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INITIAL SPECIFICATIONS FOR VAN AND AIRCRAFT TESTING OF THE SD-53 STRAPDOWN SYSTEM

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TABLE OF CONTENTS

TABLE OF CONTENTS (cont'd)

1. INTRODUCTION

The purpose of this study is to lay the groundwork and provide a basis for field evaluation of the SD-53 Strapdown Inertial System both in a van and in an aircraft. The Inertial Sensing Unit (ISU) of the system contains three Single Axis Platform (SAP) gyros and three Pendulous Integrating Gyro Accelerometers (PIGAs). For both the van and flight test programs, a Minneapolis Honeywell 516 computer will be used to process the inertial sensor outputs.

The study was divided into the following six tasks:

- Define detailed objectives of the van and aircraft tests
- Establish a philosophy of the evaluation and an approach to fulfill the objectives
- Define an evaluation system configuration and approximate performance requirements
- Define possible tests to collect data required to fulfill the objectives
- Consider how the test data is to be reduced
- Provide an approximate estimate of the evaluation system precision

Since the purpose of the study is only to perform a preliminary test design, no detailed procedures, error analyses or data reduction programs are provided. The report is organized according to the above topics, and includes conclusions and recommendations. Several related non-van or aircraft tests and analyses that can significantly support the field test program are described briefly.

A preliminary survey of the literature was made relative to field testing of inertial systems, with particular emphasis on van and flight testing of strapdown systems. Several of the more comprehensive references are [1, 2, 3, 4, 5, 6, 7].* Other references including ones in more restrictive areas, are included in Section 8.

*Numbers in brackets ([ ]) throughout the report correspond to reference numbers in the bibliography (Section 8).

2. OBJECTIVES OF TEST PROGRAM

The objectives of the field test program are defined first in general terms and then in more detail in order to better specify the test program and to provide a measure to which the results can be assessed. The five major objectives of the test program are:

- Assess accuracy performance
- Identify and characterize major system and component error sources and verify math models of SD-53 system.
- Record inertial sensor outputs under field conditions for off-line studies.
- Assess the ability of the SD-53 system to meet its design objectives and provide design information as required.
- Determine operational characteristics of the SD-53 system.

An additional objective for the van test program is to prepare for the subsequent flight test phase. The detailed objectives are discussed next.

All of the proposed test procedures and subsequent data reduction recommendations in this report are designed to fulfill the major objectives presented above and the detailed objectives discussed below. The first major objective (to assess accuracy performance) consists of characterizing the errors associated with each of the following navigation parameters* as a function of time:

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*The precision with which the navigation errors are determined should be at least several times the accuracy of the SD-53 system and sufficient to provide design information should it be needed.

Geographic Coordinates*

- Position (La, Lo, H)
- Velocity (V_N, V_E, V_V)
- Attitude (A_H, A_P, A_R)
- Attitude rate of change $(\dot{A}_H, \dot{A}_P, \dot{A}_R)$

Tangent Plane Coordinates*

- Position (X_s, Y_s, Z_s)
- Position rate of change $(\dot{X}_s, \dot{Y}_s, \dot{Z}_s)$
- Attitude (A_X, A_Y, A_Z)
- Attitude rate of change $(\dot{A}_X, \dot{A}_Y, \dot{A}_Z)$

The accuracy of each of the above parameters should also be determined as a function of:

1. vehicle environment (e. g., vibration, rate and acceleration maneuver profiles, shocks, power supply variations, etc.). This includes performance evaluation while stationary in order to establish a base-line condition.
2. System configuration (e. g., specific navigation algorithms used, compensation applied, use of external fixes (position and/or velocity), ISU mounting arrangement, etc.).

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*Symbols are defined in the Glossary (Appendix H)

3. Quality of external fixes and references
4. Quality of initial alignment and calibration.

The quality of the initial alignment is merely the errors in the above navigation parameters at $t = 0$ and should be characterized as a function not only of the first three factors above, but also of the alignment procedure and alignment time. It is very desirable to demonstrate the navigation function and accuracy performance in real time (vs. using recorded sensor data), for at least one of the two coordinate frames shown above in order to provide as realistic operational conditions as possible, to minimize the risk of relying on intermediate test equipments and extra handling of data, and to provide real time monitoring of the system in the event of abnormal behavior.

The second major objective (to identify and characterize major error sources and verify math models of SD-53 system) is intended to suppport analytical studies of the system, to verify and establish confidence in the math models used, and to provide basic understanding of system error propagation characteristics. Such information is necessary not only to support current analyses but also to specify system configurations for the ultimate applications of the SD-53 inertial system and to provide creditable estimates of expected performance under conditions peculiar to each application. Since it is most likely that candidate systems for a given space mission must be evaluated and selected without having <u>demonstrated</u> performance in the actual application, trade-off studies must be based on <u>predicted</u> performance (which can be believed only to the extent that existing math models are able to predict performance in the van (and flight) tests). A by-product of this objective is to determine which calibration terms may be estimated in the van or aircraft and thereby provide a basis for possible field calibration.

The third major objective (to record inertial sensor outputs under field conditions) consists of digitally recording during each test the counts obtained from each SAP and PIGA before any compensation.* The recorded outputs are then to be used to perform off-line (i.e., post-test time) studies that include the following:

- Design and evaluation of algorithms (i.e., coordinate transformation matrix, velocity transformation, navigation, attitude transformation, initial alignment and calibration, and yaw monitor).
- Determination of the effect of and need to compensate for such errors as SAP misalignments, mass unbalance, OA angular acceleration error, etc. and PIGA misalignments, scale factor error, etc.
- Determination of system accuracy in navigation coordinate frames other than the one(s) used in real time.

The inertial sensor data should be recorded at least as frequently as the maximum anticipated update rate of the coordinate transformation matrix (CTM) and the velocity transformation operation, in order to fulfill all objectives.

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*Except for gyro bias torquing

The fourth major objective (to assess the ability of the SD-53 system to meet its design objectives and provide engineering design information) is intended to provide assurance that the system under test will meet the requirements of the final application(s) for which it is intended. Also that required engineering design information be obtained, as necessary, to continue development of the system. If improvements or changes are deemed necessary, it is desirable to have obtained sufficient engineering information from the test program to support hardware and/or software design changes as well as improvements in procedures and other operational characteristics. This objective is in contrast to that of determining the existing accuracy and operational characteristics of the system, whether they be better or worse than the design objectives.

The fifth major objective (to determine operational characteristics of the SD-53 system) is intended to accumulate operational experience with the SD-53 system in the following areas:

- Reliability
- Availability
- Repairability
- Reaction time
- Operability

Each of these terms is defined in the Data Reduction Section (6.4.6). It is also desirable to determine which inertial system calibration terms can be estimated in the van and in the aircraft. Since the van and flight test programs are largely evaluations of an engineering model of the SD-53 system, most operational characteristics observed during testing cannot be directly extrapolated to be the same as for a production model. However, the information is desired for the following reasons:

- To provide a basis for improvements, where necessary.
- To provide a more realistic prediction of characteristics in a given final application, when the SD-53 system is compared to other candidate systems.

The additional major objective for the van test program (to prepare for the flight test program) is intended to provide experience with the inertial and evaluation systems* under field conditions, to provide base line performance of the system, and to minimize the time that an aircraft must be made available, both for equipment installation and checkout and to perform the flight tests. A significant savings in time and effort** is anticipated since many of the equipment integration, checkout and calibration activities for the inertial and evaluation systems are common to both the van and flight test programs. Furthermore, it is intended that most test procedures and data reduction techniques be checked out as part of the van test program and that operational and data gathering problems be solved before installing the equipment in the aircraft. Checkout of computer programs and algorithms to be used during flight tests is also intended, as well as operation of the altimeter input and change in vertical position determination. Operational experience during the van test phase can indicate improvements to minimize flight test time and inefficiencies, including spares required, preventive maintenance, etc.

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*The evaluation system consists of all equipments and procedures used to evaluate the SD-53 inertial system.

**The U. S. Naval Air Development Center estimates the time and cost are reduced about 60 to 70% [7].

3. PHILOSOPHY OF SYSTEM EVALUATION

3.1 GROUND RULES

In addition to the objectives discussed in Section 2, there are several ground rules to be followed in specifying the field test program. Standard test instrumentation is to be used wherever possible, and there should be a minimum reliance on special equipments not readily available. Furthermore, the relative complexity of the evaluation system, equipments, procedures and techniques is to be minimized in order to increase the probability of obtaining usable data at minimum cost with flexibility to make changes, if required. For this reason, the initial flight test program will not be conducted on a specially instrumented test range.

A Minneapolis Honeywell 516 computer is to be used in both the van and flight test programs to process all the inertial sensor outputs and to act as a buffer in recording them on tape. The NASA Coordinate Transformation Matrix Computer (CTMC) [8] will at most be used in parallel with the 516 computer, since it still requires further development. Therefore, the test program discussed in this report will not consider the CTMC as part of the system at the present time.

Evaluation of pitch, roll and yaw angle accuracy, as indicated by the SD-53 system under dynamic inputs, is considered to be of secondary importance at the present time. Not only because of this is little attention to be given to the problem, but also the test equipment and related data collection and reduction functions are somewhat complex and would require not insignificant development efforts. Photographic methods to obtain heading to approximately 6 $\widehat{\min}$ (1σ) and attitude (pitch and roll) to 3 $\widehat{\min}$ (1σ) are described in [3]*, however considerable time and effort is required. Not attempt will be made to evaluate the attitude rate accuracy of the SD-53 system.

Although both digital and analog recorders are available, their characteristics are such that any requirement to accurately time synchronize data from the two recorders is to be avoided. However, time synchronization of data between channels from the analog recorder is less difficult and can normally be done with sufficient accuracy.

3.2 APPROACH

The approach described here provides a means to fulfill all objectives discussed in Section 2. The basic approach is to perform all functions that are required to align and navigate the SD-53 system under various conditions and to measure the errors in its indicated outputs, all in real time. The emphasis is on demonstrating the performance during operation (in contrast to post-test processing of recorded inertial sensor data). It is not recommended to use the latter approach to demonstrate system operation and performance since extra data handling is required,and recording problems or interrupts could cause a large increase in the test time required to obtain satisfactory data. It is desirable,

*In [9], a 1 $\widehat{\min}$ accuracy is claimed.

however, to record the inertial sensor data simultaneously with the computation of the alignment and navigation functions, if possible*, to minimize test time, to allow computation of the effect of certain error sources, to provide data in the event problems are detected, etc.

The field test programs have been developed such that most of the inertial and evaluation systems, equipments, procedures, and techniques used are the same for the van and flight test programs. Therefore, a good deal of the equipment checkout, integration and calibration activities required for the flight test program will already be accomplished as part of the van tests. Included in this category is a computer program to initially align the SD-53 system analytically, before entering the navigate mode. Although externally provided vertical and heading references can be used in the van [1, 5, 4], such an approach is not considered feasible in an aircraft. It is recommended that analytical alignment be developed and checked out as part of the van test program.

It is recommended that long term navigation performance of the system be demonstrated using a geographic (North, East, Vertical - NEV) coordinate system, and that short term navigation and guidance be based on a Tangent Plane** (TP) coordinate system. It is desirable to demonstrate performance in each coordinate system in real time, if possible; separately if not. If computer requirements and available test time and schedules are such that a choice must be made, it is recommended that

*If not possible, test runs should be repeated performing the recording and computation functions separately.

**Defined as the NEV coordinate system just prior to entering the navigate mode and nonrotating relative to inertial space thereafter.

NASA
FORMAL
REPORT

FFNo 665 Aug 65

long term navigation in NEV coordinates be demonstrated in real time for the following reasons:

- Equations for navigation in NEV coordinates will already be programmed since they are required for inertial (analytical) alignment, whether TP or NEV coordinates are used following the alignment.
- Navigation in NEV coordinates requires calibration of the gravity vector only in the vertical acceleration loop. This eliminates an error source regarding navigation in the horizontal plane, thereby improving the precision with which system long term performance may be determined. Although this is more critical during the flight tests, the programs should be checked out during the van tests.
- Operation in NEV coordinates provides feedback corrections (by way of the Schuler and earth loops) which tend to limit the growth of errors and assures more linear operation.

If the computer is unable to perform the navigation and alignment functions in real time, recorded inertial sensor outputs may be processed post-test as a last resort.

Evaluation of system accuracy performance is recommended on the following two levels:

- Direct measurement of system indicated outputs.
- Indirect estimation of system internal errors and error sources.

Direct measurement of vehicle position and velocity* is generally possible at discrete times. Also the ISU orientation when in the van can be measured directly using facilities in the NASA alignment building at MSFC. These direct measurements can also be used in various combinations to estimate certain errors and error sources within the inertial system,** using appropriate math models and estimation filters (and/or smoothers). Two such programs are discussed in Appendices C and E (viz., analytical alignment and reset). The effect of other error sources on the navigation and attitude errors can be estimated using the inertial sensor outputs directly (e.g., effect of mass unbalance, compliance, scale factor errors, etc.). It is significant to note that the above references are not based on inertial principles, thereby assuring an independent measurement of the SD-53 inertial system errors. If attitude and attitude rate errors were to be measured directly, a second inertial system (one already developed and proven) could be used. Attitude rate errors can also be measured using separate rate gyros.

It is considered important in the assessment of SD-53 system accuracy to determine the errors associated with the test equipment and evaluation system so that confidence levels on the test results can be ascertained. Similarly, the van and aircraft test environmental conditions

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*Generally, the velocity reference measures the average vehicle velocity over a short period of time and hence the inertial system velocity would have to be measured over the same period in order to make a direct comparison.

**e.g., gyro bias errors, attitude and relative heading errors, earth loop misalignments, etc.

should also be determined. This includes not only vehicle motions (maneuvers, vibrations and shocks), but also potentially significant errors in the inertial system support equipments (e.g., power supply variations, grounding problems, ISU temperature, etc.)

Special van routes, aircraft flight paths, and maneuvers can be specified to checkout specific error sensitivities or system functions. However, such tests generally involve supporting analyses and more extensive data reduction and so are considered to be of lower priority.

There are certain non-field special tests performed on a subsystem level, along with supporting analyses, that can be integrated with results from the field tests. Such tests and analyses are only briefly considered since they are more involved with possible uses of the field test data than with the field tests themselves.

It is considered important to maintain a test program log in order to correlate problems and unusual equipment behavior and results with significant events, conditions, etc. This can provide realistic inputs into the assessment of system operating characteristics. In addition, sufficient information should be made available during each run to assure usable data is being obtained. This implies the definition and utilization of run abort criteria.

4. SYSTEM CONFIGURATION AND REQUIREMENTS

This section describes both the SD-53 Inertial System and an evaluation system consisting of various test equipments, references and procedures. Figure 4-1 is a functional diagram of these two systems and shows the relationships involved. It will be noticed that not all functions need be performed in real time (i. e., when the inertial system is operating in the van or aircraft). Definitions of symbols used are contained in Appendix H.

Each of the major systems of Fig. 4-1 are described below in Sections 4.1 and 4.2. The remaining sections describe the various references that can be used to evaluate the SD-53 system.

For purposes of test design, certain assumptions will be made concerning errors associated with the inertial and evaluation systems. Assuming the inertial system is near the upper limit of the state-of-the-art of aircraft systems, a 1/3 n. m. per hr. buildup of RMS position error with time is compatible with the following inertial system error sources:

gyro bias:	$\sim$ 4 mdh (millidegrees/hr)
random* gyro drift:	$\sim$ 4 mdh, 3 hr correlation time
accelerometer bias:	$\sim 10\ \mu g$
random* accelerometer drift:	$\sim 5\ \mu g$, 2/3 hr correlation time

*Random drifts are assumed to be Markovian processes with RMS error and correlation times as shown.

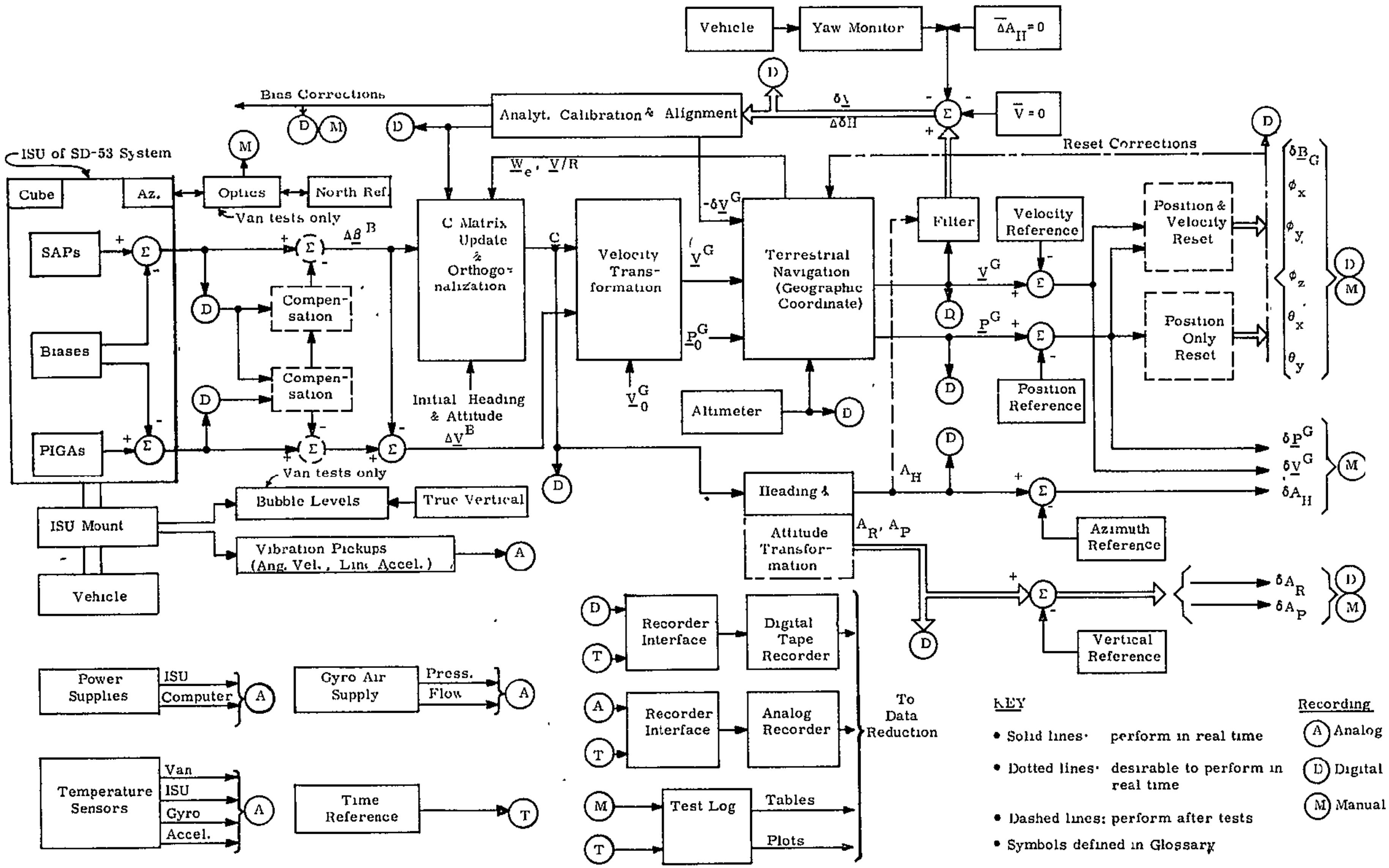

Figure 4-1 Functional Diagram of SD-53 and Evaluation Systems

initial vertical error: 1-2 sec

initial heading error: < 6 sec (with optics); ~30 sec La sec (using analytical alignment)

initial velocity error: < .1 kt.

initial position error: < 50 ft.

The corresponding RMS velocity error is approximately .5 kt with a superimposed Schuler error of about .1 kt. The above numbers are based on a simplified preliminary analysis shown in Appendix E. 2, and in no way are they to be interpreted as reflecting actual performance of the SD-53 inertial system. They are intended merely to aid in the initial design of the test program and specification of the evaluation system. It is desirable that the evaluation system measure the above quantities with a precision at least as good and preferably several times better than the errors shown.

4.1 SD-53 INERTIAL SYSTEM

The basic element of the SD-53 Inertial System is the Inertial Sensing Unit (ISU) and associated support equipment. The ISU is described in [10] and consists of three Single Axis Platform (SAP) gyros* and three Pendulous Integrating Gyro Accelerometers (PIGAs) all mounted rigidly to a support structure. The ISU also contains associated electronics, temperature systems, etc., as well as an optical cube and an azimuth porro prism. The SAP input axes (IAs) are mutually perpendicular and form equal angles with a normal to the ISU base, and the PIGA IAs are nominally parallel to the SAP IAs. The associated support equipment

*Currently gyros using output axis air bearings are installed in the SAPs. Although acceptable for short term applications (say < 15 minutes), the air requirements would be excessive for long term applications. However, for the van and flight test programs this is not a serious limitation.

includes power supplies, air supplies, air conditioning, monitoring and operating equipment, etc.

The other major element of the SD-53 Inertial Navigation System is a digital computer. For the van and flight test programs, a Minneapolis Honeywell 516 computer will be used to process the inertial sensor outputs. The functions to be performed are as follows:

- Generation of the coordinate transformation matrix (perhaps using a four parameter, third order algorithm).
- Velocity transformation into the stable coordinate frame.
- Generation of navigation parameters (latitude, longitude, vertical position, velocities North, East and vertical)*.
- Euler angle attitude transformation (roll, pitch and heading relative to North)*
- Analytical alignment (using position and velocity as references, with option to include measurements of the change in heading error**).
- Compensation for SAP and PIGA drift rates and PIGA angular rate input, and possibly other errors as well (see below).

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*The parameters listed are for a geographic NEV coordinate system.

**If it is found necessary to measure the change in vehicle heading, a separate "yaw monitor" may be required. This is considered in more detail in Section 4. 5.

The computer program must also be capable of accepting certain data inputs and instructions from the operator, as well as display navigation data outputs, system status, key parameters, certain groups of data, etc. Computer programming considerations are included in Refs [4, 8, 11]. The 516 computer may also be programmed to perform some evaluation system functions, as discussed in the next section.

The only external reference required to perform the basic navigation function is altitude. An altimeter is essential to provide accurate vertical position change and velocity over the long term since the integration of vertical acceleration would be open loop otherwise. It is recommended that the altimeter be provided for the van tests as well as flight tests so as to check out the system.

Position and/or velocity references are not required while navigating, although inertial system accuracy can be improved significantly if corrections based on such references are applied periodically. For the basic test program, it is recommended that the navigation program be capable of accepting position, velocity, heading and attitude corrections at discrete times. A second phase of the test program may include a mechanization option in which the Schuler loops are damped using an external velocity reference.

The primary purpose of the analytical alignment algorithm is to initialize the coordinate transformation matrix prior to entering the navigate mode (i. e., drive to null the heading and vertical tilt errors of the stable coordinate frame*), as well as provide initial position and velocity. Generally, initial position is no problem; however, if the vehicle is being

Assuming a geographic NEV coordinate system vibrated and/or buffeted during alignment, the instantaneous velocity may be much larger than the allowable initial velocity error and so cannot be set directly to zero (assuming the vehicle to be "stationary"). A properly designed alignment algorithm, however, can provide an accurate initialization of velocity. A by-product of the recommended alignment process is to provide SAP gyro bias error estimates for each SAP. Estimation of the vertical component of the SAP bias errors is improved significantly (in time and precision) by periodically measuring the <u>change</u> in indicated heading. If the <u>change</u> in actual heading can be assumed to be zero, (or actually measured, as discussed in Section 4.5 below), the change in heading error can be determined and used by the alignment algorithm. A discussion of the procedure and the expected accuracy is included in Section 5.4, as well as an alternate method of alignment (viz., physical alignment). It should be noted that compensation for SAP (and PIGA) bias drifts can be made in the 516 computer by adding counts at the proper rate to the counts received from the ISU.

