SAMMSOR II - A FINITE ELEMENT PROGRAM TO DETERMINE STIFFNESS AND MASS MATRICES OF SHELLS OF REVOLUTION

aerospace
engineering
department

TEXAS A&M UNIVERSITY

JOE R. TILLERSON AND WALTER E. HAISLER

TEES-RPT-70-18
OCTOBER 15, 1970

TEXAS ENGINEERING EXPERIMENT STATION

SAMMSOR II - A FINITE ELEMENT PROGRAM TO DETERMINE STIFFNESS AND MASS MATRICES OF SHELLS OF REVOLUTION

Joe R. Tillerson* and Walter E. Haisler**

October 15, 1970

TEES-RPT-70-18
Texas A&M University
College Station, Texas

* Research Assistant, Aerospace Engineering Department
** Assistant Professor, Aerospace Engineering Department

ABSTRACT

A user's guide for the SAMMSOR II (Stiffness And Mass Matrices for Shells Of Revolution) computer code is presented in this report. The finite element method of analysis is employed using a curved shell of revolution element to determine structural stiffness and mass matrices. Documentation of the analysis technique is included along with user hints and a discussion of the program limitations. Instructions for preparing the input data is included along with procedures for modifying the code. This program has the capability of internally generating the geometry of a number of important classes of shell configurations (such as shallow caps, cylinders, cones, hemispheres, etc.). Several example problems are presented to aid the user.

ACKNOWLEDGEMENTS

This research was supported under Sandia Contract 82-3683, Sandia Corporation, Albequerque, New Mexico and NASA Grant NGL-44-001-044, Manned Spacecraft Center, Houston, Texas. Appreciation is expressed to Dr. Walter Von Riesemann of Sandia and Dr. Fred Stebbins of the Manned Spacecraft Center who served technical monitors for this research effort.

Appreciation is also expressed to the principal investigator for the research grants, Dr. James A. Stricklin, Professor, Aerospace Engineering Department, Texas A&M University, for his supervision and guidance in the development of this program.

SOR - Shell Of Revolution
Computer Programs

A family of compatible computer codes for the analysis of the shell of revolution (SOR) structures has been developed by researchers at Texas A&M University. These analyses employ the matrix displacement method of structural analysis utilizing a curved shell element. Geometrically nonlinear static and dynamic analyses can be conducted using these codes. The important natural frequencies and mode shapes can also be determined by employing another of the codes. Efficient programming provides codes capable of performing these desired analyses in relatively small amounts of computer time.

Each of these programs has been extensively tested using problems the solutions to which have been reported by other researchers in order to establish the validity of the codes. In addition, the capabilities of the codes have been demonstrated in a number of publications by presenting solutions to problems which were unsolved by other researchers.

SAMMSOR II - Stiffness And Mass Matrices for Shells Of Revolution are generated utilizing the first member of this family. This program accepts a description of the structure in terms of the coordinates and slopes of the nodes and the properties of the elements joining the nodes. For shells with simple geometries (such as cylinders, shallow caps, hemispheres, etc.) the shell geometry can be internally generated. Utilizing the element properties, the structural stiffness and mass matrices are generated for as many as twenty harmonics and stored on magnetic tape. Each of the other SOR programs utilizes the output tape generated by SAMMSOR as input data for the respective analyses. One advantage of creating the stiffness and mass matrices in a separate program is that a variety of analyses can be performed on the same shell configuration without having to create the matrices more than once. Obviously, a variety of boundary and loading conditions can be employed without having to create new mass and stiffness matrices for each case.

SNASOR II - The Static Nonlinear Analysis of Shells Of Revolution subjected to arbitrary mechanical and thermal loading is performed using the second computer code. Utilizing the stiffness matrices generated by SAMMSOR and the loading conditions and boundary conditions input to SNASOR II, the equilibrium equations for the structure are generated. The nonlinear strain energy terms result in pseudo generalized forces (as functions of the displacements) which are combined with the applied generalized forces. The resulting set of nonlinear algebraic equilibrium equations is solved by one of several methods: Newton-Raphson type iteration, incremental stiffness method, or a modified incremental stiffness method. In general, the Newton-Raphson procedure is the best and yields accurate results for highly nonlinear problems.

DYNASOR II - The third code is used for the DYnamic Nonlinear Analysis of Shells Of Revolution. The equations of motion of the shell are solved using Houbolt's numerical procedure with the nonlinear terms being moved to the right-hand side of the equilibrium equations and again treated as generalized loads. The displacements and stress resultants can be determined for both symmetrical and asymmetrical loading conditions. Asymmetrical dynamic buckling can be investigated using this program. Solutions can be obtained for highly nonlinear problems in reasonable periods of time on the computer utilizing as many as five of the harmonics generated in SAMMSOR. A restart capability is incorporated in this code which allows the user to restart the program at a specified time without having to expend the computer time necessary to regenerate the prior response.

FAMSOR - Frequencies And Modes for Shells Of Revolution can be determined using the fourth code. Using the stiffness matrix generated by SAMMSOR and a lumped mass representation developed from the consistent mass matrix generated by SAMMSOR, a specified number of natural frequencies (beginning with the lowest or fundamental frequency) are obtained using the inverse iteration method. The mode shapes for each of the frequencies are also obtained.

Table of Contents

NOMENCLATURE

Variable

a_1, a_2, a_3 = coefficients in the expansion for shell slope representation

C_1 = $E_s t/(1 - \nu_{s\Theta}\nu_{\Theta s})$

C_2 = $E_\Theta t/(1 - \nu_{s\Theta}\nu_{\Theta s})$

D_1 = $E_s t^3/[12(1 - \nu_{s\Theta}\nu_{\Theta s})]$

D_2 = $E\ t^3/[12(1 - \nu_{s\Theta}\nu_{\Theta s})]$

E = modulus of elasticity

e = linear strains and rotations of shell middle surface

G = shear modulus

G_1 = Gt

G_2 = $Gt^3/12$

IA = number of Fourier cosine harmonics

[K] = structural stiffness matrix

[M] = structural mass matrix

q = generalized nodal displacement (cylindrical coordinates)

r = radial coordinate normal to the axis of revolution

s = meridional coordinate

T = kinetic energy

U = strain energy

u, v, w = meridional, tangential, and normal displacements, respectively

z = axial coordinate

α = generalized coefficient of a displacement function

Θ = circumferential angular coordinate

ν = Poisson's ratio

ϕ = angle between meridian and axis of revolution in the undeformed shell

χ = changes in curvature

σ = stress component

INTRODUCTION

The SAMMSOR (Stiffness And Mass Matrices for Shells Of Revolution) II code has been developed as the basic code used by the three SOR family analysis programs --- SNASOR II, DYNASOR II, and FAMSOR. This report is a user's guide for the SAMMSOR II code and is divided into four self-contained sections with an extended appendix.

The first section of the report describes the method of analysis used to obtain the output matrices. This documentation is not intended to be a detailed derivation of the element and structural properties but is intended to show the basic sets of equations employed in calculating the element and structural properties.

A section providing guidelines for the user and enumerating many of the limitations of the code is then presented. The hints contained in this second section concern mainly the selection of an element idealization for the shell. The limitations presented result partly from the method of analysis and partly from the programming procedures and storage capacity utilized.

A description of the input data required by the code is presented in the third section. Limitations placed upon the input parameters are once again presented and examples are provided in cases where the wording might, at first glance, seem to be unclear or insufficient.

A final section containing a number of example problems follows the description of the input data. Copies of the input data required for each case are presented along with selected output that may be scrutinized by a user who desires to check his output. The user should study and understand the example problems before attempting to input to the code.

While the appendix which follows the main report may seem to be quite extensive, it is believed that the user will find the various sections extremely useful, especially if a thorough understanding of the program is desired. A description of the various subroutines, a glossary defining the significant FORTRAN variables, a subroutine call map and a flow chart of the basic operations are included in this appendix. A discussion of the program output is also included. Since undoubtedly some users will find it neccessary to modify the element capacity of the code, a section in the appendix describes this procedure. A sample coding sheet for use in preparing input data is provided to aid the user.

An extended effort has been made to anticipate the questions which a user might ask before running the program. In particular, the report attempts to provide guidelines and hints for the successful use of the options for inputting the shell geometry and for the selection of the element idealization.

SECTION I

METHOD OF ANALYSIS

Introduction

The purpose of this section is to provide theoretical documentation of the equations and procedures employed in the SAMMSOR II code to calculate the mass and stiffness matrices used by the Shell Of Revolution (SOR) analysis programs. This program uses the curved shell element of Stricklin, Navaratna, and Pian[1] and the displacement function investigated by Mebane.[2] Since the nonlinear terms are treated as pseudo generalized forces in both the static[3] and dynamic[4] nonlinear analyses, the stiffness and mass matrices remain the same as in the linear analysis of shells of revolution. Detailed presentations of the methods of linear analysis can be found in Refs. 1, 2, and 5.

Structural Idealization

The shell of revolution is idealized as a sequence of consecutively numbered curved elements. The slope of each element (Fig. 1) is represented by a second-order polynomial in the meridional distance s:

$$\phi = a_1 + a_2 s + a_3 s^2 \tag{1}$$

where ϕ is the slope between the vertical axis and a tangent to the shell in the meridional direction, s is the meridional distance along the element, and a_1, a_2, and a_3 are coefficients that are evaluated by requiring the slopes of the structural idealization and the actual shell to be the same at the nodal points.

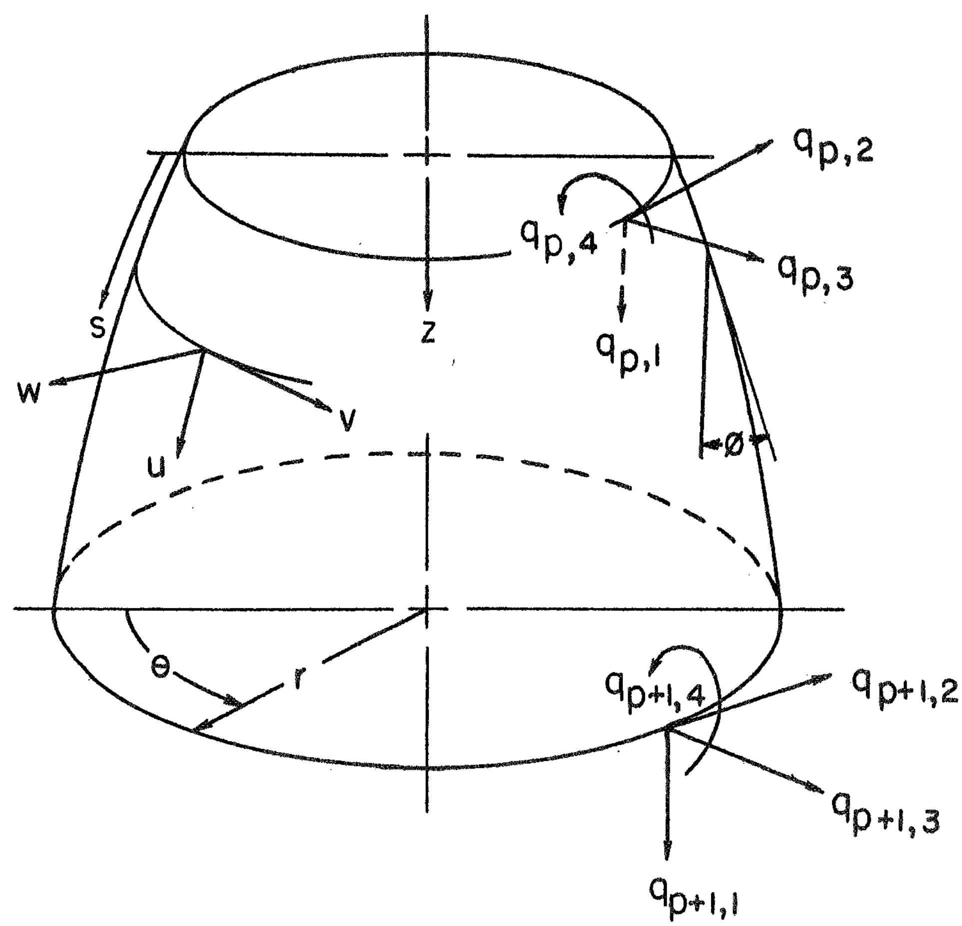


Fig. 1 GENERALIZED COORDINATES OF SHELL ELEMENT

Displacement Functions

The displacements of an element in the normal, meridional, and circumferential directions are represented by cubic polynomials in the meridional distance s and by a Fourier expansion in the circumferential angle Θ:

$$w = \sum_{i=0}^{IA} [\alpha_1^i + \alpha_2^i s + \alpha_3^i s^2 + \alpha_4^i s^3]\cos i\Theta$$

$$u = \sum_{i=0}^{IA} [\alpha_5^i + \alpha_6^i s + \alpha_9^i s(s-\ell) + \alpha_{10}^i s^2(s-\ell)]\cos i\Theta \qquad (2)$$

$$v = \sum_{i=0}^{IA} [\alpha_7^i + \alpha_8^i s + \alpha_{11}^i s(s-\ell) + \alpha_{12}^i s^2(s-\ell)]\sin i\Theta$$

In these expressions, the α's are the generalized coefficients; ℓ is the length of an element in the meridional direction; and i is the harmonic number.

Four degrees of freedom of the element are eliminated using the technique of static condensation.[6] After condensation each element has an 8 x 8 element stiffness matrix. The net result is therefore a stiffness matrix based upon higher order displacement functions without requiring additional geometrical data about the elements.

Linear expressions for u and v are used in the SAMMSOR I code,[4] but it has been recently shown[2] that the cubic polynomials better represent rigid-body motion and converge somewhat faster than the linear displacement fields. The input data for the SAMMSOR I and II codes is identical.

In both the SNASOR II and DYNASOR II codes, linear displacement functions are used for evaluating the nonlinear terms; justification for this is provided in Ref. 4.

The generalized coordinates, q_i, of the shell are defined as the displacements and rotations at the nodes in cylindrical coordinates for each harmonic (Fig. 1). The shell generalized coordinates are related to the generalized coefficients, α, through the relation

$$\{\alpha\} = [A]\{q\} \tag{3}$$

where

$$[A] = [B]^{-1}[\Psi]'$$

The matrices $[B]^{-1}$ and $[\Psi]'$ are presented as expressions (5) and (6) in Ref. 3.

<u>Strain-Displacement Relations</u>

While the nonlinear shell theory of Novozhilov[7] is applied in the SNASOR and DYNASOR codes the method of treating the nonlinear terms allows calculation of the stiffness matrix for each harmonic based upon the following linear strain-displacement relations for the midsurface strains:

$$\begin{aligned} e_s &= (\partial u/\partial s) - \phi' w \\ e_\theta &= (1/r)[(\partial v/\partial \theta) + u \sin\phi + w \cos\phi] \\ e_{s\theta} &= (1/r)(\partial u/\partial \theta) - (v/r)\sin\phi + \partial v/\partial s \end{aligned} \tag{4}$$

where

$$\phi' = \frac{\partial \phi}{\partial s}$$

The changes in curvature of the shell element are given by:

$$\begin{aligned} \chi_s &= -\partial \hat{e}_{13}/\partial s \\ \chi_\theta &= -(1/r)(\partial \hat{e}_{23}/\partial \theta) - (1/r)\sin\phi\, \hat{e}_{13} \\ \chi_{s\theta} &= -(1/r)(\partial \hat{e}_{13}/\partial \theta) + (1/r)\sin\phi\, \hat{e}_{23} - \partial \hat{e}_{23}/\partial s \end{aligned} \tag{5}$$

where

$$\hat{e}_{13} = (\partial w/\partial s) + u\phi'$$

$$\hat{e}_{23} = (1/r)(\partial w/\partial \theta) - (v \cos\phi)/r$$

The strains at any point through the thickness of the shell can be written as

$$\begin{aligned} \varepsilon_s &= e_s + \chi_s z \\ \varepsilon_\theta &= e_\theta + \chi_\theta z \\ \varepsilon_{s\theta} &= e_s + \chi_{s\theta} z \end{aligned} \tag{6}$$

Stress-Strain Relations

The stress-strain relationships used in this analysis are valid for an orthotropic material whose principal lines of orthotropy are the s and θ directions. These expressions can be written as:

$$\begin{aligned} \sigma_s &= \frac{E_s}{1-\nu_{s\theta}\nu_{\theta s}} (\varepsilon_s + \nu_{s\theta}\varepsilon_\theta) \\ \sigma_\theta &= \frac{E_\theta}{1-\nu_{s\theta}\nu_{\theta s}} (\varepsilon_\theta + \nu_{\theta s}\varepsilon_s) \\ \sigma_{s\theta} &= G\, \varepsilon_{s\theta} \end{aligned} \tag{7}$$

Strain Energy

The strain energy expression for orthotropic shells is given by

$$U = \tfrac{1}{2} \iiint [\sigma_s\varepsilon_s + \sigma_\theta\varepsilon_\theta + \sigma_{s\theta}\varepsilon_{s\theta}] r\,d\theta\,ds\,dz \tag{8}$$

Substituting Eqs. 7 into Eqs. 8 and integrating through the thickness, the strain energy can be expressed in the following form

$$U = \tfrac{1}{2} \iint (C_1\varepsilon_s^2 + C_2\varepsilon_\theta^2 + 2\nu_{s\theta}C_1\varepsilon_s\varepsilon_\theta + G_1\varepsilon_{s\theta}^2 + D_1\chi_s^2 + D_2\chi_\theta^2 + 2\nu_{s\theta}D_1\chi_s\chi_\theta + G_2\chi_{s\theta}^2)r\,d\theta\,ds \tag{9}$$

The strain energy can also be written as a quadratic form and the element stiffness matrix calculated using

$$U^i = \lfloor q^i \rfloor [k^i]\{q^i\} \tag{10}$$

where i denotes the harmonic number. This procedure has been explained in detail in Refs. 1 and 3.

Mass Matrix

The element mass matrices, which include the effects of rotary inertia, can be obtained by considering the expression for the kinetic energy:

$$T = \tfrac{1}{2} \int_{Vol} (\dot{u}^2 + \dot{v}^2 + \dot{w}^2)dm \tag{11}$$

Substituting the proper derivatives of the displacement functions, the kinetic energy can be written as a quadratic form in the generalized velocity coefficients.

$$T = \tfrac{1}{2} \lfloor \dot{\alpha} \rfloor [\bar{M}]\{\dot{\alpha}\} \tag{12}$$

The $\{\dot{\alpha}\}$ matrix is related to the velocities of the generalized structural coordinates, $\{\dot{q}\}$, by the transformation

$$\{\dot{\alpha}\} = [A]\{\dot{q}\} \tag{13}$$

The terms in $[\bar{M}]$ are listed in the appendix of Ref. 4. Because of the inclusion of the effects of rotary inertia, the mass matrix is a function of the harmonics being used. The mass matrix in structural coordinates is then obtained from

$$[M^e] = [A]^T[\bar{M}][A] \tag{14}$$

The mass matrices for the individual elements are then combined (in the same way as for element stiffness matrices) to obtain the structural mass matrix.

SECTION II

Guidelines and Program Limitations

Guidelines for the use of the SAMMSOR II code along with the limitations placed on the analysis are enumerated in this section. Some of these limitations result from the method of analysis utilized while others result from the programming procedures which are used. Most of these limitations are minor in nature and hence this code, when coupled with the other compatible SOR programs, can be used to analyze a large number of practical shell of revolution problems.

First of all, the shell must be idealized as a sequence of curved elements. As is the case with most computer codes using the finite element method, the selection of the element breakdown of the structure requires the application of engineering judgement. The following considerations should prove most helpful in the selection of the idealization:

(1) Elements should be closely spaced in regions where rapid variations of the displacements and stresses are anticipated.

(2) Elements should be concentrated in regions of rapidly varying material properties.

(3) The change in slope over an element should, in general, be kept less than about 10°.

