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COMPOUND DIFFUSION OF COSMIC RAYS

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Abstract

The combined effects of the one-dimensional diffusion of cosmic rays along interstellar magnetic field lines and the three-dimensional random walk of these field lines due to interstellar turbulence are considered. The resulting compound diffusion can account for the observed low anisotropy of cosmic rays in terms of a correlation length of a few tens of parsecs which is consistent with the observational data on the scale of the interstellar turbulence.

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Compound Diffusion of Cosmic Rays

The observed low anisotropy of cosmic rays can place significant bounds on most models of cosmic ray propagation in the interstellar medium. The present upper limits on the anisotropy are $\delta \leq 3\times10^{-4}$ at 10^{11} to 10^{12} ev (Elliot et al., 1970) and $\delta \leq 7\times10^{-4}$ at about 2×10^{13} ev (Cachon, 1962). Since the gyroradius of 10^{11} to 10^{12} ev particles in the interplanetary magnetic field is on the order of 1 A.U., the low anisotropy observed at these energies may not be representative of its interstellar value. Nevertheless, the cosmic ray anisotropy in the interstellar medium should not exceed the value of 7×10^{-4} determined at about 2×10^{13} ev where the gyroradius in the interplanetary field is about 100 A.U. so that the effects of this field can be neglected.

If the motion of the cosmic rays in the interstellar medium is one-dimensional along uniform magnetic field lines, then, as Jones (1970a,b) has shown, small values of the anisotropy are no less probable than large values and the observed anisotropy could only be an accident of our particular position in space and time. There is, however, considerable observational evidence (Morris and Berge, 1964; Hornby, 1966; Davies, 1967; Mathewson and Nicholls, 1968) that the interstellar magnetic field is quite disordered on a scale less than ~ 1 kpc, although on a larger scale the field appears to lie nearly along the spiral arms. As discussed by Jokipii et al. (1969) the fluctuations in intensity and direction are quite large with root mean square values of the same order as the means. Jokipii and Parker (1969) have suggested that these fluctuations result from the turbulent motions of the interstellar gas which cause the field lines to random walk in space. Therefore, even through the gyroradii of the particles in the interstellar field are sufficiently small so that their motion is essentially one-dimensional along the field lines, some allowance must be made for the three-dimensional nature of the field distribution.

Thus the cosmic rays might propagate by streaming along the field lines which have random walked in three dimensions. Ramaty et al. (1970) have shown that if the motion of cosmic rays can be treated as three-dimensional diffusion the observed anisotropy can be understood if the cosmic rays have an effective diffusion mean free path of $\lesssim 0.1$ pc and a mean life of $\gtrsim 10^7$ years for escape from the galactic disk. These values are also consistent (Ramaty et al., 1971) with the measurements of the relative abundances of positrons, deuterium, helium-3 and the light elements, Li, Be, B.

There is, however, some question as to whether irregularities of the appropriate scale size (~ 0.1 pc) exist in the interstellar medium. The major observed irregularities are interstellar gas clouds which have a mean diameter of around 10 pc and a mean separation of about 40 pc or a mean distance between clouds of roughly 100 pc along an arbitrary line of sight (Allen, 1963). Wentzel (1969) and Kulsrud and Pearce (1969) have suggested that the cosmic rays themselves might generate smaller scale irregularities. But Wentzel (1971) and Kulsrud and Cesarsky (1971) show that because of the steep cosmic ray energy spectrum, the density of cosmic rays of energy greater than about 10^{11} ev is so low that this mechanism can not produce sufficient irregularities to effectively reduce the cosmic ray streaming velocity to less than that of the speed of light. There may of course be other more efficient mechanisms for generating sufficient small scale irregularities, and, in fact, there is some experimental (Downs and Reichley, 1971) and theoretical (Scheuer and Tsytovich, 1970) evidence for the existence of irregularities on a much smaller scale size (10^{-4} to 10^{-8} pc).