It is recommended that compensation for SAP and PIGA gyro bias errors and PIGA angular rate input be provided as a minimum. Preliminary analysis also indicates that compensation for SAP misalignments should be provided. For example, a 180° turn with a SAP nonorthogonality (δ) of 60 $\widehat{\sec}$ would cause an error ($\delta\theta$) in the SAP of:

*This assumes alignment that includes a 90° rotation of the vehicle, as discussed in Section 5.4. If in an actual application the vehicle cannot be rotated, the strapdown system could be mounted on a rotatable table.

$$\delta\theta = \frac{180°}{57.3}\,\delta$$

$$= 3.14\,\delta$$

$$= 188\ \widehat{\sec} \qquad (4.1\text{-}1)$$

Errors of this order of magnitude are probably not compatible with the accuracy potential of the rest of the system, as well as not being acceptable for many applications. Compensation for gyro g-sensitive drift may also be required [4. 12].

Another source of error for which compensation may be required is the sensitivity of gyros to angular accelerations about their output axes (OAs) [4, 12]. The sensitivity is proportional to the gyro I/H ratio (i. e., float inertial to wheel angular momentum ratio), and is shown in Appendix A to be potentially critical when components of the angular motion about the gyro input axis (IA) are synchronized with components about the gyro OA*. Such a condition may exist in an environment of angular vibrations, and any compensation would have to be performed more often than the compensations discussed above. Components of the motion about the OA that are not synchronized with components about the IA can produce relatively large apparent input rate errors; however, the resulting navigation errors are generally small. The OA acceleration sensitivity can also produce pseudo-coning effects, which should be analyzed for the van vibration conditions expected (or measured). Refs. [13,

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*In this case a rectification effect occurs and the SAP gyro physically drifts about the SAP IA.

12, 4, 1] consider these errors as well as other potential error sources, including coning and orthogonality errors.

If any compensations are to be provided, they should be incorporated in the design before system test data is taken in order to evaluate the system as close to its final configuration as possible (see Appendix F. 2).

4.2 EVALUATION SYSTEM

The evaluation system consists of the following five major elements:

- Van (or aircraft)
- Test equipment within the van (or aircraft)
- References to which the inertial system outputs are compared
- Test procedures
- Data reduction

The first two subsystems are described in this Section (4. 2), and the recommended references are discussed in Sections 4. 3, 4. 4 and 4. 5. The last two items are considered in Sections 5 and 6, respectively.

A picture of the van is shown in Fig. 4-2. The vehicle is shown parked in the alignment building on the alignment table* and the opening in the right side of the van for viewing the SD-53 system azimuth porro prism

*A fluid floated, floor level rotatable precision table capable of supporting the van with a stability of a few arc seconds.

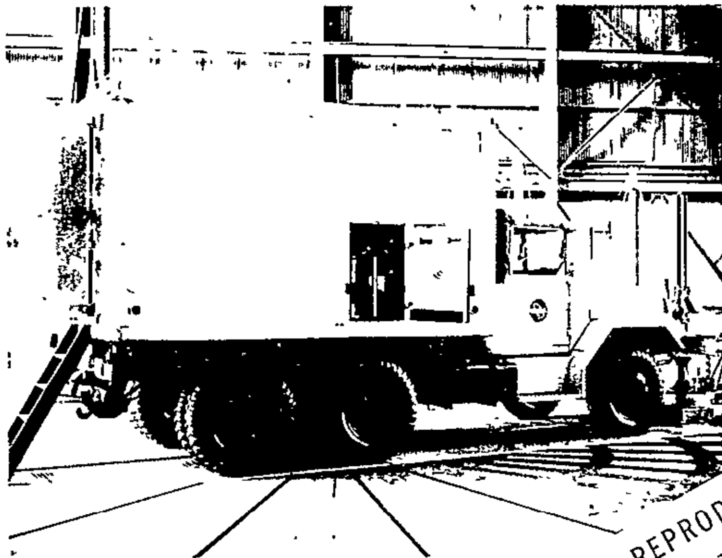

Figure 4-2 Photograph of Van (without Generator Trailer)

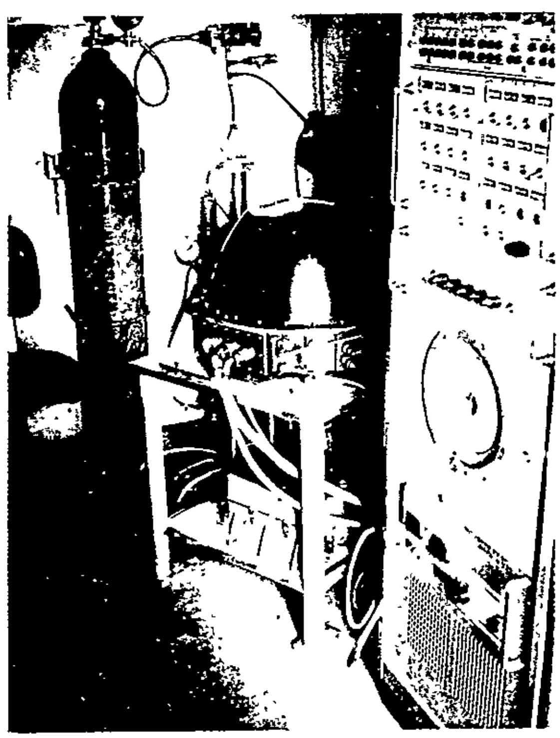

Figure 4-3 Photograph of SD-53 ISU and Some Support Equipment (Inside Van)

is shown. The cargo space of the van is air conditioned. Not shown in the picture is the auxiliary generator that will be towed behind the van. The generator provides regulated 208V, 3 phase power as well as regulated 28V dc. During the initial road tests, van speed was limited to approximately 20 mph due to lateral oscillations of the 4-wheel generator trailer. Vibrations of the van would be expected to be lower at this speed (based on results from other van test programs [1, 11]), and the motion of the trailer will cause significant motions and possibly nonrepresentative effects. Although the van/trailer has been driven at higher speeds more recently with no such oscillations, it is recommended that the van vibration measurement tests (described in Appendix B) be run and analyzed to determine whether or not any changes in the van/trailer configuration are desirable.

Figure 4-3 is a picture of the SD-53 system Inertial Sensing Unit (ISU), supporting structure, air supply and support console, as mounted in the van. The ISU structure is rigidly mounted to the floor of the van, and a fixture on the structure top provides limited (~1°) adjustment of the ISU. Bubble levels and an azimuth mirror mounted on the fixture are calibrated to the ISU optical cube such that one of the SAP axes can be aligned parallel to the earth's polar axis within a few arc-seconds (the SAPs are about 36° from the horizontal, which is approximately the latitude at Huntsville). This is done by physically jacking the van and turning the alignment table until the bubble levels are at null and a theodolite system indicates the desired angle between the azimuth mirror and a true North reference in the alignment building.

There are three linear and three angular vibration sensors mounted on the ISU base, and a FM Analog tape recorder system is available to record the data. Appendix B considers requirements of the vibration pick-up and recording system. During the tests, it is desirable to be able to monitor any signal being recorded. Test procedure and data reduction considerations are discussed in Sections 5. 4, 5. 5 and 6. 4. 5 respectively.

It is recommended that the analog tape recorder system be capable of receiving certain critical analog signals, such as power supply voltages, temperatures, etc. This is discussed more fully in Section 5. 1.

The Minneapolis Honeywell 516 computer is considered part of the evaluation system (as well as of the inertial system) since it performs certain evaluation functions as described in Section 6. In conjunction with obtaining velocity fixes, as described in Section 4. 4, the computer must be capable of accepting discrete inputs for controlling the averaging (or filtering) of the position fix taking process, as discussed in Section 4. 3.

In conjunction with the recording of the inertial sensor data, the computer is currently intended to act as an interface between the SAP and PIGA outputs and the digital tape recorder. It is planned to accumulate SAP and PIGA counts over 10 ms periods and form 40 bit words in the computer at the rate of 240 words/sec. When 2000 of these words have been formed, they are recorded on 2400 ft. reels with a density of 556 bits/inch. At the above rate, data can be recorded for approximately 27 minutes. Although this is adequate to assess short term performance, other arrangements would be necessary to record data for the long term (e. g., 4 to 6 hours). Preliminary calculations indicate that fewer bits per word may be adequate for the inputs expected and hence longer runs can be made. However, to record for 3 to 4 hours (or longer), a second tape handler and associated switching may be required.

In the flight test program, additional equipments are required in the aircraft to obtain position (and possibly velocity) fixes. These are discussed in Sections 4. 3. 2 and 4. 4. 2 below. Consideration should be given to including some of these equipments in the van in order to detect and correct equipment integration problems and to determine error characteristics of the equipments.

4. 3 POSITION REFERENCES

4. 3. 1 Van

The position reference used in the van test program consists of surveyed checkpoints alongside the road. There are two different precisions required to assure satisfactory results. For short term navigation (say <10 to 15 minutes), the change in position from the starting point is of interest, and the precision requirement of the reference points is high. Surveyed position checkpoints with errors less than about 25 ft., relative to the starting point, should be adequate and are considered feasible [5].

For long term navigation (up to 4 to 6 hours and more), the absolute position of the vehicle relative to the earth's coordinate frame is of interest, and the precision requirement of the checkpoints need not be as good as that above. Measurement of vehicle position within 5 to 10% would probably be sufficient to evaluate position accuracy of the inertial system. However, to initially align the system, absolute position errors of several sec (which is equivalent to several hundred feet) would cause errors comparable to the PIGA error. A more stringent requirement is to be able to measure the inertial system position error (on an absolute basis) to about 50 - 100 ft, in order to estimate errors internal to the inertial system. The technique to do this is called "Reset" and is described in Section 6 (Data Reduction Requirements) and in Appendix E. U. S. Coast and Geodetic Survey Charts may be adequate for this degree of precision, since accuracy of the charts is about 50 to 200 ft. [7].

Assuming a pressure altimeter will be used as the altitude reference in the flight test program (vs. a radar altimeter), van tests should include checkout of the instrument and its interface with the inertial system, as well as performance of the total altitude and vertical velocity system. Assuming a Mark 2 or 3 altitude computer system is used [14], altitude error should be less than approximately 25 to 75 ft. (95% of the time). In Ref. [18], 1σ errors of about 25 ft. to 50 ft. are considered possible.

4.3.2 Aircraft

There are a number of references that can be used to determine the position of an aircraft. Since the basic flight test program will not be conducted on a specially instrumented test range, references such as precision radars and tracking theodolites are not assumed to be available.

The most accurate position reference consists of surveyed checkpoints at the runway and landing the aircraft to make the comparison. Although this is acceptable for certain tests, generally the desired maneuver, shock and vibration acceleration profiles preclude widespread use of such a reference. However, the reference can be used at the end of each flight.

The next most accurate reference is obtained from airborne photographs of known checkpoints on the ground. This technique is used extensively [3, 6, 7, 15, 16, 17, 9] and has an expected precision of 20 to 100 ft.*, when a good quality, stabilized camera system is used,** and the aircraft altitude is less than about 5000 ft. The possibility of using a hardmounted camera instead of a stabilized one may help reduce costs. Compensation for aircraft attitude could be based on the indicated roll and pitch outputs of the inertial system. For example, an alignment error between the ISU and the camera system*** of 10 min (the verticality spec of the camera system used at the Central Inertial Guidance Test Facility at Holloman AFB [16], would cause a 15 ft. error at 5000 ft., in addition to other errors. Additional analyses would be required to develop such a hard mounted camera system, as referred to in Appendix F. 1.

For these flight paths that can be constrained to operation near airports, fixes can be taken at low altitudes and speeds by flying over the center line of the runway and actuating a mark switch at a known position along the runway (assuming the required approvals can be obtained). The same technique can be used for any known landmark (such as crossroads, towers, etc.), again subject to approval of local authorities. A variation of the method would use the outer and/or middle markers of an ILS system (after having determined their accuracy by say, photographic fixes).

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*150 ft. is reported in [17], and in [9] an accuracy of 20 ft. at 40,000 ft. is claimed.

**Assuming photos are not taken soon after turns, since transients in the camera system vertical normally occur due to the velocity changes.

***The inertial system error in indicating the attitude of its optical reference (OR) should be small compared to the ISU alignment to the camera system (viz., less than 10 sec physical misalignment between the gyros and the OR, and less than 1 to 3 min in the attitude computation, depending on the algorithm used).

Accuracies of 150 to 600 ft. may be obtainable, depending on the equipment used and degree of calibration [18]. Some of the newer landing systems (such as the Scanning Beam Microwave system) can provide position information anywhere in the vicinity of the airport (say out to 25 miles) with distance accuracy of 75 to 100 ft. (1σ), bearing to 2.5 to 6 ft/nm and elevation to about 3 ft./nm [18, 19].

Somewhat more flexibility in flight path planning is possible by using fly-overs of VOR stations [7]. Fixes can be taken relatively frequently (every 10 to 20 minutes or so). Although lateral errors should be small (from 20 to 35 ft. per 1000 ft. from the station [18, 20],. further studies (or tests) are required to determine longitudinal accuracy.

Essentially unlimited flight operations are possible by using one or more long-range radio navigation aids (such as Decca, Loran, satellites Omega, etc.). Typical accuracies of Decca range from 200 ft. (1σ) [1] to .1 to 25 nm [15]. For Loran C, accuracies of .25 nm are reported [3, 15] and Omega is even less accurate [20]. In [21], satellite position accuracy is reported as about 500 ft. Shorter range navigation systems based on various combinations of bearing and distance measurements to one or more ground stations include VOR (VHF omnirange radio), radar beacons, TACAN (Tactical Air Navigation that uses UHF radio signals), and VORTAC (a combination of VOR and TACAN). The bearing errors are given in Table 4-1 and the distance measurement error ranges from 150 to 600 ft. (1σ) [18, 20]. A brief description of the various radio aids (including an extensive bibliography) is contained in [23], and error models for VOR, DME, TACAN and Decca are included in [24]. A more detailed analysis of VOR and DME errors is included in [20].

System	Bearing Error
Precision VOR	3' to 13'/1000'
Present Modern Equipment	25'/1000'
Conventional Equipment	35'/1000'

Based on data from [18]. The 3'/1000' figure is from [22].

Table 4-1 Accuracies of VOR

Altitude error from the inertial/altimeter system can be checked using the ILS system (or Scanning Beam Microwave system, if available). A radio altimeter can also be used to provide an independent measurement. In [25], accuracy of 10 to 30 ft. is implied. In [9], a laser altimeter is described with accuracy better than 5 ft. (for altitudes from 2000 to 6000 ft., depending on reflectivity of ground).

4.4 VELOCITY REFERENCES

4.4.1 Van

The most accurate velocity reference (relative to the earth) consists merely of stopping the van. However, because of van engine vibrations, wind buffeting, movement of personnel, etc., the inertial system indicated velocity should be averaged (or filtered) to make a more precise measurement of inertial system velocity error. Preliminary calculations indicate that averaging for 36 seconds should be quite adequate for the more precise requirements (viz., analytical alignment and reset).* The analyses in Appendices C and E indicate acceptable performance using a .02 ft/sec precision in measuring the velocity error.

It is possible to measure the inertial system velocity error while moving, by measuring the time to go past two landmarks alongside the road. If the van speed is 25 mph, and 36 seconds of averaging is acceptable, a distance of 1300 ft will be traversed. The resulting precision in measuring the velocity error equals .04 ft/sec per ft of error in knowing the distance between the landmarks and .14 ft/sec per 0.1 seconds in timing the passing of each landmark (for initializing velocity averaging of the inertial system output). Such a technique is feasible, but does require a timing scheme and accurate survey of the checkpoints. Velocity should be measured to at least .1 to .15 ft/sec to evaluate the velocity accuracy of the inertial system (exclusive of the alignment and reset requirements of about .02 ft/sec -- see above).

A second method of measuring velocity of the van relative to the road is by means of a "5th wheel" towed by the van [4, 5]. One or more pulses are generated for each revolution of the wheel, and the indicated velocity is given by:

$$V = \frac{C}{\Delta T} \qquad (4.4\text{-}1)$$

where C is the circumference of the wheel and ΔT is the time to go through one revolution of the wheel. A clock interrogated by the wheel

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*In [5], velocity is averaged for 10 sec.

pulses and knowledge of the wheel circumference are sufficient to determine the average van velocity over the interval between pulses. In essence the 5th wheel is a distance measuring device and velocity is determined by differentiation. The components of the vehicle velocity (in NEV coordinates) may be determined by using the inertial system heading, pitch and roll indications to resolve the 5th wheel indicated velocity.

The major errors in the technique are due to bounce of the wheel, uncertainties in the effective circumference of the wheel, and timing accuracy. Exclusive of wheel bounce, a .1 ft./sec precision at 40 kts (= 67.6 ft/sec) is compatible with a .1% measurement of the effective wheel circumference (say 1/8" for a 3 ft. diameter wheel) and a .1% measurement of time (say .14 msec per .140 sec for each revolution of the wheel, at 40 kts.). Since velocity may be averaged over considerably longer intervals, with little loss in system evaluation precision, the timing accuracy can be relaxed proportionately. To achieve .1% precision, the inertial system heading error and alignment of the 5th wheel to the inertial system should be within 3.5 min. This would require careful alignment and calibration of the inertial system, good mechanical alignment and stability of the wheel (e.g., .018" for a 1 1/2 ft. wheel radius), and an accurately navigating inertial system. Pitch and roll errors are not as critical.

4.4.2 Aircraft

The most accurate velocity reference (besides landing the aircraft)* that relies on the inertial system to a minimum is a doppler radar. The

*Velocity as well as position fixes at the beginning and end of each flight are recommended, as discussed in Sections 5.4 and 5.5.

velocity outputs are generally resolved through the indicated aircraft heading to obtain the North and East components of velocity, which are then compared to those from the inertial system. Accuracies from 1.5 to 3.0 ft/sec over land have been reported [3, 18]. The accuracy is degraded over water depending upon movement of the water and surface condition.

Accurate position fixes can be used in conjunction with a Kalman filter, as discussed in Appendix E, to estimate the inertial system velocity error. This approach has been discussed in the literature [26], and appears to be attractive (to the extent the system can be modelled and knowledge of high frequency velocity error variations is not required). Preliminary analyses summarized in Appendix E indicate that velocity error can be estimated within .4 ft/sec using position fixes every 24 minutes (with an accuracy of 240 ft.). Smoothing of the data, rather than filtering, can increase the precision even more, as shown in Appendix E.

Accurate position fixes can also be used to make an independent measurement of aircraft speed* which can then be compared to the filtered inertial system velocity over the same time period [16]. Two or more accurate photographic fixes would probably provide the most accuracy. The basic error equation for averaging using two position fixes is derived from the simple relationship

$$V = \Delta D/\Delta T \qquad (4.4\text{-}2)$$

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*Filtered over a period of time

to obtain

$$\frac{\delta V}{V} = \frac{\delta(\Delta D)}{\Delta D} - \frac{\delta(\Delta T)}{\Delta T} \tag{4.4-3}$$

where the prefix δ denotes error and ΔD refers to the distance travelled by the aircraft over the time ΔT, at an average velocity V. The sensitivity of the method to position and timing errors ($\delta(\Delta D)$ and $\delta(\Delta T)$, respectively) is illustrated by assuming an aircraft speed (V) of 200 kts and an averaging time (ΔT) of 1 minute. The corresponding distanced travelled is 3.3 nm. Considering ΔD and ΔT to be determined from two measurements for each quantity, and assuming the measurement errors to be uncorrelated random variables, the resulting precision in measuring the inertial system velocity error is determined from Eq. (4.4-3) to be .024 ft/sec per ft. of position measurement error and .781 ft/sec per 0.1 seconds in timing each fix. The best precision available (without special test range instrumentation) would require good quality airborne photographs. Assuming a position measurement accuracy of 20 ft., as discussed in Section 4.3.2, the average velocity error over 1 minute would be about .5 ft/sec (assuming the effect of the timing error would be negligible).*

There are other methods of measuring the speed of the aircraft accurately [3, 16, 15], including the use of tracking radars, cinetheodolites, and transponders. However, they require flights over

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*To obtain a 20 ft. position accuracy at 200 kts, the time error in the photo fix would have to be less than about 1/50 sec, which at .78 ft/sec per .1 sec would cause .16 ft/sec velocity error (a negligible amount).

specially instrumented test ranges, and therefore they will not be discussed in detail here*.

4.5 OTHER REFERENCES

Bubble levels are used as a vertical reference in the van test program during physical alignment of the ISU. To be compatible with the accuracy potential of the PIGAs, they should be accurate to several arc-seconds.

The azimuth alignment system** used in the van test program need not be quite as accurate; however, for some applications it may be desirable to align to 6 $\widehat{\text{sec}}$. For long term navigation, alignment to 30 $\widehat{\text{sec}}$ is probably acceptable, assuming gyro bias and random drifts about 4 mdh each.

It is recommended that the analytical alignment technique utilize measurements of the change in inertial system heading error ($\Delta\delta A_H$), both in the van and aircraft test programs. The measurement is made as follows:

$$\begin{aligned}\Delta(\delta A_H) &= (A_{HI} - A_{HT})_2 - (A_{HI} - A_{HT})_1 \\ &= (A_{HI2} - A_{HI1}) - (A_{HT2} - A_{HT1}) \\ &= \Delta A_{HI} - \Delta A_{HT} \qquad (4.5\text{-}1)\end{aligned}$$

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*According to the ground rules (Section 3.1), the initial flight test program will not be conducted over specially instrumented ranges.

**I. e., azimuth mirror calibration to ISU cube, optical alignment theodolites and survey of the North benchmark.

where subscripts I and T refer to indicated and true, respectively, and 1 and 2 refer to the two different times at which the measurements are made. If the change in the true heading of the vehicle is negligible* (say $<20\ \widehat{\sec}$), the change-in-heading reference may be considered to be zero (i.e., $\Delta A_{HT} \cong 0$). However, if it cannot be assumed that the vehicle heading is not changing (even when stopped on the road or in a parked aircraft), a separate yaw monitor may be necessary to measure ΔA_{HT}. Note that no absolute heading measurements are required, only the change. Preliminary studies of the analytical alignment technique (see Appendix C) indicate that a white noise of 20 $\widehat{\sec}$ can be accepted, and that possibly larger vehicle motions may be tolerated, even after filtering.

A yaw monitor could be implemented in many ways. However, conceptually a technique of measuring the change in lateral displacement of the vehicle at two different points, relative to the ground (while parked), can provide the desired information. For example, measurement of lateral displacement to .016 inches at two points 20 ft apart on the vehicle would provide the following precision:

$$\delta(\Delta A_{HT}) = \frac{.016 \times 2}{20 \times 12}(57.3)(3600)$$

$$= 19.5\ \widehat{\sec} \qquad (4.5\text{-}2)$$

Actually, a larger error can be tolerated since multiple measurements over the 36 second interval can be averaged (or pre-filtered) before being processed by the analytical alignment algorithm. A more detailed study would be required to establish more firm requirements.

*Based on preliminary studies of the analytical alignment technique (Appendix C).

5. PROCEDURES

The purpose of this section is to discuss the various procedures that are recommended to obtain the required test data. The procedures are in accordance with the approach outlined in Section 3 and are designed to provide the necessary information for fulfilling the objectives of Section 2. The intent of this section is to lay the groundwork and provide a basis for generating the detailed procedures at a later time.

A second test phase is referred to for the purpose of identifying certain tests that may be performed depending upon the outcome of the initial test phase. In addition, several special tests that are recommended in support of the field test program are listed in Appendix D. Since they are not directly a part of the field tests, they are given only nominal consideration here. Second phase tests and special tests are referred to explicitly: all other tests are considered to be part of the basic test program.