Examples of judicious choices of element breakdown can be seen by considering the example problems presented in Section IV. If doubt still exists about how many elements to use, it is recommended that the maximum number of elements allowed by the program (50) be employed.

The material used in the shell may be isotropic or orthotropic. For an orthotropic material, the principal directions of orthotropy must be in the meridional and circumferential directions. It is also assumed that the material properties for any given element are constant in the two directions, but these properties may vary from element to element. The properties of an element must be constant through the thickness of the element. The thickness of an individual element is constant, but thickness variations from element to element are allowed.

Slope discontinuities are not allowed between elements, i.e. the slope at the end of element i must equal the initial slope of element i + 1.

The code requires that the units for the input, and hence output, data be given in inch-pound-seconds. A different set of consistent units may be utilized if the value of the FØRTRAN variable GRAVTY is changed in subroutines AZERO and NONAB.

The integrals which are required for the calculation of the element stiffness and mass matrices are evaluated exactly in the circumferential direction, but Simpson's rule is applied at twenty-nine (29) equally spaced stations along the element in the meridional direction. Fewer stations could probably be used for most problems, but good results have been obtained in all cases using 29 stations.

Stiffness and mass matrices may be generated for the first 20 Fourier harmonics. In general, it has been found that few problems require more than 3-5 of the harmonics. Unless the exact number of harmonics to be used by the solution programs is known, it is better to generate a relatively large number of harmonics say (6-8) in SAMMSOR so that they will be available if needed.

Utilizing only the cosine harmonics necessitates that the meridional line traced by $\theta = 0$ be selected so that the displacements, u and w, of the shell will be symmetrical about this line in the θ - direction.

A maximum of fifty (50) elements are allowed in the SAMMSOR code. It is believed that 50 elements is sufficient for a very large number of shell of revolution problems. More elements may be needed for analyzing geometrically complex structures such as the shell with negative Gaussian curvature depicted in Fig. 2-d of Ref. 3. Since undoubtedly some users will find it necessary to change the maximum allowable number of elements in the program, instructions for changing the capacity are given in Appendix 6.

A different output tape unit number (NT - Card III) must be specified for each of the cases of a particular run. Logical units 5 and 6 are used for the card reader and printer. Three scratch tapes are used by the code.

Utilizing FORTRAN IV language, it has been noted that double-precision arithmetic is necessary for operation on an IBM 360/65 computer. This additional word length is not believed necessary when the program is employed on machines with a longer built-in word length (such as a CDC 6600 machine).

The following limitation is placed upon the values of the Poisson's ratios of the materials:

$$\nu_{s\theta}\nu_{\theta s} < 1$$

If this criterion is violated, the stiffness matrix is no longer positive definite.

The slope, ϕ, at a node is considered positive if the tangent drawn in the direction of increasing node numbers is rotated counterclockwise from the positive z-axis. A negative slope exists if the tangent is rotated clockwise from the positive z-axis. The magnitude of ϕ is thus limited to the range $-180° \leq \phi \leq 180°$. Considering Fig. 2 it should be noted that at nodes a and b the value of ϕ is positive while nodes c and d give rise to negative values of ϕ. Reversing the direction of node numbering therefore changes both the sign and the magnitude of the slope at a node.

Restrictions or limitations placed upon input geometry are:

1. The magnitude of ϕ must be in the range $-180° \leq \phi \leq 180°$.

2. A generating segment cannot have a change in slope greater than 180° (this restriction is easily circumvented by using a greater number of segments to generate the element data).

3. The generating segments must always be placed in the r - z coordinate system so that r and z are always positive (first quadrant).

4. The circular and parabolic segments cannot be degenerated to flat elements (linear segments).

5. A segment cannot contain, except at an end point, the "discontinuity" in slope at ± 180°. To circumvent this restriction use one segment whose slope is -180° at its final node and a second segment whose initial slope is +180°.

6. Only elements which are consecutively numbered may intersect. This restriction is imposed to maintain the banding which is present in the stiffness and mass matrices.

In addition to the major limitations presented here, the section describing the input to the code enumerates a number of minor requirements which are necessary for valid input data. Other limitations and guidelines are also presented in the user's manuals for the solution programs: SNASOR II, DYNASOR II, and FAMSOR.

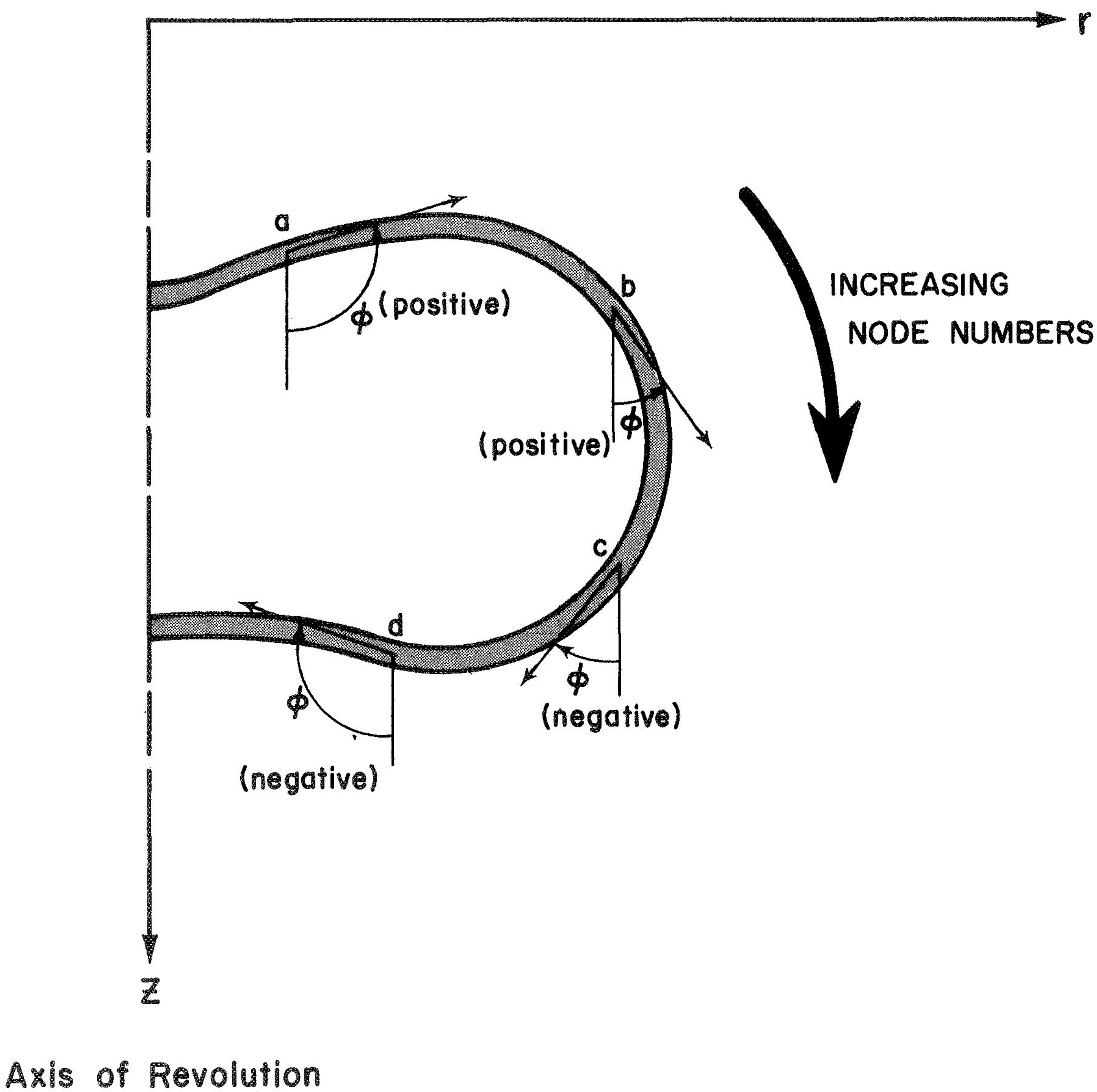


FIG. 2 SIGN CONVENTION FOR SLOPE SPECIFICATION

SECTION III

Program Input

The SAMMSOR code has been written so that the program can readily be utilized by investigators who are not familiar with the inner workings of the program. Using the hints and adherring to the limitations presented in the previous section, it is believed that a user with only a superficial knowledge of the finite element method can effectively employ the SAMMSOR II code.

The double-precision version of the code requires a storage space of about 190K bytes on the IBM 360/65. The single-precision code requires approximately 110K bytes. The code is written in the FORTRAN IV language.

The input data for the program consists of one card I (card types will be explained on the following pages) followed by a complete data set for each of the cases to be run. A final card VIII is added at the end of the last data set. A case consists of cards of type II through VII and is all the input data required to generate the desired stiffness and mass matrices for a particular shell. The set of cards II - VII for a case is also referred to as a data set. There is no limit on the number of cases which can be included per run, but the logical unit number of the output tape (NT from Card II) must be different for each case of a run.

I. RUN CONTROL CARD

This card is used to identify the number of cases to be run and the logical unit numbers of the scratch tapes used in the run. (ONLY ONE CARD I IS USED PER RUN.)

Card Type I Format (4I5)

Columns	Variable	Description
1-5	NCASES	The number of different data sets utilized for this run.
6-10	ND	Logical unit number for scratch tape. All the input data for the run is read onto this unit at the start of the run and is stored here until needed in the program.
11-15	NS	Logical unit number for scratch tape used by the program.
16-20	NS2	Logical unit number of a second scratch tape used by the code.

II. CASE IDENTIFICATION CARDS

These cards allow the user to print out comments which identify the problem being run.

A. Control Card (ONE CARD II-A PER DATA SET)

Card Type II-A Format (2I5)		
Columns	Variable	Description
1-5	NCARDS	Number of comment cards (Type II-B) which follow.
6-10	NT	Unit number of the tape on which stiffness and mass will be stored. The value of NT must be different for each case of a run.

B. Identification Cards

The information printed on these cards is printed as output for the SOR programs and should identify the particular shell being analyzed. (IF NCARDS=0, OMIT CARDS II-B, OTHERWISE INCLUDE NCARDS OF TYPE II-B.)

Card Type II-B Format (20A4)		
Columns	Variable	Description
1-80	COMENT	Any desired alphanumeric information may be printed on these cards.

III. CASE CONTROL CARD

Identifies the number of elements and harmonics used and gives the unit number for the output tape. (ONE CARD III PER DATA SET)

Card Type III Format (3I5)

Columns	Variable	Description
1-5	NELEMS	Total number of elements used to idealize the shell (NELEMS ≤ 50).
6-10	IA	Total number of harmonics for which mass and stiffness matrices are to be calculated (1 ≤ IA ≤ 20).
11-15	NPRNMS	If the mass and stiffness matrices are to be printed, set NPRNMS = 1. If not, set NPRNMS = 0.

IV. MATERIAL PROPERTIES

The elastic and mass properties in the meridional and circumferential directions are constant over an element, but the properties can vary from element to element. (THE NUMBER OF CARDS OF TYPE IV $\leq$ NELEMS (CARD III) PER DATA SET.)

Card Type IV	Format (2I5, 5F10.0)	
Columns	Variable	Description
1-5	IELM1	Number of the first element to which the properties on this card apply.
6-10	IELM2	Number of the last element to which the properties on this card apply.
11-20	EE1	Young's Modulus of elasticity in the meridional direction (psi).
21-30	EE2	Young's Modulus of elasticity in the circumferential direction (psi).
31-40	GG	Shear modulus (psi). For an isotropic material, $GG=E/2(1+\nu)$.
41-50	FFNU1	Poisson's ratio. The contraction in the meridional direction due to stress in the circumferential direction.
51-60	RHO	Density of the material (lb./in.^{3}).

NOTE: The elastic and mass properties can be read in for each individual element using NELEMS cards of type IV. If the properties of a consecutively numbered group of elements are the same, the input data can be simplified considerably by using only one card to describe the properties of this group of elements. In particular, if the elastic and mass properties of all the elements are the same, only one card of type IV is necessary per data set.

EXAMPLE: Consider a shell that is idealized using 20 elements. Assuming that the elastic properties of elements 1 through 10 are the same, different properties exist for the eleventh element and elements 12 through 20 have the same properties as the first group. Three cards should be used to input the properties of the shell. The following values for IELM1 and IELM2 should be used.

Card 1:	IELM1 = 1	IELM2 = 10
Card 2:	IELM1 = 11	IELM2 = 11
Card 3:	IELM1 = 12	IELM2 = 20

A less sophisticated procedure would be to use one card for each element. Note that the properties for the elements 12-20 must be read in on a different card from those of elements 1-10.

V. THICKNESS

It is assumed that the thickness is a constant over the element in the meridional and the circumferential directions. Essentially the same procedure is used here as is employed with cards IV for the material properties. (THE NUMBER OF CARDS OF TYPE V MUST BE $\leq$ NELEMS FOR ANY GIVEN DATA SET.)

Card Type V Format (2I5, F10.0)

Columns	Variable	Description
1-5	IELM1	Number of the first element to which thickness properties on this card apply.
6-10	IELM2	Number of the last element to which thickness properties on this card apply.
11-20	TT	Thickness of the shell element or group of elements (in).

VI. SHELL GEOMETRY

A most important part of any shell of revolution analysis is the generation of the geometry and the element idealization of the shell. For this reason, an extended discussion follows which describes in detail the procedure used to generate the geometry of the idealized shell. Slope discontinuities between elements are not allowed.

The shell nodal coordinates and slopes can be generated using either of two options available (or a combination of the two):

OPTION 1: The user may specify the coordinates and slopes of the nodes by inputting the values of these parameters at each of the nodes.

OPTION 2: For shells of relatively simple geometry (shallow caps, hemispheres, cylinders, etc.) the program has the capability of generating internally the desired nodal point data.

The shell is considered as being composed of one or more segments, with the segments being divided, in turn, into one or more sub-segments. The sub-segments are then divided into the desired number of finite elements. A segment may contain only one element or the entire shell may in some cases be considered as a single segment. The choice of the segment breakdown depends upon the option selected for inputting the shell geometry as well as the geometry of the shell itself.

If the geometry for a particular segment is to be generated internally, the profile of the segment must be linear, circular or parabolic. A segment is divided into a maximum of five sub-segments with each sub-segment being divided, inturn, into a specified number of equally spaced elements. Obviously, if the segment consists of only one element, no division into sub-segments is necessary. A set of cards (type VI) is required for each segment, with a set consisting of one card of types VI-A, VI-B, and VI-C -- in that order.

In order to generate the required nodal point data for a given shell, the following procedure must be used:

1. Select a breakdown of the shell into segments.
2. Establish the desired breakdown of the segments into sub-segments and determine the number of elements to be used for each sub-segment.
3. Input the required three data cards for each segment until the data for all the segments has been input.

The total number of data cards necessary for establishing the geometry of the entire shell is equal to three times the number of segments. If the first option is used for generating the geometry, the user will find that the total number of cards required will equal (3* NELEMS) where NELEMS was defined on card III. However, only (NELEMS + 1) of these cards will be different since duplicate cards and blank cards are used. The procedure may therefore seem somewhat cumbersome when the user desires to specify the slopes and coordinates of each node. The procedure is, however, extremely simple when the nodal point data is to be internally generated. In addition, it has been found that fewer input errors arise when using this simple but repetitious procedure. It is for these reasons that the procedure is used in the code. An example of the effective use of this scheme can be seen by considering the first example problem presented where only three cards are required to generate the desired nodal data.

A. Segment initial node data

The data furnished by this card provides the node number, slope and coordinates of the initial node of a given segment.

Card Type VI-A	Format (I5, 3F10.0)	
Columns	Variable	Description
1-5	NN1	Number of the node at the beginning of this shell segment.
6-15	Z1C	Axial coordinate of node NN1.
16-25	R1	Radial coordinate of node NN1.
26-35	PHI1	Slope of the shell in the meridional direction at node NN1 (degrees).

***The slopes PHI1 and PHI2 are measured from the positive axis of revolution to the tangent to the meridian (see Fig. 2). Refer to Fig. 3 for an explanation of the input parameters.

B. Segment terminal node data

The data furnished by this card provides the node number, slope, and coordinates of the final node of a given segment.

Card Type VI-B	Format (I5, 3F10.0)	
Columns	Variable	Description
1-5	NN2	Number of the node at the end of the segment (NN2 must be > NN1).
6-15	Z2C	Axial coordinate of node NN2.
16-25	R2	Radial coordinate of node NN2.
26-35	PHI2	Slope of the shell in the meridional direction at node NN2 (degrees).

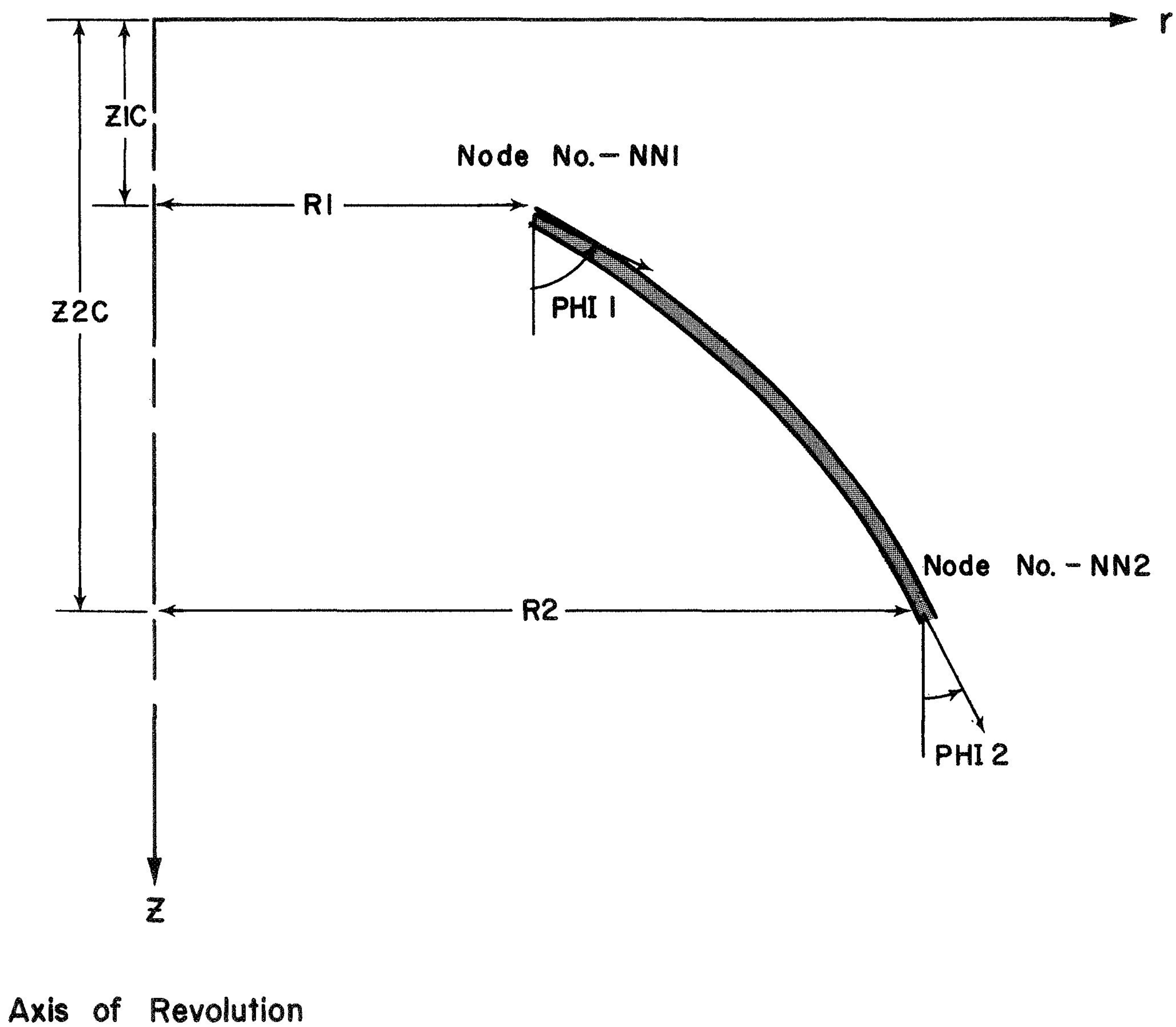


FIG. 3 INPUT VARIABLES SPECIFYING THE GEOMETRY OF A SHELL SEGMENT

C. Coordinate generation control card

This card controls the internal generation of data for the shell segment whose end coordinates are described on cards III-A and III-B of this set by describing the type of shell profile and the element breakdown to be used in the idealization of the segment. Each segment can be divided into a maximum of five sub-segments.