However, as we shall show, if the cosmic rays propagate by compound diffusion, resulting from one-dimensional diffusion along the field lines together with three-dimensional random walk of the fields, the streaming velocity of the particles will be much smaller than that predicted by ordinary diffusion with identical parameters. Thus, the observed low anisotropy, as well as the positron, deuterium, helium-3 and light element abundances, can be understood if the characteristic length of the random field distribution is about 30 pc and the mean free path for one-dimensional diffusion is of the same order.

The combined effects of one-dimensional diffusion along interstellar magnetic field lines and the three-dimensional random walk of these field lines were first considered by Getmantsev (1962). The probability density of particle displacement along the field lines is given by

$$f_1\ (s,t) = \left[3/(\pi \ell_1 ct)\right]^{\frac{1}{2}} \exp\left[-3s^2/(4\ell_1 ct)\right] \qquad , \qquad (1)$$

where s is the linear distance measured along the field line from the source and ℓ_1 is the scattering mean free path for one-dimensional diffusion. The probability density for a three-dimensional displacement r from the source for particles which have traversed a distance s in the random interstellar field is given by

$$f_2\ (r,s) = \left[3/(4\pi \ell_2 s)\right]^{3/2} \exp\left[-3r^2/(4\ell_2 s)\right] \qquad , \qquad (2)$$

where ℓ_2 is the characteristic length of the random interstellar field distribution.

The compound probability density f for the displacement of a particle over a distance r in a time t is the product of f_1 and f_2 integrated over all permissible values of s. These have obviously to satisfy $r \leq s \leq ct$. However, it can be shown that as long as r and ct are larger than both ℓ_1 and ℓ_2, this range of s can be replaced by $0 < s < \infty$ without significantly changing the value of the integral. In addition, because of the finite dimensions of the galaxy and the inherent ordering of the field along spiral arms, not all values of s are permissible and equally probable. The finite size of the galaxy leads to escape of cosmic rays in some characteristic time τ_e. Furthermore, we shall assume that in the trapping volume of the cosmic rays the field is sufficiently random so that the ordering can be ignored. Thus

$$f(r,t) = f_o(r,t) \exp(-t/\tau_e) \quad , \quad (3)$$

where

$$f_o(r,t) = \int_o^\infty ds\, f_1(s,t)\, f_2(r,s) \quad . \quad (4)$$

As has been suggested by Jokipii and Parker (1969) cosmic rays can escape from the galaxy by following field lines that have random walked to the surface of the disk. They estimate that a line of force originally lying in the equatorial plane of the disk is displaced by a scale height above this plane over a distance d of about 500 pc to 1 kpc, measured in the plane of the disk. This leads to an escape time τ_e which is roughly the propagation time of cosmic rays over the distance d. In addition, because of the turbulent velocity of the interstellar gas which is of the order 10 km/sec (Allen, 1963), individual field lines, and hence the cosmic rays moving along them, may be directly convected out of the galaxy in a time which is independent of the propagation velocity of the cosmic rays. If we use a convection velocity of 10 km/sec and a scale height, or semithickness, b , for the galactic disk of 100 pc, the escape time is on the order of 10^7 years.

We consider also nuclear destruction, characterized by a destruction time $\tau_d = (cn_H\sigma_d)^{-1}$, where n_H is the ambient hydrogen density in interstellar space and σ_d is the destruction cross section of the particular nucleus considered. Thus, we have to multiply the propagation function by $\exp(-t/\tau_d)$. We shall limit our discussion to relativistic nuclei so that ionization losses are negligible.