5.1 TEST DESIGN, DATA COLLECTION AND DISPLAY CONSIDERATIONS

It is important that each test be designed to provide statistical significance to the results obtained, either based on data from the individual test or in conjunction with data from other tests. To this end, the accuracy objectives discussed at the beginning of Section 4 (System Configuration and Requirements) are intended to be used as a guide in designing accurate tests. Each test should be designed with explicit consideration of specific objectives to be achieved and how the resulting data is to be processed. Consideration of test conditions, particularly changes from those of other tests, is important as well as specification of the number of runs to be made (to obtain statistical significance) and the length of each run. In certain cases it may be desirable to design the tests so as to separate the effects of measurement repeatability from system or component stability. In all cases, rules for aborting runs should be specified in order to minimize test time, to facilitate the analysis of operational characteristics and to achieve a maximum of usable results. Procedures for monitoring system operation are necessary*, and must be correlated with available system outputs, displays, and computer program specifications [16, 11]. If performance is excessively erratic or unexplainable, component and/or system level tests in the lab. may be necessary. Several references relative to test design considerations are [4, 5, 1, 2, 27, 3, 16, 17].

It is strongly recommended that a comprehensive log be maintained during the entire van and flight test programs and that it be keyed to real time and include at least the following:

- Identification of system** configuration for each test, including references used (viz., position, velocity, etc.), modes, initial conditions and instrument settings, etc.
- Changes of system** components or equipments that may affect system performance.
- All significant events that may affect system** accuracy and operation.
- All erratic system** behavior, failures, time to diagnose repair and return of equipment to on-line operation.
- Tabulation of all significant test conditions during each test.

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*For both normal and abnormal behavior.

**"System" as used here includes both the SD-53 Inertial system and the evaluation system.

Faithful maintenance of a comprehensive log can make the difference between efficient data taking, cataloging, and troubleshooting vs. unnecessary repetition of runs and abandonment of potentially useful data already taken. Included as part of the significant test conditions is a characterization of the "motion profile" of the inertial system. The low frequency aspects consist of vehicle maneuvers and ground track, whereas the high frequencies are due to linear and angular vibrations of the vehicle and the inertial system. The low frequency characteristics are specified by the detailed test procedure for any given run. To characterize the van vibration environment, it is recommended that a series of tests be performed as specified in Appendix B. It is also recommended [16] that the analog recorder be used to routinely record and monitor certain critical quantities that could adversely affect system operation should they become faulty (e. g., system power supplies, ISU and vehicle internal ambient temperaturs, gyro air bearing supply pressure and flow rate, etc.). Such monitoring could be most useful should problems or erratic operation develop. It would also be desirable to record the vibration sensors outputs during all tests, if possible, to provide reasonableness checks on the PIGA and SAP outputs, as well as to monitor input motion to the vehicle. Again, if performance is excessively erratic or unexplainable, lab. tests may be necessary.

Other data collection requirements have been discussed in Sections 3 and 4.

5.2 EQUIPMENT INSTALLATION AND CHECKOUT

Before navigation performance tests can be initiated, it is necessary to perform equipment installation tests in both the van and aircraft. This includes debugging, grooming and functional checkout of all equipments (including power supplies and other support equipment), integration of equipments, support and test equipment alignments and calibrations (as necessary), and preliminary performance tests to checkout procedures and data gathering operations. Included in this phase are installation type tests on the SD-53 inertial system, including gross operation of the system in the alignment and navigate modes (both stationary and moving) under conditions anticipated during the final test phase.

Calibration of the evaluation system equipments is important in order to realize the precision potential of the equipments and better evaluate the inertial system. Procedures to do this are beyond the scope of this study, however, the accuracy goals are as discussed in Section 4. Sufficient tests and data should be specified to determine the error characteristics of critical evaluation system equipments (including the optical alignment equipment and associated azimuth surveys, surveys of each checkpoint on the road, surveys of checkpoints used in the flight test program, aircraft position fixing using photographs, digital data recorder system, and the vibration pickup and recording system). It is recommended that certain equipments be recalibrated periodically, including the bubble levels and azimuth mirror on the ISU support fixture, recorder systems, optical equipment, etc.

5.3 PRE-NAVIGATE SD-53 SYSTEM CALIBRATION PROCEDURES

Although some calibration terms can be determined with the system in the vehicle, most terms are best determined using the laboratory test stand. Laboratory tests should be done periodically to check calibrations in the vehicle and to detect and/or verify abnormal situations should they occur.

A by-product of the pre-navigate alignment of the system in the vehicle, which is described in the next section (5.4), are estimates of the three SAP apparent gyro bias errors. Actually, the estimates are the sum total of error sources such as mass unbalance, compliance, misalignments, OA acceleration error (if the vehicle is vibrating), as well as constant SAP gyro drift. To the extent that these terms remain constant, the bias corrections applied to the system are proper. Appendix C contains a preliminary analysis of the analytical alignment procedure, where it is shown that the gyro bias estimation errors are expected to be approximately 4 mdh for the level components of the gyro drifts and 4.5 mdh for the vertical component. These values are essentially the same as the assumed random gyro drift rates, and can be interpreted as being about the best estimates possible in the time taken (viz., 36 min.). Since the assumed gyro correlation times are 3 hours, alignment times of that order would be required to estimate the true bias error. It is recommended that the SAP gyro bias corrections estimated by the analytical alignment algorithm be applied by the 516 computer, as described in Section 4.1.

Additional calibration terms may be estimated with the system in the van by utilizing the bubble levels on the ISU mounting fixture and the optical North reference in the alignment building, as described in Section 4.2. With the orientation of the ISU cube known precisely (within several arc seconds) relative to the earth's rotation and gravity vectors, the apparent bias errors of the PIGAs can be estimated to approximately that accuracy. Actually, the estimates are the sum total of errors due to misalignment and scale factor errors, as well as constant PIGA gyro drift. Review of post-test data can establish stability characteristics of the terms and determine which terms are more likely to change. The technique can also provide a fairly direct measure of SAP gyro apparent drift.

It is recommended that tests be defined to check the calibration portion of the analytical alignment algorithm. This is further considered in the next section. Once the algorithm is proven to perform properly, it may be used to calculate calibrations and alignments on the road and in the aircraft.

It is likely that a more detailed investigation could establish procedures to determine more calibration terms while the system is in the vehicle.

5.4 PRE-NAVIGATE SD-53 SYSTEM ALIGNMENT PROCEDURES

The first step in the pre-navigate alignment process is to zero all SAP and PIGA counters. This is recommended so that the individual gyro OA/SA orientations relative to the ISU cube axes will be known (in case significant sensitivities exist) and to provide controlled initial conditions for all tests. The PIGA counters are easily initialized since each PIGA IA is nominally 36° above horizontal, and consequently each PIGA gyro is always rotating (at 11.95°/sec per g input, less than 1 minute is required to go through a full revolution and pick up the zeroing pulse). The SAP counters are also easily initialized (following turn-on of the system) by merely rotating the vehicle through several rotations until the zeroing pulses are picked up.

There are two procedures that can be used to initially align the inertial system just prior to entering the navigate mode. The first procedure is more applicable to the van test program and is referred to as "physical alignment". It utilizes bubble levels mounted on the ISU support fixture along with an azimuth mirror that is referenced to true North (see Fig. 4-1). The van is driven onto a fluid floated alignment table in the alignment building (see Fig. 4-2) and physically jacked until the bubble levels read nulls. Next the table is rotated until the desired ISU azimuth mirror angle relative to North has been achieved, as indicated by a theodolite system. The bubble levels and azimuth mirror have been previously calibrated relative to the ISU cube such that when the alignment is completed, one of the SAP IAs is parallel to the earth's polar axis. Sufficient information is then available to initialize the Coordinate Transformation Matrix (CTM) and bias all three gyros. Although the method can be used on the road (with reduced accuracy) [1, 5, 4], it is not considered practical for the flight test phase due to excessive aircraft motions.

The second method is applicable to both the van and aircraft test programs and is referred to as "analytical alignment". It utilizes a computer algorithm that processes the PIGA outputs (after transformation through the CTM) to determine the alignment parameters (viz., the elements of the CTM). A velocity reference is required during the process* (see Fig. 4-1). The algorithm also provides estimates of initial velocity error and gyro bias errors, as discussed earlier. The procedure consists of initially inserting coarse estimates of vehicle pitch, roll and heading into the computer. Following a brief period of coarse alignment, the system enters a fine align mode which establishes the system vertical and heading (via gyrocompassing). The primary component of the heading error will most likely be the East component of gyro drift. The algorithm also provides an accurate estimate of the North gyro drift, a less accurate estimate of vertical gyro drift, and no information on the East gyro drift. Initial

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*In addition, it is recommended that change in heading error be measured in order to obtain a more accurate estimate of the vertical component of gyro drift.

studies have been based on an alignment time for this phase of about 15 min. Although the system has been aligned at this point, the initial heading error can be reduced significantly (and the other level component of gyro drift estimated) by rotating the vehicle 90° and repeating the fine align process.

The recommended alignment procedure consists of rotating the vehicle 90° and performing an analytical alignment prior to and immediately following the rotation. The inertial sensor outputs should be recorded during the process. Appendix C contains a more detailed discussion of the analytical alignment technique, including a preliminary analysis to determine its accuracy potential. It is shown that the vertical tilt errors can be reduced to an amount approximately equal to the total accelerometer drift error (i. e., bias plus random drift, each equal to about 1 $\widehat{\text{sec}}$). Following the 90° rotation of the vehicle, the heading error is reduced to an amount approximately proportional to the estimation error of the East gyro bias, which in turn is about equal to the random drift of the gyro (viz., 39 $\widehat{\text{sec}}$ of heading error is caused by 4 mdh bias error of the East gyro). The other level component of gyro drift is also estimated to approximately 4 mdh, and the vertical component is estimated to about 4.5 mdh. Without the change in heading error measurement, the vertical gyro bias estimation error is increased significantly, as shown in Fig. C-6.

Several navigation runs should be made following alignment without the 90° rotation in order to measure actual performance under such conditions. All other runs preceded by an alignment with the 90° rotation can be degraded analytically post-test time using empirically determined statistics of the difference in the alignment techniques. This method is described further in Section 6.5.4.

The analytical alignment procedure can be checked under benign conditions as part of the van test program by first performing a physical alignment in the alignment building and then immediately initiating an analytical alignment (without moving the van). Changes in the Coordinate Transformation Matrix (CTM) elements are readily available from the computer and can easily be transformed into tilt and heading errors. The feasibility of the 90° rotation procedure (and use of heading error change measurements) can also be evaluated and the resulting accuracy determined. The first series of tests should be conducted with the van engine off and the generator disconnected. A second series of tests with the engine on and the generator connected is also recommended.

Analytical alignment under road conditions can be evaluated using the following procedure:

- perform a physical alignment in the alignment building
- place the system into the navigate mode and drive out onto the road
- park the van and <u>calculate</u> an analytical alignment

The CTM corrections calculated are indicative of the quality of the analytical alignment (as well as the amount of error build-up while in the navigate mode). An alternative procedure that may be more accurate is to park the van alongside the road (for the entire test), perform the pre-navigate alignment analytically, enter the navigate mode, and take many position and velocity fixes over a 1/2 to 1 1/2 hour period. The reset technique described in Appendix E can then be used to make a smoothed estimate of the system state vector at the time the analytical alignment was completed and the navigate mode initiated. Absolute azimuth error can be determined adequately by returning the van to the alignment building after all fixes have been taken with the system still in the navigate mode, and making an optical azimuth measurement. The effect of generator and van engine vibrations can be determined by running series of tests with and without the vibrations.

Consideration should be given to performing analytical alignments at various attitudes and headings of the van to determine if any sensitivity and/or linearity problems exist. Accuracy can be assessed by placing the system into navigate mode following the initial analytical alignment and then immediately performing a physical alignment to check the elements of the coordinate transformation matrix.

If possible, the inertial sensor outputs should be recorded during all of the above tests in case design changes are required and reprocessing of the data is desired. This could save significant amounts of van test time and data handling, as well as provide controlled (realistic) conditions for evaluating different algorithms. Once the analytical alignment algorithm is proven to perform properly, it may be used to calculate alignments (and gyro bias calibrations) on the road and in the aircraft.

5.5 NAVIGATION TESTS WHILE SYSTEM IS STATIONARY

It is recommended that several navigation runs (in geographic coordinates) be made with the SD-53 inertial system in the van, while parked on the alignment table with the trailer/generator disconnected. This will establish a baseline of performance under the most ideal field conditions. Use of the bubble levels and optical azimuth reference in the initial alignment as well as during the navigate mode will provide even more information on the baseline performance. Several runs should also be made (either simultaneously with the above, or separately, if necessary) to record the inertial sensor outputs. This can provide information on random drift characteristics of the SAPs and PIGAs under the van conditions, but with a somewhat benign environment. As mentioned in Appendix B, the van vibration environment should also be recorded under these conditions. Runs should be repeated using Tangent Plane coordinates. Also runs should be made with the generator connected and the van engine running.

A second series of runs identical to the above, with the van rotated 90°, is recommended to determine if any heading sensitivities exist (for the benign conditions) and to check the various navigation algorithms. Runs at each cardinal heading are desirable, if test time permits, as well as at intercardinal headings.

Since essentially perfect position and velocity fixes are available continuously, the reset technique described in Appendix E can be checked for proper and accurate performance. The physical alignment monitoring provides attitude and gyro drift information which, in addition to the position and velocity errors, is sufficient to check each estimate provided by the reset technique.

If sufficient test time is available, a third series of tests should be considered in which position fixes and the reset technique are used to compute and apply corrections to the inertial system. Not only does this show potential accuracy performance, but measured results can be compared to predicted results, as discussed in Appendix F. 2, to increase confidence in math models.

The above runs should also be made with the inertial system in the aircraft. Although optical alignment will probably not be feasible, baseline performance can be established, including the measurement of the aircraft vibration environment.

5.6 NAVIGATION TESTS WHILE SYSTEM IS MOVING

The tests described in this section are generally applicable to both the van and aircraft test programs. Exceptions are discussed where significant differences exist. However, tests involving measurements in the alignment building (viz., physical alignment) apply only to the van test program, and the velocity averaging time is about 1/2 minute for the van tests and about 1 minute for the aircraft tests. Because flight test time is considerably more expensive than van test time, flight tests must be planned very efficiently and fewer flight tests should be scheduled. Depending on van test results, some of the tests described below may be eliminated, or at least reduced in scope.

The normal test procedure is to initiate and terminate all van test runs in the alignment building so that alignments may be made before and after each run. Following initial alignment*, the van is driven out onto the road. In the case of the aircraft tests, analytical alignment is performed after each flight, as well as before. Periodically the vehicle (van or aircraft) should be stopped (or landed), for some of the tests, and position

*The analytical alignment is used for certain tests and physical alignment for others. An optical azimuth measurement should be made in all cases, if time permits, to check the alignment and provide additional information should problems develop.

and velocity fixes taken. The time between stops may range from 10 to 25 minutes. For other tests, the vehicle should not be stopped (so as not to change the acceleration profile) and just position fixes taken periodically (also velocity, if possible, as explained in Section 4.4). Runs should generally last 3 to 4 hours, with several runs allowed to go for 6 to 8 hours in order to assess performance over extended periods and to collect information required to more accurately analyze errors within the inertial system's earth loop (using the reset process)*. Runs should be made over several different courses to provide a variety of conditions (frequency and magnitude of turns, rates, vibrations, etc.). If possible, runs for the same conditions should be repeated at least 5 times to establish repeatability characteristics and reasonable confidence in the resulting statistics. The reset process is checked by estimating the system errors for those times that an analytical or physical alignment is made or computed.

Although it is difficult to define special vehicle paths to accentuate specific error sources (for design purposes) [2] the effect of groups of error sources that are a function of vehicle direction (e.g., certain misalignments, mass unbalances, PIGA bias error, etc.) can be detected by reversing the vehicle direction every 42 minutes in order to excite the Schuler loops [16]. To a certain extent the effects can be isolated by operating at cardinal headings.

One series of tests should be run in which the vehicle is stopped for periods of 15 to 30 minutes, during which time an analytical alignment should be calculated (but not applied to the system). Results from such tests could then be compared to results from the reset calculations* to

*The reset process is described in Appendix E.

verify proper operation of both techniques. In conjunction with this, some effects of OA acceleration error may be check by performing an analytical alignment (or reset computation) with the engine running, then rotating the vehicle 180° and repeating the alignment or reset. Since the OA/SA axes of each SAP will have rotated relative to the body axes, any rectified drifts due to OA acceleration error may change and hence be detected as apparent gyro bias changes. A 90° rotation along with an alignment or reset can also provide useful information in the same area, as well as for other vehicle headings.

A series of tests should be performed with the vehicle traveling in a relatively constant direction for at least several hours, during which time frequent (say every 10 minutes) position fixes should be taken without stopping. The vehicle should then be turned approximately 90° and run for another several hours. The vibration environment should be essentially the same (statistically) for the whole time. From reset calculations, any apparent gyro bias changes can be detected. The course of the vehicle should include altitude changes in order to provide changes in vertical distance.

Another series of tests are recommended to check accuracy over the short term (<10-15 min). The system should be programmed in Tangent Plane coordinates (if possible); otherwise the inertial sensor outputs must be recorded. The system is aligned physically in the alignment building (in the case of the van) and then moved in an approximately constant direction for 30 to 60 min, during which time very accurate position fixes should be taken (without stopping) every 5 min. or so (more often, if possible). If possible, velocity fixes should also be taken (without stopping)*. This information will provide assessment of system performance over the short run. If accurate fixes are taken often enough, internal system errors may be determined using the reset technique. A series of tests should also be run using the analytical alignment technique instead of physical alignment, to determine the resulting accuracy degradation for short term runs. If all short runs in Tangent Plane coordinates are initiated from the same starting point, the conversion of the position references from geographic coordinates into Tangent Plane coordinates need only be done once.

If the SAP head angle sensitivity studies mentioned in Appendix F. 2 have not been made, several of the above tests should be repeated using other combinations of initial head angles of the SAPs. If performance changes significantly, the studies discussed in Appendix F. 2 should be undertaken.

If initial test results indicate that SAP and/or PIGA compensations are required, several of the above tests should be repeated as a second phase of the test program, using the compensation algorithms in the real time processing of the inertial sensor data.

Consideration should be given to a series of tests in which position fixes and the reset algorithm are used to compute and apply corrections to the inertial system in real time (these tests may be performed as part

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*A fifth wheel velocity reference would be particularly useful here, in the case of the van tests.

of the second phase of the test program, depending upon test time available, results of the first test phase, and results of special analyses involving analytical resets of the data (see Appendix F. 2 item 8). The effect of Schuler loop velocity damping can be approximated by taking frequent fixes and using the reset process to compute and apply appropriate corrections. If a "continuous" velocity reference is made available (e. g., a 5th wheel for the van or a doppler radar for the aircraft)*, and the Schuler loop mechanization equations suitably changed, damping can be applied without using the reset process. Even with this configuration, the further improvement in performance possible by position fixing and reset corrections can be demonstrated in real time by a series of (second phase) tests.

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*Essentially equivalent performance may be possible by taking noncontinuous velocity fixes every 2 to 10 minutes, using other types of velocity references.

6. DATA REDUCTION REQUIREMENTS AND EXPECTED PRECISION

The purpose of this section is to indicate ways in which the test data may be processed in order to evaluate the SD-53 Inertial system and fulfill the objectives discussed in Section 2. The intent of this section is to lay the groundwork and provide a basis for generating detailed data reduction procedures at a later time. An indication of evaluation precision is also included.

The first two parts of this section (viz., approach and performance criteria) establish a perspective for the remaining three subsections (viz., real time data evaluation, post-test data reduction, and special analyses). The special analyses involve checkout of certain special techniques, as well as analyses used for design purposes. Generally, the special studies require a minimum of supporting analytical analyses. Additional studies that involve more information than just that obtained from the test program are discussed in Appendix F. 2.

The data reduction recommendations generally apply to both van and aircraft test data. However, in certain cases procedures are discussed which are applicable to only specific types of test data.

6.1 APPROACH

The first step in the data reduction process is to edit all data to be processed so as to reject wild points*, as well as to reject blocks of data

*Wild points can be validly rejected only if they could have been detected by some operational scheme (in real time), or if they have no significant effect (good or bad) on actual performance of the inertial system.

that can be detected as unwanted according to previously defined rules. An interesting data editing technique used successfuly at Edwards AFB is descirbed in [28]. Whenever blocks of data are rejected, an input to an appropriate reliability figure should be made. In the case of real time data monitoring and processing, run abortion rules should be defined, as well as data rejection criteria.

Consideration of the following parameters is recommended in the evaluation of inertial system performance.

- direct accuracy measurement of system indicated position, velocity and heading/attitude outputs
- indirect estimation of system internal errors and error effects (e.g., gyro drift, vertical tilt, algorithm and update errors, etc.)*
- determination of system operational characteristics (viz., reliability, operability, and maintainability)

Each of these categories are considered in more detail below. The system performance characteristics should be presented as functions of the following conditions:

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*Many of the internal errors can be estimated using the reset technique described in Appendix E. The analytical alignment technique can also be used (see Appendix C and Sections 4.1, 5.3 and 5.4), as well as the physical alignment method (see Section 5.4). Certain algorithm error effects can be estimated using "perfect references", as described in Appendices A.2 and G.

- inertial system configuration
- key evaluation system characteristics (e. g., quality and frequency of position and velocity fixes)
- type of alignment
- elapsed time from completion of alignment
- high frequency motion profile of vehicle (i. e., angular and linear vibrations)
- low frequency motion profile of vehicle (i. e., course, speed, maneuvers, etc.)
- prefiltering time used in determining velocity errors

It is recommended that the following three performance qualities be determined for each parameter, wherever possible:

- repeatability
- stability
- absolute accuracy

The performance criteria recommended for the data reduction program are defined and discussed next.

6.2 PERFORMANCE CRITERIA

The following ten performance criteria are recommended in the data reduction process for the evaluation and characterization of the inertial system accuracy performance:

1. Error
2. Average
3. Peak-to-Peak
4. Ramp
5. Ensemble Root Mean Square
6. Time RMS
7. Sample Variance
8. Power Spectral Density function
9. Cross Power Spectral Density Function
10. Frequency Function

The term "Error" is defined as an indicated value minus true value. The ensemble RMS is based on an ensemble of measurements at a given time (or elapsed time from a starting point), whereas the time RMS is based on measurements taken as a function of time.

6.3 REAL TIME DATA EVALUATION

It is recommended that sufficient raw and semi-processed data be made available during each test run to assure with some reasonable confidence that usuable data is being obtained. Therefore, run abortion criteria must be defined and implemented, as well as means to output and display the data.

In the case of the analog recordings, it would be desirable, if possible, to produce strip chart recordings (in addition to the tape recording) for comparison by the operator to acceptable values. If this is not possible, other means (such as panel meters) should be provided for monitoring on a spot check basis. Another alternative would be to digitize the voltages and compare them periodically, within the 516 computer, to acceptable values. If vibrations are measured, they could be compared (perhaps statistically) to the PIGA outputs, if sufficient computer space and time were available.