If (NN2 = NN1 + 1), obviously, the segment under consideration consists of only one element and the coordinates and slopes at the two adjacent nodes completely define the geometry of the shell segment. If this is the case, the user must include a card III-C in the set; but it must be blank.

Card Type VI-C   Format (I5, 5(I5, F10.0))

Columns	Variable	Description
1-5	ICLASS	Control parameter which defines the shell profile ICLASS = 1 - linear profile ICLASS = 2 - circular profile ICLASS = 3 - parabolic profile
6-10	NERAT (1)	This data specifies the number of elements
11-20	RA (1)	and the relative lengths (decimal frac-
21-25	NERAT (2)	tions) into which the sub-segments of the
26-35	RA (2)	shell segment are divided. The sum of the
36-40	NERAT (3)	RA (I)'s must equal 1.0.
41-50	RA (3)	
51-55	NERAT (4)	
56-65	RA (4)	
66-70	NERAT (5)	
71-80	RA (5)	

An understanding of the use of this card can be obtained by studying the input data for the example problems and by considering the following example. Consider a circular segment to be divided into four sub-segments which are each divided into five elements. Sub-segment lengths are given as 1/10, 2/10, 1/2, and 2/10 respectively of the length of the segment. The following values would be used on card VI-C for this segment:

ICLASS = 2 (circular profile)

NERAT(1) = 5	RA(1) = 0.10
NERAT(2) = 5	RA(2) = 0.20
NERAT(3) = 5	RA(3) = 0.50
NERAT(4) = 5	RA(4) = 0.20
NERAT(5) = 0	RA(5) = 0.00
	Σ = 1.0

If the segment was divided instead into 20 equally spaced elements, then the data would be:

ICLASS = 2

NERAT(1) = 20	RA(1) = 1.0
NERAT(2) = 0	RA(2) = 0.0
NERAT(3) = 0	RA(3) = 0.0
NERAT(4) = 0	RA(4) = 0.0
NERAT(5) = 0	RA(5) = 0.0
	Σ = 1.0

VII. FINAL DATA CARD FOR A CASE

Place this card after the last card VI-C of each data set. This signifies the end of the input data for a case. (USE ONE CARD VII PER DATA SET.)

Card Type VII	
Columns	Punch
1-11	END ØF CASE

VIII. FINAL DATA CARD FOR A RUN

This card must be placed after the card VII of the last case to be run. This card denotes the end of the input data for a run. (ONLY ONE CARD VIII IS USED PER RUN.)

Card Type VIII	
Columns	Punch
1-10	END ØF RUN

SECTION IV

EXAMPLE PROBLEMS

The example problems which follow were chosen to demonstrate the various facets of the input data required to run a given case and to acquaint the user with typical problems which have been analyzed using the SOR programs. As stated in a prior section, the choice of the element idealization must be jucicíously selected. The example problems should provide the user with a "feel" for the selection of the element idealization for an analysis. The input data for the various shell geometries should answer any questions the user might have concerning the input procedure for cards VI of the input section.

Stiffness and mass matrices for selected harmonics are presented as output for the example problems. The output was generated using double-precision arithmetic on an IBM 360/65 system.

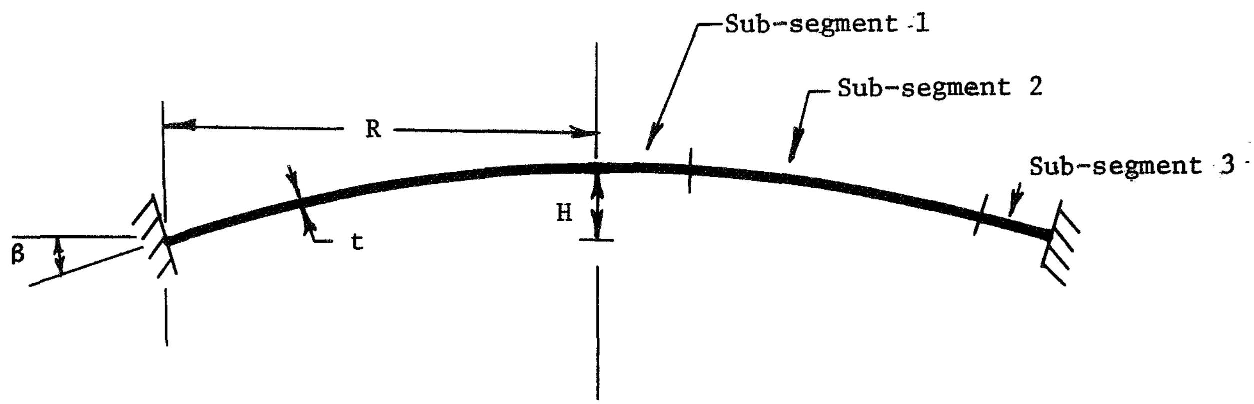
Example Problem 1

The first example problem is a shallow spherical cap (λ = 6) that has been analyzed extensively using the SNASOR II, DYNASOR II and the FAMSOR codes. The geometric and material properties for the shell are presented in Fig. 4.

The selection of the element breakdown and the number of harmonics to be generated depends not only upon the geometry of the shell, but also upon the loading to be applied to the shell. In order to be certain that the data for a sufficient number of harmonics will be generated, six (6) cosine harmonics will be used. The static and dynamic analyses previously conducted on this shell have shown that thirty (30) elements provide a good idealization for the loadings used providing these elements are concentrated near the apex of the shell, where a singularity obviously exists, and near the clamped edge.

A set of input data for this case is presented in Fig. 5. The geometry is input using the second option for inputting the data and the shell is considered as a single segment with a circular profile. Obviously, the geometry could have been input for each node, but the process would have been tedious.

A selected portion of the output generated for this case is presented in Figs. 6-10.



SHELL DESCRIPTION

R = 0.9 in.

H = 0.0859 in.

t = 0.01576 in.

λ = 6.0

β = 10.9°

MATERIAL PROPERTIES
ALUMINUM

E = 10.0 x 10^6 psi

G = 3.85 x 10^6 psi

ν = 0.30

ρ = 0.0942 lb./in.^{3}

ELEMENT BREAKDOWN

Sub-segment 1 - 6 elements

Sub-segment 2 - 12 elements

Sub-segment 3 - 12 elements

Fig. 4 SHALLOW SPHERICAL CAP

```
NCASF= 1

                              PRINTOUT OF INPUT DATA
CARD
TYPE                10        20        30        40        50        60        70        80
           12345678901234567890123456789012345678901234567890123456789012345678901234567890

II  - A        9    4
    - B    ********************************************************************************
    - B          EXAMPLE PROBLEM NO. 1                       SAMMSOR USER'S MANUAL
    - B                        SHALLOW SPHERICAL CAP  (LAMBDA = 6)
    - B             ALL ELEMENTS HAVE THE SAME MATERIAL PROPERTIES AND THICKNESS
    - B         30 ELEMENT IDEALIZATION -----  1 SEGMENT -----   3 SUB-SEGMENTS
    - B       FIRST SUB-SEGMENT  6 ELEMENTS ALONG FIRST 0.20 OF SHELL MERIDIAN
    - B       SECOND    '         12    '       '   NEXT  0.65 '    '     '
    - B        THIRD    '         12    '       '   FINAL 0.15 '    '     '
    - B    ********************************************************************************
III           30    6    1
IV             1   30 10000000. 10000000.  3850000.       0.3     0.0942
V              1   30   0.01576
VI  - A        1      0.0        0.0       90.0
    - B       31    0.0859       0.9       79.1
    - C        2    6       0.2   12       0.65   12       0.15    0       0.0     0       0.0
VII        END OF CASE
```

Fig. 5 INPUT DATA - EXAMPLE PROBLEM 1

GEOMETRIC AND ELASTIC PROPERTIES OF STRUCTURE

ELEM. NO.	NODE NOS.	COORDINATES Z	R	SLOPE	S	THICKNESS	ELASTIC CONSTANTS E1	E2	G	NU1	RHO
	1	0.0	0.10000D-05	0.90000D 02							
1					0.1509D-01	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	2	0.95697D-04	0.30182D-01	0.89637D 02							
	2	0.95697D-04	0.30182D-01	0.89637D 02							
2					0.4527D-01	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	3	0.38278D-03	0.60362D-01	0.89273D 02							
	3	0.38278D-03	0.60362D-01	0.89273D 02							
3					0.7545D-01	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	4	0.86125D-03	0.90540D-01	0.88910D 02							
	4	0.86125D-03	0.90540D-01	0.88910D 02							
4					0.1056D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	5	0.15311D-02	0.12071D 00	0.88547D 02							
	5	0.15311D-02	0.12071D 00	0.88547D 02							
5					0.1358D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	6	0.23922D-02	0.15088D 00	0.88183D 02							
	6	0.23922D-02	0.15088D 00	0.88183D 02							
6					0.1660D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	7	0.34447D-02	0.18105D 00	0.87820D 02							
	7	0.34447D-02	0.18105D 00	0.87820D 02							
7					0.2056D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	8	0.55628D-02	0.23005D 00	0.87230D 02							
	8	0.55628D-02	0.23005D 00	0.87230D 02							
8					0.2547D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	9	0.81857D-02	0.27902D 00	0.86639D 02							
	9	0.81857D-02	0.27902D 00	0.86639D 02							
9					0.3037D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	10	0.11313D-01	0.32797D 00	0.86049D 02							
	10	0.11313D-01	0.32797D 00	0.86049D 02							
10					0.3527D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	11	0.14945D-01	0.37688D 00	0.85458D 02							
	11	0.14945D-01	0.37688D 00	0.85458D 02							
11					0.4018D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	12	0.19080D-01	0.42575D 00	0.84868D 02							
	12	0.19080D-01	0.42575D 00	0.84868D 02							
12					0.4508D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	13	0.23719D-01	0.47457D 00	0.84277D 02							
	13	0.23719D-01	0.47457D 00	0.84277D 02							
13					0.4999D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	14	0.28861D-01	0.52335D 00	0.83687D 02							
	14	0.28861D-01	0.52335D 00	0.83687D 02							
14					0.5489D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	15	0.34505D-01	0.57207D 00	0.83097D 02							
	15	0.34505D-01	0.57207D 00	0.83097D 02							
15					0.5980D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	16	0.40651D-01	0.62073D 00	0.82506D 02							
	16	0.40651D-01	0.62073D 00	0.82506D 02							
16					0.6470D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	17	0.47297D-01	0.66932D 00	0.81916D 02							
	17	0.47297D-01	0.66932D 00	0.81916D 02							
17					0.6961D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	18	0.54445D-01	0.71784D 00	0.81325D 02							
	18	0.54445D-01	0.71784D 00	0.81325D 02							
18					0.7451D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	19	0.62091D-01	0.76629D 00	0.80735D 02							
	19	0.62091D-01	0.76629D 00	0.80735D 02							
19					0.7753D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	20	0.63927D-01	0.77746D 00	0.80599D 02							
	20	0.63927D-01	0.77746D 00	0.80599D 02							
20					0.7866D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	21	0.65789D-01	0.78862D 00	0.80462D 02							
	21	0.65789D-01	0.78862D 00	0.80462D 02							
21					0.7979D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	22	0.67678D-01	0.79978D 00	0.80326D 02							
	22	0.67678D-01	0.79978D 00	0.80326D 02							
22					0.8092D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	23	0.69593D-01	0.81093D 00	0.80190D 02							
	23	0.69593D-01	0.81093D 00	0.80190D 02							
23					0.8206D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	24	0.71534D-01	0.82208D 00	0.80054D 02							
	24	0.71534D-01	0.82208D 00	0.80054D 02							
24					0.8319D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	25	0.73503D-01	0.83323D 00	0.79917D 02							
	25	0.73503D-01	0.83323D 00	0.79917D 02							
25					0.8432D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	26	0.75497D-01	0.84437D 00	0.79781D 02							
	26	0.75497D-01	0.84437D 00	0.79781D 02							
26					0.8545D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	27	0.77519D-01	0.85551D 00	0.79645D 02							
	27	0.77519D-01	0.85551D 00	0.79645D 02							
27					0.8658D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	28	0.79566D-01	0.86664D 00	0.79509D 02							
	28	0.79566D-01	0.86664D 00	0.79509D 02							
28					0.8772D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	29	0.81640D-01	0.87776D 00	0.79372D 02							
	29	0.81640D-01	0.87776D 00	0.79372D 02							
29					0.8885D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	30	0.83741D-01	0.88889D 00	0.79236D 02							
	30	0.83741D-01	0.88889D 00	0.79236D 02							
30					0.8998D 00	0.1576D-01	0.100D 08	0.100D 08	0.385D 07	0.300	0.09420
	31	0.85868D-01	0.90000D 00	0.79100D 02							

Fig. 6 ELEMENT PROPERTIES (OUTPUT) EXAMPLE PROBLEM 1

HARMONIC NUMBER 0 HAS THE FOLLOWING STIFFNESS MATRIX

0.22254333D 06							
0.0	0.0						
-0.85459270D 02	0.0	0.39240221D 09					
-0.11731336D 04	0.0	0.22653566D 01	0.81416195D 04				
-0.22254333D 06	0.0	0.85459270D 02	0.11731336D 04	0.68779767D 06			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.19857257D 04	0.0	-0.13044700D 05	0.14330393D 02	0.11101088D 05	0.0	0.29020736D 07	
-0.33582030D 04	0.0	0.21322918D 01	0.17432026D 02	-0.26431589D 04	0.0	-0.45479700D 01	0.18832598D 03
-0.46525434D 06	0.0	-0.91153623D 04	0.60013619D 04	0.12194024D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.12661150D 05	0.0	-0.14509649D 07	-0.13542286D 02	0.40893723D 05	0.0	0.46422839D 07	
-0.75279022D 04	0.0	0.92097630D 02	0.67023339D 02	-0.29422875D 04	0.0	0.20552160D 01	0.36348552D 03
-0.75414809D 06	0.0	-0.28232573D 05	0.10470190D 05	0.18023722D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.33674829D 05	0.0	-0.26110155D 07	-0.99237899D 02	0.91004613D 05	0.0	0.67133565D 07	
-0.11972645D 05	0.0	0.23370593D 03	0.11229853D 03	-0.29673478D 04	0.0	0.51046256D 01	0.54255680D 03
-0.10482241D 07	0.0	-0.57329785D 05	0.14939993D 05	0.23926880D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.64765973D 05	0.0	-0.37291984D 07	-0.24202088D 03	0.16119006D 06	0.0	0.88383924D 07	
-0.16434369D 05	0.0	0.43160727D 03	0.15742666D 03	-0.29751826D 04	0.0	0.76894268D 01	0.72217632D 03
-0.13444639D 07	0.0	-0.96424089D 05	0.19409551D 05	0.29866650D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.10588739D 06	0.0	-0.48327531D 07	-0.44166073D 03	0.25139537D 06	0.0	0.10981660D 08	
-0.20898718D 05	0.0	0.68608077D 03	0.20250637D 03	-0.29785947D 04	0.0	0.10109438D 02	0.90198006D 03
-0.16422011D 07	0.0	-0.14550798D 06	0.23877312D 05	0.21234108D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.15701336D 06	0.0	-0.59289735D 07	-0.69806558D 03	0.32563637D 06	0.0	0.10645907D 08	
-0.25361791D 05	0.0	0.99716493D 03	0.24755959D 03	0.14262825D 05	0.0	-0.75826573D 03	0.87429782D 03
-0.48120969D 06	0.0	-0.16862302D 06	0.11098966D 05	0.10809841D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.18301904D 06	0.0	-0.44880595D 07	-0.20470430D 03	0.44398605D 06	0.0	0.10303486D 08	
-0.11996684D 05	0.0	0.74119448D 03	0.18858314D 03	-0.18492766D 04	0.0	-0.33825093D 03	0.84648086D 03
-0.59977442D 06	0.0	-0.26096702D 06	0.13845961D 05	0.13214323D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.27866585D 06	0.0	-0.55606055D 07	-0.40654608D 03	0.65179085D 06	0.0	0.12438867D 08	
-0.14730341D 05	0.0	0.10714547D 04	0.23360595D 03	-0.18583378D 04	0.0	-0.40936014D 03	0.10261867D 04
-0.72165786D 06	0.0	-0.37312500D 06	0.16588679D 05	0.15690465D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.39414323D 06	0.0	-0.66653034D 07	-0.66504780D 03	0.89912318D 06	0.0	0.14572287D 08	
-0.17457496D 05	0.0	0.14579625D 04	0.27859232D 03	-0.18682848D 04	0.0	-0.48039030D 03	0.12058529D 04
-0.84738866D 06	0.0	-0.50497995D 06	0.19325781D 05	0.18249146D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.52932614D 06	0.0	-0.77431101D 07	-0.98007542D 03	0.11857182D 07	0.0	0.16699752D 08	
-0.20176602D 05	0.0	0.19005836D 04	0.32354242D 03	-0.18794014D 04	0.0	-0.55127358D 03	0.13854329D 04
-0.97752593D 06	0.0	-0.65639211D 06	0.22056004D 05	0.20901657D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.68407051D 06	0.0	-0.88141651D 07	-0.13514825D 04	0.15112706D 07	0.0	0.18818474D 08	
-0.22886316D 05	0.0	0.23991451D 04	0.36845394D 03	-0.18918131D 04	0.0	-0.62195992D 03	0.15648931D 04
-0.11126397D 07	0.0	-0.82720006D 06	0.24778129D 05	0.23659425D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.85821236D 06	0.0	-0.98782375D 07	-0.17791042D 04	0.18754336D 07	0.0	0.20926242D 08	
-0.25585381D 05	0.0	0.29534436D 04	0.41332351D 03	-0.19055805D 04	0.0	-0.69240772D 03	0.17442061D 04
-0.12533028D 07	0.0	-0.10172213D 07	0.27490962D 05	0.26533889D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.10515674D 07	0.0	-0.10934911D 08	-0.22627544D 04	0.22778200D 07	0.0	0.23021151D 08	
-0.28272591D 05	0.0	0.35632492D 04	0.45814721D 03	-0.19207341D 04	0.0	-0.76257985D 03	0.19233473D 04
-0.14000861D 07	0.0	-0.12262525D 07	0.30193325D 05	0.29536430D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.12639313D 07	0.0	-0.11983670D 08	-0.28022248D 04	0.27180015D 07	0.0	0.25101480D 08	
-0.30946764D 05	0.0	0.42283056D 04	0.50292080D 03	-0.19372887D 04	0.0	-0.83244181D 03	0.21022943D 04
-0.15535569D 07	0.0	-0.14540702D 07	0.32884053D 05	0.32678335D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.14950794D 07	0.0	-0.13023943D 08	-0.33972844D 04	0.31955097D 07	0.0	0.27165621D 08	
-0.33606740D 05	0.0	0.49483327D 04	0.54763989D 03	-0.19552507D 04	0.0	-0.90196070D 03	0.22810258D 04
-0.17142766D 07	0.0	-0.17004304D 07	0.35561990D 05	0.35970765D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.17447670D 07	0.0	-0.14055127D 08	-0.40476790D 04	0.37098364D 07	0.0	0.29212045D 08	
-0.36251370D 05	0.0	0.57230262D 04	0.59229998D 03	-0.19746211D 04	0.0	-0.97110473D 03	0.24595211D 04
-0.18827999D 07	0.0	-0.19650694D 07	0.38225991D 05	0.39424736D 07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.20127299D 07	0.0	-0.15076602D 08	-0.47531315D 04	0.42604339D 07	0.0	0.31239283D 08	
-0.38879519D 05	0.0	0.65520580D 04	0.63689651D 03	-0.19953972D 04	0.0	-0.10398429D 04	0.26377599D 04
-0.20596737D 07	0.0	-0.22477039D 07	0.40874916D 05	0.14411709D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.22986841D 07	0.0	-0.16087741D 08	-0.55132414D 04	-0.88782153D 07	0.0	0.92170669D 08	
-0.41490061D 05	0.0	0.74350770D 04	0.68142488D 03	-0.76010040D 06	0.0	0.12402642D 06	0.75074868D 04
-0.14205742D 09	0.0	0.11176899D 08	0.80159046D 06	0.28611562D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.11123959D 08	0.0	-0.76038664D 08	-0.13151848D 06	-0.22621663D 08	0.0	0.15324759D 09	
-0.80539561D 06	0.0	0.13258237D 06	0.30720018D 04	-0.75000571D 04	0.0	0.27768517D 04	0.12377045D 05
-0.14405821D 09	0.0	0.11497704D 08	0.81289567D 06	0.29011407D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.11443998D 08	0.0	-0.77192957D 08	-0.13535930D 06	-0.23266716D 08	0.0	0.15555783D 09	
-0.81669571D 06	0.0	0.13643816D 06	0.31164428D 04	-0.74867603D 04	0.0	0.28156102D 04	0.12554772D 05
-0.14605587D 09	0.0	0.11822718D 08	0.82418247D 06	0.29410623D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.11768246D 08	0.0	-0.78349123D 08	-0.13925385D 06	-0.23920168D 08	0.0	0.15787184D 09	
-0.82797736D 06	0.0	0.14034765D 06	0.31608661D 04	-0.74732793D 04	0.0	0.28543050D 04	0.12732429D 05
-0.14805036D 09	0.0	0.12151922D 08	0.83545064D 06	0.29809201D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.12096635D 08	0.0	-0.79507187D 08	-0.14320203D 06	-0.24581983D 08	0.0	0.16018968D 09	
-0.83924028D 06	0.0	0.14431074D 06	0.32052716D 04	-0.74596143D 04	0.0	0.28929352D 04	0.12910013D 05
-0.15004165D 09	0.0	0.12485296D 08	0.84665989D 06	0.30207134D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.12429296D 08	0.0	-0.80667172D 08	-0.14720375D 06	-0.25252123D 08	0.0	0.16251139D 09	
-0.85048422D 06	0.0	0.14832736D 06	0.32496589D 04	-0.74457656D 04	0.0	0.29314999D 04	0.13087524D 05
-0.15202969D 09	0.0	0.12822827D 08	0.85792999D 06	0.30604412D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.12766061D 08	0.0	-0.81829102D 08	-0.15125893D 06	-0.25930551D 08	0.0	0.16483702D 09	
-0.86170894D 06	0.0	0.15239741D 06	0.32940279D 04	-0.74317336D 04	0.0	0.29699984D 04	0.13264961D 05
-0.15401443D 09	0.0	0.13164490D 08	0.86914068D 06	0.31001027D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.13106959D 08	0.0	-0.82993001D 08	-0.15536749D 06	-0.26617227D 08	0.0	0.16716662D 09	
-0.87291418D 06	0.0	0.15652080D 06	0.33383782D 04	-0.74175186D 04	0.0	0.30084296D 04	0.13442323D 05
-0.15599584D 09	0.0	0.13510267D 08	0.88033170D 06	0.31396973D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.13451972D 08	0.0	-0.84158892D 08	-0.15952930D 06	-0.27312112D 08	0.0	0.16950023D 09	
-0.86409967D 06	0.0	0.16069744D 06	0.33827096D 04	-0.74031209D 04	0.0	0.30467928D 04	0.13619610D 05
-0.15797386D 09	0.0	0.13860140D 08	0.89150280D 06	0.31792239D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.13801081D 08	0.0	-0.85326798D 08	-0.16374430D 06	-0.28015168D 08	0.0	0.17183789D 09	
-0.89526518D 06	0.0	0.16492722D 06	0.34270219D 04	-0.73885408D 04	0.0	0.30850871D 04	0.13796819D 05
-0.15994851D 09	0.0	0.14214087D 08	0.90265372D 06	0.32186818D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.14154265D 08	0.0	-0.86496743D 08	-0.16801238D 06	-0.28726353D 08	0.0	0.17417967D 09	
-0.90641045D 06	0.0	0.16921006D 06	0.34713149D 04	-0.73737787D 04	0.0	0.31233115D 04	0.13973950D 05
-0.16191968D 09	0.0	0.14572089D 08	0.91378423D 06	0.32580702D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.14511504D 08	0.0	-0.87668749D 08	-0.17233344D 06	-0.29445629D 08	0.0	0.17652559D 09	
-0.91753522D 06	0.0	0.17354586D 06	0.35155881D 04	-0.73588349D 04	0.0	0.31614653D 04	0.14151003D 05
-0.16388735D 09	0.0	0.14934125D 08	0.92489406D 06	0.16388735D 09			
0.0	0.0	0.0	0.0	0.0	0.0		
0.14872777D 08	0.0	-0.88842840D 08	-0.17670739D 06	-0.14872777D 08	0.0	0.89170358D 08	
-0.92863925D 06	0.0	0.17793452D 06	0.35598416D 04	0.92863925D 06	0.0	-0.17793773D 06	0.71485059D 04