The streaming $\vec{J}$ and the anisotropy $\delta = 3|\vec{J}|/(cf)$ can be obtained from the continuity equation

$$\frac{\partial f}{\partial t} + \nabla\cdot\vec{J} + \frac{f}{T} = 0 \quad , \tag{5}$$

where $T^{-1} = \tau_e^{-1} + \tau_d^{-1}$. Substituting Equation (3) into (5) and assuming spherical symmetry we get

$$J(r,t) = r^{-2} \exp(-t/T) \int_0^r dr' \; r'^2 \, \partial f_o(r',t)/\partial t \quad . \tag{6}$$

We have integrated Equations (4) and (6) numerically. The quantity $r^3 f_o(r,t)$ is shown in Figure 1 as a function of t/t_o, where $t_o = r^4/\ell_2^2 \ell_1 c$. As can be seen, the time to maximum is $\sim t_o$. Since $t_o \propto r^4$, the net displacement r of particles undergoing compound diffusion is proportional to $t^{\frac{1}{4}}$ and not to $t^{\frac{1}{2}}$ as in ordinary diffusion.

The assymptotic form of Equation (4) can be obtained analytically and is given by

$$f_o \simeq r^{-3}(3/\pi)^{3/2} (t_o/t)^{\frac{1}{2}} \quad ; \quad t >> t_o \quad . \tag{7}$$

Using Equations (6) and (7) we find that the assymptotic form of the anisotropy is

$$\delta \simeq 3r/4ct \quad , \tag{8}$$

which can be compared with $\delta = 3r/2ct$ for ordinary diffusion. From the numerical integration of Equation (6) we find the assymptotic form (8) is a good approximation of δ for all values of $t > t_o$.

We shall now outline the calculation of the intensity, composition and anisotropy of the galactic cosmic rays at the earth using compound diffusion for the propagation of the cosmic rays in the interstellar medium.

As discussed by Ramaty et al. (1970), if the cosmic ray sources are discrete events in space-time, such as supernova explosions or pulsars, a statistical treatment is needed to take into account the inherent uncertainty that is introduced into the theory by the unknown positions and ages of the cosmic ray sources. We shall use such a theory for the computation of the total energy density and anisotropy. However, the abundances of secondary particles which are produced continuously in interstellar space by nuclear collisions of primary cosmic rays with the interstellar gas may be calculated by using a time independent source distribution. The density of a given nuclear component of the cosmic rays at earth is given by

$$n = q \int_0^\infty dt \, \exp(-t/\tau_d) \, P(t) \quad , \tag{9}$$

where the age distribution is defined

$$P(t) = P_o(t) \, \exp(-t/\tau_e) \quad , \tag{10}$$

and

$$P_o(t) = \int \rho(r) \, f_o(r,t) \, d^3r \quad . \tag{11}$$

Here $q\rho(r)$ is the source distribution; q is the production rate per unit volume; and $\rho(r)$ is a non-dimensional function of position.

We have evaluated Equation (11) numerically for a uniform source distribution

$$\rho(x,y,x) = 1 \;;\; |x| < \infty \;;\; |y| < \infty \;;\; |z| \leq b = 100 \text{ pc}, \tag{12}$$

and $f_o(r,t)$ given by Equation (4). The quantity $P_o(t)$ is plotted in Figure (1) as a function of t/t_o, where $t_o = b^4/\ell_2^2\ell_1 c$. As can be seen, $P_o(t)$ is a slowly varying function of t for $t >> t_o$, so that the variation of $P(t)$ with time is determined essentially by τ_e.

As discussed above, if the cosmic rays are convected out of the galaxy by the random motion of the interstellar clouds, τ_e is of the order 10^7 years. If the cosmic rays escape by following field lines over a distance d in the galactic plane, $\tau_e \sim d^4/\ell_2^2\ell_1 c = 7\times10^6$ years for $d = 500$ pc and $\ell_1 = \ell_2 =$ 30 pc. Since these estimates are quite crude and, moreover, since both mechanisms can operate simultaneously, the best way to determine the escape time is to evaluate the abundances of the secondary nuclei and compare them with observations.