During normal navigation runs, generation of error plots vs. time of at least the following parameters are recommended [3, 5, 16] whenever available:

$$\left.\begin{array}{l} \bullet\ \delta La,\ \delta Lo,\ \delta H \\ \bullet\ \delta V_N,\ \delta V_E,\ \delta V_V \end{array}\right\} \text{ for outputs in Geographic (NEV) coordinates}$$

$$\left.\begin{array}{l} \bullet\ \delta \dot{X}_s,\ \delta \dot{Y}_s,\ \delta \dot{Z}_s \\ \bullet\ \delta X_s,\ \delta Y_s,\ \delta Z_s \end{array}\right\} \text{ for outputs in Tangent Plane (TP) coordinates}$$

To the extent that external position and velocity references are available in real time in inertial system coordinates, they should be subtracted from the corresponding inertial system position and velocity outputs to form the above errors. Precision of the measurements depends upon the accuracy of the references used, as discussed in Sections 4. 3 and 4. 4. If possible, the computer should compute the radial position and velocity errors* vs. time. If the reset and/or alignment algorithms are provided in the 516 computer, time plots of the following quantities should also be maintained

*Radial position error is defined here as $(\delta La^2 + \delta Lo^2)^{1/2}$ for NEV coordinates and $(\delta X_s^2 + \delta Y_s^2 + \delta Z_s^2)^{1/2}$ for T. P. coordinates. The radial velocity error is defined as the time rate of change of the radial position error.

in real time, as applicable and depending upon the availability of position and/or velocity fixes:

- δA_H, δA_P, δA_R
- $\delta \underline{B}_G$, $\delta \underline{R}_G$
- $\underline{\phi}$

The estimation precision of these terms is presented in Appendices C and E. Tabulations of all the calibration terms, whether used in the navigation process or not, should be maintained.

A measure of system performance that can be checked easily during each test run is the degree of orthonormality maintained in the C (Coordinate Transformation) matrix. Derivations of CC^T from the identity matrix are indicative of computer algorithm errors (such as roundoff, truncation and commutativity errors).

6.4 POST-TEST DATA REDUCTION

The recommendations in this section are concerned primarily with processing of the test data to evaluate and characterize the SD-53 inertial system. This is done in nine different areas, each of which is discussed below. Checkout of special techniques and analyses used for design purposes are described in Section 6.5 (Special Analyses).

6.4.1 Pre-Processing of Data

Before any of the performance criteria listed in Section 6.2 can be calculated, it is necessary to edit the data, as discussed in Section 6.1. In addition, any position and velocity reference information not expressed in inertial system coordinates must be so transformed. This includes position fixing of the aircraft from aerial photographs, [6] as well as transformations into NEV or TP coordinates, as required.

6.4.2 Determination of Navigation Accuracy

The navigation accuracy of the SD-53 inertial system is defined as the time varying errors of the following indicated output quantities:

$$\left.\begin{array}{l} \bullet\ \delta La,\ \delta Lo,\ \delta H \\ \bullet\ \delta V_N,\ \delta V_E,\ \delta V_V \\ \bullet\ \delta A_H,\ \delta A_P,\ \delta A_R \end{array}\right\} \text{ for outputs in Geographic (NEV) Coordinates}$$

$$\left.\begin{array}{l} \bullet\ \delta X_s,\ \delta Y_s,\ \delta Z_s \\ \bullet\ \delta \dot{X}_s,\ \delta \dot{Y}_s,\ \delta \dot{Z}_s \\ \bullet\ \delta A_x,\ \delta A_Y,\ \delta A_Z \end{array}\right\} \text{ for outputs in Tangent Plane (TP) Coordinates}$$

Computation of the above quantities, as well as the radial error, are normally computed during each run as part of the real time data evaluation (see Section 6.3). If necessary, the computations can be performed post-test time, as is the case for the processing of recorded inertial sensor data. The determination of attitude errors was considered as a ground rule to be of secondary importance (see Section 3.1), and is determined only when an alignment (physical or analytical) is performed. No tests or references are provided to evaluate attitude rate accuracy, as discussed under the ground rules.

A statistical measure of the above quantities is determined by computing the average and RMS of each parameter as a function of time for runs having the same test conditions. The Student's t and F tests are used to establish 95% confidence limits to account for limited sample sizes. If the average is significantly different from zero, a bias in the system is indicated (probably due to error source(s) that are constants for one system, but random with zero mean if many systems were to be tested).

A single figure of merit (FOM) traditionally used to characterize the positional accuracy of an inertial system for periods less than about 6 hours is the CEP rate. If the CEP rate for different test conditions is not statistically different, as determined by the F test, a combined value can be computed to provide greater precision. This technique can also be applied to the other parameters listed above. System sensitivities are indicated whenever results for different test conditions are found to be significantly different (statistically). Several references relative to data reduction in this area are [27, 5, 29, 30, 31, 3, 16].

6.4.3 Determination of Calibration Accuracy and Stability

Gyro and accelerometer calibration data obtained from physical and/or analytical alignment at the beginning and end of each run should be plotted vs. time for general monitoring purposes. Changes from system turn-off to turn-on should be computed and plotted to establish stability.

Computation of the average, ramp and sample variance performance criteria provides a statistical measure of the stability characteristics. Gyro bias changes as a function of time during runs should be computed using the reset technique described in Appendix E, and the same performance criteria computed. Calibration precision using the reset technique is shown in Fig. E-6 of Appendix E to be approximately 3 mdh for the level gyros (after taking 6 fixes over 1/2 hour). In the case of the flight tests, data over a period of about an hour is required to obtain 3 mdh precision. Estimation of vertical gyro bias error takes considerably longer for both the van and flight test conditions (see Fig. E-6).

Potentially more accurate estimates of the gyro bias errors during navigation runs are possible for those periods when analytical alignments were computed. Performance better than that shown in Appendix C can be expected since initial bias errors will be smaller. The most precise estimates for the van tests are obtained from the physical alignment process since attitude of the ISU can be measured to several arc-seconds. The corresponding cross-coupling of earth rate is less than .5 mdh.

Evaluation of the analytical alignment process to estimate gyro bias errors is as described in the next section.

6.4.4 Determination of Alignment Accuracy Using the Analytical Alignment Algorithm

The absolute accuracy of the analytical alignment technique is determined during the van test program by comparing each of the following alignment parameters to those obtained from a physical alignment performed simultaneously:

- attitude relative to vertical and true North
- initial velocity error (3 components)
- initial position error (3 components)

From repetitive runs under the same conditions, the average and sample variance are computed to determine absolute accuracy and repeatability of the analytical alignment algorithm. The Students' t and F tests are used to establish 95% confidence limits to account for limited sample size. If the average is significantly different from zero, a bias in the alignment process is indicated. The level of precision is determined by the accuracy of the physical alignment (approximately several arc-seconds) and by the resolution of the inertial system indicated outputs. The above data reduction procedure is repeated for the other test conditions described in Section 5.4 (Pre-Navigate SD-53 System Alignment Procedures). Comparison of the statistics can identify possible sensitivities in the analytical alignment technique.

Accuracy of the analytical alignment technique under road conditions is checked in two ways, as described in Section 5.4. In the first method, physical alignment data is obtained, and data processing is as described above. In the second method, many position fixes (and velocity fixes, if available) obtained in the navigate mode following alignment are used in conjunction with the reset technique to make a smoothed estimate of the state vector (of errors) at the time the analytical alignment was completed and the navigate mode initiated.* Absolute azimuth error is measured using the optical reference in the alignment building. The precision of the

*This second method can also be used in the aircraft test program.

estimation process is expected to be somewhat better than that indicated in Appendix E since many more fixes will be taken. The form of the data processing is the same as that described earlier in this section.

6.4.5 Determination of Operational Conditions

Data reduction in this area is primarily concerned with the linear and angular vibration measurement tests described in Sections 4.2, 5.1 and Appendix B. It is recommended that the following statistics be computed for each of the six vibration acceleration parameters and for the various test conditions:

- power spectral density
- RMS of vibration amplitude
- peak-to-peak values of vibrations
- principal frequency components and RMS magnitudes

In addition, 3 angular and 3 linear cross power spectral density functions should be computed to determine the degree of correlation between axes [11, 12]. Vibration data from both the van and aircraft tests hould be processed.

The various quantities* routinely recorded on the analog recorder for each run should be plotted out to determine typical ranges of variation for each quantity. If this is not possible for each run, it may be adequate to examine the data only when abnormal or poor inertial system performance is experienced. In addition, as a minimum, representative blocks of data should be processed to determine the average and sample variance for

*Such as system power supply voltages, temperatures, etc.

each parameter in order to establish the nominal conditions under which the inertial system was evaluated.

6.4.6 Determination of Operational Characteristics

The primary operational characteristics are defined in terms of reliability, availability, repairability, reaction time, and operability. As mentioned in Sections 5.1 and 6.1, categories of system failures must be identified, and are closely related to the run abort and data rejection criteria. The definition and determination of each of the operational characteristics is as follows:

(1) Reliability: probability that system performance is satisfactory for a given time

$$\hat{R}(T) = \frac{n_s}{n_s + n_f}$$

where

n_s = the number of runs satisfactory over time T

n_f = number of runs that fail before time T

$\hat{R}$ = estimate of reliability over time T

The reliability should be determined for various values of T, considering only inertial system hardware failures. Separate reliability figures could also be computed to include operator faults and failures of the evaluation system, as well as various definitions of "satisfactory system performance" (depending on the run abort and data rejection criteria).

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*Including the time at which the navigate mode is initiated.

For $T = 0'$, several "system turn-on" reliabilities can be defined depending upon conditions prior to system turn-on (e. g., number of hours since last system shut-down, last repair, etc.).

(2) Availability: ratio of all navigation run times during which inertial system performed satisfactorily to sum of total run time and time to repair.

$$\hat{A} = \frac{\Sigma T_u}{\Sigma T_u + \Sigma T_R}$$

where

ΣT_u = sum of up times (in navigate mode) from each run

ΣT_R = sum of all repair times required to allow inertial system to be operational (exclusive of preventive maintenance time and time from system turn-on to entering navigate mode)

$\hat{A}$ = estimate of availability

(3) Repairability: mean time to repair

$$\hat{R} = \frac{1}{n_r} \sum_{i=1}^{n_r} (T_R)_i$$

where

ΣT_R = sum of all repair times (see above)

n_r = number of times system repaired

$\hat{R}$ = estimate of repairability

Several variations of $\hat{R}$ are possible depending on the definition of "repair" (e. g., time to detect failure, time to fix system, time to checkout, total time). The ability to identify and localize failures and the complexity of executing repairs is best described qualitatively.

(4) Reaction Time: mean time from system turn-on to entering the navigate mode.

Several components of the mean reaction time could be computed considering the times to warm-up and times to align analytically (or physically); however, one number for the total process is probably adequate for an initial field test program.

(5) Operability: Usefulness of displays, computer I/O and operator functions, complexity and time for turn on/off, calibration, alignment, monitoring and mode changing, fix taking (position and velocity), reset corrections, etc. [3]

These operational characteristics are best described qualitatively.

Analysis of the operational characteristics could be extended to include failure analyses, tabulations of number of system starts, total number of system operating hours, etc. Frequency function plots (histograms) should be considered as a means to graphically illustrate the time functions of some of the above characteristics.

It is to be noted that Mean Time Before Failure (MTBF) cannot be adequately estimated if system shut-downs are intentional rather than due to failures. The reliability figure of merit, as a function of time, adequately characterizes the system in this respect.

6.4.7 Determination of Internal Inertial System Errors

The reset technique is used to make indirect estimations of the following errors and error sources that are internal to the inertial system:

(1) gyro drift

(2) attitude and relative heading errors

(3) earth loop misalignments

(4) Schuler loop misalignments

As shown in Appendix E, estimates are made of each element in the state vector, which represents the variables used to model the system. The precision with which this can be done is indicated in Appendix E; however, more detailed analyses are required to include other error sources, refine the math models, determine acceptable number and spacing of fixes, etc. Time plots of the estimates should be made, if not already done as part of the real time data evaluation (see Section 6.3). Data from all runs having the same test conditions can be combined statistically and checked for significance, as discussed in Section 6.4.2. Significant changes* from one group of fixes to another should be correlated with changes in operational and/or environmental conditions (including moving vs. stopped, in flight vs. onroad vs. in alignment building, different vibration levels, changes in heading, and different SAP head angles).

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*Particularly of gyro drift and attitude errors.

The gyro and accelerometer random drift characteristics can be determined from the sensor output recordings obtained during physical alignment. A number of statistics can be computed, including auto and cross correlations, frequency functions, ramps, etc. Time series analysis should also be considered to better determine the math models. Such statistical information is important to improve the analytical alignment and reset techniques and to provide more accurate analyses in general.

6.4.8 Effect of Certain Error Sources

Data obtained from the special test sequences described in Sections 5.4, 5.5 and 5.6 are processed to determine the effect, if any, of certain error sources. Possible sensitivities of the alignment process to the attitude of the van could be checked by performing analytical alignments at various attitudes and checking them using the physical alignment method. Any significant differences, as determined by the Student t test, would indicate a sensitivity. Similarly, heading sensitivity in the navigate mode can be checked using the physical and/or analytical alignment or reset techniques, as available and appropriate.

A preliminary check of the OA acceleration error effect due to engine (van or aircraft) vibration is performed by computing resets (or analytical alignments) at various cardinal (and other) headings. Any significant gyro drift changes may be indicative of a significant OA acceleration error effect, depending on the correlation of the accelerometer outputs. The tests can be performed with the vehicle stationary or moving, as described in Sections 5.5 and 5.6, as well as for various SAP head angles. Similar checks of coning effects may also be possible.

6.4.9 Effect of Reset Corrections and Velocity Damping

The determination of navigation accuracy when resets are applied is different from the methods described in Section 6.4.2, which apply primarily when no corrections are made. Since CEP rate has meaning only over the interval before the first reset is applied, and an equivalent CEP rate over a given number of resets can be misleading, a different figure of merit (FOM) is desirable. It is suggested that the time RMS* of the measured navigation outputs be computed, not only when resets are applied but also for those conditions when corrections are not made but for which comparisons are desired. Such a FOM will tend to become stationary over multiple reset intervals, and the precision of its estimate can be improved by considering an ensemble of time RMSs.

The CEP rate figure of merit is recommended for the evaluation of the inertial system when external velocity information is used to damp the Schuler loops, although the time RSM FOM is also valid. If reset corrections are also applied, the CEP rate FOM is not particularly meaningful, as discussed above, and therefore the time RMS FOM should be used.

6.5 SPECIAL ANALYSES OF TEST DATA

The data reduction described in this section is concerned with checkout of certain special techniques, as well as analyses used for

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*The time RMS is based on error measurements taken over a period of time.

design purposes. The analyses are based primarily on test data obtained from the field tests as described in Section 5, and require a minimum of supporting analytical analyses. Additional studies that require more information than just that obtained from the test program are discussed in Appendix F. 2.

6.5.1 Evaluation of Reset Technique

The reset technique can be evaluated several ways. Direct checks of the estimates can be made by comparison to estimates using the analytical alignment algorithm (both at the time the navigate mode is initiated and whenever an analytical alignment is calculated while navigating). In the former case, more accurate comparisons can be made in the van test program whenever a physical alignment has been performed. Data reduction details are discussed in Section 6.4.4.

Indirect checks of the reset technique math models can be made in both the van and aircraft test programs by using a series of fixes to estimate the future propagation of the output errors and comparing them to the actual errors. Also, the expected covariance can be computed and compared to the sample variance actually measured. If compatible performance can be demonstrated, confidence in the math models and reset algorithms is increased.

6.5.2 Data Processing of Analog Recordings

If unexpected performance of the inertial system is suspected, the analog recordings should be analyzed to determine if any of the parameters recorded are not normal. To establish a norm, recordings under similar conditions but when system performance is normal must also be analyzed. Various performance criteria can be computed including power spectral density, RMS of amplitude, peak-to-peak variations and principle frequency components and RMS magnitudes. Histograms may also provide useful information.

6.5.3 Effect of Changes in Computer Algorithms

The data processing considered here is limited to the comparison of navigation accuracies as a function of different computer algorithms (viz., compensation, alignment, CTM, VTM, navigation and attitude algorithms).* This includes both real time and post-test processing of the inertial sensors outputs. Other methods of determining the effect of compensation, based on more information and analyses than that available from the basic test program, are discussed Appendix F. 2.

The mean and RMS of each navigation parameter are computed as a function of time as discussed in Section 6.4.2, for runs having the same test conditions and combination of algorithms. The Student's t and F tests are then used to determine if there are any statistically

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*CTM and VTM are the Coordinate Transformation Matrix and Velocity Transformation Matrix algorithms, respectively.

significant differences between the results obtained when different combinations of algorithms are used.*

If the navigation performance using more extensive compensation is shown to be significantly better, confidence in the quality of the math models is increased. If no significant improvement results, it is more likely that the error effects are small, since incorrect math models would tend to degrade performance.

It may be possible to compare different combinations of algorithms statistically using less test data by computing the time RMS errors in the navigation parameters, as discussed in Section 6. 4. 9.

6. 5. 4 Evaluation of Alternate Alignment Techniques

It was recommended in Section 5. 4 that alignment consist of rotating the vehicle 90° and performing an analytical alignment prior to and immediately following the rotation. The effect on navigation accuracy should the alignment be made without the 90° rotation can be estimated by comparing alignment errors using only data from the first half of the alignment procedure with that from the full procedure. The error propagation models (used as a basis for the reset technique) can be used to estimate the degradation of navigation accuracy due to the additional alignment errors. The method can be checked by comparing the predicted performance to that actually measured from several test runs (as referred to in Section 5. 4). The recorded inertial sensor data can also be processed

*Test data is selected such that all other test conditions are the same, statistically.

with and without the 90° rotation, and the resultant navigation errors examined.

Similar comparisons regarding alignment accuracy and resulting navigation performance can be made considering alignment with and without the use of the yaw monitor and with and without measurement of vehicle heading changes during alignment, both for the van and aircraft test programs.

7. CONCLUSIONS AND RECOMMENDATIONS

Since most of the recommendations to be made are based on conclusions drawn from the study, the conclusions and recommendations are listed together. The items are organized into specific categories, and those items that contain recommendations are preceeded by the letter "R".

7.1 INERTIAL SYSTEM

R(1) The preliminary study indicates that sufficient measurements can be made on the SD-53 Inertial System, both in the van and in the aircraft, to adequately evaluate it in accordance with the test objectives discussed in Section 2. Therefore, it is recommended that the inertial system be implemented as discussed in Section 4.1 and that the computer be programmed to provide the functions listed.

R(2) It is recommended that long term navigation in NEV coordinates be demonstrated in real time. If possible, performance should also be demonstrated in real time using TP (Tangent Plane) coordinates. If this is not possible in real time, recorded inertial sensor outputs may be processed post-test time.

R(3) If the inertial sensor data cannot be recorded while the system is navigating in real time, consider additional runs during which only the sensor data is recorded (for representative conditions).

R(4) Preliminary studies indicate that compensation most likely will be required for at least gyro and accelerometer drifts, misalignments, and OA acceleration induced errors. It is recommended that appropriate compensation algorithms be developed and included in the system mechanization before field testing begins.

R(5) If either analytical studies or field test results indicate significantly poor performance that may be corrected by additional compensation, consider providing the required algorithms and rerunning some of the field tests to <u>demonstrate</u> improved accuracy. Some of the special studies that should be completed early in the field test program are discussed in Appendix F. 2.

R(6) The inertial system computer should be capable of accepting and applying discrete reset corrections, as well as certain other inputs as described in Section 4. 1. The ability to continuously damp the Schuler loops using an external velocity reference is not considered important for the basic field test program, particularly since the process can be approximated by frequent reset corrections.

(7) Preliminary investigations indicate that only a few calibration terms can be determined with the inertial system in the van or aircraft. Further study of the physical alignment process would very likely identify test procedures to determine additional calibration terms with the system in the van.

7.2 PRE-NAVIGATE ALIGNMENT

R(1) An analytical alignment technique is required in most final applications and therefore should be developed and tested as part of the van and flight test programs. Preliminary analyses indicate that development of such a method is feasible and should be adequate. It is recommended that the method described in Sections 4.1 and 5.4 be developed further.

R(2) Use of a 90° rotation of the inertial system about the nominal vertical during alignment is recommended since it provides bias estimates to the two level components of gyro drift, and significantly improves the gyro compassing heading accuracy.

R(3) Use of heading error change measurement is recommended since the biasing accuracy of the Z component of gyro drift can be improved* significantly.

R(4) It is recommended that a yaw monitor be developed, should further analyses and tests so indicate.

(5) The 1° wedge for physically aligning the ISU relative to the earth's polar axis is no longer required since the original CTMC (Coordinate Transformation Matrix Computer) [8] is being replaced in the test

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*By increasing accuracy and reducing test time.

program by another computer (the Minneapolis Honeywell 516), and the 516 can be programmed to account for earth rate components along any axis. This is true for physical as well as analytical alignment.

7.3 EVALUATION SYSTEM

R(1) The taking of accurate position fixes is not expected to be a significant problem in the van test program. However, in the flight test program, accurate position fixes probably will be available only from photographs and landing the aircraft*. Less accurate, but adequate position fixes can be expected if the flight path is restricted.

(2) Velocity fixes are expected to be more difficult to obtain (aside from stopping the vehicle). Adequate velocity fixes are possible with the van moving, but accurate velocity measurements in flight are more difficult without special equipment (e. g., Doppler radar).

R(3) It is recommended that a fifth wheel velocity reference not be provided in the basic van test program since sufficient test data can be obtained without it.

(4) Although pre-navigate alignment of the inertial system is possible with only a velocity reference, the fluid floated table in the

*Assuming tests are not performed over highly instrumented, special test ranges.

alignment building is useful as a check on the analytical alignment scheme and can be used to accurately measure some of the calibration terms with the inertial system in the van.

R(5) It is recommended that the significant error characteristics of the test equipment be determined as additional inputs into the data reduction process in order to properly interpret the results.

R(6) Since preliminary calculations indicate that the inertial sensor outputs can be recorded digitally on one reel for only 1/2 hour, consider alternatives to increase the recording capacity (including more efficient tape use, smaller word sizes per unit time, longer sampling intervals, longer tapes and the use of two tape recorders so reels can be changed with no loss of information).

7.4 TEST CONDITIONS

R(1) Preliminary studies indicate the importance of measuring the vibration characteristics of both the van and aircraft, since the resulting navigation errors can potentially be critical. It is recommended that the measurements be made as early in the test program as possible since they influence the test design and are needed to support related analyses and to determine compensation algorithms, as required.

(2) If maximum van speeds are limited to about 20 mph, vibration levels can be expected to be lower. From the MISER test program [1],

linear vibration accelerations are from 25 to 50% higher for speeds greater than 20 mph, and the angular accelerations are from 200 to 400% higher.

R(3) If van/trailer vibrations are determined to be significantly non-representative of those expected in the final application(s), the present van generator configuration should be reviewed for possible changes that may provide a more representative environment.

7.5 TEST DESIGN

R(1) To fulfill the test objectives with a minimum of test time, and yet provide sufficient redundancy, a methodical definition of test and data reduction procedures is required, as well as specification of certain special analyses and nonfield supporting tests. Associated with this is the careful definition of run abort and data rejection criteria, which can also contribute to a more meaningful determination of the operational characteristics of the inertial system.

R(2) Faithful maintenance of a comprehensive test log is recommended since it can make the difference between efficient data taking, cataloging and troubleshooting vs. unnecessary repetition of runs and abandonment of potentially useful data already collected.

R(3) If the effect of the calibration terms is significant, further study of the test procedures may provide techniques to isolate the effect of additional individual error sources.

R(4) A second level of testing should be considered if sufficient changes are indicated as a result of van and/or flight tests (particularly if this is the case regarding compensation.)

7.6 RESET

R(1) Development of the reset algorithm is recommended since it is very useful in the estimation of internal errors and as a check on the system math models, without interruption of the navigation process. This is particularly advantageous in the flight test program. When velocity fixes are not available, the inertial system velocity error can be estimated by the reset algorithm.

R(2) Although the resets can be computed post-test time, real time estimation is recommended in order to improve monitoring of the system.

(3) The reset technique can be adapted to process real test data so as to determine the expected system performance that would result if reset corrections had been applied to the system.

R(4) A second level of testing is recommended if reset corrections are to be actually applied in real time to demonstrate improved performance, since modifications to the reset algorithm would be required to account for control being applied to the system.

