit (specified in Field 7) + 30 * (cell width (specified by Field 8)) (F6.2,1).
- Field 9 Statistical quantities to be collected (A1,1)
- F. The time of first realizations of the node.
 - A. The time of all realizations of the node.
 - B. The time between realizations of the node.
 - I. The time interval required to go between two nodes.
 - D. The time delay from first activity completion on the node until the node is realized.

The last card of this type must have a zero in Field 1.

For GERTS IIIQ

- Field 2 Code for a Q-node is 5.
- Field 3 For a Q-node, the initial number in the queue. If greater than zero, the service activity is assumed busy and an end of service activity event is defined automatically.
- Field 4 For a Q-node, -1 to indicate maximum number in queue is 0, 0 to indicate no limit on number in the queue, otherwise the maximum number allowed in the queue.
- Field 7, 8 Lower limit of cell 2 and width of each cell for statistics on the average number in the queue.
- Field 10 Priority Ranking Procedure for the Q-nodes (I3,1).
- 0 - First-in - first-out (FIFO)
 - 1 - Last-in - first out (LIFO)
- Field 11 Node that is transferred to when an activity is completed that is incident to the Q-node and the maximum number allowed is in the queue (the node to which an item balks) (I3,1).

For GERTS IIIC

- Fields 10, 11 Lower limit and cell width for cost histograms (F10.4,2)

DATA CARD 4

The parameters associated with the distribution of the time to perform each activity. One card is required for each activity with a different time characterization. The number of cards is specified by Data Card 1, Field 7.

A maximum of 300 is permitted. The cards must be arranged by ascending parameter number and the parameters must be numbered consecutively or blank cards appropriately placed. Nine distribution types are available which are:

1. Constant
2. Normal
3. Uniform
4. Erlang
5. Lognormal
6. Poisson
7. Beta
8. Gamma
9. Beta fitted to three parameters as in PERT.

The fields required are dependent on the distribution type of the activity. Appendix B describes these fields.

DATE CARD 5

One data card for each activity associated with the network.

- | | |
|---------|--------------------------------------|
| Field 1 | Probability of realization (F8.3,1). |
| Field 2 | Start node (I3,1). |
| Field 3 | End node (I3,1). |
| Field 4 | Parameter number (I3,1). |
| Field 5 | The distribution type (I3,1). |
| Field 6 | Count type (I3,1). |
| Field 7 | Activity number (I3,1). |

The last data card of this type must have a zero (or blank) in Field 2.

For GERTS IIIR

- | | |
|----------|---|
| Field 8 | Number of resources of type 1 required for the activity (I3,1). |
| Field 9 | Number of resources of type 2 required for the activity (I3,1). |
| Field 10 | Number of resources of type 3 required for the activity (I3,1). |

For GERTS IIIC

- Field 8 Start up cost for the activity (F9.2,1).
Field 9 Variable cost per unit time for the activity (F9.2,1).

DATA CARD 6

Only required if number of nodes modified is greater than zero. (Field 7, Data Card 2).

- Field 1 An activity number (I3,1).
Field 2 The number of the node to be replaced if the activity given in Field 1 is realized (I3,1).
Field 3 The number of the node to be inserted into the network in place of the node specified in Field 2 when the activity in Field 1 is realized (I3,1).
Fields 4-21 Fields 2 and 3 are repeated if the activity given in Field 1 affects multiple nodes. A zero in an even-numbered field indicates the end of the data on the card.

The last card of this type must have a zero in Field 1.

DATA CARD 7

Only used if the project number is negative. (Field 2, Data Card 1).

- Field 1 The run number for which tracing of the end of activity events should begin (I3,1).
Field 2 The run number for which tracing of the end of activity events should terminate (I3,1).

Multiple networks can be analyzed by stacking the data cards as described above, one after another. No blank cards should separate the data cards for each network. A blank card is required to indicate the end of all networks to be simulated.

APPENDIX B

DEFINITIONS OF PARAMETERS FOR RANDOM DEVIATE SAMPLING

The parameters required on Data Card Type 4 to sample from the nine distributions available in GERTS III are described below.

For distribution type 1 (Constant):

Field 1 The constant time (F10.4,1).

For distribution type 2 (Normal); 5 (Lognormal); 7 (Beta); and 8 (Gamma):

Field 1 The mean value (F10.4,1).
Field 2 The minimum value (F10.4,1).
Field 3 The maximum value (F10.4,1)
Field 4 The standard deviation (F10.4,1).

For distribution type 3 (Uniform):

Field 1 Not used (F10.4,1).
Field 2 The minimum value (F10.4,1).
Field 3 The maximum value (F10.4,1).
Field 4 Not used (F10.4,1).

For distribution type 4 (Erlang):

Field 1 The mean time for the Erlang variable divided by the value given to Field 4 (F10.4,1).
Field 2 The minimum value (F10.4,1).
Field 3 The maximum value (F10.4,1).
Field 4 The number of exponential deviates to be included in the sample obtained from the Erlang distribution (F10.4,1).

If Field 4 is set equal to 1, an exponential deviate will be obtained from distribution type 4.

For distribution type 6 (Poisson):

Field 1 The mean minus the minimum value

Field 2 The minimum value
Field 3 The maximum value
Field 4 Not used

Care is required when using the Poisson since it is not usually used to represent an interval of time. The interpretation of the mean should be the mean number of time units per time period.

For distribution type 9 (Beta fitted to 3 values as in PERT):

Field 1 The most likely value, m (F10.4,1).
Field 2 The optimistic value, a (F10.4,1).
Field 3 The pessimistic value, b (F10.4,1).
Field 4 Not used.

Samples are obtained from the distributions such that if a sample is less than the minimum value, the sample value is given the minimum value. Similarly, if the sample is greater than the maximum value, the sample value is assigned the maximum value. This is not sampling from a truncated distribution but sampling from a distribution with a given probability of obtaining the minimum and maximum values.