Fig. 7 ZEROTH (0) HARMONIC STIFFNESS MATRIX

HARMONIC NUMBER 0 HAS THE FOLLOWING MASS MATRIX

0.21865939D-08							
0.0	0.0						
-0.32777373D-11	0.0	0.18341141D-08					
-0.12580001D-10	0.0	0.19830098D-13	0.86780194D-13				
0.11147204D-08	0.0	0.10255087D-10	-0.95608616D-11	0.17548356D-07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.60267746D-11	0.0	0.18339557D-08	0.50373513D-13	-0.24797289D-10	0.0	0.14671889D-07	
0.94885822D-11	0.0	-0.15043879D-13	-0.79185228D-13	-0.25159471D-10	0.0	0.24934956D-12	0.50333229D-12
0.33443496D-08	0.0	0.44778852D-10	-0.28609476D-10	0.35095564D-07			
0.0	0.0	0.0	0.0	0.0	0.0		
-0.40661785D-11	0.0	0.55016197D-08	0.32730445D-12	-0.79515784D-10	0.0	0.29344126D-07	
0.28537985D-10	0.0	-0.22131625D-12	-0.23756673D-12	-0.25153572D-10	0.0	0.49865136D-12	0.10066653D-11
0.55743449D-08	0.0	0.97536245D-10	-0.47652626D-10	0.52639846D-07			
0.0	0.0	0.0	0.0	0.0	0.0		
0.16130044D-10	0.0	0.91685420D-08	0.84578886D-12	-0.17068970D-09	0.0	0.44016449D-07	
0.47582928D-10	0.0	-0.66916080D-12	-0.39593394D-12	-0.25145667D-10	0.0	0.74787679D-12	0.15099474D-11
0.78049833D-08	0.0	0.16851207D-09	-0.66687670D-10	0.70179931D-07			
0.0	0.0	0.0	0.0	0.0	0.0		
0.54547970D-10	0.0	0.12834228D-07	0.16057442D-11	-0.29828185D-09	0.0	0.58689083D-07	
0.66620660D-10	0.0	-0.13585046D-11	-0.55428522D-12	-0.25134602D-10	0.0	0.99698192D-12	0.20131687D-11
0.10036522D-07	0.0	0.25768367D-09	-0.85711545D-10	0.87714420D-07			
0.0	0.0	0.0	0.0	0.0	0.0		
0.11116626D-09	0.0	0.16498185D-07	0.26070481D-11	-0.46224092D-09	0.0	0.73362130D-07	
0.85648118D-10	0.0	-0.22892369D-11	-0.71261422D-12	-0.25120379D-10	0.0	0.12459267D-11	0.25163091D-11
0.12269216D-07	0.0	0.36502109D-09	-0.10472119D-09	0.13758745D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.18595625D-09	0.0	0.20159918D-07	0.38495396D-11	-0.81256647D-09	0.0	0.11855427D-06	
0.10466224D-09	0.0	-0.34612079D-11	-0.87091456D-12	-0.40455571D-09	0.0	0.16847306D-10	0.74685929D-11
0.28820488D-07	0.0	0.80092991D-09	-0.36124075D-09	0.20805596D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.21531780D-09	0.0	0.40565696D-07	0.16659745D-10	-0.12972922D-08	0.0	0.18178694D-06	
0.35700281D-09	0.0	-0.14423048D-10	-0.43527382D-11	-0.99547865D-10	0.0	0.75630516D-11	0.13994083D-10
0.35703701D-07	0.0	0.11417645D-08	-0.44658146D-09	0.25231413D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.41658558D-09	0.0	0.50223995D-07	0.25206852D-10	-0.18965599D-08	0.0	0.22052312D-06	
0.44239509D-09	0.0	-0.22438472D-10	-0.53901106D-11	-0.99375449D-10	0.0	0.91681020D-11	0.16973309D-10
0.42592391D-07	0.0	0.15397205D-08	-0.53172790D-09	0.29652521D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.67505177D-09	0.0	0.59863693D-07	0.35510842D-10	-0.26097998D-08	0.0	0.25925619D-06	
0.52760406D-09	0.0	-0.32211956D-10	-0.64269106D-11	-0.99169993D-10	0.0	0.10769258D-10	0.19950732D-10
0.49487591D-07	0.0	0.19944419D-08	-0.61664392D-09	0.34068097D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.99037513D-09	0.0	0.69486001D-07	0.47567340D-10	-0.34362542D-08	0.0	0.29798554D-06	
0.61259352D-09	0.0	-0.43739348D-10	-0.74630281D-11	-0.98931584D-10	0.0	0.12365841D-10	0.22926036D-10
0.56390323D-07	0.0	0.25055121D-08	-0.70129345D-09	0.38477325D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.13621539D-08	0.0	0.79088143D-07	0.61371225D-10	-0.43750446D-08	0.0	0.33671058D-06	
0.69732738D-09	0.0	-0.57015752D-10	-0.84983532D-11	-0.98660323D-10	0.0	0.13957171D-10	0.25898906D-10
0.63301597D-07	0.0	0.30724546D-08	-0.78564052D-09	0.42879388D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.17899264D-08	0.0	0.88667358D-07	0.76916632D-10	-0.54251734D-08	0.0	0.37543064D-06	
0.78176964D-09	0.0	-0.72035529D-10	-0.95327759D-11	-0.98356326D-10	0.0	0.15542573D-10	0.28869026D-10
0.70222407D-07	0.0	0.36947332D-08	-0.86964933D-09	0.47273475D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.22731713D-08	0.0	0.98220900D-07	0.94196961D-10	-0.65855238D-08	0.0	0.41414507D-06	
0.86588444D-09	0.0	-0.88792301D-10	-0.10566186D-10	-0.98019721D-10	0.0	0.17121373D-10	0.31836081D-10
0.77153731D-07	0.0	0.43717529D-08	-0.95328418D-09	0.51658776D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.28113081D-08	0.0	0.10774604D-06	0.11320487D-09	-0.78548617D-08	0.0	0.45285316D-06	
0.94963606D-09	0.0	-0.10727895D-09	-0.11598475D-10	-0.97650652D-10	0.0	0.18692902D-10	0.34799754D-10
0.84096528D-07	0.0	0.51028598D-08	-0.10365096D-08	0.56034495D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.34036979D-08	0.0	0.11724008D-06	0.13393229D-09	-0.92318367D-08	0.0	0.49155417D-06	
0.10329889D-08	0.0	-0.12748762D-09	-0.12629532D-10	-0.97249276D-10	0.0	0.20256491D-10	0.37759733D-10
0.91051738D-07	0.0	0.58873425D-08	-0.11192901D-08	0.60399828D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.40496439D-08	0.0	0.12670032D-06	0.15637041D-09	-0.10714983D-07	0.0	0.53024733D-06	
0.11159076D-08	0.0	-0.14940974D-09	-0.13659247D-10	-0.96815762D-10	0.0	0.21811476D-10	0.40715702D-10
0.98020279D-07	0.0	0.67244322D-08	-0.12015907D-08	0.64753984D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.47483920D-08	0.0	0.13612411D-06	0.18050971D-09	-0.12302722D-07	0.0	0.56893161D-06	
0.11983569D-08	0.0	-0.17303598D-09	-0.14687513D-10	-0.96350295D-10	0.0	0.23357198D-10	0.43667348D-10
0.10500304D-06	0.0	0.76133037D-08	-0.12833763D-08	0.45817117D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.54991318D-08	0.0	0.14550881D-06	0.20633992D-09	-0.14260687D-07	0.0	0.37023237D-06	
0.12803018D-08	0.0	-0.19835632D-09	-0.15714219D-10	0.21305531D-08	0.0	-0.34294289D-09	0.23612287D-10
-0.12499106D-07	0.0	0.78940860D-08	-0.34629929D-10	0.23772891D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.77769745D-08	0.0	0.33889060D-07	0.57394520D-11	-0.15896353D-07	0.0	0.14435767D-06	
0.35115138D-10	0.0	-0.57232476D-11	-0.33873082D-12	-0.22053003D-11	0.0	0.56931887D-12	0.16921294D-11
-0.12641852D-07	0.0	0.81201018D-08	-0.35120442D-10	0.24106538D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.80012961D-08	0.0	0.34341240D-07	0.59065874D-11	-0.16349649D-07	0.0	0.14650781D-06	
0.35605729D-10	0.0	-0.58901550D-11	-0.34363101D-12	-0.22025737D-11	0.0	0.57726506D-12	0.17164278D-11
-0.12782901D-07	0.0	0.83490751D-08	-0.35610155D-10	0.24439723D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.82285759D-08	0.0	0.34791601D-07	0.60760538D-11	-0.16808847D-07	0.0	0.14866039D-06	
0.36095520D-10	0.0	-0.60593938D-11	-0.34852927D-12	-0.21998094D-11	0.0	0.58519819D-12	0.17407164D-11
-0.12922232D-07	0.0	0.85809930D-08	-0.36099056D-10	0.24772437D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.84588010D-08	0.0	0.35240120D-07	0.62478473D-11	-0.17273922D-07	0.0	0.15081545D-06	
0.36584501D-10	0.0	-0.62309601D-11	-0.35342555D-12	-0.21970073D-11	0.0	0.59311809D-12	0.17649952D-11
-0.13059823D-07	0.0	0.88158424D-08	-0.36587135D-10	0.25104676D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.86919583D-08	0.0	0.35686772D-07	0.64219641D-11	-0.17744847D-07	0.0	0.15297301D-06	
0.37072661D-10	0.0	-0.64048501D-11	-0.35831984D-12	-0.21941676D-11	0.0	0.60102457D-12	0.17892640D-11
-0.13195652D-07	0.0	0.90536100D-08	-0.37074380D-10	0.25436434D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.89260344D-08	0.0	0.36131534D-07	0.65984002D-11	-0.18221596D-07	0.0	0.15513310D-06	
0.37559988D-10	0.0	-0.65810598D-11	-0.36321210D-12	-0.21912904D-11	0.0	0.60891745D-12	0.18135226D-11
-0.13329696D-07	0.0	0.92942823D-08	-0.37560780D-10	0.25767703D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.91670159D-08	0.0	0.36574382D-07	0.67771517D-11	-0.18704141D-07	0.0	0.15729575D-06	
0.38046471D-10	0.0	-0.67595852D-11	-0.36810230D-12	-0.21883756D-11	0.0	0.61679656D-12	0.18377711D-11
-0.13461934D-07	0.0	0.95378456D-08	-0.38046325D-10	0.26098479D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.94088893D-08	0.0	0.37015292D-07	0.69582145D-11	-0.19192455D-07	0.0	0.15946100D-06	
0.38532100D-10	0.0	-0.69404224D-11	-0.37299043D-12	-0.21854234D-11	0.0	0.62466172D-12	0.18620091D-11
-0.13592345D-07	0.0	0.97842662D-08	-0.38531003D-10	0.26428754D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.96536406D-08	0.0	0.37454242D-07	0.71415844D-11	-0.19686511D-07	0.0	0.16162886D-06	
0.39016863D-10	0.0	-0.71235671D-11	-0.37787644D-12	-0.21824338D-11	0.0	0.63251275D-12	0.18862366D-11
-0.13720907D-07	0.0	0.10033590D-07	-0.39014804D-10	0.26758524D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.99012560D-08	0.0	0.37891207D-07	0.73272575D-11	-0.20186280D-07	0.0	0.16379938D-06	
0.39500750D-10	0.0	-0.73090153D-11	-0.38276032D-12	-0.21794069D-11	0.0	0.64034947D-12	0.19104534D-11
-0.13847597D-07	0.0	0.10285743D-07	-0.39497716D-10	0.27087781D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.10151721D-07	0.0	0.38326165D-07	0.75152294D-11	-0.20691734D-07	0.0	0.16597258D-06	
0.39983749D-10	0.0	-0.74967628D-11	-0.38764203D-12	-0.21763428D-11	0.0	0.64817171D-12	0.19346595D-11
-0.13972396D-07	0.0	0.10540731D-07	-0.39979728D-10	0.13657074D-06			
0.0	0.0	0.0	0.0	0.0	0.0		
0.10405022D-07	0.0	0.38759093D-07	0.77054959D-11	-0.10464231D-07	0.0	0.83777748D-07	
0.40465850D-10	0.0	-0.76868053D-11	-0.39252155D-12	0.18637688D-09	0.0	-0.35772761D-10	0.97594022D-12