7.7 SPECIAL TESTS AND ANALYSES

R(1) The supporting tests and analyses described in Appendices D and F. 2 are recommended in order to make the most efficient use of the field test time and to make significant contributions in fulfilling the test objectives.

R(2) An error analysis of the inertial system under van and flight test conditions is recommended after tests have been completed successfully and it is desired to increase confidence in the math models for additional analyses. Such an analysis could also be used to rank the error sources in order to identify the critical ones.

(3) The techniques discussed in Appendix G can be used to evaluate the various transformation and navigation algorithms, particularly if improvements are deemed to be necessary.

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APPENDIX A

EFFECT OF ANGULAR ACCELERATIONS ABOUT GYRO OA

A.1 ANALYSIS OF ERROR SOURCE

The analysis in this section is considered preliminary since the intention is only to determine the form and potential seriousness of the error propagation. Although representative of the total effect, the analysis is not complete and only one simplified vibration model is considered.

Figure A-1 indicates a typical SAP configuration, in which it is to be noted that rotation of the body about the IA of SAP #1 causes a change in the orientation of IA_2 and IA_3 relative to OA_1.

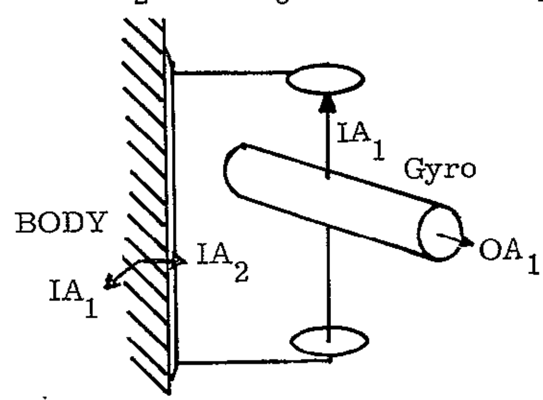

Figure A-1 SAP Configuration

Neglecting the dynamics of the gyro and the feedback loop, the equations for the SAP outputs are:

$$(w_{pb})_1 = (w_{ib})_1 - \frac{I}{H} (\dot{w}_{ib})_{01} \qquad \text{(A-1)}$$

$$(w_{pb})_2 = (w_{ib})_2 - \frac{I}{H}(\dot{w}_{ib})_{02} \quad \text{(A-2)}$$

$$(w_{pb})_3 = (w_{ib})_3 - \frac{I}{H}(\dot{w}_{ib})_{03} \quad \text{(A-3)}$$

where

w_{pb} is indicated angular velocity

w_{ib} is true angular velocity

0 letter subscript is output axis of gyro

numerical subscript identifies each individual gyro

$\frac{I}{H}$ is output axis moment of inertia to spin angular momentum ratio ($\cong$ .001 sec)

The SAP IA orientations are mutually orthogonal and the angles of the OA's about IA are defined relative to the other SAP OA's as shown in Fig. A-2.

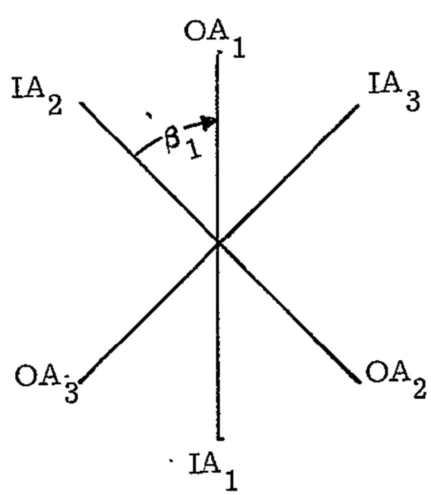

Figure A-2 IA-OA Configuration

The error in the output of each SAP (1, 2, 3) due to output axis angular acceleration is therefore

$$\delta w_1 = -\frac{1}{H}\frac{d}{dt}[(w_{ib}^b)_2 \sin\beta_1 + (w_{ib}^b)_3 \cos\beta_1] \quad \text{(A-4)}$$

$$\delta w_2 = -\frac{1}{H}\frac{d}{dt}[(w_{ib}^b)_3 \sin\beta_2 + (w_{ib}^b)_1 \cos\beta_2] \quad \text{(A-5)}$$

$$\delta w_3 = -\frac{1}{H}\frac{d}{dt}[(w_{ib}^b)_1 \sin\beta_3 + (w_{ib}^b)_2 \cos\beta_3] \quad \text{(A-6)}$$

where

$$\beta_i = \alpha_i + 45° \quad \text{(A-7)}$$

and

$$\alpha_i \equiv \int_0^t (w_{ib}^b)\, d\tau \quad \text{(A-8)}$$

Considering the first term in Eq. (A-4),

$$\frac{d}{dt}\ \frac{I}{H}(w_{ib}^b)_2 \sin\beta_1, \quad \text{(A-9)}$$

the angular rate term is expanded as follows:

$$(w_{ib}^b)_2 = (w_{ie}^b)_2 + w_2 \quad \text{(A-10)}$$

where $(w_{ie}^b)_2$ is the component of earth rate along the IA_2 axis, and w_2 is the angular rate about the axis defined by the IA of gyro 2, relative to the earth.

Consider $\dot{w}_i$ to be a sinusoidal angular acceleration of the body about axis i (i = 1, 2, 3), defined as follows:

$$\dot{w}_i = \dot{w}_{io} \sin(w_n t + \phi_i) \tag{A-11}$$

$$w_i = -\frac{\dot{w}_{io}}{w_n} \cos(w_n t + \phi_i) \tag{A-12}$$

where

$$\dot{w}_{io} = 18 \text{ rad/sec}^2 \tag{A-13}$$

$$w_n = 20 \text{ Hz} \tag{A-14}$$

From Eqs. (A-8, -10, and 12),

$$\alpha_i = -\frac{\dot{w}_{io}}{w_n^2} \sin(w_n t + \phi_i) + (w_{ie}^b)_i^t \tag{A-15}$$

Substituting Eqs. (A-10, -7 and -8) into Eq. (A-9) yields:

$$\frac{d}{dt}\left[\frac{1}{H}(w_{ib}^b)_2 \sin\beta_1\right] = \frac{1}{H}\left[\dot{w}_2 \sin\beta_1 + [(w_{ib}^b)_2 + w_2][(w_{ie}^b)_1 + w_1] \cos\beta_1\right] \tag{A-16}$$

The term $\dot{w}_2 \sin\beta_1$ contributes virtually nothing ($<10^{-5}$ft.) to the errors because of the high frequency (w_n) content of the term. This is easily seen since the transfer functions of the latitude and longitude errors caused by gyro output rate errors at w_n are:

$$\frac{\delta La}{\delta w_N} = \frac{w_s^2 w_{ie} \sin L}{(s^2+w_s^2)(s^2+w_{ie}^2)} \cong \frac{w_s^2 w_{ie} \sin L}{w_n^4} \tag{A-17}$$

$$\frac{\delta La}{\delta w_E} = \frac{s\, w_s^2}{(s^2+w_s^2)(s^2+w_{ie}^2)} \cong \frac{w_s^2}{w_n^3} \tag{A-18}$$

$$\frac{\delta La}{\delta w_V} = \frac{w_s^2 w_{ie} \cos L}{(s^2+w_s^2)(s^2+w_{ie}^2)} \cong \frac{w_s^2 w_{ie} \cos L}{w_n^4} \tag{A-19}$$

$$\frac{\delta Lo}{\delta w_N} = -\frac{w_s^2 \sec L(s^2+w_{ie}^2 \cos^2 L)}{s(s^2+w_s^2)(s^2+w_{ie}^2)} \cong \frac{w_s^2 \sec L}{w_n^3} \tag{A-20}$$

$$\frac{\delta Lo}{\delta w_V} = \frac{w_s^2 w_{ie}^2 \sin L}{s(s^2+w_s^2)(s^2+w_{ie}^2)} \cong \frac{w_s^2 w_{ie}^2 \sin L}{w_n^5} \tag{A-21}$$

where

$\delta La, \delta Lo$ are latitude and longitude errors

δw_N, δw_E, δw_V are the North, East and down components of the gyro output rate errors

w_s Schuler frequency

w_{ie} earth rate

L latitude

Coning motion can also produce an indicated constant drift rate. The drift rate is

$$\delta w = \Omega(1 - \cos\beta) \tag{A-28}$$

where β is the coning half angle

Ω is the coning angular frequency

$$\delta w \cong \Omega(1 - (1 - \frac{\beta^2}{2})) = \frac{\Omega\beta^2}{2} \tag{A-29}$$

$$\Omega = w_n \tag{A-30}$$

$$\beta = \frac{1}{2}\,\frac{I}{H} \cdot \frac{\dot{w}_{io}}{w_n} \tag{A-31}$$

$$\delta w = w_n \cdot \left(\frac{I}{H}\right)^2 \cdot \frac{\dot{w}_{io}^2}{8\,w_n^2} \tag{A-32}$$

For the above parameters

$$\delta w = .07°/hr \tag{A-33}$$

A. 2 VAN TEST OF OA COMPENSATION SCHEMES

If the recorded SAP outputs are to be used to test OA compensation schemes (including the effect of no compensation at all), then the procedure shown in Fig. A-3 can be used.

The products of earth rate and vibration rates in Eq. (A-16) cause even smaller position errors (in Eqs. A-17 through 21) than the $\dot{w}_2$ term.

The dominant rectified error term in Eq. (A-16) is $w_1 w_2$.

$$w_1 w_2 = \frac{\dot{w}_{10}\,\dot{w}_{20}}{w_n^2} \cos(w_n t + \phi_1) \cos(w_n t + \phi_2) \tag{A-22}$$

The rectified component is

$$w_1 w_2 \doteq \frac{\dot{w}_{10}\,\dot{w}_{20}}{w_n^2} \cdot \frac{1}{2} \cos(\phi_1 - \phi_2) \tag{A-23}$$

Assume worst case phase, i. e., $\cos(\phi_1 - \phi_2) = \pm 1$, the constant output rate error of gyro 1 due to motion about the IAs of gyros 1 and 2 is:

$$\delta w_1 = \frac{1}{2} \frac{1}{H} \frac{\dot{w}_{10}\,\dot{w}_{20}}{w_n^2} = 2.12°/\text{hr} \tag{A-24}$$

for

$$\frac{1}{H} = .001 \text{ sec} \tag{A-25}$$

$$\dot{w}_{10} = \dot{w}_{20} = 18 \text{ rad/sec} \tag{A-26}$$

$$w_n = 20 \text{ Hz} \tag{A-27}$$

An error of this magnitude would cause completely unsatisfactory performance of the inertial system.

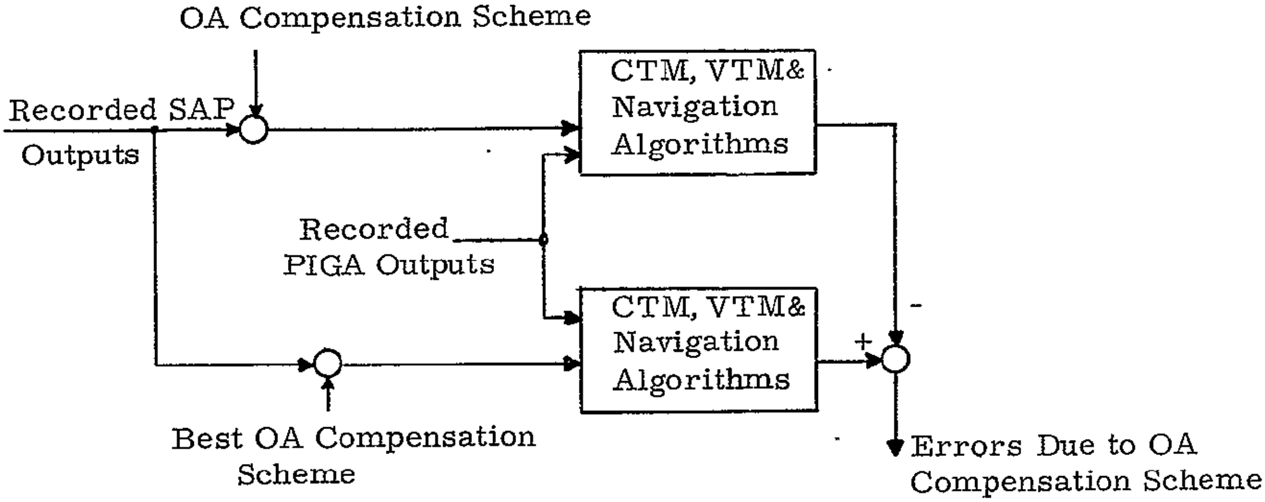

Figure A-3 Evaluation of SAP OA Compensation Schemes

Providing super position theory can be applied, it makes no difference whether the recorded SAP outputs have or have not already been compensated for the effects of other error sources. Similarly, the PIGA outputs may be simulated, if desired. For design purposes, it may be useful to directly compare the outputs of the compensation schemes.

The OA compensation scheme to be tested is compared to a "best" OA compensation scheme*. The best OA compensation scheme should <u>not</u> be a perfect compensation scheme since a perfect OA compensation scheme is unstable. This can be shown by the following simplified analysis. Consider the β's to be constant at 45°. Then

*The performance degradation due to no compensation at all is determined by using only the "Best" compensation scheme in Fig. A-3.

$$
\begin{bmatrix} (w^b_{pb})_1 \\ (w^b_{pb})_2 \\ (w^b_{pb})_3 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{I}{\sqrt{2}\,H}p & -\frac{I}{\sqrt{2}\,H}p \\ -\frac{I}{\sqrt{2}\,H}p & 1 & -\frac{I}{\sqrt{2}\,H}p \\ -\frac{I}{\sqrt{2}\,H}p & -\frac{I}{\sqrt{2}\,H}p & 1 \end{bmatrix} \begin{bmatrix} (w^b_{ib})_1 \\ (w^b_{ib})_2 \\ (w^b_{ib})_3 \end{bmatrix} \tag{A-34}
$$

where p is the Laplace transform operator. Eq. (A-34) is solved for the actual body rate vector to obtain:

$$
\begin{bmatrix} (w^b_{ib})_1 \\ (w^b_{ib})_2 \\ (w^b_{ib})_3 \end{bmatrix} = \frac{1}{1 - \left[\frac{I}{\sqrt{2}\,H}p\right]^3} \begin{bmatrix} 1 & \frac{I}{\sqrt{2}\,H}p & \left(\frac{I}{\sqrt{2}\,H}p\right)^2 \\ \left(\frac{I}{\sqrt{2}\,H}p\right)^2 & 1 & \frac{I}{\sqrt{2}\,H}p \\ \frac{I}{\sqrt{2}\,H}p & \left(\frac{I}{\sqrt{2}\,H}p\right)^2 & 1 \end{bmatrix} \begin{bmatrix} (w^b_{pb})_1 \\ (w^b_{pb})_2 \\ (w^b_{pb})_3 \end{bmatrix} \tag{A-35}
$$

The presence of the term $\frac{1}{1 - \left[\frac{I}{\sqrt{2}\,H}p\right]^3}$ shows the instability (a pole is in the right half plane). Hence a perfect OA compensation scheme should not be used for long term navigation. A technique that can be used is given in Ref. [32].

The procedure shown in Fig. A-3 and discussed in this section can also be used to evaluate algorithms for compensation of other error sources (such as coning and inter-axis coupling).

APPENDIX B

MEASUREMENT OF VAN VIBRATION ENVIRONMENT

The van angular vibration environment should be measured about three nominally orthogonal axes, according to the following considerations:

1. Accuracy of measuring magnitude of the vibrations should be at least 7 to 10%.
2. Phase error between any two pairs of axes should be less than 2% (7°). At 40 Hz, this corresponds to a time synchronization requirement of .4 ms.
3. For each test condition, data should be taken for at least 20 sec, and preferably 2 minutes. There is no need to take data for longer than 5 minutes.
4. It is desirable to be able to measure any angular rates greater than approximately 1°/sec about any axes, whatever the frequency. For example, a .5° vibration at 1 Hz corresponds to an angular rate and acceleration of 3.14°/sec and .33 rad/sec^2, respectively. If this were characteristic of a van vibration component due to the trailer/ generator, the effect would be significant.
5. Measurements from the MISER van test program [1] indicated angular accelerations were below 20 rad/sec^2 and frequencies less than 25 Hz.

Measurements should be made under a variety of conditions, including the following:

1. Van parked in the alignment building, engine off and trailer/generator disconnected physically.

2. Van parked on the roadside, engine off but generator connected. Repeat with engine on and for a range of different wind conditions.

3. Van and generator moving along a smooth paved road at 20 mph with normal tire pressures. Repeat for other speeds and lower tire pressure.

4. Repeat No. 3 above for more rough roads, railroad tracks, bumps, and a range of wind conditions.

5. Repeat No. 3 above for a range of turning rates and accelerations and decelerations.

6. Van started abruptly from a stop by racing the van engine and the quickly engaging the clutch.

The effect of the trailer/generator can be determined by repeating several of the tests for the same conditions but with the trailer disconnected (assuming a separate power source sufficient to excite the vibration pickups and drive the recorder can be carried within the van).

Some considerations in making vibration measurements are included in Refs. [33, 1, 11, 12] particularly regarding problems arising due to vibration sensor mounting resonances.

APPENDIX C

ANALYTICAL ALIGNMENT CONSIDERATIONS

C.1 INTRODUCTION

Analytical alignment of inertial systems has been used in various forms in a number of applications [5, 37, 38, 39]. For the field test programs on the SD-53 inertial system, the analytic alignment scheme can serve two purposes:

- The obvious use of analytical alignment is to align the system prior to entry into the navigation mode, using velocity error measurements over a 10 to 20 minute period. This serves as an alternative procedure to the physical alignment method described in Section 5.4.
- Under certain conditions, the alignment algorithms can be used to estimate the attitude errors and equivalent gyro bias errors while the system is in the navigate mode. The basic procedure is to stop the vehicle for about 10 to 20 minutes while recording the velocity error data. The data may then be processed at the completion of the test period (if necessary) using alignment algorithms.

Several variations of the above basic analytical alignment procedure are as follows:

- A 90° rotation of the vehicle at the end of the 10 to 20 minute period followed by a second alignment run, for better attitude estimation and estimation of all gyro biases. This procedure is described in detail in this appendix and in Section 5.4.

- In addition to utilizing velocity data, the change-in-heading error can be observed and used to significantly improve estimation of the vertical component of gyro bias.

C.2 ALIGNMENT PROCEDURES

The basic block diagram of fine alignment is shown below in Fig. C-1.

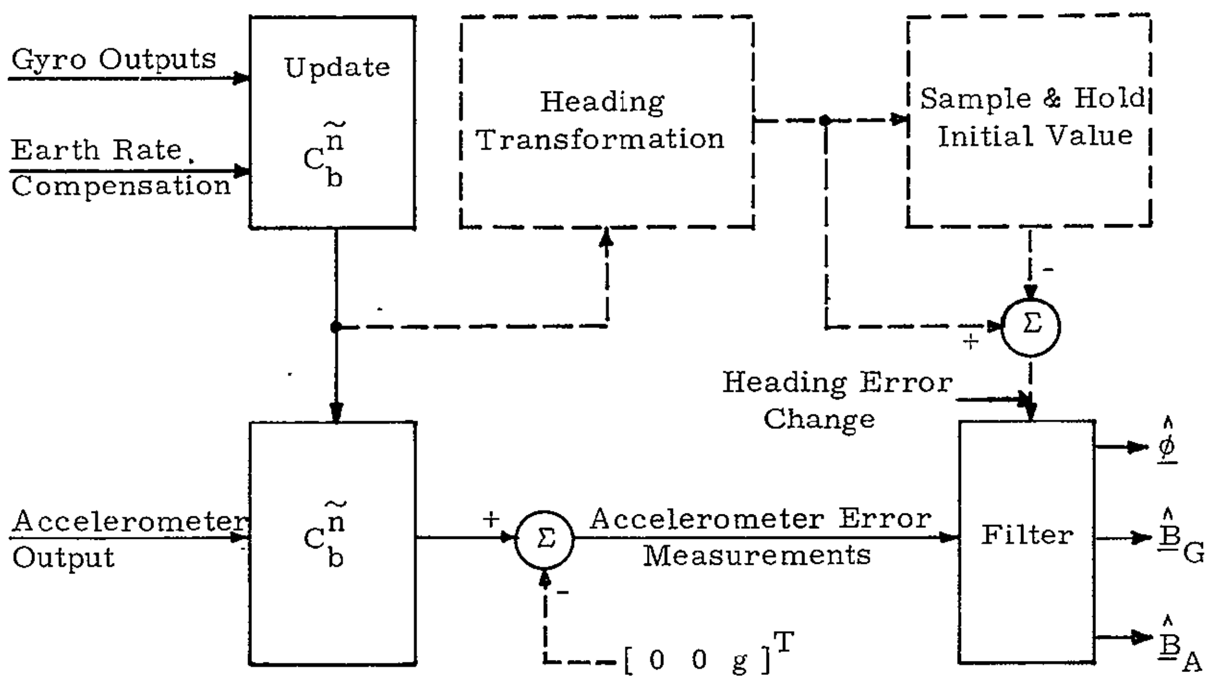

Figure C-1 Block Diagram of Fine Align

If instrument biases are to be estimated, they are required in the b frame. Although they are estimated in the n frame, they can be transformed into the b frame since for the present problem the biases can be assumed constant in the n or $\tilde{n}$ frame (the changes in the elements of

$\tilde{C}^n_b$ are small). With this assumption, the filter can be designed on the basis of a time invariant system and measurement.

Vertical deflections can be compensated (if they are known) at the end of the last stage of fine align, since they appear as equivalent accelerometer biases.

The block entitled "filter" in Fig. C-1 can be a recursive optimal filter, a finite memory filter or a least squares filter. The choice involves tradeoff studies involving mostly on-board computer limitations (scaling, memory and speed tradeoffs).

The two horizontal components of gyro drift rate can be very effectively estimated by reorienting the vehicle by a nominal 90° rotation about vertical and repeating the alignment (see Fig. C-2). The vertical component of gyro bias can be estimated with a significant increase in precision

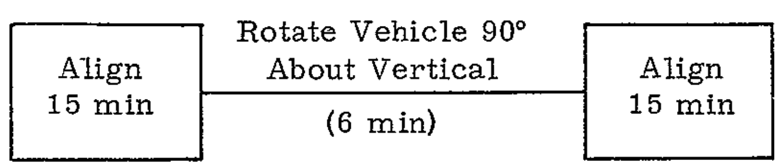

Figure C-2 Alignment Sequence

by measuring change in heading error as shown by dashed lines in Fig. C-1. Another possibility is to rotate the vehicle again about the East axis (about 30°).

C.3 PRELIMINARY ANALYSIS OF EXPECTED ACCURACY

A sampling rate of once every 36 secs was assumed to attenuate the effects of vibration. To obtain this rate the ΔV pulses would be summed for 36 sec.* The noise due to vibrations was assumed independent after 36 sec, and at an RMS level of 10 μg.

The parameters used for the alignment schemes are:**

- Initial Conditions

$$\sigma[\phi_N(0)] = \sigma[\phi_E(0)] = \sigma[\phi_V(0)] = 1°$$

- Gyro Errors
  - Bias $= \sigma[B_{GN}] = \sigma[B_{GE}] = \sigma[B_{GV}] = .050°/hr$
  - Random drift $= \sigma[R_{GN}] = \sigma[R_{GE}] = \sigma[R_{GV}] = .004°/hr$
  - Correlation time $(\tau_G) = 3$ hr
- Accelerometer Errors
  - Random drift $= \sigma[R_{AN}] = \sigma[R_{AE}] = 5\ \mu g$
  - Correlation time $(\tau_A) = 2/3$ hr

---

*In [5], velocity is averaged for 10 sec.