We find that the observed (Shapiro and Silberberg, 1970) ratio of light $(3 \le z \le 5)$ to medium $(6 \le z \le 8)$ nuclei of 0.23, can be obtained for cosmic ray propagation by compound diffusion if $\ell_1 = \ell_2 = 30$ pc and $\tau_e = 1.5 \times 10^7$ years for $n_H = 1$ cm^{-3}, or $\tau_e = 5 \times 10^6$ years for $n_H = 2$ cm^{-3}. A detailed discussion of these calculations will be published elsewhere (Ramaty et al., 1971). Here we present only the age distributions. These are shown as solid lines in Figure 2 for $\ell_1 = \ell_2 = 30$ pc and $\tau_e = 5 \times 10^6$ years and 1.5×10^7 years.

The dashed line in Figure 2 is the age distribution for ordinary three-dimensional diffusion in a random field and escape along static field lines which have random walked to the surface of the disk. This distribution was obtained from Equations (10) and (11) with $\rho(r)$ given by Equation (12) and $f(r,t)$ given by

$$f(r,t) = \left[3/(4\pi\lambda ct)\right]^{3/2} \exp\left[-3r^2/(4\lambda ct)\right] \exp\left[-2\lambda ct/(3d^2)\right] \qquad (13)$$

This three-dimensional diffusion model yields the same light-to medium ratio of 0.23 for $d = 500$ pc, $\lambda = 0.06$ pc and $n_H = 1$ cm^{-3} or for $\lambda = 0.12$ pc and $n_H = 2$ cm^{-3}.

The dotted line in Figure 2 is an exponential age distribution, $P(t) = \exp(-t/\tau_e)$, which has been widely used in past studies of the propagation. For $n_H \tau_e$ corresponding to 4.3 g cm^{-2}, this distribution also yields an light-to-medium ratio of 0.23 but it does not take into account the disk geometry of the cosmic ray sources. The shape of the other age distributions in Figure 2, in fact, results from the disk geometry and the exponential escape which is position independent. The propagation time t_o to the top of the disk is roughly given by $t_o \sim b^4/\ell_2^2\ell_1 c$ for compound diffusion and by $t_o \sim b^2/\lambda c$ for ordinary three-dimensional diffusion. For the parameters that we have used $t_o < \tau_e$. Thus for $t \ll \tau_e$, $P(t)$ will decrease rapidly with a characteristic time scale t_o, whereas for $t \gtrsim \tau_e$, $P(t) \sim \exp(-t/\tau_e)$.

Thus, unlike the exponential model, the integral under $P(t)$ for both three-dimensional and compound diffusion is smaller than τ_e, thereby allowing the long escape time demanded by the anisotropy without leading to the production of more secondary nuclei than observed. In addition, as can be seen from Equation (9), nuclear destruction will further limit the contribution of cosmic rays which were produced at large values of t.

We now consider the proton energy density and anisotropy at earth for cosmic ray propagation by compound diffusion. We assume that the cosmic rays are produced by point sources in space-time with a local rate corresponding to 1 supernova per 100 years in the galaxy. Using the same statistical method as used by Ramaty et al. (1970) and the propagation function and anisotropy given by Equations (3) and (8), we have computed the means and 1σ levels of the total proton energy density and anisotropy at earth. These are given in Table 1. The fluctuations in the proton energy density are small. A total cosmic ray output per supernova, W_{SN}, of about 10^{50} ergs is required to account for the local cosmic ray energy density of $\sim 10^{-12}$ erg cm^{-3}. The fluctuations in the proton anisotropy are much larger and reflect the uncertainty in the positions and ages of the closest cosmic ray sources. The calculated mean anisotropies, however, are lower than the observed upper limits and thus do not require that the solar system be located in a region of unusually low anisotropy.