Fig. 8 ZEROTH (0) HARMONIC MASS MATRIX

```
                              HARMONIC NUMBER    1    HAS THE FOLLCWING STIFFNESS MATRIX

 0.97165701D 16
 0.59926630D 09  0.26489106D 05
 0.59925617D 09  0.26479901D 09  0.26487129D 09
 0.97163957D 10  0.59994753D C3  0.60177931D 03  0.97387555D 04
-0.83916245D 07  0.23013349D C3 -0.54884670D 02  0.11114365D 04  0.37443739D 06
 0.16072308D 03 -0.17056595D C4  0.13448059D 04  0.22877404D 01  0.46656034D 02  C.76923879D 06
 0.10291256D C5  0.90345223D C5 -C.70933264D 05  0.45600527D 00  0.46215927D 04  0.35436097D 06  0.11225720D 07
-0.84430892D 05 -0.25447209D 01  0.27367820D 01  0.11179842D 02 -0.13248290D 04 -0.19311449D 01  0.71926515D 01  0.99358215D 02
-0.25429500D 06  0.18472539D 04 -0.40578533D 04  0.30424570D C4  0.63783130D 06
-0.12391453D 04 -0.19811530D C6 -0.80922811D 05  0.49370491D 01  0.37290540D C3  0.97587736D 06
-0.55290787D 04  0.21371205D 05 -0.68785532D 06 -0.94687442D C1  0.19974408D 05  0.23049194D 06  0.22878777D 07
-0.38424299D 04 -0.53822474D 01  0.45716583D 02  0.33045563D 02 -0.14367097D 04 -0.21792721D 01  0.36217029D 01  0.18476627D 03

-0.38932368D 06  0.20786110D 04 -0.13499618D C5  0.52608702D 04  C.92055479D 06
-0.23539416D 04 -0.41756162D 06 -0.11987229D 06  0.55252147D 01  0.44248772D C3  0.12852009D 07
-0.16050337D 05  0.21063131D 06 -0.12756907D 07 -0.51374590D 02  0.45143856D 05  0.15947236D 06  0.33503340D 07
-0.60245034D 04 -0.58336989D 01  C.11660073D C3  0.55855866D 02 -0.14675604D 04 -0.16458708D 01  0.40121952D 01  0.27328890D 03

-0.53271762D C6  0.42427126D 04 -C.28006279D C5  0.74889667D 04  0.12110116D 07
-0.34819034D 04 -0.62362079D C6 -0.13714703D 06  0.57249667D 01  0.46577355D 03  0.16304833D 07
-0.31605043D 05  0.20441841D 06 -0.18416867D C7 -0.12236753D 03  0.80277446D C5  0.12093017D 06  0.44184990D 07
-0.82423932D 04 -0.59545941D 01  0.21568454D 03  0.78501231D 02 -0.14783844D C4 -0.12851870D C1  0.48667678D 01  0.36259401D 03

-0.67867441D 06  0.53656977D C4 -0.47532779D C5  0.97198386D 04  0.15051150D 07
-0.46097313D 04 -0.82339131D 06 -C.14655918D 06  0.58172877D 01  0.47623791D 03  0.19902031D 07
-0.52172412D C5  0.19968611D 06 -C.23980229D 07 -0.22197380D 03  0.12539913D 06  0.97202406D 05  0.54917589D 07
-0.10468184D C5 -0.59989702D C1  C.34302163D 03  0.10108742D 03 -0.14833590D 04 -0.10468043D 01  0.58450910D 01  0.45219392D 03

-0.82650960D 06  0.65201631D C4 -C.72062664D 05  0.11951161D 05  0.10696347D 07
-0.57361867D 04 -0.10199592D C7 -0.15240440D 06  0.58683025D 01  0.63665916D 03  0.19331142D 07
-0.77740400D 05  0.19620937D 06 -0.29492499D 07 -0.35004439D 03  G.16272001D 06  0.10246395D C6  0.53283318D 07
-0.12695921D 05 -0.60182405D 01  0.49863759D 03  0.12364384D 03  0.71368890D C4  C.79108447D 01 -0.37794451D 03  0.43843244D 03

-0.24327465D 06  0.81571485D 04 -0.83393723D C5  0.55593153D 04  C.54524266D 06
-0.69560735D 04 -0.76221650D C6 -C.14407352D 06  0.14273436D 02  0.82745872D 03  0.18780468D 07
-0.90456601D 05  0.20087905D 06 -0.22244821D C7 -0.10452704D 03  0.22187911D 06  0.10346049D 06  0.51523488D 07
-0.60109492D 04 -0.14360874D C2  0.36989302D C3  0.94113904D 02 -C.92041123D 03 -0.21938669D 01 -0.16699465D 03  0.42452359D 03

-0.30201528D C6  0.10001348D C5 -0.12954569D C6  0.69316367D 04  0.66464271D 06
-0.87858519D 04 -0.95943821D 06 -C.15071731D 06  0.14384795D 02  0.83707245D 03  0.22427842D 07
-0.13828663D 06  0.19700513D C6 -C.27742445D 07 -0.20508831D 03  C.32580197D 06  C.85537732D 05  0.62207728D 07
-0.73750732D 04 -0.14472807D 02  0.53520272D 03  0.11665909D C3 -0.92625341D 03 -0.18340897D 01 -0.20295788D 03  0.51415079D 03

-0.36259117D C6  0.11836111D 05 -C.16561233D C6  0.83011636D 04  C.78786817D 06
-0.10612915D 05 -0.11544420D 07 -0.15508C11D 06  0.14448694D 02  0.84236381D 03  0.26120935D 07
-0.19603077D 06  0.19409356D C6 -0.33190656D C7 -0.33410304D 03  0.44948015D 06  0.72878870D 05  0.72877003D 07
-0.87369054D 04 -0.14523076D 02  0.72859157D C3  0.13917529D C3 -0.43200082D 03 -0.15724476D 01 -0.23874779D 03  0.60382579D 03

-0.42519055D 06  0.13664994D C5 -0.25153165D 06  0.96688077D 04  0.91537004D 06
-0.12437472D 05 -0.13480494D 07 -0.15812914D 06  0.14488895D 02  0.84540964D 03  0.29840457D 07
-0.26362681D 06  0.19180333D C6 -0.38598014D 07 -0.49145041D 03  0.59278534D 06  0.63471247D 05  0.83514615D 07
-0.10095239D 05 -0.14553232D C2  0.95000621D 03  0.16166702D 03 -0.93805323D 03 -0.13745747D 01 -0.27438757D 03  0.69349894D 03

-0.49005627D 06  0.15485619D C5 -0.32723210D C6  0.11033223D C5  C.10476620D 07
-0.14259678D C5 -0.15406906D 07 -0.16035248D C6  0.14515929D 02  0.64715180D 03  0.33575878D 07
-0.34100297D 06  0.18996365D C6 -0.43967380D C7 -0.67703053D 03  0.75556668D 06  0.56208517D 05  0.94107786D 07
-0.11449195D 05 -0.14572570D C2  0.11993653D C4  0.18413542D 03 -0.94459347D 03 -0.12200543D 01 -0.30988068D 03  0.78313916D 03

-0.55745254D 06  0.17310772D C5 -C.41283212D C6  0.12393735D C5  C.11852852D C7
-0.16079597D 05 -0.17326066D 07 -0.16202162D 06  0.14535045D C2  0.84806814D 03  0.37320875D 07
-0.42807741D 06  0.18843464D C6 -0.49298901D C7 -0.89074623D 03  C.93765190D 06  0.50433837D 05  0.10464592D 08
-0.12798035D 05 -0.14585610D 02  C.14768853D 04  0.26658010D 03 -0.95171389D 03 -0.10961992D 01 -0.34522218D 03  0.87272431D 03

-0.62765314D 06  0.19128633D C5 -0.50763594D C6  0.13745704D 05  0.13287924D 07
-0.17897219D C5 -0.19239387D C7 -0.16329512D 06  0.14549165D 02  0.84842227D 03  0.41071354D 07
-0.52475815D 06  C.18713CC7D C6 -0.54591321D C7 -0.11324958D C4  C.11388478D 07  0.45733188D 05  0.11511968D 08
-0.14141090D 05 -0.14594755D 02  0.17815429D 04  0.22899991D 03 -0.95946434D 03 -0.99477386D 00 -0.38040306D 03  0.96223698D 03

-0.70093562D 06  0.20543966D C5 -C.61215364D C6  0.15100514C C5  0.14787404D 07
-0.19712483D 05 -0.21147717D C7 -0.16428805D 06  0.14555779D 02  0.84836937D 03  0.44824490D 07
-0.63094307D 06  0.18559094D C6 -0.59842605D C7 -0.14021696D C4  0.13589406C 07  0.41832860D 05  0.12552054D C8
-0.15477730D C5 -0.14601370D 02  0.21141172D 04  0.25139325D 03 -0.96787284D 03 -C.91022234C 00 -0.41541216D 03  0.10516625D 04

-0.77757816D 06  0.22756213D C5 -C.72606119D 06  0.16445566D 05  0.16356846D 07
-0.21525242D C5 -0.23151568D 07 -0.16505684D 06  0.14568095D 02  0.64800570D 03  0.48578227D 07
-0.74651993D 06  0.18497566D C6 -0.65050269D 07 -0.16996465D C4  0.15976464D 07  0.38544747D 05  0.13584051D C8
-0.16807347D 05 -0.14662279D 02  0.24741699D 04  0.27375827D 03 -0.97695530D 03 -C.83867041D 00 -0.45023713D 03  0.11409878D 04

-0.85785775D 06  0.24565540D 05 -C.84924C54D 06  0.17784268D 05  C.18001770D C7
-0.23335527D 05 -0.24951237D 07 -0.16565271D 06  0.14574715D C2  0.84793535D 03  0.52330992D 07
-0.87136647D 06  0.18405422D C6 -0.70211564D C7 -0.20248030D 04  C.18548610D 07  0.35735346D 05  0.14607195D 08
-0.18129350D 05 -0.14605999D C2  0.28C15503D 04  0.29609255D 03 -C.98672055D C3 -0.77733842D 00 -0.48486491D 03  0.12302010D 04

-0.94204907D 06  0.26371866D C5 -C.98155973D C6  0.19116037D C5  0.19727639D 07
-0.25143045D 05 -0.26848884D C7 -0.16610926D 06  0.14580083D 02  0.84657473D 03  0.56081536D 07
-0.10053506D 07  0.18320442D 06 -0.75323580D C7 -0.23774927D C4  0.21301608D 07  0.33307309D 05  0.15620751D 08
-0.19443158D C5 -0.14612868D 02  0.32760955D C4  0.31839521D 03 -C.99717295D 03 -0.72418250D 00 -0.51928205D 03  0.13192906D 04

-0.10304236D C7  0.28175C82D C5 -0.11228770D C7  0.20446298D C5  C.72062300D 08
-0.26947707D 05 -0.28738575D 07 -0.16645086D 06  0.14584504D 02  0.45734734D 03  0.15900746D 08
-0.11483303D 07  0.18240953D C6 -0.80383322D C7 -0.27575660D 04 -0.44396517D 07  G.19532587D 05  0.46085650D C8
-0.20748199D 05 -0.14615114D 02  0.37176307D 04  0.34066265D 03 -0.38004992D 06 -0.15434215D 02  0.62013547D 05  0.37539830D 04

-0.71031691D C8  0.28500771D C5  C.55891452D 07  C.40079804D 06  0.14306380D 09
-0.28480257D 05 -0.12997482D C8 -0.17280299D C6  0.14995357D 01  0.58791913D 02  0.26193604D 08
 0.55026809D 07  0.17632457D C6 -0.38018178D 08 -0.65759728D 05 -C.11311781D 08  0.71205127D 04  0.76624207D C8
-0.40270071D C6 -0.15144860D C1  0.66291648D C5  0.15359899D 04 -0.37499424D 04 -0.37197990D-01  0.13884363D 04  0.61886102D 04

-0.72032038D 08  0.29315785D C5  0.57495473D C7  0.40645065D 06  0.14506294D 09
-0.28895435D 05 -0.13185574D C8 -0.17276C83D C6  0.14996488D 01  0.58770404D 02  0.26569560D 08
 0.57226999D 07  0.17623021D C6 -0.38593345D 08 -0.67680137D 05 -0.11634307D 08  0.70194422D 04  0.77779324D 08
-0.40835072D 06 -0.15143801D 01  0.68219542C C5  0.15582105D 04 -0.37432964D 04 -C.36656938D-01  0.14078154D 04  0.62774725D 04

-0.73030825D C8  0.29730633D C5  0.59120539D 07  0.41205401D C6  0.14705893D 09
-0.29310449D 05 -0.13373591D C8 -0.17271694C C6  0.14997587D 01  C.58748685D 02  0.26945366D 08
 0.58848237D 07  0.17613558C C6 -0.39173447D 08 -0.69627408D 05 -0.11961033D 08  0.69212278D 04  0.78936330D 08
-0.41399150D 06 -0.15142772D C1  0.70174285D C5  0.15804224D 04 -0.37365582D 04 -C.36130985D-01  0.14271626D C4  0.63662594D 04

-0.74026029D 08  0.30145312D C5  C.60766558D C7  0.41772805D 06  0.14905174D C5
-0.29725297D 05 -0.13561532D C8 -0.17267134D C6  0.14998655D 01  0.58726161D 02  0.27321026D 08
 0.60490428D 07  0.17604065D 06 -0.39752498D 08 -0.71601498D 05 -0.12291940D C8  0.68257505D 04  0.80095248D 08
-0.41962292D 06 -0.15141771D 01  0.72155833D 05  0.16026252D 04 -0.37297278D 04 -0.35619501D-01  0.14464776D 04  0.64550903D 04

-0.75023630D C8  0.30559820D C5  C.62433436D 07  0.42335264D 06  0.15104131D 09
-0.30139976D 05 -0.13749394D C8 -0.17262406D 06  0.14999693D 01  0.58703436D 02  0.27696532D 08
 0.62153481D 07  0.17594538D 06 -0.40332508D 08 -0.73602361D 05 -0.12627010D 08  0.67328979D 04  0.81256103D 08
-0.42524485D 06 -0.15140797D 01  0.74164143D 05  0.16248191D C4 -0.37228056D 04 -0.35121892D-01  0.14657598D C4  0.65438447D 04

-0.76017607D 08  0.30974154D C5  0.64121079D C7  0.42896766D 06  0.15302763D 09
-0.30554484D 05 -0.13937179D C8 -0.17257514D C6  0.15000703D 01  0.58680314D 02  C.28071883D 08
 0.63837300D 07  0.17584975D 06 -0.40913491D 08 -0.75629952D 05 -0.12966223D 08  0.66425636D 04  0.82418917D 08
-0.43085717D 06 -0.15139848D 01  0.76199168D 05  0.16470037D 04 -0.37157916D 04 -0.34637596D-01  0.14850089D 04  0.66325623D 04

-0.77009940D 08  0.31386312D C5  0.65829391D C7  0.43457296D 06  0.15501063D 09
-0.30968819D 05 -0.14124683D C8 -0.17252461D C6  0.15001685D 01  0.58656798D C2  C.28447078D 08
 0.65541789D 07  0.17575373D 06 -0.41495458D 08 -0.77684225D 05 -0.13309560D 08  0.65546469D 04  0.83583715D 08
-0.43645975D 06 -0.15138925D C1  0.78260863C 05  0.16691790D 04 -0.37086860D 04 -0.34166081D-01  0.15042244D 04  0.67212423D 04

-0.78000607D 08  0.31802293D 05  0.67558275D C7  0.44016844D 06  0.15699028D 09
-0.31382978D 05 -0.14312507D C8 -0.17247248D 06  0.15002642D C1  0.58632892D 02  0.28822114D 08
 0.67266853D 07  0.17565731D 06 -0.42078421D 08 -0.79765135D 05 -0.13657003D 08  0.64690523D 04  0.84750518D 08
-0.44205247D 06 -0.15138025D 01  C.80349180D 05  0.16913448D C4 -0.37014890D 04 -0.33706841D-01  0.15234059D 04  0.68098844D 04

-0.78989588D 08  0.32216093D C5  0.69307633D 07  0.44575395D 06  0.15896653D 09
-0.31796959D 05 -0.14500050D 08 -0.17241878D C6  0.15003573D C1  0.58608599D 02  0.29196989D 08
 0.69012392D 07  0.17556046D 06 -0.42662390D 06 -0.81872633D 05 -0.14008530D 08  0.63856895D 04  0.85919352D 08
-0.44763519D 06 -0.15137148D 01  0.82464073D 05  0.17135011D 04 -0.36942008D 04 -0.33259399D-01  0.15425529D 04  0.68984880D 04

-0.79976863D 08  0.32629710D C5  C.71077366D 07  0.45132938D 06  0.16093936D 09
-0.32210760D 05 -0.14687509D C8 -0.17236354D 06  0.15004481D 01  0.58583921D 02  0.29571700D 08
 0.70778307D 07  0.17546316D 06 -0.43247379D C8 -0.84006673D C5 -0.14364122D 08  C.63044726D 04  0.87090237D 08
-0.45320778D 06 -0.15136293D 01  0.84605494D 05  0.17356477D 04 -0.36868214D 04 -0.32823300D-01  0.15616650D 04  0.69870526D 04

-0.80962412D 08  0.33043142D 05  0.72867373D C7  0.45689460D C6  0.16290871D 09
-0.32624378D 05 -0.14874885D 08 -0.17230678D C6  0.15005365D 01  0.58558862D 02  0.29946247D 08
 0.72564498D 07  0.17536539D C6 -0.43833398D C8 -0.86167205D 05 -0.14723760D 08  0.62253202D 04  0.88263198D 08
-0.45877014D 06 -0.15135459D C1  0.86773394D 05  0.17577844D 04 -0.36793512D 04 -0.32398116D-01  0.15807418D 04  0.70755778D 04

-0.81946213D 08  0.33456387D C5  0.74677552D C7  0.46244948D C6  0.81946219D 08
-0.33037811D 05 -0.15062176D C8 -0.17224851D C6  0.15006226D 01 -0.25592413D 04  0.14879564D 08
 0.74370863D 07  0.17526713D 06 -0.44420458D C8 -0.88354181D 05 -0.74368580D 07 -0.10363037D 05  0.44585362D 08
-0.46432212D 06 -0.15134646D 01  0.88967725D 05  0.17799113D 04  0.46432580D 06 -0.80513390D 00 -0.88970043D 05  0.35742909D 04
```

Fig. 9 FIRST (1) HARMONIC STIFFNESS MATRIX

HARMONIC NUMBER 1 HAS THE FOLLOWING MASS MATRIX

0.91674467D-07							
-0.53591442D-12	0.91705481D-09						
-0.20426746D-11	0.50271813D-15	0.91705735D-09					
-0.82118219D-11	0.23923796D-14	0.11509967D-13	0.50980136D-13				
0.63652545D-09	-0.23781842D-12	0.50218420D-11	-0.52834535D-11	0.89801097D-08			
-0.23781556D-12	0.91699282D-09	0.33516870D-15	0.15950303D-14	-0.11098244D-11	0.73358375D-08		
-0.33833113D-11	0.11730041D-14	0.91657833D-09	0.27579248D-13	-0.13636427D-10	0.70379543D-14	0.73359523D-08	
0.53730441D-11	-0.15950303D-14	-0.83194821D-14	-0.43388030D-13	-0.12255466D-10	0.13224689D-18	0.12375190D-12	0.25706101D-12
0.16939397D-08	-0.23781322D-12	0.22219078D-10	-0.14458596D-10	0.17642632D-07			
-0.23782119D-12	0.27509416D-08	0.18432839D-14	0.15951361D-14	-0.11097706D-11	0.14671502D-07		
-0.22750690D-11	0.26911080D-14	0.27508117D-08	0.16537288D-12	-0.40947025D-10	0.14075671D-13	0.14672078D-07	
0.14431154D-10	-0.15551361D-14	-0.11191725D-12	-0.11986175D-12	-0.12510661D-10	0.25401903D-22	0.24896322D-12	0.50559908D-12

0.28000991D-08	-0.23779144D-12	0.46584743D-10	-0.23918761D-10	0.26382389D-07			
-0.23780472D-12	0.45847797D-08	0.33513473D-14	0.15951362D-14	-0.11096591D-11	0.22006515D-07		
0.78391150D-11	0.41851039D-14	0.45842742D-08	0.42451601D-12	-0.86527338D-10	0.21112799D-13	0.22008247D-07	
0.23888683D-10	-0.15951362D-14	-0.33592733D-12	-0.19862066D-12	-0.12544273D-10	0.11282877D-22	0.37370472D-12	0.75644754D-12

0.39116982D-08	-0.23776010D-12	0.84066564D-10	-0.33409955D-10	0.35136613D-07			
-0.23777869D-12	0.64184335D-08	0.48592759D-14	0.15951362D-14	-0.11095029D-11	0.29340643D-07		
0.27054623D-10	0.56969315D-14	0.64171186D-08	0.80445268D-12	-0.15032107D-09	0.28149078D-13	0.29344571D-07	
0.33377971D-10	-0.15951362D-14	-0.68063719D-12	-0.27760852D-12	-0.12551390D-10	0.64699690D-23	0.49831767D-12	0.10076806D-11

0.50254117D-08	-0.23771919D-12	0.12664459D-09	-0.42907298D-10	0.43894448D-07			
-0.23774310D-12	0.82518292D-08	0.63670091D-14	0.15951362D-14	-0.11093021D-11	0.36673592D-07		
0.55367344D-10	0.72045299D-14	0.82490981D-08	0.13050822D-11	-0.23229941D-09	0.35184226D-13	0.36681102D-07	
0.42876481D-10	-0.15951362D-14	-0.11460245D-11	-0.35666918D-12	-0.12550051D-10	0.42426021D-23	0.62282543D-12	0.12590282D-11