**σ denotes standard deviation.

- Measurement Errors

    - Vibration caused errors $= \sigma[V_1] = \sigma[V_2] = 10\mu g$ (assumed independent or white every 36 $\widehat{sec}$).
    - For the measurement of heading error change, a 20 $\widehat{sec}$ white noise error $\sigma[V_3]$ is assumed.

For the analysis, the analytical alignment was assumed to be performed at a latitude of 45°.

Using the models shown in Fig. C-3, the expected accuracy of analytical alignment with and without measurements of heading error change are shown in Figs. C-4 and C-5. The precision improvement in biasing the vertical component of gyro drift, due to measuring heading error change, is about an order of magnitude, and approaches the precision with which the other components of gyro drift can be estimated. A slight improvement in the precision of the initial heading alignment also results. The effect of the 90° vehicle rotation is also apparent. During the first alignment period (before the vehicle is rotated), the error in the East component of gyro drift is equal to the assumed bias σ (viz., .050°/hr), since the term is unobservable, and the corresponding heading error is almost 1000 $\widehat{sec}$. Upon rotating the vehicle 90°, the component of gyro drift originally in the E/W direction now lies in the N/S direction, where it can be observed and estimated, as shown in Fig. C-5. The heading error also decreases radically, as shown in Fig. C-4, by more than a factor of 20.

$$
\begin{bmatrix} \dot{\phi}_N \\ \dot{\phi}_E \\ \dot{\phi}_V \\ \delta\dot{B}_{GN} \\ \delta\dot{B}_{GE} \\ \delta\dot{B}_{GV} \\ \delta\dot{R}_{GN} \\ \delta\dot{R}_{GE} \\ \delta\dot{R}_{GV} \\ \delta\dot{R}_{AN} \\ \delta\dot{R}_{AE} \\ \dot{\phi}_V(o) \end{bmatrix}
=
\begin{bmatrix}
 & -w_{ie}\sin La & & -1 & & & -1 & & & & & \\
w_{ie}\sin La & & w_{ie}\cos La & & -1 & & & -1 & & & & \\
 & -w_{ie}\cos La & & & & -1 & & & -1 & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & & & & & & \\
 & & & & & & -\frac{1}{\tau_G} & & & & & \\
 & & & & & & & -\frac{1}{\tau_G} & & & & \\
 & & & & & & & & -\frac{1}{\tau_G} & & & \\
 & & & & & & & & & -\frac{1}{\tau_A} & & \\
 & & & & & & & & & & -\frac{1}{\tau_A} & \\
 & & & & & & & & & & &
\end{bmatrix}
\begin{bmatrix} \phi_N \\ \phi_E \\ \phi_V \\ \delta B_{GN} \\ \delta B_{GE} \\ \delta B_{GV} \\ \delta R_{GN} \\ \delta R_{GE} \\ \delta R_{GV} \\ \delta R_{AN} \\ \delta R_{AE} \\ \phi_V(o) \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ W_{GN} \\ W_{GE} \\ W_{GV} \\ W_{AN} \\ W_{AE} \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix}
=
\begin{bmatrix}
0 & g & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}
\begin{bmatrix} \phi_N \\ \phi_E \\ \phi_V \\ \delta B_{GN} \\ \delta B_{GE} \\ \delta B_{GV} \\ \delta R_{GN} \\ \delta R_{GE} \\ \delta R_{GV} \\ \delta R_{AN} \\ \delta R_{AE} \\ \phi_V(o) \end{bmatrix}
+
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}
$$

Note: For analytical alignment without measurements of the change in heading error, the $\phi_V(o)$ state and Z_3 measurement are deleted.

Fig. C-3 Math Model for Analytical Alignment

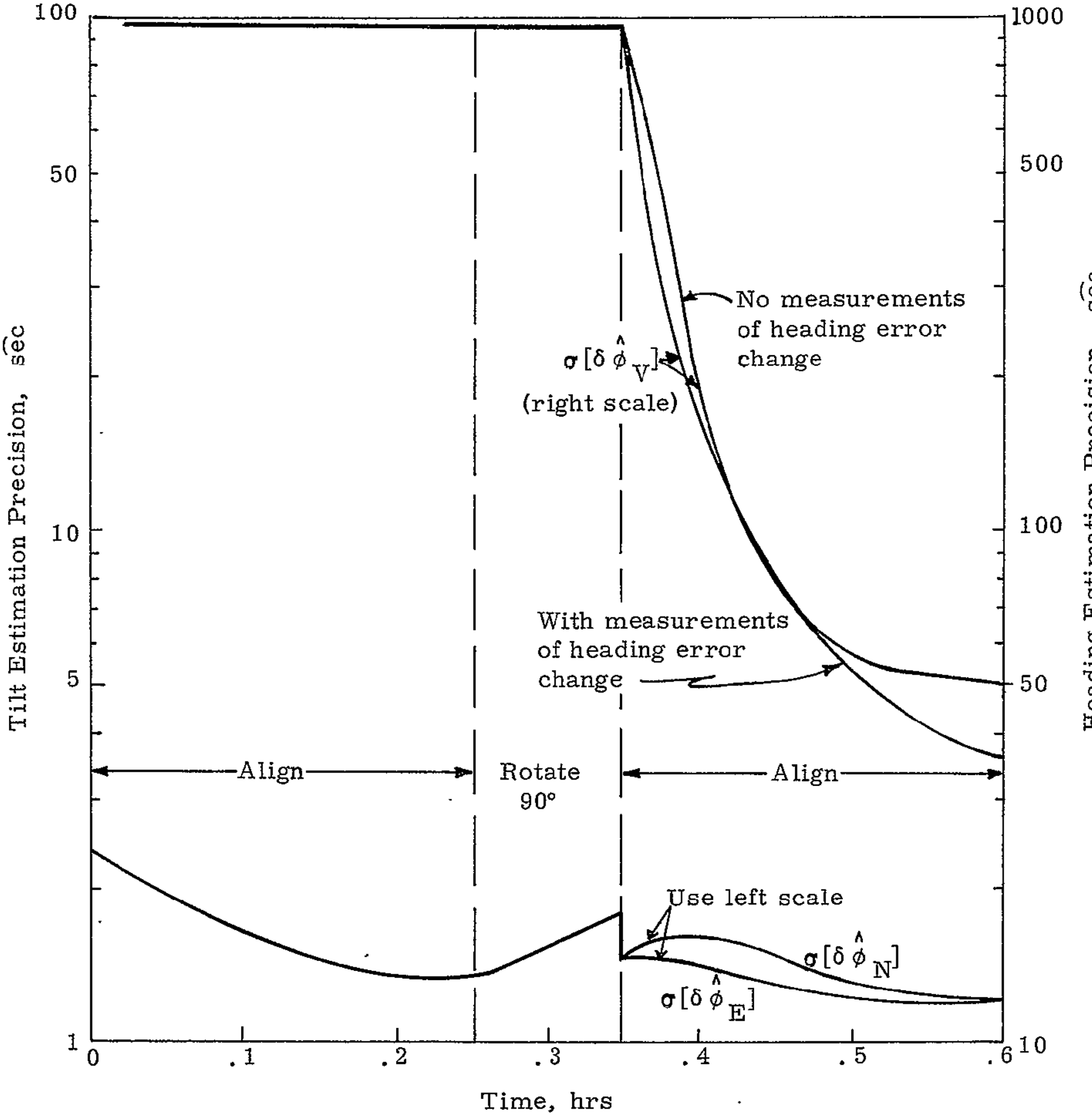

Figure C-4 Heading and Tilt Estimation Precision Using Analytical Alignment

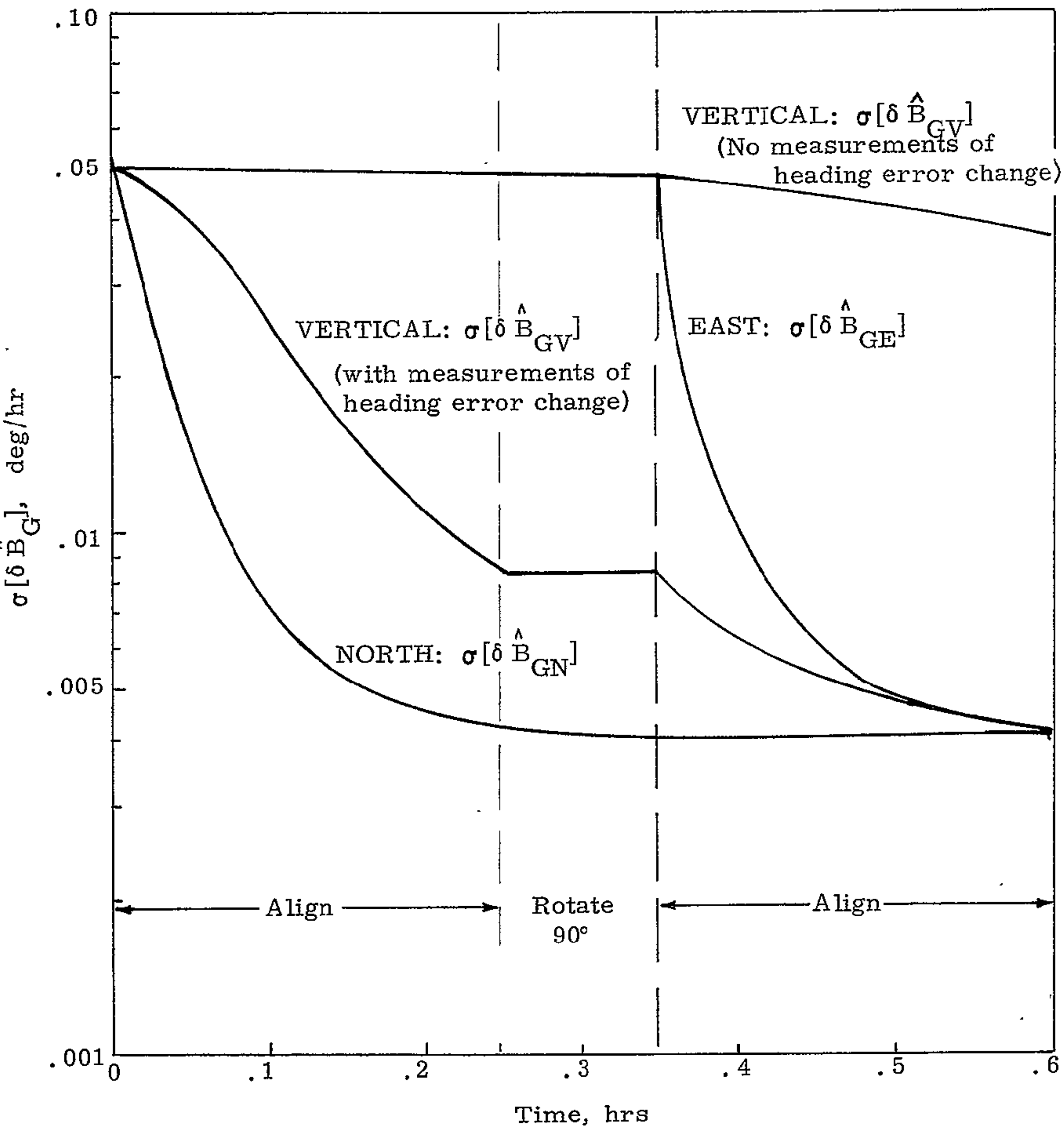

Figure C-5 Gyro Bias Estimation Precision Using Analytical Alignment

APPENDIX D

SPECIAL TESTS RECOMMENDED IN SUPPORT OF FIELD TEST PROGRAM

There are several special tests that should be performed in support of the field test program. Although not considered in depth here, since they are not directly a part of the field tests, the results are expected to contribute significantly to the total SD-53 system evaluation effort. Some of the tests are as follows:

1. Determine in the laboratory the SAP OA to IA frequency response, in support of the OA angular acceleration sensitivity studies.
2. Check sign conventions of certain calibration terms by performing the calibrations; then inserting intentional changes wherever possible and repeating the calibrations. Considerable test time in the field may be saved by avoiding inadvertent sign reversals.
3. Cross coupling of SAP and PIGA gyro internal misalignments due to encoder zeroing error can be checked by biasing the gyro pickoff signal an amount proportional to the IA about OA internal misalignments. Repeating the calibration tests that estimate the internal misalignments will provide a measure of the cross-coupling effect of the IA about SA internal misalignment, due to encoder zeroing error.

4. The SD-53 inertial system attitude algorithm should be checked on the laboratory test stand to verify it for use in the analytical alignment process when indicated heading changes are required to estimate the vertical component of gyro drift. The check should also verify the algorithm for the flight test program, where attitude of the aircraft may be required at the time photographs are taken for position fixing.

5. Perform tests to verify proper operation and adequate accuracy of the yaw monitor, should such a reference be required for the analytical alignment technique (see Section 4.5 and Appendix C).

6. Perform tests to determine error characteristics of photograph technique for determining position of aircraft.

APPENDIX E

RESET TECHNIQUE FOR ESTIMATION OF INTERNAL INERTIAL SYSTEM ERRORS

During the van and aircraft tests precise external position information (fixes) are available periodically, and at times velocity of the vehicle is also known. This information can be combined at discrete times with the inertial system outputs to estimate the error states (x) as depicted in Fig. E-1*. The process is referred to as "reset" since the estimates of the error states can be used to correct, or reset, the system to provide improved performance. In this application, reset will be used only to estimate internal errors in the inertial navigation system (INS). The purpose of this appendix is to indicate (in a preliminary way) the precision with which the INS errors may be estimated for several different field test conditions and characteristics of the fix taking devices. The nominal performance of the INS is determined first, for the same conditions assumed in the reset analysis that follows, in order to illustrate the significantly better precision of the reset process compared to the basic accuracy of the INS itself.

Since simplified math models were used, a more complete analysis would be required to more fully evaluate the technique. Analyses should also be made to determine the trade-off between estimation precision vs. amount of data used to make the estimates so that real or equivalent parameter shifts can be readily detected.

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*The process has been used in various forms in a number of other applications [30, 5, 34, 17], including the use of Kalman filters and least squares estimation.

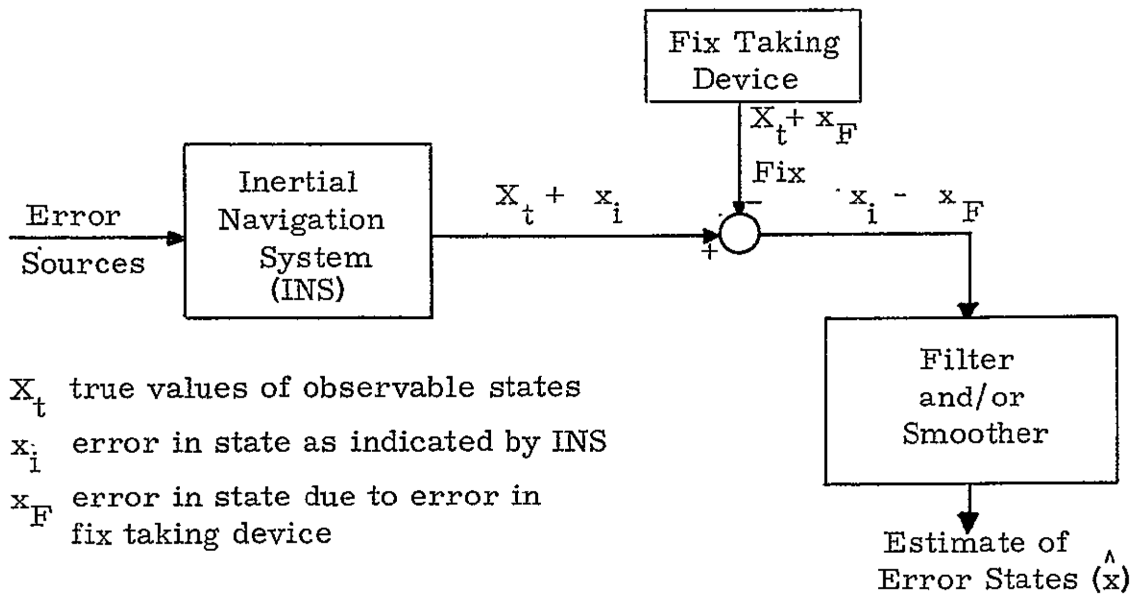


Figure E-1 Error State Estimation (Reset)

E.1 SYSTEM MATH MODEL

The basic error equations of the inertial navigation system (INS) for the van type of environment are:*

$$\dot{\epsilon}_N = -w_{ie} \sin L\, \epsilon_E - w_{ie} \sin L\, \delta La + \delta V_E + u(w)_N \qquad \text{(E-1)}$$

$$\dot{\epsilon}_E = w_{ie} \sin L\, \epsilon_N + w_{ie} \cos La\, \epsilon_V - \delta V_N + u(w)_E \qquad \text{(E-2)}$$

---

*The equations for the aircraft application are similar, except additional terms proportional to the aircraft velocity must be included. For the relatively low van speeds, the terms can be neglected.

$$\dot{\epsilon}_V = -w_{ie} \cos La\, \epsilon_E - w_{ie} \cos La\, \delta La - \sin L \frac{\delta V_E}{r} + u(w)_V \tag{E-3}$$

$$\delta \dot{V}_N = g\epsilon_E - 2w_{ie} \sin La\, \delta V_E + u(a)_N \tag{E-4}$$

$$\delta \dot{V}_E = -g\epsilon_N + 2w_{ie} \sin La\, \delta V_N + u(a)_E \tag{E-5}$$

$$\delta V_E = r\delta \dot{L}a \tag{E-6}$$

$$\delta V_E = r\delta \dot{L}o \cos La \tag{E-7}$$

where

ϵ_N, ϵ_E	North and East tilt errors
ϵ_V	azimuth error
δLa	North position error (angle units)
δV_N	North velocity error
δLo	East position error (angle units)
δV_E	East velocity error
r	radius of earth
$u(w)$	gyro drift rate errors
$u(a)$	acceleration type errors
w_{ie}	earth rate
La	Latitude
g	gravity

The inertial system is assumed to be undamped. For the results presented in this report the error state diagram is determined from Eqs. (E-1 through E-6).

E. 2 NOMINAL PERFORMANCE OF INS

To illustrate the significantly better precision of the reset process compared to the basic accuracy of the INS itself, the nominal performance of the INS must first be determined. For the results presented in this appendix, the system math model of Section E. 1 was used, assuming the following conditions and error sources:

Initial Conditions

Tilts	$\sigma\epsilon_E(0) = \sigma\epsilon_N(0) = 2 \widehat{\sec}$
Azimuth	$\sigma\epsilon_V(0) = 60 \widehat{\sec}$
Velocity	$\sigma[\delta V_N(0)] = \sigma[\delta V_E(0)] = 0$
Position	$\sigma[\delta La(0)] = \sigma[\delta Lo(0)] = 0$
Latitude	45°

Gyro Errors [u(w)]

bias	$\sigma[B_G] = .004°/hr$
random	$\sigma[R_G] = .004°/hr$, correlation time $(\tau_G) = 3$ hr.

Accelerometer Errors [u(a)]

random	$\sigma[R_A] = 5\ \mu g$, correlation time $(\tau_A) = 2/3$ hr.

The deflections of the vertical are assumed known along the test path and hence do not appear as error sources in the equations.

Results of the analysis are shown in Figs. E-2 through E-5 for the RMS navigation errors (position, velocity, platform tilt and heading error) under the van test conditions. The reset estimation errors for the various parameters and several test conditions, as determined in Section E. 3, are also included on the curves to illustrate the efficacy of the estimation process. The precision with which the gyro bias errors can be estimated is shown in Fig. E-6.

E. 3 RESET PRECISION USING POSITION AND VELOCITY FIXES

The reset scheme used to estimate the performance of the INS is based on combining INS outputs with external information (position and/or velocity) via Kalman filters and/or smoothers using DRC's state variable programs. The external information need not be uniformly spaced in time and can also be continuous. Studies of smoothing aircraft INS outputs with sparse position fixes have been studied by DRC [26].

The system and Kalman filter models used in the analysis of the reset parameter estimation precision are given in Section E. 1, and assumed conditions are as shown in Table E. 1. Several combinations of van and aircraft fix conditions were chosen to represent the effect of frequent good position (and velocity) fixes, as well as the effect of less frequent, poorer quality position fixes. The RMS estimation errors are presented in Figs. E-2 through E-6 for each of the three combinations of fix conditions. In addition, the effect of smoothing the information is shown for the aircraft tests.

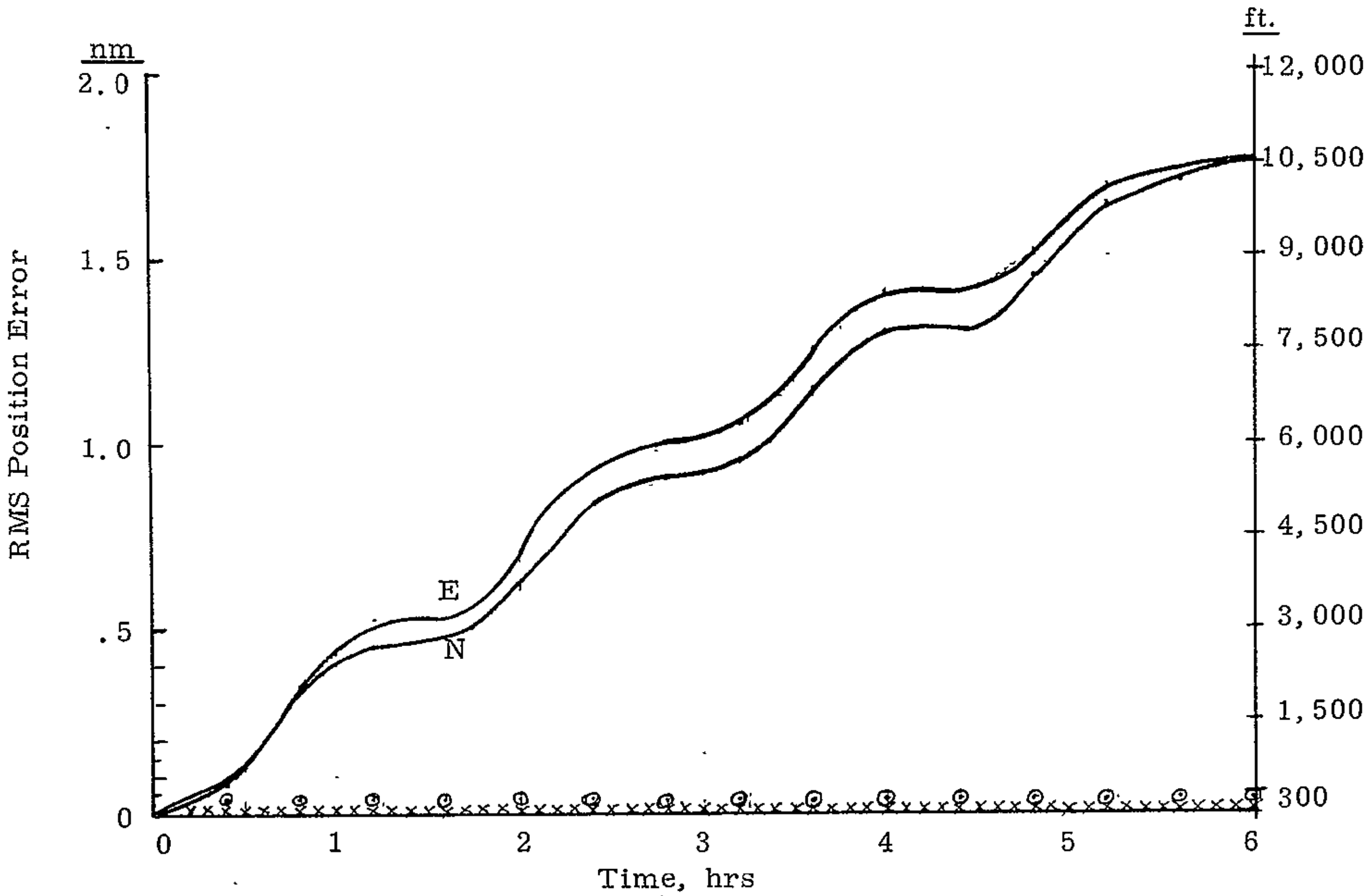
The reset estimation precision is significantly better than the basic velocity and tilt accuracy of the INS itself. In the case of heading, the estimation precision improves somewhat following the analytical alignment, until the effect of random East gyro drift predominates. In Fig. E-6, the estimation precision of the North and Vertical gyro drifts improves as more fixes are taken, whereas the East gyro drift cannot be separated from the initial heading error.