Finally, we consider the possible effect of compound diffusion with a diffusion mean free path along field lines $\ell_1 << \ell_2$, the characteristic length of the random field distribution. Such low values of ℓ_1 could be appropriate for particles with energies $<< 10^{11}$ ev where wave particle interactions (Wentzel, 1969, Kulsrud and Pearce, 1969) or small scale density fluctuations (Downs and Reichley, 1971; Scheuer and Tsytovich, 1970) may be important. For these values of ℓ_1 the particles will propagate very slowly along static field lines which lead to the edge of the disk, e.g. $\tau_e \simeq d^4/\ell_2^2\ell_1 c \geq 2\times10^8$ years with $d = 500$ pc, $\ell_2 = 30$ pc, and $\ell_1 \leq 1$ pc. Hence escape will result primarily from the particles being convected out of the galaxy along with the field lines, due to the random motion of interstellar clouds, on a time scale of $\sim 10^7$ years. As can be deduced from Figure 1, a change in ℓ_1 from 30 pc to 1 pc leads to a variation in $P_o(t)$ of only a factor of 2. Hence, even if particles undergo appreciable scattering along field lines the total age distribution $P(t)$ will remain essentially unchanged and have the same form as given in Figure 2. The anisotropy will of course become even smaller.

In summary, the available data on both the cosmic ray composition and anisotropy can be understood in terms of any model of cosmic ray propagation which leads to a low streaming velocity ($\lesssim 20$ km/sec) and an escape time from the galaxy of the order of 10^7 years which may or may not depend on the propagation mechanism. Both a compound diffusion model with $\ell_1 \lesssim \ell_2 \sim 30$ pc, and an ordinary three-dimensional diffusion model with $\lambda \sim 0.1$ pc have the required properties.

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Table 1

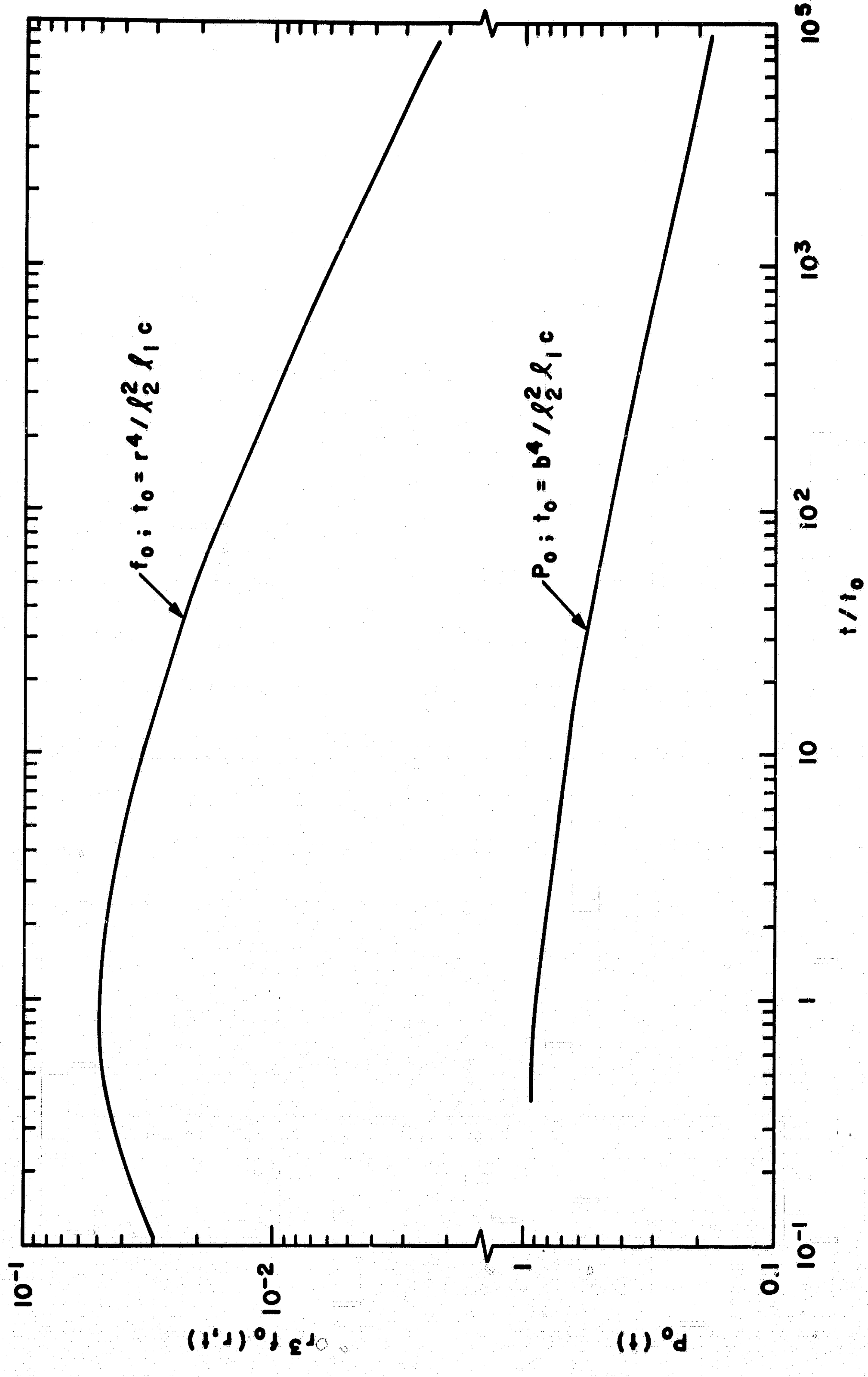
Cosmic ray energy density and anisotropy for compound diffusion with $\ell_1 = \ell_2 = 30$ pc