0.61404536D-08	-0.23766872D-12	0.18231627D-09	-0.52402791D-10	0.68833500D-07			
-0.23769794D-12	0.10084893D-07	0.78744863D-14	0.15951362D-14	-0.14556055D-11	0.59259880D-07		
0.92764609D-10	0.87118386D-14	0.10079966D-07	0.19263138D-11	-0.40781664D-09	0.56235239D-13	0.59277195D-07	
0.52373917D-10	-0.15951362D-14	-0.17320236D-11	-0.43575340D-12	-0.20236861D-09	0.39254879D-14	0.84274857D-11	0.37360934D-11

0.14417915D-07	-0.38606175D-12	0.40015407D-09	-0.18071018D-09	0.10406763D-06			
-0.38615730D-12	0.20296480D-07	0.15585313D-13	0.42121565D-14	-0.18014144D-11	0.90861259D-07		
0.10730695D-09	0.17795731D-13	0.20284862D-07	0.83340020D-11	-0.65056098D-09	0.87171544D-13	0.90893564D-07	
0.17859289D-09	-0.42121565D-14	-0.72152227D-11	-0.21773957D-11	-0.49755209D-10	0.11199330D-22	0.37811375D-11	0.69995028D-11

0.17858035D-07	-0.38585787D-12	0.57056773D-09	-0.22336325D-09	0.12618968D-06			
-0.38597620D-12	0.25133667D-07	0.19563656D-13	0.42121565D-14	-0.18004205D-11	0.11020487D-06		
0.20794513D-09	0.21772972D-13	0.25112015D-07	0.12607513D-10	-0.95019276D-09	0.10572965D-12	0.11026167D-06	
0.22127122D-09	-0.42121565D-14	-0.11222975D-10	-0.26958639D-11	-0.49675039D-10	0.82346784D-23	0.45837320D-11	0.84886785D-11

0.21301374D-07	-0.38561303D-12	0.76954335D-09	-0.26592482D-09	0.14829030D-06			
-0.38575411D-12	0.29968185D-07	0.23539921D-13	0.42121565D-14	-0.17992354D-11	0.12953679D-06		
0.33718119D-09	0.25747901D-13	0.29931868D-07	0.17759479D-10	-0.13068109D-08	0.12427653D-12	0.12962823D-06	
0.26386368D-09	-0.42121565D-14	-0.16109745D-10	-0.32141500D-11	-0.49575835D-10	0.64139046D-23	0.53843585D-11	0.99770854D-11

0.24748247D-07	-0.38532723D-12	0.99690235D-09	-0.30837438D-09	0.17036454D-06			
-0.38549106D-12	0.34799521D-07	0.27513687D-13	0.42121565D-14	-0.17978593D-11	0.14885494D-06		
0.49484509D-09	0.29720096D-13	0.34743025D-07	0.23787708D-10	-0.17200362D-08	0.14281021D-12	0.14899292D-06	
0.30634976D-09	-0.42121565D-14	-0.21873461D-10	-0.37321122D-11	-0.49458866D-10	0.52007170D-23	0.61826852D-11	0.11464513D-10

0.28199064D-07	-0.38500052D-12	0.12524363D-08	-0.35069274D-09	0.19240787D-06			
-0.38518708D-12	0.39627161D-07	0.31484532D-13	0.42121565D-14	-0.17962923D-11	0.16815730D-06		
0.68073626D-09	0.33689135D-13	0.39544100D-07	0.30689634D-10	-0.21894295D-08	0.16132873D-12	0.16835546D-06	
0.34871016D-09	-0.42121565D-14	-0.28511678D-10	-0.42497018D-11	-0.49324742D-10	0.43432891D-23	0.69783776D-11	0.12950776D-10

0.31654271D-07	-0.38463293D-12	0.15359068D-08	-0.39286126D-09	0.21441596D-06			
-0.38484219D-12	0.44450594D-07	0.35452033D-13	0.42121565D-14	-0.17945345D-11	0.18744179D-06		
0.89462399D-09	0.37654597D-13	0.44333710D-07	0.38462326D-10	-0.27144918D-08	0.17983011D-12	0.18771551D-06	
0.39092618D-09	-0.42121565D-14	-0.36021579D-10	-0.47668562D-11	-0.49173807D-10	0.37123581D-23	0.77711003D-11	0.14435699D-10

0.35114330D-07	-0.38422450D-12	0.18470455D-08	-0.43486163D-09	0.23638458D-06			
-0.38445645D-12	0.49269307D-07	0.39415770D-13	0.42121565D-14	-0.17925861D-11	0.20670638D-06		
0.11362477D-08	0.41616061D-13	0.49110485D-07	0.47102480D-10	-0.32946648D-08	0.19831240D-12	0.20707274D-06	
0.43297948D-09	-0.42121565D-14	-0.44399974D-10	-0.52835156D-11	-0.49006283D-10	0.32283036D-23	0.85605182D-11	0.15919115D-10

0.38579707D-07	-0.38377526D-12	0.21855550D-08	-0.47667574D-09	0.25830959D-06			
-0.38402987D-12	0.54082787D-07	0.43375321D-13	0.42121565D-14	-0.17904474D-11	0.22594903D-06		
0.14053172D-08	0.45573105D-13	0.53873059D-07	0.56606427D-10	-0.39293315D-08	0.21677364D-12	0.22642681D-06	
0.47485192D-09	-0.42121565D-14	-0.53643306D-10	-0.57996222D-11	-0.48822336D-10	0.28501410D-23	0.93462970D-11	0.17400861D-10

0.42050867D-07	-0.38328528D-12	0.25511083D-08	-0.51828565D-09	0.28018691D-06			
-0.38356252D-12	0.58890526D-07	0.47330266D-13	0.42121565D-14	-0.17881186D-11	0.24516767D-06		
0.17015132D-08	0.49525310D-13	0.58620080D-07	0.66970129D-10	-0.46178164D-08	0.23521185D-12	0.24577733D-06	
0.51652554D-09	-0.42121565D-14	-0.63747649D-10	-0.63151192D-11	-0.48622102D-10	0.25446389D-23	0.10128104D-10	0.18880773D-10

0.45528269D-07	-0.38275459D-12	0.29433495D-08	-0.55967356D-09	0.30201249D-06			
-0.38305444D-12	0.63692010D-07	0.51280186D-13	0.42121565D-14	-0.17855999D-11	0.26436029D-06		
0.20244872D-08	0.53472256D-13	0.63350205D-07	0.78189185D-10	-0.53593869D-08	0.25362509D-12	0.26512393D-06	
0.55798250D-09	-0.42121565D-14	-0.74708712D-10	-0.68299504D-11	-0.48405703D-10	0.22948800D-23	0.10905607D-10	0.20358691D-10

0.49012365D-07	-0.38218327D-12	0.33618944D-08	-0.60082182D-09	0.32378232D-06			
-0.38250568D-12	0.68486732D-07	0.55224660D-13	0.42121565D-14	-0.17828917D-11	0.28352484D-06		
0.23738622D-08	0.57413524D-13	0.68062103D-07	0.90258828D-10	-0.61532532D-08	0.27201140D-12	0.28446620D-06	
0.59920510D-09	-0.42121565D-14	-0.86521839D-10	-0.73440602D-11	-0.48173256D-10	0.20869773D-23	0.11678477D-10	0.21834457D-10

0.52503596D-07	-0.38157136D-12	0.38063303D-08	-0.64171287D-09	0.22909279D-06			
-0.38191631D-12	0.73274181D-07	0.59163270D-13	0.42121565D-14	-0.10955833D-11	0.18398946D-06		
0.27492330D-08	0.61348696D-13	0.72754457D-07	0.10317393D-09	-0.71315114D-08	0.17744811D-12	0.18511638D-06	
0.64017578D-09	-0.42121565D-14	-0.99182014D-10	-0.78573933D-11	0.10653158D-08	-0.59816959D-14	-0.17147777D-09	0.11806524D-10

-0.62490942D-08	-0.87995351D-13	0.39469679D-08	-0.17316231D-10	0.11886709D-06			
-0.88014459D-13	0.17588460D-07	0.14405472D-13	0.22431603D-15	-0.41060735D-12	0.70862762D-07		
0.38884115D-08	0.14521737D-13	0.16944543D-07	0.28699350D-11	-0.79486124D-08	0.67984809D-13	0.72178507D-07	
0.17558837D-10	-0.22431603D-15	-0.28618314D-11	-0.16936876D-12	-0.11026304D-11	0.79588177D-26	0.28465805D-12	0.84607360D-12

-0.63204737D-08	-0.87960569D-13	0.40599758D-08	-0.17561469D-10	0.12053528D-06			
-0.87979954D-13	0.17842901D-07	0.14614731D-13	0.22431603D-15	-0.41044452D-12	0.71880324D-07		
0.40005723D-08	0.14730949D-13	0.17170633D-07	0.29535027D-11	-0.81752599D-08	0.68961045D-13	0.73253979D-07	
0.17804114D-10	-0.22431603D-15	-0.29452851D-11	-0.17181881D-12	-0.11012677D-11	0.76415477D-26	0.28863117D-12	0.85822264D-12

-0.63910051D-08	-0.87925290D-13	0.41744625D-08	-0.17806308D-10	0.12220117D-06			
-0.87944951D-13	0.18097241D-07	0.14823906D-13	0.22431603D-15	-0.41027937D-12	0.72897479D-07		
0.41142123D-08	0.14940079D-13	0.17395813D-07	0.30382358D-11	-0.84048588D-08	0.69936891D-13	0.74330271D-07	
0.18048992D-10	-0.22431603D-15	-0.30299045D-11	-0.17426789D-12	-0.10998860D-11	0.75434331D-26	0.29259775D-12	0.87036683D-12

-0.64606772D-08	-0.87889514D-13	0.42904215D-08	-0.18050741D-10	0.12386470D-06			
-0.87909451D-13	0.18351478D-07	0.15032998D-13	0.22431603D-15	-0.41011190D-12	0.73914222D-07		
0.42293249D-08	0.15149124D-13	0.17620073D-07	0.31241326D-11	-0.86373961D-08	0.70912341D-13	0.75407799D-07	
0.18293465D-10	-0.22431603D-15	-0.31156877D-11	-0.17671599D-12	-0.10984855D-11	0.72634368D-26	0.29655772D-12	0.88250610D-12

-0.65294790D-08	-0.87853241D-13	0.44078462D-08	-0.18294763D-10	0.12552586D-06			
-0.87873454D-13	0.18605612D-07	0.15242005D-13	0.22431603D-15	-0.40994211D-12	0.74930548D-07		
0.43459035D-08	0.15358083D-13	0.17643399D-07	0.32111910D-11	-0.88728585D-08	0.71887390D-13	0.76486579D-07	
0.18537527D-10	-0.22431603D-15	-0.32026327D-11	-0.17916309D-12	-0.10970662D-11	0.71781412D-26	0.30051098D-12	0.89464038D-12

-0.65973995D-08	-0.87816471D-13	0.45267300D-08	-0.18538369D-10	0.12718461D-06			
-0.87836960D-13	0.18859641D-07	0.15450926D-13	0.22431603D-15	-0.40977000D-12	0.75946449D-07		
0.44639416D-08	0.15566955D-13	0.18065780D-07	0.32994090D-11	-0.91112327D-08	0.72862033D-13	0.77566625D-07	
0.18781174D-10	-0.22431603D-15	-0.32907375D-11	-0.18160918D-12	-0.10956280D-11	0.68972081D-26	0.30445744D-12	0.90676961D-12

-0.66644277D-08	-0.87779204D-13	0.46470662D-08	-0.18781553D-10	0.12884093D-06			
-0.87799969D-13	0.19113563D-07	0.15659759D-13	0.22431603D-15	-0.40959557D-12	0.76961921D-07		
0.45834325D-08	0.15775739D-13	0.18287204D-07	0.33887648D-11	-0.93525052D-08	0.73836263D-13	0.78647953D-07	
0.19024400D-10	-0.22431603D-15	-0.33800003D-11	-0.18405424D-12	-0.10941711D-11	0.68039253D-26	0.30839701D-12	0.91889371D-12

-0.67305528D-08	-0.87741441D-13	0.47688478D-08	-0.19024309D-10	0.13049477D-06			
-0.87762482D-13	0.19367376D-07	0.15868504D-13	0.22431603D-15	-0.40941883D-12	0.77976958D-07		
0.47043692D-08	0.15984435D-13	0.18507660D-07	0.34793162D-11	-0.95966622D-08	0.74810077D-13	0.79730577D-07	
0.19267198D-10	-0.22431603D-15	-0.34704188D-11	-0.18649826D-12	-0.10926954D-11	0.65559272D-26	0.31232961D-12	0.93101262D-12

-0.67957639D-08	-0.87703182D-13	0.48920682D-08	-0.19266633D-10	0.13214612D-06			
-0.87724498D-13	0.19621081D-07	0.16077159D-13	0.22431603D-15	-0.40923978D-12	0.78991554D-07		
0.48267449D-08	0.16193039D-13	0.18727135D-07	0.35710011D-11	-0.98436898D-08	0.75783467D-13	0.80814512D-07	
0.19509565D-10	-0.22431603D-15	-0.35619912D-11	-0.18894123D-12	-0.10912010D-11	0.63859276D-26	0.31625514D-12	0.94312627D-12

-0.68600502D-08	-0.87664427D-13	0.50167202D-08	-0.19508518D-10	0.13379493D-06			
-0.87686019D-13	0.19874674D-07	0.16285723D-13	0.22431603D-15	-0.40905841D-12	0.80005703D-07		
0.49505526D-08	0.16401552D-13	0.18945618D-07	0.36638376D-11	-0.10093574D-07	0.76756428D-13	0.81899773D-07	
0.19751493D-10	-0.22431603D-15	-0.36547153D-11	-0.19138313D-12	-0.10896880D-11	0.63395328D-26	0.32017352D-12	0.95523458D-12

-0.69234010D-08	-0.87625176D-13	0.51427967D-08	-0.19749560D-10	0.13544119D-06			
-0.87647043D-13	0.20128155D-07	0.16494196D-13	0.22431603D-15	-0.40887472D-12	0.81019400D-07		
0.50757853D-08	0.16609973D-13	0.19163097D-07	0.37578236D-11	-0.10346301D-07	0.77728956D-13	0.82986375D-07	
0.19992978D-10	-0.22431603D-15	-0.37485891D-11	-0.19382395D-12	-0.10881563D-11	0.61328512D-26	0.32406465D-12	0.96733750D-12

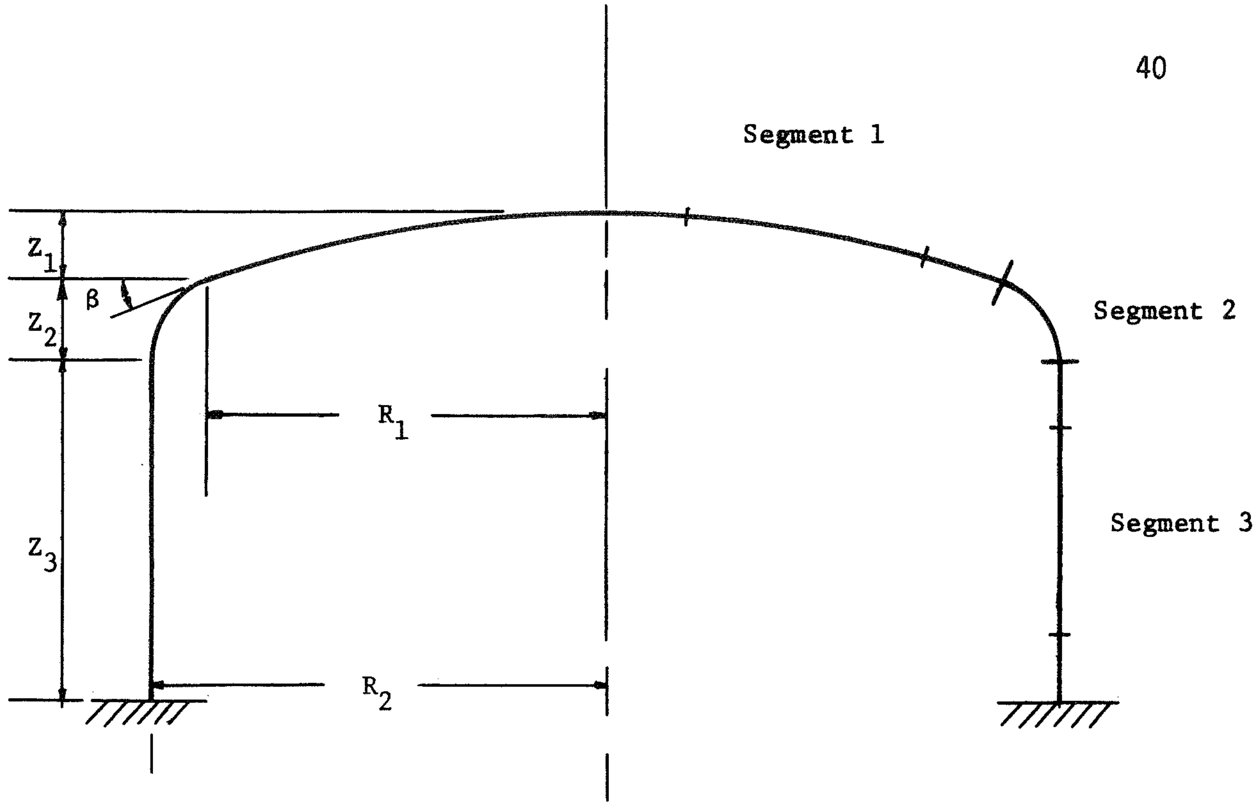
-0.69858055D-08	-0.87585429D-13	0.52702908D-08	-0.19990952D-10	0.68286499D-07			
-0.87607572D-13	0.20381522D-07	0.16702574D-13	0.22431603D-15	-0.20435773D-12	0.40889693D-07		
0.52024358D-08	0.16818299D-13	0.19379561D-07	0.38529568D-11	-0.52323327D-08	0.39281103D-13	0.41888916D-07	
0.20234014D-10	-0.22431603D-15	-0.38436103D-11	-0.19626367D-12	0.93190277D-10	-0.33647039D-15	-0.17886733D-10	0.48797396D-12

Fig. 10 FIRST (1) HARMONIC MASS MATRIX

Example Problem 2

The second example problem, a little more geometrically involved than the first problem, is the cap-torus-cylinder configuration shown in Fig. 11. To generate the required geometry, the shell is considered to be the combination of a spherical cap, a torus, and a cylinder. Hence, three segments are used for inputting the shell geometry. The profiles of the first two segments are circular and obviously, the profile of the cylinder is linear. The elements are concentrated in the area of the apex of the shell, close to the cap-torus intersection, around the torus, and near the clamped end of the cylinder with the maximum number of elements (50) being utilized. Obviously, for this case the advantage of using the segmentized procedure for inputting the geometry is quite evident since only nine cards are required to input the geometry for this complex configuration.

Since the thickness is constant for the shell, only one card is necessary for this data. The same is true for the mass and elastic properties. Again, a set of input data (Fig. 12) is presented and a portion of the output is included (Fig. 13).



SHELL DESCRIPTION

R_1 = 3.00 in.

R_2 = 5.44 in.

Z_1 = 0.0755 in.

Z_2 = 2.5495 in.

Z_3 = 3.00 in.

β = 2.88 deg.

t = 0.125 in.