Assumed Conditions	Van Tests (No Vel. Fixes)	Van Tests (with Vel. Fixes)	Aircraft Tests
Fix Conditions			
Position	60 feet	60 feet	240 feet
Velocity	none	.02 feet/sec	none
Fix Frequency	6 minutes	6 minutes	24 minutes
Initial Conditions			
Heading	60 $\widehat{\text{sec}}$	60 $\widehat{\text{sec}}$	60 $\widehat{\text{sec}}$
Tilts	2 $\widehat{\text{sec}}$	2 $\widehat{\text{sec}}$	2 $\widehat{\text{sec}}$
Instrument Errors			
Gyros	.004°/hr bias .004°/hr random, C. T. 3 hrs.	.004°/hr bias .004°/hr random, C. T. 3 hr.	.004°/hr bias .004°/hr random, C. T. 3 hr.
Accelerometers	5 μg random, C. T. $\frac{2}{3}$ hr	5 μg random, C. T. $\frac{2}{3}$ hr.	5 μg C. T. $\frac{2}{3}$ hr.
Test Conditions			
Latitude	45°	45°	45°
Vehicle Velocity	(negl)	(negl)	200 kts. East

Notes: (1) The initial system conditions and inertial instrument errors are the same as those used to determine the nominal INS performance (see Section E. 2).

(2) Unless noted otherwise, all errors are assumed to be white noise.

Table E-1 Conditions Assumed in Analysis of Reset Precision

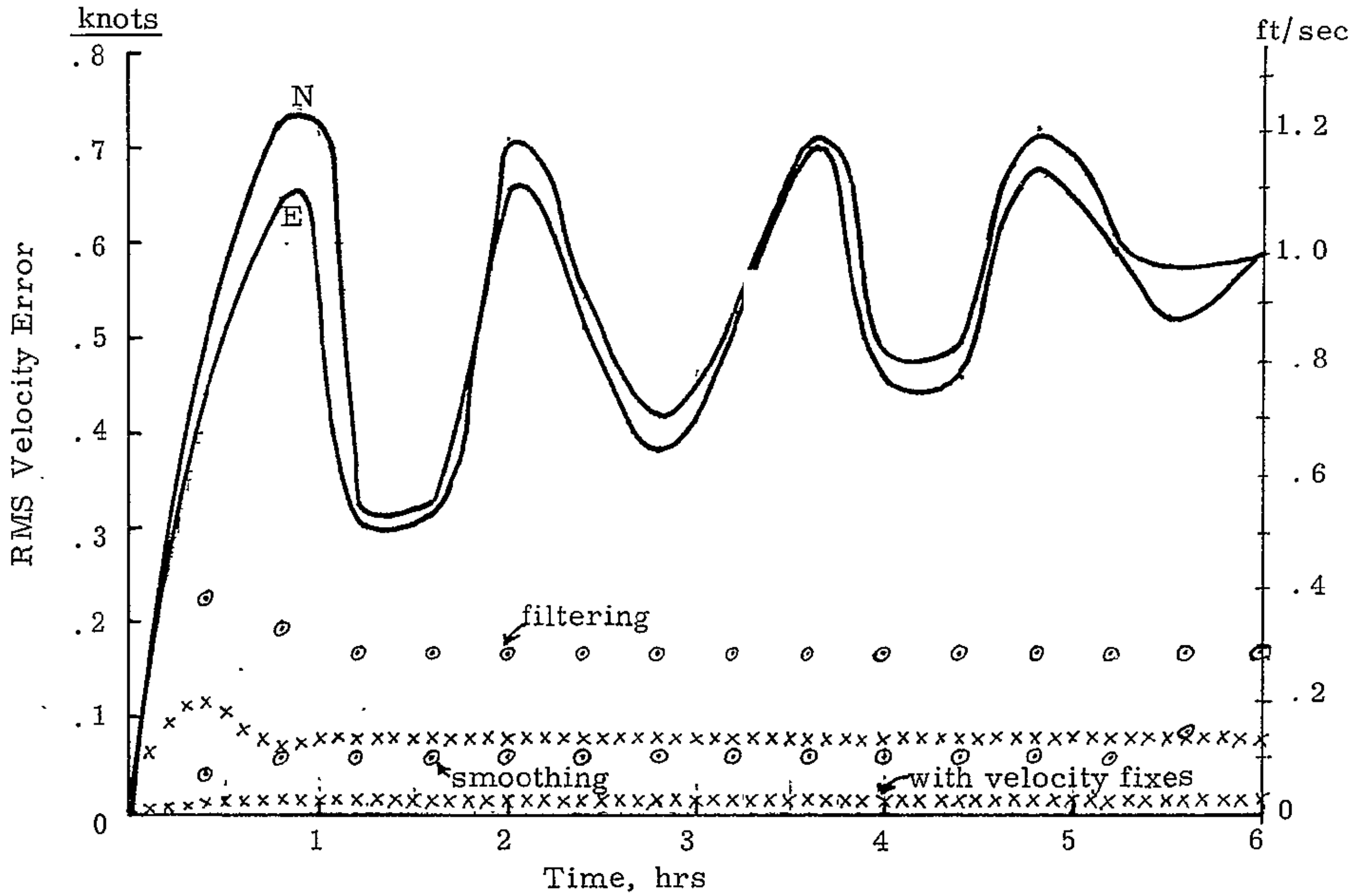


Legend

⊙ aircraft position fix accuracy

x van position fix accuracy

Figure E-2 Inertial System Position Errors for Van Test Conditions



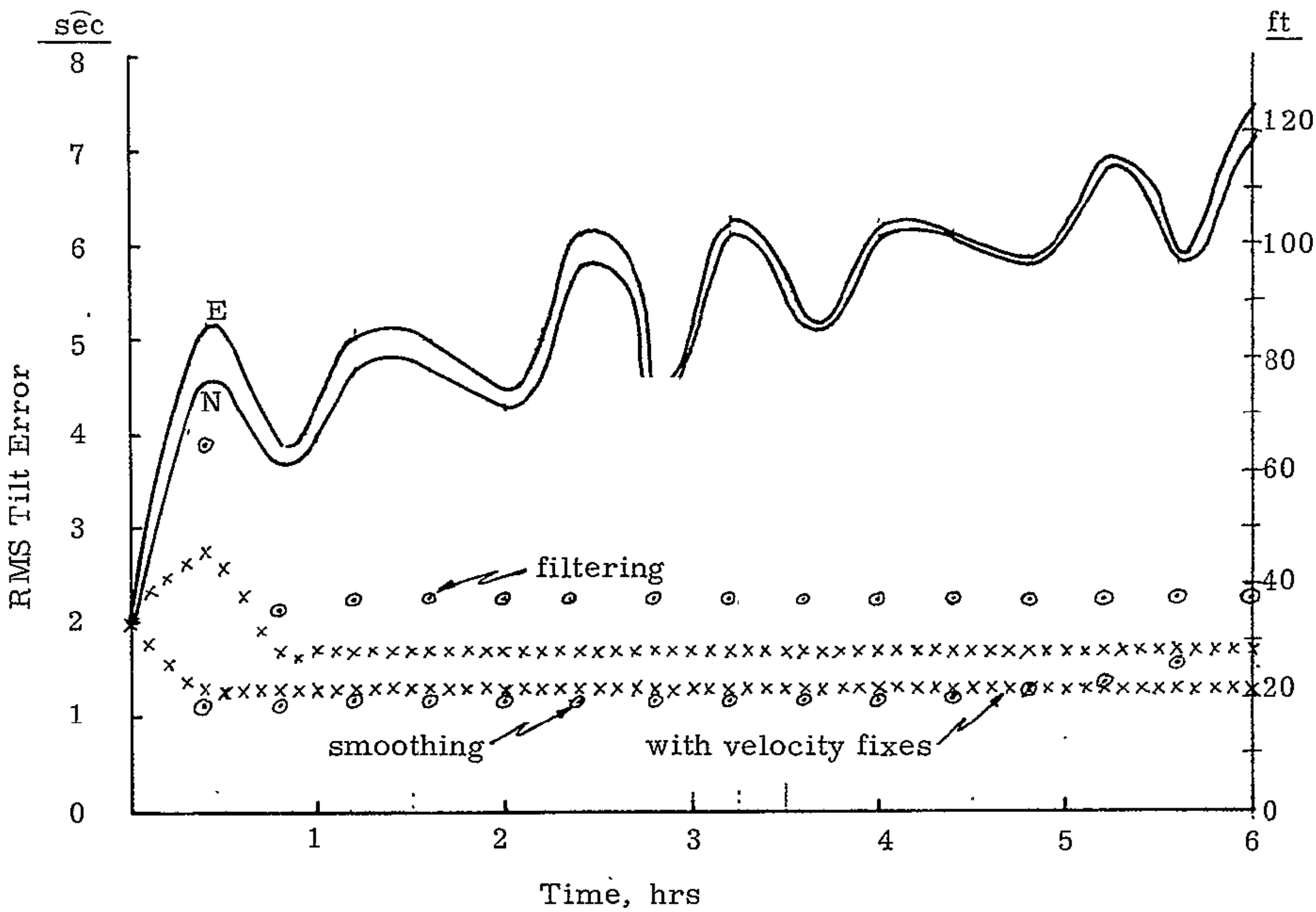
Notes:

⊙ Velocity estimation error using reset (aircraft conditions)

x Velocity estimation error using reset (van conditions)

No velocity fixes taken, except as noted.

Figure E-3 North and East Velocity Errors of Inertial System and Reset Estimation Precision



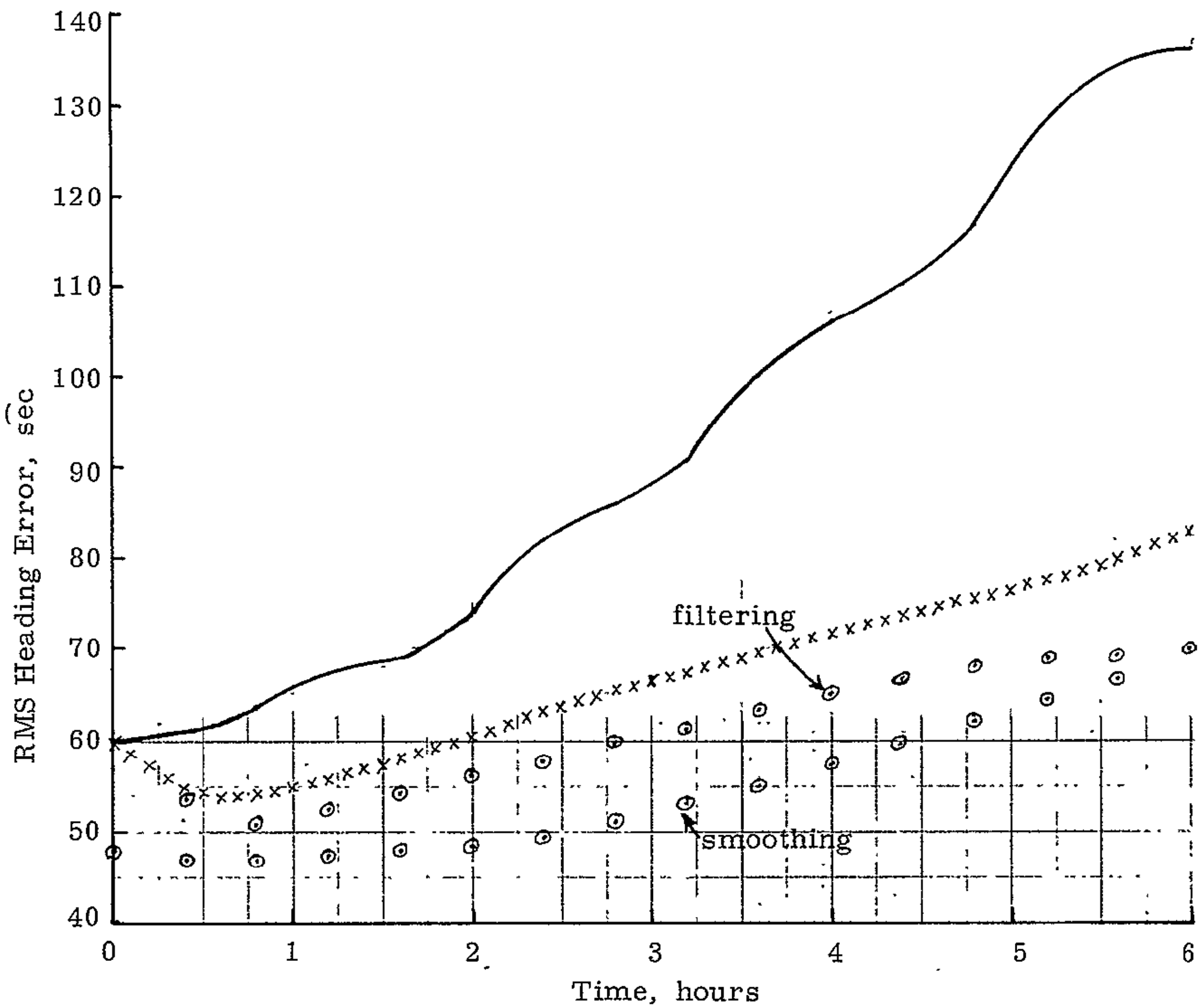
Notes:

⊙ Tilt estimation error using reset (aircraft conditions)

x Tilt estimation error using reset (van conditions)

No velocity fixes taken, except as noted.

Figure E-4 Inertial System North and East Tilts and Reset Estimation Precision



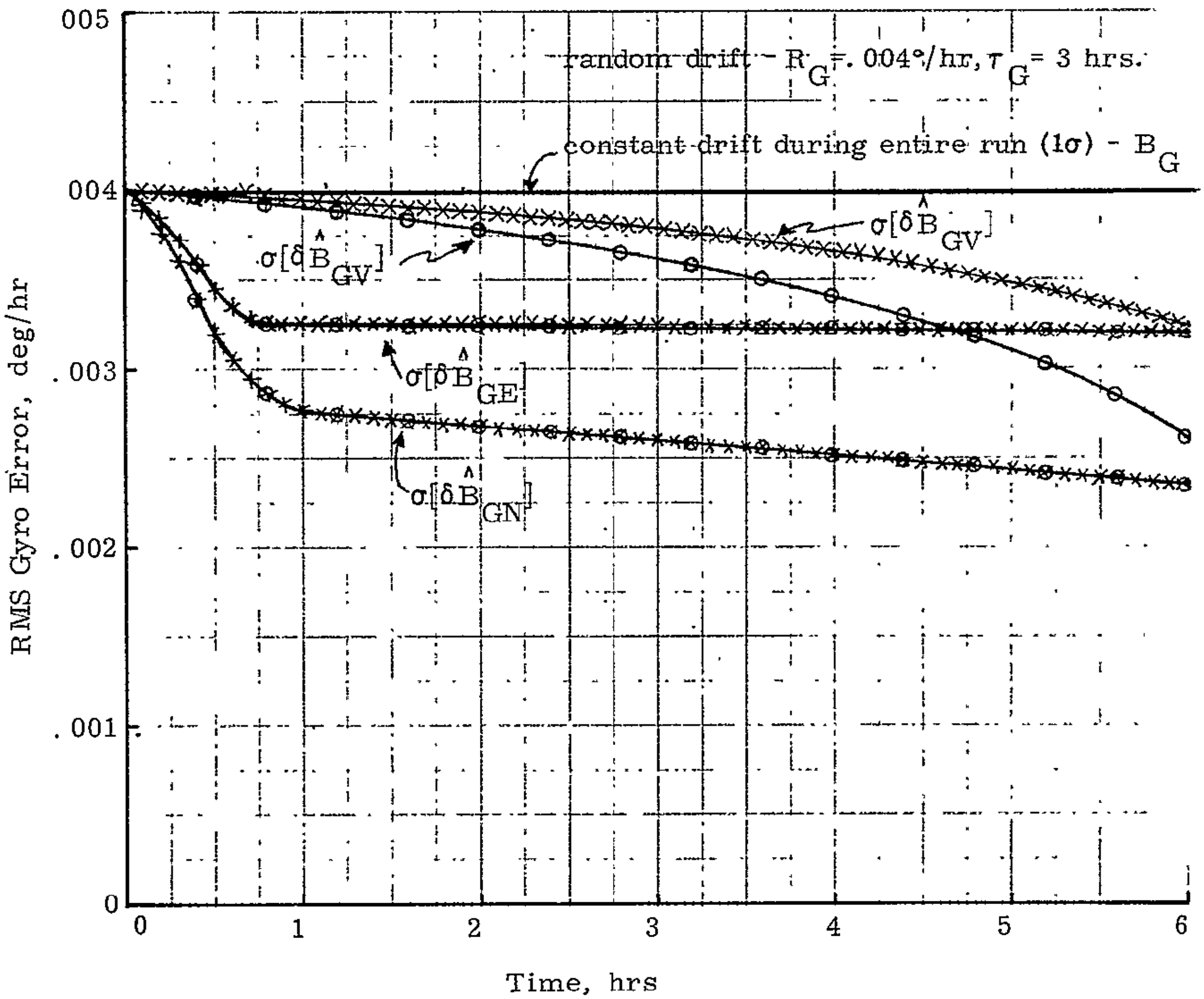
Notes:

⊙ Heading estimation error using reset (aircraft conditions)

x Heading estimation error using reset (van conditions)

Velocity fixes have little effect on estimation accuracy.

Figure E-5 Inertial System Heading Error and Reset Estimation Precision



Notes:

⊙ Gyro bias estimation errors using reset (aircraft conditions)

x Gyro bias estimation errors using reset (van conditions)

Figure E-6 Inertial System Gyro Errors and Reset Estimation Precision

APPENDIX F

SPECIAL ANALYSES IN SUPPORT OF THE FIELD TEST PROGRAM

There are several special analyses that should be performed in support of the field test program analyses. The first group of analyses are required to implement the test program as outlined. The second group of analyses are not directly a part of the van and flight test programs, and so are not considered in depth here. However, the results are expected to contribute significantly to the total SD-53 system evaluation effort.

F.1 REQUIRED ANALYSES

The special analyses required to implement the test program as outlined include the following:

1. Development of a suitable reset process to characterize errors and error sources within the inertial system (when operating in its normal navigate mode)* using external fix information. Aspects of the design and analysis are considered in Section 6 and Appendix E.

2. Development of a yaw monitor, if yaw motion of the van or aircraft is excessive to achieve satisfactory biasing of the vertical gyro (see Sections 4.5 and 5.4 and Appendix C).

3. Derivation of equations and a technique to obtain position of aircraft from photographs and inertial system indicated

*The process should be developed primarily for operations in NEV coordinates, and secondarily in Tangent Plane coordinates.

attitude, as discussed in Section 4.3.2. The analysis is to include consideration of calibration and alignment of the camera system.

4. Derivation of necessary equations to convert position references into geographic or tangent plane coordinates, as necessary.

5. If the effect of alternate alignment techniques is to be estimated as described in Section 6.5.4, a small statistical analysis would be required to determine the degradation of navigation performance due to degradations in the pre-navigate alignment process. Alternate alignment techniques include no 90° rotation of the vehicle, no yaw monitor, and no measurement of heading error change during the alignment.

6. If a fifth wheel speed reference is to be provided, analyses are required to specify equipment functional and performance requirements, data processing algorithms, etc.

Although both the reset and the analytical alignment algorithms can provide estimates of errors internal to the inertial system, the reset algorithm is not limited to the vehicle being stationary, it utilizes position fixes (primarily), and it can perform smoothing as well as filtering. It is expected to be particularly useful during the flight test phase.

F. 2 ANCILLARY ANALYSES

Although the following analyses are not required to implement the basic field test program, they are recommended in order to check certain aspects of the system design and to increase understanding of the system. The following three analyses should be performed before most of the inertial system tests are run in order to determine if large accuracy degradations are to be expected, and if so, how to provide significant improvements:

1. Using the real time recordings of van (and aircraft) vibration data, for various combinations of vehicle maneuvers and initial head angles of the SAPs, generate a real time simulation of the response of each SAP to at least each of the following error sources:*

   - OA angular acceleration sensitivity
   - coning
   - g and g^2 calibration terms
   - misalignments

   The resulting SAP error functions can then be used to determine the degradation in SD-53 system performance (separately, for each error source). Potential problem areas can be identified, and recommendations made as to which terms should be compensated and to what precision.

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*Potential error sources are discussed in [13, 4, 12, 11].

2. If serious performance degradations are detected, generate compensation algorithms, as required, and/or specify the recommended initial head angle of each SAP. The degree of performance improvement can be estimated by repeating the real time simulations of item 1 above after incorporating the comparison algorithms in the SAP models.

3. Even if no serious degradations are detected (from 1 above), a covariance error analysis may be desirable to determine the effect of other vibration models (e. g., expected booster characteristics), as well as the effect of other error sources.* If the van or aircraft vibrations are significantly different from those expected in the final application(s), studies may be in order for determining how a more representative test environment can be provided. The relative importance of each calibration term and error source can be established, and in conjunction with results from item 1 above, recommendations made concerning terms to be compensated and associated precision requirements.

4. If inertial sensor data has been recorded during real time tests, compensation algorithms for individual error sources can be evaluated using the method outlined in Appendix A. 2

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*Consideration should also be given to a simulation analysis using statistically generated inputs.

(for OA acceleration compensation). The processed output of a particular compensation algorithm to be tested is compared to that from a "perfect"* compensation algorithm (for the same error source), each having been provided with the identical input (from the recorded inertial sensor data). Error sources that can be evaluated this way include:

- OA angular acceleration sensitivity
- coning effects
- g and g^2 sensitivities
- internal and external misalignments
- scale factor errors
- bias errors

The effect of no compensation for the particular error source can be determined using the same method.

Other recommended analyses are as follows:

5. The effect of different transformation and navigation algorithms can be determined using the methods described in Appendix G. If inertial sensor data has been recorded, the algorithm errors can be determined for real time functions of angular rate and linear accelerations. To the extent the system is linear, gyro and accelerometer

---

*Perfect in the sense that the algorithm introduces negligible errors.

errors will cancel, as shown in Fig. G-1. The appendix also describes algorithm evaluation using analytical solutions and simulation.

6. Perform error analyses of SD-53 system under van (and flight) test conditions. Compare predicted performance to measured results in order to verify math models used and to establish confidence and creditability in future predictions of performance expected from the final system design in the final application.

7. If vibration effects are small statistically, combine the measured performance of the inertial system with the predicted effect of the vibration profile (based on covariance or simulation studies) to determine a more realistic estimate of system performance.

8. Determine effect on navigation accuracy of computing and applying resets to real data (post-test time)*. Define a reset regime of fix taking and application of corrections that is expected to be representative of conditions in the final application of the SD-53 system. Note that the effect of Schuler loop velocity damping can be approximated by including frequent velocity fixes. Compare analytic results

*It is recommended that this be done only for NEV coordinates since very accurate position and/or velocity fixes would be required in Tangent Plane coordinates (over the short term). Second phase tests may investigate such possibilities.

statistically to runs during which resets were computed and applied in real time, if data is available. If results are compatible, confidence in math models and analyses are increased.