τ_e(years)	w(erg cm^{-3})	δ
5×10^6	$\left(0.69 \begin{smallmatrix} +0.18 \\ -0.15 \end{smallmatrix}\right) \times 10^{-62}\ W_{SN}$	$\left(1.4 \begin{smallmatrix} +3.1 \\ -0.9 \end{smallmatrix}\right) \times 10^{-4}$
15×10^6	$\left(1.3 \begin{smallmatrix} +0.24 \\ -0.21 \end{smallmatrix}\right) \times 10^{-62}\ W_{SN}$	$\left(0.75 \begin{smallmatrix} +1.9 \\ -0.5 \end{smallmatrix}\right) \times 10^{-4}$

Figure Captions

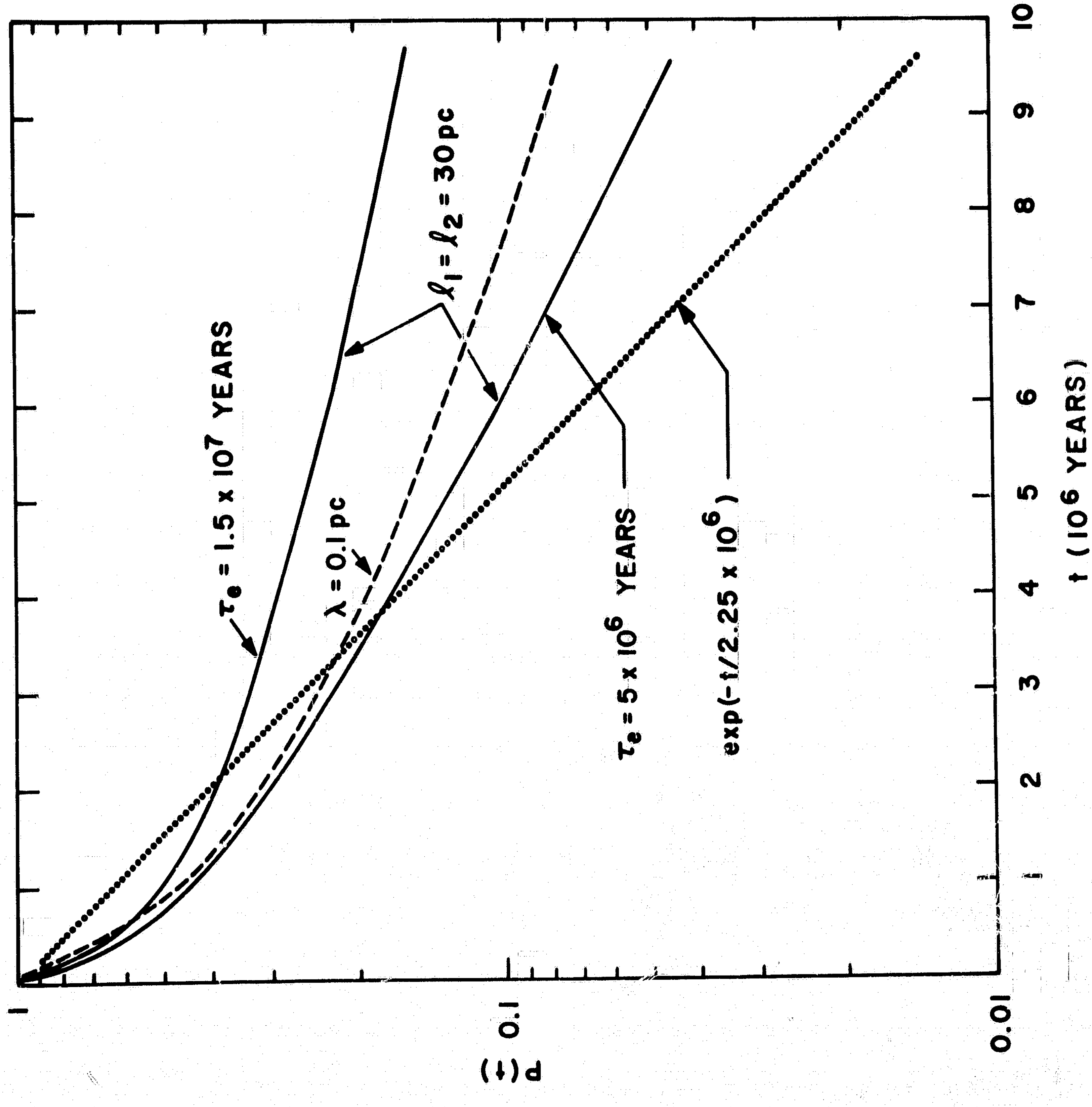
Figure 1. The propagation function from a single source, f_o , and the age distribution, P_o , for a uniform distribution of cosmic ray sources in the galactic disk. The characteristic time, t_o , is defined in terms of the distance to the source, r , or the semithickness of the galactic disk, b , for f_o and P_o, respectively. The quantities ℓ_1 and ℓ_2 are the characteristic lengths for one-dimensional diffusion along field lines and the three-dimensional distribution of the random magnetic field, respectively. Both f_o and P_o do not include the effects of escape from the galaxy.

Figure 2. Age distributions as a function of time. Solid lines - compound diffusion with various escape times; dashed line - ordinary three-dimensional diffusion with a mean free path λ ; dotted line - exponential distribution.



f_o ; $t_o = r^4/\ell_2^2 \ell_1 c$
P_o ; $t_o = b^4/\ell_2^2 \lambda_1 c$
10^{-1}
10^{-2}
1
0.1
10^{-1}
1
10
10^2
10^3
10^5
t/t_o
$r^3 f_o(r,t)$
$P_o(t)$

FIGURE 1



$\tau_e = 1.5 \times 10^7$ YEARS
$\ell_1 = \ell_2 = 30$pc
$\lambda = 0.1$ pc
$\tau_e = 5 \times 10^6$ YEARS
$\exp(-t/2.25 \times 10^6)$
P(t)
t (10^6 YEARS)
1
0.1
0.01
1
2
3
4
5
6
7
8
9
10

FIGURE 2