MATERIAL PROPERTIES
ALUMINUM

$E = 10.0 \times 10^6$ psi

$G = 3.75 \times 10^6$ psi

$\nu = 0.333$

$\rho = 0.0942$ lb/in^3

ELEMENT BREAKDOWN

Segment 1 -	Sub-segment 1	4 elements
	Sub-segment 2	8 elements
	Sub-segment 3	8 elements
Segment 2 -	Sub-segment 1	18 elements
Segment 3 -	Sub-segment 1	4 elements
	Sub-segment 2	4 elements
	Sub-segment 3	4 elements

Fig. 11 CAP-TORUS - CYLINDER CONFIGURATION

```
NCASE=1

                           PRINTOUT OF INPUT DATA

         10        20        30        40        50        60        70        80
12345678901234567890123456789012345678901234567890123456789012345678901234567890

    5    4
********************************************************************************
       EXAMPLE PROBLEM NO. 2                   SAMMSOR USER'S MANUAL
                      CAP-TORUS-CYLINDER CONFIGURATION
         ALL ELEMENTS HAVE THE SAME MATERIAL PROPERTIES AND THICKNESS
********************************************************************************
   50    6    1
    1   50 10000000. 10000000.  3750000.     0.333    0.0942
    1   50      0.125
    1       0.0       0.0     90.00
   21    0.0755      3.00     87.12
    2    4      0.2    8        0.6    8      0.2    0       0.0    0       0.0
   21    0.0755      3.00     87.12
   39     2.625      5.44       0.0
    2   18      1.0    0        0.0    0      0.0    0       0.0    0       0.0
   39     2.625      5.44       0.0
   51     5.625    5.4400      0.00
    1    4      0.2    4        0.6    4      0.2    0       0.0    0       0.0
END OF CASE
```

Fig. 12 INPUT DATA - EXAMPLE PROBLEM 2

GEOMETRIC AND ELASTIC PROPERTIES OF STRUCTURE

ELEM. NO.	NODE NOS.	COORDINATES Z	R	SLOPE	S	THICKNESS	ELASTIC CONSTANTS E1	E2	G	NU1	RHO
	1	0.0	0.10000D-05	0.90000D 02							
1					0.7503D-01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	2	0.18857D-03	0.15006D 00	0.89856D 02							
	2	0.18857D-03	0.15006D 00	0.89856D 02							
2					0.2251D 00	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	3	0.75430D-03	0.30013D 00	0.89712D 02							
	3	0.75430D-03	0.30013D 00	0.89712D 02							
3					0.3752D 00	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	4	0.16972D-02	0.45019D 00	0.89568D 02							
	4	0.16972D-02	C.45019D 00	0.89568D 02							
4					0.5252D 00	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	5	0.30172D-02	0.60024D 00	0.89424D 02							
	5	0.30172D-02	0.60024D 00	0.89424D 02							
5					0.7128D 00	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	6	0.57043D-02	0.82532D 00	0.89208D 02							
	6	0.57043D-02	0.82532D 00	0.89208D 02							
6					0.9379D 00	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	7	0.92399D-02	0.10504D 01	0.88992D 02							
	7	0.92399D-02	0.10504D 01	0.88992D 02							
7					0.1163D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	8	0.13624D-01	0.12754D 01	0.88776D 02							
	8	0.13624D-01	0.12754D 01	0.88776D 02							
8					0.1388D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	9	0.18857D-01	0.15005D 01	0.88560D 02							
	9	0.18857D-01	0.15005D 01	0.88560D 02							
9					0.1613D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	10	0.24937D-01	0.17255D 01	0.88344D 02							
	10	0.24937D-01	0.17255D 01	0.88344D 02							
10					0.1838D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	11	0.31866D-01	0.19505D 01	0.88128D 02							
	11	0.31866D-01	0.19505D 01	0.88128D 02							
11					0.2063D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	12	0.39644D-01	0.21754D 01	0.87912D 02							
	12	0.39644D-01	0.21754D 01	0.87912D 02							
12					0.2288D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	13	0.48269D-01	0.24004D 01	0.87696D 02							
	13	0.48269D-01	0.24004D 01	0.87696D 02							
13					0.2439D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	14	0.51332D-01	0.24753D 01	0.87624D 02							
	14	0.51332D-01	0.24753D 01	0.87624D 02							
14					0.2514D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	15	0.54490D-01	0.25503D 01	0.87552D 02							
	15	0.54490D-01	0.25503D 01	0.87552D 02							
15					0.2589D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	16	0.57742D-01	0.26253D 01	0.87480D 02							
	16	0.57742D-01	0.26253D 01	0.87480D 02							
16					0.2664D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	17	0.61088D-01	0.27002D 01	0.87408D 02							
	17	0.61088D-01	0.27002D 01	0.87408D 02							
17					0.2739D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	18	0.64528D-01	0.27752D 01	0.87336D 02							
	18	0.64528D-01	0.27752D 01	0.87336D 02							
18					0.2814D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	19	0.68063D-01	0.28501D 01	0.87264D 02							
	19	0.68063D-01	0.28501D 01	0.87264D 02							
19					0.2889D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	20	0.71691D-01	0.29251D 01	0.87192D 02							
	20	0.71691D-01	0.29251D 01	0.87192D 02							
20					0.2964D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	21	0.75414D-01	0.30000D 01	0.87120D 02							
	21	0.75500D-01	0.30000D 01	0.87120D 02							
21					0.3109D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	22	0.95473D-01	0.32153D 01	0.82280D 02							
	22	0.95473D-01	0.32153D 01	0.82280D 02							
22					0.3326D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	23	0.13354D 00	0.34282D 01	0.77440D 02							
	23	0.13354D 00	0.34282D 01	0.77440D 02							
23					0.3542D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	24	0.18943D 00	0.36370D 01	0.72600D 02							
	24	0.18943D 00	0.36370D 01	0.72600D 02							
24					0.3758D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	25	0.26275D 00	0.38405D 01	0.67760D 02							
	25	0.26275D 00	0.38405D 01	0.67760D 02							
25					0.3975D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	26	0.35297D 00	0.40370D 01	0.62920D 02							
	26	0.35297D 00	0.40370D 01	0.62920D 02							
26					0.4191D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	27	0.45945D 00	0.42252D 01	0.58080D 02							
	27	0.45945D 00	0.42252D 01	0.58080D 02							
27					0.4407D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	28	0.58142D 00	0.44037D 01	0.53240D 02							
	28	0.58142D 00	0.44037D 01	0.53240D 02							
28					0.4623D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	29	0.71803D 00	0.45713D 01	0.48400D 02							
	29	0.71803D 00	0.45713D 01	0.48400D 02							
29					0.4840D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	30	0.86829D 00	0.47268D 01	0.43560D 02							
	30	0.86829D 00	0.47268D 01	0.43560D 02							
30					0.5056D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	31	0.10311D 01	0.48691D 01	0.38720D 02							
	31	0.10311D 01	0.48691D 01	0.38720D 02							
31					0.5272D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	32	0.12054D 01	0.49971D 01	0.33880D 02							
	32	0.12054D 01	0.49971D 01	0.33880D 02							
32					0.5489D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	33	0.13899D 01	0.51100D 01	0.29040D 02							
	33	0.13899D 01	0.51100D 01	0.29040D 02							
33					0.5705D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	34	0.15832D 01	0.52068D 01	0.24200D 02							
	34	0.15832D 01	0.52068D 01	0.24200D 02							
34					0.5921D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	35	0.17840D 01	0.52871D 01	0.19360D 02							
	35	0.17840D 01	0.52871D 01	0.19360D 02							
35					0.6138D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	36	0.19908D 01	0.53501D 01	0.14520D 02							
	36	0.19908D 01	0.53501D 01	0.14520D 02							
36					0.6354D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	37	0.22022D 01	0.53954D 01	0.96800D 01							
	37	0.22022D 01	0.53954D 01	0.96800D 01							
37					0.6570D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	38	0.24167D 01	0.54227D 01	0.48400D 01							
	38	0.24167D 01	0.54227D 01	0.48400D 01							
38					0.6786D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	39	0.26328D 01	0.54319D 01	-0.22264D-13							
	39	0.26250D 01	0.54400D 01	0.0							
39					0.6970D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	40	0.27750D 01	0.54400D 01	0.0							
	40	0.27750D 01	0.54400D 01	0.0							
40					0.7120D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	41	0.29250D 01	0.54400D 01	0.0							
	41	0.29250D 01	0.54400D 01	0.0							
41					0.7270D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	42	0.30750D 01	0.54400D 01	0.0							
	42	0.30750D 01	0.54400D 01	0.0							
42					0.7420D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	43	0.32250D 01	0.54400D 01	0.0							
	43	0.32250D 01	0.54400D 01	0.0							
43					0.7720D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	44	0.36750D 01	0.54400D 01	0.0							
	44	0.36750D 01	0.54400D 01	0.0							
44					0.8170D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	45	0.41250D 01	0.54400D 01	0.0							
	45	0.41250D 01	0.54400D 01	0.0							
45					0.8620D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	46	0.45750D 01	0.54400D 01	0.0							
	46	0.45750D 01	0.54400D 01	0.0							
46					0.9070D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	47	0.50250D 01	0.54400D 01	0.0							
	47	0.50250D 01	0.54400D 01	0.0							
47					0.9370D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	48	0.51750D 01	0.54400D 01	0.0							
	48	0.51750D 01	0.54400D 01	0.0							
48					0.9520D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	49	0.53250D 01	0.54400D 01	0.0							
	49	0.53250D 01	0.54400D 01	0.0							
49					0.9670D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	50	0.54750D 01	0.54400D 01	0.0							
	50	0.54750D 01	0.54400D 01	0.0							
50					0.9820D 01	0.1250D 00	0.100D 08	0.100D 08	0.375D 07	0.333	0.09420
	51	0.56250D 01	0.54400D 01	0.0							

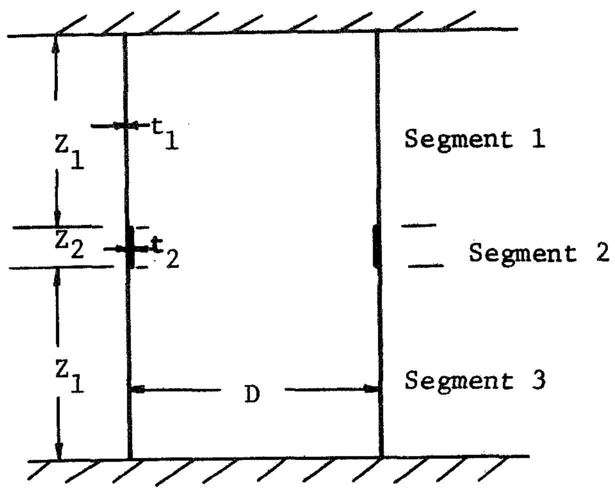
Fig. 13 ELEMENT PROPERTIES (OUTPUT) EXAMPLE PROBLEM 2

Example Problem 3

For illustrative purposes, a cylindrical shell (Fig. 14) with a stiffening ring at its midpoint has been selected as the third example. The input data for this shell should serve to illustrate many of the features of the code. In particular, the procedure used for a variable thickness shell will be demonstrated along with the option for inputting the nodal point geometry at each individual node.

To demonstrate these input procedures the cylinder will be idealized using nine elements: four on each side of the ring, and one containing the ring. If this shell were to actually be analyzed, many more elements would obviously be necessary. The elements on each side of the ring are considered to be five inches in length (s-direction) and one-eighth inch thick while the element for the stiffener is three inches long and one-fourth inch thick.

Two sets of input data are presented in Figures 15 and 16. The first set inputs separately the thickness of each element and the geometry for each node. The second data set effectively employs the program capabilities and inputs the thickness and geometry for the three segments of the cylinder.



SHELL DESCRIPTION

$D = 10$ in.

$Z_1 = 20$ in.

$Z_2 = 3$ in.

$t_1 = 0.125$ in.

$t_2 = 0.250$ in.

MATERIAL PROPERTIES
STEEL

$E = 30.0 \times 10^6$ psi

$G = 11.55 \times 10^6$ psi

$\nu = 0.3$

$\rho = 0.289$ lb/in.^{3}

ELEMENT BREAKDOWN

Segment 1 - 4 elements

Segment 2 - 1 element

Segment 3 - 4 elements

Fig. 14 STIFFENED CYLINDRICAL SHELL

```
      NCASE= 3

                                      PRINTOUT OF INPUT DATA
CARD
TYPE             10        20        30        40        50        60        70        80
        12345678901234567890123456789012345678901234567890123456789012345678901234567890

II - A       6    4
   - B  ********************************************************************************
   - B            EXAMPLE  PROBLEM  NO. 3               SAMMSOR  USER'S  MANUAL
   - B        THIS EXAMPLE DEMCNSTRATES THE PROCEDURE USED TO SPECIFY VARIABLE
   - B        THICKNESSES AND ALSO SHOWS THE PROCEDURE FOR SPECIFYING THE GEOMETRY OF
   - B        EACH NODE OF THE IDEALIZED SHELL
   - B  ********************************************************************************
III          9    5    1
IV           1    9 30C00000. 30000000. 11550000.        0.3      0.289
V            1    4      0.125
V            5    5      0.250
V            6    9      0.125
VI - A       1       0.0          5.0          0.0
   - B       2       5.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
   - A       2       5.0          5.0          0.0
   - B       3      10.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
   - A       3      1C.0          5.0          0.0
   - B       4      15.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
   - A       4      15.0          5.0          0.0
   - B       5      20.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
   - A       5      20.0          5.0          0.0
   - B       6      23.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
   - A       6      23.0          5.0          0.0
   - B       7      28.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
   - A       7      28.0          5.0          0.0
   - B       8      33.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
   - A       8      33.0          5.0          0.0
   - B       9      38.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
   - A       9      38.0          5.0          0.0
   - B      10      43.0          5.0          0.0
   - C       1    1         1.0    0           0.0    0         0.0    0         0.0    0         0.0
VII     END OF CASE
```

Fig. 15 INPUT DATA (SET #1) EXAMPLE PROBLEM 3

NCASE= 4

```
                              PRINTOUT OF INPUT DATA
CARD
TYPE              10        20        30        40        50        60        70        80
         12345678901234567890123456789012345678901234567890123456789012345678901234567890

II - A       6    4
   - B   ********************************************************************************
   - B            EXAMPLE  PROBLEM  NO.  3                SAMMSOR  USER'S  MANUAL
   - B         THIS EXAMPLE DEMONSTRATES THE PROCEDURE USED TO SPECIFY VARIABLE
   - B         THICKNESSES AND SHOWS THE MOST EFFICIENT PROCEDURE FOR SPECIFYING THE
   - B         GEOMETRY OF THE IDEALIZED SHELL
   - B   ********************************************************************************
III          9    5    1
IV           1    9 30000000. 30000000. 11550000.      0.3       0.289
V            1    4      0.125
V            5    5      0.250
V            6    9      0.125
VI - A       1      0.0        5.0       0.0
   - B       5     20.0        5.0       0.0
   - C       1    4      1.0     0       0.0      0       0.0       0       0.0       0       0.0
   - A       5     20.0        5.0       0.0
   - B       6     23.0        5.0       0.0
   - C       1    1      1.0     0       0.0      0       0.0       0       0.0       0       0.0
   - A       6     23.0        5.0       0.0
   - B      10     43.0        5.0       0.0
   - C       1    4      1.0     0       0.0      0       0.0       0       0.0       0       0.0
VII      END OF CASE
```

Fig. 16 INPUT DATA (SET #2) EXAMPLE PROBLEM 3

REFERENCES

1. Stricklin, J. A., Navaratna, D. R., and Pian, T. H. H., "Improvements on the Analysis of Shells of Revolution by the Matrix Displacement Method," AIAA Journal, Vol. 4, No. 11, Nov. 1966, pp. 2069-2072.

2. Mebane, P. M., "An Improved Shell of Revolution Element Utilizing Cubic Displacement Functions," Masters Thesis, Texas A&M University, Aug. 1970.

3. Stricklin, J. A., Haisler, W. E., MacDougall, H. R., and Stebbins, F. J., "Nonlinear Analysis of Shells of Revolution by the Matrix Displacement Method," AIAA Journal, Vol. 6, No. 12, Dec. 1968, pp. 2306-2312.

4. Stricklin, J. A., Martinez, J. E., Tillerson, J. R., Hong, H. H., and Haisler, W. E., "Nonlinear Dynamic Analysis of Shells of Revolution by Matrix Displacement Method," Report 69-77, Aerospace Engineering Department, Texas A&M University, Feb. 1970.

5. Percy, J. H., Pian, T. H. H., Klein, S., and Navaratna, D. R., "Application of Matrix Displacement Method to Linear Elastic Analysis of Shells of Revolution," AIAA Journal, Vol. 3, No. 11, Nov. 1965, pp. 2138-2145.

6. Pian, T. H. H., "Derivation of Element Stiffness Matrices," AIAA Journal, Vol. 2, No. 3, March 1964, pp. 576-577.

7. Novozhilov, V. V. Foundations of the Nonlinear Theory of Elasticity, Graylock Press, Rochester, N. Y., 1956.

APPENDIX

Appendix 1 Description of SAMMSOR II Subroutines

Subroutine	Description
MAIN	Controls the flow of the program by calling the other subroutines to both obtain the structural stiffness and mass matrices for the shell and prepare the input tape for use in the other compatible programs.
INPUT	Reads input data and program control parameters and prints a description of the idealized structure.
SLOCOR	Reads coordinates of shell segments and controls the calculation of the z,r,ϕ coordinates for the specified element breakdown.
FLAT	Calculates the z,r,ϕ nodal coordinates for a shell segment with a linear profile.
CIRCUL	Calculates the z,r,ϕ nodal coordinates for shell segment with a circular profile.
PARABO	Calculates the z,r,ϕ nodal coordinates for a shell segment with a parabolic profile.
FUS	Used by subroutine PARABO to calculate arc lengths for the elements of the segment.
ELEMCA	Calculates geometrical parameters i.e., r,ϕ, ϕ',ϕ'', $\sin\phi$, $\cos$ineϕ, etc. for each element between the nodal points. In addition, this subroutine calculates the transformation matrix [A] which relates the shell generalized coordinates, q_i, to the generalized coefficients, α_i, of the displacement functions.
CRETAL	Calculates the values of integrals in the meridional direction which are to be used in calculating the terms in the mass and stiffness matrices.

Appendix 1 Continued

Subroutines	Description
CREATL	Evaluates the element stiffness matrices based on generalized coefficients. The element stiffness matrices are condensed in this subroutine.
CREMAS	Controls the calculation of the element mass matrices according to harmonic number.
AZERO	Generates the element mass matrices based on generalized coefficients for the zero harmonic only.
NONAB	Generates the element mass matrices based on generalized coefficients for harmonics other than zero.
MMPLT3	Premultiplies the element mass and stiffness matrices based on generalized coefficients by $[A]^t$ and post-multiplies by[A] to form the element stiffness and mass matrices[k] and [m].
SIMP	Evaluates the integral of a given function using Simpson's integration.
ASEMBL	Assembles element stiffness and mass matrices to form structural stiffness and mass matrices, [K] and [M], for each harmonic.

Appendix 2 Glossary of Significant FORTRAN Variables in SAMMSOR II

Variable	Subroutine where variable is defined or calculated	Description
AK	MMPLT3	Element stiffness matrix [k]
AKK	MMPLT3	Element mass matrix, [m]
AL	CRETAL	Integral of functions used in the [L] and [EMASS] matrices.
AM	ELEMCA	Matrix $[B]^{-1}$ which relates the generalized α coordinates to the generalized shell coordinates.
AR	ELEMCA	Matrix of the radial coordinates, r, of the NET stations between the nodes of an element.
ARCL	ELEMCA	Arc length of an element (s-direction).
AS	ELEMCA	Matrix of the meridional distances, s, of the NET stations between the nodes of an element.
CHECK	ELEMCA	Matrix [A], which relates the shell generalized coordinates to the generalized coefficients of the displacement functions.
COMENT	INPUT	Alphanumeric data printed as output for problem identification.
COSINE	ELEMCA	Matrix whose elements are the cosine of ϕ for NET stations along the element.
E1	INPUT	Matrix of Young's Moduli in the meridional direction, E_s.
E2	INPUT	Matrix of Young's Moduli in the circumferential direction, E_θ.
EMASS	CREMAS	Element mass matrix based on generalized coefficients.