APPENDIX G

EVALUATION OF TRANSFORMATION AND NAVIGATION ALGORITHMS

The recorded inertial sensor outputs can be used to evaluate the following algorithms:

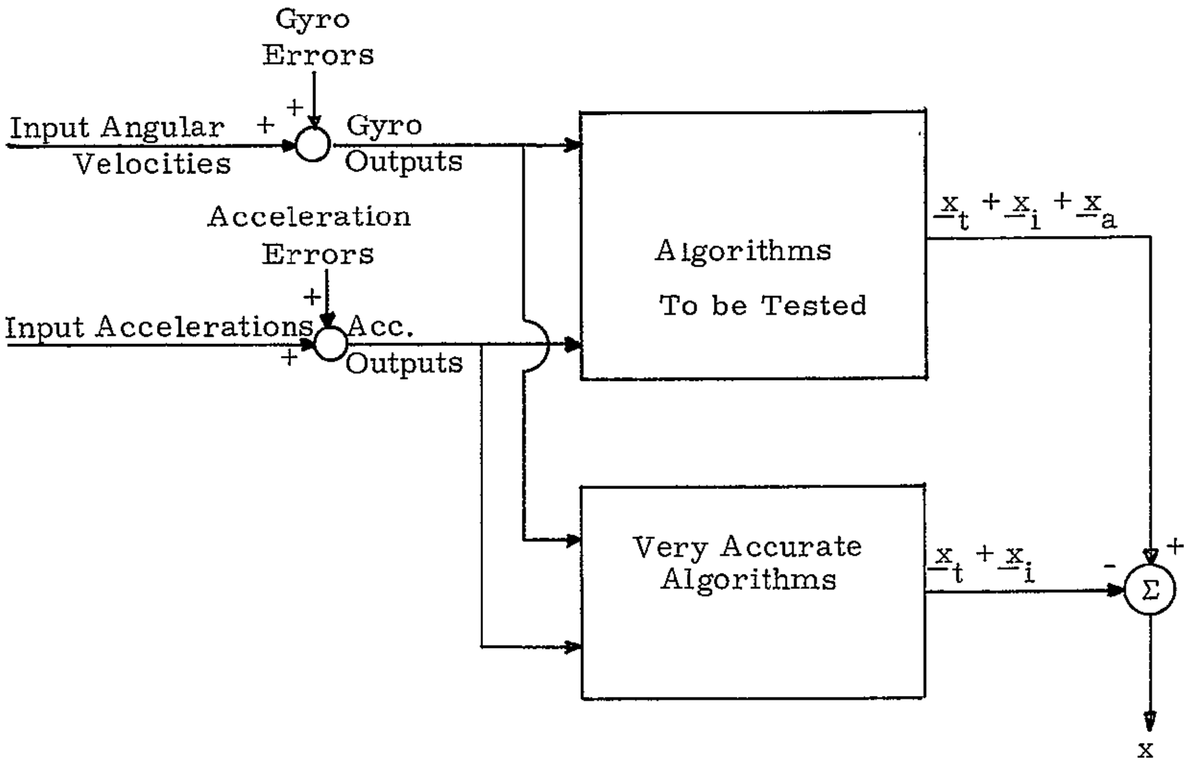
1. Coordinate Transformation Matrix (CTM) update
2. Velocity Transformation Matrix (VTM)
3. Navigation Algorithm

The procedure is to process the recorded gyro and accelerometer outputs using a proposed set of algorithms and then to process the same data using very accurate algorithms. The difference in output can be attributed to the combined errors of the algorithms. The errors caused by the gyros and accelerometers will cancel in the subtraction process, if the system is assumed to be linear. The procedure is depicted pictorially in Fig. G-1. Note that three combinations of the above algorithms can be checked this way (viz., 1, 1 and 2, and 1,2 and 3).

A second method of evaluating algorithm errors is by analysis and simulation. In this procedure the only error source is the algorithm in question. Each of the algorithms listed above can be evaluated sepatately. Detailed procedures of evaluating the coordinate transformation matrix (CTM) algorithm are discussed here.

G.1 EVALUATION OF COORDINATE TRANSFORMATION MATRIX (CTM) UPDATE ALGORITHM

The verification that the CTM update algorithm meets the specification can be done by analytic solution and simulation.



where x_t are true outputs

x_i are errors caused by the inertial instruments (gyros and accelerometers)

x_a are errors caused by the algorithms

Figure G-1 Field Test Evaluation of Algorithm Caused Errors

Various methods of evaluating algorithm errors are discussed after the block diagrams describing the analytic solutions (Fig. G-2) and simulations, (Figs. G-3, G-4, and G-5.)

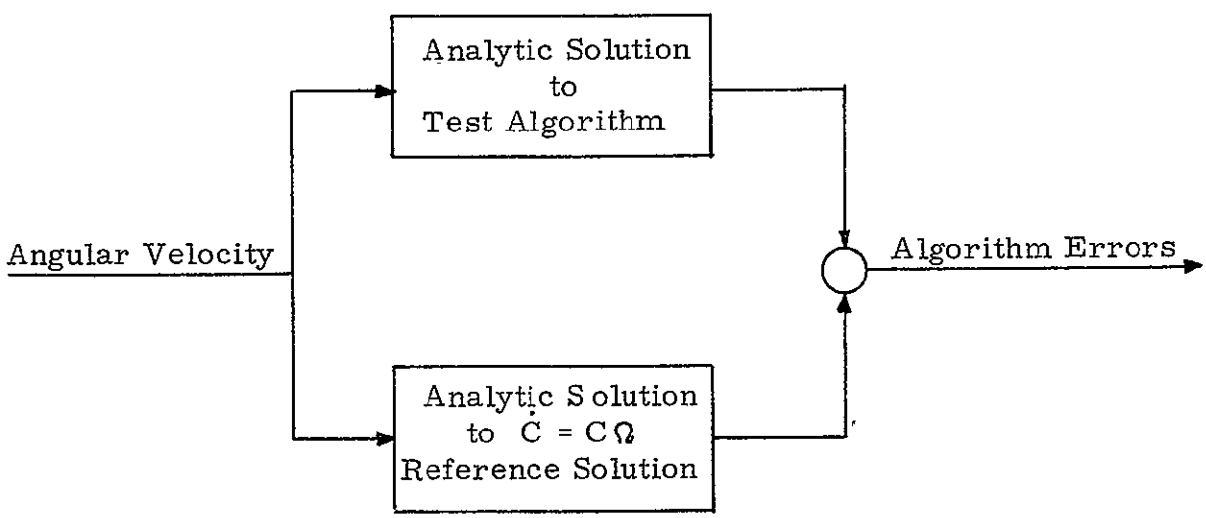


Figure G-2 Analytic Solution for CTM Algorithm Errors

The simplest type of simulation is to compare a digital computer solution of the algorithm to be tested with an analytic solution of the reference, as depicted in Fig. G-3. Since analytical solutions of the CTM differential equation appear in the literature, [40, 41, 42] they will not be reproduced here.

A full simulation generates a computer solution for both the test algorithm and the reference (i. e., true) solution, as shown in Figs. G-4 and G-5.

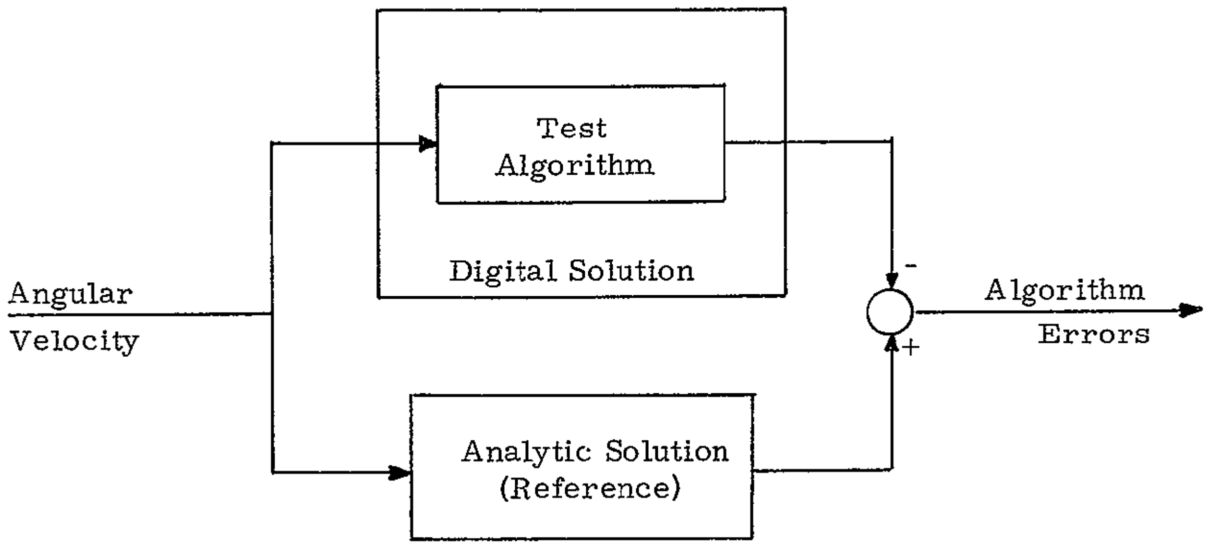


Figure G-3 Simple Simulation for CTM Algorithm Errors

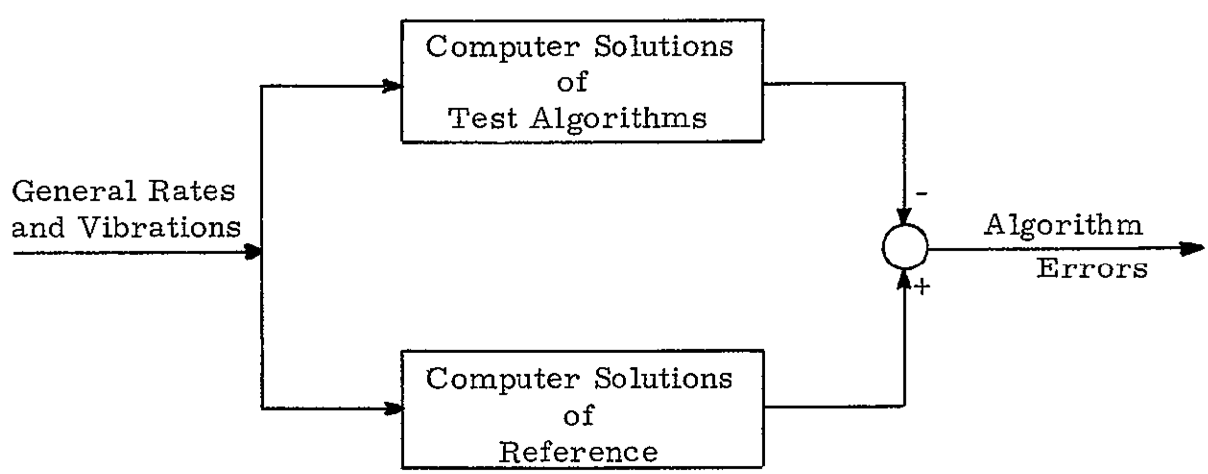


Figure G-4 Full Simulation for CTM Algorithm Error

In general the true or reference solution is generated by an integration algorithm. For a meaningful simulation this algorithm must be significantly more accurate than the update algorithm. An expanded diagram is shown in Fig. G-5.

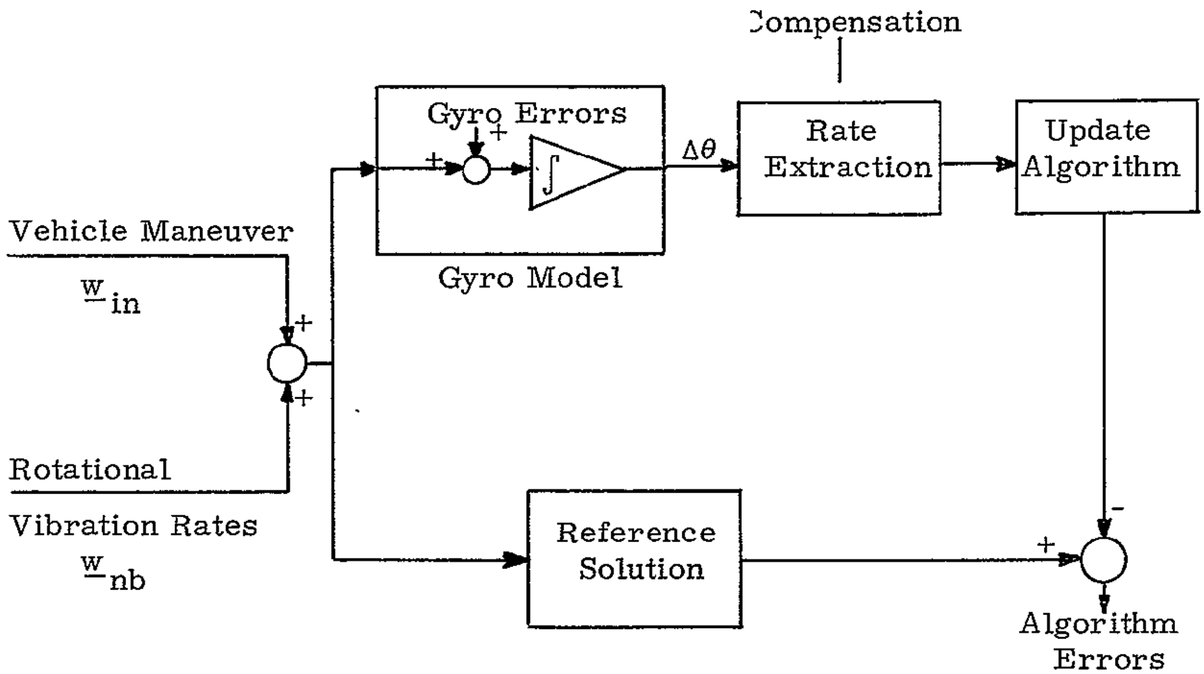


Figure G-5 Expanded Diagram of Full Simulation

The algorithm induced drift is the upper envelope of the algorithm error as depicted in Fig. G-6.

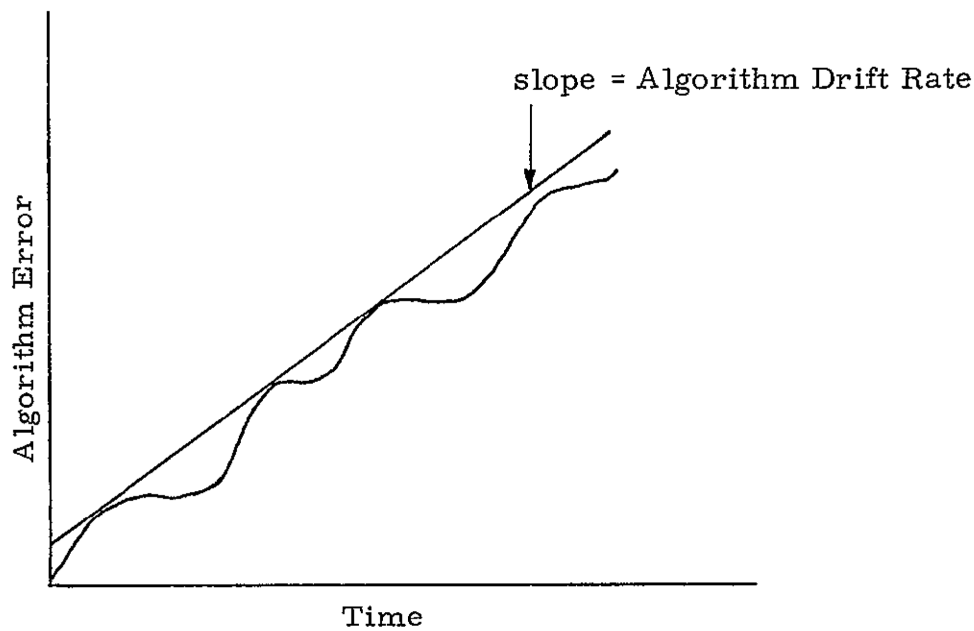


Figure G-6 CTM Algorithm Drift Rate Error

G.2 EVALUATION OF NAVIGATION ALGORITHMS

Two methods can be used to evaluate the Navigation Algorithms. The first method depends upon models of the error sources. Evaluation of the system errors is accomplished by the use of the system error equations. Models for errors in numerical integration schemes and round-off errors are discussed in the references [36, 35]. The system errors due to terms dropped in determining the navigation set (e.g., Coriolis terms or the gravity model) can be evaluated by considering the dropped terms as error terms and propagating these terms through the error equations. The error equations for the navigation set can be written in space state form as

$$\underline{\delta \dot{x}} = F(t)\, \underline{\delta x} + w(t) \tag{G-1}$$

$$\delta y = C(t)\, \underline{\delta x} \tag{G-2}$$

where

δx	are the error states
δy	are the output errors
$w(t)$	are the error forcing functions
$F(t)$	is in general a time varying matrix evaluated along a nominal vehicle path

A solution to Eq. (G-1) can be expressed as

$$\underline{\delta x}(t) = \phi(t, 0)\, \underline{\delta x}(0) + \int_0^t \phi(t, \tau)\, w(\tau) d\tau \tag{G-3}$$

where the transition matrix ϕ satisfies

$$\dot{\phi} = F\phi \tag{G-4}$$

A diagram showing the procedure is contained in Fig. G-7.

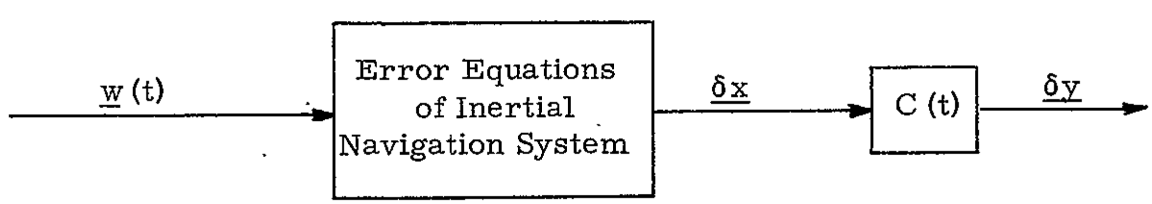


Figure G-7 Propagation of Errors Through Inertial Navigation System

If the forcing function models can be generated, then the system error can be evaluated by the existing state space computer programs at DRC.

The second method involves generating a reference solution using an assumed vehicle path and comparing its output, as shown in Fig. G-8, with the output of the Navigation Algorithm for the same vehicle path input.

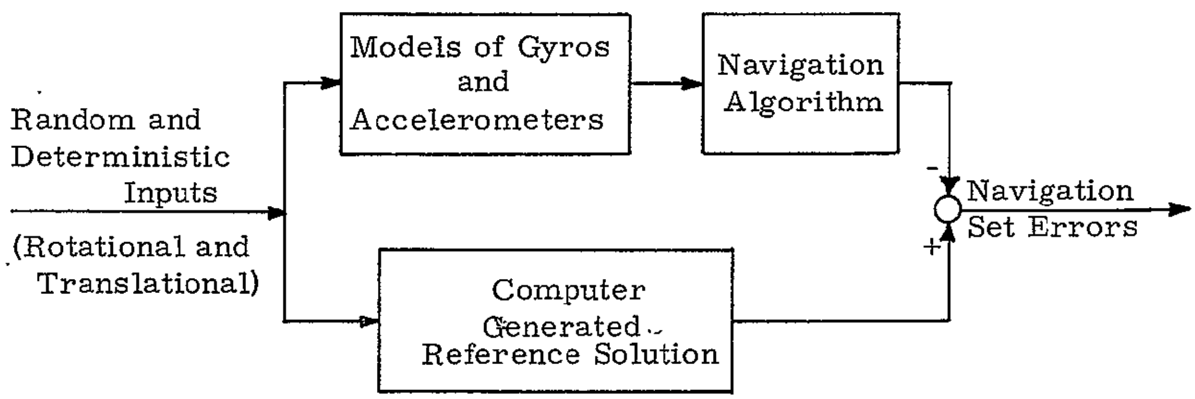


Figure G-8 Full Simulation of Navigation Set

A preliminary evaluation of the Navigation Algorithms is possible using the first method, and the second method can be used to verify results.

APPENDIX H

GLOSSARY

Formation of Symbols

$\delta(\cdot)$	error in $(\cdot)$
$\Delta(\cdot)$	change in $(\cdot)$
$[\ ]^T$	transpose of matrix
$(\dot{\ })$	denotes time rate of change of variable
$(\bar{\ })$	denotes average

Major Symbols and Abbreviations

$\left.\begin{matrix} A_R \\ A_P \\ A_H \end{matrix}\right\}$	Vehicle Euler angles defined as rotations through A_R, A_P and A_H of the ISU cube about its X, Y and Z axes, in that order. When $A_H = A_P = A_R = 0$, the X, Y, Z cube and NEV geographic coordinate frames are coincident.
$\left.\begin{matrix} A_X \\ A_Y \\ A_Z \end{matrix}\right\}$	Vehicle Euler angles defined as rotations through A_X, A_Y and A_Z of the ISU cube about its X, Y and Z axes, in that order. When $A_X = A_Y = A_Z$, the XYZ cube and Tangent Plane coordinate frames are coincident.
$\underline{B}_A$, $\underline{B}_G$	Vectors of accelerometer (PIGA) and gyro (SAP) biases, respectively.

$\underline{C}$, C_b^n, CTM	Coordinate Transformation Matrix (from body to "nominal" coordinate frames)
CEP	Circular Error Probability
g	Gravity
H	Height of vehicle above starting point
IA	Input axis of inertial sensor
INS	Inertial Navigation System
La	Geographic Latitude
Lo	Geographic Longitude
NEV	North, East Vertical geographic coordinate frame (V is plus down)
OA	Output axis of inertial sensor
$\underline{P}^G$	Position vector in geographic coordinates $= [La, Lo, H]^T$
PIGA	Pendulous Integrating Gyro Accelerometer
R	Radius of Earth
$\underline{R}_A$, $\underline{R}_G$	Vectors of random accelerometer and gyro drifts, respectively
SA	Spin Axis of Inertial Sensor
SAP	Single Axis Platform (contains gyro for measuring inertial rate of vehicle)
t	Time
V	White noise associated with measurement of system outputs
$\underline{V}^B$	Velocity vector of vehicle relative to inertial space, in body coordinates
$\underline{V}^G$	Velocity vector of vehicle relative to the earth in geographic coordinates $= [V_N, V_E, V_V]^T$
VTM	Velocity Transformation Matrix

$\underline{W}_e$, W_{ie}	Earth rate vector, magnitude of earth rate relative to inertial space
W_A, W_G	White noise associated with accelerometer and gyro random drift, respectively
XYZ, b	XYZ coordinate frame defined by ISU cube axes (down is positive).
X_s	Indicated change in vehicle position in space (Tangent Plane) X direction
Y_s	Indicated change in vehicle position in space (Tangent Plane) Y direction
Z_s	Indicated change in vehicle position in space (Tangent Plane) Z direction
$\underline{Z}$	Vector of measurements of system outputs
$\Delta\underline{\beta}^B$	Incremental angle changes in body coordinates, as derived from the SAP outputs
ϵ	Gyro drift rate error
θ_N, θ_E	Tilt of computed local level coordinate frame relative to vertical
τ_A, τ_G	Markoff time constants associated with accelerometer and gyro random drift, respectively
ϕ_N, ϕ_E, ϕ_V	Misalignments of computed earth's rotation vector, coordinatized in the NEV geographic coordinate frame

Subscripts

b	body coordinate frame (XYZ)
E	East
F	Fix
H	Heading
n	Nominal coordinate frame (NEV)

N	North
o	Initial
P	Pitch
R	Roll
S	System
V	Vertical
X, Y, Z	Tangent Plane coordinate axes

Abbreviations of Units

kt	Knots
mdh	Millidegrees/hour