Appendix 2 Continued

Variable	Subroutine where variable is defined or calculated	Description
F	CREATL	Element stiffness matrix based on generalized coefficients.
FNU1	INPUT	Poisson's Ratio, $\nu_{s\theta}$.
FNU2	INPUT	Poisson's Ratio, $\nu_{\theta s}$.
G	INPUT	Shear Modulus, G [for an isotropic material $G = E/2(1+\nu)$].
IA	INPUT	Total number of harmonics for which the mass and stiffness matrices are to be determined.
ICLASS	SLOCOR	Designates the type of profile for a given segment of the shell.
NCARDS	INPUT	Number of comment cards used for problem identification.
ND	MAIN	Logical unit number of the binary scratch tape on which the input data is stored.
NELEMS	INPUT	Total number of elements used to idealize the structure.
NELEMT	SLOCOR	Element number of the first element of a segment.
NEQ	INPUT	Number of equilibrium equations (degrees of freedom) per harmonic.

Appendix 2 Continued

Variable	Subroutine where variable is defined or calculated	Description
NET	INPUT	Number of Simpson's integration stations used over the meridional length of the shell elements. NET is set equal to 29 in the program.
NH	INPUT	Total number of harmonics for which the mass and stiffness matrices are to be determined.
NNODES	INPUT	Total number of nodes, equal to NELEMS + 1.
NS	MAIN	Logical unit number of a binary scratch tape.
NSIZE	INPUT	The number of terms in the structural stiffness or mass matrix (in vector form) for a particular harmonic.
NT	INPUT	Logical unit number of the tape on which stiffness and mass matrices will be stored.
PHI1	SLOCOR	Slope of the shell at the first node of a segment.
PHI2	SLOCOR	Slope of the shell at the last node of a segment.
PHIMAT	ELEMCA	Matrix [Ψ] which relates the generalized shell coordinates for an element to the global or cylindrical coordinates of the structure.
PHP	ELEMCA	$d\phi/ds$ for NET stations along an element.
PHPP	ELEMCA	$d^2\phi/ds^2$ for NET stations along the element (constant).
PHPRIM	ELEMCA	$d\phi/ds$ at the middle of an element.

Appendix 2 Continued

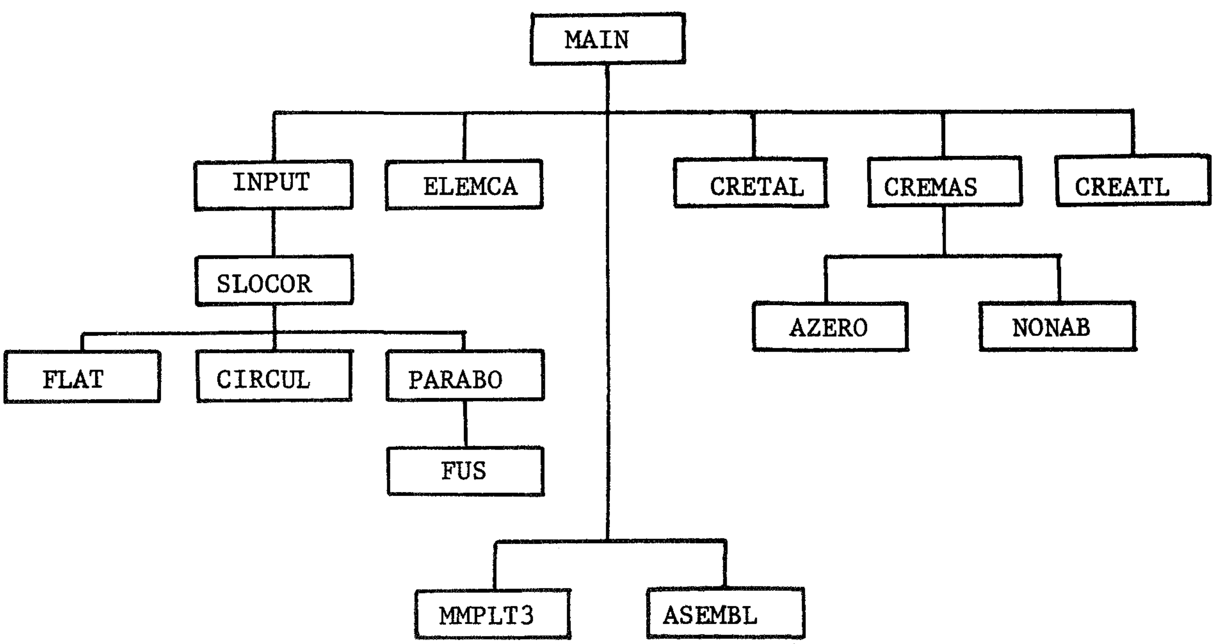
Variable	Subroutine where variable is defined or calculated	Description
PHY1	INPUT	Slope at node i of element i.
PHY2	INPUT	Slope at node (i + 1) of element i.
R1	SLOCOR	Radial coordinate of the first node of a segment.
R2	SLOCOR	Radial coordinate of the last node of a segment.
RA	SLOCOR	Decimal fraction corresponding to the lengths of the sub-segments into which a segment is being divided.
RHOS	INPUT	Matrix of the material densities of the elements, lbs/in^3.
RO1	INPUT	Radial coordinate of node i for element i.
RO2	INPUT	Radial coordinate of node (i+1) for element i.
SINE	ELEMCA	Matrix whose elements i are the sine of ϕ for the NET stations along the element.
STIFM	ASEMBL	Structural stiffness matrix, [K], for a given harmonic.
THICK	INPUT	Matrix of element thicknesses.
TMASS	ASEMBL	Structural mass matrix, [M], for a given harmonic.
Z1	INPUT	Axial coordiante of node i for element i.
Z2	INPUT	Axial coordinate of node (i+1) for element i.
Z1C	SLOCOR	Axial coordinate of the first node of a segment.

Appendix 2 Continued

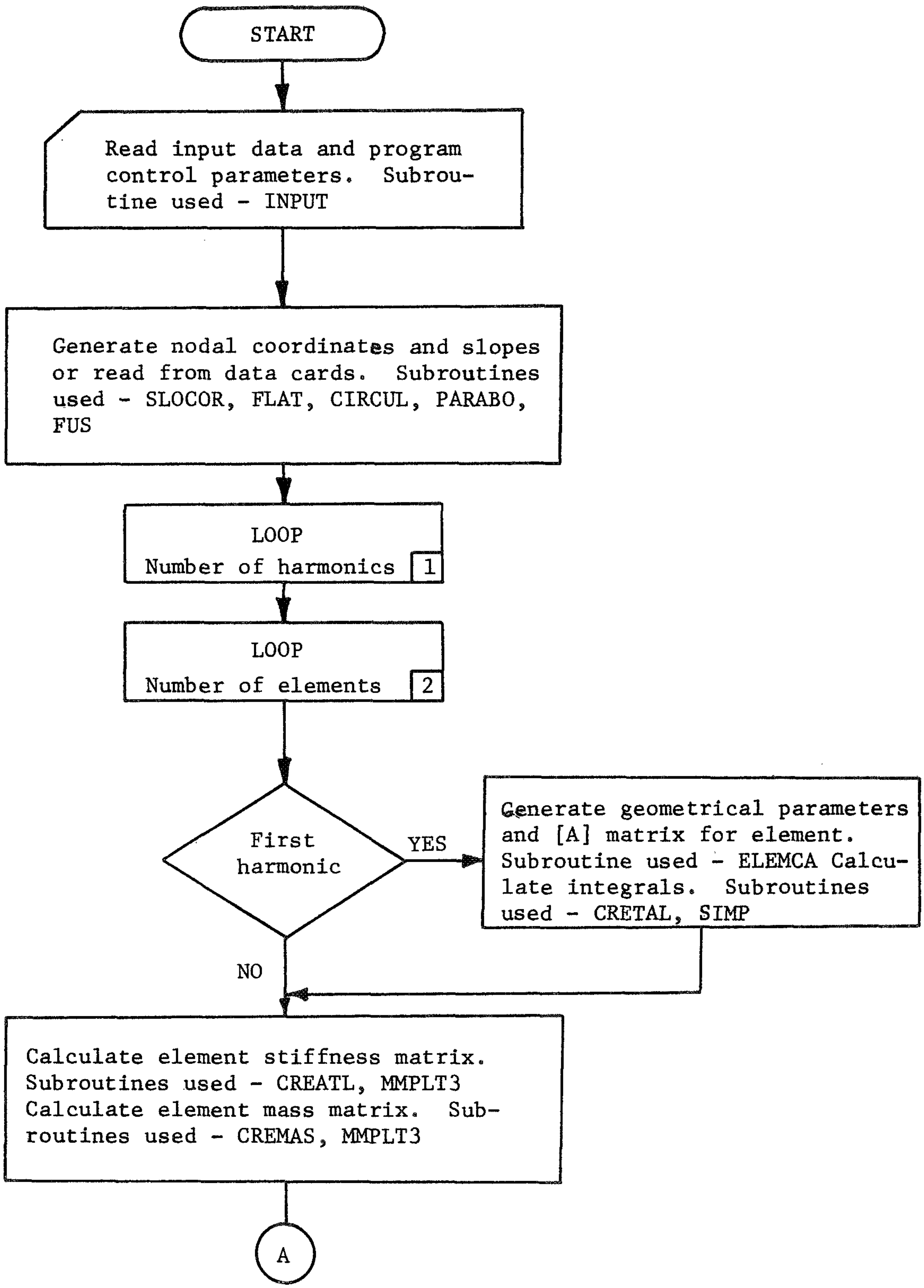
Variable	Subroutine where variable is defined or calculated	Description
Z2C	SLOCOR	Axial coordinate of the last node of a segment.

....

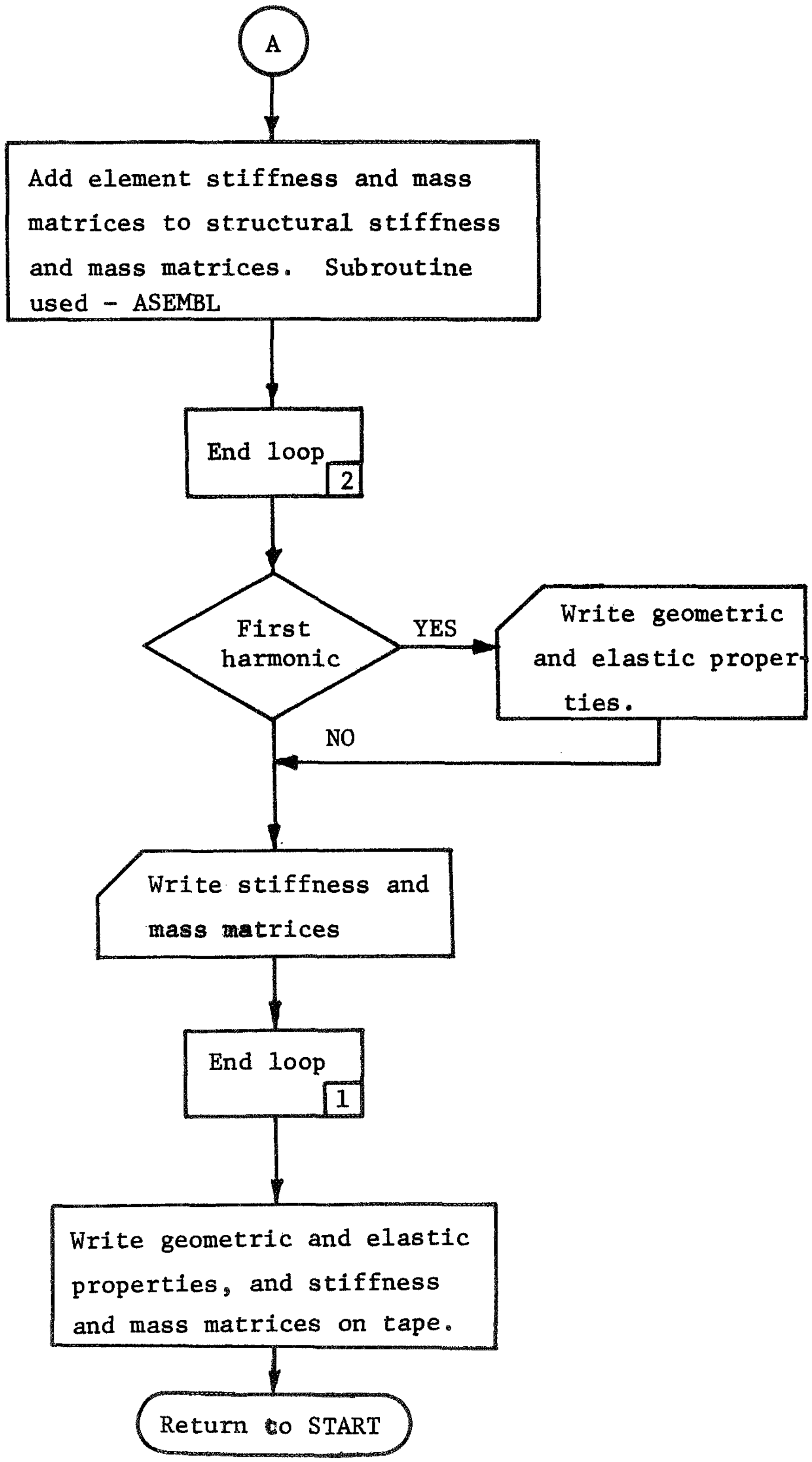
Appendix 3 Subroutine Call Map SAMMSOR II



Appendix 4 Flow Chart of Basic Operations SAMMSOR II



Appendix 4 Continued



Appendix 5 Program Output

The output for the SAMMSOR code consists basically of four parts:

1. The input data for all cases comprising the run.
2. Alphanumeric data used for case identification.
3. Listing of the element properties and nodal point geometry.
4. The upper half of the diagonal band of the stiffness and mass matrices for each harmonic (if desired).

Section 2-4 are repeated for each of the cases of the run.

The output of part 1 is extremely helpful to the user since the input data for all cases is printed at the start of a run. If an error occurs in the program as a result of an error in the input data, then this printout should readily show the source of the error.

The comments of the user which describe or identify the case or problem considered are printed in the second section of the output. This information is stored on the output tape and is also printed as identifying data in the other SOR programs.

The fourth part of the output lists the lower half of the diagonal band of the structural stiffness and mass matrices. The program output and the vector form of storage used in the program contain only those terms within the band of the matrix which are on or above the diagonal. One line of output is printed for each degree of freedom when writing the stiffness and mass matrices. The last term on a particular line of output is the diagonal term corresponding to the degree of freedom. The terms which appear to the left of this diagonal term are the terms on the row (corresponding to the degree of freedom) which are within the band of the matrix. In other words, the terms for the stiffness matrix of a particular harmonic with N degrees of freedom would appear as shown in Fig. A5-1. As is shown in Fig. A5-1, the maximum number of nonzero terms which appears in any row of the banded matrix is eight (8).

Terms whose value is zero will appear on the diagonals in the print-out of the stiffness and mass matrices for harmonic zero. These terms correspond to the V-direction displacements which are identically zero for this harmonic. Since diagonal terms of zero value cause problems when solving a system of equations, these diagonal terms are set equal to one in the analysis programs to provide equations of the form

$$(1.0)q_n = 0.0$$

In addition to the four major parts of the printout, several checks have been included which generate output remarks if an error is encountered.

K_{11}							
K_{21}	K_{22}						
K_{31}	K_{32}	K_{33}					
K_{41}	K_{42}	K_{43}	K_{44}				
K_{51}	K_{52}	K_{53}	K_{54}	K_{55}			
K_{61}	K_{62}	K_{63}	K_{64}	K_{65}	K_{66}		
K_{71}	K_{72}	K_{73}	K_{74}	K_{75}	K_{76}	K_{77}	
K_{81}	K_{82}	K_{83}	K_{84}	K_{85}	K_{86}	K_{87}	K_{88}
K_{95}	K_{96}	K_{97}	K_{98}	K_{99}			
$K_{10,5}$	$K_{10,6}$	$K_{10,7}$	$K_{10,8}$	$K_{10,9}$	$K_{10,10}$		
-	-	-	-	-	-	-	
$K_{12,5}$	-	-	-	-	-	-	$K_{12,12}$
$K_{13,9}$	$K_{13,10}$	$K_{13,11}$	$K_{13,12}$	$K_{13,13}$			
-							
-							
-							
-							
-							
-							
$K_{N,N-7}$	$K_{N,N-6}$	-	-	-	-	-	$K_{N,N}$

Fig. A5-1 Banded Stiffness Matrix Output Format

Appendix 6 Modification of Program Capacity

The procedure for modifying the maximum number of elements which can be utilized in the SAMMSOR code is presented in this section. With only one exception, the modifications consist of changing only DIMENSION and COMMON statements. The following terms are defined to facilitate the modifications:

NE = Maximum number of elements

N1 = (26 * NE) + 10

N2 = 6 * NE

Having calculated these constants for the desired maximum number of elements, the following cards in the specified subroutines must be changed by substituting the values of the above constants.

<u>Main Program</u>

```
CØMMØN /B2/  RO1(NE), RO2(NE), Z1(NE), Z2(NE), PHY1(NE), PHY2(NE)
CØMMØN /B3/  E1(NE), E2(NE), FNU1(NE), FNU2(NE), G(NE), THICK(NE),
             RHOS(NE)
CØMMØN /B21/ F(8,8), AK(8,8), STIFM(N1)
CØMMØN /B30/ EMASS(8,8), AKK(8,8), TMASS(N1)
DIMENSIØN    XZ(N2), CØMENT(20), JUNK(20), CARD(20), SL(NE)
```

<u>Subroutine INPUT</u>

```
*CØMMØN /B2/
*CØMMØN /B3/
 CØMMØN /B13/ SS(NE)
```

*Elements in this block are previously shown.

**NELMAX = NE

Subroutine SLØCØR

DIMENSION RA(5), NERAT(5), RØ1(NE), RØ2(NE), Z1(NE), Z2(NE), PHII1(NE), PHII2(NE)

Subroutines FLAT, CIRCUL, and PARABØ

DIMENSION RØ1(NE), RØ2(NE), Z1(NE),Z2(NE), PHII1(NE), PHII2(NE)

Subroutine ELEMCA

*CØMMØN /B2/

*CØMMØN /B3/

*CØMMØN /B13/

Subroutine CRETAL

***CØMMØN /B3/

Subroutine AZERØ and NONAB

*CØMMØN /B3/

*CØMMØN /B30/

Subroutines MMPLT3 and ASEMBL

DIMENSIØN F(8,8), AK(8,8), STIFM(N1)

** This is the only card that must be changed which is not a CØMMØN or DIMENSIØN statement.

***Replace G(NE) by GG(NE) in this block.

Appendix 7 Coding Sheet for Input Data

SHELL DESCRIPTION and comments	DATA SET NAME/NUMBER ______ DATE CREATED ______ CREATED BY ______ SAMMSOR ______ VERSION ______ DOUBLE/SINGLE PREC. ______	Status: Operational Deleted To be deleted on ______ ___, ____

CARD TYPE	INPUT DATA

II - A NCARDS NT (2I5)

______

- B DESCRIPTIVE COMMENTS (20A4)

______
______
______
______
______
______

III NELEMS IA NPRNMS (3I5)

______

IV IELM1 IELM2 EE1 EE2 GG FFNU1 RHO (2I5,5F10.0)

______
______
______
______
______
______

V IELM1 IELM2 THICKNESS (2I5, F10.0)

______
______
______
______
______
______
______
______
______

Appendix 7 Continued

VI SHELL GEOMETRY

- A NN1 Z R ϕ (I5, 3F10.0)

- B NN2 Z R ϕ

- C ICLASS NERAT 1 RA 1 NERAT 2 RA 2 NERAT 3 RA 3 NERAT 4 RA 4 NERAT 5 RA 5

A
B
C

A
B
C

A
B
C

A
B
C

A
B
C

A
B
C

A
B
C

VII END ØF CASE

******** Add an END ØF RUN card at the end of the final case

****** Add a Card I at the start of the first data set: This card contains NCASES, ND, NS, and NS2 -- in that order